Computer algebra independent integration tests

Summer 2022 edition

4-Trig-functions/4.5-Secant/123-4.5.3.1-a+b-sec-^m-d-sec-^n-A+B-sec-

Nasser M. Abbasi

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Contents

Chapter 1

Introduction

Local contents

This report gives the result of running the computer algebra independent integration test.

The download section in the appendix contains links to download the problems in plain text format used for all CAS systems.

The number of integrals in this report is [634]. This is test number [123].

1.1 Listing of CAS systems tested

The following are the CAS systems tested:

1. Mathematica 13.1 (June 29, 2022) on windows 10.
2. Rubi 4.16.1 (Dec 19, 2018) on Mathematica 13.0.1 on windows 10.
3. Maple 2022.1 (June 1, 2022) on windows 10.
4. Maxima 5.46 (April 13, 2022) using Lisp SBCL 2.1.11.debian on Linux via sagemath 9.6.
5. Fricas 1.3.8 (June 21, 2022) based on sbcl 2.1.11.debian on Linux via sagemath 9.6.
6. Giac/Xcas 1.9.0-13 (July 3, 2022) on Linux via sagemath 9.6.
7. Sympy 1.10.1 (March 20, 2022) Using Python 3.10.4 on Linux.
8. Mupad using Matlab 2021a with Symbolic Math Toolbox Version 8.7 on windows 10.

Maxima and Fricas and Giac are called using Sagemath. This was done using Sagemath `integrate` command by changing the name of the algorithm to use the different CAS systems.

Sympy was called directly from Python.

1.2 Results

Important note: A number of problems in this test suite have no antiderivative in closed form. This means the antiderivative of these integrals can not be expressed in terms of elementary, special functions or `Hypergeometric2F1` functions. `RootSum` and `RootOf` are not allowed.

If a CAS returns the above integral unevaluated within the time limit, then the result is counted as passed and assigned an A grade.

However, if CAS times out, then it is assigned an F grade even if the integral is not integrable, as this implies CAS could not determine that the integral is not integrable in the time limit.

If a CAS returns an antiderivative to such an integral, it is assigned an A grade automatically and this special result is listed in the introduction section of each individual test report to make it easy to identify as this can be important result to investigate.

The results given in in the table below reflects the above.

System	% solved	% Failed
Rubi	100.00 (634)	0.00 (0)
Mathematica	100.00 (634)	0.00 (0)
Maple	92.43 (586)	7.57 (48)
Fricas	72.24 (458)	27.76 (176)
Maxima	33.44 (212)	66.56 (422)
Giac	32.97 (209)	67.03 (425)
Mupad	30.76 (195)	69.24 (439)
Sympy	1.26 (8)	98.74 (626)

Table 1.1: Percentage solved for each CAS

The table below gives additional break down of the grading of quality of the antiderivatives generated by each CAS. The grading is given using the letters A,B,C and F with A being the best quality. The grading is accomplished by comparing the antiderivative generated with the optimal antiderivatives included in the test suite. The following table describes the meaning of these grades.

grade	description
A	Integral was solved and antiderivative is optimal in quality and leaf size.
B	Integral was solved and antiderivative is optimal in quality but leaf size is larger than twice the optimal antiderivatives leaf size.
C	Integral was solved and antiderivative is non-optimal in quality. This can be due to one or more of the following reasons 1. antiderivative contains a hypergeometric function and the optimal antiderivative does not. 2. antiderivative contains a special function and the optimal antiderivative does not. 3. antiderivative contains the imaginary unit and the optimal antiderivative does not.
F	Integral was not solved. Either the integral was returned unevaluated within the time limit, or it timed out, or CAS hanged or crashed or an exception was raised.

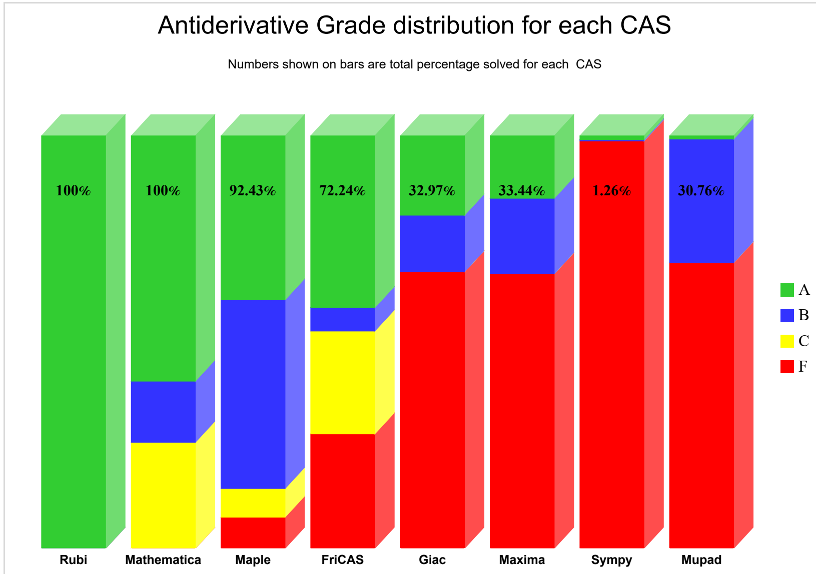
Table 1.2: Description of grading applied to integration result

Grading is implemented for all CAS systems. Based on the above, the following table summarizes the grading for this test suite.

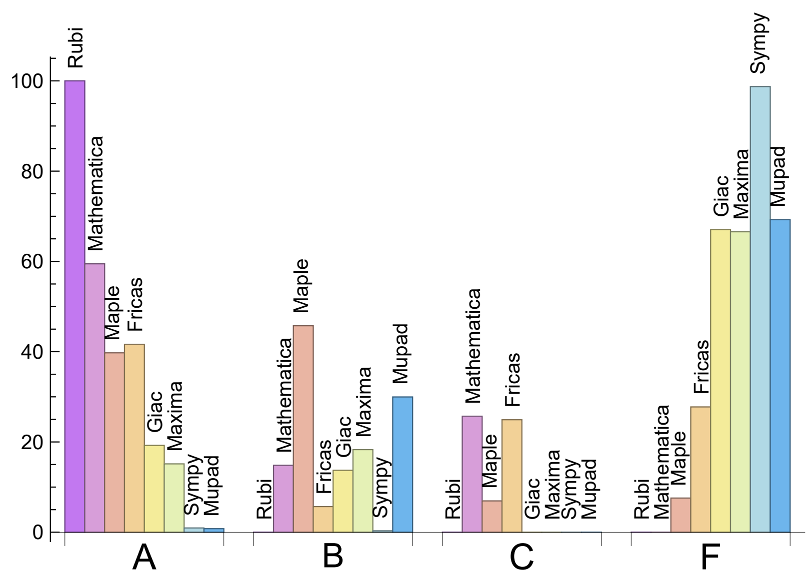
System	% A grade	% B grade	% C grade	% F grade
Rubi	100.00	0.00	0.00	0.00
Mathematica	59.46	14.83	25.71	0.00
Fricas	41.64	5.68	24.92	27.76
Maple	39.75	45.74	6.94	7.57
Giac	19.24	13.72	0.00	67.03
Maxima	15.14	18.30	0.00	66.56
Sympy	0.95	0.32	0.00	98.74
Mupad	N/A	29.97	0.00	69.24

Table 1.3: Antiderivative Grade distribution of each CAS

The following is a Bar chart illustration of the data in the above table.



The figure below compares the CAS systems for each grade level.



The following table shows the distribution of the different types of failure for each CAS. There are 3 types of reasons why it can fail. The first is when CAS returns back the input within the time limit, which means it could not solve it. This the typical normal failure **F** .

The second is due to time out. CAS could not solve the integral within the 3 minutes time limit which is assigned **F(-1)**.

The third is due to an exception generated. Assigned **F(-2)**. This most likely indicates an interface problem between sagemath and the CAS (applicable only to FriCAS, Maxima and

Giac) or it could be an indication of an internal error in CAS. This type of error requires more investigations to determine the cause.

System	Number failed	Percentage normal failure	Percentage timeout failure	Percentage exception failure
Rubi	0	0.00 %	0.00 %	0.00 %
Mathematica	0	0.00 %	0.00 %	0.00 %
Maple	48	100.00 %	0.00 %	0.00 %
Fricas	176	56.25 %	43.75 %	0.00 %
Giac	425	99.29 %	0.00 %	0.71 %
Maxima	422	73.46 %	16.59 %	9.95 %
Sympy	626	58.15 %	22.68 %	19.17 %
Mupad	439	100.00 %	0.00 %	0.00 %

Table 1.4: Failure statistics for each CAS

1.3 Time and leaf size Performance

The table below summarizes the performance of each CAS system in terms of time used and leaf size of results.

Mean size is the average leaf size produced by the CAS (before any normalization). The Normalized mean is relative to the mean size of the optimal anti-derivative given in the input files.

For example, if CAS has **Normalized mean** of 3, then the mean size of its leaf size is 3 times as large as the mean size of the optimal leaf size.

Median size is value of leaf size where half the values are larger than this and half are smaller (before any normalization). i.e. The Middle value.

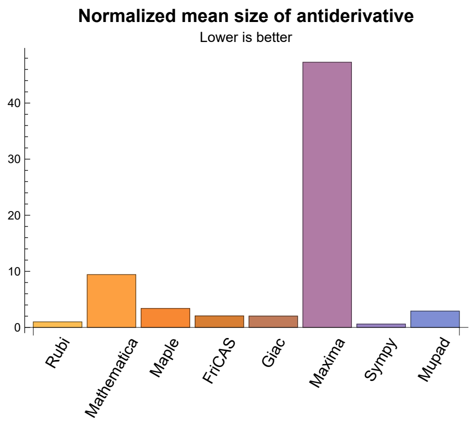
Similarly the **Normalized median** is relative to the median leaf size of the optimal.

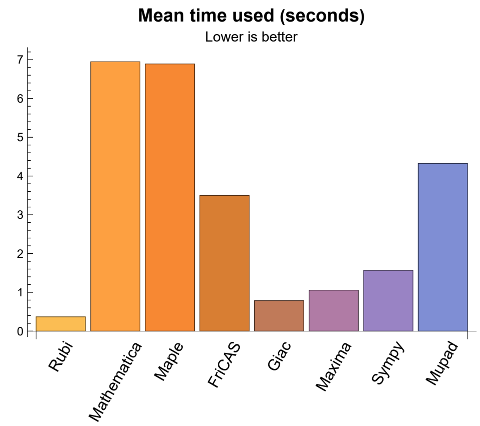
For example, if a CAS has Normalized median of 1.2, then its median is 1.2 as large as the median leaf size of the optimal.

System	Mean time (sec)	Mean size	Normalized mean	Median size	Normalized median
Rubi	0.37	217.71	0.99	187.00	1.00
Mathematica	6.95	3084.66	9.42	222.50	1.15
Maple	6.89	958.97	3.38	409.00	2.39
Maxima	1.06	10904.40	47.29	312.00	2.08
Fricas	3.50	398.08	2.05	307.00	1.79
Sympy	1.57	18.13	0.59	0.00	0.00
Giac	0.79	336.96	2.02	224.00	1.67
Mupad	4.32	515.35	2.91	198.00	1.42

Table 1.5: Time and leaf size performance for each CAS

The following are bar charts for the normalized leafsize and time used from the above table.


Normalized mean size of antiderivative
Lower is better
0
10
20
30
40
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad


Mean time used (seconds)
Lower is better
0
1
2
3
4
5
6
7
Rubi
Mathematica
Maple
FriCAS
Giac
Maxima
Sympy
Mupad

1.4 list of integrals that has no closed form antiderivative

$\{474, 475, 476, 477, 478\}$

1.5 List of integrals solved by CAS but has no known antiderivative

Rubi {}

Mathematica {}

Maple {}

Maxima {}

Fricas {}

Sympy {}

Giac {}

Mupad {}

1.6 list of integrals solved by CAS but failed verification

The following are integrals solved by CAS but the verification phase failed to verify the anti-derivative produced is correct. This does not mean necessarily that the anti-derivative is wrong, as additional methods of verification might be needed, or more time is needed (3 minutes time limit was used). These integrals are listed here to make it easier to do further investigation to determine why it was not possible to verify the result produced.

Rubi {269, 271, 272, 273, 274}

Mathematica {149, 156, 157, 164, 165, 168, 175, 193, 194, 215, 223, 256, 269, 270, 271, 272, 273, 274, 275, 348, 349, 350, 351, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 375, 377, 378, 379, 380, 382, 384, 385, 386, 387, 388, 390, 391, 392, 393, 415, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 441, 442, 443, 448, 449, 450, 451, 452, 468, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 535, 536, 549, 554, 556, 586, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634}

Maple Verification phase not implemented yet.

Maxima Verification phase not implemented yet.

Fricas Verification phase not implemented yet.

Sympy Verification phase not implemented yet.

Giac Verification phase not implemented yet.

Mupad Verification phase not implemented yet.

1.7 Timing

The command `AbsoluteTiming[]` was used in Mathematica to obtain the elapsed time for each integrate call. In Maple, the command `Usage` was used as in the following example

```
cpu_time := Usage(assign ('result_of_int',int(expr,x)),output='realtime'
```

For all other CAS systems, the elapsed time to complete each integral was found by taking the difference between the time after the call completed from the time before the call was made. This was done using Python's `time.time()` call.

All elapsed times shown are in seconds. A time limit of 3 CPU minutes was used for each integral. If the integrate command did not complete within this time limit, the integral was aborted and considered to have failed and assigned an F grade. The time used by failed integrals due to time out was not counted in the final statistics.

1.8 Verification

A verification phase was applied on the result of integration for `Rubi` and `Mathematica`.

Future version of this report will implement verification for the other CAS systems. For the integrals whose result was not run through a verification phase, it is assumed that the antiderivative was correct.

Verification phase also had 3 minutes time out. An integral whose result was not verified could still be correct, but further investigation is needed on those integrals. These integrals were marked in the summary table below and also in each integral separate section so they are easy to identify and locate.

1.9 Important notes about some of the results

1.9.1 Important note about Maxima results

Since tests were run in a batch mode, and using an automated script, then any integral where Maxima needed an interactive response from the user to answer a question during the evaluation of the integral will fail.

The exception raised is `ValueError`. Therefore Maxima results is lower than what would result if Maxima was run directly and each question was answered correctly.

The percentage of such failures were not counted for each test file, but for an example, for the `Timofeev` test file, there were about 14 such integrals out of total 705, or about 2 percent. This percentage can be higher or lower depending on the specific input test file.

Such integrals can be identified by looking at the output of the integration in each section for Maxima. The exception message will indicate the cause of error.

Maxima integrate was run using SageMath with the following settings set by default

```
'besselexpand : true'
'display2d : false'
'domain : complex'
'keepfloat : true'
'load(to_poly_solve)'
'load(simplify_sum)'
'load(abs_integrate)' 'load(diag)'
```

SageMath automatic loading of Maxima `abs_integrate` was found to cause some problems. So the following code was added to disable this effect.

```
from sage.interfaces.maxima_lib import maxima_lib
maxima_lib.set('extra_definite_integration_methods', '[]')
```

```
maxima_lib.set('extra_integration_methods', '[]')
```

See https://ask.sagemath.org/question/43088/integrate-results-that-are-different-from-using-maxima/ for reference.

1.9.2 Important note about FriCAS result

There were few integrals which failed due to SageMath interface and not because FriCAS system could not do the integration.

These will fail With error `Exception raised: NotImplementedError`.

The number of such cases seems to be very small. About 1 or 2 percent of all integrals. These can be identified by looking at the exception message given in the result.

1.9.3 Important note about finding leaf size of antiderivative

For Mathematica, Rubi, and Maple, the builtin system function `LeafSize` was used to find the leaf size of each antiderivative.

The other CAS systems (SageMath and Sympy) do not have special builtin function for this purpose at this time. Therefore the leaf size for Fricas and Sympy antiderivative was determined using the following function, thanks to user `slelievre` at https://ask.sagemath.org/question/57123/could-we-have-a-leaf_count-function-in-base-sagemath/

```
def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)
```

For Sympy, which was called directly from Python, the following code was used to obtain the leafsize of its result

```
try:
  # 1.7 is a fudge factor since it is low side from actual leaf count
  leafCount = round(1.7*count_ops(anti))

  except Exception as ee:
        leafCount =1
```

1.9.4 Important note about Mupad results

Matlab's symbolic toolbox does not have a leaf count function to measure the size of the antiderivative. Maple was used to determine the leaf size of Mupad output by post processing Mupad result.

Currently no grading of the antiderivative for Mupad is implemented. If it can integrate the problem, it was assigned a B grade automatically as a placeholder. In the future, when grading function is implemented for Mupad, the tests will be rerun again.

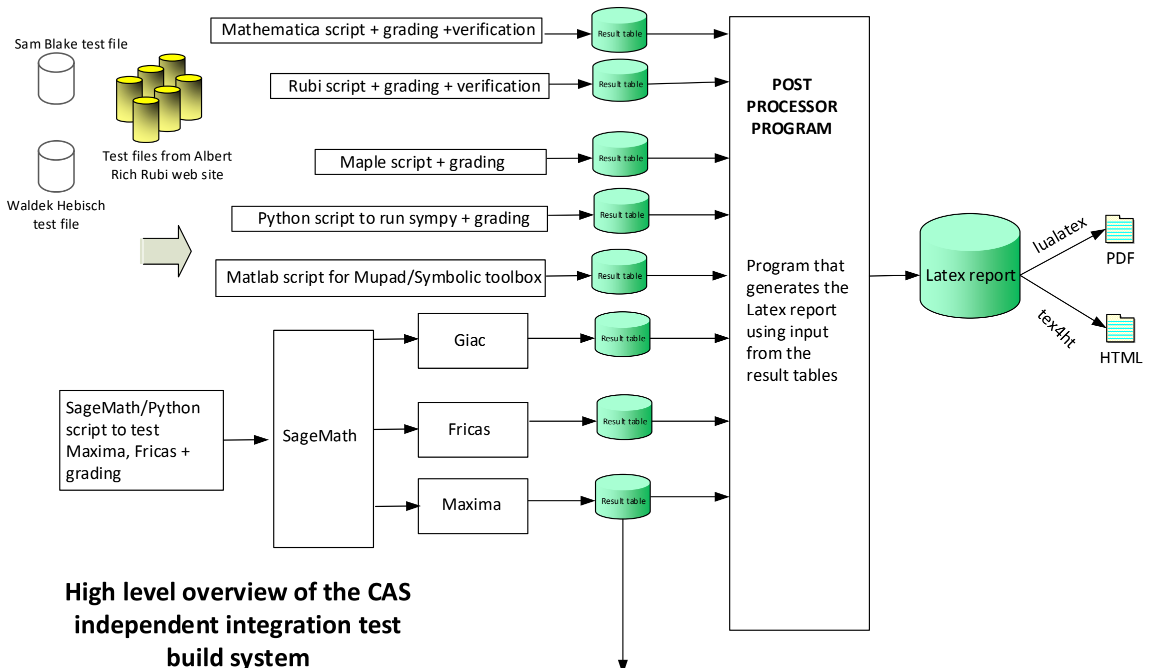
The following is an example of using Matlab's symbolic toolbox (Mupad) to solve an integral

```
integrand = evalin(symengine,'cos(x)*sin(x)')
the_variable = evalin(symengine,'x')
anti = int(integrand,the_variable)
```

Which gives `sin(x)^2/2`

1.10 Design of the test system

The following diagram gives a high level view of the current test build system.



One record (line) per one integral result. The line is CSV comma separated. This is description of each record

1. integer, the problem number.
2. integer. 0 for failed, 1 for passed, -1 for timeout, -2 for CAS specific exception. (this is not the grade field)
3. integer. Leaf size of result.
4. integer. Leaf size of the optimal antiderivative.
5. number. CPU time used to solve this integral. 0 if failed.
6. string. The integral in Latex format
7. string. The input used in CAS own syntax.
8. string. The result (antiderivative) produced by CAS in Latex format
9. string. The optimal antiderivative in Latex format.
10. integer. 0 or 1. Indicates if problem has known antiderivative or not
11. String. The result (antiderivative) in CAS own syntax.
12. String. The grade of the antiderivative. Can be "A", "B", "C", or "F"
13. String. Small string description of why the grade was given.
14. integer. 1 if result was verified or 0 if not verified.

*The following fields are present only in **Rubi** Table file*

15. integer. Number of steps used.
16. integer. Number of rules used.
17. integer. Integrand leaf size.
18. real number. Ratio. Field 16 over field 17
19. String of form "{n,n,..}" which is list of the rules used by Rubi
20. String. The optimal antiderivative in Mathematica syntx

Nasser M. Abbasi
August 26, 2022

Chapter 2

detailed summary tables of results

Local contents

2.1 List of integrals sorted by grade for each CAS

Local contents

2.1.1 Rubi

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634 }

B grade: { }

C grade: { }

F grade: { }

2.1.2 Mathematica

A grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 58, 59, 60, 61, 62, 63, 68, 69, 70, 71, 72, 73, 78, 79, 80, 81, 87, 94, 101, 102, 103, 110, 111, 112, 113, 118, 119, 120, 121, 122, 123, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 144, 145, 146, 147, 148, 150, 151, 152, 153, 154, 155, 160, 161, 162, 179, 180, 181, 182, 183, 184, 185, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 257, 258, 259, 260, 261, 264, 265, 266, 267, 268, 276, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 298, 299, 300, 301, 302, 303, 304, 306, 307, 308, 309, 310, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 344, 345, 346, 347, 350, 351, 358, 372, 373, 374, 379, 380, 388, 391, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 416, 417, 418, 419, 422, 427, 428, 429, 430, 433, 434, 437, 438, 439, 440, 445, 446, 447, 453, 454, 455, 458, 459, 460, 461, 464, 465, 466, 467, 469, 470, 471, 472, 473, 474, 475, 476, 477, 478, 479, 480, 481, 482, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593 }

B grade: { 55, 56, 57, 64, 65, 66, 67, 74, 75, 76, 77, 82, 83, 84, 85, 86, 88, 89, 90, 91, 92, 93, 95, 96, 97, 98, 99, 100, 104, 105, 106, 107, 108, 109, 114, 115, 116, 117, 255, 262, 263, 269, 270, 271, 272, 273, 274, 275, 297, 305, 311, 312, 341, 342, 343, 348, 349, 354, 355, 356, 357, 359, 360, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 377, 378, 382, 384, 385, 386, 387, 390, 392, 415, 420, 421, 423, 424, 425, 426, 431, 432, 560, 561, 578 }

C grade: { 124, 125, 126, 141, 142, 143, 149, 156, 157, 158, 159, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 256, 352, 353, 361, 375, 376, 381, 383, 389, 435, 436, 441, 442, 443, 444, 448, 449, 450, 451, 452, 456, 457, 462, 463, 468, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634 }

F grade: { }

2.1.3 Maple

A grade: { 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 127, 128, 129, 135, 136, 137, 167, 168, 182, 185, 189, 190, 191, 192, 198, 199, 200, 203, 204, 205, 206, 207, 208, 211, 212, 213, 214, 215, 218, 219, 220, 221, 222, 223, 228, 229, 230, 235, 236, 237, 238, 244, 245, 246, 247, 251, 252, 253, 254, 260, 261, 268, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 374, 396, 399, 416, 417, 418, 419, 474, 475, 476, 477, 478, 489, 490, 492, 496, 497, 498, 499, 500, 502, 509, 515, 516, 517, 518, 523, 524, 525, 526, 527, 532, 533, 534, 535, 541, 542, 543, 544, 545, 548, 549, 550, 551, 552, 555, 556, 557, 558, 576, 577, 579 }

B grade: { 72, 73, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 183, 184, 186, 187, 188, 193, 194, 195, 196, 197, 201, 202, 209, 210, 216, 217, 224, 225, 226, 227, 231, 232, 233, 234, 239, 240, 241, 242, 243, 248, 249, 250, 255, 256, 257, 258, 259, 262, 263, 264, 265, 266, 267, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 397, 398, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 438, 439, 440, 445, 446, 447, 453, 454, 455, 459, 460, 461, 464, 465, 466, 467, 469, 470, 471, 472, 473, 483, 484, 485, 486, 487, 488, 491, 493, 494, 495, 501, 503, 504, 505, 506, 507, 508, 510, 511, 512, 513, 514, 519, 520, 521, 522, 528, 529, 530, 531, 536, 537, 538, 539, 540, 546, 547, 553, 554, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 578, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 600, 601, 602, 607, 608, 609, 615, 616, 617, 621, 622, 623, 624, 628, 629, 630, 631, 632 }

C grade: { 1, 2, 3, 4, 5, 6, 435, 436, 437, 441, 442, 443, 444, 448, 449, 450, 451, 452, 456, 457, 458, 462, 463, 468, 597, 598, 599, 603, 604, 605, 606, 610, 611, 612, 613, 614, 618, 619, 620, 625, 626, 627, 633, 634 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 269, 270, 271, 272, 273, 274, 275, 276, 479, 480, 481, 482 }

2.1.4 Maxima

A grade: { 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 65, 66, 67, 68, 69, 70, 71, 75, 76, 77, 78, 79, 80, 81, 93, 94, 95, 99, 100, 101, 102, 103, 104, 105, 106, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 228, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 347, 474, 475, 476, 477, 478, 544 }

B grade: { 62, 63, 64, 72, 73, 74, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 96, 97, 98, 107, 122, 123, 124, 125, 126, 130, 131, 138, 139, 224, 225, 226, 227, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 262, 263, 264, 265, 266, 267, 268, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 545, 546, 547, 549, 550, 551, 553, 554, 555, 556, 557, 558, 559, 561 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 118, 119, 120, 121, 127, 128, 129, 132, 133, 134, 135, 136, 137, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 261, 269, 270, 271, 272, 273, 274, 275, 276, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 548, 552, 560, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634 }

2.1.5 FriCAS

A grade: { 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 157, 158, 159, 160, 161, 162, 163, 165, 166, 167, 168, 169, 170, 172, 173, 174, 176, 177, 178, 224, 225, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 258, 259, 260, 261, 262, 264, 265, 266, 267, 268, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 314, 315, 316, 317, 318, 319, 323, 326, 327, 344, 345, 346, 347, 478, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561 }

B grade: { 47, 84, 155, 156, 164, 171, 175, 226, 227, 257, 263, 280, 311, 312, 313, 320, 321, 322, 324, 325, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343 }

C grade: { 1, 2, 3, 4, 5, 6, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 438, 439, 440, 445, 446, 447, 453, 454, 455, 459, 460, 461, 464, 465, 466, 467, 469, 470, 471, 472, 473, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 594, 595, 596, 600, 601, 602, 607, 608, 609, 615, 616, 617, 621, 622, 623, 624, 628, 629, 630, 631, 632 }

F grade: { 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 269, 270, 271, 272, 273, 274, 275, 276, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 441, 442, 443, 444, 448, 449, 450, 451, 452, 456, 457, 458, 462, 463, 468, 474, 475, 476, 477, 479, 480, 481, 482, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 597, 598, 599, 603, 604, 605, 606, 610, 611, 612, 613, 614, 618, 619, 620, 625, 626, 627, 633, 634 }

2.1.6 Sympy

A grade: { 345, 474, 475, 476, 477, 478 }

B grade: { 47, 280 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 43, 44, 45, 46, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 122, 123, 124, 125, 126, 127, 128, 129, 130, 131, 132, 133, 134, 135, 136, 137, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 277, 278, 279, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 346, 347, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634 }

2.1.7 Giac

A grade: { 43, 44, 45, 50, 51, 52, 53, 54, 55, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 127, 128, 129, 135, 136, 137, 144, 145, 146, 147, 152, 153, 154, 155, 160, 161, 162, 163, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 298, 307, 313, 314, 316, 317, 320, 322, 323, 324, 326, 327, 334, 346, 347, 474, 475, 476, 477, 478 }

B grade: { 46, 47, 48, 49, 56, 57, 121, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 138, 139, 140, 141, 142, 143, 149, 150, 151, 157, 158, 159, 165, 166, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 299, 300, 301, 302, 303, 304, 305, 306, 308, 309, 310, 311, 312, 315, 318, 319, 321, 325, 328, 329, 330, 331, 332, 333, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 148, 156, 164, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 228, 229, 230, 231, 232, 233, 234, 235, 236, 237, 238, 239, 240, 241, 242, 243, 244, 245, 246, 247, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 479, 480, 481, 482, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634 }

2.1.8 Mupad

A grade: { 474, 475, 476, 477, 478 }

B grade: { 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61, 62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76, 77, 78, 79, 80, 81, 82, 83, 84, 85, 86, 87, 88, 89, 90, 91, 92, 93, 94, 95, 96, 97, 98, 99, 100, 101, 102, 103, 104, 105, 106, 107, 108, 109, 110, 111, 112, 113, 114, 115, 116, 117, 118, 119, 120, 121, 127, 128, 129, 135, 136, 137, 228, 229, 230, 236, 237, 238, 245, 246, 247, 277, 278, 279, 280, 281, 282, 283, 284, 285, 286, 287, 288, 289, 290, 291, 292, 293, 294, 295, 296, 297, 298, 299, 300, 301, 302, 303, 304, 305, 306, 307, 308, 309, 310, 311, 312, 313, 314, 315, 316, 317, 318, 319, 320, 321, 322, 323, 324, 325, 326, 327, 328, 329, 330, 331, 332, 333, 334, 335, 336, 337, 338, 339, 340, 341, 342, 343, 344, 345, 346, 347, 483, 484, 485, 486, 487, 488, 489, 490, 491, 492, 493, 494, 495, 562, 563, 564, 565, 566, 567, 568, 569, 570, 571, 572, 573 }

C grade: { }

F grade: { 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21, 22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41, 42, 122, 123, 124, 125, 126, 130, 131, 132, 133, 134, 138, 139, 140, 141, 142, 143, 144, 145, 146, 147, 148, 149, 150, 151, 152, 153, 154, 155, 156, 157, 158, 159, 160, 161, 162, 163, 164, 165, 166, 167, 168, 169, 170, 171, 172, 173, 174, 175, 176, 177, 178, 179, 180, 181, 182, 183, 184, 185, 186, 187, 188, 189, 190, 191, 192, 193, 194, 195, 196, 197, 198, 199, 200, 201, 202, 203, 204, 205, 206, 207, 208, 209, 210, 211, 212, 213, 214, 215, 216, 217, 218, 219, 220, 221, 222, 223, 224, 225, 226, 227, 231, 232, 233, 234, 235, 239, 240, 241, 242, 243, 244, 248, 249, 250, 251, 252, 253, 254, 255, 256, 257, 258, 259, 260, 261, 262, 263, 264, 265, 266, 267, 268, 269, 270, 271, 272, 273, 274, 275, 276, 348, 349, 350, 351, 352, 353, 354, 355, 356, 357, 358, 359, 360, 361, 362, 363, 364, 365, 366, 367, 368, 369, 370, 371, 372, 373, 374, 375, 376, 377, 378, 379, 380, 381, 382, 383, 384, 385, 386, 387, 388, 389, 390, 391, 392, 393, 394, 395, 396, 397, 398, 399, 400, 401, 402, 403, 404, 405, 406, 407, 408, 409, 410, 411, 412, 413, 414, 415, 416, 417, 418, 419, 420, 421, 422, 423, 424, 425, 426, 427, 428, 429, 430, 431, 432, 433, 434, 435, 436, 437, 438, 439, 440, 441, 442, 443, 444, 445, 446, 447, 448, 449, 450, 451, 452, 453, 454, 455, 456, 457, 458, 459, 460, 461, 462, 463, 464, 465, 466, 467, 468, 469, 470, 471, 472, 473, 479, 480, 481, 482, 496, 497, 498, 499, 500, 501, 502, 503, 504, 505, 506, 507, 508, 509, 510, 511, 512, 513, 514, 515, 516, 517, 518, 519, 520, 521, 522, 523, 524, 525, 526, 527, 528, 529, 530, 531, 532, 533, 534, 535, 536, 537, 538, 539, 540, 541, 542, 543, 544, 545, 546, 547, 548, 549, 550, 551, 552, 553, 554, 555, 556, 557, 558, 559, 560, 561, 574, 575, 576, 577, 578, 579, 580, 581, 582, 583, 584, 585, 586, 587, 588, 589, 590, 591, 592, 593, 594, 595, 596, 597, 598, 599, 600, 601, 602, 603, 604, 605, 606, 607, 608, 609, 610, 611, 612, 613, 614, 615, 616, 617, 618, 619, 620, 621, 622, 623, 624, 625, 626, 627, 628, 629, 630, 631, 632, 633, 634 }

2.2 Detailed conclusion table per each integral for all CAS systems

Detailed conclusion table per each integral is given by table below. The elapsed time is in seconds. For failed result it is given as F(-1) if the failure was due to timeout. It is given as F(-2) if the failure was due to an exception being raised, which could indicate a bug in the system. If the failure was due to integral not being evaluated within the time limit, then it is given just an F.

In this table,the column `N.S.` in the table below, which stands for **normalized size** is defined as $\frac{\text{antiderivative leaf size}}{\text{optimal antiderivative leaf size}}$. To help make the table fit, `Mathematica` was abbreviated to `MMA`.

Problem 1	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	99	518	0	213	0	0	-1
N.S.	1	1.00	0.58	3.03	0.00	1.25	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.493	12.200	0.000	0.930	0.000	0.000	0.000

Problem 2	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	87	499	0	186	0	0	-1
N.S.	1	1.00	0.64	3.67	0.00	1.37	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.290	11.806	0.000	0.465	0.000	0.000	0.000

Problem 3	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	104	104	73	453	0	140	0	0	-1
N.S.	1	1.00	0.70	4.36	0.00	1.35	0.00	0.00	-0.01
time (sec)	N/A	0.061	0.130	11.455	0.000	0.444	0.000	0.000	0.000

Problem 4	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	54	445	0	122	0	0	-1
N.S.	1	1.00	0.66	5.43	0.00	1.49	0.00	0.00	-0.01
time (sec)	N/A	0.050	0.096	7.858	0.000	0.435	0.000	0.000	0.000

Problem 5	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	86	470	0	150	0	0	-1
N.S.	1	1.00	0.74	4.05	0.00	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.063	0.193	8.306	0.000	0.460	0.000	0.000	0.000

Problem 6	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	147	147	88	482	0	164	0	0	-1
N.S.	1	1.00	0.60	3.28	0.00	1.12	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.545	8.128	0.000	0.545	0.000	0.000	0.000

Problem 7	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	90	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.280	0.155	0.000	0.000	0.000	0.000	0.000

Problem 8	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	91	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.077	0.128	0.121	0.000	0.000	0.000	0.000	0.000

Problem 9	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	88	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.090	0.145	0.000	0.000	0.000	0.000	0.000

Problem 10	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	88	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.080	0.102	0.200	0.000	0.000	0.000	0.000	0.000

Problem 11	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	88	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.150	0.320	0.000	0.000	0.000	0.000	0.000

Problem 12	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	90	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.318	0.146	0.000	0.000	0.000	0.000	0.000

Problem 13	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	91	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.156	0.126	0.000	0.000	0.000	0.000	0.000

Problem 14	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	112	112	88	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.066	0.117	0.139	0.000	0.000	0.000	0.000	0.000

Problem 15	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	87	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.078	0.122	0.197	0.000	0.000	0.000	0.000	0.000

Problem 16	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	88	0	0	0	0	0	-1
N.S.	1	1.00	0.74	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.090	0.106	0.296	0.000	0.000	0.000	0.000	0.000

Problem 17	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	90	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.257	0.125	0.000	0.000	0.000	0.000	0.000

Problem 18	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	90	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.108	0.105	0.000	0.000	0.000	0.000	0.000

Problem 19	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	87	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.067	0.090	0.131	0.000	0.000	0.000	0.000	0.000

Problem 20	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	90	0	0	0	0	0	-1
N.S.	1	1.00	0.79	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.073	0.026	0.006	0.000	0.000	0.000	0.000	0.000

Problem 21	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	90	0	0	0	0	0	-1
N.S.	1	1.00	0.77	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.025	0.004	0.000	0.000	0.000	0.000	0.000

Problem 22	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	91	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.228	0.129	0.000	0.000	0.000	0.000	0.000

Problem 23	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	91	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.074	0.094	0.125	0.000	0.000	0.000	0.000	0.000

Problem 24	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	87	0	0	0	0	0	-1
N.S.	1	1.00	0.76	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.064	0.147	0.126	0.000	0.000	0.000	0.000	0.000

Problem 25	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	91	0	0	0	0	0	-1
N.S.	1	1.00	0.80	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.069	0.027	0.006	0.000	0.000	0.000	0.000	0.000

Problem 26	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	91	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.071	0.025	0.005	0.000	0.000	0.000	0.000	0.000

Problem 27	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	167	167	140	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.389	0.234	0.000	0.000	0.000	0.000	0.000

Problem 28	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	140	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.091	0.219	0.214	0.000	0.000	0.000	0.000	0.000

Problem 29	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	140	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.271	0.197	0.000	0.000	0.000	0.000	0.000

Problem 30	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	140	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.085	0.247	0.181	0.000	0.000	0.000	0.000	0.000

Problem 31	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	140	0	0	0	0	0	-1
N.S.	1	1.00	0.85	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.230	0.177	0.000	0.000	0.000	0.000	0.000

Problem 32	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	140	0	0	0	0	0	-1
N.S.	1	1.00	0.81	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.092	0.330	0.176	0.000	0.000	0.000	0.000	0.000

Problem 33	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	126	0	0	0	0	0	-1
N.S.	1	1.00	0.73	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.088	0.218	0.198	0.000	0.000	0.000	0.000	0.000

Problem 34	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	119	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.224	0.119	0.000	0.000	0.000	0.000	0.000

Problem 35	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	119	0	0	0	0	0	-1
N.S.	1	1.00	0.88	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.255	0.102	0.000	0.000	0.000	0.000	0.000

Problem 36	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	107	0	0	0	0	0	-1
N.S.	1	1.00	0.78	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.076	0.156	0.165	0.000	0.000	0.000	0.000	0.000

Problem 37	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	107	0	0	0	0	0	-1
N.S.	1	1.00	0.71	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.097	0.159	0.213	0.000	0.000	0.000	0.000	0.000

Problem 38	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	114	0	0	0	0	0	-1
N.S.	1	1.00	0.75	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.109	0.306	0.275	0.000	0.000	0.000	0.000	0.000

Problem 39	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	140	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.089	0.271	0.260	0.000	0.000	0.000	0.000	0.000

Problem 40	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	140	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.084	0.239	0.209	0.000	0.000	0.000	0.000	0.000

Problem 41	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	135	0	0	0	0	0	-1
N.S.	1	1.00	0.83	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.083	0.349	0.271	0.000	0.000	0.000	0.000	0.000

Problem 42	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	140	0	0	0	0	0	-1
N.S.	1	1.00	0.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.087	0.327	0.270	0.000	0.000	0.000	0.000	0.000

Problem 43	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	87	154	200	137	0	214	198
N.S.	1	1.00	0.65	1.15	1.49	1.02	0.00	1.60	1.48
time (sec)	N/A	0.104	0.759	0.330	0.389	1.061	0.000	0.471	4.751

Problem 44	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	77	131	163	127	0	188	166
N.S.	1	1.00	0.73	1.24	1.54	1.20	0.00	1.77	1.57
time (sec)	N/A	0.092	0.430	0.302	0.358	2.225	0.000	0.485	4.617

Problem 45	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	56	105	127	105	0	154	126
N.S.	1	1.00	0.65	1.22	1.48	1.22	0.00	1.79	1.47
time (sec)	N/A	0.085	0.360	0.266	0.365	2.502	0.000	0.469	3.988

Problem 46	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	56	56	75	75	88	89	0	124	94
N.S.	1	1.00	1.34	1.34	1.57	1.59	0.00	2.21	1.68
time (sec)	N/A	0.049	0.039	0.204	0.373	1.492	0.000	0.453	2.732

Problem 47	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	43	57	56	79	71	84	100
N.S.	1	1.00	1.34	1.78	1.75	2.47	2.22	2.62	3.12
time (sec)	N/A	0.023	0.024	0.187	0.362	2.312	4.653	0.455	2.234

Problem 48	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	32	32	46	48	58	51	0	79	100
N.S.	1	1.00	1.44	1.50	1.81	1.59	0.00	2.47	3.12
time (sec)	N/A	0.033	0.033	0.192	0.358	1.816	0.000	0.450	2.149

Problem 49	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	47	47	44	57	55	38	0	93	50
N.S.	1	1.00	0.94	1.21	1.17	0.81	0.00	1.98	1.06
time (sec)	N/A	0.062	0.115	0.189	0.346	1.495	0.000	0.474	2.087

Problem 50	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	77	77	65	85	79	56	0	124	84
N.S.	1	1.00	0.84	1.10	1.03	0.73	0.00	1.61	1.09
time (sec)	N/A	0.080	0.201	0.266	0.370	1.507	0.000	0.443	2.102

Problem 51	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	97	97	75	107	101	74	0	156	184
N.S.	1	1.00	0.77	1.10	1.04	0.76	0.00	1.61	1.90
time (sec)	N/A	0.092	0.265	0.338	0.361	2.298	0.000	0.457	4.668

Problem 52	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	110	128	124	88	0	184	212
N.S.	1	1.00	0.88	1.02	0.99	0.70	0.00	1.47	1.70
time (sec)	N/A	0.102	0.276	0.414	0.376	2.203	0.000	0.457	4.808

Problem 53	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	280	223	278	165	0	246	224
N.S.	1	1.00	1.66	1.32	1.64	0.98	0.00	1.46	1.33
time (sec)	N/A	0.175	1.361	0.385	0.403	2.485	0.000	0.511	4.606

Problem 54	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	262	187	230	145	0	212	183
N.S.	1	1.00	1.90	1.36	1.67	1.05	0.00	1.54	1.33
time (sec)	N/A	0.162	1.172	0.362	0.409	2.613	0.000	0.511	4.465

Problem 55	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	481	145	167	125	0	178	145
N.S.	1	1.00	4.67	1.41	1.62	1.21	0.00	1.73	1.41
time (sec)	N/A	0.081	6.054	0.283	0.387	2.145	0.000	0.527	3.809

Problem 56	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	307	114	128	119	0	154	162
N.S.	1	1.00	3.74	1.39	1.56	1.45	0.00	1.88	1.98
time (sec)	N/A	0.062	1.335	0.243	0.338	1.458	0.000	0.476	2.005

Problem 57	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	73	73	258	88	105	108	0	157	161
N.S.	1	1.00	3.53	1.21	1.44	1.48	0.00	2.15	2.21
time (sec)	N/A	0.093	1.697	0.256	0.374	1.752	0.000	0.479	2.006

Problem 58	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	96	96	101	79	0	145	141
N.S.	1	1.00	1.09	1.09	1.15	0.90	0.00	1.65	1.60
time (sec)	N/A	0.102	0.166	0.218	0.358	1.382	0.000	0.489	2.049

Problem 59	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	61	116	110	70	0	142	98
N.S.	1	1.00	0.60	1.14	1.08	0.69	0.00	1.39	0.96
time (sec)	N/A	0.109	0.183	0.295	0.379	1.587	0.000	0.475	1.890

Problem 60	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	86	154	144	90	0	176	134
N.S.	1	1.00	0.64	1.14	1.07	0.67	0.00	1.30	0.99
time (sec)	N/A	0.166	0.389	0.375	0.367	0.993	0.000	0.487	1.933

Problem 61	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	108	186	178	110	0	210	247
N.S.	1	1.00	0.68	1.16	1.11	0.69	0.00	1.31	1.54
time (sec)	N/A	0.179	0.488	0.404	0.390	1.683	0.000	0.514	4.680

Problem 62	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	346	317	405	185	0	280	262
N.S.	1	1.00	1.65	1.51	1.93	0.88	0.00	1.33	1.25
time (sec)	N/A	0.287	1.932	0.445	0.378	1.719	0.000	0.553	4.626

Problem 63	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	294	271	337	165	0	246	224
N.S.	1	1.00	1.80	1.66	2.07	1.01	0.00	1.51	1.37
time (sec)	N/A	0.195	1.495	0.424	0.286	3.309	0.000	0.513	4.596

Problem 64	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	273	219	262	145	0	212	185
N.S.	1	1.00	2.18	1.75	2.10	1.16	0.00	1.70	1.48
time (sec)	N/A	0.101	1.302	0.347	0.274	2.207	0.000	0.528	4.474

Problem 65	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	1056	176	198	141	0	189	209
N.S.	1	1.00	9.51	1.59	1.78	1.27	0.00	1.70	1.88
time (sec)	N/A	0.109	6.413	0.288	0.274	2.029	0.000	0.497	2.078

Problem 66	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	335	137	165	137	0	192	207
N.S.	1	1.00	3.10	1.27	1.53	1.27	0.00	1.78	1.92
time (sec)	N/A	0.168	2.649	0.322	0.274	1.849	0.000	0.511	2.123

Problem 67	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	302	128	140	127	0	192	197
N.S.	1	1.00	2.58	1.09	1.20	1.09	0.00	1.64	1.68
time (sec)	N/A	0.185	4.540	0.280	0.267	2.001	0.000	0.508	2.076

Problem 68	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	113	147	148	102	0	180	178
N.S.	1	1.00	0.90	1.18	1.18	0.82	0.00	1.44	1.42
time (sec)	N/A	0.183	0.247	0.296	0.264	1.361	0.000	0.527	2.152

Problem 69	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	86	176	167	90	0	176	134
N.S.	1	1.00	0.69	1.42	1.35	0.73	0.00	1.42	1.08
time (sec)	N/A	0.121	0.284	0.359	0.289	2.142	0.000	0.539	2.005

Problem 70	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	108	223	213	110	0	210	247
N.S.	1	1.00	0.61	1.27	1.21	0.62	0.00	1.19	1.40
time (sec)	N/A	0.261	0.469	0.449	0.289	1.594	0.000	0.492	4.710

Problem 71	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	134	266	262	130	0	244	285
N.S.	1	1.00	0.67	1.32	1.30	0.65	0.00	1.21	1.42
time (sec)	N/A	0.290	0.586	0.431	0.276	2.502	0.000	0.518	4.681

Problem 72	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	358	365	464	185	0	280	262
N.S.	1	1.00	1.85	1.88	2.39	0.95	0.00	1.44	1.35
time (sec)	N/A	0.233	2.288	0.488	0.271	2.350	0.000	0.531	4.606

Problem 73	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	306	303	369	165	0	246	224
N.S.	1	1.00	1.92	1.91	2.32	1.04	0.00	1.55	1.41
time (sec)	N/A	0.133	1.629	0.406	0.303	2.924	0.000	0.538	4.542

Problem 74	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	326	250	293	157	0	223	255
N.S.	1	1.00	2.16	1.66	1.94	1.04	0.00	1.48	1.69
time (sec)	N/A	0.155	1.876	0.375	0.306	3.777	0.000	0.523	2.097

Problem 75	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	1202	199	235	159	0	227	254
N.S.	1	1.00	7.96	1.32	1.56	1.05	0.00	1.50	1.68
time (sec)	N/A	0.264	6.484	0.396	0.290	6.207	0.000	0.554	2.100

Problem 76	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	373	177	199	156	0	230	243
N.S.	1	1.00	2.33	1.11	1.24	0.98	0.00	1.44	1.52
time (sec)	N/A	0.269	4.814	0.325	0.271	4.399	0.000	0.526	2.172

Problem 77	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	342	179	187	150	0	226	242
N.S.	1	1.00	2.07	1.08	1.13	0.91	0.00	1.37	1.47
time (sec)	N/A	0.290	2.031	0.322	0.276	4.238	0.000	0.498	2.230

Problem 78	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	173	173	138	208	205	118	0	214	188
N.S.	1	1.00	0.80	1.20	1.18	0.68	0.00	1.24	1.09
time (sec)	N/A	0.276	0.354	0.348	0.287	4.594	0.000	0.510	2.450

Problem 79	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	108	248	236	110	0	210	248
N.S.	1	1.00	0.68	1.57	1.49	0.70	0.00	1.33	1.57
time (sec)	N/A	0.148	0.331	0.365	0.290	6.633	0.000	0.504	4.699

Problem 80	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	134	306	297	130	0	244	286
N.S.	1	1.00	0.61	1.39	1.35	0.59	0.00	1.11	1.30
time (sec)	N/A	0.376	0.625	0.434	0.276	2.721	0.000	0.521	4.624

Problem 81	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	241	241	156	358	356	150	0	278	323
N.S.	1	1.00	0.65	1.49	1.48	0.62	0.00	1.15	1.34
time (sec)	N/A	0.395	0.767	0.494	0.282	1.744	0.000	0.525	4.128

Problem 82	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	489	190	368	170	0	182	152
N.S.	1	1.00	3.73	1.45	2.81	1.30	0.00	1.39	1.16
time (sec)	N/A	0.124	6.053	0.233	0.265	2.580	0.000	0.489	2.437

Problem 83	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	311	142	282	156	0	156	119
N.S.	1	1.00	2.88	1.31	2.61	1.44	0.00	1.44	1.10
time (sec)	N/A	0.113	3.776	0.207	0.288	2.447	0.000	0.480	2.114

Problem 84	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	224	100	196	127	0	109	79
N.S.	1	1.00	3.61	1.61	3.16	2.05	0.00	1.76	1.27
time (sec)	N/A	0.082	1.322	0.173	0.279	4.723	0.000	0.488	2.033

Problem 85	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	43	43	109	61	99	74	0	70	41
N.S.	1	1.00	2.53	1.42	2.30	1.72	0.00	1.63	0.95
time (sec)	N/A	0.056	0.272	0.179	0.306	2.742	0.000	0.473	1.931

Problem 86	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	72	45	73	44	0	44	32
N.S.	1	1.00	2.06	1.29	2.09	1.26	0.00	1.26	0.91
time (sec)	N/A	0.041	0.151	0.198	0.489	2.751	0.000	0.465	1.900

Problem 87	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	76	76	143	63	0	79	65
N.S.	1	1.00	1.27	1.27	2.38	1.05	0.00	1.32	1.08
time (sec)	N/A	0.079	0.375	0.286	0.484	2.544	0.000	0.466	2.002

Problem 88	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	197	100	225	81	0	123	107
N.S.	1	1.00	2.01	1.02	2.30	0.83	0.00	1.26	1.09
time (sec)	N/A	0.109	0.450	0.307	0.499	3.141	0.000	0.459	2.213

Problem 89	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	122	122	249	122	310	97	0	151	138
N.S.	1	1.00	2.04	1.00	2.54	0.80	0.00	1.24	1.13
time (sec)	N/A	0.116	0.682	0.318	0.486	4.932	0.000	0.445	3.024

Problem 90	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	764	222	425	245	0	226	202
N.S.	1	1.00	4.27	1.24	2.37	1.37	0.00	1.26	1.13
time (sec)	N/A	0.229	6.371	0.259	0.289	2.734	0.000	0.507	1.980

Problem 91	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	496	177	336	228	0	198	166
N.S.	1	1.00	3.18	1.13	2.15	1.46	0.00	1.27	1.06
time (sec)	N/A	0.218	4.082	0.234	0.284	3.054	0.000	0.517	1.929

Problem 92	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	292	134	244	195	0	151	120
N.S.	1	1.00	2.70	1.24	2.26	1.81	0.00	1.40	1.11
time (sec)	N/A	0.181	1.925	0.204	0.280	6.806	0.000	0.483	1.929

Problem 93	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	79	79	169	91	145	129	0	112	74
N.S.	1	1.00	2.14	1.15	1.84	1.63	0.00	1.42	0.94
time (sec)	N/A	0.129	0.568	0.217	0.263	2.388	0.000	0.488	1.918

Problem 94	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	65	65	76	60	93	58	0	60	45
N.S.	1	1.00	1.17	0.92	1.43	0.89	0.00	0.92	0.69
time (sec)	N/A	0.057	0.217	0.207	0.275	3.258	0.000	0.477	1.892

Problem 95	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	153	74	120	94	0	85	65
N.S.	1	1.00	2.19	1.06	1.71	1.34	0.00	1.21	0.93
time (sec)	N/A	0.081	0.388	0.233	0.475	4.858	0.000	0.471	1.926

Problem 96	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	245	108	191	123	0	121	109
N.S.	1	1.00	2.50	1.10	1.95	1.26	0.00	1.23	1.11
time (sec)	N/A	0.166	0.625	0.288	0.493	1.952	0.000	0.439	2.040

Problem 97	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	315	131	283	138	0	164	154
N.S.	1	1.00	2.20	0.92	1.98	0.97	0.00	1.15	1.08
time (sec)	N/A	0.215	0.783	0.300	0.490	4.762	0.000	0.459	2.065

Problem 98	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	369	154	372	157	0	192	187
N.S.	1	1.00	2.17	0.91	2.19	0.92	0.00	1.13	1.10
time (sec)	N/A	0.226	0.753	0.333	0.507	2.846	0.000	0.477	2.082

Problem 99	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	610	206	377	295	0	233	216
N.S.	1	1.00	3.02	1.02	1.87	1.46	0.00	1.15	1.07
time (sec)	N/A	0.328	6.270	0.279	0.298	3.580	0.000	0.500	2.026

Problem 100	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	480	162	286	256	0	186	168
N.S.	1	1.00	3.08	1.04	1.83	1.64	0.00	1.19	1.08
time (sec)	N/A	0.299	4.025	0.247	0.289	2.958	0.000	0.530	2.038

Problem 101	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	197	119	187	183	0	147	124
N.S.	1	1.00	1.58	0.95	1.50	1.46	0.00	1.18	0.99
time (sec)	N/A	0.221	0.944	0.231	0.303	4.236	0.000	0.492	1.977

Problem 102	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	96	64	115	93	0	75	66
N.S.	1	1.00	0.94	0.63	1.13	0.91	0.00	0.74	0.65
time (sec)	N/A	0.138	0.318	0.247	0.268	5.445	0.000	0.503	1.918

Problem 103	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	102	102	135	64	115	93	0	75	66
N.S.	1	1.00	1.32	0.63	1.13	0.91	0.00	0.74	0.65
time (sec)	N/A	0.080	0.346	0.246	0.281	3.582	0.000	0.484	1.923

Problem 104	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	241	102	160	138	0	121	133
N.S.	1	1.00	2.23	0.94	1.48	1.28	0.00	1.12	1.23
time (sec)	N/A	0.134	0.603	0.244	0.492	4.238	0.000	0.511	2.127

Problem 105	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	365	136	231	173	0	157	155
N.S.	1	1.00	2.68	1.00	1.70	1.27	0.00	1.15	1.14
time (sec)	N/A	0.261	1.067	0.304	0.481	7.965	0.000	0.512	1.982

Problem 106	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	435	163	322	190	0	200	204
N.S.	1	1.00	2.33	0.87	1.72	1.02	0.00	1.07	1.09
time (sec)	N/A	0.325	0.781	0.339	0.486	4.317	0.000	0.493	1.997

Problem 107	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	B	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	218	218	491	182	412	205	0	228	237
N.S.	1	1.00	2.25	0.83	1.89	0.94	0.00	1.05	1.09
time (sec)	N/A	0.344	1.077	0.370	0.493	3.110	0.000	0.524	2.050

Problem 108	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	238	238	880	234	419	358	0	267	272
N.S.	1	1.00	3.70	0.98	1.76	1.50	0.00	1.12	1.14
time (sec)	N/A	0.447	6.515	0.226	0.278	3.191	0.000	0.507	2.055

Problem 109	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	754	190	326	317	0	220	237
N.S.	1	1.00	3.89	0.98	1.68	1.63	0.00	1.13	1.22
time (sec)	N/A	0.410	6.416	0.215	0.282	3.612	0.000	0.500	2.031

Problem 110	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	163	163	239	146	228	236	0	181	198
N.S.	1	1.00	1.47	0.90	1.40	1.45	0.00	1.11	1.21
time (sec)	N/A	0.320	1.409	0.265	0.279	4.094	0.000	0.502	2.128

Problem 111	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	109	88	175	125	0	117	85
N.S.	1	1.00	0.75	0.60	1.20	0.86	0.00	0.80	0.58
time (sec)	N/A	0.156	0.368	0.259	0.275	3.324	0.000	0.472	2.033

Problem 112	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	163	88	174	124	0	117	84
N.S.	1	1.00	1.18	0.64	1.26	0.90	0.00	0.85	0.61
time (sec)	N/A	0.173	0.408	0.251	0.282	3.430	0.000	0.472	1.984

Problem 113	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	193	90	175	124	0	117	88
N.S.	1	1.00	1.40	0.65	1.27	0.90	0.00	0.85	0.64
time (sec)	N/A	0.110	0.467	0.246	0.273	2.687	0.000	0.478	1.988

Problem 114	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	329	130	201	181	0	154	163
N.S.	1	1.00	2.38	0.94	1.46	1.31	0.00	1.12	1.18
time (sec)	N/A	0.186	0.777	0.249	0.507	2.286	0.000	0.489	2.034

Problem 115	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	485	164	271	223	0	190	202
N.S.	1	1.00	2.92	0.99	1.63	1.34	0.00	1.14	1.22
time (sec)	N/A	0.356	1.067	0.339	0.500	3.629	0.000	0.501	2.056

Problem 116	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	555	187	364	240	0	233	179
N.S.	1	1.00	2.49	0.84	1.63	1.08	0.00	1.04	0.80
time (sec)	N/A	0.441	1.148	0.319	0.494	3.508	0.000	0.505	1.998

Problem 117	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	611	210	452	257	0	261	300
N.S.	1	1.00	2.39	0.82	1.77	1.00	0.00	1.02	1.17
time (sec)	N/A	0.472	1.581	0.368	0.493	3.310	0.000	0.552	2.050

Problem 118	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	98	138	0	122	0	268	512
N.S.	1	1.00	0.52	0.74	0.00	0.65	0.00	1.43	2.74
time (sec)	N/A	0.225	0.603	3.925	0.000	3.553	0.000	1.130	10.047

Problem 119	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	81	116	0	105	0	222	407
N.S.	1	1.00	0.56	0.81	0.00	0.73	0.00	1.54	2.83
time (sec)	N/A	0.175	0.285	3.966	0.000	3.614	0.000	1.096	6.163

Problem 120	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	80	94	0	87	0	176	212
N.S.	1	1.00	0.79	0.93	0.00	0.86	0.00	1.74	2.10
time (sec)	N/A	0.152	0.332	3.700	0.000	2.778	0.000	1.014	6.158

Problem 121	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	62	62	53	70	0	66	0	129	159
N.S.	1	1.00	0.85	1.13	0.00	1.06	0.00	2.08	2.56
time (sec)	N/A	0.063	0.166	3.836	0.000	3.602	0.000	0.944	1.966

Problem 122	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	76	118	147	235	0	193	-1
N.S.	1	1.00	1.15	1.79	2.23	3.56	0.00	2.92	-0.02
time (sec)	N/A	0.062	0.311	4.280	0.552	3.271	0.000	1.138	0.000

Problem 123	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	68	68	93	198	939	261	0	336	-1
N.S.	1	1.00	1.37	2.91	13.81	3.84	0.00	4.94	-0.01
time (sec)	N/A	0.073	0.244	7.203	0.621	2.450	0.000	1.619	0.000

Problem 124	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	117	117	117	398	1851	308	0	630	-1
N.S.	1	1.00	1.00	3.40	15.82	2.63	0.00	5.38	-0.01
time (sec)	N/A	0.121	0.396	7.543	0.690	3.485	0.000	1.646	0.000

Problem 125	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	70	580	2981	346	0	889	-1
N.S.	1	1.00	0.44	3.62	18.63	2.16	0.00	5.56	-0.01
time (sec)	N/A	0.170	0.174	7.086	0.822	4.744	0.000	1.696	0.000

Problem 126	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	70	762	8561	380	0	1080	-1
N.S.	1	1.00	0.34	3.75	42.17	1.87	0.00	5.32	-0.00
time (sec)	N/A	0.215	0.171	5.829	1.033	3.455	0.000	1.863	0.000

Problem 127	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	189	189	100	139	0	127	0	258	596
N.S.	1	1.00	0.53	0.74	0.00	0.67	0.00	1.37	3.15
time (sec)	N/A	0.314	0.736	4.038	0.000	3.237	0.000	1.282	9.604

Problem 128	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	82	117	0	108	0	215	479
N.S.	1	1.00	0.59	0.85	0.00	0.78	0.00	1.56	3.47
time (sec)	N/A	0.205	0.389	3.806	0.000	3.135	0.000	1.240	6.889

Problem 129	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	70	95	0	89	0	170	213
N.S.	1	1.00	0.69	0.94	0.00	0.88	0.00	1.68	2.11
time (sec)	N/A	0.100	0.313	4.183	0.000	2.512	0.000	1.093	5.877

Problem 130	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	102	237	998	314	0	263	-1
N.S.	1	1.00	0.97	2.26	9.50	2.99	0.00	2.50	-0.01
time (sec)	N/A	0.106	0.573	4.174	0.619	2.890	0.000	1.286	0.000

Problem 131	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	97	212	1801	292	0	406	-1
N.S.	1	1.00	0.94	2.06	17.49	2.83	0.00	3.94	-0.01
time (sec)	N/A	0.168	0.422	7.803	0.680	3.092	0.000	1.760	0.000

Problem 132	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	111	399	0	320	0	639	-1
N.S.	1	1.00	0.93	3.35	0.00	2.69	0.00	5.37	-0.01
time (sec)	N/A	0.189	0.604	7.631	0.000	2.903	0.000	1.803	0.000

Problem 133	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-2)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	137	581	0	360	0	897	-1
N.S.	1	1.00	0.84	3.54	0.00	2.20	0.00	5.47	-0.01
time (sec)	N/A	0.244	0.969	7.697	0.000	4.214	0.000	1.916	0.000

Problem 134	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	154	763	0	396	0	1088	-1
N.S.	1	1.00	0.74	3.65	0.00	1.89	0.00	5.21	-0.00
time (sec)	N/A	0.314	1.279	7.514	0.000	4.077	0.000	2.039	0.000

Problem 135	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-2)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	117	163	0	157	0	306	856
N.S.	1	1.00	0.49	0.69	0.00	0.66	0.00	1.29	3.61
time (sec)	N/A	0.455	5.837	3.958	0.000	4.244	0.000	1.366	13.506

Problem 136	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	96	141	0	136	0	261	723
N.S.	1	1.00	0.55	0.81	0.00	0.78	0.00	1.49	4.13
time (sec)	N/A	0.242	0.647	4.249	0.000	3.413	0.000	1.466	10.815

Problem 137	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	89	119	0	115	0	216	590
N.S.	1	1.00	0.64	0.86	0.00	0.83	0.00	1.57	4.28
time (sec)	N/A	0.131	0.512	3.518	0.000	2.363	0.000	1.351	6.348

Problem 138	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	128	341	1396	378	0	309	-1
N.S.	1	1.00	0.90	2.40	9.83	2.66	0.00	2.18	-0.01
time (sec)	N/A	0.158	0.938	4.069	0.610	3.907	0.000	1.495	0.000

Problem 139	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	126	256	2780	386	0	480	-1
N.S.	1	1.00	0.88	1.79	19.44	2.70	0.00	3.36	-0.01
time (sec)	N/A	0.273	0.803	7.904	0.699	2.878	0.000	1.950	0.000

Problem 140	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	154	154	116	410	0	348	0	709	-1
N.S.	1	1.00	0.75	2.66	0.00	2.26	0.00	4.60	-0.01
time (sec)	N/A	0.292	0.828	8.083	0.000	2.934	0.000	1.990	0.000

Problem 141	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	312	583	0	380	0	905	-1
N.S.	1	1.00	1.90	3.55	0.00	2.32	0.00	5.52	-0.01
time (sec)	N/A	0.314	1.072	7.641	0.000	3.240	0.000	2.294	0.000

Problem 142	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	366	765	0	420	0	1096	-1
N.S.	1	1.00	1.75	3.66	0.00	2.01	0.00	5.24	-0.00
time (sec)	N/A	0.383	1.325	8.155	0.000	3.882	0.000	2.223	0.000

Problem 143	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	A	F(-1)	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	416	947	0	460	0	1377	-1
N.S.	1	1.00	1.64	3.73	0.00	1.81	0.00	5.42	-0.00
time (sec)	N/A	0.441	1.844	7.776	0.000	2.466	0.000	2.349	0.000

Problem 144	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	202	202	140	785	0	432	0	252	-1
N.S.	1	1.00	0.69	3.89	0.00	2.14	0.00	1.25	-0.00
time (sec)	N/A	0.416	0.562	4.353	0.000	1.674	0.000	1.592	0.000

Problem 145	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	123	595	0	397	0	229	-1
N.S.	1	1.00	0.77	3.74	0.00	2.50	0.00	1.44	-0.01
time (sec)	N/A	0.280	0.404	4.311	0.000	1.528	0.000	1.463	0.000

Problem 146	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	106	405	0	352	0	165	-1
N.S.	1	1.00	0.90	3.43	0.00	2.98	0.00	1.40	-0.01
time (sec)	N/A	0.167	0.315	4.157	0.000	2.401	0.000	1.353	0.000

Problem 147	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	88	200	0	287	0	131	-1
N.S.	1	1.00	1.13	2.56	0.00	3.68	0.00	1.68	-0.01
time (sec)	N/A	0.075	0.173	4.297	0.000	2.776	0.000	1.226	0.000

Problem 148	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	A	F	F(-2)	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	91	91	92	194	0	307	0	0	-1
N.S.	1	1.00	1.01	2.13	0.00	3.37	0.00	0.00	-0.01
time (sec)	N/A	0.072	0.294	4.151	0.000	4.583	0.000	0.000	0.000

Problem 149	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	11162	353	0	458	0	365	-1
N.S.	1	1.00	93.80	2.97	0.00	3.85	0.00	3.07	-0.01
time (sec)	N/A	0.153	26.847	7.614	0.000	4.585	0.000	1.852	0.000

Problem 150	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	165	165	135	717	0	506	0	621	-1
N.S.	1	1.00	0.82	4.35	0.00	3.07	0.00	3.76	-0.01
time (sec)	N/A	0.250	0.436	8.832	0.000	6.136	0.000	1.926	0.000

Problem 151	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	206	206	150	1067	0	539	0	818	-1
N.S.	1	1.00	0.73	5.18	0.00	2.62	0.00	3.97	-0.00
time (sec)	N/A	0.377	0.705	9.024	0.000	6.574	0.000	2.037	0.000

Problem 152	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	160	793	0	504	0	277	-1
N.S.	1	1.00	0.74	3.67	0.00	2.33	0.00	1.28	-0.00
time (sec)	N/A	0.421	2.412	4.335	0.000	2.292	0.000	1.805	0.000

Problem 153	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	141	603	0	459	0	247	-1
N.S.	1	1.00	0.82	3.53	0.00	2.68	0.00	1.44	-0.01
time (sec)	N/A	0.303	1.376	4.890	0.000	1.645	0.000	1.774	0.000

Problem 154	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	118	118	125	405	0	386	0	187	-1
N.S.	1	1.00	1.06	3.43	0.00	3.27	0.00	1.58	-0.01
time (sec)	N/A	0.175	0.765	4.519	0.000	1.944	0.000	1.655	0.000

Problem 155	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	127	402	0	367	0	133	-1
N.S.	1	1.00	1.46	4.62	0.00	4.22	0.00	1.53	-0.01
time (sec)	N/A	0.082	0.824	4.536	0.000	1.631	0.000	1.390	0.000

Problem 156	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	11183	554	0	548	0	0	-1
N.S.	1	1.00	88.06	4.36	0.00	4.31	0.00	0.00	-0.01
time (sec)	N/A	0.126	26.854	4.598	0.000	4.635	0.000	0.000	0.000

Problem 157	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	170	170	11954	713	0	609	0	418	-1
N.S.	1	1.00	70.32	4.19	0.00	3.58	0.00	2.46	-0.01
time (sec)	N/A	0.261	27.046	8.178	0.000	5.499	0.000	2.152	0.000

Problem 158	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	395	1075	0	644	0	638	-1
N.S.	1	1.00	1.79	4.86	0.00	2.91	0.00	2.89	-0.00
time (sec)	N/A	0.410	2.420	8.654	0.000	8.020	0.000	2.194	0.000

Problem 159	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	268	268	502	1425	0	675	0	816	-1
N.S.	1	1.00	1.87	5.32	0.00	2.52	0.00	3.04	-0.00
time (sec)	N/A	0.533	6.148	8.333	0.000	7.954	0.000	2.358	0.000

Problem 160	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	161	795	0	557	0	276	-1
N.S.	1	1.00	0.75	3.68	0.00	2.58	0.00	1.28	-0.00
time (sec)	N/A	0.447	2.581	4.557	0.000	1.768	0.000	1.551	0.000

Problem 161	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	169	169	144	597	0	484	0	258	-1
N.S.	1	1.00	0.85	3.53	0.00	2.86	0.00	1.53	-0.01
time (sec)	N/A	0.311	1.466	4.175	0.000	1.512	0.000	1.477	0.000

Problem 162	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	131	602	0	475	0	170	-1
N.S.	1	1.00	1.04	4.78	0.00	3.77	0.00	1.35	-0.01
time (sec)	N/A	0.187	1.416	4.487	0.000	1.721	0.000	1.605	0.000

Problem 163	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	206	594	0	475	0	170	-1
N.S.	1	1.00	1.63	4.71	0.00	3.77	0.00	1.35	-0.01
time (sec)	N/A	0.115	1.508	4.348	0.000	1.342	0.000	1.550	0.000

Problem 164	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	F(-2)	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	11243	824	0	670	0	0	-1
N.S.	1	1.00	68.55	5.02	0.00	4.09	0.00	0.00	-0.01
time (sec)	N/A	0.183	26.880	4.286	0.000	7.607	0.000	0.000	0.000

Problem 165	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	B	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	12012	1065	0	739	0	464	-1
N.S.	1	1.00	58.03	5.14	0.00	3.57	0.00	2.24	-0.00
time (sec)	N/A	0.379	27.067	8.411	0.000	10.802	0.000	2.111	0.000

Problem 166	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	B	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	264	264	512	1427	0	776	0	685	-1
N.S.	1	1.00	1.94	5.41	0.00	2.94	0.00	2.59	-0.00
time (sec)	N/A	0.527	6.154	9.104	0.000	10.601	0.000	2.208	0.000

Problem 167	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	140	120	0	305	0	71	-1
N.S.	1	1.00	1.57	1.35	0.00	3.43	0.00	0.80	-0.01
time (sec)	N/A	0.102	0.543	4.667	0.000	1.567	0.000	1.008	0.000

Problem 168	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	A	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	382	154	0	435	0	114	-1
N.S.	1	1.00	3.32	1.34	0.00	3.78	0.00	0.99	-0.01
time (sec)	N/A	0.146	1.517	8.533	0.000	1.973	0.000	1.049	0.000

Problem 169	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	155	155	332	367	0	462	0	141	-1
N.S.	1	1.00	2.14	2.37	0.00	2.98	0.00	0.91	-0.01
time (sec)	N/A	0.237	2.265	8.877	0.000	1.800	0.000	1.065	0.000

Problem 170	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	362	625	0	484	0	166	-1
N.S.	1	1.00	1.89	3.26	0.00	2.52	0.00	0.86	-0.01
time (sec)	N/A	0.352	2.446	8.974	0.000	1.719	0.000	1.060	0.000

Problem 171	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	133	265	298	0	502	0	112	-1
N.S.	1	1.15	2.28	2.57	0.00	4.33	0.00	0.97	-0.01
time (sec)	N/A	0.139	1.548	4.628	0.000	1.490	0.000	1.075	0.000

Problem 172	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	146	146	281	462	0	526	0	177	-1
N.S.	1	1.00	1.92	3.16	0.00	3.60	0.00	1.21	-0.01
time (sec)	N/A	0.224	1.727	8.774	0.000	1.246	0.000	1.145	0.000

Problem 173	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	296	883	0	550	0	183	-1
N.S.	1	1.00	1.53	4.55	0.00	2.84	0.00	0.94	-0.01
time (sec)	N/A	0.347	1.939	8.417	0.000	1.891	0.000	1.211	0.000

Problem 174	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	314	1104	0	572	0	208	-1
N.S.	1	1.00	1.33	4.68	0.00	2.42	0.00	0.88	-0.00
time (sec)	N/A	0.464	1.873	8.847	0.000	1.525	0.000	1.160	0.000

Problem 175	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	B	F	A	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	185	421	695	0	590	0	138	-1
N.S.	1	1.22	2.77	4.57	0.00	3.88	0.00	0.91	-0.01
time (sec)	N/A	0.136	6.890	4.702	0.000	1.287	0.000	1.279	0.000

Problem 176	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	184	184	298	788	0	612	0	183	-1
N.S.	1	1.00	1.62	4.28	0.00	3.33	0.00	0.99	-0.01
time (sec)	N/A	0.332	2.036	8.950	0.000	1.764	0.000	1.241	0.000

Problem 177	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	308	1475	0	634	0	224	-1
N.S.	1	1.00	1.31	6.25	0.00	2.69	0.00	0.95	-0.00
time (sec)	N/A	0.470	2.083	9.093	0.000	1.605	0.000	1.282	0.000

Problem 178	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	A	F(-1)	A	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	323	1964	0	656	0	234	-1
N.S.	1	1.00	1.15	7.01	0.00	2.34	0.00	0.84	-0.00
time (sec)	N/A	0.588	2.446	8.644	0.000	2.374	0.000	1.301	0.000

Problem 179	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	200	691	0	233	0	0	-1
N.S.	1	1.00	1.01	3.47	0.00	1.17	0.00	0.00	-0.01
time (sec)	N/A	0.130	0.812	3.713	0.000	0.614	0.000	0.000	0.000

Problem 180	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	168	635	0	219	0	0	-1
N.S.	1	1.00	0.98	3.69	0.00	1.27	0.00	0.00	-0.01
time (sec)	N/A	0.117	0.693	3.231	0.000	0.557	0.000	0.000	0.000

Problem 181	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	135	135	94	400	0	188	0	0	-1
N.S.	1	1.00	0.70	2.96	0.00	1.39	0.00	0.00	-0.01
time (sec)	N/A	0.104	0.559	2.277	0.000	0.773	0.000	0.000	0.000

Problem 182	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	106	106	77	240	0	141	0	0	-1
N.S.	1	1.00	0.73	2.26	0.00	1.33	0.00	0.00	-0.01
time (sec)	N/A	0.095	0.326	1.216	0.000	0.419	0.000	0.000	0.000

Problem 183	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	110	110	83	321	0	142	0	0	-1
N.S.	1	1.00	0.75	2.92	0.00	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.099	0.329	1.114	0.000	0.890	0.000	0.000	0.000

Problem 184	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	99	355	0	169	0	0	-1
N.S.	1	1.00	0.70	2.52	0.00	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.111	0.605	1.047	0.000	0.810	0.000	0.000	0.000

Problem 185	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	113	383	0	187	0	0	-1
N.S.	1	1.00	0.66	2.23	0.00	1.09	0.00	0.00	-0.01
time (sec)	N/A	0.122	1.039	1.164	0.000	0.908	0.000	0.000	0.000

Problem 186	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	463	825	0	263	0	0	-1
N.S.	1	1.00	1.98	3.53	0.00	1.12	0.00	0.00	-0.00
time (sec)	N/A	0.202	4.737	3.981	0.000	0.919	0.000	0.000	0.000

Problem 187	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	321	716	0	239	0	0	-1
N.S.	1	1.00	1.61	3.60	0.00	1.20	0.00	0.00	-0.01
time (sec)	N/A	0.190	6.321	3.667	0.000	0.418	0.000	0.000	0.000

Problem 188	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	160	160	295	513	0	202	0	0	-1
N.S.	1	1.00	1.84	3.21	0.00	1.26	0.00	0.00	-0.01
time (sec)	N/A	0.173	3.429	1.502	0.000	0.674	0.000	0.000	0.000

Problem 189	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	158	158	299	245	0	166	0	0	-1
N.S.	1	1.00	1.89	1.55	0.00	1.05	0.00	0.00	-0.01
time (sec)	N/A	0.178	3.090	1.342	0.000	0.811	0.000	0.000	0.000

Problem 190	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	166	166	153	357	0	187	0	0	-1
N.S.	1	1.00	0.92	2.15	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.181	1.903	1.190	0.000	0.772	0.000	0.000	0.000

Problem 191	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	193	385	0	211	0	0	-1
N.S.	1	1.00	0.96	1.92	0.00	1.05	0.00	0.00	-0.00
time (sec)	N/A	0.201	2.512	1.096	0.000	1.021	0.000	0.000	0.000

Problem 192	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	217	413	0	231	0	0	-1
N.S.	1	1.00	0.93	1.76	0.00	0.99	0.00	0.00	-0.00
time (sec)	N/A	0.218	3.068	1.257	0.000	0.701	0.000	0.000	0.000

Problem 193	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	793	1153	0	283	0	0	-1
N.S.	1	1.00	2.86	4.16	0.00	1.02	0.00	0.00	-0.00
time (sec)	N/A	0.310	6.912	5.440	0.000	0.848	0.000	0.000	0.000

Problem 194	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	465	904	0	263	0	0	-1
N.S.	1	1.00	1.91	3.70	0.00	1.08	0.00	0.00	-0.00
time (sec)	N/A	0.279	5.493	4.142	0.000	0.484	0.000	0.000	0.000

Problem 195	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	244	916	0	243	0	0	-1
N.S.	1	1.00	1.16	4.34	0.00	1.15	0.00	0.00	-0.00
time (sec)	N/A	0.273	2.426	3.376	0.000	0.683	0.000	0.000	0.000

Problem 196	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	199	199	202	654	0	215	0	0	-1
N.S.	1	1.00	1.02	3.29	0.00	1.08	0.00	0.00	-0.01
time (sec)	N/A	0.269	2.004	1.698	0.000	0.654	0.000	0.000	0.000

Problem 197	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	207	519	0	197	0	0	-1
N.S.	1	1.00	0.98	2.46	0.00	0.93	0.00	0.00	-0.00
time (sec)	N/A	0.274	1.799	1.568	0.000	0.614	0.000	0.000	0.000

Problem 198	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	194	385	0	211	0	0	-1
N.S.	1	1.00	0.92	1.82	0.00	1.00	0.00	0.00	-0.00
time (sec)	N/A	0.279	2.626	1.103	0.000	0.808	0.000	0.000	0.000

Problem 199	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	196	413	0	231	0	0	-1
N.S.	1	1.00	0.80	1.69	0.00	0.95	0.00	0.00	-0.00
time (sec)	N/A	0.304	2.920	1.269	0.000	0.761	0.000	0.000	0.000

Problem 200	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	239	441	0	251	0	0	-1
N.S.	1	1.00	0.86	1.59	0.00	0.91	0.00	0.00	-0.00
time (sec)	N/A	0.324	3.648	1.154	0.000	0.612	0.000	0.000	0.000

Problem 201	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	229	229	814	779	0	339	0	0	-1
N.S.	1	1.00	3.55	3.40	0.00	1.48	0.00	0.00	-0.00
time (sec)	N/A	0.159	7.709	3.761	0.000	0.558	0.000	0.000	0.000

Problem 202	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	372	466	0	308	0	0	-1
N.S.	1	1.00	1.94	2.43	0.00	1.60	0.00	0.00	-0.01
time (sec)	N/A	0.144	3.615	2.823	0.000	0.576	0.000	0.000	0.000

Problem 203	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	420	318	0	248	0	0	-1
N.S.	1	1.00	2.75	2.08	0.00	1.62	0.00	0.00	-0.01
time (sec)	N/A	0.133	4.673	1.667	0.000	0.454	0.000	0.000	0.000

Problem 204	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	123	123	200	243	0	237	0	0	-1
N.S.	1	1.00	1.63	1.98	0.00	1.93	0.00	0.00	-0.01
time (sec)	N/A	0.119	1.219	1.103	0.000	0.582	0.000	0.000	0.000

Problem 205	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	128	128	445	244	0	241	0	0	-1
N.S.	1	1.00	3.48	1.91	0.00	1.88	0.00	0.00	-0.01
time (sec)	N/A	0.119	2.795	1.110	0.000	0.534	0.000	0.000	0.000

Problem 206	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	232	262	0	260	0	0	-1
N.S.	1	1.00	1.41	1.60	0.00	1.59	0.00	0.00	-0.01
time (sec)	N/A	0.137	2.522	1.122	0.000	0.597	0.000	0.000	0.000

Problem 207	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	540	282	0	278	0	0	-1
N.S.	1	1.00	2.74	1.43	0.00	1.41	0.00	0.00	-0.01
time (sec)	N/A	0.152	3.866	1.182	0.000	0.627	0.000	0.000	0.000

Problem 208	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	568	300	0	297	0	0	-1
N.S.	1	1.00	2.47	1.30	0.00	1.29	0.00	0.00	-0.00
time (sec)	N/A	0.177	4.123	1.189	0.000	0.785	0.000	0.000	0.000

Problem 209	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	865	723	0	426	0	0	-1
N.S.	1	1.00	3.65	3.05	0.00	1.80	0.00	0.00	-0.00
time (sec)	N/A	0.253	8.044	3.490	0.000	0.489	0.000	0.000	0.000

Problem 210	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	455	492	0	365	0	0	-1
N.S.	1	1.00	2.23	2.41	0.00	1.79	0.00	0.00	-0.00
time (sec)	N/A	0.235	7.055	1.703	0.000	0.403	0.000	0.000	0.000

Problem 211	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	256	350	0	325	0	0	-1
N.S.	1	1.00	1.59	2.17	0.00	2.02	0.00	0.00	-0.01
time (sec)	N/A	0.210	2.732	1.373	0.000	0.916	0.000	0.000	0.000

Problem 212	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	168	168	256	350	0	324	0	0	-1
N.S.	1	1.00	1.52	2.08	0.00	1.93	0.00	0.00	-0.01
time (sec)	N/A	0.208	3.398	1.368	0.000	0.599	0.000	0.000	0.000

Problem 213	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	854	421	0	364	0	0	-1
N.S.	1	1.00	4.82	2.38	0.00	2.06	0.00	0.00	-0.01
time (sec)	N/A	0.220	6.824	1.587	0.000	0.696	0.000	0.000	0.000

Problem 214	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	211	211	899	435	0	377	0	0	-1
N.S.	1	1.00	4.26	2.06	0.00	1.79	0.00	0.00	-0.00
time (sec)	N/A	0.245	6.921	1.593	0.000	0.981	0.000	0.000	0.000

Problem 215	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	946	465	0	395	0	0	-1
N.S.	1	1.00	3.88	1.91	0.00	1.62	0.00	0.00	-0.00
time (sec)	N/A	0.265	7.007	1.612	0.000	0.984	0.000	0.000	0.000

Problem 216	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	953	876	0	536	0	0	-1
N.S.	1	1.00	3.26	3.00	0.00	1.84	0.00	0.00	-0.00
time (sec)	N/A	0.390	8.285	2.695	0.000	0.760	0.000	0.000	0.000

Problem 217	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	924	685	0	481	0	0	-1
N.S.	1	1.00	3.54	2.62	0.00	1.84	0.00	0.00	-0.00
time (sec)	N/A	0.351	7.437	1.975	0.000	0.617	0.000	0.000	0.000

Problem 218	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	919	451	0	474	0	0	-1
N.S.	1	1.00	4.18	2.05	0.00	2.15	0.00	0.00	-0.00
time (sec)	N/A	0.332	7.021	1.490	0.000	0.513	0.000	0.000	0.000

Problem 219	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	918	451	0	472	0	0	-1
N.S.	1	1.00	4.25	2.09	0.00	2.19	0.00	0.00	-0.00
time (sec)	N/A	0.328	6.970	1.448	0.000	0.705	0.000	0.000	0.000

Problem 220	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	222	222	919	451	0	474	0	0	-1
N.S.	1	1.00	4.14	2.03	0.00	2.14	0.00	0.00	-0.00
time (sec)	N/A	0.331	7.071	1.504	0.000	0.495	0.000	0.000	0.000

Problem 221	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	364	451	0	478	0	0	-1
N.S.	1	1.00	1.60	1.98	0.00	2.10	0.00	0.00	-0.00
time (sec)	N/A	0.340	6.625	1.682	0.000	0.906	0.000	0.000	0.000

Problem 222	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-2)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	377	465	0	489	0	0	-1
N.S.	1	1.00	1.44	1.78	0.00	1.87	0.00	0.00	-0.00
time (sec)	N/A	0.374	7.012	1.651	0.000	0.503	0.000	0.000	0.000

Problem 223	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	1032	493	0	506	0	0	-1
N.S.	1	1.00	3.51	1.68	0.00	1.72	0.00	0.00	-0.00
time (sec)	N/A	0.394	7.483	1.595	0.000	0.999	0.000	0.000	0.000

Problem 224	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	131	408	3342	448	0	0	-1
N.S.	1	1.00	0.74	2.32	18.99	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.190	1.484	7.114	0.780	1.516	0.000	0.000	0.000

Problem 225	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	106	344	1927	402	0	0	-1
N.S.	1	1.00	0.81	2.63	14.71	3.07	0.00	0.00	-0.01
time (sec)	N/A	0.150	0.523	7.656	0.701	2.216	0.000	0.000	0.000

Problem 226	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	78	78	89	278	905	322	0	0	-1
N.S.	1	1.00	1.14	3.56	11.60	4.13	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.281	8.003	0.688	1.580	0.000	0.000	0.000

Problem 227	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	83	177	262	307	0	0	-1
N.S.	1	1.00	1.09	2.33	3.45	4.04	0.00	0.00	-0.01
time (sec)	N/A	0.103	0.393	6.862	0.625	1.773	0.000	0.000	0.000

Problem 228	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	56	75	134	74	0	0	81
N.S.	1	1.00	0.68	0.91	1.63	0.90	0.00	0.00	0.99
time (sec)	N/A	0.109	0.229	7.651	0.656	1.266	0.000	0.000	2.785

Problem 229	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	71	96	317	92	0	0	106
N.S.	1	1.00	0.55	0.74	2.44	0.71	0.00	0.00	0.82
time (sec)	N/A	0.155	0.297	7.261	0.612	2.189	0.000	0.000	3.380

Problem 230	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	91	118	498	110	0	0	130
N.S.	1	1.00	0.52	0.67	2.85	0.63	0.00	0.00	0.74
time (sec)	N/A	0.193	0.326	7.540	0.631	2.689	0.000	0.000	4.188

Problem 231	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	153	477	5879	494	0	0	-1
N.S.	1	1.00	0.67	2.10	25.90	2.18	0.00	0.00	-0.00
time (sec)	N/A	0.327	1.419	7.257	1.072	1.784	0.000	0.000	0.000

Problem 232	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	134	417	4606	458	0	0	-1
N.S.	1	1.00	0.74	2.32	25.59	2.54	0.00	0.00	-0.01
time (sec)	N/A	0.263	1.211	6.977	0.842	2.046	0.000	0.000	0.000

Problem 233	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	133	133	107	353	3389	410	0	0	-1
N.S.	1	1.00	0.80	2.65	25.48	3.08	0.00	0.00	-0.01
time (sec)	N/A	0.212	0.691	7.637	0.726	2.778	0.000	0.000	0.000

Problem 234	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	107	346	1417	364	0	0	-1
N.S.	1	1.00	0.86	2.79	11.43	2.94	0.00	0.00	-0.01
time (sec)	N/A	0.202	1.584	8.145	0.646	3.188	0.000	0.000	0.000

Problem 235	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	125	125	109	211	314	368	0	0	-1
N.S.	1	1.00	0.87	1.69	2.51	2.94	0.00	0.00	-0.01
time (sec)	N/A	0.203	0.631	7.281	0.622	2.158	0.000	0.000	0.000

Problem 236	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	73	97	250	94	0	0	107
N.S.	1	1.00	0.56	0.74	1.91	0.72	0.00	0.00	0.82
time (sec)	N/A	0.159	0.518	7.079	0.610	1.153	0.000	0.000	3.405

Problem 237	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	92	119	514	113	0	0	131
N.S.	1	1.00	0.51	0.66	2.84	0.62	0.00	0.00	0.72
time (sec)	N/A	0.283	0.471	7.121	0.626	2.687	0.000	0.000	4.338

Problem 238	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	110	141	700	132	0	0	155
N.S.	1	1.00	0.48	0.62	3.07	0.58	0.00	0.00	0.68
time (sec)	N/A	0.339	0.673	7.581	0.640	1.969	0.000	0.000	5.299

Problem 239	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	178	543	9242	558	0	0	-1
N.S.	1	1.00	0.65	1.98	33.73	2.04	0.00	0.00	-0.00
time (sec)	N/A	0.459	2.171	7.931	1.654	3.634	0.000	0.000	0.000

Problem 240	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	227	227	154	479	7331	518	0	0	-1
N.S.	1	1.00	0.68	2.11	32.30	2.28	0.00	0.00	-0.00
time (sec)	N/A	0.397	1.485	7.628	1.159	3.423	0.000	0.000	0.000

Problem 241	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	133	419	6297	478	0	0	-1
N.S.	1	1.00	0.74	2.33	34.98	2.66	0.00	0.00	-0.01
time (sec)	N/A	0.333	1.288	7.301	3.605	2.912	0.000	0.000	0.000

Problem 242	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	137	386	14322	454	0	0	-1
N.S.	1	1.00	0.76	2.14	79.57	2.52	0.00	0.00	-0.01
time (sec)	N/A	0.321	1.937	7.786	3.600	3.282	0.000	0.000	0.000

Problem 243	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	133	376	12088	424	0	0	-1
N.S.	1	1.00	0.75	2.12	68.29	2.40	0.00	0.00	-0.01
time (sec)	N/A	0.319	0.921	7.342	0.796	3.900	0.000	0.000	0.000

Problem 244	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	127	235	655	424	0	0	-1
N.S.	1	1.00	0.74	1.37	3.81	2.47	0.00	0.00	-0.01
time (sec)	N/A	0.316	1.920	7.508	0.642	2.746	0.000	0.000	0.000

Problem 245	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	91	121	385	120	0	0	133
N.S.	1	1.00	0.51	0.68	2.16	0.67	0.00	0.00	0.75
time (sec)	N/A	0.210	0.560	7.448	0.626	1.392	0.000	0.000	4.347

Problem 246	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	108	143	746	141	0	0	157
N.S.	1	1.00	0.47	0.63	3.27	0.62	0.00	0.00	0.69
time (sec)	N/A	0.410	0.713	7.407	0.647	2.203	0.000	0.000	5.549

Problem 247	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	127	165	945	162	0	0	392
N.S.	1	1.00	0.46	0.60	3.44	0.59	0.00	0.00	1.43
time (sec)	N/A	0.464	3.786	7.723	0.652	1.701	0.000	0.000	8.768

Problem 248	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	190	190	125	423	2524	617	0	0	-1
N.S.	1	1.00	0.66	2.23	13.28	3.25	0.00	0.00	-0.01
time (sec)	N/A	0.370	0.890	7.671	0.716	2.678	0.000	0.000	0.000

Problem 249	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	141	141	106	350	1353	531	0	0	-1
N.S.	1	1.00	0.75	2.48	9.60	3.77	0.00	0.00	-0.01
time (sec)	N/A	0.251	0.411	7.727	0.667	3.059	0.000	0.000	0.000

Problem 250	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	95	211	567	366	0	0	-1
N.S.	1	1.00	0.95	2.11	5.67	3.66	0.00	0.00	-0.01
time (sec)	N/A	0.148	0.198	7.791	0.650	3.235	0.000	0.000	0.000

Problem 251	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	99	99	114	150	195	306	0	0	-1
N.S.	1	1.00	1.15	1.52	1.97	3.09	0.00	0.00	-0.01
time (sec)	N/A	0.116	0.280	7.388	0.607	1.769	0.000	0.000	0.000

Problem 252	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	142	142	132	183	387	354	0	0	-1
N.S.	1	1.00	0.93	1.29	2.73	2.49	0.00	0.00	-0.01
time (sec)	N/A	0.211	0.416	8.054	0.627	2.324	0.000	0.000	0.000

Problem 253	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	133	205	640	388	0	0	-1
N.S.	1	1.00	0.71	1.10	3.42	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.332	1.140	7.746	0.656	2.732	0.000	0.000	0.000

Problem 254	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	152	227	805	422	0	0	-1
N.S.	1	1.00	0.66	0.99	3.50	1.83	0.00	0.00	-0.00
time (sec)	N/A	0.451	1.533	7.561	0.661	1.700	0.000	0.000	0.000

Problem 255	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	497	543	13364	761	0	0	-1
N.S.	1	1.00	2.01	2.20	54.11	3.08	0.00	0.00	-0.00
time (sec)	N/A	0.501	4.440	7.796	2.076	4.870	0.000	0.000	0.000

Problem 256	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	B	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	668	479	7057	669	0	0	-1
N.S.	1	1.00	3.39	2.43	35.82	3.40	0.00	0.00	-0.01
time (sec)	N/A	0.385	5.158	7.685	1.010	3.340	0.000	0.000	0.000

Problem 257	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	113	316	17843	601	0	0	-1
N.S.	1	1.00	0.78	2.18	123.06	4.14	0.00	0.00	-0.01
time (sec)	N/A	0.261	0.818	7.737	1.277	2.300	0.000	0.000	0.000

Problem 258	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	84	219	16752	376	0	0	-1
N.S.	1	1.00	0.79	2.05	156.56	3.51	0.00	0.00	-0.01
time (sec)	N/A	0.121	0.218	8.414	1.268	2.002	0.000	0.000	0.000

Problem 259	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	174	287	8208	430	0	0	-1
N.S.	1	1.00	1.12	1.84	52.62	2.76	0.00	0.00	-0.01
time (sec)	N/A	0.242	1.432	8.289	0.736	1.429	0.000	0.000	0.000

Problem 260	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	173	317	41138	464	0	0	-1
N.S.	1	1.00	0.85	1.56	202.65	2.29	0.00	0.00	-0.00
time (sec)	N/A	0.354	1.809	7.703	1.137	1.537	0.000	0.000	0.000

Problem 261	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	171	339	0	500	0	0	-1
N.S.	1	1.00	0.68	1.36	0.00	2.00	0.00	0.00	-0.00
time (sec)	N/A	0.465	1.431	8.279	0.000	2.092	0.000	0.000	0.000

Problem 262	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	246	246	941	831	14037	803	0	0	-1
N.S.	1	1.00	3.83	3.38	57.06	3.26	0.00	0.00	-0.00
time (sec)	N/A	0.532	6.174	8.412	4.931	2.453	0.000	0.000	0.000

Problem 263	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	B	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	570	550	89320	749	0	0	-1
N.S.	1	1.00	2.94	2.84	460.41	3.86	0.00	0.00	-0.01
time (sec)	N/A	0.377	5.392	8.345	23.616	1.810	0.000	0.000	0.000

Problem 264	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	156	106	350	87207	498	0	0	-1
N.S.	1	1.00	0.68	2.24	559.02	3.19	0.00	0.00	-0.01
time (sec)	N/A	0.176	0.692	7.977	23.812	2.252	0.000	0.000	0.000

Problem 265	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	156	203	103	347	5924	502	0	0	-1
N.S.	1	1.30	0.66	2.22	37.97	3.22	0.00	0.00	-0.01
time (sec)	N/A	0.365	1.029	7.902	1.426	3.782	0.000	0.000	0.000

Problem 266	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	206	419	261506	524	0	0	-1
N.S.	1	1.00	1.01	2.06	1288.21	2.58	0.00	0.00	-0.00
time (sec)	N/A	0.364	2.319	7.658	3.652	2.706	0.000	0.000	0.000

Problem 267	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	193	449	407281	564	0	0	-1
N.S.	1	1.00	0.77	1.80	1629.12	2.26	0.00	0.00	-0.00
time (sec)	N/A	0.498	1.794	7.938	6.314	2.993	0.000	0.000	0.000

Problem 268	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	297	297	196	471	387118	592	0	0	-1
N.S.	1	1.00	0.66	1.59	1303.43	1.99	0.00	0.00	-0.00
time (sec)	N/A	0.610	2.244	7.891	8.843	2.361	0.000	0.000	0.000

Problem 269	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	4445	0	0	0	0	0	-1
N.S.	1	1.00	10.95	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.428	20.287	0.292	0.000	0.000	0.000	0.000	0.000

Problem 270	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	354	354	2709	0	0	0	0	0	-1
N.S.	1	1.00	7.65	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.281	19.270	0.283	0.000	0.000	0.000	0.000	0.000

Problem 271	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	415	415	2901	0	0	0	0	0	-1
N.S.	1	1.00	6.99	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.305	19.253	0.308	0.000	0.000	0.000	0.000	0.000

Problem 272	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	787	787	4110	0	0	0	0	0	-1
N.S.	1	1.00	5.22	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.594	19.304	0.268	0.000	0.000	0.000	0.000	0.000

Problem 273	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	739	739	5094	0	0	0	0	0	-1
N.S.	1	1.00	6.89	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.497	20.849	0.247	0.000	0.000	0.000	0.000	0.000

Problem 274	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F(-1)	F	F	F
verified	N/A	NO	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	764	764	4066	0	0	0	0	0	-1
N.S.	1	1.00	5.32	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.516	19.154	0.247	0.000	0.000	0.000	0.000	0.000

Problem 275	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	F	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	4897	0	0	0	0	0	-1
N.S.	1	1.00	24.86	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.245	21.838	0.407	0.000	0.000	0.000	0.000	0.000

Problem 276	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	111	0	0	0	0	0	-1
N.S.	1	1.00	0.68	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.165	1.161	0.518	0.000	0.000	0.000	0.000	0.000

Problem 277	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	114	114	85	131	163	136	0	304	194
N.S.	1	1.00	0.75	1.15	1.43	1.19	0.00	2.67	1.70
time (sec)	N/A	0.097	0.666	0.375	0.277	4.309	0.000	0.473	5.727

Problem 278	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	93	93	67	105	127	115	0	210	145
N.S.	1	1.00	0.72	1.13	1.37	1.24	0.00	2.26	1.56
time (sec)	N/A	0.088	0.314	0.309	0.269	4.152	0.000	0.501	4.385

Problem 279	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	75	75	88	96	0	153	104
N.S.	1	1.00	1.23	1.23	1.44	1.57	0.00	2.51	1.70
time (sec)	N/A	0.050	0.036	0.251	0.274	3.414	0.000	0.447	3.143

Problem 280	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	B	B	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	43	57	56	85	71	84	114
N.S.	1	1.00	1.23	1.63	1.60	2.43	2.03	2.40	3.26
time (sec)	N/A	0.022	0.020	0.191	0.272	4.396	4.609	0.475	2.244

Problem 281	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	35	35	46	48	58	54	0	79	100
N.S.	1	1.00	1.31	1.37	1.66	1.54	0.00	2.26	2.86
time (sec)	N/A	0.034	0.035	0.209	0.297	3.721	0.000	0.462	2.191

Problem 282	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	52	52	51	57	55	42	0	121	50
N.S.	1	1.00	0.98	1.10	1.06	0.81	0.00	2.33	0.96
time (sec)	N/A	0.063	0.104	0.214	0.287	4.798	0.000	0.478	2.034

Problem 283	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	84	84	75	85	79	60	0	180	84
N.S.	1	1.00	0.89	1.01	0.94	0.71	0.00	2.14	1.00
time (sec)	N/A	0.083	0.189	0.316	0.267	5.036	0.000	0.476	2.067

Problem 284	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	91	107	101	81	0	272	117
N.S.	1	1.00	0.87	1.02	0.96	0.77	0.00	2.59	1.11
time (sec)	N/A	0.097	0.270	0.414	0.305	4.310	0.000	0.465	2.126

Problem 285	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	150	221	276	208	0	528	359
N.S.	1	1.00	0.76	1.12	1.39	1.05	0.00	2.67	1.81
time (sec)	N/A	0.198	1.598	0.441	0.276	4.613	0.000	0.536	5.707

Problem 286	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	120	185	228	180	0	478	317
N.S.	1	1.00	0.67	1.03	1.27	1.01	0.00	2.67	1.77
time (sec)	N/A	0.218	0.778	0.447	0.279	4.804	0.000	0.518	5.687

Problem 287	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	92	143	165	150	0	294	227
N.S.	1	1.00	0.79	1.23	1.42	1.29	0.00	2.53	1.96
time (sec)	N/A	0.122	0.508	0.328	0.265	3.806	0.000	0.518	5.444

Problem 288	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	67	112	126	136	0	192	176
N.S.	1	1.00	0.78	1.30	1.47	1.58	0.00	2.23	2.05
time (sec)	N/A	0.055	0.292	0.296	0.266	4.054	0.000	0.493	2.737

Problem 289	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	60	60	109	86	103	117	0	154	163
N.S.	1	1.00	1.82	1.43	1.72	1.95	0.00	2.57	2.72
time (sec)	N/A	0.068	0.518	0.288	0.289	3.873	0.000	0.494	2.547

Problem 290	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	80	80	120	94	99	87	0	178	169
N.S.	1	1.00	1.50	1.18	1.24	1.09	0.00	2.22	2.11
time (sec)	N/A	0.129	0.235	0.228	0.276	4.411	0.000	0.520	2.381

Problem 291	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	90	114	108	85	0	254	115
N.S.	1	1.00	0.84	1.07	1.01	0.79	0.00	2.37	1.07
time (sec)	N/A	0.149	0.251	0.316	0.274	4.196	0.000	0.456	2.101

Problem 292	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	118	152	142	114	0	437	169
N.S.	1	1.00	0.87	1.12	1.04	0.84	0.00	3.21	1.24
time (sec)	N/A	0.177	0.472	0.438	0.292	4.292	0.000	0.482	2.183

Problem 293	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	146	184	176	142	0	487	307
N.S.	1	1.00	0.81	1.02	0.98	0.79	0.00	2.71	1.71
time (sec)	N/A	0.188	0.550	0.507	0.283	2.906	0.000	0.501	5.808

Problem 294	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	252	252	181	275	341	249	0	722	470
N.S.	1	1.00	0.72	1.09	1.35	0.99	0.00	2.87	1.87
time (sec)	N/A	0.323	3.472	0.527	0.270	2.447	0.000	0.512	5.787

Problem 295	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	140	223	266	211	0	586	395
N.S.	1	1.00	0.78	1.24	1.48	1.17	0.00	3.26	2.19
time (sec)	N/A	0.221	0.950	0.461	0.275	4.540	0.000	0.531	6.015

Problem 296	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	108	180	202	189	0	336	526
N.S.	1	1.00	0.79	1.31	1.47	1.38	0.00	2.45	3.84
time (sec)	N/A	0.125	0.607	0.363	0.268	5.164	0.000	0.507	4.026

Problem 297	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	131	399	141	169	167	0	241	249
N.S.	1	1.10	3.35	1.18	1.42	1.40	0.00	2.03	2.09
time (sec)	N/A	0.140	0.978	0.385	0.282	3.146	0.000	0.507	3.598

Problem 298	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	217	132	144	152	0	234	236
N.S.	1	1.00	1.75	1.06	1.16	1.23	0.00	1.89	1.90
time (sec)	N/A	0.221	0.710	0.289	0.289	1.454	0.000	0.554	3.335

Problem 299	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	159	151	152	131	0	314	1924
N.S.	1	1.00	1.10	1.04	1.05	0.90	0.00	2.17	13.27
time (sec)	N/A	0.231	0.375	0.326	0.272	1.493	0.000	0.494	3.902

Problem 300	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	179	179	140	180	171	136	0	536	202
N.S.	1	1.00	0.78	1.01	0.96	0.76	0.00	2.99	1.13
time (sec)	N/A	0.279	0.441	0.421	0.276	1.678	0.000	0.516	2.490

Problem 301	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	176	227	217	174	0	672	277
N.S.	1	1.00	0.80	1.03	0.98	0.79	0.00	3.04	1.25
time (sec)	N/A	0.324	0.750	0.536	0.272	2.766	0.000	0.508	2.728

Problem 302	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	334	334	244	375	474	327	0	1186	709
N.S.	1	1.00	0.73	1.12	1.42	0.98	0.00	3.55	2.12
time (sec)	N/A	0.459	2.912	0.608	0.307	2.127	0.000	0.550	5.737

Problem 303	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	198	313	379	281	0	850	555
N.S.	1	1.00	0.79	1.25	1.52	1.12	0.00	3.40	2.22
time (sec)	N/A	0.341	4.380	0.521	0.290	2.299	0.000	0.553	6.014

Problem 304	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	160	260	303	250	0	635	1969
N.S.	1	1.00	0.80	1.30	1.52	1.25	0.00	3.18	9.84
time (sec)	N/A	0.219	1.089	0.447	0.281	1.752	0.000	0.515	5.006

Problem 305	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	A	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	195	195	1051	209	245	219	0	387	636
N.S.	1	1.00	5.39	1.07	1.26	1.12	0.00	1.98	3.26
time (sec)	N/A	0.245	6.315	0.458	0.276	1.269	0.000	0.513	4.902

Problem 306	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	209	209	310	187	209	202	0	528	330
N.S.	1	1.00	1.48	0.89	1.00	0.97	0.00	2.53	1.58
time (sec)	N/A	0.303	1.850	0.427	0.285	2.335	0.000	0.522	4.390

Problem 307	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-1)	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	198	198	257	189	197	196	0	371	2523
N.S.	1	1.00	1.30	0.95	0.99	0.99	0.00	1.87	12.74
time (sec)	N/A	0.389	1.092	0.397	0.282	1.914	0.000	0.553	4.308

Problem 308	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	216	216	210	218	215	183	0	603	369
N.S.	1	1.00	0.97	1.01	1.00	0.85	0.00	2.79	1.71
time (sec)	N/A	0.392	0.610	0.425	0.271	4.433	0.000	0.535	3.384

Problem 309	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	258	258	263	258	246	197	0	791	307
N.S.	1	1.00	1.02	1.00	0.95	0.76	0.00	3.07	1.19
time (sec)	N/A	0.459	0.636	0.424	0.278	2.306	0.000	0.529	2.713

Problem 310	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F(-2)	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	309	309	333	316	307	243	0	1127	403
N.S.	1	1.00	1.08	1.02	0.99	0.79	0.00	3.65	1.30
time (sec)	N/A	0.528	1.220	0.568	0.288	2.024	0.000	0.512	3.183

Problem 311	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	187	187	422	335	0	743	0	412	2500
N.S.	1	1.00	2.26	1.79	0.00	3.97	0.00	2.20	13.37
time (sec)	N/A	0.441	2.739	0.508	0.000	3.317	0.000	0.496	6.931

Problem 312	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	300	229	0	609	0	269	2500
N.S.	1	1.00	2.10	1.60	0.00	4.26	0.00	1.88	17.48
time (sec)	N/A	0.266	1.863	0.420	0.000	11.543	0.000	0.498	6.083

Problem 313	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	130	144	0	472	0	176	719
N.S.	1	1.00	1.33	1.47	0.00	4.82	0.00	1.80	7.34
time (sec)	N/A	0.147	0.601	0.368	0.000	2.498	0.000	0.495	2.971

Problem 314	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	76	76	112	92	0	316	0	127	573
N.S.	1	1.00	1.47	1.21	0.00	4.16	0.00	1.67	7.54
time (sec)	N/A	0.085	0.192	0.349	0.000	3.017	0.000	0.508	3.085

Problem 315	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	67	67	68	73	0	250	0	274	573
N.S.	1	1.00	1.01	1.09	0.00	3.73	0.00	4.09	8.55
time (sec)	N/A	0.070	0.138	0.308	0.000	2.587	0.000	0.487	3.196

Problem 316	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	90	90	85	111	0	328	0	141	740
N.S.	1	1.00	0.94	1.23	0.00	3.64	0.00	1.57	8.22
time (sec)	N/A	0.104	0.227	0.412	0.000	2.982	0.000	0.506	3.359

Problem 317	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	134	134	121	168	0	427	0	227	2500
N.S.	1	1.00	0.90	1.25	0.00	3.19	0.00	1.69	18.66
time (sec)	N/A	0.265	0.364	0.422	0.000	2.709	0.000	0.488	6.019

Problem 318	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	152	242	0	547	0	360	2500
N.S.	1	1.00	0.85	1.36	0.00	3.07	0.00	2.02	14.04
time (sec)	N/A	0.423	0.521	0.469	0.000	2.754	0.000	0.459	6.904

Problem 319	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	240	240	202	381	0	685	0	642	2500
N.S.	1	1.00	0.84	1.59	0.00	2.85	0.00	2.68	10.42
time (sec)	N/A	0.649	0.669	0.485	0.000	5.906	0.000	0.501	8.636

Problem 320	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	438	330	0	1343	0	384	2500
N.S.	1	1.00	1.61	1.21	0.00	4.94	0.00	1.41	9.19
time (sec)	N/A	0.578	6.295	0.713	0.000	27.305	0.000	0.522	11.171

Problem 321	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	240	245	0	1114	0	404	2500
N.S.	1	1.00	1.46	1.49	0.00	6.79	0.00	2.46	15.24
time (sec)	N/A	0.377	2.166	0.576	0.000	24.715	0.000	0.538	10.257

Problem 322	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	191	185	0	694	0	231	2500
N.S.	1	1.00	1.46	1.41	0.00	5.30	0.00	1.76	19.08
time (sec)	N/A	0.197	0.727	0.486	0.000	9.437	0.000	0.511	9.654

Problem 323	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	100	100	97	132	0	389	0	172	106
N.S.	1	1.00	0.97	1.32	0.00	3.89	0.00	1.72	1.06
time (sec)	N/A	0.097	0.379	0.345	0.000	4.116	0.000	0.483	2.422

Problem 324	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	155	168	0	561	0	201	2500
N.S.	1	1.00	1.25	1.35	0.00	4.52	0.00	1.62	20.16
time (sec)	N/A	0.141	0.704	0.354	0.000	5.282	0.000	0.474	9.656

Problem 325	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	221	213	0	788	0	1107	2500
N.S.	1	1.00	1.23	1.18	0.00	4.38	0.00	6.15	13.89
time (sec)	N/A	0.376	1.138	0.475	0.000	5.199	0.000	0.585	7.073

Problem 326	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	184	270	0	970	0	340	2500
N.S.	1	1.00	0.70	1.03	0.00	3.72	0.00	1.30	9.58
time (sec)	N/A	0.591	1.111	0.535	0.000	5.368	0.000	0.504	11.106

Problem 327	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	224	346	0	1167	0	473	2500
N.S.	1	1.00	0.65	1.00	0.00	3.37	0.00	1.37	7.23
time (sec)	N/A	0.844	1.361	0.565	0.000	4.081	0.000	0.492	11.663

Problem 328	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	407	407	507	464	0	2444	0	1391	2500
N.S.	1	1.00	1.25	1.14	0.00	6.00	0.00	3.42	6.14
time (sec)	N/A	1.316	2.993	1.145	0.000	113.821	0.000	0.596	14.229

Problem 329	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	289	289	418	380	0	2111	0	581	2500
N.S.	1	1.00	1.45	1.31	0.00	7.30	0.00	2.01	8.65
time (sec)	N/A	0.968	6.496	1.026	0.000	116.017	0.000	0.546	14.543

Problem 330	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	270	305	0	1419	0	486	2500
N.S.	1	1.00	1.23	1.39	0.00	6.45	0.00	2.21	11.36
time (sec)	N/A	0.473	1.751	0.812	0.000	28.620	0.000	0.573	11.530

Problem 331	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	157	238	0	750	0	400	251
N.S.	1	1.00	0.87	1.32	0.00	4.17	0.00	2.22	1.39
time (sec)	N/A	0.240	0.746	0.483	0.000	4.277	0.000	0.521	5.417

Problem 332	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	172	236	0	752	0	399	251
N.S.	1	1.00	1.05	1.44	0.00	4.59	0.00	2.43	1.53
time (sec)	N/A	0.187	0.939	0.447	0.000	3.867	0.000	0.536	5.352

Problem 333	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	205	205	267	287	0	1152	0	457	2500
N.S.	1	1.00	1.30	1.40	0.00	5.62	0.00	2.23	12.20
time (sec)	N/A	0.373	1.492	0.478	0.000	3.639	0.000	0.542	11.785

Problem 334	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	290	290	306	347	0	1568	0	546	2500
N.S.	1	1.00	1.06	1.20	0.00	5.41	0.00	1.88	8.62
time (sec)	N/A	1.032	2.072	0.620	0.000	3.925	0.000	0.549	9.730

Problem 335	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	393	393	734	403	0	1811	0	2700	2500
N.S.	1	1.00	1.87	1.03	0.00	4.61	0.00	6.87	6.36
time (sec)	N/A	1.369	4.678	0.738	0.000	3.726	0.000	0.806	14.170

Problem 336	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	418	418	548	591	0	3434	0	1005	2500
N.S.	1	1.00	1.31	1.41	0.00	8.22	0.00	2.40	5.98
time (sec)	N/A	3.516	3.115	1.079	0.000	185.937	0.000	0.584	19.961

Problem 337	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	310	310	369	482	0	2278	0	844	2500
N.S.	1	1.00	1.19	1.55	0.00	7.35	0.00	2.72	8.06
time (sec)	N/A	0.929	1.901	1.259	0.000	70.958	0.000	0.567	14.145

Problem 338	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	274	274	226	375	0	1230	0	693	439
N.S.	1	1.00	0.82	1.37	0.00	4.49	0.00	2.53	1.60
time (sec)	N/A	0.481	2.326	0.698	0.000	7.039	0.000	0.544	6.799

Problem 339	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	252	388	0	1242	0	726	451
N.S.	1	1.00	0.96	1.48	0.00	4.72	0.00	2.76	1.71
time (sec)	N/A	0.416	1.265	0.682	0.000	3.388	0.000	0.594	6.646

Problem 340	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	404	376	0	1238	0	693	439
N.S.	1	1.00	1.70	1.59	0.00	5.22	0.00	2.92	1.85
time (sec)	N/A	0.349	1.122	0.605	0.000	5.375	0.000	0.549	6.673

Problem 341	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	292	292	769	464	0	1867	0	814	2500
N.S.	1	1.00	2.63	1.59	0.00	6.39	0.00	2.79	8.56
time (sec)	N/A	0.742	3.502	0.622	0.000	5.843	0.000	0.560	14.414

Problem 342	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	411	411	1205	558	0	2560	0	966	2500
N.S.	1	1.00	2.93	1.36	0.00	6.23	0.00	2.35	6.08
time (sec)	N/A	3.623	6.145	0.772	0.000	6.020	0.000	0.569	14.203

Problem 343	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	A	F(-2)	B	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	538	538	1452	615	0	2890	0	1052	2500
N.S.	1	1.00	2.70	1.14	0.00	5.37	0.00	1.96	4.65
time (sec)	N/A	4.378	5.993	0.896	0.000	2.978	0.000	0.568	15.817

Problem 344	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	61	76	0	197	0	187	91
N.S.	1	1.00	1.00	1.25	0.00	3.23	0.00	3.07	1.49
time (sec)	N/A	0.081	0.155	0.351	0.000	2.579	0.000	0.488	2.559

Problem 345	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	A	B	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	6	6	6	7	0	6	3	13	6
N.S.	1	1.00	1.00	1.17	0.00	1.00	0.50	2.17	1.00
time (sec)	N/A	0.001	0.001	0.118	0.000	2.039	3.273	0.509	2.231

Problem 346	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	97	120	0	279	0	139	444
N.S.	1	1.00	1.13	1.40	0.00	3.24	0.00	1.62	5.16
time (sec)	N/A	0.124	0.394	0.333	0.000	3.997	0.000	0.493	2.574

Problem 347	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	A	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	87	87	39	37	80	84	0	58	26
N.S.	1	1.00	0.45	0.43	0.92	0.97	0.00	0.67	0.30
time (sec)	N/A	0.050	0.075	0.302	0.488	3.714	0.000	0.499	2.298

Problem 348	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	485	485	3734	4395	0	0	0	0	-1
N.S.	1	1.00	7.70	9.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.931	25.788	11.520	0.000	0.000	0.000	0.000	0.000

Problem 349	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	397	397	3330	3438	0	0	0	0	-1
N.S.	1	1.00	8.39	8.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.599	23.742	10.825	0.000	0.000	0.000	0.000	0.000

Problem 350	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	314	314	434	2498	0	0	0	0	-1
N.S.	1	1.00	1.38	7.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.359	17.409	10.714	0.000	0.000	0.000	0.000	0.000

Problem 351	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	408	1752	0	0	0	0	-1
N.S.	1	1.00	1.59	6.84	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.214	14.381	9.638	0.000	0.000	0.000	0.000	0.000

Problem 352	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	320	320	913	1372	0	0	0	0	-1
N.S.	1	1.00	2.85	4.29	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.184	17.715	10.293	0.000	0.000	0.000	0.000	0.000

Problem 353	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	1107	1386	0	0	0	0	-1
N.S.	1	1.00	3.22	4.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.238	17.714	7.459	0.000	0.000	0.000	0.000	0.000

Problem 354	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	429	429	1149	2065	0	0	0	0	-1
N.S.	1	1.00	2.68	4.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.470	18.017	7.554	0.000	0.000	0.000	0.000	0.000

Problem 355	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	509	509	1548	2954	0	0	0	0	-1
N.S.	1	1.00	3.04	5.80	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.704	19.526	8.254	0.000	0.000	0.000	0.000	0.000

Problem 356	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	475	475	3766	4394	0	0	0	0	-1
N.S.	1	1.00	7.93	9.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.786	26.167	11.224	0.000	0.000	0.000	0.000	0.000

Problem 357	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	388	388	3342	3424	0	0	0	0	-1
N.S.	1	1.00	8.61	8.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.532	24.228	11.401	0.000	0.000	0.000	0.000	0.000

Problem 358	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	312	312	502	2683	0	0	0	0	-1
N.S.	1	1.00	1.61	8.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.360	18.369	10.985	0.000	0.000	0.000	0.000	0.000

Problem 359	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	381	381	6063	2337	0	0	0	0	-1
N.S.	1	1.00	15.91	6.13	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.300	24.391	10.273	0.000	0.000	0.000	0.000	0.000

Problem 360	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	361	361	971	2196	0	0	0	0	-1
N.S.	1	1.00	2.69	6.08	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.290	16.179	10.023	0.000	0.000	0.000	0.000	0.000

Problem 361	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	428	428	1598	2439	0	0	0	0	-1
N.S.	1	1.00	3.73	5.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.510	19.214	8.331	0.000	0.000	0.000	0.000	0.000

Problem 362	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	520	520	1535	3142	0	0	0	0	-1
N.S.	1	1.00	2.95	6.04	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.815	19.043	9.016	0.000	0.000	0.000	0.000	0.000

Problem 363	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	566	566	4227	5368	0	0	0	0	-1
N.S.	1	1.00	7.47	9.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.153	26.886	11.266	0.000	0.000	0.000	0.000	0.000

Problem 364	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	469	469	3781	4395	0	0	0	0	-1
N.S.	1	1.00	8.06	9.37	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.760	26.067	11.705	0.000	0.000	0.000	0.000	0.000

Problem 365	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	384	384	2957	3637	0	0	0	0	-1
N.S.	1	1.00	7.70	9.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.509	22.734	11.257	0.000	0.000	0.000	0.000	0.000

Problem 366	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	442	442	7138	3285	0	0	0	0	-1
N.S.	1	1.00	16.15	7.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.423	25.455	11.081	0.000	0.000	0.000	0.000	0.000

Problem 367	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	433	433	7745	3215	0	0	0	0	-1
N.S.	1	1.00	17.89	7.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.442	25.669	11.121	0.000	0.000	0.000	0.000	0.000

Problem 368	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	450	450	1326	3271	0	0	0	0	-1
N.S.	1	1.00	2.95	7.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.522	19.248	10.897	0.000	0.000	0.000	0.000	0.000

Problem 369	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	518	518	1551	3511	0	0	0	0	-1
N.S.	1	1.00	2.99	6.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.871	19.384	8.857	0.000	0.000	0.000	0.000	0.000

Problem 370	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	617	617	5172	4231	0	0	0	0	-1
N.S.	1	1.00	8.38	6.86	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.152	24.090	8.134	0.000	0.000	0.000	0.000	0.000

Problem 371	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	3000	2499	0	0	0	0	-1
N.S.	1	1.00	9.12	7.60	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.390	21.852	11.081	0.000	0.000	0.000	0.000	0.000

Problem 372	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	372	1563	0	0	0	0	-1
N.S.	1	1.00	1.43	5.99	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.244	15.634	11.151	0.000	0.000	0.000	0.000	0.000

Problem 373	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	356	829	0	0	0	0	-1
N.S.	1	1.00	1.70	3.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.126	13.519	10.915	0.000	0.000	0.000	0.000	0.000

Problem 374	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	145	215	0	0	0	0	-1
N.S.	1	1.00	0.70	1.03	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.080	2.389	7.964	0.000	0.000	0.000	0.000	0.000

Problem 375	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	348	348	1027	1025	0	0	0	0	-1
N.S.	1	1.00	2.95	2.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.266	16.323	9.354	0.000	0.000	0.000	0.000	0.000

Problem 376	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	435	435	1639	1886	0	0	0	0	-1
N.S.	1	1.00	3.77	4.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.472	14.980	8.957	0.000	0.000	0.000	0.000	0.000

Problem 377	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	525	525	1569	2954	0	0	0	0	-1
N.S.	1	1.00	2.99	5.63	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.766	19.528	8.766	0.000	0.000	0.000	0.000	0.000

Problem 378	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	3460	3329	0	0	0	0	-1
N.S.	1	1.00	10.52	10.12	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.462	24.396	17.638	0.000	0.000	0.000	0.000	0.000

Problem 379	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	467	2275	0	0	0	0	-1
N.S.	1	1.00	1.70	8.27	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.289	17.621	17.279	0.000	0.000	0.000	0.000	0.000

Problem 380	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	468	1634	0	0	0	0	-1
N.S.	1	1.00	1.84	6.43	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.226	14.937	9.279	0.000	0.000	0.000	0.000	0.000

Problem 381	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	376	376	1491	2009	0	0	0	0	-1
N.S.	1	1.00	3.97	5.34	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.281	14.289	9.452	0.000	0.000	0.000	0.000	0.000

Problem 382	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	1597	2874	0	0	0	0	-1
N.S.	1	1.00	3.74	6.73	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.452	19.228	8.941	0.000	0.000	0.000	0.000	0.000

Problem 383	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	531	531	2667	3980	0	0	0	0	-1
N.S.	1	1.00	5.02	7.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.724	17.626	9.355	0.000	0.000	0.000	0.000	0.000

Problem 384	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	630	630	2319	5086	0	0	0	0	-1
N.S.	1	1.00	3.68	8.07	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.083	21.878	9.306	0.000	0.000	0.000	0.000	0.000

Problem 385	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	510	510	4342	8044	0	0	0	0	-1
N.S.	1	1.00	8.51	15.77	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.980	27.101	18.332	0.000	0.000	0.000	0.000	0.000

Problem 386	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	417	417	3920	6455	0	0	0	0	-1
N.S.	1	1.00	9.40	15.48	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.603	26.213	17.881	0.000	0.000	0.000	0.000	0.000

Problem 387	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	387	387	3514	5170	0	0	0	0	-1
N.S.	1	1.00	9.08	13.36	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.440	23.777	9.595	0.000	0.000	0.000	0.000	0.000

Problem 388	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	353	353	603	4213	0	0	0	0	-1
N.S.	1	1.00	1.71	11.93	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.389	18.146	9.354	0.000	0.000	0.000	0.000	0.000

Problem 389	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	495	495	2083	5712	0	0	0	0	-1
N.S.	1	1.00	4.21	11.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.511	16.484	9.980	0.000	0.000	0.000	0.000	0.000

Problem 390	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	582	582	2366	8545	0	0	0	0	-1
N.S.	1	1.00	4.07	14.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.770	21.488	9.478	0.000	0.000	0.000	0.000	0.000

Problem 391	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	686	686	821	10322	0	0	0	0	-1
N.S.	1	1.00	1.20	15.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.284	15.013	9.595	0.000	0.000	0.000	0.000	0.000

Problem 392	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	105	105	248	642	0	0	0	0	-1
N.S.	1	1.00	2.36	6.11	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.050	9.848	11.811	0.000	0.000	0.000	0.000	0.000

Problem 393	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	107	107	211	457	0	0	0	0	-1
N.S.	1	1.00	1.97	4.27	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.058	7.403	11.471	0.000	0.000	0.000	0.000	0.000

Problem 394	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	132	636	0	235	0	0	-1
N.S.	1	1.00	0.73	3.53	0.00	1.31	0.00	0.00	-0.01
time (sec)	N/A	0.121	1.924	4.524	0.000	0.657	0.000	0.000	0.000

Problem 395	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	143	143	104	401	0	205	0	0	-1
N.S.	1	1.00	0.73	2.80	0.00	1.43	0.00	0.00	-0.01
time (sec)	N/A	0.102	0.917	3.295	0.000	0.475	0.000	0.000	0.000

Problem 396	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	111	111	84	244	0	153	0	0	-1
N.S.	1	1.00	0.76	2.20	0.00	1.38	0.00	0.00	-0.01
time (sec)	N/A	0.093	0.297	1.547	0.000	0.822	0.000	0.000	0.000

Problem 397	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	115	115	90	326	0	156	0	0	-1
N.S.	1	1.00	0.78	2.83	0.00	1.36	0.00	0.00	-0.01
time (sec)	N/A	0.100	0.272	1.345	0.000	0.698	0.000	0.000	0.000

Problem 398	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	148	148	108	371	0	184	0	0	-1
N.S.	1	1.00	0.73	2.51	0.00	1.24	0.00	0.00	-0.01
time (sec)	N/A	0.115	0.617	1.372	0.000	0.567	0.000	0.000	0.000

Problem 399	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	125	413	0	203	0	0	-1
N.S.	1	1.00	0.69	2.29	0.00	1.13	0.00	0.00	-0.01
time (sec)	N/A	0.127	1.101	1.594	0.000	0.996	0.000	0.000	0.000

Problem 400	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	221	832	0	314	0	0	-1
N.S.	1	1.00	0.84	3.16	0.00	1.19	0.00	0.00	-0.00
time (sec)	N/A	0.245	4.367	6.467	0.000	0.518	0.000	0.000	0.000

Problem 401	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	171	723	0	286	0	0	-1
N.S.	1	1.00	0.77	3.27	0.00	1.29	0.00	0.00	-0.00
time (sec)	N/A	0.212	2.655	5.272	0.000	0.668	0.000	0.000	0.000

Problem 402	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	125	650	0	247	0	0	-1
N.S.	1	1.00	0.71	3.67	0.00	1.40	0.00	0.00	-0.01
time (sec)	N/A	0.180	1.259	3.348	0.000	0.409	0.000	0.000	0.000

Problem 403	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	124	404	0	208	0	0	-1
N.S.	1	1.00	0.77	2.51	0.00	1.29	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.767	1.823	0.000	0.607	0.000	0.000	0.000

Problem 404	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	128	487	0	226	0	0	-1
N.S.	1	1.00	0.75	2.85	0.00	1.32	0.00	0.00	-0.01
time (sec)	N/A	0.178	0.980	1.794	0.000	0.533	0.000	0.000	0.000

Problem 405	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	213	213	161	548	0	254	0	0	-1
N.S.	1	1.00	0.76	2.57	0.00	1.19	0.00	0.00	-0.00
time (sec)	N/A	0.196	1.485	1.606	0.000	0.709	0.000	0.000	0.000

Problem 406	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	254	254	189	610	0	282	0	0	-1
N.S.	1	1.00	0.74	2.40	0.00	1.11	0.00	0.00	-0.00
time (sec)	N/A	0.225	1.890	1.820	0.000	0.916	0.000	0.000	0.000

Problem 407	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	452	1166	0	401	0	0	-1
N.S.	1	1.00	1.31	3.38	0.00	1.16	0.00	0.00	-0.00
time (sec)	N/A	0.372	6.556	9.208	0.000	1.200	0.000	0.000	0.000

Problem 408	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	225	917	0	364	0	0	-1
N.S.	1	1.00	0.76	3.11	0.00	1.23	0.00	0.00	-0.00
time (sec)	N/A	0.335	3.884	6.648	0.000	0.974	0.000	0.000	0.000

Problem 409	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	244	244	190	970	0	326	0	0	-1
N.S.	1	1.00	0.78	3.98	0.00	1.34	0.00	0.00	-0.00
time (sec)	N/A	0.314	2.459	5.296	0.000	0.932	0.000	0.000	0.000

Problem 410	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	239	239	166	1212	0	298	0	0	-1
N.S.	1	1.00	0.69	5.07	0.00	1.25	0.00	0.00	-0.00
time (sec)	N/A	0.330	1.978	3.895	0.000	0.780	0.000	0.000	0.000

Problem 411	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	236	236	172	641	0	270	0	0	-1
N.S.	1	1.00	0.73	2.72	0.00	1.14	0.00	0.00	-0.00
time (sec)	N/A	0.305	1.543	2.201	0.000	0.768	0.000	0.000	0.000

Problem 412	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	245	245	180	664	0	295	0	0	-1
N.S.	1	1.00	0.73	2.71	0.00	1.20	0.00	0.00	-0.00
time (sec)	N/A	0.305	1.387	1.897	0.000	0.666	0.000	0.000	0.000

Problem 413	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	295	295	219	745	0	332	0	0	-1
N.S.	1	1.00	0.74	2.53	0.00	1.13	0.00	0.00	-0.00
time (sec)	N/A	0.317	2.053	1.939	0.000	1.114	0.000	0.000	0.000

Problem 414	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	345	345	256	825	0	369	0	0	-1
N.S.	1	1.00	0.74	2.39	0.00	1.07	0.00	0.00	-0.00
time (sec)	N/A	0.373	3.058	1.804	0.000	0.648	0.000	0.000	0.000

Problem 415	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	277	277	664	758	0	0	0	0	-1
N.S.	1	1.00	2.40	2.74	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.679	36.983	6.202	0.000	0.000	0.000	0.000	0.000

Problem 416	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	210	210	225	439	0	0	0	0	-1
N.S.	1	1.00	1.07	2.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.456	33.699	3.740	0.000	0.000	0.000	0.000	0.000

Problem 417	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	123	298	0	0	0	0	-1
N.S.	1	1.00	0.98	2.37	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.231	31.402	2.251	0.000	0.000	0.000	0.000	0.000

Problem 418	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	76	217	0	0	0	0	-1
N.S.	1	1.00	0.75	2.15	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.128	20.639	1.455	0.000	0.000	0.000	0.000	0.000

Problem 419	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	149	149	220	295	0	0	0	0	-1
N.S.	1	1.00	1.48	1.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.179	37.122	1.523	0.000	0.000	0.000	0.000	0.000

Problem 420	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	540	822	0	0	0	0	-1
N.S.	1	1.00	2.76	4.19	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.300	36.854	1.957	0.000	0.000	0.000	0.000	0.000

Problem 421	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	242	242	612	1074	0	0	0	0	-1
N.S.	1	1.00	2.53	4.44	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.501	36.965	1.954	0.000	0.000	0.000	0.000	0.000

Problem 422	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	406	406	733	997	0	0	0	0	-1
N.S.	1	1.00	1.81	2.46	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.778	37.248	8.394	0.000	0.000	0.000	0.000	0.000

Problem 423	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	315	315	680	850	0	0	0	0	-1
N.S.	1	1.00	2.16	2.70	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.546	37.060	4.346	0.000	0.000	0.000	0.000	0.000

Problem 424	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	257	257	638	715	0	0	0	0	-1
N.S.	1	1.00	2.48	2.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.353	36.860	3.530	0.000	0.000	0.000	0.000	0.000

Problem 425	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	263	263	722	802	0	0	0	0	-1
N.S.	1	1.00	2.75	3.05	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.329	36.961	3.652	0.000	0.000	0.000	0.000	0.000

Problem 426	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	283	283	652	843	0	0	0	0	-1
N.S.	1	1.00	2.30	2.98	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.390	36.995	4.303	0.000	0.000	0.000	0.000	0.000

Problem 427	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	365	365	699	1059	0	0	0	0	-1
N.S.	1	1.00	1.92	2.90	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.565	37.180	4.999	0.000	0.000	0.000	0.000	0.000

Problem 428	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	583	583	897	2151	0	0	0	0	-1
N.S.	1	1.00	1.54	3.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.165	37.647	18.158	0.000	0.000	0.000	0.000	0.000

Problem 429	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	480	480	842	1997	0	0	0	0	-1
N.S.	1	1.00	1.75	4.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.857	37.312	9.677	0.000	0.000	0.000	0.000	0.000

Problem 430	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	795	1768	0	0	0	0	-1
N.S.	1	1.00	1.98	4.40	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.623	37.151	7.404	0.000	0.000	0.000	0.000	0.000

Problem 431	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	882	1872	0	0	0	0	-1
N.S.	1	1.00	2.19	4.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.598	37.167	7.734	0.000	0.000	0.000	0.000	0.000

Problem 432	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	402	402	885	1959	0	0	0	0	-1
N.S.	1	1.00	2.20	4.87	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.559	37.192	8.074	0.000	0.000	0.000	0.000	0.000

Problem 433	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	818	2000	0	0	0	0	-1
N.S.	1	1.00	1.92	4.68	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.670	37.352	8.641	0.000	0.000	0.000	0.000	0.000

Problem 434	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-2)	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	521	521	863	2216	0	0	0	0	-1
N.S.	1	1.00	1.66	4.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.945	37.562	9.870	0.000	0.000	0.000	0.000	0.000

Problem 435	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	422	2521	0	0	0	0	-1
N.S.	1	1.00	1.26	7.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.735	15.493	12.968	0.000	0.000	0.000	0.000	0.000

Problem 436	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	377	1431	0	0	0	0	-1
N.S.	1	1.00	1.49	5.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.520	15.011	13.500	0.000	0.000	0.000	0.000	0.000

Problem 437	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	122	1549	0	0	0	0	-1
N.S.	1	1.00	0.59	7.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.359	2.549	13.146	0.000	0.000	0.000	0.000	0.000

Problem 438	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	165	1926	0	452	0	0	-1
N.S.	1	1.00	0.82	9.58	0.00	2.25	0.00	0.00	-0.00
time (sec)	N/A	0.316	0.813	13.128	0.000	0.620	0.000	0.000	0.000

Problem 439	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	200	2739	0	519	0	0	-1
N.S.	1	1.00	0.75	10.26	0.00	1.94	0.00	0.00	-0.00
time (sec)	N/A	0.496	1.280	13.333	0.000	1.165	0.000	0.000	0.000

Problem 440	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	208	3778	0	588	0	0	-1
N.S.	1	1.00	0.61	11.01	0.00	1.71	0.00	0.00	-0.00
time (sec)	N/A	0.697	1.414	14.096	0.000	0.651	0.000	0.000	0.000

Problem 441	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	673	4051	0	0	0	0	-1
N.S.	1	1.00	1.60	9.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.044	16.824	13.318	0.000	0.000	0.000	0.000	0.000

Problem 442	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	595	2947	0	0	0	0	-1
N.S.	1	1.00	1.76	8.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.801	16.757	12.969	0.000	0.000	0.000	0.000	0.000

Problem 443	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	554	2595	0	0	0	0	-1
N.S.	1	1.00	2.04	9.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.559	16.782	13.549	0.000	0.000	0.000	0.000	0.000

Problem 444	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	437	2552	0	0	0	0	-1
N.S.	1	1.00	1.58	9.25	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.597	14.446	13.112	0.000	0.000	0.000	0.000	0.000

Problem 445	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	201	2915	0	520	0	0	-1
N.S.	1	1.00	0.76	10.96	0.00	1.95	0.00	0.00	-0.00
time (sec)	N/A	0.526	1.584	14.085	0.000	1.224	0.000	0.000	0.000

Problem 446	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	255	3752	0	589	0	0	-1
N.S.	1	1.00	0.75	10.97	0.00	1.72	0.00	0.00	-0.00
time (sec)	N/A	0.741	1.962	14.165	0.000	1.020	0.000	0.000	0.000

Problem 447	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	313	4846	0	666	0	0	-1
N.S.	1	1.00	0.73	11.35	0.00	1.56	0.00	0.00	-0.00
time (sec)	N/A	1.000	2.110	14.027	0.000	0.919	0.000	0.000	0.000

Problem 448	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	768	5392	0	0	0	0	-1
N.S.	1	1.00	1.50	10.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.326	16.966	13.913	0.000	0.000	0.000	0.000	0.000

Problem 449	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	678	4258	0	0	0	0	-1
N.S.	1	1.00	1.61	10.09	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.034	16.873	13.375	0.000	0.000	0.000	0.000	0.000

Problem 450	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	628	3939	0	0	0	0	-1
N.S.	1	1.00	1.75	10.97	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.796	16.874	13.608	0.000	0.000	0.000	0.000	0.000

Problem 451	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	599	3663	0	0	0	0	-1
N.S.	1	1.00	1.72	10.50	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.786	16.919	14.095	0.000	0.000	0.000	0.000	0.000

Problem 452	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	616	3564	0	0	0	0	-1
N.S.	1	1.00	1.80	10.42	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.778	16.883	13.687	0.000	0.000	0.000	0.000	0.000

Problem 453	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	257	3980	0	589	0	0	-1
N.S.	1	1.00	0.76	11.71	0.00	1.73	0.00	0.00	-0.00
time (sec)	N/A	0.744	1.886	13.947	0.000	0.561	0.000	0.000	0.000

Problem 454	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	313	4847	0	666	0	0	-1
N.S.	1	1.00	0.74	11.40	0.00	1.57	0.00	0.00	-0.00
time (sec)	N/A	0.994	2.569	15.124	0.000	0.901	0.000	0.000	0.000

Problem 455	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	380	5946	0	753	0	0	-1
N.S.	1	1.00	0.73	11.46	0.00	1.45	0.00	0.00	-0.00
time (sec)	N/A	1.270	3.517	14.378	0.000	1.497	0.000	0.000	0.000

Problem 456	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	451	2737	0	0	0	0	-1
N.S.	1	1.00	1.31	7.96	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.720	13.908	12.737	0.000	0.000	0.000	0.000	0.000

Problem 457	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	339	1439	0	0	0	0	-1
N.S.	1	1.00	1.32	5.62	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.472	14.725	13.727	0.000	0.000	0.000	0.000	0.000

Problem 458	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	C	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	91	265	0	0	0	0	-1
N.S.	1	1.00	0.66	1.92	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.252	0.266	13.121	0.000	0.000	0.000	0.000	0.000

Problem 459	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	103	940	0	371	0	0	-1
N.S.	1	1.00	0.69	6.27	0.00	2.47	0.00	0.00	-0.01
time (sec)	N/A	0.203	3.778	14.080	0.000	0.864	0.000	0.000	0.000

Problem 460	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	161	1731	0	452	0	0	-1
N.S.	1	1.00	0.76	8.17	0.00	2.13	0.00	0.00	-0.00
time (sec)	N/A	0.318	0.901	14.431	0.000	1.129	0.000	0.000	0.000

Problem 461	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	198	2739	0	520	0	0	-1
N.S.	1	1.00	0.71	9.78	0.00	1.86	0.00	0.00	-0.00
time (sec)	N/A	0.488	1.298	13.656	0.000	0.870	0.000	0.000	0.000

Problem 462	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	518	2656	0	0	0	0	-1
N.S.	1	1.00	1.40	7.16	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.825	15.678	20.758	0.000	0.000	0.000	0.000	0.000

Problem 463	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	464	1585	0	0	0	0	-1
N.S.	1	1.00	2.11	7.20	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.409	14.442	24.046	0.000	0.000	0.000	0.000	0.000

Problem 464	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	161	945	0	606	0	0	-1
N.S.	1	1.00	0.75	4.40	0.00	2.82	0.00	0.00	-0.00
time (sec)	N/A	0.324	0.740	20.414	0.000	0.954	0.000	0.000	0.000

Problem 465	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	178	1448	0	679	0	0	-1
N.S.	1	1.00	0.76	6.16	0.00	2.89	0.00	0.00	-0.00
time (sec)	N/A	0.364	1.071	22.763	0.000	0.765	0.000	0.000	0.000

Problem 466	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	252	2287	0	792	0	0	-1
N.S.	1	1.00	0.77	7.02	0.00	2.43	0.00	0.00	-0.00
time (sec)	N/A	0.547	1.656	22.735	0.000	1.054	0.000	0.000	0.000

Problem 467	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	423	423	316	3156	0	916	0	0	-1
N.S.	1	1.00	0.75	7.46	0.00	2.17	0.00	0.00	-0.00
time (sec)	N/A	0.788	2.312	22.514	0.000	1.364	0.000	0.000	0.000

Problem 468	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	726	5195	0	0	0	0	-1
N.S.	1	1.00	1.82	13.02	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.841	16.917	21.628	0.000	0.000	0.000	0.000	0.000

Problem 469	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	217	3138	0	951	0	0	-1
N.S.	1	1.00	0.66	9.54	0.00	2.89	0.00	0.00	-0.00
time (sec)	N/A	0.548	1.874	21.479	0.000	0.838	0.000	0.000	0.000

Problem 470	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	245	3865	0	1071	0	0	-1
N.S.	1	1.00	0.71	11.17	0.00	3.10	0.00	0.00	-0.00
time (sec)	N/A	0.525	2.190	22.298	0.000	1.497	0.000	0.000	0.000

Problem 471	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	297	5169	0	1186	0	0	-1
N.S.	1	1.00	0.81	14.05	0.00	3.22	0.00	0.00	-0.00
time (sec)	N/A	0.594	2.487	24.294	0.000	1.628	0.000	0.000	0.000

Problem 472	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	472	472	353	6745	0	1343	0	0	-1
N.S.	1	1.00	0.75	14.29	0.00	2.85	0.00	0.00	-0.00
time (sec)	N/A	0.863	3.105	22.661	0.000	2.940	0.000	0.000	0.000

Problem 473	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	588	588	392	8251	0	1531	0	0	-1
N.S.	1	1.00	0.67	14.03	0.00	2.60	0.00	0.00	-0.00
time (sec)	N/A	1.187	3.972	16.869	0.000	2.475	0.000	0.000	0.000

Problem 474	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.115	41.891	0.298	0.000	0.000	0.000	0.000	0.000

Problem 475	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.094	42.309	0.336	0.000	0.000	0.000	0.000	0.000

Problem 476	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.095	12.533	0.287	0.000	0.000	0.000	0.000	0.000

Problem 477	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	F(-1)	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	126	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.101	15.923	0.300	0.000	0.000	0.000	0.000	0.000

Problem 478	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	N/A	A	A	A	A	A	A	A	A
verified	N/A	N/A	N/A	TBD	TBD	TBD	TBD	TBD	TBD
size	36	0	0	0	0	0	0	0	-1
N.S.	1	0.00	0.00	0.00	0.00	0.00	0.00	0.00	-0.03
time (sec)	N/A	0.054	4.463	0.476	0.000	0.000	0.000	0.000	0.000

Problem 479	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	544	544	365	0	0	0	0	0	-1
N.S.	1	1.00	0.67	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.069	4.825	0.415	0.000	0.000	0.000	0.000	0.000

Problem 480	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	366	366	307	0	0	0	0	0	-1
N.S.	1	1.00	0.84	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.523	2.360	0.331	0.000	0.000	0.000	0.000	0.000

Problem 481	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	261	261	239	0	0	0	0	0	-1
N.S.	1	1.00	0.92	0.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.261	1.038	0.322	0.000	0.000	0.000	0.000	0.000

Problem 482	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	F	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	177	177	168	0	0	0	0	0	-1
N.S.	1	1.00	0.95	0.00	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.136	0.432	0.231	0.000	0.000	0.000	0.000	0.000

Problem 483	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	872	383	0	179	0	0	166
N.S.	1	1.00	6.61	2.90	0.00	1.36	0.00	0.00	1.26
time (sec)	N/A	0.154	6.310	1.993	0.000	0.611	0.000	0.000	0.891

Problem 484	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	101	101	830	355	0	161	0	0	128
N.S.	1	1.00	8.22	3.51	0.00	1.59	0.00	0.00	1.27
time (sec)	N/A	0.142	6.226	1.849	0.000	0.597	0.000	0.000	0.358

Problem 485	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	70	70	309	321	0	142	0	0	85
N.S.	1	1.00	4.41	4.59	0.00	2.03	0.00	0.00	1.21
time (sec)	N/A	0.125	5.965	1.770	0.000	0.913	0.000	0.000	2.661

Problem 486	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	66	66	252	240	0	173	0	0	96
N.S.	1	1.00	3.82	3.64	0.00	2.62	0.00	0.00	1.45
time (sec)	N/A	0.129	5.936	1.915	0.000	0.677	0.000	0.000	3.059

Problem 487	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	95	95	813	399	0	196	0	0	150
N.S.	1	1.00	8.56	4.20	0.00	2.06	0.00	0.00	1.58
time (sec)	N/A	0.145	6.372	3.520	0.000	0.521	0.000	0.000	3.293

Problem 488	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	132	132	865	634	0	219	0	0	177
N.S.	1	1.00	6.55	4.80	0.00	1.66	0.00	0.00	1.34
time (sec)	N/A	0.160	6.435	5.141	0.000	0.583	0.000	0.000	3.553

Problem 489	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	1086	413	0	223	0	0	266
N.S.	1	1.00	5.60	2.13	0.00	1.15	0.00	0.00	1.37
time (sec)	N/A	0.259	6.318	1.767	0.000	0.804	0.000	0.000	3.189

Problem 490	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	161	161	1040	385	0	203	0	0	231
N.S.	1	1.00	6.46	2.39	0.00	1.26	0.00	0.00	1.43
time (sec)	N/A	0.239	6.280	1.740	0.000	0.800	0.000	0.000	3.012

Problem 491	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	994	357	0	179	0	0	153
N.S.	1	1.00	7.89	2.83	0.00	1.42	0.00	0.00	1.21
time (sec)	N/A	0.226	6.320	1.862	0.000	0.599	0.000	0.000	2.915

Problem 492	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-2)	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	116	116	735	245	0	198	0	0	134
N.S.	1	1.00	6.34	2.11	0.00	1.71	0.00	0.00	1.16
time (sec)	N/A	0.219	6.413	1.863	0.000	0.689	0.000	0.000	3.162

Problem 493	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	120	120	736	513	0	210	0	0	196
N.S.	1	1.00	6.13	4.28	0.00	1.75	0.00	0.00	1.63
time (sec)	N/A	0.231	6.467	2.241	0.000	0.803	0.000	0.000	3.938

Problem 494	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	159	159	1025	714	0	239	0	0	229
N.S.	1	1.00	6.45	4.49	0.00	1.50	0.00	0.00	1.44
time (sec)	N/A	0.251	6.552	4.899	0.000	0.718	0.000	0.000	4.153

Problem 495	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F	F	B
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	194	194	1067	824	0	263	0	0	235
N.S.	1	1.00	5.50	4.25	0.00	1.36	0.00	0.00	1.21
time (sec)	N/A	0.271	6.640	6.447	0.000	0.940	0.000	0.000	4.692

Problem 496	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	157	157	1292	282	0	269	0	0	-1
N.S.	1	1.00	8.23	1.80	0.00	1.71	0.00	0.00	-0.01
time (sec)	N/A	0.175	6.686	1.720	0.000	0.790	0.000	0.000	0.000

Problem 497	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	124	124	1239	262	0	250	0	0	-1
N.S.	1	1.00	9.99	2.11	0.00	2.02	0.00	0.00	-0.01
time (sec)	N/A	0.163	6.597	1.935	0.000	0.659	0.000	0.000	0.000

Problem 498	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	88	88	1208	244	0	241	0	0	-1
N.S.	1	1.00	13.73	2.77	0.00	2.74	0.00	0.00	-0.01
time (sec)	N/A	0.143	6.491	1.596	0.000	0.849	0.000	0.000	0.000

Problem 499	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	83	83	1204	243	0	237	0	0	-1
N.S.	1	1.00	14.51	2.93	0.00	2.86	0.00	0.00	-0.01
time (sec)	N/A	0.145	6.517	1.596	0.000	0.752	0.000	0.000	0.000

Problem 500	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	113	113	1240	318	0	292	0	0	-1
N.S.	1	1.00	10.97	2.81	0.00	2.58	0.00	0.00	-0.01
time (sec)	N/A	0.163	6.732	2.562	0.000	1.086	0.000	0.000	0.000

Problem 501	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	152	152	1277	466	0	318	0	0	-1
N.S.	1	1.00	8.40	3.07	0.00	2.09	0.00	0.00	-0.01
time (sec)	N/A	0.170	7.158	4.279	0.000	0.803	0.000	0.000	0.000

Problem 502	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	204	204	1396	465	0	384	0	0	-1
N.S.	1	1.00	6.84	2.28	0.00	1.88	0.00	0.00	-0.00
time (sec)	N/A	0.274	6.927	2.017	0.000	0.554	0.000	0.000	0.000

Problem 503	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	171	171	1352	435	0	366	0	0	-1
N.S.	1	1.00	7.91	2.54	0.00	2.14	0.00	0.00	-0.01
time (sec)	N/A	0.262	6.839	2.299	0.000	0.870	0.000	0.000	0.000

Problem 504	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	137	137	1318	421	0	356	0	0	-1
N.S.	1	1.00	9.62	3.07	0.00	2.60	0.00	0.00	-0.01
time (sec)	N/A	0.238	6.709	2.049	0.000	0.819	0.000	0.000	0.000

Problem 505	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	921	350	0	314	0	0	-1
N.S.	1	1.00	7.61	2.89	0.00	2.60	0.00	0.00	-0.01
time (sec)	N/A	0.233	6.566	1.865	0.000	1.008	0.000	0.000	0.000

Problem 506	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	921	350	0	318	0	0	-1
N.S.	1	1.00	7.61	2.89	0.00	2.63	0.00	0.00	-0.01
time (sec)	N/A	0.227	6.594	2.187	0.000	0.878	0.000	0.000	0.000

Problem 507	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	164	164	1351	492	0	407	0	0	-1
N.S.	1	1.00	8.24	3.00	0.00	2.48	0.00	0.00	-0.01
time (sec)	N/A	0.269	6.851	2.634	0.000	0.609	0.000	0.000	0.000

Problem 508	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	1392	723	0	436	0	0	-1
N.S.	1	1.00	7.07	3.67	0.00	2.21	0.00	0.00	-0.01
time (sec)	N/A	0.272	7.489	5.572	0.000	1.255	0.000	0.000	0.000

Problem 509	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	A	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	1448	465	0	478	0	0	-1
N.S.	1	1.00	6.55	2.10	0.00	2.16	0.00	0.00	-0.00
time (sec)	N/A	0.373	7.074	2.395	0.000	1.296	0.000	0.000	0.000

Problem 510	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	188	188	1415	451	0	467	0	0	-1
N.S.	1	1.00	7.53	2.40	0.00	2.48	0.00	0.00	-0.01
time (sec)	N/A	0.350	6.952	2.347	0.000	0.723	0.000	0.000	0.000

Problem 511	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	1407	451	0	465	0	0	-1
N.S.	1	1.00	7.73	2.48	0.00	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.358	6.895	2.105	0.000	0.800	0.000	0.000	0.000

Problem 512	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	1406	451	0	465	0	0	-1
N.S.	1	1.00	7.90	2.53	0.00	2.61	0.00	0.00	-0.01
time (sec)	N/A	0.338	6.810	2.064	0.000	0.818	0.000	0.000	0.000

Problem 513	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	180	180	1407	451	0	465	0	0	-1
N.S.	1	1.00	7.82	2.51	0.00	2.58	0.00	0.00	-0.01
time (sec)	N/A	0.347	6.853	2.241	0.000	0.841	0.000	0.000	0.000

Problem 514	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	221	221	1447	685	0	521	0	0	-1
N.S.	1	1.00	6.55	3.10	0.00	2.36	0.00	0.00	-0.00
time (sec)	N/A	0.385	7.171	3.144	0.000	0.558	0.000	0.000	0.000

Problem 515	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	119	130	547	116	0	0	-1
N.S.	1	1.00	0.54	0.59	2.49	0.53	0.00	0.00	-0.00
time (sec)	N/A	0.309	0.503	12.105	0.855	2.219	0.000	0.000	0.000

Problem 516	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	175	175	96	108	418	99	0	0	-1
N.S.	1	1.00	0.55	0.62	2.39	0.57	0.00	0.00	-0.01
time (sec)	N/A	0.271	0.348	11.261	0.791	1.804	0.000	0.000	0.000

Problem 517	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	130	130	79	86	296	81	0	0	-1
N.S.	1	1.00	0.61	0.66	2.28	0.62	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.172	12.559	0.801	2.074	0.000	0.000	0.000

Problem 518	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	82	82	56	65	141	64	0	0	-1
N.S.	1	1.00	0.68	0.79	1.72	0.78	0.00	0.00	-0.01
time (sec)	N/A	0.167	0.199	11.901	0.853	2.180	0.000	0.000	0.000

Problem 519	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	96	96	94	169	262	298	0	0	-1
N.S.	1	1.00	0.98	1.76	2.73	3.10	0.00	0.00	-0.01
time (sec)	N/A	0.165	0.334	11.963	0.775	2.412	0.000	0.000	0.000

Problem 520	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	98	98	89	274	905	351	0	0	-1
N.S.	1	1.00	0.91	2.80	9.23	3.58	0.00	0.00	-0.01
time (sec)	N/A	0.170	0.405	12.610	0.906	2.756	0.000	0.000	0.000

Problem 521	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	151	151	106	342	1927	401	0	0	-1
N.S.	1	1.00	0.70	2.26	12.76	2.66	0.00	0.00	-0.01
time (sec)	N/A	0.213	0.649	12.500	1.012	2.839	0.000	0.000	0.000

Problem 522	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	196	196	131	404	3342	439	0	0	-1
N.S.	1	1.00	0.67	2.06	17.05	2.24	0.00	0.00	-0.01
time (sec)	N/A	0.262	1.046	12.102	1.051	3.093	0.000	0.000	0.000

Problem 523	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	131	153	703	142	0	0	-1
N.S.	1	1.00	0.48	0.56	2.56	0.52	0.00	0.00	-0.00
time (sec)	N/A	0.470	0.539	13.450	0.971	1.956	0.000	0.000	0.000

Problem 524	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	118	131	558	124	0	0	-1
N.S.	1	1.00	0.52	0.57	2.45	0.54	0.00	0.00	-0.00
time (sec)	N/A	0.412	0.427	12.279	0.914	2.334	0.000	0.000	0.000

Problem 525	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	100	109	451	105	0	0	-1
N.S.	1	1.00	0.55	0.60	2.49	0.58	0.00	0.00	-0.01
time (sec)	N/A	0.356	0.323	11.852	0.904	2.534	0.000	0.000	0.000

Problem 526	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	131	131	80	87	276	86	0	0	-1
N.S.	1	1.00	0.61	0.66	2.11	0.66	0.00	0.00	-0.01
time (sec)	N/A	0.234	0.273	11.778	0.890	2.874	0.000	0.000	0.000

Problem 527	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	145	145	101	201	583	343	0	0	-1
N.S.	1	1.00	0.70	1.39	4.02	2.37	0.00	0.00	-0.01
time (sec)	N/A	0.280	0.451	12.480	0.919	2.824	0.000	0.000	0.000

Problem 528	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	144	144	133	306	1417	389	0	0	-1
N.S.	1	1.00	0.92	2.12	9.84	2.70	0.00	0.00	-0.01
time (sec)	N/A	0.272	0.821	12.974	0.862	3.153	0.000	0.000	0.000

Problem 529	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	153	153	107	343	3389	409	0	0	-1
N.S.	1	1.00	0.70	2.24	22.15	2.67	0.00	0.00	-0.01
time (sec)	N/A	0.289	0.829	12.650	1.089	2.483	0.000	0.000	0.000

Problem 530	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	134	405	4606	449	0	0	-1
N.S.	1	1.00	0.67	2.02	23.03	2.24	0.00	0.00	-0.00
time (sec)	N/A	0.347	1.259	13.181	1.225	2.673	0.000	0.000	0.000

Problem 531	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	153	467	5879	485	0	0	-1
N.S.	1	1.00	0.62	1.89	23.80	1.96	0.00	0.00	-0.00
time (sec)	N/A	0.402	1.829	12.553	1.470	2.843	0.000	0.000	0.000

Problem 532	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	275	275	137	155	754	154	0	0	-1
N.S.	1	1.00	0.50	0.56	2.74	0.56	0.00	0.00	-0.00
time (sec)	N/A	0.547	0.581	12.644	1.075	2.382	0.000	0.000	0.000

Problem 533	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	228	228	116	133	596	133	0	0	-1
N.S.	1	1.00	0.51	0.58	2.61	0.58	0.00	0.00	-0.00
time (sec)	N/A	0.479	0.459	12.626	0.908	2.597	0.000	0.000	0.000

Problem 534	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	178	178	99	111	482	112	0	0	-1
N.S.	1	1.00	0.56	0.62	2.71	0.63	0.00	0.00	-0.01
time (sec)	N/A	0.286	0.358	12.733	0.891	2.136	0.000	0.000	0.000

Problem 535	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	192	192	118	225	352	399	0	0	-1
N.S.	1	1.00	0.61	1.17	1.83	2.08	0.00	0.00	-0.01
time (sec)	N/A	0.382	0.658	13.629	0.824	2.072	0.000	0.000	0.000

Problem 536	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	117	368	2589	449	0	0	-1
N.S.	1	1.00	0.59	1.87	13.14	2.28	0.00	0.00	-0.01
time (sec)	N/A	0.404	0.677	12.911	0.984	2.150	0.000	0.000	0.000

Problem 537	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	173	376	14322	453	0	0	-1
N.S.	1	1.00	0.86	1.88	71.61	2.26	0.00	0.00	-0.00
time (sec)	N/A	0.407	0.994	12.980	4.686	2.373	0.000	0.000	0.000

Problem 538	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	200	200	133	407	6297	469	0	0	-1
N.S.	1	1.00	0.66	2.04	31.48	2.34	0.00	0.00	-0.00
time (sec)	N/A	0.415	1.321	13.452	4.776	2.089	0.000	0.000	0.000

Problem 539	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	247	247	154	469	7331	509	0	0	-1
N.S.	1	1.00	0.62	1.90	29.68	2.06	0.00	0.00	-0.00
time (sec)	N/A	0.478	1.900	12.579	1.648	2.863	0.000	0.000	0.000

Problem 540	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	294	294	178	531	9242	549	0	0	-1
N.S.	1	1.00	0.61	1.81	31.44	1.87	0.00	0.00	-0.00
time (sec)	N/A	0.539	2.751	12.716	2.482	3.362	0.000	0.000	0.000

Problem 541	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	250	250	170	217	749	400	0	0	-1
N.S.	1	1.00	0.68	0.87	3.00	1.60	0.00	0.00	-0.00
time (sec)	N/A	0.538	1.155	13.342	0.921	2.793	0.000	0.000	0.000

Problem 542	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	207	207	154	195	581	368	0	0	-1
N.S.	1	1.00	0.74	0.94	2.81	1.78	0.00	0.00	-0.00
time (sec)	N/A	0.414	0.768	13.448	0.918	2.569	0.000	0.000	0.000

Problem 543	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	162	162	124	173	478	336	0	0	-1
N.S.	1	1.00	0.77	1.07	2.95	2.07	0.00	0.00	-0.01
time (sec)	N/A	0.289	0.329	13.043	0.857	2.688	0.000	0.000	0.000

Problem 544	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	A	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	119	119	140	142	195	306	0	0	-1
N.S.	1	1.00	1.18	1.19	1.64	2.57	0.00	0.00	-0.01
time (sec)	N/A	0.187	0.307	13.441	0.862	1.880	0.000	0.000	0.000

Problem 545	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	115	201	699	357	0	0	-1
N.S.	1	1.00	0.82	1.44	4.99	2.55	0.00	0.00	-0.01
time (sec)	N/A	0.216	0.205	12.645	0.891	2.018	0.000	0.000	0.000

Problem 546	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	181	181	114	343	1509	575	0	0	-1
N.S.	1	1.00	0.63	1.90	8.34	3.18	0.00	0.00	-0.01
time (sec)	N/A	0.329	0.520	12.675	0.980	3.185	0.000	0.000	0.000

Problem 547	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	230	230	137	413	2704	621	0	0	-1
N.S.	1	1.00	0.60	1.80	11.76	2.70	0.00	0.00	-0.00
time (sec)	N/A	0.453	0.987	12.523	0.993	3.460	0.000	0.000	0.000

Problem 548	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-2)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	178	329	0	480	0	0	-1
N.S.	1	1.00	0.66	1.22	0.00	1.78	0.00	0.00	-0.00
time (sec)	N/A	0.568	1.260	13.671	0.000	1.736	0.000	0.000	0.000

Problem 549	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	155	307	36231	442	0	0	-1
N.S.	1	1.00	0.70	1.38	162.47	1.98	0.00	0.00	-0.00
time (sec)	N/A	0.447	1.274	12.922	1.814	1.720	0.000	0.000	0.000

Problem 550	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	198	235	8208	410	0	0	-1
N.S.	1	1.00	1.12	1.34	46.64	2.33	0.00	0.00	-0.01
time (sec)	N/A	0.320	1.929	12.319	1.014	1.615	0.000	0.000	0.000

Problem 551	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	127	127	86	209	2166	376	0	0	-1
N.S.	1	1.00	0.68	1.65	17.06	2.96	0.00	0.00	-0.01
time (sec)	N/A	0.200	0.552	13.145	0.920	1.783	0.000	0.000	0.000

Problem 552	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F(-1)	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	185	185	113	303	0	592	0	0	-1
N.S.	1	1.00	0.61	1.64	0.00	3.20	0.00	0.00	-0.01
time (sec)	N/A	0.336	0.959	13.270	0.000	2.324	0.000	0.000	0.000

Problem 553	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	237	237	288	468	7057	716	0	0	-1
N.S.	1	1.00	1.22	1.97	29.78	3.02	0.00	0.00	-0.00
time (sec)	N/A	0.468	2.309	13.017	1.052	2.547	0.000	0.000	0.000

Problem 554	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	287	287	328	531	13364	764	0	0	-1
N.S.	1	1.00	1.14	1.85	46.56	2.66	0.00	0.00	-0.00
time (sec)	N/A	0.588	3.363	13.461	2.125	1.635	0.000	0.000	0.000

Problem 555	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	317	317	207	461	244922	572	0	0	-1
N.S.	1	1.00	0.65	1.45	772.62	1.80	0.00	0.00	-0.00
time (sec)	N/A	0.709	2.118	13.855	8.039	2.288	0.000	0.000	0.000

Problem 556	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	270	270	183	439	154245	542	0	0	-1
N.S.	1	1.00	0.68	1.63	571.28	2.01	0.00	0.00	-0.00
time (sec)	N/A	0.588	1.731	13.260	5.266	1.852	0.000	0.000	0.000

Problem 557	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	228	365	261506	504	0	0	-1
N.S.	1	1.00	1.02	1.64	1172.67	2.26	0.00	0.00	-0.00
time (sec)	N/A	0.446	2.612	13.001	3.743	1.661	0.000	0.000	0.000

Problem 558	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	B	A	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	108	339	5924	482	0	0	-1
N.S.	1	1.00	0.48	1.52	26.57	2.16	0.00	0.00	-0.00
time (sec)	N/A	0.457	1.039	13.573	1.505	2.255	0.000	0.000	0.000

Problem 559	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	176	176	108	339	5356	478	0	0	-1
N.S.	1	1.00	0.61	1.93	30.43	2.72	0.00	0.00	-0.01
time (sec)	N/A	0.251	0.890	13.454	1.289	2.345	0.000	0.000	0.000

Problem 560	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F(-1)	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	234	234	965	540	0	720	0	0	-1
N.S.	1	1.00	4.12	2.31	0.00	3.08	0.00	0.00	-0.00
time (sec)	N/A	0.466	6.166	14.398	0.000	2.402	0.000	0.000	0.000

Problem 561	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	B	A	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	286	286	1061	821	14037	850	0	0	-1
N.S.	1	1.00	3.71	2.87	49.08	2.97	0.00	0.00	-0.00
time (sec)	N/A	0.611	6.179	14.116	4.946	2.427	0.000	0.000	0.000

Problem 562	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	103	413	0	192	0	0	166
N.S.	1	1.00	0.74	2.95	0.00	1.37	0.00	0.00	1.19
time (sec)	N/A	0.158	0.952	1.833	0.000	0.716	0.000	0.000	0.765

Problem 563	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	108	108	86	371	0	175	0	0	128
N.S.	1	1.00	0.80	3.44	0.00	1.62	0.00	0.00	1.19
time (sec)	N/A	0.147	0.458	1.803	0.000	0.624	0.000	0.000	0.622

Problem 564	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	75	75	67	326	0	156	0	0	85
N.S.	1	1.00	0.89	4.35	0.00	2.08	0.00	0.00	1.13
time (sec)	N/A	0.129	0.250	1.737	0.000	0.616	0.000	0.000	0.624

Problem 565	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	71	71	64	244	0	185	0	0	96
N.S.	1	1.00	0.90	3.44	0.00	2.61	0.00	0.00	1.35
time (sec)	N/A	0.134	0.380	2.309	0.000	0.749	0.000	0.000	3.341

Problem 566	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	103	103	107	401	0	213	0	0	150
N.S.	1	1.00	1.04	3.89	0.00	2.07	0.00	0.00	1.46
time (sec)	N/A	0.147	0.540	4.254	0.000	0.601	0.000	0.000	3.858

Problem 567	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	134	636	0	235	0	0	177
N.S.	1	1.00	0.96	4.54	0.00	1.68	0.00	0.00	1.26
time (sec)	N/A	0.160	0.873	5.905	0.000	1.044	0.000	0.000	4.301

Problem 568	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	139	548	0	243	0	0	229
N.S.	1	1.00	0.76	3.01	0.00	1.34	0.00	0.00	1.26
time (sec)	N/A	0.241	1.262	2.213	0.000	0.735	0.000	0.000	3.170

Problem 569	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-1)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	140	140	106	487	0	216	0	0	177
N.S.	1	1.00	0.76	3.48	0.00	1.54	0.00	0.00	1.26
time (sec)	N/A	0.228	0.675	2.490	0.000	0.465	0.000	0.000	3.037

Problem 570	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F(-2)	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	121	121	102	404	0	240	0	0	158
N.S.	1	1.00	0.84	3.34	0.00	1.98	0.00	0.00	1.31
time (sec)	N/A	0.216	0.703	2.731	0.000	0.809	0.000	0.000	3.314

Problem 571	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	126	126	105	650	0	255	0	0	194
N.S.	1	1.00	0.83	5.16	0.00	2.02	0.00	0.00	1.54
time (sec)	N/A	0.225	1.265	4.481	0.000	0.847	0.000	0.000	4.274

Problem 572	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	172	172	175	723	0	286	0	0	227
N.S.	1	1.00	1.02	4.20	0.00	1.66	0.00	0.00	1.32
time (sec)	N/A	0.247	1.212	6.575	0.000	0.630	0.000	0.000	4.635

Problem 573	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	C	F	F	B
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	214	214	191	832	0	314	0	0	233
N.S.	1	1.00	0.89	3.89	0.00	1.47	0.00	0.00	1.09
time (sec)	N/A	0.257	4.795	8.915	0.000	0.592	0.000	0.000	4.976

Problem 574	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	182	182	260	1074	0	0	0	0	-1
N.S.	1	1.00	1.43	5.90	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.553	12.575	3.043	0.000	0.000	0.000	0.000	0.000

Problem 575	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	136	136	207	822	0	0	0	0	-1
N.S.	1	1.00	1.52	6.04	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.373	11.540	2.697	0.000	0.000	0.000	0.000	0.000

Problem 576	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	89	89	128	295	0	0	0	0	-1
N.S.	1	1.00	1.44	3.31	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.178	10.986	2.204	0.000	0.000	0.000	0.000	0.000

Problem 577	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	61	61	58	217	0	0	0	0	-1
N.S.	1	1.00	0.95	3.56	0.00	0.00	0.00	0.00	-0.02
time (sec)	N/A	0.141	0.235	1.820	0.000	0.000	0.000	0.000	0.000

Problem 578	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	B	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	86	86	206	298	0	0	0	0	-1
N.S.	1	1.00	2.40	3.47	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.257	12.557	2.787	0.000	0.000	0.000	0.000	0.000

Problem 579	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	A	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	260	439	0	0	0	0	-1
N.S.	1	1.00	1.73	2.93	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.543	12.263	5.358	0.000	0.000	0.000	0.000	0.000

Problem 580	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	217	217	326	758	0	0	0	0	-1
N.S.	1	1.00	1.50	3.49	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.765	14.602	8.671	0.000	0.000	0.000	0.000	0.000

Problem 581	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	305	305	318	1059	0	0	0	0	-1
N.S.	1	1.00	1.04	3.47	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.665	13.384	6.829	0.000	0.000	0.000	0.000	0.000

Problem 582	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	223	223	281	843	0	0	0	0	-1
N.S.	1	1.00	1.26	3.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.450	12.887	5.759	0.000	0.000	0.000	0.000	0.000

Problem 583	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	203	203	260	802	0	0	0	0	-1
N.S.	1	1.00	1.28	3.95	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.393	12.466	5.371	0.000	0.000	0.000	0.000	0.000

Problem 584	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	197	197	273	715	0	0	0	0	-1
N.S.	1	1.00	1.39	3.63	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.426	12.674	4.695	0.000	0.000	0.000	0.000	0.000

Problem 585	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-2)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	255	255	317	850	0	0	0	0	-1
N.S.	1	1.00	1.24	3.33	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.615	14.431	6.090	0.000	0.000	0.000	0.000	0.000

Problem 586	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	427	997	0	0	0	0	-1
N.S.	1	1.00	1.23	2.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.854	16.963	12.052	0.000	0.000	0.000	0.000	0.000

Problem 587	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	461	461	461	2216	0	0	0	0	-1
N.S.	1	1.00	1.00	4.81	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.041	15.844	14.195	0.000	0.000	0.000	0.000	0.000

Problem 588	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	367	367	390	2000	0	0	0	0	-1
N.S.	1	1.00	1.06	5.45	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.711	15.069	12.319	0.000	0.000	0.000	0.000	0.000

Problem 589	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	361	1959	0	0	0	0	-1
N.S.	1	1.00	1.04	5.66	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.727	14.730	10.928	0.000	0.000	0.000	0.000	0.000

Problem 590	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	338	338	364	1872	0	0	0	0	-1
N.S.	1	1.00	1.08	5.54	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.655	14.861	10.990	0.000	0.000	0.000	0.000	0.000

Problem 591	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	383	1768	0	0	0	0	-1
N.S.	1	1.00	1.12	5.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.712	14.900	11.059	0.000	0.000	0.000	0.000	0.000

Problem 592	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	420	420	458	1997	0	0	0	0	-1
N.S.	1	1.00	1.09	4.75	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.968	15.855	13.971	0.000	0.000	0.000	0.000	0.000

Problem 593	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	A	B	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	523	523	570	2151	0	0	0	0	-1
N.S.	1	1.00	1.09	4.11	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.292	17.352	25.923	0.000	0.000	0.000	0.000	0.000

Problem 594	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	343	343	455	2364	0	578	0	0	-1
N.S.	1	1.00	1.33	6.89	0.00	1.69	0.00	0.00	-0.00
time (sec)	N/A	0.778	17.683	19.068	0.000	0.878	0.000	0.000	0.000

Problem 595	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	267	267	353	1701	0	509	0	0	-1
N.S.	1	1.00	1.32	6.37	0.00	1.91	0.00	0.00	-0.00
time (sec)	N/A	0.571	14.398	19.050	0.000	1.309	0.000	0.000	0.000

Problem 596	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	201	201	305	1162	0	452	0	0	-1
N.S.	1	1.00	1.52	5.78	0.00	2.25	0.00	0.00	-0.00
time (sec)	N/A	0.391	8.489	19.819	0.000	0.692	0.000	0.000	0.000

Problem 597	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	208	208	25347	1563	0	0	0	0	-1
N.S.	1	1.00	121.86	7.51	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.446	29.897	19.525	0.000	0.000	0.000	0.000	0.000

Problem 598	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	253	253	52603	774	0	0	0	0	-1
N.S.	1	1.00	207.92	3.06	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.600	32.474	19.055	0.000	0.000	0.000	0.000	0.000

Problem 599	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	336	336	77879	1474	0	0	0	0	-1
N.S.	1	1.00	231.78	4.39	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.796	33.192	19.668	0.000	0.000	0.000	0.000	0.000

Problem 600	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	427	427	540	3069	0	656	0	0	-1
N.S.	1	1.00	1.26	7.19	0.00	1.54	0.00	0.00	-0.00
time (sec)	N/A	1.090	18.719	20.935	0.000	0.812	0.000	0.000	0.000

Problem 601	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	466	2326	0	579	0	0	-1
N.S.	1	1.00	1.36	6.80	0.00	1.69	0.00	0.00	-0.00
time (sec)	N/A	0.840	17.392	19.035	0.000	1.580	0.000	0.000	0.000

Problem 602	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	266	266	369	1749	0	510	0	0	-1
N.S.	1	1.00	1.39	6.58	0.00	1.92	0.00	0.00	-0.00
time (sec)	N/A	0.624	14.622	20.332	0.000	1.340	0.000	0.000	0.000

Problem 603	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	276	276	45958	1429	0	0	0	0	-1
N.S.	1	1.00	166.51	5.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.718	34.427	20.468	0.000	0.000	0.000	0.000	0.000

Problem 604	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	272	272	66581	1409	0	0	0	0	-1
N.S.	1	1.00	244.78	5.18	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.654	33.234	19.379	0.000	0.000	0.000	0.000	0.000

Problem 605	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	339	339	79375	1659	0	0	0	0	-1
N.S.	1	1.00	234.14	4.89	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.891	33.268	20.912	0.000	0.000	0.000	0.000	0.000

Problem 606	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	421	421	104716	2351	0	0	0	0	-1
N.S.	1	1.00	248.73	5.58	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.157	33.906	19.636	0.000	0.000	0.000	0.000	0.000

Problem 607	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	519	519	626	3816	0	743	0	0	-1
N.S.	1	1.00	1.21	7.35	0.00	1.43	0.00	0.00	-0.00
time (sec)	N/A	1.421	20.529	21.128	0.000	1.167	0.000	0.000	0.000

Problem 608	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	425	425	542	3069	0	656	0	0	-1
N.S.	1	1.00	1.28	7.22	0.00	1.54	0.00	0.00	-0.00
time (sec)	N/A	1.111	19.003	21.030	0.000	0.879	0.000	0.000	0.000

Problem 609	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	340	340	470	2450	0	579	0	0	-1
N.S.	1	1.00	1.38	7.21	0.00	1.70	0.00	0.00	-0.00
time (sec)	N/A	0.855	17.819	20.954	0.000	0.610	0.000	0.000	0.000

Problem 610	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	342	342	49609	2052	0	0	0	0	-1
N.S.	1	1.00	145.06	6.00	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.907	35.138	19.461	0.000	0.000	0.000	0.000	0.000

Problem 611	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	349	349	73332	2073	0	0	0	0	-1
N.S.	1	1.00	210.12	5.94	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.897	33.956	19.813	0.000	0.000	0.000	0.000	0.000

Problem 612	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	359	359	97208	2216	0	0	0	0	-1
N.S.	1	1.00	270.77	6.17	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.916	32.818	19.691	0.000	0.000	0.000	0.000	0.000

Problem 613	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	422	422	106199	2441	0	0	0	0	-1
N.S.	1	1.00	251.66	5.78	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.158	32.670	20.020	0.000	0.000	0.000	0.000	0.000

Problem 614	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	513	513	131553	3175	0	0	0	0	-1
N.S.	1	1.00	256.44	6.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.423	33.096	20.074	0.000	0.000	0.000	0.000	0.000

Problem 615	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	280	280	363	1700	0	510	0	0	-1
N.S.	1	1.00	1.30	6.07	0.00	1.82	0.00	0.00	-0.00
time (sec)	N/A	0.590	9.553	20.527	0.000	1.897	0.000	0.000	0.000

Problem 616	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	212	212	311	1080	0	452	0	0	-1
N.S.	1	1.00	1.47	5.09	0.00	2.13	0.00	0.00	-0.00
time (sec)	N/A	0.407	5.200	20.463	0.000	0.782	0.000	0.000	0.000

Problem 617	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	Yes	TBD	TBD	TBD	TBD	TBD	TBD
size	150	150	260	954	0	371	0	0	-1
N.S.	1	1.00	1.73	6.36	0.00	2.47	0.00	0.00	-0.01
time (sec)	N/A	0.280	4.204	21.036	0.000	0.663	0.000	0.000	0.000

Problem 618	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	138	138	9363	273	0	0	0	0	-1
N.S.	1	1.00	67.85	1.98	0.00	0.00	0.00	0.00	-0.01
time (sec)	N/A	0.342	26.499	20.148	0.000	0.000	0.000	0.000	0.000

Problem 619	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	256	256	37262	689	0	0	0	0	-1
N.S.	1	1.00	145.55	2.69	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.572	30.183	21.498	0.000	0.000	0.000	0.000	0.000

Problem 620	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	344	344	77909	1568	0	0	0	0	-1
N.S.	1	1.00	226.48	4.56	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.814	31.894	20.497	0.000	0.000	0.000	0.000	0.000

Problem 621	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	423	423	533	2084	0	928	0	0	-1
N.S.	1	1.00	1.26	4.93	0.00	2.19	0.00	0.00	-0.00
time (sec)	N/A	0.908	15.229	37.362	0.000	2.008	0.000	0.000	0.000

Problem 622	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	326	326	417	1460	0	804	0	0	-1
N.S.	1	1.00	1.28	4.48	0.00	2.47	0.00	0.00	-0.00
time (sec)	N/A	0.645	11.407	35.345	0.000	0.798	0.000	0.000	0.000

Problem 623	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F(-1)	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	235	235	365	889	0	699	0	0	-1
N.S.	1	1.00	1.55	3.78	0.00	2.97	0.00	0.00	-0.00
time (sec)	N/A	0.459	9.446	33.046	0.000	1.640	0.000	0.000	0.000

Problem 624	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	215	215	328	566	0	626	0	0	-1
N.S.	1	1.00	1.53	2.63	0.00	2.91	0.00	0.00	-0.00
time (sec)	N/A	0.416	6.557	33.620	0.000	1.133	0.000	0.000	0.000

Problem 625	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	220	220	37303	840	0	0	0	0	-1
N.S.	1	1.00	169.56	3.82	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.505	30.532	21.437	0.000	0.000	0.000	0.000	0.000

Problem 626	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	371	371	95694	1441	0	0	0	0	-1
N.S.	1	1.00	257.94	3.88	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.912	32.481	21.498	0.000	0.000	0.000	0.000	0.000

Problem 627	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F(-1)	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	487	487	140027	2295	0	0	0	0	-1
N.S.	1	1.00	287.53	4.71	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.198	33.424	20.723	0.000	0.000	0.000	0.000	0.000

Problem 628	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	588	588	4179	5675	0	1553	0	0	-1
N.S.	1	1.00	7.11	9.65	0.00	2.64	0.00	0.00	-0.00
time (sec)	N/A	1.328	22.042	22.739	0.000	1.518	0.000	0.000	0.000

Problem 629	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	472	472	626	4480	0	1365	0	0	-1
N.S.	1	1.00	1.33	9.49	0.00	2.89	0.00	0.00	-0.00
time (sec)	N/A	0.983	16.950	23.263	0.000	1.537	0.000	0.000	0.000

Problem 630	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	368	368	621	3337	0	1210	0	0	-1
N.S.	1	1.00	1.69	9.07	0.00	3.29	0.00	0.00	-0.00
time (sec)	N/A	0.698	15.624	22.661	0.000	0.934	0.000	0.000	0.000

Problem 631	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-2)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	346	346	463	2416	0	1093	0	0	-1
N.S.	1	1.00	1.34	6.98	0.00	3.16	0.00	0.00	-0.00
time (sec)	N/A	0.644	11.021	22.773	0.000	0.828	0.000	0.000	0.000

Problem 632	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	B	F	C	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	329	329	487	1921	0	973	0	0	-1
N.S.	1	1.00	1.48	5.84	0.00	2.96	0.00	0.00	-0.00
time (sec)	N/A	0.666	12.367	21.076	0.000	1.482	0.000	0.000	0.000

Problem 633	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	399	399	97528	3159	0	0	0	0	-1
N.S.	1	1.00	244.43	7.92	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	0.967	32.582	20.501	0.000	0.000	0.000	0.000	0.000

Problem 634	Optimal	Rubi	MMA	Maple	Maxima	Fricas	Sympy	Giac	Mupad
grade	A	A	C	C	F	F(-1)	F(-1)	F	F
verified	N/A	Yes	NO	TBD	TBD	TBD	TBD	TBD	TBD
size	526	526	184379	5358	0	0	0	0	-1
N.S.	1	1.00	350.53	10.19	0.00	0.00	0.00	0.00	-0.00
time (sec)	N/A	1.263	34.323	22.551	0.000	0.000	0.000	0.000	0.000

2.3 Detailed conclusion table specific for Rubi results

The following table is specific to Rubi. It gives additional statistics for each integral. the column **steps** is the number of steps used by Rubi to obtain the antiderivative. The **rules** column is the number of unique rules used. The **integrand size** column is the leaf size of the integrand. Finally the ratio $\frac{\text{number of rules}}{\text{integrand size}}$ is given. The larger this ratio is, the harder the integral was to solve. In this test, problem number [224] had the largest ratio of [35]

Table 2.1: Rubi specific breakdown of results for each integral

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
1	A	8	5	1.00	23	0.217
2	A	7	5	1.00	23	0.217
3	A	6	5	1.00	23	0.217
4	A	5	4	1.00	23	0.174
5	A	6	5	1.00	23	0.217
6	A	7	5	1.00	23	0.217
7	A	6	4	1.00	31	0.129
8	A	6	4	1.00	29	0.138
9	A	5	3	1.00	23	0.130
10	A	6	4	1.00	29	0.138
11	A	6	4	1.00	31	0.129
12	A	6	4	1.00	31	0.129
13	A	6	4	1.00	29	0.138
14	A	5	3	1.00	23	0.130
15	A	6	4	1.00	29	0.138
16	A	6	4	1.00	31	0.129
17	A	6	4	1.00	31	0.129
18	A	6	4	1.00	29	0.138
19	A	5	3	1.00	23	0.130
20	A	6	4	1.00	29	0.138
21	A	6	4	1.00	31	0.129
22	A	6	4	1.00	31	0.129
23	A	6	4	1.00	29	0.138
24	A	5	3	1.00	23	0.130
25	A	6	4	1.00	29	0.138

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
26	A	6	4	1.00	31	0.129
27	A	6	4	1.00	31	0.129
28	A	6	4	1.00	31	0.129
29	A	6	4	1.00	31	0.129
30	A	6	4	1.00	31	0.129
31	A	6	4	1.00	31	0.129
32	A	6	4	1.00	31	0.129
33	A	6	4	1.00	29	0.138
34	A	6	4	1.00	29	0.138
35	A	6	4	1.00	27	0.148
36	A	5	3	1.00	21	0.143
37	A	6	4	1.00	27	0.148
38	A	6	4	1.00	29	0.138
39	A	6	4	1.00	31	0.129
40	A	6	4	1.00	31	0.129
41	A	6	4	1.00	31	0.129
42	A	6	4	1.00	31	0.129
43	A	7	5	1.00	29	0.172
44	A	6	5	1.00	29	0.172
45	A	6	6	1.00	29	0.207
46	A	5	5	1.00	27	0.185
47	A	4	4	1.00	21	0.190
48	A	3	2	1.00	27	0.074
49	A	4	4	1.00	29	0.138
50	A	5	5	1.00	29	0.172
51	A	6	5	1.00	29	0.172
52	A	7	5	1.00	29	0.172
53	A	7	6	1.00	31	0.194
54	A	7	7	1.00	31	0.226
55	A	6	6	1.00	29	0.207
56	A	5	5	1.00	23	0.217
57	A	4	3	1.00	29	0.103
58	A	4	3	1.00	31	0.097
59	A	5	5	1.00	31	0.161
60	A	6	6	1.00	31	0.194

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
61	A	7	6	1.00	31	0.194
62	A	8	6	1.00	31	0.194
63	A	11	7	1.00	31	0.226
64	A	10	6	1.00	29	0.207
65	A	6	5	1.00	23	0.217
66	A	5	3	1.00	29	0.103
67	A	5	4	1.00	31	0.129
68	A	5	3	1.00	31	0.097
69	A	8	6	1.00	31	0.194
70	A	7	6	1.00	31	0.194
71	A	8	6	1.00	31	0.194
72	A	14	7	1.00	31	0.226
73	A	13	6	1.00	29	0.207
74	A	7	5	1.00	23	0.217
75	A	6	3	1.00	29	0.103
76	A	6	4	1.00	31	0.129
77	A	6	4	1.00	31	0.129
78	A	6	3	1.00	31	0.097
79	A	11	6	1.00	31	0.194
80	A	8	6	1.00	31	0.194
81	A	9	6	1.00	31	0.194
82	A	6	5	1.00	31	0.161
83	A	6	6	1.00	31	0.194
84	A	5	5	1.00	31	0.161
85	A	3	3	1.00	29	0.103
86	A	2	2	1.00	23	0.087
87	A	4	4	1.00	29	0.138
88	A	5	5	1.00	31	0.161
89	A	6	5	1.00	31	0.161
90	A	7	5	1.00	31	0.161
91	A	7	6	1.00	31	0.194
92	A	6	6	1.00	31	0.194
93	A	4	4	1.00	31	0.129
94	A	2	2	1.00	29	0.069
95	A	3	3	1.00	23	0.130

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
96	A	5	4	1.00	29	0.138
97	A	6	5	1.00	31	0.161
98	A	7	5	1.00	31	0.161
99	A	8	6	1.00	31	0.194
100	A	7	6	1.00	31	0.194
101	A	5	5	1.00	31	0.161
102	A	3	3	1.00	31	0.097
103	A	3	3	1.00	29	0.103
104	A	4	3	1.00	23	0.130
105	A	6	4	1.00	29	0.138
106	A	7	5	1.00	31	0.161
107	A	8	5	1.00	31	0.161
108	A	9	6	1.00	31	0.194
109	A	8	6	1.00	31	0.194
110	A	6	5	1.00	31	0.161
111	A	4	4	1.00	31	0.129
112	A	4	4	1.00	31	0.129
113	A	4	3	1.00	29	0.103
114	A	5	3	1.00	23	0.130
115	A	7	4	1.00	29	0.138
116	A	8	5	1.00	31	0.161
117	A	9	5	1.00	31	0.161
118	A	5	5	1.00	33	0.152
119	A	4	4	1.00	33	0.121
120	A	3	3	1.00	33	0.091
121	A	2	2	1.00	31	0.065
122	A	4	4	1.00	25	0.160
123	A	3	3	1.00	31	0.097
124	A	4	4	1.00	33	0.121
125	A	5	4	1.00	33	0.121
126	A	6	4	1.00	33	0.121
127	A	5	5	1.00	33	0.152
128	A	4	4	1.00	33	0.121
129	A	3	3	1.00	31	0.097
130	A	5	5	1.00	25	0.200

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
131	A	4	4	1.00	31	0.129
132	A	4	4	1.00	33	0.121
133	A	5	5	1.00	33	0.152
134	A	6	5	1.00	33	0.152
135	A	6	5	1.00	33	0.152
136	A	5	4	1.00	33	0.121
137	A	4	3	1.00	31	0.097
138	A	6	5	1.00	25	0.200
139	A	5	4	1.00	31	0.129
140	A	5	5	1.00	33	0.152
141	A	5	4	1.00	33	0.121
142	A	6	5	1.00	33	0.152
143	A	7	5	1.00	33	0.152
144	A	6	5	1.00	33	0.152
145	A	5	5	1.00	33	0.152
146	A	4	4	1.00	33	0.121
147	A	3	3	1.00	31	0.097
148	A	5	4	1.00	25	0.160
149	A	6	5	1.00	31	0.161
150	A	7	5	1.00	33	0.152
151	A	8	5	1.00	33	0.152
152	A	6	6	1.00	33	0.182
153	A	5	5	1.00	33	0.152
154	A	4	4	1.00	33	0.121
155	A	3	3	1.00	31	0.097
156	A	6	5	1.00	25	0.200
157	A	7	6	1.00	31	0.194
158	A	8	6	1.00	33	0.182
159	A	9	6	1.00	33	0.182
160	A	6	5	1.00	33	0.152
161	A	5	5	1.00	33	0.152
162	A	4	4	1.00	33	0.121
163	A	4	4	1.00	31	0.129
164	A	7	5	1.00	25	0.200
165	A	8	6	1.00	31	0.194

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
166	A	9	6	1.00	33	0.182
167	A	5	4	1.00	26	0.154
168	A	6	5	1.00	32	0.156
169	A	7	5	1.00	34	0.147
170	A	8	5	1.00	34	0.147
171	A	6	5	1.15	26	0.192
172	A	7	6	1.00	32	0.188
173	A	8	6	1.00	34	0.176
174	A	9	6	1.00	34	0.176
175	A	7	6	1.22	26	0.231
176	A	8	6	1.00	32	0.188
177	A	9	6	1.00	34	0.176
178	A	10	6	1.00	34	0.176
179	A	9	6	1.00	31	0.194
180	A	8	6	1.00	31	0.194
181	A	7	6	1.00	31	0.194
182	A	6	5	1.00	31	0.161
183	A	6	5	1.00	31	0.161
184	A	7	6	1.00	31	0.194
185	A	8	6	1.00	31	0.194
186	A	9	7	1.00	33	0.212
187	A	8	7	1.00	33	0.212
188	A	7	6	1.00	33	0.182
189	A	7	6	1.00	33	0.182
190	A	7	6	1.00	33	0.182
191	A	8	7	1.00	33	0.212
192	A	9	7	1.00	33	0.212
193	A	10	7	1.00	33	0.212
194	A	9	7	1.00	33	0.212
195	A	8	6	1.00	33	0.182
196	A	8	7	1.00	33	0.212
197	A	8	6	1.00	33	0.182
198	A	8	6	1.00	33	0.182
199	A	9	7	1.00	33	0.212
200	A	10	7	1.00	33	0.212

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
201	A	9	6	1.00	33	0.182
202	A	8	6	1.00	33	0.182
203	A	7	6	1.00	33	0.182
204	A	6	5	1.00	33	0.152
205	A	6	5	1.00	33	0.152
206	A	7	6	1.00	33	0.182
207	A	8	6	1.00	33	0.182
208	A	9	6	1.00	33	0.182
209	A	9	6	1.00	33	0.182
210	A	8	6	1.00	33	0.182
211	A	7	5	1.00	33	0.152
212	A	7	6	1.00	33	0.182
213	A	7	5	1.00	33	0.152
214	A	8	6	1.00	33	0.182
215	A	9	6	1.00	33	0.182
216	A	10	6	1.00	33	0.182
217	A	9	6	1.00	33	0.182
218	A	8	5	1.00	33	0.152
219	A	8	6	1.00	33	0.182
220	A	8	6	1.00	33	0.182
221	A	8	5	1.00	33	0.152
222	A	9	6	1.00	33	0.182
223	A	10	6	1.00	33	0.182
224	A	5	4	1.00	35	0.114
225	A	4	4	1.00	35	0.114
226	A	3	3	1.00	35	0.086
227	A	3	3	1.00	35	0.086
228	A	2	2	1.00	35	0.057
229	A	3	3	1.00	35	0.086
230	A	4	3	1.00	35	0.086
231	A	6	5	1.00	35	0.143
232	A	5	5	1.00	35	0.143
233	A	4	4	1.00	35	0.114
234	A	4	4	1.00	35	0.114
235	A	4	4	1.00	35	0.114

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
236	A	3	3	1.00	35	0.086
237	A	4	4	1.00	35	0.114
238	A	5	4	1.00	35	0.114
239	A	7	5	1.00	35	0.143
240	A	6	5	1.00	35	0.143
241	A	5	4	1.00	35	0.114
242	A	5	4	1.00	35	0.114
243	A	5	5	1.00	35	0.143
244	A	5	4	1.00	35	0.114
245	A	4	3	1.00	35	0.086
246	A	5	4	1.00	35	0.114
247	A	6	4	1.00	35	0.114
248	A	7	6	1.00	35	0.171
249	A	6	6	1.00	35	0.171
250	A	5	5	1.00	35	0.143
251	A	3	3	1.00	35	0.086
252	A	4	4	1.00	35	0.114
253	A	5	4	1.00	35	0.114
254	A	6	4	1.00	35	0.114
255	A	8	7	1.00	35	0.200
256	A	7	7	1.00	35	0.200
257	A	6	6	1.00	35	0.171
258	A	3	3	1.00	35	0.086
259	A	4	4	1.00	35	0.114
260	A	5	5	1.00	35	0.143
261	A	6	5	1.00	35	0.143
262	A	8	7	1.00	35	0.200
263	A	7	6	1.00	35	0.171
264	A	4	4	1.00	35	0.114
265	A	5	5	1.30	35	0.143
266	A	5	4	1.00	35	0.114
267	A	6	5	1.00	35	0.143
268	A	7	5	1.00	35	0.143
269	A	9	9	1.00	25	0.360
270	A	8	8	1.00	25	0.320

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
271	A	9	9	1.00	25	0.360
272	A	11	11	1.00	25	0.440
273	A	10	10	1.00	25	0.400
274	A	11	11	1.00	25	0.440
275	A	7	4	1.00	33	0.121
276	A	4	4	1.00	35	0.114
277	A	6	5	1.00	29	0.172
278	A	6	6	1.00	29	0.207
279	A	5	5	1.00	27	0.185
280	A	4	4	1.00	21	0.190
281	A	3	2	1.00	27	0.074
282	A	4	4	1.00	29	0.138
283	A	5	5	1.00	29	0.172
284	A	6	5	1.00	29	0.172
285	A	7	6	1.00	31	0.194
286	A	7	7	1.00	31	0.226
287	A	6	6	1.00	29	0.207
288	A	5	4	1.00	23	0.174
289	A	5	4	1.00	29	0.138
290	A	5	5	1.00	31	0.161
291	A	5	5	1.00	31	0.161
292	A	7	6	1.00	31	0.194
293	A	7	6	1.00	31	0.194
294	A	8	7	1.00	31	0.226
295	A	7	6	1.00	29	0.207
296	A	6	5	1.00	23	0.217
297	A	6	5	1.10	29	0.172
298	A	6	6	1.00	31	0.194
299	A	6	6	1.00	31	0.194
300	A	6	6	1.00	31	0.194
301	A	8	7	1.00	31	0.226
302	A	9	7	1.00	31	0.226
303	A	8	6	1.00	29	0.207
304	A	7	6	1.00	23	0.261
305	A	7	6	1.00	29	0.207

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
306	A	7	6	1.00	31	0.194
307	A	7	7	1.00	31	0.226
308	A	7	7	1.00	31	0.226
309	A	7	7	1.00	31	0.226
310	A	9	8	1.00	31	0.258
311	A	8	8	1.00	31	0.258
312	A	7	7	1.00	31	0.226
313	A	7	7	1.00	31	0.226
314	A	5	5	1.00	29	0.172
315	A	4	4	1.00	23	0.174
316	A	5	5	1.00	29	0.172
317	A	6	6	1.00	31	0.194
318	A	7	6	1.00	31	0.194
319	A	8	6	1.00	31	0.194
320	A	8	8	1.00	31	0.258
321	A	7	7	1.00	31	0.226
322	A	6	6	1.00	31	0.194
323	A	5	5	1.00	29	0.172
324	A	5	5	1.00	23	0.217
325	A	6	6	1.00	29	0.207
326	A	7	6	1.00	31	0.194
327	A	8	6	1.00	31	0.194
328	A	9	9	1.00	31	0.290
329	A	8	8	1.00	31	0.258
330	A	7	7	1.00	31	0.226
331	A	6	6	1.00	31	0.194
332	A	6	5	1.00	29	0.172
333	A	6	6	1.00	23	0.261
334	A	7	7	1.00	29	0.241
335	A	8	7	1.00	31	0.226
336	A	9	9	1.00	31	0.290
337	A	8	8	1.00	31	0.258
338	A	7	7	1.00	31	0.226
339	A	7	6	1.00	31	0.194
340	A	7	5	1.00	29	0.172

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
341	A	7	6	1.00	23	0.261
342	A	8	7	1.00	29	0.241
343	A	9	7	1.00	31	0.226
344	A	4	4	1.00	28	0.143
345	A	2	2	1.00	28	0.071
346	A	5	5	1.00	23	0.217
347	A	4	4	1.00	21	0.190
348	A	7	7	1.00	33	0.212
349	A	6	6	1.00	33	0.182
350	A	5	5	1.00	33	0.152
351	A	4	4	1.00	31	0.129
352	A	5	5	1.00	25	0.200
353	A	6	6	1.00	31	0.194
354	A	7	7	1.00	33	0.212
355	A	8	7	1.00	33	0.212
356	A	7	6	1.00	33	0.182
357	A	6	5	1.00	33	0.152
358	A	5	4	1.00	31	0.129
359	A	6	6	1.00	25	0.240
360	A	6	6	1.00	31	0.194
361	A	7	7	1.00	33	0.212
362	A	8	7	1.00	33	0.212
363	A	8	6	1.00	33	0.182
364	A	7	5	1.00	33	0.152
365	A	6	4	1.00	31	0.129
366	A	7	7	1.00	25	0.280
367	A	7	7	1.00	31	0.226
368	A	7	7	1.00	33	0.212
369	A	8	8	1.00	33	0.242
370	A	9	8	1.00	33	0.242
371	A	5	5	1.00	33	0.152
372	A	4	4	1.00	33	0.121
373	A	3	3	1.00	31	0.097
374	A	3	3	1.00	25	0.120
375	A	6	6	1.00	31	0.194

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
376	A	7	7	1.00	33	0.212
377	A	8	7	1.00	33	0.212
378	A	5	5	1.00	33	0.152
379	A	4	4	1.00	33	0.121
380	A	4	4	1.00	31	0.129
381	A	6	6	1.00	25	0.240
382	A	7	7	1.00	31	0.226
383	A	8	8	1.00	33	0.242
384	A	9	8	1.00	33	0.242
385	A	6	6	1.00	33	0.182
386	A	5	5	1.00	33	0.152
387	A	5	5	1.00	33	0.152
388	A	5	4	1.00	31	0.129
389	A	7	7	1.00	25	0.280
390	A	8	8	1.00	31	0.258
391	A	9	8	1.00	33	0.242
392	A	1	1	1.00	31	0.032
393	A	1	1	1.00	32	0.031
394	A	8	6	1.00	31	0.194
395	A	7	6	1.00	31	0.194
396	A	6	5	1.00	31	0.161
397	A	6	5	1.00	31	0.161
398	A	7	6	1.00	31	0.194
399	A	8	6	1.00	31	0.194
400	A	9	7	1.00	33	0.212
401	A	8	7	1.00	33	0.212
402	A	7	6	1.00	33	0.182
403	A	7	6	1.00	33	0.182
404	A	7	6	1.00	33	0.182
405	A	8	7	1.00	33	0.212
406	A	9	7	1.00	33	0.212
407	A	10	8	1.00	33	0.242
408	A	9	8	1.00	33	0.242
409	A	8	7	1.00	33	0.212
410	A	8	7	1.00	33	0.212

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
411	A	8	7	1.00	33	0.212
412	A	8	7	1.00	33	0.212
413	A	9	8	1.00	33	0.242
414	A	10	8	1.00	33	0.242
415	A	11	9	1.00	33	0.273
416	A	10	9	1.00	33	0.273
417	A	7	7	1.00	33	0.212
418	A	5	5	1.00	33	0.152
419	A	7	7	1.00	33	0.212
420	A	9	8	1.00	33	0.242
421	A	10	9	1.00	33	0.273
422	A	11	9	1.00	33	0.273
423	A	10	9	1.00	33	0.273
424	A	9	8	1.00	33	0.242
425	A	9	8	1.00	33	0.242
426	A	9	8	1.00	33	0.242
427	A	10	9	1.00	33	0.273
428	A	12	10	1.00	33	0.303
429	A	11	10	1.00	33	0.303
430	A	10	9	1.00	33	0.273
431	A	10	9	1.00	33	0.273
432	A	10	9	1.00	33	0.273
433	A	10	9	1.00	33	0.273
434	A	11	10	1.00	33	0.303
435	A	13	13	1.00	35	0.371
436	A	12	12	1.00	35	0.343
437	A	11	11	1.00	35	0.314
438	A	8	8	1.00	35	0.229
439	A	9	9	1.00	35	0.257
440	A	10	9	1.00	35	0.257
441	A	14	13	1.00	35	0.371
442	A	13	13	1.00	35	0.371
443	A	12	12	1.00	35	0.343
444	A	12	12	1.00	35	0.343
445	A	9	9	1.00	35	0.257

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
446	A	10	9	1.00	35	0.257
447	A	11	9	1.00	35	0.257
448	A	15	14	1.00	35	0.400
449	A	14	14	1.00	35	0.400
450	A	13	13	1.00	35	0.371
451	A	13	13	1.00	35	0.371
452	A	13	13	1.00	35	0.371
453	A	10	10	1.00	35	0.286
454	A	11	10	1.00	35	0.286
455	A	12	10	1.00	35	0.286
456	A	13	13	1.00	35	0.371
457	A	12	12	1.00	35	0.343
458	A	7	7	1.00	35	0.200
459	A	7	7	1.00	35	0.200
460	A	8	8	1.00	35	0.229
461	A	9	9	1.00	35	0.257
462	A	13	13	1.00	35	0.371
463	A	9	9	1.00	35	0.257
464	A	8	8	1.00	35	0.229
465	A	8	8	1.00	35	0.229
466	A	9	9	1.00	35	0.257
467	A	10	9	1.00	35	0.257
468	A	13	13	1.00	35	0.371
469	A	9	9	1.00	35	0.257
470	A	9	9	1.00	35	0.257
471	A	9	9	1.00	35	0.257
472	A	10	10	1.00	35	0.286
473	A	11	10	1.00	35	0.286
474	A	0	0	0.00	0	0.000
475	A	0	0	0.00	0	0.000
476	A	0	0	0.00	0	0.000
477	A	0	0	0.00	0	0.000
478	A	0	0	0.00	0	0.000
479	A	9	7	1.00	31	0.226
480	A	8	6	1.00	31	0.194

Continued on next page

Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
481	A	7	5	1.00	31	0.161
482	A	6	4	1.00	29	0.138
483	A	8	7	1.00	31	0.226
484	A	7	7	1.00	31	0.226
485	A	6	6	1.00	31	0.194
486	A	6	6	1.00	31	0.194
487	A	7	7	1.00	31	0.226
488	A	8	7	1.00	31	0.226
489	A	9	8	1.00	33	0.242
490	A	8	8	1.00	33	0.242
491	A	7	7	1.00	33	0.212
492	A	7	7	1.00	33	0.212
493	A	7	7	1.00	33	0.212
494	A	8	8	1.00	33	0.242
495	A	9	8	1.00	33	0.242
496	A	7	6	1.00	33	0.182
497	A	6	6	1.00	33	0.182
498	A	5	5	1.00	33	0.152
499	A	5	5	1.00	33	0.152
500	A	6	6	1.00	33	0.182
501	A	7	6	1.00	33	0.182
502	A	8	6	1.00	33	0.182
503	A	7	6	1.00	33	0.182
504	A	6	5	1.00	33	0.152
505	A	6	6	1.00	33	0.182
506	A	6	5	1.00	33	0.152
507	A	7	6	1.00	33	0.182
508	A	8	6	1.00	33	0.182
509	A	8	6	1.00	33	0.182
510	A	7	5	1.00	33	0.152
511	A	7	6	1.00	33	0.182
512	A	7	6	1.00	33	0.182
513	A	7	5	1.00	33	0.152
514	A	8	6	1.00	33	0.182
515	A	6	4	1.00	35	0.114

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
516	A	5	4	1.00	35	0.114
517	A	4	4	1.00	35	0.114
518	A	3	3	1.00	35	0.086
519	A	4	4	1.00	35	0.114
520	A	4	4	1.00	35	0.114
521	A	5	5	1.00	35	0.143
522	A	6	5	1.00	35	0.143
523	A	7	5	1.00	35	0.143
524	A	6	5	1.00	35	0.143
525	A	5	5	1.00	35	0.143
526	A	4	4	1.00	35	0.114
527	A	5	5	1.00	35	0.143
528	A	5	5	1.00	35	0.143
529	A	5	5	1.00	35	0.143
530	A	6	6	1.00	35	0.171
531	A	7	6	1.00	35	0.171
532	A	7	5	1.00	35	0.143
533	A	6	5	1.00	35	0.143
534	A	5	4	1.00	35	0.114
535	A	6	5	1.00	35	0.143
536	A	6	6	1.00	35	0.171
537	A	6	5	1.00	35	0.143
538	A	6	5	1.00	35	0.143
539	A	7	6	1.00	35	0.171
540	A	8	6	1.00	35	0.171
541	A	7	5	1.00	35	0.143
542	A	6	5	1.00	35	0.143
543	A	5	5	1.00	35	0.143
544	A	4	4	1.00	35	0.114
545	A	6	6	1.00	35	0.171
546	A	7	7	1.00	35	0.200
547	A	8	7	1.00	35	0.200
548	A	7	6	1.00	35	0.171
549	A	6	6	1.00	35	0.171
550	A	5	5	1.00	35	0.143

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
551	A	4	4	1.00	35	0.114
552	A	7	7	1.00	35	0.200
553	A	8	8	1.00	35	0.229
554	A	9	8	1.00	35	0.229
555	A	8	6	1.00	35	0.171
556	A	7	6	1.00	35	0.171
557	A	6	5	1.00	35	0.143
558	A	6	6	1.00	35	0.171
559	A	5	5	1.00	35	0.143
560	A	8	7	1.00	35	0.200
561	A	9	8	1.00	35	0.229
562	A	8	7	1.00	31	0.226
563	A	7	7	1.00	31	0.226
564	A	6	6	1.00	31	0.194
565	A	6	6	1.00	31	0.194
566	A	7	7	1.00	31	0.226
567	A	8	7	1.00	31	0.226
568	A	7	7	1.00	33	0.212
569	A	6	6	1.00	33	0.182
570	A	6	6	1.00	33	0.182
571	A	6	6	1.00	33	0.182
572	A	7	7	1.00	33	0.212
573	A	8	7	1.00	33	0.212
574	A	8	8	1.00	33	0.242
575	A	7	7	1.00	33	0.212
576	A	6	6	1.00	33	0.182
577	A	4	4	1.00	33	0.121
578	A	6	6	1.00	33	0.182
579	A	8	8	1.00	33	0.242
580	A	9	8	1.00	33	0.242
581	A	8	8	1.00	33	0.242
582	A	7	7	1.00	33	0.212
583	A	7	7	1.00	33	0.212
584	A	7	7	1.00	33	0.212
585	A	8	8	1.00	33	0.242

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
586	A	9	8	1.00	33	0.242
587	A	9	9	1.00	33	0.273
588	A	8	8	1.00	33	0.242
589	A	8	8	1.00	33	0.242
590	A	8	8	1.00	33	0.242
591	A	8	8	1.00	33	0.242
592	A	9	8	1.00	33	0.242
593	A	10	8	1.00	33	0.242
594	A	11	10	1.00	35	0.286
595	A	10	10	1.00	35	0.286
596	A	9	9	1.00	35	0.257
597	A	12	12	1.00	35	0.343
598	A	13	13	1.00	35	0.371
599	A	14	14	1.00	35	0.400
600	A	12	10	1.00	35	0.286
601	A	11	10	1.00	35	0.286
602	A	10	10	1.00	35	0.286
603	A	13	13	1.00	35	0.371
604	A	13	13	1.00	35	0.371
605	A	14	14	1.00	35	0.400
606	A	15	14	1.00	35	0.400
607	A	13	11	1.00	35	0.314
608	A	12	11	1.00	35	0.314
609	A	11	11	1.00	35	0.314
610	A	14	14	1.00	35	0.400
611	A	14	14	1.00	35	0.400
612	A	14	14	1.00	35	0.400
613	A	15	15	1.00	35	0.429
614	A	16	15	1.00	35	0.429
615	A	10	10	1.00	35	0.286
616	A	9	9	1.00	35	0.257
617	A	8	8	1.00	35	0.229
618	A	8	8	1.00	35	0.229
619	A	13	13	1.00	35	0.371
620	A	14	14	1.00	35	0.400

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Table 2.1 – continued from previous page

#	grade	number of steps used	number of unique rules	normalized antiderivative leaf size	integrand leaf size	$\frac{\text{number of rules}}{\text{integrand leaf size}}$
621	A	11	10	1.00	35	0.286
622	A	10	10	1.00	35	0.286
623	A	9	9	1.00	35	0.257
624	A	9	9	1.00	35	0.257
625	A	10	10	1.00	35	0.286
626	A	14	14	1.00	35	0.400
627	A	15	14	1.00	35	0.400
628	A	12	11	1.00	35	0.314
629	A	11	11	1.00	35	0.314
630	A	10	10	1.00	35	0.286
631	A	10	10	1.00	35	0.286
632	A	10	10	1.00	35	0.286
633	A	14	14	1.00	35	0.400
634	A	15	15	1.00	35	0.429

Chapter 3

Listing of integrals

Local contents

3.1 $\int (b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=171

$$-\frac{6b^3BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2Ab^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{6b^2B\sqrt{b\sec(c+}}{5}$$

```
[Out] 2/3*A*b*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/5*B*(b*sec(d*x+c))^(5/2)*sin(d*
x+c)/d-6/5*b^3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+6/5*b^2
*B*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d+2/3*A*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.09, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3872, 3853, 3856, 2720, 2719}

$$\frac{2Ab^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2Ab\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}-\frac{6b^3BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{6b^2B\sin(c+dx)\sqrt{b\sec(c+dx)}}{5d}+\frac{2B\sin(c+dx)(b\sec(c+dx))^{5/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-6*b^3*B*EllipticE[(c + d*x)/2, 2])/(5*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
 d*x]]) + (2*A*b^2*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[
c + d*x]])/(3*d) + (6*b^2*B*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*A
*b*(b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d) + (2*B*(b*Sec[c + d*x])^(5/2)
*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int (b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = A\int (b\sec(c+dx))^{5/2}\,dx + \frac{B\int (b\sec(c+dx))^{7/2}\,dx}{b}$$
$$= \frac{2Ab(b\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{2B(b\sec(c+dx))^{5/2}\sin(c}{5d}$$
$$= \frac{6b^2B\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2Ab(b\sec(c+dx))^{3/2}\sin(}{3d}$$
$$= \frac{2Ab^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d} + \frac{6b^2B}{}$$
$$= -\frac{6b^3BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}} + \frac{2Ab^2\sqrt{\cos(c+dx)}\,F(}{}$$

Mathematica [A]
time = 0.49, size = 99, normalized size = 0.58

$$\frac{(b\sec(c+dx))^{5/2}\left(-36B\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20A\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+21B\sin(c+dx)+10A\sin(2(c+dx))+9B\sin(3(c+dx))\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((b*Sec[c + d*x])^(5/2)*(-36*B*Cos[c + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2]
 + 20*A*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 21*B*Sin[c + d*x] +
10*A*Sin[2*(c + d*x)] + 9*B*Sin[3*(c + d*x)]))/(30*d)
```

Maple [C] Result contains complex when optimal does not.
time = 12.20, size = 518, normalized size = 3.03

method	result
default	$\frac{2(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(5iA\left(\cos^3(dx+c)\right)\sin(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \text{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/15/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(5*I*A*cos(d*x+c)^3*sin(d*x+c)*(1
/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+co
s(d*x+c))/sin(d*x+c),I)+9*I*B*cos(d*x+c)^3*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c
),I)-9*I*B*cos(d*x+c)^3*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+5*I*A*cos(d*x+
c)^2*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+9*I*B*cos(d*x+c)^2*sin(d*x+c)*(1/
(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos
(d*x+c))/sin(d*x+c),I)-9*I*B*cos(d*x+c)^2*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/
2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c)
,I)-5*A*cos(d*x+c)^3-9*B*cos(d*x+c)^3+6*B*cos(d*x+c)^2+5*A*cos(d*x+c)+3*B)*
(b/cos(d*x+c))^(5/2)/sin(d*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.93, size = 213, normalized size = 1.25

$$\frac{-5i\sqrt{2}\,Ab^{\frac{5}{2}}\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5i\sqrt{2}\,Ab^{\frac{5}{2}}\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-9i\sqrt{2}\,Bb^{\frac{5}{2}}\cos(dx+c)^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+9i\sqrt{2}\,Bb^{\frac{5}{2}}\cos(dx+c)^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\left(9\,Bb^2\cos(dx+c)^2+5\,Ab^2\cos(dx+c)+3\,Bb^2\right)\sqrt{\frac{b}{\cos(dx+c)}}\,\sin(dx+c)}{15\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*A*b^(5/2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*A*b^(5/2)*cos(d*x + c)^2*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*I*sqrt(2)*B*b^(5/2)*co
s(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
```

```
 + I*sin(d*x + c))) + 9*I*sqrt(2)*B*b^(5/2)*cos(d*x + c)^2*weierstrassZeta(
-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(9*B*
b^2*cos(d*x + c)^2 + 5*A*b^2*cos(d*x + c) + 3*B*b^2)*sqrt(b/cos(d*x + c))*s
in(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(b\sec \left(c+dx\right)\right)^{\frac{5}{2}} \left(A+B\sec \left(c+dx\right)\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
[Out] Integral((b*sec(c + d*x))**(5/2)*(A + B*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(5/2),x)
[Out] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(5/2), x)
```

3.2 $\int (b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=136

$$-\frac{2Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2bB\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}+\frac{2Ab\sqrt{b\sec(c+dx}}{d}$$

```
[Out] 2/3*B*(b*sec(d*x+c))^(3/2)*sin(d*x+c)/d-2*A*b^2*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/
2)/(b*sec(d*x+c))^(1/2)+2*A*b*sin(d*x+c)*(b*sec(d*x+c))^(1/2)/d+2/3*b*B*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.08, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3872, 3853, 3856, 2719, 2720}

$$-\frac{2Ab^2E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2Ab\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}+\frac{2B\sin(c+dx)(b\sec(c+dx))^{3/2}}{3d}+\frac{2bB\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*A*b^2*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d
*x]]) + (2*b*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c +
d*x]])/(3*d) + (2*A*b*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d + (2*B*(b*Sec[c
+ d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int (b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = A\int (b\sec(c+dx))^{3/2}\,dx + \frac{B\int (b\sec(c+dx))^{5/2}\,dx}{b}$$
$$= \frac{2Ab\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2B(b\sec(c+dx))^{3/2}\sin(c}{3d}$$
$$= \frac{2Ab\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{2B(b\sec(c+dx))^{3/2}\sin(c}{3d}$$
$$= -\frac{2Ab^2E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}} + \frac{2bB\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}\right.}{}$$

Mathematica [A]
time = 0.29, size = 87, normalized size = 0.64

$$\frac{(b\sec(c+dx))^{3/2}\left(-6A\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+2B\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+2(B+3A\cos(c+dx))\sin(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
[Out] ((b*Sec[c + d*x])^(3/2)*(-6*A*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2]
+ 2*B*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*(B + 3*A*Cos[c + d*x
])*Sin[c + d*x]))/(3*d)
```

Maple [C] Result contains complex when optimal does not.
time = 11.81, size = 499, normalized size = 3.67

method	result
default	$\frac{2(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(3iA\left(\cos^2(dx+c)\right)\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticE}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sin(dx+c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/3/d*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(3*I*A*cos(d*x+c)^2*(1/(1+cos(d*x+
c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+cos(d*x+c))/si
n(d*x+c),I)*sin(d*x+c)-3*I*A*cos(d*x+c)^2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x
+c)+I*B*cos(d*x+c)^2*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)+3*I*A*cos(d*x+c)*
(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticE(I*(-1+
cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-3*I*A*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1
/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c
),I)*sin(d*x+c)+I*B*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)-3*A*cos
(d*x+c)^2-B*cos(d*x+c)^2+3*A*cos(d*x+c)+B)*(b/cos(d*x+c))^(3/2)/sin(d*x+c)^
5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.46, size = 186, normalized size = 1.37

$$\frac{-i\sqrt{2}\,Bb^{\frac{3}{2}}\cos\left(dx+c\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\sqrt{2}\,Bb^{\frac{3}{2}}\cos\left(dx+c\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))-3i\sqrt{2}\,Ab^{\frac{3}{2}}\cos\left(dx+c\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+3i\sqrt{2}\,Ab^{\frac{3}{2}}\cos\left(dx+c\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))+2\left(3\,Ab\cos\left(dx+c\right)+Bb\right)\sqrt{\frac{b}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*B*b^(3/2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + I*sqrt(2)*B*b^(3/2)*cos(d*x + c)*weierstrassPInvers
e(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*A*b^(3/2)*cos(d*x + c
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) + 3*I*sqrt(2)*A*b^(3/2)*cos(d*x + c)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*A*b*cos(d*x + c
) + B*b)*sqrt(b/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec(c + dx))^{\frac{3}{2}} (A + B \sec(c + dx)) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
[Out] Integral((b*sec(c + d*x))**(3/2)*(A + B*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + d\,x)}\right) \left(\frac{b}{\cos(c + d\,x)}\right)^{3/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(3/2),x)
[Out] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(3/2), x)
```

3.3 $\int \sqrt{b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=104

$$-\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}+\frac{2B\sqrt{b\sec(c+dx)}}{d}$$

```
[Out] -2*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2*B*sin(d*x+c)*(b
*sec(d*x+c))^(1/2)/d+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.06, antiderivative size = 104, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3872, 3856, 2720, 3853, 2719}

$$\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}+\frac{2B\sin(c+dx)\sqrt{b\sec(c+dx)}}{d}-\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*b*B*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x
]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]
])/d + (2*B*Sqrt[b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \sqrt{b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx &= A\int \sqrt{b\sec(c+dx)}\,dx+\frac{B\int (b\sec(c+dx))^{3/2}\,dx}{b}\\ &= \frac{2B\sqrt{b\sec(c+dx)}\,\sin(c+dx)}{d}-(bB)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx\\ &= \frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{d}+\frac{2B\sqrt{b\mathrm{s}}}{}\\ &= -\frac{2bBE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 73, normalized size = 0.70

$$\frac{2\sqrt{b\sec(c+dx)}\left(-B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+B\sin(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
[Out] (2*Sqrt[b*Sec[c + d*x]]*(-(B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2])
+ A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + B*Sin[c + d*x]))/d
```

Maple [C] Result contains complex when optimal does not.
time = 11.46, size = 453, normalized size = 4.36

method	result
default	$\frac{2\sqrt{\frac{b}{\cos(dx+c)}}\,(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(iA\cos(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(b/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*(I*A*cos(d*x+c)
*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-I*B*cos(d*x+c)*EllipticF(I*(-1+cos(d*
x+c))/sin(d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)+I*B*cos(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/
(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+I*A*Elli
pticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-I*B*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))
/sin(d*x+c),I)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+I
*B*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(1+cos(d*x+c)))^
(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-B*cos(d*x+c)+B)/sin(d*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.44, size = 140, normalized size = 1.35

$$\frac{-i\sqrt{2}\,A\sqrt{b}\ \mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,A\sqrt{b}\ \mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-i\sqrt{2}\,B\sqrt{b}\ \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+i\sqrt{2}\,B\sqrt{b}\ \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\,B\sqrt{\frac{b}{\cos(dx+c)}}\,\sin(dx+c)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
 c)) + I*sqrt(2)*A*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c)) - I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) + I*sqrt(2)*B*sqrt(b)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*B*s
qrt(b/cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(b*sec(c + d*x))*(A + B*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(1/2),x)
```

```
[Out] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(1/2), x)
```

3.4 $\int \frac{A+B\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=82

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{bd}$$

```
[Out] 2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b/d
```

Rubi [A]
time = 0.05, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {3872, 3856, 2719, 2720}

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/Sqrt[b*Sec[c + d*x]],x]
```

```
[Out] (2*A*EllipticE[(c + d*x)/2, 2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]])
 + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]])/
(b*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sec(c+dx)}{\sqrt{b\sec(c+dx)}}\,dx &= A\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx+\frac{B\int \sqrt{b\sec(c+dx)}\,dx}{b}\\ &= \frac{A\int \sqrt{\cos(c+dx)}\,dx}{\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{\left(B\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}\right)\int \frac{}{\sqrt{\cos(}}}{b}\\ &= \frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c}}{bd}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 54, normalized size = 0.66

$$\frac{2\left(AE\left(\frac{1}{2}(c+dx)\middle|2\right)+BF\left(\frac{1}{2}(c+dx)\middle|2\right)\right)}{d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/Sqrt[b*Sec[c + d*x]],x]
[Out] (2*(A*EllipticE[(c + d*x)/2, 2] + B*EllipticF[(c + d*x)/2, 2]))/(d*Sqrt[Cos
[c + d*x]]*Sqrt[b*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 7.86, size = 445, normalized size = 5.43

method	result
risch	$-\frac{iA\sqrt{2}}{d\sqrt{\frac{b\,e^{i(dx+c)}}{e^{2i(dx+c)}+1}}}-\frac{i\left(\frac{iB\sqrt{-i\left(e^{i(dx+c)}+i\right)}\sqrt{2}\sqrt{i\left(e^{i(dx+c)}-i\right)}\sqrt{ie^{i(dx+c)}}\,\mathrm{EllipticF}\left(\sqrt{-i\left(e^{i(dx+c)}\right.}\right.}{\sqrt{b\,e^{3i(dx+c)}+b\,e^{i(dx+c)}}}\right.}{}$
default	$\frac{2\left(iA\cos(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sin(dx+c)-iA\cos(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*(I*A*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(1+cos(d*x
+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-I*A*EllipticE(I*(-
1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+I*B*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*
sin(d*x+c)+I*A*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*(1/(1+cos(d*x+c)))
^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-I*A*EllipticE(I*(-1+cos
(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)+I*B*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*
(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-A*cos(d*x+c)^2+A
*cos(d*x+c))*(b/cos(d*x+c))^(1/2)/sin(d*x+c)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/sqrt(b*sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.43, size = 122, normalized size = 1.49

$$\frac{-i\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)})+i\sqrt{2}\,B\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}-i\,\sin{(dx+c)})+i\sqrt{2}\,A\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}+i\,\sin{(dx+c)}))-i\sqrt{2}\,A\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos{(dx+c)}-i\,\sin{(dx+c)}))}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] (-I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
 c)) + I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c)) + I*sqrt(2)*A*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(2)*A*sqrt(b)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec{\left(c + dx \right)}}{\sqrt{b\sec{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/sqrt(b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((B*sec(d*x + c) + A)/sqrt(b*sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\sqrt{\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(b/cos(c + d*x))^(1/2),x)
[Out] int((A + B/cos(c + d*x))/(b/cos(c + d*x))^(1/2), x)
```

3.5 $\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=116

$$\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2A\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}$$

```
[Out] 2/3*A*sin(d*x+c)/b/d/(b*sec(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b/d/cos(d*x+c)^(1/
2)/(b*sec(d*x+c))^(1/2)+2/3*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1
/2)/b^2/d
```

Rubi [A]
time = 0.06, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3872, 3854, 3856, 2720, 2719}

$$\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^2d}+\frac{2A\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}}+\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*B*EllipticE[(c + d*x)/2, 2])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c + d*x]
]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d*x]]
)/(3*b^2*d) + (2*A*Sin[c + d*x])/(3*b*d*Sqrt[b*Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{3/2}}\,dx = A\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx + \frac{B\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{b}$$
$$= \frac{2A\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}} + \frac{A\int \sqrt{b\sec(c+dx)}\,dx}{3b^2} + \frac{B\int \sqrt{\cos(c+dx)}\,dx}{b\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+da}$$
$$= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}} + \frac{2A\sin(c+dx)}{3bd\sqrt{b\sec(c+dx)}} + \frac{\left(A\sqrt{\cos(c+dx)}\,\sqrt{\right.}{}$$
$$= \frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(}}{3b^2d}$$

Mathematica [A]
time = 0.19, size = 86, normalized size = 0.74

$$\frac{\sec^2(c+dx)\left(6B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+A\left(2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sin(2(c+dx))\right)\right)}{3d(b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (Sec[c + d*x]^2*(6*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + A*(2*Sq
rt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + Sin[2*(c + d*x)])))/(3*d*(b*Se
c[c + d*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 8.31, size = 470, normalized size = 4.05

method	result
default	$-\frac{2\left(-iA\cos(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sin(dx+c)-3iB\cos(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(-I*A*cos(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-3*I*B*cos(d*x+
c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*El
lipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)+3*I*B*cos(d*x+c)*(1/(1+cos(d*x+c)))
^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x
+c))/sin(d*x+c),I)-I*A*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)-3*I*B*(1/(1+cos
(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*EllipticF(I*(-
1+cos(d*x+c))/sin(d*x+c),I)+3*I*B*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*sin(d*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)+A*co
s(d*x+c)^3+3*B*cos(d*x+c)^2-A*cos(d*x+c)-3*B*cos(d*x+c))/cos(d*x+c)^2/sin(d
*x+c)/(b/cos(d*x+c))^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.46, size = 150, normalized size = 1.29

$$\frac{2A\sqrt{\frac{b}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) - i\sqrt{2}A\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) + i\sqrt{2}A\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)) + 3i\sqrt{2}B\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) - 3i\sqrt{2}B\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{3b^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] 1/3*(2*A*sqrt(b/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - I*sqrt(2)*A*sqrt(
b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*A*
sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqr
t(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
```

```
c) + I*sin(d*x + c))) - 3*I*sqrt(2)*B*sqrt(b)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(b^2*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec{(c + dx)}}{(b\sec{(c + dx)})^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))**(3/2),x)
[Out] Integral((A + B*sec(c + d*x))/(b*sec(c + d*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(3/2),x, algorithm="giac")
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+d\,x)}}{\left(\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(b/cos(c + d*x))^(3/2),x)
[Out] int((A + B/cos(c + d*x))/(b/cos(c + d*x))^(3/2), x)
```

3.6 $\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=147

$$\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^3d}+\frac{2A\sin(c+dx)}{5bd(b\sec(c+dx))}$$

```
[Out] 2/5*A*sin(d*x+c)/b/d/(b*sec(d*x+c))^(3/2)+2/3*B*sin(d*x+c)/b^2/d/(b*sec(d*x
+c))^(1/2)+6/5*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/b^2/d/cos(d*x+c)^(1/2)/(b*sec(d*x+c))^(1/2)+2/3
*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*(b*sec(d*x+c))^(1/2)/b^3/d
```

Rubi [A]
time = 0.08, antiderivative size = 147, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {3872, 3854, 3856, 2719, 2720}

$$\frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\,\sqrt{b\sec(c+dx)}}+\frac{2A\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\sec(c+dx)}}{3b^3d}+\frac{2B\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (6*A*EllipticE[(c + d*x)/2, 2])/(5*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[b*Sec[c +
d*x]]) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[b*Sec[c + d
*x]])/(3*b^3*d) + (2*A*Sin[c + d*x])/(5*b*d*(b*Sec[c + d*x])^(3/2)) + (2*B*
Sin[c + d*x])/(3*b^2*d*Sqrt[b*Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{5/2}}\,dx = A\int \frac{1}{(b\sec(c+dx))^{5/2}}\,dx + \frac{B\int \frac{1}{(b\sec(c+dx))^{3/2}}\,dx}{b}$$

$$= \frac{2A\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}} + \frac{(3A)\int \frac{1}{\sqrt{b\sec(c+dx)}}\,dx}{5b^2} + \frac{B}{}$$

$$= \frac{2A\sin(c+dx)}{5bd(b\sec(c+dx))^{3/2}} + \frac{2B\sin(c+dx)}{3b^2d\sqrt{b\sec(c+dx)}} + \frac{(3A)\int \sqrt{\cos(c+dx)}\,dx}{5b^2\sqrt{\cos(c+dx)}\sqrt{b\sec(c+d}}$$

$$= \frac{6AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^2d\sqrt{\cos(c+dx)}\sqrt{b\sec(c+dx)}} + \frac{2B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{b\,\text{se}}}{3b^3d}$$

Mathematica [A]
time = 0.54, size = 88, normalized size = 0.60

$$\frac{2\left(9AE\left(\frac{1}{2}(c+dx)\middle|2\right)+5BF\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(5B+3A\cos(c+dx))\sin(c+dx)\right)}{15d\cos^{\frac{5}{2}}(c+dx)(b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(9*A*EllipticE[(c + d*x)/2, 2] + 5*B*EllipticF[(c + d*x)/2, 2] + Sqrt[Co
s[c + d*x]]*(5*B + 3*A*Cos[c + d*x])*Sin[c + d*x]))/(15*d*Cos[c + d*x]^(5/2
)*(b*Sec[c + d*x])^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 8.13, size = 482, normalized size = 3.28

method	result
default	$-\frac{2\left(-9iA\cos(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\,\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\ \mathrm{EllipticF}\left(\frac{i(-1+\cos(dx+c))}{\sin(dx+c)},i\right)\sin(dx+c)+9iA\cos(dx+c)\sqrt{\frac{1}{1+\cos(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/15/d*(-9*I*A*cos(d*x+c)*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*
x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+9*I*A*cos(d
*x+c)*EllipticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*(1/(1+cos(d*x+c)
))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-5*I*B*cos(d*x+c)*EllipticF(I*(-1
+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)-9*I*A*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d
*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+9*I*A*Elli
pticE(I*(-1+cos(d*x+c))/sin(d*x+c),I)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-5*I*B*EllipticF(I*(-1+cos(d*x+c))/sin(d*x+
c),I)*sin(d*x+c)*(1/(1+cos(d*x+c)))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
+3*A*cos(d*x+c)^4+5*B*cos(d*x+c)^3+6*A*cos(d*x+c)^2-9*A*cos(d*x+c)-5*B*cos(
d*x+c))/cos(d*x+c)^3/sin(d*x+c)/(b/cos(d*x+c))^(5/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.55, size = 164, normalized size = 1.12

$$\frac{-5i\sqrt{2}B\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}B\sqrt{b}\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+9i\sqrt{2}A\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-9i\sqrt{2}A\sqrt{b}\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(3A\cos(dx+c)^2+5B\cos(dx+c)\right)\sqrt{\frac{b}{\cos(dx+c)}}\sin(dx+c)}{15b^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*I*sqrt(2)*B*sqrt(b)*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) + 9*I*sqrt(2)*A*sqrt(b)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*I*sqrt(2)*A*sqrt(b)*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
```

```
c))) + 2*(3*A*cos(d*x + c)^2 + 5*B*cos(d*x + c))*sqrt(b/cos(d*x + c))*sin(
d*x + c))/(b^3*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec{\left(c + dx \right)}}{\left(b\sec{\left(c + dx \right)}\right)^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))**(5/2),x)
[Out] Integral((A + B*sec(c + d*x))/(b*sec(c + d*x))**(5/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos\left(c+d\,x\right)}}{{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(b/cos(c + d*x))^(5/2),x)
[Out] int((A + B/cos(c + d*x))/(b/cos(c + d*x))^(5/2), x)
```

3.7 $\int \sec^2(c+dx)(b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=119

$$\frac{3A\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{5/3}\sin(c+dx)}{5bd\sqrt{\sin^2(c+dx)}}+\frac{3B\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};\cos^2(c+dx)\right)(b\sec(c+dx)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/5*A*hypergeom([-5/6, 1/2],[1/6],cos(d*x+c)^2)*(b*sec(d*x+c))^(5/3)*sin(d*
x+c)/b/d/(sin(d*x+c)^2)^(1/2)+3/8*B*hypergeom([-4/3, 1/2],[-1/3],cos(d*x+c)
^2)*(b*sec(d*x+c))^(8/3)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3872, 3857, 2722}

$$\frac{3A\sin(c+dx)(b\sec(c+dx))^{5/3}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{8/3}\,{}_2F_1\left(-\frac{4}{3},\frac{1}{2};-\frac{1}{3};\cos^2(c+dx)\right)}{8b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*A*Hypergeometric2F1[-5/6, 1/2, 1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(5/
3)*Sin[c + d*x])/(5*b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-4/3
, 1/2, -1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(8/3)*Sin[c + d*x])/(8*b^2*d*
Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec^2(c+dx)(b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx = \frac{\int (b\sec(c+dx))^{8/3}(A+B\sec(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\sec(c+dx))^{8/3}\,dx}{b^2} + \frac{B\int (b\sec(c+dx))^{11/}}{b^3}$$
$$= \frac{\left(A\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)}}{b^2}$$
$$= \frac{3A\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{5/3}}{5bd\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.28, size = 90, normalized size = 0.76

$$-\frac{3\csc^3(c+dx)\left(11A\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\sec^2(c+dx)\right)+8B\,{}_2F_1\left(\frac{1}{2},\frac{11}{6};\frac{17}{6};\sec^2(c+dx)\right)\right)(b\sec(c+dx))^{2/3}\left(-\tan^2(c+dx)\right)^{3/2}}{88d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*Csc[c + d*x]^3*(11*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 4/3, 7/3, Sec[
c + d*x]^2] + 8*B*Hypergeometric2F1[1/2, 11/6, 17/6, Sec[c + d*x]^2])*(b*Se
c[c + d*x])^(2/3)*(-Tan[c + d*x]^2)^(3/2))/(88*d)
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int \left(\sec^2(dx+c)\right)(b\sec(dx+c))^{\frac{2}{3}}(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*(b*sec(d*x + c))^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^{\frac{2}{3}}(A+B\sec(c+dx))\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)),x)
[Out] Integral((b*sec(c + d*x))**(2/3)*(A + B*sec(c + d*x))*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="
giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{b}{\cos(c+d\,x)}\right)}^{2/3}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3))/cos(c + d*x)^2,x)
[Out] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3))/cos(c + d*x)^2, x)
```

3.8 $\int \sec(c+dx)(b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=116

$$\frac{3A\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{2d\sqrt{\sin^2(c+dx)}}+\frac{3B\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{5}}{5bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/2*A*hypergeom([-1/3, 1/2],[2/3],cos(d*x+c)^2)*(b*sec(d*x+c))^(2/3)*sin(d*
x+c)/d/(sin(d*x+c)^2)^(1/2)+3/5*B*hypergeom([-5/6, 1/2],[1/6],cos(d*x+c)^2)
*(b*sec(d*x+c))^(5/3)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {16, 3872, 3857, 2722}

$$\frac{3A\sin(c+dx)(b\sec(c+dx))^{2/3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{5/3}\,{}_2F_1\left(-\frac{5}{6},\frac{1}{2};\frac{1}{6};\cos^2(c+dx)\right)}{5bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*A*Hypergeometric2F1[-1/3, 1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/
3)*Sin[c + d*x])/(2*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-5/6,
1/2, 1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(5/3)*Sin[c + d*x])/(5*b*d*Sqrt[
Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx &= \frac{\int (b\sec(c+dx))^{5/3}(A+B\sec(c+dx))\,dx}{b}\\ &= \frac{A\int (b\sec(c+dx))^{5/3}\,dx}{b}+\frac{B\int (b\sec(c+dx))^{8/3}\,d}{b^2}\\ &= \frac{\left(A\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{5/}}}{b}\\ &= \frac{3A\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\,\mathrm{s}}{2d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.13, size = 91, normalized size = 0.78

$$\frac{3\csc(c+dx)\left(8A\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\sec^2(c+dx)\right)+5B\,{}_2F_1\left(\frac{1}{2},\frac{4}{3};\frac{7}{3};\sec^2(c+dx)\right)\right)(b\sec(c+dx))^{5/3}\sqrt{-\tan^2(c+dx)}}{40bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*Csc[c + d*x]*(8*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[c +
 d*x]^2] + 5*B*Hypergeometric2F1[1/2, 4/3, 7/3, Sec[c + d*x]^2])*(b*Sec[c +
 d*x])^(5/3)*Sqrt[-Tan[c + d*x]^2])/(40*b*d)
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \sec(dx+c)\,(b\sec(dx+c))^{\frac{2}{3}}\,(A+B\sec(dx+c))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x + c))^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^{\frac{2}{3}}(A+B\sec(c+dx))\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)),x)
[Out] Integral((b*sec(c + d*x))**(2/3)*(A + B*sec(c + d*x))*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\left(\frac{b}{\cos(c+d\,x)}\right)^{2/3}}{\cos(c+d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3))/cos(c + d*x),x)
[Out] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3))/cos(c + d*x), x)
```

3.9 $\int (b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=112

$$-\frac{3Ab\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt[3]{b\sec(c+dx)}\,\sqrt{\sin^2(c+dx)}}+\frac{3B\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*A*b*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+c))
^(1/3)/(sin(d*x+c)^2)^(1/2)+3/2*B*hypergeom([-1/3, 1/2],[2/3],cos(d*x+c)^2)
*(b*sec(d*x+c))^(2/3)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3872, 3857, 2722}

$$\frac{3B\sin(c+dx)(b\sec(c+dx))^{2/3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}}-\frac{3Ab\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\,\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*A*b*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(
b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/3,
1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(2*d*Sqrt[Si
n[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int (b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx = A\int (b\sec(c+dx))^{2/3}\,dx + \frac{B\int (b\sec(c+dx))^{5/3}\,dx}{b}$$

$$= \left(A\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}}\,dx$$

$$= \frac{3B\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [**A**]
time = 0.09, size = 88, normalized size = 0.79

$$\frac{3\csc(c+dx)\left(5A\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sec^2(c+dx)\right)+2B\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\sec^2(c+dx)\right)\right)(b\sec(c+dx))^{2/3}\sqrt{-\tan^2(c+dx)}}{10d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
[Out] (3*Csc[c + d*x]*(5*A*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c +
d*x]^2] + 2*B*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[c + d*x]^2])*(b*Sec[c +
 d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(10*d)
```

Maple [**F**]
time = 0.14, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c))^{\frac{2}{3}}(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
[Out] int((b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^{\frac{2}{3}} (A+B\sec(c+dx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)),x)
[Out] Integral((b*sec(c + d*x))**(2/3)*(A + B*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{b}{\cos(c+d\,x)}\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3),x)
[Out] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3), x)
```

3.10 $\int \cos(c+dx)(b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=115

$$-\frac{3Ab^2\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\sec(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}-\frac{3bB\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{d\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/4*A*b^2*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x
+c))^(4/3)/(sin(d*x+c)^2)^(1/2)-3*b*B*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)
^2)*sin(d*x+c)/d/(b*sec(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {16, 3872, 3857, 2722}

$$-\frac{3Ab^2\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{4/3}}-\frac{3bB\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*A*b^2*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4
*d*(b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) - (3*b*B*Hypergeometric2F1[
1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(d*(b*Sec[c + d*x])^(1/3)*Sqrt
[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
```

```
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx &= b\int \frac{A+B\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx\\ &= (Ab)\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx+B\int (b\sec(c+dx))^{2/}\\ &= \left(Ab\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \sqrt[3]{\frac{\cos}{}}\\ &= -\frac{3B\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)(b\sec(c}{d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.10, size = 88, normalized size = 0.77

$$-\frac{3\cot(c+dx)\left(2A\cos(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\sec^2(c+dx)\right)-B\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sec^2(c+dx)\right)\right)(b\sec(c+dx))^{2/3}\sqrt{-\tan^2(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*Cot[c + d*x]*(2*A*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c
+ d*x]^2] - B*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d*x]^2])*(b*Sec[c +
d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(2*d)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\,(b\sec(dx+c))^{\frac{2}{3}}\,(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*cos(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*(b*sec(d*x + c))^(2
/3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3),x)
[Out] int(cos(c + d*x)*(A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3), x)
```

3.11 $\int \cos^2(c+dx)(b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=119

$$-\frac{3Ab^3\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7d(b\sec(c+dx))^{7/3}\sqrt{\sin^2(c+dx)}}-\frac{3b^2B\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\sec(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/7*A*b^3*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*
x+c))^(7/3)/(sin(d*x+c)^2)^(1/2)-3/4*b^2*B*hypergeom([1/2, 2/3],[5/3],cos(d
*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+c))^(4/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3872, 3857, 2722}

$$-\frac{3Ab^3\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{7/3}}-\frac{3b^2B\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*A*b^3*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(
7*d*(b*Sec[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2]) - (3*b^2*B*Hypergeometric2
F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Sec[c + d*x])^(4/3)
*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
```

```
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx = b^2\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$$
$$= \left(Ab^2\right)\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx + (bB)\int \frac{1}{\sqrt[3]{b\sec(c}$$
$$= \left(Ab^2\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \Bigg(\frac{\mathrm{co}}{}$$
$$= -\frac{3B\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)(b\sec(}{4d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.15, size = 88, normalized size = 0.74

$$-\frac{3b\cot(c+dx)\left(A\cos(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\sec^2(c+dx)\right)+4B\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\sec^2(c+dx)\right)\right)\sqrt{-\tan^2(c+dx)}}{4d\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*b*Cot[c + d*x]*(A*Cos[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[c
+ d*x]^2] + 4*B*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d*x]^2])*Sqrt[-Ta
n[c + d*x]^2])/(4*d*(b*Sec[c + d*x])^(1/3))
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int \left(\cos^2(dx+c)\right)(b\sec(dx+c))^{\frac{2}{3}}(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*cos(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*(b*sec(d*x + c)
)^(2/3), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^2 \left(A+\frac{B}{\cos(c+dx)}\right) \left(\frac{b}{\cos(c+dx)}\right)^{2/3} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3),x)
[Out] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3), x)
```

3.12 $\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=119

$$\frac{3A\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{7/3}\sin(c+dx)}{7bd\sqrt{\sin^2(c+dx)}}+\frac{3B\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+}{10b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/7*A*hypergeom([-7/6, 1/2],[-1/6],cos(d*x+c)^2)*(b*sec(d*x+c))^(7/3)*sin(d
*x+c)/b/d/(sin(d*x+c)^2)^(1/2)+3/10*B*hypergeom([-5/3, 1/2],[-2/3],cos(d*x+
c)^2)*(b*sec(d*x+c))^(10/3)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3872, 3857, 2722}

$$\frac{3A\sin(c+dx)(b\sec(c+dx))^{7/3}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{10/3}\,{}_2F_1\left(-\frac{5}{3},\frac{1}{2};-\frac{2}{3};\cos^2(c+dx)\right)}{10b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*A*Hypergeometric2F1[-7/6, 1/2, -1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(7
/3)*Sin[c + d*x])/(7*b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-5/
3, 1/2, -2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(10/3)*Sin[c + d*x])/(10*b^2
*d*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx &= \frac{\int (b\sec(c+dx))^{10/3}(A+B\sec(c+dx))\,dx}{b^2}\\ &= \frac{A\int (b\sec(c+dx))^{10/3}\,dx}{b^2}+\frac{B\int (b\sec(c+dx))^{13/3}}{b^3}\\ &= \frac{\left(A\sqrt[3]{\frac{\cos(c+dx)}{b}}\ \sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{10}}}{b^2}\\ &= \frac{3Ab\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)\sec(c+dx)\sqrt[3]{b\,\mathrm{s}}}{7d\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.32, size = 90, normalized size = 0.76

$$-\frac{3\csc^3(c+dx)\left(13A\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\sec^2(c+dx)\right)+10B\,{}_2F_1\left(\frac{1}{2},\frac{13}{6};\frac{19}{6};\sec^2(c+dx)\right)\right)(b\sec(c+dx))^{4/3}\left(-\tan^2(c+dx)\right)^{3/2}}{130d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
[Out] (-3*Csc[c + d*x]^3*(13*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 5/3, 8/3, Sec[
c + d*x]^2] + 10*B*Hypergeometric2F1[1/2, 13/6, 19/6, Sec[c + d*x]^2])*(b*S
ec[c + d*x])^(4/3)*(-Tan[c + d*x]^2)^(3/2))/(130*d)
```

Maple [F]
time = 0.15, size = 0, normalized size = 0.00

$$\int \left(\sec^2(dx+c)\right)(b\sec(dx+c))^{\frac{4}{3}}(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
[Out] integral((B*b*sec(d*x + c)^4 + A*b*sec(d*x + c)^3)*(b*sec(d*x + c))^(1/3),
x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="
giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*sec(d*x + c)^2, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(\frac{b}{\cos(c+d\,x)}\right)^{4/3}}{\cos\left(c+d\,x\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3))/cos(c + d*x)^2,x)
[Out] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3))/cos(c + d*x)^2, x)
```

3.13 $\int \sec(c+dx)(b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=116

$$\frac{3A\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sin(c+dx)}{4d\sqrt{\sin^2(c+dx)}}+\frac{3B\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)(b\sec(c+dx}{7bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3/4*A*hypergeom([-2/3, 1/2],[1/3],cos(d*x+c)^2)*(b*sec(d*x+c))^(4/3)*sin(d*
x+c)/d/(sin(d*x+c)^2)^(1/2)+3/7*B*hypergeom([-7/6, 1/2],[-1/6],cos(d*x+c)^2
)*(b*sec(d*x+c))^(7/3)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {16, 3872, 3857, 2722}

$$\frac{3A\sin(c+dx)(b\sec(c+dx))^{4/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{7/3}\,{}_2F_1\left(-\frac{7}{6},\frac{1}{2};-\frac{1}{6};\cos^2(c+dx)\right)}{7bd\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*A*Hypergeometric2F1[-2/3, 1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/
3)*Sin[c + d*x])/(4*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-7/6,
1/2, -1/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(7/3)*Sin[c + d*x])/(7*b*d*Sqrt
[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec(c+dx)(b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx = \frac{\int (b\sec(c+dx))^{7/3}(A+B\sec(c+dx))\,dx}{b}$$
$$= \frac{A\int (b\sec(c+dx))^{7/3}\,dx}{b} + \frac{B\int (b\sec(c+dx))^{10/3}\,d}{b^2}$$
$$= \frac{\left(A\sqrt[3]{\frac{\cos(c+dx)}{b}}\,\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{7/3}}}{b}$$
$$= \frac{3A\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{4/3}\sin}{4d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.16, size = 91, normalized size = 0.78

$$\frac{3\csc(c+dx)\left(10A\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\sec^2(c+dx)\right)+7B\,{}_2F_1\left(\frac{1}{2},\frac{5}{3};\frac{8}{3};\sec^2(c+dx)\right)\right)(b\sec(c+dx))^{7/3}\sqrt{-\tan^2(c+dx)}}{70bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
[Out] (3*Csc[c + d*x]*(10*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 7/6, 13/6, Sec[c
+ d*x]^2] + 7*B*Hypergeometric2F1[1/2, 5/3, 8/3, Sec[c + d*x]^2])*(b*Sec[c
+ d*x])^(7/3)*Sqrt[-Tan[c + d*x]^2])/(70*b*d)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \sec(dx+c)\,(b\sec(dx+c))^{\frac{4}{3}}\,(A+B\sec(dx+c))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
[Out] int(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] integral((B*b*sec(d*x + c)^3 + A*b*sec(d*x + c)^2)*(b*sec(d*x + c))^(1/3),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^{\frac{4}{3}}(A+B\sec(c+dx))\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((b*sec(c + d*x))**(4/3)*(A + B*sec(c + d*x))*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(\frac{b}{\cos(c+d\,x)}\right)^{4/3}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3))/cos(c + d*x),x)
[Out] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3))/cos(c + d*x), x)
```

3.14 $\int (b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=112

$$\frac{3Ab\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}+\frac{3B\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)(b\sec(c+dx))}{4d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*A*b*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*(b*sec(d*x+c))^(1/3)*sin(d*
x+c)/d/(sin(d*x+c)^2)^(1/2)+3/4*B*hypergeom([-2/3, 1/2],[1/3],cos(d*x+c)^2)
*(b*sec(d*x+c))^(4/3)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 112, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3872, 3857, 2722}

$$\frac{3Ab\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{4/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*A*b*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(
1/3)*Sin[c + d*x])/(d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-2/3,
1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)*Sin[c + d*x])/(4*d*Sqrt[Si
n[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int (b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx = A\int (b\sec(c+dx))^{4/3}\,dx + \frac{B\int (b\sec(c+dx))^{7/3}\,dx}{b}$$

$$= \left(A\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{4/3}}\,dx + \frac{\left(\right.}{}$$

$$= \frac{3Ab\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}} +$$

Mathematica [A]
time = 0.12, size = 88, normalized size = 0.79

$$\frac{3\csc(c+dx)\left(7A\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sec^2(c+dx)\right)+4B\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\sec^2(c+dx)\right)\right)(b\sec(c+dx))^{4/3}\sqrt{-\tan^2(c+dx)}}{28d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*Csc[c + d*x]*(7*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c +
d*x]^2] + 4*B*Hypergeometric2F1[1/2, 7/6, 13/6, Sec[c + d*x]^2])*(b*Sec[c +
 d*x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/(28*d)
```

Maple [F]
time = 0.14, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c))^{\frac{4}{3}}\,(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int((b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")
[Out] integral((B*b*sec(d*x + c)^2 + A*b*sec(d*x + c))*(b*sec(d*x + c))^(1/3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^{\frac{4}{3}}\,(A+B\sec(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)),x)
[Out] Integral((b*sec(c + d*x))**(4/3)*(A + B*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{b}{\cos(c+d\,x)}\right)}^{4/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3),x)
[Out] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3), x)
```

3.15 $\int \cos(c+dx)(b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=115

$$-\frac{3Ab^2\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\sec(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}+\frac{3bB\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/2*A*b^2*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x
+c))^(2/3)/(sin(d*x+c)^2)^(1/2)+3*b*B*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c
)^2)*(b*sec(d*x+c))^(1/3)*sin(d*x+c)/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {16, 3872, 3857, 2722}

$$\frac{3bB\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{d\sqrt{\sin^2(c+dx)}}-\frac{3Ab^2\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*A*b^2*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2
*d*(b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*b*B*Hypergeometric2F1[
-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*Sqr
t[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
```

```
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \cos(c+dx)(b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx = b\int \sqrt[3]{b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$$
$$= (Ab)\int \sqrt[3]{b\sec(c+dx)}\,dx + B\int (b\sec(c+dx))^{4}$$
$$= \left(Ab\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\sqrt[3]{\frac{\cos(c}{}}}$$
$$= \frac{3bB\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\ \mathrm{s}}{d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.12, size = 87, normalized size = 0.76

$$\frac{3\cot(c+dx)\left(4A\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sec^2(c+dx)\right)+B\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sec^2(c+dx)\right)\right)(b\sec(c+dx))^{4/3}\sqrt{-\tan^2(c+dx)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*Cot[c + d*x]*(4*A*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c +
d*x]^2] + B*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2])*(b*Sec[c + d*
x])^(4/3)*Sqrt[-Tan[c + d*x]^2])/(4*d)
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \cos(dx+c)\,(b\sec(dx+c))^{\frac{4}{3}}\,(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*cos(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
[Out] integral((B*b*cos(d*x + c)*sec(d*x + c)^2 + A*b*cos(d*x + c)*sec(d*x + c))*
(b*sec(d*x + c))^(1/3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)\left(A+\frac{B}{\cos(c+dx)}\right)\left(\frac{b}{\cos(c+dx)}\right)^{4/3}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3),x)
[Out] int(cos(c + d*x)*(A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3), x)
```

3.16 $\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=119

$$-\frac{3Ab^3\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\sec(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}}-\frac{3b^2B\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\sec(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/5*A*b^3*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*
x+c))^(5/3)/(sin(d*x+c)^2)^(1/2)-3/2*b^2*B*hypergeom([1/3, 1/2],[4/3],cos(d
*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3872, 3857, 2722}

$$-\frac{3Ab^3\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{5/3}}-\frac{3b^2B\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*A*b^3*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(
5*d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) - (3*b^2*B*Hypergeometric2
F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Sec[c + d*x])^(2/3)
*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
```

```
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx = b^2\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx$$
$$= \left(Ab^2\right)\int \frac{1}{(b\sec(c+dx))^{2/3}}\,dx + (bB)\int \sqrt[3]{b\sec(}$$
$$= \left(Ab^2\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \left(\frac{\cos(}{}\right.$$
$$= -\frac{3bB\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sqrt[3]{b\,\mathrm{s}}}{2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.11, size = 88, normalized size = 0.74

$$-\frac{3b\cot(c+dx)\left(A\cos(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\sec^2(c+dx)\right)-2B\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sec^2(c+dx)\right)\right)\sqrt[3]{b\sec(c+dx)}\sqrt{-\tan^2(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*b*Cot[c + d*x]*(A*Cos[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[c
+ d*x]^2] - 2*B*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c + d*x]^2])*(b*Sec[c
+ d*x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/(2*d)
```

Maple [F]
time = 0.30, size = 0, normalized size = 0.00

$$\int \left(\cos^2(dx+c)\right)(b\sec(dx+c))^{\frac{4}{3}}(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
[Out] integral((B*b*cos(d*x + c)^2*sec(d*x + c)^2 + A*b*cos(d*x + c)^2*sec(d*x +
c))*(b*sec(d*x + c))^(1/3), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)),x)
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="
giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{4/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3),x)
[Out] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3), x)
```

3.17 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=117

$$\frac{3A\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\,\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}}+\frac{3B\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{4/3}}{4b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*A*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*(b*sec(d*x+c))^(1/3)*sin(d*x+
c)/b/d/(sin(d*x+c)^2)^(1/2)+3/4*B*hypergeom([-2/3, 1/2],[1/3],cos(d*x+c)^2)
*(b*sec(d*x+c))^(4/3)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3872, 3857, 2722}

$$\frac{3A\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{4/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/
3)*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-2/3,
1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)*Sin[c + d*x])/(4*b^2*d*Sqr
t[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{2/3}}\,dx = \frac{\int (b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\sec(c+dx))^{4/3}\,dx}{b^2} + \frac{B\int (b\sec(c+dx))^{7/3}\,dx}{b^3}$$
$$= \frac{\left(A\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{4/3}}\,dx}{b^2} + \frac{\left(B\sqrt[3]{\frac{c}{}}\right.}{}$$
$$= \frac{3A\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\,\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}} + \frac{3B}{}$$

Mathematica [**A**]
time = 0.26, size = 90, normalized size = 0.77

$$-\frac{3\csc^3(c+dx)\left(7A\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sec^2(c+dx)\right)+4B\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\sec^2(c+dx)\right)\right)\left(-\tan^2(c+dx)\right)^{3/2}}{28d(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-3*Csc[c + d*x]^3*(7*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c
 + d*x]^2] + 4*B*Hypergeometric2F1[1/2, 7/6, 13/6, Sec[c + d*x]^2])*(-Tan[c
 + d*x]^2)^(3/2))/(28*d*(b*Sec[c + d*x])^(2/3))
```

Maple [**F**]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\left(\sec^2(dx+c)\right)\left(A+B\sec(dx+c)\right)}{\left(b\sec(dx+c)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x + c))^(1/3)/b, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\sec^{2}{(c + dx)}}{(b\sec{(c + dx)})^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(b*sec(c + d*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^2\,{\left(\frac{b}{\cos(c+d\,x)}\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(b/cos(c + d*x))^(2/3)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(b/cos(c + d*x))^(2/3)), x)
```

3.18 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=114

$$-\frac{3A\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\sec(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}+\frac{3B\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\,\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/2*A*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+c))
^(2/3)/(sin(d*x+c)^2)^(1/2)+3*B*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*(
b*sec(d*x+c))^(1/3)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {16, 3872, 3857, 2722}

$$\frac{3B\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}-\frac{3A\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(
b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/6,
1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sqrt[Si
n[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{2/3}}\,dx = \frac{\int \sqrt[3]{b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx}{b}$$
$$= \frac{A\int \sqrt[3]{b\sec(c+dx)}\,dx}{b} + \frac{B\int (b\sec(c+dx))^{4/3}\,dx}{b^2}$$
$$= \frac{\left(A\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\sqrt[3]{\frac{\cos(c+dx)}{b}}}\,dx}{b} + \frac{\left(B\sqrt[3]{\right.}{}}{}$$
$$= \frac{3B\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\,\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}} - \frac{3A\,c}{}$$

Mathematica [A]
time = 0.11, size = 90, normalized size = 0.79

$$\frac{3\csc(c+dx)\left(4A\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sec^2(c+dx)\right)+B\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sec^2(c+dx)\right)\right)\sqrt[3]{b\sec(c+dx)}\sqrt{-\tan^2(c+dx)}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*Csc[c + d*x]*(4*A*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c +
d*x]^2] + B*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2])*(b*Sec[c + d*
x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/(4*b*d)
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)\left(A+B\sec(dx+c)\right)}{\left(b\sec(dx+c)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)/b, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec(c + dx)}{(b\sec(c + dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(b*sec(c + d*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)\,\left(\frac{b}{\cos(c+d\,x)}\right)^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(b/cos(c + d*x))^(2/3)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)*(b/cos(c + d*x))^(2/3)), x)
```

3.19 $\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=114

$$-\frac{3Ab\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)\sin(c+dx)}{5d(b\sec(c+dx))^{5/3}\sqrt{\sin^2(c+dx)}}-\frac{3B\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\sec(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/5*A*b*hypergeom([1/2, 5/6],[11/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+
c))^(5/3)/(sin(d*x+c)^2)^(1/2)-3/2*B*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^
2)*sin(d*x+c)/d/(b*sec(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3872, 3857, 2722}

$$-\frac{3Ab\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\cos^2(c+dx)\right)}{5d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{5/3}}-\frac{3B\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-3*A*b*Hypergeometric2F1[1/2, 5/6, 11/6, Cos[c + d*x]^2]*Sin[c + d*x])/(5*
d*(b*Sec[c + d*x])^(5/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/3
, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(b*Sec[c + d*x])^(2/3)*Sqrt[
Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{2/3}}\,dx = A\int \frac{1}{(b\sec(c+dx))^{2/3}}\,dx + \frac{B\int \sqrt[3]{b\sec(c+dx)}\,dx}{b}$$

$$= \left(A\sqrt[3]{\frac{\cos(c+dx)}{b}}\sqrt[3]{b\sec(c+dx)}\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{2/3}dx + \frac{\left(B\sqrt[3]{\frac{\cos(c+\ldots}{b}}\right.}{\ldots}$$

$$= -\frac{3B\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\sin(c+dx)}{2bd\sqrt{\sin^2(c+dx)}} - \frac{3A\,\mathrm{co}\ldots}{\ldots}$$

Mathematica [**A**]
time = 0.09, size = 87, normalized size = 0.76

$$-\frac{3\csc(c+dx)\left(A\cos(c+dx)\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\sec^2(c+dx)\right)-2B\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sec^2(c+dx)\right)\right)\sqrt{-\tan^2(c+dx)}}{2d(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(2/3),x]
[Out] (-3*Csc[c + d*x]*(A*Cos[c + d*x]*Hypergeometric2F1[-1/3, 1/2, 2/3, Sec[c +
d*x]^2] - 2*B*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c + d*x]^2])*Sqrt[-Tan[c
 + d*x]^2])/(2*d*(b*Sec[c + d*x])^(2/3))
```

Maple [**F**]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec(dx+c)}{(b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
[Out] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="maxima")
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)/(b*sec(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec{\left(c + dx \right)}}{\left(b \sec{\left(c + dx \right)}\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))**(2/3),x)
[Out] Integral((A + B*sec(c + d*x))/(b*sec(c + d*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos\left(c+d\,x\right)}}{{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(b/cos(c + d*x))^(2/3),x)
[Out] int((A + B/cos(c + d*x))/(b/cos(c + d*x))^(2/3), x)
```

3.20 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=114

$$-\frac{3A\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)\sin(c+dx)}{2d(b\sec(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}+\frac{3B\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\,\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/2*A*hypergeom([1/3, 1/2],[4/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+c))
^(2/3)/(sin(d*x+c)^2)^(1/2)+3*B*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*(
b*sec(d*x+c))^(1/3)*sin(d*x+c)/b/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {16, 3872, 3857, 2722}

$$\frac{3B\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}-\frac{3A\sin(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\cos^2(c+dx)\right)}{2d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/3, 1/2, 4/3, Cos[c + d*x]^2]*Sin[c + d*x])/(2*d*(
b*Sec[c + d*x])^(2/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/6,
1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(b*d*Sqrt[Si
n[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{2/3}}\,dx = \frac{\int \sqrt[3]{b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx}{b}$$
$$= \frac{A\int \sqrt[3]{b\sec(c+dx)}\,dx}{b} + \frac{B\int (b\sec(c+dx))^{4/3}\,dx}{b^2}$$
$$= \frac{\left(A\sqrt[3]{\frac{\cos(c+dx)}{b}}\,\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\sqrt[3]{\frac{\cos(c+dx)}{b}}}\,dx}{b} + \frac{\left(B\sqrt[3]{\right.}{}$$
$$= \frac{3B\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\,\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}} - \frac{3A\,\mathrm{co}}{}$$

Mathematica [A]
time = 0.03, size = 90, normalized size = 0.79

$$\frac{3\csc(c+dx)\left(4A\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\sec^2(c+dx)\right)+B\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sec^2(c+dx)\right)\right)\sqrt[3]{b\sec(c+dx)}\,\sqrt{-\tan^2(c+dx)}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]
[Out] (3*Csc[c + d*x]*(4*A*Cos[c + d*x]*Hypergeometric2F1[1/6, 1/2, 7/6, Sec[c +
d*x]^2] + B*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c + d*x]^2])*(b*Sec[c + d*
x])^(1/3)*Sqrt[-Tan[c + d*x]^2])/(4*b*d)
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)\,(A+B\sec(dx+c))}{(b\sec(dx+c))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)/b, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec(c + dx)}{(b\sec(c + dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(b*sec(c + d*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)\,{\left(\frac{b}{\cos(c+d\,x)}\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(b/cos(c + d*x))^(2/3)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)*(b/cos(c + d*x))^(2/3)), x)
```

3.21 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=117

$$\frac{3A\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\,\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}}+\frac{3B\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{4}}{4b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] 3*A*hypergeom([-1/6, 1/2],[5/6],cos(d*x+c)^2)*(b*sec(d*x+c))^(1/3)*sin(d*x+
c)/b/d/(sin(d*x+c)^2)^(1/2)+3/4*B*hypergeom([-2/3, 1/2],[1/3],cos(d*x+c)^2)
*(b*sec(d*x+c))^(4/3)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3872, 3857, 2722}

$$\frac{3A\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}}+\frac{3B\sin(c+dx)(b\sec(c+dx))^{4/3}\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\cos^2(c+dx)\right)}{4b^2d\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*A*Hypergeometric2F1[-1/6, 1/2, 5/6, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1/
3)*Sin[c + d*x])/(b*d*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-2/3,
1/2, 1/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(4/3)*Sin[c + d*x])/(4*b^2*d*Sqr
t[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{2/3}}\,dx = \frac{\int (b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\sec(c+dx))^{4/3}\,dx}{b^2} + \frac{B\int (b\sec(c+dx))^{7/3}\,dx}{b^3}$$
$$= \frac{\left(A\sqrt[3]{\frac{\cos(c+dx)}{b}}\,\sqrt[3]{b\sec(c+dx)}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{4/3}}\,dx}{b^2} + \frac{\left(B\sqrt[3]{\frac{\cos}{}}\right.}{}$$
$$= \frac{3A\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\cos^2(c+dx)\right)\sqrt[3]{b\sec(c+dx)}\,\sin(c+dx)}{bd\sqrt{\sin^2(c+dx)}} + \frac{3B\,{}_2}{}$$

Mathematica [A]
time = 0.03, size = 90, normalized size = 0.77

$$-\frac{3\csc^3(c+dx)\left(7A\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\sec^2(c+dx)\right)+4B\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\sec^2(c+dx)\right)\right)\left(-\tan^2(c+dx)\right)^{3/2}}{28d(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-3*Csc[c + d*x]^3*(7*A*Cos[c + d*x]*Hypergeometric2F1[1/2, 2/3, 5/3, Sec[c
 + d*x]^2] + 4*B*Hypergeometric2F1[1/2, 7/6, 13/6, Sec[c + d*x]^2])*(-Tan[c
 + d*x]^2)^(3/2))/(28*d*(b*Sec[c + d*x])^(2/3))
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sec^2(dx+c)\right)\left(A+B\sec(dx+c)\right)}{\left(b\sec(dx+c)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="
maxima")
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="
fricas")
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x + c))^(1/3)/b, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec (c + dx)) \sec^2 (c + dx)}{(b \sec (c + dx))^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(2/3),x)
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(b*sec(c + d*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="
giac")
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^2\,{\left(\frac{b}{\cos(c+d\,x)}\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(b/cos(c + d*x))^(2/3)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(b/cos(c + d*x))^(2/3)), x)
```

3.22 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=117

$$-\frac{3A\,_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}}+\frac{3B\,_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{2b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*A*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(b*sec(d*x+c))
^(1/3)/(sin(d*x+c)^2)^(1/2)+3/2*B*hypergeom([-1/3, 1/2],[2/3],cos(d*x+c)^2)
*(b*sec(d*x+c))^(2/3)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3872, 3857, 2722}

$$\frac{3B\sin(c+dx)(b\sec(c+dx))^{2/3}\,_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}-\frac{3A\sin(c+dx)\,_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(
b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/3,
1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(2*b^2*d*Sqr
t[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int (b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx}{b^2}$$
$$= \frac{A\int (b\sec(c+dx))^{2/3}\,dx}{b^2} + \frac{B\int (b\sec(c+dx))^{5/3}\,dx}{b^3}$$
$$= \frac{\left(A\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}}\,dx}{b^2} + \frac{\left(B\left(\frac{\cos(c\text{-}}{b}\right.\right.}{}$$
$$= \frac{3B\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{2b^2d\sqrt{\sin^2(c+dx)}} - \frac{3A}{}$$

Mathematica [A]
time = 0.23, size = 91, normalized size = 0.78

$$\frac{3\csc(c+dx)\left(5A\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sec^2(c+dx)\right)+2B\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\sec^2(c+dx)\right)\right)(b\sec(c+dx))^{2/3}\sqrt{-\tan^2(c+dx)}}{10b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]
[Out] (3*Csc[c + d*x]*(5*A*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c +
d*x]^2] + 2*B*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[c + d*x]^2])*(b*Sec[c +
 d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(10*b^2*d)
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{(\sec^2(dx+c))\,(A+B\sec(dx+c))}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
[Out] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="
maxima")
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="
fricas")
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)/b^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec^2(c + dx)}{(b\sec(c + dx))^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(4/3),x)
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(b*sec(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="
giac")
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^2\,{\left(\frac{b}{\cos(c+d\,x)}\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(b/cos(c + d*x))^(4/3)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(b/cos(c + d*x))^(4/3)), x)
```

3.23 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=114

$$-\frac{3A\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\sec(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}-\frac{3B\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/4*A*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+c))
^(4/3)/(sin(d*x+c)^2)^(1/2)-3*B*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*si
n(d*x+c)/b/d/(b*sec(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {16, 3872, 3857, 2722}

$$-\frac{3A\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{4/3}}-\frac{3B\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(
b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/6, 1
/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin
[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int \frac{A+B\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx}{b}$$

$$= \frac{A\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx}{b} + \frac{B\int (b\sec(c+dx))^{2/3}\,dx}{b^2}$$

$$= \frac{\left(A\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \sqrt[3]{\frac{\cos(c+dx)}{b}}\,dx}{b} + \frac{\left(B\left(\frac{\mathrm{co}}{}\right.\right.}{}$$

$$= -\frac{3B\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+}{b^2 d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.09, size = 91, normalized size = 0.80

$$-\frac{3\csc(c+dx)\left(2A\cos(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\sec^2(c+dx)\right)-B\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sec^2(c+dx)\right)\right)\sqrt{-\tan^2(c+dx)}}{2bd\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*Csc[c + d*x]*(2*A*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c
+ d*x]^2] - B*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d*x]^2])*Sqrt[-Tan[c
 + d*x]^2])/(2*b*d*(b*Sec[c + d*x])^(1/3))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)\,(A+B\sec(dx+c))}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)/(b^2*sec(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\sec{(c + dx)}}{(b\sec{(c + dx)})^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(b*sec(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)\,{\left(\frac{b}{\cos(c+d\,x)}\right)}^{4/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(b/cos(c + d*x))^(4/3)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)*(b/cos(c + d*x))^(4/3)), x)
```

3.24 $\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=114

$$-\frac{3Ab\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)\sin(c+dx)}{7d(b\sec(c+dx))^{7/3}\sqrt{\sin^2(c+dx)}}-\frac{3B\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\sec(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/7*A*b*hypergeom([1/2, 7/6],[13/6],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+
c))^(7/3)/(sin(d*x+c)^2)^(1/2)-3/4*B*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^
2)*sin(d*x+c)/d/(b*sec(d*x+c))^(4/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {3872, 3857, 2722}

$$-\frac{3Ab\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{7}{6};\frac{13}{6};\cos^2(c+dx)\right)}{7d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{7/3}}-\frac{3B\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*A*b*Hypergeometric2F1[1/2, 7/6, 13/6, Cos[c + d*x]^2]*Sin[c + d*x])/(7*
d*(b*Sec[c + d*x])^(7/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/2
, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(b*Sec[c + d*x])^(4/3)*Sqrt[
Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(b\sec(c+dx))^{4/3}}\,dx = A\int \frac{1}{(b\sec(c+dx))^{4/3}}\,dx + \frac{B\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx}{b}$$

$$= \left(A\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{4/3} dx + \frac{\left(B\left(\frac{\cos(c+dx}{b}\right.\right.}{}$$

$$= -\frac{3B\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{4b^2d\sqrt{\sin^2(c+dx)}} - \frac{3A\,c}{}$$

Mathematica [A]
time = 0.15, size = 87, normalized size = 0.76

$$-\frac{3\csc(c+dx)\left(A\cos(c+dx)\,{}_2F_1\left(-\frac{2}{3},\frac{1}{2};\frac{1}{3};\sec^2(c+dx)\right)+4B\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\sec^2(c+dx)\right)\right)\sqrt{-\tan^2(c+dx)}}{4d(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*Csc[c + d*x]*(A*Cos[c + d*x]*Hypergeometric2F1[-2/3, 1/2, 1/3, Sec[c +
d*x]^2] + 4*B*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c + d*x]^2])*Sqrt[-Tan[
c + d*x]^2])/(4*d*(b*Sec[c + d*x])^(4/3))
```

Maple [F]
time = 0.13, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec(dx+c)}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int((A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(4/3), x)

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="fricas")

[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)/(b^2*sec(d*x + c)^2), x)

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec{(c + dx)}}{(b \sec{(c + dx)})^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))**(4/3),x)

[Out] Integral((A + B*sec(c + d*x))/(b*sec(c + d*x))**(4/3), x)

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="giac")

[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c))^(4/3), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+d\,x)}}{{\left(\frac{b}{\cos(c+d\,x)}\right)}^{4/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(b/cos(c + d*x))^(4/3),x)

[Out] int((A + B/cos(c + d*x))/(b/cos(c + d*x))^(4/3), x)

3.25 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=114

$$-\frac{3A\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)\sin(c+dx)}{4d(b\sec(c+dx))^{4/3}\sqrt{\sin^2(c+dx)}}-\frac{3B\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3/4*A*hypergeom([1/2, 2/3],[5/3],cos(d*x+c)^2)*sin(d*x+c)/d/(b*sec(d*x+c))
^(4/3)/(sin(d*x+c)^2)^(1/2)-3*B*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*si
n(d*x+c)/b/d/(b*sec(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {16, 3872, 3857, 2722}

$$-\frac{3A\sin(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2}{3};\frac{5}{3};\cos^2(c+dx)\right)}{4d\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{4/3}}-\frac{3B\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/2, 2/3, 5/3, Cos[c + d*x]^2]*Sin[c + d*x])/(4*d*(
b*Sec[c + d*x])^(4/3)*Sqrt[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/6, 1
/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(b*Sec[c + d*x])^(1/3)*Sqrt[Sin
[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\int \frac{A+B\sec(c+dx)}{\sqrt[3]{b\sec(c+dx)}}\,dx}{b}$$
$$= \frac{A\int \frac{1}{\sqrt[3]{b\sec(c+dx)}}\,dx}{b} + \frac{B\int (b\sec(c+dx))^{2/3}\,dx}{b^2}$$
$$= \frac{\left(A\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \sqrt[3]{\frac{\cos(c+dx)}{b}}\,dx}{b} + \frac{\left(B\left(\right.\right.}{}$$
$$= -\frac{3B\cos(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c}{b^2d\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.03, size = 91, normalized size = 0.80

$$-\frac{3\csc(c+dx)\left(2A\cos(c+dx)\,{}_2F_1\left(-\frac{1}{6},\frac{1}{2};\frac{5}{6};\sec^2(c+dx)\right)-B\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sec^2(c+dx)\right)\right)\sqrt{-\tan^2(c+dx)}}{2bd\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*Csc[c + d*x]*(2*A*Cos[c + d*x]*Hypergeometric2F1[-1/6, 1/2, 5/6, Sec[c
+ d*x]^2] - B*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c + d*x]^2])*Sqrt[-Tan[c
 + d*x]^2])/(2*b*d*(b*Sec[c + d*x])^(1/3))
```

Maple [F]
time = 0.01, size = 0, normalized size = 0.00

$$\int \frac{\sec(dx+c)\,(A+B\sec(dx+c))}{(b\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="fr
icas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)/(b^2*sec(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\sec{(c + dx)}}{(b\sec{(c + dx)})^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(b*sec(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)\,{\left(\frac{b}{\cos(c+d\,x)}\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(b/cos(c + d*x))^(4/3)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)*(b/cos(c + d*x))^(4/3)), x)
```

3.26 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=117

$$-\frac{3A\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)\sin(c+dx)}{bd\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}}+\frac{3B\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{2b^2d\sqrt{\sin^2(c+dx)}}$$

```
[Out] -3*A*hypergeom([1/6, 1/2],[7/6],cos(d*x+c)^2)*sin(d*x+c)/b/d/(b*sec(d*x+c))
^(1/3)/(sin(d*x+c)^2)^(1/2)+3/2*B*hypergeom([-1/3, 1/2],[2/3],cos(d*x+c)^2)
*(b*sec(d*x+c))^(2/3)*sin(d*x+c)/b^2/d/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {16, 3872, 3857, 2722}

$$\frac{3B\sin(c+dx)(b\sec(c+dx))^{2/3}\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)}{2b^2d\sqrt{\sin^2(c+dx)}}-\frac{3A\sin(c+dx)\,{}_2F_1\left(\frac{1}{6},\frac{1}{2};\frac{7}{6};\cos^2(c+dx)\right)}{bd\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/6, 1/2, 7/6, Cos[c + d*x]^2]*Sin[c + d*x])/(b*d*(
b*Sec[c + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2]) + (3*B*Hypergeometric2F1[-1/3,
1/2, 2/3, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(2*b^2*d*Sqr
t[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{4/3}}\,dx &= \frac{\int (b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx}{b^2}\\ &= \frac{A\int (b\sec(c+dx))^{2/3}\,dx}{b^2}+\frac{B\int (b\sec(c+dx))^{5/3}\,dx}{b^3}\\ &= \frac{\left(A\left(\frac{\cos(c+dx)}{b}\right)^{2/3}(b\sec(c+dx))^{2/3}\right)\int \frac{1}{\left(\frac{\cos(c+dx)}{b}\right)^{2/3}}\,dx}{b^2}+\frac{\left(B\left(\frac{\cos(\right.\right.}{}\\ &= \frac{3B\,{}_2F_1\left(-\frac{1}{3},\frac{1}{2};\frac{2}{3};\cos^2(c+dx)\right)(b\sec(c+dx))^{2/3}\sin(c+dx)}{2b^2d\sqrt{\sin^2(c+dx)}}-\frac{3A}{}\end{aligned}$$

Mathematica [A]
time = 0.03, size = 91, normalized size = 0.78

$$\frac{3\csc(c+dx)\left(5A\cos(c+dx)\,{}_2F_1\left(\frac{1}{3},\frac{1}{2};\frac{4}{3};\sec^2(c+dx)\right)+2B\,{}_2F_1\left(\frac{1}{2},\frac{5}{6};\frac{11}{6};\sec^2(c+dx)\right)\right)(b\sec(c+dx))^{2/3}\sqrt{-\tan^2(c+dx)}}{10b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*Csc[c + d*x]*(5*A*Cos[c + d*x]*Hypergeometric2F1[1/3, 1/2, 4/3, Sec[c +
d*x]^2] + 2*B*Hypergeometric2F1[1/2, 5/6, 11/6, Sec[c + d*x]^2])*(b*Sec[c +
 d*x])^(2/3)*Sqrt[-Tan[c + d*x]^2])/(10*b^2*d)
```

Maple [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(\sec^2(dx+c)\right)\left(A+B\sec(dx+c)\right)}{\left(b\sec(dx+c)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)/b^2, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \sec^2(c + dx)}{(b \sec(c + dx))^{\frac{4}{3}}} \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(b*sec(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c))^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^2\,{\left(\frac{b}{\cos(c+d\,x)}\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(b/cos(c + d*x))^(4/3)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(b/cos(c + d*x))^(4/3)), x)
```

3.27 $\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=167

$$\frac{3Ab\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-1-3m);\frac{1}{6}(5-3m);\cos^2(c+dx)\right)\sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\,\sin(c+dx)}{d(1+3m)\sqrt{\sin^2(c+dx)}}+\frac{3bB\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-\right.}{}$$

```
[Out] 3*A*b*hypergeom([1/2, -1/6-1/2*m],[5/6-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^m*(b
*sec(d*x+c))^(1/3)*sin(d*x+c)/d/(1+3*m)/(sin(d*x+c)^2)^(1/2)+3*b*B*hypergeo
m([1/2, -2/3-1/2*m],[1/3-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(1+m)*(b*sec(d*x+c
))^(1/3)*sin(d*x+c)/d/(4+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 167, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {20, 3872, 3857, 2722}

$$\frac{3Ab\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,\sec^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-3m-1);\frac{1}{6}(5-3m);\cos^2(c+dx)\right)}{d(3m+1)\sqrt{\sin^2(c+dx)}}+\frac{3bB\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,\sec^{m+1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-3m-4);\frac{1}{6}(2-3m);\cos^2(c+dx)\right)}{d(3m+4)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*A*b*Hypergeometric2F1[1/2, (-1 - 3*m)/6, (5 - 3*m)/6, Cos[c + d*x]^2]*Se
c[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(1 + 3*m)*Sqrt[Sin[c +
 d*x]^2]) + (3*b*B*Hypergeometric2F1[1/2, (-4 - 3*m)/6, (2 - 3*m)/6, Cos[c
+ d*x]^2]*Sec[c + d*x]^(1 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(4 +
 3*m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec^m(c+dx)(b\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx = \frac{\left(b\sqrt[3]{b\sec(c+dx)}\right)\int \sec^{\frac{4}{3}+m}(c+dx)(A+B\sec\ldots}{\sqrt[3]{\sec(c+dx)}}$$

$$= \frac{\left(Ab\sqrt[3]{b\sec(c+dx)}\right)\int \sec^{\frac{4}{3}+m}(c+dx)\,dx}{\sqrt[3]{\sec(c+dx)}} + \ldots$$

$$= \left(Ab\cos^{\frac{1}{3}+m}(c+dx)\sec^m(c+dx)\sqrt[3]{b\sec(c+dx\ldots}\right.$$

$$= \frac{3Ab\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-1-3m);\frac{1}{6}(5-3m);\cos^2(c+dx\ldots}{d(1+3m)\sqrt{\sin\ldots}}$$

Mathematica [A]
time = 0.39, size = 140, normalized size = 0.84

$$\frac{3\csc(c+dx)\left(A(7+3m)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4+3m);\frac{5}{3}+\frac{m}{2};\sec^2(c+dx)\right)+B(4+3m)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7+3m);\frac{1}{6}(13+3m);\sec^2(c+dx)\right)\right)\sec^m(c+dx)(b\sec(c+dx))^{4/3}\sqrt{-\tan^2(c+dx)}}{d(4+3m)(7+3m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*Csc[c + d*x]*(A*(7 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (4 + 3*m)/
6, 5/3 + m/2, Sec[c + d*x]^2] + B*(4 + 3*m)*Hypergeometric2F1[1/2, (7 + 3*m
)/6, (13 + 3*m)/6, Sec[c + d*x]^2])*Sec[c + d*x]^m*(b*Sec[c + d*x])^(4/3)*S
qrt[-Tan[c + d*x]^2])/(d*(4 + 3*m)*(7 + 3*m))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \left(\sec^m(dx+c)\right)\left(b\sec(dx+c)\right)^{\frac{4}{3}}\left(A+B\sec(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*sec(d*x + c)^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*b*sec(d*x + c))*(b*sec(d*x + c))^(1/3)*sec
(d*x + c)^m, x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(4/3)*sec(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\left(\frac{b}{\cos\left(c+d\,x\right)}\right)^{4/3}\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3)*(1/cos(c + d*x))^m,x)
[Out] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(4/3)*(1/cos(c + d*x))^m, x)
```

3.28 $\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=165

$$-\frac{3A\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1-3m);\frac{1}{6}(7-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)(b\sec(c+dx))^{2/3}\sin(c+dx)}{d(1-3m)\sqrt{\sin^2(c+dx)}}+\frac{3B\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}\right.}{}$$

```
[Out] -3*A*hypergeom([1/2, 1/6-1/2*m],[7/6-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-1+m)
*(b*sec(d*x+c))^(2/3)*sin(d*x+c)/d/(1-3*m)/(sin(d*x+c)^2)^(1/2)+3*B*hyperge
om([1/2, -1/3-1/2*m],[2/3-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^m*(b*sec(d*x+c))^
(2/3)*sin(d*x+c)/d/(2+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {20, 3872, 3857, 2722}

$$\frac{3B\sin(c+dx)(b\sec(c+dx))^{2/3}\sec^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-3m-2);\frac{1}{6}(4-3m);\cos^2(c+dx)\right)}{d(3m+2)\sqrt{\sin^2(c+dx)}}-\frac{3A\sin(c+dx)(b\sec(c+dx))^{2/3}\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1-3m);\frac{1}{6}(7-3m);\cos^2(c+dx)\right)}{d(1-3m)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/2, (1 - 3*m)/6, (7 - 3*m)/6, Cos[c + d*x]^2]*Sec[
c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(d*(1 - 3*m)*Sqrt[Si
n[c + d*x]^2]) + (3*B*Hypergeometric2F1[1/2, (-2 - 3*m)/6, (4 - 3*m)/6, Cos
[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c + d*x])^(2/3)*Sin[c + d*x])/(d*(2 + 3*
m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec^m(c+dx)(b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx = \frac{(b\sec(c+dx))^{2/3}\int \sec^{\frac{2}{3}+m}(c+dx)(A+B\sec(c

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^{\frac{2}{3}}(A+B\sec(c+dx))\sec^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((b*sec(c + d*x))**(2/3)*(A + B*sec(c + d*x))*sec(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{2/3}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^m,x)
[Out] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(2/3)*(1/cos(c + d*x))^m, x)
```

3.29 $\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=165

$$-\frac{3A\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2-3m);\frac{1}{6}(8-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)\sqrt[3]{b\sec(c+dx)}\,\sin(c+dx)}{d(2-3m)\sqrt{\sin^2(c+dx)}}+\frac{3B\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(\right.}{}$$

```
[Out] -3*A*hypergeom([1/2, 1/3-1/2*m],[4/3-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-1+m)
*(b*sec(d*x+c))^(1/3)*sin(d*x+c)/d/(2-3*m)/(sin(d*x+c)^2)^(1/2)+3*B*hyperge
om([1/2, -1/6-1/2*m],[5/6-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^m*(b*sec(d*x+c))^
(1/3)*sin(d*x+c)/d/(1+3*m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {20, 3872, 3857, 2722}

$$\frac{3B\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,\sec^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-3m-1);\frac{1}{6}(5-3m);\cos^2(c+dx)\right)}{d(3m+1)\sqrt{\sin^2(c+dx)}}-\frac{3A\sin(c+dx)\sqrt[3]{b\sec(c+dx)}\,\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2-3m);\frac{1}{6}(8-3m);\cos^2(c+dx)\right)}{d(2-3m)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/2, (2 - 3*m)/6, (8 - 3*m)/6, Cos[c + d*x]^2]*Sec[
c + d*x]^(-1 + m)*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(2 - 3*m)*Sqrt[Si
n[c + d*x]^2]) + (3*B*Hypergeometric2F1[1/2, (-1 - 3*m)/6, (5 - 3*m)/6, Cos
[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*Sin[c + d*x])/(d*(1 + 3*
m)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \frac{\sqrt[3]{b\sec(c+dx)}\int \sec^{\frac{1}{3}+m}(c+dx)(A+B\sec(c+ }{\sqrt[3]{\sec(c+dx)}}$$

$$= \frac{\left(A\sqrt[3]{b\sec(c+dx)}\right)\int \sec^{\frac{1}{3}+m}(c+dx)\,dx}{\sqrt[3]{\sec(c+dx)}} + \frac{\Big(B}{}$$

$$= \Big(A\cos^{\frac{1}{3}+m}(c+dx)\sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}$$

$$= -\frac{3A\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2-3m);\frac{1}{6}(8-3m);\cos^2(c+dx)\right)}{d(2-3m)\sqrt{\mathrm{si}}}$$

Mathematica [A]
time = 0.27, size = 140, normalized size = 0.85

$$\frac{3\csc(c+dx)\left(A(4+3m)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1+3m);\frac{1}{6}(7+3m);\sec^2(c+dx)\right)+B(1+3m)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4+3m);\frac{5}{3}+\frac{m}{2};\sec^2(c+dx)\right)\right)\sec^m(c+dx)\sqrt[3]{b\sec(c+dx)}\sqrt{-\tan^2(c+dx)}}{d(1+3m)(4+3m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*Csc[c + d*x]*(A*(4 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + 3*m)/
6, (7 + 3*m)/6, Sec[c + d*x]^2] + B*(1 + 3*m)*Hypergeometric2F1[1/2, (4 + 3
*m)/6, 5/3 + m/2, Sec[c + d*x]^2])*Sec[c + d*x]^m*(b*Sec[c + d*x])^(1/3)*Sq
rt[-Tan[c + d*x]^2])/(d*(1 + 3*m)*(4 + 3*m))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \left(\sec^m(dx+c)\right)\left(b\sec(dx+c)\right)^{\frac{1}{3}}\left(A+B\sec(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{b\sec{(c+dx)}}\,(A+B\sec{(c+dx)})\sec^m{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**(1/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((b*sec(c + d*x))**(1/3)*(A + B*sec(c + d*x))*sec(c + d*x)**m, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)*sec(d*x + c)^m, x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^{1/3}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(1/3)*(1/cos(c + d*x))^m,x)
[Out] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^(1/3)*(1/cos(c + d*x))^m, x)
```

3.30 $\int \frac{\sec^m(c+dx)(A+B\sec(c+dx))}{\sqrt[3]{b\sec(c+dx)}}\,dx$

Optimal. Leaf size=165

$$-\frac{3A\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4-3m);\frac{1}{6}(10-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)\sin(c+dx)}{d(4-3m)\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}}-\frac{3B\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1-3m);\frac{1}{6}(7-\right.}{d(1-3m)\sqrt[3]{}}$$

```
[Out] -3*A*hypergeom([1/2, 2/3-1/2*m],[5/3-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-1+m)
*sin(d*x+c)/d/(4-3*m)/(b*sec(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3*B*hyperge
om([1/2, 1/6-1/2*m],[7/6-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^m*sin(d*x+c)/d/(1-
3*m)/(b*sec(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {20, 3872, 3857, 2722}

$$-\frac{3A\sin(c+dx)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4-3m);\frac{1}{6}(10-3m);\cos^2(c+dx)\right)}{d(4-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}-\frac{3B\sin(c+dx)\sec^m(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1-3m);\frac{1}{6}(7-3m);\cos^2(c+dx)\right)}{d(1-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^m*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/2, (4 - 3*m)/6, (10 - 3*m)/6, Cos[c + d*x]^2]*Sec
[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(4 - 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt[S
in[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/2, (1 - 3*m)/6, (7 - 3*m)/6, Cos
[c + d*x]^2]*Sec[c + d*x]^m*Sin[c + d*x])/(d*(1 - 3*m)*(b*Sec[c + d*x])^(1/
3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{\sec^m(c+dx)(A+B\sec(c+dx))}{\sqrt[3]{b\sec(c+dx)}}\,dx = \frac{\sqrt[3]{\sec(c+dx)}\int \sec^{-\frac{1}{3}+m}(c+dx)(A+B\sec(c+dx))\,dx}{\sqrt[3]{b\sec(c+dx)}}$$
$$= \frac{\left(A\sqrt[3]{\sec(c+dx)}\right)\int \sec^{-\frac{1}{3}+m}(c+dx)\,dx}{\sqrt[3]{b\sec(c+dx)}} + \frac{\left(B\sqrt[3]{\sec(c+dx)}\right)}{\sqrt[3]{b\sec(}}$$
$$= \frac{\left(A\cos^{\frac{2}{3}+m}(c+dx)\sec^{1+m}(c+dx)\right)\int \cos^{\frac{1}{3}-m}(c+dx)\,dx}{\sqrt[3]{b\sec(c+dx)}} + \frac{(B}{}$$
$$= -\frac{3A\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4-3m);\frac{1}{6}(10-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+d}{d(4-3m)\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.25, size = 140, normalized size = 0.85

$$\frac{3\csc(c+dx)\left(A(2+3m)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-1+3m);\frac{1}{6}(5+3m);\sec^2(c+dx)\right)+B(-1+3m)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2+3m);\frac{1}{6}(8+3m);\sec^2(c+dx)\right)\right)\sec^m(c+dx)\sqrt{-\tan^2(c+dx)}}{d(-1+3m)(2+3m)\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^m*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Csc[c + d*x]*(A*(2 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + 3*m)
/6, (5 + 3*m)/6, Sec[c + d*x]^2] + B*(-1 + 3*m)*Hypergeometric2F1[1/2, (2 +
 3*m)/6, (8 + 3*m)/6, Sec[c + d*x]^2])*Sec[c + d*x]^m*Sqrt[-Tan[c + d*x]^2]
)/(d*(-1 + 3*m)*(2 + 3*m)*(b*Sec[c + d*x])^(1/3))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{(\sec^m(dx+c))(A+B\sec(dx+c))}{(b\sec(dx+c))^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/3),x)
```

```
[Out] int(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(1/3), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/3),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^m/(b*sec(
d*x + c)), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\sec^{m}{(c + dx)}}{\sqrt[3]{b\sec{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**m/(b*sec(c + d*x))**(1/3), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(1/3),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^m}{{\left(\frac{b}{\cos(c+d\,x)}\right)}^{1/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^m)/(b/cos(c + d*x))^(1/3),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^m)/(b/cos(c + d*x))^(1/3), x)
```

3.31 $\int \frac{\sec^m(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=165

$$-\frac{3A\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5-3m);\frac{1}{6}(11-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)\sin(c+dx)}{d(5-3m)(b\sec(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}-\frac{3B\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2-3m);\frac{1}{6}(8-\right.}{d(2-3m)(b\,\mathrm{s}}$$

```
[Out] -3*A*hypergeom([1/2, 5/6-1/2*m],[11/6-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-1+m
)*sin(d*x+c)/d/(5-3*m)/(b*sec(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)-3*B*hyperg
eom([1/2, 1/3-1/2*m],[4/3-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^m*sin(d*x+c)/d/(2
-3*m)/(b*sec(d*x+c))^(2/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.09, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {20, 3872, 3857, 2722}

$$-\frac{3A\sin(c+dx)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5-3m);\frac{1}{6}(11-3m);\cos^2(c+dx)\right)}{d(5-3m)\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{2/3}}-\frac{3B\sin(c+dx)\sec^{m}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(2-3m);\frac{1}{6}(8-3m);\cos^2(c+dx)\right)}{d(2-3m)\sqrt{\sin^2(c+dx)}\,(b\sec(c+dx))^{2/3}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^m*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/2, (5 - 3*m)/6, (11 - 3*m)/6, Cos[c + d*x]^2]*Sec
[c + d*x]^(-1 + m)*Sin[c + d*x])/(d*(5 - 3*m)*(b*Sec[c + d*x])^(2/3)*Sqrt[S
in[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/2, (2 - 3*m)/6, (8 - 3*m)/6, Cos
[c + d*x]^2]*Sec[c + d*x]^m*Sin[c + d*x])/(d*(2 - 3*m)*(b*Sec[c + d*x])^(2/
3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] && !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^m(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{2/3}}\,dx &= \frac{\sec^{\frac{2}{3}}(c+dx)\int \sec^{-\frac{2}{3}+m}(c+dx)(A+B\sec(c+dx))\,dx}{(b\sec(c+dx))^{2/3}} \\
&= \frac{\left(A\sec^{\frac{2}{3}}(c+dx)\right)\int \sec^{-\frac{2}{3}+m}(c+dx)\,dx}{(b\sec(c+dx))^{2/3}} + \frac{\left(B\sec^{\frac{2}{3}}(c+dx)\right)\int \mathrm{s}}{(b\sec(c+}\\
&= \frac{\left(A\cos^{\frac{1}{3}+m}(c+dx)\sec^{1+m}(c+dx)\right)\int \cos^{\frac{2}{3}-m}(c+dx)\,dx}{(b\sec(c+dx))^{2/3}} + \frac{\left(B\right.}{} \\
&= -\frac{3A\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(5-3m);\frac{1}{6}(11-3m);\cos^2(c+dx)\right)\sec^{-1+m}(c+d}{d(5-3m)(b\sec(c+dx))^{2/3}\sqrt{\sin^2(c+dx)}}
\end{aligned}
$$

Mathematica [A]
time = 0.23, size = 140, normalized size = 0.85

$$
\frac{3\csc(c+dx)\left(A(1+3m)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-2+3m);\frac{1}{6}(4+3m);\sec^2(c+dx)\right)+B(-2+3m)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(1+3m);\frac{1}{6}(7+3m);\sec^2(c+dx)\right)\right)\sec^m(c+dx)\sqrt{-\tan^2(c+dx)}}{d(-2+3m)(1+3m)(b\sec(c+dx))^{2/3}}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^m*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*Csc[c + d*x]*(A*(1 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (-2 + 3*m)
/6, (4 + 3*m)/6, Sec[c + d*x]^2] + B*(-2 + 3*m)*Hypergeometric2F1[1/2, (1 +
 3*m)/6, (7 + 3*m)/6, Sec[c + d*x]^2])*Sec[c + d*x]^m*Sqrt[-Tan[c + d*x]^2]
)/(d*(-2 + 3*m)*(1 + 3*m)*(b*Sec[c + d*x])^(2/3))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$
\int \frac{(\sec^m(dx+c))\,(A+B\sec(dx+c))}{(b\sec(dx+c))^{\frac{2}{3}}}\,dx
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

```
[Out] int(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(2/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(1/3)*sec(d*x + c)^m/(b*sec(
d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec^m(c + dx)}{(b\sec(c + dx))^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**m/(b*sec(c + d*x))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(2/3),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^m}{{\left(\frac{b}{\cos(c+d\,x)}\right)}^{2/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^m)/(b/cos(c + d*x))^(2/3),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^m)/(b/cos(c + d*x))^(2/3), x)
```

3.32 $\int \frac{\sec^m(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=173

$$-\frac{3A\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7-3m);\frac{1}{6}(13-3m);\cos^2(c+dx)\right)\sec^{-2+m}(c+dx)\sin(c+dx)}{bd(7-3m)\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}}-\frac{3B\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4-3m);\frac{1}{6}(10-\ldots}{bd(4-3m)\ldots}$$

```
[Out] -3*A*hypergeom([1/2, 7/6-1/2*m],[13/6-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-2+m
)*sin(d*x+c)/b/d/(7-3*m)/(b*sec(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)-3*B*hype
rgeom([1/2, 2/3-1/2*m],[5/3-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-1+m)*sin(d*x+
c)/b/d/(4-3*m)/(b*sec(d*x+c))^(1/3)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {20, 3872, 3857, 2722}

$$-\frac{3A\sin(c+dx)\sec^{m-2}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7-3m);\frac{1}{6}(13-3m);\cos^2(c+dx)\right)}{bd(7-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}-\frac{3B\sin(c+dx)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(4-3m);\frac{1}{6}(10-3m);\cos^2(c+dx)\right)}{bd(4-3m)\sqrt{\sin^2(c+dx)}\sqrt[3]{b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^m*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (-3*A*Hypergeometric2F1[1/2, (7 - 3*m)/6, (13 - 3*m)/6, Cos[c + d*x]^2]*Sec
[c + d*x]^(-2 + m)*Sin[c + d*x])/(b*d*(7 - 3*m)*(b*Sec[c + d*x])^(1/3)*Sqrt
[Sin[c + d*x]^2]) - (3*B*Hypergeometric2F1[1/2, (4 - 3*m)/6, (10 - 3*m)/6,
Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(b*d*(4 - 3*m)*(b*Sec[c
 + d*x])^(1/3)*Sqrt[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{\sec^m(c+dx)(A+B\sec(c+dx))}{(b\sec(c+dx))^{4/3}}\,dx = \frac{\sqrt[3]{\sec(c+dx)}\int \sec^{-\frac{4}{3}+m}(c+dx)(A+B\sec(c+dx))\,dx}{b\sqrt[3]{b\sec(c+dx)}}$$
$$= \frac{\left(A\sqrt[3]{\sec(c+dx)}\right)\int \sec^{-\frac{4}{3}+m}(c+dx)\,dx}{b\sqrt[3]{b\sec(c+dx)}} + \frac{\left(B\sqrt[3]{\sec(c+dx)}\right)}{b\sqrt[3]{b\sec}}$$
$$= \frac{\left(A\cos^{\frac{2}{3}+m}(c+dx)\sec^{1+m}(c+dx)\right)\int \cos^{\frac{4}{3}-m}(c+dx)\,dx}{b\sqrt[3]{b\sec(c+dx)}} + \frac{(B}{}$$
$$= -\frac{3A\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(7-3m);\frac{1}{6}(13-3m);\cos^2(c+dx)\right)\sec^{-2+m}(c+d}{bd(7-3m)\sqrt[3]{b\sec(c+dx)}\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.33, size = 140, normalized size = 0.81

$$\frac{3\csc(c+dx)\left(A(-1+3m)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-4+3m);\frac{1}{6}(2+3m);\sec^2(c+dx)\right)+B(-4+3m)\,{}_2F_1\left(\frac{1}{2},\frac{1}{6}(-1+3m);\frac{1}{6}(5+3m);\sec^2(c+dx)\right)\right)\sec^m(c+dx)\sqrt{-\tan^2(c+dx)}}{d(-4+3m)(-1+3m)(b\sec(c+dx))^{4/3}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^m*(A + B*Sec[c + d*x]))/(b*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*Csc[c + d*x]*(A*(-1 + 3*m)*Cos[c + d*x]*Hypergeometric2F1[1/2, (-4 + 3*m
)/6, (2 + 3*m)/6, Sec[c + d*x]^2] + B*(-4 + 3*m)*Hypergeometric2F1[1/2, (-1
 + 3*m)/6, (5 + 3*m)/6, Sec[c + d*x]^2])*Sec[c + d*x]^m*Sqrt[-Tan[c + d*x]^
2])/(d*(-4 + 3*m)*(-1 + 3*m)*(b*Sec[c + d*x])^(4/3))
```

Maple [F]
time = 0.18, size = 0, normalized size = 0.00

$$\int \frac{\left(\sec^m(dx+c)\right)\left(A+B\sec(dx+c)\right)}{\left(b\sec(dx+c)\right)^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

```
[Out] int(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(4/3), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^(2/3)*sec(d*x + c)^m/(b^2*se
c(d*x + c)^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\sec^{m}{(c + dx)}}{(b\sec{(c + dx)})^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(A+B*sec(d*x+c))/(b*sec(d*x+c))**(4/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**m/(b*sec(c + d*x))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(A+B*sec(d*x+c))/(b*sec(d*x+c))^(4/3),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^m/(b*sec(d*x + c))^(4/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^m}{{\left(\frac{b}{\cos(c+d\,x)}\right)}^{4/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^m)/(b/cos(c + d*x))^(4/3),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^m)/(b/cos(c + d*x))^(4/3), x)
```

3.33 $\int \sec^m(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=172

$$-\frac{A\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-m-n);\frac{1}{2}(3-m-n);\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)(b\sec(c+dx))^n\sin(c+dx)}{d(1-m-n)\sqrt{\sin^2(c+dx)}}+\frac{B\,{}_2F_1\left(\frac{1}{2}\right.}{}$$

```
[Out] -A*hypergeom([1/2, 1/2-1/2*m-1/2*n],[3/2-1/2*m-1/2*n],cos(d*x+c)^2)*sec(d*x
+c)^(-1+m)*(b*sec(d*x+c))^n*sin(d*x+c)/d/(1-m-n)/(sin(d*x+c)^2)^(1/2)+B*hyp
ergeom([1/2, -1/2*m-1/2*n],[1-1/2*m-1/2*n],cos(d*x+c)^2)*sec(d*x+c)^m*(b*se
c(d*x+c))^n*sin(d*x+c)/d/(n+m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {20, 3872, 3857, 2722}

$$\frac{B\sin(c+dx)\sec^m(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-n);\frac{1}{2}(-m-n+2);\cos^2(c+dx)\right)}{d(m+n)\sqrt{\sin^2(c+dx)}}-\frac{A\sin(c+dx)\sec^{m-1}(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-m-n+1);\frac{1}{2}(-m-n+3);\cos^2(c+dx)\right)}{d(-m-n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] -((A*Hypergeometric2F1[1/2, (1 - m - n)/2, (3 - m - n)/2, Cos[c + d*x]^2]*S
ec[c + d*x]^(-1 + m)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - m - n)*Sqrt[S
in[c + d*x]^2])) + (B*Hypergeometric2F1[1/2, (-m - n)/2, (2 - m - n)/2, Cos
[c + d*x]^2]*Sec[c + d*x]^m*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(m + n)*Sqr
t[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec^m(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{m+n}(c+dx$$
$$= \left(A\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{m+n}(c+$$
$$= \left(A\cos^{m+n}(c+dx)\sec^m(c+dx)(b\sec(c+dx))^n\right)$$
$$= -\frac{A\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1-m-n);\frac{1}{2}(3-m-n);\cos^2(c+\right.}{d(1-m-n)}$$

Mathematica [A]
time = 0.22, size = 126, normalized size = 0.73

$$\frac{\csc(c+dx)\left(A(1+m+n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m+n}{2};\frac{1}{2}(2+m+n);\sec^2(c+dx)\right)+B(m+n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(1+m+n);\frac{1}{2}(3+m+n);\sec^2(c+dx)\right)\right)\sec^m(c+dx)(b\sec(c+dx))^n\sqrt{-\tan^2(c+dx)}}{d(m+n)(1+m+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]*(A*(1 + m + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (m + n)/2,
 (2 + m + n)/2, Sec[c + d*x]^2] + B*(m + n)*Hypergeometric2F1[1/2, (1 + m +
 n)/2, (3 + m + n)/2, Sec[c + d*x]^2])*Sec[c + d*x]^m*(b*Sec[c + d*x])^n*Sq
rt[-Tan[c + d*x]^2])/(d*(m + n)*(1 + m + n))
```

Maple [F]
time = 0.20, size = 0, normalized size = 0.00

$$\int \left(\sec^m(dx+c)\right)(b\sec(dx+c))^n\left(A+B\sec(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b \sec (c+dx))^n (A+B \sec (c+dx)) \sec^m (c+dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))*sec(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\left(\frac{b}{\cos\left(c+d\,x\right)}\right)^n\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^n*(1/cos(c + d*x))^m,x)
[Out] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^n*(1/cos(c + d*x))^m, x)
```

3.34 $\int \sec^2(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=143

$$\frac{A\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-n);\frac{1-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^{1+n}\sin(c+dx)}{bd(1+n)\sqrt{\sin^2(c+dx)}}+\frac{B\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2-n);-\frac{n}{2};\cos^2(c+dx)\right.}{b^2d(2+n)\sqrt{\sin}}$$

```
[Out] A*hypergeom([1/2, -1/2-1/2*n],[1/2-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^(1+n
)*sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)+B*hypergeom([1/2, -1-1/2*n],[-1
/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^(2+n)*sin(d*x+c)/b^2/d/(2+n)/(sin(d*x+c)
^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {16, 3872, 3857, 2722}

$$\frac{A\sin(c+dx)(b\sec(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-n-1);\frac{1-n}{2};\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}+\frac{B\sin(c+dx)(b\sec(c+dx))^{n+2}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-n-2);-\frac{n}{2};\cos^2(c+dx)\right)}{b^2d(n+2)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (A*Hypergeometric2F1[1/2, (-1 - n)/2, (1 - n)/2, Cos[c + d*x]^2]*(b*Sec[c +
 d*x])^(1 + n)*Sin[c + d*x])/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2]) + (B*Hyperg
eometric2F1[1/2, (-2 - n)/2, -1/2*n, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(2 +
n)*Sin[c + d*x])/(b^2*d*(2 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
```

```
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx &= \frac{\int (b\sec(c+dx))^{2+n}(A+B\sec(c+dx))\,dx}{b^2}\\ &= \frac{A\int (b\sec(c+dx))^{2+n}\,dx}{b^2}+\frac{B\int (b\sec(c+dx))^{3+n}}{b^3}\\ &= \frac{\left(A\left(\frac{\cos(c+dx)}{b}\right)^n(b\sec(c+dx))^n\right)\int\left(\frac{\cos(c+dx)}{b}\right)^{-2-}}{b^2}\\ &= \frac{A\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-n);\frac{1-n}{2};\cos^2(c+dx)\right)(b\sec(c+}{bd(1+n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 119, normalized size = 0.83

$$\frac{\csc(c+dx)\sec(c+dx)(b\sec(c+dx))^n\left(A(3+n)\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\sec^2(c+dx)\right)+B(2+n)\,{}_2F_1\left(\frac{1}{2},\frac{3+n}{2};\frac{5+n}{2};\sec^2(c+dx)\right)\sec(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{d(2+n)(3+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]*Sec[c + d*x]*(b*Sec[c + d*x])^n*(A*(3 + n)*Hypergeometric2F1[
1/2, (2 + n)/2, (4 + n)/2, Sec[c + d*x]^2] + B*(2 + n)*Hypergeometric2F1[1/
2, (3 + n)/2, (5 + n)/2, Sec[c + d*x]^2]*Sec[c + d*x])*Sqrt[-Tan[c + d*x]^2
])/(d*(2 + n)*(3 + n))
```

Maple [F]
time = 0.12, size = 0, normalized size = 0.00

$$\int \left(\sec^2(dx+c)\right)(b\sec(dx+c))^n(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*(b*sec(d*x + c))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^n (A+B\sec(c+dx))\sec^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))*sec(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(\frac{b}{\cos(c+d\,x)}\right)^n}{\cos\left(c+d\,x\right)^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^n)/cos(c + d*x)^2,x)
[Out] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^n)/cos(c + d*x)^2, x)
```

3.35 $\int \sec(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=136

$$\frac{A\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{dn\sqrt{\sin^2(c+dx)}}+\frac{B\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1-n);\frac{1-n}{2};\cos^2(c+dx)\right)(b\sec(c}{bd(1+n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] A*hypergeom([1/2, -1/2*n],[1-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^n*sin(d*x+
c)/d/n/(sin(d*x+c)^2)^(1/2)+B*hypergeom([1/2, -1/2-1/2*n],[1/2-1/2*n],cos(d
*x+c)^2)*(b*sec(d*x+c))^(1+n)*sin(d*x+c)/b/d/(1+n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {16, 3872, 3857, 2722}

$$\frac{A\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}+\frac{B\sin(c+dx)(b\sec(c+dx))^{n+1}\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-n-1);\frac{1-n}{2};\cos^2(c+dx)\right)}{bd(n+1)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (A*Hypergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d*x
])^n*Sin[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2]) + (B*Hypergeometric2F1[1/2, (
-1 - n)/2, (1 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d*x])^(1 + n)*Sin[c + d*x]
)/(b*d*(1 + n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx = \frac{\int (b\sec(c+dx))^{1+n}(A+B\sec(c+dx))\,dx}{b}$$
$$= \frac{A\int (b\sec(c+dx))^{1+n}\,dx}{b} + \frac{B\int (b\sec(c+dx))^{2+n}\,c}{b^2}$$
$$= \frac{\left(A\left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{-1-n}}{b}$$
$$= \frac{A\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^n \sin}{dn\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.26, size = 119, normalized size = 0.88

$$\frac{\csc(c+dx)\left(A(2+n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sec^2(c+dx)\right)+B(1+n)\,{}_2F_1\left(\frac{1}{2},\frac{2+n}{2};\frac{4+n}{2};\sec^2(c+dx)\right)\right)\sec(c+dx)(b\sec(c+dx))^n\sqrt{-\tan^2(c+dx)}}{d(1+n)(2+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]*(A*(2 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (1 + n)/2, (3
+ n)/2, Sec[c + d*x]^2] + B*(1 + n)*Hypergeometric2F1[1/2, (2 + n)/2, (4 +
n)/2, Sec[c + d*x]^2])*Sec[c + d*x]*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2
])/(d*(1 + n)*(2 + n))
```

Maple [F]
time = 0.10, size = 0, normalized size = 0.00

$$\int \sec(dx+c)\,(b\sec(dx+c))^n\,(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="maxima
")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="fricas
")
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x + c))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^n (A+B\sec(c+dx))\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)),x)
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{b}{\cos(c+d\,x)}\right)}^n}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^n)/cos(c + d*x),x)
[Out] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^n)/cos(c + d*x), x)
```

3.36 $\int (b\sec(c+dx))^n(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=137

$$-\frac{Ab\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^{-1+n}\sin(c+dx)}{d(1-n)\sqrt{\sin^2(c+dx)}}+\frac{B\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(c+dx)\right)(b\sec(c}{dn\sqrt{\sin^2(c+dx)}}$$

```
[Out] -A*b*hypergeom([1/2, 1/2-1/2*n],[3/2-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^(-
1+n)*sin(d*x+c)/d/(1-n)/(sin(d*x+c)^2)^(1/2)+B*hypergeom([1/2, -1/2*n],[1-1
/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^n*sin(d*x+c)/d/n/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3872, 3857, 2722}

$$\frac{B\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},-\frac{n}{2};\frac{2-n}{2};\cos^2(c+dx)\right)}{dn\sqrt{\sin^2(c+dx)}}-\frac{Ab\sin(c+dx)(b\sec(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] -((A*b*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[c + d*x]^2]*(b*Sec[
c + d*x])^(-1 + n)*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])) + (B*Hyp
ergeometric2F1[1/2, -1/2*n, (2 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d*x])^n*S
in[c + d*x])/(d*n*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int (b\sec(c+dx))^n(A+B\sec(c+dx))\,dx = A\int (b\sec(c+dx))^n\,dx + \frac{B\int (b\sec(c+dx))^{1+n}\,dx}{b}$$

$$= \left(A\left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n\right)\int \left(\frac{\cos(c+dx)}{b}\right)^{-n}\,dx$$

$$= -\frac{A\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^n \ldots}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.16, size = 107, normalized size = 0.78

$$\frac{\csc(c+dx)\left(A(1+n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\sec^2(c+dx)\right)+Bn\,{}_2F_1\left(\frac{1}{2},\frac{1+n}{2};\frac{3+n}{2};\sec^2(c+dx)\right)\right)(b\sec(c+dx))^n\sqrt{-\tan^2(c+dx)}}{dn(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]*(A*(1 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, n/2, (2 + n)/2
, Sec[c + d*x]^2] + B*n*Hypergeometric2F1[1/2, (1 + n)/2, (3 + n)/2, Sec[c
+ d*x]^2])*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*n*(1 + n))
```

Maple [F]
time = 0.16, size = 0, normalized size = 0.00

$$\int (b\sec(dx+c))^n\,(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

```
[Out] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="fricas")
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^n (A+B\sec(c+dx))\ dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n*(A+B*sec(d*x+c)),x)
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{b}{\cos(c+d\,x)}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^n,x)
[Out] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^n, x)
```

3.37 $\int \cos(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=151

$$-\frac{Ab^2\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^{-2+n}\sin(c+dx)}{d(2-n)\sqrt{\sin^2(c+dx)}}-\frac{bB\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(c+dx)\right)(b\se$$

```
[Out] -A*b^2*hypergeom([1/2, 1-1/2*n],[2-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^(-2+
n)*sin(d*x+c)/d/(2-n)/(sin(d*x+c)^2)^(1/2)-b*B*hypergeom([1/2, 1/2-1/2*n],[
3/2-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^(-1+n)*sin(d*x+c)/d/(1-n)/(sin(d*x+
c)^2)^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.148$, Rules used = {16, 3872, 3857, 2722}

$$-\frac{Ab^2\sin(c+dx)(b\sec(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}}-\frac{bB\sin(c+dx)(b\sec(c+dx))^{n-1}\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(c+dx)\right)}{d(1-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] -((A*b^2*Hypergeometric2F1[1/2, (2 - n)/2, (4 - n)/2, Cos[c + d*x]^2]*(b*Se
c[c + d*x])^(-2 + n)*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2])) - (b*B
*Hypergeometric2F1[1/2, (1 - n)/2, (3 - n)/2, Cos[c + d*x]^2]*(b*Sec[c + d*
x])^(-1 + n)*Sin[c + d*x])/(d*(1 - n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
```

```
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx &= b\int (b\sec(c+dx))^{-1+n}(A+B\sec(c+dx))\,dx\\ &= (Ab)\int (b\sec(c+dx))^{-1+n}\,dx + B\int (b\sec(c+dx))^{n}\\ &= \left(Ab\left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n\right)\int \left(\frac{\cos(c+}{b}\right.\\ &= -\frac{B\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-n}{2};\frac{3-n}{2};\cos^2(c+dx)\right)(b\sec(}{d(1-n)\sqrt{\sin^2(c+dx)}}\end{aligned}$$

Mathematica [A]
time = 0.16, size = 107, normalized size = 0.71

$$\frac{\cot(c+dx)\left(An\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+n);\frac{1+n}{2};\sec^2(c+dx)\right)+B(-1+n)\,{}_2F_1\left(\frac{1}{2},\frac{n}{2};\frac{2+n}{2};\sec^2(c+dx)\right)\right)(b\sec(c+dx))^n\sqrt{-\tan^2(c+dx)}}{d(-1+n)n}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
[Out] (Cot[c + d*x]*(A*n*Cos[c + d*x]*Hypergeometric2F1[1/2, (-1 + n)/2, (1 + n)/
2, Sec[c + d*x]^2] + B*(-1 + n)*Hypergeometric2F1[1/2, n/2, (2 + n)/2, Sec[
c + d*x]^2])*(b*Sec[c + d*x])^n*Sqrt[-Tan[c + d*x]^2])/(d*(-1 + n)*n)
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \cos\left(dx+c\right)\left(b\sec\left(dx+c\right)\right)^{n}\left(A+B\sec\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
[Out] int(cos(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="maxima
")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*cos(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="fricas
")
```

```
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*(b*sec(d*x + c))^n,
 x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^n (A+B\sec(c+dx))\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))*cos(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)\,\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{b}{\cos(c+d\,x)}\right)}^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(b/cos(c + d*x))^n,x)
[Out] int(cos(c + d*x)*(A + B/cos(c + d*x))*(b/cos(c + d*x))^n, x)
```

3.38 $\int \cos^2(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=153

$$-\frac{Ab^3\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^{-3+n}\sin(c+dx)}{d(3-n)\sqrt{\sin^2(c+dx)}}-\frac{b^2B\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(c+dx)\right)(b\sec(c+dx))^{-2+n}\sin(c+dx)}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

```
[Out] -A*b^3*hypergeom([1/2, 3/2-1/2*n],[5/2-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^
(-3+n)*sin(d*x+c)/d/(3-n)/(sin(d*x+c)^2)^(1/2)-b^2*B*hypergeom([1/2, 1-1/2*
n],[2-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^(-2+n)*sin(d*x+c)/d/(2-n)/(sin(d*
x+c)^2)^(1/2)
```

Rubi [A]

time = 0.11, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {16, 3872, 3857, 2722}

$$-\frac{Ab^3\sin(c+dx)(b\sec(c+dx))^{n-3}\,{}_2F_1\left(\frac{1}{2},\frac{3-n}{2};\frac{5-n}{2};\cos^2(c+dx)\right)}{d(3-n)\sqrt{\sin^2(c+dx)}}-\frac{b^2B\sin(c+dx)(b\sec(c+dx))^{n-2}\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(c+dx)\right)}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] -((A*b^3*Hypergeometric2F1[1/2, (3 - n)/2, (5 - n)/2, Cos[c + d*x]^2]*(b*Se
c[c + d*x])^(-3 + n)*Sin[c + d*x])/(d*(3 - n)*Sqrt[Sin[c + d*x]^2])) - (b^2
*B*Hypergeometric2F1[1/2, (2 - n)/2, (4 - n)/2, Cos[c + d*x]^2]*(b*Sec[c +
d*x])^(-2 + n)*Sin[c + d*x])/(d*(2 - n)*Sqrt[Sin[c + d*x]^2])
```

Rule 16

```
Int[(u_.)*(v_)^(m_.)*((b_)*(v_))^(n_), x_Symbol] :> Dist[1/b^m, Int[u*(b*v)
^(m + n), x], x] /; FreeQ[{b, n}, x] && IntegerQ[m]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
```

```
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx = b^2\int (b\sec(c+dx))^{-2+n}(A+B\sec(c+dx))\,dx$$
$$= \left(Ab^2\right)\int (b\sec(c+dx))^{-2+n}\,dx + (bB)\int (b\sec(c+$$
$$= \left(Ab^2\left(\frac{\cos(c+dx)}{b}\right)^n (b\sec(c+dx))^n\right)\int \left(\frac{\cos(c-}{b}\right.$$
$$= -\frac{B\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2-n}{2};\frac{4-n}{2};\cos^2(c+dx)\right)(b\,\mathrm{se}}{d(2-n)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.31, size = 114, normalized size = 0.75

$$\frac{b\cot(c+dx)\left(A(-1+n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-2+n);\frac{n}{2};\sec^2(c+dx)\right)+B(-2+n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{2}(-1+n);\frac{1+n}{2};\sec^2(c+dx)\right)\right)(b\sec(c+dx))^{-1+n}\sqrt{-\tan^2(c+dx)}}{d(-2+n)(-1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (b*Cot[c + d*x]*(A*(-1 + n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (-2 + n)/2,
 n/2, Sec[c + d*x]^2] + B*(-2 + n)*Hypergeometric2F1[1/2, (-1 + n)/2, (1 +
n)/2, Sec[c + d*x]^2])*(b*Sec[c + d*x])^(-1 + n)*Sqrt[-Tan[c + d*x]^2])/(d*
(-2 + n)*(-1 + n))
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \left(\cos^2(dx+c)\right)(b\sec(dx+c))^n\left(A+B\sec(dx+c)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

```
[Out] int(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*cos(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*(b*sec(d*x + c)
)^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec (c+dx))^n (A+B\sec (c+dx))\cos^2 (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))*cos(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(b/cos(c + d*x))^n,x)
[Out] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(b/cos(c + d*x))^n, x)
```

3.39 $\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=163

$$\frac{2A\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1-2n);\frac{1}{4}(3-2n);\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\,(b\sec(c+dx))^n\sin(c+dx)}{d(1+2n)\sqrt{\sin^2(c+dx)}}+\frac{2B\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-\right.}{}$$

```
[Out] 2*B*hypergeom([1/2, -3/4-1/2*n],[1/4-1/2*n],cos(d*x+c)^2)*sec(d*x+c)^(3/2)*
(b*sec(d*x+c))^n*sin(d*x+c)/d/(3+2*n)/(sin(d*x+c)^2)^(1/2)+2*A*hypergeom([1
/2, -1/4-1/2*n],[3/4-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^n*sin(d*x+c)*sec(d
*x+c)^(1/2)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {20, 3872, 3857, 2722}

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}\,(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-2n-1);\frac{1}{4}(3-2n);\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}+\frac{2B\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-2n-3);\frac{1}{4}(1-2n);\cos^2(c+dx)\right)}{d(2n+3)\sqrt{\sin^2(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*A*Hypergeometric2F1[1/2, (-1 - 2*n)/4, (3 - 2*n)/4, Cos[c + d*x]^2]*Sqrt
[Sec[c + d*x]]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt[Sin[c + d
*x]^2]) + (2*B*Hypergeometric2F1[1/2, (-3 - 2*n)/4, (1 - 2*n)/4, Cos[c + d*
x]^2]*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 + 2*n)*Sqrt
[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{\frac{3}{2}+n}(c+dx)($$

$$= \left(A\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{\frac{3}{2}+n}(c+dx$$

$$= \left(A\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\sec(c+dx)}\,(b\sec(c+dx))^n\right)$$

$$= \frac{2A\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1-2n);\frac{1}{4}(3-2n);\cos^2(c+dx)\right)\sqrt{s}}{d(1+2n)\sqrt{\sin^2(c}}$$

Mathematica [A]
time = 0.27, size = 140, normalized size = 0.86

$$\frac{2\csc(c+dx)\left(A(5+2n)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3+2n);\frac{1}{4}(7+2n);\sec^2(c+dx)\right)+B(3+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5+2n);\frac{1}{4}(9+2n);\sec^2(c+dx)\right)\right)\sec^{\frac{3}{2}}(c+dx)(b\sec(c+dx))^n\sqrt{-\tan^2(c+dx)}}{d(3+2n)(5+2n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*Csc[c + d*x]*(A*(5 + 2*n)*Cos[c + d*x]*Hypergeometric2F1[1/2, (3 + 2*n)/
4, (7 + 2*n)/4, Sec[c + d*x]^2] + B*(3 + 2*n)*Hypergeometric2F1[1/2, (5 + 2
*n)/4, (9 + 2*n)/4, Sec[c + d*x]^2])*Sec[c + d*x]^(3/2)*(b*Sec[c + d*x])^n*
Sqrt[-Tan[c + d*x]^2])/(d*(3 + 2*n)*(5 + 2*n))
```

Maple [F]
time = 0.26, size = 0, normalized size = 0.00

$$\int \left(\sec^{\frac{3}{2}}(dx+c)\right)(b\sec(dx+c))^n\left(A+B\sec(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*(b*sec(d*x + c))^n*sqrt(sec(d*
x + c)), x)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(b*sec(d*x+c))**n*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(b*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^n\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^n*(1/cos(c + d*x))^(3/2),x)
[Out] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^n*(1/cos(c + d*x))^(3/2), x)
```

3.40 $\int \sqrt{\sec(c+dx)}\,(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=163

$$-\frac{2A\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1-2n);\frac{1}{4}(5-2n);\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(1-2n)\sqrt{\sec(c+dx)}\sqrt{\sin^2(c+dx)}}+\frac{2B\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1-2n);\frac{1}{4}(3-\right.}{}$$

```
[Out] -2*A*hypergeom([1/2, 1/4-1/2*n],[5/4-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^n*
sin(d*x+c)/d/(1-2*n)/sec(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)+2*B*hypergeom([1
/2, -1/4-1/2*n],[3/4-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^n*sin(d*x+c)*sec(d
*x+c)^(1/2)/d/(1+2*n)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.08, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {20, 3872, 3857, 2722}

$$\frac{2B\sin(c+dx)\sqrt{\sec(c+dx)}\,(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-2n-1);\frac{1}{4}(3-2n);\cos^2(c+dx)\right)}{d(2n+1)\sqrt{\sin^2(c+dx)}}-\frac{2A\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1-2n);\frac{1}{4}(5-2n);\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*A*Hypergeometric2F1[1/2, (1 - 2*n)/4, (5 - 2*n)/4, Cos[c + d*x]^2]*(b*S
ec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Sec[c + d*x]]*Sqrt[Sin[c + d
*x]^2]) + (2*B*Hypergeometric2F1[1/2, (-1 - 2*n)/4, (3 - 2*n)/4, Cos[c + d*
x]^2]*Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + 2*n)*Sqrt
[Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(b\sec(c+dx))^n(A+B\sec(c+dx))\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{\frac{1}{2}+n}(c+dx$$

$$= \left(A\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{\frac{1}{2}+n}(c+$$

$$= \Big(A\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\sec(c+dx)}\,(b\sec(c+dx))^n$$

$$= -\frac{2A\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1-2n);\frac{1}{4}(5-2n);\cos^2(c+dx)\right)(}{d(1-2n)\sqrt{\sec(c+dx)}\,\sqrt{\sin^2}}$$

Mathematica [A]
time = 0.24, size = 140, normalized size = 0.86

$$\frac{2\csc(c+dx)(b\sec(c+dx))^n\left(A(3+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1+2n);\frac{1}{4}(5+2n);\sec^2(c+dx)\right)+B(1+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3+2n);\frac{1}{4}(7+2n);\sec^2(c+dx)\right)\sec(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{d(1+2n)(3+2n)\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*Csc[c + d*x]*(b*Sec[c + d*x])^n*(A*(3 + 2*n)*Hypergeometric2F1[1/2, (1 +
 2*n)/4, (5 + 2*n)/4, Sec[c + d*x]^2] + B*(1 + 2*n)*Hypergeometric2F1[1/2,
(3 + 2*n)/4, (7 + 2*n)/4, Sec[c + d*x]^2]*Sec[c + d*x])*Sqrt[-Tan[c + d*x]^
2])/(d*(1 + 2*n)*(3 + 2*n)*Sqrt[Sec[c + d*x]])
```

Maple [F]
time = 0.21, size = 0, normalized size = 0.00

$$\int \left(b\sec\left(dx+c\right)\right)^{n}\left(A+B\sec\left(dx+c\right)\right)\left(\sqrt{\sec}\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x)
```

```
[Out] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (b\sec(c+dx))^n (A+B\sec(c+dx))\sqrt{\sec(c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))*sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{b}{\cos\left(c+d\,x\right)}\right)}^n\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^n*(1/cos(c + d*x))^(1/2),x)
[Out] int((A + B/cos(c + d*x))*(b/cos(c + d*x))^n*(1/cos(c + d*x))^(1/2), x)
```

3.41 $\int \frac{(b\sec(c+dx))^n(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=163

$$-\frac{2A\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3-2n);\frac{1}{4}(7-2n);\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(3-2n)\sec^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}-\frac{2B\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1-2n);\frac{1}{4}(5-2n\right.}{d(1-2n)\sqrt{s}}$$

```
[Out] -2*A*hypergeom([1/2, 3/4-1/2*n],[7/4-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^n*
sin(d*x+c)/d/(3-2*n)/sec(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)-2*B*hypergeom([1
/2, 1/4-1/2*n],[5/4-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^n*sin(d*x+c)/d/(1-2
*n)/sec(d*x+c)^(1/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {20, 3872, 3857, 2722}

$$-\frac{2A\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3-2n);\frac{1}{4}(7-2n);\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)}-\frac{2B\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1-2n);\frac{1}{4}(5-2n);\cos^2(c+dx)\right)}{d(1-2n)\sqrt{\sin^2(c+dx)}\,\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (-2*A*Hypergeometric2F1[1/2, (3 - 2*n)/4, (7 - 2*n)/4, Cos[c + d*x]^2]*(b*S
ec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*Sec[c + d*x]^(3/2)*Sqrt[Sin[c + d
*x]^2]) - (2*B*Hypergeometric2F1[1/2, (1 - 2*n)/4, (5 - 2*n)/4, Cos[c + d*x
]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 - 2*n)*Sqrt[Sec[c + d*x]]*Sqrt[
Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{(b\sec(c+dx))^n(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{-\frac{1}{2}+n}(c+dx)(A+B\sec($$

$$= \left(A\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{-\frac{1}{2}+n}(c+dx)\,dx + \left(B\,\text{s}\right.$$

$$= \left(A\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\sec(c+dx)}\,(b\sec(c+dx))^n\right)\int \cos^{\frac{1}{2}-n}(c$$

$$= -\frac{2A\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3-2n);\frac{1}{4}(7-2n);\cos^2(c+dx)\right)(b\sec(c+dx))^n}{d(3-2n)\sec^{\frac{3}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.35, size = 135, normalized size = 0.83

$$\frac{2\csc(c+dx)(b\sec(c+dx))^n\left(A(1+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1+2n);\frac{1}{4}(3+2n);\sec^2(c+dx)\right)+B(-1+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(1+2n);\frac{1}{4}(5+2n);\sec^2(c+dx)\right)\sec(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{d\left(-1+4n^2\right)\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Csc[c + d*x]*(b*Sec[c + d*x])^n*(A*(1 + 2*n)*Hypergeometric2F1[1/2, (-1
+ 2*n)/4, (3 + 2*n)/4, Sec[c + d*x]^2] + B*(-1 + 2*n)*Hypergeometric2F1[1/2
, (1 + 2*n)/4, (5 + 2*n)/4, Sec[c + d*x]^2]*Sec[c + d*x])*Sqrt[-Tan[c + d*x
]^2])/(d*(-1 + 4*n^2)*Sec[c + d*x]^(3/2))
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(dx+c))^n(A+B\sec(dx+c))}{\sqrt{\sec(dx+c)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

```
[Out] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b \sec(c + dx))^n (A + B \sec(c + dx))}{\sqrt{\sec(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))/sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(\frac{b}{\cos(c+d\,x)}\right)^n}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^n)/(1/cos(c + d*x))^(1/2),x)
[Out] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^n)/(1/cos(c + d*x))^(1/2), x)
```

3.42 $\int \frac{(b\sec(c+dx))^n(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=163

$$-\frac{2A\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5-2n);\frac{1}{4}(9-2n);\cos^2(c+dx)\right)(b\sec(c+dx))^n\sin(c+dx)}{d(5-2n)\sec^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}-\frac{2B\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3-2n);\frac{1}{4}(7-2\right.}{d(3-2n)\,\mathrm{se}}$$

```
[Out] -2*A*hypergeom([1/2, 5/4-1/2*n],[9/4-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^n*
sin(d*x+c)/d/(5-2*n)/sec(d*x+c)^(5/2)/(sin(d*x+c)^2)^(1/2)-2*B*hypergeom([1
/2, 3/4-1/2*n],[7/4-1/2*n],cos(d*x+c)^2)*(b*sec(d*x+c))^n*sin(d*x+c)/d/(3-2
*n)/sec(d*x+c)^(3/2)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.09, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {20, 3872, 3857, 2722}

$$-\frac{2A\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5-2n);\frac{1}{4}(9-2n);\cos^2(c+dx)\right)}{d(5-2n)\sqrt{\sin^2(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)}-\frac{2B\sin(c+dx)(b\sec(c+dx))^n\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(3-2n);\frac{1}{4}(7-2n);\cos^2(c+dx)\right)}{d(3-2n)\sqrt{\sin^2(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (-2*A*Hypergeometric2F1[1/2, (5 - 2*n)/4, (9 - 2*n)/4, Cos[c + d*x]^2]*(b*S
ec[c + d*x])^n*Sin[c + d*x])/(d*(5 - 2*n)*Sec[c + d*x]^(5/2)*Sqrt[Sin[c + d
*x]^2]) - (2*B*Hypergeometric2F1[1/2, (3 - 2*n)/4, (7 - 2*n)/4, Cos[c + d*x
]^2]*(b*Sec[c + d*x])^n*Sin[c + d*x])/(d*(3 - 2*n)*Sec[c + d*x]^(3/2)*Sqrt[
Sin[c + d*x]^2])
```

Rule 20

```
Int[(u_.)*((a_.)*(v_))^(m_)*((b_.)*(v_))^(n_), x_Symbol] :> Dist[b^IntPart[
n]*((b*v)^FracPart[n]/(a^IntPart[n]*(a*v)^FracPart[n])), Int[u*(a*v)^(m + n
), x], x] /; FreeQ[{a, b, m, n}, x] && !IntegerQ[m] && !IntegerQ[n] && !
IntegerQ[m + n]
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 && !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rubi steps

$$\int \frac{(b\sec(c+dx))^n(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \left(\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{-\frac{3}{2}+n}(c+dx)(A+B\sec($$

$$= \left(A\sec^{-n}(c+dx)(b\sec(c+dx))^n\right)\int \sec^{-\frac{3}{2}+n}(c+dx)\,dx + \left(B\,\mathrm{s}\right.$$

$$= \left(A\cos^{\frac{1}{2}+n}(c+dx)\sqrt{\sec(c+dx)}\,(b\sec(c+dx))^n\right)\int \cos^{\frac{3}{2}-n}(c$$

$$= -\frac{2A\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(5-2n);\frac{1}{4}(9-2n);\cos^2(c+dx)\right)(b\sec(c+dx))^n}{d(5-2n)\sec^{\frac{5}{2}}(c+dx)\sqrt{\sin^2(c+dx)}}$$

Mathematica [A]
time = 0.33, size = 140, normalized size = 0.86

$$\frac{2\csc(c+dx)(b\sec(c+dx))^n\left(A(-1+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-3+2n);\frac{1}{4}(1+2n);\sec^2(c+dx)\right)+B(-3+2n)\,{}_2F_1\left(\frac{1}{2},\frac{1}{4}(-1+2n);\frac{1}{4}(3+2n);\sec^2(c+dx)\right)\sec(c+dx)\right)\sqrt{-\tan^2(c+dx)}}{d(-3+2n)(-1+2n)\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*Sec[c + d*x])^n*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*Csc[c + d*x]*(b*Sec[c + d*x])^n*(A*(-1 + 2*n)*Hypergeometric2F1[1/2, (-3
 + 2*n)/4, (1 + 2*n)/4, Sec[c + d*x]^2] + B*(-3 + 2*n)*Hypergeometric2F1[1/
2, (-1 + 2*n)/4, (3 + 2*n)/4, Sec[c + d*x]^2]*Sec[c + d*x])*Sqrt[-Tan[c + d
*x]^2])/(d*(-3 + 2*n)*(-1 + 2*n)*Sec[c + d*x]^(5/2))
```

Maple [F]
time = 0.27, size = 0, normalized size = 0.00

$$\int \frac{\left(b\sec\left(dx+c\right)\right)^{n}\left(A+B\sec\left(dx+c\right)\right)}{\sec\left(dx+c\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

```
[Out] int((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(b\sec(c+dx))^n (A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))**n*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((b*sec(c + d*x))**n*(A + B*sec(c + d*x))/sec(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*sec(d*x+c))^n*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c))^n/sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(\frac{b}{\cos(c+d\,x)}\right)^n}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^n)/(1/cos(c + d*x))^(3/2),x)
[Out] int(((A + B/cos(c + d*x))*(b/cos(c + d*x))^n)/(1/cos(c + d*x))^(3/2), x)
```

3.43 $\int \sec^4(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=134

$$\frac{3a(A+B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(5A+4B)\tan(c+dx)}{5d}+\frac{3a(A+B)\sec(c+dx)\tan(c+dx)}{8d}+\frac{a(A+B)}{}$$

```
[Out] 3/8*a*(A+B)*arctanh(sin(d*x+c))/d+1/5*a*(5*A+4*B)*tan(d*x+c)/d+3/8*a*(A+B)*
sec(d*x+c)*tan(d*x+c)/d+1/4*a*(A+B)*sec(d*x+c)^3*tan(d*x+c)/d+1/5*a*B*sec(d
*x+c)^4*tan(d*x+c)/d+1/15*a*(5*A+4*B)*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4082, 3872, 3852, 3853, 3855}

$$\frac{a(5A+4B)\tan^3(c+dx)}{15d}+\frac{a(5A+4B)\tan(c+dx)}{5d}+\frac{3a(A+B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(A+B)\tan(c+dx)\sec^3(c+dx)}{4d}+\frac{3a(A+B)\tan(c+dx)\sec(c+dx)}{8d}+\frac{aB\tan(c+dx)\sec^4(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*a*(A + B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(5*A + 4*B)*Tan[c + d*x])/(5
*d) + (3*a*(A + B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*(A + B)*Sec[c + d*
x]^3*Tan[c + d*x])/(4*d) + (a*B*Sec[c + d*x]^4*Tan[c + d*x])/(5*d) + (a*(5*
A + 4*B)*Tan[c + d*x]^3)/(15*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^4(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx &= \frac{aB\sec^4(c+dx)\tan(c+dx)}{5d}+\frac{1}{5}\int \sec^4(c+dx)\\ &= \frac{aB\sec^4(c+dx)\tan(c+dx)}{5d}+(a(A+B))\int \sec\\ &= \frac{a(A+B)\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{aB\sec^4(c\\ &= \frac{a(5A+4B)\tan(c+dx)}{5d}+\frac{3a(A+B)\sec(c+d}{8d}\\ &= \frac{3a(A+B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(5A+4B)\,t}{5d}\end{aligned}$$

Mathematica [A]
time = 0.76, size = 87, normalized size = 0.65

$$\frac{a\left(45(A+B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(45(A+B)\sec(c+dx)+30(A+B)\sec^3(c+dx)+8\left(15(A+B)+5(A+2B)\tan^2(c+dx)+3B\tan^4(c+dx)\right)\right)\right)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(45*(A + B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(45*(A + B)*Sec[c + d*x
] + 30*(A + B)*Sec[c + d*x]^3 + 8*(15*(A + B) + 5*(A + 2*B)*Tan[c + d*x]^2
+ 3*B*Tan[c + d*x]^4))))/(120*d)
```

Maple [A]
time = 0.33, size = 154, normalized size = 1.15

method	result
derivativedivides	$\frac{Aa\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-Ba\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right.}{15}\right.}{}$
default	$\frac{Aa\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-Ba\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c)\right.}{15}\right.}{}$
norman	$\frac{-\frac{4a(25A+29B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{a(29A+13B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{a(35A+19B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{13(A+B)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{3(A+}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}$
risch	$-\frac{ia\left(45A\,e^{9i(dx+c)}+45B\,e^{9i(dx+c)}+210A\,e^{7i(dx+c)}+210B\,e^{7i(dx+c)}-240A\,e^{6i(dx+c)}-560A\,e^{4i(dx+c)}-640B\,e^{4i(dx+c)}\right.}{60d\left(e^{2i(dx+c}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+
tan(d*x+c)))-B*a*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)-A*a*
(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+B*a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))
*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]

time = 0.39, size = 200, normalized size = 1.49

$$\frac{80\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Aa+16\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)Ba-15\,Aa\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)-15\,Ba\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/240*(80*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a + 16*(3*tan(d*x + c)^5 + 10
*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a - 15*A*a*(2*(3*sin(d*x + c)^3 - 5*si
n(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) +
1) + 3*log(sin(d*x + c) - 1)) - 15*B*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c
))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*lo
g(sin(d*x + c) - 1)))/d
```

Fricas [A]

time = 1.06, size = 137, normalized size = 1.02

$$\frac{45\left(A+B\right)a\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-45\left(A+B\right)a\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\left(5\,A+4\,B\right)a\cos\left(dx+c\right)^4+45\left(A+B\right)a\cos\left(dx+c\right)^3+8\left(5\,A+4\,B\right)a\cos\left(dx+c\right)^2+30\left(A+B\right)a\cos\left(dx+c\right)+24\,Ba\right)\sin\left(dx+c\right)}{240\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/240*(45*(A + B)*a*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 45*(A + B)*a*cos
(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(5*A + 4*B)*a*cos(d*x + c)^4 + 4
5*(A + B)*a*cos(d*x + c)^3 + 8*(5*A + 4*B)*a*cos(d*x + c)^2 + 30*(A + B)*a*
cos(d*x + c) + 24*B*a)*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec^4(c+dx)\,dx+\int A\sec^5(c+dx)\,dx+\int B\sec^5(c+dx)\,dx+\int B\sec^6(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**4, x) + Integral(A*sec(c + d*x)**5, x) + Integr
al(B*sec(c + d*x)**5, x) + Integral(B*sec(c + d*x)**6, x))
```

Giac [A]
time = 0.47, size = 214, normalized size = 1.60

$$\frac{45\,(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-45\,(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(45\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+45\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-290\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-130\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+400\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+464\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-350\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-190\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+195\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+195\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/120*(45*(A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 45*(A*a + B*a)*l
og(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(45*A*a*tan(1/2*d*x + 1/2*c)^9 + 45*B
*a*tan(1/2*d*x + 1/2*c)^9 - 290*A*a*tan(1/2*d*x + 1/2*c)^7 - 130*B*a*tan(1/
2*d*x + 1/2*c)^7 + 400*A*a*tan(1/2*d*x + 1/2*c)^5 + 464*B*a*tan(1/2*d*x + 1
/2*c)^5 - 350*A*a*tan(1/2*d*x + 1/2*c)^3 - 190*B*a*tan(1/2*d*x + 1/2*c)^3 +
 195*A*a*tan(1/2*d*x + 1/2*c) + 195*B*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x
+ 1/2*c)^2 - 1)^5)/d
```

Mupad [B]
time = 4.75, size = 198, normalized size = 1.48

$$\frac{3\,a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(A+B)}{4\,d}-\frac{\left(\frac{3\,A\,a}{4}+\frac{3\,B\,a}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{29\,A\,a}{6}-\frac{13\,B\,a}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{20\,A\,a}{3}+\frac{116\,B\,a}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{35\,A\,a}{6}-\frac{19\,B\,a}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{13\,A\,a}{4}+\frac{13\,B\,a}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/cos(c + d*x)^4,x)
```

```
[Out] (3*a*atanh(tan(c/2 + (d*x)/2))*(A + B))/(4*d) - (tan(c/2 + (d*x)/2)*((13*A*
a)/4 + (13*B*a)/4) + tan(c/2 + (d*x)/2)^9*((3*A*a)/4 + (3*B*a)/4) - tan(c/2
 + (d*x)/2)^7*((29*A*a)/6 + (13*B*a)/6) - tan(c/2 + (d*x)/2)^3*((35*A*a)/6
+ (19*B*a)/6) + tan(c/2 + (d*x)/2)^5*((20*A*a)/3 + (116*B*a)/15))/(d*(5*tan
(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*t
an(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.44 $\int \sec^3(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=106

$$\frac{a(4A+3B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(A+B)\tan(c+dx)}{d}+\frac{a(4A+3B)\sec(c+dx)\tan(c+dx)}{8d}+\frac{aB\sec^3(c$$

```
[Out] 1/8*a*(4*A+3*B)*arctanh(sin(d*x+c))/d+a*(A+B)*tan(d*x+c)/d+1/8*a*(4*A+3*B)*
sec(d*x+c)*tan(d*x+c)/d+1/4*a*B*sec(d*x+c)^3*tan(d*x+c)/d+1/3*a*(A+B)*tan(d
*x+c)^3/d
```

Rubi [A]
time = 0.09, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4082, 3872, 3853, 3855, 3852}

$$\frac{a(A+B)\tan^3(c+dx)}{3d}+\frac{a(A+B)\tan(c+dx)}{d}+\frac{a(4A+3B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(4A+3B)\tan(c+dx)\sec(c+dx)}{8d}+\frac{aB\tan(c+dx)\sec^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(4*A + 3*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a*(A + B)*Tan[c + d*x])/d +
(a*(4*A + 3*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a*B*Sec[c + d*x]^3*Tan[c
 + d*x])/(4*d) + (a*(A + B)*Tan[c + d*x]^3)/(3*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^3(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx &= \frac{aB\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int \sec^3(c+dx)\\ &= \frac{aB\sec^3(c+dx)\tan(c+dx)}{4d}+(a(A+B))\int \sec\\ &= \frac{a(4A+3B)\sec(c+dx)\tan(c+dx)}{8d}+\frac{aB\sec^3(}{}\\ &= \frac{a(4A+3B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a(A+B)\tan}{d}\end{aligned}$$

Mathematica [A]
time = 0.43, size = 77, normalized size = 0.73

$$\frac{a\left(3(4A+3B)\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\left(12A+9B+8(A+B)(2+\cos(2(c+dx)))\sec(c+dx)+6B\sec^2(c+dx)\right)\tan(c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
[Out] (a*(3*(4*A + 3*B)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(12*A + 9*B + 8*(A +
 B)*(2 + Cos[2*(c + d*x)])*Sec[c + d*x] + 6*B*Sec[c + d*x]^2)*Tan[c + d*x])
)/(24*d)
```

Maple [A]
time = 0.30, size = 131, normalized size = 1.24

method	result

derivativedivides	$\frac{-Aa\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)+Ba\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+}{d}$
default	$\frac{-Aa\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)+Ba\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)+}{d}$
norman	$\frac{-\frac{a(4A+3B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{7a(4A+7B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{a(12A+13B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{a(52A+31B)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}-\frac{a(4A}{}$
risch	$-\frac{ia\left(12A\,e^{7i(dx+c)}+9B\,e^{7i(dx+c)}+12A\,e^{5i(dx+c)}+33B\,e^{5i(dx+c)}-48A\,e^{4i(dx+c)}-48B\,e^{4i(dx+c)}-12A\,e^{3i(dx+c)}-33B\right.}{12d\left(e^{2i(dx+c)}+1\right)^{4}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-A*a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+B*a*(-(-1/4*sec(d*x+c)^3-3/8*s
ec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+A*a*(1/2*sec(d*x+c)*ta
n(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-B*a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x
+c))
```

Maxima [A]
time = 0.36, size = 163, normalized size = 1.54

$$\frac{16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa+16\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba-3\,Ba\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-12\,Aa\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a + 16*(tan(d*x + c)^3 + 3*tan
(d*x + c))*B*a - 3*B*a*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 12*A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
+ log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 2.22, size = 127, normalized size = 1.20

$$\frac{3\left(4\,A+3\,B\right)a\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left(4\,A+3\,B\right)a\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\left(A+B\right)a\cos\left(dx+c\right)^{3}+3\left(4\,A+3\,B\right)a\cos\left(dx+c\right)^{2}+8\left(A+B\right)a\cos\left(dx+c\right)+6\,Ba\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/48*(3*(4*A + 3*B)*a*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*A + 3*B)*
a*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*(A + B)*a*cos(d*x + c)^3 +
```

```
3*(4*A + 3*B)*a*cos(d*x + c)^2 + 8*(A + B)*a*cos(d*x + c) + 6*B*a)*sin(d*x
+ c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec^3\left(c+dx\right)dx+\int A\sec^4\left(c+dx\right)dx+\int B\sec^4\left(c+dx\right)dx+\int B\sec^5\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**3, x) + Integral(A*sec(c + d*x)**4, x) + Integr
al(B*sec(c + d*x)**4, x) + Integral(B*sec(c + d*x)**5, x))
```

Giac [A]
time = 0.49, size = 188, normalized size = 1.77

$$\frac{3\left(4\,Aa+3\,Ba\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(4\,Aa+3\,Ba\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(12\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+9\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-28\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-49\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+52\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+31\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-36\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-39\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/24*(3*(4*A*a + 3*B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*A*a + 3*B
*a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(12*A*a*tan(1/2*d*x + 1/2*c)^7 +
 9*B*a*tan(1/2*d*x + 1/2*c)^7 - 28*A*a*tan(1/2*d*x + 1/2*c)^5 - 49*B*a*tan(
1/2*d*x + 1/2*c)^5 + 52*A*a*tan(1/2*d*x + 1/2*c)^3 + 31*B*a*tan(1/2*d*x + 1
/2*c)^3 - 36*A*a*tan(1/2*d*x + 1/2*c) - 39*B*a*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 4.62, size = 166, normalized size = 1.57

$$\frac{\left(-A\,a-\frac{3\,B\,a}{4}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{7\,A\,a}{3}+\frac{49\,B\,a}{12}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{13\,A\,a}{3}-\frac{31\,B\,a}{12}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(3\,A\,a+\frac{13\,B\,a}{4}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{a\,\mathrm{atanh}\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(4\,A+3\,B\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/cos(c + d*x)^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(3*A*a + (13*B*a)/4) - tan(c/2 + (d*x)/2)^7*(A*a + (3*B
*a)/4) - tan(c/2 + (d*x)/2)^3*((13*A*a)/3 + (31*B*a)/12) + tan(c/2 + (d*x)/
2)^5*((7*A*a)/3 + (49*B*a)/12))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d
*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (a*atanh(t
an(c/2 + (d*x)/2))*(4*A + 3*B))/(4*d)
```

3.45 $\int \sec^2(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=86

$$\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(3A+2B)\tan(c+dx)}{3d}+\frac{a(A+B)\sec(c+dx)\tan(c+dx)}{2d}+\frac{aB\sec^2(c+$$

```
[Out] 1/2*a*(A+B)*arctanh(sin(d*x+c))/d+1/3*a*(3*A+2*B)*tan(d*x+c)/d+1/2*a*(A+B)*
sec(d*x+c)*tan(d*x+c)/d+1/3*a*B*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4082, 3872, 3852, 8, 3853, 3855}

$$\frac{a(3A+2B)\tan(c+dx)}{3d}+\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(A+B)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aB\tan(c+dx)\sec^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(A + B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(3*A + 2*B)*Tan[c + d*x])/(3*d
) + (a*(A + B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (a*B*Sec[c + d*x]^2*Tan[c
 + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx &= \frac{aB\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \sec^2(c+dx)\\ &= \frac{aB\sec^2(c+dx)\tan(c+dx)}{3d}+(a(A+B))\int \sec\\ &= \frac{a(A+B)\sec(c+dx)\tan(c+dx)}{2d}+\frac{aB\sec^2(c+}{}\\ &= \frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(3A+2B)\tan}{3d}\end{aligned}$$

Mathematica [A]
time = 0.36, size = 56, normalized size = 0.65

$$\frac{a\left(3(A+B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(6(A+B)+3(A+B)\sec(c+dx)+2B\tan^2(c+dx)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(3*(A + B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*(A + B) + 3*(A + B)*S
ec[c + d*x] + 2*B*Tan[c + d*x]^2)))/(6*d)
```

Maple [A]
time = 0.27, size = 105, normalized size = 1.22

method	result

derivativedivides	$\frac{Aa\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-Ba\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+Aa\tan(dx+c)+Ba\left(\frac{\sec(dx+c}{}\right.}{d}$
default	$\frac{Aa\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-Ba\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+Aa\tan(dx+c)+Ba\left(\frac{\sec(dx+c}{}\right.}{d}$
norman	$\frac{\frac{4a(3A+B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{3(A+B)a\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{(A+B)a\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{(A+B)a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{(A+B)}{}$
risch	$-\frac{ia\left(3A\,e^{5i(dx+c)}+3B\,e^{5i(dx+c)}-6A\,e^{4i(dx+c)}-12A\,e^{2i(dx+c)}-12B\,e^{2i(dx+c)}-3\,e^{i(dx+c)}A-3B\,e^{i(dx+c)}-6A-4B\right)}{3d\left(e^{2i(dx+c)}+1\right)^3}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-B*a*(-2/
3-1/3*sec(d*x+c)^2)*tan(d*x+c)+A*a*tan(d*x+c)+B*a*(1/2*sec(d*x+c)*tan(d*x+c
)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.36, size = 127, normalized size = 1.48

$$\frac{4\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Ba-3\,Aa\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-3\,Ba\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+12\,Aa\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a - 3*A*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*B*a*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 12*A*a*tan(d*x + c))/d
```

Fricas [A]
time = 2.50, size = 105, normalized size = 1.22

$$\frac{3\left(A+B\right)a\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(A+B\right)a\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\left(3\,A+2\,B\right)a\cos\left(dx+c\right)^2+3\left(A+B\right)a\cos\left(dx+c\right)+2\,Ba\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(A + B)*a*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(A + B)*a*cos(d*
x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(3*A + 2*B)*a*cos(d*x + c)^2 + 3*(A
+ B)*a*cos(d*x + c) + 2*B*a)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec^2(c+dx)\,dx+\int A\sec^3(c+dx)\,dx+\int B\sec^3(c+dx)\,dx+\int B\sec^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] a*(Integral(A*sec(c + d*x)**2, x) + Integral(A*sec(c + d*x)**3, x) + Integr
al(B*sec(c + d*x)**3, x) + Integral(B*sec(c + d*x)**4, x))
```

Giac [A]
time = 0.47, size = 154, normalized size = 1.79

$$\frac{3\,(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(3\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(A*a + B*a)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(3*A*a*tan(1/2*d*x + 1/2*c)^5 + 3*B*a*tan
(1/2*d*x + 1/2*c)^5 - 12*A*a*tan(1/2*d*x + 1/2*c)^3 - 4*B*a*tan(1/2*d*x + 1
/2*c)^3 + 9*A*a*tan(1/2*d*x + 1/2*c) + 9*B*a*tan(1/2*d*x + 1/2*c))/(tan(1/2
*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 3.99, size = 126, normalized size = 1.47

$$\frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(A+B)}{d}-\frac{(A\,a+B\,a)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-4\,A\,a-\frac{4\,B\,a}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+(3\,A\,a+3\,B\,a)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] (a*atanh(tan(c/2 + (d*x)/2))*(A + B))/d - (tan(c/2 + (d*x)/2)*(3*A*a + 3*B*
a) + tan(c/2 + (d*x)/2)^5*(A*a + B*a) - tan(c/2 + (d*x)/2)^3*(4*A*a + (4*B*
a)/3))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x
)/2)^6 - 1))
```

3.46 $\int \sec(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=56

$$\frac{a(2A+B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(A+B)\tan(c+dx)}{d}+\frac{aB\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 1/2*a*(2*A+B)*arctanh(sin(d*x+c))/d+a*(A+B)*tan(d*x+c)/d+1/2*a*B*sec(d*x+c)
*tan(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 56, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4082, 3872, 3855, 3852, 8}

$$\frac{a(A+B)\tan(c+dx)}{d}+\frac{a(2A+B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aB\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(2*A + B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a*(A + B)*Tan[c + d*x])/d + (a
*B*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{aB\sec(c+dx)\tan(c+dx)}{2d} + \frac{1}{2}\int \sec(c+dx)(a($$
$$= \frac{aB\sec(c+dx)\tan(c+dx)}{2d} + (a(A+B))\int \sec^2($$
$$= \frac{a(2A+B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{aB\sec(c+dx)}{2d}$$
$$= \frac{a(2A+B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{a(A+B)\tan(}{d}$$

Mathematica [A]
time = 0.04, size = 75, normalized size = 1.34

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{d} + \frac{aB\tanh^{-1}(\sin(c+dx))}{2d} + \frac{aA\tan(c+dx)}{d} + \frac{aB\tan(c+dx)}{d} + \frac{aB\sec(c+dx)\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])/(2*d) + (a*A*Ta
n[c + d*x])/d + (a*B*Tan[c + d*x])/d + (a*B*Sec[c + d*x]*Tan[c + d*x])/(2*d
)
```

Maple [A]
time = 0.20, size = 75, normalized size = 1.34

method	result
derivativedivides	$\frac{Aa\tan(dx+c)+Ba\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+Aa\ln(\sec(dx+c)+\tan(dx+c))+Ba\tan(dx+c)}{d}$
default	$\frac{Aa\tan(dx+c)+Ba\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+Aa\ln(\sec(dx+c)+\tan(dx+c))+Ba\tan(dx+c)}{d}$

norman	$\frac{\frac{a(2A+3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{a(2A+B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a(2A+B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{a(2A+B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2d}$
risch	$-\frac{ia\left(B\,e^{3i(dx+c)}-2A\,e^{2i(dx+c)}-2B\,e^{2i(dx+c)}-B\,e^{i(dx+c)}-2A-2B\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{a\ln\left(e^{i(dx+c)}+i\right)A}{d}+\frac{a\ln\left(e^{i(dx+c)}+i\right)B}{2d}-$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(A*a*tan(d*x+c)+B*a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+A*a*ln(sec(d*x+c)+tan(d*x+c))+B*a*tan(d*x+c))

Maxima [A]
time = 0.37, size = 88, normalized size = 1.57

$$-\frac{Ba\left(\frac{2\sin\left(dx+c\right)}{\sin\left(dx+c\right)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-4\,Aa\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)-4\,Aa\tan\left(dx+c\right)-4\,Ba\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] -1/4*(B*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 4*A*a*log(sec(d*x + c) + tan(d*x + c)) - 4*A*a*tan(d*x + c) - 4*B*a*tan(d*x + c))/d

Fricas [A]
time = 1.49, size = 89, normalized size = 1.59

$$\frac{(2\,A+B)a\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(2\,A+B)a\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,(A+B)a\cos\left(dx+c\right)+Ba\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/4*((2*A + B)*a*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*A + B)*a*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*(A + B)*a*cos(d*x + c) + B*a)*sin(d*x + c))/(d*cos(d*x + c)^2)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sec\left(c+dx\right)dx+\int A\sec^2\left(c+dx\right)dx+\int B\sec^2\left(c+dx\right)dx+\int B\sec^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] a*(Integral(A*sec(c + d*x), x) + Integral(A*sec(c + d*x)**2, x) + Integral(
B*sec(c + d*x)**2, x) + Integral(B*sec(c + d*x)**3, x))
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 124 vs. $2(52) = 104$.
time = 0.45, size = 124, normalized size = 2.21

$$\frac{(2\,Aa + Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - (2\,Aa + Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(2\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 2\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 3\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((2*A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*A*a + B*a)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*A*a*tan(1/2*d*x + 1/2*c)^3 + B*a*tan(1
/2*d*x + 1/2*c)^3 - 2*A*a*tan(1/2*d*x + 1/2*c) - 3*B*a*tan(1/2*d*x + 1/2*c)
)/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [**B**]
time = 2.73, size = 94, normalized size = 1.68

$$\frac{\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\,\left(2\,A\,a + 3\,B\,a\right) - \tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^3\left(2\,A\,a + B\,a\right)}{d\,\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^4 - 2\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)^2 + 1\right)} + \frac{a\,\mathrm{atanh}\left(\tan\left(\frac{c}{2} + \frac{d\,x}{2}\right)\right)\,\left(2\,A + B\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/cos(c + d*x),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(2*A*a + 3*B*a) - tan(c/2 + (d*x)/2)^3*(2*A*a + B*a))/(
d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1)) + (a*atanh(tan(c/2 +
 (d*x)/2))*(2*A + B))/d
```

3.47 $\int (a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=32

$$aAx+\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\tan(c+dx)}{d}$$

```
[Out] a*A*x+a*(A+B)*arctanh(sin(d*x+c))/d+a*B*tan(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}}=0.190$, Rules used = {3999, 3852, 8, 3855}

$$\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{d}+aAx+\frac{aB\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
[Out] a*A*x + (a*(A + B)*ArcTanh[Sin[c + d*x]])/d + (a*B*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3999

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rubi steps

$$
\begin{aligned}
\int (a+a\sec(c+dx))(A+B\sec(c+dx))\,dx &= aAx+(aB)\int \sec^2(c+dx)\,dx+(a(A+B))\int \sec(c+dx)\,d\\
&= aAx+\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(aB)\text{Subst}(\int 1\,dx,x}{d}\\
&= aAx+\frac{a(A+B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\tan(c+dx)}{d}
\end{aligned}
$$

Mathematica [A]
time = 0.02, size = 43, normalized size = 1.34

$$
aAx+\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\tan(c+dx)}{d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] a*A*x + (a*A*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*
B*Tan[c + d*x])/d
```

Maple [A]
time = 0.19, size = 57, normalized size = 1.78

method	result
derivativedivides	$\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))+Ba\tan(dx+c)+Aa(dx+c)+Ba\ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))+Ba\tan(dx+c)+Aa(dx+c)+Ba\ln(\sec(dx+c)+\tan(dx+c))}{d}$
norman	$\frac{aAx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-aAx-\frac{2Ba\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{(A+B)a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{(A+B)a\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$
risch	$aAx+\frac{2iBa}{d\left(e^{2i(dx+c)}+1\right)}+\frac{a\ln\left(e^{i(dx+c)}+i\right)A}{d}+\frac{a\ln\left(e^{i(dx+c)}+i\right)B}{d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)A}{d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)B}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a*ln(sec(d*x+c)+tan(d*x+c))+B*a*tan(d*x+c)+A*a*(d*x+c)+B*a*ln(sec(d*
x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.36, size = 56, normalized size = 1.75

$$
\frac{(dx+c)Aa+Aa\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+Ba\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+Ba\tan\left(dx+c\right)}{d}
$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((d*x + c)*A*a + A*a*log(sec(d*x + c) + tan(d*x + c)) + B*a*log(sec(d*x + c
) + tan(d*x + c)) + B*a*tan(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. $2(32) = 64$.
time = 2.31, size = 79, normalized size = 2.47

$$\frac{2\,Aadx\cos\left(dx+c\right)+(A+B)a\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-(A+B)a\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,Ba\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*A*a*d*x*cos(d*x + c) + (A + B)*a*cos(d*x + c)*log(sin(d*x + c) + 1)
- (A + B)*a*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*B*a*sin(d*x + c))/(d*co
s(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(31) = 62$.
time = 4.65, size = 71, normalized size = 2.22

$$\begin{cases} \frac{Aa(c+dx)+Aa\log\left(\tan\left(c+dx\right)+\sec\left(c+dx\right)\right)+Ba\log\left(\tan\left(c+dx\right)+\sec\left(c+dx\right)\right)+Ba\tan\left(c+dx\right)}{d} & \text{for}\: d\neq 0 \\ x(A+B\sec\left(c\right))\left(a\sec\left(c\right)+a\right) & \text{otherwise} \end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Piecewise(((A*a*(c + d*x) + A*a*log(tan(c + d*x) + sec(c + d*x)) + B*a*log(
tan(c + d*x) + sec(c + d*x)) + B*a*tan(c + d*x))/d, Ne(d, 0)), (x*(A + B*se
c(c))*(a*sec(c) + a), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(32) = 64$.
time = 0.46, size = 84, normalized size = 2.62

$$\frac{(dx+c)Aa+(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Aa+Ba)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*A*a + (A*a + B*a)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (A*a + B*
a)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*B*a*tan(1/2*d*x + 1/2*c)/(tan(1/2
*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B]
time = 2.23, size = 100, normalized size = 3.12

$$\frac{B\,a\,\tan(c+d\,x)}{d}+\frac{2\,A\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,A\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x)),x)
```

```
[Out] (B*a*tan(c + d*x))/d + (2*A*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
d + (2*A*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a*atanh(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.48 $\int \cos(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=32

$$a(A+B)x+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\sin(c+dx)}{d}$$

```
[Out] a*(A+B)*x+a*B*arctanh(sin(d*x+c))/d+a*A*sin(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 32, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}}=0.074$, Rules used = {4081, 3855}

$$ax(A+B)+\frac{aA\sin(c+dx)}{d}+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] a*(A + B)*x + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*A*Sin[c + d*x])/d
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx=\frac{aA\sin(c+dx)}{d}-\int(-a(A+B)-aB\sec(c+$$

$$=a(A+B)x+\frac{aA\sin(c+dx)}{d}+(aB)\int\sec(c+$$

$$=a(A+B)x+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\sin($$

Mathematica [A]
time = 0.03, size = 46, normalized size = 1.44

$$aAx + aBx + \frac{aB\tanh^{-1}(\sin(c+dx))}{d} + \frac{aA\cos(dx)\sin(c)}{d} + \frac{aA\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] a*A*x + a*B*x + (a*B*ArcTanh[Sin[c + d*x]])/d + (a*A*Cos[d*x]*Sin[c])/d + (
a*A*Cos[c]*Sin[d*x])/d
```

Maple [A]
time = 0.19, size = 48, normalized size = 1.50

method	result
derivativedivides	$\frac{Aa(dx+c)+Ba\ln(\sec(dx+c)+\tan(dx+c))+Aa\sin(dx+c)+Ba(dx+c)}{d}$
default	$\frac{Aa(dx+c)+Ba\ln(\sec(dx+c)+\tan(dx+c))+Aa\sin(dx+c)+Ba(dx+c)}{d}$
risch	$aAx + axB - \frac{iAa\,e^{i(dx+c)}}{2d} + \frac{iAa\,e^{-i(dx+c)}}{2d} + \frac{a\ln(e^{i(dx+c)}+i)B}{d} - \frac{a\ln(e^{i(dx+c)}-i)B}{d}$
norman	$\frac{(-Aa-Ba)x+(Aa+Ba)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2Aa\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2Aa\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} + \frac{Ba\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d} - \frac{Ba\ln(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a*(d*x+c)+B*a*ln(sec(d*x+c)+tan(d*x+c))+A*a*sin(d*x+c)+B*a*(d*x+c))
```

Maxima [A]
time = 0.36, size = 58, normalized size = 1.81

$$\frac{2(dx+c)Aa + 2(dx+c)Ba + Ba(\log(\sin(dx+c)+1) - \log(\sin(dx+c)-1)) + 2Aa\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima
")
```

```
[Out] 1/2*(2*(d*x + c)*A*a + 2*(d*x + c)*B*a + B*a*(log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + 2*A*a*sin(d*x + c))/d
```

Fricas [A]
time = 1.82, size = 51, normalized size = 1.59

$$\frac{2(A+B)adx + Ba\log(\sin(dx+c)+1) - Ba\log(-\sin(dx+c)+1) + 2Aa\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/2*(2*(A + B)*a*d*x + B*a*log(sin(d*x + c) + 1) - B*a*log(-sin(d*x + c) +
1) + 2*A*a*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\cos(c+dx)\,dx + \int A\cos(c+dx)\sec(c+dx)\,dx + \int B\cos(c+dx)\sec(c+dx)\,dx + \int B\cos(c+dx)\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] a*(Integral(A*cos(c + d*x), x) + Integral(A*cos(c + d*x)*sec(c + d*x), x) +
 Integral(B*cos(c + d*x)*sec(c + d*x), x) + Integral(B*cos(c + d*x)*sec(c +
 d*x)**2, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(32) = 64.
time = 0.45, size = 79, normalized size = 2.47

$$\frac{Ba\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - Ba\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) + (Aa+Ba)(dx+c) + \frac{2\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (B*a*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - B*a*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + (A*a + B*a)*(d*x + c) + 2*A*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1
/2*c)^2 + 1))/d
```

Mupad [B]
time = 2.15, size = 100, normalized size = 3.12

$$\frac{A\,a\,\sin\left(c+d\,x\right)}{d}+\frac{2\,A\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x)),x)
```

```
[Out] (A*a*sin(c + d*x))/d + (2*A*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
d + (2*B*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a*atanh(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.49 $\int \cos^2(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=47

$$\frac{1}{2}a(A+2B)x+\frac{a(A+B)\sin(c+dx)}{d}+\frac{aA\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*a*(A+2*B)*x+a*(A+B)*sin(d*x+c)/d+1/2*a*A*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 47, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4081, 3872, 2717, 8}

$$\frac{a(A+B)\sin(c+dx)}{d}+\frac{1}{2}ax(A+2B)+\frac{aA\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(A + 2*B)*x)/2 + (a*(A + B)*Sin[c + d*x])/d + (a*A*Cos[c + d*x]*Sin[c +
d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{aA\cos(c+dx)\sin(c+dx)}{2d} - \frac{1}{2}\int \cos(c+dx)($$
$$= \frac{aA\cos(c+dx)\sin(c+dx)}{2d} + (a(A+B))\int \mathrm{co}$$
$$= \frac{1}{2}a(A+2B)x + \frac{a(A+B)\sin(c+dx)}{d} + \frac{aA\,\mathrm{co}}{}$$

Mathematica [A]
time = 0.11, size = 44, normalized size = 0.94

$$\frac{a(2Ac+2Adx+4Bdx+4(A+B)\sin(c+dx)+A\sin(2(c+dx)))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(2*A*c + 2*A*d*x + 4*B*d*x + 4*(A + B)*Sin[c + d*x] + A*Sin[2*(c + d*x)]
))/(4*d)
```

Maple [A]
time = 0.19, size = 57, normalized size = 1.21

method	result
risch	$\frac{aAx}{2}+axB+\frac{aA\sin(dx+c)}{d}+\frac{\sin(dx+c)Ba}{d}+\frac{Aa\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{Aa\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Aa\sin(dx+c)+Ba\sin(dx+c)+Ba(dx+c)}{d}$
default	$\frac{Aa\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Aa\sin(dx+c)+Ba\sin(dx+c)+Ba(dx+c)}{d}$
norman	$\frac{\frac{a(A+2B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a(A+2B)x}{2}+\frac{2Aa\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a(A+2B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{a(A+2B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{a(A+2}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*a*sin(d*x+c)+B*a*sin(d
*x+c)+B*a*(d*x+c))
```

Maxima [A]
time = 0.35, size = 55, normalized size = 1.17

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa+4\,(dx+c)Ba+4\,Aa\sin(dx+c)+4\,Ba\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a + 4*(d*x + c)*B*a + 4*A*a*sin(d*x
 + c) + 4*B*a*sin(d*x + c))/d
```

Fricas [A]
time = 1.49, size = 38, normalized size = 0.81

$$\frac{(A+2B)adx+(Aa\cos(dx+c)+2(A+B)a)\sin(dx+c)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*((A + 2*B)*a*d*x + (A*a*cos(d*x + c) + 2*(A + B)*a)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\cos^2(c+dx)\,dx+\int A\cos^2(c+dx)\sec(c+dx)\,dx+\int B\cos^2(c+dx)\sec(c+dx)\,dx+\int B\cos^2(c+dx)\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] a*(Integral(A*cos(c + d*x)**2, x) + Integral(A*cos(c + d*x)**2*sec(c + d*x)
, x) + Integral(B*cos(c + d*x)**2*sec(c + d*x), x) + Integral(B*cos(c + d*x
)**2*sec(c + d*x)**2, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 93 vs. $2(43) = 86$.
time = 0.47, size = 93, normalized size = 1.98

$$\frac{(Aa+2Ba)(dx+c)+\frac{2\left(Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+2Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((A*a + 2*B*a)*(d*x + c) + 2*(A*a*tan(1/2*d*x + 1/2*c)^3 + 2*B*a*tan(1/
2*d*x + 1/2*c)^3 + 3*A*a*tan(1/2*d*x + 1/2*c) + 2*B*a*tan(1/2*d*x + 1/2*c))
/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 2.09, size = 50, normalized size = 1.06

$$\frac{A\,a\,x}{2}+B\,a\,x+\frac{A\,a\,\sin\left(c+d\,x\right)}{d}+\frac{B\,a\,\sin\left(c+d\,x\right)}{d}+\frac{A\,a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + a/cos(c + d*x)),x)
```

```
[Out] (A*a*x)/2 + B*a*x + (A*a*sin(c + d*x))/d + (B*a*sin(c + d*x))/d + (A*a*sin(
2*c + 2*d*x))/(4*d)
```

3.50 $\int \cos^3(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=77

$$\frac{1}{2}a(A+B)x+\frac{a(2A+3B)\sin(c+dx)}{3d}+\frac{a(A+B)\cos(c+dx)\sin(c+dx)}{2d}+\frac{aA\cos^2(c+dx)\sin(c+dx)}{3d}$$

```
[Out] 1/2*a*(A+B)*x+1/3*a*(2*A+3*B)*sin(d*x+c)/d+1/2*a*(A+B)*cos(d*x+c)*sin(d*x+c
)/d+1/3*a*A*cos(d*x+c)^2*sin(d*x+c)/d
```

Rubi [A]

time = 0.08, antiderivative size = 77, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4081, 3872, 2715, 8, 2717}

$$\frac{a(2A+3B)\sin(c+dx)}{3d}+\frac{a(A+B)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}ax(A+B)+\frac{aA\sin(c+dx)\cos^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(A + B)*x)/2 + (a*(2*A + 3*B)*Sin[c + d*x])/(3*d) + (a*(A + B)*Cos[c + d
*x]*Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{aA\cos^2(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\int \cos^2(c+dx$$
$$= \frac{aA\cos^2(c+dx)\sin(c+dx)}{3d} + (a(A+B))\int c$$
$$= \frac{a(2A+3B)\sin(c+dx)}{3d} + \frac{a(A+B)\cos(c+d}{2d}$$
$$= \frac{1}{2}a(A+B)x + \frac{a(2A+3B)\sin(c+dx)}{3d} + \frac{a(A}{}$$

Mathematica [A]
time = 0.20, size = 65, normalized size = 0.84

$$\frac{a(6Ac+6Bc+6Adx+6Bdx+3(3A+4B)\sin(c+dx)+3(A+B)\sin(2(c+dx))+A\sin(3(c+dx)))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(6*A*c + 6*B*c + 6*A*d*x + 6*B*d*x + 3*(3*A + 4*B)*Sin[c + d*x] + 3*(A +
 B)*Sin[2*(c + d*x)] + A*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A]
time = 0.27, size = 85, normalized size = 1.10

method	result
derivativedivides	$\frac{\frac{Aa\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+Aa\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ba\sin(dx+c)}{d}$
default	$\frac{\frac{Aa\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+Aa\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ba\sin(dx+c)}{d}$
risch	$\frac{aAx}{2}+\frac{axB}{2}+\frac{3aA\sin(dx+c)}{4d}+\frac{\sin(dx+c)Ba}{d}+\frac{Aa\sin(3dx+3c)}{12d}+\frac{Aa\sin(2dx+2c)}{4d}+\frac{\sin(2dx+2c)Ba}{4d}$

norman	$\frac{a(A+B)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{(A+B)a\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a(A+B)x}{2}-a(A+B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{a(A+B)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}+\frac{a(\ldots}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*A*a*(2+cos(d*x+c)^2)*sin(d*x+c)+A*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+B*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*a*sin(d*x+c))
```

Maxima [A]
time = 0.37, size = 79, normalized size = 1.03

$$-\frac{4\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)Aa-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ba-12\,Ba\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 3*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*A*a - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a - 12*B*a*sin(d*x + c)
)/d
```

Fricas [A]
time = 1.51, size = 56, normalized size = 0.73

$$\frac{3\,(A+B)adx+\left(2\,Aa\cos\left(dx+c\right)^2+3\,(A+B)a\cos\left(dx+c\right)+2\,(2\,A+3\,B)a\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*(A + B)*a*d*x + (2*A*a*cos(d*x + c)^2 + 3*(A + B)*a*cos(d*x + c) + 2
*(2*A + 3*B)*a)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\cos^3\left(c+dx\right)dx+\int A\cos^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\cos^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\cos^3\left(c+dx\right)\sec^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] a*(Integral(A*cos(c + d*x)**3, x) + Integral(A*cos(c + d*x)**3*sec(c + d*x)
, x) + Integral(B*cos(c + d*x)**3*sec(c + d*x), x) + Integral(B*cos(c + d*x
)**3*sec(c + d*x)**2, x))
```

Giac [A]
time = 0.44, size = 124, normalized size = 1.61

$$\frac{3\,(Aa+Ba)(dx+c)+\frac{2\left(3\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+3\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+12\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+9\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(A*a + B*a)*(d*x + c) + 2*(3*A*a*tan(1/2*d*x + 1/2*c)^5 + 3*B*a*tan(
1/2*d*x + 1/2*c)^5 + 4*A*a*tan(1/2*d*x + 1/2*c)^3 + 12*B*a*tan(1/2*d*x + 1/
2*c)^3 + 9*A*a*tan(1/2*d*x + 1/2*c) + 9*B*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*
d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 2.10, size = 84, normalized size = 1.09

$$\frac{A\,a\,x}{2}+\frac{B\,a\,x}{2}+\frac{3\,A\,a\,\sin\left(c+d\,x\right)}{4\,d}+\frac{B\,a\,\sin\left(c+d\,x\right)}{d}+\frac{A\,a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{A\,a\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{B\,a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + a/cos(c + d*x)),x)
```

```
[Out] (A*a*x)/2 + (B*a*x)/2 + (3*A*a*sin(c + d*x))/(4*d) + (B*a*sin(c + d*x))/d +
 (A*a*sin(2*c + 2*d*x))/(4*d) + (A*a*sin(3*c + 3*d*x))/(12*d) + (B*a*sin(2*
c + 2*d*x))/(4*d)
```

3.51 $\int \cos^4(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=97

$$\frac{1}{8}a(3A+4B)x+\frac{a(A+B)\sin(c+dx)}{d}+\frac{a(3A+4B)\cos(c+dx)\sin(c+dx)}{8d}+\frac{aA\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{a}{}$$

```
[Out] 1/8*a*(3*A+4*B)*x+a*(A+B)*sin(d*x+c)/d+1/8*a*(3*A+4*B)*cos(d*x+c)*sin(d*x+c
)/d+1/4*a*A*cos(d*x+c)^3*sin(d*x+c)/d-1/3*a*(A+B)*sin(d*x+c)^3/d
```

Rubi [A]

time = 0.09, antiderivative size = 97, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4081, 3872, 2713, 2715, 8}

$$-\frac{a(A+B)\sin^3(c+dx)}{3d}+\frac{a(A+B)\sin(c+dx)}{d}+\frac{a(3A+4B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}ax(3A+4B)+\frac{aA\sin(c+dx)\cos^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(3*A + 4*B)*x)/8 + (a*(A + B)*Sin[c + d*x])/d + (a*(3*A + 4*B)*Cos[c + d
*x]*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*(A +
 B)*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{aA\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{1}{4}\int \cos^3(c+dx$$
$$= \frac{aA\cos^3(c+dx)\sin(c+dx)}{4d} + (a(A+B))\int c$$
$$= \frac{a(3A+4B)\cos(c+dx)\sin(c+dx)}{8d} + \frac{aA\cos^3}{}$$
$$= \frac{1}{8}a(3A+4B)x + \frac{a(A+B)\sin(c+dx)}{d} + \frac{a(3A}{}$$

Mathematica [A]
time = 0.27, size = 75, normalized size = 0.77

$$\frac{a(36Ac+48Bc+36Adx+48Bdx+96(A+B)\sin(c+dx)-32(A+B)\sin^3(c+dx)+24(A+B)\sin(2(c+dx))+3A\sin(4(c+dx)))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(36*A*c + 48*B*c + 36*A*d*x + 48*B*d*x + 96*(A + B)*Sin[c + d*x] - 32*(A
 + B)*Sin[c + d*x]^3 + 24*(A + B)*Sin[2*(c + d*x)] + 3*A*Sin[4*(c + d*x)]))
/(96*d)
```

Maple [A]
time = 0.34, size = 107, normalized size = 1.10

method	result
derivativedivides	$\frac{Aa\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Aa\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+\frac{Ba\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+Ba\Big(}{d}$

default	$\frac{Aa\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Aa\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+\frac{Ba\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+Ba\left(\cos\right.}{d}$
risch	$\frac{3aAx}{8}+\frac{axB}{2}+\frac{3aA\sin(dx+c)}{4d}+\frac{3\sin(dx+c)Ba}{4d}+\frac{Aa\sin(4dx+4c)}{32d}+\frac{Aa\sin(3dx+3c)}{12d}+\frac{\sin(3dx+3c)Ba}{12d}+A$
norman	$-\frac{a(3A+4B)x}{8}+\frac{2a(A-2B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{a(3A-4B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d}+\frac{a(3A+4B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{3a(3A+4B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*A
*a*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*B*a*(2+cos(d*x+c)^2)*sin(d*x+c)+B*a*(1/2
*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.36, size = 101, normalized size = 1.04

$$-\frac{32\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)Aa-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\sin\left(2\,dx+2\,c\right)\right)Aa+32\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)Ba-24\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ba}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/96*(32*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 3*(12*d*x + 12*c + sin(4*
d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))
*B*a - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a)/d
```

Fricas [A]
time = 2.30, size = 74, normalized size = 0.76

$$\frac{3\left(3\,A+4\,B\right)adx+\left(6\,Aa\cos\left(dx+c\right)^3+8\left(A+B\right)a\cos\left(dx+c\right)^2+3\left(3\,A+4\,B\right)a\cos\left(dx+c\right)+16\left(A+B\right)a\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(3*A + 4*B)*a*d*x + (6*A*a*cos(d*x + c)^3 + 8*(A + B)*a*cos(d*x + c
)^2 + 3*(3*A + 4*B)*a*cos(d*x + c) + 16*(A + B)*a)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\cos^4\left(c+dx\right)dx+\int A\cos^4\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\cos^4\left(c+dx\right)\sec\left(c+dx\right)dx+\int B\cos^4\left(c+dx\right)\sec^2\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] a*(Integral(A*cos(c + d*x)**4, x) + Integral(A*cos(c + d*x)**4*sec(c + d*x)
, x) + Integral(B*cos(c + d*x)**4*sec(c + d*x), x) + Integral(B*cos(c + d*x
)**4*sec(c + d*x)**2, x))
```

<u>Giac</u> [A]
time = 0.46, size = 156, normalized size = 1.61

$$\frac{3\,(3\,Aa+4\,Ba)(dx+c)+\frac{2\,\left(9\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+12\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+49\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+28\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+31\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+52\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+39\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/24*(3*(3*A*a + 4*B*a)*(d*x + c) + 2*(9*A*a*tan(1/2*d*x + 1/2*c)^7 + 12*B*
a*tan(1/2*d*x + 1/2*c)^7 + 49*A*a*tan(1/2*d*x + 1/2*c)^5 + 28*B*a*tan(1/2*d
*x + 1/2*c)^5 + 31*A*a*tan(1/2*d*x + 1/2*c)^3 + 52*B*a*tan(1/2*d*x + 1/2*c)
^3 + 39*A*a*tan(1/2*d*x + 1/2*c) + 36*B*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*
x + 1/2*c)^2 + 1)^4)/d
```

<u>Mupad</u> [B]
time = 4.67, size = 184, normalized size = 1.90

$$\frac{\left(\frac{3\,A\,a}{4}+B\,a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{49\,A\,a}{12}+\frac{7\,B\,a}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{31\,A\,a}{12}+\frac{13\,B\,a}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{13\,A\,a}{4}+3\,B\,a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{a\,\mathrm{atan}\left(\frac{a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(3\,A+4\,B\right)}{4\,\left(\frac{3\,A\,a}{4}+B\,a\right)}\right)\,\left(3\,A+4\,B\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + a/cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((13*A*a)/4 + 3*B*a) + tan(c/2 + (d*x)/2)^7*((3*A*a)/4
+ B*a) + tan(c/2 + (d*x)/2)^3*((31*A*a)/12 + (13*B*a)/3) + tan(c/2 + (d*x)/
2)^5*((49*A*a)/12 + (7*B*a)/3))/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d
*x)/2)^4 + 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (a*atan((a
*tan(c/2 + (d*x)/2)*(3*A + 4*B))/(4*((3*A*a)/4 + B*a)))*(3*A + 4*B))/(4*d)
```

3.52 $\int \cos^5(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=125

$$\frac{3}{8}a(A+B)x+\frac{a(4A+5B)\sin(c+dx)}{5d}+\frac{3a(A+B)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a(A+B)\cos^3(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 3/8*a*(A+B)*x+1/5*a*(4*A+5*B)*sin(d*x+c)/d+3/8*a*(A+B)*cos(d*x+c)*sin(d*x+c
)/d+1/4*a*(A+B)*cos(d*x+c)^3*sin(d*x+c)/d+1/5*a*A*cos(d*x+c)^4*sin(d*x+c)/d
-1/15*a*(4*A+5*B)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4081, 3872, 2715, 8, 2713}

$$-\frac{a(4A+5B)\sin^3(c+dx)}{15d}+\frac{a(4A+5B)\sin(c+dx)}{5d}+\frac{a(A+B)\sin(c+dx)\cos^3(c+dx)}{4d}+\frac{3a(A+B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{3}{8}ax(A+B)+\frac{aA\sin(c+dx)\cos^4(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*a*(A + B)*x)/8 + (a*(4*A + 5*B)*Sin[c + d*x])/(5*d) + (3*a*(A + B)*Cos[c
 + d*x]*Sin[c + d*x])/(8*d) + (a*(A + B)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d)
 + (a*A*Cos[c + d*x]^4*Sin[c + d*x])/(5*d) - (a*(4*A + 5*B)*Sin[c + d*x]^3)
/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{aA\cos^4(c+dx)\sin(c+dx)}{5d} - \frac{1}{5}\int \cos^4(c+dx$$

$$= \frac{aA\cos^4(c+dx)\sin(c+dx)}{5d} + (a(A+B))\int \mathrm{c}$$

$$= \frac{a(A+B)\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{aA\cos^4(c}{}$$

$$= \frac{a(4A+5B)\sin(c+dx)}{5d} + \frac{3a(A+B)\cos(c+}{8d}$$

$$= \frac{3}{8}a(A+B)x + \frac{a(4A+5B)\sin(c+dx)}{5d} + \frac{3a(}{}$$

Mathematica [A]
time = 0.28, size = 110, normalized size = 0.88

$$\frac{a(180Ac+180Bc+180Adx+180Bdx+60(5A+8B)\sin(c+dx)-160B\sin^3(c+dx)+120(A+B)\sin(2(c+dx))+50A\sin(3(c+dx))+15A\sin(4(c+dx))+15B\sin(4(c+dx))+6A\sin(5(c+dx)))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(180*A*c + 180*B*c + 180*A*d*x + 180*B*d*x + 60*(5*A + 8*B)*Sin[c + d*x]
 - 160*B*Sin[c + d*x]^3 + 120*(A + B)*Sin[2*(c + d*x)] + 50*A*Sin[3*(c + d*
x)] + 15*A*Sin[4*(c + d*x)] + 15*B*Sin[4*(c + d*x)] + 6*A*Sin[5*(c + d*x)])
)/(480*d)
```

Maple [A]
time = 0.41, size = 128, normalized size = 1.02

method	result
derivativedivides	$\frac{\frac{Aa\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+Aa\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+Ba\left(\frac{\left(\cos^3(dx+c)+3\right.}{}\right.}{d}$
default	$\frac{\frac{Aa\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+Aa\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+Ba\left(\frac{\left(\cos^3(dx+c)+3\right.}{}\right.}{d}$
risch	$\frac{3aAx}{8}+\frac{3axB}{8}+\frac{5aA\sin(dx+c)}{8d}+\frac{3\sin(dx+c)Ba}{4d}+\frac{Aa\sin(5dx+5c)}{80d}+\frac{Aa\sin(4dx+4c)}{32d}+\frac{\sin(4dx+4c)Ba}{32d}+$
norman	$-\frac{3a(A+B)x}{8}+\frac{a(A-31B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{3a(A+B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2}-\frac{15a(A+B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}+\frac{15a(A+B)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/5*A*a*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+A*a*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+B*a*(1/4*(cos(d*x+c)^3+3/
2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*B*a*(2+cos(d*x+c)^2)*sin(d*x+c)
)
```

Maxima [A]
time = 0.38, size = 124, normalized size = 0.99

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa+15\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Aa-160\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba+15\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Ba}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a + 15
*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a - 160*(sin(d*x
 + c)^3 - 3*sin(d*x + c))*B*a + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*si
n(2*d*x + 2*c))*B*a)/d
```

Fricas [A]
time = 2.20, size = 88, normalized size = 0.70

$$\frac{45(A+B)adx+\left(24Aa\cos(dx+c)^4+30(A+B)a\cos(dx+c)^3+8(4A+5B)a\cos(dx+c)^2+45(A+B)a\cos(dx+c)+16(4A+5B)a\right)\sin(dx+c)}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/120*(45*(A + B)*a*d*x + (24*A*a*cos(d*x + c)^4 + 30*(A + B)*a*cos(d*x + c
)^3 + 8*(4*A + 5*B)*a*cos(d*x + c)^2 + 45*(A + B)*a*cos(d*x + c) + 16*(4*A
+ 5*B)*a)*sin(d*x + c))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.46, size = 184, normalized size = 1.47

$$\frac{45\,(Aa+Ba)(dx+c)+\frac{2\left(45\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+45\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+130\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+290\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+464\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+400\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+190\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+350\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+195\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+195\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/120*(45*(A*a + B*a)*(d*x + c) + 2*(45*A*a*tan(1/2*d*x + 1/2*c)^9 + 45*B*a
*tan(1/2*d*x + 1/2*c)^9 + 130*A*a*tan(1/2*d*x + 1/2*c)^7 + 290*B*a*tan(1/2*
d*x + 1/2*c)^7 + 464*A*a*tan(1/2*d*x + 1/2*c)^5 + 400*B*a*tan(1/2*d*x + 1/2
*c)^5 + 190*A*a*tan(1/2*d*x + 1/2*c)^3 + 350*B*a*tan(1/2*d*x + 1/2*c)^3 + 1
95*A*a*tan(1/2*d*x + 1/2*c) + 195*B*a*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 + 1)^5)/d
```

Mupad [**B**]
time = 4.81, size = 212, normalized size = 1.70

$$\frac{\left(\frac{3\,A\,a}{4}+\frac{3\,B\,a}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{13\,A\,a}{6}+\frac{29\,B\,a}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{116\,A\,a}{15}+\frac{20\,B\,a}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{19\,A\,a}{6}+\frac{35\,B\,a}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{13\,A\,a}{4}+\frac{13\,B\,a}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{3\,a\,\mathrm{atan}\left(\frac{3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A+B\right)}{4\,\left(\frac{3\,A\,a}{4}+\frac{3\,B\,a}{4}\right)}\right)\,\left(A+B\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(A + B/cos(c + d*x))*(a + a/cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((13*A*a)/4 + (13*B*a)/4) + tan(c/2 + (d*x)/2)^9*((3*A*
a)/4 + (3*B*a)/4) + tan(c/2 + (d*x)/2)^7*((13*A*a)/6 + (29*B*a)/6) + tan(c/
2 + (d*x)/2)^3*((19*A*a)/6 + (35*B*a)/6) + tan(c/2 + (d*x)/2)^5*((116*A*a)/
15 + (20*B*a)/3))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2)^4 + 10
*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 + 1)
) + (3*a*atan((3*a*tan(c/2 + (d*x)/2)*(A + B))/(4*((3*A*a)/4 + (3*B*a)/4)))
*(A + B))/(4*d)
```

3.53 $\int \sec^3(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=169

$$\frac{a^2(7A+6B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(10A+9B)\tan(c+dx)}{5d}+\frac{a^2(7A+6B)\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^2(5A$$

```
[Out] 1/8*a^2*(7*A+6*B)*arctanh(sin(d*x+c))/d+1/5*a^2*(10*A+9*B)*tan(d*x+c)/d+1/8
*a^2*(7*A+6*B)*sec(d*x+c)*tan(d*x+c)/d+1/20*a^2*(5*A+6*B)*sec(d*x+c)^3*tan(
d*x+c)/d+1/5*B*sec(d*x+c)^3*(a^2+a^2*sec(d*x+c))*tan(d*x+c)/d+1/15*a^2*(10*
A+9*B)*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.18, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4103, 4082, 3872, 3853, 3855, 3852}

$$\frac{a^2(10A+9B)\tan^3(c+dx)}{15d}+\frac{a^2(10A+9B)\tan(c+dx)}{5d}+\frac{a^2(7A+6B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(5A+6B)\tan(c+dx)\sec^3(c+dx)}{20d}+\frac{a^2(7A+6B)\tan(c+dx)\sec(c+dx)}{8d}+\frac{B\tan(c+dx)\sec^3(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(7*A + 6*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(10*A + 9*B)*Tan[c + d
*x])/(5*d) + (a^2*(7*A + 6*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (a^2*(5*A
+ 6*B)*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (B*Sec[c + d*x]^3*(a^2 + a^2*S
ec[c + d*x])*Tan[c + d*x])/(5*d) + (a^2*(10*A + 9*B)*Tan[c + d*x]^3)/(15*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \sec^3(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx &= \frac{B\sec^3(c+dx)\left(a^2+a^2\sec(c+dx)\right)\tan(c+d\\
&= \frac{a^2(5A+6B)\sec^3(c+dx)\tan(c+dx)}{20d}+\frac{B\sec\\
&= \frac{a^2(5A+6B)\sec^3(c+dx)\tan(c+dx)}{20d}+\frac{B\sec\\
&= \frac{a^2(7A+6B)\sec(c+dx)\tan(c+dx)}{8d}+\frac{a^2(5A\\
&= \frac{a^2(7A+6B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(10A+
\end{aligned}
$$

Mathematica [A]
time = 1.36, size = 280, normalized size = 1.66

$$
-\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(240(7A+6B)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\sec(c)(80(14A+15B)\sin(dx)-240(2A+B)\sin(2c+dx)+330A\sin(c+2dx)+420B\sin(c+2dx)+330A\sin(3c+2dx)+420B\sin(3c+2dx)+800A\sin(2c+3dx)+720B\sin(2c+3dx)+105A\sin(3c+4dx)+90B\sin(3c+4dx)+105A\sin(5c+4dx)+90B\sin(5c+4dx)+160A\sin(4c+5dx)+144B\sin(4c+5dx))\right)}{7680d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] -1/7680*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*Sec[c + d*x]^5*(240*(7
*A + 6*B)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(80*(14*A + 15*B)*Sin[d*x] - 2
40*(2*A + B)*Sin[2*c + d*x] + 330*A*Sin[c + 2*d*x] + 420*B*Sin[c + 2*d*x] +
 330*A*Sin[3*c + 2*d*x] + 420*B*Sin[3*c + 2*d*x] + 800*A*Sin[2*c + 3*d*x] +
 720*B*Sin[2*c + 3*d*x] + 105*A*Sin[3*c + 4*d*x] + 90*B*Sin[3*c + 4*d*x] +
105*A*Sin[5*c + 4*d*x] + 90*B*Sin[5*c + 4*d*x] + 160*A*Sin[4*c + 5*d*x] + 1
44*B*Sin[4*c + 5*d*x]))))/d
```

Maple [A]
time = 0.38, size = 223, normalized size = 1.32

method	result
norman	$\frac{\frac{7a^{2}(7A+6B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{a^{2}(7A+6B)\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{a^{2}(25A+26B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{8a^{2}(25A+27B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{a^{2}(79A}{}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}}$
derivativedivides	$a^{2}A\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-a^{2}B\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right.}{15}\right.$
default	$a^{2}A\left(-\left(-\frac{\left(\sec^{3}(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-a^{2}B\left(-\frac{8}{15}-\frac{\left(\sec^{4}(dx+c)\right)}{5}-\frac{4\left(\sec^{2}(dx+c)\right.}{15}\right.$
risch	$-\frac{ia^{2}\left(105A\,e^{9i(dx+c)}+90B\,e^{9i(dx+c)}+330A\,e^{7i(dx+c)}+420B\,e^{7i(dx+c)}-480A\,e^{6i(dx+c)}-240B\,e^{6i(dx+c)}-1120A\,e^{4i(dx+}\right.}{60}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c
)+tan(d*x+c)))-a^2*B*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)-
2*a^2*A*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+2*a^2*B*(-(-1/4*sec(d*x+c)^3-3/8
*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+a^2*A*(1/2*sec(d*x+c
)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-a^2*B*(-2/3-1/3*sec(d*x+c)^2)*t
an(d*x+c))
```

Maxima [A]
time = 0.40, size = 278, normalized size = 1.64

$$\frac{160\left(\tan(dx+c)^{3}+3\tan(dx+c)\right)Aa^{2}+16\left(3\tan(dx+c)^{5}+10\tan(dx+c)^{3}+15\tan(dx+c)\right)Ba^{2}+80\left(\tan(dx+c)^{3}+3\tan(dx+c)\right)Ba^{2}-15Aa^{2}\left(\frac{2\left(3\sin(dx+c)^{3}-5\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\sin(dx+c)^{2}+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-30Ba^{2}\left(\frac{2\left(3\sin(dx+c)^{3}-5\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\sin(dx+c)^{2}+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-60Aa^{2}\left(\frac{2\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/240*(160*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 + 16*(3*tan(d*x + c)^5 +
 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^2 + 80*(tan(d*x + c)^3 + 3*tan(d*
x + c))*B*a^2 - 15*A*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 30*B*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
- 60*A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 2.48, size = 165, normalized size = 0.98

$$\frac{15\,(7\,A+6\,B)a^2\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\,(7\,A+6\,B)a^2\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\,(10\,A+9\,B)a^2\cos\left(dx+c\right)^4+15\,(7\,A+6\,B)a^2\cos\left(dx+c\right)^3+8\,(10\,A+9\,B)a^2\cos\left(dx+c\right)^2+30\,(A+2\,B)a^2\cos\left(dx+c\right)+24\,Ba^2\right)\sin\left(dx+c\right)}{240\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(7*A + 6*B)*a^2*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(7*A +
6*B)*a^2*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(16*(10*A + 9*B)*a^2*cos
(d*x + c)^4 + 15*(7*A + 6*B)*a^2*cos(d*x + c)^3 + 8*(10*A + 9*B)*a^2*cos(d*
x + c)^2 + 30*(A + 2*B)*a^2*cos(d*x + c) + 24*B*a^2)*sin(d*x + c))/(d*cos(d
*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int A\sec^3\left(c+dx\right)dx+\int 2A\sec^4\left(c+dx\right)dx+\int A\sec^5\left(c+dx\right)dx+\int B\sec^4\left(c+dx\right)dx+\int 2B\sec^5\left(c+dx\right)dx+\int B\sec^6\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x)**3, x) + Integral(2*A*sec(c + d*x)**4, x) + I
ntegral(A*sec(c + d*x)**5, x) + Integral(B*sec(c + d*x)**4, x) + Integral(2
*B*sec(c + d*x)**5, x) + Integral(B*sec(c + d*x)**6, x))
```

Giac [A]
time = 0.51, size = 246, normalized size = 1.46

$$\frac{15\,(7\,Aa^2+6\,Ba^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(7\,Aa^2+6\,Ba^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(105\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+90\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-490\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-420\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+800\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+864\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-790\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-540\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+375\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+390\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(7*A*a^2 + 6*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(7*A*
a^2 + 6*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(105*A*a^2*tan(1/2*d*
```

```
x + 1/2*c)^9 + 90*B*a^2*tan(1/2*d*x + 1/2*c)^9 - 490*A*a^2*tan(1/2*d*x + 1/
2*c)^7 - 420*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 800*A*a^2*tan(1/2*d*x + 1/2*c)^
5 + 864*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 790*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 5
40*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 375*A*a^2*tan(1/2*d*x + 1/2*c) + 390*B*a^
2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B]
time = 4.61, size = 224, normalized size = 1.33

$$\frac{a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(7\,A+6\,B\right)}{4\,d}-\frac{\left(\frac{7\,A\,a^2}{4}+\frac{3\,B\,a^2}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{49\,A\,a^2}{6}-7\,B\,a^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{40\,A\,a^2}{3}+\frac{72\,B\,a^2}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{79\,A\,a^2}{6}-9\,B\,a^2\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{25\,A\,a^2}{4}+\frac{13\,B\,a^2}{2}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/cos(c + d*x)^3,x)
```

```
[Out] (a^2*atanh(tan(c/2 + (d*x)/2))*(7*A + 6*B))/(4*d) - (tan(c/2 + (d*x)/2)*((2
5*A*a^2)/4 + (13*B*a^2)/2) + tan(c/2 + (d*x)/2)^9*((7*A*a^2)/4 + (3*B*a^2)/
2) - tan(c/2 + (d*x)/2)^7*((49*A*a^2)/6 + 7*B*a^2) - tan(c/2 + (d*x)/2)^3*(
(79*A*a^2)/6 + 9*B*a^2) + tan(c/2 + (d*x)/2)^5*((40*A*a^2)/3 + (72*B*a^2)/5
))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x
)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.54 $\int \sec^2(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=138

$$\frac{a^2(8A+7B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(8A+7B)\tan(c+dx)}{6d}+\frac{a^2(8A+7B)\sec(c+dx)\tan(c+dx)}{24d}+\frac{(4A\cdots}{}$$

```
[Out] 1/8*a^2*(8*A+7*B)*arctanh(sin(d*x+c))/d+1/6*a^2*(8*A+7*B)*tan(d*x+c)/d+1/24
*a^2*(8*A+7*B)*sec(d*x+c)*tan(d*x+c)/d+1/12*(4*A-B)*(a+a*sec(d*x+c))^2*tan(
d*x+c)/d+1/4*B*(a+a*sec(d*x+c))^3*tan(d*x+c)/a/d
```

Rubi [A]
time = 0.16, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4095, 4086, 3873, 3852, 8, 4131, 3855}

$$\frac{a^2(8A+7B)\tan(c+dx)}{6d}+\frac{a^2(8A+7B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(8A+7B)\tan(c+dx)\sec(c+dx)}{24d}+\frac{(4A-B)\tan(c+dx)(a\sec(c+dx)+a)^2}{12d}+\frac{B\tan(c+dx)(a\sec(c+dx)+a)^3}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(8*A + 7*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^2*(8*A + 7*B)*Tan[c + d*
x])/(6*d) + (a^2*(8*A + 7*B)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((4*A - B)
*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + (B*(a + a*Sec[c + d*x])^3*Ta
n[c + d*x])/(4*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
```

```
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx &= \frac{B(a+a\sec(c+dx))^3\tan(c+dx)}{4ad}+\frac{\int \sec(c+\\
&= \frac{(4A-B)(a+a\sec(c+dx))^2\tan(c+dx)}{12d}+\frac{B\\
&= \frac{(4A-B)(a+a\sec(c+dx))^2\tan(c+dx)}{12d}+\frac{B\\
&= \frac{a^2(8A+7B)\sec(c+dx)\tan(c+dx)}{24d}+\frac{(4A-\\
&= \frac{a^2(8A+7B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^2(8A+7\end{aligned}$$

Mathematica [**A**]

time = 1.17, size = 262, normalized size = 1.90

$$-\frac{a^2(1+\cos(c+dx))^2\sec^4\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(24(8A+7B)\cos^4(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\sec(c)(-24(5A+4B)\sin(c)+3(8A+15B)\sin(dx)+24A\sin(2c+dx)+45B\sin(2c+dx)+136A\sin(c+2dx)+128B\sin(c+2dx)-24A\sin(3c+2dx)+24A\sin(2c+3dx)+21B\sin(2c+3dx)+24A\sin(4c+3dx)+21B\sin(4c+3dx)+40A\sin(3c+4dx)+32B\sin(3c+4dx))\right)}{768d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] -1/768*(a^2*(1 + Cos[c + d*x])^2*Sec[(c + d*x)/2]^4*Sec[c + d*x]^4*(24*(8*A
 + 7*B)*Cos[c + d*x]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[
(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-24*(5*A + 4*B)*Sin[c] + 3*(8*A
 + 15*B)*Sin[d*x] + 24*A*Sin[2*c + d*x] + 45*B*Sin[2*c + d*x] + 136*A*Sin[c
 + 2*d*x] + 128*B*Sin[c + 2*d*x] - 24*A*Sin[3*c + 2*d*x] + 24*A*Sin[2*c + 3
*d*x] + 21*B*Sin[2*c + 3*d*x] + 24*A*Sin[4*c + 3*d*x] + 21*B*Sin[4*c + 3*d*
x] + 40*A*Sin[3*c + 4*d*x] + 32*B*Sin[3*c + 4*d*x])))/d
```

Maple [A]
time = 0.36, size = 187, normalized size = 1.36

method	result
norman	$\frac{\frac{11a^2(8A+7B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{a^2(8A+7B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{a^2(24A+25B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{a^2(136A+83B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4} -$
derivativedivides	$\frac{-a^2A\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+a^2B\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right.}{}$
default	$\frac{-a^2A\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+a^2B\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right.}{}$
risch	$-\frac{ia^2\left(24A\,e^{7i(dx+c)}+21B\,e^{7i(dx+c)}-24A\,e^{6i(dx+c)}+24A\,e^{5i(dx+c)}+45B\,e^{5i(dx+c)}-120A\,e^{4i(dx+c)}-96B\,e^{4i(dx+c)}-2\right.}{12d\left(e^{2i(dx+c)}+\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-a^2*A*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^2*B*(-(-1/4*sec(d*x+c)^3-3
/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+2*a^2*A*(1/2*sec(d
*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-2*a^2*B*(-2/3-1/3*sec(d*x+c
)^2)*tan(d*x+c)+a^2*A*tan(d*x+c)+a^2*B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(se
c(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.41, size = 230, normalized size = 1.67

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^2+32\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^2-3Ba^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-24Aa^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-12Ba^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+48Aa^2\tan(dx+c)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^2 + 32*(tan(d*x + c)^3 + 3*t
an(d*x + c))*B*a^2 - 3*B*a^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 24*A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 12*B*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*A*a^2*tan(d*x +
 c))/d
```

Fricas [**A**]
time = 2.61, size = 145, normalized size = 1.05

$$\frac{3\,(8\,A+7\,B)a^2\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,(8\,A+7\,B)a^2\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\,(5\,A+4\,B)a^2\cos\left(dx+c\right)^3+3\,(8\,A+7\,B)a^2\cos\left(dx+c\right)^2+8\,(A+2\,B)a^2\cos\left(dx+c\right)+6\,Ba^2\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(8*A + 7*B)*a^2*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(8*A + 7*B
)*a^2*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(5*A + 4*B)*a^2*cos(d*x
+ c)^3 + 3*(8*A + 7*B)*a^2*cos(d*x + c)^2 + 8*(A + 2*B)*a^2*cos(d*x + c) +
6*B*a^2)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int A\sec^2\left(c+dx\right)dx+\int 2A\sec^3\left(c+dx\right)dx+\int A\sec^4\left(c+dx\right)dx+\int B\sec^3\left(c+dx\right)dx+\int 2B\sec^4\left(c+dx\right)dx+\int B\sec^5\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x)**2, x) + Integral(2*A*sec(c + d*x)**3, x) + I
ntegral(A*sec(c + d*x)**4, x) + Integral(B*sec(c + d*x)**3, x) + Integral(2
*B*sec(c + d*x)**4, x) + Integral(B*sec(c + d*x)**5, x))
```

Giac [**A**]
time = 0.51, size = 212, normalized size = 1.54

$$\frac{3\,(8\,Aa^2+7\,Ba^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(8\,Aa^2+7\,Ba^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(24\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+21\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-88\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-77\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+136\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+83\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-72\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-75\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(8*A*a^2 + 7*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(8*A*a^2
 + 7*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(24*A*a^2*tan(1/2*d*x +
1/2*c)^7 + 21*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 88*A*a^2*tan(1/2*d*x + 1/2*c)^
5 - 77*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 136*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 83
*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 72*A*a^2*tan(1/2*d*x + 1/2*c) - 75*B*a^2*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 4.47, size = 183, normalized size = 1.33

$$\frac{\left(-2\,A\,a^2-\frac{7\,B\,a^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{22\,A\,a^2}{3}+\frac{77\,B\,a^2}{12}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{34\,A\,a^2}{3}-\frac{83\,B\,a^2}{12}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(6\,A\,a^2+\frac{25\,B\,a^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{2\,a^2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(A+\frac{7\,B}{8}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/cos(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(6*A*a^2 + (25*B*a^2)/4) - tan(c/2 + (d*x)/2)^7*(2*A*a^
2 + (7*B*a^2)/4) + tan(c/2 + (d*x)/2)^5*((22*A*a^2)/3 + (77*B*a^2)/12) - ta
n(c/2 + (d*x)/2)^3*((34*A*a^2)/3 + (83*B*a^2)/12))/(d*(6*tan(c/2 + (d*x)/2)
^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8
 + 1)) + (2*a^2*atanh(tan(c/2 + (d*x)/2))*(A + (7*B)/8))/d
```

3.55 $\int \sec(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=103

$$\frac{a^2(3A+2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2a^2(3A+2B)\tan(c+dx)}{3d}+\frac{a^2(3A+2B)\sec(c+dx)\tan(c+dx)}{6d}+\frac{B(a+$$

```
[Out] 1/2*a^2*(3*A+2*B)*arctanh(sin(d*x+c))/d+2/3*a^2*(3*A+2*B)*tan(d*x+c)/d+1/6*
a^2*(3*A+2*B)*sec(d*x+c)*tan(d*x+c)/d+1/3*B*(a+a*sec(d*x+c))^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4086, 3873, 3852, 8, 4131, 3855}

$$\frac{2a^2(3A+2B)\tan(c+dx)}{3d}+\frac{a^2(3A+2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(3A+2B)\tan(c+dx)\sec(c+dx)}{6d}+\frac{B\tan(c+dx)(a\sec(c+dx)+a)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(3*A + 2*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*a^2*(3*A + 2*B)*Tan[c +
d*x])/(3*d) + (a^2*(3*A + 2*B)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (B*(a + a
*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx &= \frac{B(a+a\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{3}(3A+2\\ &= \frac{B(a+a\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{3}(3A+2\\ &= \frac{a^2(3A+2B)\sec(c+dx)\tan(c+dx)}{6d}+\frac{B(a+}{}\\ &= \frac{a^2(3A+2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2a^2(3A+}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 481 vs. 2(103) = 206.
time = 6.05, size = 481, normalized size = 4.67

$$\frac{a^2\cos^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^2(A+B\sec(c+dx))\left(-6(3A+2B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+6(3A+2B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{2B\sin\left(\frac{dx}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}+\frac{(3A+7B)\cos\left(\frac{c}{2}\right)-(3A+5B)\sin\left(\frac{c}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4(6A+5B)\sin\left(\frac{dx}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{2B\sin\left(\frac{dx}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3}-\frac{(3A+7B)\cos\left(\frac{c}{2}\right)+(3A+5B)\sin\left(\frac{c}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4(6A+5B)\sin\left(\frac{dx}{2}\right)}{\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)}{48d(B+A\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*Cos[c + d*x]^3*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + B*Sec[c +
d*x])*(-6*(3*A + 2*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 6*(3*A + 2
*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (2*B*Sin[(d*x)/2])/((Cos[c/2
] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3) + ((3*A + 7*B)*Cos[c
/2] - (3*A + 5*B)*Sin[c/2])/((Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[
(c + d*x)/2])^2) + (4*(6*A + 5*B)*Sin[(d*x)/2])/((Cos[c/2] - Sin[c/2])*(Cos
```

```
[(c + d*x)/2] - Sin[(c + d*x)/2])) + (2*B*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/
2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^3) - ((3*A + 7*B)*Cos[c/2] + (3*A
 + 5*B)*Sin[c/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2) + (4*(6*A + 5*B)*Sin[(d*x)/2])/((Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2]))))/(48*d*(B + A*Cos[c + d*x]))
```

Maple [A]
time = 0.28, size = 145, normalized size = 1.41

method	result
derivativedivides	$\frac{a^2 A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a^2 B\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+2a^2 A\tan(dx+c)+2a^2 B\left(\sec\right.}{d}$
default	$\frac{a^2 A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a^2 B\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+2a^2 A\tan(dx+c)+2a^2 B\left(\sec\right.}{d}$
norman	$\frac{\frac{8a^2(3A+2B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{a^2(3A+2B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{a^2(5A+6B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{a^2(3A+2B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+$
risch	$-\frac{ia^2\left(3A\,e^{5i(dx+c)}+6B\,e^{5i(dx+c)}-12A\,e^{4i(dx+c)}-6B\,e^{4i(dx+c)}-24A\,e^{2i(dx+c)}-24B\,e^{2i(dx+c)}-3\,e^{i(dx+c)}A-6B\,e^{i(dx+c)}\right.}{3d\left(e^{2i(dx+c)}+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*A*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-a^2*B*
(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+2*a^2*A*tan(d*x+c)+2*a^2*B*(1/2*sec(d*x+
c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+a^2*A*ln(sec(d*x+c)+tan(d*x+c)
)+a^2*B*tan(d*x+c))
```

Maxima [A]
time = 0.39, size = 167, normalized size = 1.62

$$\frac{4\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Ba^2-3\,Aa^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-6\,Ba^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+12\,Aa^2\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+24\,Aa^2\tan\left(dx+c\right)+12\,Ba^2\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2 - 3*A*a^2*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 6*B*
a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 12*A*a^2*log(sec(d*x + c) + tan(d*x + c)) + 24*A*a^2*tan(d
*x + c) + 12*B*a^2*tan(d*x + c))/d
```

Fricas [A]
time = 2.15, size = 125, normalized size = 1.21

$$\frac{3\,(3\,A+2\,B)a^2\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,(3\,A+2\,B)a^2\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(2\,(6\,A+5\,B)a^2\cos\left(dx+c\right)^2+3\,(A+2\,B)a^2\cos\left(dx+c\right)+2\,Ba^2\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(3*A + 2*B)*a^2*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(3*A + 2*B
)*a^2*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(6*A + 5*B)*a^2*cos(d*x
+ c)^2 + 3*(A + 2*B)*a^2*cos(d*x + c) + 2*B*a^2)*sin(d*x + c))/(d*cos(d*x +
 c)^3)
```

Sympy [**F**]

time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int A\sec\left(c+dx\right)dx+\int 2A\sec^{2}\left(c+dx\right)dx+\int A\sec^{3}\left(c+dx\right)dx+\int B\sec^{2}\left(c+dx\right)dx+\int 2B\sec^{3}\left(c+dx\right)dx+\int B\sec^{4}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] a**2*(Integral(A*sec(c + d*x), x) + Integral(2*A*sec(c + d*x)**2, x) + Inte
gral(A*sec(c + d*x)**3, x) + Integral(B*sec(c + d*x)**2, x) + Integral(2*B*
sec(c + d*x)**3, x) + Integral(B*sec(c + d*x)**4, x))
```

Giac [**A**]

time = 0.53, size = 178, normalized size = 1.73

$$\frac{3\left(3\,Aa^{2}+2\,Ba^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(3\,Aa^{2}+2\,Ba^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(9\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-24\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-16\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,Aa^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+18\,Ba^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(3*A*a^2 + 2*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(3*A*a^2
+ 2*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(9*A*a^2*tan(1/2*d*x + 1/
2*c)^5 + 6*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 24*A*a^2*tan(1/2*d*x + 1/2*c)^3 -
 16*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 15*A*a^2*tan(1/2*d*x + 1/2*c) + 18*B*a^2
*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [**B**]

time = 3.81, size = 145, normalized size = 1.41

$$\frac{2\,a^2\,\mathrm{atanh}\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(\frac{3\,A}{2}+B\right)}{d}-\frac{\left(3\,A\,a^2+2\,B\,a^2\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5+\left(-8\,A\,a^2-\frac{16\,B\,a^2}{3}\right)\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3+\left(5\,A\,a^2+6\,B\,a^2\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+3\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/cos(c + d*x),x)
```

```
[Out] (2*a^2*atanh(tan(c/2 + (d*x)/2))*((3*A)/2 + B))/d - (tan(c/2 + (d*x)/2)*(5*
A*a^2 + 6*B*a^2) + tan(c/2 + (d*x)/2)^5*(3*A*a^2 + 2*B*a^2) - tan(c/2 + (d*
x)/2)^3*(8*A*a^2 + (16*B*a^2)/3))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 +
(d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.56 $\int (a + a\sec(c + dx))^2 (A + B\sec(c + dx))\,dx$

Optimal. Leaf size=82

$$a^2Ax+\frac{a^2(4A+3B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(2A+3B)\tan(c+dx)}{2d}+\frac{B(a^2+a^2\sec(c+dx))\tan(c+dx)}{2d}$$

```
[Out] a^2*A*x+1/2*a^2*(4*A+3*B)*arctanh(sin(d*x+c))/d+1/2*a^2*(2*A+3*B)*tan(d*x+c
)/d+1/2*B*(a^2+a^2*sec(d*x+c))*tan(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4002, 3999, 3852, 8, 3855}

$$\frac{a^2(2A+3B)\tan(c+dx)}{2d}+\frac{a^2(4A+3B)\tanh^{-1}(\sin(c+dx))}{2d}+a^2Ax+\frac{B\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] a^2*A*x + (a^2*(4*A + 3*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (a^2*(2*A + 3*B)*
Tan[c + d*x])/(2*d) + (B*(a^2 + a^2*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3999

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 4002

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m +
(b*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*
m]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx &= \frac{B(a^2+a^2\sec(c+dx))\tan(c+dx)}{2d}+\frac{1}{2}\int (a+a\sec(c+d\\ &= a^2Ax+\frac{B(a^2+a^2\sec(c+dx))\tan(c+dx)}{2d}+\frac{1}{2}(a^2(2A+3\\ &= a^2Ax+\frac{a^2(4A+3B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{B(a^2+a^2\sec\\ &= a^2Ax+\frac{a^2(4A+3B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(2A+3B)}{2d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 307 vs. 2(82) = 164.
time = 1.34, size = 307, normalized size = 3.74

$$\frac{a^2\cos^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^2(A+B\sec(c+dx))\left(4Ax-\frac{2(4A+3B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{2(4A+3B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{B}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4(A+2B)\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{B}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4(A+2B)\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)}{16(B+A\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*Cos[c + d*x]^3*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + B*Sec[c +
d*x])*(4*A*x - (2*(4*A + 3*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d +
 (2*(4*A + 3*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + B/(d*(Cos[(c
+ d*x)/2] - Sin[(c + d*x)/2])^2) + (4*(A + 2*B)*Sin[(d*x)/2])/(d*(Cos[c/2]
- Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - B/(d*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])^2) + (4*(A + 2*B)*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2
])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(16*(B + A*Cos[c + d*x]))
```

Maple [A]
time = 0.24, size = 114, normalized size = 1.39

method	result
derivativedivides	$\frac{a^2A\tan(dx+c)+a^2B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2a^2A\ln(\sec(dx+c)+\tan(dx+c))+2a^2B\tan(dx+}{d}$

default	$\frac{a^2 A\tan(dx+c)+a^2 B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2a^2 A\ln(\sec(dx+c)+\tan(dx+c))+2a^2 B\tan(dx+c)}{d}$
norman	$\frac{a^2 Ax+a^2 Ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{a^2(2A+5B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-2a^2 Ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a^2(2A+3B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{a^2(4A+3}{}$
risch	$a^2Ax-\frac{ia^2\left(B\,e^{3i(dx+c)}-2A\,e^{2i(dx+c)}-4B\,e^{2i(dx+c)}-B\,e^{i(dx+c)}-2A-4B\right)}{d\left(e^{2i(dx+c)}+1\right)^2}+\frac{2a^2\ln\left(e^{i(dx+c)}+i\right)A}{d}+\frac{3a^2\ln\left(e^{i(dx}\right.}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*A*tan(d*x+c)+a^2*B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+ta
n(d*x+c)))+2*a^2*A*ln(sec(d*x+c)+tan(d*x+c))+2*a^2*B*tan(d*x+c)+a^2*A*(d*x+
c)+a^2*B*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.34, size = 128, normalized size = 1.56

$$\frac{4(dx+c)Aa^2-Ba^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+8Aa^2\log(\sec(dx+c)+\tan(dx+c))+4Ba^2\log(\sec(dx+c)+\tan(dx+c))+4Aa^2\tan(dx+c)+8Ba^2\tan(dx+c)}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*A*a^2 - B*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 8*A*a^2*log(sec(d*x + c) + tan(
d*x + c)) + 4*B*a^2*log(sec(d*x + c) + tan(d*x + c)) + 4*A*a^2*tan(d*x + c)
 + 8*B*a^2*tan(d*x + c))/d
```

Fricas [A]
time = 1.46, size = 119, normalized size = 1.45

$$\frac{4Aa^2dx\cos(dx+c)^2+(4A+3B)a^2\cos(dx+c)^2\log(\sin(dx+c)+1)-(4A+3B)a^2\cos(dx+c)^2\log(-\sin(dx+c)+1)+2\left(2(A+2B)a^2\cos(dx+c)+Ba^2\right)\sin(dx+c)}{4d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*A*a^2*d*x*cos(d*x + c)^2 + (4*A + 3*B)*a^2*cos(d*x + c)^2*log(sin(d*
x + c) + 1) - (4*A + 3*B)*a^2*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*
(A + 2*B)*a^2*cos(d*x + c) + B*a^2)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int A\,dx+\int 2A\sec(c+dx)\,dx+\int A\sec^2(c+dx)\,dx+\int B\sec(c+dx)\,dx+\int 2B\sec^2(c+dx)\,dx+\int B\sec^3(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] a**2*(Integral(A, x) + Integral(2*A*sec(c + d*x), x) + Integral(A*sec(c + d
*x)**2, x) + Integral(B*sec(c + d*x), x) + Integral(2*B*sec(c + d*x)**2, x)
 + Integral(B*sec(c + d*x)**3, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(76) = 152.
time = 0.48, size = 154, normalized size = 1.88

$$\frac{2\,(dx+c)Aa^2+(4\,Aa^2+3\,Ba^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(4\,Aa^2+3\,Ba^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(2\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*A*a^2 + (4*A*a^2 + 3*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) +
 1)) - (4*A*a^2 + 3*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*A*a^2*
tan(1/2*d*x + 1/2*c)^3 + 3*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 2*A*a^2*tan(1/2*d
*x + 1/2*c) - 5*B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)
/d
```

Mupad [B]
time = 2.01, size = 162, normalized size = 1.98

$$\frac{2\,A\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,A\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,B\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{A\,a^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{2\,B\,a^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{B\,a^2\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*A*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*A*a^2*atanh(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*B*a^2*atanh(sin(c/2 + (d*x)/2)
/cos(c/2 + (d*x)/2)))/d + (A*a^2*sin(c + d*x))/(d*cos(c + d*x)) + (2*B*a^2*
sin(c + d*x))/(d*cos(c + d*x)) + (B*a^2*sin(c + d*x))/(2*d*cos(c + d*x)^2)
```

3.57 $\int \cos(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=73

$$a^2(2A+B)x+\frac{a^2(A+2B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2(A-B)\sin(c+dx)}{d}+\frac{B(a^2+a^2\sec(c+dx))\sin(c+dx)}{d}$$

```
[Out] a^2*(2*A+B)*x+a^2*(A+2*B)*arctanh(sin(d*x+c))/d+a^2*(A-B)*sin(d*x+c)/d+B*(a
^2+a^2*sec(d*x+c))*sin(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 73, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4103, 4081, 3855}

$$\frac{a^2(A-B)\sin(c+dx)}{d}+\frac{a^2(A+2B)\tanh^{-1}(\sin(c+dx))}{d}+a^2x(2A+B)+\frac{B\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] a^2*(2*A + B)*x + (a^2*(A + 2*B)*ArcTanh[Sin[c + d*x]])/d + (a^2*(A - B)*Si
n[c + d*x])/d + (B*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/d
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
```

```
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{B(a^2+a^2\sec(c+dx))\sin(c+dx)}{d} + \int \cos(c$$
$$= \frac{a^2(A-B)\sin(c+dx)}{d} + \frac{B(a^2+a^2\sec(c+dx}{d}$$
$$= a^2(2A+B)x + \frac{a^2(A-B)\sin(c+dx)}{d} + \frac{B(a^2}{}$$
$$= a^2(2A+B)x + \frac{a^2(A+2B)\tanh^{-1}(\sin(c+dx}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 258 vs. $2(73) = 146$.
time = 1.70, size = 258, normalized size = 3.53

$$\frac{a^2\cos^3(c+dx)\sec^4\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^2(A+B\sec(c+dx))\left((2A+B)x - \frac{(A+2B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{(A+2B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d} + \frac{A\cos(dx)\sin(c)}{d} + \frac{A\cos(c)\sin(dx)}{d} + \frac{B\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{B\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)}{4(B+A\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*Cos[c + d*x]^3*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + B*Sec[c +
d*x])*((2*A + B)*x - ((A + 2*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/d
 + ((A + 2*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (A*Cos[d*x]*Sin
[c])/d + (A*Cos[c]*Sin[d*x])/d + (B*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*
(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (B*Sin[(d*x)/2])/(d*(Cos[c/2] + Si
n[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(4*(B + A*Cos[c + d*x]))
```

Maple [A]
time = 0.26, size = 88, normalized size = 1.21

method	result
derivativedivides	$\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))+a^2B\tan(dx+c)+2a^2A(dx+c)+2a^2B\ln(\sec(dx+c)+\tan(dx+c))+a^2A\sin(dx+c)+a^2B}{d}$
default	$\frac{a^2A\ln(\sec(dx+c)+\tan(dx+c))+a^2B\tan(dx+c)+2a^2A(dx+c)+2a^2B\ln(\sec(dx+c)+\tan(dx+c))+a^2A\sin(dx+c)+a^2B}{d}$
risch	$2a^2Ax+a^2xB-\frac{ia^2A\,e^{i(dx+c)}}{2d}+\frac{ia^2A\,e^{-i(dx+c)}}{2d}+\frac{2ia^2B}{d(e^{2i(dx+c)}+1)}+\frac{a^2\ln(e^{i(dx+c)}+i)A}{d}+\frac{2a^2\ln(e^{i(dx}{d}$
norman	$\frac{(2a^2A+a^2B)x+(-2a^2A-a^2B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-2a^2A-a^2B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(2a^2A+a^2B)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*A*ln(sec(d*x+c)+tan(d*x+c))+a^2*B*tan(d*x+c)+2*a^2*A*(d*x+c)+2*a^2
*B*ln(sec(d*x+c)+tan(d*x+c))+a^2*A*sin(d*x+c)+a^2*B*(d*x+c))
```

Maxima [A]
time = 0.37, size = 105, normalized size = 1.44

$$\frac{4\,(dx+c)Aa^2+2\,(dx+c)Ba^2+Aa^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+2\,Ba^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+2\,Aa^2\sin\left(dx+c\right)+2\,Ba^2\tan\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/2*(4*(d*x + c)*A*a^2 + 2*(d*x + c)*B*a^2 + A*a^2*(log(sin(d*x + c) + 1) -
 log(sin(d*x + c) - 1)) + 2*B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 2*A*a^2*sin(d*x + c) + 2*B*a^2*tan(d*x + c))/d
```

Fricas [A]
time = 1.75, size = 108, normalized size = 1.48

$$\frac{2\,(2\,A+B)a^2dx\cos\left(dx+c\right)+(A+2\,B)a^2\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-(A+2\,B)a^2\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,(Aa^2\cos\left(dx+c\right)+Ba^2)\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*(2*(2*A + B)*a^2*d*x*cos(d*x + c) + (A + 2*B)*a^2*cos(d*x + c)*log(sin(
d*x + c) + 1) - (A + 2*B)*a^2*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(A*a^
2*cos(d*x + c) + B*a^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int A\cos\left(c+dx\right)dx+\int 2A\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int A\cos\left(c+dx\right)\sec^{2}\left(c+dx\right)dx+\int B\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int 2B\cos\left(c+dx\right)\sec^{2}\left(c+dx\right)dx+\int B\cos\left(c+dx\right)\sec^{3}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] a**2*(Integral(A*cos(c + d*x), x) + Integral(2*A*cos(c + d*x)*sec(c + d*x),
 x) + Integral(A*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(B*cos(c + d*x)
*sec(c + d*x), x) + Integral(2*B*cos(c + d*x)*sec(c + d*x)**2, x) + Integra
l(B*cos(c + d*x)*sec(c + d*x)**3, x))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 157 vs. 2(73) = 146.
time = 0.48, size = 157, normalized size = 2.15

$$\frac{(2\,Aa^2+Ba^2)(dx+c)+(Aa^2+2\,Ba^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Aa^2+2\,Ba^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] ((2*A*a^2 + B*a^2)*(d*x + c) + (A*a^2 + 2*B*a^2)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - (A*a^2 + 2*B*a^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a^2*
tan(1/2*d*x + 1/2*c)^3 - B*a^2*tan(1/2*d*x + 1/2*c)^3 - A*a^2*tan(1/2*d*x +
 1/2*c) - B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

Mupad [B]
time = 2.01, size = 161, normalized size = 2.21

$$\frac{A\,a^2\,\sin\left(c+d\,x\right)}{d}+\frac{4\,A\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,A\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,B\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{B\,a^2\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (A*a^2*sin(c + d*x))/d + (4*A*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/d + (2*A*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a^2
*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*B*a^2*atanh(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/d + (B*a^2*sin(c + d*x))/(d*cos(c + d*x))
```

3.58 $\int \cos^2(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=88

$$\frac{1}{2}a^2(3A+4B)x+\frac{a^2B\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2(3A+2B)\sin(c+dx)}{2d}+\frac{A\cos(c+dx)\left(a^2+a^2\sec(c+dx)\right)\sin}{2d}$$

```
[Out] 1/2*a^2*(3*A+4*B)*x+a^2*B*arctanh(sin(d*x+c))/d+1/2*a^2*(3*A+2*B)*sin(d*x+c
)/d+1/2*A*cos(d*x+c)*(a^2+a^2*sec(d*x+c))*sin(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4102, 4081, 3855}

$$\frac{a^2(3A+2B)\sin(c+dx)}{2d}+\frac{1}{2}a^2x(3A+4B)+\frac{A\sin(c+dx)\cos(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{2d}+\frac{a^2B\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(3*A + 4*B)*x)/2 + (a^2*B*ArcTanh[Sin[c + d*x]])/d + (a^2*(3*A + 2*B)*
Sin[c + d*x])/(2*d) + (A*Cos[c + d*x]*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x]
)/(2*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

```
&& GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{A\cos(c+dx)\left(a^2+a^2\sec(c+dx)\right)\sin(c+dx}{2d}$$
$$= \frac{a^2(3A+2B)\sin(c+dx)}{2d} + \frac{A\cos(c+dx)\left(a^2\right.}{}$$
$$= \frac{1}{2}a^2(3A+4B)x + \frac{a^2(3A+2B)\sin(c+dx)}{2d} +$$
$$= \frac{1}{2}a^2(3A+4B)x + \frac{a^2B\tanh^{-1}(\sin(c+dx))}{d} +$$

Mathematica [A]
time = 0.17, size = 96, normalized size = 1.09

$$\frac{a^2\left(6Adx+8Bdx-4B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+4(2A+B)\sin(c+dx)+A\sin(2(c+dx))\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
[Out] (a^2*(6*A*d*x + 8*B*d*x - 4*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 4*
B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*(2*A + B)*Sin[c + d*x] + A*S
in[2*(c + d*x)]))/(4*d)
```

Maple [A]
time = 0.22, size = 96, normalized size = 1.09

method	result
derivativedivides	$\frac{a^2A(dx+c)+a^2B\ln(\sec(dx+c)+\tan(dx+c))+2a^2A\sin(dx+c)+2a^2B(dx+c)+a^2A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2B}{d}$
default	$\frac{a^2A(dx+c)+a^2B\ln(\sec(dx+c)+\tan(dx+c))+2a^2A\sin(dx+c)+2a^2B(dx+c)+a^2A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2B}{d}$
risch	$\frac{3a^2Ax}{2}+2a^2xB-\frac{ia^2A\,e^{i(dx+c)}}{d}-\frac{ie^{i(dx+c)}a^2B}{2d}+\frac{ia^2A\,e^{-i(dx+c)}}{d}+\frac{ie^{-i(dx+c)}a^2B}{2d}+\frac{a^2\ln\left(e^{i(dx+c)}+i\right)B}{d}$
norman	$\frac{\left(\frac{3}{2}a^2A+2a^2B\right)x+\left(\frac{3}{2}a^2A+2a^2B\right)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-3a^2A-4a^2B\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{a^2(3A+2B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*(d*x+c)+a^2*B*ln(sec(d*x+c)+tan(d*x+c))+2*a^2*A*sin(d*x+c)+2*a^2
*B*(d*x+c)+a^2*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^2*B*sin(d*x+c)
)
```

Maxima [A]
time = 0.36, size = 101, normalized size = 1.15

$$\frac{(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Aa^2+4\,(dx+c)Aa^2+8\,(dx+c)Ba^2+2\,Ba^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+8\,Aa^2\sin\left(dx+c\right)+4\,Ba^2\sin\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 + 4*(d*x + c)*A*a^2 + 8*(d*x +
c)*B*a^2 + 2*B*a^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 8*A*a^
2*sin(d*x + c) + 4*B*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 1.38, size = 79, normalized size = 0.90

$$\frac{(3\,A+4\,B)a^2dx+Ba^2\log\left(\sin\left(dx+c\right)+1\right)-Ba^2\log\left(-\sin\left(dx+c\right)+1\right)+(Aa^2\cos\left(dx+c\right)+2\,(2\,A+B)a^2)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2*((3*A + 4*B)*a^2*d*x + B*a^2*log(sin(d*x + c) + 1) - B*a^2*log(-sin(d*x
 + c) + 1) + (A*a^2*cos(d*x + c) + 2*(2*A + B)*a^2)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int A\cos^2\left(c+dx\right)dx+\int 2A\cos^2\left(c+dx\right)\sec\left(c+dx\right)dx+\int A\cos^2\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int B\cos^2\left(c+dx\right)\sec\left(c+dx\right)dx+\int 2B\cos^2\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int B\cos^2\left(c+dx\right)\sec^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] a**2*(Integral(A*cos(c + d*x)**2, x) + Integral(2*A*cos(c + d*x)**2*sec(c +
 d*x), x) + Integral(A*cos(c + d*x)**2*sec(c + d*x)**2, x) + Integral(B*cos
(c + d*x)**2*sec(c + d*x), x) + Integral(2*B*cos(c + d*x)**2*sec(c + d*x)**
2, x) + Integral(B*cos(c + d*x)**2*sec(c + d*x)**3, x))
```

Giac [A]
time = 0.49, size = 145, normalized size = 1.65

$$\frac{2\,Ba^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,Ba^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(3\,Aa^2+4\,Ba^2)(dx+c)+\frac{2\left(3\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*B*a^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*B*a^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + (3*A*a^2 + 4*B*a^2)*(d*x + c) + 2*(3*A*a^2*tan(1/2*d*x
+ 1/2*c)^3 + 2*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^2*tan(1/2*d*x + 1/2*c)
+ 2*B*a^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 2.05, size = 141, normalized size = 1.60

$$\frac{2\,A\,a^2\,\sin\left(c+d\,x\right)}{d}+\frac{B\,a^2\,\sin\left(c+d\,x\right)}{d}+\frac{3\,A\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,B\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{A\,a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*A*a^2*sin(c + d*x))/d + (B*a^2*sin(c + d*x))/d + (3*A*a^2*atan(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*B*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2
 + (d*x)/2)))/d + (2*B*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
+ (A*a^2*sin(2*c + 2*d*x))/(4*d)
```

3.59 $\int \cos^3(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=102

$$\frac{1}{2}a^2(2A+3B)x+\frac{2a^2(2A+3B)\sin(c+dx)}{3d}+\frac{a^2(2A+3B)\cos(c+dx)\sin(c+dx)}{6d}+\frac{A\cos^2(c+dx)(a+a\sec($$

```
[Out] 1/2*a^2*(2*A+3*B)*x+2/3*a^2*(2*A+3*B)*sin(d*x+c)/d+1/6*a^2*(2*A+3*B)*cos(d*
x+c)*sin(d*x+c)/d+1/3*A*cos(d*x+c)^2*(a+a*sec(d*x+c))^2*sin(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4098, 3873, 2717, 4130, 8}

$$\frac{2a^2(2A+3B)\sin(c+dx)}{3d}+\frac{a^2(2A+3B)\sin(c+dx)\cos(c+dx)}{6d}+\frac{1}{2}a^2x(2A+3B)+\frac{A\sin(c+dx)\cos^2(c+dx)(a\sec(c+dx)+a)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(2*A + 3*B)*x)/2 + (2*a^2*(2*A + 3*B)*Sin[c + d*x])/(3*d) + (a^2*(2*A
+ 3*B)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (A*Cos[c + d*x]^2*(a + a*Sec[c +
d*x])^2*Sin[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3873

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^2, x_Symbol] :> Dist[2*a*(b/d), Int[(d*Csc[e + f*x])^(n + 1), x], x]
+ Int[(d*Csc[e + f*x])^n*(a^2 + b^2*Csc[e + f*x]^2), x] /; FreeQ[{a, b, d,
e, f, n}, x]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
```

```
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^2\sin(c+dx}{3d}$$
$$= \frac{A\cos^2(c+dx)(a+a\sec(c+dx))^2\sin(c+dx}{3d}$$
$$= \frac{2a^2(2A+3B)\sin(c+dx)}{3d} + \frac{a^2(2A+3B)\cos}{}$$
$$= \frac{1}{2}a^2(2A+3B)x + \frac{2a^2(2A+3B)\sin(c+dx)}{3d}$$

Mathematica [A]
time = 0.18, size = 61, normalized size = 0.60

$$\frac{a^2(12Adx+18Bdx+3(7A+8B)\sin(c+dx)+3(2A+B)\sin(2(c+dx))+A\sin(3(c+dx)))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
[Out] (a^2*(12*A*d*x + 18*B*d*x + 3*(7*A + 8*B)*Sin[c + d*x] + 3*(2*A + B)*Sin[2*
(c + d*x)] + A*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A]
time = 0.30, size = 116, normalized size = 1.14

method	result
risch	$a^2Ax + \frac{3a^2xB}{2} + \frac{7a^2A\sin(dx+c)}{4d} + \frac{2\sin(dx+c)a^2B}{d} + \frac{a^2A\sin(3dx+3c)}{12d} + \frac{a^2A\sin(2dx+2c)}{2d} + \frac{\sin(2dx+2c}{4d}$
derivativedivides	$\frac{\frac{a^2A\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+2a^2A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2A\sin(dx+c)}{d}$

default	$\frac{\frac{a^2 A\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+2a^2A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2A\sin(dx+c)+2}{d}$
norman	$\frac{\frac{a^2(2A+3B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}+\frac{a^2(6A+5B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{a^2(2A+3B)x}{2}-\frac{8a^2(A+3B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a^2(2A+3B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/3*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)+2*a^2*A*(1/2*cos(d*x+c)*sin(d*x+
c)+1/2*d*x+1/2*c)+a^2*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^2*A*sin
(d*x+c)+2*a^2*B*sin(d*x+c)+a^2*B*(d*x+c))
```

Maxima [A]
time = 0.38, size = 110, normalized size = 1.08

$$-\frac{4\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)Aa^2-6\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^2-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ba^2-12\left(dx+c\right)Ba^2-12\,Aa^2\sin\left(dx+c\right)-24\,Ba^2\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 6*(2*d*x + 2*c + sin(2*d
*x + 2*c))*A*a^2 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2 - 12*(d*x + c)*
B*a^2 - 12*A*a^2*sin(d*x + c) - 24*B*a^2*sin(d*x + c))/d
```

Fricas [A]
time = 1.59, size = 70, normalized size = 0.69

$$\frac{3\left(2\,A+3\,B\right)a^2dx+\left(2\,Aa^2\cos\left(dx+c\right)^2+3\left(2\,A+B\right)a^2\cos\left(dx+c\right)+2\left(5\,A+6\,B\right)a^2\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(2*A + 3*B)*a^2*d*x + (2*A*a^2*cos(d*x + c)^2 + 3*(2*A + B)*a^2*cos(
d*x + c) + 2*(5*A + 6*B)*a^2)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int A\cos^3\left(c+dx\right)dx+\int 2A\cos^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int A\cos^3\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int B\cos^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int 2B\cos^3\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int B\cos^3\left(c+dx\right)\sec^3\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] a**2*(Integral(A*cos(c + d*x)**3, x) + Integral(2*A*cos(c + d*x)**3*sec(c +
 d*x), x) + Integral(A*cos(c + d*x)**3*sec(c + d*x)**2, x) + Integral(B*cos
(c + d*x)**3*sec(c + d*x), x) + Integral(2*B*cos(c + d*x)**3*sec(c + d*x)**
2, x) + Integral(B*cos(c + d*x)**3*sec(c + d*x)**3, x))
```

Giac [A]
time = 0.47, size = 142, normalized size = 1.39

$$\frac{3\,(2\,Aa^2+3\,Ba^2)(dx+c)+\frac{2\left(6\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+16\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+18\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(2*A*a^2 + 3*B*a^2)*(d*x + c) + 2*(6*A*a^2*tan(1/2*d*x + 1/2*c)^5 +
9*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 16*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 24*B*a^2
*tan(1/2*d*x + 1/2*c)^3 + 18*A*a^2*tan(1/2*d*x + 1/2*c) + 15*B*a^2*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 1.89, size = 98, normalized size = 0.96

$$A\,a^2\,x+\frac{3\,B\,a^2\,x}{2}+\frac{7\,A\,a^2\,\sin\left(c+d\,x\right)}{4\,d}+\frac{2\,B\,a^2\,\sin\left(c+d\,x\right)}{d}+\frac{A\,a^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{A\,a^2\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{B\,a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)
```

```
[Out] A*a^2*x + (3*B*a^2*x)/2 + (7*A*a^2*sin(c + d*x))/(4*d) + (2*B*a^2*sin(c + d
*x))/d + (A*a^2*sin(2*c + 2*d*x))/(2*d) + (A*a^2*sin(3*c + 3*d*x))/(12*d) +
 (B*a^2*sin(2*c + 2*d*x))/(4*d)
```

3.60 $\int \cos^4(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=135

$$\frac{1}{8}a^2(7A+8B)x+\frac{a^2(4A+5B)\sin(c+dx)}{3d}+\frac{a^2(7A+8B)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2(5A+4B)\cos^2(c+dx)}{12d}$$

```
[Out] 1/8*a^2*(7*A+8*B)*x+1/3*a^2*(4*A+5*B)*sin(d*x+c)/d+1/8*a^2*(7*A+8*B)*cos(d*
x+c)*sin(d*x+c)/d+1/12*a^2*(5*A+4*B)*cos(d*x+c)^2*sin(d*x+c)/d+1/4*A*cos(d*
x+c)^3*(a^2+a^2*sec(d*x+c))*sin(d*x+c)/d
```

Rubi [A]

time = 0.17, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4102, 4081, 3872, 2715, 8, 2717}

$$\frac{a^2(4A+5B)\sin(c+dx)}{3d}+\frac{a^2(5A+4B)\sin(c+dx)\cos^2(c+dx)}{12d}+\frac{a^2(7A+8B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}a^2x(7A+8B)+\frac{A\sin(c+dx)\cos^3(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(7*A + 8*B)*x)/8 + (a^2*(4*A + 5*B)*Sin[c + d*x])/(3*d) + (a^2*(7*A +
8*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(5*A + 4*B)*Cos[c + d*x]^2*Sin
[c + d*x])/(12*d) + (A*Cos[c + d*x]^3*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x]
)/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \cos^4(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx &= \frac{A\cos^3(c+dx)\left(a^2+a^2\sec(c+dx)\right)\sin(c+d\\
&= \frac{a^2(5A+4B)\cos^2(c+dx)\sin(c+dx)}{12d}+\frac{A\cos\\
&= \frac{a^2(5A+4B)\cos^2(c+dx)\sin(c+dx)}{12d}+\frac{A\cos\\
&= \frac{a^2(4A+5B)\sin(c+dx)}{3d}+\frac{a^2(7A+8B)\cos(\\
&= \frac{1}{8}a^2(7A+8B)x+\frac{a^2(4A+5B)\sin(c+dx)}{3d}+
\end{aligned}
$$

Mathematica [A]
time = 0.39, size = 86, normalized size = 0.64

$$\frac{a^2(84Ac+84Adx+96Bdx+24(6A+7B)\sin(c+dx)+48(A+B)\sin(2(c+dx))+16A\sin(3(c+dx))+8B\sin(3(c+dx))+3A\sin(4(c+dx)))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

[Out] (a^2*(84*A*c + 84*A*d*x + 96*B*d*x + 24*(6*A + 7*B)*Sin[c + d*x] + 48*(A + B)*Sin[2*(c + d*x)] + 16*A*Sin[3*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*A*Sin[4*(c + d*x)]))/(96*d)

Maple [A]
time = 0.38, size = 154, normalized size = 1.14

method	result
risch	$\frac{7a^2Ax}{8}+a^2xB+\frac{3a^2A\sin(dx+c)}{2d}+\frac{7\sin(dx+c)a^2B}{4d}+\frac{a^2A\sin(4dx+4c)}{32d}+\frac{a^2A\sin(3dx+3c)}{6d}+\frac{\sin(3dx+3c)a}{12d}$
derivativedivides	$\frac{a^2A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2B\sin(dx+c)+\frac{2a^2A\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+2a^2B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+}{d}$
default	$\frac{a^2A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2B\sin(dx+c)+\frac{2a^2A\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+2a^2B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+}{d}$
norman	$\frac{a^2(7A+8B)x}{8}-\frac{7a^2(A+8B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{5a^2(7A+8B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{a^2(7A+8B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{a^2(7A+8B)x\left(\tan^2\left(\frac{dx}{2}\right.\right.}{4}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(a^2*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^2*B*sin(d*x+c)+2/3*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)+2*a^2*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*a^2*B*(2+cos(d*x+c)^2)*sin(d*x+c))

Maxima [A]
time = 0.37, size = 144, normalized size = 1.07

$$-\frac{64\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^2-3\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Aa^2-24\left(2dx+2c+\sin(2dx+2c)\right)Aa^2+32\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba^2-48\left(2dx+2c+\sin(2dx+2c)\right)Ba^2-96Ba^2\sin(dx+c)}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] -1/96*(64*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 48*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2 - 96*B*a^2*sin(d*x + c))/d

Fricas [A]
time = 0.99, size = 90, normalized size = 0.67

$$\frac{3(7A+8B)a^2dx+\left(6Aa^2\cos(dx+c)^3+8(2A+B)a^2\cos(dx+c)^2+3(7A+8B)a^2\cos(dx+c)+8(4A+5B)a^2\right)\sin(dx+c)}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/24*(3*(7*A + 8*B)*a^2*d*x + (6*A*a^2*cos(d*x + c)^3 + 8*(2*A + B)*a^2*cos
(d*x + c)^2 + 3*(7*A + 8*B)*a^2*cos(d*x + c) + 8*(4*A + 5*B)*a^2)*sin(d*x +
 c))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.49, size = 176, normalized size = 1.30

$$\frac{3\,(7\,Aa^2+8\,Ba^2)(dx+c)+\frac{2\left(21\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+24\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+77\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+88\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+83\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+136\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+75\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+72\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(7*A*a^2 + 8*B*a^2)*(d*x + c) + 2*(21*A*a^2*tan(1/2*d*x + 1/2*c)^7
+ 24*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 77*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 88*B*
a^2*tan(1/2*d*x + 1/2*c)^5 + 83*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 136*B*a^2*ta
n(1/2*d*x + 1/2*c)^3 + 75*A*a^2*tan(1/2*d*x + 1/2*c) + 72*B*a^2*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [**B**]
time = 1.93, size = 134, normalized size = 0.99

$$\frac{7\,A\,a^2\,x}{8}+B\,a^2\,x+\frac{3\,A\,a^2\,\sin\left(c+d\,x\right)}{2\,d}+\frac{7\,B\,a^2\,\sin\left(c+d\,x\right)}{4\,d}+\frac{A\,a^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{A\,a^2\,\sin\left(3\,c+3\,d\,x\right)}{6\,d}+\frac{A\,a^2\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{B\,a^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{B\,a^2\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (7*A*a^2*x)/8 + B*a^2*x + (3*A*a^2*sin(c + d*x))/(2*d) + (7*B*a^2*sin(c + d
*x))/(4*d) + (A*a^2*sin(2*c + 2*d*x))/(2*d) + (A*a^2*sin(3*c + 3*d*x))/(6*d
) + (A*a^2*sin(4*c + 4*d*x))/(32*d) + (B*a^2*sin(2*c + 2*d*x))/(2*d) + (B*a
^2*sin(3*c + 3*d*x))/(12*d)
```

3.61 $\int \cos^5(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=160

$$\frac{1}{8}a^2(6A+7B)x+\frac{a^2(9A+10B)\sin(c+dx)}{5d}+\frac{a^2(6A+7B)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^2(6A+5B)\cos^3(c+dx)\sin(c+dx)}{20d}$$

```
[Out] 1/8*a^2*(6*A+7*B)*x+1/5*a^2*(9*A+10*B)*sin(d*x+c)/d+1/8*a^2*(6*A+7*B)*cos(d
*x+c)*sin(d*x+c)/d+1/20*a^2*(6*A+5*B)*cos(d*x+c)^3*sin(d*x+c)/d+1/5*A*cos(d
*x+c)^4*(a^2+a^2*sec(d*x+c))*sin(d*x+c)/d-1/15*a^2*(9*A+10*B)*sin(d*x+c)^3/
d
```

Rubi [A]
time = 0.18, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4102, 4081, 3872, 2713, 2715, 8}

$$-\frac{a^2(9A+10B)\sin^3(c+dx)}{15d}+\frac{a^2(9A+10B)\sin(c+dx)}{5d}+\frac{a^2(6A+5B)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{a^2(6A+7B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}a^2x(6A+7B)+\frac{A\sin(c+dx)\cos^4(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(6*A + 7*B)*x)/8 + (a^2*(9*A + 10*B)*Sin[c + d*x])/(5*d) + (a^2*(6*A +
 7*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(6*A + 5*B)*Cos[c + d*x]^3*Si
n[c + d*x])/(20*d) + (A*Cos[c + d*x]^4*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x
])/(5*d) - (a^2*(9*A + 10*B)*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{A\cos^4(c+dx)\left(a^2+a^2\sec(c+dx)\right)\sin(c+d$$
$$= \frac{a^2(6A+5B)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{A\cos$$
$$= \frac{a^2(6A+5B)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{A\cos$$
$$= \frac{a^2(6A+7B)\cos(c+dx)\sin(c+dx)}{8d} + \frac{a^2(6A$$
$$= \frac{1}{8}a^2(6A+7B)x + \frac{a^2(9A+10B)\sin(c+dx)}{5d}$$

Mathematica [A]
time = 0.49, size = 108, normalized size = 0.68

$$\frac{a^2(360Ac+360Adx+420Bdx+60(11A+12B)\sin(c+dx)+240(A+B)\sin(2(c+dx))+90A\sin(3(c+dx))+80B\sin(3(c+dx))+30A\sin(4(c+dx))+15B\sin(4(c+dx))+6A\sin(5(c+dx)))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(360*A*c + 360*A*d*x + 420*B*d*x + 60*(11*A + 12*B)*Sin[c + d*x] + 240
*(A + B)*Sin[2*(c + d*x)] + 90*A*Sin[3*(c + d*x)] + 80*B*Sin[3*(c + d*x)] +
 30*A*Sin[4*(c + d*x)] + 15*B*Sin[4*(c + d*x)] + 6*A*Sin[5*(c + d*x)]))/(48
0*d)
```

Maple [A]
time = 0.40, size = 186, normalized size = 1.16

method	result
risch	$\frac{3a^2Ax}{4}+\frac{7a^2xB}{8}+\frac{11a^2A\sin(dx+c)}{8d}+\frac{3\sin(dx+c)a^2B}{2d}+\frac{a^2A\sin(5dx+5c)}{80d}+\frac{a^2A\sin(4dx+4c)}{16d}+\frac{\sin(4dx+4c)}{32d}$
derivativedivides	$\frac{\frac{a^2A(2+\cos^2(dx+c))\sin(dx+c)}{3}+a^2B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^2A\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{}$
default	$\frac{\frac{a^2A(2+\cos^2(dx+c))\sin(dx+c)}{3}+a^2B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+2a^2A\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)}{}$
norman	$\frac{a^2(6A+7B)x}{8}-\frac{2a^2(6A-B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{2a^2(6A+7B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}+\frac{a^2(6A+7B)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{3a^2(6A+7B)x\left(\tan^2\left(\right.\right.}{8}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/3*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)+a^2*B*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+2*a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d
*x+3/8*c)+2/3*a^2*B*(2+cos(d*x+c)^2)*sin(d*x+c)+1/5*a^2*A*(8/3+cos(d*x+c)^4
+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^2*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(
d*x+c)+3/8*d*x+3/8*c))
```

Maxima [A]
time = 0.39, size = 178, normalized size = 1.11

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^2-160\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^2+30\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Aa^2-320\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba^2+15\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Ba^2+120\left(2dx+2c+\sin(2dx+2c)\right)Ba^2}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^2 -
160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 + 30*(12*d*x + 12*c + sin(4*d*x
 + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))
```

```
*B*a^2 + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2 +
 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2)/d
```

Fricas [A]
time = 1.68, size = 110, normalized size = 0.69

$$\frac{15\,(6\,A+7\,B)a^2dx+\left(24\,Aa^2\cos\left(dx+c\right)^4+30\,(2\,A+B)a^2\cos\left(dx+c\right)^3+8\,(9\,A+10\,B)a^2\cos\left(dx+c\right)^2+15\,(6\,A+7\,B)a^2\cos\left(dx+c\right)+16\,(9\,A+10\,B)a^2\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(6*A + 7*B)*a^2*d*x + (24*A*a^2*cos(d*x + c)^4 + 30*(2*A + B)*a^2
*cos(d*x + c)^3 + 8*(9*A + 10*B)*a^2*cos(d*x + c)^2 + 15*(6*A + 7*B)*a^2*co
s(d*x + c) + 16*(9*A + 10*B)*a^2)*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.51, size = 210, normalized size = 1.31

$$\frac{15\,(6\,Aa^2+7\,Ba^2)(dx+c)+\frac{2\left(90\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+105\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+420\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+490\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+864\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+800\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+540\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+790\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+390\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+375\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(6*A*a^2 + 7*B*a^2)*(d*x + c) + 2*(90*A*a^2*tan(1/2*d*x + 1/2*c)^
9 + 105*B*a^2*tan(1/2*d*x + 1/2*c)^9 + 420*A*a^2*tan(1/2*d*x + 1/2*c)^7 + 4
90*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 864*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 800*B*
a^2*tan(1/2*d*x + 1/2*c)^5 + 540*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 790*B*a^2*t
an(1/2*d*x + 1/2*c)^3 + 390*A*a^2*tan(1/2*d*x + 1/2*c) + 375*B*a^2*tan(1/2*
d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B]
time = 4.68, size = 247, normalized size = 1.54

$$\frac{\left(\frac{3\,A\,a^2}{2}+\frac{7\,B\,a^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(7\,A\,a^2+\frac{49\,B\,a^2}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{72\,A\,a^2}{5}+\frac{40\,B\,a^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(9\,A\,a^2+\frac{79\,B\,a^2}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{13\,A\,a^2}{2}+\frac{25\,B\,a^2}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{a^2\,\mathrm{atan}\left(\frac{a^2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(6\,A+7\,B\right)}{4\,\left(\frac{3\,A\,a^2}{2}+\frac{7\,B\,a^2}{4}\right)}\right)\,\left(6\,A+7\,B\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((13*A*a^2)/2 + (25*B*a^2)/4) + tan(c/2 + (d*x)/2)^9*((
3*A*a^2)/2 + (7*B*a^2)/4) + tan(c/2 + (d*x)/2)^7*(7*A*a^2 + (49*B*a^2)/6) +
 tan(c/2 + (d*x)/2)^3*(9*A*a^2 + (79*B*a^2)/6) + tan(c/2 + (d*x)/2)^5*((72*
A*a^2)/5 + (40*B*a^2)/3))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 + (d*x)/2
)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)
^10 + 1)) + (a^2*atan((a^2*tan(c/2 + (d*x)/2)*(6*A + 7*B))/(4*((3*A*a^2)/2
+ (7*B*a^2)/4)))*(6*A + 7*B))/(4*d)
```

3.62 $\int \sec^3(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=210

$$\frac{a^3(26A+23B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^3(19A+17B)\tan(c+dx)}{5d}+\frac{a^3(26A+23B)\sec(c+dx)\tan(c+dx)}{16d}$$

```
[Out] 1/16*a^3*(26*A+23*B)*arctanh(sin(d*x+c))/d+1/5*a^3*(19*A+17*B)*tan(d*x+c)/d
+1/16*a^3*(26*A+23*B)*sec(d*x+c)*tan(d*x+c)/d+1/40*a^3*(22*A+21*B)*sec(d*x+
c)^3*tan(d*x+c)/d+1/6*a*B*sec(d*x+c)^3*(a+a*sec(d*x+c))^2*tan(d*x+c)/d+1/15
*(3*A+4*B)*sec(d*x+c)^3*(a^3+a^3*sec(d*x+c))*tan(d*x+c)/d+1/15*a^3*(19*A+17
*B)*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.29, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4103, 4082, 3872, 3853, 3855, 3852}

$$\frac{a^3(19A+17B)\tan^3(c+dx)}{15d}+\frac{a^3(19A+17B)\tan(c+dx)}{5d}+\frac{a^3(26A+23B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^3(22A+21B)\tan(c+dx)\sec^3(c+dx)}{40d}+\frac{(3A+4B)\tan(c+dx)\sec^3(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{15d}+\frac{a^3(26A+23B)\tan(c+dx)\sec(c+dx)}{16d}+\frac{aB\tan(c+dx)\sec^3(c+dx)(a\sec(c+dx)+a)^2}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*(26*A + 23*B)*ArcTanh[Sin[c + d*x]])/(16*d) + (a^3*(19*A + 17*B)*Tan[c
 + d*x])/(5*d) + (a^3*(26*A + 23*B)*Sec[c + d*x]*Tan[c + d*x])/(16*d) + (a^
3*(22*A + 21*B)*Sec[c + d*x]^3*Tan[c + d*x])/(40*d) + (a*B*Sec[c + d*x]^3*(
a + a*Sec[c + d*x])^2*Tan[c + d*x])/(6*d) + ((3*A + 4*B)*Sec[c + d*x]^3*(a^
3 + a^3*Sec[c + d*x])*Tan[c + d*x])/(15*d) + (a^3*(19*A + 17*B)*Tan[c + d*x
]^3)/(15*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \sec^3(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx &= \frac{aB\sec^3(c+dx)(a+a\sec(c+dx))^2\tan(c+dx}{6d} \\
&= \frac{aB\sec^3(c+dx)(a+a\sec(c+dx))^2\tan(c+dx}{6d} \\
&= \frac{a^3(22A+21B)\sec^3(c+dx)\tan(c+dx)}{40d} + \frac{aB}{} \\
&= \frac{a^3(22A+21B)\sec^3(c+dx)\tan(c+dx)}{40d} + \frac{aB}{} \\
&= \frac{a^3(26A+23B)\sec(c+dx)\tan(c+dx)}{16d} + \frac{a^3(2}{} \\
&= \frac{a^3(26A+23B)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{a^3(19A}{}
\end{aligned}
$$

Mathematica [A]
time = 1.93, size = 346, normalized size = 1.65

$$-\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(480(26A+23B)\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\sec(c)(-320(19A+17B)\sin(c)+750(2A+3B)\sin(dx)+1500A\sin(2c+dx)+2250B\sin(2c+dx)+7680A\sin(c+2dx)+7680B\sin(c+2dx)-1440A\sin(3c+2dx)-480B\sin(3c+2dx)+1890A\sin(2c+3dx)+1955B\sin(2c+3dx)+1890A\sin(4c+3dx)+1955B\sin(4c+3dx)+3648A\sin(3c+4dx)+3264B\sin(3c+4dx)+390A\sin(4c+5dx)+345B\sin(4c+5dx)+390A\sin(6c+5dx)+345B\sin(6c+5dx)+608A\sin(5c+6dx)+544B\sin(5c+6dx))\right)}{61440d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] -1/61440*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^6*(480*(
26*A + 23*B)*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-320*(19*A + 17*B)*Sin[c]
+ 750*(2*A + 3*B)*Sin[d*x] + 1500*A*Sin[2*c + d*x] + 2250*B*Sin[2*c + d*x]
+ 7680*A*Sin[c + 2*d*x] + 7680*B*Sin[c + 2*d*x] - 1440*A*Sin[3*c + 2*d*x] -
 480*B*Sin[3*c + 2*d*x] + 1890*A*Sin[2*c + 3*d*x] + 1955*B*Sin[2*c + 3*d*x]
 + 1890*A*Sin[4*c + 3*d*x] + 1955*B*Sin[4*c + 3*d*x] + 3648*A*Sin[3*c + 4*d
*x] + 3264*B*Sin[3*c + 4*d*x] + 390*A*Sin[4*c + 5*d*x] + 345*B*Sin[4*c + 5*
d*x] + 390*A*Sin[6*c + 5*d*x] + 345*B*Sin[6*c + 5*d*x] + 608*A*Sin[5*c + 6*
d*x] + 544*B*Sin[5*c + 6*d*x])))/d
```

Maple [A]
time = 0.44, size = 317, normalized size = 1.51 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-A*a^3*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+a^3*B*(-(
-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)+5/16*ln(sec
(d*x+c)+tan(d*x+c)))+3*A*a^3*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c
)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-3*a^3*B*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d
*x+c)^2)*tan(d*x+c)-3*A*a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3*a^3*B*(-(-
1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+
A*a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-a^3*B*(-2/3
-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 405 vs. 2(196) = 392.
time = 0.38, size = 405, normalized size = 1.93

$$\frac{32\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Aa^3+480\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^3+96\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Ba^3+160\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^3-5Ba^3\left(\frac{2\left(15\sin(dx+c)^5-40\sin(dx+c)^3+33\sin(dx+c)\right)}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-1}-15\log(\sin(dx+c)+1)+15\log(\sin(dx+c)-1)\right)-90Aa^3\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-90Ba^3\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-120Aa^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(32*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^3 +
480*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 96*(3*tan(d*x + c)^5 + 10*tan
```

```
(d*x + c)^3 + 15*tan(d*x + c))*B*a^3 + 160*(tan(d*x + c)^3 + 3*tan(d*x + c)
)*B*a^3 - 5*B*a^3*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x +
c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(sin
(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 90*A*a^3*(2*(3*sin(d*x + c)^3
- 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x +
 c) + 1) + 3*log(sin(d*x + c) - 1)) - 90*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin
(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1
) + 3*log(sin(d*x + c) - 1)) - 120*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 -
1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 1.72, size = 185, normalized size = 0.88

$$\frac{15\,(26\,A+23\,B)a^3\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-15\,(26\,A+23\,B)a^3\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(32\,(19\,A+17\,B)a^3\cos\left(dx+c\right)^5+15\,(26\,A+23\,B)a^3\cos\left(dx+c\right)^4+16\,(19\,A+17\,B)a^3\cos\left(dx+c\right)^3+10\,(18\,A+23\,B)a^3\cos\left(dx+c\right)^2+48\,(A+3\,B)a^3\cos\left(dx+c\right)+40\,Ba^3\right)\sin\left(dx+c\right)}{480\,d\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/480*(15*(26*A + 23*B)*a^3*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 15*(26*A
 + 23*B)*a^3*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(32*(19*A + 17*B)*a^
3*cos(d*x + c)^5 + 15*(26*A + 23*B)*a^3*cos(d*x + c)^4 + 16*(19*A + 17*B)*a
^3*cos(d*x + c)^3 + 10*(18*A + 23*B)*a^3*cos(d*x + c)^2 + 48*(A + 3*B)*a^3*
cos(d*x + c) + 40*B*a^3)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int A\sec^3\left(c+dx\right)dx+\int 3A\sec^4\left(c+dx\right)dx+\int 3A\sec^5\left(c+dx\right)dx+\int A\sec^6\left(c+dx\right)dx+\int B\sec^4\left(c+dx\right)dx+\int 3B\sec^5\left(c+dx\right)dx+\int 3B\sec^6\left(c+dx\right)dx+\int B\sec^7\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] a**3*(Integral(A*sec(c + d*x)**3, x) + Integral(3*A*sec(c + d*x)**4, x) + I
ntegral(3*A*sec(c + d*x)**5, x) + Integral(A*sec(c + d*x)**6, x) + Integral
(B*sec(c + d*x)**4, x) + Integral(3*B*sec(c + d*x)**5, x) + Integral(3*B*se
c(c + d*x)**6, x) + Integral(B*sec(c + d*x)**7, x))
```

Giac [A]
time = 0.55, size = 280, normalized size = 1.33

$$\frac{15\,(26\,Aa^3+23\,Ba^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(26\,Aa^3+23\,Ba^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(390\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+345\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}-2210\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-1955\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+5148\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+4554\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-5988\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-5814\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4190\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3165\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1530\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1575\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

[Out] 1/240*(15*(26*A*a^3 + 23*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(26 *A*a^3 + 23*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(390*A*a^3*tan(1/ 2*d*x + 1/2*c)^11 + 345*B*a^3*tan(1/2*d*x + 1/2*c)^11 - 2210*A*a^3*tan(1/2* d*x + 1/2*c)^9 - 1955*B*a^3*tan(1/2*d*x + 1/2*c)^9 + 5148*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 4554*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 5988*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 5814*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 4190*A*a^3*tan(1/2*d*x + 1/2 *c)^3 + 3165*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 1530*A*a^3*tan(1/2*d*x + 1/2*c) - 1575*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d

Mupad [B]

time = 4.63, size = 262, normalized size = 1.25

$$\frac{\left(-\frac{13\,A\,a^3}{4}-\frac{23\,B\,a^3}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{221\,A\,a^3}{12}+\frac{391\,B\,a^3}{24}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{429\,A\,a^3}{10}-\frac{759\,B\,a^3}{20}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{499\,A\,a^3}{10}+\frac{969\,B\,a^3}{20}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{419\,A\,a^3}{12}-\frac{211\,B\,a^3}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{51\,A\,a^3}{4}+\frac{105\,B\,a^3}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}-6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-20\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(26\,A+23\,B\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/cos(c + d*x)^3,x)

[Out] (tan(c/2 + (d*x)/2)*((51*A*a^3)/4 + (105*B*a^3)/8) - tan(c/2 + (d*x)/2)^11* ((13*A*a^3)/4 + (23*B*a^3)/8) - tan(c/2 + (d*x)/2)^3*((419*A*a^3)/12 + (211 *B*a^3)/8) + tan(c/2 + (d*x)/2)^9*((221*A*a^3)/12 + (391*B*a^3)/24) - tan(c /2 + (d*x)/2)^7*((429*A*a^3)/10 + (759*B*a^3)/20) + tan(c/2 + (d*x)/2)^5*((499*A*a^3)/10 + (969*B*a^3)/20))/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (a^3*atanh(tan(c/2 + (d*x)/2))*(26*A + 23*B))/(8*d)

3.63 $\int \sec^2(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=163

$$\frac{a^3(15A+13B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(15A+13B)\tan(c+dx)}{5d}+\frac{3a^3(15A+13B)\sec(c+dx)\tan(c+dx)}{40d}$$

```
[Out] 1/8*a^3*(15*A+13*B)*arctanh(sin(d*x+c))/d+1/5*a^3*(15*A+13*B)*tan(d*x+c)/d+
3/40*a^3*(15*A+13*B)*sec(d*x+c)*tan(d*x+c)/d+1/20*(5*A-B)*(a+a*sec(d*x+c))^
3*tan(d*x+c)/d+1/5*B*(a+a*sec(d*x+c))^4*tan(d*x+c)/a/d+1/60*a^3*(15*A+13*B)
*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.19, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4095, 4086, 3876, 3855, 3852, 8, 3853}

$$\frac{a^3(15A+13B)\tan^3(c+dx)}{60d}+\frac{a^3(15A+13B)\tan(c+dx)}{5d}+\frac{a^3(15A+13B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^3(15A+13B)\tan(c+dx)\sec(c+dx)}{40d}+\frac{(5A-B)\tan(c+dx)(a\sec(c+dx)+a)^3}{20d}+\frac{B\tan(c+dx)(a\sec(c+dx)+a)^4}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*(15*A + 13*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(15*A + 13*B)*Tan[c
+ d*x])/(5*d) + (3*a^3*(15*A + 13*B)*Sec[c + d*x]*Tan[c + d*x])/(40*d) + ((
5*A - B)*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(20*d) + (B*(a + a*Sec[c + d*
x])^4*Tan[c + d*x])/(5*a*d) + (a^3*(15*A + 13*B)*Tan[c + d*x]^3)/(60*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{B(a+a\sec(c+dx))^4\tan(c+dx)}{5ad} + \frac{\int \sec(c+$$

$$= \frac{(5A-B)(a+a\sec(c+dx))^3\tan(c+dx)}{20d} + \frac{I}{}$$

$$= \frac{(5A-B)(a+a\sec(c+dx))^3\tan(c+dx)}{20d} + \frac{I}{}$$

$$= \frac{(5A-B)(a+a\sec(c+dx))^3\tan(c+dx)}{20d} + \frac{I}{}$$

$$= \frac{a^3(15A+13B)\tanh^{-1}(\sin(c+dx))}{20d} + \frac{3a^3(15}{}$$

$$= \frac{a^3(15A+13B)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{a^3(15A}{}$$

Mathematica [A]
time = 1.49, size = 294, normalized size = 1.80

$$-\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(240(15A+13B)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\sec(c)(80(30A+29B)\sin(dx)-240(5A+3B)\sin(2c+dx)+570A\sin(c+2dx)+750B\sin(c+2dx)+570A\sin(3c+2dx)+750B\sin(3c+2dx)+1680A\sin(2c+3dx)+1520B\sin(2c+3dx)-120A\sin(4c+3dx)+225A\sin(3c+4dx)+195B\sin(3c+4dx)+225A\sin(5c+4dx)+195B\sin(5c+4dx)+360A\sin(4c+5dx)+304B\sin(4c+5dx))\right)}{15360d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] -1/15360*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^5*(240*(
15*A + 13*B)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(80*(30*A + 29*B)*Sin[d*x]
- 240*(5*A + 3*B)*Sin[2*c + d*x] + 570*A*Sin[c + 2*d*x] + 750*B*Sin[c + 2*d
*x] + 570*A*Sin[3*c + 2*d*x] + 750*B*Sin[3*c + 2*d*x] + 1680*A*Sin[2*c + 3*
d*x] + 1520*B*Sin[2*c + 3*d*x] - 120*A*Sin[4*c + 3*d*x] + 225*A*Sin[3*c + 4
*d*x] + 195*B*Sin[3*c + 4*d*x] + 225*A*Sin[5*c + 4*d*x] + 195*B*Sin[5*c + 4
*d*x] + 360*A*Sin[4*c + 5*d*x] + 304*B*Sin[4*c + 5*d*x])))/d
```

Maple [A]
time = 0.42, size = 271, normalized size = 1.66

method	result
norman	$\frac{-\frac{32a^3(15A+13B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{7a^3(15A+13B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{a^3(15A+13B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{a^3(49A+51B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}$
derivativedivides	$\frac{Aa^3\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-a^3B\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c\right.}{15}\right.}{}$
default	$\frac{Aa^3\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-a^3B\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c\right.}{15}\right.}{}$
risch	$-\frac{ia^3\left(225A\,e^{9i(dx+c)}+195B\,e^{9i(dx+c)}-120A\,e^{8i(dx+c)}+570A\,e^{7i(dx+c)}+750B\,e^{7i(dx+c)}-1200A\,e^{6i(dx+c)}-720B\,e^{6i(dx\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^3*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c
)+tan(d*x+c)))-a^3*B*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)-
3*A*a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3*a^3*B*(-(-1/4*sec(d*x+c)^3-3/8
*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+3*A*a^3*(1/2*sec(d*x
+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-3*a^3*B*(-2/3-1/3*sec(d*x+c)^
2)*tan(d*x+c)+A*a^3*tan(d*x+c)+a^3*B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(
d*x+c)+tan(d*x+c))))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 337 vs. 2(151) = 302.

time = 0.29, size = 337, normalized size = 2.07

$$\frac{240\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa^{3}+16\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)Ba^{3}+240\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba^{3}-15\,Aa^{3}\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-45\,Ba^{3}\left(\frac{2\left(3\,\sin(dx+c)^{3}-5\,\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\,\sin(dx+c)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-180\,Aa^{3}\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-60\,Ba^{3}\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+240\,Aa^{3}\tan\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/240*(240*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 16*(3*tan(d*x + c)^5 +
 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^3 + 240*(tan(d*x + c)^3 + 3*tan(d
*x + c))*B*a^3 - 15*A*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
 c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 45*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
 - 180*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
 log(sin(d*x + c) - 1)) - 60*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - l
og(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*A*a^3*tan(d*x + c))/d
```

Fricas [A]
time = 3.31, size = 165, normalized size = 1.01

$$\frac{15\left(15\,A+13\,B\right)a^{3}\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-15\left(15\,A+13\,B\right)a^{3}\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\left(45\,A+38\,B\right)a^{3}\cos\left(dx+c\right)^{4}+15\left(15\,A+13\,B\right)a^{3}\cos\left(dx+c\right)^{3}+8\left(15\,A+19\,B\right)a^{3}\cos\left(dx+c\right)^{2}+30\left(A+3\,B\right)a^{3}\cos\left(dx+c\right)+24\,Ba^{3}\right)\sin\left(dx+c\right)}{240\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(15*A + 13*B)*a^3*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 15*(15*A
 + 13*B)*a^3*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(45*A + 38*B)*a^3
*cos(d*x + c)^4 + 15*(15*A + 13*B)*a^3*cos(d*x + c)^3 + 8*(15*A + 19*B)*a^3
*cos(d*x + c)^2 + 30*(A + 3*B)*a^3*cos(d*x + c) + 24*B*a^3)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int A\sec^{2}\left(c+dx\right)dx+\int 3A\sec^{3}\left(c+dx\right)dx+\int 3A\sec^{4}\left(c+dx\right)dx+\int A\sec^{5}\left(c+dx\right)dx+\int B\sec^{3}\left(c+dx\right)dx+\int 3B\sec^{4}\left(c+dx\right)dx+\int 3B\sec^{5}\left(c+dx\right)dx+\int B\sec^{6}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] a**3*(Integral(A*sec(c + d*x)**2, x) + Integral(3*A*sec(c + d*x)**3, x) + I
ntegral(3*A*sec(c + d*x)**4, x) + Integral(A*sec(c + d*x)**5, x) + Integral
(B*sec(c + d*x)**3, x) + Integral(3*B*sec(c + d*x)**4, x) + Integral(3*B*se
c(c + d*x)**5, x) + Integral(B*sec(c + d*x)**6, x))
```

Giac [A]
time = 0.51, size = 246, normalized size = 1.51

$$\frac{15\,(15\,Aa^3+13\,Ba^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(15\,Aa^3+13\,Ba^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(225\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+195\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-1050\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-910\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1920\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1664\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-1830\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1330\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+735\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+765\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(15*A*a^3 + 13*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(15
*A*a^3 + 13*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(225*A*a^3*tan(1/
2*d*x + 1/2*c)^9 + 195*B*a^3*tan(1/2*d*x + 1/2*c)^9 - 1050*A*a^3*tan(1/2*d*
x + 1/2*c)^7 - 910*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 1920*A*a^3*tan(1/2*d*x +
1/2*c)^5 + 1664*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 1830*A*a^3*tan(1/2*d*x + 1/2
*c)^3 - 1330*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 735*A*a^3*tan(1/2*d*x + 1/2*c)
+ 765*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B]
time = 4.60, size = 224, normalized size = 1.37

$$\frac{a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(15\,A+13\,B\right)}{4\,d}-\frac{\left(\frac{15\,A\,a^3}{4}+\frac{13\,B\,a^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{35\,A\,a^3}{2}-\frac{91\,B\,a^3}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(32\,A\,a^3+\frac{416\,B\,a^3}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{61\,A\,a^3}{2}-\frac{133\,B\,a^3}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{49\,A\,a^3}{4}+\frac{51\,B\,a^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}-5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-10\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+5\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] (a^3*atanh(tan(c/2 + (d*x)/2))*(15*A + 13*B))/(4*d) - (tan(c/2 + (d*x)/2)*(
(49*A*a^3)/4 + (51*B*a^3)/4) + tan(c/2 + (d*x)/2)^9*((15*A*a^3)/4 + (13*B*a
^3)/4) - tan(c/2 + (d*x)/2)^7*((35*A*a^3)/2 + (91*B*a^3)/6) - tan(c/2 + (d*
x)/2)^3*((61*A*a^3)/2 + (133*B*a^3)/6) + tan(c/2 + (d*x)/2)^5*(32*A*a^3 + (
416*B*a^3)/15))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*t
an(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.64 $\int \sec(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=125

$$\frac{5a^3(4A+3B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(4A+3B)\tan(c+dx)}{d}+\frac{3a^3(4A+3B)\sec(c+dx)\tan(c+dx)}{8d}+\frac{B(}{}$$

```
[Out] 5/8*a^3*(4*A+3*B)*arctanh(sin(d*x+c))/d+a^3*(4*A+3*B)*tan(d*x+c)/d+3/8*a^3*
(4*A+3*B)*sec(d*x+c)*tan(d*x+c)/d+1/4*B*(a+a*sec(d*x+c))^3*tan(d*x+c)/d+1/1
2*a^3*(4*A+3*B)*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4086, 3876, 3855, 3852, 8, 3853}

$$\frac{a^3(4A+3B)\tan^3(c+dx)}{12d}+\frac{a^3(4A+3B)\tan(c+dx)}{d}+\frac{5a^3(4A+3B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{3a^3(4A+3B)\tan(c+dx)\sec(c+dx)}{8d}+\frac{B\tan(c+dx)(a\sec(c+dx)+a)^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (5*a^3*(4*A + 3*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (a^3*(4*A + 3*B)*Tan[c +
d*x])/d + (3*a^3*(4*A + 3*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (B*(a + a*S
ec[c + d*x])^3*Tan[c + d*x])/(4*d) + (a^3*(4*A + 3*B)*Tan[c + d*x]^3)/(12*d
)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx &= \frac{B(a+a\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{1}{4}(4A+3B\\ &= \frac{B(a+a\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{1}{4}(4A+3B\\ &= \frac{B(a+a\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{1}{4}\left(a^3(4A+\right.\\ &= \frac{a^3(4A+3B)\tanh^{-1}(\sin(c+dx))}{4d}+\frac{3a^3(4A+3}{}\\ &= \frac{5a^3(4A+3B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^3(4A+3}{}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 273 vs. 2(125) = 250.
time = 1.30, size = 273, normalized size = 2.18

$$-\frac{a^3(1+\cos(c+dx))^3\sec^6\left(\frac{1}{2}(c+dx)\right)\sec^4(c+dx)\left(120(4A+3B)\cos^4(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\sec(c)(-24(11A+9B)\sin(c)+(36A+69B)\sin(dx)+36A\sin(2c+dx)+69B\sin(2c+dx)+280A\sin(c+2dx)+264B\sin(c+2dx)-72A\sin(3c+2dx)-24B\sin(3c+2dx)+36A\sin(2c+3dx)+45B\sin(2c+3dx)+36A\sin(4c+3dx)+45B\sin(4c+3dx)+88A\sin(3c+4dx)+72B\sin(3c+4dx))\right)}{1536d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] -1/1536*(a^3*(1 + Cos[c + d*x])^3*Sec[(c + d*x)/2]^6*Sec[c + d*x]^4*(120*(4
*A + 3*B)*Cos[c + d*x]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Co
s[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-24*(11*A + 9*B)*Sin[c] + (36
*A + 69*B)*Sin[d*x] + 36*A*Sin[2*c + d*x] + 69*B*Sin[2*c + d*x] + 280*A*Sin
[c + 2*d*x] + 264*B*Sin[c + 2*d*x] - 72*A*Sin[3*c + 2*d*x] - 24*B*Sin[3*c +
 2*d*x] + 36*A*Sin[2*c + 3*d*x] + 45*B*Sin[2*c + 3*d*x] + 36*A*Sin[4*c + 3*
d*x] + 45*B*Sin[4*c + 3*d*x] + 88*A*Sin[3*c + 4*d*x] + 72*B*Sin[3*c + 4*d*x
])))/d
```

Maple [A]
time = 0.35, size = 219, normalized size = 1.75

method	result
norman	$\frac{-\frac{73a^3(4A+3B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{55a^3(4A+3B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{5a^3(4A+3B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{a^3(44A+49B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}$ —
derivativedivides	$-Aa^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+a^3B\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right.$
default	$-Aa^3\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+a^3B\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right.$
risch	$-\frac{ia^3(36A\,e^{7i(dx+c)}+45B\,e^{7i(dx+c)}-72A\,e^{6i(dx+c)}-24B\,e^{6i(dx+c)}+36A\,e^{5i(dx+c)}+69B\,e^{5i(dx+c)}-264A\,e^{4i(dx+c)}-2}{12d(e^{2i}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-A*a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^3*B*(-(-1/4*sec(d*x+c)^3-3
/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+3*A*a^3*(1/2*sec(d
*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-3*a^3*B*(-2/3-1/3*sec(d*x+c
)^2)*tan(d*x+c)+3*A*a^3*tan(d*x+c)+3*a^3*B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*l
n(sec(d*x+c)+tan(d*x+c)))+A*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*B*tan(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. 2(117) = 234.
time = 0.27, size = 262, normalized size = 2.10

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^3+48\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^3-3Ba^3\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-36Aa^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-36Ba^3\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+48Aa^3\log(\sec(dx+c)+\tan(dx+c))+144Aa^3\tan(dx+c)+48Ba^3\tan(dx+c)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^3 + 48*(tan(d*x + c)^3 + 3*t
an(d*x + c))*B*a^3 - 3*B*a^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
```

```
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 36*A*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 36*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*A*a^3*log(sec(d
*x + c) + tan(d*x + c)) + 144*A*a^3*tan(d*x + c) + 48*B*a^3*tan(d*x + c))/d
```

Fricas [A]
time = 2.21, size = 145, normalized size = 1.16

$$\frac{15\,(4\,A+3\,B)a^{3}\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-15\,(4\,A+3\,B)a^{3}\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\,(11\,A+9\,B)a^{3}\cos\left(dx+c\right)^{3}+9\,(4\,A+5\,B)a^{3}\cos\left(dx+c\right)^{2}+8\,(A+3\,B)a^{3}\cos\left(dx+c\right)+6\,Ba^{3}\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/48*(15*(4*A + 3*B)*a^3*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 15*(4*A + 3
*B)*a^3*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(8*(11*A + 9*B)*a^3*cos(d
*x + c)^3 + 9*(4*A + 5*B)*a^3*cos(d*x + c)^2 + 8*(A + 3*B)*a^3*cos(d*x + c)
 + 6*B*a^3)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int A\sec\left(c+dx\right)dx+\int 3A\sec^{2}\left(c+dx\right)dx+\int 3A\sec^{3}\left(c+dx\right)dx+\int A\sec^{4}\left(c+dx\right)dx+\int B\sec^{2}\left(c+dx\right)dx+\int 3B\sec^{3}\left(c+dx\right)dx+\int 3B\sec^{4}\left(c+dx\right)dx+\int B\sec^{5}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] a**3*(Integral(A*sec(c + d*x), x) + Integral(3*A*sec(c + d*x)**2, x) + Inte
gral(3*A*sec(c + d*x)**3, x) + Integral(A*sec(c + d*x)**4, x) + Integral(B*
sec(c + d*x)**2, x) + Integral(3*B*sec(c + d*x)**3, x) + Integral(3*B*sec(c
 + d*x)**4, x) + Integral(B*sec(c + d*x)**5, x))
```

Giac [A]
time = 0.53, size = 212, normalized size = 1.70

$$\frac{15\,(4\,Aa^{3}+3\,Ba^{3})\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-15\,(4\,Aa^{3}+3\,Ba^{3})\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(60\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+45\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-220\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-165\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+292\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+219\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-132\,Aa^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-147\,Ba^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/24*(15*(4*A*a^3 + 3*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(4*A*a
^3 + 3*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(60*A*a^3*tan(1/2*d*x
+ 1/2*c)^7 + 45*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 220*A*a^3*tan(1/2*d*x + 1/2*
c)^5 - 165*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 292*A*a^3*tan(1/2*d*x + 1/2*c)^3
```

```
+ 219*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 132*A*a^3*tan(1/2*d*x + 1/2*c) - 147*B
*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 4.47, size = 185, normalized size = 1.48

$$\frac{\left(-5\,A\,a^3-\frac{15\,B\,a^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{55\,A\,a^3}{3}+\frac{55\,B\,a^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{73\,A\,a^3}{3}-\frac{73\,B\,a^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(11\,A\,a^3+\frac{49\,B\,a^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{5\,a^3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(4\,A+3\,B\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/cos(c + d*x),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(11*A*a^3 + (49*B*a^3)/4) - tan(c/2 + (d*x)/2)^7*(5*A*a
^3 + (15*B*a^3)/4) + tan(c/2 + (d*x)/2)^5*((55*A*a^3)/3 + (55*B*a^3)/4) - t
an(c/2 + (d*x)/2)^3*((73*A*a^3)/3 + (73*B*a^3)/4))/(d*(6*tan(c/2 + (d*x)/2)
^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8
 + 1)) + (5*a^3*atanh(tan(c/2 + (d*x)/2))*(4*A + 3*B))/(4*d)
```

3.65 $\int (a + a\sec(c + dx))^3 (A + B\sec(c + dx))\, dx$

Optimal. Leaf size=111

$$a^3Ax+\frac{a^3(7A+5B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5a^3(A+B)\tan(c+dx)}{2d}+\frac{aB(a+a\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{(3$$

```
[Out] a^3*A*x+1/2*a^3*(7*A+5*B)*arctanh(sin(d*x+c))/d+5/2*a^3*(A+B)*tan(d*x+c)/d+
1/3*a*B*(a+a*sec(d*x+c))^2*tan(d*x+c)/d+1/6*(3*A+5*B)*(a^3+a^3*sec(d*x+c))*
tan(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4002, 3999, 3852, 8, 3855}

$$\frac{5a^3(A+B)\tan(c+dx)}{2d}+\frac{a^3(7A+5B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(3A+5B)\tan(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{6d}+a^3Ax+\frac{aB\tan(c+dx)(a\sec(c+dx)+a)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] a^3*A*x + (a^3*(7*A + 5*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (5*a^3*(A + B)*Ta
n[c + d*x])/(2*d) + (a*B*(a + a*Sec[c + d*x])^2*Tan[c + d*x])/(3*d) + ((3*A
 + 5*B)*(a^3 + a^3*Sec[c + d*x])*Tan[c + d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3999

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 4002

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m +
(b*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*
m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aB(a+a\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{3}\int (a+a\sec(c+d$$

$$= \frac{aB(a+a\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{(3A+5B)\left(a^3+a^3\right.}{}$$

$$= a^3Ax+\frac{aB(a+a\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{(3A+5B)\,(}{}$$

$$= a^3Ax+\frac{a^3(7A+5B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aB(a+a\sec(}{}$$

$$= a^3Ax+\frac{a^3(7A+5B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5a^3(A+B)\,\mathrm{ta}}{2d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1056 vs. $2(111) = 222$.
time = 6.41, size = 1056, normalized size = 9.51

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (A*x*Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[
c + d*x]))/(8*(B + A*Cos[c + d*x])) + ((-7*A - 5*B)*Cos[c + d*x]^4*Log[Cos[
c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*
x])^3*(A + B*Sec[c + d*x]))/(16*d*(B + A*Cos[c + d*x])) + ((7*A + 5*B)*Cos[
c + d*x]^4*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^
6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/(16*d*(B + A*Cos[c + d*x]))
+ (B*Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[
c + d*x])*Sin[(d*x)/2])/(48*d*(B + A*Cos[c + d*x])*(Cos[c/2] - Sin[c/2])*(C
os[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^3) + (Cos[c + d*x]^4*Sec[c/2 + (d*x
)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x])*(3*A*Cos[c/2] + 10*B*Cos
[c/2] - 3*A*Sin[c/2] - 8*B*Sin[c/2]))/(96*d*(B + A*Cos[c + d*x])*(Cos[c/2]
```

```
- Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])^2) + (Cos[c + d*x]^4*
Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x])*(9*A*Sin[(
d*x)/2] + 11*B*Sin[(d*x)/2]))/(24*d*(B + A*Cos[c + d*x])*(Cos[c/2] - Sin[c/
2])*(Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2])) + (B*Cos[c + d*x]^4*Sec[c/2
+ (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x])*Sin[(d*x)/2])/(48*
d*(B + A*Cos[c + d*x])*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2
+ (d*x)/2])^3) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^
3*(A + B*Sec[c + d*x])*(-3*A*Cos[c/2] - 10*B*Cos[c/2] - 3*A*Sin[c/2] - 8*B*
Sin[c/2]))/(96*d*(B + A*Cos[c + d*x])*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x
)/2] + Sin[c/2 + (d*x)/2])^2) + (Cos[c + d*x]^4*Sec[c/2 + (d*x)/2]^6*(a + a
*Sec[c + d*x])^3*(A + B*Sec[c + d*x])*(9*A*Sin[(d*x)/2] + 11*B*Sin[(d*x)/2]
))/(24*d*(B + A*Cos[c + d*x])*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + S
in[c/2 + (d*x)/2]))
```

Maple [A]
time = 0.29, size = 176, normalized size = 1.59

method	result
derivativedivides	$\frac{A a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a^3 B\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+3A a^3\tan(dx+c)+3a^3 B\left(\sec\cdots\right.}{\cdots}$
default	$\frac{A a^3\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a^3 B\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+3A a^3\tan(dx+c)+3a^3 B\left(\sec\cdots\right.}{\cdots}$
norman	$\frac{a^3Ax\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^3Ax+3a^3Ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3a^3Ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{a^3(7A+11B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{4a^3(9A+10B)(\cdots}{3\cdots}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$
risch	$a^3Ax-\frac{ia^3\left(3A\,e^{5i(dx+c)}+9B\,e^{5i(dx+c)}-18A\,e^{4i(dx+c)}-18B\,e^{4i(dx+c)}-36A\,e^{2i(dx+c)}-48B\,e^{2i(dx+c)}-3\,e^{i(dx+c)}A-\cdots\right.}{3d\left(e^{2i(dx+c)}+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-a^3*B*
(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+3*A*a^3*tan(d*x+c)+3*a^3*B*(1/2*sec(d*x+
c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*A*a^3*ln(sec(d*x+c)+tan(d*x+
c))+3*a^3*B*tan(d*x+c)+A*a^3*(d*x+c)+a^3*B*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.27, size = 198, normalized size = 1.78

$$\frac{12\,(dx+c)Aa^3+4\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ba^3-3\,Aa^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-9\,Ba^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+36\,Aa^3\log(\sec(dx+c)+\tan(dx+c))+12\,Ba^3\log(\sec(dx+c)+\tan(dx+c))+36\,Aa^3\tan(dx+c)+36\,Ba^3\tan(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*A*a^3 + 4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3 - 3*A*
a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) - 9*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) + 36*A*a^3*log(sec(d*x + c) + tan(d*x +
 c)) + 12*B*a^3*log(sec(d*x + c) + tan(d*x + c)) + 36*A*a^3*tan(d*x + c) +
36*B*a^3*tan(d*x + c))/d
```

Fricas [A]
time = 2.03, size = 141, normalized size = 1.27

$$\frac{12\,Aa^3dx\cos\left(dx+c\right)^3+3\,(7\,A+5\,B)a^3\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,(7\,A+5\,B)a^3\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(2\,(9\,A+11\,B)a^3\cos\left(dx+c\right)^2+3\,(A+3\,B)a^3\cos\left(dx+c\right)+2\,Ba^3\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(12*A*a^3*d*x*cos(d*x + c)^3 + 3*(7*A + 5*B)*a^3*cos(d*x + c)^3*log(si
n(d*x + c) + 1) - 3*(7*A + 5*B)*a^3*cos(d*x + c)^3*log(-sin(d*x + c) + 1) +
 2*(2*(9*A + 11*B)*a^3*cos(d*x + c)^2 + 3*(A + 3*B)*a^3*cos(d*x + c) + 2*B*
a^3)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int A\,dx+\int 3A\sec\left(c+dx\right)dx+\int 3A\sec^2\left(c+dx\right)dx+\int A\sec^3\left(c+dx\right)dx+\int B\sec\left(c+dx\right)dx+\int 3B\sec^2\left(c+dx\right)dx+\int 3B\sec^3\left(c+dx\right)dx+\int B\sec^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] a**3*(Integral(A, x) + Integral(3*A*sec(c + d*x), x) + Integral(3*A*sec(c +
 d*x)**2, x) + Integral(A*sec(c + d*x)**3, x) + Integral(B*sec(c + d*x), x)
 + Integral(3*B*sec(c + d*x)**2, x) + Integral(3*B*sec(c + d*x)**3, x) + In
tegral(B*sec(c + d*x)**4, x))
```

Giac [A]
time = 0.50, size = 189, normalized size = 1.70

$$\frac{6\,(dx+c)Aa^3+3\,(7\,Aa^3+5\,Ba^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(7\,Aa^3+5\,Ba^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(15\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-36\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+21\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+33\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*A*a^3 + 3*(7*A*a^3 + 5*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1)) - 3*(7*A*a^3 + 5*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(15*A
*a^3*tan(1/2*d*x + 1/2*c)^5 + 15*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 36*A*a^3*ta
n(1/2*d*x + 1/2*c)^3 - 40*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 21*A*a^3*tan(1/2*d
```

```
*x + 1/2*c) + 33*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3
)/d
```

Mupad [B]
time = 2.08, size = 209, normalized size = 1.88

$$\frac{2\,A\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{7\,A\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{5\,B\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,A\,a^3\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{A\,a^3\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2}+\frac{11\,B\,a^3\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)}+\frac{3\,B\,a^3\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2}+\frac{B\,a^3\,\sin\left(c+d\,x\right)}{3\,d\,{\cos\left(c+d\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (2*A*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (7*A*a^3*atanh(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (5*B*a^3*atanh(sin(c/2 + (d*x)/2)
/cos(c/2 + (d*x)/2)))/d + (3*A*a^3*sin(c + d*x))/(d*cos(c + d*x)) + (A*a^3*
sin(c + d*x))/(2*d*cos(c + d*x)^2) + (11*B*a^3*sin(c + d*x))/(3*d*cos(c + d
*x)) + (3*B*a^3*sin(c + d*x))/(2*d*cos(c + d*x)^2) + (B*a^3*sin(c + d*x))/(
3*d*cos(c + d*x)^3)
```

3.66 $\int \cos(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=108

$$a^3(3A+B)x+\frac{a^3(6A+7B)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{5a^3B\sin(c+dx)}{2d}+\frac{aB(a+a\sec(c+dx))^2\sin(c+dx)}{2d}+(\ldots$$

```
[Out] a^3*(3*A+B)*x+1/2*a^3*(6*A+7*B)*arctanh(sin(d*x+c))/d-5/2*a^3*B*sin(d*x+c)/
d+1/2*a*B*(a+a*sec(d*x+c))^2*sin(d*x+c)/d+(A+2*B)*(a^3+a^3*sec(d*x+c))*sin(
d*x+c)/d
```

Rubi [A]
time = 0.17, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4103, 4081, 3855}

$$\frac{a^3(6A+7B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(A+2B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{d}+a^3x(3A+B)-\frac{5a^3B\sin(c+dx)}{2d}+\frac{aB\sin(c+dx)(a\sec(c+dx)+a)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] a^3*(3*A + B)*x + (a^3*(6*A + 7*B)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^3*B*
Sin[c + d*x])/(2*d) + (a*B*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) + ((A
 + 2*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/d
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
```

```
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \cos(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx &= \frac{aB(a+a\sec(c+dx))^2\sin(c+dx)}{2d}+\frac{1}{2}\int \cos(c\\
&= \frac{aB(a+a\sec(c+dx))^2\sin(c+dx)}{2d}+\frac{(A+2B)\\
&= -\frac{5a^3B\sin(c+dx)}{2d}+\frac{aB(a+a\sec(c+dx))^2\sin\\
&= a^3(3A+B)x-\frac{5a^3B\sin(c+dx)}{2d}+\frac{aB(a+a\operatorname{se}\\
&= a^3(3A+B)x+\frac{a^3(6A+7B)\tanh^{-1}(\sin(c+dx)}{2d}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 335 vs. 2(108) = 216.

time = 2.65, size = 335, normalized size = 3.10

$$\frac{a^3\cos^4(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^3(A+B\sec(c+dx))\left(4(3A+B)x-\frac{2(6A+7B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{2(6A+7B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{4A\cos(dx)\sin(c)}{d}+\frac{4A\cos(c)\sin(dx)}{d}+\frac{B}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4(A+3B)\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{B}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4(A+3B)\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)}{32(B+A\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*Cos[c + d*x]^4*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(A + B*Sec[c +
d*x])*(4*(3*A + B)*x - (2*(6*A + 7*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]])/d + (2*(6*A + 7*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (4*A*
Cos[d*x]*Sin[c])/d + (4*A*Cos[c]*Sin[d*x])/d + B/(d*(Cos[(c + d*x)/2] - Sin
[(c + d*x)/2])^2) + (4*(A + 3*B)*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(Co
s[(c + d*x)/2] - Sin[(c + d*x)/2])) - B/(d*(Cos[(c + d*x)/2] + Sin[(c + d*x
)/2])^2) + (4*(A + 3*B)*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2]))))/(32*(B + A*Cos[c + d*x]))
```

Maple [A]
time = 0.32, size = 137, normalized size = 1.27

method	result
derivativedivides	$\frac{A\,a^3\tan(dx+c)+a^3B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3A\,a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3B\tan(dx+c}{d}$
default	$\frac{A\,a^3\tan(dx+c)+a^3B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+3A\,a^3\ln(\sec(dx+c)+\tan(dx+c))+3a^3B\tan(dx+c}{d}$

risch	$3a^3Ax + a^3xB - \frac{iA\,a^3\mathrm{e}^{i(dx+c)}}{2d} + \frac{iA\,a^3\mathrm{e}^{-i(dx+c)}}{2d} - \frac{ia^3\left(B\,\mathrm{e}^{3i(dx+c)} - 2A\,\mathrm{e}^{2i(dx+c)} - 6B\,\mathrm{e}^{2i(dx+c)} - B\,\mathrm{e}^{i(dx+c}}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^2}$
norman	$\frac{(-3A\,a^3-a^3B)x+(-6A\,a^3-2a^3B)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(3A\,a^3+a^3B)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(6A\,a^3+2a^3B)x\left(\tan^2\left(\frac{dx}{2}+\right.\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a^3*tan(d*x+c)+a^3*B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+ta
n(d*x+c)))+3*A*a^3*ln(sec(d*x+c)+tan(d*x+c))+3*a^3*B*tan(d*x+c)+3*A*a^3*(d*
x+c)+3*a^3*B*ln(sec(d*x+c)+tan(d*x+c))+A*a^3*sin(d*x+c)+a^3*B*(d*x+c))
```

Maxima [A]
time = 0.27, size = 165, normalized size = 1.53

$$\frac{12\,(dx+c)Aa^3 + 4\,(dx+c)Ba^3 - Ba^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1} - \log\left(\sin\left(dx+c\right)+1\right) + \log\left(\sin\left(dx+c\right)-1\right)\right) + 6\,Aa^3(\log\left(\sin\left(dx+c\right)+1\right) - \log\left(\sin\left(dx+c\right)-1\right)) + 6\,Ba^3(\log\left(\sin\left(dx+c\right)+1\right) - \log\left(\sin\left(dx+c\right)-1\right)) + 4\,Aa^3\sin\left(dx+c\right) + 4\,Aa^3\tan\left(dx+c\right) + 12\,Ba^3\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*(12*(d*x + c)*A*a^3 + 4*(d*x + c)*B*a^3 - B*a^3*(2*sin(d*x + c)/(sin(d*
x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*A*a^3*(l
og(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*B*a^3*(log(sin(d*x + c) +
 1) - log(sin(d*x + c) - 1)) + 4*A*a^3*sin(d*x + c) + 4*A*a^3*tan(d*x + c)
+ 12*B*a^3*tan(d*x + c))/d
```

Fricas [A]
time = 1.85, size = 137, normalized size = 1.27

$$\frac{4\,(3\,A+B)a^3dx\cos\left(dx+c\right)^2 + (6\,A+7\,B)a^3\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right) - (6\,A+7\,B)a^3\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right) + 2\left(2\,Aa^3\cos\left(dx+c\right)^2 + 2\,(A+3\,B)a^3\cos\left(dx+c\right) + Ba^3\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/4*(4*(3*A + B)*a^3*d*x*cos(d*x + c)^2 + (6*A + 7*B)*a^3*cos(d*x + c)^2*lo
g(sin(d*x + c) + 1) - (6*A + 7*B)*a^3*cos(d*x + c)^2*log(-sin(d*x + c) + 1)
 + 2*(2*A*a^3*cos(d*x + c)^2 + 2*(A + 3*B)*a^3*cos(d*x + c) + B*a^3)*sin(d*
x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int A\cos\left(c+dx\right)dx + \int 3A\cos\left(c+dx\right)\sec\left(c+dx\right)dx + \int 3A\cos\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int A\cos\left(c+dx\right)\sec^3\left(c+dx\right)dx + \int B\cos\left(c+dx\right)\sec\left(c+dx\right)dx + \int 3B\cos\left(c+dx\right)\sec^2\left(c+dx\right)dx + \int 3B\cos\left(c+dx\right)\sec^3\left(c+dx\right)dx + \int B\cos\left(c+dx\right)\sec^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] a**3*(Integral(A*cos(c + d*x), x) + Integral(3*A*cos(c + d*x)*sec(c + d*x),
 x) + Integral(3*A*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(A*cos(c + d*
x)*sec(c + d*x)**3, x) + Integral(B*cos(c + d*x)*sec(c + d*x), x) + Integra
l(3*B*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(3*B*cos(c + d*x)*sec(c +
d*x)**3, x) + Integral(B*cos(c + d*x)*sec(c + d*x)**4, x))
```

Giac [A]
time = 0.51, size = 192, normalized size = 1.78

$$\frac{\frac{4\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+2\,(3\,Aa^3+Ba^3)(dx+c)+(6\,Aa^3+7\,Ba^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(6\,Aa^3+7\,Ba^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(2\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-7\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*(4*A*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(3*A*a^3
 + B*a^3)*(d*x + c) + (6*A*a^3 + 7*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1)
) - (6*A*a^3 + 7*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*A*a^3*tan
(1/2*d*x + 1/2*c)^3 + 5*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 2*A*a^3*tan(1/2*d*x
+ 1/2*c) - 7*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2)/d
```

Mupad [B]
time = 2.12, size = 207, normalized size = 1.92

$$\frac{A\,a^3\,\sin\left(c+d\,x\right)}{d}+\frac{6\,A\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{6\,A\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{7\,B\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{A\,a^3\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{3\,B\,a^3\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{B\,a^3\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (A*a^3*sin(c + d*x))/d + (6*A*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/d + (6*A*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a^3
*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (7*B*a^3*atanh(sin(c/2 +
(d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*a^3*sin(c + d*x))/(d*cos(c + d*x)) + (
3*B*a^3*sin(c + d*x))/(d*cos(c + d*x)) + (B*a^3*sin(c + d*x))/(2*d*cos(c +
d*x)^2)
```

3.67 $\int \cos^2(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=117

$$\frac{1}{2}a^3(7A+6B)x+\frac{a^3(A+3B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^3A\sin(c+dx)}{2d}+\frac{aA\cos(c+dx)(a+a\sec(c+dx))^2\,\mathrm{si}}{2d}$$

```
[Out] 1/2*a^3*(7*A+6*B)*x+a^3*(A+3*B)*arctanh(sin(d*x+c))/d+5/2*a^3*A*sin(d*x+c)/
d+1/2*a*A*cos(d*x+c)*(a+a*sec(d*x+c))^2*sin(d*x+c)/d-1/2*(A-2*B)*(a^3+a^3*s
ec(d*x+c))*sin(d*x+c)/d
```

Rubi [A]
time = 0.18, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4102, 4103, 4081, 3855}

$$\frac{a^3(A+3B)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(A-2B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{2d}+\frac{1}{2}a^3x(7A+6B)+\frac{5a^3A\sin(c+dx)}{2d}+\frac{aA\sin(c+dx)\cos(c+dx)(a\sec(c+dx)+a)^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*(7*A + 6*B)*x)/2 + (a^3*(A + 3*B)*ArcTanh[Sin[c + d*x]])/d + (5*a^3*A*
Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]*(a + a*Sec[c + d*x])^2*Sin[c + d*x]
)/(2*d) - ((A - 2*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(2*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
```

```
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aA\cos(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{2d}$$
$$= \frac{aA\cos(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{2d}$$
$$= \frac{5a^3A\sin(c+dx)}{2d} + \frac{aA\cos(c+dx)(a+a\sec(c}{2d}$$
$$= \frac{1}{2}a^3(7A+6B)x + \frac{5a^3A\sin(c+dx)}{2d} + \frac{aA\cos(c}{}$$
$$= \frac{1}{2}a^3(7A+6B)x + \frac{a^3(A+3B)\tanh^{-1}(\sin(c+}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 302 vs. 2(117) = 234.
time = 4.54, size = 302, normalized size = 2.58

$$\frac{a^3\cos^4(c+dx)\sec^6\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^3(A+B\sec(c+dx))\left(2(7A+6B)x-\frac{4(A+3B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{4(A+3B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{4(3A+B)\cos(dx)\sin(c)}{d}+\frac{A\cos(2dx)\sin(2c)}{d}+\frac{4(3A+B)\cos(c)\sin(dx)}{d}+\frac{A\cos(2c)\sin(2dx)}{d}+\frac{4B\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{4B\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)}{32(B+A\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*Cos[c + d*x]^4*Sec[(c + d*x)/2]^6*(1 + Sec[c + d*x])^3*(A + B*Sec[c +
d*x])*(2*(7*A + 6*B)*x - (4*(A + 3*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/
2]])/d + (4*(A + 3*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (4*(3*A
 + B)*Cos[d*x]*Sin[c])/d + (A*Cos[2*d*x]*Sin[2*c])/d + (4*(3*A + B)*Cos[c]*
Sin[d*x])/d + (A*Cos[2*c]*Sin[2*d*x])/d + (4*B*Sin[(d*x)/2])/(d*(Cos[c/2] -
 Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (4*B*Sin[(d*x)/2])/(d*(
```

```
Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/(32*(B + A*Co
s[c + d*x]))
```

Maple [A]

time = 0.28, size = 128, normalized size = 1.09

method	result
derivativedivides	$\frac{A\,a^3 \ln(\sec(dx+c)+\tan(dx+c))+a^3 B \tan(dx+c)+3A\,a^3 (dx+c)+3a^3 B \ln(\sec(dx+c)+\tan(dx+c))+3A\,a^3 \sin(dx+c)+3a}{d}$
default	$\frac{A\,a^3 \ln(\sec(dx+c)+\tan(dx+c))+a^3 B \tan(dx+c)+3A\,a^3 (dx+c)+3a^3 B \ln(\sec(dx+c)+\tan(dx+c))+3A\,a^3 \sin(dx+c)+3a}{d}$
risch	$\frac{7a^3 Ax}{2}+3a^3 xB-\frac{iA\,a^3 e^{2i(dx+c)}}{8d}-\frac{3iA\,a^3 e^{i(dx+c)}}{2d}-\frac{i e^{i(dx+c)} a^3 B}{2d}+\frac{3iA\,a^3 e^{-i(dx+c)}}{2d}+\frac{i e^{-i(dx+c)} a^3 B}{2d}$
norman	$\frac{(-\frac{7}{2}A\,a^3-3a^3 B)x+(-\frac{7}{2}A\,a^3-3a^3 B)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{7}{2}A\,a^3+3a^3 B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{7}{2}A\,a^3+3a^3 B)x\left(\tan^{10}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*B*tan(d*x+c)+3*A*a^3*(d*x+c)+3*a^3
*B*ln(sec(d*x+c)+tan(d*x+c))+3*A*a^3*sin(d*x+c)+3*a^3*B*(d*x+c)+A*a^3*(1/2*
cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^3*B*sin(d*x+c))
```

Maxima [A]

time = 0.27, size = 140, normalized size = 1.20

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^3+12\,(dx+c)Aa^3+12\,(dx+c)Ba^3+2\,Aa^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+6\,Ba^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+12\,Aa^3\sin(dx+c)+4\,Ba^3\sin(dx+c)+4\,Ba^3\tan(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 + 12*(d*x + c)*A*a^3 + 12*(d*x
+ c)*B*a^3 + 2*A*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*B*
a^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 12*A*a^3*sin(d*x + c)
 + 4*B*a^3*sin(d*x + c) + 4*B*a^3*tan(d*x + c))/d
```

Fricas [A]

time = 2.00, size = 127, normalized size = 1.09

$$\frac{(7\,A+6\,B)a^3dx\cos(dx+c)+(A+3\,B)a^3\cos(dx+c)\log(\sin(dx+c)+1)-(A+3\,B)a^3\cos(dx+c)\log(-\sin(dx+c)+1)+\left(Aa^3\cos(dx+c)^2+2\,(3\,A+B)a^3\cos(dx+c)+2\,Ba^3\right)\sin(dx+c)}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2*((7*A + 6*B)*a^3*d*x*cos(d*x + c) + (A + 3*B)*a^3*cos(d*x + c)*log(sin(
d*x + c) + 1) - (A + 3*B)*a^3*cos(d*x + c)*log(-sin(d*x + c) + 1) + (A*a^3*
cos(d*x + c)^2 + 2*(3*A + B)*a^3*cos(d*x + c) + 2*B*a^3)*sin(d*x + c))/(d*c
os(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int A\cos^2(c+dx)\,dx+\int 3A\cos^2(c+dx)\sec(c+dx)\,dx+\int 3A\cos^2(c+dx)\sec^2(c+dx)\,dx+\int A\cos^2(c+dx)\sec^3(c+dx)\,dx+\int B\cos^2(c+dx)\sec(c+dx)\,dx+\int 3B\cos^2(c+dx)\sec^2(c+dx)\,dx+\int 3B\cos^2(c+dx)\sec^3(c+dx)\,dx+\int B\cos^2(c+dx)\sec^4(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] a**3*(Integral(A*cos(c + d*x)**2, x) + Integral(3*A*cos(c + d*x)**2*sec(c +
 d*x), x) + Integral(3*A*cos(c + d*x)**2*sec(c + d*x)**2, x) + Integral(A*c
os(c + d*x)**2*sec(c + d*x)**3, x) + Integral(B*cos(c + d*x)**2*sec(c + d*x
), x) + Integral(3*B*cos(c + d*x)**2*sec(c + d*x)**2, x) + Integral(3*B*cos
(c + d*x)**2*sec(c + d*x)**3, x) + Integral(B*cos(c + d*x)**2*sec(c + d*x)*
*4, x))
```

Giac [A]
time = 0.51, size = 192, normalized size = 1.64

$$-\frac{\frac{4\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-(7\,Aa^3+6\,Ba^3)(dx+c)-2\,(Aa^3+3\,Ba^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+2\,(Aa^3+3\,Ba^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(5\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*B*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (7*A*a^3
+ 6*B*a^3)*(d*x + c) - 2*(A*a^3 + 3*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) + 1
)) + 2*(A*a^3 + 3*B*a^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(5*A*a^3*ta
n(1/2*d*x + 1/2*c)^3 + 2*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 7*A*a^3*tan(1/2*d*x
 + 1/2*c) + 2*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 2.08, size = 197, normalized size = 1.68

$$\frac{3\,A\,a^3\,\sin\left(c+d\,x\right)}{d}+\frac{B\,a^3\,\sin\left(c+d\,x\right)}{d}+\frac{7\,A\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,A\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{6\,B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{6\,B\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{B\,a^3\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{A\,a^3\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (3*A*a^3*sin(c + d*x))/d + (B*a^3*sin(c + d*x))/d + (7*A*a^3*atan(sin(c/2 +
 (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*A*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/
2 + (d*x)/2)))/d + (6*B*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
+ (6*B*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (B*a^3*sin(c +
 d*x))/(d*cos(c + d*x)) + (A*a^3*cos(c + d*x)*sin(c + d*x))/(2*d)
```

3.68 $\int \cos^3(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=125

$$\frac{1}{2}a^3(5A+7B)x+\frac{a^3B\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^3(A+B)\sin(c+dx)}{2d}+\frac{aA\cos^2(c+dx)(a+a\sec(c+dx))^2\sin}{3d}$$

```
[Out] 1/2*a^3*(5*A+7*B)*x+a^3*B*arctanh(sin(d*x+c))/d+5/2*a^3*(A+B)*sin(d*x+c)/d+
1/3*a*A*cos(d*x+c)^2*(a+a*sec(d*x+c))^2*sin(d*x+c)/d+1/6*(5*A+3*B)*cos(d*x+
c)*(a^3+a^3*sec(d*x+c))*sin(d*x+c)/d
```

Rubi [A]
time = 0.18, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4102, 4081, 3855}

$$\frac{5a^3(A+B)\sin(c+dx)}{2d}+\frac{(5A+3B)\sin(c+dx)\cos(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{6d}+\frac{1}{2}a^3x(5A+7B)+\frac{a^3B\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\sin(c+dx)\cos^2(c+dx)(a\sec(c+dx)+a)^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*(5*A + 7*B)*x)/2 + (a^3*B*ArcTanh[Sin[c + d*x]])/d + (5*a^3*(A + B)*Si
n[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^2*Sin[c + d*x]
)/(3*d) + ((5*A + 3*B)*Cos[c + d*x]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/
(6*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
```

```
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aA\cos^2(c+dx)(a+a\sec(c+dx))^2\sin(c+dx}{3d}$$
$$= \frac{aA\cos^2(c+dx)(a+a\sec(c+dx))^2\sin(c+dx}{3d}$$
$$= \frac{5a^3(A+B)\sin(c+dx)}{2d} + \frac{aA\cos^2(c+dx)(a+}{}$$
$$= \frac{1}{2}a^3(5A+7B)x + \frac{5a^3(A+B)\sin(c+dx)}{2d} +$$
$$= \frac{1}{2}a^3(5A+7B)x + \frac{a^3B\tanh^{-1}(\sin(c+dx))}{d} +$$

Mathematica [A]
time = 0.25, size = 113, normalized size = 0.90

$$\frac{a^3\left(30Adx+42Bdx-12B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+9(5A+4B)\sin(c+dx)+3(3A+B)\sin(2(c+dx))+A\sin(3(c+dx))\right)}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
[Out] (a^3*(30*A*d*x + 42*B*d*x - 12*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] +
 12*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 9*(5*A + 4*B)*Sin[c + d*x]
 + 3*(3*A + B)*Sin[2*(c + d*x)] + A*Sin[3*(c + d*x)]))/(12*d)
```

Maple [A]
time = 0.30, size = 147, normalized size = 1.18

method	result
derivativedivides	$\frac{A\,a^3(dx+c)+a^3B\ln(\sec(dx+c)+\tan(dx+c))+3A\,a^3\sin(dx+c)+3a^3B(dx+c)+3A\,a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a}{d}$
default	$\frac{A\,a^3(dx+c)+a^3B\ln(\sec(dx+c)+\tan(dx+c))+3A\,a^3\sin(dx+c)+3a^3B(dx+c)+3A\,a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3a}{d}$
risch	$\frac{5a^3Ax}{2}+\frac{7a^3xB}{2}-\frac{15iA\,a^3e^{i(dx+c)}}{8d}-\frac{3ie^{i(dx+c)}a^3B}{2d}+\frac{15iA\,a^3e^{-i(dx+c)}}{8d}+\frac{3ie^{-i(dx+c)}a^3B}{2d}+\frac{a^3\ln(e^{i(dx+c}}{d}$

norman	$\frac{(-\frac{5}{2}A\,a^3-\frac{7}{2}a^3B)x+(-\frac{15}{2}A\,a^3-\frac{21}{2}a^3B)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{5}{2}A\,a^3+\frac{7}{2}a^3B)x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{15}{2}A\,a^3+\frac{21}{2}a^3B)x\left(\text{ta}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^3*(d*x+c)+a^3*B*ln(sec(d*x+c)+tan(d*x+c))+3*A*a^3*sin(d*x+c)+3*a^3
*B*(d*x+c)+3*A*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^3*B*sin(d*
x+c)+1/3*A*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+a^3*B*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.26, size = 148, normalized size = 1.18

$$-\frac{4\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Aa^3-9\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^3-12\left(dx+c\right)Aa^3-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ba^3-36\left(dx+c\right)Ba^3-6\,Ba^3\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-36\,Aa^3\sin\left(dx+c\right)-36\,Ba^3\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 9*(2*d*x + 2*c + sin(2*d
*x + 2*c))*A*a^3 - 12*(d*x + c)*A*a^3 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*
B*a^3 - 36*(d*x + c)*B*a^3 - 6*B*a^3*(log(sin(d*x + c) + 1) - log(sin(d*x +
 c) - 1)) - 36*A*a^3*sin(d*x + c) - 36*B*a^3*sin(d*x + c))/d
```

Fricas [A]
time = 1.36, size = 102, normalized size = 0.82

$$\frac{3\left(5\,A+7\,B\right)a^3dx+3\,Ba^3\log\left(\sin\left(dx+c\right)+1\right)-3\,Ba^3\log\left(-\sin\left(dx+c\right)+1\right)+\left(2\,Aa^3\cos\left(dx+c\right)^2+3\left(3\,A+B\right)a^3\cos\left(dx+c\right)+2\left(11\,A+9\,B\right)a^3\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(5*A + 7*B)*a^3*d*x + 3*B*a^3*log(sin(d*x + c) + 1) - 3*B*a^3*log(-s
in(d*x + c) + 1) + (2*A*a^3*cos(d*x + c)^2 + 3*(3*A + B)*a^3*cos(d*x + c) +
 2*(11*A + 9*B)*a^3)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int A\cos^3\left(c+dx\right)dx+\int 3A\cos^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3A\cos^3\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int A\cos^3\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int B\cos^3\left(c+dx\right)\sec\left(c+dx\right)dx+\int 3B\cos^3\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int 3B\cos^3\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int B\cos^3\left(c+dx\right)\sec^4\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] a**3*(Integral(A*cos(c + d*x)**3, x) + Integral(3*A*cos(c + d*x)**3*sec(c +
 d*x), x) + Integral(3*A*cos(c + d*x)**3*sec(c + d*x)**2, x) + Integral(A*c
os(c + d*x)**3*sec(c + d*x)**3, x) + Integral(B*cos(c + d*x)**3*sec(c + d*x
), x) + Integral(3*B*cos(c + d*x)**3*sec(c + d*x)**2, x) + Integral(3*B*cos
(c + d*x)**3*sec(c + d*x)**3, x) + Integral(B*cos(c + d*x)**3*sec(c + d*x)*
*4, x))
```

Giac [A]
time = 0.53, size = 180, normalized size = 1.44

$$\frac{6\,Ba^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,Ba^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\,(5\,Aa^3+7\,Ba^3)(dx+c)+\frac{2\left(15\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+15\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+40\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+36\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+33\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+21\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*B*a^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*B*a^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(5*A*a^3 + 7*B*a^3)*(d*x + c) + 2*(15*A*a^3*tan(1/2*d
*x + 1/2*c)^5 + 15*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 40*A*a^3*tan(1/2*d*x + 1/
2*c)^3 + 36*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 33*A*a^3*tan(1/2*d*x + 1/2*c) +
21*B*a^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 2.15, size = 178, normalized size = 1.42

$$\frac{15\,A\,a^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{3\,B\,a^3\,\sin\left(c+d\,x\right)}{d}+\frac{5\,A\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{7\,B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,A\,a^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{A\,a^3\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{B\,a^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (15*A*a^3*sin(c + d*x))/(4*d) + (3*B*a^3*sin(c + d*x))/d + (5*A*a^3*atan(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (7*B*a^3*atan(sin(c/2 + (d*x)/2)/
cos(c/2 + (d*x)/2)))/d + (2*B*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/
2)))/d + (3*A*a^3*sin(2*c + 2*d*x))/(4*d) + (A*a^3*sin(3*c + 3*d*x))/(12*d)
 + (B*a^3*sin(2*c + 2*d*x))/(4*d)
```

3.69 $\int \cos^4(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=124

$$\frac{5}{8}a^3(3A+4B)x+\frac{a^3(3A+4B)\sin(c+dx)}{d}+\frac{3a^3(3A+4B)\cos(c+dx)\sin(c+dx)}{8d}+\frac{A\cos^3(c+dx)(a+a\sec(}{4d}$$

```
[Out] 5/8*a^3*(3*A+4*B)*x+a^3*(3*A+4*B)*sin(d*x+c)/d+3/8*a^3*(3*A+4*B)*cos(d*x+c)
*sin(d*x+c)/d+1/4*A*cos(d*x+c)^3*(a+a*sec(d*x+c))^3*sin(d*x+c)/d-1/12*a^3*(
3*A+4*B)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.12, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4098, 3876, 2717, 2715, 8, 2713}

$$-\frac{a^3(3A+4B)\sin^3(c+dx)}{12d}+\frac{a^3(3A+4B)\sin(c+dx)}{d}+\frac{3a^3(3A+4B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{5}{8}a^3x(3A+4B)+\frac{A\sin(c+dx)\cos^3(c+dx)(a\sec(c+dx)+a)^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (5*a^3*(3*A + 4*B)*x)/8 + (a^3*(3*A + 4*B)*Sin[c + d*x])/d + (3*a^3*(3*A +
4*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (A*Cos[c + d*x]^3*(a + a*Sec[c + d*
x])^3*Sin[c + d*x])/(4*d) - (a^3*(3*A + 4*B)*Sin[c + d*x]^3)/(12*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{A\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{4d}$$
$$= \frac{A\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{4d}$$
$$= \frac{1}{4}a^3(3A+4B)x + \frac{A\cos^3(c+dx)(a+a\sec(c}{4d}$$
$$= \frac{1}{4}a^3(3A+4B)x + \frac{3a^3(3A+4B)\sin(c+dx)}{4d}$$
$$= \frac{5}{8}a^3(3A+4B)x + \frac{a^3(3A+4B)\sin(c+dx)}{d} +$$

Mathematica [A]
time = 0.28, size = 86, normalized size = 0.69

$$\frac{a^3(180Adx+240Bdx+24(13A+15B)\sin(c+dx)+24(4A+3B)\sin(2(c+dx))+24A\sin(3(c+dx))+8B\sin(3(c+dx))+3A\sin(4(c+dx)))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*(180*A*d*x + 240*B*d*x + 24*(13*A + 15*B)*Sin[c + d*x] + 24*(4*A + 3*B
)*Sin[2*(c + d*x)] + 24*A*Sin[3*(c + d*x)] + 8*B*Sin[3*(c + d*x)] + 3*A*Sin
[4*(c + d*x)]))/(96*d)
```

Maple [A]
time = 0.36, size = 176, normalized size = 1.42

method	result
risch	$\frac{15a^3Ax}{8}+\frac{5a^3xB}{2}+\frac{13a^3A\sin(dx+c)}{4d}+\frac{15a^3B\sin(dx+c)}{4d}+\frac{A\,a^3\sin(4dx+4c)}{32d}+\frac{A\,a^3\sin(3dx+3c)}{4d}+\frac{\sin(3dx+}{12d}$
derivativedivides	$\frac{A\,a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+A\,a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)+\frac{a^3B\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+3}{d}$
default	$\frac{A\,a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+A\,a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)+\frac{a^3B\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+3}{d}$
norman	$\frac{-\frac{5a^3(3A+4B)x}{8}-\frac{47a^3(3A+4B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{5a^3(3A+4B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}+\frac{5a^3(3A+4B)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{5a^3(3A+4B)x(}{8}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*a
^3*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*a^3*B*(2+cos(d*x+c)^2)*sin(d*x+c)+3*A*a^
3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*a^3*B*(1/2*cos(d*x+c)*sin(d*x
+c)+1/2*d*x+1/2*c)+A*a^3*sin(d*x+c)+3*a^3*B*sin(d*x+c)+a^3*B*(d*x+c))
```

Maxima [A]

time = 0.29, size = 167, normalized size = 1.35

$$-\frac{96\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Aa^3-3\left(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right)\right)Aa^3-72\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^3+32\left(\sin\left(dx+c\right)^3-3\,\sin\left(dx+c\right)\right)Ba^3-72\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ba^3-96\left(dx+c\right)Ba^3-96\,Aa^3\sin\left(dx+c\right)-288\,Ba^3\sin\left(dx+c\right)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/96*(96*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 3*(12*d*x + 12*c + sin(
4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 - 72*(2*d*x + 2*c + sin(2*d*x + 2*
c))*A*a^3 + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 - 72*(2*d*x + 2*c +
sin(2*d*x + 2*c))*B*a^3 - 96*(d*x + c)*B*a^3 - 96*A*a^3*sin(d*x + c) - 288*
B*a^3*sin(d*x + c))/d
```

Fricas [A]

time = 2.14, size = 90, normalized size = 0.73

$$\frac{15\,(3\,A+4\,B)a^3dx+\left(6\,Aa^3\cos\left(dx+c\right)^3+8\,(3\,A+B)a^3\cos\left(dx+c\right)^2+9\,(5\,A+4\,B)a^3\cos\left(dx+c\right)+8\,(9\,A+11\,B)a^3\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/24*(15*(3*A + 4*B)*a^3*d*x + (6*A*a^3*cos(d*x + c)^3 + 8*(3*A + B)*a^3*co
s(d*x + c)^2 + 9*(5*A + 4*B)*a^3*cos(d*x + c) + 8*(9*A + 11*B)*a^3)*sin(d*x
 + c))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 0.54, size = 176, normalized size = 1.42

$$\frac{15\,(3\,Aa^3+4\,Ba^3)(dx+c)+\frac{2\left(45\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+60\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+165\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+220\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+219\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+292\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+147\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+132\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(15*(3*A*a^3 + 4*B*a^3)*(d*x + c) + 2*(45*A*a^3*tan(1/2*d*x + 1/2*c)^7
 + 60*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 165*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 220
*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 219*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 292*B*a^
3*tan(1/2*d*x + 1/2*c)^3 + 147*A*a^3*tan(1/2*d*x + 1/2*c) + 132*B*a^3*tan(1
/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [**B**]
time = 2.01, size = 134, normalized size = 1.08

$$\frac{15\,A\,a^3\,x}{8}+\frac{5\,B\,a^3\,x}{2}+\frac{13\,A\,a^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{15\,B\,a^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{A\,a^3\,\sin\left(2\,c+2\,d\,x\right)}{d}+\frac{A\,a^3\,\sin\left(3\,c+3\,d\,x\right)}{4\,d}+\frac{A\,a^3\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{3\,B\,a^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,a^3\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (15*A*a^3*x)/8 + (5*B*a^3*x)/2 + (13*A*a^3*sin(c + d*x))/(4*d) + (15*B*a^3*
sin(c + d*x))/(4*d) + (A*a^3*sin(2*c + 2*d*x))/d + (A*a^3*sin(3*c + 3*d*x))
/(4*d) + (A*a^3*sin(4*c + 4*d*x))/(32*d) + (3*B*a^3*sin(2*c + 2*d*x))/(4*d)
 + (B*a^3*sin(3*c + 3*d*x))/(12*d)
```

3.70 $\int \cos^5(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=176

$$\frac{1}{8}a^3(13A+15B)x+\frac{a^3(38A+45B)\sin(c+dx)}{15d}+\frac{a^3(13A+15B)\cos(c+dx)\sin(c+dx)}{8d}+\frac{a^3(43A+45B)\cos$$

```
[Out] 1/8*a^3*(13*A+15*B)*x+1/15*a^3*(38*A+45*B)*sin(d*x+c)/d+1/8*a^3*(13*A+15*B)
*cos(d*x+c)*sin(d*x+c)/d+1/60*a^3*(43*A+45*B)*cos(d*x+c)^2*sin(d*x+c)/d+1/5
*a*A*cos(d*x+c)^4*(a+a*sec(d*x+c))^2*sin(d*x+c)/d+1/20*(7*A+5*B)*cos(d*x+c)
^3*(a^3+a^3*sec(d*x+c))*sin(d*x+c)/d
```

Rubi [A]
time = 0.26, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4102, 4081, 3872, 2715, 8, 2717}

$$\frac{a^3(38A+45B)\sin(c+dx)}{15d}+\frac{a^3(43A+45B)\sin(c+dx)\cos^2(c+dx)}{60d}+\frac{a^3(13A+15B)\sin(c+dx)\cos(c+dx)}{8d}+\frac{(7A+5B)\sin(c+dx)\cos^3(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{20d}+\frac{1}{8}a^3x(13A+15B)+\frac{aA\sin(c+dx)\cos^4(c+dx)(a\sec(c+dx)+a)^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*(13*A + 15*B)*x)/8 + (a^3*(38*A + 45*B)*Sin[c + d*x])/(15*d) + (a^3*(1
3*A + 15*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^3*(43*A + 45*B)*Cos[c + d
*x]^2*Sin[c + d*x])/(60*d) + (a*A*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^2*Sin
[c + d*x])/(5*d) + ((7*A + 5*B)*Cos[c + d*x]^3*(a^3 + a^3*Sec[c + d*x])*Sin
[c + d*x])/(20*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aA\cos^4(c+dx)(a+a\sec(c+dx))^2\sin(c+dx}{5d}$$
$$= \frac{aA\cos^4(c+dx)(a+a\sec(c+dx))^2\sin(c+dx}{5d}$$
$$= \frac{a^3(43A+45B)\cos^2(c+dx)\sin(c+dx)}{60d} + \frac{aA}{}$$
$$= \frac{a^3(43A+45B)\cos^2(c+dx)\sin(c+dx)}{60d} + \frac{aA}{}$$
$$= \frac{a^3(38A+45B)\sin(c+dx)}{15d} + \frac{a^3(13A+15B)}{}$$
$$= \frac{1}{8}a^3(13A+15B)x + \frac{a^3(38A+45B)\sin(c+d}{15d}$$

Mathematica [A]
time = 0.47, size = 108, normalized size = 0.61

$$\frac{a^3(780Ac+780Adx+900Bdx+60(23A+26B)\sin(c+dx)+480(A+B)\sin(2(c+dx))+170A\sin(3(c+dx))+120B\sin(3(c+dx))+45A\sin(4(c+dx))+15B\sin(4(c+dx))+6A\sin(5(c+dx)))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*(780*A*c + 780*A*d*x + 900*B*d*x + 60*(23*A + 26*B)*Sin[c + d*x] + 480
*(A + B)*Sin[2*(c + d*x)] + 170*A*Sin[3*(c + d*x)] + 120*B*Sin[3*(c + d*x)]
 + 45*A*Sin[4*(c + d*x)] + 15*B*Sin[4*(c + d*x)] + 6*A*Sin[5*(c + d*x)]))/(
480*d)
```

Maple [A]
time = 0.45, size = 223, normalized size = 1.27

method	result
risch	$\frac{13a^3Ax}{8}+\frac{15a^3xB}{8}+\frac{23a^3A\sin(dx+c)}{8d}+\frac{13a^3B\sin(dx+c)}{4d}+\frac{A\,a^3\sin(5dx+5c)}{80d}+\frac{3A\,a^3\sin(4dx+4c)}{32d}+\frac{\sin(4dx$
derivativedivides	$A\,a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^3B\sin(dx+c)+A\,a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)+3a^3B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)$
default	$A\,a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^3B\sin(dx+c)+A\,a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)+3a^3B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)$
norman	$-\frac{a^3(13A+15B)x}{8}-\frac{37a^3(13A+15B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60d}+\frac{5a^3(13A+15B)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}+\frac{a^3(13A+15B)\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{a^3(13A$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^3*B*sin(d*x+c)+A*a^3
*(2+cos(d*x+c)^2)*sin(d*x+c)+3*a^3*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2
*c)+3*A*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^
3*B*(2+cos(d*x+c)^2)*sin(d*x+c)+1/5*A*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^
2)*sin(d*x+c)+a^3*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3
/8*c))
```

Maxima [A]
time = 0.29, size = 213, normalized size = 1.21

$$\frac{32\,(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c))Aa^3-480\,(\sin(dx+c)^3-3\,\sin(dx+c))Aa^3+45\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Aa^3+120\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^3-480\,(\sin(dx+c)^3-3\,\sin(dx+c))Ba^3+15\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Ba^3+360\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^3+480\,Ba^3\sin(dx+c)}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^3 -
480*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 + 45*(12*d*x + 12*c + sin(4*d*x
```

```
 + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 + 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*
A*a^3 - 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 + 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 + 360*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*B*a^3 + 480*B*a^3*sin(d*x + c))/d
```

Fricas [A]
time = 1.59, size = 110, normalized size = 0.62

$$\frac{15\,(13\,A+15\,B)a^3dx+\left(24\,Aa^3\cos\left(dx+c\right)^4+30\,(3\,A+B)a^3\cos\left(dx+c\right)^3+8\,(19\,A+15\,B)a^3\cos\left(dx+c\right)^2+15\,(13\,A+15\,B)a^3\cos\left(dx+c\right)+8\,(38\,A+45\,B)a^3\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(13*A + 15*B)*a^3*d*x + (24*A*a^3*cos(d*x + c)^4 + 30*(3*A + B)*a
^3*cos(d*x + c)^3 + 8*(19*A + 15*B)*a^3*cos(d*x + c)^2 + 15*(13*A + 15*B)*a
^3*cos(d*x + c) + 8*(38*A + 45*B)*a^3)*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.49, size = 210, normalized size = 1.19

$$\frac{15\,(13\,Aa^3+15\,Ba^3)(dx+c)+\frac{2\left(195\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+225\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+910\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1050\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1664\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1920\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1330\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1830\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+765\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+735\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(13*A*a^3 + 15*B*a^3)*(d*x + c) + 2*(195*A*a^3*tan(1/2*d*x + 1/2*
c)^9 + 225*B*a^3*tan(1/2*d*x + 1/2*c)^9 + 910*A*a^3*tan(1/2*d*x + 1/2*c)^7
+ 1050*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 1664*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 1
920*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 1330*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 1830
*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 765*A*a^3*tan(1/2*d*x + 1/2*c) + 735*B*a^3*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B]
time = 4.71, size = 247, normalized size = 1.40

$$\frac{\left(\frac{13\,A\,a^3}{4}+\frac{15\,B\,a^3}{4}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{91\,A\,a^3}{6}+\frac{35\,B\,a^3}{2}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{416\,A\,a^3}{15}+32\,B\,a^3\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{133\,A\,a^3}{6}+\frac{61\,B\,a^3}{2}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{51\,A\,a^3}{4}+\frac{49\,B\,a^3}{4}\right)\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+5\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+10\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{a^3\,\mathrm{atan}\left(\frac{a^3\,\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(13\,A+15\,B\right)}{4\,\left(\frac{13\,A\,a^3}{4}+\frac{15\,B\,a^3}{4}\right)}\right)\,\left(13\,A+15\,B\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((51*A*a^3)/4 + (49*B*a^3)/4) + tan(c/2 + (d*x)/2)^9*((
13*A*a^3)/4 + (15*B*a^3)/4) + tan(c/2 + (d*x)/2)^7*((91*A*a^3)/6 + (35*B*a^
3)/2) + tan(c/2 + (d*x)/2)^3*((133*A*a^3)/6 + (61*B*a^3)/2) + tan(c/2 + (d*
x)/2)^5*((416*A*a^3)/15 + 32*B*a^3))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/
2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2
 + (d*x)/2)^10 + 1)) + (a^3*atan((a^3*tan(c/2 + (d*x)/2)*(13*A + 15*B))/(4*
((13*A*a^3)/4 + (15*B*a^3)/4)))*(13*A + 15*B))/(4*d)
```

3.71 $\int \cos^6(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=201

$$\frac{1}{16}a^3(23A+26B)x+\frac{a^3(17A+19B)\sin(c+dx)}{5d}+\frac{a^3(23A+26B)\cos(c+dx)\sin(c+dx)}{16d}+\frac{a^3(21A+22B)\cos\ldots}{}$$

```
[Out] 1/16*a^3*(23*A+26*B)*x+1/5*a^3*(17*A+19*B)*sin(d*x+c)/d+1/16*a^3*(23*A+26*B
)*cos(d*x+c)*sin(d*x+c)/d+1/40*a^3*(21*A+22*B)*cos(d*x+c)^3*sin(d*x+c)/d+1/
6*a*A*cos(d*x+c)^5*(a+a*sec(d*x+c))^2*sin(d*x+c)/d+1/15*(4*A+3*B)*cos(d*x+c
)^4*(a^3+a^3*sec(d*x+c))*sin(d*x+c)/d-1/15*a^3*(17*A+19*B)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.29, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4102, 4081, 3872, 2713, 2715, 8}

$$-\frac{a^3(17A+19B)\sin^3(c+dx)}{15d}+\frac{a^3(17A+19B)\sin(c+dx)}{5d}+\frac{a^3(21A+22B)\sin(c+dx)\cos^3(c+dx)}{40d}+\frac{a^3(23A+26B)\sin(c+dx)\cos(c+dx)}{16d}+\frac{(4A+3B)\sin(c+dx)\cos^4(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{15d}+\frac{1}{16}a^3x(23A+26B)+\frac{aA\sin(c+dx)\cos^5(c+dx)(a\sec(c+dx)+a)^2}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*(23*A + 26*B)*x)/16 + (a^3*(17*A + 19*B)*Sin[c + d*x])/(5*d) + (a^3*(2
3*A + 26*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^3*(21*A + 22*B)*Cos[c +
d*x]^3*Sin[c + d*x])/(40*d) + (a*A*Cos[c + d*x]^5*(a + a*Sec[c + d*x])^2*Si
n[c + d*x])/(6*d) + ((4*A + 3*B)*Cos[c + d*x]^4*(a^3 + a^3*Sec[c + d*x])*Si
n[c + d*x])/(15*d) - (a^3*(17*A + 19*B)*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^6(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aA\cos^5(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{6d}$$
$$= \frac{aA\cos^5(c+dx)(a+a\sec(c+dx))^2\sin(c+dx)}{6d}$$
$$= \frac{a^3(21A+22B)\cos^3(c+dx)\sin(c+dx)}{40d} + \frac{aA}{}$$
$$= \frac{a^3(21A+22B)\cos^3(c+dx)\sin(c+dx)}{40d} + \frac{aA}{}$$
$$= \frac{a^3(23A+26B)\cos(c+dx)\sin(c+dx)}{16d} + \frac{a^3(21}{}$$
$$= \frac{1}{16}a^3(23A+26B)x + \frac{a^3(17A+19B)\sin(c+d}{5d}$$

Mathematica [A]
time = 0.59, size = 134, normalized size = 0.67

$$\frac{a^3(1380Ac+1380Adx+1560Bdx+120(21A+23B)\sin(c+dx)+15(63A+64B)\sin(2(c+dx))+380A\sin(3(c+dx))+340B\sin(3(c+dx))+135A\sin(4(c+dx))+90B\sin(4(c+dx))+36A\sin(5(c+dx))+12B\sin(5(c+dx))+5A\sin(6(c+dx)))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*(1380*A*c + 1380*A*d*x + 1560*B*d*x + 120*(21*A + 23*B)*Sin[c + d*x] +
 15*(63*A + 64*B)*Sin[2*(c + d*x)] + 380*A*Sin[3*(c + d*x)] + 340*B*Sin[3*(
c + d*x)] + 135*A*Sin[4*(c + d*x)] + 90*B*Sin[4*(c + d*x)] + 36*A*Sin[5*(c
+ d*x)] + 12*B*Sin[5*(c + d*x)] + 5*A*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A]
time = 0.43, size = 266, normalized size = 1.32

method	result
risch	$\frac{23a^3Ax}{16}+\frac{13a^3xB}{8}+\frac{21a^3A\sin(dx+c)}{8d}+\frac{23a^3B\sin(dx+c)}{8d}+\frac{Aa^3\sin(6dx+6c)}{192d}+\frac{3Aa^3\sin(5dx+5c)}{80d}+\sin(5$
derivativedivides	$\frac{\frac{Aa^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+a^3B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3Aa^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3}{}\right.}{}$
default	$\frac{\frac{Aa^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+a^3B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3Aa^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/3*A*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+a^3*B*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+3*A*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d
*x+3/8*c)+a^3*B*(2+cos(d*x+c)^2)*sin(d*x+c)+3/5*A*a^3*(8/3+cos(d*x+c)^4+4/3
*cos(d*x+c)^2)*sin(d*x+c)+3*a^3*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*
x+c)+3/8*d*x+3/8*c)+A*a^3*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+
c))*sin(d*x+c)+5/16*d*x+5/16*c)+1/5*a^3*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^
2)*sin(d*x+c))
```

Maxima [A]
time = 0.28, size = 262, normalized size = 1.30

$$\frac{192\,(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c))Aa^3-5\,(4\,\sin(2\,dx+2\,c)^3-60\,dx-60\,c-9\,\sin(4\,dx+4\,c)-48\,\sin(2\,dx+2\,c))Aa^3-320\,(\sin(dx+c)^3-3\,\sin(dx+c))Aa^3+90\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Aa^3+64\,(3\,\sin(dx+c)^5-10\,\sin(dx+c)^3+15\,\sin(dx+c))Ba^3-960\,(\sin(dx+c)^3-3\,\sin(dx+c))Ba^3+90\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Ba^3+240\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^3}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/960*(192*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^3 -
 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*
```

```
x + 2*c))*A*a^3 - 320*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 + 90*(12*d*x
+ 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 + 64*(3*sin(d*x + c)^
5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^3 - 960*(sin(d*x + c)^3 - 3*si
n(d*x + c))*B*a^3 + 90*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*
c))*B*a^3 + 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3)/d
```

Fricas [A]
time = 2.50, size = 130, normalized size = 0.65

$$\frac{15\,(23\,A+26\,B)a^3dx+\left(40\,Aa^3\cos\left(dx+c\right)^5+48\,(3\,A+B)a^3\cos\left(dx+c\right)^4+10\,(23\,A+18\,B)a^3\cos\left(dx+c\right)^3+16\,(17\,A+19\,B)a^3\cos\left(dx+c\right)^2+15\,(23\,A+26\,B)a^3\cos\left(dx+c\right)+32\,(17\,A+19\,B)a^3\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(23*A + 26*B)*a^3*d*x + (40*A*a^3*cos(d*x + c)^5 + 48*(3*A + B)*a
^3*cos(d*x + c)^4 + 10*(23*A + 18*B)*a^3*cos(d*x + c)^3 + 16*(17*A + 19*B)*
a^3*cos(d*x + c)^2 + 15*(23*A + 26*B)*a^3*cos(d*x + c) + 32*(17*A + 19*B)*a
^3)*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.52, size = 244, normalized size = 1.21

$$\frac{15\,(23\,Aa^3+26\,Ba^3)(dx+c)+\frac{2\left(345\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+390\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+1955\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+2210\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+4554\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+5148\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+5814\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+5988\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3165\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4190\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1575\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1530\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/240*(15*(23*A*a^3 + 26*B*a^3)*(d*x + c) + 2*(345*A*a^3*tan(1/2*d*x + 1/2*
c)^11 + 390*B*a^3*tan(1/2*d*x + 1/2*c)^11 + 1955*A*a^3*tan(1/2*d*x + 1/2*c)
^9 + 2210*B*a^3*tan(1/2*d*x + 1/2*c)^9 + 4554*A*a^3*tan(1/2*d*x + 1/2*c)^7
+ 5148*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 5814*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 5
988*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 3165*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 4190
*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 1575*A*a^3*tan(1/2*d*x + 1/2*c) + 1530*B*a^
3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B]
time = 4.68, size = 285, normalized size = 1.42

$$\frac{\left(\frac{23\,A\,a^3}{8}+\frac{13\,B\,a^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{391\,A\,a^3}{24}+\frac{221\,B\,a^3}{12}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{759\,A\,a^3}{20}+\frac{429\,B\,a^3}{10}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{969\,A\,a^3}{20}+\frac{499\,B\,a^3}{10}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{211\,A\,a^3}{8}+\frac{419\,B\,a^3}{12}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{105\,A\,a^3}{8}+\frac{51\,B\,a^3}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{12}+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{10}+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8+20\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+15\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}+\frac{a^3\,\mathrm{atan}\left(\frac{a^3\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(23\,A+26\,B\right)}{8\,\left(\frac{23\,A\,a^3}{8}+\frac{13\,B\,a^3}{4}\right)}\right)\,\left(23\,A+26\,B\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((105*A*a^3)/8 + (51*B*a^3)/4) + tan(c/2 + (d*x)/2)^11*
((23*A*a^3)/8 + (13*B*a^3)/4) + tan(c/2 + (d*x)/2)^3*((211*A*a^3)/8 + (419*
B*a^3)/12) + tan(c/2 + (d*x)/2)^9*((391*A*a^3)/24 + (221*B*a^3)/12) + tan(c
/2 + (d*x)/2)^7*((759*A*a^3)/20 + (429*B*a^3)/10) + tan(c/2 + (d*x)/2)^5*((
969*A*a^3)/20 + (499*B*a^3)/10))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 +
(d*x)/2)^4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2
+ (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (a^3*atan((a^3*tan(c/2 + (d*x
)/2)*(23*A + 26*B))/(8*((23*A*a^3)/8 + (13*B*a^3)/4)))*(23*A + 26*B))/(8*d)
```

3.72 $\int \sec^2(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=194

$$\frac{7a^4(8A+7B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{4a^4(8A+7B)\tan(c+dx)}{5d}+\frac{27a^4(8A+7B)\sec(c+dx)\tan(c+dx)}{80d}+\frac{a^4}{}$$

```
[Out] 7/16*a^4*(8*A+7*B)*arctanh(sin(d*x+c))/d+4/5*a^4*(8*A+7*B)*tan(d*x+c)/d+27/
80*a^4*(8*A+7*B)*sec(d*x+c)*tan(d*x+c)/d+1/40*a^4*(8*A+7*B)*sec(d*x+c)^3*ta
n(d*x+c)/d+1/30*(6*A-B)*(a+a*sec(d*x+c))^4*tan(d*x+c)/d+1/6*B*(a+a*sec(d*x+
c))^5*tan(d*x+c)/a/d+2/15*a^4*(8*A+7*B)*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.23, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.226, Rules used = {4095, 4086, 3876, 3855, 3852, 8, 3853}

$$\frac{2a^4(8A+7B)\tan^3(c+dx)}{15d}+\frac{4a^4(8A+7B)\tan(c+dx)}{5d}+\frac{7a^4(8A+7B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{a^4(8A+7B)\tan(c+dx)\sec^3(c+dx)}{40d}+\frac{27a^4(8A+7B)\tan(c+dx)\sec(c+dx)}{80d}+\frac{(6A-B)\tan(c+dx)(a\sec(c+dx)+a)^4}{30d}+\frac{B\tan(c+dx)(a\sec(c+dx)+a)^5}{6ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (7*a^4*(8*A + 7*B)*ArcTanh[Sin[c + d*x]])/(16*d) + (4*a^4*(8*A + 7*B)*Tan[c
 + d*x])/(5*d) + (27*a^4*(8*A + 7*B)*Sec[c + d*x]*Tan[c + d*x])/(80*d) + (a
^4*(8*A + 7*B)*Sec[c + d*x]^3*Tan[c + d*x])/(40*d) + ((6*A - B)*(a + a*Sec[
c + d*x])^4*Tan[c + d*x])/(30*d) + (B*(a + a*Sec[c + d*x])^5*Tan[c + d*x])/
(6*a*d) + (2*a^4*(8*A + 7*B)*Tan[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{B(a+a\sec(c+dx))^5\tan(c+dx)}{6ad} + \frac{\int \sec(c+$$

$$= \frac{(6A-B)(a+a\sec(c+dx))^4\tan(c+dx)}{30d} + \frac{B$$

$$= \frac{(6A-B)(a+a\sec(c+dx))^4\tan(c+dx)}{30d} + \frac{B$$

$$= \frac{(6A-B)(a+a\sec(c+dx))^4\tan(c+dx)}{30d} + \frac{B$$

$$= \frac{a^4(8A+7B)\tanh^{-1}(\sin(c+dx))}{10d} + \frac{3a^4(8A+$$

$$= \frac{2a^4(8A+7B)\tanh^{-1}(\sin(c+dx))}{5d} + \frac{4a^4(8A+$$

$$= \frac{7a^4(8A+7B)\tanh^{-1}(\sin(c+dx))}{16d} + \frac{4a^4(8A+$$

Mathematica [**A**]
time = 2.29, size = 358, normalized size = 1.85

$$-\frac{a^4(1+\cos(c+dx))^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^6(c+dx)\left(3360(8A+7B)\cos^6(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\sec(c)(-160(83A+72B)\sin(c)+30(88A+125B)\sin(dx)+2640A\sin(2c+dx)+3750B\sin(2c+dx)+15840A\sin(c+2dx)+15360B\sin(c+2dx)-4080A\sin(3c+2dx)-1920B\sin(3c+2dx)+3480A\sin(2c+3dx)+3845B\sin(2c+3dx)+3480A\sin(4c+3dx)+3845B\sin(4c+3dx)+7728A\sin(3c+4dx)+6912B\sin(3c+4dx)-240A\sin(5c+4dx)+840A\sin(4c+5dx)+735B\sin(4c+5dx)+840A\sin(6c+5dx)+735B\sin(6c+5dx)+1328A\sin(5c+6dx)+1152B\sin(5c+6dx))\right)}{122880d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] -1/122880*(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^6*(3360
*(8*A + 7*B)*Cos[c + d*x]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(-160*(83*A + 72*B)*Sin[c]
+ 30*(88*A + 125*B)*Sin[d*x] + 2640*A*Sin[2*c + d*x] + 3750*B*Sin[2*c + d*x
] + 15840*A*Sin[c + 2*d*x] + 15360*B*Sin[c + 2*d*x] - 4080*A*Sin[3*c + 2*d*
x] - 1920*B*Sin[3*c + 2*d*x] + 3480*A*Sin[2*c + 3*d*x] + 3845*B*Sin[2*c + 3
*d*x] + 3480*A*Sin[4*c + 3*d*x] + 3845*B*Sin[4*c + 3*d*x] + 7728*A*Sin[3*c
+ 4*d*x] + 6912*B*Sin[3*c + 4*d*x] - 240*A*Sin[5*c + 4*d*x] + 840*A*Sin[4*c
 + 5*d*x] + 735*B*Sin[4*c + 5*d*x] + 840*A*Sin[6*c + 5*d*x] + 735*B*Sin[6*c
 + 5*d*x] + 1328*A*Sin[5*c + 6*d*x] + 1152*B*Sin[5*c + 6*d*x])))/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(180) = 360$.
time = 0.49, size = 365, normalized size = 1.88 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-A*a^4*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)+a^4*B*(-(
-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*sec(d*x+c))*tan(d*x+c)+5/16*ln(sec
(d*x+c)+tan(d*x+c)))+4*A*a^4*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c
)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-4*a^4*B*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d
*x+c)^2)*tan(d*x+c)-6*A*a^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+6*a^4*B*(-(-
1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+
4*A*a^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-4*a^4*B*(
-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+A*a^4*tan(d*x+c)+a^4*B*(1/2*sec(d*x+c)*ta
n(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 464 vs. $2(180) = 360$.
time = 0.27, size = 464, normalized size = 2.39

$$\frac{32\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)Aa^4+960\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Aa^4+128\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)Ba^4+640\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Ba^4-5Ba^4\left(\frac{2\left(15\sin(dx+c)^5-40\sin(dx+c)^3+33\sin(dx+c)\right)}{\sin(dx+c)^6-3\sin(dx+c)^4+3\sin(dx+c)^2-1}-15\log\left(\sin\left(dx+c\right)+1\right)+15\log\left(\sin\left(dx+c\right)-1\right)\right)-120Aa^4\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)-180Ba^4\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)-480Aa^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-120Ba^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+480Aa^4\tan\left(dx+c\right)}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(32*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*x + c))*A*a^4 +
960*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 128*(3*tan(d*x + c)^5 + 10*ta
n(d*x + c)^3 + 15*tan(d*x + c))*B*a^4 + 640*(tan(d*x + c)^3 + 3*tan(d*x + c
))*B*a^4 - 5*B*a^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*sin(d*x +
 c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 15*log(si
n(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 120*A*a^4*(2*(3*sin(d*x + c)^
3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x
 + c) + 1) + 3*log(sin(d*x + c) - 1)) - 180*B*a^4*(2*(3*sin(d*x + c)^3 - 5*
sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)) - 480*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 120*B*a^4*(2*sin(d
*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1
)) + 480*A*a^4*tan(d*x + c))/d
```

Fricas [**A**]
time = 2.35, size = 185, normalized size = 0.95

$$\frac{105\left(8A+7B\right)a^4\cos\left(dx+c\right)^6\log\left(\sin\left(dx+c\right)+1\right)-105\left(8A+7B\right)a^4\cos\left(dx+c\right)^6\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\left(83A+72B\right)a^4\cos\left(dx+c\right)^5+105\left(8A+7B\right)a^4\cos\left(dx+c\right)^4+32\left(17A+18B\right)a^4\cos\left(dx+c\right)^3+10\left(24A+41B\right)a^4\cos\left(dx+c\right)^2+48\left(A+4B\right)a^4\cos\left(dx+c\right)+40Ba^4\right)\sin\left(dx+c\right)}{480d\cos\left(dx+c\right)^6}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/480*(105*(8*A + 7*B)*a^4*cos(d*x + c)^6*log(sin(d*x + c) + 1) - 105*(8*A
+ 7*B)*a^4*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(83*A + 72*B)*a^4*
cos(d*x + c)^5 + 105*(8*A + 7*B)*a^4*cos(d*x + c)^4 + 32*(17*A + 18*B)*a^4*
```

```
cos(d*x + c)^3 + 10*(24*A + 41*B)*a^4*cos(d*x + c)^2 + 48*(A + 4*B)*a^4*cos
(d*x + c) + 40*B*a^4)*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int A\sec^2(c+dx)\,dx+\int 4A\sec^3(c+dx)\,dx+\int 6A\sec^4(c+dx)\,dx+\int 4A\sec^5(c+dx)\,dx+\int A\sec^6(c+dx)\,dx+\int B\sec^3(c+dx)\,dx+\int 4B\sec^4(c+dx)\,dx+\int 6B\sec^5(c+dx)\,dx+\int 4B\sec^6(c+dx)\,dx+\int B\sec^7(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] a**4*(Integral(A*sec(c + d*x)**2, x) + Integral(4*A*sec(c + d*x)**3, x) + I
ntegral(6*A*sec(c + d*x)**4, x) + Integral(4*A*sec(c + d*x)**5, x) + Integr
al(A*sec(c + d*x)**6, x) + Integral(B*sec(c + d*x)**3, x) + Integral(4*B*se
c(c + d*x)**4, x) + Integral(6*B*sec(c + d*x)**5, x) + Integral(4*B*sec(c +
 d*x)**6, x) + Integral(B*sec(c + d*x)**7, x))
```

Giac [A]
time = 0.53, size = 280, normalized size = 1.44

$$\frac{105\left(8Aa^4+7Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-105\left(8Aa^4+7Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(840Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}+735Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{11}-4760Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-4165Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+11088Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+9702Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-13488Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-11802Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+9320Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+7355Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3000Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3105Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^6}}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/240*(105*(8*A*a^4 + 7*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(8*
A*a^4 + 7*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(840*A*a^4*tan(1/2*
d*x + 1/2*c)^11 + 735*B*a^4*tan(1/2*d*x + 1/2*c)^11 - 4760*A*a^4*tan(1/2*d*
x + 1/2*c)^9 - 4165*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 11088*A*a^4*tan(1/2*d*x
+ 1/2*c)^7 + 9702*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 13488*A*a^4*tan(1/2*d*x +
1/2*c)^5 - 11802*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 9320*A*a^4*tan(1/2*d*x + 1/
2*c)^3 + 7355*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 3000*A*a^4*tan(1/2*d*x + 1/2*c
) - 3105*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B]
time = 4.61, size = 262, normalized size = 1.35

$$\frac{\left(-7Aa^4-\frac{49Ba^4}{8}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{11}+\left(\frac{119Aa^4}{3}+\frac{833Ba^4}{24}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(-\frac{462Aa^4}{5}-\frac{1617Ba^4}{20}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{562Aa^4}{5}+\frac{1967Ba^4}{20}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-\frac{233Aa^4}{3}-\frac{1471Ba^4}{24}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(25Aa^4+\frac{207Ba^4}{8}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{12}-6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}+15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-20\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+15\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}+\frac{7a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(8A+7B)}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^4)/cos(c + d*x)^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(25*A*a^4 + (207*B*a^4)/8) - tan(c/2 + (d*x)/2)^11*(7*A
*a^4 + (49*B*a^4)/8) + tan(c/2 + (d*x)/2)^9*((119*A*a^4)/3 + (833*B*a^4)/24
) - tan(c/2 + (d*x)/2)^3*((233*A*a^4)/3 + (1471*B*a^4)/24) - tan(c/2 + (d*x
```

```
)/2)^7*((462*A*a^4)/5 + (1617*B*a^4)/20) + tan(c/2 + (d*x)/2)^5*((562*A*a^4
)/5 + (1967*B*a^4)/20))/(d*(15*tan(c/2 + (d*x)/2)^4 - 6*tan(c/2 + (d*x)/2)^
2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 - 6*tan(c/2 + (d*x)/2
)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (7*a^4*atanh(tan(c/2 + (d*x)/2))*(8*A
+ 7*B))/(8*d)
```

3.73 $\int \sec(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=159

$$\frac{7a^4(5A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{8a^4(5A+4B)\tan(c+dx)}{5d}+\frac{27a^4(5A+4B)\sec(c+dx)\tan(c+dx)}{40d}+\frac{a^4}{}$$

```
[Out] 7/8*a^4*(5*A+4*B)*arctanh(sin(d*x+c))/d+8/5*a^4*(5*A+4*B)*tan(d*x+c)/d+27/4
0*a^4*(5*A+4*B)*sec(d*x+c)*tan(d*x+c)/d+1/20*a^4*(5*A+4*B)*sec(d*x+c)^3*tan
(d*x+c)/d+1/5*B*(a+a*sec(d*x+c))^4*tan(d*x+c)/d+4/15*a^4*(5*A+4*B)*tan(d*x+
c)^3/d
```

Rubi [A]
time = 0.13, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4086, 3876, 3855, 3852, 8, 3853}

$$\frac{4a^4(5A+4B)\tan^3(c+dx)}{15d}+\frac{8a^4(5A+4B)\tan(c+dx)}{5d}+\frac{7a^4(5A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{a^4(5A+4B)\tan(c+dx)\sec^3(c+dx)}{20d}+\frac{27a^4(5A+4B)\tan(c+dx)\sec(c+dx)}{40d}+\frac{B\tan(c+dx)(a\sec(c+dx)+a)^4}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (7*a^4*(5*A + 4*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (8*a^4*(5*A + 4*B)*Tan[c
+ d*x])/(5*d) + (27*a^4*(5*A + 4*B)*Sec[c + d*x]*Tan[c + d*x])/(40*d) + (a^
4*(5*A + 4*B)*Sec[c + d*x]^3*Tan[c + d*x])/(20*d) + (B*(a + a*Sec[c + d*x])
^4*Tan[c + d*x])/(5*d) + (4*a^4*(5*A + 4*B)*Tan[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$
\begin{aligned}
\int \sec(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx &= \frac{B(a+a\sec(c+dx))^4\tan(c+dx)}{5d}+\frac{1}{5}(5A+4 \\
&= \frac{B(a+a\sec(c+dx))^4\tan(c+dx)}{5d}+\frac{1}{5}(5A+4 \\
&= \frac{B(a+a\sec(c+dx))^4\tan(c+dx)}{5d}+\frac{1}{5}\big(a^4(5A \\
&= \frac{a^4(5A+4B)\tanh^{-1}(\sin(c+dx))}{5d}+\frac{3a^4(5A+ \\
&= \frac{4a^4(5A+4B)\tanh^{-1}(\sin(c+dx))}{5d}+\frac{8a^4(5A+ \\
&= \frac{7a^4(5A+4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{8a^4(5A+
\end{aligned}
$$

Mathematica [A]
time = 1.63, size = 306, normalized size = 1.92

$$
-\frac{a^4(1+\cos(c+dx))^4\sec^8\left(\frac{1}{2}(c+dx)\right)\sec^5(c+dx)\left(1680(5A+4B)\cos^5(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)-\sec(c)(80(64A+59B)\sin(dx)-960(3A+2B)\sin(2c+dx)+930A\sin(c+2dx)+1320B\sin(c+2dx)+930A\sin(3c+2dx)+1320B\sin(3c+2dx)+3520A\sin(2c+3dx)+3200B\sin(2c+3dx)-480A\sin(4c+3dx)-120B\sin(4c+3dx)+405A\sin(3c+4dx)+420B\sin(3c+4dx)+405A\sin(5c+4dx)+420B\sin(5c+4dx)+800A\sin(4c+5dx)+664B\sin(4c+5dx))\right)}{30720d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] -1/30720*(a^4*(1 + Cos[c + d*x])^4*Sec[(c + d*x)/2]^8*Sec[c + d*x]^5*(1680*
(5*A + 4*B)*Cos[c + d*x]^5*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) - Sec[c]*(80*(64*A + 59*B)*Sin[d*x] -
 960*(3*A + 2*B)*Sin[2*c + d*x] + 930*A*Sin[c + 2*d*x] + 1320*B*Sin[c + 2*d
*x] + 930*A*Sin[3*c + 2*d*x] + 1320*B*Sin[3*c + 2*d*x] + 3520*A*Sin[2*c + 3
*d*x] + 3200*B*Sin[2*c + 3*d*x] - 480*A*Sin[4*c + 3*d*x] - 120*B*Sin[4*c +
3*d*x] + 405*A*Sin[3*c + 4*d*x] + 420*B*Sin[3*c + 4*d*x] + 405*A*Sin[5*c +
4*d*x] + 420*B*Sin[5*c + 4*d*x] + 800*A*Sin[4*c + 5*d*x] + 664*B*Sin[4*c +
5*d*x])))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 302 vs. $2(147) = 294$.

time = 0.41, size = 303, normalized size = 1.91

method	result
norman	$\frac{\frac{79a^4(5A+4B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{224a^4(5A+4B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{49a^4(5A+4B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6d}-\frac{7a^4(5A+4B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\cdots}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^5}$
derivativedivides	$\frac{Aa^4\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-a^4B\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c\cdots\right.}{15}\cdots\right.}{\cdots}$
default	$\frac{Aa^4\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right)-a^4B\left(-\frac{8}{15}-\frac{\left(\sec^4(dx+c)\right)}{5}-\frac{4\left(\sec^2(dx+c\cdots\right.}{15}\cdots\right.}{\cdots}$
risch	$-\frac{ia^4\left(405A\,e^{9i(dx+c)}+420B\,e^{9i(dx+c)}-480A\,e^{8i(dx+c)}-120B\,e^{8i(dx+c)}+930A\,e^{7i(dx+c)}+1320B\,e^{7i(dx+c)}-2880A\,e^{6i(dx\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a^4*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c
)+tan(d*x+c)))-a^4*B*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)-
4*A*a^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+4*a^4*B*(-(-1/4*sec(d*x+c)^3-3/8
*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+6*A*a^4*(1/2*sec(d*x
+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-6*a^4*B*(-2/3-1/3*sec(d*x+c)^
2)*tan(d*x+c)+4*A*a^4*tan(d*x+c)+4*a^4*B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(
sec(d*x+c)+tan(d*x+c)))+A*a^4*ln(sec(d*x+c)+tan(d*x+c))+a^4*B*tan(d*x+c))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 369 vs. $2(147) = 294$.

time = 0.30, size = 369, normalized size = 2.32

$$\frac{320\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^4+16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Ba^4+480\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^4-15Aa^4\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-60Ba^4\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-360Aa^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-240Ba^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+240Aa^4\log(\sec(dx+c)+\tan(dx+c))+960Aa^4\tan(dx+c)+240Ba^4\tan(dx+c)}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/240*(320*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 16*(3*tan(d*x + c)^5 +
 10*tan(d*x + c)^3 + 15*tan(d*x + c))*B*a^4 + 480*(tan(d*x + c)^3 + 3*tan(d
*x + c))*B*a^4 - 15*A*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
 c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c
) - 1)) - 60*B*a^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
 - 360*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) +
 log(sin(d*x + c) - 1)) - 240*B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) -
log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*A*a^4*log(sec(d*x + c)
 + tan(d*x + c)) + 960*A*a^4*tan(d*x + c) + 240*B*a^4*tan(d*x + c))/d
```

Fricas [A]
time = 2.92, size = 165, normalized size = 1.04

$$\frac{105\,(5\,A+4\,B)a^4\cos(dx+c)^5\log(\sin(dx+c)+1)-105\,(5\,A+4\,B)a^4\cos(dx+c)^5\log(-\sin(dx+c)+1)+2\left(8\,(100\,A+83\,B)a^4\cos(dx+c)^4+15\,(27\,A+28\,B)a^4\cos(dx+c)^3+16\,(10\,A+17\,B)a^4\cos(dx+c)^2+30\,(A+4\,B)a^4\cos(dx+c)+24\,Ba^4\right)\sin(dx+c)}{240\,d\cos(dx+c)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/240*(105*(5*A + 4*B)*a^4*cos(d*x + c)^5*log(sin(d*x + c) + 1) - 105*(5*A
+ 4*B)*a^4*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(8*(100*A + 83*B)*a^4*
cos(d*x + c)^4 + 15*(27*A + 28*B)*a^4*cos(d*x + c)^3 + 16*(10*A + 17*B)*a^4
*cos(d*x + c)^2 + 30*(A + 4*B)*a^4*cos(d*x + c) + 24*B*a^4)*sin(d*x + c))/(
d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int A\sec(c+dx)\,dx+\int 4A\sec^2(c+dx)\,dx+\int 6A\sec^3(c+dx)\,dx+\int 4A\sec^4(c+dx)\,dx+\int A\sec^5(c+dx)\,dx+\int B\sec^2(c+dx)\,dx+\int 4B\sec^3(c+dx)\,dx+\int 6B\sec^4(c+dx)\,dx+\int 4B\sec^5(c+dx)\,dx+\int B\sec^6(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] a**4*(Integral(A*sec(c + d*x), x) + Integral(4*A*sec(c + d*x)**2, x) + Inte
gral(6*A*sec(c + d*x)**3, x) + Integral(4*A*sec(c + d*x)**4, x) + Integral(
A*sec(c + d*x)**5, x) + Integral(B*sec(c + d*x)**2, x) + Integral(4*B*sec(c
 + d*x)**3, x) + Integral(6*B*sec(c + d*x)**4, x) + Integral(4*B*sec(c + d*
x)**5, x) + Integral(B*sec(c + d*x)**6, x))
```

Giac [A]
time = 0.54, size = 246, normalized size = 1.55

$$\frac{105\,(5\,Aa^4+4\,Ba^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-105\,(5\,Aa^4+4\,Ba^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(525\,Aa^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^9+420\,Ba^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^9-2450\,Aa^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^7-1960\,Ba^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^7+4480\,Aa^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5+3584\,Ba^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^5-3950\,Aa^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3-3160\,Ba^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^3+1395\,Aa^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)+1500\,Ba^4\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)\right)}{\left(\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)^2-1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/120*(105*(5*A*a^4 + 4*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 105*(5*
A*a^4 + 4*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(525*A*a^4*tan(1/2*
d*x + 1/2*c)^9 + 420*B*a^4*tan(1/2*d*x + 1/2*c)^9 - 2450*A*a^4*tan(1/2*d*x
+ 1/2*c)^7 - 1960*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 4480*A*a^4*tan(1/2*d*x + 1
/2*c)^5 + 3584*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 3950*A*a^4*tan(1/2*d*x + 1/2*
c)^3 - 3160*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 1395*A*a^4*tan(1/2*d*x + 1/2*c)
+ 1500*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B]
time = 4.54, size = 224, normalized size = 1.41

$$\frac{7\,a^4\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(5\,A+4\,B\right)}{4\,d}-\frac{\left(\frac{35\,A\,a^4}{4}+7\,B\,a^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(-\frac{245\,A\,a^4}{6}-\frac{98\,B\,a^4}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{224\,A\,a^4}{3}+\frac{896\,B\,a^4}{15}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-\frac{395\,A\,a^4}{6}-\frac{158\,B\,a^4}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{93\,A\,a^4}{4}+25\,B\,a^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}-5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6-10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^4)/cos(c + d*x),x)
```

```
[Out] (7*a^4*atanh(tan(c/2 + (d*x)/2))*(5*A + 4*B))/(4*d) - (tan(c/2 + (d*x)/2)*(
(93*A*a^4)/4 + 25*B*a^4) + tan(c/2 + (d*x)/2)^9*((35*A*a^4)/4 + 7*B*a^4) -
tan(c/2 + (d*x)/2)^7*((245*A*a^4)/6 + (98*B*a^4)/3) - tan(c/2 + (d*x)/2)^3*
((395*A*a^4)/6 + (158*B*a^4)/3) + tan(c/2 + (d*x)/2)^5*((224*A*a^4)/3 + (89
6*B*a^4)/15))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan
(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.74 $\int (a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=151

$$a^4Ax+\frac{a^4(48A+35B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{5a^4(8A+7B)\tan(c+dx)}{8d}+\frac{aB(a+a\sec(c+dx))^3\tan(c+d}{4d}$$

```
[Out] a^4*A*x+1/8*a^4*(48*A+35*B)*arctanh(sin(d*x+c))/d+5/8*a^4*(8*A+7*B)*tan(d*x
+c)/d+1/4*a*B*(a+a*sec(d*x+c))^3*tan(d*x+c)/d+1/12*(4*A+7*B)*(a^2+a^2*sec(d
*x+c))^2*tan(d*x+c)/d+1/24*(32*A+35*B)*(a^4+a^4*sec(d*x+c))*tan(d*x+c)/d
```

Rubi [A]
time = 0.16, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4002, 3999, 3852, 8, 3855}

$$\frac{5a^4(8A+7B)\tan(c+dx)}{8d}+\frac{a^4(48A+35B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(32A+35B)\tan(c+dx)\left(a^4\sec(c+dx)+a^4\right)}{24d}+a^4Ax+\frac{(4A+7B)\tan(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{12d}+\frac{aB\tan(c+dx)(a\sec(c+dx)+a)^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] a^4*A*x + (a^4*(48*A + 35*B)*ArcTanh[Sin[c + d*x]])/(8*d) + (5*a^4*(8*A + 7
*B)*Tan[c + d*x])/(8*d) + (a*B*(a + a*Sec[c + d*x])^3*Tan[c + d*x])/(4*d) +
 ((4*A + 7*B)*(a^2 + a^2*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + ((32*A + 35
*B)*(a^4 + a^4*Sec[c + d*x])*Tan[c + d*x])/(24*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3999

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
```

```
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rule 4002

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m +
(b*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*
m]
```

Rubi steps

$$\begin{aligned}\int (a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx &= \frac{aB(a+a\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{1}{4}\int (a+a\sec(c+dx\\ &= \frac{aB(a+a\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{(4A+7B)\left(a^2+a^2\,\mathrm{se}\right.}{1}\\ &= \frac{aB(a+a\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{(4A+7B)\left(a^2+a^2\,\mathrm{se}\right.}{1}\\ &= a^4Ax+\frac{aB(a+a\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{(4A+7B)\,(a}{}\\ &= a^4Ax+\frac{a^4(48A+35B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{aB(a+a\,\mathrm{sec}}{}\\ &= a^4Ax+\frac{a^4(48A+35B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{5a^4(8A+7B}{8}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 326 vs. 2(151) = 302.
time = 1.88, size = 326, normalized size = 2.16

$$\frac{a^4\sec^8\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^4\left(-24(48A+35B)\cos^4(c+dx)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\sec(c)(72Adx\cos(c)+48Adx\cos(c+2dx)+48Adx\cos(3c+2dx)+12Adx\cos(3c+4dx)+12Adx\cos(5c+4dx)-480A\sin(c)-480B\sin(c)+48A\sin(dx)+105B\sin(dx)+48A\sin(2c+dx)+105B\sin(2c+dx)+496A\sin(c+2dx)+544B\sin(c+2dx)-144A\sin(3c+2dx)-96B\sin(3c+2dx)+48A\sin(2c+3dx)+81B\sin(2c+3dx)+48A\sin(4c+3dx)+81B\sin(4c+3dx)+160A\sin(3c+4dx)+160B\sin(3c+4dx))\right)}{3072d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^4*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(-24*(48*A + 35*B)*Cos[c + d*x
]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]]) + Sec[c]*(72*A*d*x*Cos[c] + 48*A*d*x*Cos[c + 2*d*x] + 48*A*d
*x*Cos[3*c + 2*d*x] + 12*A*d*x*Cos[3*c + 4*d*x] + 12*A*d*x*Cos[5*c + 4*d*x]
 - 480*A*Sin[c] - 480*B*Sin[c] + 48*A*Sin[d*x] + 105*B*Sin[d*x] + 48*A*Sin[
2*c + d*x] + 105*B*Sin[2*c + d*x] + 496*A*Sin[c + 2*d*x] + 544*B*Sin[c + 2*
```

```
d*x] - 144*A*Sin[3*c + 2*d*x] - 96*B*Sin[3*c + 2*d*x] + 48*A*Sin[2*c + 3*d*
x] + 81*B*Sin[2*c + 3*d*x] + 48*A*Sin[4*c + 3*d*x] + 81*B*Sin[4*c + 3*d*x]
+ 160*A*Sin[3*c + 4*d*x] + 160*B*Sin[3*c + 4*d*x])))/(3072*d)
```

Maple [A]
time = 0.38, size = 250, normalized size = 1.66

method	result
derivativedivides	$\frac{-A\,a^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+a^4B\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right.}{}$
default	$\frac{-A\,a^4\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)+a^4B\left(-\left(-\frac{\left(\sec^3(dx+c)\right)}{4}-\frac{3\sec(dx+c)}{8}\right)\tan(dx+c)+\frac{3\ln(\sec(dx+c)+\tan(dx+c))}{8}\right.}{}$
norman	$\frac{a^4Ax+a^4Ax\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a^4Ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+6a^4Ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-4a^4Ax\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{3a^4(24A+31B}{4}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}$
risch	$a^4Ax-\frac{ia^4\left(48A\,e^{7i(dx+c)}+81B\,e^{7i(dx+c)}-144A\,e^{6i(dx+c)}-96B\,e^{6i(dx+c)}+48A\,e^{5i(dx+c)}+105B\,e^{5i(dx+c)}-480A\,e\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-A*a^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+a^4*B*(-(-1/4*sec(d*x+c)^3-3
/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+4*A*a^4*(1/2*sec(d
*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-4*a^4*B*(-2/3-1/3*sec(d*x+c
)^2)*tan(d*x+c)+6*A*a^4*tan(d*x+c)+6*a^4*B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*l
n(sec(d*x+c)+tan(d*x+c)))+4*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a^4*B*tan(d*x
+c)+A*a^4*(d*x+c)+a^4*B*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 293 vs. $2(141) = 282$.
time = 0.31, size = 293, normalized size = 1.94

$$\frac{16\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aa^4+48(dx+c)Aa^4+64\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^4-3Ba^4\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-48Aa^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-72Ba^4\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+192Aa^4\log(\sec(dx+c)+\tan(dx+c))+48Ba^4\log(\sec(dx+c)+\tan(dx+c))+288Aa^4\tan(dx+c)+192Ba^4\tan(dx+c)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*a^4 + 48*(d*x + c)*A*a^4 + 64*
(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^4 - 3*B*a^4*(2*(3*sin(d*x + c)^3 - 5*
sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c)
+ 1) + 3*log(sin(d*x + c) - 1)) - 48*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2
- 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 72*B*a^4*(2*sin(d*x
 + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1))
 + 192*A*a^4*log(sec(d*x + c) + tan(d*x + c)) + 48*B*a^4*log(sec(d*x + c) +
 tan(d*x + c)) + 288*A*a^4*tan(d*x + c) + 192*B*a^4*tan(d*x + c))/d
```

Fricas [A]
time = 3.78, size = 157, normalized size = 1.04

$$\frac{48\,Aa^4dx\cos\left(dx+c\right)^4+3\,(48\,A+35\,B)a^4\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\,(48\,A+35\,B)a^4\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(160\,(A+B)a^4\cos\left(dx+c\right)^3+3\,(16\,A+27\,B)a^4\cos\left(dx+c\right)^2+8\,(A+4\,B)a^4\cos\left(dx+c\right)+6\,Ba^4\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(48*A*a^4*d*x*cos(d*x + c)^4 + 3*(48*A + 35*B)*a^4*cos(d*x + c)^4*log(
sin(d*x + c) + 1) - 3*(48*A + 35*B)*a^4*cos(d*x + c)^4*log(-sin(d*x + c) +
1) + 2*(160*(A + B)*a^4*cos(d*x + c)^3 + 3*(16*A + 27*B)*a^4*cos(d*x + c)^2
 + 8*(A + 4*B)*a^4*cos(d*x + c) + 6*B*a^4)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int A\,dx+\int 4A\sec\left(c+dx\right)dx+\int 6A\sec^2\left(c+dx\right)dx+\int 4A\sec^3\left(c+dx\right)dx+\int A\sec^4\left(c+dx\right)dx+\int B\sec\left(c+dx\right)dx+\int 4B\sec^2\left(c+dx\right)dx+\int 6B\sec^3\left(c+dx\right)dx+\int 4B\sec^4\left(c+dx\right)dx+\int B\sec^5\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] a**4*(Integral(A, x) + Integral(4*A*sec(c + d*x), x) + Integral(6*A*sec(c +
 d*x)**2, x) + Integral(4*A*sec(c + d*x)**3, x) + Integral(A*sec(c + d*x)**
4, x) + Integral(B*sec(c + d*x), x) + Integral(4*B*sec(c + d*x)**2, x) + In
tegral(6*B*sec(c + d*x)**3, x) + Integral(4*B*sec(c + d*x)**4, x) + Integra
l(B*sec(c + d*x)**5, x))
```

Giac [A]
time = 0.52, size = 223, normalized size = 1.48

$$\frac{24\,(dx+c)Aa^4+3\,(48\,Aa^4+35\,Ba^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(48\,Aa^4+35\,Ba^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,\left(120\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+105\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-424\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-385\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+520\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+511\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-216\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-279\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)*A*a^4 + 3*(48*A*a^4 + 35*B*a^4)*log(abs(tan(1/2*d*x + 1/
2*c) + 1)) - 3*(48*A*a^4 + 35*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2
*(120*A*a^4*tan(1/2*d*x + 1/2*c)^7 + 105*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 424
*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 385*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 520*A*a^
4*tan(1/2*d*x + 1/2*c)^3 + 511*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 216*A*a^4*tan
(1/2*d*x + 1/2*c) - 279*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2
 - 1)^4)/d
```

Mupad [B]
time = 2.10, size = 255, normalized size = 1.69

$$\frac{2\,A\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{12\,A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{35\,B\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,d}+\frac{20\,A\,a^4\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)}+\frac{2\,A\,a^4\,\sin\left(c+d\,x\right)}{d\,{\cos\left(c+d\,x\right)}^2}+\frac{A\,a^4\,\sin\left(c+d\,x\right)}{3\,d\,{\cos\left(c+d\,x\right)}^3}+\frac{20\,B\,a^4\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)}+\frac{27\,B\,a^4\,\sin\left(c+d\,x\right)}{8\,d\,{\cos\left(c+d\,x\right)}^2}+\frac{4\,B\,a^4\,\sin\left(c+d\,x\right)}{3\,d\,{\cos\left(c+d\,x\right)}^3}+\frac{B\,a^4\,\sin\left(c+d\,x\right)}{4\,d\,{\cos\left(c+d\,x\right)}^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^4,x)
```

```
[Out] (2*A*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (12*A*a^4*atanh(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (35*B*a^4*atanh(sin(c/2 + (d*x)/
2)/cos(c/2 + (d*x)/2)))/(4*d) + (20*A*a^4*sin(c + d*x))/(3*d*cos(c + d*x))
+ (2*A*a^4*sin(c + d*x))/(d*cos(c + d*x)^2) + (A*a^4*sin(c + d*x))/(3*d*cos
(c + d*x)^3) + (20*B*a^4*sin(c + d*x))/(3*d*cos(c + d*x)) + (27*B*a^4*sin(c
 + d*x))/(8*d*cos(c + d*x)^2) + (4*B*a^4*sin(c + d*x))/(3*d*cos(c + d*x)^3)
 + (B*a^4*sin(c + d*x))/(4*d*cos(c + d*x)^4)
```

3.75 $\int \cos(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=151

$$a^4(4A+B)x+\frac{a^4(13A+12B)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{5a^4(A+2B)\sin(c+dx)}{2d}+\frac{aB(a+a\sec(c+dx))^3\sin(c-$$

```
[Out] a^4*(4*A+B)*x+1/2*a^4*(13*A+12*B)*arctanh(sin(d*x+c))/d-5/2*a^4*(A+2*B)*sin
(d*x+c)/d+1/3*a*B*(a+a*sec(d*x+c))^3*sin(d*x+c)/d+1/2*(A+2*B)*(a^2+a^2*sec(
d*x+c))^2*sin(d*x+c)/d+1/3*(9*A+11*B)*(a^4+a^4*sec(d*x+c))*sin(d*x+c)/d
```

Rubi [A]
time = 0.26, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4103, 4081, 3855}

$$-\frac{5a^4(A+2B)\sin(c+dx)}{2d}+\frac{a^4(13A+12B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(9A+11B)\sin(c+dx)\left(a^4\sec(c+dx)+a^4\right)}{3d}+a^4x(4A+B)+\frac{(A+2B)\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{2d}+\frac{aB\sin(c+dx)(a\sec(c+dx)+a)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] a^4*(4*A + B)*x + (a^4*(13*A + 12*B)*ArcTanh[Sin[c + d*x]])/(2*d) - (5*a^4*
(A + 2*B)*Sin[c + d*x])/(2*d) + (a*B*(a + a*Sec[c + d*x])^3*Sin[c + d*x])/(
3*d) + ((A + 2*B)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(2*d) + ((9*A +
11*B)*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
```

```
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{aB(a+a\sec(c+dx))^3\sin(c+dx)}{3d} + \frac{1}{3}\int \cos($$
$$= \frac{aB(a+a\sec(c+dx))^3\sin(c+dx)}{3d} + \frac{(A+2B}{}$$
$$= \frac{aB(a+a\sec(c+dx))^3\sin(c+dx)}{3d} + \frac{(A+2B}{}$$
$$= -\frac{5a^4(A+2B)\sin(c+dx)}{2d} + \frac{aB(a+a\sec(c+}{3}$$
$$= a^4(4A+B)x - \frac{5a^4(A+2B)\sin(c+dx)}{2d} + \frac{aB}{}$$
$$= a^4(4A+B)x + \frac{a^4(13A+12B)\tanh^{-1}(\sin(c+}{2d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1202 vs. $2(151) = 302$.
time = 6.48, size = 1202, normalized size = 7.96

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((4*A + B)*x*Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A
+ B*Sec[c + d*x]))/(16*(B + A*Cos[c + d*x])) + ((-13*A - 12*B)*Cos[c + d*x]
^5*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x)/2]]*Sec[c/2 + (d*x)/2]^8*(a + a
*Sec[c + d*x])^4*(A + B*Sec[c + d*x]))/(32*d*(B + A*Cos[c + d*x])) + ((13*A
 + 12*B)*Cos[c + d*x]^5*Log[Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c/
2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]))/(32*d*(B + A*Co
s[c + d*x])) + (A*Cos[d*x]*Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c
 + d*x])^4*(A + B*Sec[c + d*x])*Sin[c])/(16*d*(B + A*Cos[c + d*x])) + (A*Co
s[c]*Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[
c + d*x])*Sin[d*x])/(16*d*(B + A*Cos[c + d*x])) + (B*Cos[c + d*x]^5*Sec[c/2
 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x])*Sin[(d*x)/2])/(96
*d*(B + A*Cos[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] - Sin[c/2
```

```
 + (d*x)/2])^3) + (Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])
^4*(A + B*Sec[c + d*x])*(3*A*Cos[c/2] + 13*B*Cos[c/2] - 3*A*Sin[c/2] - 11*B
*Sin[c/2]))/(192*d*(B + A*Cos[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d
*x)/2] - Sin[c/2 + (d*x)/2])^2) + (Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^8*(a +
 a*Sec[c + d*x])^4*(A + B*Sec[c + d*x])*(3*A*Sin[(d*x)/2] + 5*B*Sin[(d*x)/2
]))/(12*d*(B + A*Cos[c + d*x])*(Cos[c/2] - Sin[c/2])*(Cos[c/2 + (d*x)/2] -
Sin[c/2 + (d*x)/2])) + (B*Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c
+ d*x])^4*(A + B*Sec[c + d*x])*Sin[(d*x)/2])/(96*d*(B + A*Cos[c + d*x])*(Co
s[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2])^3) + (Cos[c +
d*x]^5*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x])*(-3
*A*Cos[c/2] - 13*B*Cos[c/2] - 3*A*Sin[c/2] - 11*B*Sin[c/2]))/(192*d*(B + A*
Cos[c + d*x])*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2
])^2) + (Cos[c + d*x]^5*Sec[c/2 + (d*x)/2]^8*(a + a*Sec[c + d*x])^4*(A + B*
Sec[c + d*x])*(3*A*Sin[(d*x)/2] + 5*B*Sin[(d*x)/2]))/(12*d*(B + A*Cos[c + d
*x])*(Cos[c/2] + Sin[c/2])*(Cos[c/2 + (d*x)/2] + Sin[c/2 + (d*x)/2]))
```

Maple [A]
time = 0.40, size = 199, normalized size = 1.32

method	result
derivativedivides	$A\,a^{4}\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a^{4}B\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)+4A\,a^{4}\tan(dx+c)+4a^{4}B\left(\mathrm{se}\right.$
default	$A\,a^{4}\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a^{4}B\left(-\frac{2}{3}-\frac{\left(\sec^{2}(dx+c)\right)}{3}\right)\tan(dx+c)+4A\,a^{4}\tan(dx+c)+4a^{4}B\left(\mathrm{se}\right.$
risch	$4a^{4}Ax+a^{4}xB-\frac{iA\,a^{4}{\mathrm e}^{i(dx+c)}}{2d}+\frac{iA\,a^{4}{\mathrm e}^{-i(dx+c)}}{2d}-\frac{ia^{4}\left(3A\,{\mathrm e}^{5i(dx+c)}+12B\,{\mathrm e}^{5i(dx+c)}-24A\,{\mathrm e}^{4i(dx+c)}-36B\,{\mathrm e}^{4i(}\right.}{}$
norman	$(4A\,a^{4}+a^{4}B)x+(-12A\,a^{4}-3a^{4}B)x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-12A\,a^{4}-3a^{4}B)x\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(4A\,a^{4}+a^{4}B)x\left(\tan^{10}\left(\frac{dx}{2}+\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-a^4*B*
(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+4*A*a^4*tan(d*x+c)+4*a^4*B*(1/2*sec(d*x+
c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+6*A*a^4*ln(sec(d*x+c)+tan(d*x+
c))+6*a^4*B*tan(d*x+c)+4*A*a^4*(d*x+c)+4*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+A*
a^4*sin(d*x+c)+a^4*B*(d*x+c))
```

Maxima [A]
time = 0.29, size = 235, normalized size = 1.56

$$\frac{48\,(dx+c)Aa^{4}+4\left(\tan(dx+c)^{3}+3\,\tan(dx+c)\right)Ba^{4}+12\,(dx+c)Ba^{4}-3\,Aa^{4}\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-12\,Ba^{4}\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+36\,Aa^{4}(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+24\,Ba^{4}(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+12\,Aa^{4}\sin(dx+c)+48\,Aa^{4}\tan(dx+c)+72\,Ba^{4}\tan(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(48*(d*x + c)*A*a^4 + 4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^4 + 12*(
d*x + c)*B*a^4 - 3*A*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) - 12*B*a^4*(2*sin(d*x + c)/(sin(d*x + c
)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 36*A*a^4*(log(s
in(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 24*B*a^4*(log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) + 12*A*a^4*sin(d*x + c) + 48*A*a^4*tan(d*x + c) +
 72*B*a^4*tan(d*x + c))/d
```

Fricas [**A**]
time = 6.21, size = 159, normalized size = 1.05

$$\frac{12\,(4\,A+B)a^4dx\cos\left(dx+c\right)^3+3\,(13\,A+12\,B)a^4\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,(13\,A+12\,B)a^4\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(6\,Aa^4\cos\left(dx+c\right)^3+8\,(3\,A+5\,B)a^4\cos\left(dx+c\right)^2+3\,(A+4\,B)a^4\cos\left(dx+c\right)+2\,Ba^4\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(12*(4*A + B)*a^4*d*x*cos(d*x + c)^3 + 3*(13*A + 12*B)*a^4*cos(d*x + c
)^3*log(sin(d*x + c) + 1) - 3*(13*A + 12*B)*a^4*cos(d*x + c)^3*log(-sin(d*x
 + c) + 1) + 2*(6*A*a^4*cos(d*x + c)^3 + 8*(3*A + 5*B)*a^4*cos(d*x + c)^2 +
 3*(A + 4*B)*a^4*cos(d*x + c) + 2*B*a^4)*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^4\left(\int A\cos\left(c+dx\right)dx+\int 4A\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int 6A\cos\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int 4A\cos\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int A\cos\left(c+dx\right)\sec^4\left(c+dx\right)dx+\int B\cos\left(c+dx\right)\sec\left(c+dx\right)dx+\int 4B\cos\left(c+dx\right)\sec^2\left(c+dx\right)dx+\int 6B\cos\left(c+dx\right)\sec^3\left(c+dx\right)dx+\int 4B\cos\left(c+dx\right)\sec^4\left(c+dx\right)dx+\int B\cos\left(c+dx\right)\sec^5\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] a**4*(Integral(A*cos(c + d*x), x) + Integral(4*A*cos(c + d*x)*sec(c + d*x),
 x) + Integral(6*A*cos(c + d*x)*sec(c + d*x)**2, x) + Integral(4*A*cos(c +
d*x)*sec(c + d*x)**3, x) + Integral(A*cos(c + d*x)*sec(c + d*x)**4, x) + In
tegral(B*cos(c + d*x)*sec(c + d*x), x) + Integral(4*B*cos(c + d*x)*sec(c +
d*x)**2, x) + Integral(6*B*cos(c + d*x)*sec(c + d*x)**3, x) + Integral(4*B*
cos(c + d*x)*sec(c + d*x)**4, x) + Integral(B*cos(c + d*x)*sec(c + d*x)**5,
 x))
```

Giac [**A**]
time = 0.55, size = 227, normalized size = 1.50

$$\frac{\frac{12\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1}+6\left(4\,Aa^4+Ba^4\right)(dx+c)+3\left(13\,Aa^4+12\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(13\,Aa^4+12\,Ba^4\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(21\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+30\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-48\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-76\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+27\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+54\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(12*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 6*(4*A*a^
4 + B*a^4)*(d*x + c) + 3*(13*A*a^4 + 12*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c)
 + 1)) - 3*(13*A*a^4 + 12*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(21
*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 30*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 48*A*a^4*
tan(1/2*d*x + 1/2*c)^3 - 76*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 27*A*a^4*tan(1/2
*d*x + 1/2*c) + 54*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)
^3)/d
```

Mupad [B]
time = 2.10, size = 254, normalized size = 1.68

$$\frac{A\,a^4\,\sin\left(c+d\,x\right)}{d}+\frac{8\,A\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{13\,A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{12\,B\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,A\,a^4\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{A\,a^4\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2}+\frac{20\,B\,a^4\,\sin\left(c+d\,x\right)}{3\,d\,\cos\left(c+d\,x\right)}+\frac{2\,B\,a^4\,\sin\left(c+d\,x\right)}{d\,{\cos\left(c+d\,x\right)}^2}+\frac{B\,a^4\,\sin\left(c+d\,x\right)}{3\,d\,{\cos\left(c+d\,x\right)}^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^4,x)
```

```
[Out] (A*a^4*sin(c + d*x))/d + (8*A*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/d + (13*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*a^
4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (12*B*a^4*atanh(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*A*a^4*sin(c + d*x))/(d*cos(c + d*x))
 + (A*a^4*sin(c + d*x))/(2*d*cos(c + d*x)^2) + (20*B*a^4*sin(c + d*x))/(3*d
*cos(c + d*x)) + (2*B*a^4*sin(c + d*x))/(d*cos(c + d*x)^2) + (B*a^4*sin(c +
 d*x))/(3*d*cos(c + d*x)^3)
```

3.76 $\int \cos^2(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=160

$$\frac{1}{2}a^4(13A+8B)x+\frac{a^4(8A+13B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{5a^4(A-B)\sin(c+dx)}{2d}+\frac{aA\cos(c+dx)(a+a\sec(c$$

```
[Out] 1/2*a^4*(13*A+8*B)*x+1/2*a^4*(8*A+13*B)*arctanh(sin(d*x+c))/d+5/2*a^4*(A-B)
*sin(d*x+c)/d+1/2*a*A*cos(d*x+c)*(a+a*sec(d*x+c))^3*sin(d*x+c)/d-1/2*(A-B)*
(a^2+a^2*sec(d*x+c))^2*sin(d*x+c)/d+1/2*(A+6*B)*(a^4+a^4*sec(d*x+c))*sin(d*
x+c)/d
```

Rubi [A]

time = 0.27, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4102, 4103, 4081, 3855}

$$\frac{5a^4(A-B)\sin(c+dx)}{2d}+\frac{a^4(8A+13B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(A+6B)\sin(c+dx)\left(a^4\sec(c+dx)+a^4\right)}{2d}+\frac{1}{2}a^4x(13A+8B)-\frac{(A-B)\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{2d}+\frac{aA\sin(c+dx)\cos(c+dx)(a\sec(c+dx)+a)^3}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^4*(13*A + 8*B)*x)/2 + (a^4*(8*A + 13*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (
5*a^4*(A - B)*Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]*(a + a*Sec[c + d*x])^
3*Sin[c + d*x])/(2*d) - ((A - B)*(a^2 + a^2*Sec[c + d*x])^2*Sin[c + d*x])/(
2*d) + ((A + 6*B)*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(2*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
```

```
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \cos^2(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx &= \frac{aA\cos(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{2d} \\
&= \frac{aA\cos(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{2d} \\
&= \frac{aA\cos(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{2d} \\
&= \frac{5a^4(A-B)\sin(c+dx)}{2d} + \frac{aA\cos(c+dx)(a+}{} \\
&= \frac{1}{2}a^4(13A+8B)x + \frac{5a^4(A-B)\sin(c+dx)}{2d} + \\
&= \frac{1}{2}a^4(13A+8B)x + \frac{a^4(8A+13B)\tanh^{-1}(\sin(c}{2d}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 373 vs. 2(160) = 320.
time = 4.81, size = 373, normalized size = 2.33

$$\frac{a^4\cos^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^4(A+B\sec(c+dx))\left(2(13A+8B)x-\frac{2(8A+13B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{2(8A+13B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{4(4A+B)\cos(dx)\sin(c)}{d}+\frac{A\cos(2dx)\sin(2c)}{d}+\frac{4(4A+B)\cos(c)\sin(dx)}{d}+\frac{A\cos(2c)\sin(2dx)}{d}+\frac{B}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4(A+4B)\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}-\frac{B}{d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4(A+4B)\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)}{64(B+A\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^4*Cos[c + d*x]^5*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(A + B*Sec[c +
d*x])*(2*(13*A + 8*B)*x - (2*(8*A + 13*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d
```

```
*x)/2]])/d + (2*(8*A + 13*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d +
(4*(4*A + B)*Cos[d*x]*Sin[c])/d + (A*Cos[2*d*x]*Sin[2*c])/d + (4*(4*A + B)*
Cos[c]*Sin[d*x])/d + (A*Cos[2*c]*Sin[2*d*x])/d + B/(d*(Cos[(c + d*x)/2] - S
in[(c + d*x)/2])^2) + (4*(A + 4*B)*Sin[(d*x)/2])/(d*(Cos[c/2] - Sin[c/2])*(
Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) - B/(d*(Cos[(c + d*x)/2] + Sin[(c + d
*x)/2])^2) + (4*(A + 4*B)*Sin[(d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c +
d*x)/2] + Sin[(c + d*x)/2]))))/(64*(B + A*Cos[c + d*x]))
```

Maple [A]
time = 0.32, size = 177, normalized size = 1.11

method	result
derivativedivides	$\frac{A\,a^4\tan(dx+c)+a^4B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+4A\,a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4B\tan(dx-}{}$
default	$\frac{A\,a^4\tan(dx+c)+a^4B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+4A\,a^4\ln(\sec(dx+c)+\tan(dx+c))+4a^4B\tan(dx-}{}$
risch	$\frac{13a^4Ax}{2}+4a^4xB-\frac{iA\,a^4e^{2i(dx+c)}}{8d}-\frac{2iA\,a^4e^{i(dx+c)}}{d}-\frac{ie^{i(dx+c)}a^4B}{2d}+\frac{2iA\,a^4e^{-i(dx+c)}}{d}+\frac{ie^{-i(dx+c)}a^4B}{2d}$
norman	$\frac{(\frac{13}{2}A\,a^4+4a^4B)x+(-\frac{13}{2}A\,a^4-4a^4B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-\frac{13}{2}A\,a^4-4a^4B)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{13}{2}A\,a^4+4a^4B)x\left(\tan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^4*tan(d*x+c)+a^4*B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+ta
n(d*x+c)))+4*A*a^4*ln(sec(d*x+c)+tan(d*x+c))+4*a^4*B*tan(d*x+c)+6*A*a^4*(d*
x+c)+6*a^4*B*ln(sec(d*x+c)+tan(d*x+c))+4*A*a^4*sin(d*x+c)+4*a^4*B*(d*x+c)+A
*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^4*B*sin(d*x+c))
```

Maxima [A]
time = 0.27, size = 199, normalized size = 1.24

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^4+24\,(dx+c)Aa^4+16\,(dx+c)Ba^4-Ba^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+8\,Aa^4(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+12\,Ba^4(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+16\,Aa^4\sin(dx+c)+4\,Ba^4\sin(dx+c)+4\,Aa^4\tan(dx+c)+16\,Ba^4\tan(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 + 24*(d*x + c)*A*a^4 + 16*(d*x
+ c)*B*a^4 - B*a^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c)
+ 1) + log(sin(d*x + c) - 1)) + 8*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d*
x + c) - 1)) + 12*B*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 1
6*A*a^4*sin(d*x + c) + 4*B*a^4*sin(d*x + c) + 4*A*a^4*tan(d*x + c) + 16*B*a
^4*tan(d*x + c))/d
```

Fricas [A]
time = 4.40, size = 156, normalized size = 0.98

$$\frac{2\,(13\,A+8\,B)a^4dx\cos\left(dx+c\right)^2+(8\,A+13\,B)a^4\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(8\,A+13\,B)a^4\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(Aa^4\cos\left(dx+c\right)^3+2\,(4\,A+B)a^4\cos\left(dx+c\right)^2+2\,(A+4\,B)a^4\cos\left(dx+c\right)+Ba^4\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/4*(2*(13*A + 8*B)*a^4*d*x*cos(d*x + c)^2 + (8*A + 13*B)*a^4*cos(d*x + c)^
2*log(sin(d*x + c) + 1) - (8*A + 13*B)*a^4*cos(d*x + c)^2*log(-sin(d*x + c)
 + 1) + 2*(A*a^4*cos(d*x + c)^3 + 2*(4*A + B)*a^4*cos(d*x + c)^2 + 2*(A + 4
*B)*a^4*cos(d*x + c) + B*a^4)*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.53, size = 230, normalized size = 1.44

$$\frac{(13\,Aa^4+8\,Ba^4)(dx+c)+(8\,Aa^4+13\,Ba^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(8\,Aa^4+13\,Ba^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(5\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-5\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-7\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-7\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-9\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+11\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2*((13*A*a^4 + 8*B*a^4)*(d*x + c) + (8*A*a^4 + 13*B*a^4)*log(abs(tan(1/2*
d*x + 1/2*c) + 1)) - (8*A*a^4 + 13*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)
) + 2*(5*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 5*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 7*
A*a^4*tan(1/2*d*x + 1/2*c)^5 - 7*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 9*A*a^4*tan
(1/2*d*x + 1/2*c)^3 + 9*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 11*A*a^4*tan(1/2*d*x
 + 1/2*c) + 11*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/
d
```

Mupad [B]
time = 2.17, size = 243, normalized size = 1.52

$$\frac{4\,A\,a^4\,\sin\left(c+d\,x\right)}{d}+\frac{B\,a^4\,\sin\left(c+d\,x\right)}{d}+\frac{13\,A\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{8\,A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{8\,B\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{13\,B\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{A\,a^4\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{4\,B\,a^4\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{B\,a^4\,\sin\left(c+d\,x\right)}{2\,d\,{\cos\left(c+d\,x\right)}^2}+\frac{A\,a^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^4,x)
```

```
[Out] (4*A*a^4*sin(c + d*x))/d + (B*a^4*sin(c + d*x))/d + (13*A*a^4*atan(sin(c/2
+ (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (8*A*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c
/2 + (d*x)/2)))/d + (8*B*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
 + (13*B*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*a^4*sin(c
 + d*x))/(d*cos(c + d*x)) + (4*B*a^4*sin(c + d*x))/(d*cos(c + d*x)) + (B*a^
4*sin(c + d*x))/(2*d*cos(c + d*x)^2) + (A*a^4*cos(c + d*x)*sin(c + d*x))/(2
*d)
```

3.77 $\int \cos^3(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=165

$$\frac{1}{2}a^4(12A+13B)x+\frac{a^4(A+4B)\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^4(2A+B)\sin(c+dx)}{2d}+\frac{aA\cos^2(c+dx)(a+a\sec(c}{3d}$$

```
[Out] 1/2*a^4*(12*A+13*B)*x+a^4*(A+4*B)*arctanh(sin(d*x+c))/d+5/2*a^4*(2*A+B)*sin
(d*x+c)/d+1/3*a*A*cos(d*x+c)^2*(a+a*sec(d*x+c))^3*sin(d*x+c)/d+1/2*(2*A+B)*
cos(d*x+c)*(a^2+a^2*sec(d*x+c))^2*sin(d*x+c)/d-1/6*(8*A-3*B)*(a^4+a^4*sec(d
*x+c))*sin(d*x+c)/d
```

Rubi [A]
time = 0.29, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4102, 4103, 4081, 3855}

$$\frac{5a^4(2A+B)\sin(c+dx)}{2d}+\frac{a^4(A+4B)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(8A-3B)\sin(c+dx)\left(a^4\sec(c+dx)+a^4\right)}{6d}+\frac{1}{2}a^4x(12A+13B)+\frac{(2A+B)\sin(c+dx)\cos(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{2d}+\frac{aA\sin(c+dx)\cos^2(c+dx)(a\sec(c+dx)+a)^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^4*(12*A + 13*B)*x)/2 + (a^4*(A + 4*B)*ArcTanh[Sin[c + d*x]])/d + (5*a^4*
(2*A + B)*Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^3*
Sin[c + d*x])/(3*d) + ((2*A + B)*Cos[c + d*x]*(a^2 + a^2*Sec[c + d*x])^2*Si
n[c + d*x])/(2*d) - ((8*A - 3*B)*(a^4 + a^4*Sec[c + d*x])*Sin[c + d*x])/(6*
d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
```

```
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{aA\cos^2(c+dx)(a+a\sec(c+dx))^3\sin(c+d}{3d}$$
$$= \frac{aA\cos^2(c+dx)(a+a\sec(c+dx))^3\sin(c+d}{3d}$$
$$= \frac{aA\cos^2(c+dx)(a+a\sec(c+dx))^3\sin(c+d}{3d}$$
$$= \frac{5a^4(2A+B)\sin(c+dx)}{2d} + \frac{aA\cos^2(c+dx)(a}{}$$
$$= \frac{1}{2}a^4(12A+13B)x + \frac{5a^4(2A+B)\sin(c+dx)}{2d}$$
$$= \frac{1}{2}a^4(12A+13B)x + \frac{a^4(A+4B)\tanh^{-1}(\sin(c}{d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 342 vs. 2(165) = 330.
time = 2.03, size = 342, normalized size = 2.07

$$\frac{a^4\cos^5(c+dx)\sec^8\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^4(A+B\sec(c+dx))\left(72Ax+78Bx-\frac{12(A+4B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{12(A+4B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}+\frac{3(27A+16B)\cos(dx)\sin(c)}{d}+\frac{3(4A+B)\cos(2dx)\sin(2c)}{d}+\frac{A\cos(3dx)\sin(3c)}{d}+\frac{3(27A+16B)\cos(c)\sin(dx)}{d}+\frac{3(4A+B)\cos(2c)\sin(2dx)}{d}+\frac{A\cos(3c)\sin(3dx)}{d}+\frac{12B\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}+\frac{12B\sin\left(\frac{dx}{2}\right)}{d\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)}{192(B+A\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^4*Cos[c + d*x]^5*Sec[(c + d*x)/2]^8*(1 + Sec[c + d*x])^4*(A + B*Sec[c +
d*x])*(72*A*x + 78*B*x - (12*(A + 4*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]])/d + (12*(A + 4*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/d + (3*(2
7*A + 16*B)*Cos[d*x]*Sin[c])/d + (3*(4*A + B)*Cos[2*d*x]*Sin[2*c])/d + (A*C
os[3*d*x]*Sin[3*c])/d + (3*(27*A + 16*B)*Cos[c]*Sin[d*x])/d + (3*(4*A + B)*
Cos[2*c]*Sin[2*d*x])/d + (A*Cos[3*c]*Sin[3*d*x])/d + (12*B*Sin[(d*x)/2])/(d
*(Cos[c/2] - Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (12*B*Sin[(
d*x)/2])/(d*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))))/
(192*(B + A*Cos[c + d*x]))
```

Maple [A]
time = 0.32, size = 179, normalized size = 1.08

method	result
derivativedivides	$\frac{A a^4 \ln(\sec(dx+c)+\tan(dx+c))+a^4 B \tan(dx+c)+4A a^4(dx+c)+4a^4 B \ln(\sec(dx+c)+\tan(dx+c))+6A a^4 \sin(dx+c)+6a^4}{}$
default	$\frac{A a^4 \ln(\sec(dx+c)+\tan(dx+c))+a^4 B \tan(dx+c)+4A a^4(dx+c)+4a^4 B \ln(\sec(dx+c)+\tan(dx+c))+6A a^4 \sin(dx+c)+6a^4}{}$
risch	$6a^4Ax+\frac{13a^4xB}{2}+\frac{2i\mathrm{e}^{-i(dx+c)}a^4B}{d}+\frac{i\mathrm{e}^{-2i(dx+c)}a^4B}{8d}+\frac{iAa^4\mathrm{e}^{-2i(dx+c)}}{2d}+\frac{2ia^4B}{d(\mathrm{e}^{2i(dx+c)}+1)}-\frac{2i\mathrm{e}^{i(dx+c)}a^4}{d}$
norman	$\frac{(6A a^4+\frac{13}{2}a^4B)x+(-18A a^4-\frac{39}{2}a^4B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-18A a^4-\frac{39}{2}a^4B)x\left(\tan^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-6A a^4-\frac{13}{2}a^4B)x(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^4*ln(sec(d*x+c)+tan(d*x+c))+a^4*B*tan(d*x+c)+4*A*a^4*(d*x+c)+4*a^4
*B*ln(sec(d*x+c)+tan(d*x+c))+6*A*a^4*sin(d*x+c)+6*a^4*B*(d*x+c)+4*A*a^4*(1/
2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*a^4*B*sin(d*x+c)+1/3*A*a^4*(2+cos(
d*x+c)^2)*sin(d*x+c)+a^4*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 187, normalized size = 1.13

$$-\frac{4\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^4-12\left(2dx+2c+\sin(2dx+2c)\right)Aa^4-48(dx+c)Aa^4-3\left(2dx+2c+\sin(2dx+2c)\right)Ba^4-72(dx+c)Ba^4-6Aa^4\left(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)-24Ba^4\left(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1)\right)-72Aa^4\sin(dx+c)-48Ba^4\sin(dx+c)-12Ba^4\tan(dx+c)}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 12*(2*d*x + 2*c + sin(2*
d*x + 2*c))*A*a^4 - 48*(d*x + c)*A*a^4 - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))
*B*a^4 - 72*(d*x + c)*B*a^4 - 6*A*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x
```

```
+ c) - 1)) - 24*B*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 72*
A*a^4*sin(d*x + c) - 48*B*a^4*sin(d*x + c) - 12*B*a^4*tan(d*x + c))/d
```

Fricas [A]
time = 4.24, size = 150, normalized size = 0.91

$$\frac{3\,(12\,A+13\,B)a^4dx\cos\left(dx+c\right)+3\,(A+4\,B)a^4\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3\,(A+4\,B)a^4\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+\left(2\,Aa^4\cos\left(dx+c\right)^3+3\,(4\,A+B)a^4\cos\left(dx+c\right)^2+8\,(5\,A+3\,B)a^4\cos\left(dx+c\right)+6\,Ba^4\right)\sin\left(dx+c\right)}{6\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(12*A + 13*B)*a^4*d*x*cos(d*x + c) + 3*(A + 4*B)*a^4*cos(d*x + c)*lo
g(sin(d*x + c) + 1) - 3*(A + 4*B)*a^4*cos(d*x + c)*log(-sin(d*x + c) + 1) +
 (2*A*a^4*cos(d*x + c)^3 + 3*(4*A + B)*a^4*cos(d*x + c)^2 + 8*(5*A + 3*B)*a
^4*cos(d*x + c) + 6*B*a^4)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.50, size = 226, normalized size = 1.37

$$-\frac{\frac{12\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}-3\,(12\,Aa^4+13\,Ba^4)(dx+c)-6\,(Aa^4+4\,Ba^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+6\,(Aa^4+4\,Ba^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(30\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+21\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+76\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+48\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+54\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+27\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/6*(12*B*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(12*A*
a^4 + 13*B*a^4)*(d*x + c) - 6*(A*a^4 + 4*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c
) + 1)) + 6*(A*a^4 + 4*B*a^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(30*A*
a^4*tan(1/2*d*x + 1/2*c)^5 + 21*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 76*A*a^4*tan
(1/2*d*x + 1/2*c)^3 + 48*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 54*A*a^4*tan(1/2*d*
x + 1/2*c) + 27*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)
/d
```

Mupad [B]
time = 2.23, size = 242, normalized size = 1.47

$$\frac{20\,A\,a^4\,\sin\left(c+d\,x\right)}{3\,d}+\frac{4\,B\,a^4\,\sin\left(c+d\,x\right)}{d}+\frac{12\,A\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,A\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{13\,B\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{8\,B\,a^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{A\,a^4\,\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)}{3\,d}+\frac{B\,a^4\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}+\frac{2\,A\,a^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{d}+\frac{B\,a^4\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^4,x)
```

```
[Out] (20*A*a^4*sin(c + d*x))/(3*d) + (4*B*a^4*sin(c + d*x))/d + (12*A*a^4*atan(s
in(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*A*a^4*atanh(sin(c/2 + (d*x)/2
)/cos(c/2 + (d*x)/2)))/d + (13*B*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2)))/d + (8*B*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*a^
4*cos(c + d*x)^2*sin(c + d*x))/(3*d) + (B*a^4*sin(c + d*x))/(d*cos(c + d*x)
) + (2*A*a^4*cos(c + d*x)*sin(c + d*x))/d + (B*a^4*cos(c + d*x)*sin(c + d*x
))/(2*d)
```

3.78 $\int \cos^4(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=173

$$\frac{1}{8}a^4(35A+48B)x+\frac{a^4B\tanh^{-1}(\sin(c+dx))}{d}+\frac{5a^4(7A+8B)\sin(c+dx)}{8d}+\frac{aA\cos^3(c+dx)(a+a\sec(c+dx)}{4d}$$

```
[Out] 1/8*a^4*(35*A+48*B)*x+a^4*B*arctanh(sin(d*x+c))/d+5/8*a^4*(7*A+8*B)*sin(d*x
+c)/d+1/4*a*A*cos(d*x+c)^3*(a+a*sec(d*x+c))^3*sin(d*x+c)/d+1/12*(7*A+4*B)*c
os(d*x+c)^2*(a^2+a^2*sec(d*x+c))^2*sin(d*x+c)/d+1/24*(35*A+32*B)*cos(d*x+c)
*(a^4+a^4*sec(d*x+c))*sin(d*x+c)/d
```

Rubi [A]
time = 0.28, antiderivative size = 173, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4102, 4081, 3855}

$$\frac{5a^4(7A+8B)\sin(c+dx)}{8d}+\frac{(35A+32B)\sin(c+dx)\cos(c+dx)\left(a^4\sec(c+dx)+a^4\right)}{24d}+\frac{1}{8}a^4x(35A+48B)+\frac{a^4B\tanh^{-1}(\sin(c+dx))}{d}+\frac{(7A+4B)\sin(c+dx)\cos^2(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{12d}+\frac{aA\sin(c+dx)\cos^3(c+dx)(a\sec(c+dx)+a)^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^4*(35*A + 48*B)*x)/8 + (a^4*B*ArcTanh[Sin[c + d*x]])/d + (5*a^4*(7*A + 8
*B)*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^3*Sin[c
+ d*x])/(4*d) + ((7*A + 4*B)*Cos[c + d*x]^2*(a^2 + a^2*Sec[c + d*x])^2*Sin[
c + d*x])/(12*d) + ((35*A + 32*B)*Cos[c + d*x]*(a^4 + a^4*Sec[c + d*x])*Sin
[c + d*x])/(24*d)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
```

```
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{aA\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{4d}$$

$$= \frac{aA\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{4d}$$

$$= \frac{aA\cos^3(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{4d}$$

$$= \frac{5a^4(7A+8B)\sin(c+dx)}{8d} + \frac{aA\cos^3(c+dx)(a}{}$$

$$= \frac{1}{8}a^4(35A+48B)x + \frac{5a^4(7A+8B)\sin(c+dx)}{8d}$$

$$= \frac{1}{8}a^4(35A+48B)x + \frac{a^4B\tanh^{-1}(\sin(c+dx))}{d}$$

Mathematica [A]

time = 0.35, size = 138, normalized size = 0.80

$$\frac{a^4\left(420Adx+576Bdx-96B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+96B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+24(28A+27B)\sin(c+dx)+24(7A+4B)\sin(2(c+dx))+32A\sin(3(c+dx))+8B\sin(3(c+dx))+3A\sin(4(c+dx))\right)}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
[Out] (a^4*(420*A*d*x + 576*B*d*x - 96*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 96*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 24*(28*A + 27*B)*Sin[c +
 d*x] + 24*(7*A + 4*B)*Sin[2*(c + d*x)] + 32*A*Sin[3*(c + d*x)] + 8*B*Sin[3
*(c + d*x)] + 3*A*Sin[4*(c + d*x)]))/(96*d)
```

Maple [A]

time = 0.35, size = 208, normalized size = 1.20

method	result
derivativedivides	$A\,a^4(dx+c)+a^4B\ln(\sec(dx+c)+\tan(dx+c))+4A\,a^4\sin(dx+c)+4a^4B(dx+c)+6A\,a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+6a^4$

default	$\frac{A\,a^4(dx+c)+a^4B\ln(\sec(dx+c)+\tan(dx+c))+4A\,a^4\sin(dx+c)+4a^4B(dx+c)+6A\,a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+6a}{}$
risch	$\frac{35a^4Ax}{8}+6a^4xB-\frac{7iA\,a^4\mathrm{e}^{i(dx+c)}}{2d}-\frac{27i\mathrm{e}^{i(dx+c)}a^4B}{8d}+\frac{7iA\,a^4\mathrm{e}^{-i(dx+c)}}{2d}+\frac{27i\mathrm{e}^{-i(dx+c)}a^4B}{8d}+\frac{a^4\ln(\mathrm{e}^{i(dx}}{d}$
norman	$\frac{(\frac{35}{8}A\,a^4+6a^4B)x+(-\frac{35}{2}A\,a^4-24a^4B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-\frac{35}{2}A\,a^4-24a^4B)x\left(\tan^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{35}{8}A\,a^4+6a^4B)x(}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^4*(d*x+c)+a^4*B*ln(sec(d*x+c)+tan(d*x+c))+4*A*a^4*sin(d*x+c)+4*a^4
*B*(d*x+c)+6*A*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+6*a^4*B*sin(d*
x+c)+4/3*A*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)+4*a^4*B*(1/2*cos(d*x+c)*sin(d*x+
c)+1/2*d*x+1/2*c)+A*a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d
*x+3/8*c)+1/3*a^4*B*(2+cos(d*x+c)^2)*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 205, normalized size = 1.18

$$-\frac{128\,(\sin(dx+c)^3-3\,\sin(dx+c))Aa^4-3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Aa^4-144\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^4-96\,(dx+c)Aa^4+32\,(\sin(dx+c)^3-3\,\sin(dx+c))Ba^4-96\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^4-384\,(dx+c)Ba^4-48\,Ba^4(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))-384\,Aa^4\sin(dx+c)-576\,Ba^4\sin(dx+c)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/96*(128*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 3*(12*d*x + 12*c + sin
(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 - 144*(2*d*x + 2*c + sin(2*d*x +
2*c))*A*a^4 - 96*(d*x + c)*A*a^4 + 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a
^4 - 96*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^4 - 384*(d*x + c)*B*a^4 - 48*B
*a^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) - 384*A*a^4*sin(d*x +
c) - 576*B*a^4*sin(d*x + c))/d
```

Fricas [A]
time = 4.59, size = 118, normalized size = 0.68

$$\frac{3\,(35\,A+48\,B)a^4dx+12\,Ba^4\log(\sin(dx+c)+1)-12\,Ba^4\log(-\sin(dx+c)+1)+(6\,Aa^4\cos(dx+c)^3+8\,(4\,A+B)a^4\cos(dx+c)^2+3\,(27\,A+16\,B)a^4\cos(dx+c)+160\,(A+B)a^4)\sin(dx+c)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/24*(3*(35*A + 48*B)*a^4*d*x + 12*B*a^4*log(sin(d*x + c) + 1) - 12*B*a^4*l
og(-sin(d*x + c) + 1) + (6*A*a^4*cos(d*x + c)^3 + 8*(4*A + B)*a^4*cos(d*x +
 c)^2 + 3*(27*A + 16*B)*a^4*cos(d*x + c) + 160*(A + B)*a^4)*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [A]
time = 0.51, size = 214, normalized size = 1.24

$$\frac{24\,Ba^{4}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-24\,Ba^{4}\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\,(35\,Aa^{4}+48\,Ba^{4})(dx+c)+\frac{2\left(105\,Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+120\,Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+385\,Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+424\,Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+511\,Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+520\,Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+279\,Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+216\,Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(24*B*a^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*B*a^4*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) + 3*(35*A*a^4 + 48*B*a^4)*(d*x + c) + 2*(105*A*a^4*tan
(1/2*d*x + 1/2*c)^7 + 120*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 385*A*a^4*tan(1/2*
d*x + 1/2*c)^5 + 424*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 511*A*a^4*tan(1/2*d*x +
 1/2*c)^3 + 520*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 279*A*a^4*tan(1/2*d*x + 1/2*
c) + 216*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 2.45, size = 188, normalized size = 1.09

$$\frac{105\,A\,a^{4}\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+144\,B\,a^{4}\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+24\,B\,a^{4}\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+21\,A\,a^{4}\,\sin\left(2\,c+2\,d\,x\right)+4\,A\,a^{4}\,\sin\left(3\,c+3\,d\,x\right)+\frac{3\,A\,a^{4}\,\sin\left(4\,c+4\,d\,x\right)}{8}+12\,B\,a^{4}\,\sin\left(2\,c+2\,d\,x\right)+B\,a^{4}\,\sin\left(3\,c+3\,d\,x\right)+84\,A\,a^{4}\,\sin\left(c+d\,x\right)+81\,B\,a^{4}\,\sin\left(c+d\,x\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^4,x)
```

```
[Out] (105*A*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 144*B*a^4*atan(sin
(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 24*B*a^4*atanh(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)) + 21*A*a^4*sin(2*c + 2*d*x) + 4*A*a^4*sin(3*c + 3*d*x) + (
3*A*a^4*sin(4*c + 4*d*x))/8 + 12*B*a^4*sin(2*c + 2*d*x) + B*a^4*sin(3*c + 3
*d*x) + 84*A*a^4*sin(c + d*x) + 81*B*a^4*sin(c + d*x))/(12*d)
```

3.79 $\int \cos^5(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=158

$$\frac{7}{8}a^4(4A+5B)x+\frac{8a^4(4A+5B)\sin(c+dx)}{5d}+\frac{27a^4(4A+5B)\cos(c+dx)\sin(c+dx)}{40d}+\frac{a^4(4A+5B)\cos^3(c+}{20d}$$

```
[Out] 7/8*a^4*(4*A+5*B)*x+8/5*a^4*(4*A+5*B)*sin(d*x+c)/d+27/40*a^4*(4*A+5*B)*cos(
d*x+c)*sin(d*x+c)/d+1/20*a^4*(4*A+5*B)*cos(d*x+c)^3*sin(d*x+c)/d+1/5*A*cos(
d*x+c)^4*(a+a*sec(d*x+c))^4*sin(d*x+c)/d-4/15*a^4*(4*A+5*B)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.15, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4098, 3876, 2717, 2715, 8, 2713}

$$-\frac{4a^4(4A+5B)\sin^3(c+dx)}{15d}+\frac{8a^4(4A+5B)\sin(c+dx)}{5d}+\frac{a^4(4A+5B)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{27a^4(4A+5B)\sin(c+dx)\cos(c+dx)}{40d}+\frac{7}{8}a^4x(4A+5B)+\frac{A\sin(c+dx)\cos^4(c+dx)(a\sec(c+dx)+a)^4}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (7*a^4*(4*A + 5*B)*x)/8 + (8*a^4*(4*A + 5*B)*Sin[c + d*x])/(5*d) + (27*a^4*
(4*A + 5*B)*Cos[c + d*x]*Sin[c + d*x])/(40*d) + (a^4*(4*A + 5*B)*Cos[c + d*
x]^3*Sin[c + d*x])/(20*d) + (A*Cos[c + d*x]^4*(a + a*Sec[c + d*x])^4*Sin[c
+ d*x])/(5*d) - (4*a^4*(4*A + 5*B)*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3876

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Int[ExpandTrig[(a + b*csc[e + f*x])^m*(d*csc[e + f
*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && I
GtQ[m, 0] && RationalQ[n]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{A\cos^4(c+dx)(a+a\sec(c+dx))^4\sin(c+dx)}{5d}$$

$$= \frac{A\cos^4(c+dx)(a+a\sec(c+dx))^4\sin(c+dx)}{5d}$$

$$= \frac{1}{5}a^4(4A+5B)x + \frac{A\cos^4(c+dx)(a+a\sec(c+}{5d}$$

$$= \frac{1}{5}a^4(4A+5B)x + \frac{4a^4(4A+5B)\sin(c+dx)}{5d} +$$

$$= \frac{4}{5}a^4(4A+5B)x + \frac{8a^4(4A+5B)\sin(c+dx)}{5d} +$$

$$= \frac{7}{8}a^4(4A+5B)x + \frac{8a^4(4A+5B)\sin(c+dx)}{5d} +$$

Mathematica [A]

time = 0.33, size = 108, normalized size = 0.68

$$\frac{a^4(1680Adx+2100Bdx+420(7A+8B)\sin(c+dx)+120(8A+7B)\sin(2(c+dx))+290A\sin(3(c+dx))+160B\sin(3(c+dx))+60A\sin(4(c+dx))+15B\sin(4(c+dx))+6A\sin(5(c+dx)))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^4*(1680*A*d*x + 2100*B*d*x + 420*(7*A + 8*B)*Sin[c + d*x] + 120*(8*A + 7
*B)*Sin[2*(c + d*x)] + 290*A*Sin[3*(c + d*x)] + 160*B*Sin[3*(c + d*x)] + 60
*A*Sin[4*(c + d*x)] + 15*B*Sin[4*(c + d*x)] + 6*A*Sin[5*(c + d*x)]))/(480*d
)
```

Maple [A]
time = 0.36, size = 248, normalized size = 1.57

method	result
risch	$\frac{7a^4 Ax}{2} + \frac{35a^4 xB}{8} + \frac{49\sin(dx+c)A\,a^4}{8d} + \frac{7\sin(dx+c)a^4 B}{d} + \frac{A\,a^4\sin(5dx+5c)}{80d} + \frac{A\,a^4\sin(4dx+4c)}{8d} + \frac{\sin(4dx\cdots}{3\cdots}$
derivativedivides	$\frac{\frac{A\,a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+4A\,a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a^4B\left(\frac{\left(\cos^3(\cdots\right.}{\cdots}\right.\cdots}{\cdots}$
default	$\frac{\frac{A\,a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+4A\,a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a^4B\left(\frac{\left(\cos^3(\cdots\right.}{\cdots}\right.\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/5*A*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*A*a^4*(1/4*
(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^4*B*(1/4*(cos(d*x
+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*A*a^4*(2+cos(d*x+c)^2)*si
n(d*x+c)+4/3*a^4*B*(2+cos(d*x+c)^2)*sin(d*x+c)+4*A*a^4*(1/2*cos(d*x+c)*sin(
d*x+c)+1/2*d*x+1/2*c)+6*a^4*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*a
^4*sin(d*x+c)+4*a^4*B*sin(d*x+c)+a^4*B*(d*x+c))
```

Maxima [A]
time = 0.29, size = 236, normalized size = 1.49

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^4-960\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^4+60\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Aa^4+480\left(2dx+2c+\sin(2dx+2c)\right)Aa^4-640\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba^4+15\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Ba^4+720\left(2dx+2c+\sin(2dx+2c)\right)Ba^4+480(dx+c)Ba^4+480Aa^4\sin(dx+c)+1920Ba^4\sin(dx+c)}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4 -
960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 60*(12*d*x + 12*c + sin(4*d*x
 + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 480*(2*d*x + 2*c + sin(2*d*x + 2*c))*
A*a^4 - 640*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 15*(12*d*x + 12*c + s
in(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 + 720*(2*d*x + 2*c + sin(2*d*x
+ 2*c))*B*a^4 + 480*(d*x + c)*B*a^4 + 480*A*a^4*sin(d*x + c) + 1920*B*a^4*s
in(d*x + c))/d
```

Fricas [A]
time = 6.63, size = 110, normalized size = 0.70

$$\frac{105\,(4\,A+5\,B)a^4dx+\left(24\,Aa^4\cos\left(dx+c\right)^4+30\,(4\,A+B)a^4\cos\left(dx+c\right)^3+16\,(17\,A+10\,B)a^4\cos\left(dx+c\right)^2+15\,(28\,A+27\,B)a^4\cos\left(dx+c\right)+8\,(83\,A+100\,B)a^4\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(105*(4*A + 5*B)*a^4*d*x + (24*A*a^4*cos(d*x + c)^4 + 30*(4*A + B)*a^
4*cos(d*x + c)^3 + 16*(17*A + 10*B)*a^4*cos(d*x + c)^2 + 15*(28*A + 27*B)*a
^4*cos(d*x + c) + 8*(83*A + 100*B)*a^4)*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [A]
time = 0.50, size = 210, normalized size = 1.33

$$\frac{105\,(4\,Aa^4+5\,Ba^4)(dx+c)+\frac{2\left(420\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+525\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+1960\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+2450\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+3584\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4480\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3160\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3950\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1500\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1395\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/120*(105*(4*A*a^4 + 5*B*a^4)*(d*x + c) + 2*(420*A*a^4*tan(1/2*d*x + 1/2*c
)^9 + 525*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 1960*A*a^4*tan(1/2*d*x + 1/2*c)^7
+ 2450*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 3584*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 4
480*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 3160*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 3950
*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 1500*A*a^4*tan(1/2*d*x + 1/2*c) + 1395*B*a^
4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B]
time = 4.70, size = 248, normalized size = 1.57

$$\frac{\left(7\,A\,a^4+\frac{35\,B\,a^4}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{98\,A\,a^4}{3}+\frac{245\,B\,a^4}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{896\,A\,a^4}{15}+\frac{224\,B\,a^4}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{158\,A\,a^4}{3}+\frac{395\,B\,a^4}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(25\,A\,a^4+\frac{93\,B\,a^4}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{7\,a^4\,\mathrm{atan}\left(\frac{7\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(4\,A+5\,B\right)}{4\,\left(7\,A\,a^4+\frac{35\,B\,a^4}{4}\right)}\right)\,\left(4\,A+5\,B\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^4,x)

[Out] (tan(c/2 + (d*x)/2)*(25*A*a^4 + (93*B*a^4)/4) + tan(c/2 + (d*x)/2)^9*(7*A*a
^4 + (35*B*a^4)/4) + tan(c/2 + (d*x)/2)^7*((98*A*a^4)/3 + (245*B*a^4)/6) +
tan(c/2 + (d*x)/2)^3*((158*A*a^4)/3 + (395*B*a^4)/6) + tan(c/2 + (d*x)/2)^5
*((896*A*a^4)/15 + (224*B*a^4)/3))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2
+ (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 +
 (d*x)/2)^10 + 1)) + (7*a^4*atan((7*a^4*tan(c/2 + (d*x)/2)*(4*A + 5*B))/(4*
(7*A*a^4 + (35*B*a^4)/4)))*(4*A + 5*B))/(4*d)
```

3.80 $\int \cos^6(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=220

$$\frac{7}{16}a^4(7A+8B)x+\frac{a^4(72A+83B)\sin(c+dx)}{15d}+\frac{7a^4(7A+8B)\cos(c+dx)\sin(c+dx)}{16d}+\frac{a^4(159A+176B)\cos^2}{120}$$

```
[Out] 7/16*a^4*(7*A+8*B)*x+1/15*a^4*(72*A+83*B)*sin(d*x+c)/d+7/16*a^4*(7*A+8*B)*c
os(d*x+c)*sin(d*x+c)/d+1/120*a^4*(159*A+176*B)*cos(d*x+c)^2*sin(d*x+c)/d+1/
6*a*A*cos(d*x+c)^5*(a+a*sec(d*x+c))^3*sin(d*x+c)/d+1/10*(3*A+2*B)*cos(d*x+c
)^4*(a^2+a^2*sec(d*x+c))^2*sin(d*x+c)/d+1/120*(73*A+72*B)*cos(d*x+c)^3*(a^4
+a^4*sec(d*x+c))*sin(d*x+c)/d
```

Rubi [A]
time = 0.38, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4102, 4081, 3872, 2715, 8, 2717}

$$\frac{a^4(72A+83B)\sin(c+dx)}{15d}+\frac{a^4(159A+176B)\sin(c+dx)\cos^2(c+dx)}{120d}+\frac{7a^4(7A+8B)\sin(c+dx)\cos(c+dx)}{16d}+\frac{(73A+72B)\sin(c+dx)\cos^3(c+dx)\left(a^4\sec(c+dx)+a^4\right)}{120d}+\frac{7}{16}a^4x(7A+8B)+\frac{(3A+2B)\sin(c+dx)\cos^4(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{10d}+\frac{aA\sin(c+dx)\cos^5(c+dx)(a\sec(c+dx)+a)^3}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (7*a^4*(7*A + 8*B)*x)/16 + (a^4*(72*A + 83*B)*Sin[c + d*x])/(15*d) + (7*a^4
*(7*A + 8*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^4*(159*A + 176*B)*Cos[c
 + d*x]^2*Sin[c + d*x])/(120*d) + (a*A*Cos[c + d*x]^5*(a + a*Sec[c + d*x])^
3*Sin[c + d*x])/(6*d) + ((3*A + 2*B)*Cos[c + d*x]^4*(a^2 + a^2*Sec[c + d*x]
)^2*Sin[c + d*x])/(10*d) + ((73*A + 72*B)*Cos[c + d*x]^3*(a^4 + a^4*Sec[c +
 d*x])*Sin[c + d*x])/(120*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \cos^6(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx &= \frac{aA\cos^5(c+dx)(a+a\sec(c+dx))^3\sin(c+dx}{6d} \\
&= \frac{aA\cos^5(c+dx)(a+a\sec(c+dx))^3\sin(c+dx}{6d} \\
&= \frac{aA\cos^5(c+dx)(a+a\sec(c+dx))^3\sin(c+dx}{6d} \\
&= \frac{a^4(159A+176B)\cos^2(c+dx)\sin(c+dx)}{120d} + \\
&= \frac{a^4(159A+176B)\cos^2(c+dx)\sin(c+dx)}{120d} + \\
&= \frac{a^4(72A+83B)\sin(c+dx)}{15d} + \frac{7a^4(7A+8B)\,c}{} \\
&= \frac{7}{16}a^4(7A+8B)x + \frac{a^4(72A+83B)\sin(c+dx}{15d}
\end{aligned}
$$

Mathematica [A]

time = 0.63, size = 134, normalized size = 0.61

$$\frac{a^4(2940Ac+2940Adx+3360Bdx+120(44A+49B)\sin(c+dx)+15(127A+128B)\sin(2(c+dx))+720A\sin(3(c+dx))+580B\sin(3(c+dx))+225A\sin(4(c+dx))+120B\sin(4(c+dx))+48A\sin(5(c+dx))+12B\sin(5(c+dx))+5A\sin(6(c+dx)))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^4*(2940*A*c + 2940*A*d*x + 3360*B*d*x + 120*(44*A + 49*B)*Sin[c + d*x] +
 15*(127*A + 128*B)*Sin[2*(c + d*x)] + 720*A*Sin[3*(c + d*x)] + 580*B*Sin[3
*(c + d*x)] + 225*A*Sin[4*(c + d*x)] + 120*B*Sin[4*(c + d*x)] + 48*A*Sin[5*
(c + d*x)] + 12*B*Sin[5*(c + d*x)] + 5*A*Sin[6*(c + d*x)]))/(960*d)
```

Maple [A]

time = 0.43, size = 306, normalized size = 1.39

method	result
risch	$\frac{49a^4Ax}{16}+\frac{7a^4xB}{2}+\frac{11\sin(dx+c)A\,a^4}{2d}+\frac{49\sin(dx+c)a^4B}{8d}+\frac{A\,a^4\sin(6dx+6c)}{192d}+\frac{A\,a^4\sin(5dx+5c)}{20d}+\frac{\sin(5dx+}{80}$
derivativedivides	$A\,a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^4B\sin(dx+c)+\frac{4A\,a^4\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+4a^4B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)$
default	$A\,a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^4B\sin(dx+c)+\frac{4A\,a^4\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+4a^4B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^4*B*sin(d*x+c)+4/3*A
*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)+4*a^4*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x
+1/2*c)+6*A*a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c
)+2*a^4*B*(2+cos(d*x+c)^2)*sin(d*x+c)+4/5*A*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d
*x+c)^2)*sin(d*x+c)+4*a^4*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3
/8*d*x+3/8*c)+A*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*si
n(d*x+c)+5/16*d*x+5/16*c)+1/5*a^4*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin
(d*x+c))
```

Maxima [A]

time = 0.28, size = 297, normalized size = 1.35

$$\frac{256\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^4-5\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c)-48\sin(2dx+2c)\right)Aa^4-1280\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^4+180\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Aa^4+240\left(2dx+2c+\sin(2dx+2c)\right)Aa^4+64\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Ba^4-1920\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba^4+120\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Ba^4+960\left(2dx+2c+\sin(2dx+2c)\right)Ba^4+960Ba^4\sin(dx+c)}{960d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/960*(256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4 -
 5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*
x + 2*c))*A*a^4 - 1280*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 + 180*(12*d*
x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 + 240*(2*d*x + 2*c
+ sin(2*d*x + 2*c))*A*a^4 + 64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*s
in(d*x + c))*B*a^4 - 1920*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 + 120*(12
*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 + 960*(2*d*x + 2
*c + sin(2*d*x + 2*c))*B*a^4 + 960*B*a^4*sin(d*x + c))/d
```

Fricas [A]
time = 2.72, size = 130, normalized size = 0.59

$$\frac{105\,(7\,A+8\,B)a^4dx+\left(40\,Aa^4\cos\left(dx+c\right)^5+48\,(4\,A+B)a^4\cos\left(dx+c\right)^4+10\,(41\,A+24\,B)a^4\cos\left(dx+c\right)^3+32\,(18\,A+17\,B)a^4\cos\left(dx+c\right)^2+105\,(7\,A+8\,B)a^4\cos\left(dx+c\right)+16\,(72\,A+83\,B)a^4\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(105*(7*A + 8*B)*a^4*d*x + (40*A*a^4*cos(d*x + c)^5 + 48*(4*A + B)*a^
4*cos(d*x + c)^4 + 10*(41*A + 24*B)*a^4*cos(d*x + c)^3 + 32*(18*A + 17*B)*a
^4*cos(d*x + c)^2 + 105*(7*A + 8*B)*a^4*cos(d*x + c) + 16*(72*A + 83*B)*a^4
)*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [A]
time = 0.52, size = 244, normalized size = 1.11

$$\frac{105\,(7\,Aa^4+8\,Ba^4)(dx+c)+\frac{2\left(735\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+840\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+4165\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+4760\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+9702\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+11088\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+11802\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+13488\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+7355\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9320\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3105\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3000\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^6}}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/240*(105*(7*A*a^4 + 8*B*a^4)*(d*x + c) + 2*(735*A*a^4*tan(1/2*d*x + 1/2*c
)^11 + 840*B*a^4*tan(1/2*d*x + 1/2*c)^11 + 4165*A*a^4*tan(1/2*d*x + 1/2*c)^
9 + 4760*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 9702*A*a^4*tan(1/2*d*x + 1/2*c)^7 +
 11088*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 11802*A*a^4*tan(1/2*d*x + 1/2*c)^5 +
13488*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 7355*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 93
20*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 3105*A*a^4*tan(1/2*d*x + 1/2*c) + 3000*B*
a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/d
```

Mupad [B]
time = 4.62, size = 286, normalized size = 1.30

$$\frac{\left(\frac{49\,A\,a^4}{8}+7\,B\,a^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{833\,A\,a^4}{24}+\frac{119\,B\,a^4}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{1617\,A\,a^4}{20}+\frac{462\,B\,a^4}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{1967\,A\,a^4}{20}+\frac{562\,B\,a^4}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{1471\,A\,a^4}{24}+\frac{233\,B\,a^4}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(\frac{207\,A\,a^4}{8}+25\,B\,a^4\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+20\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+15\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+6\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{7\,a^4\,\mathrm{atan}\left(\frac{7\,a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(7\,A+8\,B\right)}{8\,\left(\frac{49\,A\,a^4}{8}+7\,B\,a^4\right)}\right)\,\left(7\,A+8\,B\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((207*A*a^4)/8 + 25*B*a^4) + tan(c/2 + (d*x)/2)^11*((49
*A*a^4)/8 + 7*B*a^4) + tan(c/2 + (d*x)/2)^9*((833*A*a^4)/24 + (119*B*a^4)/3
) + tan(c/2 + (d*x)/2)^3*((1471*A*a^4)/24 + (233*B*a^4)/3) + tan(c/2 + (d*x
)/2)^7*((1617*A*a^4)/20 + (462*B*a^4)/5) + tan(c/2 + (d*x)/2)^5*((1967*A*a^
4)/20 + (562*B*a^4)/5))/(d*(6*tan(c/2 + (d*x)/2)^2 + 15*tan(c/2 + (d*x)/2)^
4 + 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (d*x)/2)^8 + 6*tan(c/2 + (d*x)/2
)^10 + tan(c/2 + (d*x)/2)^12 + 1)) + (7*a^4*atan((7*a^4*tan(c/2 + (d*x)/2)*
(7*A + 8*B))/(8*((49*A*a^4)/8 + 7*B*a^4)))*(7*A + 8*B))/(8*d)
```

3.81 $\int \cos^7(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=241

$$\frac{1}{16}a^4(44A+49B)x+\frac{a^4(227A+252B)\sin(c+dx)}{35d}+\frac{a^4(44A+49B)\cos(c+dx)\sin(c+dx)}{16d}+\frac{a^4(276A+30$$

```
[Out] 1/16*a^4*(44*A+49*B)*x+1/35*a^4*(227*A+252*B)*sin(d*x+c)/d+1/16*a^4*(44*A+4
9*B)*cos(d*x+c)*sin(d*x+c)/d+1/280*a^4*(276*A+301*B)*cos(d*x+c)^3*sin(d*x+c
)/d+1/7*a*A*cos(d*x+c)^6*(a+a*sec(d*x+c))^3*sin(d*x+c)/d+1/42*(10*A+7*B)*co
s(d*x+c)^5*(a^2+a^2*sec(d*x+c))^2*sin(d*x+c)/d+7/15*(A+B)*cos(d*x+c)^4*(a^4
+a^4*sec(d*x+c))*sin(d*x+c)/d-1/105*a^4*(227*A+252*B)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.40, antiderivative size = 241, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4102, 4081, 3872, 2713, 2715, 8}

$$-\frac{a^4(227A+252B)\sin^3(c+dx)}{105d}+\frac{a^4(227A+252B)\sin(c+dx)}{35d}+\frac{a^4(276A+301B)\sin(c+dx)\cos^3(c+dx)}{280d}+\frac{a^4(44A+49B)\sin(c+dx)\cos(c+dx)}{16d}+\frac{7(A+B)\sin(c+dx)\cos^4(c+dx)\left(a^4\sec(c+dx)+a^4\right)}{15d}+\frac{1}{16}a^4x(44A+49B)+\frac{(10A+7B)\sin(c+dx)\cos^5(c+dx)\left(a^2\sec(c+dx)+a^2\right)^2}{42d}+\frac{aA\sin(c+dx)\cos^6(c+dx)(a\sec(c+dx)+a)^3}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^4*(44*A + 49*B)*x)/16 + (a^4*(227*A + 252*B)*Sin[c + d*x])/(35*d) + (a^4
*(44*A + 49*B)*Cos[c + d*x]*Sin[c + d*x])/(16*d) + (a^4*(276*A + 301*B)*Cos
[c + d*x]^3*Sin[c + d*x])/(280*d) + (a*A*Cos[c + d*x]^6*(a + a*Sec[c + d*x]
)^3*Sin[c + d*x])/(7*d) + ((10*A + 7*B)*Cos[c + d*x]^5*(a^2 + a^2*Sec[c + d
*x])^2*Sin[c + d*x])/(42*d) + (7*(A + B)*Cos[c + d*x]^4*(a^4 + a^4*Sec[c +
d*x])*Sin[c + d*x])/(15*d) - (a^4*(227*A + 252*B)*Sin[c + d*x]^3)/(105*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

```
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^7(c+dx)(a+a\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{aA\cos^6(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{7d}$$

$$= \frac{aA\cos^6(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{7d}$$

$$= \frac{aA\cos^6(c+dx)(a+a\sec(c+dx))^3\sin(c+dx)}{7d}$$

$$= \frac{a^4(276A+301B)\cos^3(c+dx)\sin(c+dx)}{280d} + \frac{a}{}$$

$$= \frac{a^4(276A+301B)\cos^3(c+dx)\sin(c+dx)}{280d} + \frac{a}{}$$

$$= \frac{a^4(44A+49B)\cos(c+dx)\sin(c+dx)}{16d} + \frac{a^4(27}{}$$

$$= \frac{1}{16}a^4(44A+49B)x + \frac{a^4(227A+252B)\sin(c+}{35d}$$

Mathematica [A]

time = 0.77, size = 156, normalized size = 0.65

$$\frac{a^4(18480Ac+18480Adx+20580Bdx+105(323A+352B)\sin(c+dx)+105(124A+127B)\sin(2(c+dx))+5495A\sin(3(c+dx))+5040B\sin(3(c+dx))+2100A\sin(4(c+dx))+1575B\sin(4(c+dx))+651A\sin(5(c+dx))+336B\sin(5(c+dx))+140A\sin(6(c+dx))+35B\sin(6(c+dx))+15A\sin(7(c+dx)))}{6720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^7*(a + a*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^4*(18480*A*c + 18480*A*d*x + 20580*B*d*x + 105*(323*A + 352*B)*Sin[c + d
*x] + 105*(124*A + 127*B)*Sin[2*(c + d*x)] + 5495*A*Sin[3*(c + d*x)] + 5040
*B*Sin[3*(c + d*x)] + 2100*A*Sin[4*(c + d*x)] + 1575*B*Sin[4*(c + d*x)] + 6
51*A*Sin[5*(c + d*x)] + 336*B*Sin[5*(c + d*x)] + 140*A*Sin[6*(c + d*x)] + 3
5*B*Sin[6*(c + d*x)] + 15*A*Sin[7*(c + d*x)]))/(6720*d)
```

Maple [A]

time = 0.49, size = 358, normalized size = 1.49

method	result
risch	$\frac{11a^4Ax}{4}+\frac{49a^4xB}{16}+\frac{323\sin(dx+c)A\,a^4}{64d}+\frac{11\sin(dx+c)a^4B}{2d}+\frac{A\,a^4\sin(7dx+7c)}{448d}+\frac{A\,a^4\sin(6dx+6c)}{48d}+\sin(6$
derivativedivides	$\frac{\frac{A\,a^4\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+a^4B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4A\,a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3}{8}\right.}{}$
default	$\frac{\frac{A\,a^4\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+a^4B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+4A\,a^4\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3}{8}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^7*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/3*A*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)+a^4*B*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+4*A*a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d
*x+3/8*c)+4/3*a^4*B*(2+cos(d*x+c)^2)*sin(d*x+c)+6/5*A*a^4*(8/3+cos(d*x+c)^4
+4/3*cos(d*x+c)^2)*sin(d*x+c)+6*a^4*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*si
n(d*x+c)+3/8*d*x+3/8*c)+4*A*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*co
s(d*x+c))*sin(d*x+c)+5/16*d*x+5/16*c)+4/5*a^4*B*(8/3+cos(d*x+c)^4+4/3*cos(d
*x+c)^2)*sin(d*x+c)+1/7*A*a^4*(16/5+cos(d*x+c)^6+6/5*cos(d*x+c)^4+8/5*cos(d
*x+c)^2)*sin(d*x+c)+a^4*B*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+
c))*sin(d*x+c)+5/16*d*x+5/16*c))
```

Maxima [A]

time = 0.28, size = 356, normalized size = 1.48

$$-\frac{192\left(5\sin(dx+c)^7-21\sin(dx+c)^5+35\sin(dx+c)^3-35\sin(dx+c)\right)Aa^4-2688\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^4+140\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c)-48\sin(2dx+2c)\right)Aa^4+2240\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^4-840\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Aa^4-1792\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Ba^4+35\left(4\sin(2dx+2c)^3-60dx-60c-9\sin(4dx+4c)-48\sin(2dx+2c)\right)Ba^4+8960\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba^4-1260\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Ba^4-1680\left(2dx+2c+\sin(2dx+2c)\right)Ba^4}{6720d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/6720*(192*(5*sin(d*x + c)^7 - 21*sin(d*x + c)^5 + 35*sin(d*x + c)^3 - 35
*sin(d*x + c))*A*a^4 - 2688*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(
d*x + c))*A*a^4 + 140*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x +
 4*c) - 48*sin(2*d*x + 2*c))*A*a^4 + 2240*(sin(d*x + c)^3 - 3*sin(d*x + c))
*A*a^4 - 840*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4
- 1792*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*B*a^4 + 35*
(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*sin(2*d*x +
 2*c))*B*a^4 + 8960*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 - 1260*(12*d*x
+ 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 - 1680*(2*d*x + 2*c +
 sin(2*d*x + 2*c))*B*a^4)/d
```

Fricas [**A**]
time = 1.74, size = 150, normalized size = 0.62

$$\frac{105\,(44\,A+49\,B)a^4dx+\left(240\,Aa^4\cos\left(dx+c\right)^6+280\,(4\,A+B)a^4\cos\left(dx+c\right)^5+192\,(12\,A+7\,B)a^4\cos\left(dx+c\right)^4+70\,(44\,A+41\,B)a^4\cos\left(dx+c\right)^3+16\,(227\,A+252\,B)a^4\cos\left(dx+c\right)^2+105\,(44\,A+49\,B)a^4\cos\left(dx+c\right)+32\,(227\,A+252\,B)a^4\right)\sin\left(dx+c\right)}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/1680*(105*(44*A + 49*B)*a^4*d*x + (240*A*a^4*cos(d*x + c)^6 + 280*(4*A +
B)*a^4*cos(d*x + c)^5 + 192*(12*A + 7*B)*a^4*cos(d*x + c)^4 + 70*(44*A + 41
*B)*a^4*cos(d*x + c)^3 + 16*(227*A + 252*B)*a^4*cos(d*x + c)^2 + 105*(44*A
+ 49*B)*a^4*cos(d*x + c) + 32*(227*A + 252*B)*a^4)*sin(d*x + c))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**7*(a+a*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8570 deep
```

Giac [**A**]
time = 0.52, size = 278, normalized size = 1.15

$$\frac{105\,(44\,Aa^4+49\,Ba^4)(dx+c)+\frac{2\left(4620\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+5145\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{13}+30800\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+34300\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{11}+87164\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+97069\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{9}+135168\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+150528\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+126084\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+134099\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+58800\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+73220\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+22260\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+21735\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^7}}{1680\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^7*(a+a*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/1680*(105*(44*A*a^4 + 49*B*a^4)*(d*x + c) + 2*(4620*A*a^4*tan(1/2*d*x + 1
/2*c)^13 + 5145*B*a^4*tan(1/2*d*x + 1/2*c)^13 + 30800*A*a^4*tan(1/2*d*x + 1
/2*c)^11 + 34300*B*a^4*tan(1/2*d*x + 1/2*c)^11 + 87164*A*a^4*tan(1/2*d*x +
1/2*c)^9 + 97069*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 135168*A*a^4*tan(1/2*d*x +
1/2*c)^7 + 150528*B*a^4*tan(1/2*d*x + 1/2*c)^7 + 126084*A*a^4*tan(1/2*d*x +
 1/2*c)^5 + 134099*B*a^4*tan(1/2*d*x + 1/2*c)^5 + 58800*A*a^4*tan(1/2*d*x +
 1/2*c)^3 + 73220*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 22260*A*a^4*tan(1/2*d*x +
1/2*c) + 21735*B*a^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^7)/
d
```

Mupad [B]
time = 4.13, size = 323, normalized size = 1.34

$$\frac{\left(\frac{11\,A\,a^4}{2}+\frac{49\,B\,a^4}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{13}+\left(\frac{110\,A\,a^4}{3}+\frac{245\,B\,a^4}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{11}+\left(\frac{3113\,A\,a^4}{30}+\frac{13867\,B\,a^4}{120}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{9}+\left(\frac{5632\,A\,a^4}{35}+\frac{896\,B\,a^4}{5}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{7}+\left(\frac{1501\,A\,a^4}{10}+\frac{19157\,B\,a^4}{120}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{5}+\left(70\,A\,a^4+\frac{523\,B\,a^4}{6}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{3}+\left(\frac{53\,A\,a^4}{2}+\frac{207\,B\,a^4}{8}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{14}+7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{12}+21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{8}+35\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{6}+21\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{4}+7\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{2}+1\right)}+\frac{a^4\,\mathrm{atan}\left(\frac{a^4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(44\,A+49\,B\right)}{8\,\left(\frac{11\,A\,a^4}{2}+\frac{49\,B\,a^4}{8}\right)}\right)\,\left(44\,A+49\,B\right)}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^7*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((53*A*a^4)/2 + (207*B*a^4)/8) + tan(c/2 + (d*x)/2)^13*
((11*A*a^4)/2 + (49*B*a^4)/8) + tan(c/2 + (d*x)/2)^11*((110*A*a^4)/3 + (245
*B*a^4)/6) + tan(c/2 + (d*x)/2)^3*(70*A*a^4 + (523*B*a^4)/6) + tan(c/2 + (d
*x)/2)^7*((5632*A*a^4)/35 + (896*B*a^4)/5) + tan(c/2 + (d*x)/2)^9*((3113*A*
a^4)/30 + (13867*B*a^4)/120) + tan(c/2 + (d*x)/2)^5*((1501*A*a^4)/10 + (191
57*B*a^4)/120))/(d*(7*tan(c/2 + (d*x)/2)^2 + 21*tan(c/2 + (d*x)/2)^4 + 35*t
an(c/2 + (d*x)/2)^6 + 35*tan(c/2 + (d*x)/2)^8 + 21*tan(c/2 + (d*x)/2)^10 +
7*tan(c/2 + (d*x)/2)^12 + tan(c/2 + (d*x)/2)^14 + 1)) + (a^4*atan((a^4*tan(
c/2 + (d*x)/2)*(44*A + 49*B))/(8*((11*A*a^4)/2 + (49*B*a^4)/8)))*(44*A + 49
*B))/(8*d)
```

3.82 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=131

$$\frac{3(A-B)\tanh^{-1}(\sin(c+dx))}{2ad}-\frac{(3A-4B)\tan(c+dx)}{ad}+\frac{3(A-B)\sec(c+dx)\tan(c+dx)}{2ad}+\frac{(A-B)\sec^3(c}{d(a+a\,\mathrm{s}}$$

```
[Out] 3/2*(A-B)*arctanh(sin(d*x+c))/a/d-(3*A-4*B)*tan(d*x+c)/a/d+3/2*(A-B)*sec(d*
x+c)*tan(d*x+c)/a/d+(A-B)*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))-1/3*(3
*A-4*B)*tan(d*x+c)^3/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4104, 3872, 3853, 3855, 3852}

$$-\frac{(3A-4B)\tan^3(c+dx)}{3ad}-\frac{(3A-4B)\tan(c+dx)}{ad}+\frac{3(A-B)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(A-B)\tan(c+dx)\sec^3(c+dx)}{d(a\sec(c+dx)+a)}+\frac{3(A-B)\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*(A - B)*ArcTanh[Sin[c + d*x]])/(2*a*d) - ((3*A - 4*B)*Tan[c + d*x])/(a*d
) + (3*(A - B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) + ((A - B)*Sec[c + d*x]^3
*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])) - ((3*A - 4*B)*Tan[c + d*x]^3)/(3*a
*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx = \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sec^3(c+dx)(3a(A-B)-}{a}$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))} - \frac{(3A-4B)\int \sec^4(c+dx)\,dx}{a}$$

$$= \frac{3(A-B)\sec(c+dx)\tan(c+dx)}{2ad} + \frac{(A-B)\sec^3(c+dx)\tan(c-}{d(a+a\sec(c+dx))}$$

$$= \frac{3(A-B)\tanh^{-1}(\sin(c+dx))}{2ad} - \frac{(3A-4B)\tan(c+dx)}{ad} + \frac{3(A-}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 489 vs. 2(131) = 262.
time = 6.05, size = 489, normalized size = 3.73

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(-144*(A - B)*Cos[(c + d*x)/2]*(Log[Cos[(c + d*x)/2] - Si
n[(c + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Sec[c/2]*Sec[
c]*Sec[c + d*x]^3*(6*(A + B)*Sin[(d*x)/2] + (-27*A + 39*B)*Sin[(3*d*x)/2] +
 12*A*Sin[c - (d*x)/2] - 24*B*Sin[c - (d*x)/2] + 6*A*Sin[c + (d*x)/2] - 6*B
*Sin[c + (d*x)/2] + 24*A*Sin[2*c + (d*x)/2] - 24*B*Sin[2*c + (d*x)/2] - 9*A
*Sin[c + (3*d*x)/2] + 21*B*Sin[c + (3*d*x)/2] - 9*A*Sin[2*c + (3*d*x)/2] +
```

```
9*B*Sin[2*c + (3*d*x)/2] + 9*A*Sin[3*c + (3*d*x)/2] - 9*B*Sin[3*c + (3*d*x)
/2] - 3*A*Sin[c + (5*d*x)/2] + 7*B*Sin[c + (5*d*x)/2] + 3*A*Sin[2*c + (5*d*
x)/2] + B*Sin[2*c + (5*d*x)/2] + 3*A*Sin[3*c + (5*d*x)/2] - 3*B*Sin[3*c + (
5*d*x)/2] + 9*A*Sin[4*c + (5*d*x)/2] - 9*B*Sin[4*c + (5*d*x)/2] - 12*A*Sin[
2*c + (7*d*x)/2] + 16*B*Sin[2*c + (7*d*x)/2] - 6*A*Sin[3*c + (7*d*x)/2] + 1
0*B*Sin[3*c + (7*d*x)/2] - 6*A*Sin[4*c + (7*d*x)/2] + 6*B*Sin[4*c + (7*d*x)
/2])))/(48*a*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 0.23, size = 190, normalized size = 1.45

method	result
derivativedivides	$\frac{-A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{2B-A}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\left(\frac{3B}{2}-\frac{3A}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{\frac{5B}{2}-\frac{3A}{2}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}}{da}$
default	$\frac{-A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{2B-A}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\left(\frac{3B}{2}-\frac{3A}{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{\frac{5B}{2}-\frac{3A}{2}}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right.}}{da}$
norman	$\frac{\frac{(-9B+7A)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{2(A-2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{(A-B)\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{(27A-37B)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}-\frac{(-49B+39A)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}}$
risch	$-\frac{i\left(9A\,e^{6i(dx+c)}-9B\,e^{6i(dx+c)}+9A\,e^{5i(dx+c)}-9B\,e^{5i(dx+c)}+24A\,e^{4i(dx+c)}-24B\,e^{4i(dx+c)}+12A\,e^{3i(dx+c)}-24B\,e^{3i(dx+c)}\right.}{3ad\left(e^{2i(dx+c)}+1\right)^{3}\left(e^{i(dx+c)}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c)-1/3*B/(tan(1/2*d*x+1/2*c)
-1)^3-1/2*(2*B-A)/(tan(1/2*d*x+1/2*c)-1)^2+(3/2*B-3/2*A)*ln(tan(1/2*d*x+1/2
*c)-1)-(5/2*B-3/2*A)/(tan(1/2*d*x+1/2*c)-1)-1/3*B/(tan(1/2*d*x+1/2*c)+1)^3-
1/2*(A-2*B)/(tan(1/2*d*x+1/2*c)+1)^2-(5/2*B-3/2*A)/(tan(1/2*d*x+1/2*c)+1)+(
-3/2*B+3/2*A)*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 368 vs. 2(125) = 250.
time = 0.26, size = 368, normalized size = 2.81

$$\frac{B\left(\frac{2\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{16\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{15\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{a-\frac{3a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}-\frac{a\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}-\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{9\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{6\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-3A\left(\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a-\frac{2a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/6*(B*(2*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 16*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a - 3*a*sin(d*x + c)^
2/(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a*sin(d*
```

```
x + c)^6/(cos(d*x + c) + 1)^6) - 9*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)
/a + 9*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 6*sin(d*x + c)/(a*(cos(
d*x + c) + 1))) - 3*A*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^
3/(cos(d*x + c) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1)
+ 1)/a + 3*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(
cos(d*x + c) + 1))))/d
```

Fricas [**A**]
time = 2.58, size = 170, normalized size = 1.30

$$\frac{9\left((A-B)\cos\left(dx+c\right)^{4}+(A-B)\cos\left(dx+c\right)^{3}\right)\log\left(\sin\left(dx+c\right)+1\right)-9\left((A-B)\cos\left(dx+c\right)^{4}+(A-B)\cos\left(dx+c\right)^{3}\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(4\left(3A-4B\right)\cos\left(dx+c\right)^{3}+\left(3A-7B\right)\cos\left(dx+c\right)^{2}-\left(3A-B\right)\cos\left(dx+c\right)-2B\right)\sin\left(dx+c\right)}{12\left(ad\cos\left(dx+c\right)^{4}+ad\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(9*((A - B)*cos(d*x + c)^4 + (A - B)*cos(d*x + c)^3)*log(sin(d*x + c)
+ 1) - 9*((A - B)*cos(d*x + c)^4 + (A - B)*cos(d*x + c)^3)*log(-sin(d*x + c
) + 1) - 2*(4*(3*A - 4*B)*cos(d*x + c)^3 + (3*A - 7*B)*cos(d*x + c)^2 - (3*
A - B)*cos(d*x + c) - 2*B)*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x
+ c)^3)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^{4}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx+\int \frac{B\sec^{5}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**4/(sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x
)**5/(sec(c + d*x) + 1), x))/a
```

Giac [**A**]
time = 0.49, size = 182, normalized size = 1.39

$$\frac{\frac{9(A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{9(A-B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{6\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}+\frac{2\left(9A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-15B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-12A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+16B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+3A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-9B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{3}a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(9*(A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - 9*(A - B)*log(abs(tan
(1/2*d*x + 1/2*c) - 1))/a - 6*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2
```

```
*c))/a + 2*(9*A*tan(1/2*d*x + 1/2*c)^5 - 15*B*tan(1/2*d*x + 1/2*c)^5 - 12*A
*tan(1/2*d*x + 1/2*c)^3 + 16*B*tan(1/2*d*x + 1/2*c)^3 + 3*A*tan(1/2*d*x + 1
/2*c) - 9*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*a))/d
```

Mupad [B]
time = 2.44, size = 152, normalized size = 1.16

$$\frac{3\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(A-B\right)}{a\,d}-\frac{\left(3\,A-5\,B\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{16\,B}{3}-4\,A\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(A-3\,B\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(-a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-3\,a\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A-B\right)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + a/cos(c + d*x))),x)
```

```
[Out] (3*atanh(tan(c/2 + (d*x)/2))*(A - B))/(a*d) - (tan(c/2 + (d*x)/2)^5*(3*A -
5*B) - tan(c/2 + (d*x)/2)^3*(4*A - (16*B)/3) + tan(c/2 + (d*x)/2)*(A - 3*B)
)/(d*(a - 3*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c/2 + (d*x)/2)^4 - a*tan(c/2 +
 (d*x)/2)^6)) - (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

3.83 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=108

$$-\frac{(2A-3B)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{2(A-B)\tan(c+dx)}{ad}-\frac{(2A-3B)\sec(c+dx)\tan(c+dx)}{2ad}+\frac{(A-B)\,\mathrm{se}}{d(a}$$

```
[Out] -1/2*(2*A-3*B)*arctanh(sin(d*x+c))/a/d+2*(A-B)*tan(d*x+c)/a/d-1/2*(2*A-3*B)
*sec(d*x+c)*tan(d*x+c)/a/d+(A-B)*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4104, 3872, 3852, 8, 3853, 3855}

$$\frac{2(A-B)\tan(c+dx)}{ad}-\frac{(2A-3B)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{d(a\sec(c+dx)+a)}-\frac{(2A-3B)\tan(c+dx)\sec(c+dx)}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] -1/2*((2*A - 3*B)*ArcTanh[Sin[c + d*x]])/(a*d) + (2*(A - B)*Tan[c + d*x])/(
a*d) - ((2*A - 3*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a*d) + ((A - B)*Sec[c + d
*x]^2*Tan[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx &= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))}+\frac{\int \sec^2(c+dx)(2a(A-B)-a\cdots}{a^2}\\ &= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{d(a+a\sec(c+dx))}-\frac{(2A-3B)\int \sec^3(c+dx)\,dx}{a}\cdots\\ &= -\frac{(2A-3B)\sec(c+dx)\tan(c+dx)}{2ad}+\frac{(A-B)\sec^2(c+dx)\tan(c\cdots}{d(a+a\sec(c+dx))\cdots}\\ &= -\frac{(2A-3B)\tanh^{-1}(\sin(c+dx))}{2ad}+\frac{2(A-B)\tan(c+dx)}{ad}-\frac{(2A\cdots}{\cdots}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 311 vs. 2(108) = 216.

time = 3.78, size = 311, normalized size = 2.88

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)(A+B\sec(c+dx))\left(4(A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left((4A-6B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-4A\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+6B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{B}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}-\frac{B}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4(A-B)\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)\right)}{2ad(B+A\cos(c+dx))(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(A + B*Sec[c + d*x])*(4*(A - B)*Sec[c/2]*Sin[(d*x)/2] + C
os[(c + d*x)/2]*((4*A - 6*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 4*A
*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 6*B*Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] + B/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 - B/(Cos[(c + d*x
)/2] + Sin[(c + d*x)/2])^2 + (4*(A - B)*Sin[d*x])/((Cos[c/2] - Sin[c/2])*(C
```

```
os[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*x)/2]
 + Sin[(c + d*x)/2])))))/(2*a*d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.21, size = 142, normalized size = 1.31

method	result
derivativedivides	$\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{-\frac{3B}{2}+A}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\left(-\frac{3B}{2}+A\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{B}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{da}$
default	$\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{B}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}-\frac{-\frac{3B}{2}+A}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\left(-\frac{3B}{2}+A\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{B}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{da}$
norman	$\frac{\frac{(A-B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{7(A-B)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{(3A-2B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{(5A-6B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}+\frac{(2A-3B)\ln\left(\tan\left(\right.\right.}{2ad}$
risch	$\frac{i\left(2A\,e^{4i(dx+c)}-3B\,e^{4i(dx+c)}+2A\,e^{3i(dx+c)}-3B\,e^{3i(dx+c)}+6A\,e^{2i(dx+c)}-5B\,e^{2i(dx+c)}+2\,e^{i(dx+c)}A-B\,e^{i(dx+c)}+4A-\right.}{da\left(e^{i(dx+c)}+1\right)\left(e^{2i(dx+c)}+1\right)^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(A*tan(1/2*d*x+1/2*c)-B*tan(1/2*d*x+1/2*c)+1/2*B/(tan(1/2*d*x+1/2*c)-
1)^2-(-3/2*B+A)/(tan(1/2*d*x+1/2*c)-1)+(-3/2*B+A)*ln(tan(1/2*d*x+1/2*c)-1)-
1/2*B/(tan(1/2*d*x+1/2*c)+1)^2+(3/2*B-A)*ln(tan(1/2*d*x+1/2*c)+1)-(-3/2*B+A
)/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 282 vs. 2(104) = 208.
time = 0.29, size = 282, normalized size = 2.61

$$-\frac{B\left(\frac{2\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-\frac{3\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a-\frac{2a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}-\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}+\frac{3\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}+\frac{2\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+2A\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/2*(B*(2*(sin(d*x + c)/(cos(d*x + c) + 1) - 3*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3)/(a - 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4) - 3*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a + 3*log
(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a + 2*sin(d*x + c)/(a*(cos(d*x + c) +
 1))) + 2*A*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/
(cos(d*x + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x
+ c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A]
time = 2.45, size = 156, normalized size = 1.44

$$-\frac{\left((2A-3B)\cos(dx+c)^{3}+(2A-3B)\cos(dx+c)^{2}\right)\log(\sin(dx+c)+1)-\left((2A-3B)\cos(dx+c)^{3}+(2A-3B)\cos(dx+c)^{2}\right)\log(-\sin(dx+c)+1)-2\left(4(A-B)\cos(dx+c)^{2}+(2A-B)\cos(dx+c)+B\right)\sin(dx+c)}{4\left(ad\cos(dx+c)^{3}+ad\cos(dx+c)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/4*(((2*A - 3*B)*cos(d*x + c)^3 + (2*A - 3*B)*cos(d*x + c)^2)*log(sin(d*x
 + c) + 1) - ((2*A - 3*B)*cos(d*x + c)^3 + (2*A - 3*B)*cos(d*x + c)^2)*log(
-sin(d*x + c) + 1) - 2*(4*(A - B)*cos(d*x + c)^2 + (2*A - B)*cos(d*x + c) +
 B)*sin(d*x + c))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^3(c+dx)}{\sec(c+dx)+1}\,dx + \int \frac{B\sec^4(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x
)**4/(sec(c + d*x) + 1), x))/a
```

Giac [A]
time = 0.48, size = 156, normalized size = 1.44

$$-\frac{\frac{(2A-3B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{(2A-3B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}-\frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a}+\frac{2\left(2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/2*((2*A - 3*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (2*A - 3*B)*log(ab
s(tan(1/2*d*x + 1/2*c) - 1))/a - 2*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x
+ 1/2*c))/a + 2*(2*A*tan(1/2*d*x + 1/2*c)^3 - 3*B*tan(1/2*d*x + 1/2*c)^3 -
2*A*tan(1/2*d*x + 1/2*c) + B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2
 - 1)^2*a))/d
```

Mupad [B]
time = 2.11, size = 119, normalized size = 1.10

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A-B)}{a\,d}-\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(A-\frac{3\,B}{2}\right)}{a\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,(2\,A-3\,B)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,A-B)}{d\,\left(a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,a\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+a\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + a/cos(c + d*x))),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(A - B))/(a*d) - (2*atanh(tan(c/2 + (d*x)/2))*(A - (3*B
)/2))/(a*d) - (tan(c/2 + (d*x)/2)^3*(2*A - 3*B) - tan(c/2 + (d*x)/2)*(2*A -
 B))/(d*(a - 2*a*tan(c/2 + (d*x)/2)^2 + a*tan(c/2 + (d*x)/2)^4))
```

3.84 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=62

$$\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad}+\frac{B\tan(c+dx)}{ad}-\frac{(A-B)\tan(c+dx)}{d(a+a\sec(c+dx))}$$

```
[Out] (A-B)*arctanh(sin(d*x+c))/a/d+B*tan(d*x+c)/a/d-(A-B)*tan(d*x+c)/d/(a+a*sec(
d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4093, 3872, 3855, 3852, 8}

$$\frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B)\tan(c+dx)}{d(a\sec(c+dx)+a)}+\frac{B\tan(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((A - B)*ArcTanh[Sin[c + d*x]])/(a*d) + (B*Tan[c + d*x])/(a*d) - ((A - B)*T
an[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx &= -\frac{(A-B)\tan(c+dx)}{d(a+a\sec(c+dx))}-\frac{\int \sec(c+dx)(-a(A-B)-aB\sec(c+
\\ &= -\frac{(A-B)\tan(c+dx)}{d(a+a\sec(c+dx))}+\frac{(A-B)\int\sec(c+dx)\,dx}{a}+\frac{B\int\sec^2(c+
\\ &= \frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad}-\frac{(A-B)\tan(c+dx)}{d(a+a\sec(c+dx))}-\frac{B\text{Subst}(\int
\\ &= \frac{(A-B)\tanh^{-1}(\sin(c+dx))}{ad}+\frac{B\tan(c+dx)}{ad}-\frac{(A-B)\tan(c+
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 224 vs. $2(62) = 124$.
time = 1.32, size = 224, normalized size = 3.61

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)(A+B\sec(c+dx))\left((-A+B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left(-\left((A-B)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)\right)+\frac{B\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)\right)}{ad(B+A\cos(c+dx))(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(A + B*Sec[c + d*x])*((-A + B)*Sec[c/2]*Sin[(d*x)/2] +
Cos[(c + d*x)/2]*(-((A - B)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])) + (B*Sin[d*x])/((Cos[c/2] - Sin[c/2
])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d*
x)/2] + Sin[(c + d*x)/2])))))/(a*d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x]))
```

Maple [A]
time = 0.17, size = 100, normalized size = 1.61

method	result
derivativedivides	$\frac{-A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+(A-B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+(-A+B)\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{da}$
default	$\frac{-A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1}+(A-B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)-\frac{B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+(-A+B)\ln\left(\tan\left(\frac{dx}{2}+\right.\right.}{da}$

norman	$\frac{-\frac{(A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{2(A-2B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{(A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}+\frac{(A-B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}-\frac{(A-B)\ln(}{}$
risch	$-\frac{2i(A\,\mathrm{e}^{2i(dx+c)}-B\,\mathrm{e}^{2i(dx+c)}-B\,\mathrm{e}^{i(dx+c)}+A-2B)}{da(\mathrm{e}^{2i(dx+c)}+1)(\mathrm{e}^{i(dx+c)}+1)}-\frac{\ln(\mathrm{e}^{i(dx+c)}-i)A}{ad}+\frac{\ln(\mathrm{e}^{i(dx+c)}-i)B}{ad}+\frac{\ln(\mathrm{e}^{i(dx+c)}+i)A}{ad}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c)-B/(tan(1/2*d*x+1/2*c)+1)+
(A-B)*ln(tan(1/2*d*x+1/2*c)+1)-B/(tan(1/2*d*x+1/2*c)-1)+(-A+B)*ln(tan(1/2*d
*x+1/2*c)-1))
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 196 vs. $2(62) = 124$.
time = 0.28, size = 196, normalized size = 3.16

$$-\frac{B\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{2\sin(dx+c)}{\left(a-\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-A\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -(B*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a - 2*sin(d*x + c)/((a - a*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))) - A*(log(si
n(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x + c) + 1)
- 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 127 vs. $2(62) = 124$.
time = 4.72, size = 127, normalized size = 2.05

$$\frac{\left((A-B)\cos(dx+c)^2+(A-B)\cos(dx+c)\right)\log(\sin(dx+c)+1)-\left((A-B)\cos(dx+c)^2+(A-B)\cos(dx+c)\right)\log(-\sin(dx+c)+1)-2\left((A-2B)\cos(dx+c)-B\right)\sin(dx+c)}{2\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*(((A - B)*cos(d*x + c)^2 + (A - B)*cos(d*x + c))*log(sin(d*x + c) + 1)
- ((A - B)*cos(d*x + c)^2 + (A - B)*cos(d*x + c))*log(-sin(d*x + c) + 1) -
2*((A - 2*B)*cos(d*x + c) - B)*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(
d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^2(c+dx)}{\sec(c+dx)+1}\,dx + \int \frac{B\sec^3(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x
)**3/(sec(c + d*x) + 1), x))/a
```

Giac [A]
time = 0.49, size = 109, normalized size = 1.76

$$\frac{\frac{(A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a} - \frac{(A-B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a} - \frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a} - \frac{2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] ((A - B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - (A - B)*log(abs(tan(1/2*d*x
 + 1/2*c) - 1))/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a - 2
*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 - 1)*a))/d
```

Mupad [B]
time = 2.03, size = 79, normalized size = 1.27

$$\frac{2\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a-a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\right)} + \frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(A-B)}{a\,d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A-B)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + a/cos(c + d*x))),x)
```

```
[Out] (2*B*tan(c/2 + (d*x)/2))/(d*(a - a*tan(c/2 + (d*x)/2)^2)) + (2*atanh(tan(c/
2 + (d*x)/2))*(A - B))/(a*d) - (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

3.85 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=43

$$\frac{B\tanh^{-1}(\sin(c+dx))}{ad}+\frac{(A-B)\tan(c+dx)}{d(a+a\sec(c+dx))}$$

```
[Out] B*arctanh(sin(d*x+c))/a/d+(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 43, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4083, 3855, 3879}

$$\frac{(A-B)\tan(c+dx)}{d(a\sec(c+dx)+a)}+\frac{B\tanh^{-1}(\sin(c+dx))}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(a*d) + ((A - B)*Tan[c + d*x])/(d*(a + a*Sec[c +
d*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx &= (A-B)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx+\frac{B\int \sec(c+dx)\,dx}{a}\\ &= \frac{B\tanh^{-1}(\sin(c+dx))}{ad}+\frac{(A-B)\tan(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 109 vs. 2(43) = 86.
time = 0.27, size = 109, normalized size = 2.53

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(B\cos\left(\frac{1}{2}(c+dx)\right)\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+(A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)\right)}{ad(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (2*Cos[(c + d*x)/2]*(B*Cos[(c + d*x)/2]*(-Log[Cos[(c + d*x)/2] - Sin[(c + d
*x)/2]] + Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (A - B)*Sec[c/2]*Sin[
(d*x)/2]))/(a*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 0.18, size = 61, normalized size = 1.42

method	result	size
derivativedivides	$\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	61
default	$\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{da}$	61
risch	$\frac{2iA}{da\left(e^{i(dx+c)}+1\right)}-\frac{2iB}{da\left(e^{i(dx+c)}+1\right)}+\frac{\ln\left(e^{i(dx+c)}+i\right)B}{ad}-\frac{\ln\left(e^{i(dx+c)}-i\right)B}{ad}$	91
norman	$\frac{\frac{(A-B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}}{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{ad}-\frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{ad}$	105

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(A*tan(1/2*d*x+1/2*c)-B*tan(1/2*d*x+1/2*c)-B*ln(tan(1/2*d*x+1/2*c)-1)
+B*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 99 vs. 2(43) = 86.
time = 0.31, size = 99, normalized size = 2.30

$$\frac{B\left(\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a}-\frac{\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+\frac{A\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxima
")
```

```
[Out] (B*(log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a - log(sin(d*x + c)/(cos(d*x
+ c) + 1) - 1)/a - sin(d*x + c)/(a*(cos(d*x + c) + 1))) + A*sin(d*x + c)/(a
*(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 2.74, size = 74, normalized size = 1.72

$$\frac{(B\cos(dx+c)+B)\log(\sin(dx+c)+1)-(B\cos(dx+c)+B)\log(-\sin(dx+c)+1)+2(A-B)\sin(dx+c)}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/2*((B*cos(d*x + c) + B)*log(sin(d*x + c) + 1) - (B*cos(d*x + c) + B)*log(
-sin(d*x + c) + 1) + 2*(A - B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{A\sec(c+dx)}{\sec(c+dx)+1}\,dx+\int\frac{B\sec^2(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**
2/(sec(c + d*x) + 1), x))/a
```

Giac [A]
time = 0.47, size = 70, normalized size = 1.63

$$\frac{\frac{B\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a}+\frac{A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a - B*log(abs(tan(1/2*d*x + 1/2*c) -
1))/a + (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a)/d
```

Mupad [B]
time = 1.93, size = 41, normalized size = 0.95

$$\frac{2B\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A-B)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + a/cos(c + d*x))),x)
```

```
[Out] (2*B*atanh(tan(c/2 + (d*x)/2)))/(a*d) + (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

3.86 $\int \frac{A+B\sec(c+dx)}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=35

$$\frac{Ax}{a}-\frac{(A-B)\tan(c+dx)}{d(a+a\sec(c+dx))}$$

```
[Out] A*x/a-(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))
```

Rubi [A]
time = 0.04, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.087, Rules used = {4004, 3879}

$$\frac{Ax}{a}-\frac{(A-B)\tan(c+dx)}{d(a\sec(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x]),x]
```

```
[Out] (A*x)/a - ((A - B)*Tan[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sec(c+dx)}{a+a\sec(c+dx)}\,dx &= \frac{Ax}{a}-(A-B)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx\\ &= \frac{Ax}{a}-\frac{(A-B)\tan(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 72 vs. 2(35) = 70.

time = 0.15, size = 72, normalized size = 2.06

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(Adx\cos\left(\frac{dx}{2}\right)+Adx\cos\left(c+\frac{dx}{2}\right)+2(-A+B)\sin\left(\frac{dx}{2}\right)\right)}{ad(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(A*d*x*Cos[(d*x)/2] + A*d*x*Cos[c + (d*x)/2] + 2
*(-A + B)*Sin[(d*x)/2]))/(a*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 0.20, size = 45, normalized size = 1.29

method	result	size
norman	$\frac{Ax}{a}-\frac{(A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}$	30
derivativedivides	$\frac{-A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2A\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	45
default	$\frac{-A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+2A\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	45
risch	$\frac{Ax}{a}-\frac{2iA}{da\left(e^{i(dx+c)}+1\right)}+\frac{2iB}{da\left(e^{i(dx+c)}+1\right)}$	54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c)+2*A*arctan(tan(1/2*d*x+1/
2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 73 vs. $2(35) = 70$.
time = 0.49, size = 73, normalized size = 2.09

$$\frac{A\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+\frac{B\sin(dx+c)}{a(\cos(dx+c)+1)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] (A*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(d*x + c)/(a*(cos(d*x
+ c) + 1))) + B*sin(d*x + c)/(a*(cos(d*x + c) + 1)))/d
```

Fricas [A]
time = 2.75, size = 44, normalized size = 1.26

$$\frac{Adx\cos(dx+c)+Adx-(A-B)\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] (A*d*x*cos(d*x + c) + A*d*x - (A - B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec(c+dx)+1}\,dx + \int \frac{B\sec(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A/(sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)/(sec(c + d*x)
+ 1), x))/a
```

Giac [A]
time = 0.46, size = 44, normalized size = 1.26

$$\frac{\frac{(dx+c)A}{a} - \frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*A/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a)/d
```

Mupad [B]
time = 1.90, size = 32, normalized size = 0.91

$$-\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A-B)}{a} - \frac{A\,d\,x}{a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x)),x)
```

```
[Out] -((tan(c/2 + (d*x)/2)*(A - B))/a - (A*d*x)/a)/d
```

3.87 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=60

$$-\frac{(A-B)x}{a}+\frac{(2A-B)\sin(c+dx)}{ad}-\frac{(A-B)\sin(c+dx)}{d(a+a\sec(c+dx))}$$

```
[Out] -(A-B)*x/a+(2*A-B)*sin(d*x+c)/a/d-(A-B)*sin(d*x+c)/d/(a+a*sec(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4105, 3872, 2717, 8}

$$\frac{(2A-B)\sin(c+dx)}{ad}-\frac{(A-B)\sin(c+dx)}{d(a\sec(c+dx)+a)}-\frac{x(A-B)}{a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(((A - B)*x)/a) + ((2*A - B)*Sin[c + d*x])/(a*d) - ((A - B)*Sin[c + d*x])/
(d*(a + a*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
```

```
0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx &= -\frac{(A-B)\sin(c+dx)}{d(a+a\sec(c+dx))}+\frac{\int \cos(c+dx)(a(2A-B)-a(A-B)\sec(\\ &= -\frac{(A-B)\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{(A-B)\int 1\,dx}{a}+\frac{(2A-B)\int \cos(c+dx)}{a}\\ &= -\frac{(A-B)x}{a}+\frac{(2A-B)\sin(c+dx)}{ad}-\frac{(A-B)\sin(c+dx)}{d(a+a\sec(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.38, size = 76, normalized size = 1.27

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left((A-B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos\left(\frac{1}{2}(c+dx)\right)\left((-A+B)dx+A\sin(c+dx)\right)\right)}{ad(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
[Out] (2*Cos[(c + d*x)/2]*((A - B)*Sec[c/2]*Sin[(d*x)/2] + Cos[(c + d*x)/2]*((-A
+ B)*d*x + A*Sin[c + d*x])))/(a*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 0.29, size = 76, normalized size = 1.27

method	result	size
derivativedivides	$\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-2(A-B)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	76
default	$\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-2(A-B)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$	76
norman	$\frac{\frac{(A-B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{(3A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}-\frac{(A-B)x}{a}-\frac{(A-B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}$	97
risch	$-\frac{Ax}{a}+\frac{xB}{a}-\frac{iA\,e^{i(dx+c)}}{2ad}+\frac{iA\,e^{-i(dx+c)}}{2ad}+\frac{2iA}{da\left(e^{i(dx+c)}+1\right)}-\frac{2iB}{da\left(e^{i(dx+c)}+1\right)}$	99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d/a*(A*tan(1/2*d*x+1/2*c)-B*tan(1/2*d*x+1/2*c)+2*A*tan(1/2*d*x+1/2*c)/(1+
tan(1/2*d*x+1/2*c)^2)-2*(A-B)*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 143 vs. $2(60) = 120$.

time = 0.48, size = 143, normalized size = 2.38

$$-\frac{A\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-B\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxima
")
```

```
[Out] -(A*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - 2*sin(d*x + c)/((a + a*s
in(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) - sin(d*x + c)/(a*(
cos(d*x + c) + 1))) - B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a - sin(
d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A]

time = 2.54, size = 63, normalized size = 1.05

$$-\frac{(A-B)dx\cos(dx+c)+(A-B)dx-(A\cos(dx+c)+2A-B)\sin(dx+c)}{ad\cos(dx+c)+ad}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fricas
")
```

```
[Out] -((A - B)*d*x*cos(d*x + c) + (A - B)*d*x - (A*cos(d*x + c) + 2*A - B)*sin(d
*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{A\cos(c+dx)}{\sec(c+dx)+1}\,dx+\int\frac{B\cos(c+dx)\sec(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*cos(c + d*x)/(sec(c + d*x) + 1), x) + Integral(B*cos(c + d*x)*s
ec(c + d*x)/(sec(c + d*x) + 1), x))/a
```

Giac [A]

time = 0.47, size = 79, normalized size = 1.32

$$-\frac{\frac{(dx+c)(A-B)}{a}-\frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a}-\frac{2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -((d*x + c)*(A - B)/a - (A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1/2*c))/a
 - 2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c)^2 + 1)*a))/d
```

Mupad [B]
time = 2.00, size = 65, normalized size = 1.08

$$\frac{2\,A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a\right)}-\frac{x\,(A-B)}{a}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A-B)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x)),x)
```

```
[Out] (2*A*tan(c/2 + (d*x)/2))/(d*(a + a*tan(c/2 + (d*x)/2)^2)) - (x*(A - B))/a +
 (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

3.88 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=98

$$\frac{(3A-2B)x}{2a}-\frac{2(A-B)\sin(c+dx)}{ad}+\frac{(3A-2B)\cos(c+dx)\sin(c+dx)}{2ad}-\frac{(A-B)\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}$$

```
[Out] 1/2*(3*A-2*B)*x/a-2*(A-B)*sin(d*x+c)/a/d+1/2*(3*A-2*B)*cos(d*x+c)*sin(d*x+c
)/a/d-(A-B)*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4105, 3872, 2715, 8, 2717}

$$-\frac{2(A-B)\sin(c+dx)}{ad}+\frac{(3A-2B)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{(A-B)\sin(c+dx)\cos(c+dx)}{d(a\sec(c+dx)+a)}+\frac{x(3A-2B)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((3*A - 2*B)*x)/(2*a) - (2*(A - B)*Sin[c + d*x])/(a*d) + ((3*A - 2*B)*Cos[c
 + d*x]*Sin[c + d*x])/(2*a*d) - ((A - B)*Cos[c + d*x]*Sin[c + d*x])/(d*(a +
 a*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx &= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}+\frac{\int \cos^2(c+dx)(a(3A-2B)-}{a^2}\\ &= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}+\frac{(3A-2B)\int \cos^2(c+dx)\,dx}{a}\\ &= -\frac{2(A-B)\sin(c+dx)}{ad}+\frac{(3A-2B)\cos(c+dx)\sin(c+dx)}{2ad}-\frac{(A}{}\\ &= \frac{(3A-2B)x}{2a}-\frac{2(A-B)\sin(c+dx)}{ad}+\frac{(3A-2B)\cos(c+dx)\sin(}{2ad}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 197 vs. 2(98) = 196.
time = 0.45, size = 197, normalized size = 2.01

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(4(3A-2B)dx\cos\left(\frac{dx}{2}\right)+4(3A-2B)dx\cos\left(c+\frac{dx}{2}\right)-20A\sin\left(\frac{dx}{2}\right)+20B\sin\left(\frac{dx}{2}\right)-4A\sin\left(c+\frac{dx}{2}\right)+4B\sin\left(c+\frac{dx}{2}\right)-3A\sin\left(c+\frac{3dx}{2}\right)+4B\sin\left(c+\frac{3dx}{2}\right)-3A\sin\left(2c+\frac{3dx}{2}\right)+4B\sin\left(2c+\frac{3dx}{2}\right)+A\sin\left(2c+\frac{5dx}{2}\right)+A\sin\left(3c+\frac{5dx}{2}\right)\right)}{8ad(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(4*(3*A - 2*B)*d*x*Cos[(d*x)/2] + 4*(3*A - 2*B)*
d*x*Cos[c + (d*x)/2] - 20*A*Sin[(d*x)/2] + 20*B*Sin[(d*x)/2] - 4*A*Sin[c +
(d*x)/2] + 4*B*Sin[c + (d*x)/2] - 3*A*Sin[c + (3*d*x)/2] + 4*B*Sin[c + (3*d
*x)/2] - 3*A*Sin[2*c + (3*d*x)/2] + 4*B*Sin[2*c + (3*d*x)/2] + A*Sin[2*c +
(5*d*x)/2] + A*Sin[3*c + (5*d*x)/2]))/(8*a*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 0.31, size = 100, normalized size = 1.02

method	result

derivativedivides	$\frac{-A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2\left(-\frac{3A}{2}+B\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(-\frac{A}{2}+B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+(3A-2B)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
default	$\frac{-A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{2\left(-\frac{3A}{2}+B\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\left(-\frac{A}{2}+B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+(3A-2B)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{da}$
norman	$\frac{\frac{(3A-2B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{(3A-2B)x}{2a}-\frac{(A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{(2A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{ad}+\frac{(3A-2B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{(5A-4}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$
risch	$\frac{3Ax}{2a}-\frac{xB}{a}+\frac{iA\,\mathrm{e}^{i(dx+c)}}{2ad}-\frac{i\mathrm{e}^{i(dx+c)}B}{2ad}-\frac{iA\,\mathrm{e}^{-i(dx+c)}}{2ad}+\frac{i\mathrm{e}^{-i(dx+c)}B}{2ad}-\frac{2iA}{da\left(\mathrm{e}^{i(dx+c)}+1\right)}+\frac{2iB}{da\left(\mathrm{e}^{i(dx+c)}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(-A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c)+2*((-3/2*A+B)*tan(1/2*d*x
+1/2*c)^3+(-1/2*A+B)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+(3*A-2*
B)*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 225 vs. $2(94) = 188$.
time = 0.50, size = 225, normalized size = 2.30

$$-\frac{A\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)+B\left(\frac{2\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}-\frac{2\sin(dx+c)}{\left(a+\frac{a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}-\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -(A*((sin(d*x + c)/(cos(d*x + c) + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a + 2*a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*
x + c) + 1)^4) - 3*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)
/(a*(cos(d*x + c) + 1))) + B*(2*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a -
 2*sin(d*x + c)/((a + a*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c)
+ 1)) - sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A]
time = 3.14, size = 81, normalized size = 0.83

$$\frac{(3A-2B)dx\cos(dx+c)+(3A-2B)dx+\left(A\cos(dx+c)^2-(A-2B)\cos(dx+c)-4A+4B\right)\sin(dx+c)}{2\left(ad\cos(dx+c)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*((3*A - 2*B)*d*x*cos(d*x + c) + (3*A - 2*B)*d*x + (A*cos(d*x + c)^2 - (
A - 2*B)*cos(d*x + c) - 4*A + 4*B)*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\cos^2(c+dx)}{\sec(c+dx)+1}\,dx + \int \frac{B\cos^2(c+dx)\sec(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*cos(c + d*x)**2/(sec(c + d*x) + 1), x) + Integral(B*cos(c + d*x
)**2*sec(c + d*x)/(sec(c + d*x) + 1), x))/a
```

Giac [A]
time = 0.46, size = 123, normalized size = 1.26

$$\frac{\frac{(dx+c)(3A-2B)}{a} - \frac{2\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a} - \frac{2\left(3A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^2 a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((d*x + c)*(3*A - 2*B)/a - 2*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x +
1/2*c))/a - 2*(3*A*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(1/2*d*x + 1/2*c)^3 + A*
tan(1/2*d*x + 1/2*c) - 2*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 +
 1)^2*a))/d
```

Mupad [B]
time = 2.21, size = 107, normalized size = 1.09

$$\frac{x\,(3\,A-2\,B)}{2\,a} - \frac{(3\,A-2\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + (A-2\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 + 2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + a\right)} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A-B)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x)),x)
```

```
[Out] (x*(3*A - 2*B))/(2*a) - (tan(c/2 + (d*x)/2)^3*(3*A - 2*B) + tan(c/2 + (d*x)
/2)*(A - 2*B))/(d*(a + 2*a*tan(c/2 + (d*x)/2)^2 + a*tan(c/2 + (d*x)/2)^4))
- (tan(c/2 + (d*x)/2)*(A - B))/(a*d)
```

3.89 $\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=122

$$-\frac{3(A-B)x}{2a}+\frac{(4A-3B)\sin(c+dx)}{ad}-\frac{3(A-B)\cos(c+dx)\sin(c+dx)}{2ad}-\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))}$$

```
[Out] -3/2*(A-B)*x/a+(4*A-3*B)*sin(d*x+c)/a/d-3/2*(A-B)*cos(d*x+c)*sin(d*x+c)/a/d
-(A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))-1/3*(4*A-3*B)*sin(d*x+c)^
3/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 122, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4105, 3872, 2713, 2715, 8}

$$-\frac{(4A-3B)\sin^3(c+dx)}{3ad}+\frac{(4A-3B)\sin(c+dx)}{ad}-\frac{3(A-B)\sin(c+dx)\cos(c+dx)}{2ad}-\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{d(a\sec(c+dx)+a)}-\frac{3x(A-B)}{2a}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-3*(A - B)*x)/(2*a) + ((4*A - 3*B)*Sin[c + d*x])/(a*d) - (3*(A - B)*Cos[c
+ d*x]*Sin[c + d*x])/(2*a*d) - ((A - B)*Cos[c + d*x]^2*Sin[c + d*x])/(d*(a
+ a*Sec[c + d*x])) - ((4*A - 3*B)*Sin[c + d*x]^3)/(3*a*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx = -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \cos^3(c+dx)(a(4A-3B)\ \cdots}{a\ \cdots}$$
$$= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{(4A-3B)\int \cos^3(c+dx)\,dx\ \cdots}{a}$$
$$= -\frac{3(A-B)\cos(c+dx)\sin(c+dx)}{2ad} - \frac{(A-B)\cos^2(c+dx)\sin(c\ \cdots}{d(a+a\sec(c+dx))}$$
$$= -\frac{3(A-B)x}{2a} + \frac{(4A-3B)\sin(c+dx)}{ad} - \frac{3(A-B)\cos(c+dx)\sin\ \cdots}{2ad}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 249 vs. 2(122) = 244.
time = 0.68, size = 249, normalized size = 2.04

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(-36(A-B)dx\cos\left(\frac{dx}{2}\right)-36(A-B)dx\cos\left(c+\frac{dx}{2}\right)+69A\sin\left(\frac{dx}{2}\right)-60B\sin\left(\frac{dx}{2}\right)+21A\sin\left(c+\frac{dx}{2}\right)-12B\sin\left(c+\frac{dx}{2}\right)+18A\sin\left(c+\frac{3dx}{2}\right)-9B\sin\left(c+\frac{3dx}{2}\right)+18A\sin\left(2c+\frac{3dx}{2}\right)-9B\sin\left(2c+\frac{3dx}{2}\right)-2A\sin\left(2c+\frac{5dx}{2}\right)+3B\sin\left(2c+\frac{5dx}{2}\right)-2A\sin\left(3c+\frac{5dx}{2}\right)+3B\sin\left(3c+\frac{5dx}{2}\right)+A\sin\left(3c+\frac{7dx}{2}\right)+A\sin\left(4c+\frac{7dx}{2}\right)\right)}{24ad(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-36*(A - B)*d*x*Cos[(d*x)/2] - 36*(A - B)*d*x*C
os[c + (d*x)/2] + 69*A*Sin[(d*x)/2] - 60*B*Sin[(d*x)/2] + 21*A*Sin[c + (d*x
)/2] - 12*B*Sin[c + (d*x)/2] + 18*A*Sin[c + (3*d*x)/2] - 9*B*Sin[c + (3*d*x
)/2] + 18*A*Sin[2*c + (3*d*x)/2] - 9*B*Sin[2*c + (3*d*x)/2] - 2*A*Sin[2*c +
 (5*d*x)/2] + 3*B*Sin[2*c + (5*d*x)/2] - 2*A*Sin[3*c + (5*d*x)/2] + 3*B*Sin
[3*c + (5*d*x)/2] + A*Sin[3*c + (7*d*x)/2] + A*Sin[4*c + (7*d*x)/2]))/(24*a
*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 0.32, size = 122, normalized size = 1.00

method	result
derivativedivides	$\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2\left(\left(\frac{3B}{2}-\frac{5A}{2}\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{8A}{3}+2B\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{B}{2}-\frac{3A}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-3(A}{da}$
default	$\frac{A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2\left(\left(\frac{3B}{2}-\frac{5A}{2}\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{8A}{3}+2B\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{B}{2}-\frac{3A}{2}\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}-3(A}{da}$
risch	$-\frac{3Ax}{2a}+\frac{3xB}{2a}-\frac{7iA\,e^{i(dx+c)}}{8ad}+\frac{ie^{i(dx+c)}B}{2ad}+\frac{7iA\,e^{-i(dx+c)}}{8ad}-\frac{ie^{-i(dx+c)}B}{2ad}+\frac{2iA}{da\left(e^{i(dx+c)}+1\right)}-\frac{2iB}{da\left(e^{i(dx+c)}\right.}$
norman	$\frac{\frac{(A-B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{3(A-B)x}{2a}-\frac{9(A-B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{9(A-B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{3(A-B)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{2(2A-}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d/a*(A*tan(1/2*d*x+1/2*c)-B*tan(1/2*d*x+1/2*c)-2*((3/2*B-5/2*A)*tan(1/2*d
*x+1/2*c)^5+(-8/3*A+2*B)*tan(1/2*d*x+1/2*c)^3+(1/2*B-3/2*A)*tan(1/2*d*x+1/2
*c))/(1+tan(1/2*d*x+1/2*c)^2)^3-3*(A-B)*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 310 vs. $2(116) = 232$.

time = 0.49, size = 310, normalized size = 2.54

$$\frac{A\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{16\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a+\frac{3a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}-\frac{9\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{3\sin(dx+c)}{a(\cos(dx+c)+1)}\right)-3B\left(\frac{\frac{\sin(dx+c)}{\cos(dx+c)+1}+\frac{3\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a+\frac{2a\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}-\frac{3\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a}+\frac{\sin(dx+c)}{a(\cos(dx+c)+1)}\right)}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/3*(A*((9*sin(d*x + c)/(cos(d*x + c) + 1) + 16*sin(d*x + c)^3/(cos(d*x + c
) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a + 3*a*sin(d*x + c)^2/
(cos(d*x + c) + 1)^2 + 3*a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a*sin(d*x
+ c)^6/(cos(d*x + c) + 1)^6) - 9*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a
+ 3*sin(d*x + c)/(a*(cos(d*x + c) + 1))) - 3*B*((sin(d*x + c)/(cos(d*x + c)
 + 1) + 3*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a + 2*a*sin(d*x + c)^2/(cos
(d*x + c) + 1)^2 + a*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) - 3*arctan(sin(d*
x + c)/(cos(d*x + c) + 1))/a + sin(d*x + c)/(a*(cos(d*x + c) + 1))))/d
```

Fricas [A]

time = 4.93, size = 97, normalized size = 0.80

$$-\frac{9(A-B)dx\cos(dx+c)+9(A-B)dx-\left(2A\cos(dx+c)^3-(A-3B)\cos(dx+c)^2+(7A-3B)\cos(dx+c)+16A-12B\right)\sin(dx+c)}{6(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] -1/6*(9*(A - B)*d*x*cos(d*x + c) + 9*(A - B)*d*x - (2*A*cos(d*x + c)^3 - (A
 - 3*B)*cos(d*x + c)^2 + (7*A - 3*B)*cos(d*x + c) + 16*A - 12*B)*sin(d*x +
c))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\cos^3(c+dx)}{\sec(c+dx)+1}\,dx + \int \frac{B\cos^3(c+dx)\sec(c+dx)}{\sec(c+dx)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*cos(c + d*x)**3/(sec(c + d*x) + 1), x) + Integral(B*cos(c + d*x
)**3*sec(c + d*x)/(sec(c + d*x) + 1), x))/a
```

Giac [A]
time = 0.45, size = 151, normalized size = 1.24

$$-\frac{\frac{9(dx+c)(A-B)}{a} - \frac{6\left(A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{a} - \frac{2\left(15A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-9B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+16A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-12B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+9A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+1\right)^3 a}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/6*(9*(d*x + c)*(A - B)/a - 6*(A*tan(1/2*d*x + 1/2*c) - B*tan(1/2*d*x + 1
/2*c))/a - 2*(15*A*tan(1/2*d*x + 1/2*c)^5 - 9*B*tan(1/2*d*x + 1/2*c)^5 + 16
*A*tan(1/2*d*x + 1/2*c)^3 - 12*B*tan(1/2*d*x + 1/2*c)^3 + 9*A*tan(1/2*d*x +
 1/2*c) - 3*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a))/d
```

Mupad [B]
time = 3.02, size = 138, normalized size = 1.13

$$\frac{(5A-3B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(\frac{16A}{3}-4B\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(3A-B)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3a\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a\right)} - \frac{3x(A-B)}{2a} + \frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)(A-B)}{a\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x)),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(5*A - 3*B) + tan(c/2 + (d*x)/2)^3*((16*A)/3 - 4*B) +
 tan(c/2 + (d*x)/2)*(3*A - B))/(d*(a + 3*a*tan(c/2 + (d*x)/2)^2 + 3*a*tan(c
/2 + (d*x)/2)^4 + a*tan(c/2 + (d*x)/2)^6)) - (3*x*(A - B))/(2*a) + (tan(c/2
 + (d*x)/2)*(A - B))/(a*d)
```

3.90 $\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=179

$$\frac{(7A-10B)\tanh^{-1}(\sin(c+dx))}{2a^2d}-\frac{4(2A-3B)\tan(c+dx)}{a^2d}+\frac{(7A-10B)\sec(c+dx)\tan(c+dx)}{2a^2d}+\frac{(7A-1$$

```
[Out] 1/2*(7*A-10*B)*arctanh(sin(d*x+c))/a^2/d-4*(2*A-3*B)*tan(d*x+c)/a^2/d+1/2*(
7*A-10*B)*sec(d*x+c)*tan(d*x+c)/a^2/d+1/3*(7*A-10*B)*sec(d*x+c)^3*tan(d*x+c
)/a^2/d/(1+sec(d*x+c))+1/3*(A-B)*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))
^2-4/3*(2*A-3*B)*tan(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.23, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.161$, Rules used = {4104, 3872, 3853, 3855, 3852}

$$-\frac{4(2A-3B)\tan^3(c+dx)}{3a^2d}-\frac{4(2A-3B)\tan(c+dx)}{a^2d}+\frac{(7A-10B)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{(7A-10B)\tan(c+dx)\sec^3(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{(7A-10B)\tan(c+dx)\sec(c+dx)}{2a^2d}+\frac{(A-B)\tan(c+dx)\sec^4(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((7*A - 10*B)*ArcTanh[Sin[c + d*x]])/(2*a^2*d) - (4*(2*A - 3*B)*Tan[c + d*x
])/(a^2*d) + ((7*A - 10*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) + ((7*A - 1
0*B)*Sec[c + d*x]^3*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) + ((A - B)*S
ec[c + d*x]^4*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) - (4*(2*A - 3*B)*T
an[c + d*x]^3)/(3*a^2*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec^4(c+dx)(4a(A-B)-3a(A-2B)\sec}{a+a\sec(c+dx)}}{3a^2}$$

$$= \frac{(7A-10B)\sec^3(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^4(c+dx)\tan(}{3d(a+a\sec(c+dx))}$$

$$= \frac{(7A-10B)\sec^3(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^4(c+dx)\tan(}{3d(a+a\sec(c+dx))}$$

$$= \frac{(7A-10B)\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{(7A-10B)\sec^3(c+dx)\tan}{3a^2d(1+\sec(c+dx}$$

$$= \frac{(7A-10B)\tanh^{-1}(\sin(c+dx))}{2a^2d} - \frac{4(2A-3B)\tan(c+dx)}{a^2d} + \frac{(7A}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 764 vs. 2(179) = 358.
time = 6.37, size = 764, normalized size = 4.27

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*(-7*A + 10*B)*Cos[c/2 + (d*x)/2]^4*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d
*x)/2]]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(d*(B + A*Cos[c + d*x])*(a + a*S
```

```
ec[c + d*x])^2) - (2*(-7*A + 10*B)*Cos[c/2 + (d*x)/2]^4*Log[Cos[c/2 + (d*x)
/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(d*(B + A*Cos[
c + d*x])*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c]*Sec
[c + d*x]^4*(A + B*Sec[c + d*x])*(45*A*Sin[(d*x)/2] - 6*B*Sin[(d*x)/2] - 20
1*A*Sin[(3*d*x)/2] + 310*B*Sin[(3*d*x)/2] + 195*A*Sin[c - (d*x)/2] - 306*B*
Sin[c - (d*x)/2] - 51*A*Sin[c + (d*x)/2] + 42*B*Sin[c + (d*x)/2] + 189*A*Si
n[2*c + (d*x)/2] - 270*B*Sin[2*c + (d*x)/2] - A*Sin[c + (3*d*x)/2] + 50*B*S
in[c + (3*d*x)/2] - 81*A*Sin[2*c + (3*d*x)/2] + 90*B*Sin[2*c + (3*d*x)/2] +
 119*A*Sin[3*c + (3*d*x)/2] - 170*B*Sin[3*c + (3*d*x)/2] - 129*A*Sin[c + (5
*d*x)/2] + 198*B*Sin[c + (5*d*x)/2] - 9*A*Sin[2*c + (5*d*x)/2] + 42*B*Sin[2
*c + (5*d*x)/2] - 57*A*Sin[3*c + (5*d*x)/2] + 66*B*Sin[3*c + (5*d*x)/2] + 6
3*A*Sin[4*c + (5*d*x)/2] - 90*B*Sin[4*c + (5*d*x)/2] - 75*A*Sin[2*c + (7*d*
x)/2] + 114*B*Sin[2*c + (7*d*x)/2] - 15*A*Sin[3*c + (7*d*x)/2] + 36*B*Sin[3
*c + (7*d*x)/2] - 39*A*Sin[4*c + (7*d*x)/2] + 48*B*Sin[4*c + (7*d*x)/2] + 2
1*A*Sin[5*c + (7*d*x)/2] - 30*B*Sin[5*c + (7*d*x)/2] - 32*A*Sin[3*c + (9*d*
x)/2] + 48*B*Sin[3*c + (9*d*x)/2] - 12*A*Sin[4*c + (9*d*x)/2] + 22*B*Sin[4*
c + (9*d*x)/2] - 20*A*Sin[5*c + (9*d*x)/2] + 26*B*Sin[5*c + (9*d*x)/2]))/(9
6*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [A]
time = 0.26, size = 222, normalized size = 1.24

method	result
derivativedivides	$\frac{-\frac{A\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{B\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+9B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{6B-2A}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+(10B-7A)\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{}$
default	$\frac{-\frac{A\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{B\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+9B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{6B-2A}{2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+(10B-7A)\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{}$
norman	$\frac{-\frac{(A-B)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{(8A-11B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}+\frac{(13A-21B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{5(25A-37B)\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{2(77A-115B}{}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{5}a}$
risch	$-\frac{i\left(21A\,\mathrm{e}^{8i(dx+c)}-30B\,\mathrm{e}^{8i(dx+c)}+63A\,\mathrm{e}^{7i(dx+c)}-90B\,\mathrm{e}^{7i(dx+c)}+119A\,\mathrm{e}^{6i(dx+c)}-170B\,\mathrm{e}^{6i(dx+c)}+189A\,\mathrm{e}^{5i(dx+c)}-2\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/d/a^2*(-1/3*A*tan(1/2*d*x+1/2*c)^3+1/3*B*tan(1/2*d*x+1/2*c)^3-7*A*tan(1
/2*d*x+1/2*c)+9*B*tan(1/2*d*x+1/2*c)-1/2*(6*B-2*A)/(tan(1/2*d*x+1/2*c)-1)^2
+(10*B-7*A)*ln(tan(1/2*d*x+1/2*c)-1)-(10*B-5*A)/(tan(1/2*d*x+1/2*c)-1)-2/3*
B/(tan(1/2*d*x+1/2*c)-1)^3+(7*A-10*B)*ln(tan(1/2*d*x+1/2*c)+1)-1/2*(-6*B+2*
A)/(tan(1/2*d*x+1/2*c)+1)^2-(10*B-5*A)/(tan(1/2*d*x+1/2*c)+1)-2/3*B/(tan(1/
2*d*x+1/2*c)+1)^3)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 425 vs. $2(169) = 338$.

time = 0.29, size = 425, normalized size = 2.37

$$\frac{B\left(\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{15\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^2-\frac{3a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}-\frac{a^2\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{30\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-A\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(B*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 - 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 - a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1
) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 30*log(sin(d*x + c)/(cos(d*x
 + c) + 1) + 1)/a^2 + 30*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) - A*
(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x + c)^4/(
cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3
/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^
2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2))/d
```

Fricas [A]

time = 2.73, size = 245, normalized size = 1.37

$$\frac{3\left((7A-10B)\cos(dx+c)^5+2(7A-10B)\cos(dx+c)^4+(7A-10B)\cos(dx+c)^3\right)\log(\sin(dx+c)+1)-3\left((7A-10B)\cos(dx+c)^5+2(7A-10B)\cos(dx+c)^4+(7A-10B)\cos(dx+c)^3\right)\log(-\sin(dx+c)+1)-2\left(16(2A-3B)\cos(dx+c)^4+(43A-66B)\cos(dx+c)^3+6(A-2B)\cos(dx+c)^2-(3A-2B)\cos(dx+c)-2B\right)\sin(dx+c)}{12\left(a^2d\cos(dx+c)^5+2a^2d\cos(dx+c)^4+a^2d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/12*(3*((7*A - 10*B)*cos(d*x + c)^5 + 2*(7*A - 10*B)*cos(d*x + c)^4 + (7*A
 - 10*B)*cos(d*x + c)^3)*log(sin(d*x + c) + 1) - 3*((7*A - 10*B)*cos(d*x +
c)^5 + 2*(7*A - 10*B)*cos(d*x + c)^4 + (7*A - 10*B)*cos(d*x + c)^3)*log(-si
n(d*x + c) + 1) - 2*(16*(2*A - 3*B)*cos(d*x + c)^4 + (43*A - 66*B)*cos(d*x
+ c)^3 + 6*(A - 2*B)*cos(d*x + c)^2 - (3*A - 2*B)*cos(d*x + c) - 2*B)*sin(d
*x + c))/(a^2*d*cos(d*x + c)^5 + 2*a^2*d*cos(d*x + c)^4 + a^2*d*cos(d*x + c
)^3)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx+\int \frac{B\sec^6(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + In
tegral(B*sec(c + d*x)**6/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A]
time = 0.51, size = 226, normalized size = 1.26

$$\frac{\frac{3\,(7\,A-10\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{3\,(7\,A-10\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+\frac{2\left(15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-30\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-24\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-18\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3a^2}-\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+21\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-27\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(7*A - 10*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(7*A - 10*B)
*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 2*(15*A*tan(1/2*d*x + 1/2*c)^5 -
30*B*tan(1/2*d*x + 1/2*c)^5 - 24*A*tan(1/2*d*x + 1/2*c)^3 + 40*B*tan(1/2*d*
x + 1/2*c)^3 + 9*A*tan(1/2*d*x + 1/2*c) - 18*B*tan(1/2*d*x + 1/2*c))/((tan(
1/2*d*x + 1/2*c)^2 - 1)^3*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(
1/2*d*x + 1/2*c)^3 + 21*A*a^4*tan(1/2*d*x + 1/2*c) - 27*B*a^4*tan(1/2*d*x +
 1/2*c))/a^6)/d
```

Mupad [B]
time = 1.98, size = 202, normalized size = 1.13

$$\frac{(5\,A-10\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{40\,B}{3}-8\,A\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+(3\,A-6\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{2\,(A-B)}{a^2}+\frac{3\,A-5\,B}{2\,a^2}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(A-B)}{6\,a^2\,d}+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(7\,A-10\,B)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^5*(a + a/cos(c + d*x))^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(5*A - 10*B) - tan(c/2 + (d*x)/2)^3*(8*A - (40*B)/3)
+ tan(c/2 + (d*x)/2)*(3*A - 6*B))/(d*(3*a^2*tan(c/2 + (d*x)/2)^2 - 3*a^2*ta
n(c/2 + (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 - a^2)) - (tan(c/2 + (d*x)/2)
*((2*(A - B))/a^2 + (3*A - 5*B)/(2*a^2)))/d - (tan(c/2 + (d*x)/2)^3*(A - B)
)/(6*a^2*d) + (atanh(tan(c/2 + (d*x)/2))*(7*A - 10*B))/(a^2*d)
```

3.91 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=156

$$-\frac{(4A-7B)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{2(5A-8B)\tan(c+dx)}{3a^2d}-\frac{(4A-7B)\sec(c+dx)\tan(c+dx)}{2a^2d}+\frac{(5A-8B}{3a}$$

```
[Out] -1/2*(4*A-7*B)*arctanh(sin(d*x+c))/a^2/d+2/3*(5*A-8*B)*tan(d*x+c)/a^2/d-1/2
*(4*A-7*B)*sec(d*x+c)*tan(d*x+c)/a^2/d+1/3*(5*A-8*B)*sec(d*x+c)^2*tan(d*x+c
)/a^2/d/(1+sec(d*x+c))+1/3*(A-B)*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))
^2
```

Rubi [A]
time = 0.22, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4104, 3872, 3852, 8, 3853, 3855}

$$\frac{2(5A-8B)\tan(c+dx)}{3a^2d}-\frac{(4A-7B)\tanh^{-1}(\sin(c+dx))}{2a^2d}+\frac{(5A-8B)\tan(c+dx)\sec^2(c+dx)}{3a^2d(\sec(c+dx)+1)}-\frac{(4A-7B)\tan(c+dx)\sec(c+dx)}{2a^2d}+\frac{(A-B)\tan(c+dx)\sec^3(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/2*((4*A - 7*B)*ArcTanh[Sin[c + d*x]])/(a^2*d) + (2*(5*A - 8*B)*Tan[c + d
*x])/(3*a^2*d) - ((4*A - 7*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^2*d) + ((5*A
- 8*B)*Sec[c + d*x]^2*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) + ((A - B)
*Sec[c + d*x]^3*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec^3(c+dx)(3a(A-B)-a(2A-5B)\,\mathrm{se}}{a+a\sec(c+dx)}}{3a^2}$$

$$= \frac{(5A-8B)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^3(c+dx)\tan(}{3d(a+a\sec(c+dx)}$$

$$= \frac{(5A-8B)\sec^2(c+dx)\tan(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^3(c+dx)\tan(}{3d(a+a\sec(c+dx)}$$

$$= -\frac{(4A-7B)\sec(c+dx)\tan(c+dx)}{2a^2d} + \frac{(5A-8B)\sec^2(c+dx)\,\mathrm{ta}}{3a^2d(1+\sec(c+d}$$

$$= -\frac{(4A-7B)\tanh^{-1}(\sin(c+dx))}{2a^2d} + \frac{2(5A-8B)\tan(c+dx)}{3a^2d} - \frac{(}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 496 vs. 2(156) = 312.
time = 4.08, size = 496, normalized size = 3.18

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (96*(4*A - 7*B)*Cos[(c + d*x)/2]^4*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*S
ec[c]*Sec[c + d*x]^2*(-14*(A - B)*Sin[(d*x)/2] + (64*A - 97*B)*Sin[(3*d*x)/
2] - 84*A*Sin[c - (d*x)/2] + 126*B*Sin[c - (d*x)/2] + 42*A*Sin[c + (d*x)/2]
 - 42*B*Sin[c + (d*x)/2] - 56*A*Sin[2*c + (d*x)/2] + 98*B*Sin[2*c + (d*x)/2
] - 6*A*Sin[c + (3*d*x)/2] + 3*B*Sin[c + (3*d*x)/2] + 34*A*Sin[2*c + (3*d*x
)/2] - 37*B*Sin[2*c + (3*d*x)/2] - 36*A*Sin[3*c + (3*d*x)/2] + 63*B*Sin[3*c
 + (3*d*x)/2] + 48*A*Sin[c + (5*d*x)/2] - 75*B*Sin[c + (5*d*x)/2] + 6*A*Sin
[2*c + (5*d*x)/2] - 15*B*Sin[2*c + (5*d*x)/2] + 30*A*Sin[3*c + (5*d*x)/2] -
 39*B*Sin[3*c + (5*d*x)/2] - 12*A*Sin[4*c + (5*d*x)/2] + 21*B*Sin[4*c + (5*
d*x)/2] + 20*A*Sin[2*c + (7*d*x)/2] - 32*B*Sin[2*c + (7*d*x)/2] + 6*A*Sin[3
*c + (7*d*x)/2] - 12*B*Sin[3*c + (7*d*x)/2] + 14*A*Sin[4*c + (7*d*x)/2] - 2
0*B*Sin[4*c + (7*d*x)/2]))/(48*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A]
time = 0.23, size = 177, normalized size = 1.13

method	result
derivativedivides	$\frac{\frac{A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+5A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-7B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2A-5B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+(4A-7B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d\,a^2}$
default	$\frac{\frac{A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+5A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-7B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{2A-5B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+(4A-7B)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d\,a^2}$
norman	$\frac{\frac{(A-B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}+\frac{(9A-13B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{(11A-18B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{(11A-17B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}+\frac{(61A-100B)\left(\tan^5\right.}{3ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4 a}$
risch	$\frac{i\left(12A\,e^{6i(dx+c)}-21B\,e^{6i(dx+c)}+36A\,e^{5i(dx+c)}-63B\,e^{5i(dx+c)}+56A\,e^{4i(dx+c)}-98B\,e^{4i(dx+c)}+84A\,e^{3i(dx+c)}-126B\,e^{3i(\right.}{3d\,a^2\left(e^{i(dx+c)}+1\right)^3\left(e^{2i(dx+c)}+1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/d/a^2*(1/3*A*tan(1/2*d*x+1/2*c)^3-1/3*B*tan(1/2*d*x+1/2*c)^3+5*A*tan(1/
2*d*x+1/2*c)-7*B*tan(1/2*d*x+1/2*c)-(2*A-5*B)/(tan(1/2*d*x+1/2*c)-1)+(4*A-7
*B)*ln(tan(1/2*d*x+1/2*c)-1)+B/(tan(1/2*d*x+1/2*c)-1)^2-(2*A-5*B)/(tan(1/2*
d*x+1/2*c)+1)+(7*B-4*A)*ln(tan(1/2*d*x+1/2*c)+1)-B/(tan(1/2*d*x+1/2*c)+1)^2
)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. $2(146) = 292$.
time = 0.28, size = 336, normalized size = 2.15

$$-\frac{B\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{5\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2-\frac{2a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^2\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{21\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-A\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(B*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) - 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 - 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 21*log(sin(d*x + c)/(cos(d*x + c) + 1)
 + 1)/a^2 + 21*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^2) - A*((15*sin(d
*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 12*
log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(d*x + c)/(cos(d*x
 + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x + c)^2/(cos(d*x +
 c) + 1)^2)*(cos(d*x + c) + 1))))/d
```

Fricas [**A**]
time = 3.05, size = 228, normalized size = 1.46

$$-\frac{3\left((4A-7B)\cos(dx+c)^4+2(4A-7B)\cos(dx+c)^3+(4A-7B)\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-3\left((4A-7B)\cos(dx+c)^4+2(4A-7B)\cos(dx+c)^3+(4A-7B)\cos(dx+c)^2\right)\log(-\sin(dx+c)+1)-2\left(4(5A-8B)\cos(dx+c)^3+(28A-43B)\cos(dx+c)^2+6(A-B)\cos(dx+c)+3B\right)\sin(dx+c)}{12\left(a^2d\cos(dx+c)^4+2a^2d\cos(dx+c)^3+a^2d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/12*(3*((4*A - 7*B)*cos(d*x + c)^4 + 2*(4*A - 7*B)*cos(d*x + c)^3 + (4*A
- 7*B)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 3*((4*A - 7*B)*cos(d*x + c)^
4 + 2*(4*A - 7*B)*cos(d*x + c)^3 + (4*A - 7*B)*cos(d*x + c)^2)*log(-sin(d*x
 + c) + 1) - 2*(4*(5*A - 8*B)*cos(d*x + c)^3 + (28*A - 43*B)*cos(d*x + c)^2
 + 6*(A - B)*cos(d*x + c) + 3*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^4 + 2*a^
2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^4(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx+\int \frac{B\sec^5(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + In
tegral(B*sec(c + d*x)**5/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [**A**]
time = 0.52, size = 198, normalized size = 1.27

$$-\frac{\frac{3(4A-7B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^2}-\frac{3(4A-7B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^2}+\frac{6\left(2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2A\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^2a^2}-\frac{Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+15Aa^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-21Ba^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^6}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(3*(4*A - 7*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 3*(4*A - 7*B)*
log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^2 + 6*(2*A*tan(1/2*d*x + 1/2*c)^3 - 5*
B*tan(1/2*d*x + 1/2*c)^3 - 2*A*tan(1/2*d*x + 1/2*c) + 3*B*tan(1/2*d*x + 1/2
*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 -
 B*a^4*tan(1/2*d*x + 1/2*c)^3 + 15*A*a^4*tan(1/2*d*x + 1/2*c) - 21*B*a^4*ta
n(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B]
time = 1.93, size = 166, normalized size = 1.06

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,\left(A-B\right)}{2\,a^2}+\frac{2\,A-4\,B}{2\,a^2}\right)}{d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(2\,A-5\,B\right)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,A-3\,B\right)}{d\,\left(a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+a^2\right)}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(A-B\right)}{6\,a^2\,d}-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(4\,A-7\,B\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + a/cos(c + d*x))^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((3*(A - B))/(2*a^2) + (2*A - 4*B)/(2*a^2)))/d - (tan(c
/2 + (d*x)/2)^3*(2*A - 5*B) - tan(c/2 + (d*x)/2)*(2*A - 3*B))/(d*(a^2*tan(c
/2 + (d*x)/2)^4 - 2*a^2*tan(c/2 + (d*x)/2)^2 + a^2)) + (tan(c/2 + (d*x)/2)^
3*(A - B))/(6*a^2*d) - (atanh(tan(c/2 + (d*x)/2))*(4*A - 7*B))/(a^2*d)
```

3.92 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=108

$$\frac{(A-2B)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(A-4B)\tan(c+dx)}{3a^2d}-\frac{(A-2B)\tan(c+dx)}{a^2d(1+\sec(c+dx))}+\frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}$$

```
[Out] (A-2*B)*arctanh(sin(d*x+c))/a^2/d-1/3*(A-4*B)*tan(d*x+c)/a^2/d-(A-2*B)*tan(
d*x+c)/a^2/d/(1+sec(d*x+c))+1/3*(A-B)*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*
x+c))^2
```

Rubi [A]
time = 0.18, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4104, 4093, 3872, 3855, 3852, 8}

$$-\frac{(A-4B)\tan(c+dx)}{3a^2d}+\frac{(A-2B)\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(A-2B)\tan(c+dx)}{a^2d(\sec(c+dx)+1)}+\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((A - 2*B)*ArcTanh[Sin[c + d*x]])/(a^2*d) - ((A - 4*B)*Tan[c + d*x])/(3*a^2
*d) - ((A - 2*B)*Tan[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) + ((A - B)*Sec[c
+ d*x]^2*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx &= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec^2(c+dx)(2a(A-B)-a(A-4B)\sec(c}{a+a\sec(c+dx)}}{3a^2} \\
&= -\frac{(A-2B)\tan(c+dx)}{a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \int s \\
&= -\frac{(A-2B)\tan(c+dx)}{a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - (A \\
&= \frac{(A-2B)\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(A-2B)\tan(c+dx)}{a^2d(1+\sec(c+dx))} + \frac{(A-B)}{3d} \\
&= \frac{(A-2B)\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(A-4B)\tan(c+dx)}{3a^2d} - \frac{(A-2B}{a^2d(1+}
\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 292 vs. 2(108) = 216.
time = 1.93, size = 292, normalized size = 2.70

$$\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(A+B\sec(c+dx))\left((-A+B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)-2(4A-7B)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)+\cos^3\left(\frac{1}{2}(c+dx)\right)\left(-6(A-2B)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\frac{6B\sin(dx)}{\left(\cos\left(\frac{c}{2}\right)-\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{c}{2}\right)+\sin\left(\frac{c}{2}\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}\right)-(A-B)\cos\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)\right)}{3a^2d(B+A\cos(c+dx))(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (2*Cos[(c + d*x)/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((-A + B)*Sec[c/2]*Si
n[(d*x)/2] - 2*(4*A - 7*B)*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] + Cos[(
c + d*x)/2]^3*(-6*(A - 2*B)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log
[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (6*B*Sin[d*x])/((Cos[c/2] - Sin[c/
2])*(Cos[c/2] + Sin[c/2])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])*(Cos[(c + d
*x)/2] + Sin[(c + d*x)/2]))) - (A - B)*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d
*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x])^2)
```

Maple [A]
time = 0.20, size = 134, normalized size = 1.24

method	result
derivativedivides	$\frac{-\frac{A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+(4B-2A)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{2B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{2d\,a^2}$
default	$\frac{-\frac{A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+(4B-2A)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)-\frac{2B}{\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}}{2d\,a^2}$
norman	$\frac{\frac{(4A-9B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{3(A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{(A-2B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{(A-B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{(13A-34B)\left(\tan^3\left(\frac{dx}{2}+\right.\right.}{3ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3 a}$
risch	$-\frac{2i\left(3A\,e^{4i(dx+c)}-6B\,e^{4i(dx+c)}+9A\,e^{3i(dx+c)}-18B\,e^{3i(dx+c)}+7A\,e^{2i(dx+c)}-22B\,e^{2i(dx+c)}+9\,e^{i(dx+c)}A-24B\,e^{i(dx+\right.}{3d\,a^2\left(e^{i(dx+c)}+1\right)^3\left(e^{2i(dx+c)}+1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/d/a^2*(-1/3*A*tan(1/2*d*x+1/2*c)^3+1/3*B*tan(1/2*d*x+1/2*c)^3-3*A*tan(1
/2*d*x+1/2*c)+5*B*tan(1/2*d*x+1/2*c)+(4*B-2*A)*ln(tan(1/2*d*x+1/2*c)-1)-2*B
/(tan(1/2*d*x+1/2*c)-1)+(-4*B+2*A)*ln(tan(1/2*d*x+1/2*c)+1)-2*B/(tan(1/2*d*
x+1/2*c)+1))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 244 vs. $2(104) = 208$.
time = 0.28, size = 244, normalized size = 2.26

$$\frac{B\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{12\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2-\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)-A\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(B*((15*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 12*log(sin(
d*x + c)/(cos(d*x + c) + 1) - 1)/a^2 + 12*sin(d*x + c)/((a^2 - a^2*sin(d*x
```

```
+ c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) - A*((9*sin(d*x + c)/(cos
(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 6*log(sin(d*x +
 c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1
)/a^2))/d
```

Fricas [A]
time = 6.81, size = 195, normalized size = 1.81

$$\frac{3\left((A-2B)\cos\left(dx+c\right)^{3}+2\left(A-2B\right)\cos\left(dx+c\right)^{2}+\left(A-2B\right)\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left((A-2B)\cos\left(dx+c\right)^{3}+2\left(A-2B\right)\cos\left(dx+c\right)^{2}+\left(A-2B\right)\cos\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(2\left(2A-5B\right)\cos\left(dx+c\right)^{2}+\left(5A-14B\right)\cos\left(dx+c\right)-3B\right)\sin\left(dx+c\right)}{6\left(a^{2}d\cos\left(dx+c\right)^{3}+2a^{2}d\cos\left(dx+c\right)^{2}+a^{2}d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*((A - 2*B)*cos(d*x + c)^3 + 2*(A - 2*B)*cos(d*x + c)^2 + (A - 2*B)*c
os(d*x + c))*log(sin(d*x + c) + 1) - 3*((A - 2*B)*cos(d*x + c)^3 + 2*(A - 2
*B)*cos(d*x + c)^2 + (A - 2*B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*
(2*A - 5*B)*cos(d*x + c)^2 + (5*A - 14*B)*cos(d*x + c) - 3*B)*sin(d*x + c))
/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^{3}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx+\int \frac{B\sec^{4}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + In
tegral(B*sec(c + d*x)**4/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A]
time = 0.48, size = 151, normalized size = 1.40

$$\frac{\frac{6\left(A-2B\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^{2}}-\frac{6\left(A-2B\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^{2}}-\frac{12B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)a^{2}}-\frac{Aa^{4}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-Ba^{4}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+9Aa^{4}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-15Ba^{4}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{6}}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*(A - 2*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*(A - 2*B)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1))/a^2 - 12*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x
 + 1/2*c)^2 - 1)*a^2) - (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x +
 1/2*c)^3 + 9*A*a^4*tan(1/2*d*x + 1/2*c) - 15*B*a^4*tan(1/2*d*x + 1/2*c))/a
^6)/d
```

Mupad [B]
time = 1.93, size = 120, normalized size = 1.11

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(A-2\,B\right)}{a^2\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(A-B\right)}{6\,a^2\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{A-B}{a^2}+\frac{A-3\,B}{2\,a^2}\right)}{d}-\frac{2\,B\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-a^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + a/cos(c + d*x))^2),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2))*(A - 2*B))/(a^2*d) - (tan(c/2 + (d*x)/2)^3*(A
- B))/(6*a^2*d) - (tan(c/2 + (d*x)/2)*((A - B)/a^2 + (A - 3*B)/(2*a^2)))/d
- (2*B*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)/2)^2 - a^2))
```

3.93 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=79

$$\frac{B\tanh^{-1}(\sin(c+dx))}{a^2d}+\frac{(2A-5B)\tan(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}$$

```
[Out] B*arctanh(sin(d*x+c))/a^2/d+1/3*(2*A-5*B)*tan(d*x+c)/a^2/d/(1+sec(d*x+c))-1
/3*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^2
```

Rubi [A]
time = 0.13, antiderivative size = 79, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4093, 4083, 3855, 3879}

$$\frac{(2A-5B)\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{B\tanh^{-1}(\sin(c+dx))}{a^2d}-\frac{(A-B)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(a^2*d) + ((2*A - 5*B)*Tan[c + d*x])/(3*a^2*d*(1
+ Sec[c + d*x])) - ((A - B)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
```

```
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx &= -\frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)(-2a(A-B)-3aB\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}\\ &= -\frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{(2A-5B)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a} + \frac{B\int \sec}{}\\ &= \frac{B\tanh^{-1}(\sin(c+dx))}{a^2d} - \frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{(2A-5B)\tan}{3d\left(a^2+a^2\sec\right.}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 169 vs. 2(79) = 158.
time = 0.57, size = 169, normalized size = 2.14

$$-\frac{2\cos\left(\frac{1}{2}(c+dx)\right)\left(6B\cos^3\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+(-A+B)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)-2(A-4B)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)-(A-B)\cos\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{c}{2}\right)\right)}{3a^2d(1+\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (-2*Cos[(c + d*x)/2]*(6*B*Cos[(c + d*x)/2]^3*(Log[Cos[(c + d*x)/2] - Sin[(c
 + d*x)/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (-A + B)*Sec[c/2]
*Sin[(d*x)/2] - 2*(A - 4*B)*Cos[(c + d*x)/2]^2*Sec[c/2]*Sin[(d*x)/2] - (A -
 B)*Cos[(c + d*x)/2]*Tan[c/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A]
time = 0.22, size = 91, normalized size = 1.15

method	result
derivativedivides	$\frac{\frac{A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{2d\,a^2}$
default	$\frac{\frac{A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-2B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+2B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\right.}{2d\,a^2}$
risch	$-\frac{2i\left(3B\,e^{2i(dx+c)}-3\,e^{i(dx+c)}A+9B\,e^{i(dx+c)}-A+4B\right)}{3d\,a^2\left(e^{i(dx+c)}+1\right)^3}+\frac{\ln\left(e^{i(dx+c)}+i\right)B}{a^2d}-\frac{\ln\left(e^{i(dx+c)}-i\right)B}{a^2d}$

norman	$\frac{\frac{(A-7B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}+\frac{(A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{(A-B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{(5A-17B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2 a}+\frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right.}{a^2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/d/a^2*(1/3*A*tan(1/2*d*x+1/2*c)^3-1/3*B*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*
d*x+1/2*c)-3*B*tan(1/2*d*x+1/2*c)-2*B*ln(tan(1/2*d*x+1/2*c)-1)+2*B*ln(tan(1
/2*d*x+1/2*c)+1))
```

Maxima [A]
time = 0.26, size = 145, normalized size = 1.84

$$-\frac{B\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^2}+\frac{6\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^2}\right)-\frac{A\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(B*((9*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 6*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^2 + 6*log(sin(d*
x + c)/(cos(d*x + c) + 1) - 1)/a^2) - A*(3*sin(d*x + c)/(cos(d*x + c) + 1)
+ sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A]
time = 2.39, size = 129, normalized size = 1.63

$$\frac{3\left(B\cos(dx+c)^2+2B\cos(dx+c)+B\right)\log(\sin(dx+c)+1)-3\left(B\cos(dx+c)^2+2B\cos(dx+c)+B\right)\log(-\sin(dx+c)+1)+2\left((A-4B)\cos(dx+c)+2A-5B\right)\sin(dx+c)}{6\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(B*cos(d*x + c)^2 + 2*B*cos(d*x + c) + B)*log(sin(d*x + c) + 1) - 3*
(B*cos(d*x + c)^2 + 2*B*cos(d*x + c) + B)*log(-sin(d*x + c) + 1) + 2*((A -
4*B)*cos(d*x + c) + 2*A - 5*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*
d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{A\sec^2(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx+\int\frac{B\sec^3(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + In
tegral(B*sec(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A]
time = 0.49, size = 112, normalized size = 1.42

$$\frac{\frac{6\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^2}-\frac{6\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^2}+\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-9\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^2 - 6*B*log(abs(tan(1/2*d*x +
 1/2*c) - 1))/a^2 + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1/2
*c)^3 + 3*A*a^4*tan(1/2*d*x + 1/2*c) - 9*B*a^4*tan(1/2*d*x + 1/2*c))/a^6)/d
```

Mupad [B]
time = 1.92, size = 74, normalized size = 0.94

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{A-B}{2\,a^2}-\frac{B}{a^2}\right)}{d}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(A-B\right)}{6\,a^2\,d}+\frac{2\,B\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + a/cos(c + d*x))^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((A - B)/(2*a^2) - B/a^2))/d + (tan(c/2 + (d*x)/2)^3*(A
 - B))/(6*a^2*d) + (2*B*atanh(tan(c/2 + (d*x)/2)))/(a^2*d)
```

3.94 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=65

$$\frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{(A+2B)\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}$$

```
[Out] 1/3*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^2+1/3*(A+2*B)*tan(d*x+c)/d/(a^2+a^2
*sec(d*x+c))
```

Rubi [A]
time = 0.06, antiderivative size = 65, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}=0.069$, Rules used = {4085, 3879}

$$\frac{(A+2B)\tan(c+dx)}{3d\left(a^2\sec(c+dx)+a^2\right)}+\frac{(A-B)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((A - B)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2) + ((A + 2*B)*Tan[c + d*
x])/(3*d*(a^2 + a^2*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx &= \frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{(A+2B)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a}\\ &= \frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{(A+2B)\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}\end{aligned}$$

Mathematica [A]
time = 0.22, size = 76, normalized size = 1.17

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(3(A+B)\sin\left(\frac{dx}{2}\right)-3A\sin\left(c+\frac{dx}{2}\right)+(2A+B)\sin\left(c+\frac{3dx}{2}\right)\right)}{3a^2d(1+\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(3*(A + B)*Sin[(d*x)/2] - 3*A*Sin[c + (d*x)/2] +
 (2*A + B)*Sin[c + (3*d*x)/2]))/(3*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A]
time = 0.21, size = 60, normalized size = 0.92

method	result	size
derivativedivides	$\frac{-\frac{A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}$	60
default	$\frac{-\frac{A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2d\,a^2}$	60
risch	$\frac{2i\left(3A\,e^{2i(dx+c)}+3\,e^{i(dx+c)}A+3B\,e^{i(dx+c)}+2A+B\right)}{3d\,a^2\left(e^{i(dx+c)}+1\right)^3}$	64
norman	$\frac{-\frac{(A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{(A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{(2A+B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$	89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^2*(-1/3*A*tan(1/2*d*x+1/2*c)^3+1/3*B*tan(1/2*d*x+1/2*c)^3+A*tan(1/2
*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 93, normalized size = 1.43

$$\frac{\frac{B\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}+\frac{A\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/6*(B*(3*sin(d*x + c)/(cos(d*x + c) + 1) + sin(d*x + c)^3/(cos(d*x + c) +
1)^3)/a^2 + A*(3*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3)/a^2)/d
```

Fricas [A]
time = 3.26, size = 58, normalized size = 0.89

$$\frac{((2\,A+B)\cos\left(dx+c\right)+A+2\,B)\sin\left(dx+c\right)}{3\,\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] 1/3*((2*A + B)*cos(d*x + c) + A + 2*B)*sin(d*x + c)/(a^2*d*cos(d*x + c)^2 +
 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{A\sec\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx+\int\frac{B\sec^2\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integ
ral(B*sec(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A]
time = 0.48, size = 60, normalized size = 0.92

$$-\frac{A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{6\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] -1/6*(A*tan(1/2*d*x + 1/2*c)^3 - B*tan(1/2*d*x + 1/2*c)^3 - 3*A*tan(1/2*d*x
 + 1/2*c) - 3*B*tan(1/2*d*x + 1/2*c))/(a^2*d)
```

Mupad [B]
time = 1.89, size = 45, normalized size = 0.69

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A+B\right)}{2\,a^2\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(A-B\right)}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(A + B))/(2*a^2*d) - (tan(c/2 + (d*x)/2)^3*(A - B))/(6*
a^2*d)
```

3.95 $\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=70

$$\frac{Ax}{a^2}-\frac{(4A-B)\tan(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}$$

```
[Out] A*x/a^2-1/3*(4*A-B)*tan(d*x+c)/a^2/d/(1+sec(d*x+c))-1/3*(A-B)*tan(d*x+c)/d/
(a+a*sec(d*x+c))^2
```

Rubi [A]
time = 0.08, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {4007, 4004, 3879}

$$-\frac{(4A-B)\tan(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{Ax}{a^2}-\frac{(A-B)\tan(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (A*x)/a^2 - ((4*A - B)*Tan[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A - B
)*Tan[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx &= -\frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{\int \frac{-3aA+a(A-B)\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a^2}\\ &= \frac{Ax}{a^2}-\frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{(4A-B)\int \frac{\sec(c+dx)}{a+a\sec(c+dx)}\,dx}{3a}\\ &= \frac{Ax}{a^2}-\frac{(A-B)\tan(c+dx)}{3d(a+a\sec(c+dx))^2}-\frac{(4A-B)\tan(c+dx)}{3d\left(a^2+a^2\sec(c+dx)\right)}\end{aligned}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 153 vs. 2(70) = 140.

time = 0.39, size = 153, normalized size = 2.19

$$\frac{\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{1}{2}(c+dx)\right)\left(9Adx\cos\left(\frac{dx}{2}\right)+9Adx\cos\left(c+\frac{dx}{2}\right)+3Adx\cos\left(c+\frac{3dx}{2}\right)+3Adx\cos\left(2c+\frac{3dx}{2}\right)-18A\sin\left(\frac{dx}{2}\right)+6B\sin\left(\frac{dx}{2}\right)+12A\sin\left(c+\frac{dx}{2}\right)-6B\sin\left(c+\frac{dx}{2}\right)-10A\sin\left(c+\frac{3dx}{2}\right)+4B\sin\left(c+\frac{3dx}{2}\right)\right)}{24a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^3*(9*A*d*x*Cos[(d*x)/2] + 9*A*d*x*Cos[c + (d*x)/
2] + 3*A*d*x*Cos[c + (3*d*x)/2] + 3*A*d*x*Cos[2*c + (3*d*x)/2] - 18*A*Sin[(
d*x)/2] + 6*B*Sin[(d*x)/2] + 12*A*Sin[c + (d*x)/2] - 6*B*Sin[c + (d*x)/2] -
 10*A*Sin[c + (3*d*x)/2] + 4*B*Sin[c + (3*d*x)/2]))/(24*a^2*d)
```

Maple [A]

time = 0.23, size = 74, normalized size = 1.06

method	result	size
norman	$\frac{\frac{Ax}{a}+\frac{(A-B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{(3A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}}{a}$	60
derivativedivides	$\frac{\frac{A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4A\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}$	74
default	$\frac{\frac{A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-3A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+4A\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2d\,a^2}$	74
risch	$\frac{Ax}{a^2}-\frac{2i\left(6A\,e^{2i(dx+c)}-3B\,e^{2i(dx+c)}+9\,e^{i(dx+c)}A-3B\,e^{i(dx+c)}+5A-2B\right)}{3d\,a^2\left(e^{i(dx+c)}+1\right)^3}$	85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^2*(1/3*A*tan(1/2*d*x+1/2*c)^3-1/3*B*tan(1/2*d*x+1/2*c)^3-3*A*tan(1/
2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c)+4*A*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.47, size = 120, normalized size = 1.71

$$-\frac{A\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)-\frac{B\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^2}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="maxima")
```

```
[Out] -1/6*(A*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2) - B*(3*sin(d*
x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2)/d
```

Fricas [A]
time = 4.86, size = 94, normalized size = 1.34

$$\frac{3\,Adx\cos\left(dx+c\right)^2+6\,Adx\cos\left(dx+c\right)+3\,Adx-\left(\left(5\,A-2\,B\right)\cos\left(dx+c\right)+4\,A-B\right)\sin\left(dx+c\right)}{3\,\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] 1/3*(3*A*d*x*cos(d*x + c)^2 + 6*A*d*x*cos(d*x + c) + 3*A*d*x - ((5*A - 2*B)
*cos(d*x + c) + 4*A - B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx+\int \frac{B\sec\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integral(B*sec(c +
 d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))/a**2
```

Giac [A]
time = 0.47, size = 85, normalized size = 1.21

$$\frac{\frac{6\,(dx+c)A}{a^2}+\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*A/a^2 + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x
+ 1/2*c)^3 - 9*A*a^4*tan(1/2*d*x + 1/2*c) + 3*B*a^4*tan(1/2*d*x + 1/2*c))/a
^6)/d
```

Mupad [B]
time = 1.93, size = 65, normalized size = 0.93

$$\frac{3\,B\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)-9\,A\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)+A\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3-B\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+6\,A\,d\,x}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (3*B*tan(c/2 + (d*x)/2) - 9*A*tan(c/2 + (d*x)/2) + A*tan(c/2 + (d*x)/2)^3 -
 B*tan(c/2 + (d*x)/2)^3 + 6*A*d*x)/(6*a^2*d)
```

3.96 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=98

$$-\frac{(2A-B)x}{a^2}+\frac{2(5A-2B)\sin(c+dx)}{3a^2d}-\frac{(2A-B)\sin(c+dx)}{a^2d(1+\sec(c+dx))}-\frac{(A-B)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}$$

```
[Out] -(2*A-B)*x/a^2+2/3*(5*A-2*B)*sin(d*x+c)/a^2/d-(2*A-B)*sin(d*x+c)/a^2/d/(1+s
ec(d*x+c))-1/3*(A-B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2
```

Rubi [A]
time = 0.17, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4105, 3872, 2717, 8}

$$\frac{2(5A-2B)\sin(c+dx)}{3a^2d}-\frac{(2A-B)\sin(c+dx)}{a^2d(\sec(c+dx)+1)}-\frac{x(2A-B)}{a^2}-\frac{(A-B)\sin(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(((2*A - B)*x)/a^2) + (2*(5*A - 2*B)*Sin[c + d*x])/(3*a^2*d) - ((2*A - B)*
Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x])) - ((A - B)*Sin[c + d*x])/(3*d*(a +
 a*Sec[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
```

```
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx &= -\frac{(A-B)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\int \frac{\cos(c+dx)(a(4A-B)-2a(A-B)\sec(c+dx))}{a+a\sec(c+dx)}\,dx}{3a^2}\\ &= -\frac{(2A-B)\sin(c+dx)}{a^2d(1+\sec(c+dx))}-\frac{(A-B)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\int \cos(c+dx)\,(2\end{aligned}$$

$$= -\frac{(2A-B)\sin(c+dx)}{a^2d(1+\sec(c+dx))}-\frac{(A-B)\sin(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{(2(5A-2B))\int}{3a}$$

$$= -\frac{(2A-B)x}{a^2}+\frac{2(5A-2B)\sin(c+dx)}{3a^2d}-\frac{(2A-B)\sin(c+dx)}{a^2d(1+\sec(c+dx))}-$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 245 vs. 2(98) = 196.
time = 0.63, size = 245, normalized size = 2.50

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(-18(2A-B)dx\cos\left(\frac{dx}{2}\right)-18(2A-B)dx\cos\left(c+\frac{dx}{2}\right)-12Adx\cos\left(c+\frac{3dx}{2}\right)+6Bdx\cos\left(c+\frac{3dx}{2}\right)-12Adx\cos\left(2c+\frac{3dx}{2}\right)+6Bdx\cos\left(2c+\frac{3dx}{2}\right)+66A\sin\left(\frac{dx}{2}\right)-36B\sin\left(\frac{dx}{2}\right)-30A\sin\left(c+\frac{dx}{2}\right)+24B\sin\left(c+\frac{dx}{2}\right)+41A\sin\left(c+\frac{3dx}{2}\right)-20B\sin\left(c+\frac{3dx}{2}\right)+9A\sin\left(2c+\frac{3dx}{2}\right)+3A\sin\left(2c+\frac{5dx}{2}\right)+3A\sin\left(3c+\frac{5dx}{2}\right)\right)}{12a^2d(1+\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-18*(2*A - B)*d*x*Cos[(d*x)/2] - 18*(2*A - B)*d
*x*Cos[c + (d*x)/2] - 12*A*d*x*Cos[c + (3*d*x)/2] + 6*B*d*x*Cos[c + (3*d*x)
/2] - 12*A*d*x*Cos[2*c + (3*d*x)/2] + 6*B*d*x*Cos[2*c + (3*d*x)/2] + 66*A*S
in[(d*x)/2] - 36*B*Sin[(d*x)/2] - 30*A*Sin[c + (d*x)/2] + 24*B*Sin[c + (d*x
)/2] + 41*A*Sin[c + (3*d*x)/2] - 20*B*Sin[c + (3*d*x)/2] + 9*A*Sin[2*c + (3
*d*x)/2] + 3*A*Sin[2*c + (5*d*x)/2] + 3*A*Sin[3*c + (5*d*x)/2]))/(12*a^2*d*
(1 + Cos[c + d*x])^2)
```

Maple [A]
time = 0.29, size = 108, normalized size = 1.10

method	result
derivativedivides	$\frac{-\frac{A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+5A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-4(2A-B)\arctan\left(\tan\left(\frac{dx}{2}+\right.\right.}{2d\,a^2}$
default	$\frac{-\frac{A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+5A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-3B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}-4(2A-B)\arctan\left(\tan\left(\frac{dx}{2}+\right.\right.}{2d\,a^2}$

risch	$-\frac{2Ax}{a^2}+\frac{xB}{a^2}-\frac{iA\,\mathrm{e}^{i(dx+c)}}{2a^2d}+\frac{iA\,\mathrm{e}^{-i(dx+c)}}{2a^2d}+\frac{2i\left(9A\,\mathrm{e}^{2i(dx+c)}-6B\,\mathrm{e}^{2i(dx+c)}+15\,\mathrm{e}^{i(dx+c)}A-9B\,\mathrm{e}^{i(dx+c)}+8A-5\right.}{3d\,a^2\left(\mathrm{e}^{i(dx+c)}+1\right)^3}$
norman	$\frac{-\frac{(2A-B)x}{a}-\frac{(A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{(2A-B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{3(3A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}+\frac{(7A-4B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3ad}}{a\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/2/d/a^2*(-1/3*A*tan(1/2*d*x+1/2*c)^3+1/3*B*tan(1/2*d*x+1/2*c)^3+5*A*tan(1
/2*d*x+1/2*c)-3*B*tan(1/2*d*x+1/2*c)+4*A*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+
1/2*c)^2)-4*(2*A-B)*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 191 vs. $2(94) = 188$.
time = 0.49, size = 191, normalized size = 1.95

$$\frac{A\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{24\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}+\frac{12\sin(dx+c)}{\left(a^2+\frac{a^2\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}\right)-B\left(\frac{\frac{9\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^3}{(\cos(dx+c)+1)^3}}{a^2}-\frac{12\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^2}\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] 1/6*(A*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*sin(d*x +
 c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1))) -
 B*((9*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/a^2 - 12*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2))/d
```

Fricas [A]
time = 1.95, size = 123, normalized size = 1.26

$$-\frac{3\,(2\,A-B)dx\cos\left(dx+c\right)^2+6\,(2\,A-B)dx\cos\left(dx+c\right)+3\,(2\,A-B)dx-\left(3\,A\cos\left(dx+c\right)^2+(14\,A-5\,B)\cos\left(dx+c\right)+10\,A-4\,B\right)\sin\left(dx+c\right)}{3\,\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] -1/3*(3*(2*A - B)*d*x*cos(d*x + c)^2 + 6*(2*A - B)*d*x*cos(d*x + c) + 3*(2*
A - B)*d*x - (3*A*cos(d*x + c)^2 + (14*A - 5*B)*cos(d*x + c) + 10*A - 4*B)*
sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{A\cos\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx+\int\frac{B\cos\left(c+dx\right)\sec\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*cos(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + Integ
ral(B*cos(c + d*x)*sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))
/a**2
```

Giac [A]
time = 0.44, size = 121, normalized size = 1.23

$$-\frac{\frac{6\,(dx+c)(2\,A-B)}{a^2}-\frac{12\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^2}+\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] -1/6*(6*(d*x + c)*(2*A - B)/a^2 - 12*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x +
 1/2*c)^2 + 1)*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d*x + 1
/2*c)^3 - 15*A*a^4*tan(1/2*d*x + 1/2*c) + 9*B*a^4*tan(1/2*d*x + 1/2*c))/a^6
)/d
```

Mupad [B]
time = 2.04, size = 109, normalized size = 1.11

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{A-B}{a^2}+\frac{3\,A-B}{2\,a^2}\right)}{d}-\frac{x\,(2\,A-B)}{a^2}+\frac{2\,A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(A-B)}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((A - B)/a^2 + (3*A - B)/(2*a^2)))/d - (x*(2*A - B))/a^
2 + (2*A*tan(c/2 + (d*x)/2))/(d*(a^2*tan(c/2 + (d*x)/2)^2 + a^2)) - (tan(c/
2 + (d*x)/2)^3*(A - B))/(6*a^2*d)
```

3.97 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=143

$$\frac{(7A-4B)x}{2a^2}-\frac{2(8A-5B)\sin(c+dx)}{3a^2d}+\frac{(7A-4B)\cos(c+dx)\sin(c+dx)}{2a^2d}-\frac{(8A-5B)\cos(c+dx)\sin(c+}{3a^2d(1+\sec(c+dx))}$$

```
[Out] 1/2*(7*A-4*B)*x/a^2-2/3*(8*A-5*B)*sin(d*x+c)/a^2/d+1/2*(7*A-4*B)*cos(d*x+c)
*sin(d*x+c)/a^2/d-1/3*(8*A-5*B)*cos(d*x+c)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))-
1/3*(A-B)*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2
```

Rubi [A]
time = 0.21, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4105, 3872, 2715, 8, 2717}

$$-\frac{2(8A-5B)\sin(c+dx)}{3a^2d}+\frac{(7A-4B)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{(8A-5B)\sin(c+dx)\cos(c+dx)}{3a^2d(\sec(c+dx)+1)}+\frac{x(7A-4B)}{2a^2}-\frac{(A-B)\sin(c+dx)\cos(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((7*A - 4*B)*x)/(2*a^2) - (2*(8*A - 5*B)*Sin[c + d*x])/(3*a^2*d) + ((7*A -
4*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - ((8*A - 5*B)*Cos[c + d*x]*Sin[c
 + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A - B)*Cos[c + d*x]*Sin[c + d*x])
/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\cos^2(c+dx)(a(5A-2B)-3a(A-B)\mathrm{se}}{a+a\sec(c+dx)}}{3a^2}$$
$$= -\frac{(8A-5B)\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\cos(c+dx)\sin(c-}{3d(a+a\sec(c+dx))^2}$$
$$= -\frac{(8A-5B)\cos(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\cos(c+dx)\sin(c-}{3d(a+a\sec(c+dx))^2}$$
$$= -\frac{2(8A-5B)\sin(c+dx)}{3a^2d} + \frac{(7A-4B)\cos(c+dx)\sin(c+dx)}{2a^2d} -$$
$$= \frac{(7A-4B)x}{2a^2} - \frac{2(8A-5B)\sin(c+dx)}{3a^2d} + \frac{(7A-4B)\cos(c+dx)\mathrm{s}}{2a^2d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 315 vs. 2(143) = 286.
time = 0.78, size = 315, normalized size = 2.20

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(36(7A-4B)dx\cos\left(\frac{dx}{2}\right)+36(7A-4B)dx\cos\left(c+\frac{dx}{2}\right)+84Adx\cos\left(c+\frac{3dx}{2}\right)-48Bdx\cos\left(c+\frac{3dx}{2}\right)+84Adx\cos\left(2c+\frac{3dx}{2}\right)-48Bdx\cos\left(2c+\frac{3dx}{2}\right)-381A\sin\left(\frac{dx}{2}\right)+264B\sin\left(\frac{dx}{2}\right)+147A\sin\left(c+\frac{dx}{2}\right)-120B\sin\left(c+\frac{dx}{2}\right)-239A\sin\left(c+\frac{3dx}{2}\right)+164B\sin\left(c+\frac{3dx}{2}\right)-63A\sin\left(2c+\frac{3dx}{2}\right)+36B\sin\left(2c+\frac{3dx}{2}\right)-15A\sin\left(2c+\frac{5dx}{2}\right)+12B\sin\left(2c+\frac{5dx}{2}\right)-15A\sin\left(3c+\frac{5dx}{2}\right)+12B\sin\left(3c+\frac{5dx}{2}\right)+3A\sin\left(3c+\frac{7dx}{2}\right)+3A\sin\left(4c+\frac{7dx}{2}\right)\right)}{48a^2d(1+\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(36*(7*A - 4*B)*d*x*Cos[(d*x)/2] + 36*(7*A - 4*B
)*d*x*Cos[c + (d*x)/2] + 84*A*d*x*Cos[c + (3*d*x)/2] - 48*B*d*x*Cos[c + (3*
d*x)/2] + 84*A*d*x*Cos[2*c + (3*d*x)/2] - 48*B*d*x*Cos[2*c + (3*d*x)/2] - 3
81*A*Sin[(d*x)/2] + 264*B*Sin[(d*x)/2] + 147*A*Sin[c + (d*x)/2] - 120*B*Sin
[c + (d*x)/2] - 239*A*Sin[c + (3*d*x)/2] + 164*B*Sin[c + (3*d*x)/2] - 63*A*
Sin[2*c + (3*d*x)/2] + 36*B*Sin[2*c + (3*d*x)/2] - 15*A*Sin[2*c + (5*d*x)/2
] + 12*B*Sin[2*c + (5*d*x)/2] - 15*A*Sin[3*c + (5*d*x)/2] + 12*B*Sin[3*c +
```

```
(5*d*x)/2] + 3*A*Sin[3*c + (7*d*x)/2] + 3*A*Sin[4*c + (7*d*x)/2]))/(48*a^2*
d*(1 + Cos[c + d*x])^2)
```

Maple [A]
time = 0.30, size = 131, normalized size = 0.92

method	result
derivativedivides	$\frac{\frac{A\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{B\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4\left(-\frac{5A}{2}+B\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\left(-\frac{3A}{2}+B\right)\tan\left(\frac{d}{}\right.}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}}{2d\,a^{2}}$
default	$\frac{\frac{A\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{B\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+5B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{4\left(-\frac{5A}{2}+B\right)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4\left(-\frac{3A}{2}+B\right)\tan\left(\frac{d}{}\right.}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}}}{2d\,a^{2}}$
norman	$\frac{\frac{(7A-4B)x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{(7A-4B)x}{2a}+\frac{(A-B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}+\frac{(7A-4B)x\left(\tan^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{(13A-9B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2ad}-\frac{(19A-}{}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^{2}a}$
risch	$\frac{7Ax}{2a^{2}}-\frac{2xB}{a^{2}}-\frac{iA\,\mathrm{e}^{2i(dx+c)}}{8a^{2}d}+\frac{iA\,\mathrm{e}^{i(dx+c)}}{a^{2}d}-\frac{i\mathrm{e}^{i(dx+c)}B}{2a^{2}d}-\frac{iA\,\mathrm{e}^{-i(dx+c)}}{a^{2}d}+\frac{i\mathrm{e}^{-i(dx+c)}B}{2a^{2}d}+\frac{iA\,\mathrm{e}^{-2i(dx+c)}}{8a^{2}d}-$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/d/a^2*(1/3*A*tan(1/2*d*x+1/2*c)^3-1/3*B*tan(1/2*d*x+1/2*c)^3-7*A*tan(1/
2*d*x+1/2*c)+5*B*tan(1/2*d*x+1/2*c)+4*((-5/2*A+B)*tan(1/2*d*x+1/2*c)^3+(-3/
2*A+B)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+2*(7*A-4*B)*arctan(ta
n(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 283 vs. $2(133) = 266$.
time = 0.49, size = 283, normalized size = 1.98

$$-\frac{A\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a^{2}+\frac{2a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{a^{2}}-\frac{42\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}\right)-B\left(\frac{\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{a^{2}}-\frac{24\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}+\frac{12\sin(dx+c)}{\left(a^{2}+\frac{a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)(\cos(dx+c)+1)}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] -1/6*(A*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^2*sin(d*x
+ c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*
x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)/(cos(d*x + c) +
 1))/a^2) - B*((15*sin(d*x + c)/(cos(d*x + c) + 1) - sin(d*x + c)^3/(cos(d*
x + c) + 1)^3)/a^2 - 24*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^2 + 12*si
n(d*x + c)/((a^2 + a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) +
 1))))/d
```

Fricas [A]

time = 4.76, size = 138, normalized size = 0.97

$$\frac{3\,(7\,A-4\,B)dx\cos\left(dx+c\right)^2+6\,(7\,A-4\,B)dx\cos\left(dx+c\right)+3\,(7\,A-4\,B)dx+\left(3\,A\cos\left(dx+c\right)^3-6\,(A-B)\cos\left(dx+c\right)^2-(43\,A-28\,B)\cos\left(dx+c\right)-32\,A+20\,B\right)\sin\left(dx+c\right)}{6\,\left(a^2d\cos\left(dx+c\right)^2+2\,a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(7*A - 4*B)*d*x*cos(d*x + c)^2 + 6*(7*A - 4*B)*d*x*cos(d*x + c) + 3*
(7*A - 4*B)*d*x + (3*A*cos(d*x + c)^3 - 6*(A - B)*cos(d*x + c)^2 - (43*A -
28*B)*cos(d*x + c) - 32*A + 20*B)*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a
^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\cos^2\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx+\int \frac{B\cos^2\left(c+dx\right)\sec\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*cos(c + d*x)**2/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + In
tegral(B*cos(c + d*x)**2*sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1
), x))/a**2
```

Giac [A]

time = 0.46, size = 164, normalized size = 1.15

$$\frac{\frac{3\,(dx+c)(7\,A-4\,B)}{a^2}-\frac{6\left(5\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}+\frac{Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-21\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^6}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(d*x + c)*(7*A - 4*B)/a^2 - 6*(5*A*tan(1/2*d*x + 1/2*c)^3 - 2*B*tan(
1/2*d*x + 1/2*c)^3 + 3*A*tan(1/2*d*x + 1/2*c) - 2*B*tan(1/2*d*x + 1/2*c))/(
(tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4
*tan(1/2*d*x + 1/2*c)^3 - 21*A*a^4*tan(1/2*d*x + 1/2*c) + 15*B*a^4*tan(1/2*
d*x + 1/2*c))/a^6)/d
```

Mupad [B]

time = 2.07, size = 154, normalized size = 1.08

$$\frac{x\,(7\,A-4\,B)}{2\,a^2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,(A-B)}{2\,a^2}+\frac{4\,A-2\,B}{2\,a^2}\right)}{d}-\frac{(5\,A-2\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+(3\,A-2\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^2\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(A-B)}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (x*(7*A - 4*B))/(2*a^2) - (tan(c/2 + (d*x)/2)*((3*(A - B))/(2*a^2) + (4*A -
 2*B)/(2*a^2)))/d - (tan(c/2 + (d*x)/2)^3*(5*A - 2*B) + tan(c/2 + (d*x)/2)*
(3*A - 2*B))/(d*(2*a^2*tan(c/2 + (d*x)/2)^2 + a^2*tan(c/2 + (d*x)/2)^4 + a^
2)) + (tan(c/2 + (d*x)/2)^3*(A - B))/(6*a^2*d)
```

3.98 $\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=170

$$-\frac{(10A-7B)x}{2a^2}+\frac{4(3A-2B)\sin(c+dx)}{a^2d}-\frac{(10A-7B)\cos(c+dx)\sin(c+dx)}{2a^2d}-\frac{(10A-7B)\cos^2(c+dx)\sin}{3a^2d(1+\sec(c+dx}$$

```
[Out] -1/2*(10*A-7*B)*x/a^2+4*(3*A-2*B)*sin(d*x+c)/a^2/d-1/2*(10*A-7*B)*cos(d*x+c
)*sin(d*x+c)/a^2/d-1/3*(10*A-7*B)*cos(d*x+c)^2*sin(d*x+c)/a^2/d/(1+sec(d*x+
c))-1/3*(A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-4/3*(3*A-2*B)*si
n(d*x+c)^3/a^2/d
```

Rubi [A]
time = 0.23, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4105, 3872, 2713, 2715, 8}

$$-\frac{4(3A-2B)\sin^3(c+dx)}{3a^2d}+\frac{4(3A-2B)\sin(c+dx)}{a^2d}-\frac{(10A-7B)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{(10A-7B)\sin(c+dx)\cos^2(c+dx)}{3a^2d(\sec(c+dx)+1)}-\frac{x(10A-7B)}{2a^2}-\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -1/2*((10*A - 7*B)*x)/a^2 + (4*(3*A - 2*B)*Sin[c + d*x])/(a^2*d) - ((10*A -
 7*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^2*d) - ((10*A - 7*B)*Cos[c + d*x]^2*S
in[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) - ((A - B)*Cos[c + d*x]^2*Sin[c +
 d*x])/(3*d*(a + a*Sec[c + d*x])^2) - (4*(3*A - 2*B)*Sin[c + d*x]^3)/(3*a^2
*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\cos^3(c+dx)(3a(2A-B)-4a(A-B}{a+a\sec(c+dx)}}{3a^2}$$
$$= -\frac{(10A-7B)\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\cos^2(c+dx)\,\mathrm{si}}{3d(a+a\sec(c+d}$$
$$= -\frac{(10A-7B)\cos^2(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} - \frac{(A-B)\cos^2(c+dx)\,\mathrm{si}}{3d(a+a\sec(c+d}$$
$$= -\frac{(10A-7B)\cos(c+dx)\sin(c+dx)}{2a^2d} - \frac{(10A-7B)\cos^2(c+dx}{3a^2d(1+\sec(c+}$$
$$= -\frac{(10A-7B)x}{2a^2} + \frac{4(3A-2B)\sin(c+dx)}{a^2d} - \frac{(10A-7B)\cos(c+}{2a^2c}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 369 vs. 2(170) = 340.
time = 0.75, size = 369, normalized size = 2.17

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(-36(10A-7B)dx\cos\left(\frac{dx}{2}\right)-36(10A-7B)dx\cos\left(c+\frac{dx}{2}\right)-120Adx\cos\left(c+\frac{3dx}{2}\right)+84Bdx\cos\left(c+\frac{3dx}{2}\right)-120Adx\cos\left(2c+\frac{3dx}{2}\right)+84Bdx\cos\left(2c+\frac{3dx}{2}\right)+516A\sin\left(\frac{dx}{2}\right)-381B\sin\left(\frac{dx}{2}\right)-156A\sin\left(c+\frac{dx}{2}\right)+147B\sin\left(c+\frac{dx}{2}\right)+342A\sin\left(c+\frac{3dx}{2}\right)-239B\sin\left(c+\frac{3dx}{2}\right)+118A\sin\left(2c+\frac{3dx}{2}\right)-63B\sin\left(2c+\frac{3dx}{2}\right)+30A\sin\left(2c+\frac{5dx}{2}\right)-15B\sin\left(2c+\frac{5dx}{2}\right)+30A\sin\left(3c+\frac{5dx}{2}\right)-15B\sin\left(3c+\frac{5dx}{2}\right)-3A\sin\left(3c+\frac{7dx}{2}\right)+3B\sin\left(3c+\frac{7dx}{2}\right)-3A\sin\left(4c+\frac{7dx}{2}\right)+3B\sin\left(4c+\frac{7dx}{2}\right)+A\sin\left(4c+\frac{9dx}{2}\right)+A\sin\left(5c+\frac{9dx}{2}\right)\right)}{48a^2d(1+\cos(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-36*(10*A - 7*B)*d*x*Cos[(d*x)/2] - 36*(10*A -
7*B)*d*x*Cos[c + (d*x)/2] - 120*A*d*x*Cos[c + (3*d*x)/2] + 84*B*d*x*Cos[c +
 (3*d*x)/2] - 120*A*d*x*Cos[2*c + (3*d*x)/2] + 84*B*d*x*Cos[2*c + (3*d*x)/2
] + 516*A*Sin[(d*x)/2] - 381*B*Sin[(d*x)/2] - 156*A*Sin[c + (d*x)/2] + 147*
```

```
B*Sin[c + (d*x)/2] + 342*A*Sin[c + (3*d*x)/2] - 239*B*Sin[c + (3*d*x)/2] +
118*A*Sin[2*c + (3*d*x)/2] - 63*B*Sin[2*c + (3*d*x)/2] + 30*A*Sin[2*c + (5*
d*x)/2] - 15*B*Sin[2*c + (5*d*x)/2] + 30*A*Sin[3*c + (5*d*x)/2] - 15*B*Sin[
3*c + (5*d*x)/2] - 3*A*Sin[3*c + (7*d*x)/2] + 3*B*Sin[3*c + (7*d*x)/2] - 3*
A*Sin[4*c + (7*d*x)/2] + 3*B*Sin[4*c + (7*d*x)/2] + A*Sin[4*c + (9*d*x)/2]
+ A*Sin[5*c + (9*d*x)/2]))/(48*a^2*d*(1 + Cos[c + d*x])^2)
```

Maple [A]
time = 0.33, size = 154, normalized size = 0.91

method	result
derivativedivides	$\frac{-\frac{A\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{B\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+9A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-7B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8\left(\left(-\frac{5A}{2}+\frac{5B}{4}\right)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{10A}{3}+2B\right)\left(\mathrm{ta}\right.\right.}{\left(1+\tan^{2}\left(\frac{dx}{2}\right.\right.}}{2d\,a^{2}}$
default	$\frac{-\frac{A\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{B\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+9A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-7B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{8\left(\left(-\frac{5A}{2}+\frac{5B}{4}\right)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{10A}{3}+2B\right)\left(\mathrm{ta}\right.\right.}{\left(1+\tan^{2}\left(\frac{dx}{2}\right.\right.}}{2d\,a^{2}}$
norman	$\frac{\frac{(4A-3B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}+\frac{(23A-15B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{ad}-\frac{(10A-7B)x}{2a}-\frac{(A-B)\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{3(10A-7B)x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{3(}{}}{\left(1+\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}$
risch	$-\frac{5Ax}{a^{2}}+\frac{7xB}{2a^{2}}+\frac{iA\,\mathrm{e}^{2i(dx+c)}}{4a^{2}d}-\frac{i\mathrm{e}^{2i(dx+c)}B}{8a^{2}d}-\frac{15iA\,\mathrm{e}^{i(dx+c)}}{8a^{2}d}+\frac{i\mathrm{e}^{i(dx+c)}B}{a^{2}d}+\frac{15iA\,\mathrm{e}^{-i(dx+c)}}{8a^{2}d}-\frac{i\mathrm{e}^{-i(dx+c)}B}{a^{2}d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/2/d/a^2*(-1/3*A*tan(1/2*d*x+1/2*c)^3+1/3*B*tan(1/2*d*x+1/2*c)^3+9*A*tan(1
/2*d*x+1/2*c)-7*B*tan(1/2*d*x+1/2*c)-8*((-5/2*A+5/4*B)*tan(1/2*d*x+1/2*c)^5
+(-10/3*A+2*B)*tan(1/2*d*x+1/2*c)^3+(-3/2*A+3/4*B)*tan(1/2*d*x+1/2*c))/(1+t
an(1/2*d*x+1/2*c)^2)^3-2*(10*A-7*B)*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 372 vs. $2(160) = 320$.
time = 0.51, size = 372, normalized size = 2.19

$$\frac{A\left(\frac{4\left(\frac{9\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{15\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{a^{2}+\frac{3a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{3a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}+\frac{a^{2}\sin(dx+c)^{6}}{(\cos(dx+c)+1)^{6}}}+\frac{\frac{27\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{a^{2}}-\frac{60\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}\right)-B\left(\frac{6\left(\frac{3\sin(dx+c)}{\cos(dx+c)+1}+\frac{5\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}\right)}{a^{2}+\frac{2a^{2}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}+\frac{a^{2}\sin(dx+c)^{4}}{(\cos(dx+c)+1)^{4}}}+\frac{\frac{21\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}}{a^{2}}-\frac{42\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^{2}}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] 1/6*(A*(4*(9*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 15*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^2 + 3*a^2*sin(d*x +
 c)^2/(cos(d*x + c) + 1)^2 + 3*a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 + a^
2*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (27*sin(d*x + c)/(cos(d*x + c) + 1
```

```
) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 60*arctan(sin(d*x + c)/(cos(
d*x + c) + 1))/a^2) - B*(6*(3*sin(d*x + c)/(cos(d*x + c) + 1) + 5*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3)/(a^2 + 2*a^2*sin(d*x + c)^2/(cos(d*x + c) + 1)^
2 + a^2*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) + (21*sin(d*x + c)/(cos(d*x +
c) + 1) - sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/a^2 - 42*arctan(sin(d*x + c)
/(cos(d*x + c) + 1))/a^2))/d
```

Fricas [A]
time = 2.85, size = 157, normalized size = 0.92

$$-\frac{3\,(10\,A-7\,B)dx\cos\left(dx+c\right)^{2}+6\,(10\,A-7\,B)dx\cos\left(dx+c\right)+3\,(10\,A-7\,B)dx-\left(2\,A\cos\left(dx+c\right)^{4}-(2\,A-3\,B)\cos\left(dx+c\right)^{3}+6\,(2\,A-B)\cos\left(dx+c\right)^{2}+(66\,A-43\,B)\cos\left(dx+c\right)+48\,A-32\,B\right)\sin\left(dx+c\right)}{6\,\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] -1/6*(3*(10*A - 7*B)*d*x*cos(d*x + c)^2 + 6*(10*A - 7*B)*d*x*cos(d*x + c) +
 3*(10*A - 7*B)*d*x - (2*A*cos(d*x + c)^4 - (2*A - 3*B)*cos(d*x + c)^3 + 6*
(2*A - B)*cos(d*x + c)^2 + (66*A - 43*B)*cos(d*x + c) + 48*A - 32*B)*sin(d*
x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\cos^{3}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx+\int \frac{B\cos^{3}\left(c+dx\right)\sec\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*cos(c + d*x)**3/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) + In
tegral(B*cos(c + d*x)**3*sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1
), x))/a**2
```

Giac [A]
time = 0.48, size = 192, normalized size = 1.13

$$-\frac{\frac{3\,(dx+c)(10\,A-7\,B)}{a^{2}}-\frac{2\,\left(30\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+40\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-24\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+18\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-9\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}a^{2}}+\frac{Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-27\,Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+21\,Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{6}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(3*(d*x + c)*(10*A - 7*B)/a^2 - 2*(30*A*tan(1/2*d*x + 1/2*c)^5 - 15*B*
tan(1/2*d*x + 1/2*c)^5 + 40*A*tan(1/2*d*x + 1/2*c)^3 - 24*B*tan(1/2*d*x + 1
```

```
/2*c)^3 + 18*A*tan(1/2*d*x + 1/2*c) - 9*B*tan(1/2*d*x + 1/2*c))/((tan(1/2*d
*x + 1/2*c)^2 + 1)^3*a^2) + (A*a^4*tan(1/2*d*x + 1/2*c)^3 - B*a^4*tan(1/2*d
*x + 1/2*c)^3 - 27*A*a^4*tan(1/2*d*x + 1/2*c) + 21*B*a^4*tan(1/2*d*x + 1/2*
c))/a^6)/d
```

Mupad [B]
time = 2.08, size = 187, normalized size = 1.10

$$\frac{(10\,A-5\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{40\,A}{3}-8\,B\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+(6\,A-3\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^2\right)}-\frac{x\,(10\,A-7\,B)}{2\,a^2}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{2\,(A-B)}{a^2}+\frac{5\,A-3\,B}{2\,a^2}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(A-B)}{6\,a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^2,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(10*A - 5*B) + tan(c/2 + (d*x)/2)^3*((40*A)/3 - 8*B)
+ tan(c/2 + (d*x)/2)*(6*A - 3*B))/(d*(3*a^2*tan(c/2 + (d*x)/2)^2 + 3*a^2*ta
n(c/2 + (d*x)/2)^4 + a^2*tan(c/2 + (d*x)/2)^6 + a^2)) - (x*(10*A - 7*B))/(2
*a^2) + (tan(c/2 + (d*x)/2)*((2*(A - B))/a^2 + (5*A - 3*B)/(2*a^2)))/d - (t
an(c/2 + (d*x)/2)^3*(A - B))/(6*a^2*d)
```

3.99 $\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=202

$$-\frac{(6A-13B)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{8(9A-19B)\tan(c+dx)}{15a^3d}-\frac{(6A-13B)\sec(c+dx)\tan(c+dx)}{2a^3d}+\frac{(A-$$

```
[Out] -1/2*(6*A-13*B)*arctanh(sin(d*x+c))/a^3/d+8/15*(9*A-19*B)*tan(d*x+c)/a^3/d-
1/2*(6*A-13*B)*sec(d*x+c)*tan(d*x+c)/a^3/d+1/5*(A-B)*sec(d*x+c)^4*tan(d*x+c
)/d/(a+a*sec(d*x+c))^3+1/15*(6*A-11*B)*sec(d*x+c)^3*tan(d*x+c)/a/d/(a+a*sec
(d*x+c))^2+4/15*(9*A-19*B)*sec(d*x+c)^2*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A]

time = 0.33, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4104, 3872, 3852, 8, 3853, 3855}

$$\frac{8(9A-19B)\tan(c+dx)}{15a^3d}-\frac{(6A-13B)\tanh^{-1}(\sin(c+dx))}{2a^3d}+\frac{4(9A-19B)\tan(c+dx)\sec^2(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(6A-13B)\tan(c+dx)\sec(c+dx)}{2a^3d}+\frac{(A-B)\tan(c+dx)\sec^4(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{(6A-11B)\tan(c+dx)\sec^3(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/2*((6*A - 13*B)*ArcTanh[Sin[c + d*x]])/(a^3*d) + (8*(9*A - 19*B)*Tan[c +
 d*x])/(15*a^3*d) - ((6*A - 13*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^3*d) + ((
A - B)*Sec[c + d*x]^4*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((6*A -
11*B)*Sec[c + d*x]^3*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + (4*(9*
A - 19*B)*Sec[c + d*x]^2*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^4(c+dx)(4a(A-B)-a(2A-7B)\sec}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(6A-11B)\sec^3(c+dx)\tan(c}{15ad(a+a\sec(c+dx))}$$

$$= \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(6A-11B)\sec^3(c+dx)\tan(c}{15ad(a+a\sec(c+dx))}$$

$$= \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(6A-11B)\sec^3(c+dx)\tan(c}{15ad(a+a\sec(c+dx))}$$

$$= -\frac{(6A-13B)\sec(c+dx)\tan(c+dx)}{2a^3d} + \frac{(A-B)\sec^4(c+dx)\tan(}{5d(a+a\sec(c+dx)}$$

$$= -\frac{(6A-13B)\tanh^{-1}(\sin(c+dx))}{2a^3d} + \frac{8(9A-19B)\tan(c+dx)}{15a^3d} -$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 610 vs. 2(202) = 404.
time = 6.27, size = 610, normalized size = 3.02

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (1920*(6*A - 13*B)*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)
/2]] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2
]*Sec[c]*Sec[c + d*x]^2*((-870*A + 1235*B)*Sin[(d*x)/2] + 5*(366*A - 761*B)
*Sin[(3*d*x)/2] - 2094*A*Sin[c - (d*x)/2] + 4329*B*Sin[c - (d*x)/2] + 1314*
A*Sin[c + (d*x)/2] - 1989*B*Sin[c + (d*x)/2] - 1650*A*Sin[2*c + (d*x)/2] +
3575*B*Sin[2*c + (d*x)/2] - 450*A*Sin[c + (3*d*x)/2] + 475*B*Sin[c + (3*d*x
)/2] + 1230*A*Sin[2*c + (3*d*x)/2] - 2005*B*Sin[2*c + (3*d*x)/2] - 1050*A*S
in[3*c + (3*d*x)/2] + 2275*B*Sin[3*c + (3*d*x)/2] + 1278*A*Sin[c + (5*d*x)/
2] - 2673*B*Sin[c + (5*d*x)/2] - 90*A*Sin[2*c + (5*d*x)/2] - 105*B*Sin[2*c
+ (5*d*x)/2] + 918*A*Sin[3*c + (5*d*x)/2] - 1593*B*Sin[3*c + (5*d*x)/2] - 4
50*A*Sin[4*c + (5*d*x)/2] + 975*B*Sin[4*c + (5*d*x)/2] + 630*A*Sin[2*c + (7
*d*x)/2] - 1325*B*Sin[2*c + (7*d*x)/2] + 60*A*Sin[3*c + (7*d*x)/2] - 255*B*
Sin[3*c + (7*d*x)/2] + 480*A*Sin[4*c + (7*d*x)/2] - 875*B*Sin[4*c + (7*d*x)
/2] - 90*A*Sin[5*c + (7*d*x)/2] + 195*B*Sin[5*c + (7*d*x)/2] + 144*A*Sin[3*
c + (9*d*x)/2] - 304*B*Sin[3*c + (9*d*x)/2] + 30*A*Sin[4*c + (9*d*x)/2] - 9
0*B*Sin[4*c + (9*d*x)/2] + 114*A*Sin[5*c + (9*d*x)/2] - 214*B*Sin[5*c + (9*
d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A]

time = 0.28, size = 206, normalized size = 1.02

method	result
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}+2A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{8B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+17A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-31B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\cdots}{\cdots}$
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}+2A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{8B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+17A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-31B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\cdots}{\cdots}$
norman	$\frac{\frac{(A-B)\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad}+\frac{(3A-5B)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}-\frac{25(9A-19B)\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}-\frac{(25A-51B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{(27A-59B)\left(\tan\cdots\right.}{12a\cdots}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\cdots\right.}$
risch	$\frac{i\left(90A\,e^{8i(dx+c)}-195B\,e^{8i(dx+c)}+450A\,e^{7i(dx+c)}-975B\,e^{7i(dx+c)}+1050A\,e^{6i(dx+c)}-2275B\,e^{6i(dx+c)}+1650A\,e^{5i(dx+c)}\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5*A-1/5*tan(1/2*d*x+1/2*c)^5*B+2*A*tan(1/
2*d*x+1/2*c)^3-8/3*B*tan(1/2*d*x+1/2*c)^3+17*A*tan(1/2*d*x+1/2*c)-31*B*tan(
1/2*d*x+1/2*c)-(-14*B+4*A)/(tan(1/2*d*x+1/2*c)+1)+(26*B-12*A)*ln(tan(1/2*d*
x+1/2*c)+1)-2*B/(tan(1/2*d*x+1/2*c)+1)^2+(-26*B+12*A)*ln(tan(1/2*d*x+1/2*c)
-1)-(-14*B+4*A)/(tan(1/2*d*x+1/2*c)-1)+2*B/(tan(1/2*d*x+1/2*c)-1)^2)
```

Maxima [A]
time = 0.30, size = 377, normalized size = 1.87

$$-\frac{B\left(\frac{60\left(\frac{5\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{7\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}\right)}{a^{3}-\frac{2a^{3}\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}+\frac{a^{3}\sin\left(dx+c\right)^{4}}{\left(\cos\left(dx+c\right)+1\right)^{4}}}+\frac{\frac{465\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{40\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}+\frac{3\sin\left(dx+c\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}}{a^{3}}-\frac{390\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1\right)}{a^{3}}+\frac{390\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-1\right)}{a^{3}}\right)-3A\left(\frac{40\sin\left(dx+c\right)}{\left(a^{3}-\frac{a^{3}\sin\left(dx+c\right)^{2}}{\left(\cos\left(dx+c\right)+1\right)^{2}}\right)\left(\cos\left(dx+c\right)+1\right)}+\frac{\frac{85\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{10\sin\left(dx+c\right)^{3}}{\left(\cos\left(dx+c\right)+1\right)^{3}}+\frac{\sin\left(dx+c\right)^{5}}{\left(\cos\left(dx+c\right)+1\right)^{5}}}{a^{3}}-\frac{60\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1\right)}{a^{3}}+\frac{60\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-1\right)}{a^{3}}\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(B*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) - 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 - 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) + 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 390*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 390*log(sin(d*x
+ c)/(cos(d*x + c) + 1) - 1)/a^3) - 3*A*(40*sin(d*x + c)/((a^3 - a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(
d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(co
s(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 +
60*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^3))/d
```

Fricas [A]
time = 3.58, size = 295, normalized size = 1.46

$$-\frac{15\left(\left(6A-13B\right)\cos\left(dx+c\right)^{5}+3\left(6A-13B\right)\cos\left(dx+c\right)^{4}+3\left(6A-13B\right)\cos\left(dx+c\right)^{3}+\left(6A-13B\right)\cos\left(dx+c\right)^{2}\right)\log\left(\sin\left(dx+c\right)+1\right)-15\left(\left(6A-13B\right)\cos\left(dx+c\right)^{5}+3\left(6A-13B\right)\cos\left(dx+c\right)^{4}+3\left(6A-13B\right)\cos\left(dx+c\right)^{3}+\left(6A-13B\right)\cos\left(dx+c\right)^{2}\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\left(16\left(9A-19B\right)\cos\left(dx+c\right)^{4}+3\left(114A-239B\right)\cos\left(dx+c\right)^{3}+\left(234A-479B\right)\cos\left(dx+c\right)^{2}+15\left(2A-3B\right)\cos\left(dx+c\right)+15B\right)\sin\left(dx+c\right)}{60\left(a^{3}d\cos\left(dx+c\right)^{5}+3a^{3}d\cos\left(dx+c\right)^{4}+3a^{3}d\cos\left(dx+c\right)^{3}+a^{3}d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/60*(15*((6*A - 13*B)*cos(d*x + c)^5 + 3*(6*A - 13*B)*cos(d*x + c)^4 + 3*
(6*A - 13*B)*cos(d*x + c)^3 + (6*A - 13*B)*cos(d*x + c)^2)*log(sin(d*x + c)
 + 1) - 15*((6*A - 13*B)*cos(d*x + c)^5 + 3*(6*A - 13*B)*cos(d*x + c)^4 + 3
*(6*A - 13*B)*cos(d*x + c)^3 + (6*A - 13*B)*cos(d*x + c)^2)*log(-sin(d*x +
c) + 1) - 2*(16*(9*A - 19*B)*cos(d*x + c)^4 + 3*(114*A - 239*B)*cos(d*x + c
)^3 + (234*A - 479*B)*cos(d*x + c)^2 + 15*(2*A - 3*B)*cos(d*x + c) + 15*B)*
sin(d*x + c))/(a^3*d*cos(d*x + c)^5 + 3*a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(
d*x + c)^3 + a^3*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^{5}\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx+\int \frac{B\sec^{6}\left(c+dx\right)}{\sec^{3}\left(c+dx\right)+3\sec^{2}\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(B*sec(c + d*x)**6/(sec(c + d*x)**3 + 3*sec(c + d
*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A]
time = 0.50, size = 233, normalized size = 1.15

$$-\frac{\frac{30\,(6\,A-13\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{30\,(6\,A-13\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{60\left(2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-7\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+30\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+255\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-465\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(30*(6*A - 13*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 30*(6*A - 1
3*B)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 + 60*(2*A*tan(1/2*d*x + 1/2*c)^
3 - 7*B*tan(1/2*d*x + 1/2*c)^3 - 2*A*tan(1/2*d*x + 1/2*c) + 5*B*tan(1/2*d*x
 + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^3) - (3*A*a^12*tan(1/2*d*x + 1
/2*c)^5 - 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 + 30*A*a^12*tan(1/2*d*x + 1/2*c)^
3 - 40*B*a^12*tan(1/2*d*x + 1/2*c)^3 + 255*A*a^12*tan(1/2*d*x + 1/2*c) - 46
5*B*a^12*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B]
time = 2.03, size = 216, normalized size = 1.07

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,(A-B)}{2\,a^3}+\frac{3\,(3\,A-5\,B)}{4\,a^3}+\frac{2\,A-10\,B}{4\,a^3}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(2\,A-7\,B)-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,A-5\,B)}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{A-B}{4\,a^3}+\frac{3\,A-5\,B}{12\,a^3}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(A-B)}{20\,a^3\,d}-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(6\,A-13\,B)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^5*(a + a/cos(c + d*x))^3),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((3*(A - B))/(2*a^3) + (3*(3*A - 5*B))/(4*a^3) + (2*A -
 10*B)/(4*a^3)))/d - (tan(c/2 + (d*x)/2)^3*(2*A - 7*B) - tan(c/2 + (d*x)/2)
*(2*A - 5*B))/(d*(a^3*tan(c/2 + (d*x)/2)^4 - 2*a^3*tan(c/2 + (d*x)/2)^2 + a
^3)) + (tan(c/2 + (d*x)/2)^3*((A - B)/(4*a^3) + (3*A - 5*B)/(12*a^3)))/d +
(tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d) - (atanh(tan(c/2 + (d*x)/2))*(6*A
 - 13*B))/(a^3*d)
```

3.100 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=156

$$\frac{(A-3B)\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(7A-27B)\tan(c+dx)}{15a^3d}+\frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{(4A-9B)\sec}{15ad(a+}$$

```
[Out] (A-3*B)*arctanh(sin(d*x+c))/a^3/d-1/15*(7*A-27*B)*tan(d*x+c)/a^3/d+1/5*(A-B
)*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^3+1/15*(4*A-9*B)*sec(d*x+c)^2*
tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2-(A-3*B)*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A]

time = 0.30, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4104, 4093, 3872, 3855, 3852, 8}

$$-\frac{(7A-27B)\tan(c+dx)}{15a^3d}+\frac{(A-3B)\tanh^{-1}(\sin(c+dx))}{a^3d}-\frac{(A-3B)\tan(c+dx)}{d\,(a^3\sec(c+dx)+a^3)}+\frac{(A-B)\tan(c+dx)\sec^3(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{(4A-9B)\tan(c+dx)\sec^2(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((A - 3*B)*ArcTanh[Sin[c + d*x]])/(a^3*d) - ((7*A - 27*B)*Tan[c + d*x])/(15
*a^3*d) + ((A - B)*Sec[c + d*x]^3*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3
) + ((4*A - 9*B)*Sec[c + d*x]^2*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^
2) - ((A - 3*B)*Tan[c + d*x])/(d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^3(c+dx)(3a(A-B)-a(A-6B)\sec}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(4A-9B)\sec^2(c+dx)\tan(}{15ad(a+a\sec(c+dx)}$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(4A-9B)\sec^2(c+dx)\tan(}{15ad(a+a\sec(c+dx)}$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(4A-9B)\sec^2(c+dx)\tan(}{15ad(a+a\sec(c+dx)}$$

$$= \frac{(A-3B)\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{(A-B)\sec^3(c+dx)\tan(c+dx}{5d(a+a\sec(c+dx))^3}$$

$$= \frac{(A-3B)\tanh^{-1}(\sin(c+dx))}{a^3d} - \frac{(7A-27B)\tan(c+dx)}{15a^3d} + \frac{(A-}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 480 vs. 2(156) = 312.
time = 4.02, size = 480, normalized size = 3.08

$$\frac{-960(A-3B)\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\sec(c)\sec(c+dx)\left(5(32A-51B)\sin\left(\frac{dx}{2}\right)+(-167A+567B)\sin\left(\frac{3dx}{2}\right)+170A\sin\left(c-\frac{dx}{2}\right)-600B\sin\left(c-\frac{dx}{2}\right)-170A\sin\left(c+\frac{dx}{2}\right)+375B\sin\left(c+\frac{dx}{2}\right)+160A\sin\left(2c+\frac{dx}{2}\right)-480B\sin\left(2c+\frac{dx}{2}\right)+75A\sin\left(c+\frac{3dx}{2}\right)-60B\sin\left(c+\frac{3dx}{2}\right)-167A\sin\left(2c+\frac{3dx}{2}\right)+402B\sin\left(2c+\frac{3dx}{2}\right)+75A\sin\left(3c+\frac{3dx}{2}\right)-225B\sin\left(3c+\frac{3dx}{2}\right)-95A\sin\left(c+\frac{5dx}{2}\right)+315B\sin\left(c+\frac{5dx}{2}\right)+15A\sin\left(2c+\frac{5dx}{2}\right)+30B\sin\left(2c+\frac{5dx}{2}\right)-95A\sin\left(3c+\frac{5dx}{2}\right)+240B\sin\left(3c+\frac{5dx}{2}\right)+15A\sin\left(4c+\frac{5dx}{2}\right)-45B\sin\left(4c+\frac{5dx}{2}\right)-22A\sin\left(2c+\frac{7dx}{2}\right)+72B\sin\left(2c+\frac{7dx}{2}\right)+15B\sin\left(3c+\frac{7dx}{2}\right)-22A\sin\left(4c+\frac{7dx}{2}\right)+57B\sin\left(4c+\frac{7dx}{2}\right)\right)}{120a^3d(1+\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-960*(A - 3*B)*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
] - Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*S
ec[c]*Sec[c + d*x]*(5*(32*A - 51*B)*Sin[(d*x)/2] + (-167*A + 567*B)*Sin[(3*
d*x)/2] + 170*A*Sin[c - (d*x)/2] - 600*B*Sin[c - (d*x)/2] - 170*A*Sin[c + (
d*x)/2] + 375*B*Sin[c + (d*x)/2] + 160*A*Sin[2*c + (d*x)/2] - 480*B*Sin[2*c
 + (d*x)/2] + 75*A*Sin[c + (3*d*x)/2] - 60*B*Sin[c + (3*d*x)/2] - 167*A*Sin
[2*c + (3*d*x)/2] + 402*B*Sin[2*c + (3*d*x)/2] + 75*A*Sin[3*c + (3*d*x)/2]
- 225*B*Sin[3*c + (3*d*x)/2] - 95*A*Sin[c + (5*d*x)/2] + 315*B*Sin[c + (5*d
*x)/2] + 15*A*Sin[2*c + (5*d*x)/2] + 30*B*Sin[2*c + (5*d*x)/2] - 95*A*Sin[3
*c + (5*d*x)/2] + 240*B*Sin[3*c + (5*d*x)/2] + 15*A*Sin[4*c + (5*d*x)/2] -
45*B*Sin[4*c + (5*d*x)/2] - 22*A*Sin[2*c + (7*d*x)/2] + 72*B*Sin[2*c + (7*d
*x)/2] + 15*B*Sin[3*c + (7*d*x)/2] - 22*A*Sin[4*c + (7*d*x)/2] + 57*B*Sin[4
*c + (7*d*x)/2]))/(120*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A]

time = 0.25, size = 162, normalized size = 1.04

method	result
derivativedivides	$-\frac{\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}+\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}-\frac{4A\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+2B\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+17B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+(-}{4d\,a^{3}}$
default	$-\frac{\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}+\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}-\frac{4A\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+2B\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-7A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+17B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+(-}{4d\,a^{3}}$
norman	$\frac{-\frac{(A-B)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad}-\frac{(4A-9B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{30ad}-\frac{(7A-25B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{5(8A-27B)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}+\frac{(26A-81B)\left(\tan^{7}\right.}{5ad}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{4}a^{2}}$
risch	$-\frac{2i\left(15A\,e^{6i(dx+c)}-45B\,e^{6i(dx+c)}+75A\,e^{5i(dx+c)}-225B\,e^{5i(dx+c)}+160A\,e^{4i(dx+c)}-480B\,e^{4i(dx+c)}+170A\,e^{3i(dx+c)}-6\right.}{15d\,a^{3}\left(e^{i(dx+c)}+1\right)^{5}\left(e^{2i(dx+c)}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/d/a^3*(-1/5*tan(1/2*d*x+1/2*c)^5*A+1/5*tan(1/2*d*x+1/2*c)^5*B-4/3*A*tan
(1/2*d*x+1/2*c)^3+2*B*tan(1/2*d*x+1/2*c)^3-7*A*tan(1/2*d*x+1/2*c)+17*B*tan(
1/2*d*x+1/2*c)+(-12*B+4*A)*ln(tan(1/2*d*x+1/2*c)+1)-4*B/(tan(1/2*d*x+1/2*c)
+1)+(12*B-4*A)*ln(tan(1/2*d*x+1/2*c)-1)-4*B/(tan(1/2*d*x+1/2*c)-1))
```

Maxima [A]

time = 0.29, size = 286, normalized size = 1.83

$$\frac{3B\left(\frac{40\sin(dx+c)}{\left(a^{3}-\frac{a^{3}\sin(dx+c)^{2}}{(\cos(dx+c)+1)^{2}}\right)(\cos(dx+c)+1)}+\frac{\frac{85\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}}{a^{3}}-\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{3}}+\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{3}}\right)-A\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}}{a^{3}}-\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{3}}+\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{3}}\right)}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(3*B*(40*sin(d*x + c)/((a^3 - a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*lo
g(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x +
 c) + 1) - 1)/a^3) - A*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x +
 c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 6
0*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d
*x + c) + 1) - 1)/a^3))/d
```

Fricas [A]
time = 2.96, size = 256, normalized size = 1.64

$$\frac{15\left((A-3B)\cos(dx+c)^4+3(A-3B)\cos(dx+c)^3+3(A-3B)\cos(dx+c)^2+(A-3B)\cos(dx+c)\right)\log(\sin(dx+c)+1)-15\left((A-3B)\cos(dx+c)^4+3(A-3B)\cos(dx+c)^3+3(A-3B)\cos(dx+c)^2+(A-3B)\cos(dx+c)\right)\log(-\sin(dx+c)+1)-2\left(2(11A-36B)\cos(dx+c)^3+3(17A-57B)\cos(dx+c)^2+(32A-117B)\cos(dx+c)-15B\right)\sin(dx+c)}{30\left(a^3d\cos(dx+c)^4+3a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+a^3d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(15*((A - 3*B)*cos(d*x + c)^4 + 3*(A - 3*B)*cos(d*x + c)^3 + 3*(A - 3*
B)*cos(d*x + c)^2 + (A - 3*B)*cos(d*x + c))*log(sin(d*x + c) + 1) - 15*((A
- 3*B)*cos(d*x + c)^4 + 3*(A - 3*B)*cos(d*x + c)^3 + 3*(A - 3*B)*cos(d*x +
c)^2 + (A - 3*B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*(11*A - 36*B)*
cos(d*x + c)^3 + 3*(17*A - 57*B)*cos(d*x + c)^2 + (32*A - 117*B)*cos(d*x +
c) - 15*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3
*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int \frac{B\sec^5(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(B*sec(c + d*x)**5/(sec(c + d*x)**3 + 3*sec(c + d
*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A]
time = 0.53, size = 186, normalized size = 1.19

$$\frac{\frac{60(A-3B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{a^3}-\frac{60(A-3B)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{a^3}-\frac{120B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)a^3}-\frac{3Aa^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3Ba^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+20Aa^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-30Ba^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+105Aa^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-255Ba^{12}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^{15}}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(60*(A - 3*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*(A - 3*B)*lo
g(abs(tan(1/2*d*x + 1/2*c) - 1))/a^3 - 120*B*tan(1/2*d*x + 1/2*c)/((tan(1/2
*d*x + 1/2*c)^2 - 1)*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan
(1/2*d*x + 1/2*c)^5 + 20*A*a^12*tan(1/2*d*x + 1/2*c)^3 - 30*B*a^12*tan(1/2*
d*x + 1/2*c)^3 + 105*A*a^12*tan(1/2*d*x + 1/2*c) - 255*B*a^12*tan(1/2*d*x +
 1/2*c))/a^15)/d
```

Mupad [B]
time = 2.04, size = 168, normalized size = 1.08

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(A-3\,B\right)}{a^3\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,\left(A-B\right)}{4\,a^3}-\frac{3\,B}{2\,a^3}+\frac{2\,A-4\,B}{2\,a^3}\right)}{d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(A-B\right)}{20\,a^3\,d}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{A-B}{6\,a^3}+\frac{2\,A-4\,B}{12\,a^3}\right)}{d}-\frac{2\,B\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^3\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-a^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + a/cos(c + d*x))^3),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2))*(A - 3*B))/(a^3*d) - (tan(c/2 + (d*x)/2)*((3*(
A - B))/(4*a^3) - (3*B)/(2*a^3) + (2*A - 4*B)/(2*a^3)))/d - (tan(c/2 + (d*x
)/2)^5*(A - B))/(20*a^3*d) - (tan(c/2 + (d*x)/2)^3*((A - B)/(6*a^3) + (2*A
- 4*B)/(12*a^3)))/d - (2*B*tan(c/2 + (d*x)/2))/(d*(a^3*tan(c/2 + (d*x)/2)^2
 - a^3))
```

3.101 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=125

$$\frac{B\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{(2A-7B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{(4A-29B)\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}$$

```
[Out] B*arctanh(sin(d*x+c))/a^3/d+1/5*(A-B)*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*
x+c))^3-1/15*(2*A-7*B)*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^2+1/15*(4*A-29*B)*ta
n(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A]
time = 0.22, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4104, 4093, 4083, 3855, 3879}

$$\frac{(4A-29B)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{B\tanh^{-1}(\sin(c+dx))}{a^3d}+\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{(2A-7B)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(a^3*d) + ((A - B)*Sec[c + d*x]^2*Tan[c + d*x])/(
5*d*(a + a*Sec[c + d*x])^3) - ((2*A - 7*B)*Tan[c + d*x])/(15*a*d*(a + a*Sec
[c + d*x])^2) + ((4*A - 29*B)*Tan[c + d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^2(c+dx)(2a(A-B)+5aB\sec(c+dx}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(2A-7B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \int$$

$$= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(2A-7B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(4}{}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{a^3d} + \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(2A}{15ad}$$

Mathematica [**A**]
time = 0.94, size = 197, normalized size = 1.58

$$\frac{-240B\cos^6\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(5(4A-29B)\sin\left(\frac{dx}{2}\right)+75B\sin\left(c+\frac{dx}{2}\right)+10A\sin\left(c+\frac{3dx}{2}\right)-95B\sin\left(c+\frac{3dx}{2}\right)+15B\sin\left(2c+\frac{3dx}{2}\right)+2A\sin\left(2c+\frac{5dx}{2}\right)-22B\sin\left(2c+\frac{5dx}{2}\right)\right)}{30a^3d(1+\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-240*B*Cos[(c + d*x)/2]^6*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log[
Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(5*(4*A -
 29*B)*Sin[(d*x)/2] + 75*B*Sin[c + (d*x)/2] + 10*A*Sin[c + (3*d*x)/2] - 95*
```

```
B*Sin[c + (3*d*x)/2] + 15*B*Sin[2*c + (3*d*x)/2] + 2*A*Sin[2*c + (5*d*x)/2]
 - 22*B*Sin[2*c + (5*d*x)/2]))/(30*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A]
time = 0.23, size = 119, normalized size = 0.95

method	result
derivativedivides	$\frac{\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}-\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}-\frac{4B\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{2A\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-}{4d\,a^{3}}$
default	$\frac{\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}-\frac{\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}-\frac{4B\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{2A\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-4B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-}{4d\,a^{3}}$
risch	$-\frac{2i\left(15B\,e^{4i(dx+c)}+75B\,e^{3i(dx+c)}-20A\,e^{2i(dx+c)}+145B\,e^{2i(dx+c)}-10\,e^{i(dx+c)}A+95B\,e^{i(dx+c)}-2A+22B\right)}{15d\,a^{3}\left(e^{i(dx+c)}+1\right)^{5}}+\ln(e^{i(}$
norman	$\frac{\frac{(A-11B)\left(\tan^{9}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60ad}-\frac{(A-7B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{(A-B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad}-\frac{(A+9B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{10ad}-\frac{(3A-43B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{10ad}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}a^{2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5*A-1/5*tan(1/2*d*x+1/2*c)^5*B-4/3*B*tan(
1/2*d*x+1/2*c)^3+2/3*A*tan(1/2*d*x+1/2*c)^3-4*B*ln(tan(1/2*d*x+1/2*c)-1)+A*
tan(1/2*d*x+1/2*c)-7*B*tan(1/2*d*x+1/2*c)+4*B*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [A]
time = 0.30, size = 187, normalized size = 1.50

$$-\frac{B\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{20\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}}{a^{3}}-\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^{3}}+\frac{60\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^{3}}\right)-\frac{A\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}+\frac{3\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}\right)}{a^{3}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(B*((105*sin(d*x + c)/(cos(d*x + c) + 1) + 20*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 60*log(sin(d*x
+ c)/(cos(d*x + c) + 1) + 1)/a^3 + 60*log(sin(d*x + c)/(cos(d*x + c) + 1) -
 1)/a^3) - A*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d
*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A]
time = 4.24, size = 183, normalized size = 1.46

$$\frac{15\left(B\cos(dx+c)^{3}+3B\cos(dx+c)^{2}+3B\cos(dx+c)+B\right)\log(\sin(dx+c)+1)-15\left(B\cos(dx+c)^{3}+3B\cos(dx+c)^{2}+3B\cos(dx+c)+B\right)\log(-\sin(dx+c)+1)+2\left(2(A-11B)\cos(dx+c)^{2}+3(2A-17B)\cos(dx+c)+7A-32B\right)\sin(dx+c)}{30\left(a^{3}d\cos(dx+c)^{3}+3a^{3}d\cos(dx+c)^{2}+3a^{3}d\cos(dx+c)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(15*(B*cos(d*x + c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c) + B)*log
(sin(d*x + c) + 1) - 15*(B*cos(d*x + c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*
x + c) + B)*log(-sin(d*x + c) + 1) + 2*(2*(A - 11*B)*cos(d*x + c)^2 + 3*(2*
A - 17*B)*cos(d*x + c) + 7*A - 32*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 +
3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^3(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int \frac{B\sec^4(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(B*sec(c + d*x)**4/(sec(c + d*x)**3 + 3*sec(c + d
*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A]
time = 0.49, size = 147, normalized size = 1.18

$$\frac{\frac{60\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^3}-\frac{60\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^3}+\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+10\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-20\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+15\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-105\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(60*B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^3 - 60*B*log(abs(tan(1/2*d*
x + 1/2*c) - 1))/a^3 + (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/2*
d*x + 1/2*c)^5 + 10*A*a^12*tan(1/2*d*x + 1/2*c)^3 - 20*B*a^12*tan(1/2*d*x +
 1/2*c)^3 + 15*A*a^12*tan(1/2*d*x + 1/2*c) - 105*B*a^12*tan(1/2*d*x + 1/2*c
))/a^15)/d
```

Mupad [B]
time = 1.98, size = 124, normalized size = 0.99

$$\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(\frac{A-B}{12\,a^3}+\frac{A-3\,B}{12\,a^3}\right)}{d}+\frac{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{A-B}{4\,a^3}+\frac{A-3\,B}{4\,a^3}-\frac{A+3\,B}{4\,a^3}\right)}{d}+\frac{{\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(A-B\right)}{20\,a^3\,d}+\frac{2\,B\,\mathrm{atanh}\left(\mathrm{tan}\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + a/cos(c + d*x))^3),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*((A - B)/(12*a^3) + (A - 3*B)/(12*a^3)))/d + (tan(c/2
 + (d*x)/2)*((A - B)/(4*a^3) + (A - 3*B)/(4*a^3) - (A + 3*B)/(4*a^3)))/d +
(tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d) + (2*B*atanh(tan(c/2 + (d*x)/2)))
/(a^3*d)
```

3.102 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$-\frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{(3A-8B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{(3A+7B)\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}$$

```
[Out] -1/5*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^3+1/15*(3*A-8*B)*tan(d*x+c)/a/d/(a
+a*sec(d*x+c))^2+1/15*(3*A+7*B)*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A]
time = 0.14, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4093, 4085, 3879}

$$\frac{(3A+7B)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(3A-8B)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(A-B)\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/5*((A - B)*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^3) + ((3*A - 8*B)*Tan[c
 + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((3*A + 7*B)*Tan[c + d*x])/(15*d
*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
```

```
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)(-3a(A-B)-5aB\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$
$$= -\frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(3A+7B}{}$$
$$= -\frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(3A+7}{15d\,(a^3+}$$

Mathematica [A]
time = 0.32, size = 96, normalized size = 0.94

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(5(3A+4B)\sin\left(\frac{dx}{2}\right)-15A\sin\left(c+\frac{dx}{2}\right)+(3A+2B)\left(5\sin\left(c+\frac{3dx}{2}\right)+\sin\left(2c+\frac{5dx}{2}\right)\right)\right)}{30a^3d(1+\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(5*(3*A + 4*B)*Sin[(d*x)/2] - 15*A*Sin[c + (d*x)
/2] + (3*A + 2*B)*(5*Sin[c + (3*d*x)/2] + Sin[2*c + (5*d*x)/2])))/(30*a^3*d
*(1 + Cos[c + d*x])^3)
```

Maple [A]
time = 0.25, size = 64, normalized size = 0.63

method	result
derivativedivides	$\frac{\frac{(-A+B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{2B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$
default	$\frac{\frac{(-A+B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{2B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$
risch	$\frac{2i\left(15A\,e^{3i(dx+c)}+15A\,e^{2i(dx+c)}+20B\,e^{2i(dx+c)}+15\,e^{i(dx+c)}A+10B\,e^{i(dx+c)}+3A+2B\right)}{15d\,a^3\left(e^{i(dx+c)}+1\right)^5}$
norman	$\frac{-\frac{(A-B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad}+\frac{(A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{(3A+2B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}+\frac{(3A+2B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{30ad}+\frac{(6A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{30ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/d/a^3*(1/5*(-A+B)*tan(1/2*d*x+1/2*c)^5+2/3*B*tan(1/2*d*x+1/2*c)^3+A*tan
(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.27, size = 115, normalized size = 1.13

$$\frac{\frac{B\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}+\frac{3\,A\left(\frac{5\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(B*(15*sin(d*x + c)/(cos(d*x + c) + 1) + 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + 3*A*(5*sin(d*x + c
)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A]
time = 5.45, size = 93, normalized size = 0.91

$$\frac{\left((3\,A+2\,B)\cos\left(dx+c\right)^2+3\,(3\,A+2\,B)\cos\left(dx+c\right)+3\,A+7\,B\right)\sin\left(dx+c\right)}{15\,\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/15*((3*A + 2*B)*cos(d*x + c)^2 + 3*(3*A + 2*B)*cos(d*x + c) + 3*A + 7*B)*
sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{A\sec^2\left(c+dx\right)}{\sec^3\left(c+dx\right)+3\sec^2\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx+\int\frac{B\sec^3\left(c+dx\right)}{\sec^3\left(c+dx\right)+3\sec^2\left(c+dx\right)+3\sec\left(c+dx\right)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(B*sec(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d
*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A]
time = 0.50, size = 75, normalized size = 0.74

$$-\frac{3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-10\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(3*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*c)^5 - 10*B*tan(1
/2*d*x + 1/2*c)^3 - 15*A*tan(1/2*d*x + 1/2*c) - 15*B*tan(1/2*d*x + 1/2*c))/
(a^3*d)
```

Mupad [B]
time = 1.92, size = 66, normalized size = 0.65

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(15\,A+15\,B-3\,A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+10\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+3\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\right)}{60\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + a/cos(c + d*x))^3),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(15*A + 15*B - 3*A*tan(c/2 + (d*x)/2)^4 + 10*B*tan(c/2
+ (d*x)/2)^2 + 3*B*tan(c/2 + (d*x)/2)^4))/(60*a^3*d)
```

3.103 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=102

$$\frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{(2A+3B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}+\frac{(2A+3B)\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}$$

```
[Out] 1/5*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^3+1/15*(2*A+3*B)*tan(d*x+c)/a/d/(a+
a*sec(d*x+c))^2+1/15*(2*A+3*B)*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A]
time = 0.08, antiderivative size = 102, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4085, 3881, 3879}

$$\frac{(2A+3B)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(2A+3B)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}+\frac{(A-B)\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((A - B)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) + ((2*A + 3*B)*Tan[c +
d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) + ((2*A + 3*B)*Tan[c + d*x])/(15*d*(a
^3 + a^3*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
```

```
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2A+3B)\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{5a}$$

$$= \frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2A+3B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(2A+3B)\int}{1}$$

$$= \frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(2A+3B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{(2A+3B)}{15d\,(a^3+a^3}$$

Mathematica [A]
time = 0.35, size = 135, normalized size = 1.32

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(5(8A+3B)\sin\left(\frac{dx}{2}\right)-15(2A+B)\sin\left(c+\frac{dx}{2}\right)+20A\sin\left(c+\frac{3dx}{2}\right)+15B\sin\left(c+\frac{3dx}{2}\right)-15A\sin\left(2c+\frac{3dx}{2}\right)+7A\sin\left(2c+\frac{5dx}{2}\right)+3B\sin\left(2c+\frac{5dx}{2}\right)\right)}{30a^3d(1+\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(5*(8*A + 3*B)*Sin[(d*x)/2] - 15*(2*A + B)*Sin[c
 + (d*x)/2] + 20*A*Sin[c + (3*d*x)/2] + 15*B*Sin[c + (3*d*x)/2] - 15*A*Sin[
2*c + (3*d*x)/2] + 7*A*Sin[2*c + (5*d*x)/2] + 3*B*Sin[2*c + (5*d*x)/2]))/(3
0*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A]
time = 0.25, size = 64, normalized size = 0.63

method	result
derivativedivides	$\frac{\frac{(A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$
default	$\frac{\frac{(A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}-\frac{2A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d\,a^3}$
risch	$\frac{2i\left(15A\,e^{4i(dx+c)}+30A\,e^{3i(dx+c)}+15B\,e^{3i(dx+c)}+40A\,e^{2i(dx+c)}+15B\,e^{2i(dx+c)}+20\,e^{i(dx+c)}A+15B\,e^{i(dx+c)}+7A+3B\right)}{15d\,a^3\left(e^{i(dx+c)}+1\right)^5}$
norman	$\frac{\frac{(A-B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad}-\frac{(A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}+\frac{(5A+3B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}-\frac{(13A-3B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^3*(1/5*(A-B)*tan(1/2*d*x+1/2*c)^5-2/3*A*tan(1/2*d*x+1/2*c)^3+A*tan(
1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.28, size = 115, normalized size = 1.13

$$\frac{\frac{A\left(\frac{15\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}+\frac{3B\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}-\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/60*(A*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x +
c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 + 3*B*(5*sin(d*x + c
)/(cos(d*x + c) + 1) - sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d
```

Fricas [A]
time = 3.58, size = 93, normalized size = 0.91

$$\frac{\left((7A+3B)\cos(dx+c)^2+3(2A+3B)\cos(dx+c)+2A+3B\right)\sin(dx+c)}{15\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] 1/15*((7*A + 3*B)*cos(d*x + c)^2 + 3*(2*A + 3*B)*cos(d*x + c) + 2*A + 3*B)*
sin(d*x + c)/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int \frac{B\sec^2(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x) + Integral(B*sec(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)
**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A]
time = 0.48, size = 75, normalized size = 0.74

$$\frac{3\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-10\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{60\,a^{3}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] 1/60*(3*A*tan(1/2*d*x + 1/2*c)^5 - 3*B*tan(1/2*d*x + 1/2*c)^5 - 10*A*tan(1/
2*d*x + 1/2*c)^3 + 15*A*tan(1/2*d*x + 1/2*c) + 15*B*tan(1/2*d*x + 1/2*c))/(
a^3*d)
```

Mupad [B]
time = 1.92, size = 66, normalized size = 0.65

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(15\,A+15\,B-10\,A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{2}+3\,A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}-3\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^{4}\right)}{60\,a^{3}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + a/cos(c + d*x))^3),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(15*A + 15*B - 10*A*tan(c/2 + (d*x)/2)^2 + 3*A*tan(c/2
+ (d*x)/2)^4 - 3*B*tan(c/2 + (d*x)/2)^4))/(60*a^3*d)
```

3.104 $\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=108

$$\frac{Ax}{a^3}-\frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{(7A-2B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2}-\frac{2(11A-B)\tan(c+dx)}{15d\left(a^3+a^3\sec(c+dx)\right)}$$

```
[Out] A*x/a^3-1/5*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^3-1/15*(7*A-2*B)*tan(d*x+c)
/a/d/(a+a*sec(d*x+c))^2-2/15*(11*A-B)*tan(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A]
time = 0.13, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}}=0.130$, Rules used = {4007, 4004, 3879}

$$-\frac{2(11A-B)\tan(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{Ax}{a^3}-\frac{(7A-2B)\tan(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(A-B)\tan(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (A*x)/a^3 - ((A - B)*Tan[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((7*A - 2
*B)*Tan[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (2*(11*A - B)*Tan[c + d
*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{\int \frac{-5aA+2a(A-B)\sec(c+dx)}{(a+a\sec(c+dx))^2}\,dx}{5a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(7A-2B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} + \frac{\int \frac{15a^2A-a^2(7A-2B)\sec(c+}{a+a\sec(c+dx)}}{15a^4}$$

$$= \frac{Ax}{a^3} - \frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(7A-2B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(2(11A-B))\int \frac{}{a}}{15a^2}$$

$$= \frac{Ax}{a^3} - \frac{(A-B)\tan(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(7A-2B)\tan(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{2(11A-B)\tan(}{15d\,(a^3+a^3\sec(}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 241 vs. 2(108) = 216.
time = 0.60, size = 241, normalized size = 2.23

$$\frac{\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{1}{2}(c+dx)\right)\left(150Adx\cos\left(\frac{dx}{2}\right)+150Adx\cos\left(c+\frac{dx}{2}\right)+75Adx\cos\left(c+\frac{3dx}{2}\right)+75Adx\cos\left(2c+\frac{3dx}{2}\right)+15Adx\cos\left(2c+\frac{5dx}{2}\right)+15Adx\cos\left(3c+\frac{5dx}{2}\right)-370A\sin\left(\frac{dx}{2}\right)+80B\sin\left(\frac{dx}{2}\right)+270A\sin\left(c+\frac{dx}{2}\right)-60B\sin\left(c+\frac{dx}{2}\right)-230A\sin\left(c+\frac{3dx}{2}\right)+40B\sin\left(c+\frac{3dx}{2}\right)+90A\sin\left(2c+\frac{3dx}{2}\right)-30B\sin\left(2c+\frac{3dx}{2}\right)-64A\sin\left(2c+\frac{5dx}{2}\right)+14B\sin\left(2c+\frac{5dx}{2}\right)\right)}{480a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^5*(150*A*d*x*Cos[(d*x)/2] + 150*A*d*x*Cos[c + (d
*x)/2] + 75*A*d*x*Cos[c + (3*d*x)/2] + 75*A*d*x*Cos[2*c + (3*d*x)/2] + 15*A
*d*x*Cos[2*c + (5*d*x)/2] + 15*A*d*x*Cos[3*c + (5*d*x)/2] - 370*A*Sin[(d*x)
/2] + 80*B*Sin[(d*x)/2] + 270*A*Sin[c + (d*x)/2] - 60*B*Sin[c + (d*x)/2] -
230*A*Sin[c + (3*d*x)/2] + 40*B*Sin[c + (3*d*x)/2] + 90*A*Sin[2*c + (3*d*x)
/2] - 30*B*Sin[2*c + (3*d*x)/2] - 64*A*Sin[2*c + (5*d*x)/2] + 14*B*Sin[2*c
+ (5*d*x)/2]))/(480*a^3*d)
```

Maple [A]
time = 0.24, size = 102, normalized size = 0.94

method	result
norman	$\frac{\frac{Ax}{a}-\frac{(A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad}+\frac{(2A-B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{6ad}-\frac{(7A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}}{a^2}$
derivativedivides	$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}+\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}+\frac{4A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{2B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8A\,\mathrm{ar}}{4d\,a^3}$
default	$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}+\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}+\frac{4A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-\frac{2B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-7A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+8A\,\mathrm{ar}}{4d\,a^3}$
risch	$\frac{Ax}{a^3}-\frac{2i\left(45A\,e^{4i(dx+c)}-15B\,e^{4i(dx+c)}+135A\,e^{3i(dx+c)}-30B\,e^{3i(dx+c)}+185A\,e^{2i(dx+c)}-40B\,e^{2i(dx+c)}+115\,e^{i(dx+c)}\right.}{15d\,a^3\left(e^{i(dx+c)}+1\right)^5}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/4/d/a^3*(-1/5*tan(1/2*d*x+1/2*c)^5*A+1/5*tan(1/2*d*x+1/2*c)^5*B+4/3*A*tan(1/2*d*x+1/2*c)^3-2/3*B*tan(1/2*d*x+1/2*c)^3-7*A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c)+8*A*arctan(tan(1/2*d*x+1/2*c)))

Maxima [A]
time = 0.49, size = 160, normalized size = 1.48

$$-\frac{A\left(\frac{\frac{105\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-\frac{B\left(\frac{15\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="maxima")

[Out] -1/60*(A*((105*sin(d*x + c)/(cos(d*x + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - B*(15*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3)/d

Fricas [A]
time = 4.24, size = 138, normalized size = 1.28

$$\frac{15\,Adx\cos\left(dx+c\right)^3+45\,Adx\cos\left(dx+c\right)^2+45\,Adx\cos\left(dx+c\right)+15\,Adx-\left(\left(32\,A-7\,B\right)\cos\left(dx+c\right)^2+3\,\left(17\,A-2\,B\right)\cos\left(dx+c\right)+22\,A-2\,B\right)\sin\left(dx+c\right)}{15\,\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fricas")

[Out] 1/15*(15*A*d*x*cos(d*x + c)^3 + 45*A*d*x*cos(d*x + c)^2 + 45*A*d*x*cos(d*x + c) + 15*A*d*x - ((32*A - 7*B)*cos(d*x + c)^2 + 3*(17*A - 2*B)*cos(d*x + c) + 22*A - 2*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx+\int \frac{B\sec(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x)
+ Integral(B*sec(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c +
d*x) + 1), x))/a**3
```

Giac [A]
time = 0.51, size = 121, normalized size = 1.12

$$\frac{\frac{60\,(dx+c)A}{a^3} - \frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 20\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 10\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 105\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 15\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] 1/60*(60*(d*x + c)*A/a^3 - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(
1/2*d*x + 1/2*c)^5 - 20*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 10*B*a^12*tan(1/2*d
*x + 1/2*c)^3 + 105*A*a^12*tan(1/2*d*x + 1/2*c) - 15*B*a^12*tan(1/2*d*x + 1
/2*c))/a^15)/d
```

Mupad [B]
time = 2.13, size = 133, normalized size = 1.23

$$\frac{A\,x}{a^3} + \frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\left(\frac{A\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{3} - \frac{B\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{6}\right) - \cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\left(\frac{7\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4} - \frac{B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{4}\right) - \frac{A\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{20} + \frac{B\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{20}}{a^3\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (A*x)/a^3 + (cos(c/2 + (d*x)/2)^2*((A*sin(c/2 + (d*x)/2)^3)/3 - (B*sin(c/2
+ (d*x)/2)^3)/6) - cos(c/2 + (d*x)/2)^4*((7*A*sin(c/2 + (d*x)/2))/4 - (B*si
n(c/2 + (d*x)/2))/4) - (A*sin(c/2 + (d*x)/2)^5)/20 + (B*sin(c/2 + (d*x)/2)^
5)/20)/(a^3*d*cos(c/2 + (d*x)/2)^5)
```

3.105 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=136

$$-\frac{(3A-B)x}{a^3}+\frac{2(36A-11B)\sin(c+dx)}{15a^3d}-\frac{(A-B)\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{(9A-4B)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2}-\frac{(3A-B)\sin}{d\,(a^3+a^3\sec}$$

```
[Out] -(3*A-B)*x/a^3+2/15*(36*A-11*B)*sin(d*x+c)/a^3/d-1/5*(A-B)*sin(d*x+c)/d/(a+
a*sec(d*x+c))^3-1/15*(9*A-4*B)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-(3*A-B)*si
n(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A]
time = 0.26, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4105, 3872, 2717, 8}

$$\frac{2(36A-11B)\sin(c+dx)}{15a^3d}-\frac{(3A-B)\sin(c+dx)}{d\,(a^3\sec(c+dx)+a^3)}-\frac{x(3A-B)}{a^3}-\frac{(9A-4B)\sin(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -(((3*A - B)*x)/a^3) + (2*(36*A - 11*B)*Sin[c + d*x])/(15*a^3*d) - ((A - B)
*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((9*A - 4*B)*Sin[c + d*x])/(1
5*a*d*(a + a*Sec[c + d*x])^2) - ((3*A - B)*Sin[c + d*x])/(d*(a^3 + a^3*Sec[
c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
```

```
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx &= -\frac{(A-B)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\cos(c+dx)(a(6A-B)-3a(A-B)\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx}{5a^2} \\
&= -\frac{(A-B)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(9A-4B)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} + \int \frac{\cos(c+dx)(\cdots}{\cdots} \\
&= -\frac{(A-B)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(9A-4B)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(3A-B)\cdots}{d\,(a^3+a^3\cdots} \\
&= -\frac{(A-B)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(9A-4B)\sin(c+dx)}{15ad(a+a\sec(c+dx))^2} - \frac{(3A-B)\cdots}{d\,(a^3+a^3\cdots} \\
&= -\frac{(3A-B)x}{a^3} + \frac{2(36A-11B)\sin(c+dx)}{15a^3d} - \frac{(A-B)\sin(c+dx\cdots}{5d(a+a\sec(c+dx)\cdots}
\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 365 vs. 2(136) = 272.
time = 1.07, size = 365, normalized size = 2.68

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(-300(3A-B)dx\cos\left(\frac{dx}{2}\right)-300(3A-B)dx\cos\left(c+\frac{dx}{2}\right)-450Adx\cos\left(c+\frac{3dx}{2}\right)+150Bdx\cos\left(c+\frac{3dx}{2}\right)-450Adx\cos\left(2c+\frac{3dx}{2}\right)+150Bdx\cos\left(2c+\frac{3dx}{2}\right)-90Adx\cos\left(2c+\frac{5dx}{2}\right)+30Bdx\cos\left(2c+\frac{5dx}{2}\right)-90Adx\cos\left(3c+\frac{5dx}{2}\right)+30Bdx\cos\left(3c+\frac{5dx}{2}\right)+1755A\sin\left(\frac{dx}{2}\right)-740B\sin\left(\frac{dx}{2}\right)-1125A\sin\left(c+\frac{dx}{2}\right)+540B\sin\left(c+\frac{dx}{2}\right)+1215A\sin\left(c+\frac{3dx}{2}\right)-460B\sin\left(c+\frac{3dx}{2}\right)-225A\sin\left(2c+\frac{3dx}{2}\right)+180B\sin\left(2c+\frac{3dx}{2}\right)+363A\sin\left(2c+\frac{5dx}{2}\right)-128B\sin\left(2c+\frac{5dx}{2}\right)+75A\sin\left(3c+\frac{5dx}{2}\right)+15A\sin\left(3c+\frac{7dx}{2}\right)+15A\sin\left(4c+\frac{7dx}{2}\right)\right)}{120a^3d(1+\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-300*(3*A - B)*d*x*Cos[(d*x)/2] - 300*(3*A - B)
*d*x*Cos[c + (d*x)/2] - 450*A*d*x*Cos[c + (3*d*x)/2] + 150*B*d*x*Cos[c + (3
*d*x)/2] - 450*A*d*x*Cos[2*c + (3*d*x)/2] + 150*B*d*x*Cos[2*c + (3*d*x)/2]
- 90*A*d*x*Cos[2*c + (5*d*x)/2] + 30*B*d*x*Cos[2*c + (5*d*x)/2] - 90*A*d*x*
Cos[3*c + (5*d*x)/2] + 30*B*d*x*Cos[3*c + (5*d*x)/2] + 1755*A*Sin[(d*x)/2]
- 740*B*Sin[(d*x)/2] - 1125*A*Sin[c + (d*x)/2] + 540*B*Sin[c + (d*x)/2] + 1
215*A*Sin[c + (3*d*x)/2] - 460*B*Sin[c + (3*d*x)/2] - 225*A*Sin[2*c + (3*d*
x)/2] + 180*B*Sin[2*c + (3*d*x)/2] + 363*A*Sin[2*c + (5*d*x)/2] - 128*B*Sin
[2*c + (5*d*x)/2] + 75*A*Sin[3*c + (5*d*x)/2] + 15*A*Sin[3*c + (7*d*x)/2] +
 15*A*Sin[4*c + (7*d*x)/2]))/(120*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [**A**]
time = 0.30, size = 136, normalized size = 1.00

method	result
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}-2A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+17A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-7B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{8A\,\mathrm{ta}}{1+\mathrm{ta}}}{4d\,a^3}$
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}-2A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+17A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-7B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+\frac{8A\,\mathrm{ta}}{1+\mathrm{ta}}}{4d\,a^3}$
norman	$\frac{-\frac{(3A-B)x}{a}+\frac{(A-B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad}-\frac{(3A-B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{(25A-7B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4ad}-\frac{(27A-17B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60ad}+\frac{(45A-1}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2}$
risch	$-\frac{3Ax}{a^3}+\frac{xB}{a^3}-\frac{iA\,\mathrm{e}^{i(dx+c)}}{2a^3d}+\frac{iA\,\mathrm{e}^{-i(dx+c)}}{2a^3d}+\frac{2i\left(90A\,\mathrm{e}^{4i(dx+c)}-45B\,\mathrm{e}^{4i(dx+c)}+300A\,\mathrm{e}^{3i(dx+c)}-135B\,\mathrm{e}^{3i(dx+c)}+\right.}{15d\,a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5*A-1/5*tan(1/2*d*x+1/2*c)^5*B-2*A*tan(1/
2*d*x+1/2*c)^3+4/3*B*tan(1/2*d*x+1/2*c)^3+17*A*tan(1/2*d*x+1/2*c)-7*B*tan(1
/2*d*x+1/2*c)+8*A*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-8*(3*A-B)*arc
tan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]

time = 0.48, size = 231, normalized size = 1.70

$$\frac{3A\left(\frac{40\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-B\left(\frac{\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{20\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] 1/60*(3*A*(40*sin(d*x + c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)
*(cos(d*x + c) + 1)) + (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - B*((105*sin(d*x + c)/(cos(d*x
 + c) + 1) - 20*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos
(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Fricas [A]

time = 7.96, size = 173, normalized size = 1.27

$$-\frac{15\,(3\,A-B)dx\cos\left(dx+c\right)^3+45\,(3\,A-B)dx\cos\left(dx+c\right)^2+45\,(3\,A-B)dx\cos\left(dx+c\right)+15\,(3\,A-B)dx-\left(15\,A\cos\left(dx+c\right)^3+(117\,A-32\,B)\cos\left(dx+c\right)^2+3\,(57\,A-17\,B)\cos\left(dx+c\right)+72\,A-22\,B\right)\sin\left(dx+c\right)}{15\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] -1/15*(15*(3*A - B)*d*x*cos(d*x + c)^3 + 45*(3*A - B)*d*x*cos(d*x + c)^2 +
45*(3*A - B)*d*x*cos(d*x + c) + 15*(3*A - B)*d*x - (15*A*cos(d*x + c)^3 + (
117*A - 32*B)*cos(d*x + c)^2 + 3*(57*A - 17*B)*cos(d*x + c) + 72*A - 22*B)*
sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(
d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\cos(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx + \int \frac{B\cos(c+dx)\sec(c+dx)}{\sec^3(c+dx)+3\sec^2(c+dx)+3\sec(c+dx)+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*cos(c + d*x)/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d
*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x)/(sec(c + d*x)**3 + 3*se
c(c + d*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A]
time = 0.51, size = 157, normalized size = 1.15

$$-\frac{\frac{60\,(dx+c)(3\,A-B)}{a^3} - \frac{120\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3} - \frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 30\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 20\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 255\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 105\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] -1/60*(60*(d*x + c)*(3*A - B)/a^3 - 120*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*
x + 1/2*c)^2 + 1)*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^12*tan(1/
2*d*x + 1/2*c)^5 - 30*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 20*B*a^12*tan(1/2*d*x
 + 1/2*c)^3 + 255*A*a^12*tan(1/2*d*x + 1/2*c) - 105*B*a^12*tan(1/2*d*x + 1/
2*c))/a^15)/d
```

Mupad [B]
time = 1.98, size = 155, normalized size = 1.14

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,A}{2\,a^3}+\frac{3\,(A-B)}{4\,a^3}+\frac{4\,A-2\,B}{2\,a^3}\right)}{d} - \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{A-B}{6\,a^3}+\frac{4\,A-2\,B}{12\,a^3}\right)}{d} - \frac{x\,(3\,A-B)}{a^3} + \frac{2\,A\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)} + \frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(A-B)}{20\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)*((3*A)/(2*a^3) + (3*(A - B))/(4*a^3) + (4*A - 2*B)/(2*a
^3)))/d - (tan(c/2 + (d*x)/2)^3*((A - B)/(6*a^3) + (4*A - 2*B)/(12*a^3)))/d
 - (x*(3*A - B))/a^3 + (2*A*tan(c/2 + (d*x)/2))/(d*(a^3*tan(c/2 + (d*x)/2)^
2 + a^3)) + (tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d)
```

3.106 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=187

$$\frac{(13A-6B)x}{2a^3}-\frac{8(19A-9B)\sin(c+dx)}{15a^3d}+\frac{(13A-6B)\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{(A-B)\cos(c+dx)\sin(c+}{5d(a+a\sec(c+dx))^3}$$

```
[Out] 1/2*(13*A-6*B)*x/a^3-8/15*(19*A-9*B)*sin(d*x+c)/a^3/d+1/2*(13*A-6*B)*cos(d*
x+c)*sin(d*x+c)/a^3/d-1/5*(A-B)*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-
1/15*(11*A-6*B)*cos(d*x+c)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-4/15*(19*A-9*B
)*cos(d*x+c)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))
```

Rubi [A]
time = 0.33, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4105, 3872, 2715, 8, 2717}

$$-\frac{8(19A-9B)\sin(c+dx)}{15a^3d}+\frac{(13A-6B)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{4(19A-9B)\sin(c+dx)\cos(c+dx)}{15d\left(a^3\sec(c+dx)+a^3\right)}+\frac{x(13A-6B)}{2a^3}-\frac{(11A-6B)\sin(c+dx)\cos(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)\cos(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((13*A - 6*B)*x)/(2*a^3) - (8*(19*A - 9*B)*Sin[c + d*x])/(15*a^3*d) + ((13*
A - 6*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A - B)*Cos[c + d*x]*Sin[c
 + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((11*A - 6*B)*Cos[c + d*x]*Sin[c +
d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - (4*(19*A - 9*B)*Cos[c + d*x]*Sin[c
+ d*x])/(15*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\cos^2(c+dx)(a(7A-2B)-4a(A-B)}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(11A-6B)\cos(c+dx)\sin(}{15ad(a+a\sec(c+dx)}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(11A-6B)\cos(c+dx)\sin(}{15ad(a+a\sec(c+dx)}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(11A-6B)\cos(c+dx)\sin(}{15ad(a+a\sec(c+dx)}$$

$$= -\frac{8(19A-9B)\sin(c+dx)}{15a^3d} + \frac{(13A-6B)\cos(c+dx)\sin(c+dx)}{2a^3d}$$

$$= \frac{(13A-6B)x}{2a^3} - \frac{8(19A-9B)\sin(c+dx)}{15a^3d} + \frac{(13A-6B)\cos(c+}{2a^3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 435 vs. 2(187) = 374.
time = 0.78, size = 435, normalized size = 2.33

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(600(13A-6B)dx\cos\left(\frac{dx}{2}\right)+600(13A-6B)dx\cos\left(c+\frac{dx}{2}\right)+3900Adx\cos\left(c+\frac{3dx}{2}\right)-1800Bdx\cos\left(c+\frac{3dx}{2}\right)+3900Adx\cos\left(2c+\frac{3dx}{2}\right)-1800Bdx\cos\left(2c+\frac{3dx}{2}\right)+780Adx\cos\left(2c+\frac{5dx}{2}\right)-360Bdx\cos\left(2c+\frac{5dx}{2}\right)+780Adx\cos\left(3c+\frac{5dx}{2}\right)-360Bdx\cos\left(3c+\frac{5dx}{2}\right)-12760A\sin\left(\frac{dx}{2}\right)+7020B\sin\left(\frac{dx}{2}\right)+7560A\sin\left(c+\frac{dx}{2}\right)-4500B\sin\left(c+\frac{dx}{2}\right)-9230A\sin\left(c+\frac{3dx}{2}\right)+4860B\sin\left(c+\frac{3dx}{2}\right)+930A\sin\left(2c+\frac{3dx}{2}\right)-900B\sin\left(2c+\frac{3dx}{2}\right)-2782A\sin\left(2c+\frac{5dx}{2}\right)+1452B\sin\left(2c+\frac{5dx}{2}\right)-750A\sin\left(3c+\frac{5dx}{2}\right)+300B\sin\left(3c+\frac{5dx}{2}\right)-105A\sin\left(3c+\frac{7dx}{2}\right)+60B\sin\left(3c+\frac{7dx}{2}\right)-105A\sin\left(4c+\frac{7dx}{2}\right)+60B\sin\left(4c+\frac{7dx}{2}\right)+15A\sin\left(4c+\frac{9dx}{2}\right)+15A\sin\left(5c+\frac{9dx}{2}\right)\right)}{480a^3d(1+\cos(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(600*(13*A - 6*B)*d*x*Cos[(d*x)/2] + 600*(13*A -
 6*B)*d*x*Cos[c + (d*x)/2] + 3900*A*d*x*Cos[c + (3*d*x)/2] - 1800*B*d*x*Cos
```

```
[c + (3*d*x)/2] + 3900*A*d*x*Cos[2*c + (3*d*x)/2] - 1800*B*d*x*Cos[2*c + (3
*d*x)/2] + 780*A*d*x*Cos[2*c + (5*d*x)/2] - 360*B*d*x*Cos[2*c + (5*d*x)/2]
+ 780*A*d*x*Cos[3*c + (5*d*x)/2] - 360*B*d*x*Cos[3*c + (5*d*x)/2] - 12760*A
*Sin[(d*x)/2] + 7020*B*Sin[(d*x)/2] + 7560*A*Sin[c + (d*x)/2] - 4500*B*Sin[
c + (d*x)/2] - 9230*A*Sin[c + (3*d*x)/2] + 4860*B*Sin[c + (3*d*x)/2] + 930*
A*Sin[2*c + (3*d*x)/2] - 900*B*Sin[2*c + (3*d*x)/2] - 2782*A*Sin[2*c + (5*d
*x)/2] + 1452*B*Sin[2*c + (5*d*x)/2] - 750*A*Sin[3*c + (5*d*x)/2] + 300*B*S
in[3*c + (5*d*x)/2] - 105*A*Sin[3*c + (7*d*x)/2] + 60*B*Sin[3*c + (7*d*x)/2
] - 105*A*Sin[4*c + (7*d*x)/2] + 60*B*Sin[4*c + (7*d*x)/2] + 15*A*Sin[4*c +
 (9*d*x)/2] + 15*A*Sin[5*c + (9*d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)
```

Maple [A]
time = 0.34, size = 163, normalized size = 0.87

method	result
derivativedivides	$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}+\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}+\frac{8A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-31A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+17B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+}{4d\,a^3}$
default	$\frac{-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}+\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}+\frac{8A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-2B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-31A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+17B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+}{4d\,a^3}$
norman	$\frac{\frac{(13A-6B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{(13A-6B)x}{2a}-\frac{(A-B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad}+\frac{(13A-6B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}+\frac{(17A-12B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{30ad}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2a^2}$
risch	$\frac{13Ax}{2a^3}-\frac{3xB}{a^3}-\frac{iA\,\mathrm{e}^{2i(dx+c)}}{8a^3d}+\frac{3iA\,\mathrm{e}^{i(dx+c)}}{2a^3d}-\frac{i\mathrm{e}^{i(dx+c)}B}{2a^3d}-\frac{3iA\,\mathrm{e}^{-i(dx+c)}}{2a^3d}+\frac{i\mathrm{e}^{-i(dx+c)}B}{2a^3d}+\frac{iA\,\mathrm{e}^{-2i(dx+c)}}{8a^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/4/d/a^3*(-1/5*tan(1/2*d*x+1/2*c)^5*A+1/5*tan(1/2*d*x+1/2*c)^5*B+8/3*A*tan
(1/2*d*x+1/2*c)^3-2*B*tan(1/2*d*x+1/2*c)^3-31*A*tan(1/2*d*x+1/2*c)+17*B*tan
(1/2*d*x+1/2*c)+16*((-7/4*A+1/2*B)*tan(1/2*d*x+1/2*c)^3+(-5/4*A+1/2*B)*tan(
1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+4*(13*A-6*B)*arctan(tan(1/2*d*x+
1/2*c)))
```

Maxima [A]
time = 0.49, size = 322, normalized size = 1.72

$$-\frac{A\left(\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{780\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-3B\left(\frac{40\sin(dx+c)}{\left(a^3+\frac{a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{85\sin(dx+c)}{\cos(dx+c)+1}-\frac{10\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{120\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] -1/60*(A*(60*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x
 + c) + 1)^3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*
x + c)^4/(cos(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*
sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5
)/a^3 - 780*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - 3*B*(40*sin(d*x
+ c)/((a^3 + a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) +
 (85*sin(d*x + c)/(cos(d*x + c) + 1) - 10*sin(d*x + c)^3/(cos(d*x + c) + 1)
^3 + sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 120*arctan(sin(d*x + c)/(co
s(d*x + c) + 1))/a^3))/d
```

Fricas [A]
time = 4.32, size = 190, normalized size = 1.02

$$\frac{15 (13 A - 6 B)dx \cos (dx + c)^3 + 45 (13 A - 6 B)dx \cos (dx + c)^2 + 45 (13 A - 6 B)dx \cos (dx + c) + 15 (13 A - 6 B)dx + \left(15 A \cos (dx + c)^4 - 15 (3 A - 2 B) \cos (dx + c)^3 - (479 A - 234 B) \cos (dx + c)^2 - 3 (239 A - 114 B) \cos (dx + c) - 304 A + 144 B\right) \sin (dx + c)}{30 \left(a^3 d \cos (dx + c)^3 + 3 a^3 d \cos (dx + c)^2 + 3 a^3 d \cos (dx + c) + a^3 d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] 1/30*(15*(13*A - 6*B)*d*x*cos(d*x + c)^3 + 45*(13*A - 6*B)*d*x*cos(d*x + c)
^2 + 45*(13*A - 6*B)*d*x*cos(d*x + c) + 15*(13*A - 6*B)*d*x + (15*A*cos(d*x
 + c)^4 - 15*(3*A - 2*B)*cos(d*x + c)^3 - (479*A - 234*B)*cos(d*x + c)^2 -
3*(239*A - 114*B)*cos(d*x + c) - 304*A + 144*B)*sin(d*x + c))/(a^3*d*cos(d*
x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A \cos^2 (c+dx)}{\sec^3 (c+dx)+3 \sec^2 (c+dx)+3 \sec (c+dx)+1}\, dx + \int \frac{B \cos^2 (c+dx) \sec (c+dx)}{\sec^3 (c+dx)+3 \sec^2 (c+dx)+3 \sec (c+dx)+1}\, dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*cos(c + d*x)**2/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(B*cos(c + d*x)**2*sec(c + d*x)/(sec(c + d*x)**3
+ 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A]
time = 0.49, size = 200, normalized size = 1.07

$$\frac{\frac{30 (dx+c)(13 A-6 B)}{a^3} - \frac{60 \left(7 A \tan(\frac{1}{2} dx+\frac{1}{2} c)^3-2 B \tan(\frac{1}{2} dx+\frac{1}{2} c)^3+5 A \tan(\frac{1}{2} dx+\frac{1}{2} c)-2 B \tan(\frac{1}{2} dx+\frac{1}{2} c)\right)}{\left(\tan(\frac{1}{2} dx+\frac{1}{2} c)^2+1\right)^2 a^3} - \frac{3 A a^{12} \tan(\frac{1}{2} dx+\frac{1}{2} c)^5-3 B a^{12} \tan(\frac{1}{2} dx+\frac{1}{2} c)^5-40 A a^{12} \tan(\frac{1}{2} dx+\frac{1}{2} c)^3+30 B a^{12} \tan(\frac{1}{2} dx+\frac{1}{2} c)^3+465 A a^{12} \tan(\frac{1}{2} dx+\frac{1}{2} c)-255 B a^{12} \tan(\frac{1}{2} dx+\frac{1}{2} c)}{a^{15}}}{60 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] 1/60*(30*(d*x + c)*(13*A - 6*B)/a^3 - 60*(7*A*tan(1/2*d*x + 1/2*c)^3 - 2*B*
tan(1/2*d*x + 1/2*c)^3 + 5*A*tan(1/2*d*x + 1/2*c) - 2*B*tan(1/2*d*x + 1/2*c
))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5
- 3*B*a^12*tan(1/2*d*x + 1/2*c)^5 - 40*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 30*B
*a^12*tan(1/2*d*x + 1/2*c)^3 + 465*A*a^12*tan(1/2*d*x + 1/2*c) - 255*B*a^12
*tan(1/2*d*x + 1/2*c))/a^15)/d
```

Mupad [B]
time = 2.00, size = 204, normalized size = 1.09

$$\frac{x\,(13\,A-6\,B)}{2\,a^3}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{3\,(A-B)}{2\,a^3}+\frac{3\,(5\,A-3\,B)}{4\,a^3}+\frac{10\,A-2\,B}{4\,a^3}\right)}{d}-\frac{(7\,A-2\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+(5\,A-2\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+2\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{A-B}{4\,a^3}+\frac{5\,A-3\,B}{12\,a^3}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(A-B)}{20\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (x*(13*A - 6*B))/(2*a^3) - (tan(c/2 + (d*x)/2)*((3*(A - B))/(2*a^3) + (3*(5
*A - 3*B))/(4*a^3) + (10*A - 2*B)/(4*a^3)))/d - (tan(c/2 + (d*x)/2)^3*(7*A
- 2*B) + tan(c/2 + (d*x)/2)*(5*A - 2*B))/(d*(2*a^3*tan(c/2 + (d*x)/2)^2 + a
^3*tan(c/2 + (d*x)/2)^4 + a^3)) + (tan(c/2 + (d*x)/2)^3*((A - B)/(4*a^3) +
(5*A - 3*B)/(12*a^3)))/d - (tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d)
```

3.107 $\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=218

$$-\frac{(23A-13B)x}{2a^3}+\frac{4(34A-19B)\sin(c+dx)}{5a^3d}-\frac{(23A-13B)\cos(c+dx)\sin(c+dx)}{2a^3d}-\frac{(A-B)\cos^2(c+dx)}{5d(a+a\sec(c+$$

```
[Out] -1/2*(23*A-13*B)*x/a^3+4/5*(34*A-19*B)*sin(d*x+c)/a^3/d-1/2*(23*A-13*B)*cos
(d*x+c)*sin(d*x+c)/a^3/d-1/5*(A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c
))^3-1/15*(13*A-8*B)*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-1/3*(23
*A-13*B)*cos(d*x+c)^2*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))-4/15*(34*A-19*B)*si
n(d*x+c)^3/a^3/d
```

Rubi [A]
time = 0.34, antiderivative size = 218, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4105, 3872, 2713, 2715, 8}

$$-\frac{4(34A-19B)\sin^3(c+dx)}{15a^3d}+\frac{4(34A-19B)\sin(c+dx)}{5a^3d}-\frac{(23A-13B)\sin(c+dx)\cos(c+dx)}{2a^3d}-\frac{(23A-13B)\sin(c+dx)\cos^2(c+dx)}{3d\left(a^3\sec(c+dx)+a^3\right)}-\frac{x(23A-13B)}{2a^3}-\frac{(13A-8B)\sin(c+dx)\cos^2(c+dx)}{15ad(a\sec(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/2*((23*A - 13*B)*x)/a^3 + (4*(34*A - 19*B)*Sin[c + d*x])/(5*a^3*d) - ((2
3*A - 13*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*d) - ((A - B)*Cos[c + d*x]^2*
Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x])^3) - ((13*A - 8*B)*Cos[c + d*x]^2*S
in[c + d*x])/(15*a*d*(a + a*Sec[c + d*x])^2) - ((23*A - 13*B)*Cos[c + d*x]^
2*Sin[c + d*x])/(3*d*(a^3 + a^3*Sec[c + d*x])) - (4*(34*A - 19*B)*Sin[c + d
*x]^3)/(15*a^3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
```

```
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\cos^3(c+dx)(a(8A-3B)-5a(A-B)\mathrm{s}}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(13A-8B)\cos^2(c+dx)\sin}{15ad(a+a\sec(c+dx)}$$

$$= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(13A-8B)\cos^2(c+dx)\sin}{15ad(a+a\sec(c+dx)}$$

$$= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(13A-8B)\cos^2(c+dx)\sin}{15ad(a+a\sec(c+dx)}$$

$$= -\frac{(23A-13B)\cos(c+dx)\sin(c+dx)}{2a^3d} - \frac{(A-B)\cos^2(c+dx)\sin}{5d(a+a\sec(c+dx}$$

$$= -\frac{(23A-13B)x}{2a^3} + \frac{4(34A-19B)\sin(c+dx)}{5a^3d} - \frac{(23A-13B)\cos(}{2}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 491 vs. 2(218) = 436.
time = 1.08, size = 491, normalized size = 2.25

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(-600(23A-13B)dx\cos\left(\frac{dx}{2}\right)-600(23A-13B)dx\cos\left(c+\frac{dx}{2}\right)-6900Adx\cos\left(c+\frac{3dx}{2}\right)+3900Bdx\cos\left(c+\frac{3dx}{2}\right)-6900Adx\cos\left(2c+\frac{3dx}{2}\right)+3900Bdx\cos\left(2c+\frac{3dx}{2}\right)-1380Adx\cos\left(2c+\frac{5dx}{2}\right)+780Bdx\cos\left(2c+\frac{5dx}{2}\right)-1380Adx\cos\left(3c+\frac{5dx}{2}\right)+780Bdx\cos\left(3c+\frac{5dx}{2}\right)+20410A\sin\left(\frac{dx}{2}\right)-12760B\sin\left(\frac{dx}{2}\right)-11110A\sin\left(c+\frac{dx}{2}\right)+7560B\sin\left(c+\frac{dx}{2}\right)+15380A\sin\left(c+\frac{3dx}{2}\right)-9230B\sin\left(c+\frac{3dx}{2}\right)-380A\sin\left(2c+\frac{3dx}{2}\right)+930B\sin\left(2c+\frac{3dx}{2}\right)+4777A\sin\left(2c+\frac{5dx}{2}\right)-2782B\sin\left(2c+\frac{5dx}{2}\right)+1625A\sin\left(3c+\frac{5dx}{2}\right)-750B\sin\left(3c+\frac{5dx}{2}\right)+230A\sin\left(3c+\frac{7dx}{2}\right)-105B\sin\left(3c+\frac{7dx}{2}\right)+230A\sin\left(4c+\frac{7dx}{2}\right)-105B\sin\left(4c+\frac{7dx}{2}\right)-20A\sin\left(4c+\frac{9dx}{2}\right)+15B\sin\left(4c+\frac{9dx}{2}\right)-20A\sin\left(5c+\frac{9dx}{2}\right)+15B\sin\left(5c+\frac{9dx}{2}\right)+5A\sin\left(5c+\frac{11dx}{2}\right)+5A\sin\left(6c+\frac{11dx}{2}\right)\right)}{480a^3d(1+\cos(c+dx))^3}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]

[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-600*(23*A - 13*B)*d*x*Cos[(d*x)/2] - 600*(23*A - 13*B)*d*x*Cos[c + (d*x)/2] - 6900*A*d*x*Cos[c + (3*d*x)/2] + 3900*B*d*x*Cos[c + (3*d*x)/2] - 6900*A*d*x*Cos[2*c + (3*d*x)/2] + 3900*B*d*x*Cos[2*c + (3*d*x)/2] - 1380*A*d*x*Cos[2*c + (5*d*x)/2] + 780*B*d*x*Cos[2*c + (5*d*x)/2] - 1380*A*d*x*Cos[3*c + (5*d*x)/2] + 780*B*d*x*Cos[3*c + (5*d*x)/2] + 20410*A*Sin[(d*x)/2] - 12760*B*Sin[(d*x)/2] - 11110*A*Sin[c + (d*x)/2] + 7560*B*Sin[c + (d*x)/2] + 15380*A*Sin[c + (3*d*x)/2] - 9230*B*Sin[c + (3*d*x)/2] - 380*A*Sin[2*c + (3*d*x)/2] + 930*B*Sin[2*c + (3*d*x)/2] + 4777*A*Sin[2*c + (5*d*x)/2] - 2782*B*Sin[2*c + (5*d*x)/2] + 1625*A*Sin[3*c + (5*d*x)/2] - 750*B*Sin[3*c + (5*d*x)/2] + 230*A*Sin[3*c + (7*d*x)/2] - 105*B*Sin[3*c + (7*d*x)/2] + 230*A*Sin[4*c + (7*d*x)/2] - 105*B*Sin[4*c + (7*d*x)/2] - 20*A*Sin[4*c + (9*d*x)/2] + 15*B*Sin[4*c + (9*d*x)/2] - 20*A*Sin[5*c + (9*d*x)/2] + 15*B*Sin[5*c + (9*d*x)/2] + 5*A*Sin[5*c + (11*d*x)/2] + 5*A*Sin[6*c + (11*d*x)/2]))/(480*a^3*d*(1 + Cos[c + d*x])^3)

Maple [A]
time = 0.37, size = 182, normalized size = 0.83

method	result
derivativedivides	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}-\frac{10A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{8B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+49A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-31B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-16}{4}$
default	$\frac{\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}-\frac{\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}-\frac{10A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{8B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+49A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-31B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-16}{4}$
norman	$-\frac{(23A-13B)x}{2a}+\frac{(A-B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{20ad}-\frac{3(23A-13B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{3(23A-13B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{(23A-13B)x\left(\tan^6\right.}{2a}$
risch	$-\frac{23Ax}{2a^3}+\frac{13xB}{2a^3}-\frac{iA\,e^{3i(dx+c)}}{24a^3d}+\frac{3iA\,e^{2i(dx+c)}}{8a^3d}-\frac{i e^{2i(dx+c)}B}{8a^3d}-\frac{27iA\,e^{i(dx+c)}}{8a^3d}+\frac{3ie^{i(dx+c)}B}{2a^3d}+\frac{27iA\,e^{-i(}}{8a^3}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/4/d/a^3*(1/5*tan(1/2*d*x+1/2*c)^5*A-1/5*tan(1/2*d*x+1/2*c)^5*B-10/3*A*tan(1/2*d*x+1/2*c)^3+8/3*B*tan(1/2*d*x+1/2*c)^3+49*A*tan(1/2*d*x+1/2*c)-31*B*tan(1/2*d*x+1/2*c)-16*((-17/4*A+7/4*B)*tan(1/2*d*x+1/2*c)^5+(-19/3*A+3*B)*tan(1/2*d*x+1/2*c)^3+(-11/4*A+5/4*B)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^3-4*(23*A-13*B)*arctan(tan(1/2*d*x+1/2*c)))

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(204) = 408$.
time = 0.49, size = 412, normalized size = 1.89

$$\frac{A\left(\frac{20\left(\frac{33\sin(dx+c)}{\cos(dx+c)+1}+\frac{76\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{51\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^3+\frac{3a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^3\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{735\sin(dx+c)}{\cos(dx+c)+1}-\frac{50\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{1380\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)-B\left(\frac{60\left(\frac{5\sin(dx+c)}{\cos(dx+c)+1}+\frac{7\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^3+\frac{2a^3\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^3\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{465\sin(dx+c)}{\cos(dx+c)+1}-\frac{40\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{3\sin(dx+c)^5}{(\cos(dx+c)+1)^5}}{a^3}-\frac{780\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^3}\right)}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] 1/60*(A*(20*(33*sin(d*x + c)/(cos(d*x + c) + 1) + 76*sin(d*x + c)^3/(cos(d*
x + c) + 1)^3 + 51*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^3 + 3*a^3*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^3*sin(d*x + c)^4/(cos(d*x + c) + 1)^4 +
 a^3*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (735*sin(d*x + c)/(cos(d*x + c)
 + 1) - 50*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x
+ c) + 1)^5)/a^3 - 1380*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3) - B*(6
0*(5*sin(d*x + c)/(cos(d*x + c) + 1) + 7*sin(d*x + c)^3/(cos(d*x + c) + 1)^
3)/(a^3 + 2*a^3*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^3*sin(d*x + c)^4/(c
os(d*x + c) + 1)^4) + (465*sin(d*x + c)/(cos(d*x + c) + 1) - 40*sin(d*x + c
)^3/(cos(d*x + c) + 1)^3 + 3*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/a^3 - 780
*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^3))/d
```

Fricas [A]
time = 3.11, size = 205, normalized size = 0.94

$$-\frac{15\,(23\,A-13\,B)dx\cos\left(dx+c\right)^{3}+45\,(23\,A-13\,B)dx\cos\left(dx+c\right)^{2}+45\,(23\,A-13\,B)dx\cos\left(dx+c\right)+15\,(23\,A-13\,B)dx-\left(10\,A\cos\left(dx+c\right)^{5}-15\,(A-B)\cos\left(dx+c\right)^{4}+5\,(19\,A-9\,B)\cos\left(dx+c\right)^{3}+(869\,A-479\,B)\cos\left(dx+c\right)^{2}+3\,(429\,A-239\,B)\cos\left(dx+c\right)+544\,A-304\,B\right)\sin\left(dx+c\right)}{30\,\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] -1/30*(15*(23*A - 13*B)*d*x*cos(d*x + c)^3 + 45*(23*A - 13*B)*d*x*cos(d*x +
 c)^2 + 45*(23*A - 13*B)*d*x*cos(d*x + c) + 15*(23*A - 13*B)*d*x - (10*A*co
s(d*x + c)^5 - 15*(A - B)*cos(d*x + c)^4 + 5*(19*A - 9*B)*cos(d*x + c)^3 +
(869*A - 479*B)*cos(d*x + c)^2 + 3*(429*A - 239*B)*cos(d*x + c) + 544*A - 3
04*B)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*
d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\cos^{3}(c+dx)}{\sec^{3}(c+dx)+3\sec^{2}(c+dx)+3\sec(c+dx)+1}\,dx+\int \frac{B\cos^{3}(c+dx)\sec(c+dx)}{\sec^{3}(c+dx)+3\sec^{2}(c+dx)+3\sec(c+dx)+1}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*cos(c + d*x)**3/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec(c
+ d*x) + 1), x) + Integral(B*cos(c + d*x)**3*sec(c + d*x)/(sec(c + d*x)**3
+ 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [A]
time = 0.52, size = 228, normalized size = 1.05

$$-\frac{\frac{30\,(dx+c)(23\,A-13\,B)}{a^3}-\frac{20\left(51\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-21\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+76\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-36\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+33\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^3}-\frac{3\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-50\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+735\,Aa^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-465\,Ba^{12}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{15}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/60*(30*(d*x + c)*(23*A - 13*B)/a^3 - 20*(51*A*tan(1/2*d*x + 1/2*c)^5 - 2
1*B*tan(1/2*d*x + 1/2*c)^5 + 76*A*tan(1/2*d*x + 1/2*c)^3 - 36*B*tan(1/2*d*x
 + 1/2*c)^3 + 33*A*tan(1/2*d*x + 1/2*c) - 15*B*tan(1/2*d*x + 1/2*c))/((tan(
1/2*d*x + 1/2*c)^2 + 1)^3*a^3) - (3*A*a^12*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^1
2*tan(1/2*d*x + 1/2*c)^5 - 50*A*a^12*tan(1/2*d*x + 1/2*c)^3 + 40*B*a^12*tan
(1/2*d*x + 1/2*c)^3 + 735*A*a^12*tan(1/2*d*x + 1/2*c) - 465*B*a^12*tan(1/2*
d*x + 1/2*c))/a^15)/d
```

Mupad [B]
time = 2.05, size = 237, normalized size = 1.09

$$\frac{(17\,A-7\,B)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{76\,A}{3}-12\,B\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+(11\,A-5\,B)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^3\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{5\,(A-B)}{2\,a^3}+\frac{6\,A-4\,B}{a^3}+\frac{15\,A-5\,B}{4\,a^3}\right)}{d}-\frac{x\,(23\,A-13\,B)}{2\,a^3}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{A-B}{3\,a^3}+\frac{6\,A-4\,B}{12\,a^3}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5(A-B)}{20\,a^3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^3,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(17*A - 7*B) + tan(c/2 + (d*x)/2)^3*((76*A)/3 - 12*B)
 + tan(c/2 + (d*x)/2)*(11*A - 5*B))/(d*(3*a^3*tan(c/2 + (d*x)/2)^2 + 3*a^3*
tan(c/2 + (d*x)/2)^4 + a^3*tan(c/2 + (d*x)/2)^6 + a^3)) + (tan(c/2 + (d*x)/
2)*((5*(A - B))/(2*a^3) + (6*A - 4*B)/a^3 + (15*A - 5*B)/(4*a^3)))/d - (x*(
23*A - 13*B))/(2*a^3) - (tan(c/2 + (d*x)/2)^3*((A - B)/(3*a^3) + (6*A - 4*B
)/(12*a^3)))/d + (tan(c/2 + (d*x)/2)^5*(A - B))/(20*a^3*d)
```

3.108 $\int \frac{\sec^6(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=238

$$-\frac{(8A-21B)\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{8(83A-216B)\tan(c+dx)}{105a^4d}-\frac{(8A-21B)\sec(c+dx)\tan(c+dx)}{2a^4d}+\frac{(52A$$

```
[Out] -1/2*(8*A-21*B)*arctanh(sin(d*x+c))/a^4/d+8/105*(83*A-216*B)*tan(d*x+c)/a^4
/d-1/2*(8*A-21*B)*sec(d*x+c)*tan(d*x+c)/a^4/d+1/105*(52*A-129*B)*sec(d*x+c)
^3*tan(d*x+c)/a^4/d/(1+sec(d*x+c))^2+4/105*(83*A-216*B)*sec(d*x+c)^2*tan(d*
x+c)/a^4/d/(1+sec(d*x+c))+1/7*(A-B)*sec(d*x+c)^5*tan(d*x+c)/d/(a+a*sec(d*x+
c))^4+1/5*(A-2*B)*sec(d*x+c)^4*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3
```

Rubi [A]
time = 0.45, antiderivative size = 238, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4104, 3872, 3852, 8, 3853, 3855}

$$\frac{8(83A-216B)\tan(c+dx)}{105a^4d}-\frac{(8A-21B)\tanh^{-1}(\sin(c+dx))}{2a^4d}+\frac{(52A-129B)\tan(c+dx)\sec^3(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{4(83A-216B)\tan(c+dx)\sec^2(c+dx)}{105a^4d(\sec(c+dx)+1)}-\frac{(8A-21B)\tan(c+dx)\sec(c+dx)}{2a^4d}+\frac{(A-B)\tan(c+dx)\sec^5(c+dx)}{7d(a\sec(c+dx)+a)^4}+\frac{(A-2B)\tan(c+dx)\sec^4(c+dx)}{5ad(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^6*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -1/2*((8*A - 21*B)*ArcTanh[Sin[c + d*x]])/(a^4*d) + (8*(83*A - 216*B)*Tan[c
 + d*x])/(105*a^4*d) - ((8*A - 21*B)*Sec[c + d*x]*Tan[c + d*x])/(2*a^4*d) +
 ((52*A - 129*B)*Sec[c + d*x]^3*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])
^2) + (4*(83*A - 216*B)*Sec[c + d*x]^2*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c
+ d*x])) + ((A - B)*Sec[c + d*x]^5*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^
4) + ((A - 2*B)*Sec[c + d*x]^4*Tan[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^6(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx = \frac{(A-B)\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\sec^5(c+dx)(5a(A-B)-a(2A-9B)\,\mathrm{se}}{(a+a\sec(c+dx))^3}}{7a^2}$$

$$= \frac{(A-B)\sec^5(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(A-2B)\sec^4(c+dx)\tan(c}{5ad(a+a\sec(c+dx))^3}$$

$$= \frac{(52A-129B)\sec^3(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^5(c+dx)\,\mathrm{t}}{7d(a+a\sec(c+}$$

$$= \frac{(52A-129B)\sec^3(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^5(c+dx)\,\mathrm{t}}{7d(a+a\sec(c+}$$

$$= \frac{(52A-129B)\sec^3(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^5(c+dx)\,\mathrm{t}}{7d(a+a\sec(c+}$$

$$= -\frac{(8A-21B)\sec(c+dx)\tan(c+dx)}{2a^4d} + \frac{(52A-129B)\sec^3(c+}{105a^4d(1+\sec(c}$$

$$= -\frac{(8A-21B)\tanh^{-1}(\sin(c+dx))}{2a^4d} + \frac{8(83A-216B)\tan(c+dx)}{105a^4d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 880 vs. $2(238) = 476$.

time = 6.52, size = 880, normalized size = 3.70

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^6*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (-8*(-8*A + 21*B)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (
d*x)/2]]*Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(d*(B + A*Cos[c + d*x])*(a +
a*Sec[c + d*x])^4) + (8*(-8*A + 21*B)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d
*x)/2] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(d*(B + A
*Cos[c + d*x])*(a + a*Sec[c + d*x])^4) + (Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c
]*Sec[c + d*x]^5*(A + B*Sec[c + d*x])*(-38668*A*Sin[(d*x)/2] + 73206*B*Sin[
(d*x)/2] + 64384*A*Sin[(3*d*x)/2] - 166668*B*Sin[(3*d*x)/2] - 70896*A*Sin[c
 - (d*x)/2] + 183162*B*Sin[c - (d*x)/2] + 50316*A*Sin[c + (d*x)/2] - 100842
*B*Sin[c + (d*x)/2] - 59248*A*Sin[2*c + (d*x)/2] + 155526*B*Sin[2*c + (d*x)
/2] - 22820*A*Sin[c + (3*d*x)/2] + 37380*B*Sin[c + (3*d*x)/2] + 48004*A*Sin
[2*c + (3*d*x)/2] - 101148*B*Sin[2*c + (3*d*x)/2] - 39200*A*Sin[3*c + (3*d*
x)/2] + 102900*B*Sin[3*c + (3*d*x)/2] + 46032*A*Sin[c + (5*d*x)/2] - 119364
*B*Sin[c + (5*d*x)/2] - 8750*A*Sin[2*c + (5*d*x)/2] + 8820*B*Sin[2*c + (5*d
*x)/2] + 35742*A*Sin[3*c + (5*d*x)/2] - 78204*B*Sin[3*c + (5*d*x)/2] - 1904
0*A*Sin[4*c + (5*d*x)/2] + 49980*B*Sin[4*c + (5*d*x)/2] + 24664*A*Sin[2*c +
 (7*d*x)/2] - 64053*B*Sin[2*c + (7*d*x)/2] - 1050*A*Sin[3*c + (7*d*x)/2] -
3885*B*Sin[3*c + (7*d*x)/2] + 19834*A*Sin[4*c + (7*d*x)/2] - 44733*B*Sin[4*
c + (7*d*x)/2] - 5880*A*Sin[5*c + (7*d*x)/2] + 15435*B*Sin[5*c + (7*d*x)/2]
 + 8456*A*Sin[3*c + (9*d*x)/2] - 21987*B*Sin[3*c + (9*d*x)/2] + 630*A*Sin[4
*c + (9*d*x)/2] - 3675*B*Sin[4*c + (9*d*x)/2] + 6986*A*Sin[5*c + (9*d*x)/2]
 - 16107*B*Sin[5*c + (9*d*x)/2] - 840*A*Sin[6*c + (9*d*x)/2] + 2205*B*Sin[6
*c + (9*d*x)/2] + 1328*A*Sin[4*c + (11*d*x)/2] - 3456*B*Sin[4*c + (11*d*x)/
2] + 210*A*Sin[5*c + (11*d*x)/2] - 840*B*Sin[5*c + (11*d*x)/2] + 1118*A*Sin
[6*c + (11*d*x)/2] - 2616*B*Sin[6*c + (11*d*x)/2]))/(6720*d*(B + A*Cos[c +
d*x])*(a + a*Sec[c + d*x])^4)
```

Maple [A]

time = 0.23, size = 234, normalized size = 0.98

method	result
derivativedivides	$\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}+\frac{7\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}-\frac{9\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}+\frac{23A\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-13B\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}{}$
default	$\frac{\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}+\frac{7\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}-\frac{9\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}+\frac{23A\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-13B\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}{}$

risch	$\frac{i\left(840A\,e^{10i(dx+c)}-2205\,e^{10i(dx+c)}B+5880A\,e^{9i(dx+c)}-15435B\,e^{9i(dx+c)}+19040A\,e^{8i(dx+c)}-49980B\,e^{8i(dx+c)}+3920\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^6*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7*A-1/7*tan(1/2*d*x+1/2*c)^7*B+7/5*tan(1/
2*d*x+1/2*c)^5*A-9/5*tan(1/2*d*x+1/2*c)^5*B+23/3*A*tan(1/2*d*x+1/2*c)^3-13*
B*tan(1/2*d*x+1/2*c)^3+49*A*tan(1/2*d*x+1/2*c)-111*B*tan(1/2*d*x+1/2*c)+(-8
4*B+32*A)*ln(tan(1/2*d*x+1/2*c)-1)-(-36*B+8*A)/(tan(1/2*d*x+1/2*c)-1)+4*B/(
tan(1/2*d*x+1/2*c)-1)^2-(-36*B+8*A)/(tan(1/2*d*x+1/2*c)+1)+(84*B-32*A)*ln(t
an(1/2*d*x+1/2*c)+1)-4*B/(tan(1/2*d*x+1/2*c)+1)^2)
```

Maxima [A]
time = 0.28, size = 419, normalized size = 1.76

$$-\frac{3B\left(\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}-\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4-\frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}+\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{2940\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{2940\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}\right)-A\left(\frac{1680\sin(dx+c)}{\left(a^4-\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\sin(dx+c)}{\cos(dx+c)+1}+\frac{805\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{3360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{3360\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}\right)}{840d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] -1/840*(3*B*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) - 9*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3)/(a^4 - 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1)
+ 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 2940*log(sin(d*x + c
)/(cos(d*x + c) + 1) + 1)/a^4 + 2940*log(sin(d*x + c)/(cos(d*x + c) + 1) -
1)/a^4) - A*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1
)^2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d
*x + c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4))
/d
```

Fricas [A]
time = 3.19, size = 358, normalized size = 1.50

$$-\frac{105\left((8A-21B)\cos(dx+c)^6+4(8A-21B)\cos(dx+c)^5+6(8A-21B)\cos(dx+c)^4+4(8A-21B)\cos(dx+c)^3+(8A-21B)\cos(dx+c)^2\right)\log(\sin(dx+c)+1)-105\left((8A-21B)\cos(dx+c)^6+4(8A-21B)\cos(dx+c)^5+6(8A-21B)\cos(dx+c)^4+4(8A-21B)\cos(dx+c)^3+(8A-21B)\cos(dx+c)^2\right)\log(-\sin(dx+c)+1)-2\left(16(83A-216B)\cos(dx+c)^5+(4472A-11619B)\cos(dx+c)^4+4(1318A-3411B)\cos(dx+c)^3+4(592A-1509B)\cos(dx+c)^2+210(A-2B)\cos(dx+c)+105B\right)\sin(dx+c)}{420\left(a^4d\cos(dx+c)^6+4a^4d\cos(dx+c)^5+6a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+a^4d\cos(dx+c)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] -1/420*(105*((8*A - 21*B)*cos(d*x + c)^6 + 4*(8*A - 21*B)*cos(d*x + c)^5 +
6*(8*A - 21*B)*cos(d*x + c)^4 + 4*(8*A - 21*B)*cos(d*x + c)^3 + (8*A - 21*B
)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - 105*((8*A - 21*B)*cos(d*x + c)^6
+ 4*(8*A - 21*B)*cos(d*x + c)^5 + 6*(8*A - 21*B)*cos(d*x + c)^4 + 4*(8*A -
21*B)*cos(d*x + c)^3 + (8*A - 21*B)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1)
- 2*(16*(83*A - 216*B)*cos(d*x + c)^5 + (4472*A - 11619*B)*cos(d*x + c)^4 +
 4*(1318*A - 3411*B)*cos(d*x + c)^3 + 4*(592*A - 1509*B)*cos(d*x + c)^2 + 2
10*(A - 2*B)*cos(d*x + c) + 105*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^6 + 4*
a^4*d*cos(d*x + c)^5 + 6*a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + a^
4*d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^6(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int \frac{B\sec^7(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**6*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A*sec(c + d*x)**6/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c
+ d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**7/(sec(c + d
*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a
**4
```

Giac [A]
time = 0.51, size = 267, normalized size = 1.12

$$-\frac{\frac{420\,(8\,A-21\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{420\,(8\,A-21\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}+\frac{840\left(2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2a^4}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+147\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-189\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+805\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1365\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5145\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-11655\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^6*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/840*(420*(8*A - 21*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 420*(8*A
- 21*B)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 + 840*(2*A*tan(1/2*d*x + 1/2
*c)^3 - 9*B*tan(1/2*d*x + 1/2*c)^3 - 2*A*tan(1/2*d*x + 1/2*c) + 7*B*tan(1/2
*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*a^4) - (15*A*a^24*tan(1/2*d*
x + 1/2*c)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c)^7 + 147*A*a^24*tan(1/2*d*x +
1/2*c)^5 - 189*B*a^24*tan(1/2*d*x + 1/2*c)^5 + 805*A*a^24*tan(1/2*d*x + 1/2
*c)^3 - 1365*B*a^24*tan(1/2*d*x + 1/2*c)^3 + 5145*A*a^24*tan(1/2*d*x + 1/2*
c) - 11655*B*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

Mupad [B]
time = 2.05, size = 272, normalized size = 1.14

$$\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\left(\frac{A-B}{4\,a^4}+\frac{4\,A-6\,B}{8\,a^4}+\frac{5\,A-15\,B}{24\,a^4}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(\frac{5\,(A-B)}{4\,a^4}-\frac{5\,B}{2\,a^4}+\frac{3\,(4\,A-6\,B)}{4\,a^4}+\frac{3\,(5\,A-15\,B)}{8\,a^4}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3(2\,A-9\,B)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,(2\,A-7\,B)}{d\,\left(a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-2\,a^4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+a^4\right)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\,\left(\frac{3\,(A-B)}{40\,a^4}+\frac{4\,A-6\,B}{40\,a^4}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7(A-B)}{56\,a^4\,d}-\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)\,(8\,A-21\,B)}{a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^6*(a + a/cos(c + d*x))^4),x)
```

```
[Out] (tan(c/2 + (d*x)/2)^3*((A - B)/(4*a^4) + (4*A - 6*B)/(8*a^4) + (5*A - 15*B)
/(24*a^4)))/d + (tan(c/2 + (d*x)/2)*((5*(A - B))/(4*a^4) - (5*B)/(2*a^4) +
(3*(4*A - 6*B))/(4*a^4) + (3*(5*A - 15*B))/(8*a^4)))/d - (tan(c/2 + (d*x)/2
)^3*(2*A - 9*B) - tan(c/2 + (d*x)/2)*(2*A - 7*B))/(d*(a^4*tan(c/2 + (d*x)/2
)^4 - 2*a^4*tan(c/2 + (d*x)/2)^2 + a^4)) + (tan(c/2 + (d*x)/2)^5*((3*(A - B
))/(40*a^4) + (4*A - 6*B)/(40*a^4)))/d + (tan(c/2 + (d*x)/2)^7*(A - B))/(56
*a^4*d) - (atanh(tan(c/2 + (d*x)/2))*(8*A - 21*B))/(a^4*d)
```

3.109 $\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=194

$$\frac{(A-4B)\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(55A-244B)\tan(c+dx)}{105a^4d}+\frac{(25A-88B)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2}-\frac{(A-4}{a^4d(1+}$$

```
[Out] (A-4*B)*arctanh(sin(d*x+c))/a^4/d-1/105*(55*A-244*B)*tan(d*x+c)/a^4/d+1/105
*(25*A-88*B)*sec(d*x+c)^2*tan(d*x+c)/a^4/d/(1+sec(d*x+c))^2-(A-4*B)*tan(d*x
+c)/a^4/d/(1+sec(d*x+c))+1/7*(A-B)*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c
))^4+1/35*(5*A-12*B)*sec(d*x+c)^3*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3
```

Rubi [A]
time = 0.41, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4104, 4093, 3872, 3855, 3852, 8}

$$-\frac{(55A-244B)\tan(c+dx)}{105a^4d}+\frac{(A-4B)\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{(25A-88B)\tan(c+dx)\sec^2(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{(A-4B)\tan(c+dx)}{a^4d(\sec(c+dx)+1)}+\frac{(A-B)\tan(c+dx)\sec^4(c+dx)}{7d(a\sec(c+dx)+a)^4}+\frac{(5A-12B)\tan(c+dx)\sec^3(c+dx)}{35ad(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] ((A - 4*B)*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((55*A - 244*B)*Tan[c + d*x])/(
105*a^4*d) + ((25*A - 88*B)*Sec[c + d*x]^2*Tan[c + d*x])/(105*a^4*d*(1 + Se
c[c + d*x])^2) - ((A - 4*B)*Tan[c + d*x])/(a^4*d*(1 + Sec[c + d*x])) + ((A
- B)*Sec[c + d*x]^4*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((5*A - 12
*B)*Sec[c + d*x]^3*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx = \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\sec^4(c+dx)(4a(A-B)-a(A-8B)\sec}{(a+a\sec(c+dx))^3}}{7a^2}$$

$$= \frac{(A-B)\sec^4(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(5A-12B)\sec^3(c+dx)\tan}{35ad(a+a\sec(c+dx)}$$

$$= \frac{(25A-88B)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^4(c+dx)\tan}{7d(a+a\sec(c+d}$$

$$= \frac{(25A-88B)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^4(c+dx)\tan}{7d(a+a\sec(c+d}$$

$$= \frac{(25A-88B)\sec^2(c+dx)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^4(c+dx)\tan}{7d(a+a\sec(c+d}$$

$$= \frac{(A-4B)\tanh^{-1}(\sin(c+dx))}{a^4d} + \frac{(25A-88B)\sec^2(c+dx)\tan(c}{105a^4d(1+\sec(c+dx))^2}$$

$$= \frac{(A-4B)\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{(55A-244B)\tan(c+dx)}{105a^4d} + \frac{(2}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 754 vs. 2(194) = 388.

time = 6.42, size = 754, normalized size = 3.89

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (16*(-A + 4*B)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2] - Sin[c/2 + (d*x
)/2]]*Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(d*(B + A*Cos[c + d*x])*(a + a*S
ec[c + d*x])^4) - (16*(-A + 4*B)*Cos[c/2 + (d*x)/2]^8*Log[Cos[c/2 + (d*x)/2
] + Sin[c/2 + (d*x)/2]]*Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(d*(B + A*Cos[
c + d*x])*(a + a*Sec[c + d*x])^4) + (Cos[c/2 + (d*x)/2]*Sec[c/2]*Sec[c]*Sec
[c + d*x]^4*(A + B*Sec[c + d*x])*(4165*A*Sin[(d*x)/2] - 10780*B*Sin[(d*x)/2
] - 4445*A*Sin[(3*d*x)/2] + 18788*B*Sin[(3*d*x)/2] + 4795*A*Sin[c - (d*x)/2
] - 20524*B*Sin[c - (d*x)/2] - 4795*A*Sin[c + (d*x)/2] + 14644*B*Sin[c + (d
*x)/2] + 4165*A*Sin[2*c + (d*x)/2] - 16660*B*Sin[2*c + (d*x)/2] + 2275*A*Si
n[c + (3*d*x)/2] - 4690*B*Sin[c + (3*d*x)/2] - 4445*A*Sin[2*c + (3*d*x)/2]
+ 14378*B*Sin[2*c + (3*d*x)/2] + 2275*A*Sin[3*c + (3*d*x)/2] - 9100*B*Sin[3
*c + (3*d*x)/2] - 2785*A*Sin[c + (5*d*x)/2] + 11668*B*Sin[c + (5*d*x)/2] +
735*A*Sin[2*c + (5*d*x)/2] - 630*B*Sin[2*c + (5*d*x)/2] - 2785*A*Sin[3*c +
(5*d*x)/2] + 9358*B*Sin[3*c + (5*d*x)/2] + 735*A*Sin[4*c + (5*d*x)/2] - 294
0*B*Sin[4*c + (5*d*x)/2] - 1015*A*Sin[2*c + (7*d*x)/2] + 4228*B*Sin[2*c + (
7*d*x)/2] + 105*A*Sin[3*c + (7*d*x)/2] + 315*B*Sin[3*c + (7*d*x)/2] - 1015*
A*Sin[4*c + (7*d*x)/2] + 3493*B*Sin[4*c + (7*d*x)/2] + 105*A*Sin[5*c + (7*d
*x)/2] - 420*B*Sin[5*c + (7*d*x)/2] - 160*A*Sin[3*c + (9*d*x)/2] + 664*B*Si
n[3*c + (9*d*x)/2] + 105*B*Sin[4*c + (9*d*x)/2] - 160*A*Sin[5*c + (9*d*x)/2
] + 559*B*Sin[5*c + (9*d*x)/2]))/(1680*d*(B + A*Cos[c + d*x])*(a + a*Sec[c
+ d*x])^4)
```

Maple [A]

time = 0.22, size = 190, normalized size = 0.98

method	result
derivativedivides	$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}+\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A+\frac{7\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}-\frac{11A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{23B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-}{}$
default	$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}+\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A+\frac{7\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}-\frac{11A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{23B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-}{}$
risch	$-\frac{2i\left(105A\,e^{8i(dx+c)}-420B\,e^{8i(dx+c)}+735A\,e^{7i(dx+c)}-2940B\,e^{7i(dx+c)}+2275A\,e^{6i(dx+c)}-9100B\,e^{6i(dx+c)}+4165A\,e^{5i(}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7*A+1/7*tan(1/2*d*x+1/2*c)^7*B-tan(1/2*d
*x+1/2*c)^5*A+7/5*tan(1/2*d*x+1/2*c)^5*B-11/3*A*tan(1/2*d*x+1/2*c)^3+23/3*B
*tan(1/2*d*x+1/2*c)^3-15*A*tan(1/2*d*x+1/2*c)+49*B*tan(1/2*d*x+1/2*c)+(32*B
-8*A)*ln(tan(1/2*d*x+1/2*c)-1)-8*B/(tan(1/2*d*x+1/2*c)-1)+(-32*B+8*A)*ln(ta
n(1/2*d*x+1/2*c)+1)-8*B/(tan(1/2*d*x+1/2*c)+1))
```

Maxima [A]
time = 0.28, size = 326, normalized size = 1.68

$$\frac{B\left(\frac{1680\,\sin\left(dx+c\right)}{\left(a^4-\frac{a^4\sin\left(dx+c\right)^2}{\left(\cos\left(dx+c\right)+1\right)^2}\right)\left(\cos\left(dx+c\right)+1\right)}+\frac{\frac{5145\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{805\,\sin\left(dx+c\right)^3}{\left(\cos\left(dx+c\right)+1\right)^3}+\frac{147\,\sin\left(dx+c\right)^5}{\left(\cos\left(dx+c\right)+1\right)^5}+\frac{15\,\sin\left(dx+c\right)^7}{\left(\cos\left(dx+c\right)+1\right)^7}}{a^4}-\frac{3360\,\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1\right)}{a^4}+\frac{3360\,\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-1\right)}{a^4}\right)-5\,A\left(\frac{\frac{315\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+\frac{77\,\sin\left(dx+c\right)^3}{\left(\cos\left(dx+c\right)+1\right)^3}+\frac{21\,\sin\left(dx+c\right)^5}{\left(\cos\left(dx+c\right)+1\right)^5}+\frac{3\,\sin\left(dx+c\right)^7}{\left(\cos\left(dx+c\right)+1\right)^7}}{a^4}-\frac{168\,\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}+1\right)}{a^4}+\frac{168\,\log\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-1\right)}{a^4}\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(B*(1680*sin(d*x + c)/((a^4 - a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) + 805*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 3360*log(sin(d*x + c)/(cos(d*x
+ c) + 1) + 1)/a^4 + 3360*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) - 5
*A*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x + c)
+ 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7/(cos(d*x
 + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 + 168*
log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4))/d
```

Fricas [A]
time = 3.61, size = 317, normalized size = 1.63

$$\frac{105\,\left(\left(A-4\,B\right)\cos\left(dx+c\right)^5+4\,\left(A-4\,B\right)\cos\left(dx+c\right)^4+6\,\left(A-4\,B\right)\cos\left(dx+c\right)^3+4\,\left(A-4\,B\right)\cos\left(dx+c\right)^2+\left(A-4\,B\right)\cos\left(dx+c\right)\right)\log\left(\sin\left(dx+c\right)+1\right)-105\,\left(\left(A-4\,B\right)\cos\left(dx+c\right)^5+4\,\left(A-4\,B\right)\cos\left(dx+c\right)^4+6\,\left(A-4\,B\right)\cos\left(dx+c\right)^3+4\,\left(A-4\,B\right)\cos\left(dx+c\right)^2+\left(A-4\,B\right)\cos\left(dx+c\right)\right)\log\left(-\sin\left(dx+c\right)+1\right)-2\,\left(8\,\left(20\,A-83\,B\right)\cos\left(dx+c\right)^4+\left(535\,A-2236\,B\right)\cos\left(dx+c\right)^3+4\,\left(155\,A-659\,B\right)\cos\left(dx+c\right)^2+4\,\left(65\,A-296\,B\right)\cos\left(dx+c\right)-105\,B\right)\sin\left(dx+c\right)}{210\,\left(a^4d\cos\left(dx+c\right)^5+4\,a^4d\cos\left(dx+c\right)^4+6\,a^4d\cos\left(dx+c\right)^3+4\,a^4d\cos\left(dx+c\right)^2+a^4d\cos\left(dx+c\right)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/210*(105*((A - 4*B)*cos(d*x + c)^5 + 4*(A - 4*B)*cos(d*x + c)^4 + 6*(A -
4*B)*cos(d*x + c)^3 + 4*(A - 4*B)*cos(d*x + c)^2 + (A - 4*B)*cos(d*x + c))*
log(sin(d*x + c) + 1) - 105*((A - 4*B)*cos(d*x + c)^5 + 4*(A - 4*B)*cos(d*x
 + c)^4 + 6*(A - 4*B)*cos(d*x + c)^3 + 4*(A - 4*B)*cos(d*x + c)^2 + (A - 4*
B)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(8*(20*A - 83*B)*cos(d*x + c)^4
 + (535*A - 2236*B)*cos(d*x + c)^3 + 4*(155*A - 659*B)*cos(d*x + c)^2 + 4*(
65*A - 296*B)*cos(d*x + c) - 105*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^5 + 4
*a^4*d*cos(d*x + c)^4 + 6*a^4*d*cos(d*x + c)^3 + 4*a^4*d*cos(d*x + c)^2 + a
^4*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^{5}\left(c+dx\right)}{\sec^{4}\left(c+dx\right)+4\sec^{3}\left(c+dx\right)+6\sec^{2}\left(c+dx\right)+4\sec\left(c+dx\right)+1}\,dx+\int \frac{B\sec^{6}\left(c+dx\right)}{\sec^{4}\left(c+dx\right)+4\sec^{3}\left(c+dx\right)+6\sec^{2}\left(c+dx\right)+4\sec\left(c+dx\right)+1}\,dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A*sec(c + d*x)**5/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c
+ d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**6/(sec(c + d
*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a
**4
```

Giac [A]
time = 0.50, size = 220, normalized size = 1.13

$$\frac{\frac{840\,(A-4\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^4}-\frac{840\,(A-4\,B)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^4}-\frac{1680\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)a^4}-\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+105\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-147\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+385\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-805\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1575\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5145\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/840*(840*(A - 4*B)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 840*(A - 4*B)
*log(abs(tan(1/2*d*x + 1/2*c) - 1))/a^4 - 1680*B*tan(1/2*d*x + 1/2*c)/((tan
(1/2*d*x + 1/2*c)^2 - 1)*a^4) - (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^
24*tan(1/2*d*x + 1/2*c)^7 + 105*A*a^24*tan(1/2*d*x + 1/2*c)^5 - 147*B*a^24*
tan(1/2*d*x + 1/2*c)^5 + 385*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 805*B*a^24*tan
(1/2*d*x + 1/2*c)^3 + 1575*A*a^24*tan(1/2*d*x + 1/2*c) - 5145*B*a^24*tan(1/
2*d*x + 1/2*c))/a^28)/d
```

Mupad [B]
time = 2.03, size = 237, normalized size = 1.22

$$\frac{2\,\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,(A-4\,B)}{a^4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{A-B}{20\,a^4}+\frac{3\,A-5\,B}{40\,a^4}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{A-B}{2\,a^4}+\frac{3\,(3\,A-5\,B)}{8\,a^4}+\frac{2\,A-10\,B}{4\,a^4}-\frac{2\,A+10\,B}{8\,a^4}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,(A-B)}{56\,a^4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{A-B}{8\,a^4}+\frac{3\,A-5\,B}{12\,a^4}+\frac{2\,A-10\,B}{24\,a^4}\right)}{d}-\frac{2\,B\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-a^4\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^5*(a + a/cos(c + d*x))^4),x)
```

```
[Out] (2*atanh(tan(c/2 + (d*x)/2))*(A - 4*B))/(a^4*d) - (tan(c/2 + (d*x)/2)^5*((A
 - B)/(20*a^4) + (3*A - 5*B)/(40*a^4)))/d - (tan(c/2 + (d*x)/2)*((A - B)/(2
*a^4) + (3*(3*A - 5*B))/(8*a^4) + (2*A - 10*B)/(4*a^4) - (2*A + 10*B)/(8*a^
4)))/d - (tan(c/2 + (d*x)/2)^7*(A - B))/(56*a^4*d) - (tan(c/2 + (d*x)/2)^3*
((A - B)/(8*a^4) + (3*A - 5*B)/(12*a^4) + (2*A - 10*B)/(24*a^4)))/d - (2*B*
tan(c/2 + (d*x)/2))/(d*(a^4*tan(c/2 + (d*x)/2)^2 - a^4))
```

3.110 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=163

$$\frac{B\tanh^{-1}(\sin(c+dx))}{a^4d}-\frac{(6A-55B)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2}+\frac{(12A-215B)\tan(c+dx)}{105a^4d(1+\sec(c+dx))}+\frac{(A-B)\sec^3(c+dx)\tan(}{7d(a+a\sec(c+dx)}$$

```
[Out] B*arctanh(sin(d*x+c))/a^4/d-1/105*(6*A-55*B)*tan(d*x+c)/a^4/d/(1+sec(d*x+c)
)^2+1/105*(12*A-215*B)*tan(d*x+c)/a^4/d/(1+sec(d*x+c))+1/7*(A-B)*sec(d*x+c)
^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^4+1/35*(3*A-10*B)*sec(d*x+c)^2*tan(d*x+c)/
a/d/(a+a*sec(d*x+c))^3
```

Rubi [A]
time = 0.32, antiderivative size = 163, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4104, 4093, 4083, 3855, 3879}

$$\frac{(12A-215B)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)}-\frac{(6A-55B)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{B\tanh^{-1}(\sin(c+dx))}{a^4d}+\frac{(A-B)\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}+\frac{(3A-10B)\tan(c+dx)\sec^2(c+dx)}{35ad(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(a^4*d) - ((6*A - 55*B)*Tan[c + d*x])/(105*a^4*d*
(1 + Sec[c + d*x])^2) + ((12*A - 215*B)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c
 + d*x])) + ((A - B)*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])
^4) + ((3*A - 10*B)*Sec[c + d*x]^2*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x
])^3)
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx = \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\sec^3(c+dx)(3a(A-B)+7aB\sec(c+dx)}{(a+a\sec(c+dx))^3}}{7a^2}$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(3A-10B)\sec^2(c+dx)\tan(c}{35ad(a+a\sec(c+dx))}$$

$$= -\frac{(6A-55B)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} +$$

$$= -\frac{(6A-55B)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} +$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{a^4d} - \frac{(6A-55B)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2} + \frac{(A-B)\sec^3}{7d(a+}$$

Mathematica [A]
time = 1.41, size = 239, normalized size = 1.47

$$\frac{-6720B\cos^8\left(\frac{1}{2}(c+dx)\right)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(70(3A-49B)\sin\left(\frac{dx}{2}\right)+2170B\sin\left(c+\frac{dx}{2}\right)+126A\sin\left(c+\frac{3dx}{2}\right)-2625B\sin\left(c+\frac{3dx}{2}\right)+735B\sin\left(2c+\frac{3dx}{2}\right)+42A\sin\left(2c+\frac{5dx}{2}\right)-1015B\sin\left(2c+\frac{5dx}{2}\right)+105B\sin\left(3c+\frac{5dx}{2}\right)+6A\sin\left(3c+\frac{7dx}{2}\right)-160B\sin\left(3c+\frac{7dx}{2}\right)\right)}{420a^4d(1+\cos(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

[Out] (-6720*B*Cos[(c + d*x)/2]^8*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - Log [Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + Cos[(c + d*x)/2]*Sec[c/2]*(70*(3*A - 49*B)*Sin[(d*x)/2] + 2170*B*Sin[c + (d*x)/2] + 126*A*Sin[c + (3*d*x)/2] - 2625*B*Sin[c + (3*d*x)/2] + 735*B*Sin[2*c + (3*d*x)/2] + 42*A*Sin[2*c + (5*d*x)/2] - 1015*B*Sin[2*c + (5*d*x)/2] + 105*B*Sin[3*c + (5*d*x)/2] + 6*A* Sin[3*c + (7*d*x)/2] - 160*B*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 + Cos[c + d*x])^4)

Maple [A]
time = 0.26, size = 146, normalized size = 0.90

method	result
derivativedivides	$\frac{-8B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B+A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{11B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{\left(\tan^7\left(\frac{dx}{2}\cdots\right.\right.}{7}}{8d\,a^4}$
default	$\frac{-8B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B+A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{11B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{\left(\tan^7\left(\frac{dx}{2}\cdots\right.\right.}{7}}{8d\,a^4}$
risch	$-\frac{2i\left(105B\,e^{6i(dx+c)}+735B\,e^{5i(dx+c)}+2170B\,e^{4i(dx+c)}-210A\,e^{3i(dx+c)}+3430B\,e^{3i(dx+c)}-126A\,e^{2i(dx+c)}+2625B\,e^{2i\cdots}\right.}{105d\,a^4\left(e^{i(dx+c)}+1\right)^7}$
norman	$\frac{\frac{(A-15B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{(A-15B)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{280ad}+\frac{(A-B)\left(\tan^{15}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56ad}+\frac{(3A-605B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{840ad}-\frac{(9A-1465B)\left(\tan^7\cdots\right.}{280ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4 a\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)

[Out] 1/8/d/a^4*(-8*B*ln(tan(1/2*d*x+1/2*c)-1)+3/5*tan(1/2*d*x+1/2*c)^5*A-tan(1/2*d*x+1/2*c)^5*B+A*tan(1/2*d*x+1/2*c)^3-11/3*B*tan(1/2*d*x+1/2*c)^3+1/7*tan(1/2*d*x+1/2*c)^7*A-1/7*tan(1/2*d*x+1/2*c)^7*B+A*tan(1/2*d*x+1/2*c)-15*B*tan(1/2*d*x+1/2*c)+8*B*ln(tan(1/2*d*x+1/2*c)+1))

Maxima [A]
time = 0.28, size = 228, normalized size = 1.40

$$-\frac{5B\left(\frac{\frac{315\sin(dx+c)}{\cos(dx+c)+1}+\frac{77\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{3\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{168\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}+1\right)}{a^4}+\frac{168\log\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}-1\right)}{a^4}\right)-\frac{3A\left(\frac{35\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="maxima")

[Out] -1/840*(5*B*((315*sin(d*x + c)/(cos(d*x + c) + 1) + 77*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 3*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 168*log(sin(d*x + c)/(cos(d*x + c) + 1) + 1)/a^4 + 168*log(sin(d*x + c)/(cos(d*x + c) + 1) - 1)/a^4) - 3*A*(35*sin(d*x +

```
c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x
 + c)^5/(cos(d*x + c) + 1)^5 + 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/
d
```

Fricas [A]
time = 4.09, size = 236, normalized size = 1.45

$$\frac{105\,\left(B\cos\left(dx+c\right)^{4}+4\,B\cos\left(dx+c\right)^{3}+6\,B\cos\left(dx+c\right)^{2}+4\,B\cos\left(dx+c\right)+B\right)\log\left(\sin\left(dx+c\right)+1\right)-105\,\left(B\cos\left(dx+c\right)^{4}+4\,B\cos\left(dx+c\right)^{3}+6\,B\cos\left(dx+c\right)^{2}+4\,B\cos\left(dx+c\right)+B\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(2\,\left(3\,A-80\,B\right)\cos\left(dx+c\right)^{3}+\left(24\,A-535\,B\right)\cos\left(dx+c\right)^{2}+\left(39\,A-620\,B\right)\cos\left(dx+c\right)+36\,A-260\,B\right)\sin\left(dx+c\right)}{210\,\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/210*(105*(B*cos(d*x + c)^4 + 4*B*cos(d*x + c)^3 + 6*B*cos(d*x + c)^2 + 4*
B*cos(d*x + c) + B)*log(sin(d*x + c) + 1) - 105*(B*cos(d*x + c)^4 + 4*B*cos
(d*x + c)^3 + 6*B*cos(d*x + c)^2 + 4*B*cos(d*x + c) + B)*log(-sin(d*x + c)
+ 1) + 2*(2*(3*A - 80*B)*cos(d*x + c)^3 + (24*A - 535*B)*cos(d*x + c)^2 + (
39*A - 620*B)*cos(d*x + c) + 36*A - 260*B)*sin(d*x + c))/(a^4*d*cos(d*x + c
)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c
) + a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^{4}\left(c+dx\right)}{\sec^{4}\left(c+dx\right)+4\sec^{3}\left(c+dx\right)+6\sec^{2}\left(c+dx\right)+4\sec\left(c+dx\right)+1}\,dx+\int \frac{B\sec^{5}\left(c+dx\right)}{\sec^{4}\left(c+dx\right)+4\sec^{3}\left(c+dx\right)+6\sec^{2}\left(c+dx\right)+4\sec\left(c+dx\right)+1}\,dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A*sec(c + d*x)**4/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c
+ d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**5/(sec(c + d
*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a
**4
```

Giac [A]
time = 0.50, size = 181, normalized size = 1.11

$$\frac{\frac{840\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{a^{4}}-\frac{840\,B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{a^{4}}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+63\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-105\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+105\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-385\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+105\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1575\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/840*(840*B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/a^4 - 840*B*log(abs(tan(1/2
*d*x + 1/2*c) - 1))/a^4 + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*tan
```

```
(1/2*d*x + 1/2*c)^7 + 63*A*a^24*tan(1/2*d*x + 1/2*c)^5 - 105*B*a^24*tan(1/2
*d*x + 1/2*c)^5 + 105*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 385*B*a^24*tan(1/2*d*
x + 1/2*c)^3 + 105*A*a^24*tan(1/2*d*x + 1/2*c) - 1575*B*a^24*tan(1/2*d*x +
1/2*c))/a^28)/d
```

Mupad [B]
time = 2.13, size = 198, normalized size = 1.21

$$\frac{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,\left(\frac{A\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{8}-\frac{11\,B\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3}{24}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,\left(\frac{3\,A\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{40}-\frac{B\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5}{8}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6\,\left(\frac{A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}-\frac{15\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{8}\right)+\frac{A\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{56}-\frac{B\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}{56}}{a^4\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}+\frac{2\,B\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + a/cos(c + d*x))^4),x)
```

```
[Out] (cos(c/2 + (d*x)/2)^4*((A*sin(c/2 + (d*x)/2)^3)/8 - (11*B*sin(c/2 + (d*x)/2
)^3)/24) + cos(c/2 + (d*x)/2)^2*((3*A*sin(c/2 + (d*x)/2)^5)/40 - (B*sin(c/2
 + (d*x)/2)^5)/8) + cos(c/2 + (d*x)/2)^6*((A*sin(c/2 + (d*x)/2))/8 - (15*B*
sin(c/2 + (d*x)/2))/8) + (A*sin(c/2 + (d*x)/2)^7)/56 - (B*sin(c/2 + (d*x)/2
)^7)/56)/(a^4*d*cos(c/2 + (d*x)/2)^7) + (2*B*atanh(sin(c/2 + (d*x)/2)/cos(c
/2 + (d*x)/2)))/(a^4*d)
```

3.111 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=146

$$-\frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{(4A+3B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}-\frac{8(4A+3B)\tan(c+dx)}{105d\left(a^2+a^2\sec(c+dx)\right)^2}+\frac{(4A+3B)\tan(c+dx)}{15d\left(a^4+a^4\sec(c+dx)\right)}$$

```
[Out] -1/7*(A-B)*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^4+1/35*(4*A+3*B)*tan(
d*x+c)/a/d/(a+a*sec(d*x+c))^3-8/105*(4*A+3*B)*tan(d*x+c)/d/(a^2+a^2*sec(d*x
+c))^2+1/15*(4*A+3*B)*tan(d*x+c)/d/(a^4+a^4*sec(d*x+c))
```

Rubi [A]
time = 0.16, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4097, 3884, 4085, 3879}

$$\frac{(4A+3B)\tan(c+dx)}{15d\left(a^4\sec(c+dx)+a^4\right)}-\frac{8(4A+3B)\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}-\frac{(A-B)\tan(c+dx)\sec^3(c+dx)}{7d(a\sec(c+dx)+a)^4}+\frac{(4A+3B)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -1/7*((A - B)*Sec[c + d*x]^3*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^4) + ((4
*A + 3*B)*Tan[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3) - (8*(4*A + 3*B)*Ta
n[c + d*x])/(105*d*(a^2 + a^2*Sec[c + d*x])^2) + ((4*A + 3*B)*Tan[c + d*x])
/(15*d*(a^4 + a^4*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3884

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))),
x] - Dist[1/(a^2*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
(a*m - b*(2*m + 1)*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a
^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
```

```
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 4097

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n},
 x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m
, -1]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(4A+3B)\int \frac{\sec^3(c+dx)}{(a+a\sec(c+dx))^3}}{7a}$$

$$= -\frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(4A+3B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} +$$

$$= -\frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(4A+3B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} -$$

$$= -\frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(4A+3B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} -$$

Mathematica [A]
time = 0.37, size = 109, normalized size = 0.75

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(35(2A+3B)\sin\left(\frac{dx}{2}\right)-70A\sin\left(c+\frac{dx}{2}\right)+(4A+3B)\left(21\sin\left(c+\frac{3dx}{2}\right)+7\sin\left(2c+\frac{5dx}{2}\right)+\sin\left(3c+\frac{7dx}{2}\right)\right)\right)}{210a^4d(1+\cos(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(35*(2*A + 3*B)*Sin[(d*x)/2] - 70*A*Sin[c + (d*x
)/2] + (4*A + 3*B)*(21*Sin[c + (3*d*x)/2] + 7*Sin[2*c + (5*d*x)/2] + Sin[3*
c + (7*d*x)/2])))/(210*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A]
time = 0.26, size = 88, normalized size = 0.60

method	result
derivativedivides	$\frac{\frac{(-A+B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{(-A+3B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{(A+3B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$
default	$\frac{\frac{(-A+B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{(-A+3B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{(A+3B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$
risch	$\frac{4i\left(70A\,e^{4i(dx+c)}+70A\,e^{3i(dx+c)}+105B\,e^{3i(dx+c)}+84A\,e^{2i(dx+c)}+63B\,e^{2i(dx+c)}+28\,e^{i(dx+c)}A+21B\,e^{i(dx+c)}+4A+3B\right)}{105d\,a^4\left(e^{i(dx+c)}+1\right)^7}$
norman	$\frac{-\frac{(A-B)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56ad}-\frac{(A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{3(3A+B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40ad}+\frac{(4A+3B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}-\frac{(4A+3B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{70ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3a^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/8/d/a^4*(1/7*(-A+B)*tan(1/2*d*x+1/2*c)^7+1/5*(-A+3*B)*tan(1/2*d*x+1/2*c)^
5+1/3*(A+3*B)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c
))
```

Maxima [A]

time = 0.28, size = 175, normalized size = 1.20

$$\frac{\frac{A\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{3B\left(\frac{35\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(A*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + 3*B*(35*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 +
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A]

time = 3.32, size = 125, normalized size = 0.86

$$\frac{\left(2\left(4A+3B\right)\cos\left(dx+c\right)^3+8\left(4A+3B\right)\cos\left(dx+c\right)^2+13\left(4A+3B\right)\cos\left(dx+c\right)+13A+36B\right)\sin\left(dx+c\right)}{105\left(a^4d\cos\left(dx+c\right)^4+4a^4d\cos\left(dx+c\right)^3+6a^4d\cos\left(dx+c\right)^2+4a^4d\cos\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/105*(2*(4*A + 3*B)*cos(d*x + c)^3 + 8*(4*A + 3*B)*cos(d*x + c)^2 + 13*(4*
A + 3*B)*cos(d*x + c) + 13*A + 36*B)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4
*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4
*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^3(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int \frac{B\sec^4(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A*sec(c + d*x)**3/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c
+ d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**4/(sec(c + d
*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a
**4
```

Giac [A]
time = 0.47, size = 117, normalized size = 0.80

$$-\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+21\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-63\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-105\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-105\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-105\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{840\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 + 21*A*ta
n(1/2*d*x + 1/2*c)^5 - 63*B*tan(1/2*d*x + 1/2*c)^5 - 35*A*tan(1/2*d*x + 1/2
*c)^3 - 105*B*tan(1/2*d*x + 1/2*c)^3 - 105*A*tan(1/2*d*x + 1/2*c) - 105*B*t
an(1/2*d*x + 1/2*c))/(a^4*d)
```

Mupad [B]
time = 2.03, size = 85, normalized size = 0.58

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(A+3\,B)}{24\,a^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(A-3\,B)}{40\,a^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,(A-B)}{56\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A+B)}{8\,a^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + a/cos(c + d*x))^4),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(A + 3*B))/(24*a^4) - (tan(c/2 + (d*x)/2)^5*(A - 3*B
))/(40*a^4) - (tan(c/2 + (d*x)/2)^7*(A - B))/(56*a^4) + (tan(c/2 + (d*x)/2)
*(A + B))/(8*a^4))/d
```

3.112 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=138

$$-\frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{(4A-11B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}+\frac{(8A+13B)\tan(c+dx)}{105d\,(a^2+a^2\sec(c+dx))^2}+\frac{(8A+13B)\tan(c+dx)}{105d\,(a^4+a^4\sec(c+dx))}$$

```
[Out] -1/7*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^4+1/35*(4*A-11*B)*tan(d*x+c)/a/d/(
a+a*sec(d*x+c))^3+1/105*(8*A+13*B)*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))^2+1/10
5*(8*A+13*B)*tan(d*x+c)/d/(a^4+a^4*sec(d*x+c))
```

Rubi [A]
time = 0.17, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = $\{4093, 4085, 3881, 3879\}$

$$\frac{(8A+13B)\tan(c+dx)}{105d\,(a^4\sec(c+dx)+a^4)}+\frac{(8A+13B)\tan(c+dx)}{105d\,(a^2\sec(c+dx)+a^2)^2}+\frac{(4A-11B)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{(A-B)\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -1/7*((A - B)*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^4) + ((4*A - 11*B)*Tan[
c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3) + ((8*A + 13*B)*Tan[c + d*x])/(10
5*d*(a^2 + a^2*Sec[c + d*x])^2) + ((8*A + 13*B)*Tan[c + d*x])/(105*d*(a^4 +
 a^4*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
```

```
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{\int \frac{\sec(c+dx)(-4a(A-B)-7aB\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(4A-11B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{(8A+13\ldots}{\ldots}$$

$$= -\frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(4A-11B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{(8A+1\ldots}{105d\,(a^2\ldots}$$

$$= -\frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(4A-11B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{(8A+1\ldots}{105d\,(a^2\ldots}$$

Mathematica [A]
time = 0.41, size = 163, normalized size = 1.18

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(140(2A+B)\sin\left(\frac{dx}{2}\right)-35(5A+4B)\sin\left(c+\frac{dx}{2}\right)+168A\sin\left(c+\frac{3dx}{2}\right)+168B\sin\left(c+\frac{3dx}{2}\right)-105A\sin\left(2c+\frac{3dx}{2}\right)+91A\sin\left(2c+\frac{5dx}{2}\right)+56B\sin\left(2c+\frac{5dx}{2}\right)+13A\sin\left(3c+\frac{7dx}{2}\right)+8B\sin\left(3c+\frac{7dx}{2}\right)\right)}{420a^4d(1+\cos(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(140*(2*A + B)*Sin[(d*x)/2] - 35*(5*A + 4*B)*Sin
[c + (d*x)/2] + 168*A*Sin[c + (3*d*x)/2] + 168*B*Sin[c + (3*d*x)/2] - 105*A
*Sin[2*c + (3*d*x)/2] + 91*A*Sin[2*c + (5*d*x)/2] + 56*B*Sin[2*c + (5*d*x)/
2] + 13*A*Sin[3*c + (7*d*x)/2] + 8*B*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 +
 Cos[c + d*x])^4)
```

Maple [A]
time = 0.25, size = 88, normalized size = 0.64

method	result
derivativedivides	$\frac{\frac{(A-B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{(-A-B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{(-A+B)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$
default	$\frac{\frac{(A-B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{(-A-B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{(-A+B)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^{4}}$
risch	$\frac{2i\left(105A\,e^{5i(dx+c)}+175A\,e^{4i(dx+c)}+140B\,e^{4i(dx+c)}+280A\,e^{3i(dx+c)}+140B\,e^{3i(dx+c)}+168A\,e^{2i(dx+c)}+168B\,e^{2i(dx+c)}+\right.}{105d\,a^{4}\left(e^{i(dx+c)}+1\right)^{7}}$
norman	$\frac{\frac{(A-B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56ad}+\frac{(A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}-\frac{(7A+5B)\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}+\frac{(11A+B)\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60ad}+\frac{(11A+31B)\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{420ad}}{\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}a^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/8/d/a^4*(1/7*(A-B)*tan(1/2*d*x+1/2*c)^7+1/5*(-A-B)*tan(1/2*d*x+1/2*c)^5+1
/3*(-A+B)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c))
```

Maxima [A]

time = 0.28, size = 174, normalized size = 1.26

$$\frac{\frac{B\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}+\frac{35\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{21\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}-\frac{15\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)}{a^{4}}+\frac{A\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sin(dx+c)^{3}}{(\cos(dx+c)+1)^{3}}-\frac{21\sin(dx+c)^{5}}{(\cos(dx+c)+1)^{5}}+\frac{15\sin(dx+c)^{7}}{(\cos(dx+c)+1)^{7}}\right)}{a^{4}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(B*(105*sin(d*x + c)/(cos(d*x + c) + 1) + 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + A*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d
*x + c)^3/(cos(d*x + c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 1
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A]

time = 3.43, size = 124, normalized size = 0.90

$$\frac{\left((13\,A+8\,B)\cos\left(dx+c\right)^{3}+4\,(13\,A+8\,B)\cos\left(dx+c\right)^{2}+4\,(8\,A+13\,B)\cos\left(dx+c\right)+8\,A+13\,B\right)\sin\left(dx+c\right)}{105\,\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/105*((13*A + 8*B)*cos(d*x + c)^3 + 4*(13*A + 8*B)*cos(d*x + c)^2 + 4*(8*A
 + 13*B)*cos(d*x + c) + 8*A + 13*B)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*
a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*
d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^2(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int \frac{B\sec^3(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A*sec(c + d*x)**2/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c
+ d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**3/(sec(c + d
*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a
**4
```

Giac [A]
time = 0.47, size = 117, normalized size = 0.85

$$\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-21\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-21\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-35\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+35\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+105\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+105\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{840\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 - 21*A*tan
(1/2*d*x + 1/2*c)^5 - 21*B*tan(1/2*d*x + 1/2*c)^5 - 35*A*tan(1/2*d*x + 1/2*
c)^3 + 35*B*tan(1/2*d*x + 1/2*c)^3 + 105*A*tan(1/2*d*x + 1/2*c) + 105*B*tan
(1/2*d*x + 1/2*c))/(a^4*d)
```

Mupad [B]
time = 1.98, size = 84, normalized size = 0.61

$$-\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,(A+B)}{40\,a^4}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(A-B)}{24\,a^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,(A-B)}{56\,a^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(A+B)}{8\,a^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + a/cos(c + d*x))^4),x)
```

```
[Out] -((tan(c/2 + (d*x)/2)^5*(A + B))/(40*a^4) + (tan(c/2 + (d*x)/2)^3*(A - B))/
(24*a^4) - (tan(c/2 + (d*x)/2)^7*(A - B))/(56*a^4) - (tan(c/2 + (d*x)/2)*(A
 + B))/(8*a^4))/d
```

3.113 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=138

$$\frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}+\frac{(3A+4B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}+\frac{2(3A+4B)\tan(c+dx)}{105d\left(a^2+a^2\sec(c+dx)\right)^2}+\frac{2(3A+4B)\tan(c+dx)}{105d\left(a^4+a^4\sec(c+dx)\right)}$$

```
[Out] 1/7*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^4+1/35*(3*A+4*B)*tan(d*x+c)/a/d/(a+
a*sec(d*x+c))^3+2/105*(3*A+4*B)*tan(d*x+c)/d/(a^2+a^2*sec(d*x+c))^2+2/105*(
3*A+4*B)*tan(d*x+c)/d/(a^4+a^4*sec(d*x+c))
```

Rubi [A]
time = 0.11, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.103$, Rules used = {4085, 3881, 3879}

$$\frac{2(3A+4B)\tan(c+dx)}{105d\left(a^4\sec(c+dx)+a^4\right)}+\frac{2(3A+4B)\tan(c+dx)}{105d\left(a^2\sec(c+dx)+a^2\right)^2}+\frac{(3A+4B)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}+\frac{(A-B)\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] ((A - B)*Tan[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) + ((3*A + 4*B)*Tan[c +
d*x])/(35*a*d*(a + a*Sec[c + d*x])^3) + (2*(3*A + 4*B)*Tan[c + d*x])/(105*d
*(a^2 + a^2*Sec[c + d*x])^2) + (2*(3*A + 4*B)*Tan[c + d*x])/(105*d*(a^4 + a
^4*Sec[c + d*x]))
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
 && IntegerQ[2*m]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
```

```
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx = \frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(3A+4B)\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{7a}$$

$$= \frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(3A+4B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{(2(3A+4B)}{}$$

$$= \frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(3A+4B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{2(3A+4B}{105d\,(a^2+a}$$

$$= \frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{(3A+4B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3} + \frac{2(3A+4B}{105d\,(a^2+a}$$

Mathematica [A]
time = 0.47, size = 193, normalized size = 1.40

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(70(9A+4B)\sin\left(\frac{dx}{2}\right)-35(18A+5B)\sin\left(c+\frac{dx}{2}\right)+441A\sin\left(c+\frac{3dx}{2}\right)+168B\sin\left(c+\frac{3dx}{2}\right)-315A\sin\left(2c+\frac{3dx}{2}\right)-105B\sin\left(2c+\frac{3dx}{2}\right)+147A\sin\left(2c+\frac{5dx}{2}\right)+91B\sin\left(2c+\frac{5dx}{2}\right)-105A\sin\left(3c+\frac{5dx}{2}\right)+36A\sin\left(3c+\frac{7dx}{2}\right)+13B\sin\left(3c+\frac{7dx}{2}\right)\right)}{420a^4d(1+\cos(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(70*(9*A + 4*B)*Sin[(d*x)/2] - 35*(18*A + 5*B)*S
in[c + (d*x)/2] + 441*A*Sin[c + (3*d*x)/2] + 168*B*Sin[c + (3*d*x)/2] - 315
*A*Sin[2*c + (3*d*x)/2] - 105*B*Sin[2*c + (3*d*x)/2] + 147*A*Sin[2*c + (5*d
*x)/2] + 91*B*Sin[2*c + (5*d*x)/2] - 105*A*Sin[3*c + (5*d*x)/2] + 36*A*Sin[
3*c + (7*d*x)/2] + 13*B*Sin[3*c + (7*d*x)/2]))/(420*a^4*d*(1 + Cos[c + d*x]
)^4)
```

Maple [A]
time = 0.25, size = 90, normalized size = 0.65

method	result
derivativedivides	$\frac{\frac{(-A+B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{(3A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{(-3A-B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$
default	$\frac{\frac{(-A+B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{7}+\frac{(3A-B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{5}+\frac{(-3A-B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+A\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+B\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8d\,a^4}$

norman	$\frac{-\frac{(A-B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56ad}-\frac{(A+B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{(3A+2B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12ad}-\frac{(12A+B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{60ad}+\frac{(13A-6B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}{140ad}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)a^3}$
risch	$\frac{2i\left(105A\,e^{6i(dx+c)}+315A\,e^{5i(dx+c)}+105B\,e^{5i(dx+c)}+630A\,e^{4i(dx+c)}+175B\,e^{4i(dx+c)}+630A\,e^{3i(dx+c)}+280B\,e^{3i(dx+c)}+\right.}{105d\,a^4\left(e^{i(dx+c)}+1\right)^7}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^4*(1/7*(-A+B)*tan(1/2*d*x+1/2*c)^7+1/5*(3*A-B)*tan(1/2*d*x+1/2*c)^5
+1/3*(-3*A-B)*tan(1/2*d*x+1/2*c)^3+A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c
))
```

Maxima [A]
time = 0.27, size = 175, normalized size = 1.27

$$\frac{\frac{B\left(\frac{105\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}-\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}+\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}+\frac{3A\left(\frac{35\sin(dx+c)}{\cos(dx+c)+1}-\frac{35\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/840*(B*(105*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x
+ c) + 1)^3 - 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 + 15*sin(d*x + c)^7/(c
os(d*x + c) + 1)^7)/a^4 + 3*A*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(
d*x + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 -
5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A]
time = 2.69, size = 124, normalized size = 0.90

$$\frac{\left((36\,A+13\,B)\cos(dx+c)^3+13\,(3\,A+4\,B)\cos(dx+c)^2+8\,(3\,A+4\,B)\cos(dx+c)+6\,A+8\,B\right)\sin(dx+c)}{105\left(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4d\cos(dx+c)^2+4\,a^4d\cos(dx+c)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fric
as")
```

```
[Out] 1/105*((36*A + 13*B)*cos(d*x + c)^3 + 13*(3*A + 4*B)*cos(d*x + c)^2 + 8*(3*
A + 4*B)*cos(d*x + c) + 6*A + 8*B)*sin(d*x + c)/(a^4*d*cos(d*x + c)^4 + 4*a
^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int\frac{A\sec(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int\frac{B\sec^2(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A*sec(c + d*x)/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*sec(c + d*x)**2/(sec(c + d*x)
**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a**4
```

Giac [A]
time = 0.48, size = 117, normalized size = 0.85

$$-\frac{15\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-63\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+21\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+105\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+35\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-105\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-105\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{840\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="giac
")
```

```
[Out] -1/840*(15*A*tan(1/2*d*x + 1/2*c)^7 - 15*B*tan(1/2*d*x + 1/2*c)^7 - 63*A*ta
n(1/2*d*x + 1/2*c)^5 + 21*B*tan(1/2*d*x + 1/2*c)^5 + 105*A*tan(1/2*d*x + 1/
2*c)^3 + 35*B*tan(1/2*d*x + 1/2*c)^3 - 105*A*tan(1/2*d*x + 1/2*c) - 105*B*t
an(1/2*d*x + 1/2*c))/(a^4*d)
```

Mupad [B]
time = 1.99, size = 88, normalized size = 0.64

$$-\frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(3\,A+B\right)}{24\,a^4}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7\,\left(A-B\right)}{56\,a^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A+B\right)}{8\,a^4}-\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(3\,A-B\right)}{40\,a^4}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + a/cos(c + d*x))^4),x)
```

```
[Out] -((tan(c/2 + (d*x)/2)^3*(3*A + B))/(24*a^4) + (tan(c/2 + (d*x)/2)^7*(A - B)
)/(56*a^4) - (tan(c/2 + (d*x)/2)*(A + B))/(8*a^4) - (tan(c/2 + (d*x)/2)^5*(
3*A - B))/(40*a^4))/d
```

3.114 $\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=138

$$\frac{Ax}{a^4}-\frac{(55A-6B)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2}-\frac{2(80A-3B)\tan(c+dx)}{105a^4d(1+\sec(c+dx))}-\frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{(10A-3B)\tan(c+dx}{35ad(a+a\sec(c+dx)}$$

```
[Out] A*x/a^4-1/105*(55*A-6*B)*tan(d*x+c)/a^4/d/(1+sec(d*x+c))^2-2/105*(80*A-3*B)
*tan(d*x+c)/a^4/d/(1+sec(d*x+c))-1/7*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^4-
1/35*(10*A-3*B)*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^3
```

Rubi [A]
time = 0.19, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 3, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.130$, Rules used = {4007, 4004, 3879}

$$-\frac{2(80A-3B)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)}-\frac{(55A-6B)\tan(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{Ax}{a^4}-\frac{(10A-3B)\tan(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{(A-B)\tan(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (A*x)/a^4 - ((55*A - 6*B)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) -
(2*(80*A - 3*B)*Tan[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])) - ((A - B)*Tan
[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - ((10*A - 3*B)*Tan[c + d*x])/(35*a
*d*(a + a*Sec[c + d*x])^3)
```

Rule 3879

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Simp[-Cot[e + f*x]/(f*(b + a*Csc[e + f*x])), x] /; FreeQ[{a, b, e, f}
, x] && EqQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
```

```
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^4}\,dx &= -\frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{\int \frac{-7aA+3a(A-B)\sec(c+dx)}{(a+a\sec(c+dx))^3}\,dx}{7a^2}\\
&= -\frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{(10A-3B)\tan(c+dx)}{35ad(a+a\sec(c+dx))^3}+\frac{\int \frac{35a^2A-2a^2(10A-3B)\sec(\cdots}{(a+a\sec(c+dx))^2}}{35a^4}\\
&= -\frac{(55A-6B)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2}-\frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{(10A-3B)\tan(c+\cdots}{35ad(a+a\sec(c+dx\cdots}\\
&= \frac{Ax}{a^4}-\frac{(55A-6B)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2}-\frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{(10A-3B)\tan(\cdots}{35ad(a+a\sec(c\cdots}\\
&= \frac{Ax}{a^4}-\frac{(55A-6B)\tan(c+dx)}{105a^4d(1+\sec(c+dx))^2}-\frac{(A-B)\tan(c+dx)}{7d(a+a\sec(c+dx))^4}-\frac{(10A-3B)\tan(\cdots}{35ad(a+a\sec(c\cdots}\end{aligned}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 329 vs. 2(138) = 276.
time = 0.78, size = 329, normalized size = 2.38

$$\frac{\sec\left(\frac{c}{2}\right)\sec^7\left(\frac{1}{2}(c+dx)\right)\left(3675Adx\cos\left(\frac{dx}{2}\right)+3675Adx\cos\left(c+\frac{dx}{2}\right)+2205Adx\cos\left(c+\frac{3dx}{2}\right)+2205Adx\cos\left(2c+\frac{3dx}{2}\right)+735Adx\cos\left(2c+\frac{5dx}{2}\right)+735Adx\cos\left(3c+\frac{5dx}{2}\right)+105Adx\cos\left(3c+\frac{7dx}{2}\right)+105Adx\cos\left(4c+\frac{7dx}{2}\right)-9940A\sin\left(\frac{dx}{2}\right)+1260B\sin\left(\frac{dx}{2}\right)+8260A\sin\left(c+\frac{dx}{2}\right)-1260B\sin\left(c+\frac{dx}{2}\right)-7140A\sin\left(c+\frac{3dx}{2}\right)+882B\sin\left(c+\frac{3dx}{2}\right)+3780A\sin\left(2c+\frac{3dx}{2}\right)-630B\sin\left(2c+\frac{3dx}{2}\right)-2800A\sin\left(2c+\frac{5dx}{2}\right)+294B\sin\left(2c+\frac{5dx}{2}\right)+840A\sin\left(3c+\frac{5dx}{2}\right)-210B\sin\left(3c+\frac{5dx}{2}\right)-520A\sin\left(3c+\frac{7dx}{2}\right)+72B\sin\left(3c+\frac{7dx}{2}\right)\right)}{13440a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Sec[c/2]*Sec[(c + d*x)/2]^7*(3675*A*d*x*Cos[(d*x)/2] + 3675*A*d*x*Cos[c +
(d*x)/2] + 2205*A*d*x*Cos[c + (3*d*x)/2] + 2205*A*d*x*Cos[2*c + (3*d*x)/2]
+ 735*A*d*x*Cos[2*c + (5*d*x)/2] + 735*A*d*x*Cos[3*c + (5*d*x)/2] + 105*A*d
*x*Cos[3*c + (7*d*x)/2] + 105*A*d*x*Cos[4*c + (7*d*x)/2] - 9940*A*Sin[(d*x)
/2] + 1260*B*Sin[(d*x)/2] + 8260*A*Sin[c + (d*x)/2] - 1260*B*Sin[c + (d*x)/
2] - 7140*A*Sin[c + (3*d*x)/2] + 882*B*Sin[c + (3*d*x)/2] + 3780*A*Sin[2*c
+ (3*d*x)/2] - 630*B*Sin[2*c + (3*d*x)/2] - 2800*A*Sin[2*c + (5*d*x)/2] + 2
94*B*Sin[2*c + (5*d*x)/2] + 840*A*Sin[3*c + (5*d*x)/2] - 210*B*Sin[3*c + (5
*d*x)/2] - 520*A*Sin[3*c + (7*d*x)/2] + 72*B*Sin[3*c + (7*d*x)/2]))/(13440*
a^4*d)
```

Maple [**A**]
time = 0.25, size = 130, normalized size = 0.94

method	result

norman	$\frac{\frac{Ax}{a}+\frac{(A-B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56ad}-\frac{(5A-3B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{40ad}+\frac{(11A-3B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{24ad}-\frac{(15A-B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}}{a^3}$
derivativedivides	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}+\frac{11A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-15}{8d\,a^4}$
default	$\frac{\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A+\frac{3\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}+\frac{11A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}-B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-15}{8d\,a^4}$
risch	$\frac{Ax}{a^4}-\frac{2i\left(420A\,e^{6i(dx+c)}-105B\,e^{6i(dx+c)}+1890A\,e^{5i(dx+c)}-315B\,e^{5i(dx+c)}+4130A\,e^{4i(dx+c)}-630B\,e^{4i(dx+c)}+4970A}{105d\,a^4\left(e^{i(dx}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7*A-1/7*tan(1/2*d*x+1/2*c)^7*B-tan(1/2*d*
x+1/2*c)^5*A+3/5*tan(1/2*d*x+1/2*c)^5*B+11/3*A*tan(1/2*d*x+1/2*c)^3-B*tan(1
/2*d*x+1/2*c)^3-15*A*tan(1/2*d*x+1/2*c)+B*tan(1/2*d*x+1/2*c)+16*A*arctan(ta
n(1/2*d*x+1/2*c)))
```

Maxima [A]
time = 0.51, size = 201, normalized size = 1.46

$$-\frac{5\,A\left(\frac{\frac{315\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{77\,\sin\left(dx+c\right)^3}{\left(\cos\left(dx+c\right)+1\right)^3}+\frac{21\,\sin\left(dx+c\right)^5}{\left(\cos\left(dx+c\right)+1\right)^5}-\frac{3\,\sin\left(dx+c\right)^7}{\left(\cos\left(dx+c\right)+1\right)^7}}{a^4}-\frac{336\,\arctan\left(\frac{\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}\right)}{a^4}\right)-\frac{3\,B\left(\frac{35\,\sin\left(dx+c\right)}{\cos\left(dx+c\right)+1}-\frac{35\,\sin\left(dx+c\right)^3}{\left(\cos\left(dx+c\right)+1\right)^3}+\frac{21\,\sin\left(dx+c\right)^5}{\left(\cos\left(dx+c\right)+1\right)^5}-\frac{5\,\sin\left(dx+c\right)^7}{\left(\cos\left(dx+c\right)+1\right)^7}\right)}{a^4}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] -1/840*(5*A*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*sin(d*x + c)^7
/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^
4) - 3*B*(35*sin(d*x + c)/(cos(d*x + c) + 1) - 35*sin(d*x + c)^3/(cos(d*x +
 c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos
(d*x + c) + 1)^7)/a^4)/d
```

Fricas [A]
time = 2.29, size = 181, normalized size = 1.31

$$\frac{105\,Adx\cos\left(dx+c\right)^4+420\,Adx\cos\left(dx+c\right)^3+630\,Adx\cos\left(dx+c\right)^2+420\,Adx\cos\left(dx+c\right)+105\,Adx-\left(4\left(65\,A-9\,B\right)\cos\left(dx+c\right)^3+\left(620\,A-39\,B\right)\cos\left(dx+c\right)^2+\left(535\,A-24\,B\right)\cos\left(dx+c\right)+160\,A-6\,B\right)\sin\left(dx+c\right)}{105\left(a^4d\cos\left(dx+c\right)^4+4\,a^4d\cos\left(dx+c\right)^3+6\,a^4d\cos\left(dx+c\right)^2+4\,a^4d\cos\left(dx+c\right)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] 1/105*(105*A*d*x*cos(d*x + c)^4 + 420*A*d*x*cos(d*x + c)^3 + 630*A*d*x*cos(
d*x + c)^2 + 420*A*d*x*cos(d*x + c) + 105*A*d*x - (4*(65*A - 9*B)*cos(d*x +
```

```
c)^3 + (620*A - 39*B)*cos(d*x + c)^2 + (535*A - 24*B)*cos(d*x + c) + 160*A
 - 6*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^
4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int \frac{B\sec(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*se
c(c + d*x) + 1), x) + Integral(B*sec(c + d*x)/(sec(c + d*x)**4 + 4*sec(c +
d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1), x))/a**4
```

Giac [A]
time = 0.49, size = 154, normalized size = 1.12

$$\frac{\frac{840\,(dx+c)A}{a^4}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-105\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+63\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+385\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-105\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1575\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+105\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/840*(840*(d*x + c)*A/a^4 + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*
tan(1/2*d*x + 1/2*c)^7 - 105*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 63*B*a^24*tan(
1/2*d*x + 1/2*c)^5 + 385*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 105*B*a^24*tan(1/2
*d*x + 1/2*c)^3 - 1575*A*a^24*tan(1/2*d*x + 1/2*c) + 105*B*a^24*tan(1/2*d*x
 + 1/2*c))/a^28)/d
```

Mupad [B]
time = 2.03, size = 163, normalized size = 1.18

$$\frac{A\,x}{a^4}-\frac{\left(\frac{52\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}-\frac{12\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{35}\right)\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+\left(\frac{23\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{70}-\frac{16\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}\right)\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+\left(\frac{5\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{28}-\frac{9\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{70}\right)\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2-\frac{A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{56}+\frac{B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{56}}{a^4\,d\,{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^7}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^4,x)
```

```
[Out] (A*x)/a^4 - ((B*sin(c/2 + (d*x)/2))/56 - (A*sin(c/2 + (d*x)/2))/56 + cos(c/
2 + (d*x)/2)^2*((5*A*sin(c/2 + (d*x)/2))/28 - (9*B*sin(c/2 + (d*x)/2))/70)
+ cos(c/2 + (d*x)/2)^6*((52*A*sin(c/2 + (d*x)/2))/21 - (12*B*sin(c/2 + (d*x
)/2))/35) - cos(c/2 + (d*x)/2)^4*((16*A*sin(c/2 + (d*x)/2))/21 - (23*B*sin(
c/2 + (d*x)/2))/70))/(a^4*d*cos(c/2 + (d*x)/2)^7)
```

3.115 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=166

$$-\frac{(4A-B)x}{a^4}+\frac{8(83A-20B)\sin(c+dx)}{105a^4d}-\frac{(88A-25B)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2}-\frac{(4A-B)\sin(c+dx)}{a^4d(1+\sec(c+dx))}-\frac{(A-B)\sin(}{7d(a+a\sec(c}$$

```
[Out] -(4*A-B)*x/a^4+8/105*(83*A-20*B)*sin(d*x+c)/a^4/d-1/105*(88*A-25*B)*sin(d*x
+c)/a^4/d/(1+sec(d*x+c))^2-(4*A-B)*sin(d*x+c)/a^4/d/(1+sec(d*x+c))-1/7*(A-B
)*sin(d*x+c)/d/(a+a*sec(d*x+c))^4-1/35*(12*A-5*B)*sin(d*x+c)/a/d/(a+a*sec(d
*x+c))^3
```

Rubi [A]
time = 0.36, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4105, 3872, 2717, 8}

$$\frac{8(83A-20B)\sin(c+dx)}{105a^4d}-\frac{(4A-B)\sin(c+dx)}{a^4d(\sec(c+dx)+1)}-\frac{(88A-25B)\sin(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{x(4A-B)}{a^4}-\frac{(12A-5B)\sin(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{(A-B)\sin(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -(((4*A - B)*x)/a^4) + (8*(83*A - 20*B)*Sin[c + d*x])/(105*a^4*d) - ((88*A
 - 25*B)*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) - ((4*A - B)*Sin[c +
 d*x])/(a^4*d*(1 + Sec[c + d*x])) - ((A - B)*Sin[c + d*x])/(7*d*(a + a*Sec[
c + d*x])^4) - ((12*A - 5*B)*Sin[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
```

```
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A-B)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\cos(c+dx)(a(8A-B)-4a(A-B)\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx}{7a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(12A-5B)\sin(c+dx)}{35ad(a+a\sec(c+dx))^3} + \int \frac{\cos(c+dx)(}{}$$

$$= -\frac{(88A-25B)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(12A-5}{35ad(a+}$$

$$= -\frac{(88A-25B)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(12A-5}{35ad(a+}$$

$$= -\frac{(88A-25B)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(12A-5}{35ad(a+}$$

$$= -\frac{(4A-B)x}{a^4} + \frac{8(83A-20B)\sin(c+dx)}{105a^4d} - \frac{(88A-25B)\sin(c+}{105a^4d(1+\sec(c+}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 485 vs. 2(166) = 332.
time = 1.07, size = 485, normalized size = 2.92

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec\left(\frac{c}{2}\right)\left(-7350(4A-B)dx\cos\left(\frac{dx}{2}\right)-7350(4A-B)dx\cos\left(c+\frac{dx}{2}\right)-17640Adx\cos\left(c+\frac{3dx}{2}\right)+4410Bdx\cos\left(c+\frac{3dx}{2}\right)-17640Adx\cos\left(2c+\frac{3dx}{2}\right)+4410Bdx\cos\left(2c+\frac{3dx}{2}\right)-5880Adx\cos\left(2c+\frac{5dx}{2}\right)+1470Bdx\cos\left(2c+\frac{5dx}{2}\right)-5880Adx\cos\left(3c+\frac{5dx}{2}\right)+1470Bdx\cos\left(3c+\frac{5dx}{2}\right)-840Adx\cos\left(3c+\frac{7dx}{2}\right)+210Bdx\cos\left(3c+\frac{7dx}{2}\right)-840Adx\cos\left(4c+\frac{7dx}{2}\right)+210Bdx\cos\left(4c+\frac{7dx}{2}\right)+60830A\sin\left(\frac{dx}{2}\right)-19880B\sin\left(\frac{dx}{2}\right)-46130A\sin\left(c+\frac{dx}{2}\right)+16520B\sin\left(c+\frac{dx}{2}\right)+46116A\sin\left(c+\frac{3dx}{2}\right)-14280B\sin\left(c+\frac{3dx}{2}\right)-18060A\sin\left(2c+\frac{3dx}{2}\right)+7560B\sin\left(2c+\frac{3dx}{2}\right)+19292A\sin\left(2c+\frac{5dx}{2}\right)-5600B\sin\left(2c+\frac{5dx}{2}\right)-2100A\sin\left(3c+\frac{5dx}{2}\right)+1680B\sin\left(3c+\frac{5dx}{2}\right)+3791A\sin\left(3c+\frac{7dx}{2}\right)-1040B\sin\left(3c+\frac{7dx}{2}\right)+735A\sin\left(4c+\frac{7dx}{2}\right)+105A\sin\left(4c+\frac{9dx}{2}\right)+105A\sin\left(5c+\frac{9dx}{2}\right)\right)}{1680a^4d(1+\cos(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-7350*(4*A - B)*d*x*Cos[(d*x)/2] - 7350*(4*A -
B)*d*x*Cos[c + (d*x)/2] - 17640*A*d*x*Cos[c + (3*d*x)/2] + 4410*B*d*x*Cos[c
 + (3*d*x)/2] - 17640*A*d*x*Cos[2*c + (3*d*x)/2] + 4410*B*d*x*Cos[2*c + (3*
d*x)/2] - 5880*A*d*x*Cos[2*c + (5*d*x)/2] + 1470*B*d*x*Cos[2*c + (5*d*x)/2]
 - 5880*A*d*x*Cos[3*c + (5*d*x)/2] + 1470*B*d*x*Cos[3*c + (5*d*x)/2] - 840*
A*d*x*Cos[3*c + (7*d*x)/2] + 210*B*d*x*Cos[3*c + (7*d*x)/2] - 840*A*d*x*Cos
[4*c + (7*d*x)/2] + 210*B*d*x*Cos[4*c + (7*d*x)/2] + 60830*A*Sin[(d*x)/2] -
 19880*B*Sin[(d*x)/2] - 46130*A*Sin[c + (d*x)/2] + 16520*B*Sin[c + (d*x)/2]
 + 46116*A*Sin[c + (3*d*x)/2] - 14280*B*Sin[c + (3*d*x)/2] - 18060*A*Sin[2*
```

```
c + (3*d*x)/2] + 7560*B*Sin[2*c + (3*d*x)/2] + 19292*A*Sin[2*c + (5*d*x)/2]
 - 5600*B*Sin[2*c + (5*d*x)/2] - 2100*A*Sin[3*c + (5*d*x)/2] + 1680*B*Sin[3
*c + (5*d*x)/2] + 3791*A*Sin[3*c + (7*d*x)/2] - 1040*B*Sin[3*c + (7*d*x)/2]
 + 735*A*Sin[4*c + (7*d*x)/2] + 105*A*Sin[4*c + (9*d*x)/2] + 105*A*Sin[5*c
+ (9*d*x)/2]))/(1680*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A]
time = 0.34, size = 164, normalized size = 0.99

method	result
derivativedivides	$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}+\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}+\frac{7\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B-\frac{23A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{11B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+}{8d\,a^4}$
default	$\frac{-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}+\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}+\frac{7\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}-\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B-\frac{23A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+\frac{11B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+}{8d\,a^4}$
norman	$\frac{-\frac{(4A-B)x}{a}-\frac{(A-B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56ad}-\frac{(4A-B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{5(13A-3B)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{8ad}+\frac{(22A-15B)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{140ad}-\frac{(47A-}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}$
risch	$-\frac{4Ax}{a^4}+\frac{xB}{a^4}-\frac{iA\,e^{i(dx+c)}}{2a^4d}+\frac{iA\,e^{-i(dx+c)}}{2a^4d}+\frac{4i\left(525A\,e^{6i(dx+c)}-210B\,e^{6i(dx+c)}+2625A\,e^{5i(dx+c)}-945B\,e^{5i(dx+c)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7*A+1/7*tan(1/2*d*x+1/2*c)^7*B+7/5*tan(1
/2*d*x+1/2*c)^5*A-tan(1/2*d*x+1/2*c)^5*B-23/3*A*tan(1/2*d*x+1/2*c)^3+11/3*B
*tan(1/2*d*x+1/2*c)^3+49*A*tan(1/2*d*x+1/2*c)-15*B*tan(1/2*d*x+1/2*c)+16*A*
tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)-16*(4*A-B)*arctan(tan(1/2*d*x+1
/2*c)))
```

Maxima [A]
time = 0.50, size = 271, normalized size = 1.63

$$\frac{A\left(\frac{1680\,\sin(dx+c)}{\left(a^4+\frac{a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{805\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{6720\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)-5\,B\left(\frac{\frac{315\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{77\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{21\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{3\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{336\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="maxi
ma")
```

```
[Out] 1/840*(A*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2
)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*x + c) + 1) - 805*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 15
*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*arctan(sin(d*x + c)/(cos(d
*x + c) + 1))/a^4) - 5*B*((315*sin(d*x + c)/(cos(d*x + c) + 1) - 77*sin(d*x
 + c)^3/(cos(d*x + c) + 1)^3 + 21*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 3*s
```

```
in(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 336*arctan(sin(d*x + c)/(cos(d*x
+ c) + 1))/a^4))/d
```

Fricas [A]
time = 3.63, size = 223, normalized size = 1.34

$$-\frac{105\,(4\,A-B)dx\cos\left(dx+c\right)^{4}+420\,(4\,A-B)dx\cos\left(dx+c\right)^{3}+630\,(4\,A-B)dx\cos\left(dx+c\right)^{2}+420\,(4\,A-B)dx\cos\left(dx+c\right)+105\,(4\,A-B)dx-\left(105\,A\cos\left(dx+c\right)^{4}+4\,(296\,A-65\,B)\cos\left(dx+c\right)^{3}+4\,(659\,A-155\,B)\cos\left(dx+c\right)^{2}+(2236\,A-535\,B)\cos\left(dx+c\right)+664\,A-160\,B\right)\sin\left(dx+c\right)}{105\,\left(a^{4}d\cos\left(dx+c\right)^{4}+4\,a^{4}d\cos\left(dx+c\right)^{3}+6\,a^{4}d\cos\left(dx+c\right)^{2}+4\,a^{4}d\cos\left(dx+c\right)+a^{4}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fric
as")
```

```
[Out] -1/105*(105*(4*A - B)*d*x*cos(d*x + c)^4 + 420*(4*A - B)*d*x*cos(d*x + c)^3
 + 630*(4*A - B)*d*x*cos(d*x + c)^2 + 420*(4*A - B)*d*x*cos(d*x + c) + 105*
(4*A - B)*d*x - (105*A*cos(d*x + c)^4 + 4*(296*A - 65*B)*cos(d*x + c)^3 + 4
*(659*A - 155*B)*cos(d*x + c)^2 + (2236*A - 535*B)*cos(d*x + c) + 664*A - 1
60*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*
d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\cos\left(c+dx\right)}{\sec^{4}\left(c+dx\right)+4\sec^{3}\left(c+dx\right)+6\sec^{2}\left(c+dx\right)+4\sec\left(c+dx\right)+1}\,dx+\int \frac{B\cos\left(c+dx\right)\sec\left(c+dx\right)}{\sec^{4}\left(c+dx\right)+4\sec^{3}\left(c+dx\right)+6\sec^{2}\left(c+dx\right)+4\sec\left(c+dx\right)+1}\,dx}{a^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A*cos(c + d*x)/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d
*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*cos(c + d*x)*sec(c + d*x)/(se
c(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x) + 1)
, x))/a**4
```

Giac [A]
time = 0.50, size = 190, normalized size = 1.14

$$-\frac{\frac{840\,(dx+c)(4\,A-B)}{a^{4}}-\frac{1680\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)a^{4}}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-147\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+105\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+805\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-385\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-5145\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1575\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="giac
")
```

```
[Out] -1/840*(840*(d*x + c)*(4*A - B)/a^4 - 1680*A*tan(1/2*d*x + 1/2*c)/((tan(1/2
*d*x + 1/2*c)^2 + 1)*a^4) + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15*B*a^24*t
an(1/2*d*x + 1/2*c)^7 - 147*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 105*B*a^24*tan(
1/2*d*x + 1/2*c)^5 + 805*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 385*B*a^24*tan(1/2
```

```
*d*x + 1/2*c)^3 - 5145*A*a^24*tan(1/2*d*x + 1/2*c) + 1575*B*a^24*tan(1/2*d*
x + 1/2*c))/a^28)/d
```

Mupad [B]
time = 2.06, size = 202, normalized size = 1.22

$$\frac{\left(\frac{764\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{105}-\frac{52\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+\left(\frac{16\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{21}-\frac{143\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{105}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+\left(\frac{8\,A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{35}-\frac{5\,B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{28}\right)\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-\frac{A\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{56}+\frac{B\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{56}}{a^4\,d\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7}-\frac{4\,A\,d\,x-B\,d\,x}{a^4\,d}+\frac{2\,A\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^4,x)
```

```
[Out] ((B*sin(c/2 + (d*x)/2))/56 - (A*sin(c/2 + (d*x)/2))/56 + cos(c/2 + (d*x)/2)
^2*((8*A*sin(c/2 + (d*x)/2))/35 - (5*B*sin(c/2 + (d*x)/2))/28) - cos(c/2 +
(d*x)/2)^4*((143*A*sin(c/2 + (d*x)/2))/105 - (16*B*sin(c/2 + (d*x)/2))/21)
+ cos(c/2 + (d*x)/2)^6*((764*A*sin(c/2 + (d*x)/2))/105 - (52*B*sin(c/2 + (d
*x)/2))/21))/(a^4*d*cos(c/2 + (d*x)/2)^7) - (4*A*d*x - B*d*x)/(a^4*d) + (2*
A*cos(c/2 + (d*x)/2)*sin(c/2 + (d*x)/2))/(a^4*d)
```

3.116 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=223

$$\frac{(21A-8B)x}{2a^4}-\frac{8(216A-83B)\sin(c+dx)}{105a^4d}+\frac{(21A-8B)\cos(c+dx)\sin(c+dx)}{2a^4d}-\frac{(129A-52B)\cos(c+d}{105a^4d(1+\sec(c}$$

```
[Out] 1/2*(21*A-8*B)*x/a^4-8/105*(216*A-83*B)*sin(d*x+c)/a^4/d+1/2*(21*A-8*B)*cos
(d*x+c)*sin(d*x+c)/a^4/d-1/105*(129*A-52*B)*cos(d*x+c)*sin(d*x+c)/a^4/d/(1+
sec(d*x+c))^2-4/105*(216*A-83*B)*cos(d*x+c)*sin(d*x+c)/a^4/d/(1+sec(d*x+c))
-1/7*(A-B)*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^4-1/5*(2*A-B)*cos(d*x+c
)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^3
```

Rubi [A]
time = 0.44, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4105, 3872, 2715, 8, 2717}

$$-\frac{8(216A-83B)\sin(c+dx)}{105a^4d}+\frac{(21A-8B)\sin(c+dx)\cos(c+dx)}{2a^4d}-\frac{4(216A-83B)\sin(c+dx)\cos(c+dx)}{105a^4d(\sec(c+dx)+1)}-\frac{(129A-52B)\sin(c+dx)\cos(c+dx)}{105a^4d(\sec(c+dx)+1)^2}+\frac{x(21A-8B)}{2a^4}-\frac{(2A-B)\sin(c+dx)\cos(c+dx)}{5ad(a\sec(c+dx)+a)^3}-\frac{(A-B)\sin(c+dx)\cos(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] ((21*A - 8*B)*x)/(2*a^4) - (8*(216*A - 83*B)*Sin[c + d*x])/(105*a^4*d) + ((
21*A - 8*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*d) - ((129*A - 52*B)*Cos[c +
d*x]*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) - (4*(216*A - 83*B)*Cos
[c + d*x]*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])) - ((A - B)*Cos[c + d
*x]*Sin[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - ((2*A - B)*Cos[c + d*x]*Si
n[c + d*x])/(5*a*d*(a + a*Sec[c + d*x])^3)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\cos^2(c+dx)(a(9A-2B)-5a(A-B)\,\mathrm{se}}{(a+a\sec(c+dx))^3}}{7a^2}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(2A-B)\cos(c+dx)\sin(c+}{5ad(a+a\sec(c+dx))^3}$$

$$= -\frac{(129A-52B)\cos(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\cos(c+dx)\sin}{7d(a+a\sec(c+d}$$

$$= -\frac{(129A-52B)\cos(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\cos(c+dx)\sin}{7d(a+a\sec(c+d}$$

$$= -\frac{(129A-52B)\cos(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\cos(c+dx)\sin}{7d(a+a\sec(c+d}$$

$$= -\frac{8(216A-83B)\sin(c+dx)}{105a^4d} + \frac{(21A-8B)\cos(c+dx)\sin(c+dx}{2a^4d}$$

$$= \frac{(21A-8B)x}{2a^4} - \frac{8(216A-83B)\sin(c+dx)}{105a^4d} + \frac{(21A-8B)\cos(c+}{2a^4d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 555 vs. 2(223) = 446.
time = 1.15, size = 555, normalized size = 2.49

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(14700*(21*A - 8*B)*d*x*Cos[(d*x)/2] + 14700*(21
*A - 8*B)*d*x*Cos[c + (d*x)/2] + 185220*A*d*x*Cos[c + (3*d*x)/2] - 70560*B*
d*x*Cos[c + (3*d*x)/2] + 185220*A*d*x*Cos[2*c + (3*d*x)/2] - 70560*B*d*x*Co
s[2*c + (3*d*x)/2] + 61740*A*d*x*Cos[2*c + (5*d*x)/2] - 23520*B*d*x*Cos[2*c
 + (5*d*x)/2] + 61740*A*d*x*Cos[3*c + (5*d*x)/2] - 23520*B*d*x*Cos[3*c + (5
*d*x)/2] + 8820*A*d*x*Cos[3*c + (7*d*x)/2] - 3360*B*d*x*Cos[3*c + (7*d*x)/2
] + 8820*A*d*x*Cos[4*c + (7*d*x)/2] - 3360*B*d*x*Cos[4*c + (7*d*x)/2] - 539
490*A*Sin[(d*x)/2] + 243320*B*Sin[(d*x)/2] + 386190*A*Sin[c + (d*x)/2] - 18
4520*B*Sin[c + (d*x)/2] - 422478*A*Sin[c + (3*d*x)/2] + 184464*B*Sin[c + (3
*d*x)/2] + 132930*A*Sin[2*c + (3*d*x)/2] - 72240*B*Sin[2*c + (3*d*x)/2] - 1
81461*A*Sin[2*c + (5*d*x)/2] + 77168*B*Sin[2*c + (5*d*x)/2] + 3675*A*Sin[3*
c + (5*d*x)/2] - 8400*B*Sin[3*c + (5*d*x)/2] - 36003*A*Sin[3*c + (7*d*x)/2]
 + 15164*B*Sin[3*c + (7*d*x)/2] - 9555*A*Sin[4*c + (7*d*x)/2] + 2940*B*Sin[
4*c + (7*d*x)/2] - 945*A*Sin[4*c + (9*d*x)/2] + 420*B*Sin[4*c + (9*d*x)/2]
- 945*A*Sin[5*c + (9*d*x)/2] + 420*B*Sin[5*c + (9*d*x)/2] + 105*A*Sin[5*c +
 (11*d*x)/2] + 105*A*Sin[6*c + (11*d*x)/2]))/(6720*a^4*d*(1 + Cos[c + d*x])
^4)
```

Maple [A]
time = 0.32, size = 187, normalized size = 0.84

method	result
derivativedivides	$\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}-\frac{9\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}+\frac{7\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}+13A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{23B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}$
default	$\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}-\frac{\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}-\frac{9\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}+\frac{7\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}+13A\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{23B\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}$
norman	$\frac{\frac{(21A-8B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{(21A-8B)x}{2a}+\frac{(A-B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56ad}+\frac{(21A-8B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{(53A-39B)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{280ad}}{\left(1+\tan^2\left(\frac{dx}{2}\right.\right.}$
risch	$\frac{21Ax}{2a^4}-\frac{4xB}{a^4}-\frac{iA\,e^{2i(dx+c)}}{8a^4d}+\frac{2iA\,e^{i(dx+c)}}{a^4d}-\frac{ie^{i(dx+c)}B}{2a^4d}-\frac{2iA\,e^{-i(dx+c)}}{a^4d}+\frac{ie^{-i(dx+c)}B}{2a^4d}+\frac{iA\,e^{-2i(dx+c)}}{8a^4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/8/d/a^4*(1/7*tan(1/2*d*x+1/2*c)^7*A-1/7*tan(1/2*d*x+1/2*c)^7*B-9/5*tan(1/
2*d*x+1/2*c)^5*A+7/5*tan(1/2*d*x+1/2*c)^5*B+13*A*tan(1/2*d*x+1/2*c)^3-23/3*
B*tan(1/2*d*x+1/2*c)^3-111*A*tan(1/2*d*x+1/2*c)+49*B*tan(1/2*d*x+1/2*c)+16*
((-9/2*A+B)*tan(1/2*d*x+1/2*c)^3+(-7/2*A+B)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*
d*x+1/2*c)^2)^2+8*(21*A-8*B)*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]

time = 0.49, size = 364, normalized size = 1.63

$$-\frac{3\,A\left(\frac{280\left(\frac{7\,\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4+\frac{2\,a^4\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\,\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{455\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{5880\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)-B\left(\frac{1680\,\sin(dx+c)}{\left(a^4+\frac{a^4\,\sin(dx+c)^2}{(\cos(dx+c)+1)^2}\right)(\cos(dx+c)+1)}+\frac{\frac{5145\,\sin(dx+c)}{\cos(dx+c)+1}-\frac{805\,\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{147\,\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\,\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{6720\,\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] -1/840*(3*A*(280*(7*sin(d*x + c)/(cos(d*x + c) + 1) + 9*sin(d*x + c)^3/(cos
(d*x + c) + 1)^3)/(a^4 + 2*a^4*sin(d*x + c)^2/(cos(d*x + c) + 1)^2 + a^4*si
n(d*x + c)^4/(cos(d*x + c) + 1)^4) + (3885*sin(d*x + c)/(cos(d*x + c) + 1)
- 455*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c)
 + 1)^5 - 5*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 5880*arctan(sin(d*x
+ c)/(cos(d*x + c) + 1))/a^4) - B*(1680*sin(d*x + c)/((a^4 + a^4*sin(d*x +
c)^2/(cos(d*x + c) + 1)^2)*(cos(d*x + c) + 1)) + (5145*sin(d*x + c)/(cos(d*
x + c) + 1) - 805*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 147*sin(d*x + c)^5/
(cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 6720*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4))/d
```

Fricas [A]
time = 3.51, size = 240, normalized size = 1.08

$$\frac{105\,(21\,A-8\,B)dx\cos(dx+c)^4+420\,(21\,A-8\,B)dx\cos(dx+c)^3+630\,(21\,A-8\,B)dx\cos(dx+c)^2+420\,(21\,A-8\,B)dx\cos(dx+c)+105\,(21\,A-8\,B)dx+(105\,A\cos(dx+c)^5-210\,(2\,A-B)\cos(dx+c)^4-4\,(1509\,A-592\,B)\cos(dx+c)^3-4\,(3411\,A-1318\,B)\cos(dx+c)^2-(11619\,A-4472\,B)\cos(dx+c)-3456\,A+1328\,B)\sin(dx+c)}{210\,(a^4d\cos(dx+c)^4+4\,a^4d\cos(dx+c)^3+6\,a^4d\cos(dx+c)^2+4\,a^4d\cos(dx+c)+a^4d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] 1/210*(105*(21*A - 8*B)*d*x*cos(d*x + c)^4 + 420*(21*A - 8*B)*d*x*cos(d*x +
 c)^3 + 630*(21*A - 8*B)*d*x*cos(d*x + c)^2 + 420*(21*A - 8*B)*d*x*cos(d*x
+ c) + 105*(21*A - 8*B)*d*x + (105*A*cos(d*x + c)^5 - 210*(2*A - B)*cos(d*x
 + c)^4 - 4*(1509*A - 592*B)*cos(d*x + c)^3 - 4*(3411*A - 1318*B)*cos(d*x +
 c)^2 - (11619*A - 4472*B)*cos(d*x + c) - 3456*A + 1328*B)*sin(d*x + c))/(a
^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a
^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\cos^2(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int \frac{B\cos^2(c+dx)\sec(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A*cos(c + d*x)**2/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c
+ d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*cos(c + d*x)**2*sec(c + d*
x)/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x
) + 1), x))/a**4
```

Giac [A]
time = 0.51, size = 233, normalized size = 1.04

$$\frac{\frac{420\,(dx+c)(21\,A-8\,B)}{a^4}-\frac{840\,\left(9\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^4}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-189\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+147\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1365\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-805\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-11655\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5145\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/840*(420*(d*x + c)*(21*A - 8*B)/a^4 - 840*(9*A*tan(1/2*d*x + 1/2*c)^3 - 2
*B*tan(1/2*d*x + 1/2*c)^3 + 7*A*tan(1/2*d*x + 1/2*c) - 2*B*tan(1/2*d*x + 1/
2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^4) + (15*A*a^24*tan(1/2*d*x + 1/2*c
)^7 - 15*B*a^24*tan(1/2*d*x + 1/2*c)^7 - 189*A*a^24*tan(1/2*d*x + 1/2*c)^5
+ 147*B*a^24*tan(1/2*d*x + 1/2*c)^5 + 1365*A*a^24*tan(1/2*d*x + 1/2*c)^3 -
805*B*a^24*tan(1/2*d*x + 1/2*c)^3 - 11655*A*a^24*tan(1/2*d*x + 1/2*c) + 514
5*B*a^24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

Mupad [B]
time = 2.00, size = 179, normalized size = 0.80

$$\frac{\frac{21\,A\,d\,x}{2}-4\,B\,d\,x}{a^4\,d}-\frac{\left(9\,A-2\,B\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(7\,A-2\,B\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{a^4\,d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)^2}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(\frac{111\,A}{8}-\frac{49\,B}{8}\right)}{a^4\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(\frac{13\,A}{8}-\frac{23\,B}{24}\right)}{a^4\,d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(\frac{9\,A}{40}-\frac{7\,B}{40}\right)}{a^4\,d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7\,\left(\frac{A}{56}-\frac{B}{56}\right)}{a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^4,x)
```

```
[Out] ((21*A*d*x)/2 - 4*B*d*x)/(a^4*d) - (tan(c/2 + (d*x)/2)^3*(9*A - 2*B) + tan(
c/2 + (d*x)/2)*(7*A - 2*B))/(a^4*d*(tan(c/2 + (d*x)/2)^2 + 1)^2) - (tan(c/2
 + (d*x)/2)*((111*A)/8 - (49*B)/8))/(a^4*d) + (tan(c/2 + (d*x)/2)^3*((13*A)
/8 - (23*B)/24))/(a^4*d) - (tan(c/2 + (d*x)/2)^5*((9*A)/40 - (7*B)/40))/(a^
4*d) + (tan(c/2 + (d*x)/2)^7*(A/56 - B/56))/(a^4*d)
```

3.117 $\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx$

Optimal. Leaf size=256

$$-\frac{(44A-21B)x}{2a^4}+\frac{8(227A-108B)\sin(c+dx)}{35a^4d}-\frac{(44A-21B)\cos(c+dx)\sin(c+dx)}{2a^4d}-\frac{(178A-87B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2}$$

```
[Out] -1/2*(44*A-21*B)*x/a^4+8/35*(227*A-108*B)*sin(d*x+c)/a^4/d-1/2*(44*A-21*B)*
cos(d*x+c)*sin(d*x+c)/a^4/d-1/105*(178*A-87*B)*cos(d*x+c)^2*sin(d*x+c)/a^4/
d/(1+sec(d*x+c))^2-1/3*(44*A-21*B)*cos(d*x+c)^2*sin(d*x+c)/a^4/d/(1+sec(d*x
+c))-1/7*(A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^4-1/35*(16*A-9*B)
*cos(d*x+c)^2*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^3-8/105*(227*A-108*B)*sin(d*x
+c)^3/a^4/d
```

Rubi [A]
time = 0.47, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4105, 3872, 2713, 2715, 8}

$$-\frac{8(227A-108B)\sin^3(c+dx)}{105a^4d}+\frac{8(227A-108B)\sin(c+dx)}{35a^4d}-\frac{(44A-21B)\sin(c+dx)\cos(c+dx)}{2a^4d}-\frac{(44A-21B)\sin(c+dx)\cos^2(c+dx)}{3a^4d(\sec(c+dx)+1)}-\frac{(178A-87B)\sin(c+dx)\cos^2(c+dx)}{105a^4d(\sec(c+dx)+1)^2}-\frac{x(44A-21B)}{2a^4}-\frac{(16A-9B)\sin(c+dx)\cos^2(c+dx)}{35ad(a\sec(c+dx)+a)^3}-\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{7d(a\sec(c+dx)+a)^4}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] -1/2*((44*A - 21*B)*x)/a^4 + (8*(227*A - 108*B)*Sin[c + d*x])/(35*a^4*d) -
((44*A - 21*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*d) - ((178*A - 87*B)*Cos[c
 + d*x]^2*Sin[c + d*x])/(105*a^4*d*(1 + Sec[c + d*x])^2) - ((44*A - 21*B)*C
os[c + d*x]^2*Sin[c + d*x])/(3*a^4*d*(1 + Sec[c + d*x])) - ((A - B)*Cos[c +
 d*x]^2*Sin[c + d*x])/(7*d*(a + a*Sec[c + d*x])^4) - ((16*A - 9*B)*Cos[c +
d*x]^2*Sin[c + d*x])/(35*a*d*(a + a*Sec[c + d*x])^3) - (8*(227*A - 108*B)*S
in[c + d*x]^3)/(105*a^4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
```

```
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^4}\,dx = -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} + \frac{\int \frac{\cos^3(c+dx)(a(10A-3B)-6a(A-}{(a+a\sec(c+dx))^3}}{7a^2}$$

$$= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{7d(a+a\sec(c+dx))^4} - \frac{(16A-9B)\cos^2(c+dx)\,\mathrm{si}}{35ad(a+a\sec(c+d}$$

$$= -\frac{(178A-87B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\cos^2(c+dx)}{7d(a+a\sec(c+}$$

$$= -\frac{(178A-87B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\cos^2(c+dx)}{7d(a+a\sec(c+}$$

$$= -\frac{(178A-87B)\cos^2(c+dx)\sin(c+dx)}{105a^4d(1+\sec(c+dx))^2} - \frac{(A-B)\cos^2(c+dx)}{7d(a+a\sec(c+}$$

$$= -\frac{(44A-21B)\cos(c+dx)\sin(c+dx)}{2a^4d} - \frac{(178A-87B)\cos^2(c+}{105a^4d(1+\sec(}$$

$$= -\frac{(44A-21B)x}{2a^4} + \frac{8(227A-108B)\sin(c+dx)}{35a^4d} - \frac{(44A-21B)}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 611 vs. 2(256) = 512.
time = 1.58, size = 611, normalized size = 2.39

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^4,x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c/2]*(-14700*(44*A - 21*B)*d*x*Cos[(d*x)/2] - 14700*(
44*A - 21*B)*d*x*Cos[c + (d*x)/2] - 388080*A*d*x*Cos[c + (3*d*x)/2] + 18522
0*B*d*x*Cos[c + (3*d*x)/2] - 388080*A*d*x*Cos[2*c + (3*d*x)/2] + 185220*B*d
*x*Cos[2*c + (3*d*x)/2] - 129360*A*d*x*Cos[2*c + (5*d*x)/2] + 61740*B*d*x*C
os[2*c + (5*d*x)/2] - 129360*A*d*x*Cos[3*c + (5*d*x)/2] + 61740*B*d*x*Cos[3
*c + (5*d*x)/2] - 18480*A*d*x*Cos[3*c + (7*d*x)/2] + 8820*B*d*x*Cos[3*c + (
7*d*x)/2] - 18480*A*d*x*Cos[4*c + (7*d*x)/2] + 8820*B*d*x*Cos[4*c + (7*d*x)
/2] + 1010660*A*Sin[(d*x)/2] - 539490*B*Sin[(d*x)/2] - 687260*A*Sin[c + (d*
x)/2] + 386190*B*Sin[c + (d*x)/2] + 814107*A*Sin[c + (3*d*x)/2] - 422478*B*
Sin[c + (3*d*x)/2] - 204645*A*Sin[2*c + (3*d*x)/2] + 132930*B*Sin[2*c + (3*
d*x)/2] + 357609*A*Sin[2*c + (5*d*x)/2] - 181461*B*Sin[2*c + (5*d*x)/2] + 1
8025*A*Sin[3*c + (5*d*x)/2] + 3675*B*Sin[3*c + (5*d*x)/2] + 72522*A*Sin[3*c
 + (7*d*x)/2] - 36003*B*Sin[3*c + (7*d*x)/2] + 24010*A*Sin[4*c + (7*d*x)/2]
 - 9555*B*Sin[4*c + (7*d*x)/2] + 2310*A*Sin[4*c + (9*d*x)/2] - 945*B*Sin[4*
c + (9*d*x)/2] + 2310*A*Sin[5*c + (9*d*x)/2] - 945*B*Sin[5*c + (9*d*x)/2] -
 175*A*Sin[5*c + (11*d*x)/2] + 105*B*Sin[5*c + (11*d*x)/2] - 175*A*Sin[6*c
+ (11*d*x)/2] + 105*B*Sin[6*c + (11*d*x)/2] + 35*A*Sin[6*c + (13*d*x)/2] +
35*A*Sin[7*c + (13*d*x)/2]))/(6720*a^4*d*(1 + Cos[c + d*x])^4)
```

Maple [A]

time = 0.37, size = 210, normalized size = 0.82

method	result
derivativedivides	$-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}+\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}+\frac{11\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}-\frac{9\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}-\frac{59A\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+13B\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$
default	$-\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{7}+\frac{\left(\tan^{7}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{7}+\frac{11\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)A}{5}-\frac{9\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)B}{5}-\frac{59A\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3}+13B\left(\tan^{3}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.$
norman	$\frac{-\frac{(44A-21B)x}{2a}-\frac{(A-B)\left(\tan^{13}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{56ad}+\frac{(31A-24B)\left(\tan^{11}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{140ad}-\frac{3(44A-21B)x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2a}-\frac{3(44A-21B)x\left(\tan^{4}\left(\frac{dx}{2}\right.\right.}{2a}}{}$
risch	$-\frac{22Ax}{a^4}+\frac{21xB}{2a^4}-\frac{iA\,e^{3i(dx+c)}}{24a^4d}+\frac{iA\,e^{2i(dx+c)}}{2a^4d}-\frac{ie^{2i(dx+c)}B}{8a^4d}-\frac{43iA\,e^{i(dx+c)}}{8a^4d}+\frac{2ie^{i(dx+c)}B}{a^4d}+\frac{43iA\,e^{-i(dx-}}{8a^4d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/8/d/a^4*(-1/7*tan(1/2*d*x+1/2*c)^7*A+1/7*tan(1/2*d*x+1/2*c)^7*B+11/5*tan(
1/2*d*x+1/2*c)^5*A-9/5*tan(1/2*d*x+1/2*c)^5*B-59/3*A*tan(1/2*d*x+1/2*c)^3+1
3*B*tan(1/2*d*x+1/2*c)^3+209*A*tan(1/2*d*x+1/2*c)-111*B*tan(1/2*d*x+1/2*c)-
16*((-13*A+9/2*B)*tan(1/2*d*x+1/2*c)^5+(-62/3*A+8*B)*tan(1/2*d*x+1/2*c)^3+(
```

```
-9*A+7/2*B)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^3-8*(44*A-21*B)*ar
ctan(tan(1/2*d*x+1/2*c)))
```

Maxima [A]

time = 0.49, size = 452, normalized size = 1.77

$$\frac{A\left(\frac{560\left(\frac{27\sin(dx+c)}{\cos(dx+c)+1}+\frac{62\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{39\sin(dx+c)^5}{(\cos(dx+c)+1)^5}\right)}{a^4+\frac{3a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{3a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}+\frac{a^4\sin(dx+c)^6}{(\cos(dx+c)+1)^6}}+\frac{\frac{21945\sin(dx+c)}{\cos(dx+c)+1}-\frac{2065\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{231\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{15\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{36960\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)-3B\left(\frac{280\left(\frac{7\sin(dx+c)}{\cos(dx+c)+1}+\frac{9\sin(dx+c)^3}{(\cos(dx+c)+1)^3}\right)}{a^4+\frac{2a^4\sin(dx+c)^2}{(\cos(dx+c)+1)^2}+\frac{a^4\sin(dx+c)^4}{(\cos(dx+c)+1)^4}}+\frac{\frac{3885\sin(dx+c)}{\cos(dx+c)+1}-\frac{455\sin(dx+c)^3}{(\cos(dx+c)+1)^3}+\frac{63\sin(dx+c)^5}{(\cos(dx+c)+1)^5}-\frac{5\sin(dx+c)^7}{(\cos(dx+c)+1)^7}}{a^4}-\frac{5880\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{a^4}\right)}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] 1/840*(A*(560*(27*sin(d*x + c)/(cos(d*x + c) + 1) + 62*sin(d*x + c)^3/(cos(
d*x + c) + 1)^3 + 39*sin(d*x + c)^5/(cos(d*x + c) + 1)^5)/(a^4 + 3*a^4*sin(
d*x + c)^2/(cos(d*x + c) + 1)^2 + 3*a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4
 + a^4*sin(d*x + c)^6/(cos(d*x + c) + 1)^6) + (21945*sin(d*x + c)/(cos(d*x
+ c) + 1) - 2065*sin(d*x + c)^3/(cos(d*x + c) + 1)^3 + 231*sin(d*x + c)^5/(
cos(d*x + c) + 1)^5 - 15*sin(d*x + c)^7/(cos(d*x + c) + 1)^7)/a^4 - 36960*a
rctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4) - 3*B*(280*(7*sin(d*x + c)/(cos
(d*x + c) + 1) + 9*sin(d*x + c)^3/(cos(d*x + c) + 1)^3)/(a^4 + 2*a^4*sin(d*
x + c)^2/(cos(d*x + c) + 1)^2 + a^4*sin(d*x + c)^4/(cos(d*x + c) + 1)^4) +
(3885*sin(d*x + c)/(cos(d*x + c) + 1) - 455*sin(d*x + c)^3/(cos(d*x + c) +
1)^3 + 63*sin(d*x + c)^5/(cos(d*x + c) + 1)^5 - 5*sin(d*x + c)^7/(cos(d*x +
 c) + 1)^7)/a^4 - 5880*arctan(sin(d*x + c)/(cos(d*x + c) + 1))/a^4))/d
```

Fricas [A]

time = 3.31, size = 257, normalized size = 1.00

$$-\frac{105\,(44A-21B)dx\cos(dx+c)^4+420\,(44A-21B)dx\cos(dx+c)^3+630\,(44A-21B)dx\cos(dx+c)^2+420\,(44A-21B)dx\cos(dx+c)+105\,(44A-21B)dx-\left(70A\cos(dx+c)^6-35\,(4A-3B)\cos(dx+c)^5+140\,(7A-3B)\cos(dx+c)^4+4\,(3196A-1509B)\cos(dx+c)^3+4\,(7184A-3411B)\cos(dx+c)^2+(24436A-11619B)\cos(dx+c)+7264A-3456B\right)\sin(dx+c)}{210\left(a^4d\cos(dx+c)^4+4a^4d\cos(dx+c)^3+6a^4d\cos(dx+c)^2+4a^4d\cos(dx+c)+a^4d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] -1/210*(105*(44*A - 21*B)*d*x*cos(d*x + c)^4 + 420*(44*A - 21*B)*d*x*cos(d*
x + c)^3 + 630*(44*A - 21*B)*d*x*cos(d*x + c)^2 + 420*(44*A - 21*B)*d*x*cos
(d*x + c) + 105*(44*A - 21*B)*d*x - (70*A*cos(d*x + c)^6 - 35*(4*A - 3*B)*c
os(d*x + c)^5 + 140*(7*A - 3*B)*cos(d*x + c)^4 + 4*(3196*A - 1509*B)*cos(d*
x + c)^3 + 4*(7184*A - 3411*B)*cos(d*x + c)^2 + (24436*A - 11619*B)*cos(d*x
 + c) + 7264*A - 3456*B)*sin(d*x + c))/(a^4*d*cos(d*x + c)^4 + 4*a^4*d*cos(
d*x + c)^3 + 6*a^4*d*cos(d*x + c)^2 + 4*a^4*d*cos(d*x + c) + a^4*d)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\cos^3(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx+\int \frac{B\cos^3(c+dx)\sec(c+dx)}{\sec^4(c+dx)+4\sec^3(c+dx)+6\sec^2(c+dx)+4\sec(c+dx)+1}\,dx}{a^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**4,x)
```

```
[Out] (Integral(A*cos(c + d*x)**3/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c
+ d*x)**2 + 4*sec(c + d*x) + 1), x) + Integral(B*cos(c + d*x)**3*sec(c + d*
x)/(sec(c + d*x)**4 + 4*sec(c + d*x)**3 + 6*sec(c + d*x)**2 + 4*sec(c + d*x
) + 1), x))/a**4
```

Giac [A]
time = 0.55, size = 261, normalized size = 1.02

$$-\frac{\frac{420\,(dx+c)(44\,A-21\,B)}{a^4}-\frac{280\left(78\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-27\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+124\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-48\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+54\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-21\,B\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^4}+\frac{15\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-231\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+189\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+2065\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-1365\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-21945\,Aa^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+11655\,Ba^{24}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{28}}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/840*(420*(d*x + c)*(44*A - 21*B)/a^4 - 280*(78*A*tan(1/2*d*x + 1/2*c)^5
- 27*B*tan(1/2*d*x + 1/2*c)^5 + 124*A*tan(1/2*d*x + 1/2*c)^3 - 48*B*tan(1/2
*d*x + 1/2*c)^3 + 54*A*tan(1/2*d*x + 1/2*c) - 21*B*tan(1/2*d*x + 1/2*c))/((
tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^4) + (15*A*a^24*tan(1/2*d*x + 1/2*c)^7 - 15
*B*a^24*tan(1/2*d*x + 1/2*c)^7 - 231*A*a^24*tan(1/2*d*x + 1/2*c)^5 + 189*B*
a^24*tan(1/2*d*x + 1/2*c)^5 + 2065*A*a^24*tan(1/2*d*x + 1/2*c)^3 - 1365*B*a
^24*tan(1/2*d*x + 1/2*c)^3 - 21945*A*a^24*tan(1/2*d*x + 1/2*c) + 11655*B*a^
24*tan(1/2*d*x + 1/2*c))/a^28)/d
```

Mupad [B]
time = 2.05, size = 300, normalized size = 1.17

$$\frac{(26\,A-9\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{124\,A}{3}-16\,B\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+(18\,A-7\,B)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+3\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a^4\right)}-\frac{x\,(44\,A-21\,B)}{2\,a^4}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\left(\frac{5\,(A-B)}{12\,a^4}+\frac{7\,A-5\,B}{6\,a^4}+\frac{21\,A-9\,B}{24\,a^4}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\left(\frac{A-B}{10\,a^4}+\frac{7\,A-5\,B}{40\,a^4}\right)}{d}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{5\,(A-B)}{2\,a^4}+\frac{5\,(7\,A-5\,B)}{4\,a^4}+\frac{21\,A-9\,B}{2\,a^4}+\frac{35\,A-5\,B}{8\,a^4}\right)}{d}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7(A-B)}{56\,a^4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^4,x)
```

```
[Out] (tan(c/2 + (d*x)/2)^5*(26*A - 9*B) + tan(c/2 + (d*x)/2)^3*((124*A)/3 - 16*B
) + tan(c/2 + (d*x)/2)*(18*A - 7*B))/(d*(3*a^4*tan(c/2 + (d*x)/2)^2 + 3*a^4
*tan(c/2 + (d*x)/2)^4 + a^4*tan(c/2 + (d*x)/2)^6 + a^4)) - (x*(44*A - 21*B)
)/(2*a^4) - (tan(c/2 + (d*x)/2)^3*((5*(A - B))/(12*a^4) + (7*A - 5*B)/(6*a^
4) + (21*A - 9*B)/(24*a^4)))/d + (tan(c/2 + (d*x)/2)^5*((A - B)/(10*a^4) +
(7*A - 5*B)/(40*a^4)))/d + (tan(c/2 + (d*x)/2)*((5*(A - B))/(2*a^4) + (5*(7
*A - 5*B))/(4*a^4) + (21*A - 9*B)/(2*a^4) + (35*A - 5*B)/(8*a^4)))/d - (tan
(c/2 + (d*x)/2)^7*(A - B))/(56*a^4*d)
```

3.118 $\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=187

$$\frac{4a(9A+8B)\tan(c+dx)}{45d\sqrt{a+a\sec(c+dx)}}+\frac{2a(9A+8B)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}}+\frac{2aB\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}}-\frac{8(9A+$$

```
[Out] 4/105*(9*A+8*B)*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/a/d+4/45*a*(9*A+8*B)*tan(
d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/63*a*(9*A+8*B)*sec(d*x+c)^3*tan(d*x+c)/d/
(a+a*sec(d*x+c))^(1/2)+2/9*a*B*sec(d*x+c)^4*tan(d*x+c)/d/(a+a*sec(d*x+c))^(
1/2)-8/315*(9*A+8*B)*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.22, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4101, 3888, 3885, 4086, 3877}

$$\frac{2a(9A+8B)\tan(c+dx)\sec^3(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{4(9A+8B)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{105ad}-\frac{8(9A+8B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}+\frac{4a(9A+8B)\tan(c+dx)}{45d\sqrt{a\sec(c+dx)+a}}+\frac{2aB\tan(c+dx)\sec^4(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (4*a*(9*A + 8*B)*Tan[c + d*x])/(45*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(9*A
+ 8*B)*Sec[c + d*x]^3*Tan[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*
B*Sec[c + d*x]^4*Tan[c + d*x])/(9*d*Sqrt[a + a*Sec[c + d*x]]) - (8*(9*A + 8
*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (4*(9*A + 8*B)*(a + a*
Sec[c + d*x])^(3/2)*Tan[c + d*x])/(105*a*d)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int \sec^4(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \frac{2aB\sec^4(c+dx)\tan(c+dx)}{9d\sqrt{a+a\sec(c+dx)}}+\frac{1}{9}(9A+8B)\int$$

$$= \frac{2a(9A+8B)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}}+\frac{2aB\,\mathrm{s}}{9d}$$

$$= \frac{2a(9A+8B)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}}+\frac{2aB\,\mathrm{s}}{9d}$$

$$= \frac{2a(9A+8B)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}}+\frac{2aB\,\mathrm{s}}{9d}$$

$$= \frac{4a(9A+8B)\tan(c+dx)}{45d\sqrt{a+a\sec(c+dx)}}+\frac{2a(9A+8B)\sec^3(}{63d\sqrt{a+a}}$$

Mathematica [A]

time = 0.60, size = 98, normalized size = 0.52

$$\frac{2a(16(9A+8B)+8(9A+8B)\sec(c+dx)+6(9A+8B)\sec^2(c+dx)+5(9A+8B)\sec^3(c+dx)+35B\sec^4(c+dx))\tan(c+dx)}{315d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*(16*(9*A + 8*B) + 8*(9*A + 8*B)*Sec[c + d*x] + 6*(9*A + 8*B)*Sec[c + d
*x]^2 + 5*(9*A + 8*B)*Sec[c + d*x]^3 + 35*B*Sec[c + d*x]^4)*Tan[c + d*x])/(
315*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 3.92, size = 138, normalized size = 0.74

method	result
default	$-\frac{2(-1+\cos(dx+c))(144A(\cos^4(dx+c))+128B(\cos^4(dx+c))+72A(\cos^3(dx+c))+64B(\cos^3(dx+c))+54A(\cos^2(dx+c))+48B(\cos^2}{315d\cos(dx+c)^4\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(144*A*cos(d*x+c)^4+128*B*cos(d*x+c)^4+72*A*cos(d*
x+c)^3+64*B*cos(d*x+c)^3+54*A*cos(d*x+c)^2+48*B*cos(d*x+c)^2+45*A*cos(d*x+c
)+40*B*cos(d*x+c)+35*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^4/si
n(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 16/315*(315*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*((A*d*cos(2*d*x + 2*c)^4 + A*d*sin(2*d*x + 2*c)^4 + 4*A*d*cos(2*d
*x + 2*c)^3 + 6*A*d*cos(2*d*x + 2*c)^2 + 4*A*d*cos(2*d*x + 2*c) + 2*(A*d*co
s(2*d*x + 2*c)^2 + 2*A*d*cos(2*d*x + 2*c) + A*d)*sin(2*d*x + 2*c)^2 + A*d)*
integrate((((cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 5*cos(10*d*x + 10*c)*cos
(2*d*x + 2*c) + 10*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 10*cos(6*d*x + 6*c)*
cos(2*d*x + 2*c) + 5*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2
 + sin(12*d*x + 12*c)*sin(2*d*x + 2*c) + 5*sin(10*d*x + 10*c)*sin(2*d*x + 2
```

```
*c) + 10*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 10*sin(6*d*x + 6*c)*sin(2*d*x
+ 2*c) + 5*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(7/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(12*d*x
 + 12*c) + 5*cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 10*cos(2*d*x + 2*c)*sin(
8*d*x + 8*c) + 10*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 5*cos(2*d*x + 2*c)*si
n(4*d*x + 4*c) - cos(12*d*x + 12*c)*sin(2*d*x + 2*c) - 5*cos(10*d*x + 10*c)
*sin(2*d*x + 2*c) - 10*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 10*cos(6*d*x + 6
*c)*sin(2*d*x + 2*c) - 5*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(7/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(12*d*x + 12*c) + 5*cos(2*d*x
+ 2*c)*sin(10*d*x + 10*c) + 10*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 10*cos(2
*d*x + 2*c)*sin(6*d*x + 6*c) + 5*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(12
*d*x + 12*c)*sin(2*d*x + 2*c) - 5*cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 10*
cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 10*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) -
5*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - (cos(12*d*x + 12*c)*cos(2*d*x + 2*c) + 5*cos(10*d*x + 10*c
)*cos(2*d*x + 2*c) + 10*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 10*cos(6*d*x +
6*c)*cos(2*d*x + 2*c) + 5*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2
*c)^2 + sin(12*d*x + 12*c)*sin(2*d*x + 2*c) + 5*sin(10*d*x + 10*c)*sin(2*d*
x + 2*c) + 10*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 10*sin(6*d*x + 6*c)*sin(2
*d*x + 2*c) + 5*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin
(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)))/(((2*(5*cos(10*d*x + 10*c) + 10*cos(8*d*x
 + 8*c) + 10*cos(6*d*x + 6*c) + 5*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(
12*d*x + 12*c) + cos(12*d*x + 12*c)^2 + 10*(10*cos(8*d*x + 8*c) + 10*cos(6*
d*x + 6*c) + 5*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 25
*cos(10*d*x + 10*c)^2 + 20*(10*cos(6*d*x + 6*c) + 5*cos(4*d*x + 4*c) + cos(
2*d*x + 2*c))*cos(8*d*x + 8*c) + 100*cos(8*d*x + 8*c)^2 + 20*(5*cos(4*d*x +
 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 100*cos(6*d*x + 6*c)^2 + 25*co
s(4*d*x + 4*c)^2 + 10*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^
2 + 2*(5*sin(10*d*x + 10*c) + 10*sin(8*d*x + 8*c) + 10*sin(6*d*x + 6*c) + 5
*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + sin(12*d*x + 12*
c)^2 + 10*(10*sin(8*d*x + 8*c) + 10*sin(6*d*x + 6*c) + 5*sin(4*d*x + 4*c) +
 sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 25*sin(10*d*x + 10*c)^2 + 20*(10*si
n(6*d*x + 6*c) + 5*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) +
100*sin(8*d*x + 8*c)^2 + 20*(5*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d
*x + 6*c) + 100*sin(6*d*x + 6*c)^2 + 25*sin(4*d*x + 4*c)^2 + 10*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))^2 + (2*(5*cos(10*d*x + 10*c) + 10*cos(8*d*x + 8*
c) + 10*cos(6*d*x + 6*c) + 5*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(12*d*
x + 12*c) + cos(12*d*x + 12*c)^2 + 10*(10*cos(8*d*x + 8*c) + 10*cos(6*d*x +
 6*c) + 5*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + 25*cos(
10*d*x + 10*c)^2 + 20*(10*cos(6*d*x + 6*c) + 5*cos(4*d*x + 4*c) + cos(2*d*x
 + 2*c))*cos(8*d*x + 8*c) + 100*cos(8*d*x + 8*c)^2 + 20*(5*cos(4*d*x + 4*c)
 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 100*cos(6*d*x + 6*c)^2 + 25*cos(4*d
```

```
*x + 4*c)^2 + 10*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2
*(5*sin(10*d*x + 10*c) + 10*sin(8*d*x + 8*c) + 10*sin(6*d*x + 6*c) + 5*sin(
4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(12*d*x + 12*c) + sin(12*d*x + 12*c)^2
+ 10*(10*sin(8*d*x + 8*c) + 10*sin(6*d*x + 6*c) + 5*sin(4*d*x + 4*c) + sin(
2*d*x + 2*c))*sin(10*d*x + 10*c) + 25*sin(10*d*x + 10*c)^2 + 20*(10*sin(6*d
*x + 6*c) + 5*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 100*s
in(8*d*x + 8*c)^2 + 20*(5*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x +
6*c) + 100*sin(6*d*x + 6*c)^2 + 25*sin(4*d*x + 4*c)^2 + 10*sin(4*d*x + 4*c)
*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)), x) + ((A + 2*B)*d*cos(2*d*x + 2*c)^4 + (A + 2*B)*d
*sin(2*d*x + 2*c)^4 + 4*(A + 2*B)*d*cos(2*d*x + 2*c)^3 + 6*(A + 2*B)*d*cos(
2*d*x + 2*c)^2 + 4*(A + 2*B)*d*cos(2*d*x + 2*c) + 2*((A + 2*B)*d*cos(2*d*x
+ 2*c)^2 + 2*(A + 2*B)*d*cos(2*d*x + 2*c) + (A ...
```

Fricas [**A**]
time = 3.55, size = 122, normalized size = 0.65

$$\frac{2\,{\left(16\,{\left(9\,A + 8\,B\right)} \cos\left(dx + c\right)^{4} + 8\,{\left(9\,A + 8\,B\right)} \cos\left(dx + c\right)^{3} + 6\,{\left(9\,A + 8\,B\right)} \cos\left(dx + c\right)^{2} + 5\,{\left(9\,A + 8\,B\right)} \cos\left(dx + c\right) + 35\,B\right)} \sqrt{\frac{a \cos\left(dx + c\right) + a}{\cos\left(dx + c\right)}} \sin\left(dx + c\right)}{315\,{\left(d \cos\left(dx + c\right)^{5} + d \cos\left(dx + c\right)^{4}\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/315*(16*(9*A + 8*B)*cos(d*x + c)^4 + 8*(9*A + 8*B)*cos(d*x + c)^3 + 6*(9*
A + 8*B)*cos(d*x + c)^2 + 5*(9*A + 8*B)*cos(d*x + c) + 35*B)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4
)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c + dx\right) + 1\right)} \left(A + B\sec\left(c + dx\right)\right) \sec^{4}\left(c + dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*sec(c + d*x)**4, x
)
```

Giac [**A**]
time = 1.13, size = 268, normalized size = 1.43

$$\frac{2\,{\left(315\,\sqrt{2} A a^{5} \mathrm{sgn}\left(\cos\left(dx + c\right)\right) + 315\,\sqrt{2} B a^{5} \mathrm{sgn}\left(\cos\left(dx + c\right)\right) - {\left(630\,\sqrt{2} A a^{5} \mathrm{sgn}\left(\cos\left(dx + c\right)\right) + 420\,\sqrt{2} B a^{5} \mathrm{sgn}\left(\cos\left(dx + c\right)\right) - {\left(756\,\sqrt{2} A a^{5} \mathrm{sgn}\left(\cos\left(dx + c\right)\right) + 882\,\sqrt{2} B a^{5} \mathrm{sgn}\left(\cos\left(dx + c\right)\right) - {\left(522\,\sqrt{2} A a^{5} \mathrm{sgn}\left(\cos\left(dx + c\right)\right) + 324\,\sqrt{2} B a^{5} \mathrm{sgn}\left(\cos\left(dx + c\right)\right) - {\left(81\,\sqrt{2} A a^{5} \mathrm{sgn}\left(\cos\left(dx + c\right)\right) + 107\,\sqrt{2} B a^{5} \mathrm{sgn}\left(\cos\left(dx + c\right)\right)\right)} \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{2}\right)} \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{2}\right)} \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{2}\right)} \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{2}\right)} \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{315\,{\left(a \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{2} - a\right)}^{4} \sqrt{-a \tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{2} + a}\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] 2/315*(315*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 315*sqrt(2)*B*a^5*sgn(cos(d*x
+ c)) - (630*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 420*sqrt(2)*B*a^5*sgn(cos(d*
x + c)) - (756*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 882*sqrt(2)*B*a^5*sgn(cos(
d*x + c)) - (522*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 324*sqrt(2)*B*a^5*sgn(co
s(d*x + c)) - (81*sqrt(2)*A*a^5*sgn(cos(d*x + c)) + 107*sqrt(2)*B*a^5*sgn(c
os(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x +
 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1
/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

Mupad [B]
time = 10.05, size = 512, normalized size = 2.74

$$\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(e^{c\,1i+d\,x\,1i}\,\left(\frac{A\,16i}{5\,d}+\frac{\left(48\,A-32\,B\right)\,1i}{105\,d}\right)+\frac{\left(336\,A+672\,B\right)\,1i}{105\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^2}-\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(e^{c\,1i+d\,x\,1i}\,\left(\frac{A\,16i}{7\,d}-\frac{B\,320i}{63\,d}\right)+\frac{B\,32i}{7\,d}+\frac{\left(144\,A+288\,B\right)\,1i}{63\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^3}+\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(-\frac{A\,16i}{9\,d}+e^{c\,1i+d\,x\,1i}\,\left(\frac{A\,16i}{9\,d}-\frac{\left(16\,A+32\,B\right)\,1i}{9\,d}\right)+\frac{\left(16\,A+32\,B\right)\,1i}{9\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^4}-\frac{e^{c\,1i+d\,x\,1i}\,\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(288\,A+256\,B\right)\,1i}{315\,d\,\left(e^{c\,1i+d\,x\,1i}+1\right)}-\frac{e^{c\,1i+d\,x\,1i}\,\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(144\,A+128\,B\right)\,1i}{315\,d\,\left(e^{c\,1i+d\,x\,1i}+1\right)\,\left(e^{c\,2i+d\,x\,2i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/cos(c + d*x)^4,x)
```

```
[Out] ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i +
d*x*1i)*((A*16i)/(5*d) + ((48*A - 32*B)*1i)/(105*d)) + ((336*A + 672*B)*1i)
/(105*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) - ((a + a/
(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i)*
((A*16i)/(7*d) - (B*320i)/(63*d)) + (B*32i)/(7*d) + ((144*A + 288*B)*1i)/(6
3*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) + ((a + a/(exp
(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i)*((A*
16i)/(9*d) - ((16*A + 32*B)*1i)/(9*d)) - (A*16i)/(9*d) + ((16*A + 32*B)*1i)
/(9*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^4) - (exp(c*1i
+ d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(28
8*A + 256*B)*1i)/(315*d*(exp(c*1i + d*x*1i) + 1)) - (exp(c*1i + d*x*1i)*(a
+ a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(144*A + 128*B)*
1i)/(315*d*(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1))
```

3.119 $\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=144

$$\frac{2a(7A+6B)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{2aB\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}}-\frac{4(7A+6B)\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{105d}+\frac{2}{}$$

```
[Out] 2/35*(7*A+6*B)*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/a/d+2/15*a*(7*A+6*B)*tan(d
*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/7*a*B*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d
*x+c))^(1/2)-4/105*(7*A+6*B)*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]

time = 0.18, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4101, 3885, 4086, 3877}

$$\frac{2(7A+6B)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35ad}-\frac{4(7A+6B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2a(7A+6B)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2aB\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*(7*A + 6*B)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*B*Sec
[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c + d*x]]) - (4*(7*A + 6*B)*S
qrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*(7*A + 6*B)*(a + a*Sec[c
 + d*x])^(3/2)*Tan[c + d*x])/(35*a*d)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
```

```
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int \sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \frac{2aB\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}}+\frac{1}{7}(7A+6B)\int$$

$$= \frac{2aB\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}}+\frac{2(7A+6B)(a}{}$$

$$= \frac{2aB\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}}-\frac{4(7A+6B)\sqrt{}}{}$$

$$= \frac{2a(7A+6B)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{2aB\sec^3(c+dx)\,\mathrm{t}}{7d\sqrt{a+a\sec(}}$$

Mathematica [A]
time = 0.28, size = 81, normalized size = 0.56

$$\frac{2a(8(7A+6B)+4(7A+6B)\sec(c+dx)+3(7A+6B)\sec^2(c+dx)+15B\sec^3(c+dx))\tan(c+dx)}{105d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*(8*(7*A + 6*B) + 4*(7*A + 6*B)*Sec[c + d*x] + 3*(7*A + 6*B)*Sec[c + d*
x]^2 + 15*B*Sec[c + d*x]^3)*Tan[c + d*x])/(105*d*Sqrt[a*(1 + Sec[c + d*x])]
)
```

Maple [A]
time = 3.97, size = 116, normalized size = 0.81

method	result
default	$-\frac{2(-1+\cos(dx+c))(56A(\cos^3(dx+c))+48B(\cos^3(dx+c))+28A(\cos^2(dx+c))+24B(\cos^2(dx+c))+21A\cos(dx+c)+18B\cos(dx+c)}{105d\cos(dx+c)^3\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(56*A*cos(d*x+c)^3+48*B*cos(d*x+c)^3+28*A*cos(d*x+
c)^2+24*B*cos(d*x+c)^2+21*A*cos(d*x+c)+18*B*cos(d*x+c)+15*B)*(a*(1+cos(d*x+
c))/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] -8/105*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(7*(5*A*sin(4*d*x + 4*c) + (7*A + 6*B)*sin(2*d*x + 2*c))*cos(7/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (35*A*cos(4*d*x + 4*c) + 7*(
7*A + 6*B)*cos(2*d*x + 2*c) + 14*A + 12*B)*sin(7/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)))*sqrt(a) - 105*(((A + 2*B)*d*cos(2*d*x + 2*c)^4 +
(A + 2*B)*d*sin(2*d*x + 2*c)^4 + 4*(A + 2*B)*d*cos(2*d*x + 2*c)^3 + 6*(A +
2*B)*d*cos(2*d*x + 2*c)^2 + 4*(A + 2*B)*d*cos(2*d*x + 2*c) + 2*((A + 2*B)*d
*cos(2*d*x + 2*c)^2 + 2*(A + 2*B)*d*cos(2*d*x + 2*c) + (A + 2*B)*d)*sin(2*d
*x + 2*c)^2 + (A + 2*B)*d)*integrate((((cos(10*d*x + 10*c)*cos(2*d*x + 2*c)
 + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c
) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x +
 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x
 + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x
+ 2*c)^2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x
 + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*
d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*
d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(
6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) + 4*c
os(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*
cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c) - 4
```

```
*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) -
4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) - (cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*c)*
cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4*c)
*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x + 2*c
) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x + 2*
c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(5/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)))/(((2*(4*cos(8*d*x + 8*c) + 6*cos(6*d*x + 6*c) + 4*
cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(10*d*x + 10*c) + cos(10*d*x + 10*c
)^2 + 8*(6*cos(6*d*x + 6*c) + 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*
d*x + 8*c) + 16*cos(8*d*x + 8*c)^2 + 12*(4*cos(4*d*x + 4*c) + cos(2*d*x + 2
*c))*cos(6*d*x + 6*c) + 36*cos(6*d*x + 6*c)^2 + 16*cos(4*d*x + 4*c)^2 + 8*c
os(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(4*sin(8*d*x + 8*
c) + 6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x
 + 10*c) + sin(10*d*x + 10*c)^2 + 8*(6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c
) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*sin(8*d*x + 8*c)^2 + 12*(4*sin(
4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 36*sin(6*d*x + 6*c)^2 +
 16*sin(4*d*x + 4*c)^2 + 8*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x +
2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (2*(4*
cos(8*d*x + 8*c) + 6*cos(6*d*x + 6*c) + 4*cos(4*d*x + 4*c) + cos(2*d*x + 2*
c))*cos(10*d*x + 10*c) + cos(10*d*x + 10*c)^2 + 8*(6*cos(6*d*x + 6*c) + 4*c
os(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 16*cos(8*d*x + 8*c)^
2 + 12*(4*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 36*cos(6*
d*x + 6*c)^2 + 16*cos(4*d*x + 4*c)^2 + 8*cos(4*d*x + 4*c)*cos(2*d*x + 2*c)
+ cos(2*d*x + 2*c)^2 + 2*(4*sin(8*d*x + 8*c) + 6*sin(6*d*x + 6*c) + 4*sin(4
*d*x + 4*c) + sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + sin(10*d*x + 10*c)^2 +
 8*(6*sin(6*d*x + 6*c) + 4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x +
 8*c) + 16*sin(8*d*x + 8*c)^2 + 12*(4*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*
sin(6*d*x + 6*c) + 36*sin(6*d*x + 6*c)^2 + 16*sin(4*d*x + 4*c)^2 + 8*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)), x) + (A*d*cos(2*d*x + 2*c)^4 + A*d*sin(2
*d*x + 2*c)^4 + 4*A*d*cos(2*d*x + 2*c)^3 + 6*A*d*cos(2*d*x + 2*c)^2 + 4*A*d
*cos(2*d*x + 2*c) + 2*(A*d*cos(2*d*x + 2*c)^2 + 2*A*d*cos(2*d*x + 2*c) + A*
d)*sin(2*d*x + 2*c)^2 + A*d)*integrate((((cos(10*d*x + 10*c)*cos(2*d*x + 2*
c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2
*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x
 + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d
*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*
x + 2*c)^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d
*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(
2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(1
0*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*co
s(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x +...
```

Fricas [A]
time = 3.61, size = 105, normalized size = 0.73

$$\frac{2\left(8\left(7A+6B\right)\cos\left(dx+c\right)^{3}+4\left(7A+6B\right)\cos\left(dx+c\right)^{2}+3\left(7A+6B\right)\cos\left(dx+c\right)+15B\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/105*(8*(7*A + 6*B)*cos(d*x + c)^3 + 4*(7*A + 6*B)*cos(d*x + c)^2 + 3*(7*A
 + 6*B)*cos(d*x + c) + 15*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)\right)\sec^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*sec(c + d*x)**3, x
)
```

Giac [A]
time = 1.10, size = 222, normalized size = 1.54

$$-\frac{2\left(105\sqrt{2}Aa^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+105\sqrt{2}Ba^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(175\sqrt{2}Aa^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+105\sqrt{2}Ba^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(119\sqrt{2}Aa^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+147\sqrt{2}Ba^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\left(49\sqrt{2}Aa^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+27\sqrt{2}Ba^{4}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{105\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)^{3}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -2/105*(105*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 105*sqrt(2)*B*a^4*sgn(cos(d*x
 + c)) - (175*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 105*sqrt(2)*B*a^4*sgn(cos(d
*x + c)) - (119*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 147*sqrt(2)*B*a^4*sgn(cos
(d*x + c)) - (49*sqrt(2)*A*a^4*sgn(cos(d*x + c)) + 27*sqrt(2)*B*a^4*sgn(cos
(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1
/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a)*d)
```

Mupad [B]
time = 6.16, size = 407, normalized size = 2.83

$$\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(-\frac{A\,8i}{5\,d}+\frac{B\,16i}{5\,d}+e^{c\,1i+d\,x\,1i}\left(\frac{B\,16i}{35\,d}+\frac{\left(56\,A+112\,B\right)\,1i}{35\,d}\right)\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^{2}}+\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{A\,8i}{7\,d}+e^{c\,1i+d\,x\,1i}\left(\frac{A\,8i}{7\,d}-\frac{\left(8\,A+16\,B\right)\,1i}{7\,d}\right)-\frac{\left(8\,A+16\,B\right)\,1i}{7\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^{3}}+\frac{\left(\frac{A\,8i}{3\,d}-\frac{e^{c\,1i+d\,x\,1i}\left(56\,A+48\,B\right)\,1i}{105\,d}\right)\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)}-\frac{e^{c\,1i+d\,x\,1i}\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(112\,A+96\,B\right)\,1i}{105\,d\left(e^{c\,1i+d\,x\,1i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/cos(c + d*x)^3,x)
```

```
[Out] ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((B*16i)/(5*
d) - (A*8i)/(5*d) + exp(c*1i + d*x*1i)*((B*16i)/(35*d) + ((56*A + 112*B)*1i
)/(35*d))))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) + ((a + a
/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((A*8i)/(7*d) + exp
(c*1i + d*x*1i)*((A*8i)/(7*d) - ((8*A + 16*B)*1i)/(7*d)) - ((8*A + 16*B)*1i
)/(7*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) + (((A*8i)/
(3*d) - (exp(c*1i + d*x*1i)*(56*A + 48*B)*1i)/(105*d))*(a + a/(exp(- c*1i -
 d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c
*2i + d*x*2i) + 1)) - (exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)/2 +
exp(c*1i + d*x*1i)/2))^(1/2)*(112*A + 96*B)*1i)/(105*d*(exp(c*1i + d*x*1i)
+ 1))
```

3.120 $\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=101

$$\frac{2a(5A+7B)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{2(5A-2B)\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{15d}+\frac{2B(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5ad}$$

```
[Out] 2/5*B*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/a/d+2/15*a*(5*A+7*B)*tan(d*x+c)/d/(
a+a*sec(d*x+c))^(1/2)+2/15*(5*A-2*B)*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.15, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.091$, Rules used = {4095, 4086, 3877}

$$\frac{2(5A-2B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2a(5A+7B)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2B\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*(5*A + 7*B)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*(5*A -
2*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*B*(a + a*Sec[c + d*
x])^(3/2)*Tan[c + d*x])/(5*a*d)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
```

```
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx &= \frac{2B(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5ad}+\frac{2\int \sec\\ &= \frac{2(5A-2B)\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{15d}+\\ &= \frac{2a(5A+7B)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{2(5A-2B)\sqrt{a+}\end{aligned}$$

Mathematica [A]
time = 0.33, size = 80, normalized size = 0.79

$$\frac{2(5A+7B+(5A+4B)\cos(c+dx)+(5A+4B)\cos(2(c+dx)))\sec(c+dx)\sqrt{a(1+\sec(c+dx))}\,\tan(c+dx)}{15d(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(5*A + 7*B + (5*A + 4*B)*Cos[c + d*x] + (5*A + 4*B)*Cos[2*(c + d*x)])*Se
c[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*Tan[c + d*x])/(15*d*(1 + Cos[c + d*x]
))
```

Maple [A]
time = 3.70, size = 94, normalized size = 0.93

method	result	size
default	$-\frac{2(-1+\cos(dx+c))\left(10A\left(\cos^2(dx+c)\right)+8B\left(\cos^2(dx+c)\right)+5A\cos(dx+c)+4B\cos(dx+c)+3B\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}}{15d\cos(dx+c)^2\sin(dx+c)}$	94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(10*A*cos(d*x+c)^2+8*B*cos(d*x+c)^2+5*A*cos(d*x+c)+
4*B*cos(d*x+c)+3*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*
x+c)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 4/15*(15*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*((A*d*cos(2*d*x + 2*c)^2 + A*d*sin(2*d*x + 2*c)^2 + 2*A*d*cos(2*d*x
+ 2*c) + A*d)*integrate((((cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 3*cos(6*d*x
+ 6*c)*cos(2*d*x + 2*c) + 3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x +
 2*c)^2 + sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d*x + 6*c)*sin(2*d*x
+ 2*c) + 3*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(5/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(8*d*x
+ 8*c) + 3*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 3*cos(2*d*x + 2*c)*sin(4*d*x
 + 4*c) - cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x
+ 2*c) - 3*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) - ((cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 3*cos(2*d*x + 2*c)*sin(6*d*x
 + 6*c) + 3*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(8*d*x + 8*c)*sin(2*d*x
+ 2*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 3*cos(4*d*x + 4*c)*sin(2*d*x
 + 2*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(8*d*x
+ 8*c)*cos(2*d*x + 2*c) + 3*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 3*cos(4*d*x
 + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(8*d*x + 8*c)*sin(2*d*x
+ 2*c) + 3*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 3*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + sin(2*d*x + 2*c)^2)*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((2*(3*
cos(6*d*x + 6*c) + 3*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x + 8*c)
+ cos(8*d*x + 8*c)^2 + 6*(3*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x
+ 6*c) + 9*cos(6*d*x + 6*c)^2 + 9*cos(4*d*x + 4*c)^2 + 6*cos(4*d*x + 4*c)*c
os(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(3*sin(6*d*x + 6*c) + 3*sin(4*d*x
+ 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 6*(3*sin
(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 9*sin(6*d*x + 6*c)^2 +
 9*sin(4*d*x + 4*c)^2 + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2
*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (2*(3*c
os(6*d*x + 6*c) + 3*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) +
 cos(8*d*x + 8*c)^2 + 6*(3*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x +
 6*c) + 9*cos(6*d*x + 6*c)^2 + 9*cos(4*d*x + 4*c)^2 + 6*cos(4*d*x + 4*c)*co
s(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(3*sin(6*d*x + 6*c) + 3*sin(4*d*x +
 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x + 8*c)^2 + 6*(3*sin(
4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 9*sin(6*d*x + 6*c)^2 +
9*sin(4*d*x + 4*c)^2 + 6*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*
c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2)*(cos(2*d*
```

```
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)), x) + ((A
+ 2*B)*d*cos(2*d*x + 2*c)^2 + (A + 2*B)*d*sin(2*d*x + 2*c)^2 + 2*(A + 2*B)*
d*cos(2*d*x + 2*c) + (A + 2*B)*d)*integrate((((cos(8*d*x + 8*c)*cos(2*d*x +
 2*c) + 3*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 3*cos(4*d*x + 4*c)*cos(2*d*x
+ 2*c) + cos(2*d*x + 2*c)^2 + sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d
*x + 6*c)*sin(2*d*x + 2*c) + 3*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*
x + 2*c)^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d
*x + 2*c)*sin(8*d*x + 8*c) + 3*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 3*cos(2*
d*x + 2*c)*sin(4*d*x + 4*c) - cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 3*cos(6*d
*x + 6*c)*sin(2*d*x + 2*c) - 3*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 3*cos(2*
d*x + 2*c)*sin(6*d*x + 6*c) + 3*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(8*d
*x + 8*c)*sin(2*d*x + 2*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 3*cos(4*
d*x + 4*c)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) - (cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 3*cos(6*d*x + 6*c)*cos(2*d*x
+ 2*c) + 3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(8*d
*x + 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 3*sin(4*
d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))/(((2*(3*cos(6*d*x + 6*c) + 3*cos(4*d*x + 4*c) + cos(2*d*x + 2*c
))*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 6*(3*cos(4*d*x + 4*c) + cos(2*d*
x + 2*c))*cos(6*d*x + 6*c) + 9*cos(6*d*x + 6*c)^2 + 9*cos(4*d*x + 4*c)^2 +
6*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(3*sin(6*d*x +
 6*c) + 3*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + sin(8*d*x
 + 8*c)^2 + 6*(3*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 9*
sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 6*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + sin(2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))^2 + (2*(3*cos(6*d*x + 6*c) + 3*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)
)*cos(8*d*x + 8*c) + cos(8*d*x + 8*c)^2 + 6*(3*cos(4*d*x + 4*c) + cos(2*d*x
 + 2*c))*cos(6*d*x + 6*c) + 9*cos(6*d*x + 6*c)^...
```

Fricas [**A**]
time = 2.78, size = 87, normalized size = 0.86

$$\frac{2\left(2\left(5A+4B\right)\cos\left(dx+c\right)^{2}+\left(5A+4B\right)\cos\left(dx+c\right)+3B\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/15*(2*(5*A + 4*B)*cos(d*x + c)^2 + (5*A + 4*B)*cos(d*x + c) + 3*B)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*cos(d*
x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\left(A+B\sec\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*sec(c + d*x)**2, x
)
```

Giac [A]
time = 1.01, size = 176, normalized size = 1.74

$$\frac{2\left(15\sqrt{2}\,Aa^{3}\mathrm{sgn}(\cos\left(dx+c\right))+15\sqrt{2}\,Ba^{3}\mathrm{sgn}(\cos\left(dx+c\right))-\left(20\sqrt{2}\,Aa^{3}\mathrm{sgn}(\cos\left(dx+c\right))+10\sqrt{2}\,Ba^{3}\mathrm{sgn}(\cos\left(dx+c\right))-\left(5\sqrt{2}\,Aa^{3}\mathrm{sgn}(\cos\left(dx+c\right))+7\sqrt{2}\,Ba^{3}\mathrm{sgn}(\cos\left(dx+c\right))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{15\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] 2/15*(15*sqrt(2)*A*a^3*sgn(cos(d*x + c)) + 15*sqrt(2)*B*a^3*sgn(cos(d*x + c
)) - (20*sqrt(2)*A*a^3*sgn(cos(d*x + c)) + 10*sqrt(2)*B*a^3*sgn(cos(d*x + c
)) - (5*sqrt(2)*A*a^3*sgn(cos(d*x + c)) + 7*sqrt(2)*B*a^3*sgn(cos(d*x + c))
)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*
tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

Mupad [B]
time = 6.16, size = 212, normalized size = 2.10

$$-\frac{4\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}-1\right)\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,\left(A\,5\mathrm{i}+B\,4\mathrm{i}+A\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,5\mathrm{i}+A\,\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,10\mathrm{i}+A\,\mathrm{e}^{c\,3\mathrm{i}+d\,x\,3\mathrm{i}}\,5\mathrm{i}+A\,\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,5\mathrm{i}+B\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,4\mathrm{i}+B\,\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}\,14\mathrm{i}+B\,\mathrm{e}^{c\,3\mathrm{i}+d\,x\,3\mathrm{i}}\,4\mathrm{i}+B\,\mathrm{e}^{c\,4\mathrm{i}+d\,x\,4\mathrm{i}}\,4\mathrm{i}\right)}{15\,d\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/cos(c + d*x)^2,x)
```

```
[Out] -(4*(exp(c*1i + d*x*1i) - 1)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*
x*1i)/2))^(1/2)*(A*5i + B*4i + A*exp(c*1i + d*x*1i)*5i + A*exp(c*2i + d*x*2
i)*10i + A*exp(c*3i + d*x*3i)*5i + A*exp(c*4i + d*x*4i)*5i + B*exp(c*1i + d
*x*1i)*4i + B*exp(c*2i + d*x*2i)*14i + B*exp(c*3i + d*x*3i)*4i + B*exp(c*4i
 + d*x*4i)*4i))/(15*d*(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2)
```

3.121 $\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=62

$$\frac{2a(3A+B)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2B\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3d}$$

```
[Out] 2/3*a*(3*A+B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*B*(a+a*sec(d*x+c))^(1
/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 62, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.065$, Rules used = {4086, 3877}

$$\frac{2a(3A+B)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2B\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*(3*A + B)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*B*Sqrt[a +
 a*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \frac{2B\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3d} + \frac{1}{3}(3A+$$
$$= \frac{2a(3A+B)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2B\sqrt{a+a\sec(c+}{3c}$$

Mathematica [**A**]
time = 0.17, size = 53, normalized size = 0.85

$$\frac{2(B+(3A+2B)\cos(c+dx))\sqrt{a(1+\sec(c+dx))}\,\tan(c+dx)}{3d(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(B + (3*A + 2*B)*Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[c + d*x])/
(3*d*(1 + Cos[c + d*x]))
```

Maple [**A**]
time = 3.84, size = 70, normalized size = 1.13

method	result	size
default	$-\frac{2(-1+\cos(dx+c))(3A\cos(dx+c)+2B\cos(dx+c)+B)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}}{3d\sin(dx+c)\cos(dx+c)}$	70

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(3*A*cos(d*x+c)+2*B*cos(d*x+c)+B)*(a*(1+cos(d*x+c))/
cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] -2/3*((3*A*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(2*d
*x + 2*c) - (3*A*cos(2*d*x + 2*c) + 3*A + 2*B)*sin(3/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) - 3*(((A + 2*B)*d*cos(2*d*x + 2*c)^2 +
(A + 2*B)*d*sin(2*d*x + 2*c)^2 + 2*(A + 2*B)*d*cos(2*d*x + 2*c) + (A + 2*B)
*d)*integrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos
(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*
cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*c
os(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - ((c
os(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - co
s(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(6*d*x + 6*c)*cos(2*
d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin
(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(
2*d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((2*(2*cos(4*d*x + 4*
c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2 + 4*cos(4*d*x
+ 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + 2*(2*
sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2
+ 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x +
2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (2*(2*
cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2
+ 4*cos(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x +
2*c)^2 + 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6
*d*x + 6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)), x) + (A*d*cos(2*d*x + 2*c)^2 + A*d*sin(2*d*x + 2*c)^2 + 2*A*d*cos(2*
d*x + 2*c) + A*d)*integrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*
d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d
*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d
*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*
x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d
*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*
x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(6*d*x
 + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x
+ 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x
 + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((2*(2*
```

```
cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + cos(6*d*x + 6*c)^2
+ 4*cos(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x +
2*c)^2 + 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + sin(6
*d*x + 6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ sin(2*d*x + 2*c)^2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1))^2 + (2*(2*cos(4*d*x + 4*c) + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + cos(6
*d*x + 6*c)^2 + 4*cos(4*d*x + 4*c)^2 + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c)
+ cos(2*d*x + 2*c)^2 + 2*(2*sin(4*d*x + 4*c) + sin(2*d*x + 2*c))*sin(6*d*x
+ 6*c) + sin(6*d*x + 6*c)^2 + 4*sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin
(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)), x))*sqrt(a))/(d*cos(2*d*x + 2*c)^2 + d*sin(2*d*x + 2*c
)^2 + 2*d*cos(2*d*x + 2*c) + d)
```

Fricas [A]
time = 3.60, size = 66, normalized size = 1.06

$$\frac{2\left((3A+2B)\cos(dx+c)+B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin(dx+c)}{3\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 2/3*((3*A + 2*B)*cos(d*x + c) + B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sin(d*x + c)/(d*cos(d*x + c)^2 + d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sec(c+dx)+1\right)}\,\left(A+B\sec(c+dx)\right)\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*sec(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 129 vs. 2(54) = 108.
time = 0.94, size = 129, normalized size = 2.08

$$-\frac{2\left(3\sqrt{2}\,Aa^2\mathrm{sgn}(\cos(dx+c))+3\sqrt{2}\,Ba^2\mathrm{sgn}(\cos(dx+c))-\left(3\sqrt{2}\,Aa^2\mathrm{sgn}(\cos(dx+c))+\sqrt{2}\,Ba^2\mathrm{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{3\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\;d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] -2/3*(3*sqrt(2)*A*a^2*sgn(cos(d*x + c)) + 3*sqrt(2)*B*a^2*sgn(cos(d*x + c))
 - (3*sqrt(2)*A*a^2*sgn(cos(d*x + c)) + sqrt(2)*B*a^2*sgn(cos(d*x + c)))*ta
n(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

Mupad [B]
time = 1.97, size = 159, normalized size = 2.56

$$\frac{2\,\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}\,\left(6\,A\,\sin\left(c+d\,x\right)+6\,B\,\sin\left(c+d\,x\right)+6\,A\,\sin\left(2\,c+2\,d\,x\right)+6\,A\,\sin\left(3\,c+3\,d\,x\right)+3\,A\,\sin\left(4\,c+4\,d\,x\right)+8\,B\,\sin\left(2\,c+2\,d\,x\right)+6\,B\,\sin\left(3\,c+3\,d\,x\right)+2\,B\,\sin\left(4\,c+4\,d\,x\right)\right)}{3\,d\,\left(12\,\cos\left(c+d\,x\right)+8\,\cos\left(2\,c+2\,d\,x\right)+4\,\cos\left(3\,c+3\,d\,x\right)+\cos\left(4\,c+4\,d\,x\right)+7\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/cos(c + d*x),x)
```

```
[Out] (2*((a*(cos(c + d*x) + 1))/cos(c + d*x))^(1/2)*(6*A*sin(c + d*x) + 6*B*sin(
c + d*x) + 6*A*sin(2*c + 2*d*x) + 6*A*sin(3*c + 3*d*x) + 3*A*sin(4*c + 4*d*
x) + 8*B*sin(2*c + 2*d*x) + 6*B*sin(3*c + 3*d*x) + 2*B*sin(4*c + 4*d*x)))/(
3*d*(12*cos(c + d*x) + 8*cos(2*c + 2*d*x) + 4*cos(3*c + 3*d*x) + cos(4*c +
4*d*x) + 7))
```

3.122 $\int \sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=66

$$\frac{2\sqrt{a}\,A\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2aB\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*A*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+2*a*B*tan(d
*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.06, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4000, 3859, 209, 3877}

$$\frac{2\sqrt{a}\,A\,\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2aB\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[a]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a*B*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = A\int \sqrt{a+a\sec(c+dx)}\,dx + B\int \sec(c+dx)\sqrt{a+a\sec$$

$$= \frac{2aB\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(2aA)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{}{\sqrt{a-}}\right.}{d}$$

$$= \frac{2\sqrt{a}\,A\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2aB\tan(c+d}{d\sqrt{a+a\sec(c+}}$$

Mathematica [A]
time = 0.31, size = 76, normalized size = 1.15

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{2}\,A\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2B\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
[Out] (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*A*ArcSin[Sqrt[2]*Sin[
(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*B*Sin[(c + d*x)/2]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 117 vs. $2(58) = 116$.
time = 4.28, size = 118, normalized size = 1.79

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(A\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)+2B\cos(dx+c)-2\right.}{d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(A*arctanh(1/2*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)+2*B*cos(d*x+c)-2*B)/sin(d*x+c)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 147 vs. $2(58) = 116$.
time = 0.55, size = 147, normalized size = 2.23

$$\frac{A\sqrt{a}\arctan\left(\left(\cos\left(2\,dx+2\,c\right)^2+\sin\left(2\,dx+2\,c\right)^2+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{4}}\sin\left(\frac{1}{2}\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)+1\right)\right)+\sin\left(dx+c\right),\left(\cos\left(2\,dx+2\,c\right)^2+\sin\left(2\,dx+2\,c\right)^2+2\,\cos\left(2\,dx+2\,c\right)+1\right)^{\frac{1}{4}}\cos\left(\frac{1}{2}\arctan\left(\sin\left(2\,dx+2\,c\right),\cos\left(2\,dx+2\,c\right)+1\right)\right)+\cos\left(dx+c\right)\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] A*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) +
sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + cos(
d*x + c))/d
```

Fricas [**A**]
time = 3.27, size = 235, normalized size = 3.56

$$\left[\frac{\left(A\cos\left(dx+c\right)+A\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\,B\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{d\cos\left(dx+c\right)+d},-\frac{2\left(\left(A\cos\left(dx+c\right)+A\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-B\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)\right)}{d\cos\left(dx+c\right)+d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [((A*cos(d*x + c) + A)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c
) - a)/(cos(d*x + c) + 1)) + 2*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c))/(d*cos(d*x + c) + d), -2*((A*cos(d*x + c) + A)*sqrt(a)*arctan(s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
 - B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
+ d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. 2(58) = 116.
time = 1.14, size = 193, normalized size = 2.92

$$-\frac{\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\,Ba\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a}+\frac{A\sqrt{-a}\,a\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-4\sqrt{2}\left|a\right|-6\,a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}+4\sqrt{2}\left|a\right|-6\,a\right|}\right)\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}{\left|a\right|}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -(2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*B*a*sgn(cos(d*x + c))*tan(1
/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + A*sqrt(-a)*a*log(abs(2*(sq
rt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sq
rt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(a) - 6*a))*sgn(cos(d*x + c))/abs(a))
/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.02

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)
```

3.123 $\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=68

$$\frac{\sqrt{a}\,(A+2B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{aA\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] (A+2*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+a*A*sin
(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 68, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4100, 3859, 209}

$$\frac{\sqrt{a}\,(A+2B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{aA\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[a]*(A + 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])
/d + (a*A*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
```

```
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \frac{aA\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}(A+2B)\int \sqrt{a+a$$

$$= \frac{aA\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(a(A+2B))\text{Subst}\Big(\int}{}$$

$$= \frac{\sqrt{a}\,(A+2B)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} +$$

Mathematica [A]
time = 0.24, size = 93, normalized size = 1.37

$$\frac{\sqrt{\cos(c+dx)}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{2}\,(A+2B)\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2A\sqrt{\cos(c+dx)}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(A
 + 2*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*A*Sqrt[Cos[c + d*x]]*Sin[(c +
d*x)/2]))/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 197 vs. $2(60) = 120$.
time = 7.20, size = 198, normalized size = 2.91

method	result
default	$-\frac{\left(A\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)+2B\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin}{2\cos(dx+c)}\right.\right.}{2d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/2/d*(A*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d
*x+c)*2^(1/2))*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*B*
arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1
/2))*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+2*A*cos(d*x+c)
^2-2*A*cos(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 939 vs. $2(60) = 120$.
time = 0.62, size = 939, normalized size = 13.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] 1/4*(4*B*sqrt(a)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + sin(d*x + c), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + cos(d*x + c)) + (2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
))*sin(d*x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1
/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*A)/d
```

Fricas [**A**]

time = 2.45, size = 261, normalized size = 3.84

$$\left[\frac{2\,A\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)+\left(\left(A+2\,B\right)\cos\left(dx+c\right)+A+2\,B\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{2\left(d\cos\left(dx+c\right)+d\right)},\ \frac{A\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)-\left(\left(A+2\,B\right)\cos\left(dx+c\right)+A+2\,B\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{d\cos\left(dx+c\right)+d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] [1/2*(2*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)
 + ((A + 2*B)*cos(d*x + c) + A + 2*B)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*
sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)
+ a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(d*cos(d*x + c) + d), (A*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - ((A + 2*B)*cos
(d*x + c) + A + 2*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(d*cos(d*x + c) + d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)\right)\cos\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*cos(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(60) = 120.
time = 1.62, size = 336, normalized size = 4.94

$$-\frac{\left(A\sqrt{-a}\,\mathrm{sgn}(\cos\left(dx+c\right))+2\,B\sqrt{-a}\,\mathrm{sgn}(\cos\left(dx+c\right))\right)\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-a\left(2\,\sqrt{2}+3\right)\right|\right)-\left(A\sqrt{-a}\,\mathrm{sgn}(\cos\left(dx+c\right))+2\,B\sqrt{-a}\,\mathrm{sgn}(\cos\left(dx+c\right))\right)\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+a\left(2\,\sqrt{2}-3\right)\right|\right)+\frac{4\left(3\sqrt{2}\left(\sqrt{-a}\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2A\sqrt{-a}\,a\mathrm{sgn}(\cos(dx+c))-\sqrt{2}\,A\sqrt{-a}\,a^2\mathrm{sgn}(\cos(dx+c))\right)}{\left(\sqrt{-a}\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^4-6\left(\sqrt{-a}\tan(\frac{1}{2}\,dx+\frac{1}{2}\,c)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2a+a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] -1/2*((A*sqrt(-a)*sgn(cos(d*x + c)) + 2*B*sqrt(-a)*sgn(cos(d*x + c)))*log(a
bs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2
- a*(2*sqrt(2) + 3))) - (A*sqrt(-a)*sgn(cos(d*x + c)) + 2*B*sqrt(-a)*sgn(co
s(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*(3*sqrt(2)*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a*sgn(cos(d*
```

```
x + c)) - sqrt(2)*A*sqrt(-a)*a^2*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x
 + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)\left(A+\frac{B}{\cos(c+dx)}\right)\sqrt{a+\frac{a}{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)
```

3.124 $\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=117

$$\frac{\sqrt{a}\,(3A+4B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{a(3A+4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{aA\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 1/4*(3*A+4*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+1
/4*a*(3*A+4*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a*A*cos(d*x+c)*sin(d
*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 117, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4100, 3890, 3859, 209}

$$\frac{\sqrt{a}\,(3A+4B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a(3A+4B)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{aA\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[a]*(3*A + 4*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(4*d) + (a*(3*A + 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (a
*A*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
```

```
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx &= \frac{aA\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{1}{4}(3A+4B)\int \\ &= \frac{a(3A+4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{aA\cos(c+dx)\sin}{2d\sqrt{a+a\sec(c}} \\ &= \frac{a(3A+4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{aA\cos(c+dx)\sin}{2d\sqrt{a+a\sec(c}} \\ &= \frac{\sqrt{a}\,(3A+4B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.40, size = 117, normalized size = 1.00

$$\frac{\left(B\left(\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)+\cos(c+dx)\sqrt{1-\sec(c+dx)}\right)+2A\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\sec(c+dx)\right)\sqrt{1-\sec(c+dx)}\right)\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((B*(ArcTanh[Sqrt[1 - Sec[c + d*x]]] + Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]])
 + 2*A*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*
x]])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqrt[1 - Sec[c + d*x]]
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 397 vs. $2(101) = 202$.
time = 7.54, size = 398, normalized size = 3.40

method	result
default	$\frac{\left(3A\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sin(dx+c)\cos(dx+c)\sqrt{2}+4B\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/16/d*(3*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)
*2^(1/2)+4*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c
)*2^(1/2)+3*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)+
4*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)-8*A*cos(d*
x+c)^4-4*A*cos(d*x+c)^3-16*B*cos(d*x+c)^3+12*A*cos(d*x+c)^2+16*B*cos(d*x+c)
^2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1851 vs. $2(101) = 202$.
time = 0.69, size = 1851, normalized size = 15.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/16*((2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*((cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2
*c) - (cos(2*d*x + 2*c) - 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1)) + ((cos(2*d*x + 2*c) - 2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - cos(2*d*x + 2*c) + 2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)))*sqrt(a) + 3*sqrt(a)*(arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
```

1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) + 1) - arctan2((cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*s
in(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))) - 1) - arctan2((cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) - 1)))*A + 4*(2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*
sin(d*x + c) - (cos(d*x + c) - 1)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c) + 1)))*sqrt(a) + sqrt(a)*(arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))) + 1) - arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))) - 1) - arctan2((cos(2*d*x + 2*c)^2 + sin(2*d
*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1)) + 1) + arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1)))*B)/d

Fricas [A]
time = 3.49, size = 308, normalized size = 2.63

$$\left[\frac{\left((3A+4B)\cos(dx+c)+3A+4B\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2A\cos(dx+c)^2+(3A+4B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{8\left(d\cos(dx+c)+d\right)},\ -\frac{\left((3A+4B)\cos(dx+c)+3A+4B\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(2A\cos(dx+c)^2+(3A+4B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{4\left(d\cos(dx+c)+d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [1/8*(((3*A + 4*B)*cos(d*x + c) + 3*A + 4*B)*sqrt(-a)*log((2*a*cos(d*x + c)
^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*A*cos(d*x + c)^2 +
(3*A + 4*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c))/(d*cos(d*x + c) + d), -1/4*(((3*A + 4*B)*cos(d*x + c) + 3*A + 4*B)*sqr
t(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*s
in(d*x + c))) - (2*A*cos(d*x + c)^2 + (3*A + 4*B)*cos(d*x + c))*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)\right)\cos^2\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*cos(c + d*x)**2, x
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 630 vs. 2(101) = 202.
time = 1.65, size = 630, normalized size = 5.38

$$-\frac{\left(3A\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))+4B\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))\right)\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)-\left(3A\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))+4B\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))\right)\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+a\left(2\sqrt{2}-3\right)\right|\right)-\frac{4\sqrt{2}\left(5\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^6A\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))-12\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^6B\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))+19\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4A\sqrt{-a}\,a\,\mathrm{sgn}(\cos(dx+c))+76\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4B\sqrt{-a}\,a\,\mathrm{sgn}(\cos(dx+c))-17\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2A\sqrt{-a}\,a^2\,\mathrm{sgn}(\cos(dx+c))-36\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2B\sqrt{-a}\,a^2\,\mathrm{sgn}(\cos(dx+c))+A\sqrt{-a}\,a^3\,\mathrm{sgn}(\cos(dx+c))+4B\sqrt{-a}\,a^3\,\mathrm{sgn}(\cos(dx+c))\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2a+a^2\right)^2}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/8*((3*A*sqrt(-a)*sgn(cos(d*x + c)) + 4*B*sqrt(-a)*sgn(cos(d*x + c)))*log
(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
2 - a*(2*sqrt(2) + 3))) - (3*A*sqrt(-a)*sgn(cos(d*x + c)) + 4*B*sqrt(-a)*sg
n(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
```

```
*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) - 4*sqrt(2)*(5*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a*sgn(co
s(d*x + c)) - 12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^6*B*sqrt(-a)*a*sgn(cos(d*x + c)) + 19*(sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^2*sgn(cos(d*x +
 c)) + 76*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^4*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 17*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^3*sgn(cos(d*x + c))
- 36*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
2*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) + A*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 4*B
*sqrt(-a)*a^4*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)^2\,\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)
```

3.125 $\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=160

$$\frac{\sqrt{a}\,(5A+6B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{a(5A+6B)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{a(5A+6B)\cos(c+dx)\sin(c+d$$

```
[Out] 1/8*(5*A+6*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+1
/8*a*(5*A+6*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/12*a*(5*A+6*B)*cos(d*x
+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/3*a*A*cos(d*x+c)^2*sin(d*x+c)/d/(
a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.17, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4100, 3890, 3859, 209}

$$\frac{\sqrt{a}\,(5A+6B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a(5A+6B)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{a(5A+6B)\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{aA\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[a]*(5*A + 6*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(8*d) + (a*(5*A + 6*B)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]) + (a
*(5*A + 6*B)*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*x]]) + (
a*A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
```

```
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \frac{aA\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{1}{6}(5A+6B)\int$$

$$= \frac{a(5A+6B)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{aA\cos}{3d\sqrt{}}$$

$$= \frac{a(5A+6B)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{a(5A+6B)\cos(c}{12d\sqrt{a+a\,s}}$$

$$= \frac{a(5A+6B)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{a(5A+6B)\cos(c}{12d\sqrt{a+a\,s}}$$

$$= \frac{\sqrt{a}\,(5A+6B)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.17, size = 70, normalized size = 0.44

$$\frac{2\left(B\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\sec(c+dx)\right)+A\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};1-\sec(c+dx)\right)\right)\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(B*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]] + A*Hypergeometric2F
1[1/2, 4, 3/2, 1 - Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/
2])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 579 vs. $2(140) = 280$.

time = 7.09, size = 580, normalized size = 3.62

method	result
default	$-\frac{\left(15A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}(\cos^2(dx+c))\sin(dx+c)+18B\sqrt{2}\,\operatorname{arctanh}\left(\sqrt{-\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/192/d*(15*A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)
^2*sin(d*x+c)+18*B*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*
x+c)^2*sin(d*x+c)+30*A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*co
s(d*x+c)*sin(d*x+c)+36*B*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*
cos(d*x+c)*sin(d*x+c)+15*A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2
)*sin(d*x+c)+18*B*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x
+c)+64*A*cos(d*x+c)^6+16*A*cos(d*x+c)^5+96*B*cos(d*x+c)^5+40*A*cos(d*x+c)^4
+48*B*cos(d*x+c)^4-120*A*cos(d*x+c)^3-144*B*cos(d*x+c)^3)*(a*(1+cos(d*x+c))
/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2981 vs. $2(140) = 280$.

time = 0.82, size = 2981, normalized size = 18.63

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/96*((4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*arcta
```

n2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(sin(
3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(
3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) +
 1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin(1/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*sin(1
/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(arc
tan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos
(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*
c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*
sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arctan2(-(cos
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c
), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
 + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*arcta
n2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin(3*
d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*
c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x
 + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (cos

```
(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c)
, cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))
+ 1)) + 1) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^
2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*c
os(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arct
an2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))*A + 6*(2*(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((cos(1/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - (cos(2*d*x + 2*c) -
2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(2*d*x + 2*c))
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + ...
```

Fricas [A]
time = 4.74, size = 346, normalized size = 2.16

$$\left[\frac{3\,\left(\left(5\,A+6\,B\right)\cos\left(dx+c\right)+5\,A+6\,B\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos\left(dx+c\right)^{2}-2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)+a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)+1}\right)+2\,\left(8\,A\cos\left(dx+c\right)^{3}+2\,\left(5\,A+6\,B\right)\cos\left(dx+c\right)^{2}+3\,\left(5\,A+6\,B\right)\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{48\,\left(d\cos\left(dx+c\right)+d\right)},\ -\frac{3\,\left(\left(5\,A+6\,B\right)\cos\left(dx+c\right)+5\,A+6\,B\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{\sqrt{a}\sin\left(dx+c\right)}\right)-\left(8\,A\cos\left(dx+c\right)^{3}+2\,\left(5\,A+6\,B\right)\cos\left(dx+c\right)^{2}+3\,\left(5\,A+6\,B\right)\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{24\,\left(d\cos\left(dx+c\right)+d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [1/48*(3*((5*A + 6*B)*cos(d*x + c) + 5*A + 6*B)*sqrt(-a)*log((2*a*cos(d*x +
 c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*A*cos(d*x + c)^3
 + 2*(5*A + 6*B)*cos(d*x + c)^2 + 3*(5*A + 6*B)*cos(d*x + c))*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/24*(3*((5
*A + 6*B)*cos(d*x + c) + 5*A + 6*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (8*A*cos(d*x + c)^3
+ 2*(5*A + 6*B)*cos(d*x + c)^2 + 3*(5*A + 6*B)*cos(d*x + c))*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\left(A+B\sec\left(c+dx\right)\right)\cos^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*cos(c + d*x)**3, x
)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 889 vs. $2(140) = 280$.
time = 1.70, size = 889, normalized size = 5.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/48*(3*(5*A*sqrt(-a)*sgn(cos(d*x + c)) + 6*B*sqrt(-a)*sgn(cos(d*x + c)))*
log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^2 - a*(2*sqrt(2) + 3))) - 3*(5*A*sqrt(-a)*sgn(cos(d*x + c)) + 6*B*sqrt(-
a)*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*(63*sqrt(2)*(sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a)*a
*sgn(cos(d*x + c)) - 30*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a*sgn(cos(d*x + c)) - 369*sqrt(2)*
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*s
qrt(-a)*a^2*sgn(cos(d*x + c)) + 66*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) +
1638*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^6*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 756*sqrt(2)*(sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^3*sgn(co
s(d*x + c)) - 1074*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 732*sqrt(2)*(sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(
-a)*a^4*sgn(cos(d*x + c)) + 171*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 138
*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^2*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 13*sqrt(2)*A*sqrt(-a)*a^6*sgn(cos(
d*x + c)) - 6*sqrt(2)*B*sqrt(-a)*a^6*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)
```

3.126 $\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=203

$$\frac{5\sqrt{a}\,(7A+8B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d}+\frac{5a(7A+8B)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}}+\frac{5a(7A+8B)\cos(c+dx)\sin(c}{96d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 5/64*(7*A+8*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+
5/64*a*(7*A+8*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+5/96*a*(7*A+8*B)*cos(d
*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/24*a*(7*A+8*B)*cos(d*x+c)^2*sin
(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/4*a*A*cos(d*x+c)^3*sin(d*x+c)/d/(a+a*sec
(d*x+c))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4100, 3890, 3859, 209}

$$\frac{5\sqrt{a}\,(7A+8B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{5a(7A+8B)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a(7A+8B)\sin(c+dx)\cos^2(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{5a(7A+8B)\sin(c+dx)\cos(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+\frac{aA\sin(c+dx)\cos^3(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (5*Sqrt[a]*(7*A + 8*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(64*d) + (5*a*(7*A + 8*B)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]
) + (5*a*(7*A + 8*B)*Cos[c + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*
x]]) + (a*(7*A + 8*B)*Cos[c + d*x]^2*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c +
 d*x]]) + (a*A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \cos^4(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \frac{aA\cos^3(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{1}{8}(7A+8B)\int$$
$$= \frac{a(7A+8B)\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{aA\,\mathrm{co}}{4d}$$
$$= \frac{5a(7A+8B)\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{a(7A}{}$$
$$= \frac{5a(7A+8B)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{5a(7A+8B)\cos}{96d\sqrt{a+}}$$
$$= \frac{5a(7A+8B)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{5a(7A+8B)\cos}{96d\sqrt{a+}}$$
$$= \frac{5\sqrt{a}\,(7A+8B)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right.}{64d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 0.17, size = 70, normalized size = 0.34

$$\frac{2\left(B\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};1-\sec(c+dx)\right)+A\,{}_2F_1\left(\frac{1}{2},5;\frac{3}{2};1-\sec(c+dx)\right)\right)\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(B*Hypergeometric2F1[1/2, 4, 3/2, 1 - Sec[c + d*x]] + A*Hypergeometric2F
1[1/2, 5, 3/2, 1 - Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/
2])/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 761 vs. $2(179) = 358$.
time = 5.83, size = 762, normalized size = 3.75

method	result
default	$\left(105A\sqrt{2}\,(\cos^3(dx+c))\sin(dx+c)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)+120B\sqrt{2}\,(\cos^3(dx+c))\,\text{s}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/3072/d*(105*A*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d
*x+c)*2^(1/2))+120*B*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/
cos(d*x+c)*2^(1/2))+315*A*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)/cos(d*x+c)*2^(1/2))+360*B*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)/cos(d*x+c)*2^(1/2))+315*A*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+360*B*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+105*A*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos
(d*x+c)*2^(1/2))*sin(d*x+c)+120*B*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7
/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*
2^(1/2))*sin(d*x+c)-768*A*cos(d*x+c)^8-128*A*cos(d*x+c)^7-1024*B*cos(d*x+c)
^7-224*A*cos(d*x+c)^6-256*B*cos(d*x+c)^6-560*A*cos(d*x+c)^5-640*B*cos(d*x+c
)^5+1680*A*cos(d*x+c)^4+1920*B*cos(d*x+c)^4)*(a*(1+cos(d*x+c))/cos(d*x+c))^
(1/2)/cos(d*x+c)^3/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 8561 vs. $2(179) = 358$.
time = 1.03, size = 8561, normalized size = 42.17

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 1/768*(8*(4*(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2
/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3
*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(3/4)*(cos(3/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1))*sin(3*d*x + 3*c) - (cos(3*d*x + 3*c) - 1)*sin(3/2*ar
ctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 6*(cos(2/3*arctan2(s
in(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), c
os(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))
) + 1)^(1/4)*((sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 5*sin
(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))))*cos(1/2*arctan2(sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))) + 1)) - (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*
x + 3*c))) + 3*cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - 4)*si
n(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3
*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)))*sqrt(a) + 15*sqrt(a)*(
arctan2(-(cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*
arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*
x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) -
cos(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/
3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x +
 3*c), cos(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2
*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2
(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), co
s(3*d*x + 3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), co
s(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) + 1) - arctan2(-(
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x +
3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c
))) + 1))*sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) - cos(1/3*ar
ctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2*arctan2(sin(2/3*arctan2(
sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos
(3*d*x + 3*c))) + 1))), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
```

```
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*(cos(1/3*arctan2(sin
(3*d*x + 3*c), cos(3*d*x + 3*c)))*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
3*c))) + 1)) + sin(1/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))*sin(1/2
*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arct
an2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1))) - 1) - arctan2((cos(2/3*arc
tan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x +
 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*
d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)), (
cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + sin(2/3*arctan2(si
n(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*arctan2(sin(3*d*x + 3*c),
cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*arctan2(sin(2/3*arctan2(sin(3*d*x + 3
*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
)) + 1)) + 1) + arctan2((cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)
))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 + 2*cos(2/3*a
rctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*sin(1/2*arctan2(sin(
2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan2(sin(3*d*x
 + 3*c), cos(3*d*x + 3*c))) + 1)), (cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3
*d*x + 3*c)))^2 + sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c)))^2 +
2*cos(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)^(1/4)*cos(1/2*a
rctan2(sin(2/3*arctan2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))), cos(2/3*arctan
2(sin(3*d*x + 3*c), cos(3*d*x + 3*c))) + 1)) - 1)))*B - (2*(cos(1/2*arctan2
(sin(4*d*x + 4*c), cos(4*d*x + 4*c)))^2 + sin(1/2*arctan2(sin(4*d*x + 4*c),
 cos(4*d*x + 4*c)))^2 + 2*cos(1/2*arctan2(sin(4*d*x + 4*c), cos(4*d*x + 4*c
))) + 1)^(3/4)*((36*(sin(4*d*x + 4*c)^3 + (cos(4*d*x + 4*c)^2 - 2*cos(4*d*x
 + 4*c) + 1)*sin(4*d*x + 4*c))*cos(1/2*arctan2(...
```

Fricas [A]
time = 3.45, size = 380, normalized size = 1.87

$$\left[\frac{15\left((7A+8B)\cos(dx+c)+7A+8B\right)\sqrt{-a}\,\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(48A\cos(dx+c)^4+8(7A+8B)\cos(dx+c)^3+10(7A+8B)\cos(dx+c)^2+15(7A+8B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{384\left(d\cos(dx+c)+d\right)},\ -\frac{15\left((7A+8B)\cos(dx+c)+7A+8B\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(48A\cos(dx+c)^4+8(7A+8B)\cos(dx+c)^3+10(7A+8B)\cos(dx+c)^2+15(7A+8B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{192\left(d\cos(dx+c)+d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [1/384*(15*((7*A + 8*B)*cos(d*x + c) + 7*A + 8*B)*sqrt(-a)*log((2*a*cos(d*x
 + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*s
in(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(48*A*cos(d*x + c
)^4 + 8*(7*A + 8*B)*cos(d*x + c)^3 + 10*(7*A + 8*B)*cos(d*x + c)^2 + 15*(7*
A + 8*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c) + d), -1/192*(15*((7*A + 8*B)*cos(d*x + c) + 7*A + 8*B)*s
qrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)
*sin(d*x + c))) - (48*A*cos(d*x + c)^4 + 8*(7*A + 8*B)*cos(d*x + c)^3 + 10*
```

```
(7*A + 8*B)*cos(d*x + c)^2 + 15*(7*A + 8*B)*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(1/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1080 vs. $2(179) = 358$.
time = 1.86, size = 1080, normalized size = 5.32

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(1/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/384*(15*(7*A*sqrt(-a)*sgn(cos(d*x + c)) + 8*B*sqrt(-a)*sgn(cos(d*x + c))
)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2 - a*(2*sqrt(2) + 3))) - 15*(7*A*sqrt(-a)*sgn(cos(d*x + c)) + 8*B*sqr
t(-a)*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) - 4*sqrt(2)*(279*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*sqrt(-
a)*a*sgn(cos(d*x + c)) - 504*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^14*B*sqrt(-a)*a*sgn(cos(d*x + c)) + 285*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*A*sqrt(-a)*a^
2*sgn(cos(d*x + c)) + 5976*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a))^12*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 4605*(sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a)*a
^3*sgn(cos(d*x + c)) - 31320*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 37281*(sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*sqrt(-a)
*a^4*sgn(cos(d*x + c)) + 90168*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 35643*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a
)*a^5*sgn(cos(d*x + c)) - 66024*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 9175*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a
)*a^6*sgn(cos(d*x + c)) + 16904*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 1311*(sqrt(
```

```
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a
)*a^7*sgn(cos(d*x + c)) - 1992*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan
(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^7*sgn(cos(d*x + c)) + 43*A*sqrt(-a
)*a^8*sgn(cos(d*x + c)) + 104*B*sqrt(-a)*a^8*sgn(cos(d*x + c)))/((sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)
*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)/
d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^4\left(A+\frac{B}{\cos(c+d\,x)}\right)\sqrt{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)
```

3.127 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=189

$$\frac{2a^2(39A+34B)\tan(c+dx)}{45d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(9A+10B)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}}-\frac{4a(39A+34B)\sqrt{a+a\sec(c+dx)}}{315d}$$

```
[Out] 2/105*(39*A+34*B)*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/45*a^2*(39*A+34*B)*
tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/63*a^2*(9*A+10*B)*sec(d*x+c)^3*tan(d*
x+c)/d/(a+a*sec(d*x+c))^(1/2)-4/315*a*(39*A+34*B)*(a+a*sec(d*x+c))^(1/2)*ta
n(d*x+c)/d+2/9*a*B*sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.31, antiderivative size = 189, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4103, 4101, 3885, 4086, 3877}

$$\frac{2a^2(9A+10B)\tan(c+dx)\sec^3(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(39A+34B)\tan(c+dx)}{45d\sqrt{a\sec(c+dx)+a}}+\frac{2(39A+34B)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{105d}-\frac{4a(39A+34B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}+\frac{2aB\tan(c+dx)\sec^3(c+dx)\sqrt{a\sec(c+dx)+a}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^2*(39*A + 34*B)*Tan[c + d*x])/(45*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2
*(9*A + 10*B)*Sec[c + d*x]^3*Tan[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]])
- (4*a*(39*A + 34*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (2*a*
B*Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(9*d) + (2*(39*A +
34*B)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(105*d)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
^2, 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \sec^3(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx &= \frac{2aB\sec^3(c+dx)\sqrt{a+a\sec(c+dx)}\,\tan(c+ \ldots}{9d} \\
&= \frac{2a^2(9A+10B)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2a \ldots}{\ldots} \\
&= \frac{2a^2(9A+10B)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2a \ldots}{\ldots} \\
&= \frac{2a^2(9A+10B)\sec^3(c+dx)\tan(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} - \frac{4a \ldots}{\ldots} \\
&= \frac{2a^2(39A+34B)\tan(c+dx)}{45d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(9A+10B \ldots}{63d\sqrt{\ldots}}
\end{aligned}
$$

Mathematica [A]
time = 0.74, size = 100, normalized size = 0.53

$$\frac{2a^2(8(39A+34B)+4(39A+34B)\sec(c+dx)+3(39A+34B)\sec^2(c+dx)+5(9A+17B)\sec^3(c+dx)+35B\sec^4(c+dx))\tan(c+dx)}{315d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^2*(8*(39*A + 34*B) + 4*(39*A + 34*B)*Sec[c + d*x] + 3*(39*A + 34*B)*Se
c[c + d*x]^2 + 5*(9*A + 17*B)*Sec[c + d*x]^3 + 35*B*Sec[c + d*x]^4)*Tan[c +
 d*x])/(315*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 4.04, size = 139, normalized size = 0.74

method	result
default	$-\frac{2(-1+\cos(dx+c))(312A(\cos^4(dx+c))+272B(\cos^4(dx+c))+156A(\cos^3(dx+c))+136B(\cos^3(dx+c))+117A(\cos^2(dx+c))+102B}{315d\cos(dx+c)^4\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(312*A*cos(d*x+c)^4+272*B*cos(d*x+c)^4+156*A*cos(d
*x+c)^3+136*B*cos(d*x+c)^3+117*A*cos(d*x+c)^2+102*B*cos(d*x+c)^2+45*A*cos(d
*x+c)+85*B*cos(d*x+c)+35*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^
4/sin(d*x+c)*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 8/315*(315*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(((3*A + 2*B)*a*d*cos(2*d*x + 2*c)^4 + (3*A + 2*B)*a*d*sin(2*d*x +
 2*c)^4 + 4*(3*A + 2*B)*a*d*cos(2*d*x + 2*c)^3 + 6*(3*A + 2*B)*a*d*cos(2*d*
x + 2*c)^2 + 4*(3*A + 2*B)*a*d*cos(2*d*x + 2*c) + (3*A + 2*B)*a*d + 2*((3*A
 + 2*B)*a*d*cos(2*d*x + 2*c)^2 + 2*(3*A + 2*B)*a*d*cos(2*d*x + 2*c) + (3*A
+ 2*B)*a*d)*sin(2*d*x + 2*c)^2)*integrate((cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(10*d*x + 10*c)*cos(2*d*x + 2
```

```
*c) + 4*cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x +
2*c) + 4*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*
x + 10*c)*sin(2*d*x + 2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*
d*x + 6*c)*sin(2*d*x + 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d
*x + 2*c)^2)*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*
d*x + 2*c)*sin(10*d*x + 10*c) + 4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos
(2*d*x + 2*c)*sin(6*d*x + 6*c) + 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(
10*d*x + 10*c)*sin(2*d*x + 2*c) - 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*c
os(6*d*x + 6*c)*sin(2*d*x + 2*c) - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin
(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(10*d*x + 10*c) +
4*cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 6*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) +
 4*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(10*d*x + 10*c)*sin(2*d*x + 2*c)
- 4*cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 6*cos(6*d*x + 6*c)*sin(2*d*x + 2*c)
 - 4*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - (cos(10*d*x + 10*c)*cos(2*d*x + 2*c) + 4*cos(8*d*x + 8*
c)*cos(2*d*x + 2*c) + 6*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 4*cos(4*d*x + 4
*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(10*d*x + 10*c)*sin(2*d*x +
2*c) + 4*sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 6*sin(6*d*x + 6*c)*sin(2*d*x +
 2*c) + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(7/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)))/((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(10*d*x
+ 10*c)^2 + 16*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c
) + 1)*cos(8*d*x + 8*c)^2 + 36*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 16*(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x + 2
*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
sin(10*d*x + 10*c)^2 + 16*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)*sin(8*d*x + 8*c)^2 + 36*(cos(2*d*x + 2*c)^2 + sin(2*d*x +
 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 16*(cos(2*d*x + 2*c)
^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*c
os(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d
*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c) + 6*(cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos
(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(10*d*x + 10*c)
 + 8*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 6*(cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos
(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) +
 12*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 4*(cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)
+ 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 8*(cos(2*d*x
```

```
+ 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos
(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^
3 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*si
n(8*d*x + 8*c) + 6*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
 2*c) + 1)*sin(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(10*d*x + 10*c) + 8*(sin(2*d*x + 2*c)^3
+ 6*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(
6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*s
in(2*d*x + 2*c))*sin(8*d*x + 8*c) + 12*(sin(2*d*x + 2*c)^3 + 4*(cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (
cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x +
6*c) + 8*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))*cos(3/2*a...
```

Fricas [A]
time = 3.24, size = 127, normalized size = 0.67

$$\frac{2 \left(8 \left(39 A + 34 B\right) a \cos \left(dx + c\right)^{4} + 4 \left(39 A + 34 B\right) a \cos \left(dx + c\right)^{3} + 3 \left(39 A + 34 B\right) a \cos \left(dx + c\right)^{2} + 5 \left(9 A + 17 B\right) a \cos \left(dx + c\right) + 35 B a\right) \sqrt{\frac{a \cos \left(dx + c\right) + a}{\cos \left(dx + c\right)}} \sin \left(dx + c\right)}{315 \left(d \cos \left(dx + c\right)^{5} + d \cos \left(dx + c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/315*(8*(39*A + 34*B)*a*cos(d*x + c)^4 + 4*(39*A + 34*B)*a*cos(d*x + c)^3
+ 3*(39*A + 34*B)*a*cos(d*x + c)^2 + 5*(9*A + 17*B)*a*cos(d*x + c) + 35*B*a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^5 +
d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a \left(\sec \left(c + dx\right) + 1\right)\right)^{\frac{3}{2}} \left(A + B \sec \left(c + dx\right)\right) \sec^{3} \left(c + dx\right)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*(A + B*sec(c + d*x))*sec(c + d*x)**3
, x)
```

Giac [A]
time = 1.28, size = 258, normalized size = 1.37

$$\frac{4 \left(\left(\left(\left(2 \sqrt{2} \left(57 A a^{6} \mathrm{sgn}\left(\cos \left(dx + c\right)\right) + 47 B a^{6} \mathrm{sgn}\left(\cos \left(dx + c\right)\right)\right) \tan \left(\frac{1}{2} dx + \frac{1}{2} c\right)^{2} - 9 \sqrt{2} \left(57 A a^{6} \mathrm{sgn}\left(\cos \left(dx + c\right)\right) + 47 B a^{6} \mathrm{sgn}\left(\cos \left(dx + c\right)\right)\right)\right) \tan \left(\frac{1}{2} dx + \frac{1}{2} c\right)^{2} + 819 \sqrt{2} \left(A a^{6} \mathrm{sgn}\left(\cos \left(dx + c\right)\right) + B a^{6} \mathrm{sgn}\left(\cos \left(dx + c\right)\right)\right)\right) \tan \left(\frac{1}{2} dx + \frac{1}{2} c\right)^{2} - 105 \sqrt{2} \left(7 A a^{6} \mathrm{sgn}\left(\cos \left(dx + c\right)\right) + 5 B a^{6} \mathrm{sgn}\left(\cos \left(dx + c\right)\right)\right)\right) \tan \left(\frac{1}{2} dx + \frac{1}{2} c\right)^{2} + 315 \sqrt{2} \left(A a^{6} \mathrm{sgn}\left(\cos \left(dx + c\right)\right) + B a^{6} \mathrm{sgn}\left(\cos \left(dx + c\right)\right)\right)\right) \tan \left(\frac{1}{2} dx + \frac{1}{2} c\right)}{315 \left(a \tan \left(\frac{1}{2} dx + \frac{1}{2} c\right)^{2} - a\right)^{4} \sqrt{-a \tan \left(\frac{1}{2} dx + \frac{1}{2} c\right)^{2} + a}\, d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] 4/315*((((2*sqrt(2)*(57*A*a^6*sgn(cos(d*x + c)) + 47*B*a^6*sgn(cos(d*x + c)
))*tan(1/2*d*x + 1/2*c)^2 - 9*sqrt(2)*(57*A*a^6*sgn(cos(d*x + c)) + 47*B*a^
6*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 819*sqrt(2)*(A*a^6*sgn(cos(d
*x + c)) + B*a^6*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 105*sqrt(2)*(
7*A*a^6*sgn(cos(d*x + c)) + 5*B*a^6*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c
)^2 + 315*sqrt(2)*(A*a^6*sgn(cos(d*x + c)) + B*a^6*sgn(cos(d*x + c))))*tan(
1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a)*d)
```

Mupad [B]
time = 9.60, size = 596, normalized size = 3.15

$$\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(e^{c\,1i+d\,x\,1i}\left(-\frac{a\,(3\,A+2\,B)\,8i}{7\,d}+\frac{a\,(A+4\,B)\,8i}{7\,d}+\frac{B\,a\,32i}{63\,d}\right)+\frac{A\,a\,8i}{7\,d}-\frac{a\,(A+2\,B)\,24i}{7\,d}-\frac{B\,a\,32i}{7\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^3}+\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(e^{c\,1i+d\,x\,1i}\left(-\frac{a\,(2\,A+3\,B)\,16i}{9\,d}+\frac{a\,(3\,A+2\,B)\,8i}{9\,d}+\frac{A\,a\,8i}{9\,d}\right)+\frac{a\,(2\,A+3\,B)\,16i}{9\,d}-\frac{a\,(3\,A+2\,B)\,8i}{9\,d}-\frac{A\,a\,8i}{9\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^4}+\frac{\left(\frac{A\,a\,8i}{3\,d}-\frac{a\,e^{c\,1i+d\,x\,1i}\,(39\,A+34\,B)\,8i}{315\,d}\right)\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)}+\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(e^{c\,1i+d\,x\,1i}\left(\frac{a\,(3\,A+2\,B)\,8i}{5\,d}+\frac{a\,(3\,A+B)\,16i}{105\,d}\right)-\frac{A\,a\,8i}{5\,d}+\frac{a\,(A+3\,B)\,16i}{5\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^2}-\frac{a\,e^{c\,1i+d\,x\,1i}\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,(39\,A+34\,B)\,16i}{315\,d\,\left(e^{c\,1i+d\,x\,1i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/cos(c + d*x)^3,x)
```

```
[Out] ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i +
d*x*1i)*((a*(A + 4*B)*8i)/(7*d) - (a*(3*A + 2*B)*8i)/(7*d) + (B*a*32i)/(63*
d)) + (A*a*8i)/(7*d) - (a*(A + 2*B)*24i)/(7*d) - (B*a*32i)/(7*d)))/((exp(c*
1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) + ((a + a/(exp(- c*1i - d*x*1
i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i)*((a*(3*A + 2*B)*8i)
/(9*d) - (a*(2*A + 3*B)*16i)/(9*d) + (A*a*8i)/(9*d)) + (a*(2*A + 3*B)*16i)/
(9*d) - (a*(3*A + 2*B)*8i)/(9*d) - (A*a*8i)/(9*d)))/((exp(c*1i + d*x*1i) +
1)*(exp(c*2i + d*x*2i) + 1)^4) + (((A*a*8i)/(3*d) - (a*exp(c*1i + d*x*1i)*(
39*A + 34*B)*8i)/(315*d))*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1
i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)) + ((a + a
/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i)
*((a*(3*A + 2*B)*8i)/(5*d) + (a*(3*A + B)*16i)/(105*d)) - (A*a*8i)/(5*d) +
(a*(A + 3*B)*16i)/(5*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1
)^2) - (a*exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*
x*1i)/2))^(1/2)*(39*A + 34*B)*16i)/(315*d*(exp(c*1i + d*x*1i) + 1))
```

3.128 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=138

$$\frac{8a^2(21A+19B)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}+\frac{2a(21A+19B)\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{105d}+\frac{2(7A-2B)(a+a\sec(c+}{35d}$$

```
[Out] 2/35*(7*A-2*B)*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/7*B*(a+a*sec(d*x+c))^(
5/2)*tan(d*x+c)/a/d+8/105*a^2*(21*A+19*B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/
2)+2/105*a*(21*A+19*B)*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.20, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4095, 4086, 3878, 3877}

$$\frac{8a^2(21A+19B)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2(7A-2B)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35d}+\frac{2a(21A+19B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2B\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (8*a^2*(21*A + 19*B)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*
(21*A + 19*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*(7*A - 2*
B)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2*B*(a + a*Sec[c + d*
x])^(5/2)*Tan[c + d*x])/(7*a*d)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3878

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[a*((2*m - 1)/m), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
```

```
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{2B(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7ad} + \frac{2\int \mathrm{se}}{}$$

$$= \frac{2(7A-2B)(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{35d}$$

$$= \frac{2a(21A+19B)\sqrt{a+a\sec(c+dx)}\,\tan(c+}{105d}$$

$$= \frac{8a^2(21A+19B)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2a(21A+19}{}$$

Mathematica [A]
time = 0.39, size = 82, normalized size = 0.59

$$\frac{2a^2(2(63A+52B)+(63A+52B)\sec(c+dx)+3(7A+13B)\sec^2(c+dx)+15B\sec^3(c+dx))\tan(c+dx)}{105d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^2*(2*(63*A + 52*B) + (63*A + 52*B)*Sec[c + d*x] + 3*(7*A + 13*B)*Sec[c
 + d*x]^2 + 15*B*Sec[c + d*x]^3)*Tan[c + d*x])/(105*d*Sqrt[a*(1 + Sec[c + d
*x])])
```

Maple [A]
time = 3.81, size = 117, normalized size = 0.85

method	result
default	$-\frac{2(-1+\cos(dx+c))(126A(\cos^3(dx+c))+104B(\cos^3(dx+c))+63A(\cos^2(dx+c))+52B(\cos^2(dx+c))+21A\cos(dx+c)+39B\cos(dx+}{105d\cos(dx+c)^3\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(126*A*cos(d*x+c)^3+104*B*cos(d*x+c)^3+63*A*cos(d*
x+c)^2+52*B*cos(d*x+c)^2+21*A*cos(d*x+c)+39*B*cos(d*x+c)+15*B)*(a*(1+cos(d*
x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] -4/105*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(7*(5*(3*A + 2*B)*a*sin(4*d*x + 4*c) + 2*(12*A + 13*B)*a*sin(2*d*x +
2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - (35*(3*A +
 2*B)*a*cos(4*d*x + 4*c) + 14*(12*A + 13*B)*a*cos(2*d*x + 2*c) + (63*A + 52
*B)*a)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) -
105*((A*a*d*cos(2*d*x + 2*c)^4 + A*a*d*sin(2*d*x + 2*c)^4 + 4*A*a*d*cos(2*d
*x + 2*c)^3 + 6*A*a*d*cos(2*d*x + 2*c)^2 + 4*A*a*d*cos(2*d*x + 2*c) + A*a*d
 + 2*(A*a*d*cos(2*d*x + 2*c)^2 + 2*A*a*d*cos(2*d*x + 2*c) + A*a*d)*sin(2*d*
x + 2*c)^2)*integrate((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*
x + 2*c) + 1)^(1/4)*(((cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 3*cos(6*d*x + 6*
c)*cos(2*d*x + 2*c) + 3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c
)^2 + sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d*x + 6*c)*sin(2*d*x + 2*
c) + 3*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(7/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(8*d*x + 8*
c) + 3*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 3*cos(2*d*x + 2*c)*sin(4*d*x + 4
*c) - cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*
c) - 3*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(7/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)) - ((cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 3*cos(2*d*x + 2*c)*sin(6*d*x + 6
*c) + 3*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(8*d*x + 8*c)*sin(2*d*x + 2*
c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 3*cos(4*d*x + 4*c)*sin(2*d*x + 2
*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (cos(8*d*x + 8*
```

c)*cos(2*d*x + 2*c) + 3*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 3*cos(4*d*x + 4
*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(8*d*x + 8*c)*sin(2*d*x + 2*
c) + 3*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 3*sin(4*d*x + 4*c)*sin(2*d*x + 2
*c) + sin(2*d*x + 2*c)^2)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/((cos(2*d*x
+ 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c)^2 + 9*(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 9*(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 +
2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)*sin(8*d*x + 8*c)^2 + 9*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c
)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 9*(cos(2*d*x + 2*c)^2 +
sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x +
2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 3*(cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c) + 3*(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*
cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 6*(cos(2*d*x + 2*
c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 3*(cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c
)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 6*(cos(2*d*x + 2*c)^3 + cos(2*d*
x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(
4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 3*(cos(2*d*x +
2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*
x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*
sin(8*d*x + 8*c) + 6*(sin(2*d*x + 2*c)^3 + 3*(cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(sin(2*d*
x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)
)*sin(4*d*x + 4*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)^2 + (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(
2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c)^2 + 9*(cos(2*d*x
+ 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2
+ 9*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(
4*d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(8*d*x + 8*c)^2 + 9*(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 9*(cos(
2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4
*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2
+ 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 3*(cos(2*d*
x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)
+ 3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(
4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) +
6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*...

Fricas [A]
time = 3.13, size = 108, normalized size = 0.78

$$\frac{2\,\left(2\,\left(63\,A+52\,B\right)a\cos\left(dx+c\right)^{3}+\left(63\,A+52\,B\right)a\cos\left(dx+c\right)^{2}+3\,\left(7\,A+13\,B\right)a\cos\left(dx+c\right)+15\,Ba\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{105\,\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/105*(2*(63*A + 52*B)*a*cos(d*x + c)^3 + (63*A + 52*B)*a*cos(d*x + c)^2 +
3*(7*A + 13*B)*a*cos(d*x + c) + 15*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\left(A+B\sec\left(c+dx\right)\right)\sec^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*(A + B*sec(c + d*x))*sec(c + d*x)**2
, x)
```

Giac [A]
time = 1.24, size = 215, normalized size = 1.56

$$\frac{4\left(\left(\left(2\sqrt{2}\left(21\,Aa^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+19\,Ba^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-7\sqrt{2}\left(21\,Aa^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+19\,Ba^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+70\sqrt{2}\left(3\,Aa^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+2\,Ba^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-105\sqrt{2}\left(Aa^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)+Ba^{5}\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\right)\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{105\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{3}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] 4/105*(((2*sqrt(2)*(21*A*a^5*sgn(cos(d*x + c)) + 19*B*a^5*sgn(cos(d*x + c))
)*tan(1/2*d*x + 1/2*c)^2 - 7*sqrt(2)*(21*A*a^5*sgn(cos(d*x + c)) + 19*B*a^5
*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 70*sqrt(2)*(3*A*a^5*sgn(cos(d
*x + c)) + 2*B*a^5*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 105*sqrt(2)
*(A*a^5*sgn(cos(d*x + c)) + B*a^5*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)/
((a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

Mupad [B]
time = 6.89, size = 479, normalized size = 3.47

$$-\frac{\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,\left(\frac{a\,\left(7\,A+13\,B\right)\,8\mathrm{i}}{105\,d}-\frac{A\,a\,4\mathrm{i}}{3\,d}\right)-\frac{a\,\left(3\,A+2\,B\right)\,4\mathrm{i}}{3\,d}\right)}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\,\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}+\frac{\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,\left(-\frac{a\,\left(2\,A+3\,B\right)\,8\mathrm{i}}{7\,d}+\frac{a\,\left(3\,A+2\,B\right)\,4\mathrm{i}}{7\,d}+\frac{A\,a\,4\mathrm{i}}{7\,d}\right)-\frac{a\,\left(2\,A+3\,B\right)\,8\mathrm{i}}{7\,d}+\frac{a\,\left(3\,A+2\,B\right)\,4\mathrm{i}}{7\,d}+\frac{A\,a\,4\mathrm{i}}{7\,d}\right)}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^3}+\frac{\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,\left(-\frac{A\,a\,4\mathrm{i}}{5\,d}+\frac{a\,\left(A+2\,B\right)\,12\mathrm{i}}{5\,d}+\frac{B\,a\,16\mathrm{i}}{35\,d}\right)-\frac{a\,\left(3\,A+2\,B\right)\,4\mathrm{i}}{5\,d}+\frac{a\,\left(A+4\,B\right)\,4\mathrm{i}}{5\,d}\right)}{\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)\,{\left(\mathrm{e}^{c\,2\mathrm{i}+d\,x\,2\mathrm{i}}+1\right)}^2}-\frac{a\,\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}\,\sqrt{a+\frac{a}{\frac{\mathrm{e}^{-c\,1\mathrm{i}-d\,x\,1\mathrm{i}}}{2}+\frac{\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}}{2}}}\,\left(63\,A+52\,B\right)\,4\mathrm{i}}{105\,d\,\left(\mathrm{e}^{c\,1\mathrm{i}+d\,x\,1\mathrm{i}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/cos(c + d*x)^2,x)

[Out] ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i)*((a*(3*A + 2*B)*4i)/(7*d) - (a*(2*A + 3*B)*8i)/(7*d) + (A*a*4i)/(7*d)) - (a*(2*A + 3*B)*8i)/(7*d) + (a*(3*A + 2*B)*4i)/(7*d) + (A*a*4i)/(7*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) - ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i)*((a*(7*A + 13*B)*8i)/(105*d) - (A*a*4i)/(3*d)) - (a*(3*A + 2*B)*4i)/(3*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)) + ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i)*((a*(A + 2*B)*12i)/(5*d) - (A*a*4i)/(5*d) + (B*a*16i)/(35*d)) - (a*(3*A + 2*B)*4i)/(5*d) + (a*(A + 4*B)*4i)/(5*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) - (a*exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(63*A + 52*B)*4i)/(105*d*(exp(c*1i + d*x*1i) + 1))

3.129 $\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=101

$$\frac{8a^2(5A+3B)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{2a(5A+3B)\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{15d}+\frac{2B(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d}$$

```
[Out] 2/5*B*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+8/15*a^2*(5*A+3*B)*tan(d*x+c)/d/(
a+a*sec(d*x+c))^(1/2)+2/15*a*(5*A+3*B)*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.10, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4086, 3878, 3877}

$$\frac{8a^2(5A+3B)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a(5A+3B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2B\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (8*a^2*(5*A + 3*B)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(5*
A + 3*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*B*(a + a*Sec[c
+ d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3878

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[a*((2*m - 1)/m), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
```

```
e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{2B(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{1}{5}(5A$$
$$= \frac{2a(5A+3B)\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{15d}$$
$$= \frac{8a^2(5A+3B)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2a(5A+3B)\sqrt{a}}{}$$

Mathematica [A]
time = 0.31, size = 70, normalized size = 0.69

$$\frac{2a\sqrt{a(1+\sec(c+dx))}\,((25A+18B)\sin(c+dx)+(5A+9B+3B\sec(c+dx))\tan(c+dx))}{15d(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*Sqrt[a*(1 + Sec[c + d*x])]*((25*A + 18*B)*Sin[c + d*x] + (5*A + 9*B +
3*B*Sec[c + d*x])*Tan[c + d*x]))/(15*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 4.18, size = 95, normalized size = 0.94

method	result	size
default	$-\frac{2(-1+\cos(dx+c))(25A(\cos^2(dx+c))+18B(\cos^2(dx+c))+5A\cos(dx+c)+9B\cos(dx+c)+3B)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,a}{15d\cos(dx+c)^2\sin(dx+c)}$	95

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(25*A*cos(d*x+c)^2+18*B*cos(d*x+c)^2+5*A*cos(d*x+c)
+9*B*cos(d*x+c)+3*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d
*x+c)*a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] 2/15*(15*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(((3*A + 2*B)*a*d*cos(2*d*x + 2*c)^2 + (3*A + 2*B)*a*d*sin(2*d*x + 2
*c)^2 + 2*(3*A + 2*B)*a*d*cos(2*d*x + 2*c) + (3*A + 2*B)*a*d)*integrate((co
s(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((co
s(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos
(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*si
n(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(5/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*si
n(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin
(2*d*x + 2*c))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin
(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(
2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) - (cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(
4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2
*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin
(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c) + 1)))/((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4
 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6
*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 4*(cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*
c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 +
 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 4*(cos(2*d*x +
 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(
2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3
 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin
(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2
*c))*sin(6*d*x + 6*c) + 4*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))*cos(3/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2 + (cos(2*d*x + 2*c)^4 + sin(2*d*x +
2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*cos(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 4*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*
x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*
```

```
c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*(cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 4*(cos(2
*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2
+ cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x +
2*c)^3 + 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + 4*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 +
 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))*sin(3/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))^2), x) + 2*((2*A + 3*B)*a*d*cos(
2*d*x + 2*c)^2 + (2*A + 3*B)*a*d*sin(2*d*x + 2*c)^2 + 2*(2*A + 3*B)*a*d*cos
(2*d*x + 2*c) + (2*A + 3*B)*a*d)*integrate((cos(2*d*x + 2*c)^2 + sin(2*d*x
+ 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(((cos(6*d*x + 6*c)*cos(2*d*x + 2*
c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x +
 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x +
2*c)^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x +
 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x +
6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(3/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x +
 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x +
4*c)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - (cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c)
 + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4
*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
1)))/((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 ...
```

Fricas [A]
time = 2.51, size = 89, normalized size = 0.88

$$\frac{2\left((25A+18B)a\cos\left(dx+c\right)^{2}+(5A+9B)a\cos\left(dx+c\right)+3Ba\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 2/15*((25*A + 18*B)*a*cos(d*x + c)^2 + (5*A + 9*B)*a*cos(d*x + c) + 3*B*a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^3 + d*
cos(d*x + c)^2)
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec (c+dx)+1))^{\frac{3}{2}} (A+B\sec (c+dx))\sec (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*(A + B*sec(c + d*x))*sec(c + d*x), x
)
```

Giac [A]
time = 1.09, size = 170, normalized size = 1.68

$$\frac{4\left(\left(2\sqrt{2}\left(5Aa^4\mathrm{sgn}(\cos(dx+c))+3Ba^4\mathrm{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-5\sqrt{2}\left(5Aa^4\mathrm{sgn}(\cos(dx+c))+3Ba^4\mathrm{sgn}(\cos(dx+c))\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+15\sqrt{2}\left(Aa^4\mathrm{sgn}(\cos(dx+c))+Ba^4\mathrm{sgn}(\cos(dx+c))\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{15\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] 4/15*((2*sqrt(2)*(5*A*a^4*sgn(cos(d*x + c)) + 3*B*a^4*sgn(cos(d*x + c)))*ta
n(1/2*d*x + 1/2*c)^2 - 5*sqrt(2)*(5*A*a^4*sgn(cos(d*x + c)) + 3*B*a^4*sgn(c
os(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 15*sqrt(2)*(A*a^4*sgn(cos(d*x + c))
 + B*a^4*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^
2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

Mupad [B]
time = 5.88, size = 213, normalized size = 2.11

$$-\frac{2a\left(e^{c\,1i+d\,x\,1i}-1\right)\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(A\,25i+B\,18i+A\,e^{c\,1i+d\,x\,1i}\,10i+A\,e^{c\,2i+d\,x\,2i}\,50i+A\,e^{c\,3i+d\,x\,3i}\,10i+A\,e^{c\,4i+d\,x\,4i}\,25i+B\,e^{c\,1i+d\,x\,1i}\,18i+B\,e^{c\,2i+d\,x\,2i}\,48i+B\,e^{c\,3i+d\,x\,3i}\,18i+B\,e^{c\,4i+d\,x\,4i}\,18i\right)}{15\,d\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/cos(c + d*x),x)
```

```
[Out] -(2*a*(exp(c*1i + d*x*1i) - 1)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i +
d*x*1i)/2))^(1/2)*(A*25i + B*18i + A*exp(c*1i + d*x*1i)*10i + A*exp(c*2i +
d*x*2i)*50i + A*exp(c*3i + d*x*3i)*10i + A*exp(c*4i + d*x*4i)*25i + B*exp(c
*1i + d*x*1i)*18i + B*exp(c*2i + d*x*2i)*48i + B*exp(c*3i + d*x*3i)*18i + B
*exp(c*4i + d*x*4i)*18i))/(15*d*(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i
) + 1)^2)
```

3.130 $\int (a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=105

$$\frac{2a^{3/2}A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2(3A+4B)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2aB\sqrt{a+a\sec(c+dx)}\tan(c+dx)}{3d}$$

```
[Out] 2*a^(3/2)*A*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2/3*a^2*(3*
A+4*B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a*B*(a+a*sec(d*x+c))^(1/2)*t
an(d*x+c)/d
```

Rubi [A]
time = 0.11, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4002, 4000, 3859, 209, 3877}

$$\frac{2a^{3/2}A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2(3A+4B)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2aB\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^(3/2)*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a^2*(3*A + 4*B)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*B*Sqr
t[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4002

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m +
(b*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*
m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{2aB\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3d}+\frac{2}{3}\int \sqrt{a+a\sec(}$$

$$= \frac{2aB\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3d}+(aA)\int \sqrt{a+a\,}$$

$$= \frac{2a^2(3A+4B)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2aB\sqrt{a+a\sec(c+dx)}\,\text{ta}}{3d}$$

$$= \frac{2a^{3/2}A\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2(3A+4B)\,\text{t}}{3d\sqrt{a+a\sec}}$$

Mathematica [A]
time = 0.57, size = 102, normalized size = 0.97

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(1+\sec(c+dx))}\left(3\sqrt{2}\,A\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)+2(B+(3A+5B)\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sec[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*A*Ar
cSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + 2*(B + (3*A + 5*B)*Cos[
c + d*x])*Sin[(c + d*x)/2]))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 236 vs. $2(91) = 182$.
time = 4.17, size = 237, normalized size = 2.26

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3A\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sin(dx+c)\cos(dx+c)\sqrt{2}+3A\left(-\frac{2\cos}{1+c}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/6/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*A*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*
x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)*2^(1/2)+3*A*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d
*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)-12*A*cos(d*x+c)^2-20*B*cos(d*x+c)^2+12*A*
cos(d*x+c)+16*B*cos(d*x+c)+4*B)/sin(d*x+c)/cos(d*x+c)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 998 vs. $2(91) = 182$.
time = 0.62, size = 998, normalized size = 9.50

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/2*((a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
```

```
), cos(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2
*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
 + 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (
cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1
)))*sqrt(a))*A/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*d)
```

Fricas [A]
time = 2.89, size = 314, normalized size = 2.99

$$\left[\frac{3\left(Aa\cos\left(dx+c\right)^2+Aa\cos\left(dx+c\right)\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left((3\,A+5\,B)a\cos\left(dx+c\right)+Ba\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)},-\frac{2\left(3\left(Aa\cos\left(dx+c\right)^2+Aa\cos\left(dx+c\right)\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left((3\,A+5\,B)a\cos\left(dx+c\right)+Ba\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)\right)}{3\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/3*(3*(A*a*cos(d*x + c)^2 + A*a*cos(d*x + c))*sqrt(-a)*log((2*a*cos(d*x +
 c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*((3*A + 5*B)*a*cos(
d*x + c) + B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c)^2 + d*cos(d*x + c)), -2/3*(3*(A*a*cos(d*x + c)^2 + A*a*cos(d*x +
 c))*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(s
qrt(a)*sin(d*x + c))) - ((3*A + 5*B)*a*cos(d*x + c) + B*a)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\left(A+B\sec\left(c+dx\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*(A + B*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 263 vs. 2(91) = 182.

time = 1.29, size = 263, normalized size = 2.50

$$-\frac{\frac{3\,A\sqrt{-a}\,a^{2}\log\left(\frac{\left|2\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}-4\,\sqrt{2}\,|a|-6\,a\right|}{\left|2\left(\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right)^{2}+4\,\sqrt{2}\,|a|-6\,a\right|}\right)\operatorname{sgn}(\cos(dx+c))}{|a|}+\frac{2\left(3\,\sqrt{2}\,Aa^{3}\operatorname{sgn}(\cos(dx+c))+6\,\sqrt{2}\,Ba^{3}\operatorname{sgn}(\cos(dx+c))-\left(3\,\sqrt{2}\,Aa^{3}\operatorname{sgn}(\cos(dx+c))+4\,\sqrt{2}\,Ba^{3}\operatorname{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/3*(3*A*sqrt(-a)*a^2*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*tan
(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*abs(
a) - 6*a))*sgn(cos(d*x + c))/abs(a) + 2*(3*sqrt(2)*A*a^3*sgn(cos(d*x + c))
+ 6*sqrt(2)*B*a^3*sgn(cos(d*x + c)) - (3*sqrt(2)*A*a^3*sgn(cos(d*x + c)) +
4*sqrt(2)*B*a^3*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/
2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/
d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)
```

3.131 $\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=103

$$\frac{a^{3/2}(3A+2B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{2aB\sqrt{a+a\sec(c+dx)}\sin(c+$$

```
[Out] a^(3/2)*(3*A+2*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+a^2*(
A-2*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2*a*B*sin(d*x+c)*(a+a*sec(d*x+c)
)^(1/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4103, 4100, 3859, 209}

$$\frac{a^{3/2}(3A+2B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}+\frac{2aB\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(3/2)*(3*A + 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/d + (a^2*(A - 2*B)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*B*S
qrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
```

```
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{2aB\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{d} + 2\int \mathrm{co}$$

$$= \frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2aB\sqrt{a+a\sec(c+}}{d}$$

$$= \frac{a^2(A-2B)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{2aB\sqrt{a+a\sec(c+}}{d}$$

$$= \frac{a^{3/2}(3A+2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

Mathematica [A]
time = 0.42, size = 97, normalized size = 0.94

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{2}\,(3A+2B)\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2(2B+A\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(3*A + 2*B)*ArcSin[
Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(2*B + A*Cos[c + d*x])*Sin
[(c + d*x)/2]))/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(93) = 186$.
time = 7.80, size = 212, normalized size = 2.06

method	result
default	$-\frac{\left(3A\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)+2B\,\text{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}{2\cos(dx+}\right.\right.}{2d\,s}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/2/d*(3*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)+2*
B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)+2*A*cos(d*x+
c)^2-2*A*cos(d*x+c)+4*B*cos(d*x+c)-4*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)
/sin(d*x+c)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1801 vs. $2(93) = 186$.
time = 0.68, size = 1801, normalized size = 17.49

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] 1/4*((2*(a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(d*x
 + c) - (a*cos(d*x + c) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)))*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*sqrt(a) + 3*(a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
+ 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos
(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))) + 1) - a*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(
```

```
2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
+ 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 1) + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2
*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (c
os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a))*A + 2*(
(a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))) + 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x
 + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))) - 1) - a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2
 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*
c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + 1)
 + a*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)), (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*(cos(2*d*x + 2*c)^2
+ sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sqrt(a) + 4*(a*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - (a*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) - a)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*s
qrt(a))*B/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4))/d
```

Fricas [A]
time = 3.09, size = 292, normalized size = 2.83

$$\left[\frac{((3A+2B)a\cos(dx+c)+(3A+2B)a)\sqrt{-a}\,\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2(Aa\cos(dx+c)+2Ba)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{2(d\cos(dx+c)+d)},\ -\frac{((3A+2B)a\cos(dx+c)+(3A+2B)a)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-(Aa\cos(dx+c)+2Ba)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{d\cos(dx+c)+d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] [1/2*(((3*A + 2*B)*a*cos(d*x + c) + (3*A + 2*B)*a)*sqrt(-a)*log((2*a*cos(d*
x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*
sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(A*a*cos(d*x + c
) + 2*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c) + d), -(((3*A + 2*B)*a*cos(d*x + c) + (3*A + 2*B)*a)*sqrt(a)*arctan(s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
 - (A*a*cos(d*x + c) + 2*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d
*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(93) = 186.
time = 1.76, size = 406, normalized size = 3.94

$$-\frac{\frac{4\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,Ba^2\operatorname{sgn}(\cos(dx+c))\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}+\left(3A\sqrt{-a}\,a\operatorname{sgn}(\cos(dx+c))+2B\sqrt{-a}\,a\operatorname{sgn}(\cos(dx+c))\right)\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)-\left(3A\sqrt{-a}\,a\operatorname{sgn}(\cos(dx+c))+2B\sqrt{-a}\,a\operatorname{sgn}(\cos(dx+c))\right)\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+a\left(2\sqrt{2}-3\right)\right|\right)+\frac{4\left(3\sqrt{2}\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2A\sqrt{-a}\,a^2\operatorname{sgn}(\cos(dx+c))-\sqrt{2}A\sqrt{-a}\,a^3\operatorname{sgn}(\cos(dx+c))\right)}{\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2a+a^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] -1/2*(4*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*B*a^2*sgn(cos(d*x + c))
*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + (3*A*sqrt(-a)*a*sgn(
cos(d*x + c)) + 2*B*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))
) - (3*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 2*B*sqrt(-a)*a*sgn(cos(d*x + c)))*l
og(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)
)^2 + a*(2*sqrt(2) - 3))) + 4*(3*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) - sq
rt(2)*A*sqrt(-a)*a^3*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)
```

3.132 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=119

$$\frac{a^{3/2}(7A+12B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{a^2(5A+4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{aA\cos(c+dx)\sqrt{a+a\sec(c+dx)}\ldots}{2d}$$

```
[Out] 1/4*a^(3/2)*(7*A+12*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+
1/4*a^2*(5*A+4*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a*A*cos(d*x+c)*si
n(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.19, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4102, 4100, 3859, 209}

$$\frac{a^{3/2}(7A+12B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a^2(5A+4B)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{aA\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(3/2)*(7*A + 12*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(4*d) + (a^2*(5*A + 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) +
 (a*A*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
```

```
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx}{2d}$$

$$= \frac{a^2(5A+4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{aA\cos(c+dx)\sqrt{}}{}$$

$$= \frac{a^2(5A+4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{aA\cos(c+dx)\sqrt{}}{}$$

$$= \frac{a^{3/2}(7A+12B)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}$$

Mathematica [A]
time = 0.60, size = 111, normalized size = 0.93

$$\frac{a\sqrt{\cos(c+dx)}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{2}\,(7A+12B)\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sqrt{\cos(c+dx)}\,(7A+4B+2A\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*
(7*A + 12*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sqrt[Cos[c + d*x]]*(7*A +
 4*B + 2*A*Cos[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(103) = 206$.
time = 7.63, size = 399, normalized size = 3.35

method	result
default	$\frac{\left(7A\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sin(dx+c)\cos(dx+c)\sqrt{2}+12B\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctan}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/16/d*(7*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)*2^(1/2)+12*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)*2^(1/2)+7*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)+12*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)-8*A*cos(d*x+c)^4-20*A*cos(d*x+c)^3-16*B*cos(d*x+c)^3+28*A*cos(d*x+c)^2+16*B*cos(d*x+c)^2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)*a

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] Timed out

Fricas [A]
time = 2.90, size = 320, normalized size = 2.69

$$\left[\frac{\left((7A+12B)a\cos(dx+c)+(7A+12B)a\right)\sqrt{-a}\,\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2Aa\cos(dx+c)^2+(7A+4B)a\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{8\left(d\cos(dx+c)+d\right)},\ -\frac{\left((7A+12B)a\cos(dx+c)+(7A+12B)a\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(2Aa\cos(dx+c)^2+(7A+4B)a\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{4\left(d\cos(dx+c)+d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] [1/8*(((7*A + 12*B)*a*cos(d*x + c) + (7*A + 12*B)*a)*sqrt(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c

```
)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*A*a*cos(d*x
 + c)^2 + (7*A + 4*B)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/4*(((7*A + 12*B)*a*cos(d*x + c) +
 (7*A + 12*B)*a)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos
(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*A*a*cos(d*x + c)^2 + (7*A + 4*B)*a*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d
*x + c) + d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 639 vs. $2(103) = 206$.
time = 1.80, size = 639, normalized size = 5.37

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/8*((7*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 12*B*sqrt(-a)*a*sgn(cos(d*x + c))
)*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2 - a*(2*sqrt(2) + 3))) - (7*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 12*B*sqr
t(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(7*(sqrt(
-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a
)*a^2*sgn(cos(d*x + c)) + 12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 95*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^3
*sgn(cos(d*x + c)) - 76*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 53*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^4*sgn(
cos(d*x + c)) + 36*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1
/2*c)^2 + a))^2*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 5*A*sqrt(-a)*a^5*sgn(cos
(d*x + c)) - 4*B*sqrt(-a)*a^5*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^2\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)
```

3.133 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=164

$$\frac{a^{3/2}(11A+14B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{a^2(11A+14B)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{a^2(7A+6B)\cos(c+dx)\sin}{12d\sqrt{a+a\sec(c+dx}}$$

```
[Out] 1/8*a^(3/2)*(11*A+14*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d
+1/8*a^2*(11*A+14*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/12*a^2*(7*A+6*B)
*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/3*a*A*cos(d*x+c)^2*sin(d*
x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.24, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4102, 4100, 3890, 3859, 209}

$$\frac{a^{3/2}(11A+14B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^2(11A+14B)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(7A+6B)\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{aA\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(3/2)*(11*A + 14*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(8*d) + (a^2*(11*A + 14*B)*Sin[c + d*x])/(8*d*Sqrt[a + a*Sec[c + d*x]]
) + (a^2*(7*A + 6*B)*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a + a*Sec[c + d*
x]]) + (a*A*Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
```

```
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos^2(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+$$

$$= \frac{a^2(7A+6B)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{aA\,c}{}$$

$$= \frac{a^2(11A+14B)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(7A+6B)}{12d\sqrt{a}}$$

$$= \frac{a^2(11A+14B)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(7A+6B)}{12d\sqrt{a}}$$

$$= \frac{a^{3/2}(11A+14B)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx}}\right.}{8d}$$

Mathematica [A]

time = 0.97, size = 137, normalized size = 0.84

$$\frac{a\cos(c+dx)\sqrt{a(1+\sec(c+dx))}\left((37A+42B+2(11A+6B)\cos(c+dx)+4A\cos(2(c+dx)))\sqrt{1-\sec(c+dx)}\sin(c+dx)+3(11A+14B)\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)\right)}{24d(1+\cos(c+dx))\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*((37*A + 42*B + 2*(11*A + 6*B)*C
os[c + d*x] + 4*A*Cos[2*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] + 3
*(11*A + 14*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]*Tan[c + d*x]))/(24*d*(1 + Co
s[c + d*x])*Sqrt[1 - Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 580 vs. $2(144) = 288$.
time = 7.70, size = 581, normalized size = 3.54

method	result
default	$-\frac{\left(33A\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}(\cos^2(dx+c))\sin(dx+c)+42B\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/192/d*(33*A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)
^2*sin(d*x+c)+42*B*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*
x+c)^2*sin(d*x+c)+66*A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*co
s(d*x+c)*sin(d*x+c)+84*B*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*
cos(d*x+c)*sin(d*x+c)+33*A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2
)*sin(d*x+c)+42*B*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x
+c)+64*A*cos(d*x+c)^6+112*A*cos(d*x+c)^5+96*B*cos(d*x+c)^5+88*A*cos(d*x+c)^
4+240*B*cos(d*x+c)^4-264*A*cos(d*x+c)^3-336*B*cos(d*x+c)^3)*(a*(1+cos(d*x+c
))/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)*a
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 4.21, size = 360, normalized size = 2.20

$$\left[\frac{3\,\left((11\,A+14\,B)a\cos\left(dx+c\right)+(11\,A+14\,B)a\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos\left(dx+c\right)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)+a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)+1}\right)+2\,\left(8\,Aa\cos\left(dx+c\right)^3+2\,(11\,A+6\,B)a\cos\left(dx+c\right)^2+3\,(11\,A+14\,B)a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{48\,\left(d\cos\left(dx+c\right)+d\right)},-\frac{3\,\left((11\,A+14\,B)a\cos\left(dx+c\right)+(11\,A+14\,B)a\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)}{\sqrt{a}\sin\left(dx+c\right)}\right)-\left(8\,Aa\cos\left(dx+c\right)^3+2\,(11\,A+6\,B)a\cos\left(dx+c\right)^2+3\,(11\,A+14\,B)a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{24\,\left(d\cos\left(dx+c\right)+d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [1/48*(3*((11*A + 14*B)*a*cos(d*x + c) + (11*A + 14*B)*a)*sqrt(-a)*log((2*a
*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*
x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*A*a*co
s(d*x + c)^3 + 2*(11*A + 6*B)*a*cos(d*x + c)^2 + 3*(11*A + 14*B)*a*cos(d*x
+ c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
 + d), -1/24*(3*((11*A + 14*B)*a*cos(d*x + c) + (11*A + 14*B)*a)*sqrt(a)*ar
ctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
+ c))) - (8*A*a*cos(d*x + c)^3 + 2*(11*A + 6*B)*a*cos(d*x + c)^2 + 3*(11*A
+ 14*B)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c) + d)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 897 vs. 2(144) = 288.
time = 1.92, size = 897, normalized size = 5.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/48*(3*(11*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 14*B*sqrt(-a)*a*sgn(cos(d*x +
 c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(11*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 1
4*B*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*(33*sqrt(2
)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*
A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 42*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a^2*sgn(cos(d*x + c)
) - 303*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a))^8*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 822*sqrt(2)*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^3*sgn
(cos(d*x + c)) + 2394*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 3780*sqrt(2)*
(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*s
qrt(-a)*a^4*sgn(cos(d*x + c)) - 1806*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^5*sgn(cos(d*x + c))
- 2508*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^4*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 309*sqrt(2)*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^6*sgn(
cos(d*x + c)) + 498*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 19*sqrt(2)*A*sq
rt(-a)*a^7*sgn(cos(d*x + c)) - 30*sqrt(2)*B*sqrt(-a)*a^7*sgn(cos(d*x + c)))
/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 -
 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*
a + a^2)^3)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^3\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)
```

3.134 $\int \cos^4(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=209

$$\frac{a^{3/2}(75A+88B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d}+\frac{a^2(75A+88B)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}}+\frac{a^2(75A+88B)\cos(c+dx}{96d\sqrt{a+a\sec(c+}$$

```
[Out] 1/64*a^(3/2)*(75*A+88*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/
d+1/64*a^2*(75*A+88*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/96*a^2*(75*A+8
8*B)*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/24*a^2*(9*A+8*B)*cos(
d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/4*a*A*cos(d*x+c)^3*sin(d*x+c
)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.31, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4102, 4100, 3890, 3859, 209}

$$\frac{a^{3/2}(75A+88B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^2(75A+88B)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(9A+8B)\sin(c+dx)\cos^2(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(75A+88B)\sin(c+dx)\cos(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+\frac{aA\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(3/2)*(75*A + 88*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(64*d) + (a^2*(75*A + 88*B)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x
]]) + (a^2*(75*A + 88*B)*Cos[c + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c
+ d*x]]) + (a^2*(9*A + 8*B)*Cos[c + d*x]^2*Sin[c + d*x])/(24*d*Sqrt[a + a*S
ec[c + d*x]]) + (a*A*Cos[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/
(4*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos^3(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+$$

$$= \frac{a^2(9A+8B)\cos^2(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{aA$$

$$= \frac{a^2(75A+88B)\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{a^2$$

$$= \frac{a^2(75A+88B)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(75A+88B}{96d\sqrt{}}$$

$$= \frac{a^2(75A+88B)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(75A+88B}{96d\sqrt{}}$$

$$= \frac{a^{3/2}(75A+88B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx}}\right.}{64d}$$

Mathematica [**A**]
time = 1.28, size = 154, normalized size = 0.74

$$\frac{a\cos(c+dx)\sqrt{a(1+\sec(c+dx))}\left((285A+296B+2(93A+88B)\cos(c+dx)+4(15A+8B)\cos(2(c+dx))+12A\cos(3(c+dx)))\sqrt{1-\sec(c+dx)}\sin(c+dx)+3(75A+88B)\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)\right)}{192d(1+\cos(c+dx))\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*((285*A + 296*B + 2*(93*A + 88*B
)*Cos[c + d*x] + 4*(15*A + 8*B)*Cos[2*(c + d*x)] + 12*A*Cos[3*(c + d*x)])*S
qrt[1 - Sec[c + d*x]]*Sin[c + d*x] + 3*(75*A + 88*B)*ArcTanh[Sqrt[1 - Sec[c
 + d*x]]]*Tan[c + d*x]))/(192*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 762 vs. 2(185) = 370.
time = 7.51, size = 763, normalized size = 3.65

method	result
default	$\left(225A\sqrt{2}\,(\cos^3(dx+c))\sin(dx+c)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)+264B\sqrt{2}\,(\cos^3(dx+c)\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/3072/d*(225*A*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d
*x+c)*2^(1/2))+264*B*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/
cos(d*x+c)*2^(1/2))+675*A*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)/cos(d*x+c)*2^(1/2))+792*B*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)/cos(d*x+c)*2^(1/2))+675*A*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+792*B*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+225*A*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos
(d*x+c)*2^(1/2))*sin(d*x+c)+264*B*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7
/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*
2^(1/2))*sin(d*x+c)-768*A*cos(d*x+c)^8-1152*A*cos(d*x+c)^7-1024*B*cos(d*x+c
)^7-480*A*cos(d*x+c)^6-1792*B*cos(d*x+c)^6-1200*A*cos(d*x+c)^5-1408*B*cos(d
*x+c)^5+3600*A*cos(d*x+c)^4+4224*B*cos(d*x+c)^4)*(a*(1+cos(d*x+c))/cos(d*x+
c))^(1/2)/sin(d*x+c)/cos(d*x+c)^3*a
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 4.08, size = 396, normalized size = 1.89

$$\left[\frac{3\left((75A+88B)a\cos(dx+c)+(75A+88B)a\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(48Aa\cos(dx+c)^4+8(15A+8B)a\cos(dx+c)^3+2(75A+88B)a\cos(dx+c)^2+3(75A+88B)a\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{384\left(d\cos(dx+c)+d\right)}, -\frac{3\left((75A+88B)a\cos(dx+c)+(75A+88B)a\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(48Aa\cos(dx+c)^4+8(15A+8B)a\cos(dx+c)^3+2(75A+88B)a\cos(dx+c)^2+3(75A+88B)a\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{192\left(d\cos(dx+c)+d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [1/384*(3*((75*A + 88*B)*a*cos(d*x + c) + (75*A + 88*B)*a)*sqrt(-a)*log((2*
a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d
```

```
*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(48*A*a*
cos(d*x + c)^4 + 8*(15*A + 8*B)*a*cos(d*x + c)^3 + 2*(75*A + 88*B)*a*cos(d*
x + c)^2 + 3*(75*A + 88*B)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/192*(3*((75*A + 88*B)*a*cos(
d*x + c) + (75*A + 88*B)*a)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (48*A*a*cos(d*x + c)^4 + 8*(
15*A + 8*B)*a*cos(d*x + c)^3 + 2*(75*A + 88*B)*a*cos(d*x + c)^2 + 3*(75*A +
 88*B)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c) + d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1088 vs. 2(185) = 370.
time = 2.04, size = 1088, normalized size = 5.21

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/384*(3*(75*A*sqrt(-a)*a*sgn(cos(d*x + c)) + 88*B*sqrt(-a)*a*sgn(cos(d*x
+ c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(75*A*sqrt(-a)*a*sgn(cos(d*x + c)) +
88*B*sqrt(-a)*a*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(
225*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^1
4*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 264*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*B*sqrt(-a)*a^2*sgn(cos(d*x + c)) - 6
261*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^1
2*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 4008*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) +
35925*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
^10*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 33960*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a^4*sgn(cos(d*x + c))
 - 127449*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^8*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 131784*(sqrt(-a)*tan(1/2*d*x + 1/
```

```
2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^5*sgn(cos(d*x +
c)) + 101667*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^6*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 108312*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^6*sgn(cos(d*x
 + c)) - 26079*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^4*A*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 29432*(sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^7*sgn(cos(d*
x + c)) + 3303*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2*A*sqrt(-a)*a^8*sgn(cos(d*x + c)) + 3384*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^8*sgn(cos(d*x
 + c)) - 147*A*sqrt(-a)*a^9*sgn(cos(d*x + c)) - 152*B*sqrt(-a)*a^9*sgn(cos(
d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^2*a + a^2)^4)/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+dx)^4\left(A+\frac{B}{\cos(c+dx)}\right)\left(a+\frac{a}{\cos(c+dx)}\right)^{3/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)
```

3.135 $\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=237

$$\frac{2a^3(803A+710B)\tan(c+dx)}{495d\sqrt{a+a\sec(c+dx)}}+\frac{2a^3(209A+194B)\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}-\frac{4a^2(803A+710B)\sqrt{a+a\,\mathrm{s}}}{3465}$$

```
[Out] 2/1155*a*(803*A+710*B)*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/11*a*B*sec(d*x
+c)^3*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/495*a^3*(803*A+710*B)*tan(d*x+c
)/d/(a+a*sec(d*x+c))^(1/2)+2/693*a^3*(209*A+194*B)*sec(d*x+c)^3*tan(d*x+c)/
d/(a+a*sec(d*x+c))^(1/2)-4/3465*a^2*(803*A+710*B)*(a+a*sec(d*x+c))^(1/2)*ta
n(d*x+c)/d+2/99*a^2*(11*A+14*B)*sec(d*x+c)^3*(a+a*sec(d*x+c))^(1/2)*tan(d*x
+c)/d
```

Rubi [A]
time = 0.46, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4103, 4101, 3885, 4086, 3877}

$$\frac{2a^3(209A+194B)\tan(c+dx)\sec^3(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(803A+710B)\tan(c+dx)}{495d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(11A+14B)\tan(c+dx)\sec^3(c+dx)\sqrt{a\sec(c+dx)+a}}{99d}-\frac{4a^2(803A+710B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3465d}+\frac{2a(803A+710B)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{1155d}+\frac{2aB\tan(c+dx)\sec^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^3*(803*A + 710*B)*Tan[c + d*x])/(495*d*Sqrt[a + a*Sec[c + d*x]]) + (2*
a^3*(209*A + 194*B)*Sec[c + d*x]^3*Tan[c + d*x])/(693*d*Sqrt[a + a*Sec[c +
d*x]]) - (4*a^2*(803*A + 710*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(346
5*d) + (2*a^2*(11*A + 14*B)*Sec[c + d*x]^3*Sqrt[a + a*Sec[c + d*x]]*Tan[c +
 d*x])/(99*d) + (2*a*(803*A + 710*B)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x
])/(1155*d) + (2*a*B*Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x]
)/(11*d)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3885

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_),
x_Symbol] :> Simp[(-Cot[e + f*x])*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2
))), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*(b*(m
 + 1) - a*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f, m}, x] && EqQ[a^2 - b
```

```
^2, 0] && !LtQ[m, -2^(-1)]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] && !LtQ[m, -2^(-1)]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && !
LtQ[n, 0]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && !LtQ[n, -1]
```

Rubi steps

$$\int \sec^3(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{2aB\sec^3(c+dx)(a+a\sec(c+dx))^{3/2}\tan(c}{11d}$$

$$= \frac{2a^2(11A+14B)\sec^3(c+dx)\sqrt{a+a\sec(c+}}{99d}$$

$$= \frac{2a^3(209A+194B)\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^3(209A+194B)\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^3(209A+194B)\sec^3(c+dx)\tan(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^3(803A+710B)\tan(c+dx)}{495d\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(209A}{6}$$

Mathematica [A]
time = 5.84, size = 117, normalized size = 0.49

$$\frac{2a^3(8(803A+710B)+4(803A+710B)\sec(c+dx)+3(803A+710B)\sec^2(c+dx)+5(286A+355B)\sec^3(c+dx)+35(11A+32B)\sec^4(c+dx)+315B\sec^5(c+dx))\tan(c+dx)}{3465d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
[Out] (2*a^3*(8*(803*A + 710*B) + 4*(803*A + 710*B)*Sec[c + d*x] + 3*(803*A + 710
*B)*Sec[c + d*x]^2 + 5*(286*A + 355*B)*Sec[c + d*x]^3 + 35*(11*A + 32*B)*Se
c[c + d*x]^4 + 315*B*Sec[c + d*x]^5)*Tan[c + d*x])/(3465*d*Sqrt[a*(1 + Sec[
c + d*x])])
```

Maple [A]
time = 3.96, size = 163, normalized size = 0.69

method	result
default	$-\frac{2(-1+\cos(dx+c))(6424A(\cos^5(dx+c))+5680B(\cos^5(dx+c))+3212A(\cos^4(dx+c))+2840B(\cos^4(dx+c))+2409A(\cos^3(dx+c))+}{3465d\cos(d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
[Out] -2/3465/d*(-1+cos(d*x+c))*(6424*A*cos(d*x+c)^5+5680*B*cos(d*x+c)^5+3212*A*c
os(d*x+c)^4+2840*B*cos(d*x+c)^4+2409*A*cos(d*x+c)^3+2130*B*cos(d*x+c)^3+143
```

```
0*A*cos(d*x+c)^2+1775*B*cos(d*x+c)^2+385*A*cos(d*x+c)+1120*B*cos(d*x+c)+315
*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^5/sin(d*x+c)*a^2
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 4.24, size = 157, normalized size = 0.66

$$\frac{2\left(8\,(803\,A+710\,B)a^2\cos\left(dx+c\right)^5+4\,(803\,A+710\,B)a^2\cos\left(dx+c\right)^4+3\,(803\,A+710\,B)a^2\cos\left(dx+c\right)^3+5\,(286\,A+355\,B)a^2\cos\left(dx+c\right)^2+35\,(11\,A+32\,B)a^2\cos\left(dx+c\right)+315\,Ba^2\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{3465\left(d\cos\left(dx+c\right)^6+d\cos\left(dx+c\right)^5\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/3465*(8*(803*A + 710*B)*a^2*cos(d*x + c)^5 + 4*(803*A + 710*B)*a^2*cos(d*
x + c)^4 + 3*(803*A + 710*B)*a^2*cos(d*x + c)^3 + 5*(286*A + 355*B)*a^2*cos
(d*x + c)^2 + 35*(11*A + 32*B)*a^2*cos(d*x + c) + 315*B*a^2)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5
)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{Exception raised: SystemError}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [**A**]
time = 1.37, size = 306, normalized size = 1.29

$$\frac{8\left(\left(\left(\left(4\left(2\sqrt{2}\,(143\,Aa^8\mathrm{sgn}(\cos\left(dx+c\right))+125\,Ba^8\mathrm{sgn}(\cos\left(dx+c\right)))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-11\sqrt{2}\,(143\,Aa^8\mathrm{sgn}(\cos\left(dx+c\right))+125\,Ba^8\mathrm{sgn}(\cos\left(dx+c\right)))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+99\sqrt{2}\,(143\,Aa^8\mathrm{sgn}(\cos\left(dx+c\right))+125\,Ba^8\mathrm{sgn}(\cos\left(dx+c\right)))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-231\sqrt{2}\,(69\,Aa^8\mathrm{sgn}(\cos\left(dx+c\right))+65\,Ba^8\mathrm{sgn}(\cos\left(dx+c\right)))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1155\sqrt{2}\,(9\,Aa^8\mathrm{sgn}(\cos\left(dx+c\right))+7\,Ba^8\mathrm{sgn}(\cos\left(dx+c\right)))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-3465\sqrt{2}\,(Aa^8\mathrm{sgn}(\cos\left(dx+c\right))+Ba^8\mathrm{sgn}(\cos\left(dx+c\right)))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{3465\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^5\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] 8/3465*((((4*(2*sqrt(2)*(143*A*a^8*sgn(cos(d*x + c)) + 125*B*a^8*sgn(cos(d*
x + c)))*tan(1/2*d*x + 1/2*c)^2 - 11*sqrt(2)*(143*A*a^8*sgn(cos(d*x + c)) +
 125*B*a^8*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 99*sqrt(2)*(143*A*a
^8*sgn(cos(d*x + c)) + 125*B*a^8*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2
 - 231*sqrt(2)*(69*A*a^8*sgn(cos(d*x + c)) + 65*B*a^8*sgn(cos(d*x + c))))*t
an(1/2*d*x + 1/2*c)^2 + 1155*sqrt(2)*(9*A*a^8*sgn(cos(d*x + c)) + 7*B*a^8*s
gn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 3465*sqrt(2)*(A*a^8*sgn(cos(d*x
 + c)) + B*a^8*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1
/2*c)^2 - a)^5*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)
```

Mupad [B]
time = 13.51, size = 856, normalized size = 3.61

$$\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(\frac{A\,a^2\,8i}{3\,d}-\frac{a^2\,e^{c\,1i+d\,x\,1i}\,\left(803\,A+710\,B\right)\,8i}{3465\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\,\left(e^{c\,2i+d\,x\,2i}+1\right)}-\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(-e^{c\,1i+d\,x\,1i}\,\left(-\frac{a^2\,\left(5\,A+2\,B\right)\,8i}{7\,d}+\frac{a^2\,\left(5\,A+16\,B\right)\,8i}{7\,d}+\frac{a^2\,\left(11\,A+50\,B\right)\,32i}{693\,d}\right)+\frac{A\,a^2\,24i}{7\,d}+\frac{a^2\,\left(9\,A+10\,B\right)\,8i}{7\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^3}+\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(e^{c\,1i+d\,x\,1i}\,\left(\frac{A\,a^2\,8i}{11\,d}+\frac{a^2\,\left(3\,A+4\,B\right)\,40i}{11\,d}-\frac{a^2\,\left(5\,A+2\,B\right)\,8i}{11\,d}-\frac{a^2\,\left(11\,A+10\,B\right)\,8i}{11\,d}\right)+\frac{A\,a^2\,8i}{11\,d}+\frac{a^2\,\left(3\,A+4\,B\right)\,40i}{11\,d}-\frac{a^2\,\left(5\,A+2\,B\right)\,8i}{11\,d}-\frac{a^2\,\left(11\,A+10\,B\right)\,8i}{11\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^5}-\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(-e^{c\,1i+d\,x\,1i}\,\left(\frac{a^2\,\left(A-8\,B\right)\,8i}{9\,d}-\frac{B\,a^2\,64i}{99\,d}+\frac{a^2\,\left(5\,A+2\,B\right)\,8i}{9\,d}-\frac{a^2\,\left(5\,A+9\,B\right)\,16i}{9\,d}\right)+\frac{A\,a^2\,8i}{9\,d}+\frac{a^2\,\left(A+2\,B\right)\,40i}{9\,d}+\frac{B\,a^2\,64i}{9\,d}-\frac{a^2\,\left(A+B\right)\,80i}{9\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^4}+\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(e^{c\,1i+d\,x\,1i}\,\left(\frac{a^2\,\left(5\,A+2\,B\right)\,8i}{5\,d}+\frac{a^2\,\left(44\,A-31\,B\right)\,16i}{1155\,d}\right)-\frac{A\,a^2\,8i}{5\,d}+\frac{a^2\,\left(4\,A+5\,B\right)\,16i}{5\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\,{\left(e^{c\,2i+d\,x\,2i}+1\right)}^2}-\frac{a^2\,e^{c\,1i+d\,x\,1i}\,\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\,\left(803\,A+710\,B\right)\,16i}{3465\,d\,\left(e^{c\,1i+d\,x\,1i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/cos(c + d*x)^3,x)
```

```
[Out] ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((A*a^2*8i)/
(3*d) - (a^2*exp(c*1i + d*x*1i)*(803*A + 710*B)*8i)/(3465*d)))/((exp(c*1i +
 d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)) - ((a + a/(exp(- c*1i - d*x*1i)/2 +
 exp(c*1i + d*x*1i)/2))^(1/2)*((A*a^2*24i)/(7*d) - exp(c*1i + d*x*1i)*((a^2
*(5*A + 16*B)*8i)/(7*d) - (a^2*(5*A + 2*B)*8i)/(7*d) + (a^2*(11*A + 50*B)*3
2i)/(693*d)) + (a^2*(9*A + 10*B)*8i)/(7*d)))/((exp(c*1i + d*x*1i) + 1)*(exp
(c*2i + d*x*2i) + 1)^3) + ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*
1i)/2))^(1/2)*(exp(c*1i + d*x*1i)*((A*a^2*8i)/(11*d) + (a^2*(3*A + 4*B)*40i
)/(11*d) - (a^2*(5*A + 2*B)*8i)/(11*d) - (a^2*(11*A + 10*B)*8i)/(11*d)) + (
A*a^2*8i)/(11*d) + (a^2*(3*A + 4*B)*40i)/(11*d) - (a^2*(5*A + 2*B)*8i)/(11*
d) - (a^2*(11*A + 10*B)*8i)/(11*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i +
d*x*2i) + 1)^5) - ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^
(1/2)*((A*a^2*8i)/(9*d) - exp(c*1i + d*x*1i)*((a^2*(A - 8*B)*8i)/(9*d) - (B
*a^2*64i)/(99*d) + (a^2*(5*A + 2*B)*8i)/(9*d) - (a^2*(5*A + 9*B)*16i)/(9*d)
) + (a^2*(A + 2*B)*40i)/(9*d) + (B*a^2*64i)/(9*d) - (a^2*(A + B)*80i)/(9*d)
))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^4) + ((a + a/(exp(- c
*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i)*((a^2*(5
*A + 2*B)*8i)/(5*d) + (a^2*(44*A - 31*B)*16i)/(1155*d)) - (A*a^2*8i)/(5*d)
+ (a^2*(4*A + 5*B)*16i)/(5*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2
i) + 1)^2) - (a^2*exp(c*1i + d*x*1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c
*1i + d*x*1i)/2))^(1/2)*(803*A + 710*B)*16i)/(3465*d*(exp(c*1i + d*x*1i) +
1))
```

3.136 $\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=175

$$\frac{64a^3(15A+13B)\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}+\frac{16a^2(15A+13B)\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{315d}+\frac{2a(15A+13B)(a+a\,\mathrm{se}}{10}$$

```
[Out] 2/105*a*(15*A+13*B)*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/63*(9*A-2*B)*(a+a
*sec(d*x+c))^(5/2)*tan(d*x+c)/d+2/9*B*(a+a*sec(d*x+c))^(7/2)*tan(d*x+c)/a/d
+64/315*a^3*(15*A+13*B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+16/315*a^2*(15*
A+13*B)*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.24, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4095, 4086, 3878, 3877}

$$\frac{64a^3(15A+13B)\tan(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(15A+13B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{315d}+\frac{2(9A-2B)\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{63d}+\frac{2a(15A+13B)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{105d}+\frac{2B\tan(c+dx)(a\sec(c+dx)+a)^{7/2}}{9ad}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (64*a^3*(15*A + 13*B)*Tan[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (16*
a^2*(15*A + 13*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(315*d) + (2*a*(15
*A + 13*B)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(105*d) + (2*(9*A - 2*B
)*(a + a*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*d) + (2*B*(a + a*Sec[c + d*x
])^(7/2)*Tan[c + d*x])/(9*a*d)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3878

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[a*((2*m - 1)/m), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx &= \frac{2B(a+a\sec(c+dx))^{7/2}\tan(c+dx)}{9ad}+\frac{2\int \\ &= \frac{2(9A-2B)(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{63d} \\ &= \frac{2a(15A+13B)(a+a\sec(c+dx))^{3/2}\tan(c}{105d} \\ &= \frac{16a^2(15A+13B)\sqrt{a+a\sec(c+dx)}\,\tan(c}{315d} \\ &= \frac{64a^3(15A+13B)\tan(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}+\frac{16a^2(15A}{}\end{aligned}$$

Mathematica [A]
time = 0.65, size = 96, normalized size = 0.55

$$\frac{2a^3(690A+584B+(345A+292B)\sec(c+dx)+3(60A+73B)\sec^2(c+dx)+5(9A+26B)\sec^3(c+dx)+35B\sec^4(c+dx))\tan(c+dx)}{315d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^3*(690*A + 584*B + (345*A + 292*B)*Sec[c + d*x] + 3*(60*A + 73*B)*Sec[
c + d*x]^2 + 5*(9*A + 26*B)*Sec[c + d*x]^3 + 35*B*Sec[c + d*x]^4)*Tan[c + d
*x])/(315*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 4.25, size = 141, normalized size = 0.81

method	result
default	$-\frac{2(-1+\cos(dx+c))(690A(\cos^4(dx+c))+584B(\cos^4(dx+c))+345A(\cos^3(dx+c))+292B(\cos^3(dx+c))+180A(\cos^2(dx+c))+219B(\text{c}}{315d\cos(dx+c)^4\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(690*A*cos(d*x+c)^4+584*B*cos(d*x+c)^4+345*A*cos(d
*x+c)^3+292*B*cos(d*x+c)^3+180*A*cos(d*x+c)^2+219*B*cos(d*x+c)^2+45*A*cos(d
*x+c)+130*B*cos(d*x+c)+35*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)
^4/sin(d*x+c)*a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] 4/315*(315*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
1)^(1/4)*((A*a^2*d*cos(2*d*x + 2*c)^4 + A*a^2*d*sin(2*d*x + 2*c)^4 + 4*A*a^
2*d*cos(2*d*x + 2*c)^3 + 6*A*a^2*d*cos(2*d*x + 2*c)^2 + 4*A*a^2*d*cos(2*d*x
 + 2*c) + A*a^2*d + 2*(A*a^2*d*cos(2*d*x + 2*c)^2 + 2*A*a^2*d*cos(2*d*x + 2
*c) + A*a^2*d)*sin(2*d*x + 2*c)^2)*integrate((((cos(8*d*x + 8*c)*cos(2*d*x
+ 2*c) + 3*cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 3*cos(4*d*x + 4*c)*cos(2*d*x
 + 2*c) + cos(2*d*x + 2*c)^2 + sin(8*d*x + 8*c)*sin(2*d*x + 2*c) + 3*sin(6*
d*x + 6*c)*sin(2*d*x + 2*c) + 3*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d
*x + 2*c)^2)*cos(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*
d*x + 2*c)*sin(8*d*x + 8*c) + 3*cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 3*cos(2
*d*x + 2*c)*sin(4*d*x + 4*c) - cos(8*d*x + 8*c)*sin(2*d*x + 2*c) - 3*cos(6*
d*x + 6*c)*sin(2*d*x + 2*c) - 3*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*sin(9/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(8*d*x + 8*c) + 3*cos(2
*d*x + 2*c)*sin(6*d*x + 6*c) + 3*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(8*
d*x + 8*c)*sin(2*d*x + 2*c) - 3*cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 3*cos(4
*d*x + 4*c)*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - (cos(8*d*x + 8*c)*cos(2*d*x + 2*c) + 3*cos(6*d*x + 6*c)*cos(2*d*x
 + 2*c) + 3*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(8*
```

d*x + 8*c)*sin(2*d*x + 2*c) + 3*sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 3*sin(4 *d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(9/2*arctan2(sin(2*d* x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))/(((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c) ^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(8*d*x + 8*c)^2 + 9*(c os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 9*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(8*d*x + 8*c)^2 + 9*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 9*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 3* (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d* x + 6*c) + 3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 3*(c os(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 6*(cos (2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^ 2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c) + 3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2* c) + 1)*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 6*(sin(2*d*x + 2*c)^3 + 3*(cos (2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + 6*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))*cos(5/2*arctan2(sin(2*d*x + 2 *c), cos(2*d*x + 2*c) + 1))^2 + (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(8*d* x + 8*c)^2 + 9*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 + 9*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2* cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d* x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(8*d*x + 8*c)^ 2 + 9*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*si n(6*d*x + 6*c)^2 + 9*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2* d*x + 2*c)^2 + 3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2 *c) + 1)*cos(6*d*x + 6*c) + 3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2* cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(8*d*x + 8*c) + 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d* x + 2*c)^2 + 3*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*

```
x + 6*c) + 6*(cos(2*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^
2 + 2*(sin(2*d*x + 2*c)^3 + 3*(cos(2*d*x + 2*c)...
```

Fricas [A]
time = 3.41, size = 136, normalized size = 0.78

$$\frac{2\,\left(2\,(345\,A+292\,B)a^{2}\cos\left(dx+c\right)^{4}+(345\,A+292\,B)a^{2}\cos\left(dx+c\right)^{3}+3\,(60\,A+73\,B)a^{2}\cos\left(dx+c\right)^{2}+5\,(9\,A+26\,B)a^{2}\cos\left(dx+c\right)+35\,Ba^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{315\,\left(d\cos\left(dx+c\right)^{5}+d\cos\left(dx+c\right)^{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 2/315*(2*(345*A + 292*B)*a^2*cos(d*x + c)^4 + (345*A + 292*B)*a^2*cos(d*x +
 c)^3 + 3*(60*A + 73*B)*a^2*cos(d*x + c)^2 + 5*(9*A + 26*B)*a^2*cos(d*x + c
) + 35*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/(d*cos(d
*x + c)^5 + d*cos(d*x + c)^4)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.47, size = 261, normalized size = 1.49

$$\frac{8\left(\left(\left(4\left(2\sqrt{2}\,(15\,Aa^{7}\mathrm{sgn}(\cos\left(dx+c\right))+13\,Ba^{7}\mathrm{sgn}(\cos\left(dx+c\right)))\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-9\sqrt{2}\,(15\,Aa^{7}\mathrm{sgn}(\cos\left(dx+c\right))+13\,Ba^{7}\mathrm{sgn}(\cos\left(dx+c\right)))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+63\sqrt{2}\,(15\,Aa^{7}\mathrm{sgn}(\cos\left(dx+c\right))+13\,Ba^{7}\mathrm{sgn}(\cos\left(dx+c\right)))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-210\sqrt{2}\,(4\,Aa^{7}\mathrm{sgn}(\cos\left(dx+c\right))+3\,Ba^{7}\mathrm{sgn}(\cos\left(dx+c\right)))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+315\sqrt{2}\,(Aa^{7}\mathrm{sgn}(\cos\left(dx+c\right))+Ba^{7}\mathrm{sgn}(\cos\left(dx+c\right)))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{315\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{4}\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] 8/315*(((4*(2*sqrt(2)*(15*A*a^7*sgn(cos(d*x + c)) + 13*B*a^7*sgn(cos(d*x +
c)))*tan(1/2*d*x + 1/2*c)^2 - 9*sqrt(2)*(15*A*a^7*sgn(cos(d*x + c)) + 13*B*
a^7*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 63*sqrt(2)*(15*A*a^7*sgn(c
os(d*x + c)) + 13*B*a^7*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 210*sq
rt(2)*(4*A*a^7*sgn(cos(d*x + c)) + 3*B*a^7*sgn(cos(d*x + c))))*tan(1/2*d*x
+ 1/2*c)^2 + 315*sqrt(2)*(A*a^7*sgn(cos(d*x + c)) + B*a^7*sgn(cos(d*x + c))
))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^4*sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a)*d)
```

Mupad [B]
time = 10.81, size = 723, normalized size = 4.13

$$\frac{\left({\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}\,\left(\frac{A\,a^2\,4{}\mathrm{i}}{3\,d}-\frac{a^2\,\left(60\,A+73\,B\right)\,8{}\mathrm{i}}{315\,d}\right)+\frac{a^2\,\left(5\,A+2\,B\right)\,4{}\mathrm{i}}{3\,d}\right)\,\sqrt{a+\frac{a}{\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}}{\left({\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}+1\right)\,\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}+\frac{\sqrt{a+\frac{a}{\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}\,\left({\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}\,\left(-\frac{A\,a^2\,4{}\mathrm{i}}{5\,d}+\frac{a^2\,\left(3\,A+4\,B\right)\,16{}\mathrm{i}}{105\,d}+\frac{a^2\,\left(9\,A+10\,B\right)\,4{}\mathrm{i}}{5\,d}\right)-\frac{a^2\,\left(5\,A+2\,B\right)\,4{}\mathrm{i}}{5\,d}+\frac{a^2\,\left(5\,A+16\,B\right)\,4{}\mathrm{i}}{5\,d}\right)}{\left({\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}+1\right)\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^2}+\frac{\sqrt{a+\frac{a}{\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}\,\left({\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}\,\left(\frac{A\,a^2\,4{}\mathrm{i}}{7\,d}+\frac{a^2\,\left(A+2\,B\right)\,20{}\mathrm{i}}{7\,d}+\frac{B\,a^2\,32{}\mathrm{i}}{63\,d}-\frac{a^2\,\left(A+B\right)\,40{}\mathrm{i}}{7\,d}\right)+\frac{a^2\,\left(A-8\,B\right)\,4{}\mathrm{i}}{7\,d}+\frac{a^2\,\left(5\,A+2\,B\right)\,4{}\mathrm{i}}{7\,d}-\frac{a^2\,\left(5\,A+9\,B\right)\,8{}\mathrm{i}}{7\,d}\right)}{\left({\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}+1\right)\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^3}-\frac{\sqrt{a+\frac{a}{\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}\,\left({\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}\,\left(\frac{A\,a^2\,4{}\mathrm{i}}{9\,d}+\frac{a^2\,\left(3\,A+4\,B\right)\,20{}\mathrm{i}}{9\,d}-\frac{a^2\,\left(5\,A+2\,B\right)\,4{}\mathrm{i}}{9\,d}-\frac{a^2\,\left(11\,A+10\,B\right)\,4{}\mathrm{i}}{9\,d}\right)-\frac{A\,a^2\,4{}\mathrm{i}}{9\,d}-\frac{a^2\,\left(3\,A+4\,B\right)\,20{}\mathrm{i}}{9\,d}+\frac{a^2\,\left(5\,A+2\,B\right)\,4{}\mathrm{i}}{9\,d}+\frac{a^2\,\left(11\,A+10\,B\right)\,4{}\mathrm{i}}{9\,d}\right)}{\left({\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}+1\right)\,{\left({\mathrm{e}}^{c\,2{}\mathrm{i}+d\,x\,2{}\mathrm{i}}+1\right)}^4}-\frac{a^2\,{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}\,\sqrt{a+\frac{a}{\frac{{\mathrm{e}}^{-c\,1{}\mathrm{i}-d\,x\,1{}\mathrm{i}}}{2}+\frac{{\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}}{2}}}\,\left(345\,A+292\,B\right)\,4{}\mathrm{i}}{315\,d\,\left({\mathrm{e}}^{c\,1{}\mathrm{i}+d\,x\,1{}\mathrm{i}}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/cos(c + d*x)^2,x)
```

```
[Out] ((exp(c*1i + d*x*1i)*((A*a^2*4i)/(3*d) - (a^2*(60*A + 73*B)*8i)/(315*d)) +
(a^2*(5*A + 2*B)*4i)/(3*d))*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x
*1i)/2))^(1/2))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)) + ((a +
 a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1
i)*((a^2*(3*A + 4*B)*16i)/(105*d) - (A*a^2*4i)/(5*d) + (a^2*(9*A + 10*B)*4i
)/(5*d)) - (a^2*(5*A + 2*B)*4i)/(5*d) + (a^2*(5*A + 16*B)*4i)/(5*d)))/((exp
(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) + ((a + a/(exp(- c*1i - d*
x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i)*((A*a^2*4i)/(7*d
) + (a^2*(A + 2*B)*20i)/(7*d) + (B*a^2*32i)/(63*d) - (a^2*(A + B)*40i)/(7*d
)) + (a^2*(A - 8*B)*4i)/(7*d) + (a^2*(5*A + 2*B)*4i)/(7*d) - (a^2*(5*A + 9*
B)*8i)/(7*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) - ((a
+ a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*
1i)*((A*a^2*4i)/(9*d) + (a^2*(3*A + 4*B)*20i)/(9*d) - (a^2*(5*A + 2*B)*4i)/
(9*d) - (a^2*(11*A + 10*B)*4i)/(9*d)) - (A*a^2*4i)/(9*d) - (a^2*(3*A + 4*B)
*20i)/(9*d) + (a^2*(5*A + 2*B)*4i)/(9*d) + (a^2*(11*A + 10*B)*4i)/(9*d)))/(
(exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^4) - (a^2*exp(c*1i + d*x*
1i)*(a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(345*A +
292*B)*4i)/(315*d*(exp(c*1i + d*x*1i) + 1))
```

3.137 $\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=138

$$\frac{64a^3(7A+5B)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}+\frac{16a^2(7A+5B)\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{105d}+\frac{2a(7A+5B)(a+a\sec(c+dx)}{35d}$$

```
[Out] 2/35*a*(7*A+5*B)*(a+a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/7*B*(a+a*sec(d*x+c))
^(5/2)*tan(d*x+c)/d+64/105*a^3*(7*A+5*B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2
)+16/105*a^2*(7*A+5*B)*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.13, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4086, 3878, 3877}

$$\frac{64a^3(7A+5B)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(7A+5B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2a(7A+5B)\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{35d}+\frac{2B\tan(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (64*a^3*(7*A + 5*B)*Tan[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (16*a^
2*(7*A + 5*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*a*(7*A +
5*B)*(a + a*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2*B*(a + a*Sec[c +
d*x])^(5/2)*Tan[c + d*x])/(7*d)
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3878

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[(-b)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m - 1)/(f*m)), x]
+ Dist[a*((2*m - 1)/m), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m - 1), x],
x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] && Integer
Q[2*m]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
```

```
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \sec(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{2B(a+a\sec(c+dx))^{5/2}\tan(c+dx)}{7d} + \frac{1}{7}(7A$$
$$= \frac{2a(7A+5B)(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{35d}$$
$$= \frac{16a^2(7A+5B)\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{105d}$$
$$= \frac{64a^3(7A+5B)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2(7A+5B}{}$$

Mathematica [A]
time = 0.51, size = 89, normalized size = 0.64

$$\frac{2a^2\sqrt{a(1+\sec(c+dx))}\,((301A+230B)\sin(c+dx)+(98A+115B+3(7A+20B)\sec(c+dx)+15B\sec^2(c+dx))\tan(c+dx))}{105d(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^2*Sqrt[a*(1 + Sec[c + d*x])]*((301*A + 230*B)*Sin[c + d*x] + (98*A + 1
15*B + 3*(7*A + 20*B)*Sec[c + d*x] + 15*B*Sec[c + d*x]^2)*Tan[c + d*x]))/(1
05*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 3.52, size = 119, normalized size = 0.86

method	result
default	$-\frac{2(-1+\cos(dx+c))(301A(\cos^3(dx+c))+230B(\cos^3(dx+c))+98A(\cos^2(dx+c))+115B(\cos^2(dx+c))+21A\cos(dx+c)+60B\cos(dx}{105d\cos(dx+c)^3\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(301*A*cos(d*x+c)^3+230*B*cos(d*x+c)^3+98*A*cos(d*
x+c)^2+115*B*cos(d*x+c)^2+21*A*cos(d*x+c)+60*B*cos(d*x+c)+15*B)*(a*(1+cos(d
*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(d*x+c)*a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] -2/105*((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(1/4)*(7*(15*A*a^2*sin(6*d*x + 6*c) + 5*(17*A + 10*B)*a^2*sin(4*d*x + 4*c)
+ (113*A + 100*B)*a^2*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1)) - (105*A*a^2*cos(6*d*x + 6*c) + 35*(17*A + 10*B)*a^2*
cos(4*d*x + 4*c) + 7*(113*A + 100*B)*a^2*cos(2*d*x + 2*c) + (301*A + 230*B)
*a^2)*sin(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) - 1
05*(((5*A + 2*B)*a^2*d*cos(2*d*x + 2*c)^4 + (5*A + 2*B)*a^2*d*sin(2*d*x + 2
*c)^4 + 4*(5*A + 2*B)*a^2*d*cos(2*d*x + 2*c)^3 + 6*(5*A + 2*B)*a^2*d*cos(2*
d*x + 2*c)^2 + 4*(5*A + 2*B)*a^2*d*cos(2*d*x + 2*c) + (5*A + 2*B)*a^2*d + 2
*((5*A + 2*B)*a^2*d*cos(2*d*x + 2*c)^2 + 2*(5*A + 2*B)*a^2*d*cos(2*d*x + 2*
c) + (5*A + 2*B)*a^2*d)*sin(2*d*x + 2*c)^2)*integrate((((cos(6*d*x + 6*c)*c
os(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(2*d*x + 2*c) + cos(2*d*x + 2*c)^2
+ sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
 sin(2*d*x + 2*c)^2)*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 (cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) -
 cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*s
in(7/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1)) - ((cos(2*d*x + 2*c)*sin(6*d*x + 6*c) +
2*cos(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + 6*c)*sin(2*d*x + 2*c) - 2
*cos(4*d*x + 4*c)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) - (cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos
(2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*sin(7/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c) + 1)))/(((cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c)^2 +
 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4
*d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*c)^2 +
 sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2*cos(2
*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x +
 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2 + sin
```

```
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*d*x +
2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^3 + cos(2
*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2*c))*c
os(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 2*(cos(2*d*x
 + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c) +
 (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(6*d*x
+ 6*c) + 4*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))*cos(5/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c) + 1))^2 + (cos(2*d*x + 2*c)^4 + sin(2*d*x + 2*c)^4 + (cos(2
*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*
c)^2 + 4*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*cos(4*d*x + 4*c)^2 + 2*cos(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + sin(2*d*
x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(6*d*x + 6*c)^2 + 4*(cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x + 4*c)^2 + (2
*cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c)^2 + 2*(cos(2
*d*x + 2*c)^3 + cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*(cos(2*d*x + 2*c)^2
 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 2*cos(2*
d*x + 2*c)^2 + cos(2*d*x + 2*c))*cos(6*d*x + 6*c) + 4*(cos(2*d*x + 2*c)^3 +
 cos(2*d*x + 2*c)*sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)^2 + cos(2*d*x + 2
*c))*cos(4*d*x + 4*c) + cos(2*d*x + 2*c)^2 + 2*(sin(2*d*x + 2*c)^3 + 2*(cos
(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(4*d*x +
4*c) + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sin(2*d*x + 2*c))*sin(
6*d*x + 6*c) + 4*(sin(2*d*x + 2*c)^3 + (cos(2*d*x + 2*c)^2 + 2*cos(2*d*x +
2*c) + 1)*sin(2*d*x + 2*c))*sin(4*d*x + 4*c))*sin(5/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))^2)*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)), x) + 5*((3*A + 4*B)*a^2*d*cos(2*d*x + 2*c)^4
 + (3*A + 4*B)*a^2*d*sin(2*d*x + 2*c)^4 + 4*(3*A + 4*B)*a^2*d*cos(2*d*x + 2
*c)^3 + 6*(3*A + 4*B)*a^2*d*cos(2*d*x + 2*c)^2 + 4*(3*A + 4*B)*a^2*d*cos(2*
d*x + 2*c) + (3*A + 4*B)*a^2*d + 2*((3*A + 4*B)*a^2*d*cos(2*d*x + 2*c)^2 +
2*(3*A + 4*B)*a^2*d*cos(2*d*x + 2*c) + (3*A + 4*B)*a^2*d)*sin(2*d*x + 2*c)^
2)*integrate((((cos(6*d*x + 6*c)*cos(2*d*x + 2*c) + 2*cos(4*d*x + 4*c)*cos(
2*d*x + 2*c) + cos(2*d*x + 2*c)^2 + sin(6*d*x + 6*c)*sin(2*d*x + 2*c) + 2*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + sin(2*d*x + 2*c)^2)*cos(5/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + (cos(2*d*x + 2*c)*sin(6*d*x + 6*c) + 2*c
os(2*d*x + 2*c)*sin(4*d*x + 4*c) - cos(6*d*x + ...
```

Fricas [A]
time = 2.36, size = 115, normalized size = 0.83

$$\frac{2\left((301A+230B)a^2\cos(dx+c)^3+(98A+115B)a^2\cos(dx+c)^2+3(7A+20B)a^2\cos(dx+c)+15Ba^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin(dx+c)}{105\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

[Out] 2/105*((301*A + 230*B)*a^2*cos(d*x + c)^3 + (98*A + 115*B)*a^2*cos(d*x + c)
^2 + 3*(7*A + 20*B)*a^2*cos(d*x + c) + 15*B*a^2)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{5}{2}}\,(A+B\sec(c+dx))\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)

[Out] Integral((a*(sec(c + d*x) + 1))**(5/2)*(A + B*sec(c + d*x))*sec(c + d*x), x
)

Giac [A]
time = 1.35, size = 216, normalized size = 1.57

$$\frac{8\left(\left(4\left(2\sqrt{2}\left(7Aa^6\mathrm{sgn}(\cos(dx+c))+5Ba^6\mathrm{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-7\sqrt{2}\left(7Aa^6\mathrm{sgn}(\cos(dx+c))+5Ba^6\mathrm{sgn}(\cos(dx+c))\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+35\sqrt{2}\left(7Aa^6\mathrm{sgn}(\cos(dx+c))+5Ba^6\mathrm{sgn}(\cos(dx+c))\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-105\sqrt{2}\left(Aa^6\mathrm{sgn}(\cos(dx+c))+Ba^6\mathrm{sgn}(\cos(dx+c))\right)\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{105\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
giac")

[Out] 8/105*((4*(2*sqrt(2)*(7*A*a^6*sgn(cos(d*x + c)) + 5*B*a^6*sgn(cos(d*x + c))
)*tan(1/2*d*x + 1/2*c)^2 - 7*sqrt(2)*(7*A*a^6*sgn(cos(d*x + c)) + 5*B*a^6*s
gn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 + 35*sqrt(2)*(7*A*a^6*sgn(cos(d*x
 + c)) + 5*B*a^6*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)^2 - 105*sqrt(2)*(
A*a^6*sgn(cos(d*x + c)) + B*a^6*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)/((
a*tan(1/2*d*x + 1/2*c)^2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*d)

Mupad [B]
time = 6.35, size = 590, normalized size = 4.28

$$\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(\frac{A\,a^2\,2i}{d}-\frac{a^2\,e^{c\,1i+d\,x\,1i}\,(301\,A+230\,B)\,2i}{105\,d}\right)}{e^{c\,1i+d\,x\,1i}+1}-\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(e^{c\,1i+d\,x\,1i}\left(\frac{A\,a^2\,2i}{7\,d}+\frac{a^2\,(3\,A+4\,B)\,10i}{7\,d}-\frac{a^2\,(5\,A+2\,B)\,2i}{7\,d}-\frac{a^2\,(11\,A+10\,B)\,2i}{7\,d}\right)+\frac{A\,a^2\,2i}{7\,d}+\frac{a^2\,(3\,A+4\,B)\,10i}{7\,d}-\frac{a^2\,(5\,A+2\,B)\,2i}{7\,d}-\frac{a^2\,(11\,A+10\,B)\,2i}{7\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^3}-\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(e^{c\,1i+d\,x\,1i}\left(\frac{a^2\,(5\,A+2\,B)\,2i}{5\,d}-\frac{a^2\,(5\,A+9\,B)\,4i}{5\,d}+\frac{a^2\,(7\,A-8\,B)\,2i}{35\,d}\right)-\frac{A\,a^2\,2i}{5\,d}-\frac{a^2\,(A+2\,B)\,2i}{d}+\frac{a^2\,(A+B)\,4i}{d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)^2}+\frac{\sqrt{a+\frac{a}{\frac{e^{-c\,1i-d\,x\,1i}}{2}+\frac{e^{c\,1i+d\,x\,1i}}{2}}}\left(e^{c\,1i+d\,x\,1i}\left(\frac{a^2\,(5\,A+2\,B)\,2i}{3\,d}-\frac{a^2\,(63\,A+80\,B)\,2i}{105\,d}\right)-\frac{A\,a^2\,2i}{3\,d}+\frac{a^2\,(9\,A+10\,B)\,2i}{3\,d}\right)}{\left(e^{c\,1i+d\,x\,1i}+1\right)\left(e^{c\,2i+d\,x\,2i}+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/cos(c + d*x),x)

[Out] ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*((A*a^2*2i)/
d - (a^2*exp(c*1i + d*x*1i)*(301*A + 230*B)*2i)/(105*d)))/(exp(c*1i + d*x*1
i) + 1) - ((a + a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(e
xp(c*1i + d*x*1i)*((A*a^2*2i)/(7*d) + (a^2*(3*A + 4*B)*10i)/(7*d) - (a^2*(5
*A + 2*B)*2i)/(7*d) - (a^2*(11*A + 10*B)*2i)/(7*d)) + (A*a^2*2i)/(7*d) + (a
^2*(3*A + 4*B)*10i)/(7*d) - (a^2*(5*A + 2*B)*2i)/(7*d) - (a^2*(11*A + 10*B)

```
*2i)/(7*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^3) - ((a +
a/(exp(- c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i
)*((a^2*(5*A + 2*B)*2i)/(5*d) - (a^2*(5*A + 9*B)*4i)/(5*d) + (a^2*(7*A - 8*
B)*2i)/(35*d)) - (A*a^2*2i)/(5*d) - (a^2*(A + 2*B)*2i)/d + (a^2*(A + B)*4i)
/d))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2i) + 1)^2) + ((a + a/(exp(-
 c*1i - d*x*1i)/2 + exp(c*1i + d*x*1i)/2))^(1/2)*(exp(c*1i + d*x*1i)*((a^2*
(5*A + 2*B)*2i)/(3*d) - (a^2*(63*A + 80*B)*2i)/(105*d)) - (A*a^2*2i)/(3*d)
+ (a^2*(9*A + 10*B)*2i)/(3*d)))/((exp(c*1i + d*x*1i) + 1)*(exp(c*2i + d*x*2
i) + 1))
```

3.138 $\int (a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=142

$$\frac{2a^{5/2}A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^3(35A+32B)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(5A+8B)\sqrt{a+a\sec(c+dx)}\,\text{ta}}{15d}$$

```
[Out] 2*a^(5/2)*A*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2/5*a*B*(a+
a*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/15*a^3*(35*A+32*B)*tan(d*x+c)/d/(a+a*sec
(d*x+c))^(1/2)+2/15*a^2*(5*A+8*B)*(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.16, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4002, 4000, 3859, 209, 3877}

$$\frac{2a^{5/2}A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^3(35A+32B)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(5A+8B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2aB\tan(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^(5/2)*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d + (
2*a^3*(35*A + 32*B)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*
(5*A + 8*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*d) + (2*a*B*(a + a*S
ec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3877

```
Int[csc[(e_.) + (f_.)*(x_)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*b*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]])), x] /; Free
Q[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4000

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[c, Int[Sqrt[a + b*Csc[e + f*x]], x], x] + Dis
t[d, Int[Sqrt[a + b*Csc[e + f*x]]*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d
, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4002

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 1)*Simp[a*c*m +
(b*c*m + a*d*(2*m - 1))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f
}, x] && NeQ[b*c - a*d, 0] && GtQ[m, 1] && EqQ[a^2 - b^2, 0] && IntegerQ[2*
m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{2aB(a+a\sec(c+dx))^{3/2}\tan(c+dx)}{5d}+\frac{2}{5}\int (a+a\sec($$

$$= \frac{2a^2(5A+8B)\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{15d}+\frac{2aB(a}{}$$

$$= \frac{2a^2(5A+8B)\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{15d}+\frac{2aB(a}{}$$

$$= \frac{2a^3(35A+32B)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(5A+8B)\sqrt{a+a\sec}}{15d}$$

$$= \frac{2a^{5/2}A\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^3(35A+32B}{15d\sqrt{a+a\sec}}$$

Mathematica [A]
time = 0.94, size = 128, normalized size = 0.90

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sqrt{a(1+\sec(c+dx))}\left(30\sqrt{2}\,A\,\mathrm{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{5}{2}}(c+dx)+2(40A+49B+2(5A+14B)\cos(c+dx)+(40A+43B)\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]^2*Sqrt[a*(1 + Sec[c + d*x])]*(30*Sqrt[2]
*A*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(5/2) + 2*(40*A + 49*B + 2
```

```
*(5*A + 14*B)*Cos[c + d*x] + (40*A + 43*B)*Cos[2*(c + d*x)])*Sin[(c + d*x)/
2]))/(30*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 340 vs. $2(124) = 248$.
time = 4.07, size = 341, normalized size = 2.40

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(15A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}(\cos^2(dx+c))\sin(dx+c)+30A\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -1/60/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(15*A*2^(1/2)*arctanh(1/2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2*sin(d*x+c)+30*A*2^(1/2)*arctanh(1/2*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)*sin(d*x+c)+15*A*2^(1/2)*arctanh(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)+320*A*cos(d*x+c)^3+344*B*cos(d*x+
c)^3-280*A*cos(d*x+c)^2-232*B*cos(d*x+c)^2-40*A*cos(d*x+c)-88*B*cos(d*x+c)-
24*B)/cos(d*x+c)^2/sin(d*x+c)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1396 vs. $2(124) = 248$.
time = 0.61, size = 1396, normalized size = 9.83

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/6*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^
(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 2*
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*((
12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c
) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2*cos
(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*((a
```

```
^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a
^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(
2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
 + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c
)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2
*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) +
 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*
d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(
2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
+ 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(
1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*sqrt(a)
)*A/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*d)
```

Fricas [A]
time = 3.91, size = 378, normalized size = 2.66

$$\left[\frac{15\left(Aa^{2}\cos\left(dx+c\right)^{3}+Aa^{2}\cos\left(dx+c\right)^{2}\right)\sqrt{-a}\log\left(\frac{2\,a\cos(dx+c)^{2}-2\,\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left((40\,A+43\,B)a^{2}\cos\left(dx+c\right)^{2}+(5\,A+14\,B)a^{2}\cos\left(dx+c\right)+3\,Ba^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)},-\frac{2\left(15\left(Aa^{2}\cos\left(dx+c\right)^{3}+Aa^{2}\cos\left(dx+c\right)^{2}\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left((40\,A+43\,B)a^{2}\cos\left(dx+c\right)^{2}+(5\,A+14\,B)a^{2}\cos\left(dx+c\right)+3\,Ba^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)\right)}{15\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/15*(15*(A*a^2*cos(d*x + c)^3 + A*a^2*cos(d*x + c)^2)*sqrt(-a)*log((2*a*c
os(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
+ c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*((40*A + 43
*B)*a^2*cos(d*x + c)^2 + (5*A + 14*B)*a^2*cos(d*x + c) + 3*B*a^2)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3 + d*cos(d*x
+ c)^2), -2/15*(15*(A*a^2*cos(d*x + c)^3 + A*a^2*cos(d*x + c)^2)*sqrt(a)*ar
```

```
ctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
+ c))) - ((40*A + 43*B)*a^2*cos(d*x + c)^2 + (5*A + 14*B)*a^2*cos(d*x + c)
+ 3*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
 + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{5}{2}}(A+B\sec(c+dx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(5/2)*(A + B*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 309 vs. 2(124) = 248.
time = 1.49, size = 309, normalized size = 2.18

$$-\frac{\frac{15 A\sqrt{-a}\,a^{3}\log\left(\frac{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}-4\sqrt{2}|a|-6a\right|}{\left|2\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}\right)^{2}+4\sqrt{2}|a|-6a\right|}\right)\mathrm{sgn}(\cos(dx+c))}{|a|}-\frac{2\left(45\sqrt{2}Aa^{5}\mathrm{sgn}(\cos(dx+c))+60\sqrt{2}Ba^{5}\mathrm{sgn}(\cos(dx+c))-\left(80\sqrt{2}Aa^{5}\mathrm{sgn}(\cos(dx+c))+80\sqrt{2}Ba^{5}\mathrm{sgn}(\cos(dx+c))-\left(35\sqrt{2}Aa^{5}\mathrm{sgn}(\cos(dx+c))+32\sqrt{2}Ba^{5}\mathrm{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-a\right)^{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}+a}}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] -1/15*(15*A*sqrt(-a)*a^3*log(abs(2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 4*sqrt(2)*abs(a) - 6*a)/abs(2*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 4*sqrt(2)*ab
s(a) - 6*a))*sgn(cos(d*x + c))/abs(a) - 2*(45*sqrt(2)*A*a^5*sgn(cos(d*x + c
)) + 60*sqrt(2)*B*a^5*sgn(cos(d*x + c)) - (80*sqrt(2)*A*a^5*sgn(cos(d*x + c
)) + 80*sqrt(2)*B*a^5*sgn(cos(d*x + c)) - (35*sqrt(2)*A*a^5*sgn(cos(d*x + c
)) + 32*sqrt(2)*B*a^5*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*
x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos(c+d\,x)}\right)\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)
```

3.139 $\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=143

$$\frac{a^{5/2}(5A+2B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}-\frac{a^3(3A+14B)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(A+2B)\sqrt{a+a\sec(c+dx)}}{d}$$

```
[Out] a^(5/2)*(5*A+2*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2/3*a
*B*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d-1/3*a^3*(3*A+14*B)*sin(d*x+c)/d/(a+a
*sec(d*x+c))^(1/2)+2*a^2*(A+2*B)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.27, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4103, 4100, 3859, 209}

$$\frac{a^{5/2}(5A+2B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}-\frac{a^3(3A+14B)\sin(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(A+2B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d}+\frac{2aB\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(5/2)*(5*A + 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/d - (a^3*(3*A + 14*B)*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]]) + (2*
a^2*(A + 2*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/d + (2*a*B*(a + a*Sec[
c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
```

```
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{2aB(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d} + \frac{2}{3}\int c$$

$$= \frac{2a^2(A+2B)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{d}$$

$$= -\frac{a^3(3A+14B)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(A+2B)\sqrt{}}{}$$

$$= -\frac{a^3(3A+14B)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(A+2B)\sqrt{}}{}$$

$$= \frac{a^{5/2}(5A+2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

Mathematica [A]
time = 0.80, size = 126, normalized size = 0.88

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sqrt{a(1+\sec(c+dx))}\left(3\sqrt{2}(5A+2B)\text{ArcSin}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^{\frac{3}{2}}(c+dx)+(3A+4B+4(3A+8B)\cos(c+dx)+3A\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sec[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(5
*A + 2*B)*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^(3/2) + (3*A + 4*B
```

```
+ 4*(3*A + 8*B)*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(6*
d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(127) = 254$.
time = 7.90, size = 256, normalized size = 1.79

method	result
default	$-\frac{\left(15 A \cos(dx+c) \sin(dx+c) \sqrt{2}\, \sqrt{-\frac{2 \cos(dx+c)}{1+\cos(dx+c)}}\, \operatorname{arctanh}\left(\frac{\sqrt{-\frac{2 \cos(dx+c)}{1+\cos(dx+c)}}\, \sin(dx+c) \sqrt{2}}{2 \cos(dx+c)}\right) + 6 B \cos(dx+c) \sin(dx+c$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/6/d*(15*A*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)
*2^(1/2))+6*B*cos(d*x+c)*sin(d*x+c)*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c
)*2^(1/2))+6*A*cos(d*x+c)^3+6*A*cos(d*x+c)^2+32*B*cos(d*x+c)^2-12*A*cos(d*x
+c)-28*B*cos(d*x+c)-4*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/cos
(d*x+c)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2780 vs. $2(127) = 254$.
time = 0.70, size = 2780, normalized size = 19.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] 1/12*(3*(18*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))
+ 2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4
)*((4*a^2*sin(3*d*x + 3*c) + 5*a^2*sin(2*d*x + 2*c) + 4*a^2*sin(d*x + c))*c
os(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) + (a^2*cos(2*d*x +
2*c)^2*sin(d*x + c) + a^2*sin(2*d*x + 2*c)^2*sin(d*x + c) + 2*a^2*cos(2*d*x
 + 2*c)*sin(d*x + c) + a^2*sin(d*x + c))*cos(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c) + 1)) - (4*a^2*cos(3*d*x + 3*c) + 5*a^2*cos(2*d*x + 2*c) +
 4*a^2*cos(d*x + c) + 5*a^2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) - ((a^2*cos(d*x + c) - a^2)*cos(2*d*x + 2*c)^2 + a^2*cos(d*x + c
```

```
) + (a^2*cos(d*x + c) - a^2)*sin(2*d*x + 2*c)^2 - a^2 + 2*(a^2*cos(d*x + c)
 - a^2)*cos(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
) + 1)))*sqrt(a) + 5*((a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*
a^2*cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c) + 1))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)
^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2(-(cos(2*d*x + 2*c)^2 + sin(2*d*x
 + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c) + 1))*sin(d*x + c) - cos(d*x + c)*sin(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c) + 1))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)
^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*(cos(d*x + c)*cos(1/2*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c) + 1)) + sin(d*x + c)*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c) + 1))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x
 + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin
(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c) + 1)) + 1) + (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*
cos(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 +
2*cos(2*d*x + 2*c) + 1)^(1/4)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c) + 1)), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
+ 1)^(1/4)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - 1))*s
qrt(a))*A/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
) + 2*(30*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1
)^(3/4)*a^(5/2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) -
2*(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)^(1/4)*
((12*a^2*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2
*c) - 3*a^2*sin(2*d*x + 2*c) - 4*(3*a^2*cos(2*d*x + 2*c) + 4*a^2)*sin(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c) + 1)) + (12*a^2*sin(2*d*x + 2*c)*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 3*a^2*cos(2*d*x + 2*c) - a^2 + 4*(3*a^2*c
os(2*d*x + 2*c) + 4*a^2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)))*sqrt(a) + 3*(
(a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*cos(2*d*x + 2*c) +
 a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c)
 + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arctan2(sin(2*d*x +
```

```
 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))) + 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2*c)^2 + 2*a^2*co
s(2*d*x + 2*c) + a^2)*arctan2((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*
cos(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1)) - cos(1/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))), (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*co
s(2*d*x + 2*c) + 1)^(1/4)*(cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c) + 1))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/2*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c) + 1))*sin(1/2*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c)))) - 1) - (a^2*cos(2*d*x + 2*c)^2 + a^2*sin(2*d*x + 2
*c)^2 + 2*a^2*cos(2*d*x + 2*c) + a^2)*arctan2((...
```

Fricas [**A**]
time = 2.88, size = 386, normalized size = 2.70

$$\left[\frac{3\,\left((5\,A+2\,B)a^2\cos\left(dx+c\right)^2+(5\,A+2\,B)a^2\cos\left(dx+c\right)\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^2-2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\,\left(3\,Aa^2\cos\left(dx+c\right)^2+2\,(3\,A+8\,B)a^2\cos\left(dx+c\right)+2\,Ba^2\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{6\,\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)},-\frac{3\,\left((5\,A+2\,B)a^2\cos\left(dx+c\right)^2+(5\,A+2\,B)a^2\cos\left(dx+c\right)\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\cos(dx+c)}{\sqrt{a}\,\sin(dx+c)}\right)-\left(3\,Aa^2\cos\left(dx+c\right)^2+2\,(3\,A+8\,B)a^2\cos\left(dx+c\right)+2\,Ba^2\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{3\,\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] [1/6*(3*((5*A + 2*B)*a^2*cos(d*x + c)^2 + (5*A + 2*B)*a^2*cos(d*x + c))*sqr
t(-a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1))
 + 2*(3*A*a^2*cos(d*x + c)^2 + 2*(3*A + 8*B)*a^2*cos(d*x + c) + 2*B*a^2)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2 + d*c
os(d*x + c)), -1/3*(3*((5*A + 2*B)*a^2*cos(d*x + c)^2 + (5*A + 2*B)*a^2*cos
(d*x + c))*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(sqrt(a)*sin(d*x + c))) - (3*A*a^2*cos(d*x + c)^2 + 2*(3*A + 8*B)*a^2*c
os(d*x + c) + 2*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
)/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 480 vs. 2(127) = 254.

time = 1.95, size = 480, normalized size = 3.36

$$-\frac{3\left(5A\sqrt{-a}a^2\mathrm{sgn}(\cos(dx+c))+2B\sqrt{-a}a^2\mathrm{sgn}(\cos(dx+c))\right)\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)-3\left(5A\sqrt{-a}a^2\mathrm{sgn}(\cos(dx+c))+2B\sqrt{-a}a^2\mathrm{sgn}(\cos(dx+c))\right)\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+a\left(2\sqrt{2}-3\right)\right|\right)+\frac{4\left(3\sqrt{2}Aa^4\mathrm{sgn}(\cos(dx+c))+9\sqrt{2}Ba^4\mathrm{sgn}(\cos(dx+c))-\left(3\sqrt{2}Aa^4\mathrm{sgn}(\cos(dx+c))+7\sqrt{2}Ba^4\mathrm{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}+\frac{12\left(3\sqrt{2}\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2A\sqrt{-a}a^3\mathrm{sgn}(\cos(dx+c))-\sqrt{2}A\sqrt{-a}a^4\mathrm{sgn}(\cos(dx+c))\right)}{\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2a+a^2}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] -1/6*(3*(5*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 2*B*sqrt(-a)*a^2*sgn(cos(d*x
+ c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(5*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) +
 2*B*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*(3*sqrt
(2)*A*a^4*sgn(cos(d*x + c)) + 9*sqrt(2)*B*a^4*sgn(cos(d*x + c)) - (3*sqrt(2
)*A*a^4*sgn(cos(d*x + c)) + 7*sqrt(2)*B*a^4*sgn(cos(d*x + c)))*tan(1/2*d*x
+ 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a)) + 12*(3*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) -
sqrt(2)*A*sqrt(-a)*a^4*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+d\,x)\,\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)
```

3.140 $\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=154

$$\frac{a^{5/2}(19A+20B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}-\frac{a^2(A-4B)\sqrt{a+a\sec(c+dx)}}{2d}$$

```
[Out] 1/4*a^(5/2)*(19*A+20*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d
+1/2*a*A*cos(d*x+c)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+1/4*a^3*(9*A-4*B)*s
in(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-1/2*a^2*(A-4*B)*sin(d*x+c)*(a+a*sec(d*x+
c))^(1/2)/d
```

Rubi [A]
time = 0.29, antiderivative size = 154, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4102, 4103, 4100, 3859, 209}

$$\frac{a^{5/2}(19A+20B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(A-4B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}+\frac{aA\sin(c+dx)\cos(c+dx)(a\sec(c+dx)+a)^{3/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(5/2)*(19*A + 20*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(4*d) + (a^3*(9*A - 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]])
- (a^2*(A - 4*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(2*d) + (a*A*Cos[c
+ d*x]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
```

```
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+d}{2d}$$

$$= -\frac{a^2(A-4B)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{2d}$$

$$= \frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} - \frac{a^2(A-4B)\sqrt{a+}}{}$$

$$= \frac{a^3(9A-4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} - \frac{a^2(A-4B)\sqrt{a+}}{}$$

$$= \frac{a^{5/2}(19A+20B)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right.}{4d}$$

Mathematica [A]

time = 0.83, size = 116, normalized size = 0.75

$$\frac{a^2 \sec\left(\frac{1}{2}(c+dx)\right) \sqrt{a(1+\sec(c+dx))} \left(\sqrt{2}\,(19A+20B)\text{ArcSin}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sqrt{\cos(c+dx)}+2(A+8B+(11A+4B)\cos(c+dx)+A\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(19*A + 20*B)*Arc
Sin[Sqrt[2]*Sin[(c + d*x)/2]]*Sqrt[Cos[c + d*x]] + 2*(A + 8*B + (11*A + 4*B
)*Cos[c + d*x] + A*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 409 vs. $2(134) = 268$.
time = 8.08, size = 410, normalized size = 2.66

method	result
default	$\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(19A\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sin(dx+c)\cos(dx+c)\sqrt{2}+20B\left(-\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/16/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(19*A*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(
d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)*2^(1/2)+20*B*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/co
s(d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)*2^(1/2)+19*A*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/
cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)+20*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c
)*2^(1/2))*2^(1/2)*sin(d*x+c)-8*A*cos(d*x+c)^4-36*A*cos(d*x+c)^3-16*B*cos(d
*x+c)^3+44*A*cos(d*x+c)^2-16*B*cos(d*x+c)^2+32*B*cos(d*x+c))/cos(d*x+c)/sin
(d*x+c)*a^2
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 2.93, size = 348, normalized size = 2.26

$$\left[\frac{\left((19A+20B)a^2\cos(dx+c)+(19A+20B)a^2\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2Aa^2\cos(dx+c)^2+(11A+4B)a^2\cos(dx+c)+8Ba^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{8(d\cos(dx+c)+d)},\ -\frac{\left((19A+20B)a^2\cos(dx+c)+(19A+20B)a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(2Aa^2\cos(dx+c)^2+(11A+4B)a^2\cos(dx+c)+8Ba^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{4(d\cos(dx+c)+d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [1/8*(((19*A + 20*B)*a^2*cos(d*x + c) + (19*A + 20*B)*a^2)*sqrt(-a)*log((2*
a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d
*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*A*a^2
*cos(d*x + c)^2 + (11*A + 4*B)*a^2*cos(d*x + c) + 8*B*a^2)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/4*(((19*A +
20*B)*a^2*cos(d*x + c) + (19*A + 20*B)*a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (2*A*a^2*cos
(d*x + c)^2 + (11*A + 4*B)*a^2*cos(d*x + c) + 8*B*a^2)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 709 vs. 2(134) = 268.
time = 1.99, size = 709, normalized size = 4.60

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/8*(16*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*B*a^3*sgn(cos(d*x + c)
)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + (19*A*sqrt(-a)*a^2*
sgn(cos(d*x + c)) + 20*B*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*
tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(2*sqrt(2
```

```
) + 3))) - (19*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 20*B*sqrt(-a)*a^2*sgn(cos
(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*sqrt(2)*(19*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^3*sgn(cos(
d*x + c)) + 12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^6*B*sqrt(-a)*a^3*sgn(cos(d*x + c)) - 171*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^4*sgn(cos(d*x
+ c)) - 76*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^4*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 89*(sqrt(-a)*tan(1/2*d*x + 1/2*c
) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^5*sgn(cos(d*x + c))
 + 36*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
^2*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 9*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) -
4*B*sqrt(-a)*a^6*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt
(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^2\left(A+\frac{B}{\cos(c+dx)}\right)\left(a+\frac{a}{\cos(c+dx)}\right)^{5/2}dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)
```

3.141 $\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=164

$$\frac{a^{5/2}(25A+38B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{a^3(49A+54B)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}}+\frac{a^2(3A+2B)\cos(c+dx)\sqrt{a}}{4}$$

```
[Out] 1/8*a^(5/2)*(25*A+38*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d
+1/3*a*A*cos(d*x+c)^2*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+1/24*a^3*(49*A+54
*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/4*a^2*(3*A+2*B)*cos(d*x+c)*sin(d*
x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.31, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4102, 4100, 3859, 209}

$$\frac{a^{5/2}(25A+38B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^3(49A+54B)\sin(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(3A+2B)\sin(c+dx)\cos(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}+\frac{aA\sin(c+dx)\cos^2(c+dx)(a\sec(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(5/2)*(25*A + 38*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(8*d) + (a^3*(49*A + 54*B)*Sin[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]
]) + (a^2*(3*A + 2*B)*Cos[c + d*x]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(
4*d) + (a*A*Cos[c + d*x]^2*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
```

```
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos^2(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c ...}{3d}$$

$$= \frac{a^2(3A+2B)\cos(c+dx)\sqrt{a+a\sec(c+dx ...}}{4d}$$

$$= \frac{a^3(49A+54B)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(3A+2B) ...}{}$$

$$= \frac{a^3(49A+54B)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{a^2(3A+2B) ...}{}$$

$$= \frac{a^{5/2}(25A+38B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx ...}}\right.}{8d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.07, size = 312, normalized size = 1.90

$$-\frac{a^2\cos(c+dx)\sqrt{a(1+\sec(c+dx))}\left(-165A\sqrt{1-\sec(c+dx)}\sin(c+dx)+18B\sqrt{1-\sec(c+dx)}\sin(c+dx)+8A\cos^2(c+dx)\sqrt{1-\sec(c+dx)}\sin(c+dx)-31A\sqrt{1-\sec(c+dx)}\sin(2(c+dx))+54B\sqrt{1-\sec(c+dx)}\sin(2(c+dx))-165A\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)-126B\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)-576B\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\sec(c+dx)\right)\sqrt{1-\sec(c+dx)}\tan(c+dx)-192A\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};1-\sec(c+dx)\right)\sqrt{1-\sec(c+dx)}\tan(c+dx)\right)}{72d(1+\cos(c+dx))\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] -1/72*(a^2*Cos[c + d*x]*Sqrt[a*(1 + Sec[c + d*x])]*(-165*A*Sqrt[1 - Sec[c +
 d*x]]*Sin[c + d*x] + 18*B*Sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] + 8*A*Cos[c
```

```
+ d*x]^2*Sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] - 31*A*Sqrt[1 - Sec[c + d*x]]*
Sin[2*(c + d*x)] + 54*B*Sqrt[1 - Sec[c + d*x]]*Sin[2*(c + d*x)] - 165*A*Arc
Tanh[Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 126*B*ArcTanh[Sqrt[1 - Sec[c +
d*x]]]*Tan[c + d*x] - 576*B*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]
]*Sqrt[1 - Sec[c + d*x]]*Tan[c + d*x] - 192*A*Hypergeometric2F1[1/2, 4, 3/2
, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]]*Tan[c + d*x]))/(d*(1 + Cos[c + d
*x])*Sqrt[1 - Sec[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 582 vs. $2(144) = 288$.
time = 7.64, size = 583, normalized size = 3.55

method	result
default	$-\frac{\left(75A\sqrt{2}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}(\cos^2(dx+c))\sin(dx+c)+114B\sqrt{2}\,\operatorname{arctanh}\left(\sqrt{\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/192/d*(75*A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)
^2*sin(d*x+c)+114*B*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d
*x+c)^2*sin(d*x+c)+150*A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*
cos(d*x+c)*sin(d*x+c)+228*B*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/
2)*cos(d*x+c)*sin(d*x+c)+75*A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
5/2)*sin(d*x+c)+114*B*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin
(d*x+c)+64*A*cos(d*x+c)^6+208*A*cos(d*x+c)^5+96*B*cos(d*x+c)^5+328*A*cos(d*
x+c)^4+432*B*cos(d*x+c)^4-600*A*cos(d*x+c)^3-528*B*cos(d*x+c)^3)*(a*(1+cos(
d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^2*a^2
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 3.24, size = 380, normalized size = 2.32

$$\left[\frac{3\left((25A+38B)a^{2}\cos(dx+c)+(25A+38B)a^{2}\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^{2}-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(8Aa^{2}\cos(dx+c)^{3}+2(17A+6B)a^{2}\cos(dx+c)^{2}+3(25A+22B)a^{2}\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{48\left(d\cos(dx+c)+d\right)}, -\frac{3\left((25A+38B)a^{2}\cos(dx+c)+(25A+38B)a^{2}\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(8Aa^{2}\cos(dx+c)^{3}+2(17A+6B)a^{2}\cos(dx+c)^{2}+3(25A+22B)a^{2}\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{24\left(d\cos(dx+c)+d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [1/48*(3*((25*A + 38*B)*a^2*cos(d*x + c) + (25*A + 38*B)*a^2)*sqrt(-a)*log(
(2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*co
s(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*A*
a^2*cos(d*x + c)^3 + 2*(17*A + 6*B)*a^2*cos(d*x + c)^2 + 3*(25*A + 22*B)*a^
2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*co
s(d*x + c) + d), -1/24*(3*((25*A + 38*B)*a^2*cos(d*x + c) + (25*A + 38*B)*a
^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sq
rt(a)*sin(d*x + c))) - (8*A*a^2*cos(d*x + c)^3 + 2*(17*A + 6*B)*a^2*cos(d*x
 + c)^2 + 3*(25*A + 22*B)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 905 vs. 2(144) = 288.
time = 2.29, size = 905, normalized size = 5.52

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/48*(3*(25*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 38*B*sqrt(-a)*a^2*sgn(cos(d
*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(25*A*sqrt(-a)*a^2*sgn(cos(d*x + c
```

```
)) + 38*B*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1
/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) + 4*(7
5*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^10*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 114*sqrt(2)*(sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a^3*sgn(cos
(d*x + c)) - 1125*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^8*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 1710*sqrt(2)*(sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(
-a)*a^4*sgn(cos(d*x + c)) + 6174*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 68
04*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^6*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 4314*sqrt(2)*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^6*sgn(cos
(d*x + c)) - 4284*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^6*sgn(cos(d*x + c)) + 807*sqrt(2)*(sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-
a)*a^7*sgn(cos(d*x + c)) + 858*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 49*s
qrt(2)*A*sqrt(-a)*a^8*sgn(cos(d*x + c)) - 54*sqrt(2)*B*sqrt(-a)*a^8*sgn(cos
(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^
2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^2*a + a^2)^3)/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos(c+dx)^3 \left(A+\frac{B}{\cos(c+dx)}\right) \left(a+\frac{a}{\cos(c+dx)}\right)^{5/2} dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)
```

3.142 $\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=209

$$\frac{a^{5/2}(163A+200B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d}+\frac{a^3(163A+200B)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}}+\frac{a^3(95A+104B)\cos(c}{96d\sqrt{a+a\text{se}}}$$

[Out] 1/64*a^(5/2)*(163*A+200*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+1/4*a*A*cos(d*x+c)^3*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+1/64*a^3*(163*A+200*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/96*a^3*(95*A+104*B)*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/24*a^2*(11*A+8*B)*cos(d*x+c)^2*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d

Rubi [A]
time = 0.38, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4102, 4100, 3890, 3859, 209}

$$\frac{a^{5/2}(163A+200B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^3(163A+200B)\sin(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^3(95A+104B)\sin(c+dx)\cos(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(11A+8B)\sin(c+dx)\cos^2(c+dx)\sqrt{a\sec(c+dx)+a}}{24d}+\frac{aA\sin(c+dx)\cos^3(c+dx)(a\sec(c+dx)+a)^{3/2}}{4d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] (a^(5/2)*(163*A + 200*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(64*d) + (a^3*(163*A + 200*B)*Sin[c + d*x])/(64*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(95*A + 104*B)*Cos[c + d*x]*Sin[c + d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(11*A + 8*B)*Cos[c + d*x]^2*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (a*A*Cos[c + d*x]^3*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d), Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^4(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos^3(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+}{4d}$$

$$= \frac{a^2(11A+8B)\cos^2(c+dx)\sqrt{a+a\sec(c+dx}}{24d}$$

$$= \frac{a^3(95A+104B)\cos(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + \frac{a^2}{}$$

$$= \frac{a^3(163A+200B)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(95A+10}{96d\sqrt{}}$$

$$= \frac{a^3(163A+200B)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(95A+10}{96d\sqrt{}}$$

$$= \frac{a^{5/2}(163A+200B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+d}}\right.}{64d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.32, size = 366, normalized size = 1.75

$$\frac{a^2\left(6075A\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)+6600B\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)+2079A\sqrt{1-\sec(c+dx)}+1240B\sqrt{1-\sec(c+dx)}+7641A\cos(c+dx)\sqrt{1-\sec(c+dx)}+6360B\cos(c+dx)\sqrt{1-\sec(c+dx)}+2097A\cos(2(c+dx))\sqrt{1-\sec(c+dx)}+1240B\cos(2(c+dx))\sqrt{1-\sec(c+dx)}+522A\cos(3(c+dx))\sqrt{1-\sec(c+dx)}-80B\cos(3(c+dx))\sqrt{1-\sec(c+dx)}+18A\cos(4(c+dx))\sqrt{1-\sec(c+dx)}+7680B\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};1-\sec(c+dx)\right)\sqrt{1-\sec(c+dx)}+4608A\,{}_2F_1\left(\frac{1}{2},5;\frac{3}{2};1-\sec(c+dx)\right)\sqrt{1-\sec(c+dx)}\right)\sqrt{a(1+\sec(c+dx))}\sin(c+dx)}{2880d(1+\cos(c+dx))\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(6075*A*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + 6600*B*ArcTanh[Sqrt[1 - Sec[
c + d*x]]] + 2079*A*Sqrt[1 - Sec[c + d*x]] + 1240*B*Sqrt[1 - Sec[c + d*x]]
+ 7641*A*Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]] + 6360*B*Cos[c + d*x]*Sqrt[1 -
 Sec[c + d*x]] + 2097*A*Cos[2*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 1240*B*Co
s[2*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 522*A*Cos[3*(c + d*x)]*Sqrt[1 - Sec
[c + d*x]] - 80*B*Cos[3*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 18*A*Cos[4*(c +
 d*x)]*Sqrt[1 - Sec[c + d*x]] + 7680*B*Hypergeometric2F1[1/2, 4, 3/2, 1 - S
ec[c + d*x]]*Sqrt[1 - Sec[c + d*x]] + 4608*A*Hypergeometric2F1[1/2, 5, 3/2,
 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c
 + d*x])/(2880*d*(1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 764 vs. $2(185) = 370$.
time = 8.16, size = 765, normalized size = 3.66

method	result
default	$\frac{\left(489A\sqrt{2}\,(\cos^3(dx+c))\sin(dx+c)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)+600B\sqrt{2}\,(\cos^3(dx+c)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/3072/d*(489*A*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d
*x+c)*2^(1/2))+600*B*2^(1/2)*cos(d*x+c)^3*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/
cos(d*x+c)*2^(1/2))+1467*A*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)/cos(d*x+c)*2^(1/2))+1800*B*2^(1/2)*cos(d*x+c)^2*sin(d*x+c)*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)/cos(d*x+c)*2^(1/2))+1467*A*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+1800*B*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(-
```

```
2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+489*A*2^(1/2)*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
/cos(d*x+c)*2^(1/2))*sin(d*x+c)+600*B*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(7/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x
+c)*2^(1/2))*sin(d*x+c)-768*A*cos(d*x+c)^8-2176*A*cos(d*x+c)^7-1024*B*cos(d
*x+c)^7-2272*A*cos(d*x+c)^6-3328*B*cos(d*x+c)^6-2608*A*cos(d*x+c)^5-5248*B*
cos(d*x+c)^5+7824*A*cos(d*x+c)^4+9600*B*cos(d*x+c)^4)*(a*(1+cos(d*x+c))/cos
(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)^3*a^2
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 3.88, size = 420, normalized size = 2.01

$$\left[\frac{3\left((163\,A + 200\,B)a^2\cos(dx+c) + (163\,A + 200\,B)a^2\right)\sqrt{-a}\,\log\left(\frac{2\,a\cos(dx+c)^2 - 2\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c) + a\cos(dx+c) - a}{\cos(dx+c)+1}\right) + 2\left(48\,Aa^2\cos(dx+c)^4 + 8\,(23\,A + 8\,B)a^2\cos(dx+c)^3 + 2\,(163\,A + 136\,B)a^2\cos(dx+c)^2 + 3\,(163\,A + 200\,B)a^2\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{384\,(d\cos(dx+c)+d)}, -\frac{3\left((163\,A + 200\,B)a^2\cos(dx+c) + (163\,A + 200\,B)a^2\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right) - \left(48\,Aa^2\cos(dx+c)^4 + 8\,(23\,A + 8\,B)a^2\cos(dx+c)^3 + 2\,(163\,A + 136\,B)a^2\cos(dx+c)^2 + 3\,(163\,A + 200\,B)a^2\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{192\,(d\cos(dx+c)+d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [1/384*(3*((163*A + 200*B)*a^2*cos(d*x + c) + (163*A + 200*B)*a^2)*sqrt(-a)
*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*
(48*A*a^2*cos(d*x + c)^4 + 8*(23*A + 8*B)*a^2*cos(d*x + c)^3 + 2*(163*A + 1
36*B)*a^2*cos(d*x + c)^2 + 3*(163*A + 200*B)*a^2*cos(d*x + c))*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d), -1/192*(3*(
(163*A + 200*B)*a^2*cos(d*x + c) + (163*A + 200*B)*a^2)*sqrt(a)*arctan(sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) -
(48*A*a^2*cos(d*x + c)^4 + 8*(23*A + 8*B)*a^2*cos(d*x + c)^3 + 2*(163*A + 1
36*B)*a^2*cos(d*x + c)^2 + 3*(163*A + 200*B)*a^2*cos(d*x + c))*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c) + d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

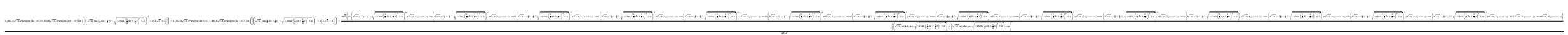
Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1096 vs. $2(185) = 370$.
time = 2.22, size = 1096, normalized size = 5.24



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/384*(3*(163*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 200*B*sqrt(-a)*a^2*sgn(co
s(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 3*(163*A*sqrt(-a)*a^2*sgn(cos(d*x
 + c)) + 200*B*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*
x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))) +
 4*sqrt(2)*(489*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a))^14*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 600*(sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*B*sqrt(-a)*a^3*sgn(cos(d
*x + c)) - 10269*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^12*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 12600*(sqrt(-a)*tan(1/2*d
*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*B*sqrt(-a)*a^4*sgn(co
s(d*x + c)) + 69885*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^10*A*sqrt(-a)*a^5*sgn(cos(d*x + c)) + 103992*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*B*sqrt(-a)*a^5*sg
n(cos(d*x + c)) - 259233*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^8*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 339864*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^6
*sgn(cos(d*x + c)) + 209979*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^7*sgn(cos(d*x + c)) + 262920*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*
a^7*sgn(cos(d*x + c)) - 55511*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^8*sgn(cos(d*x + c)) - 73640*(sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)
*a^8*sgn(cos(d*x + c)) + 6687*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^9*sgn(cos(d*x + c)) + 8808*(sqrt(-a
)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*
a^9*sgn(cos(d*x + c)) - 299*A*sqrt(-a)*a^10*sgn(cos(d*x + c)) - 392*B*sqrt(
-a)*a^10*sgn(cos(d*x + c)))/((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^4)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos (c+d\,x)^4\,\left(A+\frac{B}{\cos (c+d\,x)}\right)\,{\left(a+\frac{a}{\cos (c+d\,x)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)
```

3.143 $\int \cos^5(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=254

$$\frac{a^{5/2}(283A+326B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{128d}+\frac{a^3(283A+326B)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}}+\frac{a^3(283A+326B)\cos(}{192d\sqrt{a+a\,s}}$$

```
[Out] 1/128*a^(5/2)*(283*A+326*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2
))/d+1/5*a*A*cos(d*x+c)^4*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+1/128*a^3*(28
3*A+326*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/192*a^3*(283*A+326*B)*cos(
d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/240*a^3*(157*A+170*B)*cos(d*x+
c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/40*a^2*(13*A+10*B)*cos(d*x+c)^3*
sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.44, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4102, 4100, 3890, 3859, 209}

$$\frac{a^{5/2}(283A+326B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{128d}+\frac{a^3(283A+326B)\sin(c+dx)}{128d\sqrt{a\sec(c+dx)+a}}+\frac{a^3(157A+170B)\sin(c+dx)\cos^2(c+dx)}{240d\sqrt{a\sec(c+dx)+a}}+\frac{a^3(283A+326B)\sin(c+dx)\cos(c+dx)}{192d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(13A+10B)\sin(c+dx)\cos^3(c+dx)\sqrt{a\sec(c+dx)+a}}{40d}+\frac{aA\sin(c+dx)\cos^4(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(5/2)*(283*A + 326*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d
*x]]])/(128*d) + (a^3*(283*A + 326*B)*Sin[c + d*x])/(128*d*Sqrt[a + a*Sec[c
 + d*x]]) + (a^3*(283*A + 326*B)*Cos[c + d*x]*Sin[c + d*x])/(192*d*Sqrt[a +
 a*Sec[c + d*x]]) + (a^3*(157*A + 170*B)*Cos[c + d*x]^2*Sin[c + d*x])/(240*
d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(13*A + 10*B)*Cos[c + d*x]^3*Sqrt[a + a*
Sec[c + d*x]]*Sin[c + d*x])/(40*d) + (a*A*Cos[c + d*x]^4*(a + a*Sec[c + d*x
])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^5(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos^4(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c-}{5d}$$

$$= \frac{a^2(13A+10B)\cos^3(c+dx)\sqrt{a+a\sec(c+}}{40d}$$

$$= \frac{a^3(157A+170B)\cos^2(c+dx)\sin(c+dx)}{240d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{a^3(283A+326B)\cos(c+dx)\sin(c+dx)}{192d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{a^3(283A+326B)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(283A+}{192}$$

$$= \frac{a^3(283A+326B)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} + \frac{a^3(283A+}{192}$$

$$= \frac{a^{5/2}(283A+326B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+}}\right.}{128d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.84, size = 416, normalized size = 1.64

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(25935*A*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + 28350*B*ArcTanh[Sqrt[1 - Se
c[c + d*x]]] + 11651*A*Sqrt[1 - Sec[c + d*x]] + 9702*B*Sqrt[1 - Sec[c + d*x
]] + 37029*A*Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]] + 35658*B*Cos[c + d*x]*Sqr
t[1 - Sec[c + d*x]] + 12653*A*Cos[2*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 978
6*B*Cos[2*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 3818*A*Cos[3*(c + d*x)]*Sqrt[
1 - Sec[c + d*x]] + 2436*B*Cos[3*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 1002*A
*Cos[4*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 84*B*Cos[4*(c + d*x)]*Sqrt[1 - S
ec[c + d*x]] + 72*A*Cos[5*(c + d*x)]*Sqrt[1 - Sec[c + d*x]] + 21504*B*Hyper
geometric2F1[1/2, 5, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]] + 15360*
A*Hypergeometric2F1[1/2, 6, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]])*
Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(13440*d*(1 + Cos[c + d*x])*Sqrt[1
 - Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 946 vs. $2(226) = 452$.

time = 7.78, size = 947, normalized size = 3.73

method	result	size
default	Expression too large to display	947

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/61440/d*(4245*A*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arctanh(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*
x+c)^4*sin(d*x+c)+4890*B*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arcta
nh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*
cos(d*x+c)^4*sin(d*x+c)+16980*A*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2
)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^
(1/2))*cos(d*x+c)^3*sin(d*x+c)+19560*B*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(9/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*
x+c)*2^(1/2))*cos(d*x+c)^3*sin(d*x+c)+25470*A*2^(1/2)*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(9/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)+29340*B*2^(1/2)*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(9/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)+16980*A*2^(1/2)*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(9/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)*sin(d*x+c)+19560*B*2^(1/2)*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*cos(d*x+c)*sin(d*x+c)+4245*A*2^(1/2
)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+4890*B*2^(1/2)*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(9/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+12288*A*cos(d*x+c)^10+32256
*A*cos(d*x+c)^9+15360*B*cos(d*x+c)^9+27904*A*cos(d*x+c)^8+43520*B*cos(d*x+c
)^8+18112*A*cos(d*x+c)^7+45440*B*cos(d*x+c)^7+45280*A*cos(d*x+c)^6+52160*B*
cos(d*x+c)^6-135840*A*cos(d*x+c)^5-156480*B*cos(d*x+c)^5)*(a*(1+cos(d*x+c))
/cos(d*x+c))^(1/2)/cos(d*x+c)^4/sin(d*x+c)*a^2
```

Maxima [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

[Out] Timed out

Fricas [**A**]
time = 2.47, size = 460, normalized size = 1.81

$$\left[\frac{15\left((283A+326B)a^2\cos(dx+c)+(283A+326B)a^2\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(384Aa^2\cos(dx+c)^5+48(29A+10B)a^2\cos(dx+c)^4+8(283A+230B)a^2\cos(dx+c)^3+10(283A+326B)a^2\cos(dx+c)^2+15(283A+326B)a^2\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{3840\,(d\cos(dx+c)+d)},\ -\frac{15\left((283A+326B)a^2\cos(dx+c)+(283A+326B)a^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(384Aa^2\cos(dx+c)^5+48(29A+10B)a^2\cos(dx+c)^4+8(283A+230B)a^2\cos(dx+c)^3+10(283A+326B)a^2\cos(dx+c)^2+15(283A+326B)a^2\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{1920\,(d\cos(dx+c)+d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] [1/3840*(15*((283*A + 326*B)*a^2*cos(d*x + c) + (283*A + 326*B)*a^2)*sqrt(-
a)*log((2*a*cos(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) +
2*(384*A*a^2*cos(d*x + c)^5 + 48*(29*A + 10*B)*a^2*cos(d*x + c)^4 + 8*(283*
A + 230*B)*a^2*cos(d*x + c)^3 + 10*(283*A + 326*B)*a^2*cos(d*x + c)^2 + 15*
(283*A + 326*B)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
in(d*x + c))/(d*cos(d*x + c) + d), -1/1920*(15*((283*A + 326*B)*a^2*cos(d*x
 + c) + (283*A + 326*B)*a^2)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (384*A*a^2*cos(d*x + c)^5 +
 48*(29*A + 10*B)*a^2*cos(d*x + c)^4 + 8*(283*A + 230*B)*a^2*cos(d*x + c)^3
 + 10*(283*A + 326*B)*a^2*cos(d*x + c)^2 + 15*(283*A + 326*B)*a^2*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
+ d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1377 vs. 2(226) = 452.
time = 2.35, size = 1377, normalized size = 5.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] -1/3840*(15*(283*A*sqrt(-a)*a^2*sgn(cos(d*x + c)) + 326*B*sqrt(-a)*a^2*sgn(
cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
 + 1/2*c)^2 + a))^2 - a*(2*sqrt(2) + 3))) - 15*(283*A*sqrt(-a)*a^2*sgn(cos(
d*x + c)) + 326*B*sqrt(-a)*a^2*sgn(cos(d*x + c)))*log(abs((sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(2) - 3))
) + 4*(4245*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
1/2*c)^2 + a))^18*A*sqrt(-a)*a^3*sgn(cos(d*x + c)) + 4890*sqrt(2)*(sqrt(-a)
*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^18*B*sqrt(-a)*
a^3*sgn(cos(d*x + c)) - 114615*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^16*A*sqrt(-a)*a^4*sgn(cos(d*x + c)) - 132
030*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
 + a))^16*B*sqrt(-a)*a^4*sgn(cos(d*x + c)) + 1298820*sqrt(2)*(sqrt(-a)*tan(
1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^14*A*sqrt(-a)*a^5*s
gn(cos(d*x + c)) + 1319880*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^14*B*sqrt(-a)*a^5*sgn(cos(d*x + c)) - 6176700
*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^12*A*sqrt(-a)*a^6*sgn(cos(d*x + c)) - 6888120*sqrt(2)*(sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^12*B*sqrt(-a)*a^6*sgn(
cos(d*x + c)) + 16394598*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(-a)*a^7*sgn(cos(d*x + c)) + 18352620*
sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a
))^10*B*sqrt(-a)*a^7*sgn(cos(d*x + c)) - 14042770*sqrt(2)*(sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*A*sqrt(-a)*a^8*sgn(c
os(d*x + c)) - 15746180*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a^8*sgn(cos(d*x + c)) + 4791060*sqr
t(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
6*A*sqrt(-a)*a^9*sgn(cos(d*x + c)) + 5497320*sqrt(2)*(sqrt(-a)*tan(1/2*d*x
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqrt(-a)*a^9*sgn(cos(d*
x + c)) - 860300*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a))^4*A*sqrt(-a)*a^10*sgn(cos(d*x + c)) - 959320*sqrt(2)*(s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqr
t(-a)*a^10*sgn(cos(d*x + c)) + 75885*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^11*sgn(cos(d*x + c))
 + 84810*sqrt(2)*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a))^2*B*sqrt(-a)*a^11*sgn(cos(d*x + c)) - 2671*sqrt(2)*A*sqrt(-a)*a
^12*sgn(cos(d*x + c)) - 2990*sqrt(2)*B*sqrt(-a)*a^12*sgn(cos(d*x + c)))/((s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(
sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a +
a^2)^5)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos (c+d\,x)^5\,\left(A+\frac{B}{\cos (c+d\,x)}\right)\,\left(a+\frac{a}{\cos (c+d\,x)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^5*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)
```

3.144 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=202

$$-\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{4(49A-37B)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}+\frac{2(7A-B)\sec^2(c+dx)\tan}{35d\sqrt{a+a\sec(c+d}}$$

```
[Out] -(A-B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2
)/d/a^(1/2)+4/105*(49*A-37*B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/35*(7*A
-B)*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/7*B*sec(d*x+c)^3*tan
(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-2/105*(7*A-31*B)*(a+a*sec(d*x+c))^(1/2)*ta
n(d*x+c)/a/d
```

Rubi [A]
time = 0.42, antiderivative size = 202, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4106, 4095, 4086, 3880, 209}

$$-\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2(7A-B)\tan(c+dx)\sec^2(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}-\frac{2(7A-31B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{105ad}+\frac{4(49A-37B)\tan(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2B\tan(c+dx)\sec^3(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
+ d*x]])])/(Sqrt[a]*d)) + (4*(49*A - 37*B)*Tan[c + d*x])/(105*d*Sqrt[a + a*
Sec[c + d*x]]) + (2*(7*A - B)*Sec[c + d*x]^2*Tan[c + d*x])/(35*d*Sqrt[a + a
*Sec[c + d*x]]) + (2*B*Sec[c + d*x]^3*Tan[c + d*x])/(7*d*Sqrt[a + a*Sec[c +
 d*x]]) - (2*(7*A - 31*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(105*a*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rule 4106

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fre
eQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2B\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{\sec^3(c+dx)\left(3aB+\frac{1}{2}a(7A-B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,d}{7a}$$

$$= \frac{2(7A-B)\sec^2(c+dx)\tan(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2B\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2(7A-B)\sec^2(c+dx)\tan(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2B\sec^3(c+dx)\tan(c+dx)}{7d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{4(49A-37B)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2(7A-B)\sec^2(c+dx)\tan(c+dx)}{35d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{4(49A-37B)\tan(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2(7A-B)\sec^2(c+dx)\tan(c+dx)}{35d\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{4(49A-37B)}{105d\sqrt{a+a}}$$

Mathematica [A]
time = 0.56, size = 140, normalized size = 0.69

$$\frac{\left(-105\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)+2\sqrt{1-\sec(c+dx)}\left(91A-43B+(-7A+31B)\sec(c+dx)+3(7A-B)\sec^2(c+dx)+15B\sec^3(c+dx)\right)\right)\tan(c+dx)}{105d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((-105*Sqrt[2]*(A - B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*Sqrt[1 -
 Sec[c + d*x]]*(91*A - 43*B + (-7*A + 31*B)*Sec[c + d*x] + 3*(7*A - B)*Sec[
c + d*x]^2 + 15*B*Sec[c + d*x]^3))*Tan[c + d*x])/(105*d*Sqrt[1 - Sec[c + d*
x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 784 vs. $2(177) = 354$.
time = 4.35, size = 785, normalized size = 3.89

method	result

default	$\frac{\left(105A\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}\left(\cos^{3}(dx+c)\right)\sin(dx+c)\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)-105B\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}(\cos}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/840/d*(105*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^3*sin(d*x+c)
*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+
c))-105*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^3*sin(d*x+c)*ln((
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+3
15*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^2*sin(d*x+c)*ln(((-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-315*B*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)^2*sin(d*x+c)*ln(((-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+315*A*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)*sin(d*x+c)*ln(((-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-315*B*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(7/2)*cos(d*x+c)*sin(d*x+c)*ln(((-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+105*A*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(7/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d
*x+c)+1)/sin(d*x+c))*sin(d*x+c)-105*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)*
ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c
))*sin(d*x+c)-1456*A*cos(d*x+c)^4+688*B*cos(d*x+c)^4+1568*A*cos(d*x+c)^3-11
84*B*cos(d*x+c)^3-448*A*cos(d*x+c)^2+544*B*cos(d*x+c)^2+336*A*cos(d*x+c)-28
8*B*cos(d*x+c)+240*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^3/sin(
d*x+c)/a
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^4/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A]

time = 1.67, size = 432, normalized size = 2.14

$$\left[-\frac{105\sqrt{2}\left((A-B)a\cos(dx+c)^{4}+(A-B)a\cos(dx+c)^{3}\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^{2}-2\cos(dx+c)+1}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)-4\left((91A-43B)\cos(dx+c)^{3}-(7A-31B)\cos(dx+c)^{2}+3(7A-B)\cos(dx+c)+15B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{210\left(ad\cos(dx+c)^{4}+ad\cos(dx+c)^{3}\right)},\ \frac{2\left((91A-43B)\cos(dx+c)^{3}-(7A-31B)\cos(dx+c)^{2}+3(7A-B)\cos(dx+c)+15B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)+\frac{105\sqrt{2}\left((A-B)a\cos(dx+c)^{4}+(A-B)a\cos(dx+c)^{3}\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}}{105\left(ad\cos(dx+c)^{4}+ad\cos(dx+c)^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/210*(105*sqrt(2)*((A - B)*a*cos(d*x + c)^4 + (A - B)*a*cos(d*x + c)^3)*
sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/
a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(
d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((91*A - 43*B)*cos(d*x + c)^3 - (7*A
- 31*B)*cos(d*x + c)^2 + 3*(7*A - B)*cos(d*x + c) + 15*B)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)
^3), 1/105*(2*((91*A - 43*B)*cos(d*x + c)^3 - (7*A - 31*B)*cos(d*x + c)^2 +
 3*(7*A - B)*cos(d*x + c) + 15*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
in(d*x + c) + 105*sqrt(2)*((A - B)*a*cos(d*x + c)^4 + (A - B)*a*cos(d*x + c
)^3)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(s
qrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^4 + a*d*cos(d*x + c)^3)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec^4(c + dx)}{\sqrt{a(\sec(c + dx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/sqrt(a*(sec(c + d*x) + 1)), x
)
```

Giac [A]
time = 1.59, size = 252, normalized size = 1.25

$$\frac{\frac{105\sqrt{2}(A-B)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))} - \frac{2\left(\frac{105\sqrt{2}Aa^3}{\mathrm{sgn}(\cos(dx+c))} - \left(\left(\frac{\sqrt{2}(119Aa^3-92Ba^3)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\mathrm{sgn}(\cos(dx+c))} - \frac{7\sqrt{2}(37Aa^3-16Ba^3)}{\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + \frac{35\sqrt{2}(7Aa^3-4Ba^3)}{\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^3\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/105*(105*sqrt(2)*(A - B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(cos(d*x + c))) - 2*(105*sqrt(2
)*A*a^3/sgn(cos(d*x + c)) - ((sqrt(2)*(119*A*a^3 - 92*B*a^3)*tan(1/2*d*x +
1/2*c)^2/sgn(cos(d*x + c)) - 7*sqrt(2)*(37*A*a^3 - 16*B*a^3)/sgn(cos(d*x +
c)))*tan(1/2*d*x + 1/2*c)^2 + 35*sqrt(2)*(7*A*a^3 - 4*B*a^3)/sgn(cos(d*x +
c)))*tan(1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^
2 - a)^3*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^4\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/2)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(1/2)), x)
```

3.145 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=159

$$\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{4(5A-7B)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{2B\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}}+\cdots$$

```
[Out] (A-B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)
/d/a^(1/2)-4/15*(5*A-7*B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/5*B*sec(d*x
+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/15*(5*A-B)*(a+a*sec(d*x+c))^(1/
2)*tan(d*x+c)/a/d
```

Rubi [A]
time = 0.28, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4106, 4095, 4086, 3880, 209}

$$\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2(5A-B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{15ad}-\frac{4(5A-7B)\tan(c+dx)}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2B\tan(c+dx)\sec^2(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(Sqrt[a]*d) - (4*(5*A - 7*B)*Tan[c + d*x])/(15*d*Sqrt[a + a*Sec[c
+ d*x]]) + (2*B*Sec[c + d*x]^2*Tan[c + d*x])/(5*d*Sqrt[a + a*Sec[c + d*x]])
 + (2*(5*A - B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(15*a*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
```

```
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rule 4106

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fre
eQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2B\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{\sec^2(c+dx)\left(2aB+\frac{1}{2}a(5A-B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}}{5a}$$

$$= \frac{2B\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2(5A-B)\sqrt{a+a\sec(c+dx)}\ \text{ta}}{15ad}$$

$$= -\frac{4(5A-7B)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2B\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2(5A}{}$$

$$= -\frac{4(5A-7B)\tan(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2B\sec^2(c+dx)\tan(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{2(5A}{}$$

$$= \frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{4(5A-7B)\,\text{ta}}{15d\sqrt{a+a\,\text{se}}}$$

Mathematica [A]
time = 0.40, size = 123, normalized size = 0.77

$$\frac{\left(15\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)+2\sqrt{1-\sec(c+dx)}\,(-5A+13B+(5A-B)\sec(c+dx)+3B\sec^2(c+dx))\right)\tan(c+dx)}{15d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((15*Sqrt[2]*(A - B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*Sqrt[1 - S
ec[c + d*x]]*(-5*A + 13*B + (5*A - B)*Sec[c + d*x] + 3*B*Sec[c + d*x]^2))*T
an[c + d*x])/(15*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 594 vs. $2(138) = 276$.
time = 4.31, size = 595, normalized size = 3.74

method	result
default	$\frac{\left(15A\sin(dx+c)\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}(\cos^2(dx+c))-15B\sin(dx+c)\ln\left(\frac{\sqrt{-\frac{2\cos(}{1+\cos}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/60/d*(15*A*sin(d*x+c)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^
2-15*B*sin(d*x+c)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d
*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2+30*A
*sin(d*x+c)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+
1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)-30*B*sin(d*x
+c)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d
*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)+15*A*ln(((-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)-15*B*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(5/2)*sin(d*x+c)+40*A*cos(d*x+c)^3-104*B*cos(d*x+c)^3-80*A*cos(d*x+c)^2+112
*B*cos(d*x+c)^2+40*A*cos(d*x+c)-32*B*cos(d*x+c)+24*B)*(a*(1+cos(d*x+c))/cos
(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^3/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A]
time = 1.53, size = 397, normalized size = 2.50

$$\left[-\frac{15 \sqrt{2} \left((A-B) a \cos\left(dx+c\right)^{3} + (A-B) a \cos\left(dx+c\right)^{2}\right) \sqrt{-\frac{1}{a}} \log\left(\frac{2 \sqrt{2} \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \sqrt{-\frac{1}{a}} \cos\left(dx+c\right) \sin\left(dx+c\right) + 3 \cos\left(dx+c\right)^{2} + 2 \cos\left(dx+c\right) - 1}{\cos\left(dx+c\right)^{2} + 2 \cos\left(dx+c\right) + 1}\right) + 4 \left((5 A - 13 B) \cos\left(dx+c\right)^{2} - (5 A - B) \cos\left(dx+c\right) - 3 B\right) \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \sin\left(dx+c\right)}{30 \left(a d \cos\left(dx+c\right)^{3} + a d \cos\left(dx+c\right)^{2}\right)}, -\frac{2 \left((5 A - 13 B) \cos\left(dx+c\right)^{2} - (5 A - B) \cos\left(dx+c\right) - 3 B\right) \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \sin\left(dx+c\right) + \frac{15 \sqrt{2} \left((A-B) a \cos\left(dx+c\right)^{3} + (A-B) a \cos\left(dx+c\right)^{2}\right) \arctan\left(\frac{\sqrt{2} \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \cos\left(dx+c\right)}{\sqrt{a} \sin\left(dx+c\right)}\right)}{\sqrt{a}}}{15 \left(a d \cos\left(dx+c\right)^{3} + a d \cos\left(dx+c\right)^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/30*(15*sqrt(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*x + c)^2)*sq
rt(-1/a)*log((2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*
cos(d*x + c)*sin(d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x
 + c)^2 + 2*cos(d*x + c) + 1)) + 4*((5*A - 13*B)*cos(d*x + c)^2 - (5*A - B)
*cos(d*x + c) - 3*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/
(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2), -1/15*(2*((5*A - 13*B)*cos(d*x +
 c)^2 - (5*A - B)*cos(d*x + c) - 3*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c) + 15*sqrt(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*x
+ c)^2)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B \sec\left(c + dx\right)\right) \sec^{3}\left(c + dx\right)}{\sqrt{a \left(\sec\left(c + dx\right) + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/sqrt(a*(sec(c + d*x) + 1)), x
)
```

Giac [A]
time = 1.46, size = 229, normalized size = 1.44

$$-\frac{\frac{15 \left(\sqrt{2} A - \sqrt{2} B\right) \log\left(\left|-\sqrt{-a} \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right) + \sqrt{-a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{2} + a}\right|\right)}{\sqrt{-a} \mathrm{sgn}\left(\cos\left(dx+c\right)\right)} - \frac{2 \left(\left(\left(10 \sqrt{2} A a^{2} \mathrm{sgn}\left(\cos\left(dx+c\right)\right) - 20 \sqrt{2} B a^{2} \mathrm{sgn}\left(\cos\left(dx+c\right)\right) - \left(10 \sqrt{2} A a^{2} \mathrm{sgn}\left(\cos\left(dx+c\right)\right) - 17 \sqrt{2} B a^{2} \mathrm{sgn}\left(\cos\left(dx+c\right)\right)\right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{2}\right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{2} + \frac{15 \sqrt{2} B a^{2}}{\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}\right) \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)}{\left(a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{2} - a\right)^{2} \sqrt{-a \tan\left(\frac{1}{2} dx + \frac{1}{2} c\right)^{2} + a}}}{15 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] -1/15*(15*(sqrt(2)*A - sqrt(2)*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) +
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(cos(d*x + c))) - 2*((10
*sqrt(2)*A*a^2*sgn(cos(d*x + c)) - 20*sqrt(2)*B*a^2*sgn(cos(d*x + c)) - (10
*sqrt(2)*A*a^2*sgn(cos(d*x + c)) - 17*sqrt(2)*B*a^2*sgn(cos(d*x + c)))*tan(
1/2*d*x + 1/2*c)^2)*tan(1/2*d*x + 1/2*c)^2 + 15*sqrt(2)*B*a^2/sgn(cos(d*x +
 c)))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^3\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(1/2)), x)
```

3.146 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=118

$$-\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2(3A-2B)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2B\sqrt{a+a\sec(c+dx)}\,\tan}{3ad}$$

```
[Out] -(A-B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2
)/d/a^(1/2)+2/3*(3*A-2*B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/3*B*(a+a*se
c(d*x+c))^(1/2)*tan(d*x+c)/a/d
```

Rubi [A]
time = 0.17, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {4095, 4086, 3880, 209}

$$-\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2(3A-2B)\tan(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2B\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
+ d*x]])])/(Sqrt[a]*d)) + (2*(3*A - 2*B)*Tan[c + d*x])/(3*d*Sqrt[a + a*Sec[
c + d*x]]) + (2*B*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(3*a*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
```

```
1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2B\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3ad} + \frac{2\int \frac{\sec(c+dx)\left(\frac{aB}{2}+\frac{1}{2}a(3A-2B)\sec\right.}{\sqrt{a+a\sec(c+dx}}}{3a}$$

$$= \frac{2(3A-2B)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2B\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3ad} +$$

$$= \frac{2(3A-2B)\tan(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2B\sqrt{a+a\sec(c+dx)}\,\tan(c+dx)}{3ad} +$$

$$= -\frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2(3A-2B)\,\mathrm{ta}}{3d\sqrt{a+a\sec}}$$

Mathematica [A]
time = 0.31, size = 106, normalized size = 0.90

$$\frac{\left(-3\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)+2\sqrt{1-\sec(c+dx)}\,(3A-B+B\sec(c+dx))\right)\tan(c+dx)}{3d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((-3*Sqrt[2]*(A - B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*Sqrt[1 - S
ec[c + d*x]]*(3*A - B + B*Sec[c + d*x]))*Tan[c + d*x])/(3*d*Sqrt[1 - Sec[c
+ d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(101) = 202$.

time = 4.16, size = 405, normalized size = 3.43

method	result
default	$\frac{\left(3A\sin(dx+c)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\cos(dx+c)\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)-3B\sin(dx+c)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\mathrm{c}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/6/d*(3*A*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*ln(((
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-3*
B*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*ln(((-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+3*A*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(3/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)-3*B*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(3/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)
/sin(d*x+c))*sin(d*x+c)-12*A*cos(d*x+c)^2+4*B*cos(d*x+c)^2+12*A*cos(d*x+c)-
8*B*cos(d*x+c)+4*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+
c)/a
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A]

time = 2.40, size = 352, normalized size = 2.98

$$\left[-\frac{3\sqrt{2}\left((A-B)a\cos\left(dx+c\right)^{2}+(A-B)a\cos\left(dx+c\right)\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^{2}-2\cos(dx+c)+1}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)-4\left((3A-B)\cos\left(dx+c\right)+B\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{6\left(ad\cos\left(dx+c\right)^{2}+ad\cos\left(dx+c\right)\right)}, \frac{2\left((3A-B)\cos\left(dx+c\right)+B\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)+\frac{3\sqrt{2}\left((A-B)a\cos(dx+c)^{2}+(A-B)a\cos(dx+c)\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}}{3\left(ad\cos\left(dx+c\right)^{2}+ad\cos\left(dx+c\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm ="fricas")

[Out] [-1/6*(3*sqrt(2)*((A - B)*a*cos(d*x + c)^2 + (A - B)*a*cos(d*x + c))*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((3*A - B)*cos(d*x + c) + B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)), 1/3*(2*((3*A - B)*cos(d*x + c) + B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) + 3*sqrt(2)*((A - B)*a*cos(d*x + c)^2 + (A - B)*a*cos(d*x + c))*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec^2(c + dx)}{\sqrt{a(\sec(c + dx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x)

Giac [A]
time = 1.35, size = 165, normalized size = 1.40

$$\frac{\frac{3\sqrt{2}(A-B)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))}+\frac{2\left(\frac{\sqrt{2}(3Aa-2Ba)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{\mathrm{sgn}(\cos(dx+c))}-\frac{3\sqrt{2}Aa}{\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm ="giac")

[Out] 1/3*(3*sqrt(2)*(A - B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*sgn(cos(d*x + c))) + 2*(sqrt(2)*(3*A*a - 2*B*a)*tan(1/2*d*x + 1/2*c)^2/sgn(cos(d*x + c)) - 3*sqrt(2)*A*a/sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/d

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+d\,x)}}{\cos(c + d\,x)^2\,\sqrt{a + \frac{a}{\cos(c + d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/2)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(1/2)), x)
```

3.147 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2B\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] (A-B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)
/d/a^(1/2)+2*B*tan(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {4086, 3880, 209}

$$\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2B\tan(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
d*x]])])/(Sqrt[a]*d) + (2*B*Tan[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
```

```
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2B\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + (A-B)\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$

$$= \frac{2B\tan(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(2(A-B))\text{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{\cdots}{\sqrt{a+\cdots}}\right)}{d}$$

$$= \frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2B\tan(c+\cdots}{d\sqrt{a+a\sec(c\cdots}}$$

Mathematica [A]
time = 0.17, size = 88, normalized size = 1.13

$$\frac{\left(\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)+2B\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((Sqrt[2]*(A - B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + 2*B*Sqrt[1 - Se
c[c + d*x]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*
x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 199 vs. 2(67) = 134.
time = 4.30, size = 200, normalized size = 2.56

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(A\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-B\ln\left(\frac{\sqrt{-\frac{2\cos(d\cdots}{1+\cos(d\cdots}}\cdots}{\cdots}\right.\cdots\right.}{d\sin(dx+c)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(A*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*sin(d*x+c)-B*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-co
s(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*B
*cos(d*x+c)+2*B)/sin(d*x+c)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A]
time = 2.78, size = 287, normalized size = 3.68

$$\left[-\frac{\sqrt{2}\,((A-B)a\cos(dx+c)+(A-B)a)\sqrt{-\frac{1}{a}}\,\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4B\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{2\,(ad\cos(dx+c)+ad)},\ \frac{2B\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)-\frac{\sqrt{2}\,((A-B)a\cos(dx+c)+(A-B)a)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}}{ad\cos(dx+c)+ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] [-1/2*(sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*log((2*sqrt(
2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x
+ c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x +
 c) + 1)) - 4*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*
cos(d*x + c) + a*d), (2*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c) - sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*arctan(sqrt(2)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/sqrt(a)
)/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\sec(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [A]
time = 1.23, size = 131, normalized size = 1.68

$$-\frac{\frac{2\sqrt{2}\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\mathrm{sgn}(\cos(dx+c))}+\frac{\left(\sqrt{2}A-\sqrt{2}B\right)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] -(2*sqrt(2)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*B*tan(1/2*d*x + 1/2*c)/((a*
tan(1/2*d*x + 1/2*c)^2 - a)*sgn(cos(d*x + c))) + (sqrt(2)*A - sqrt(2)*B)*lo
g(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
)/(sqrt(-a)*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + a/cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + a/cos(c + d*x))^(1/2)), x)
```

3.148 $\int \frac{A+B\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=91

$$\frac{2A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*A*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-(A-B)*arcta
n(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A]

time = 0.07, antiderivative size = 91, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.160$, Rules used = {4005, 3859, 209, 3880}

$$\frac{2A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d) -
 (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c +
 d*x]])])/(Sqrt[a]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{A\int \sqrt{a+a\sec(c+dx)}\,dx}{a} - (A-B)\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx$$
$$= -\frac{(2A)\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{(2(A-B))\mathrm{Subst}\left(\int \frac{}{2}\right.}{}$$
$$= \frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}(A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c-}}\right.}{\sqrt{a}\,d}$$

Mathematica [A]
time = 0.29, size = 92, normalized size = 1.01

$$\frac{2\left(\sqrt{2}A\mathrm{ArcSin}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+(-A+B)\mathrm{ArcTan}\left(\frac{\sin\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\cos(c+dx)}}\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{\cos(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*(Sqrt[2]*A*ArcSin[Sqrt[2]*Sin[(c + d*x)/2]] + (-A + B)*ArcTan[Sin[(c + d
*x)/2]/Sqrt[Cos[c + d*x]]])*Cos[(c + d*x)/2])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a*
(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 193 vs. $2(76) = 152$.
time = 4.15, size = 194, normalized size = 2.13

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\left(A\sqrt{2}\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)+A\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c}{1+\cos(dx+}}}{s}\right.\right.}{da}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*(A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/c
os(d*x+c)*2^(1/2))+A*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-co
s(d*x+c)+1)/sin(d*x+c))-B*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)-cos(d*x+c)+1)/sin(d*x+c)))/a
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima:
 sign: argument cannot be imaginary; found %i
```

Fricas [A]
time = 4.58, size = 307, normalized size = 3.37

$$\left[-\frac{\sqrt{2}\,(A-B)a\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+2A\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{2ad},\ \frac{\sqrt{2}\,(A-B)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-2A\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*(A - B)*a*sqrt(-1/a)*log(-(2*sqrt(2)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c) - 3*cos(d*x + c)^2 -
2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 2*A*sqrt(-a)*l
og((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(a*d)
, (sqrt(2)*(A - B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*A*sqrt(a)*arctan(sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+B\sec(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))/sqrt(a*(sec(c + d*x) + 1)), x)

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="giac")

[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(1/2),x)

[Out] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(1/2), x)

3.149 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=119

$$-\frac{(A-2B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{A\sin(c}{d\sqrt{a+a\,\text{se}}}$$

```
[Out] -(A-2*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+(A-B)*
arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(
1/2)+A*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4107, 4005, 3859, 209, 3880}

$$-\frac{(A-2B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{A\sin(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -(((A - 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt
[a]*d)) + (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
a*Sec[c + d*x]])])/(Sqrt[a]*d) + (A*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x
]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
```

```
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{A\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{-\frac{1}{2}a(A-2B)+\frac{1}{2}aA\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{a}$$

$$= \frac{A\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(A-2B)\int \sqrt{a+a\sec(c+dx)}\,dx}{2a} + (A$$

$$= \frac{A\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{(A-2B)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan}{\sqrt{a+a\sec}}\right.}{d}$$

$$= -\frac{(A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\,(A-B)\tan^{-1}\left(\right.}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 26.85, size = 11162, normalized size = 93.80

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

[Out] Result too large to show

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. 2(102) = 204.
time = 7.61, size = 353, normalized size = 2.97

method	result
default	$\frac{\left(A\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sin(dx+c)-2B\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\operatorname{arctanh}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/2/d*(A*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-2*B*2
^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)+2*A*ln(((-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*B*ln(((-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)-2*A*cos(d*x+c)^2+2*A*cos(d*x+c))*(a*(1+cos(d*x+c))/cos(
d*x+c))^(1/2)/sin(d*x+c)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A]
time = 4.59, size = 458, normalized size = 3.85

$$\left[\frac{2A\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-\sqrt{2}\left((A-B)a\cos(dx+c)+(A-B)a\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\left((A-2B)\cos(dx+c)+A-2B\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{2\left(ad\cos(dx+c)+ad\right)},\ \frac{A\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+\left((A-2B)\cos(dx+c)+A-2B\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\frac{\sqrt{2}\left((A-B)a\cos(dx+c)+(A-B)a\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}}{ad\cos(dx+c)+ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/2*(2*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c)
 - sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*log((2*sqrt(2)*s
qrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d*x + c)
 + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d*x + c)
+ 1)) + ((A - 2*B)*cos(d*x + c) + A - 2*B)*sqrt(-a)*log((2*a*cos(d*x + c)^2
 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
+ c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)))/(a*d*cos(d*x + c) + a*d), (
A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + ((A -
 2*B)*cos(d*x + c) + A - 2*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - sqrt(2)*((A - B)*a*cos(d*x
 + c) + (A - B)*a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*c
os(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\cos(c + dx)}{\sqrt{a(\sec(c + dx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)/sqrt(a*(sec(c + d*x) + 1)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 365 vs. 2(102) = 204.
time = 1.85, size = 365, normalized size = 3.07

$$-\frac{\frac{\sqrt{2}(A-B)\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))}+\frac{(A-2B)\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))}-\frac{(A-2B)\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+a\left(2\sqrt{2}-3\right)\right|\right)}{\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))}+\frac{4\sqrt{2}\left(3\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2A\sqrt{-a}-A\sqrt{-a}\,a\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2a+a^2\right)\mathrm{sgn}(\cos(dx+c))}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] -1/2*(sqrt(2)*(A - B)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*sgn(cos(d*x + c))) + (A - 2*B)*log(abs((s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - a*(
2*sqrt(2) + 3)))/(sqrt(-a)*sgn(cos(d*x + c))) - (A - 2*B)*log(abs((sqrt(-a)
*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*sqrt(
2) - 3)))/(sqrt(-a)*sgn(cos(d*x + c))) + 4*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*x
 + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a) - A*sqrt(-a)*
a)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
```

```
4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
^2*a + a^2)*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2), x)
```

3.150 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=165

$$\frac{(7A-4B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{a}\,d}-\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{(A-4B)}{4d\sqrt{a+a}}$$

[Out] 1/4*(7*A-4*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-(A-B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)-1/4*(A-4*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*A*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)

Rubi [A]

time = 0.25, antiderivative size = 165, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4107, 4005, 3859, 209, 3880}

$$\frac{(7A-4B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}\,d}-\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{(A-4B)\sin(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{A\sin(c+dx)\cos(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]

[Out] ((7*A - 4*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*Sqrt[a]*d) - (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) - ((A - 4*B)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (A*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*ArcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d), Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
```

```
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\cos(c+dx)\left(-\frac{1}{2}a(A-4B)+\frac{3}{2}aA\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a}$$

$$= -\frac{(A-4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\frac{1}{4}a^2(7A-}{\sqrt{a}}}{}$$

$$= -\frac{(A-4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{(7A-4B}{}$$

$$= -\frac{(A-4B)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} - \frac{(7A-4B}{}$$

$$= \frac{(7A-4B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{a}\,d} - \frac{\sqrt{2}\,(A-B)\tan^{-1}\Big(}{}$$

Mathematica [A]
time = 0.44, size = 135, normalized size = 0.82

$$\frac{\left((7A-4B)\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)-4\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)+\cos(c+dx)(-A+4B+2A\cos(c+dx))\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)}{4d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (((7*A - 4*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]] - 4*Sqrt[2]*(A - B)*ArcTanh[S
qrt[1 - Sec[c + d*x]]/Sqrt[2]] + Cos[c + d*x]*(-A + 4*B + 2*A*Cos[c + d*x])
*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(
1 + Sec[c + d*x])])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 716 vs. $2(140) = 280$.
time = 8.83, size = 717, normalized size = 4.35

method	result
default	$\frac{\left(7A\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sin(dx+c)\cos(dx+c)\sqrt{2}-4B\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\operatorname{arctanh}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/16/d*(7*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c)
*2^(1/2)-4*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)*cos(d*x+c
)*2^(1/2)+7*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)+
8*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)-4*B*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)-8*B*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(3/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)*sin(d*x+c)+8*A*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(3/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(
d*x+c)-1)/sin(d*x+c))*sin(d*x+c)-8*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*l
n(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+
c))*sin(d*x+c)-8*A*cos(d*x+c)^4+12*A*cos(d*x+c)^3-16*B*cos(d*x+c)^3-4*A*cos
(d*x+c)^2+16*B*cos(d*x+c)^2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)
/sin(d*x+c)/a
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A]
time = 6.14, size = 506, normalized size = 3.07

$$\left[-\frac{4\sqrt{2}\left((A-B)a\cos(dx+c)+(A-B)a\right)\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)-3\cos(dx+c)^2-2\cos(dx+c)+1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-\left((7A-4B)\cos(dx+c)+7A-4B\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2\left(2A\cos(dx+c)^2-(A-4B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{8\left(ad\cos(dx+c)+ad\right)},\ -\frac{\left((7A-4B)\cos(dx+c)+7A-4B\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(2A\cos(dx+c)^2-(A-4B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)-\frac{4\sqrt{2}\left((A-B)a\cos(dx+c)+(A-B)a\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}}{4\left(ad\cos(dx+c)+ad\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/8*(4*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*log(-(2*sq
rt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(d
*x + c) - 3*cos(d*x + c)^2 - 2*cos(d*x + c) + 1)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) - ((7*A - 4*B)*cos(d*x + c) + 7*A - 4*B)*sqrt(-a)*log((2*a*cos
(d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(2*A*cos(d*x
+ c)^2 - (A - 4*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c))/(a*d*cos(d*x + c) + a*d), -1/4*(((7*A - 4*B)*cos(d*x + c) + 7*A
 - 4*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)
/(sqrt(a)*sin(d*x + c))) - (2*A*cos(d*x + c)^2 - (A - 4*B)*cos(d*x + c))*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 4*sqrt(2)*((A - B)*a*c
os(d*x + c) + (A - B)*a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d)
]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\cos^2(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/sqrt(a*(sec(c + d*x) + 1)), x
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 621 vs. 2(140) = 280.
time = 1.93, size = 621, normalized size = 3.76

$$\frac{\frac{4\sqrt{2}(A-B)\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))}+\frac{(7A-4B)\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-a\left(2\sqrt{2}+3\right)\right|\right)}{\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))}-\frac{(7A-4B)\log\left(\left|\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+a\left(2\sqrt{2}-3\right)\right|\right)}{\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))}+\frac{4\sqrt{2}\left(17\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^6A\sqrt{-a}-12\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^6B\sqrt{-a}-57\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4A\sqrt{-a}\,a+76\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4B\sqrt{-a}\,a+19\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2A\sqrt{-a}\,a^2-36\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2B\sqrt{-a}\,a^2-3A\sqrt{-a}\,a^3+4B\sqrt{-a}\,a^3\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2a+a^2\right)^2\mathrm{sgn}(\cos(dx+c))}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/8*(4*sqrt(2)*(A - B)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2
*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*sgn(cos(d*x + c))) + (7*A - 4*B)*log(abs
((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 -
a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(cos(d*x + c))) - (7*A - 4*B)*log(abs((sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + a*(2*
sqrt(2) - 3)))/(sqrt(-a)*sgn(cos(d*x + c))) + 4*sqrt(2)*(17*(sqrt(-a)*tan(1
/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a) - 12*(s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*sqr
t(-a) - 57*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^4*A*sqrt(-a)*a + 76*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a + 19*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^2 - 36*(sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*sqrt(-a)*a^2 - 3*A
*sqrt(-a)*a^3 + 4*B*sqrt(-a)*a^3)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-
a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(
-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2), x)
```

3.151 $\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=206

$$-\frac{(9A-14B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8\sqrt{a}\,d}+\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{(7A-2\ldots}{8d\sqrt{a+\ldots}}$$

```
[Out] -1/8*(9*A-14*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)
+(A-B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2
)/d/a^(1/2)+1/8*(7*A-2*B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-1/12*(A-6*B)*
cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/3*A*cos(d*x+c)^2*sin(d*x+c
)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.38, antiderivative size = 206, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4107, 4005, 3859, 209, 3880}

$$-\frac{(9A-14B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8\sqrt{a}\,d}+\frac{\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{(7A-2B)\sin(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}-\frac{(A-6B)\sin(c+dx)\cos(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{A\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -1/8*((9*A - 14*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])
/(Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt
[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + ((7*A - 2*B)*Sin[c + d*x])/(8*d*Sqrt[
a + a*Sec[c + d*x]]) - ((A - 6*B)*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a +
 a*Sec[c + d*x]]) + (A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c +
 d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{A\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\cos^2(c+dx)\left(-\frac{1}{2}a(A-6B)+\frac{5}{2}aA\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,d}{3a}$$

$$= -\frac{(A-6B)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(7A-2B)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} - \frac{(A-6B)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A}{3}$$

$$= \frac{(7A-2B)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} - \frac{(A-6B)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A}{3}$$

$$= \frac{(7A-2B)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} - \frac{(A-6B)\cos(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{A}{3}$$

$$= -\frac{(9A-14B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8\sqrt{a}\,d} + \frac{\sqrt{2}\,(A-B)\tan}{}$$

Mathematica [A]
time = 0.70, size = 150, normalized size = 0.73

$$\frac{\left((-27A+42B)\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)+24\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)+\cos(c+dx)\,(21A-6B-2(A-6B)\cos(c+dx)+8A\cos^2(c+dx))\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)}{24d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (((-27*A + 42*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]] + 24*Sqrt[2]*(A - B)*ArcTa
nh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] + Cos[c + d*x]*(21*A - 6*B - 2*(A - 6*B)
*Cos[c + d*x] + 8*A*Cos[c + d*x]^2)*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(
24*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1066 vs. 2(177) = 354.
time = 9.02, size = 1067, normalized size = 5.18

method	result	size
default	Expression too large to display	1067

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/192/d*(27*A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^
2*sin(d*x+c)-42*B*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x
+c)^2*sin(d*x+c)+54*A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos
(d*x+c)*sin(d*x+c)+48*A*sin(d*x+c)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*
cos(d*x+c)^2-84*B*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x
+c)*sin(d*x+c)-48*B*sin(d*x+c)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(
d*x+c)^2+27*A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)+
96*A*sin(d*x+c)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x
+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)-42*B*2^(
1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)
*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)-96*B*sin(d*x+c)*l
n(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c)
)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)+48*A*ln(((-2*cos(d*x+c)/(
```

```
1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(5/2)*sin(d*x+c)-48*B*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)
*sin(d*x+c)-64*A*cos(d*x+c)^6+80*A*cos(d*x+c)^5-96*B*cos(d*x+c)^5-184*A*cos
(d*x+c)^4+144*B*cos(d*x+c)^4+168*A*cos(d*x+c)^3-48*B*cos(d*x+c)^3)*(a*(1+co
s(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^2/sin(d*x+c)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^3/sqrt(a*sec(d*x + c) + a), x)
```

Fricas [A]
time = 6.57, size = 539, normalized size = 2.62

$$\left[-\frac{24\sqrt{2}\left((A-B)a\cos(dx+c)+(A-B)a\right)\sqrt{-\frac{1}{a}}\log\left(\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\cos(dx+c)\sin(dx+c)+3\cos(dx+c)^2+2\cos(dx+c)-1}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-3\left((9A-14B)\cos(dx+c)+9A-14B\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-2\left(8A\cos(dx+c)^3-2(A-6B)\cos(dx+c)^2+3(7A-2B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{48\left(ad\cos(dx+c)+ad\right)}, \frac{3\left((9A-14B)\cos(dx+c)+9A-14B\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+\left(8A\cos(dx+c)^3-2(A-6B)\cos(dx+c)^2+3(7A-2B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)-\frac{24\sqrt{2}\left((A-B)a\cos(dx+c)+(A-B)a\right)\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{\sqrt{a}}}{24\left(ad\cos(dx+c)+ad\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] [-1/48*(24*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*log((2*s
qrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*cos(d*x + c)*sin(
d*x + c) + 3*cos(d*x + c)^2 + 2*cos(d*x + c) - 1)/(cos(d*x + c)^2 + 2*cos(d
*x + c) + 1)) - 3*((9*A - 14*B)*cos(d*x + c) + 9*A - 14*B)*sqrt(-a)*log((2*
a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d
*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) - 2*(8*A*cos
(d*x + c)^3 - 2*(A - 6*B)*cos(d*x + c)^2 + 3*(7*A - 2*B)*cos(d*x + c))*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d),
 1/24*(3*((9*A - 14*B)*cos(d*x + c) + 9*A - 14*B)*sqrt(a)*arctan(sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + (8*A*c
os(d*x + c)^3 - 2*(A - 6*B)*cos(d*x + c)^2 + 3*(7*A - 2*B)*cos(d*x + c))*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c) - 24*sqrt(2)*((A - B)*a*
cos(d*x + c) + (A - B)*a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))/sqrt(a))/(a*d*cos(d*x + c) + a*d
)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\cos^3(c+dx)}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**3/sqrt(a*(sec(c + d*x) + 1)), x
)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 818 vs. 2(177) = 354.
time = 2.04, size = 818, normalized size = 3.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] -1/48*(24*sqrt(2)*(A - B)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*sgn(cos(d*x + c))) + 3*(9*A - 14*B)*l
og(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)
)^2 - a*(2*sqrt(2) + 3)))/(sqrt(-a)*sgn(cos(d*x + c))) - 3*(9*A - 14*B)*log
(abs((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
2 + a*(2*sqrt(2) - 3)))/(sqrt(-a)*sgn(cos(d*x + c))) + 4*sqrt(2)*(165*(sqrt
(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10*A*sqrt(
-a) - 102*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^10*B*sqrt(-a) - 1323*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*
d*x + 1/2*c)^2 + a))^8*A*sqrt(-a)*a + 954*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^8*B*sqrt(-a)*a + 3906*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A*sqrt(-a)*a^2 - 22
68*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*
B*sqrt(-a)*a^2 - 2118*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x
+ 1/2*c)^2 + a))^4*A*sqrt(-a)*a^3 + 1044*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*sqrt(-a)*a^3 + 393*(sqrt(-a)*tan(1/
2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*sqrt(-a)*a^4 - 22
2*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B
*sqrt(-a)*a^4 - 31*A*sqrt(-a)*a^5 + 18*B*sqrt(-a)*a^5)/(((sqrt(-a)*tan(1/2*
d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2
*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3*sgn(cos(d
*x + c))))/d
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^3\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2), x)
```

3.152 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=216

$$\frac{(11A-15B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}-\frac{(65A-93B)\tan(c+}{15ad\sqrt{a+a\sec(c+}}$$

```
[Out] 1/4*(11*A-15*B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2
))/a^(3/2)/d*2^(1/2)+1/2*(A-B)*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c))^(
3/2)-1/15*(65*A-93*B)*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)-1/10*(5*A-9*B)*
sec(d*x+c)^2*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)+1/30*(35*A-39*B)*(a+a*se
c(d*x+c))^(1/2)*tan(d*x+c)/a^2/d
```

Rubi [A]
time = 0.42, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4104, 4106, 4095, 4086, 3880, 209}

$$\frac{(11A-15B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(35A-39B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{30a^2d}+\frac{(A-B)\tan(c+dx)\sec^3(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{(5A-9B)\tan(c+dx)\sec^2(c+dx)}{10ad\sqrt{a\sec(c+dx)+a}}-\frac{(65A-93B)\tan(c+dx)}{15ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((11*A - 15*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sec[c + d*x]^3*Tan[c + d*x])/(2*d*(
a + a*Sec[c + d*x])^(3/2)) - ((65*A - 93*B)*Tan[c + d*x])/(15*a*d*Sqrt[a +
a*Sec[c + d*x]]) - ((5*A - 9*B)*Sec[c + d*x]^2*Tan[c + d*x])/(10*a*d*Sqrt[a
 + a*Sec[c + d*x]]) + ((35*A - 39*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])
/(30*a^2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4106

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fre
eQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& GtQ[n, 1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sec^3(c+dx)\left(3a(A-B)-\frac{1}{2}a(5A-9B)\sec \cdots\right.}{\sqrt{a+a\sec(c+dx)}}}{2a^2} \cdots$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(5A-9B)\sec^2(c+dx)\tan(c \cdots}{10ad\sqrt{a+a\sec(c+dx)}} \cdots$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(5A-9B)\sec^2(c+dx)\tan(c \cdots}{10ad\sqrt{a+a\sec(c+dx)}} \cdots$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(65A-93B)\tan(c+dx)}{15ad\sqrt{a+a\sec(c+dx)}} - \cdots$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(65A-93B)\tan(c+dx)}{15ad\sqrt{a+a\sec(c+dx)}} - \cdots$$

$$= \frac{(11A-15B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} + \frac{(A-B)\sec^3(c \cdots}{2d(a+a\,\mathrm{se} \cdots} \cdots$$

Mathematica [A]
time = 2.41, size = 160, normalized size = 0.74

$$\frac{\left(15\sqrt{2}(11A-15B)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)+\sqrt{1-\sec(c+dx)}\left(-95A+147B-12(5A-9B)\sec(c+dx)+4(5A-3B)\sec^2(c+dx)+12B\sec^3(c+dx)\right)\right)\tan(c+dx)}{30d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),
x]
```

```
[Out] ((15*Sqrt[2]*(11*A - 15*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c +
 d*x)/2]^2*Sec[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(-95*A + 147*B - 12*(5*A -
 9*B)*Sec[c + d*x] + 4*(5*A - 3*B)*Sec[c + d*x]^2 + 12*B*Sec[c + d*x]^3))*T
an[c + d*x])/(30*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 792 vs. 2(189) = 378.
time = 4.34, size = 793, normalized size = 3.67

method	result

default	$-\frac{(-1+\cos(dx+c))\left(165A\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\sin(dx+c)\left(\cos^3(dx+c)\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}-225B\ln\left(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/240/d*(-1+cos(d*x+c))*(165*A*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(5/2)-225*B*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(5/2)+495*A*sin(d*x+c)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*co
s(d*x+c)^2-675*B*sin(d*x+c)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*
x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x
+c)^2+495*A*sin(d*x+c)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-
cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)-6
75*B*sin(d*x+c)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x
+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)+165*A*ln
(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)-225*B*ln(((-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(5/2)*sin(d*x+c)+760*A*cos(d*x+c)^4-1176*B*cos(d*x+c)^4-280*A
*cos(d*x+c)^3+312*B*cos(d*x+c)^3-640*A*cos(d*x+c)^2+960*B*cos(d*x+c)^2+160*
A*cos(d*x+c)-192*B*cos(d*x+c)+96*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin
(d*x+c)^3/cos(d*x+c)^2/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^4/(a*sec(d*x + c) + a)^(3/2), x
)
```

Fricas [A]
time = 2.29, size = 504, normalized size = 2.33

$$\left[\frac{15\sqrt{2}\left((11A-15B)\cos(dx+c)^4+2(11A-15B)\cos(dx+c)^3+(11A-15B)\cos(dx+c)^2\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos(dx+c)^2-2a\cos(dx+c)+a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\left((95A-147B)\cos(dx+c)^3+12(5A-9B)\cos(dx+c)^2-4(5A-3B)\cos(dx+c)-12B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{120\left(a^2d\cos(dx+c)^4+2a^2d\cos(dx+c)^3+a^2d\cos(dx+c)^2\right)}, -\frac{15\sqrt{2}\left((11A-15B)\cos(dx+c)^4+2(11A-15B)\cos(dx+c)^3+(11A-15B)\cos(dx+c)^2\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+2\left((95A-147B)\cos(dx+c)^3+12(5A-9B)\cos(dx+c)^2-4(5A-3B)\cos(dx+c)-12B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{60\left(a^2d\cos(dx+c)^4+2a^2d\cos(dx+c)^3+a^2d\cos(dx+c)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/120*(15*sqrt(2)*((11*A - 15*B)*cos(d*x + c)^4 + 2*(11*A - 15*B)*cos(d*x
+ c)^3 + (11*A - 15*B)*cos(d*x + c)^2)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d
*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) -
4*((95*A - 147*B)*cos(d*x + c)^3 + 12*(5*A - 9*B)*cos(d*x + c)^2 - 4*(5*A -
 3*B)*cos(d*x + c) - 12*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2
), -1/60*(15*sqrt(2)*((11*A - 15*B)*cos(d*x + c)^4 + 2*(11*A - 15*B)*cos(d*
x + c)^3 + (11*A - 15*B)*cos(d*x + c)^2)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*((95*
A - 147*B)*cos(d*x + c)^3 + 12*(5*A - 9*B)*cos(d*x + c)^2 - 4*(5*A - 3*B)*c
os(d*x + c) - 12*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(
a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec^4(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(3/2)
, x)
```

Giac [A]
time = 1.80, size = 277, normalized size = 1.28

$$-\frac{\frac{15\sqrt{2}(11A-15B)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,a\,\mathrm{sgn}(\cos(dx+c))} - \frac{\left(\left(\left(\frac{15\sqrt{2}(Aa^3-Ba^3)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^2\mathrm{sgn}(\cos(dx+c))} - \frac{\sqrt{2}(245Aa^3-381Ba^3)}{a^2\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 + \frac{5\sqrt{2}(73Aa^3-105Ba^3)}{a^2\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - \frac{15\sqrt{2}(9Aa^3-17Ba^3)}{a^2\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] -1/60*(15*sqrt(2)*(11*A - 15*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(cos(d*x + c))) - (((15*
sqrt(2)*(A*a^3 - B*a^3)*tan(1/2*d*x + 1/2*c)^2/(a^2*sgn(cos(d*x + c))) - sq
rt(2)*(245*A*a^3 - 381*B*a^3)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)
```

```
^2 + 5*sqrt(2)*(73*A*a^3 - 105*B*a^3)/(a^2*sgn(cos(d*x + c))))*tan(1/2*d*x
+ 1/2*c)^2 - 15*sqrt(2)*(9*A*a^3 - 17*B*a^3)/(a^2*sgn(cos(d*x + c))))*tan(1
/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)^2*sqrt(-a*tan(1/2*d*x + 1/2
*c)^2 + a)))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^4\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(3/2)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(3/2)), x)
```

3.153 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=171

$$-\frac{(7A-11B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{(9A-13B)\tan(c+}{3ad\sqrt{a+a\sec(c+}$$

```
[Out] -1/4*(7*A-11*B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2
))/a^(3/2)/d*2^(1/2)+1/2*(A-B)*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^(
3/2)+1/3*(9*A-13*B)*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)-1/6*(3*A-7*B)*(a+
a*sec(d*x+c))^(1/2)*tan(d*x+c)/a^2/d
```

Rubi [A]

time = 0.30, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4104, 4095, 4086, 3880, 209}

$$-\frac{(7A-11B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(3A-7B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{6a^2d}+\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+\frac{(9A-13B)\tan(c+dx)}{3ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] -1/2*((7*A - 11*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c
+ d*x]])])/(Sqrt[2]*a^(3/2)*d) + ((A - B)*Sec[c + d*x]^2*Tan[c + d*x])/(2*d
*(a + a*Sec[c + d*x])^(3/2)) + ((9*A - 13*B)*Tan[c + d*x])/(3*a*d*Sqrt[a +
a*Sec[c + d*x]]) - ((3*A - 7*B)*Sqrt[a + a*Sec[c + d*x]]*Tan[c + d*x])/(6*a
^2*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
```

```
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sec^2(c+dx)\left(2a(A-B)-\frac{1}{2}a(3A-7B)\right.}{\sqrt{a+a\sec(c+dx}}}{2a^2}$$

$$= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(3A-7B)\sqrt{a+a\sec(c+d}}{6a^2d}$$

$$= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(9A-13B)\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}} - \frac{(}{}$$

$$= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(9A-13B)\tan(c+dx)}{3ad\sqrt{a+a\sec(c+dx)}} - \frac{(}{}$$

$$= -\frac{(7A-11B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{(A-B)\sec^2}{2d(a+a}$$

Mathematica [A]
time = 1.38, size = 141, normalized size = 0.82

$$\frac{\left(-3\sqrt{2}\,(7A-11B)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)+\sqrt{1-\sec(c+dx)}\,(15A-19B+12(A-B)\sec(c+dx)+4B\sec^2(c+dx))\right)\tan(c+dx)}{6d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),
x]
```

```
[Out] ((-3*Sqrt[2]*(7*A - 11*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c +
d*x)/2]^2*Sec[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(15*A - 19*B + 12*(A - B)*S
ec[c + d*x] + 4*B*Sec[c + d*x]^2))*Tan[c + d*x])/(6*d*Sqrt[1 - Sec[c + d*x]
]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 602 vs. 2(148) = 296.
time = 4.89, size = 603, normalized size = 3.53

method	result
default	$-\frac{(-1+\cos(dx+c))\left(21A\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}(\cos^2(dx+c))\sin(dx+c)-33B\ln\left(\frac{\sqrt{}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/24/d*(-1+cos(d*x+c))*(21*A*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d
*x+c)^2*sin(d*x+c)-33*B*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^
2*sin(d*x+c)+42*A*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c
)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x
+c))-66*B*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*ln(((-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+21*
A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)-33*B*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(3/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos
(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)-60*A*cos(d*x+c)^3+76*B*cos(d*x+c)^3+12*A*
cos(d*x+c)^2-28*B*cos(d*x+c)^2+48*A*cos(d*x+c)-64*B*cos(d*x+c)+16*B)*(a*(1+
cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^3/cos(d*x+c)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^3/(a*sec(d*x + c) + a)^(3/2), x
)
```

Fricas [A]
time = 1.65, size = 459, normalized size = 2.68

$$\left[\frac{3\sqrt{2}\left((7A-11B)\cos(dx+c)^3+2(7A-11B)\cos(dx+c)^2+(7A-11B)\cos(dx+c)\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\left((15A-19B)\cos(dx+c)^2+12(A-B)\cos(dx+c)+4B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{24\left(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2+a^2d\cos(dx+c)\right)}, \frac{3\sqrt{2}\left((7A-11B)\cos(dx+c)^3+2(7A-11B)\cos(dx+c)^2+(7A-11B)\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+2\left((15A-19B)\cos(dx+c)^2+12(A-B)\cos(dx+c)+4B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{12\left(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2+a^2d\cos(dx+c)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/24*(3*sqrt(2)*((7*A - 11*B)*cos(d*x + c)^3 + 2*(7*A - 11*B)*cos(d*x + c)
^2 + (7*A - 11*B)*cos(d*x + c))*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^
2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((15*A
 - 19*B)*cos(d*x + c)^2 + 12*(A - B)*cos(d*x + c) + 4*B)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x
 + c)^2 + a^2*d*cos(d*x + c)), 1/12*(3*sqrt(2)*((7*A - 11*B)*cos(d*x + c)^3
 + 2*(7*A - 11*B)*cos(d*x + c)^2 + (7*A - 11*B)*cos(d*x + c))*sqrt(a)*arcta
n(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin
(d*x + c))) + 2*((15*A - 19*B)*cos(d*x + c)^2 + 12*(A - B)*cos(d*x + c) + 4
*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x +
c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\sec^3(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2)
, x)
```

Giac [A]
time = 1.77, size = 247, normalized size = 1.44

$$-\frac{\frac{\left(\left(\frac{3\left(\sqrt{2}A\operatorname{sgn}(\cos(dx+c))-\sqrt{2}B\operatorname{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a}-\frac{2\left(15\sqrt{2}A\operatorname{sgn}(\cos(dx+c))-23\sqrt{2}B\operatorname{sgn}(\cos(dx+c))\right)}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{27\left(\sqrt{2}A\operatorname{sgn}(\cos(dx+c))-\sqrt{2}B\operatorname{sgn}(\cos(dx+c))\right)}{a}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}-\frac{3\left(7\sqrt{2}A-11\sqrt{2}B\right)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\operatorname{sgn}(\cos(dx+c))}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] -1/12*(((3*(sqrt(2)*A*a*sgn(cos(d*x + c)) - sqrt(2)*B*a*sgn(cos(d*x + c)))*
tan(1/2*d*x + 1/2*c)^2/a - 2*(15*sqrt(2)*A*a*sgn(cos(d*x + c)) - 23*sqrt(2)
*B*a*sgn(cos(d*x + c)))/a)*tan(1/2*d*x + 1/2*c)^2 + 27*(sqrt(2)*A*a*sgn(cos
(d*x + c)) - sqrt(2)*B*a*sgn(cos(d*x + c)))/a)*tan(1/2*d*x + 1/2*c)/((a*tan
(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)) - 3*(7*sqrt(2
)*A - 11*sqrt(2)*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^3\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(3/2)), x)
```

3.154 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=118

$$\frac{(3A-7B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{2B\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 1/4*(3*A-7*B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))
/a^(3/2)/d*2^(1/2)-1/2*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+2*B*tan(d*
x+c)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 118, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4093, 4086, 3880, 209}

$$\frac{(3A-7B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}+\frac{2B\tan(c+dx)}{ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((3*A - 7*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]
])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x
])^(3/2)) + (2*B*Tan[c + d*x])/(a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
```

```
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{\sec(c+dx)\left(-\frac{3}{2}a(A-B)-2aB\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{2B\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} + \frac{(3A-7B)}{}$$

$$= -\frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{2B\tan(c+dx)}{ad\sqrt{a+a\sec(c+dx)}} - \frac{(3A-7B)}{}$$

$$= \frac{(3A-7B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{(A-B)\tan(c+}{2d(a+a\sec(c+}$$

Mathematica [A]
time = 0.77, size = 125, normalized size = 1.06

$$\frac{\left(\sqrt{2}(3A-7B)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)+\sqrt{1-\sec(c+dx)}(-A+5B+4B\sec(c+dx))\right)\tan(c+dx)}{2d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),
x]
```

```
[Out] ((Sqrt[2]*(3*A - 7*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)
/2]^2*Sec[c + d*x] + Sqrt[1 - Sec[c + d*x]]*(-A + 5*B + 4*B*Sec[c + d*x]))*
Tan[c + d*x])/(2*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(101) = 202$.
time = 4.52, size = 405, normalized size = 3.43

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(3A\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)\,\mathrm{si}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(3*A*ln(((-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)-7*B*ln(((-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+3*A*ln(((-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)-7*B*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)+2*A*cos(d*x+c)^2-10*B*cos(d*x+c)^2-2*A*cos(d*x+c)+2*B*cos(d*x+c)+8*B)/sin
(d*x+c)^3/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x
)
```

Fricas [A]
time = 1.94, size = 386, normalized size = 3.27

$$\left[\frac{\sqrt{2}\left((3A-7B)\cos(dx+c)^2+2(3A-7B)\cos(dx+c)+3A-7B\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos(dx+c)^2-2a\cos(dx+c)+a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\left((A-5B)\cos(dx+c)-4B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}, -\frac{\sqrt{2}\left((3A-7B)\cos(dx+c)^2+2(3A-7B)\cos(dx+c)+3A-7B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+2\left((A-5B)\cos(dx+c)-4B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/8*(sqrt(2)*((3*A - 7*B)*cos(d*x + c)^2 + 2*(3*A - 7*B)*cos(d*x + c) + 3*
A - 7*B)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) +
 a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*((A - 5*B)*cos(d*x + c) - 4*
B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c
)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)*((3*A - 7*B)*cos(d*x + c
)^2 + 2*(3*A - 7*B)*cos(d*x + c) + 3*A - 7*B)*sqrt(a)*arctan(sqrt(2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*
((A - 5*B)*cos(d*x + c) - 4*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec^2(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2)
, x)
```

Giac [A]
time = 1.65, size = 187, normalized size = 1.58

$$-\frac{\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + a}\left(\frac{\sqrt{2}\,(Aa^2 - Ba^2)\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2}{a^3\mathrm{sgn}(\cos(dx+c))} - \frac{\sqrt{2}\,(Aa^2 - 9\,Ba^2)}{a^3\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 - a} + \frac{\sqrt{2}\,(3\,A - 7\,B)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + \sqrt{-a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + a}\right|\right)}{\sqrt{-a}\,a\mathrm{sgn}(\cos(dx+c))}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] -1/4*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(sqrt(2)*(A*a^2 - B*a^2)*tan(1/2*
d*x + 1/2*c)^2/(a^3*sgn(cos(d*x + c))) - sqrt(2)*(A*a^2 - 9*B*a^2)/(a^3*sgn
(cos(d*x + c))))*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 - a) + sqrt
(2)*(3*A - 7*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+d\,x)}}{\cos\left(c + d\,x\right)^2\left(a + \frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(3/2)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(3/2)), x)
```

3.155 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=87

$$\frac{(A+3B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

```
[Out] 1/4*(A+3*B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a
^(3/2)/d*2^(1/2)+1/2*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.08, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.097, Rules used = {4085, 3880, 209}

$$\frac{(A+3B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(A-B)\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((A + 3*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])
])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x])
^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
```

```
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(A+3B)\int \frac{\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{4a}$$

$$= \frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(A+3B)\mathrm{Subst}\left(\int \frac{1}{2a+x^2}\,dx, x, -\frac{}{\sqrt{a+}}\right.}{2ad}$$

$$= \frac{(A+3B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} + \frac{(A-B)\tan(c+}{2d(a+a\sec(c+d}$$

Mathematica [A]
time = 0.82, size = 127, normalized size = 1.46

$$\frac{2(A-B)\sqrt{1-\sec(c+dx)}\,\sin(c+dx)+2\sqrt{2}\,(A+3B)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\tan(c+dx)}{4ad(1+\cos(c+dx))\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(A - B)*Sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] + 2*Sqrt[2]*(A + 3*B)*ArcTan
h[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^2*Tan[c + d*x])/(4*a*d*(
1 + Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 401 vs. $2(72) = 144$.
time = 4.54, size = 402, normalized size = 4.62

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(A\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3B\ln\left(\sqrt{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(A*ln(((-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*cos(d*x+c)*sin(d*x+c)+3*B*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*cos(d*x+c)*sin(d*x+c)+A*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
+3*B*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(
d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*A*cos(d*x+c)^2+2*
B*cos(d*x+c)^2+2*A*cos(d*x+c)-2*B*cos(d*x+c))/(1+cos(d*x+c))/sin(d*x+c)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 159 vs. $2(72) = 144$.
time = 1.63, size = 367, normalized size = 4.22

$$\left[\frac{4\,(A-B)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)-\sqrt{2}\left((A+3\,B)\cos\left(dx+c\right)^{2}+2\,(A+3\,B)\cos\left(dx+c\right)+A+3\,B\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)\sin(dx+c)+3\,a\cos(dx+c)^{2}+2\,a\cos(dx+c)-a}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)}{8\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)},\ \frac{2\,(A-B)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos\left(dx+c\right)\sin\left(dx+c\right)-\sqrt{2}\left((A+3\,B)\cos\left(dx+c\right)^{2}+2\,(A+3\,B)\cos\left(dx+c\right)+A+3\,B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{4\left(a^{2}d\cos\left(dx+c\right)^{2}+2\,a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] [1/8*(4*(A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*
x + c) - sqrt(2)*((A + 3*B)*cos(d*x + c)^2 + 2*(A + 3*B)*cos(d*x + c) + A +
 3*B)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/
(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos
(d*x + c) + a^2*d), 1/4*(2*(A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
cos(d*x + c)*sin(d*x + c) - sqrt(2)*((A + 3*B)*cos(d*x + c)^2 + 2*(A + 3*B)
*cos(d*x + c) + A + 3*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 +
2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\sec\left(c+dx\right)\right)\sec\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x
)
```

Giac [A]
time = 1.39, size = 133, normalized size = 1.53

$$-\frac{\frac{\left(\sqrt{2}\,A+3\,\sqrt{2}\,B\right)\log\left(\left|-\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{\sqrt{-a}\,a\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}-\frac{\left(\sqrt{2}\,Aa\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\sqrt{2}\,Ba\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^{3}}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] -1/4*((sqrt(2)*A + 3*sqrt(2)*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) + sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a*sgn(cos(d*x + c))) - (sqrt(
2)*A*a*sgn(cos(d*x + c)) - sqrt(2)*B*a*sgn(cos(d*x + c)))*sqrt(-a*tan(1/2*d
*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/a^3)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + a/cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + a/cos(c + d*x))^(3/2)), x)
```

3.156 $\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=127

$$\frac{2A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}-\frac{(5A-B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

```
[Out] 2*A*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-1/4*(5*A-B)
*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^
(1/2)-1/2*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.13, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4007, 4005, 3859, 209, 3880}

$$-\frac{(5A-B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{2A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-B)\tan(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d) -
 ((5*A - B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]]
)])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Tan[c + d*x])/(2*d*(a + a*Sec[c + d*x]
)^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{\int \frac{-2aA+\frac{1}{2}a(A-B)\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{A\int \sqrt{a+a\sec(c+dx)}\,dx}{a^2} - \frac{(5A-B)\int \frac{}{\sqrt{a}}}{}$$

$$= -\frac{(A-B)\tan(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(2A)\text{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, -\frac{a\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{ad}$$

$$= \frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{(5A-B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+}}\right)}{2\sqrt{2}a^{3/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 26.85, size = 11183, normalized size = 88.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(3/2),x]
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 553 vs. $2(106) = 212$.
time = 4.60, size = 554, normalized size = 4.36

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(4A\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{2}\,\sin(dx+c)\cos(dx+c)+4A\,\mathrm{a}\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(4*A*arctanh(1/2*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)+4*A*arctanh(1/2*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*2^(1/2)*sin(d*x+c)+5*A*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)*cos(d*x+c)-B*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)*cos(d*x+c)+5*A*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
-B*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(
d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*A*cos(d*x+c)^2+2*
B*cos(d*x+c)^2+2*A*cos(d*x+c)-2*B*cos(d*x+c))/(1+cos(d*x+c))/sin(d*x+c)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 231 vs. $2(106) = 212$.
time = 4.64, size = 548, normalized size = 4.31

$$\left[-\frac{4\,(A-B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-\sqrt{2}\,((5\,A-B)\cos(dx+c)^2+2\,(5\,A-B)\cos(dx+c)+5\,A-B)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+8\,(A\cos(dx+c)^2+2\,A\cos(dx+c)+A)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)}{8\,(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d)},\ -\frac{2\,(A-B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-\sqrt{2}\,((5\,A-B)\cos(dx+c)^2+2\,(5\,A-B)\cos(dx+c)+5\,A-B)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+8\,(A\cos(dx+c)^2+2\,A\cos(dx+c)+A)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)}{4\,(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/8*(4*(A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d
*x + c) - sqrt(2)*((5*A - B)*cos(d*x + c)^2 + 2*(5*A - B)*cos(d*x + c) + 5*
A - B)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)
/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 8*(A*cos(d*x + c)^2 + 2*A*cos(d*x
 + c) + A)*sqrt(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(
d*x + c) + 1)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4
*(2*(A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
c) - sqrt(2)*((5*A - B)*cos(d*x + c)^2 + 2*(5*A - B)*cos(d*x + c) + 5*A - B
)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
c)/(sqrt(a)*sin(d*x + c))) + 8*(A*cos(d*x + c)^2 + 2*A*cos(d*x + c) + A)*sq
rt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*
sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec{(c + dx)}}{(a (\sec{(c + dx)} + 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a*(sec(c + d*x) + 1))**(3/2), x)
```

Giac [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+d\,x)}}{{\left(a + \frac{a}{\cos(c+d\,x)}\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(3/2),x)
[Out] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(3/2), x)
```

3.157 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=170

$$-\frac{(3A-2B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}+\frac{(9A-5B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)}{2d(a+a\,\text{se}}$$

```
[Out] -(3*A-2*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d-1/2*
(A-B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+1/4*(9*A-5*B)*arctan(1/2*a^(1/2)*
tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+1/2*(3*A-B)*si
n(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 170, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4105, 4107, 4005, 3859, 209, 3880}

$$-\frac{(3A-2B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(9A-5B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(3A-B)\sin(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{(A-B)\sin(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] -(((3*A - 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^
(3/2)*d)) + ((9*A - 5*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*
Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*(a + a
*Sec[c + d*x])^(3/2)) + ((3*A - B)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c +
d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
```

```
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+dx)\left(a(3A-B)-\frac{3}{2}a(A-B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}\,d}{2a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A-B)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{-a^2(3A-}{\sqrt{a}}}{}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A-B)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} + \frac{(9A-5B}{}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A-B)\sin(c+dx)}{2ad\sqrt{a+a\sec(c+dx)}} - \frac{(9A-5B}{}$$

$$= -\frac{(3A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{(9A-5B)\tan^{-1}\left(\right.}{2}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 27.05, size = 11954, normalized size = 70.32

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 712 vs. $2(145) = 290$.
time = 8.18, size = 713, normalized size = 4.19

method	result
default	$-\frac{(-1+\cos(dx+c))\left(6A\,\mathrm{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}\sin(dx+c)\cos(dx+c)-4B\cos(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))*(6*A*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/
2)*sin(d*x+c)*cos(d*x+c)-4*B*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)/cos(d*x+c)*2^(1/2))+6*A*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(
1/2)*sin(d*x+c)+9*A*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+c
os(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*co
s(d*x+c)-4*B*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*sin(d*x+c)-5
*B*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(
d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)-4*A*cos(
d*x+c)^3+9*A*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+
c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-5*B*ln(-(
-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*A*cos(d*x+c)^2+2*B*cos(d*
x+c)^2+6*A*cos(d*x+c)-2*B*cos(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/s
in(d*x+c)^3/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/(a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A]
time = 5.50, size = 609, normalized size = 3.58

$$\left[\frac{\sqrt{2}\left((9A-5B)\cos(dx+c)^2+2(9A-5B)\cos(dx+c)+9A-5B\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos(dx+c)^2-2a\cos(dx+c)+a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\left((3A-2B)\cos(dx+c)^2+2(3A-2B)\cos(dx+c)+3A-2B\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+4\left(2A\cos(dx+c)^2+(3A-B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}, -\frac{\sqrt{2}\left((9A-5B)\cos(dx+c)^2+2(9A-5B)\cos(dx+c)+9A-5B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-4\left((3A-2B)\cos(dx+c)^2+2(3A-2B)\cos(dx+c)+3A-2B\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-2\left(2A\cos(dx+c)^2+(3A-B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] [1/8*(sqrt(2)*((9*A - 5*B)*cos(d*x + c)^2 + 2*(9*A - 5*B)*cos(d*x + c) + 9*
A - 5*B)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) +
 a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((3*A - 2*B)*cos(d*x + c)^2
+ 2*(3*A - 2*B)*cos(d*x + c) + 3*A - 2*B)*sqrt(-a)*log((2*a*cos(d*x + c)^2
+ 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
```

```
c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 4*(2*A*cos(d*x + c)^2 + (3*
A - B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/
(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)*((9*A
- 5*B)*cos(d*x + c)^2 + 2*(9*A - 5*B)*cos(d*x + c) + 9*A - 5*B)*sqrt(a)*arc
tan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*s
in(d*x + c))) - 4*((3*A - 2*B)*cos(d*x + c)^2 + 2*(3*A - 2*B)*cos(d*x + c)
+ 3*A - 2*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)/(sqrt(a)*sin(d*x + c))) - 2*(2*A*cos(d*x + c)^2 + (3*A - B)*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x +
 c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\cos(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)/(a*(sec(c + d*x) + 1))**(3/2), x
)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(145) = 290.
time = 2.15, size = 418, normalized size = 2.46

$$-\frac{\frac{\sqrt{2}(9A-5B)\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}\,a\,\mathrm{sgn}(\cos(dx+c))}-\frac{4(3A-2B)\log\left(\frac{\left|-17179869184\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-34359738368\sqrt{2}|a|+51539607552a\right|}{\left|-17179869184\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+34359738368\sqrt{2}|a|+51539607552a\right|}\right)}{\sqrt{-a}\,|a|\,\mathrm{sgn}(\cos(dx+c))}-\frac{2\left(\sqrt{2}Aa\,\mathrm{sgn}(\cos(dx+c))-\sqrt{2}Ba\,\mathrm{sgn}(\cos(dx+c))\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\,\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{a^3}-\frac{16\sqrt{2}\left(3\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2A-Aa\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2a+a^2\right)\sqrt{-a}\,\mathrm{sgn}(\cos(dx+c))}}{8d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] -1/8*(sqrt(2)*(9*A - 5*B)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*a*sgn(cos(d*x + c))) - 4*(3*A - 2*B)*
log(abs(-17179869184*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x +
 1/2*c)^2 + a))^2 - 34359738368*sqrt(2)*abs(a) + 51539607552*a)/abs(-171798
69184*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
^2 + 34359738368*sqrt(2)*abs(a) + 51539607552*a))/(sqrt(-a)*abs(a)*sgn(cos(
d*x + c))) - 2*(sqrt(2)*A*a*sgn(cos(d*x + c)) - sqrt(2)*B*a*sgn(cos(d*x + c
)))*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/a^3 - 16*sqrt(
2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
^2*A - A*a)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)
^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c
)^2 + a))^2*a + a^2)*sqrt(-a)*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)\,\left(A+\frac{B}{\cos(c+d\,x)}\right)}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(3/2),x)
[Out] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(3/2), x)
```

3.158 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=221

$$\frac{(19A-12B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{3/2}d}-\frac{(13A-9B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\cdots}{2d(a\cdots}$$

```
[Out] 1/4*(19*A-12*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d
-1/2*(A-B)*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)-1/4*(13*A-9*B)*ar
ctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/
2)-1/4*(7*A-6*B)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)+1/2*(2*A-B)*cos(d*x+
c)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4105, 4107, 4005, 3859, 209, 3880}

$$\frac{(19A-12B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{3/2}d}-\frac{(13A-9B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(7A-6B)\sin(c+dx)}{4ad\sqrt{a\sec(c+dx)+a}}+\frac{(2A-B)\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{(A-B)\sin(c+dx)\cos(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((19*A - 12*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*
a^(3/2)*d) - ((13*A - 9*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Cos[c + d*x]*Sin[c + d*
x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - ((7*A - 6*B)*Sin[c + d*x])/(4*a*d*Sq
rt[a + a*Sec[c + d*x]]) + ((2*A - B)*Cos[c + d*x]*Sin[c + d*x])/(2*a*d*Sqrt
[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos^2(c+dx)\left(2a(2A-B)-\frac{5}{2}a(A-B)\dots\right.}{\sqrt{a+a\sec(c+dx}\dots}}{2a^2}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(2A-B)\cos(c+dx)\sin(c\dots}{2ad\sqrt{a+a\sec(c+dx)}\dots}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(7A-6B)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \dots$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(7A-6B)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \dots$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(7A-6B)\sin(c+dx)}{4ad\sqrt{a+a\sec(c+dx)}} + \dots$$

$$= \frac{(19A-12B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{3/2}d} - \frac{(13A-9B)\tan^{-1}\dots}{\dots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 2.42, size = 395, normalized size = 1.79

$$\frac{\sec(c+dx)\left(-52\sqrt{2}A\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\sin(c+dx)+36\sqrt{2}B\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\sin(c+dx)-13A\sqrt{1-\sec(c+dx)}\sin(c+dx)+24B\sqrt{1-\sec(c+dx)}\sin(c+dx)+18A\cos^2(c+dx)\sqrt{1-\sec(c+dx)}\sin(c+dx)+\frac{13}{2}A\sqrt{1-\sec(c+dx)}\sin(2(c+dx))+8B\sqrt{1-\sec(c+dx)}\sin(2(c+dx))-52\sqrt{2}A\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\tan(c+dx)+36\sqrt{2}B\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\tan(c+dx)+(91A-48B)\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)(\sin(c+dx)+\tan(c+dx))-40A\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\sec(c+dx)\right)\sqrt{1-\sec(c+dx)}(\sin(c+dx)+\tan(c+dx))\right)}{16d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),
x]
```

```
[Out] (Sec[c + d*x]*(-52*Sqrt[2]*A*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Sin[c
+ d*x] + 36*Sqrt[2]*B*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Sin[c + d*x]
- 13*A*Sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] + 24*B*Sqrt[1 - Sec[c + d*x]]*Si
n[c + d*x] + 18*A*Cos[c + d*x]^2*Sqrt[1 - Sec[c + d*x]]*Sin[c + d*x] + (13*
A*Sqrt[1 - Sec[c + d*x]]*Sin[2*(c + d*x)])/2 + 8*B*Sqrt[1 - Sec[c + d*x]]*S
in[2*(c + d*x)] - 52*Sqrt[2]*A*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Tan[
c + d*x] + 36*Sqrt[2]*B*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Tan[c + d*x
] + (91*A - 48*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]]*(Sin[c + d*x] + Tan[c + d
*x]) - 40*A*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Sqrt[1 - Sec[c
 + d*x]]*(Sin[c + d*x] + Tan[c + d*x])))/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1
 + Sec[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1074 vs. $2(190) = 380$.
time = 8.65, size = 1075, normalized size = 4.86

method	result	size
default	Expression too large to display	1075

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/16/d*(-1+cos(d*x+c))*(19*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)/cos(d*x+c)*2^(1/2))-12*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+38*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*
arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1
/2))*sin(d*x+c)*cos(d*x+c)*2^(1/2)+26*A*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
3/2)*cos(d*x+c)^2*sin(d*x+c)-24*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arct
anh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))
*sin(d*x+c)*cos(d*x+c)*2^(1/2)-18*B*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)
*cos(d*x+c)^2*sin(d*x+c)+19*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(
1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(
1/2)*sin(d*x+c)+52*A*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*
x+c)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(
d*x+c))-12*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)-3
6*B*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*ln(((-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+26*A*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)-8*A*cos(d*x+c)^5-18*B*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(3/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin
(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)+20*A*cos(d*x+c)^4-16*B*cos(d*x
+c)^4+16*A*cos(d*x+c)^3-8*B*cos(d*x+c)^3-28*A*cos(d*x+c)^2+24*B*cos(d*x+c)^
2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^3/cos(d*x+c)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/(a*sec(d*x + c) + a)^(3/2), x
)
```

Fricas [A]
time = 8.02, size = 644, normalized size = 2.91

$$\left[\frac{\sqrt{2}\left((13A-9B)\cos(dx+c)^2+2(13A-9B)\cos(dx+c)+13A-9B\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\left((19A-12B)\cos(dx+c)^2+2(19A-12B)\cos(dx+c)+19A-12B\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2-2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(2A\cos(dx+c)^3-(3A-4B)\cos(dx+c)^2-(7A-6B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)},\ \frac{\sqrt{2}\left((13A-9B)\cos(dx+c)^2+2(13A-9B)\cos(dx+c)+13A-9B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left((19A-12B)\cos(dx+c)^2+2(19A-12B)\cos(dx+c)+19A-12B\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+\left(2A\cos(dx+c)^3-(3A-4B)\cos(dx+c)^2-(7A-6B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/8*(sqrt(2)*((13*A - 9*B)*cos(d*x + c)^2 + 2*(13*A - 9*B)*cos(d*x + c) +
13*A - 9*B)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c)
 - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((19*A - 12*B)*cos(d*x + c)^
2 + 2*(19*A - 12*B)*cos(d*x + c) + 19*A - 12*B)*sqrt(-a)*log((2*a*cos(d*x +
 c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin
(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(2*A*cos(d*x + c)^3
 - (3*A - 4*B)*cos(d*x + c)^2 - (7*A - 6*B)*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*
x + c) + a^2*d), 1/4*(sqrt(2)*((13*A - 9*B)*cos(d*x + c)^2 + 2*(13*A - 9*B)
*cos(d*x + c) + 13*A - 9*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - ((19*A - 12*B)*cos(d
*x + c)^2 + 2*(19*A - 12*B)*cos(d*x + c) + 19*A - 12*B)*sqrt(a)*arctan(sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) +
(2*A*cos(d*x + c)^3 - (3*A - 4*B)*cos(d*x + c)^2 - (7*A - 6*B)*cos(d*x + c)
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)
^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\cos^2(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/(a*(sec(c + d*x) + 1))**(3/2)
, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 638 vs. 2(190) = 380.

time = 2.19, size = 638, normalized size = 2.89

$$\frac{\frac{\sqrt{2}\,(13\,A-9\,B)\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2\right)}{\sqrt{-a}\,a\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}+\frac{(19\,A-12\,B)\log\left(\frac{\left|147573952589676412928\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2-295147905179352825856\,\sqrt{2}\,|a|-442721857769029238784\,a\right|}{\left|147573952589676412928\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2+295147905179352825856\,\sqrt{2}\,|a|-442721857769029238784\,a\right|}\right)}{\sqrt{-a}\,|a|\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}-\frac{2\left(\sqrt{2}\,Aa\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)-\sqrt{2}\,Ba\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)\right)\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a^3}-\frac{4\,\sqrt{2}\left(29\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^6A-12\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^6B-133\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^4Aa+76\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^4Ba+55\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2Aa^2-36\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2Ba^2-7\,Aa^3+4\,Ba^3\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^4-6\left(\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a}\right)^2a+a^2\right)^2\sqrt{-a}\,\mathrm{sgn}\left(\cos\left(dx+c\right)\right)}}{8\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] 1/8*(sqrt(2)*(13*A - 9*B)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*a*sgn(cos(d*x + c))) + (19*A - 12*B)*
log(abs(147573952589676412928*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 - 295147905179352825856*sqrt(2)*abs(a) - 4427218
57769029238784*a)/abs(147573952589676412928*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 295147905179352825856*sqrt(2)*ab
s(a) - 442721857769029238784*a))/(sqrt(-a)*abs(a)*sgn(cos(d*x + c))) - 2*(s
qrt(2)*A*a*sgn(cos(d*x + c)) - sqrt(2)*B*a*sgn(cos(d*x + c)))*sqrt(-a*tan(1
/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/a^3 - 4*sqrt(2)*(29*(sqrt(-a)*t
an(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*A - 12*(sqrt(-
a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B - 133*(s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a +
 76*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4
*B*a + 55*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
 a))^2*A*a^2 - 36*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/
2*c)^2 + a))^2*B*a^2 - 7*A*a^3 + 4*B*a^3)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c)
- sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^2*sqrt(-a)*sgn(cos(d*x +
 c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(3/2),x)
[Out] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(3/2), x)
```

3.159 $\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=268

$$-\frac{(47A-38B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8a^{3/2}d}+\frac{(17A-13B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-\ldots}{2\ldots}$$

```
[Out] -1/8*(47*A-38*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/
d-1/2*(A-B)*cos(d*x+c)^2*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+1/4*(17*A-13*B
)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d*2
^(1/2)+7/8*(3*A-2*B)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)-1/12*(13*A-12*B)
*cos(d*x+c)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)+1/6*(5*A-3*B)*cos(d*x+c)^
2*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.53, antiderivative size = 268, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4105, 4107, 4005, 3859, 209, 3880}

$$-\frac{(47A-38B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8a^{3/2}d}+\frac{(17A-13B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{7(3A-2B)\sin(c+dx)}{8ad\sqrt{a\sec(c+dx)+a}}+\frac{(5A-3B)\sin(c+dx)\cos^2(c+dx)}{6ad\sqrt{a\sec(c+dx)+a}}-\frac{(A-B)\sin(c+dx)\cos^2(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{(13A-12B)\sin(c+dx)\cos(c+dx)}{12ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] -1/8*((47*A - 38*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]
)/(a^(3/2)*d) + ((17*A - 13*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Cos[c + d*x]^2*Sin[
c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + (7*(3*A - 2*B)*Sin[c + d*x])/(
8*a*d*Sqrt[a + a*Sec[c + d*x]]) - ((13*A - 12*B)*Cos[c + d*x]*Sin[c + d*x])
/(12*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((5*A - 3*B)*Cos[c + d*x]^2*Sin[c + d*
x])/(6*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos^3(c+dx)\left(a(5A-3B)-\frac{7}{2}a(A-B\ldots\right.}{\sqrt{a+a\sec(c+d\ldots}}}{2a^2}$$

$$= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(5A-3B)\cos^2(c+dx)\sin\ldots}{6ad\sqrt{a+a\sec(c+d\ldots}}$$

$$= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(13A-12B)\cos(c+dx)\sin\ldots}{12ad\sqrt{a+a\sec(c+\ldots}}$$

$$= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{7(3A-2B)\sin(c+dx)}{8ad\sqrt{a+a\sec(c+dx)}} - \ldots$$

$$= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{7(3A-2B)\sin(c+dx)}{8ad\sqrt{a+a\sec(c+dx)}} - \ldots$$

$$= -\frac{(A-B)\cos^2(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{7(3A-2B)\sin(c+dx)}{8ad\sqrt{a+a\sec(c+dx)}} - \ldots$$

$$= -\frac{(47A-38B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8a^{3/2}d} + \frac{(17A-13B)\tan\ldots}{\ldots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.15, size = 502, normalized size = 1.87

$$-\frac{B\cos(c+dx)\sin(c+dx)}{2d(a(1+\sec(c+dx)))^{3/2}} - \frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a(1+\sec(c+dx)))^{3/2}} - \frac{B(1+\sec(c+dx))^{3/2}\left(\frac{40\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}} - \frac{13\left(7\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)-4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)-\cos(c+dx)\sqrt{1-\sec(c+dx)}+2\cos^2(c+dx)\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\right)}{16(a(1+\sec(c+dx)))^{3/2}} - \frac{A(1+\sec(c+dx))^{3/2}\left(\frac{336\,{}_2F_1\left(\frac{1}{2},4;\frac{3}{2};1-\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}} + \frac{17\left(3\left(9\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)-8\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\right)-7\cos(c+dx)\sqrt{1-\sec(c+dx)}+2\cos^2(c+dx)\sqrt{1-\sec(c+dx)}-8\cos^3(c+dx)\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\right)}{96(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),
x]
```

```
[Out] -1/2*(B*Cos[c + d*x]*Sin[c + d*x])/(d*(a*(1 + Sec[c + d*x]))^(3/2)) - (A*Co
s[c + d*x]^2*Sin[c + d*x])/(2*d*(a*(1 + Sec[c + d*x]))^(3/2)) - (B*(1 + Sec
[c + d*x])^(3/2)*((40*Hypergeometric2F1[1/2, 3, 3/2, 1 - Sec[c + d*x]]*Tan[
c + d*x])/(d*Sqrt[1 + Sec[c + d*x]]) - (13*(7*ArcTanh[Sqrt[1 - Sec[c + d*x]
]] - 4*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] - Cos[c + d*x]*Sqrt[
1 - Sec[c + d*x]] + 2*Cos[c + d*x]^2*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/
(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])))/(16*(a*(1 + Sec[c + d*x
```

```
]))^(3/2)) - (A*(1 + Sec[c + d*x])^(3/2)*((336*Hypergeometric2F1[1/2, 4, 3/
2, 1 - Sec[c + d*x]]*Tan[c + d*x])/(d*Sqrt[1 + Sec[c + d*x]]) + (17*(3*(9*A
rcTanh[Sqrt[1 - Sec[c + d*x]]] - 8*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/S
qrt[2]] - 7*Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]]) + 2*Cos[c + d*x]^2*Sqrt[1
- Sec[c + d*x]] - 8*Cos[c + d*x]^3*Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(d
*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])))/(96*(a*(1 + Sec[c + d*x])
)^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1424 vs. $2(233) = 466$.

time = 8.33, size = 1425, normalized size = 5.32

method	result	size
default	Expression too large to display	1425

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/192/d*(-1+cos(d*x+c))*(204*A*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin
(d*x+c)-156*B*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c
)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)-64*A*cos(d
*x+c)^7-96*B*cos(d*x+c)^6-344*A*cos(d*x+c)^5+141*A*2^(1/2)*arctanh(1/2*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)-114*B*2^(1/2)*arctanh(1/2*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(5/2)*sin(d*x+c)+612*A*sin(d*x+c)*ln(((-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(5/2)*cos(d*x+c)^2-468*B*sin(d*x+c)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(5/2)*cos(d*x+c)^2+612*A*sin(d*x+c)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)
*cos(d*x+c)-468*B*sin(d*x+c)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*
x+c)+141*A*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(
d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)*cos(d*x+c)^
3*2^(1/2)-114*B*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*sin(d*x+c)*cos(d*
x+c)^3*2^(1/2)+423*A*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(
d*x+c)*sin(d*x+c)-342*B*2^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*c
os(d*x+c)*sin(d*x+c)+112*A*cos(d*x+c)^6+240*B*cos(d*x+c)^5+423*A*2^(1/2)*ar
ctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2
```

```
))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2*sin(d*x+c)-342*B*2^(1/
2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2
^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*cos(d*x+c)^2*sin(d*x+c)+504*A*
cos(d*x+c)^3-336*B*cos(d*x+c)^3+204*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*
ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c
))*sin(d*x+c)*cos(d*x+c)^3-156*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*ln(((
-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*si
n(d*x+c)*cos(d*x+c)^3-208*A*cos(d*x+c)^4+192*B*cos(d*x+c)^4)*(a*(1+cos(d*x+
c))/cos(d*x+c))^(1/2)/sin(d*x+c)^3/cos(d*x+c)^2/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^3/(a*sec(d*x + c) + a)^(3/2), x
)
```

Fricas [A]
time = 7.95, size = 675, normalized size = 2.52

$$\left[\frac{6\sqrt{2}\left((17A-13B)\cos(dx+c)^2+2(17A-13B)\cos(dx+c)+17A-13B\right)\sqrt{-a}\log\left(-\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)-3a\cos(dx+c)^2-2a\cos(dx+c)+a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+3\left((47A-38B)\cos(dx+c)^2+2(47A-38B)\cos(dx+c)+47A-38B\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)+2\left(8A\cos(dx+c)^4-6(A-2B)\cos(dx+c)^3+(37A-18B)\cos(dx+c)^2+21(3A-2B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{48\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}, -\frac{6\sqrt{2}\left((17A-13B)\cos(dx+c)^2+2(17A-13B)\cos(dx+c)+17A-13B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-3\left((47A-38B)\cos(dx+c)^2+2(47A-38B)\cos(dx+c)+47A-38B\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-\left(8A\cos(dx+c)^4-6(A-2B)\cos(dx+c)^3+(37A-18B)\cos(dx+c)^2+21(3A-2B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{24\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] [1/48*(6*sqrt(2)*((17*A - 13*B)*cos(d*x + c)^2 + 2*(17*A - 13*B)*cos(d*x +
c) + 17*A - 13*B)*sqrt(-a)*log(-(2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d
*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 3*((47*A - 38*B)*cos(
d*x + c)^2 + 2*(47*A - 38*B)*cos(d*x + c) + 47*A - 38*B)*sqrt(-a)*log((2*a*
cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x
 + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 2*(8*A*cos(d
*x + c)^4 - 6*(A - 2*B)*cos(d*x + c)^3 + (37*A - 18*B)*cos(d*x + c)^2 + 21*
(3*A - 2*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/24*(6*sqrt(2
)*((17*A - 13*B)*cos(d*x + c)^2 + 2*(17*A - 13*B)*cos(d*x + c) + 17*A - 13*
B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x +
 c)/(sqrt(a)*sin(d*x + c))) - 3*((47*A - 38*B)*cos(d*x + c)^2 + 2*(47*A - 3
8*B)*cos(d*x + c) + 47*A - 38*B)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/c
```

```
os(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - (8*A*cos(d*x + c)^4 - 6
*(A - 2*B)*cos(d*x + c)^3 + (37*A - 18*B)*cos(d*x + c)^2 + 21*(3*A - 2*B)*c
os(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^2*d*c
os(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\cos^3(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**3/(a*(sec(c + d*x) + 1))**(3/2)
, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 816 vs. 2(233) = 466.
time = 2.36, size = 816, normalized size = 3.04

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] -1/48*(6*sqrt(2)*(17*A - 13*B)*log((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a
*tan(1/2*d*x + 1/2*c)^2 + a))^2)/(sqrt(-a)*a*sgn(cos(d*x + c))) - 3*(47*A -
 38*B)*log(abs(-1947111321950560360698936123457536*(sqrt(-a)*tan(1/2*d*x +
1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 3894222643901120721397872
246915072*sqrt(2)*abs(a) + 5841333965851681082096808370372608*a)/abs(-19471
11321950560360698936123457536*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(
1/2*d*x + 1/2*c)^2 + a))^2 + 3894222643901120721397872246915072*sqrt(2)*abs
(a) + 5841333965851681082096808370372608*a))/(sqrt(-a)*abs(a)*sgn(cos(d*x +
 c))) - 12*(sqrt(2)*A*a*sgn(cos(d*x + c)) - sqrt(2)*B*a*sgn(cos(d*x + c)))*
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*tan(1/2*d*x + 1/2*c)/a^3 - 4*sqrt(2)*(3
39*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^10
*A - 174*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 +
a))^10*B - 3165*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*
c)^2 + a))^8*A*a + 1842*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^8*B*a + 9198*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^6*A*a^2 - 5292*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^6*B*a^2 - 4938*(sqrt(-a)*tan(1/2*d*x +
 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*A*a^3 + 2820*(sqrt(-a)*tan
```

```
(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a^3 + 975*(sqr
t(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^4 -
 582*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
2*B*a^4 - 73*A*a^5 + 42*B*a^5)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*
tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)^3*sqrt(-a)*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^3\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(3/2),x)
[Out] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(3/2), x)
```

3.160 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=216

$$-\frac{(75A-163B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{(9A-17B)\sec^2(c}{16ad(a+a\,\mathrm{se}}$$

```
[Out] -1/32*(75*A-163*B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(
1/2))/a^(5/2)/d*2^(1/2)+1/4*(A-B)*sec(d*x+c)^3*tan(d*x+c)/d/(a+a*sec(d*x+c)
)^(5/2)+1/16*(9*A-17*B)*sec(d*x+c)^2*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+
1/24*(93*A-197*B)*tan(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)-1/48*(39*A-95*B)*
(a+a*sec(d*x+c))^(1/2)*tan(d*x+c)/a^3/d
```

Rubi [A]
time = 0.45, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4104, 4095, 4086, 3880, 209}

$$-\frac{(75A-163B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(39A-95B)\tan(c+dx)\sqrt{a\sec(c+dx)+a}}{48a^3d}+\frac{(93A-197B)\tan(c+dx)}{24a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(A-B)\tan(c+dx)\sec^3(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{(9A-17B)\tan(c+dx)\sec^2(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -1/16*((75*A - 163*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec
[c + d*x]])])/(Sqrt[2]*a^(5/2)*d) + ((A - B)*Sec[c + d*x]^3*Tan[c + d*x])/(
4*d*(a + a*Sec[c + d*x])^(5/2)) + ((9*A - 17*B)*Sec[c + d*x]^2*Tan[c + d*x]
)/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + ((93*A - 197*B)*Tan[c + d*x])/(24*a
^2*d*Sqrt[a + a*Sec[c + d*x]]) - ((39*A - 95*B)*Sqrt[a + a*Sec[c + d*x]]*Ta
n[c + d*x])/(48*a^3*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\sec^3(c+dx)\left(3a(A-B)-\frac{1}{2}a(3A-11B)\right.}{(a+a\sec(c+dx))^{3/2}}}{4a^2}$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(9A-17B)\sec^2(c+dx)\tan(}{16ad(a+a\sec(c+dx))^{3}}$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(9A-17B)\sec^2(c+dx)\tan(}{16ad(a+a\sec(c+dx))^{3}}$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(9A-17B)\sec^2(c+dx)\tan(}{16ad(a+a\sec(c+dx))^{3}}$$

$$= \frac{(A-B)\sec^3(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(9A-17B)\sec^2(c+dx)\tan(}{16ad(a+a\sec(c+dx))^{3}}$$

$$= -\frac{(75A-163B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{(A-B)\sec}{4d(a+}$$

Mathematica [A]
time = 2.58, size = 161, normalized size = 0.75

$$\frac{\left(-6\sqrt{2}(75A-163B)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)+\sqrt{1-\sec(c+dx)}\left(147A-299B+(255A-503B)\sec(c+dx)+32(3A-5B)\sec^2(c+dx)+32B\sec^3(c+dx)\right)\right)\tan(c+dx)}{48d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),
x]
```

```
[Out] ((-6*Sqrt[2]*(75*A - 163*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c
+ d*x)/2]^4*Sec[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(147*A - 299*B + (255*A
 - 503*B)*Sec[c + d*x] + 32*(3*A - 5*B)*Sec[c + d*x]^2 + 32*B*Sec[c + d*x]^
3))*Tan[c + d*x])/(48*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 794 vs. $2(189) = 378$.
time = 4.56, size = 795, normalized size = 3.68

method	result

default	$-\frac{(-1+\cos(dx+c))^2\left(-225A(\cos^3(dx+c))\sin(dx+c)\ln\left(-\frac{-\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)+\cos(dx+c)-1}{\sin(dx+c)}\right)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}+489\cdots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/192/d*(-1+cos(d*x+c))^2*(-225*A*cos(d*x+c)^3*sin(d*x+c)*ln(-(-(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(3/2)+489*B*cos(d*x+c)^3*sin(d*x+c)*ln(-(-(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(3/2)-675*A*cos(d*x+c)^2*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(3/2)+1467*B*cos(d*x+c)^2*sin(d*x+c)*ln(-(-(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)-675*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*ln(-(-(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c
)*sin(d*x+c)+1467*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*ln(-(-(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*cos(d*x+c)*si
n(d*x+c)-225*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*ln(-(-(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)+489*B*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*sin(d*x+c)+588*A*cos(d*x+c)^4-1196
*B*cos(d*x+c)^4+432*A*cos(d*x+c)^3-816*B*cos(d*x+c)^3-636*A*cos(d*x+c)^2+13
72*B*cos(d*x+c)^2-384*A*cos(d*x+c)+768*B*cos(d*x+c)-128*B)*(a*(1+cos(d*x+c)
)/cos(d*x+c))^(1/2)/sin(d*x+c)^5/cos(d*x+c)/a^3
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 1.77, size = 557, normalized size = 2.58

$$\left[\frac{3\sqrt{2}\left((75A-163B)\cos(dx+c)^4+3(75A-163B)\cos(dx+c)^3+3(75A-163B)\cos(dx+c)^2+(75A-163B)\cos(dx+c)\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\left((147A-299B)\cos(dx+c)^3+(255A-503B)\cos(dx+c)^2+32(3A-5B)\cos(dx+c)+32B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{192\left(a^3d\cos(dx+c)^4+3a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+a^3d\cos(dx+c)\right)}, \frac{3\sqrt{2}\left((75A-163B)\cos(dx+c)^4+3(75A-163B)\cos(dx+c)^3+3(75A-163B)\cos(dx+c)^2+(75A-163B)\cos(dx+c)\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+2\left((147A-299B)\cos(dx+c)^3+(255A-503B)\cos(dx+c)^2+32(3A-5B)\cos(dx+c)+32B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{96\left(a^3d\cos(dx+c)^4+3a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+a^3d\cos(dx+c)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] [1/192*(3*sqrt(2)*((75*A - 163*B)*cos(d*x + c)^4 + 3*(75*A - 163*B)*cos(d*x
 + c)^3 + 3*(75*A - 163*B)*cos(d*x + c)^2 + (75*A - 163*B)*cos(d*x + c))*sq
rt(-a)*log((2*sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c)*sin(d*x + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x
 + c)^2 + 2*cos(d*x + c) + 1)) + 4*((147*A - 299*B)*cos(d*x + c)^3 + (255*A
 - 503*B)*cos(d*x + c)^2 + 32*(3*A - 5*B)*cos(d*x + c) + 32*B)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*c
os(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c)), 1/96*(3*sqrt(
2)*((75*A - 163*B)*cos(d*x + c)^4 + 3*(75*A - 163*B)*cos(d*x + c)^3 + 3*(75
*A - 163*B)*cos(d*x + c)^2 + (75*A - 163*B)*cos(d*x + c))*sqrt(a)*arctan(sq
rt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
 + c))) + 2*((147*A - 299*B)*cos(d*x + c)^3 + (255*A - 503*B)*cos(d*x + c)^
2 + 32*(3*A - 5*B)*cos(d*x + c) + 32*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d
*cos(d*x + c)^2 + a^3*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec^4(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/(a*(sec(c + d*x) + 1))**(5/2)
, x)
```

Giac [A]
time = 1.55, size = 276, normalized size = 1.28

$$-\frac{\frac{\left(\left(3\left(\frac{2\sqrt{2}\left(Aa^5-Ba^5\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^6\mathrm{sgn}(\cos(dx+c))}+\frac{\sqrt{2}\left(15Aa^5-23Ba^5\right)}{a^6\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-\frac{4\sqrt{2}\left(75Aa^5-167Ba^5\right)}{a^6\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\frac{3\sqrt{2}\left(83Aa^5-155Ba^5\right)}{a^6\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}-\frac{3\sqrt{2}(75A-163B)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right|\right)}{\sqrt{-a}\,a^2\mathrm{sgn}(\cos(dx+c))}}{96d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] -1/96*(((3*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(1/2*d*x + 1/2*c)^2/(a^6*sgn(cos(d
*x + c))) + sqrt(2)*(15*A*a^5 - 23*B*a^5)/(a^6*sgn(cos(d*x + c))))*tan(1/2*
```

```
d*x + 1/2*c)^2 - 4*sqrt(2)*(75*A*a^5 - 167*B*a^5)/(a^6*sgn(cos(d*x + c))))*
tan(1/2*d*x + 1/2*c)^2 + 3*sqrt(2)*(83*A*a^5 - 155*B*a^5)/(a^6*sgn(cos(d*x
+ c))))*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - a)*sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a)) - 3*sqrt(2)*(75*A - 163*B)*log(abs(-sqrt(-a)*tan(1/2
*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(cos
(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^4\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/2)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + a/cos(c + d*x))^(5/2)), x)
```

3.161 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=169

$$\frac{(19A-75B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}-\frac{(5A-13B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

```
[Out] 1/32*(19*A-75*B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/
2))/a^(5/2)/d*2^(1/2)+1/4*(A-B)*sec(d*x+c)^2*tan(d*x+c)/d/(a+a*sec(d*x+c))^
(5/2)-1/16*(5*A-13*B)*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)-1/4*(A-9*B)*tan
(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.31, antiderivative size = 169, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4104, 4093, 4086, 3880, 209}

$$\frac{(19A-75B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(A-9B)\tan(c+dx)}{4a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(A-B)\tan(c+dx)\sec^2(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}-\frac{(5A-13B)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((19*A - 75*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sec[c + d*x]^2*Tan[c + d*x])/(4*d*
(a + a*Sec[c + d*x])^(5/2)) - ((5*A - 13*B)*Tan[c + d*x])/(16*a*d*(a + a*Se
c[c + d*x])^(3/2)) - ((A - 9*B)*Tan[c + d*x])/(4*a^2*d*Sqrt[a + a*Sec[c + d
*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4086

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
```

```
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[(a*B*m + A*b*(m + 1))/(b*(m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m, x], x] /; FreeQ[{a, b, A, B,
 e, f, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[a*B*m + A*b*
(m + 1), 0] &&  !LtQ[m, -2^(-1)]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\sec^2(c+dx)\left(2a(A-B)-\frac{1}{2}a(A-9B)\right.}{(a+a\sec(c+dx))^{3/2}}}{4a^2}$$

$$= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5A-13B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

$$= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5A-13B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

$$= \frac{(A-B)\sec^2(c+dx)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(5A-13B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

$$= \frac{(19A-75B)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{(A-B)\sec^2(}{4d(a+a}$$

Mathematica [A]
time = 1.47, size = 144, normalized size = 0.85

$$\frac{\left(2\sqrt{2}\,(19A-75B)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)+\sqrt{1-\sec(c+dx)}\,(-9A+49B+(-13A+85B)\sec(c+dx)+32B\sec^2(c+dx))\right)\tan(c+dx)}{16d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),
x]
```

```
[Out] ((2*Sqrt[2]*(19*A - 75*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c +
d*x)/2]^4*Sec[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(-9*A + 49*B + (-13*A + 8
5*B)*Sec[c + d*x] + 32*B*Sec[c + d*x]^2))*Tan[c + d*x])/(16*d*Sqrt[1 - Sec[
c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 596 vs. $2(146) = 292$.
time = 4.18, size = 597, normalized size = 3.53

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2\left(19A\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\sin(dx+c)(\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/32/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(19*A*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2-75*B*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-
cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2+38*A*ln(((-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)-150*B*ln(((-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+19*A*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)+18*A*cos(d*x+c)^3-75*B*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)-98*B*cos(d*x+c)^3+8*A*cos(d*x+c)^2-72*B*cos(d*x+c)^2-26*A*co
s(d*x+c)+106*B*cos(d*x+c)+64*B)/sin(d*x+c)^5/a^3
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [A]
time = 1.51, size = 484, normalized size = 2.86

$$\left[\frac{\sqrt{2}\left((19\,A-75\,B)\cos\left(dx+c\right)^{3}+3\,(19\,A-75\,B)\cos\left(dx+c\right)^{2}+3\,(19\,A-75\,B)\cos\left(dx+c\right)+19\,A-75\,B\right)\sqrt{-a}\log\left(-\frac{2\,\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)\sin(dx+c)-3\,a\cos(dx+c)^{2}-2\,a\cos(dx+c)+a}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)-4\left((9\,A-49\,B)\cos\left(dx+c\right)^{2}+(13\,A-85\,B)\cos\left(dx+c\right)-32\,B\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)},\ -\frac{\sqrt{2}\left((19\,A-75\,B)\cos\left(dx+c\right)^{3}+3\,(19\,A-75\,B)\cos\left(dx+c\right)^{2}+3\,(19\,A-75\,B)\cos\left(dx+c\right)+19\,A-75\,B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)+2\left((9\,A-49\,B)\cos\left(dx+c\right)^{2}+(13\,A-85\,B)\cos\left(dx+c\right)-32\,B\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] [1/64*(sqrt(2)*((19*A - 75*B)*cos(d*x + c)^3 + 3*(19*A - 75*B)*cos(d*x + c)
^2 + 3*(19*A - 75*B)*cos(d*x + c) + 19*A - 75*B)*sqrt(-a)*log(-(2*sqrt(2)*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) -
 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x + c
) + 1)) - 4*((9*A - 49*B)*cos(d*x + c)^2 + (13*A - 85*B)*cos(d*x + c) - 32*
B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c
)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2
)*((19*A - 75*B)*cos(d*x + c)^3 + 3*(19*A - 75*B)*cos(d*x + c)^2 + 3*(19*A
- 75*B)*cos(d*x + c) + 19*A - 75*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*((9*A - 49
*B)*cos(d*x + c)^2 + (13*A - 85*B)*cos(d*x + c) - 32*B)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x
+ c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec{(c+dx)})\sec^{3}{(c+dx)}}{(a(\sec{(c+dx)}+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a*(sec(c + d*x) + 1))**(5/2)
, x)
```

Giac [A]
time = 1.48, size = 258, normalized size = 1.53

$$-\frac{\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(\left(\frac{2\left(\sqrt{2}Aa^{6}\mathrm{sgn}(\cos(dx+c))-\sqrt{2}Ba^{6}\mathrm{sgn}(\cos(dx+c))\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{8}}+\frac{9\,\sqrt{2}Aa^{6}\mathrm{sgn}(\cos(dx+c))-17\,\sqrt{2}Ba^{6}\mathrm{sgn}(\cos(dx+c))}{a^{8}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-\frac{11\,\sqrt{2}Aa^{6}\mathrm{sgn}(\cos(dx+c))-83\,\sqrt{2}Ba^{6}\mathrm{sgn}(\cos(dx+c))}{a^{8}}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a}+\frac{\left(19\,\sqrt{2}A-75\,\sqrt{2}B\right)\log\left(\left|-\sqrt{-a}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{\sqrt{-a}a^{2}\mathrm{sgn}(\cos(dx+c))}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] -1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*((2*(sqrt(2)*A*a^6*sgn(cos(d*x +
 c)) - sqrt(2)*B*a^6*sgn(cos(d*x + c)))*tan(1/2*d*x + 1/2*c)^2/a^8 + (9*sqr
t(2)*A*a^6*sgn(cos(d*x + c)) - 17*sqrt(2)*B*a^6*sgn(cos(d*x + c)))/a^8)*tan
(1/2*d*x + 1/2*c)^2 - (11*sqrt(2)*A*a^6*sgn(cos(d*x + c)) - 83*sqrt(2)*B*a^
6*sgn(cos(d*x + c)))/a^8)*tan(1/2*d*x + 1/2*c)/(a*tan(1/2*d*x + 1/2*c)^2 -
a) + (19*sqrt(2)*A - 75*sqrt(2)*B)*log(abs(-sqrt(-a)*tan(1/2*d*x + 1/2*c) +
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(-a)*a^2*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^3\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + a/cos(c + d*x))^(5/2)), x)
```

3.162 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{(5A+19B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{(5A-13B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

```
[Out] 1/32*(5*A+19*B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2
))/a^(5/2)/d*2^(1/2)-1/4*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+1/16*(5*
A-13*B)*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.19, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4093, 4085, 3880, 209}

$$\frac{(5A+19B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(5A-13B)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B)\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((5*A + 19*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x
]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Tan[c + d*x])/(4*d*(a + a*Sec[c + d
*x])^(5/2)) + ((5*A - 13*B)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2
))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
```

```
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rule 4093

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(b^2*(2*m +
 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[A*b*m - a*B*m + b*
B*(2*m + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ
[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{\sec(c+dx)\left(-\frac{5}{2}a(A-B)-4aB\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A-13B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(5A+1}{}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A-13B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(5A+1}{}$$

$$= \frac{(5A+19B)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{(A-B)\tan(c}{4d(a+a\sec(c+}$$

Mathematica [A]
time = 1.42, size = 131, normalized size = 1.04

$$\frac{\left(2\sqrt{2}\,(5A+19B)\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)+\sqrt{1-\sec(c+dx)}\,(A-9B+(5A-13B)\sec(c+dx))\right)\tan(c+dx)}{16d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),
x]
```

```
[Out] ((2*Sqrt[2]*(5*A + 19*B)*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d
*x)/2]^4*Sec[c + d*x]^2 + Sqrt[1 - Sec[c + d*x]]*(A - 9*B + (5*A - 13*B)*Se
c[c + d*x]))*Tan[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x
]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 601 vs. $2(107) = 214$.
time = 4.49, size = 602, normalized size = 4.78

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(5A\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\sin(dx+c)(c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(5*A*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x
+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2+19*B*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-co
s(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2+10*A*ln(((-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+38*B*ln(((-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*cos(d*x+c)*sin(d*x+c)+5*A*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c)-2*A*cos(d*x+c)^3+19*B*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*s
in(d*x+c)+18*B*cos(d*x+c)^3-8*A*cos(d*x+c)^2+8*B*cos(d*x+c)^2+10*A*cos(d*x+
c)-26*B*cos(d*x+c))/(1+cos(d*x+c))/sin(d*x+c)^3/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(a*sec(d*x + c) + a)^(5/2), x
)
```

Fricas [A]
time = 1.72, size = 475, normalized size = 3.77

$$\left[-\frac{\sqrt{2}\left((5A+19B)\cos(dx+c)^3+3(5A+19B)\cos(dx+c)^2+3(5A+19B)\cos(dx+c)+5A+19B\right)\sqrt{-a}\,\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\left((A-9B)\cos(dx+c)^2+(5A-13B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}, -\frac{\sqrt{2}\left((5A+19B)\cos(dx+c)^3+3(5A+19B)\cos(dx+c)^2+3(5A+19B)\cos(dx+c)+5A+19B\right)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-2\left((A-9B)\cos(dx+c)^2+(5A-13B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((5*A + 19*B)*cos(d*x + c)^3 + 3*(5*A + 19*B)*cos(d*x + c)^
2 + 3*(5*A + 19*B)*cos(d*x + c) + 5*A + 19*B)*sqrt(-a)*log((2*sqrt(2)*sqrt(
-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a
*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
1)) - 4*((A - 9*B)*cos(d*x + c)^2 + (5*A - 13*B)*cos(d*x + c))*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*c
os(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((5*A + 19*B)
*cos(d*x + c)^3 + 3*(5*A + 19*B)*cos(d*x + c)^2 + 3*(5*A + 19*B)*cos(d*x +
c) + 5*A + 19*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*((A - 9*B)*cos(d*x + c)^2 + (
5*A - 13*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c)
+ a^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\sec^2{(c + dx)}}{(a\,(\sec{(c + dx)} + 1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/2)
, x)
```

Giac [A]
time = 1.61, size = 170, normalized size = 1.35

$$\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + a}\left(\frac{2\sqrt{2}\,(Aa^5 - Ba^5)\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2}{a^8\mathrm{sgn}(\cos(dx+c))} + \frac{\sqrt{2}\,(3\,Aa^5 - 11\,Ba^5)}{a^8\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - \frac{\sqrt{2}\,(5\,A + 19\,B)\log\left(\left|-\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + \sqrt{-a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + a}\right|\right)}{\sqrt{-a}\,a^2\mathrm{sgn}(\cos(dx+c))}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] 1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(1/
2*d*x + 1/2*c)^2/(a^8*sgn(cos(d*x + c))) + sqrt(2)*(3*A*a^5 - 11*B*a^5)/(a^
8*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c) - sqrt(2)*(5*A + 19*B)*log(abs(-
sqrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt
(-a)*a^2*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^2\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/2)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + a/cos(c + d*x))^(5/2)), x)
```

3.163 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=126

$$\frac{(3A+5B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{(3A+5B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}}$$

```
[Out] 1/32*(3*A+5*B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2)
)/a^(5/2)/d*2^(1/2)+1/4*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+1/16*(3*A
+5*B)*tan(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.12, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4085, 3881, 3880, 209}

$$\frac{(3A+5B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(3A+5B)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}+\frac{(A-B)\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((3*A + 5*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]
])]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Tan[c + d*x])/(4*d*(a + a*Sec[c + d*
x])^(5/2)) + ((3*A + 5*B)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3881

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_
Symbol] :> Simp[b*Cot[e + f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x]
 + Dist[(m + 1)/(a*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
), x], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)]
```

```
 && IntegerQ[2*m]
```

Rule 4085

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(A*b - a*B)*Cot[e +
f*x]*((a + b*Csc[e + f*x])^m/(a*f*(2*m + 1))), x] + Dist[(a*B*m + A*b*(m +
1))/(a*b*(2*m + 1)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1), x], x]
/; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& NeQ[a*B*m + A*b*(m + 1), 0] && LtQ[m, -2^(-1)]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3A+5B)\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{8a}$$

$$= \frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3A+5B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(3A+5B}{}$$

$$= \frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3A+5B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(3A+5B}{}$$

$$= \frac{(3A+5B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{(A-B)\tan(c-}{4d(a+a\sec(c+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 1.51, size = 206, normalized size = 1.63

$$\frac{40\sqrt{2}\,B\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)+64A\cos^5\left(\frac{1}{2}(c+dx)\right)\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};\frac{1}{2}(1-\sec(c+dx))\right)\sqrt{1-\sec(c+dx)}\,\sec(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)+B\sqrt{1-\sec(c+dx)}\,(10\sin(c+dx)+\sin(2(c+dx)))}{32a^2d(1+\cos(c+dx))^2\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (40*Sqrt[2]*B*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]]*Cos[(c + d*x)/2]^5*Se
c[c + d*x]*Sin[(c + d*x)/2] + 64*A*Cos[(c + d*x)/2]^5*Hypergeometric2F1[1/2
, 3, 3/2, (1 - Sec[c + d*x])/2]*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]*Sin[(c
+ d*x)/2] + B*Sqrt[1 - Sec[c + d*x]]*(10*Sin[c + d*x] + Sin[2*(c + d*x)]))/
(32*a^2*d*(1 + Cos[c + d*x])^2*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d
*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 593 vs. $2(107) = 214$.
time = 4.35, size = 594, normalized size = 4.71

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3A\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \ln\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\ \sin(dx+c)-\cos(dx+c)+1}{\sin(dx+c)}\right)\sin(dx+c)\left(\cos^2(dx+c)\right)+5B\sqrt{-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/32/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*A*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)
/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2+5*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*
x+c))*sin(d*x+c)*cos(d*x+c)^2+6*A*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*c
os(d*x+c)*sin(d*x+c)+10*B*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c
)*sin(d*x+c)+3*A*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*
x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-14*A*co
s(d*x+c)^3+5*B*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+
c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-2*B*cos(d
*x+c)^3+8*A*cos(d*x+c)^2-8*B*cos(d*x+c)^2+6*A*cos(d*x+c)+10*B*cos(d*x+c))/(
1+cos(d*x+c))^2/sin(d*x+c)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A]
time = 1.34, size = 475, normalized size = 3.77

$$\left[-\frac{\sqrt{2}\left((3A+5B)\cos(dx+c)^3+3(3A+5B)\cos(dx+c)^2+3(3A+5B)\cos(dx+c)+3A+5B\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\left((7A+B)\cos(dx+c)^2+(3A+5B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}, -\frac{\sqrt{2}\left((3A+5B)\cos(dx+c)^3+3(3A+5B)\cos(dx+c)^2+3(3A+5B)\cos(dx+c)+3A+5B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-2\left((7A+B)\cos(dx+c)^2+(3A+5B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] [-1/64*(sqrt(2)*((3*A + 5*B)*cos(d*x + c)^3 + 3*(3*A + 5*B)*cos(d*x + c)^2
+ 3*(3*A + 5*B)*cos(d*x + c) + 3*A + 5*B)*sqrt(-a)*log((2*sqrt(2)*sqrt(-a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*cos
(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))
- 4*((7*A + B)*cos(d*x + c)^2 + (3*A + 5*B)*cos(d*x + c))*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*
x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((3*A + 5*B)*cos(d
*x + c)^3 + 3*(3*A + 5*B)*cos(d*x + c)^2 + 3*(3*A + 5*B)*cos(d*x + c) + 3*A
 + 5*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(
d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*((7*A + B)*cos(d*x + c)^2 + (3*A + 5*B
)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*
d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a*(sec(c + d*x) + 1))**(5/2), x
)
```

Giac [A]
time = 1.55, size = 170, normalized size = 1.35

$$-\frac{\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\left(\frac{2\sqrt{2}\,(Aa^{5}-Ba^{5})\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}{a^{8}\mathrm{sgn}(\cos(dx+c))}-\frac{\sqrt{2}\,(5\,Aa^{5}+3\,Ba^{5})}{a^{8}\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\frac{\sqrt{2}\,(3\,A+5\,B)\log\left(\left|-\sqrt{-a}\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+\sqrt{-a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a}\right|\right)}{\sqrt{-a}\,a^{2}\mathrm{sgn}(\cos(dx+c))}}{32\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] -1/32*(sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(1
/2*d*x + 1/2*c)^2/(a^8*sgn(cos(d*x + c))) - sqrt(2)*(5*A*a^5 + 3*B*a^5)/(a^
8*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c) + sqrt(2)*(3*A + 5*B)*log(abs(-s
qrt(-a)*tan(1/2*d*x + 1/2*c) + sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)))/(sqrt(
-a)*a^2*sgn(cos(d*x + c))))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + a/cos(c + d*x))^(5/2)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + a/cos(c + d*x))^(5/2)), x)
```

3.164 $\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=164

$$\frac{2A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d}-\frac{(43A-3B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}$$

```
[Out] 2*A*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-1/32*(43*A-
3*B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/
d*2^(1/2)-1/4*(A-B)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-1/16*(11*A-3*B)*tan
(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.18, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4007, 4005, 3859, 209, 3880}

$$-\frac{(43A-3B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-3B)\tan(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B)\tan(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d) -
 ((43*A - 3*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Tan[c + d*x])/(4*d*(a + a*Sec[c +
d*x])^(5/2)) - ((11*A - 3*B)*Tan[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/
2))
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
```

```
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4007

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-(b*c - a*d))*Cot[e + f*x]*((a + b*Csc[e +
f*x])^m/(b*f*(2*m + 1))), x] + Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f
*x])^(m + 1)*Simp[a*c*(2*m + 1) - (b*c - a*d)*(m + 1)*Csc[e + f*x], x], x],
 x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && E
qQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{\int \frac{-4aA+\frac{3}{2}a(A-B)\sec(c+dx)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(11A-3B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{8a^2A-\frac{1}{4}a^2(11A-3B)}{\sqrt{a+a\sec(c}}}{8a^4}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(11A-3B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{A\int\sqrt{a+a\sec(c}}{a^3}$$

$$= -\frac{(A-B)\tan(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(11A-3B)\tan(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} - \frac{(2A)\text{Subst}\left(\int \frac{1}{a+x^2}\right.}{}$$

$$= \frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{(43A-3B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+}}\right.}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 26.88, size = 11243, normalized size = 68.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(139) = 278$.
time = 4.29, size = 824, normalized size = 5.02

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(32A\operatorname{arctanh}\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sin(dx+c)\sqrt{2}}{2\cos(dx+c)}\right)\sqrt{2}\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,(\cos^2(dx+c))\sin(dx+c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] -1/32/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(32*A*arctanh(1/2*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+64*A*arctanh(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)+43*A*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d
*x+c)+1)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2-3*B*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1
)/sin(d*x+c))*sin(d*x+c)*cos(d*x+c)^2+32*A*arctanh(1/2*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*2^(1/2)*sin(d*x+c)+86*A*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*c
os(d*x+c)*sin(d*x+c)-6*B*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
)-cos(d*x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)
*sin(d*x+c)+43*A*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*
x+c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-30*A*co
s(d*x+c)^3-3*B*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+
c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+14*B*cos(
d*x+c)^3+8*A*cos(d*x+c)^2-8*B*cos(d*x+c)^2+22*A*cos(d*x+c)-6*B*cos(d*x+c))/
(1+cos(d*x+c))^2/sin(d*x+c)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 292 vs. $2(139) = 278$.
time = 7.61, size = 670, normalized size = 4.09

$$\left[\frac{\sqrt{2}\left((43A-3B)\cos(dx+c)^3+3(43A-3B)\cos(dx+c)^2+3(43A-3B)\cos(dx+c)+43A-3B\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+3a\cos(dx+c)^2+2a\cos(dx+c)-a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-64\left(A\cos(dx+c)^3+3A\cos(dx+c)^2+3A\cos(dx+c)+A\right)\sqrt{-a}\log\left(\frac{2a\cos(dx+c)^2+2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)\sin(dx+c)+a\cos(dx+c)-a}{\cos(dx+c)+1}\right)-4\left((15A-7B)\cos(dx+c)^2+(11A-3B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}, \frac{\sqrt{2}\left((43A-3B)\cos(dx+c)^3+3(43A-3B)\cos(dx+c)^2+3(43A-3B)\cos(dx+c)+43A-3B\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-64\left(A\cos(dx+c)^3+3A\cos(dx+c)^2+3A\cos(dx+c)+A\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-2\left((15A-7B)\cos(dx+c)^2+(11A-3B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((43*A - 3*B)*cos(d*x + c)^3 + 3*(43*A - 3*B)*cos(d*x + c)^2
 + 3*(43*A - 3*B)*cos(d*x + c) + 43*A - 3*B)*sqrt(-a)*log((2*sqrt(2)*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x + c) + 3*a*
cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
)) - 64*(A*cos(d*x + c)^3 + 3*A*cos(d*x + c)^2 + 3*A*cos(d*x + c) + A)*sqrt
(-a)*log((2*a*cos(d*x + c)^2 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*cos(d*x + c)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1))
- 4*((15*A - 7*B)*cos(d*x + c)^2 + (11*A - 3*B)*cos(d*x + c))*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*co
s(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(sqrt(2)*((43*A - 3*B)*c
os(d*x + c)^3 + 3*(43*A - 3*B)*cos(d*x + c)^2 + 3*(43*A - 3*B)*cos(d*x + c)
 + 43*A - 3*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 64*(A*cos(d*x + c)^3 + 3*A*cos(d*
x + c)^2 + 3*A*cos(d*x + c) + A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 2*((15*A - 7*B)*cos(d*x
 + c)^2 + (11*A - 3*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*co
s(d*x + c) + a^3*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a*(sec(c + d*x) + 1))**(5/2), x)
```

Giac [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: TypeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] Exception raised: TypeError >> An error occurred running a Giac command:INP
UT:sage2:=int(sage0,sageVARx):;OUTPUT:Warning, integration of abs or sign a
ssumes constant sign by intervals (correct if the argument is real):Check [
abs(co
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(5/2), x)
```

3.165 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=207

$$-\frac{(5A-2B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d}+\frac{(115A-43B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(A-B}{4d(a+a}$$

```
[Out] -(5*A-2*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d-1/4*
(A-B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-1/16*(15*A-7*B)*sin(d*x+c)/a/d/(a
+a*sec(d*x+c))^(3/2)+1/32*(115*A-43*B)*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2
)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+1/16*(35*A-11*B)*sin(d*x+c)/a^2
/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.38, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4105, 4107, 4005, 3859, 209, 3880}

$$-\frac{(5A-2B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{(115A-43B)\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(35A-11B)\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(15A-7B)\sin(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -(((5*A - 2*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^
(5/2)*d)) + ((115*A - 43*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a +
 a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*(a
 + a*Sec[c + d*x])^(5/2)) - ((15*A - 7*B)*Sin[c + d*x])/(16*a*d*(a + a*Sec[
c + d*x])^(3/2)) + ((35*A - 11*B)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c
+ d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\cos(c+dx)\left(a(5A-B)-\frac{5}{2}a(A-B)\sec(c+dx)\right)}{(a+a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15A-7B)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+d}{}}{}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15A-7B)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(35A-}{16a^2d\sqrt{}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15A-7B)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(35A-}{16a^2d\sqrt{}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(15A-7B)\sin(c+dx)}{16ad(a+a\sec(c+dx))^{3/2}} + \frac{(35A-}{16a^2d\sqrt{}}$$

$$= -\frac{(5A-2B)\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{(115A-43B)\tan^{-1}}{16}$$

Mathematica [C] Result contains higher order function than in optimal. Order 4 vs. order 3 in optimal.
time = 27.07, size = 12012, normalized size = 58.03

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1064 vs. 2(178) = 356.
time = 8.41, size = 1065, normalized size = 5.14

method	result	size
default	Expression too large to display	1065

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/32/d*(-1+cos(d*x+c))^2*(-80*A*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*cos(d*x+c)^2*sin(d*x+c)+32*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*a
rctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/
2))*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)-115*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin
(d*x+c))*cos(d*x+c)^2*sin(d*x+c)-160*A*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)+43*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(
d*x+c))*cos(d*x+c)^2*sin(d*x+c)+64*B*2^(1/2)*cos(d*x+c)*sin(d*x+c)*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+32*A*cos(d*x+c)^4-230*A*ln(-(-(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)-80*A*arctanh(1/2*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*2^(1/2)*sin(d*x+c)+86*B*ln(-(-(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*sin(d*x+c)*cos(d*x+c)+32*B*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+
c)*2^(1/2))*sin(d*x+c)+78*A*cos(d*x+c)^3-115*A*ln(-(-(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*sin(d*x+c)-30*B*cos(d*x+c)^3+43*B*ln(-(-(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)+cos(d*x+c)-1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)-40*A*cos(d*x+c)^2+8*B*cos(d*x+c)^2-70*A*cos(d*x+c
)+22*B*cos(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^5/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/(a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A]
time = 10.80, size = 739, normalized size = 3.57

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] [1/64*(sqrt(2)*((115*A - 43*B)*cos(d*x + c)^3 + 3*(115*A - 43*B)*cos(d*x +
c)^2 + 3*(115*A - 43*B)*cos(d*x + c) + 115*A - 43*B)*sqrt(-a)*log(-(2*sqrt(
2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x +
c) - 3*a*cos(d*x + c)^2 - 2*a*cos(d*x + c) + a)/(cos(d*x + c)^2 + 2*cos(d*x
 + c) + 1)) + 32*((5*A - 2*B)*cos(d*x + c)^3 + 3*(5*A - 2*B)*cos(d*x + c)^2
 + 3*(5*A - 2*B)*cos(d*x + c) + 5*A - 2*B)*sqrt(-a)*log((2*a*cos(d*x + c)^2
 + 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
+ c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 4*(16*A*cos(d*x + c)^3 + 5
*(11*A - 3*B)*cos(d*x + c)^2 + (35*A - 11*B)*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d
*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((115*A - 43*B)*c
os(d*x + c)^3 + 3*(115*A - 43*B)*cos(d*x + c)^2 + 3*(115*A - 43*B)*cos(d*x
+ c) + 115*A - 43*B)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - 32*((5*A - 2*B)*cos(d*x + c
)^3 + 3*(5*A - 2*B)*cos(d*x + c)^2 + 3*(5*A - 2*B)*cos(d*x + c) + 5*A - 2*B
)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt
(a)*sin(d*x + c))) - 2*(16*A*cos(d*x + c)^3 + 5*(11*A - 3*B)*cos(d*x + c)^2
 + (35*A - 11*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
+ c) + a^3*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 464 vs. 2(178) = 356.
time = 2.11, size = 464, normalized size = 2.24

$$-\frac{2\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\left(\frac{2\sqrt{2}(Aa^5-Ba^5)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}{a^8\mathrm{sgn}(\cos(dx+c))}-\frac{\sqrt{2}(21Aa^5-13Ba^5)}{a^8\mathrm{sgn}(\cos(dx+c))}\right)\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+\frac{\sqrt{2}(115A-43B)\log\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2\right)}{\sqrt{-a}\,a^2\mathrm{sgn}(\cos(dx+c))}-\frac{32(5A-2B)\log\left(\frac{\left|-562949953421312\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2-1125899906842624\sqrt{2}|a|+1688849860263936a\right|}{\left|-562949953421312\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2+1125899906842624\sqrt{2}|a|+1688849860263936a\right|}\right)}{\sqrt{-a}\,a|a|\mathrm{sgn}(\cos(dx+c))}-\frac{128\sqrt{2}\left(3\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2A-Aa\right)}{\left(\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^4-6\left(\sqrt{-a}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-\sqrt{-a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}\right)^2a+a^2\right)\sqrt{-a}\,a\,\mathrm{sgn}(\cos(dx+c))}}{64d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] -1/64*(2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5)*tan
(1/2*d*x + 1/2*c)^2/(a^8*sgn(cos(d*x + c))) - sqrt(2)*(21*A*a^5 - 13*B*a^5)
/(a^8*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c) + sqrt(2)*(115*A - 43*B)*log
```

```
((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/(
sqrt(-a)*a^2*sgn(cos(d*x + c))) - 32*(5*A - 2*B)*log(abs(-562949953421312*(
sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 - 11
25899906842624*sqrt(2)*abs(a) + 1688849860263936*a)/abs(-562949953421312*(s
qrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 112
5899906842624*sqrt(2)*abs(a) + 1688849860263936*a))/(sqrt(-a)*a*abs(a)*sgn(
cos(d*x + c))) - 128*sqrt(2)*(3*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*ta
n(1/2*d*x + 1/2*c)^2 + a))^2*A - A*a)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sq
rt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - s
qrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*a + a^2)*sqrt(-a)*a*sgn(cos(d*x + c))
))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(5/2),x)
[Out] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(5/2), x)
```

3.166 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=264

$$\frac{(39A-20B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{5/2}d}-\frac{(219A-115B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{(A-}{4d}$$

```
[Out] 1/4*(39*A-20*B)*arctan(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d
-1/4*(A-B)*cos(d*x+c)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-1/16*(19*A-11*B)*
cos(d*x+c)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)-1/32*(219*A-115*B)*arctan(
1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-7/
16*(9*A-5*B)*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)+1/16*(31*A-15*B)*cos(d
*x+c)*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 264, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4105, 4107, 4005, 3859, 209, 3880}

$$\frac{(39A-20B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{5/2}d}-\frac{(219A-115B)\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}-\frac{7(9A-5B)\sin(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(31A-15B)\sin(c+dx)\cos(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(19A-11B)\sin(c+dx)\cos(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)\cos(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((39*A - 20*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(4*
a^(5/2)*d) - ((219*A - 115*B)*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a
 + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Cos[c + d*x]*Sin[c
+ d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((19*A - 11*B)*Cos[c + d*x]*Sin[
c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - (7*(9*A - 5*B)*Sin[c + d*x]
)/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((31*A - 15*B)*Cos[c + d*x]*Sin[c +
 d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\cos^2(c+dx)\left(2a(3A-B)-\frac{7}{2}a(A-B)\,\mathrm{se}\right.}{(a+a\sec(c+dx))^{3/2}}}{4a^2}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A-11B)\cos(c+dx)\sin(}{16ad(a+a\sec(c+dx))^{3}}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A-11B)\cos(c+dx)\sin(}{16ad(a+a\sec(c+dx))^{3}}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A-11B)\cos(c+dx)\sin(}{16ad(a+a\sec(c+dx))^{3}}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A-11B)\cos(c+dx)\sin(}{16ad(a+a\sec(c+dx))^{3}}$$

$$= -\frac{(A-B)\cos(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(19A-11B)\cos(c+dx)\sin(}{16ad(a+a\sec(c+dx))^{3}}$$

$$= \frac{(39A-20B)\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{5/2}d} - \frac{(219A-115B)\tan}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 3 in optimal.
time = 6.15, size = 512, normalized size = 1.94

$$-\frac{B\sin(c+dx)}{4d(a(1+\sec(c+dx)))^{5/2}} - \frac{A\cos(c+dx)\sin(c+dx)}{4d(a(1+\sec(c+dx)))^{5/2}} - \frac{5B(1+\sec(c+dx))^{5/2}\left(\frac{6\sin(c+dx)}{d(1+\sec(c+dx))^{3/2}} + \frac{9\left(\cos(c+dx)+\frac{\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}} + \frac{23\left(\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)-\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)-\cos(c+dx)\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\right)}{32(a(1+\sec(c+dx)))^{5/2}} - \frac{A(1+\sec(c+dx))^{5/2}\left(\frac{152\cos(c+dx)\sin(c+dx)}{d(1+\sec(c+dx))^{3/2}} + \frac{760\,{}_2F_1\left(\frac{1}{2},3;\frac{3}{2};1-\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}} - \frac{219\left(7\tanh^{-1}\left(\sqrt{1-\sec(c+dx)}\right)-4\sqrt{2}\tanh^{-1}\left(\frac{\sqrt{1-\sec(c+dx)}}{\sqrt{2}}\right)-\cos(c+dx)\sqrt{1-\sec(c+dx)}+2\cos^2(c+dx)\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\sqrt{1+\sec(c+dx)}}\right)}{128(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),
x]
```

```
[Out] -1/4*(B*Sin[c + d*x])/(d*(a*(1 + Sec[c + d*x]))^(5/2)) - (A*Cos[c + d*x]*Si
n[c + d*x])/(4*d*(a*(1 + Sec[c + d*x]))^(5/2)) - (5*B*(1 + Sec[c + d*x])^(5
/2)*((6*Sin[c + d*x])/(d*(1 + Sec[c + d*x])^(3/2)) + (9*(Cos[c + d*x] + Arc
Tanh[Sqrt[1 - Sec[c + d*x]]]/Sqrt[1 - Sec[c + d*x]])*Tan[c + d*x])/(d*Sqrt[
1 + Sec[c + d*x]]) + (23*(ArcTanh[Sqrt[1 - Sec[c + d*x]]] - Sqrt[2]*ArcTanh
[Sqrt[1 - Sec[c + d*x]]/Sqrt[2]] - Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]])*Tan
[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]])))/(32*(a*(1 +
```

```
Sec[c + d*x]))^(5/2)) - (A*(1 + Sec[c + d*x])^(5/2)*((152*Cos[c + d*x]*Sin[
c + d*x])/(d*(1 + Sec[c + d*x])^(3/2)) + (760*Hypergeometric2F1[1/2, 3, 3/2
, 1 - Sec[c + d*x]]*Tan[c + d*x])/(d*Sqrt[1 + Sec[c + d*x]]) - (219*(7*ArcT
anh[Sqrt[1 - Sec[c + d*x]]] - 4*Sqrt[2]*ArcTanh[Sqrt[1 - Sec[c + d*x]]/Sqrt
[2]] - Cos[c + d*x]*Sqrt[1 - Sec[c + d*x]] + 2*Cos[c + d*x]^2*Sqrt[1 - Sec[
c + d*x]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[1 + Sec[c + d*x]]))
)/(128*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1426 vs. $2(229) = 458$.
time = 9.10, size = 1427, normalized size = 5.41

method	result	size
default	Expression too large to display	1427

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/64/d*(-1+cos(d*x+c))^2*(468*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctan
h(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*s
in(d*x+c)*cos(d*x+c)*2^(1/2)-240*B*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arc
tanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2)
)*sin(d*x+c)*cos(d*x+c)*2^(1/2)+468*A*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-252*A*cos(d*x+c)^2+156*A*2^(1/2)*sin(d
*x+c)*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))+112*A*cos(d*x+
c)^5+219*A*sin(d*x+c)*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*ln(
((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-
115*B*sin(d*x+c)*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*ln(((-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))+219*A
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)-115*B*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(3/2)*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos
(d*x+c)+1)/sin(d*x+c))*sin(d*x+c)-240*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)^2*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))-32*A*cos(d*x+c)^6-64*B*cos(d*x+c)^5-
80*B*2^(1/2)*sin(d*x+c)*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*a
rctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/
2))+657*A*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*ln(((-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-345
*B*sin(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)*ln(((-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)/sin(d*x+c))-128*A*cos(
d*x+c)^3+80*B*cos(d*x+c)^3+140*B*cos(d*x+c)^2+300*A*cos(d*x+c)^4-156*B*cos(
```

```
d*x+c)^4+657*A*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+
c)+1)/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2*sin(d*x
+c)-345*B*ln(((-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-cos(d*x+c)+1)
/sin(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*cos(d*x+c)^2*sin(d*x+c)+1
56*A*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c)-80*B*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*arctanh(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*sin(d*x+c)/cos(d*x+c)*2^(1/2))*2^(1/2)*sin(d*x+c))*(a*(1+cos(d*x+c))
/cos(d*x+c))^(1/2)/sin(d*x+c)^5/cos(d*x+c)/a^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/(a*sec(d*x + c) + a)^(5/2), x
)
```

Fricas [A]
time = 10.60, size = 776, normalized size = 2.94

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] [1/64*(sqrt(2)*((219*A - 115*B)*cos(d*x + c)^3 + 3*(219*A - 115*B)*cos(d*x
+ c)^2 + 3*(219*A - 115*B)*cos(d*x + c) + 219*A - 115*B)*sqrt(-a)*log((2*sq
rt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)*sin(d*x
 + c) + 3*a*cos(d*x + c)^2 + 2*a*cos(d*x + c) - a)/(cos(d*x + c)^2 + 2*cos(
d*x + c) + 1)) + 8*((39*A - 20*B)*cos(d*x + c)^3 + 3*(39*A - 20*B)*cos(d*x
+ c)^2 + 3*(39*A - 20*B)*cos(d*x + c) + 39*A - 20*B)*sqrt(-a)*log((2*a*cos(
d*x + c)^2 - 2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c
)*sin(d*x + c) + a*cos(d*x + c) - a)/(cos(d*x + c) + 1)) + 4*(8*A*cos(d*x +
 c)^4 - 4*(5*A - 4*B)*cos(d*x + c)^3 - 5*(19*A - 11*B)*cos(d*x + c)^2 - 7*(
9*A - 5*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) +
 a^3*d), 1/32*(sqrt(2)*((219*A - 115*B)*cos(d*x + c)^3 + 3*(219*A - 115*B)*
cos(d*x + c)^2 + 3*(219*A - 115*B)*cos(d*x + c) + 219*A - 115*B)*sqrt(a)*ar
ctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*
sin(d*x + c))) - 8*((39*A - 20*B)*cos(d*x + c)^3 + 3*(39*A - 20*B)*cos(d*x
```

```
+ c)^2 + 3*(39*A - 20*B)*cos(d*x + c) + 39*A - 20*B)*sqrt(a)*arctan(sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) + 2*(
8*A*cos(d*x + c)^4 - 4*(5*A - 4*B)*cos(d*x + c)^3 - 5*(19*A - 11*B)*cos(d*x
 + c)^2 - 7*(9*A - 5*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*c
os(d*x + c) + a^3*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\cos^2{(c + dx)}}{(a(\sec{(c + dx)} + 1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/(a*(sec(c + d*x) + 1))**(5/2)
, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 685 vs. 2(229) = 458.
time = 2.21, size = 685, normalized size = 2.59

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] 1/64*(2*sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a)*(2*sqrt(2)*(A*a^5 - B*a^5)*tan(
1/2*d*x + 1/2*c)^2/(a^8*sgn(cos(d*x + c))) - sqrt(2)*(29*A*a^5 - 21*B*a^5)/
(a^8*sgn(cos(d*x + c))))*tan(1/2*d*x + 1/2*c) + sqrt(2)*(219*A - 115*B)*log
((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2)/(
sqrt(-a)*a^2*sgn(cos(d*x + c))) + 8*(39*A - 20*B)*log(abs(30948500982134506
8724781056*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2
+ a))^2 - 618970019642690137449562112*sqrt(2)*abs(a) - 92845502946403520617
4343168*a)/abs(309485009821345068724781056*(sqrt(-a)*tan(1/2*d*x + 1/2*c) -
 sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2 + 618970019642690137449562112*sqrt(
2)*abs(a) - 928455029464035206174343168*a))/(sqrt(-a)*a*abs(a)*sgn(cos(d*x
+ c))) - 32*sqrt(2)*(41*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*
x + 1/2*c)^2 + a))^6*A - 12*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/
2*d*x + 1/2*c)^2 + a))^6*B - 209*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*t
an(1/2*d*x + 1/2*c)^2 + a))^4*A*a + 76*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqr
t(-a*tan(1/2*d*x + 1/2*c)^2 + a))^4*B*a + 91*(sqrt(-a)*tan(1/2*d*x + 1/2*c)
 - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*A*a^2 - 36*(sqrt(-a)*tan(1/2*d*x
```

```
+ 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^2*B*a^2 - 11*A*a^3 + 4*B*a^
3)/(((sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))^
4 - 6*(sqrt(-a)*tan(1/2*d*x + 1/2*c) - sqrt(-a*tan(1/2*d*x + 1/2*c)^2 + a))
^2*a + a^2)^2*sqrt(-a)*a*sgn(cos(d*x + c))))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(5/2),x)
[Out] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(5/2), x)
```

3.167 $\int \frac{A+A\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx$

Optimal. Leaf size=89

$$\frac{2A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{2\sqrt{2}\,A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*A*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/d/a^(1/2)-2*A*arctan(
1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.154$, Rules used = {3989, 3972, 492, 209}

$$\frac{2A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{2\sqrt{2}\,A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + A*Sec[c + d*x])/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(Sqrt[a]*d) -
 (2*Sqrt[2]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x
]]])])/(Sqrt[a]*d)
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 492

```
Int[((e_.)*(x_))^(m_.)/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(n_))),
 x_Symbol] :> Dist[(-a)*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(a + b*x^n), x
], x] + Dist[c*(e^n/(b*c - a*d)), Int[(e*x)^(m - n)/(c + d*x^n), x], x] /;
FreeQ[{a, b, c, d, e, m}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LeQ[n, m
, 2*n - 1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
```

```
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{A+A\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx = -\left((aA)\int \frac{\tan^2(c+dx)}{(a-a\sec(c+dx))^{3/2}}\,dx\right)$$

$$= \frac{(2aA)\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

$$= -\frac{(2A)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d} + \frac{(4A)\text{Subst}\left(\int \frac{1}{2+ax^2}\,dx\right.}{}$$

$$= \frac{2A\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{2\sqrt{2}\,A\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 0.54, size = 140, normalized size = 1.57

$$-\frac{iA\left(-1+e^{i(c+dx)}\right)\left(\sqrt{2}\,\sinh^{-1}\left(e^{i(c+dx)}\right)-4\tanh^{-1}\left(\frac{1+e^{i(c+dx)}}{\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}}\right)+\sqrt{2}\,\tanh^{-1}\left(\sqrt{1+e^{2i(c+dx)}}\right)\right)}{\sqrt{2}\,d\sqrt{1+e^{2i(c+dx)}}\,\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + A*Sec[c + d*x])/Sqrt[a - a*Sec[c + d*x]],x]
[Out] ((-I)*A*(-1 + E^(I*(c + d*x)))*(Sqrt[2]*ArcSinh[E^(I*(c + d*x))] - 4*ArcTan
h[(1 + E^(I*(c + d*x)))/(Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))])] + Sqrt[2]*
ArcTanh[Sqrt[1 + E^((2*I)*(c + d*x))]]))/(Sqrt[2]*d*Sqrt[1 + E^((2*I)*(c +
d*x))]*Sqrt[a - a*Sec[c + d*x]])
```

Maple [A]
time = 4.67, size = 120, normalized size = 1.35

method	result
default	$\frac{A(1+\cos(dx+c))\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{a(-1+\cos(dx+c))}{\cos(dx+c)}}\left(\sqrt{2}\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)+\arctan\left(\frac{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+}}}{2}\right.\right.}{da\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] A/d*(1+cos(d*x+c))*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(a*(-1+cos(d*x+c))/cos(d*x+c))^(1/2)*(2^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)))/a/sin(d*x+c)*2^(1/2)

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] Exception raised: RuntimeError >> ECL says: Error executing code in Maxima: sign: argument cannot be imaginary; found %i

Fricas [A]
time = 1.57, size = 305, normalized size = 3.43

$$\left[\frac{\sqrt{2}\,Aa\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)-A\sqrt{-a}\log\left(\frac{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}-(2a\cos(dx+c)+a)\sin(dx+c)}{\sin(dx+c)}\right)}{ad},\ \frac{2\left(\sqrt{2}\,A\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)-A\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm="fricas")

[Out] [(sqrt(2)*A*a*sqrt(-1/a)*log(-(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c) + 1)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c))) - A*sqrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c)))/(a*d), 2*(sqrt(2)*A*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))) - A*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c))))/(a*d)]

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$A\left(\int \frac{\sec(c+dx)}{\sqrt{-a\sec(c+dx)+a}}\,dx+\int \frac{1}{\sqrt{-a\sec(c+dx)+a}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(1/2),x)
```

```
[Out] A*(Integral(sec(c + d*x)/sqrt(-a*sec(c + d*x) + a), x) + Integral(1/sqrt(-a
*sec(c + d*x) + a), x))
```

Giac [A]
time = 1.01, size = 71, normalized size = 0.80

$$\frac{2\,A\left(\frac{\sqrt{2}\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a}}{2\sqrt{a}}\right)}{\sqrt{a}}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] 2*A*(sqrt(2)*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/sqrt(a) - a
rctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/sqrt(a))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{A}{\cos(c+d\,x)}}{\sqrt{a-\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + A/cos(c + d*x))/(a - a/cos(c + d*x))^(1/2),x)
```

```
[Out] int((A + A/cos(c + d*x))/(a - a/cos(c + d*x))^(1/2), x)
```

3.168 $\int \frac{\cos(c+dx)(A+A\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}}\,dx$

Optimal. Leaf size=115

$$\frac{3A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{2\sqrt{2}\,A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{A\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}$$

```
[Out] 3*A*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/d/a^(1/2)-2*A*arctan(
1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)+A*
sin(d*x+c)/d/(a-a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.156$, Rules used = {4107, 4005, 3859, 209, 3880}

$$\frac{3A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{2\sqrt{2}\,A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{A\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + A*Sec[c + d*x]))/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (3*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(Sqrt[a]*d) -
 (2*Sqrt[2]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x
]])])/(Sqrt[a]*d) + (A*Sin[c + d*x])/(d*Sqrt[a - a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\cos(c+dx)(A+A\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}}\,dx &= \frac{A\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}} - \frac{\int \frac{-\frac{3aA}{2}-\frac{1}{2}aA\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx}{a}\\ &= \frac{A\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}} + (2A)\int \frac{\sec(c+dx)}{\sqrt{a-a\sec(c+dx)}}\,dx + (3A)\int\\ &= \frac{A\sin(c+dx)}{d\sqrt{a-a\sec(c+dx)}} + \frac{(3A)\mathrm{Subst}\left(\int \frac{1}{a+x^2}\,dx, x, \frac{a\tan(c+dx)}{\sqrt{a-a\sec(c+}}\right.}{d}\\ &= \frac{3A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{2\sqrt{2}A\tan^{-1}\left(\frac{\sqrt{a}\tan(c}{\sqrt{2}\sqrt{a-a\,\mathrm{se}}}\right.}{\sqrt{a}\,d}\end{aligned}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.52, size = 382, normalized size = 3.32

$$A\left(\frac{e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}(1+\cos(c+dx))\left(-3idx+3\sinh^{-1}\left(e^{i(c+dx)}\right)+4\sqrt{2}\log\left(1-e^{i(c+dx)}\right)+3\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)-4\sqrt{2}\log\left(1+e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sqrt{2}d\sqrt{a-a\sec(c+dx)}}+\frac{(1+\cos(c+dx))\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\sec(c+dx)\left(\frac{\cos\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{2d}+\frac{\cos\left(\frac{3c}{2}\right)\cos\left(\frac{3dx}{2}\right)}{2d}-\frac{\sin\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{2d}-\frac{\sin\left(\frac{3c}{2}\right)\sin\left(\frac{3dx}{2}\right)}{2d}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{\sqrt{a-a\sec(c+dx)}}\right)$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + A*Sec[c + d*x]))/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] A*((Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
d*x))]*(1 + Cos[c + d*x])*((-3*I)*d*x + 3*ArcSinh[E^(I*(c + d*x))] + 4*Sqrt
```

```
[2]*Log[1 - E^(I*(c + d*x))] + 3*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - 4
*Sqrt[2]*Log[1 + E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])*
Sec[c/2 + (d*x)/2]*Sqrt[Sec[c + d*x]]*Tan[c/2 + (d*x)/2])/(2*Sqrt[2]*d*E^((
I/2)*(c + d*x))*Sqrt[a - a*Sec[c + d*x]]) + ((1 + Cos[c + d*x])*Sec[c/2 + (
d*x)/2]*Sec[c + d*x]*((Cos[c/2]*Cos[(d*x)/2])/(2*d) + (Cos[(3*c)/2]*Cos[(3*
d*x)/2])/(2*d) - (Sin[c/2]*Sin[(d*x)/2])/(2*d) - (Sin[(3*c)/2]*Sin[(3*d*x)/
2])/(2*d))*Tan[c/2 + (d*x)/2])/Sqrt[a - a*Sec[c + d*x]])
```

Maple [A]
time = 8.53, size = 154, normalized size = 1.34

method	result
default	$-\frac{A(1+\cos(dx+c))\left(-2\sqrt{2}\,\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right)+\sqrt{2}\,\cos(dx+c)-3\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\,\text{arc}}{2d\sin(dx+c)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/2*A/d*(1+cos(d*x+c))*(-2*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*ar
ctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+2^(1/2)*cos(d*x+c)-3*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*2^(1/2)))*(a*(-1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/a*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)/sqrt(-a*sec(d*x + c) + a), x)
```

Fricas [A]
time = 1.97, size = 435, normalized size = 3.78

$$\left[\frac{2\sqrt{2}\,Aa\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)-3\,A\sqrt{-a}\log\left(\frac{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}-(2a\cos(dx+c)+a)\sin(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)-2\left(A\cos(dx+c)^2+A\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{2ad\sin(dx+c)},\ \frac{2\sqrt{2}\,A\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)-3\,A\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)-\left(A\cos(dx+c)^2+A\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{ad\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] [1/2*(2*sqrt(2)*A*a*sqrt(-1/a)*log(-(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x +
c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c) +
1)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) - 3*A*sqrt
(-a)*log((2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) -
 a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin(
d*x + c) - 2*(A*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/
cos(d*x + c)))/(a*d*sin(d*x + c)), (2*sqrt(2)*A*sqrt(a)*arctan(sqrt(2)*sqrt
((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*si
n(d*x + c) - 3*A*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos
(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - (A*cos(d*x + c)^2 + A*cos(
d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/(a*d*sin(d*x + c))]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$A\left(\int \frac{\cos(c+dx)}{\sqrt{-a\sec(c+dx)+a}}\,dx+\int \frac{\cos(c+dx)\sec(c+dx)}{\sqrt{-a\sec(c+dx)+a}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(1/2),x)
```

```
[Out] A*(Integral(cos(c + d*x)/sqrt(-a*sec(c + d*x) + a), x) + Integral(cos(c + d
*x)*sec(c + d*x)/sqrt(-a*sec(c + d*x) + a), x))
```

Giac [A]

time = 1.05, size = 114, normalized size = 0.99

$$\frac{\frac{2\sqrt{2}\,A\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{3\,A\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{2\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}\,A}{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] (2*sqrt(2)*A*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/sqrt(a) - 3
*A*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/sqrt(a) -
 sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*A/(a*tan(1/2*d*x + 1/2*c)^2 + a
))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos(c+d\,x)\,\left(A+\frac{A}{\cos(c+d\,x)}\right)}{\sqrt{a-\frac{a}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(1/2), x)
```

3.169 $\int \frac{\cos^2(c+dx)(A+A\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}}\,dx$

Optimal. Leaf size=155

$$\frac{11A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{4\sqrt{a}\,d}-\frac{2\sqrt{2}\,A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{5A\sin(c+dx)}{4d\sqrt{a-a\sec(c+dx)}}+$$

```
[Out] 11/4*A*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/d/a^(1/2)-2*A*arct
an(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)
+5/4*A*sin(d*x+c)/d/(a-a*sec(d*x+c))^(1/2)+1/2*A*cos(d*x+c)*sin(d*x+c)/d/(a
-a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 155, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {4107, 4005, 3859, 209, 3880}

$$\frac{11A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{4\sqrt{a}\,d}-\frac{2\sqrt{2}\,A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{5A\sin(c+dx)}{4d\sqrt{a-a\sec(c+dx)}}+\frac{A\sin(c+dx)\cos(c+dx)}{2d\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + A*Sec[c + d*x]))/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (11*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(4*Sqrt[a]*d
) - (2*Sqrt[2]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c +
d*x]])])/(Sqrt[a]*d) + (5*A*Sin[c + d*x])/(4*d*Sqrt[a - a*Sec[c + d*x]]) +
(A*Cos[c + d*x]*Sin[c + d*x])/(2*d*Sqrt[a - a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
```

```
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+A\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}}\,dx = \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a-a\sec(c+dx)}} - \frac{\int \frac{\cos(c+dx)\left(-\frac{5aA}{2}-\frac{3}{2}aA\sec(c+dx)\right)}{\sqrt{a-a\sec(c+dx)}}\,dx}{2a}$$

$$= \frac{5A\sin(c+dx)}{4d\sqrt{a-a\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a-a\sec(c+dx)}} + \frac{\int \frac{\frac{11a^2A}{4}+\frac{5}{4}\cdots}{\sqrt{a-a\cdots}}\cdots}{\cdots}$$

$$= \frac{5A\sin(c+dx)}{4d\sqrt{a-a\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a-a\sec(c+dx)}} + (2A)\int \frac{\cdots}{\sqrt{\cdots}}\cdots$$

$$= \frac{5A\sin(c+dx)}{4d\sqrt{a-a\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2d\sqrt{a-a\sec(c+dx)}} + \frac{(11A)\text{Subs}\cdots}{\cdots}$$

$$= \frac{11A\tan^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{4\sqrt{a}\,d} - \frac{2\sqrt{2}\,A\tan^{-1}\left(\frac{\sqrt{a}\,\tan\cdots}{\sqrt{2}\,\sqrt{a-\cdots}}\right)}{\sqrt{a}\,d}\cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.27, size = 332, normalized size = 2.14

$$\frac{Ae^{-i(c+dx)}\left(7+6e^{-i(c+dx)}+7e^{i(c+dx)}+e^{-2i(c+dx)}+6e^{2i(c+dx)}+e^{3i(c+dx)}-11id\sqrt{1+e^{2i(c+dx)}}\,x+11\sqrt{1+e^{2i(c+dx)}}\,\sinh^{-1}\left(e^{i(c+dx)}\right)+16\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}\,\log\left(1-e^{i(c+dx)}\right)+11\sqrt{1+e^{2i(c+dx)}}\,\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)-16\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}\,\log\left(1+e^{i(c+dx)}+\sqrt{2}\,\sqrt{1+e^{2i(c+dx)}}\right)\right)\sec(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+i\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin\left(\frac{1}{2}(c+dx)\right)}{8d\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[(Cos[c + d*x]^2*(A + A*Sec[c + d*x]))/Sqrt[a - a*Sec[c + d*x]],x]

[Out] (A*(7 + 6/E^(I*(c + d*x)) + 7*E^(I*(c + d*x)) + E^((-2*I)*(c + d*x)) + 6*E^
((2*I)*(c + d*x)) + E^((3*I)*(c + d*x)) - (11*I)*d*Sqrt[1 + E^((2*I)*(c + d
*x))]*x + 11*Sqrt[1 + E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))] + 16*Sq
rt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]*Log[1 - E^(I*(c + d*x))] + 11*Sqrt[1 +
E^((2*I)*(c + d*x))]*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - 16*Sqrt[2]*Sq
rt[1 + E^((2*I)*(c + d*x))]*Log[1 + E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((
2*I)*(c + d*x))]])*Sec[c + d*x]*(Cos[(c + d*x)/2] + I*Sin[(c + d*x)/2])*Sin
[(c + d*x)/2])/(8*d*E^(I*(c + d*x))*Sqrt[a - a*Sec[c + d*x]])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 366 vs. $2(130) = 260$.
time = 8.88, size = 367, normalized size = 2.37

method	result
default	$-\frac{A(-1+\cos(dx+c))^2\left(16\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{2}\cos(dx+c)-6\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{2}\left(\cos^3(dx+c)\right)+16\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)

[Out] -1/24*A/d*(-1+cos(d*x+c))^2*(16*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2
)*cos(d*x+c)-6*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*cos(d*x+c)^3+16
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)-27*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*2^(1/2)*cos(d*x+c)^2-4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1
/2)*cos(d*x+c)-48*2^(1/2)*cos(d*x+c)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2))-15*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)-48*2^(1/2)*arctan(
1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-66*cos(d*x+c)*arctan(1/2*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-66*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*2^(1/2)))/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(a*(-1+cos(d*x+c
))/cos(d*x+c))^(1/2)/sin(d*x+c)^3*2^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)^2/sqrt(-a*sec(d*x + c) + a), x)
```

Fricas [A]
time = 1.80, size = 462, normalized size = 2.98

$$\left[\frac{8\sqrt{2}Aa\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)-11A\sqrt{-a}\log\left(\frac{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}-(2a\cos(dx+c)+a)\sin(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)-2\left(2A\cos(dx+c)^3+7A\cos(dx+c)^2+5A\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{8ad\sin(dx+c)}, \frac{8\sqrt{2}A\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)-11A\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)-\left(2A\cos(dx+c)^3+7A\cos(dx+c)^2+5A\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{4ad\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/8*(8*sqrt(2)*A*a*sqrt(-1/a)*log(-(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x +
c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c) +
1)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) - 11*A*sqr
t(-a)*log((2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c)
- a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*sin
(d*x + c) - 2*(2*A*cos(d*x + c)^3 + 7*A*cos(d*x + c)^2 + 5*A*cos(d*x + c))*
sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/(a*d*sin(d*x + c)), 1/4*(8*sqrt(2)
*A*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x +
 c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 11*A*sqrt(a)*arctan(sqrt((a*cos(
d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x +
c) - (2*A*cos(d*x + c)^3 + 7*A*cos(d*x + c)^2 + 5*A*cos(d*x + c))*sqrt((a*c
os(d*x + c) - a)/cos(d*x + c)))/(a*d*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$A\left(\int\frac{\cos^2(c+dx)}{\sqrt{-a\sec(c+dx)+a}}\,dx+\int\frac{\cos^2(c+dx)\sec(c+dx)}{\sqrt{-a\sec(c+dx)+a}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(1/2),x)
```

```
[Out] A*(Integral(cos(c + d*x)**2/sqrt(-a*sec(c + d*x) + a), x) + Integral(cos(c
+ d*x)**2*sec(c + d*x)/sqrt(-a*sec(c + d*x) + a), x))
```

Giac [A]
time = 1.06, size = 141, normalized size = 0.91

$$\frac{\frac{8\sqrt{2}A\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{11A\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{2\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{2}\left(3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^{\frac{3}{2}}A+10\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}\,Aa\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+a\right)^2}}{4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] 1/4*(8*sqrt(2)*A*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/sqrt(a)
 - 11*A*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/sqrt
(a) - sqrt(2)*(3*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(3/2)*A + 10*sqrt(a*tan(1/2
*d*x + 1/2*c)^2 - a)*A*a)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^2)/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(A+\frac{A}{\cos\left(c+d\,x\right)}\right)}{\sqrt{a-\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(1/2), x)
```

3.170 $\int \frac{\cos^3(c+dx)(A+A\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}}\,dx$

Optimal. Leaf size=192

$$\frac{23A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{a}\,d}-\frac{2\sqrt{2}\,A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{9A\sin(c+dx)}{8d\sqrt{a-a\sec(c+dx)}}$$

```
[Out] 23/8*A*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/d/a^(1/2)-2*A*arct
an(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))*2^(1/2)/d/a^(1/2)
+9/8*A*sin(d*x+c)/d/(a-a*sec(d*x+c))^(1/2)+7/12*A*cos(d*x+c)*sin(d*x+c)/d/(
a-a*sec(d*x+c))^(1/2)+1/3*A*cos(d*x+c)^2*sin(d*x+c)/d/(a-a*sec(d*x+c))^(1/2
)
```

Rubi [A]

time = 0.35, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.147$, Rules used = {4107, 4005, 3859, 209, 3880}

$$\frac{23A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{a}\,d}-\frac{2\sqrt{2}\,A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{9A\sin(c+dx)}{8d\sqrt{a-a\sec(c+dx)}}+\frac{A\sin(c+dx)\cos^2(c+dx)}{3d\sqrt{a-a\sec(c+dx)}}+\frac{7A\sin(c+dx)\cos(c+dx)}{12d\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + A*Sec[c + d*x]))/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (23*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(8*Sqrt[a]*d
) - (2*Sqrt[2]*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c +
d*x]])])/(Sqrt[a]*d) + (9*A*Sin[c + d*x])/(8*d*Sqrt[a - a*Sec[c + d*x]]) +
(7*A*Cos[c + d*x]*Sin[c + d*x])/(12*d*Sqrt[a - a*Sec[c + d*x]]) + (A*Cos[c
+ d*x]^2*Sin[c + d*x])/(3*d*Sqrt[a - a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+A\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}}\,dx = \frac{A\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a-a\sec(c+dx)}} - \frac{\int \frac{\cos^2(c+dx)\left(-\frac{7aA}{2}-\frac{5}{2}aA\sec(c+dx)\right)}{\sqrt{a-a\sec(c+dx)}}\,dx}{3a}$$

$$= \frac{7A\cos(c+dx)\sin(c+dx)}{12d\sqrt{a-a\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin(c+dx)}{3d\sqrt{a-a\sec(c+dx)}} + \frac{\int \frac{\cos(c+}{\sqrt{}}}{}$$

$$= \frac{9A\sin(c+dx)}{8d\sqrt{a-a\sec(c+dx)}} + \frac{7A\cos(c+dx)\sin(c+dx)}{12d\sqrt{a-a\sec(c+dx)}} + \frac{A\cos^2(c+}{3d\sqrt{a-}}$$

$$= \frac{9A\sin(c+dx)}{8d\sqrt{a-a\sec(c+dx)}} + \frac{7A\cos(c+dx)\sin(c+dx)}{12d\sqrt{a-a\sec(c+dx)}} + \frac{A\cos^2(c+}{3d\sqrt{a-}}$$

$$= \frac{9A\sin(c+dx)}{8d\sqrt{a-a\sec(c+dx)}} + \frac{7A\cos(c+dx)\sin(c+dx)}{12d\sqrt{a-a\sec(c+dx)}} + \frac{A\cos^2(c+}{3d\sqrt{a-}}$$

$$= \frac{23A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{a}\,d} - \frac{2\sqrt{2}\,A\tan^{-1}\left(\frac{\sqrt{a}\tan}{\sqrt{2}\sqrt{a-a}}\right.}{\sqrt{a}\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.45, size = 362, normalized size = 1.89

$$\frac{Ae^{-i(c+dx)}\left(47+40e^{-i(c+dx)}+47e^{i(c+dx)}+9e^{-2i(c+dx)}+40e^{2i(c+dx)}+2e^{-3i(c+dx)}+9e^{3i(c+dx)}+2e^{4i(c+dx)}-69id\sqrt{1+e^{2i(c+dx)}}\,x+69\sqrt{1+e^{2i(c+dx)}}\sinh^{-1}\left(e^{i(c+dx)}\right)+96\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\log\left(1-e^{i(c+dx)}\right)+69\sqrt{1+e^{2i(c+dx)}}\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)-96\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\log\left(1+e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)\right)\sec(c+dx)\left(\cos\left(\frac{1}{2}(c+dx)\right)+i\sin\left(\frac{1}{2}(c+dx)\right)\right)\sin\left(\frac{1}{2}(c+dx)\right)}{48d\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + A*Sec[c + d*x]))/Sqrt[a - a*Sec[c + d*x]],x]
```

```
[Out] (A*(47 + 40/E^(I*(c + d*x)) + 47*E^(I*(c + d*x)) + 9/E^((2*I)*(c + d*x)) +
40*E^((2*I)*(c + d*x)) + 2/E^((3*I)*(c + d*x)) + 9*E^((3*I)*(c + d*x)) + 2*
E^((4*I)*(c + d*x)) - (69*I)*d*Sqrt[1 + E^((2*I)*(c + d*x))]*x + 69*Sqrt[1
+ E^((2*I)*(c + d*x))]*ArcSinh[E^(I*(c + d*x))] + 96*Sqrt[2]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Log[1 - E^(I*(c + d*x))] + 69*Sqrt[1 + E^((2*I)*(c + d*x))]
*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - 96*Sqrt[2]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Log[1 + E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])*S
ec[c + d*x]*(Cos[(c + d*x)/2] + I*Sin[(c + d*x)/2])*Sin[(c + d*x)/2])/(48*d
*E^(I*(c + d*x))*Sqrt[a - a*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 624 vs. $2(163) = 326$.
time = 8.97, size = 625, normalized size = 3.26

method	result
default	$-\frac{A(-1+\cos(dx+c))^3\left(96\sqrt{2}\,(\cos^2(dx+c))\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}+40\sqrt{2}\,(\cos^5(dx+c))\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}+192\sqrt{2}\,\cos(dx+c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/240*A/d*(-1+cos(d*x+c))^3*(96*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(5/2)+40*2^(1/2)*cos(d*x+c)^5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)+192*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+190*2^(1/2)*c
os(d*x+c)^4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-160*2^(1/2)*cos(d*x+c)^2*(
-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+96*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(5/2)+465*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*cos(d*x+c)^3-320*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)*cos(d*x+c)+480*arctan(1/(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)*cos(d*x+c)^2-49*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*2^(1/2)*cos(d*x+c)^2-160*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
3/2)*2^(1/2)+960*2^(1/2)*cos(d*x+c)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2))+690*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*
x+c)^2+155*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*cos(d*x+c)+480*2^(1
```

```
/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+1380*cos(d*x+c)*arctan(1
/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+135*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*2^(1/2)+690*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
2^(1/2)))/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)/(a*(-1+cos(d*x+c))/cos(d*x+c
))^(1/2)/sin(d*x+c)^5*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)^3/sqrt(-a*sec(d*x + c) + a), x)
```

Fricas [A]
time = 1.72, size = 484, normalized size = 2.52

$$\left[\frac{48\sqrt{2}Aa\sqrt{-\frac{1}{a}}\log\left(-\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}-(3\cos(dx+c)+1)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)-69A\sqrt{-a}\log\left(\frac{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}-(2a\cos(dx+c)+a)\sin(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)-2\left(8A\cos(dx+c)^4+22A\cos(dx+c)^3+41A\cos(dx+c)^2+27A\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{48ad\sin(dx+c)}, \frac{48\sqrt{2}A\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)-69A\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)-\left(8A\cos(dx+c)^4+22A\cos(dx+c)^3+41A\cos(dx+c)^2+27A\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{24ad\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] [1/48*(48*sqrt(2)*A*a*sqrt(-1/a)*log(-(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x
+ c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*sqrt(-1/a) - (3*cos(d*x + c)
+ 1)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x + c) - 69*A*s
qrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c
) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*s
in(d*x + c) - 2*(8*A*cos(d*x + c)^4 + 22*A*cos(d*x + c)^3 + 41*A*cos(d*x +
c)^2 + 27*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/(a*d*sin
(d*x + c)), 1/24*(48*sqrt(2)*A*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c)
- a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 69*A
*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(
a)*sin(d*x + c)))*sin(d*x + c) - (8*A*cos(d*x + c)^4 + 22*A*cos(d*x + c)^3
+ 41*A*cos(d*x + c)^2 + 27*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*
x + c)))/(a*d*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$A\left(\int \frac{\cos^{3}{\left(c + dx \right)}}{\sqrt{- a \sec{\left(c + dx \right)} + a}}\, dx + \int \frac{\cos^{3}{\left(c + dx \right)} \sec{\left(c + dx \right)}}{\sqrt{- a \sec{\left(c + dx \right)} + a}}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)**3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(1/2),x)

[Out] A*(Integral(cos(c + d*x)**3/sqrt(-a*sec(c + d*x) + a), x) + Integral(cos(c + d*x)**3*sec(c + d*x)/sqrt(-a*sec(c + d*x) + a), x))

Giac [A]
time = 1.06, size = 166, normalized size = 0.86

$$\frac{\frac{48\sqrt{2}\,A\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}}{\sqrt{a}}\right)}{\sqrt{a}}-\frac{69\,A\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}}{2\sqrt{a}}\right)}{\sqrt{a}}-\frac{\sqrt{2}\left(21\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^{\frac{5}{2}}A+80\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^{\frac{3}{2}}Aa+108\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}\,Aa^2\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)^3}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(1/2),x, algorithm ="giac")

[Out] 1/24*(48*sqrt(2)*A*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/sqrt(a) - 69*A*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/sqrt(a) - sqrt(2)*(21*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(5/2)*A + 80*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(3/2)*A*a + 108*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*A*a^2)/(a*tan(1/2*d*x + 1/2*c)^2 + a)^3)/d

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^3\,\left(A+\frac{A}{\cos\left(c+d\,x\right)}\right)}{\sqrt{a-\frac{a}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((cos(c + d*x)^3*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(1/2),x)

[Out] int((cos(c + d*x)^3*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(1/2), x)

3.171 $\int \frac{A+A\sec(c+dx)}{(a-a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=116

$$\frac{2A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{3/2}d}-\frac{3A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}\,a^{3/2}d}-\frac{A\tan(c+dx)}{d(a-a\sec(c+dx))^{3/2}}$$

```
[Out] 2*A*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/a^(3/2)/d-3/2*A*arcta
n(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-
A*tan(d*x+c)/d/(a-a*sec(d*x+c))^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 133, normalized size of antiderivative = 1.15, number of steps used = 6, number of rules used = 5, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}} = 0.192$, Rules used = {3989, 3972, 482, 536, 209}

$$\frac{2A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{3/2}d}-\frac{3A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}\,a^{3/2}d}+\frac{A\sin(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{2ad\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + A*Sec[c + d*x])/(a - a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(a^(3/2)*d) -
 (3*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])])/(S
qrt[2]*a^(3/2)*d) + (A*Csc[(c + d*x)/2]^2*Sin[c + d*x])/(2*a*d*Sqrt[a - a*S
ec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$\int \frac{A+A\sec(c+dx)}{(a-a\sec(c+dx))^{3/2}}\,dx = -\left((aA)\int \frac{\tan^2(c+dx)}{(a-a\sec(c+dx))^{5/2}}\,dx\right)$$

$$= \frac{(2A)\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{d}$$

$$= \frac{A\csc^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{2ad\sqrt{a-a\sec(c+dx)}} - \frac{A\text{Subst}\left(\int \frac{1-ax^2}{(1+ax^2)(2+ax^2)}\,dx, x, -\frac{\tan(\cdots}{\sqrt{a-a\,\mathrm{se}\cdots}}\right.}{ad}$$

$$= \frac{A\csc^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{2ad\sqrt{a-a\sec(c+dx)}} - \frac{(2A)\text{Subst}\left(\int \frac{1}{1+ax^2}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a-a\sec(c\cdots}}\right.}{ad}$$

$$= \frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{3A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.55, size = 265, normalized size = 2.28

$$\frac{A\left(\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(2idx-2\sinh^{-1}\left(e^{i(c+dx)}\right)-3\sqrt{2}\log\left(1-e^{i(c+dx)}\right)-2\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)+3\sqrt{2}\log\left(1+e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)\right)-\left(\cos\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{3}{2}(c+dx)\right)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\right)\sec^{\frac{3}{2}}(c+dx)\sin^3\left(\frac{1}{2}(c+dx)\right)}{d(a-a\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + A*Sec[c + d*x])/(a - a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (A*((Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*((2*I)*d*x - 2*ArcSinh[E^(I*(c + d*x))] - 3*Sqrt[2]*Log[1 -
 E^(I*(c + d*x))] - 2*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] + 3*Sqrt[2]*Lo
g[1 + E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((I/2)*(
c + d*x)) - (Cos[(c + d*x)/2] + Cos[(3*(c + d*x))/2])*Csc[(c + d*x)/2]^2*Sq
rt[Sec[c + d*x]])*Sec[c + d*x]^(3/2)*Sin[(c + d*x)/2]^3)/(d*(a - a*Sec[c +
d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. $2(99) = 198$.
time = 4.63, size = 298, normalized size = 2.57

method	result
default	$-\frac{A(-1+\cos(dx+c))^2\left(\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{2}\,\cos(dx+c)+\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\sqrt{2}+3\sqrt{2}\,\cos(dx+c)\arctan\left(\frac{1}{\sqrt{-\frac{2\cos(dx+c)}{1+\cos(dx+c)}}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] -A/d*(-1+cos(d*x+c))^2*((-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)*cos(d*
x+c)+(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)+3*2^(1/2)*cos(d*x+c)*arct
an(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*2^(1/2)*cos(d*x+c)+4*cos(d*x+c)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*2^(1/2))-3*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))
-(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)-4*arctan(1/2*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*2^(1/2)))/(a*(-1+cos(d*x+c))/cos(d*x+c))^(3/2)/sin(d*x
+c)^3/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm="maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)/(-a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 211 vs. $2(99) = 198$.

time = 1.49, size = 502, normalized size = 4.33

$$\left[-\frac{3\sqrt{2}\left(A\cos\left(dx+c\right)-A\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)}}+(3a\cos(dx+c)+a)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin\left(dx+c\right)+4\left(A\cos\left(dx+c\right)-A\right)\sqrt{-a}\log\left(\frac{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)}}-(2a\cos(dx+c)+a)\sin(dx+c)}{\sin(dx+c)}\right)\sin\left(dx+c\right)-4\left(A\cos\left(dx+c\right)^2+A\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)}}}{4\left(a^2d\cos\left(dx+c\right)-a^2d\right)\sin\left(dx+c\right)}, \frac{3\sqrt{2}\left(A\cos\left(dx+c\right)-A\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin\left(dx+c\right)-4\left(A\cos\left(dx+c\right)-A\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin\left(dx+c\right)+2\left(A\cos\left(dx+c\right)^2+A\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)-a}{\cos\left(dx+c\right)}}}{2\left(a^2d\cos\left(dx+c\right)-a^2d\right)\sin\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] [-1/4*(3*sqrt(2)*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*sqrt(2)*(cos(d*x + c)
^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) + (3*a*
cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x
+ c) + 4*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*x + c
))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a
)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) - 4*(A*cos(d*x + c)^2 + A*cos(d*
x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^2*d*cos(d*x + c) - a^2
*d)*sin(d*x + c)), 1/2*(3*sqrt(2)*(A*cos(d*x + c) - A)*sqrt(a)*arctan(sqrt(
2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x +
c)))*sin(d*x + c) - 4*(A*cos(d*x + c) - A)*sqrt(a)*arctan(sqrt((a*cos(d*x +
 c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) +
2*(A*cos(d*x + c)^2 + A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c
)))/((a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$A\left(\int\frac{\sec\left(c+dx\right)}{-a\sqrt{-a\sec\left(c+dx\right)+a}\sec\left(c+dx\right)+a\sqrt{-a\sec\left(c+dx\right)+a}}\,dx+\int\frac{1}{-a\sqrt{-a\sec\left(c+dx\right)+a}\sec\left(c+dx\right)+a\sqrt{-a\sec\left(c+dx\right)+a}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(3/2),x)
```

```
[Out] A*(Integral(sec(c + d*x)/(-a*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a*sqr
t(-a*sec(c + d*x) + a)), x) + Integral(1/(-a*sqrt(-a*sec(c + d*x) + a)*sec(
c + d*x) + a*sqrt(-a*sec(c + d*x) + a)), x))
```

Giac [A]
time = 1.07, size = 112, normalized size = 0.97

$$\frac{\frac{3\sqrt{2}A\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{4A\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}\,A}{a^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] 1/2*(3*sqrt(2)*A*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/a^(3/2)
 - 4*A*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/a^(3/
2) - sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*A/(a^2*tan(1/2*d*x + 1/2*c)
^2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{A}{\cos(c+d\,x)}}{\left(a-\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + A/cos(c + d*x))/(a - a/cos(c + d*x))^(3/2),x)
```

```
[Out] int((A + A/cos(c + d*x))/(a - a/cos(c + d*x))^(3/2), x)
```

3.172 $\int \frac{\cos(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=146

$$\frac{5A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{3/2}d}-\frac{7A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}\,a^{3/2}d}-\frac{A\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}}+\frac{}{ad}$$

```
[Out] 5*A*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/a^(3/2)/d-A*sin(d*x+c
)/d/(a-a*sec(d*x+c))^(3/2)-7/2*A*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a-a
*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+2*A*sin(d*x+c)/a/d/(a-a*sec(d*x+c))^(
1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 146, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4105, 4107, 4005, 3859, 209, 3880}

$$\frac{5A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{3/2}d}-\frac{7A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}\,a^{3/2}d}+\frac{2A\sin(c+dx)}{ad\sqrt{a-a\sec(c+dx)}}-\frac{A\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (5*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(a^(3/2)*d) -
 (7*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])])/(S
qrt[2]*a^(3/2)*d) - (A*Sin[c + d*x])/(d*(a - a*Sec[c + d*x])^(3/2)) + (2*A*
Sin[c + d*x])/(a*d*Sqrt[a - a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
```

```
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}}\,dx = -\frac{A\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+dx)(4aA+3aA\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{A\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{2A\sin(c+dx)}{ad\sqrt{a-a\sec(c+dx)}} - \frac{\int \frac{-5a^2A-2a}{\sqrt{a-a\,\mathrm{s}}}}{2}$$

$$= -\frac{A\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{2A\sin(c+dx)}{ad\sqrt{a-a\sec(c+dx)}} + \frac{(5A)\int\sqrt{a}}{}$$

$$= -\frac{A\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{2A\sin(c+dx)}{ad\sqrt{a-a\sec(c+dx)}} + \frac{(5A)\mathrm{Subst}\Big(}{}$$

$$= \frac{5A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{7A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx}{\sqrt{2}\sqrt{a-a\sec(c}}\right.}{\sqrt{2}\,a^{3/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.73, size = 281, normalized size = 1.92

$$\frac{A\left(\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(5idx-5\sinh^{-1}\left(e^{i(c+dx)}\right)-7\sqrt{2}\log\left(1-e^{i(c+dx)}\right)-5\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)+7\sqrt{2}\log\left(1+e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)\right)+\frac{1}{2}\left(-2\cos\left(\frac{1}{2}(c+dx)\right)-3\cos\left(\frac{3}{2}(c+dx)\right)+\cos\left(\frac{5}{2}(c+dx)\right)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\right)\sec^{\frac{3}{2}}(c+dx)\sin^3\left(\frac{1}{2}(c+dx)\right)}{d(a-a\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (A*((Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*((5*I)*d*x - 5*ArcSinh[E^(I*(c + d*x))] - 7*Sqrt[2]*Log[1 -
 E^(I*(c + d*x))] - 5*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] + 7*Sqrt[2]*Lo
g[1 + E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((I/2)*(
c + d*x)) + ((-2*Cos[(c + d*x)/2] - 3*Cos[(3*(c + d*x))/2] + Cos[(5*(c + d*
x))/2])*Csc[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]])/2)*Sec[c + d*x]^(3/2)*Sin[(c
 + d*x)/2]^3)/(d*(a - a*Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 461 vs. $2(127) = 254$.
time = 8.77, size = 462, normalized size = 3.16

method	result

default	$\frac{A(-1+\cos(dx+c))^3\left(-3\sqrt{2}\,(\cos^2(dx+c))\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}-6\sqrt{2}\,\cos(dx+c)\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}-7\sqrt{2}\,(\cos^2(dx+c))\left(-\frac{2\cos(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/3*A/d*(-1+cos(d*x+c))^3*(-3*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(5/2)-6*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-7*2^
(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-3*2^(1/2)*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(5/2)+3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*
cos(d*x+c)^3+21*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)*cos(
d*x+c)^2-2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*cos(d*x+c)^2+7*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)+5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*2^(1/2)*cos(d*x+c)+30*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
2^(1/2))*cos(d*x+c)^2-21*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2))-6*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)-30*arctan(1/2*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)))/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)/
(a*(-1+cos(d*x+c))/cos(d*x+c))^(3/2)/sin(d*x+c)^5*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)/(-a*sec(d*x + c) + a)^(3/2), x)
```

Fricas [A]
time = 1.25, size = 526, normalized size = 3.60

$$\left[-\frac{7\sqrt{2}\,(A\cos(dx+c)-A)\sqrt{-a}\log\left(\frac{2\sqrt{2}\,(\cos(dx+c)^2+\cos(dx+c))\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}+(3a\cos(dx+c)+a)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+10\,(A\cos(dx+c)-A)\sqrt{-a}\log\left(\frac{2\,(\cos(dx+c)^2+\cos(dx+c))\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}-(2a\cos(dx+c)+a)\sin(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)+4\,(A\cos(dx+c)^3-A\cos(dx+c)^2-2A\cos(dx+c))\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{4\,(a^2d\cos(dx+c)-a^2d)\sin(dx+c)},\ \frac{7\sqrt{2}\,(A\cos(dx+c)-A)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)-10\,(A\cos(dx+c)-A)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)-2\,(A\cos(dx+c)^3-A\cos(dx+c)^2-2A\cos(dx+c))\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{2\,(a^2d\cos(dx+c)-a^2d)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] [-1/4*(7*sqrt(2)*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*sqrt(2)*(cos(d*x + c)
^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) + (3*a*
```

```
cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x
+ c) + 10*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*x +
c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) +
a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) + 4*(A*cos(d*x + c)^3 - A*cos(d
*x + c)^2 - 2*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^
2*d*cos(d*x + c) - a^2*d)*sin(d*x + c)), 1/2*(7*sqrt(2)*(A*cos(d*x + c) - A
)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x +
c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 10*(A*cos(d*x + c) - A)*sqrt(a)*a
rctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x
 + c)))*sin(d*x + c) - 2*(A*cos(d*x + c)^3 - A*cos(d*x + c)^2 - 2*A*cos(d*x
 + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^2*d*cos(d*x + c) - a^2*
d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$A\left(\int \frac{\cos(c+dx)}{-a\sqrt{-a\sec(c+dx)+a}\,\sec(c+dx)+a\sqrt{-a\sec(c+dx)+a}}\,dx+\int \frac{\cos(c+dx)\sec(c+dx)}{-a\sqrt{-a\sec(c+dx)+a}\,\sec(c+dx)+a\sqrt{-a\sec(c+dx)+a}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(3/2),x)
```

```
[Out] A*(Integral(cos(c + d*x)/(-a*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a*sqr
t(-a*sec(c + d*x) + a)), x) + Integral(cos(c + d*x)*sec(c + d*x)/(-a*sqrt(-
a*sec(c + d*x) + a)*sec(c + d*x) + a*sqrt(-a*sec(c + d*x) + a)), x))
```

Giac [A]
time = 1.14, size = 177, normalized size = 1.21

$$\frac{\frac{7\sqrt{2}\,A\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{10\,A\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{3\sqrt{2}\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^{\frac{3}{2}}A+4\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}\,Aa}{\left(\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2+3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)a+2a^2\right)a}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] 1/2*(7*sqrt(2)*A*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/a^(3/2)
 - 10*A*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/a^(3
/2) - (3*sqrt(2)*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(3/2)*A + 4*sqrt(2)*sqrt(a*
tan(1/2*d*x + 1/2*c)^2 - a)*A*a)/(((a*tan(1/2*d*x + 1/2*c)^2 - a)^2 + 3*(a*
tan(1/2*d*x + 1/2*c)^2 - a)*a + 2*a^2)*a))/d
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,\left(A+\frac{A}{\cos\left(c+d\,x\right)}\right)}{{\left(a-\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(3/2),x)
[Out] int((cos(c + d*x)*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(3/2), x)
```

3.173 $\int \frac{\cos^2(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=194

$$\frac{31A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{4a^{3/2}d}-\frac{11A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}\,a^{3/2}d}-\frac{A\cos(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}}$$

```
[Out] 31/4*A*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/a^(3/2)/d-A*cos(d*
x+c)*sin(d*x+c)/d/(a-a*sec(d*x+c))^(3/2)-11/2*A*arctan(1/2*a^(1/2)*tan(d*x+
c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+13/4*A*sin(d*x+c)/a/d/
(a-a*sec(d*x+c))^(1/2)+3/2*A*cos(d*x+c)*sin(d*x+c)/a/d/(a-a*sec(d*x+c))^(1/
2)
```

Rubi [A]
time = 0.35, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4105, 4107, 4005, 3859, 209, 3880}

$$\frac{31A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{4a^{3/2}d}-\frac{11A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}\,a^{3/2}d}+\frac{13A\sin(c+dx)}{4ad\sqrt{a-a\sec(c+dx)}}+\frac{3A\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a-a\sec(c+dx)}}-\frac{A\sin(c+dx)\cos(c+dx)}{d(a-a\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (31*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(4*a^(3/2)*d
) - (11*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])]
)/(Sqrt[2]*a^(3/2)*d) - (A*Cos[c + d*x]*Sin[c + d*x])/(d*(a - a*Sec[c + d*x
])^(3/2)) + (13*A*Sin[c + d*x])/(4*a*d*Sqrt[a - a*Sec[c + d*x]]) + (3*A*Cos
[c + d*x]*Sin[c + d*x])/(2*a*d*Sqrt[a - a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}}\,dx = -\frac{A\cos(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos^2(c+dx)(6aA+5aA\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{A\cos(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{3A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a-a\sec(c+dx)}} - \frac{\int \frac{\cos(}{}}{}$$

$$= -\frac{A\cos(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{13A\sin(c+dx)}{4ad\sqrt{a-a\sec(c+dx)}} + \frac{3A\cos(}{2ad\sqrt{}}$$

$$= -\frac{A\cos(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{13A\sin(c+dx)}{4ad\sqrt{a-a\sec(c+dx)}} + \frac{3A\cos(}{2ad\sqrt{}}$$

$$= -\frac{A\cos(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{13A\sin(c+dx)}{4ad\sqrt{a-a\sec(c+dx)}} + \frac{3A\cos(}{2ad\sqrt{}}$$

$$= \frac{31A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{4a^{3/2}d} - \frac{11A\tan^{-1}\left(\frac{\sqrt{a}\tan(}{\sqrt{2}\sqrt{a-a\,\mathrm{se}}}\right.}{\sqrt{2}\,a^{3/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.94, size = 296, normalized size = 1.53

$$\frac{A\left(\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(31idx-31\sinh^{-1}\left(e^{i(c+dx)}\right)-44\sqrt{2}\log\left(1-e^{i(c+dx)}\right)-31\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)+44\sqrt{2}\log\left(1+e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)\right)+\frac{1}{2}\left(-9\cos\left(\frac{1}{2}(c+dx)\right)-16\cos\left(\frac{3}{2}(c+dx)\right)+8\cos\left(\frac{5}{2}(c+dx)\right)+\cos\left(\frac{7}{2}(c+dx)\right)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\right)\sec^{\frac{3}{2}}(c+dx)\sin^3\left(\frac{1}{2}(c+dx)\right)}{4d(a-a\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(3/2),
x]
```

```
[Out] (A*((Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*((31*I)*d*x - 31*ArcSinh[E^(I*(c + d*x))] - 44*Sqrt[2]*Log[
1 - E^(I*(c + d*x))] - 31*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] + 44*Sqrt[
2]*Log[1 + E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((I
/2)*(c + d*x)) + ((-9*Cos[(c + d*x)/2] - 16*Cos[(3*(c + d*x))/2] + 8*Cos[(5
*(c + d*x))/2] + Cos[(7*(c + d*x))/2])*Csc[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]
])/2)*Sec[c + d*x]^(3/2)*Sin[(c + d*x)/2]^3)/(4*d*(a - a*Sec[c + d*x])^(3/2
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 882 vs. $2(167) = 334$.
time = 8.42, size = 883, normalized size = 4.55

method	result	size
default	Expression too large to display	883

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/60*A/d*(-1+cos(d*x+c))^4*(-660*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2))-195*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)-930*arctan(1
/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+60*2^(1/2)*cos(d*x+c)^3*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+180*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(7/2)+132*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(5/2)+180*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-220*2^
(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+660*2^(1/2)*cos(d*x
+c)^3*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+220*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(3/2)*2^(1/2)*cos(d*x+c)-278*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*2^(1/2)*cos(d*x+c)^3+288*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*c
os(d*x+c)^2-40*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*cos(d*x+c)-660*
2^(1/2)*cos(d*x+c)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+132*2^(1/
2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+930*cos(d*x+c)^3*arcta
n(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+60*2^(1/2)*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(7/2)+30*2^(1/2)*cos(d*x+c)^5*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)-132*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+195*
2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-220*2^(1/2)*cos(d
*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+660*arctan(1/(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2))*2^(1/2)*cos(d*x+c)^2-132*2^(1/2)*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(5/2)+930*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)
)*cos(d*x+c)^2+220*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)-930*cos(d*x
+c)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)))/(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(3/2)/(a*(-1+cos(d*x+c))/cos(d*x+c))^(3/2)/sin(d*x+c)^7*2
^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)^2/(-a*sec(d*x + c) + a)^(3/2),
x)
```

Fricas [A]
time = 1.89, size = 550, normalized size = 2.84

$$\left[-\frac{22\,\sqrt{2}\,(A\cos(dx+c)-A)\sqrt{-a}\,\log\left(\frac{2\,\sqrt{2}\,(\cos(dx+c)^2+\cos(dx+c))\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}+(3\,a\cos(dx+c)+a)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+31\,(A\cos(dx+c)-A)\sqrt{-a}\,\log\left(\frac{2\,(\cos(dx+c)^2+\cos(dx+c))\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}-(2\,a\cos(dx+c)+a)\sin(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)+2\,(2\,A\cos(dx+c)^4+9\,A\cos(dx+c)^3-6\,A\cos(dx+c)^2-13\,A\cos(dx+c))\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{8\,(a^2d\cos(dx+c)-a^2d)\sin(dx+c)},\ \frac{22\,\sqrt{2}\,(A\cos(dx+c)-A)\sqrt{a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)-31\,(A\cos(dx+c)-A)\sqrt{a}\,\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)-(2\,A\cos(dx+c)^4+9\,A\cos(dx+c)^3-6\,A\cos(dx+c)^2-13\,A\cos(dx+c))\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{4\,(a^2d\cos(dx+c)-a^2d)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] [-1/8*(22*sqrt(2)*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*sqrt(2)*(cos(d*x + c
)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) + (3*a
*cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d*x
 + c) + 31*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*x +
 c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) +
 a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) + 2*(2*A*cos(d*x + c)^4 + 9*A*
cos(d*x + c)^3 - 6*A*cos(d*x + c)^2 - 13*A*cos(d*x + c))*sqrt((a*cos(d*x +
c) - a)/cos(d*x + c)))/((a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c)), 1/4*(22
*sqrt(2)*(A*cos(d*x + c) - A)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) -
 a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 31*(A
*cos(d*x + c) - A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*c
os(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - (2*A*cos(d*x + c)^4 + 9*
A*cos(d*x + c)^3 - 6*A*cos(d*x + c)^2 - 13*A*cos(d*x + c))*sqrt((a*cos(d*x
+ c) - a)/cos(d*x + c)))/((a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$A\left(\int\frac{\cos^2(c+dx)}{-a\sqrt{-a\sec(c+dx)+a}\,\sec(c+dx)+a\sqrt{-a\sec(c+dx)+a}}\,dx+\int\frac{\cos^2(c+dx)\sec(c+dx)}{-a\sqrt{-a\sec(c+dx)+a}\,\sec(c+dx)+a\sqrt{-a\sec(c+dx)+a}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(3/2),x)
```

```
[Out] A*(Integral(cos(c + d*x)**2/(-a*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a*
sqrt(-a*sec(c + d*x) + a)), x) + Integral(cos(c + d*x)**2*sec(c + d*x)/(-a*
sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a*sqrt(-a*sec(c + d*x) + a)), x))
```

Giac [A]
time = 1.21, size = 183, normalized size = 0.94

$$\frac{\frac{22\,\sqrt{2}\,A\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{31\,A\arctan\left(\frac{\sqrt{2}\,\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}}{2\,\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{\sqrt{2}\left(7\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^{\frac{3}{2}}A+18\,\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}\,Aa\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)^2a}-\frac{2\,\sqrt{2}\,\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}\,A}{a^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2}}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] 1/4*(22*sqrt(2)*A*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/a^(3/2
) - 31*A*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/a^(
3/2) - sqrt(2)*(7*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(3/2)*A + 18*sqrt(a*tan(1/
2*d*x + 1/2*c)^2 - a)*A*a)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^2*a) - 2*sqrt(2)
*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*A/(a^2*tan(1/2*d*x + 1/2*c)^2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(A+\frac{A}{\cos\left(c+d\,x\right)}\right)}{{\left(a-\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(3/2), x)
```

3.174 $\int \frac{\cos^3(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=236

$$\frac{85A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{8a^{3/2}d}-\frac{15A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d}-\frac{A\cos^2(c+dx)\sin(c+dx}{d(a-a\sec(c+dx))^{3/2}}$$

```
[Out] 85/8*A*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/a^(3/2)/d-A*cos(d*
x+c)^2*sin(d*x+c)/d/(a-a*sec(d*x+c))^(3/2)-15/2*A*arctan(1/2*a^(1/2)*tan(d*
x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)+35/8*A*sin(d*x+c)/a/
d/(a-a*sec(d*x+c))^(1/2)+25/12*A*cos(d*x+c)*sin(d*x+c)/a/d/(a-a*sec(d*x+c))
^(1/2)+4/3*A*cos(d*x+c)^2*sin(d*x+c)/a/d/(a-a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.46, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4105, 4107, 4005, 3859, 209, 3880}

$$\frac{85A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{8a^{3/2}d}-\frac{15A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{\sqrt{2}a^{3/2}d}+\frac{35A\sin(c+dx)}{8ad\sqrt{a-a\sec(c+dx)}}+\frac{4A\sin(c+dx)\cos^2(c+dx)}{3ad\sqrt{a-a\sec(c+dx)}}-\frac{A\sin(c+dx)\cos^2(c+dx)}{d(a-a\sec(c+dx))^{3/2}}+\frac{25A\sin(c+dx)\cos(c+dx)}{12ad\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (85*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(8*a^(3/2)*d
) - (15*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])]
)/(Sqrt[2]*a^(3/2)*d) - (A*Cos[c + d*x]^2*Sin[c + d*x])/(d*(a - a*Sec[c + d
*x])^(3/2)) + (35*A*Sin[c + d*x])/(8*a*d*Sqrt[a - a*Sec[c + d*x]]) + (25*A*
Cos[c + d*x]*Sin[c + d*x])/(12*a*d*Sqrt[a - a*Sec[c + d*x]]) + (4*A*Cos[c +
 d*x]^2*Sin[c + d*x])/(3*a*d*Sqrt[a - a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}}\,dx = -\frac{A\cos^2(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos^3(c+dx)(8aA+7aA\sec(c+dx))}{\sqrt{a-a\sec(c+dx)}}\,dx}{2a^2}$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{4A\cos^2(c+dx)\sin(c+dx)}{3ad\sqrt{a-a\sec(c+dx)}} - \ldots$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{25A\cos(c+dx)\sin(c+dx)}{12ad\sqrt{a-a\sec(c+dx)}} + \frac{4A \ldots}{3 \ldots}$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{35A\sin(c+dx)}{8ad\sqrt{a-a\sec(c+dx)}} + \frac{25A\,\mathrm{co}\ldots}{12ad \ldots}$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{35A\sin(c+dx)}{8ad\sqrt{a-a\sec(c+dx)}} + \frac{25A\,\mathrm{co}\ldots}{12ad \ldots}$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{d(a-a\sec(c+dx))^{3/2}} + \frac{35A\sin(c+dx)}{8ad\sqrt{a-a\sec(c+dx)}} + \frac{25A\,\mathrm{co}\ldots}{12ad \ldots}$$

$$= \frac{85A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{8a^{3/2}d} - \frac{15A\tan^{-1}\left(\frac{\sqrt{a}\tan(\ldots}{\sqrt{2}\sqrt{a-a\,\mathrm{se}\ldots}}\right.}{\sqrt{2}\,a^{3/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 1.87, size = 314, normalized size = 1.33

$$\frac{A\left(-\frac{5\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(-17idx+17\sinh^{-1}\left(e^{i(c+dx)}\right)+24\sqrt{2}\log\left(1-e^{i(c+dx)}\right)+17\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)-24\sqrt{2}\log\left(1+e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)\right)}{d}+\frac{\left(-61\cos\left(\frac{1}{2}(c+dx)\right)-120\cos\left(\frac{3}{2}(c+dx)\right)+72\cos\left(\frac{5}{2}(c+dx)\right)+11\cos\left(\frac{7}{2}(c+dx)\right)+2\cos\left(\frac{9}{2}(c+dx)\right)\right)\csc^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}}{6d}\right)\sec^{\frac{3}{2}}(c+dx)\sin^3\left(\frac{1}{2}(c+dx)\right)}{8(a-a\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(3/2),
x]
```

```
[Out] (A*((-5*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^
((2*I)*(c + d*x))]*((-17*I)*d*x + 17*ArcSinh[E^(I*(c + d*x))] + 24*Sqrt[2]*
Log[1 - E^(I*(c + d*x))] + 17*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - 24*S
qrt[2]*Log[1 + E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/(
d*E^((I/2)*(c + d*x))) + ((-61*Cos[(c + d*x)/2] - 120*Cos[(3*(c + d*x))/2]
+ 72*Cos[(5*(c + d*x))/2] + 11*Cos[(7*(c + d*x))/2] + 2*Cos[(9*(c + d*x))/2
])*Csc[(c + d*x)/2]^2*Sqrt[Sec[c + d*x]])/(6*d))*Sec[c + d*x]^(3/2)*Sin[(c
+ d*x)/2]^3)/(8*(a - a*Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1103 vs. $2(205) = 410$.
time = 8.85, size = 1104, normalized size = 4.68

method	result	size
default	Expression too large to display	1104

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/168*A/d*(-1+cos(d*x+c))^5*(-2520*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2))-735*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)-3570*arctan
(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-720*2^(1/2)*cos(d*x+c)^3
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+1008*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(5/2)+720*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(7/2)-1680*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+5
040*2^(1/2)*cos(d*x+c)^3*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+168
0*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)*cos(d*x+c)+1130*(-2*cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*cos(d*x+c)^3+952*(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*2^(1/2)*cos(d*x+c)^2-875*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2
^(1/2)*cos(d*x+c)-5040*2^(1/2)*cos(d*x+c)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*
x+c)))^(1/2))+56*2^(1/2)*cos(d*x+c)^7*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+
7140*cos(d*x+c)^3*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+
360*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+1225*2^(1/2)*cos(d*x+c)^5*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-1008*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)
/(1+cos(d*x+c)))^(5/2)-2103*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)-504*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+350*2^(1/2)*cos
(d*x+c)^6*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-168*2^(1/2)*cos(d*x+c)^4*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)-672*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(9/2)-360*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(7/2)-1008*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)-672*
2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)+504*2^(1/2)*cos(d*x
+c)^4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-840*2^(1/2)*cos(d*x+c)^4*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(3/2)+2520*2^(1/2)*cos(d*x+c)^4*arctan(1/(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(1/2))-168*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2
)+3570*cos(d*x+c)^4*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)
)+840*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)-7140*cos(d*x+c)*arctan(1
/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)))/(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(3/2)/(a*(-1+cos(d*x+c))/cos(d*x+c))^(3/2)/sin(d*x+c)^9*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)^3/(-a*sec(d*x + c) + a)^(3/2),
x)
```

Fricas [A]
time = 1.52, size = 572, normalized size = 2.42

$$\left[-\frac{180\sqrt{2}\,(A\cos(dx+c)-A)\sqrt{-a}\log\left(\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}+(3a\cos(dx+c)+a)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+255\,(A\cos(dx+c)-A)\sqrt{-a}\log\left(\frac{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}-(2a\cos(dx+c)+a)\sin(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)+2\left(8A\cos(dx+c)^5+26A\cos(dx+c)^4+73A\cos(dx+c)^3-50A\cos(dx+c)^2-105A\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{48\left(a^2d\cos(dx+c)-a^2d\right)\sin(dx+c)},\ \frac{180\sqrt{2}\,(A\cos(dx+c)-A)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)-255\,(A\cos(dx+c)-A)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)-\left(8A\cos(dx+c)^5+26A\cos(dx+c)^4+73A\cos(dx+c)^3-50A\cos(dx+c)^2-105A\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{24\left(a^2d\cos(dx+c)-a^2d\right)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] [-1/48*(180*sqrt(2)*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*sqrt(2)*(cos(d*x +
 c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) + (3
*a*cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)*sin(d*x + c)))*sin(d
*x + c) + 255*(A*cos(d*x + c) - A)*sqrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*
x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c
) + a)*sin(d*x + c))/sin(d*x + c))*sin(d*x + c) + 2*(8*A*cos(d*x + c)^5 + 2
6*A*cos(d*x + c)^4 + 73*A*cos(d*x + c)^3 - 50*A*cos(d*x + c)^2 - 105*A*cos(
d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^2*d*cos(d*x + c) - a
^2*d)*sin(d*x + c)), 1/24*(180*sqrt(2)*(A*cos(d*x + c) - A)*sqrt(a)*arctan(
sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d
*x + c)))*sin(d*x + c) - 255*(A*cos(d*x + c) - A)*sqrt(a)*arctan(sqrt((a*co
s(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x
+ c) - (8*A*cos(d*x + c)^5 + 26*A*cos(d*x + c)^4 + 73*A*cos(d*x + c)^3 - 50
*A*cos(d*x + c)^2 - 105*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x +
 c)))/((a^2*d*cos(d*x + c) - a^2*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$A\left(\int \frac{\cos^{3}{\left(c+dx \right)}}{-a\sqrt{-a\sec{\left(c+dx \right)}+a}\sec{\left(c+dx \right)}+a\sqrt{-a\sec{\left(c+dx \right)}+a}}\,dx+\int \frac{\cos^{3}{\left(c+dx \right)}\sec{\left(c+dx \right)}}{-a\sqrt{-a\sec{\left(c+dx \right)}+a}\sec{\left(c+dx \right)}+a\sqrt{-a\sec{\left(c+dx \right)}+a}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(3/2),x)
```

```
[Out] A*(Integral(cos(c + d*x)**3/(-a*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a*
sqrt(-a*sec(c + d*x) + a)), x) + Integral(cos(c + d*x)**3*sec(c + d*x)/(-a*
sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a*sqrt(-a*sec(c + d*x) + a)), x))
```

Giac [A]
time = 1.16, size = 208, normalized size = 0.88

$$\frac{\frac{180\sqrt{2}\,A\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a}}{\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{255\,A\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a}}{2\sqrt{a}}\right)}{a^{\frac{3}{2}}}-\frac{12\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a}\,A}{a^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}}-\frac{\sqrt{2}\left(63\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{\frac{5}{2}}A+272\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a\right)^{\frac{3}{2}}Aa+324\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-a}\,Aa^{2}\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+a\right)^{3}a}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] 1/24*(180*sqrt(2)*A*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/a^(3
/2) - 255*A*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/
a^(3/2) - 12*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*A/(a^2*tan(1/2*d*x
+ 1/2*c)^2) - sqrt(2)*(63*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(5/2)*A + 272*(a*t
an(1/2*d*x + 1/2*c)^2 - a)^(3/2)*A*a + 324*sqrt(a*tan(1/2*d*x + 1/2*c)^2 -
a)*A*a^2)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^3*a))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^3\,\left(A+\frac{A}{\cos\left(c+d\,x\right)}\right)}{{\left(a-\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(3/2), x)
```

3.175 $\int \frac{A+A\sec(c+dx)}{(a-a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=152

$$\frac{2A\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{5/2}d}-\frac{23A\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}\,a^{5/2}d}-\frac{A\tan(c+dx)}{2d(a-a\sec(c+dx))^{5/2}}-$$

```
[Out] 2*A*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/a^(5/2)/d-23/16*A*arc
tan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2
)-1/2*A*tan(d*x+c)/d/(a-a*sec(d*x+c))^(5/2)-7/8*A*tan(d*x+c)/a/d/(a-a*sec(d
*x+c))^(3/2)
```

Rubi [A]
time = 0.14, antiderivative size = 185, normalized size of antiderivative = 1.22, number of steps used = 7, number of rules used = 6, integrand size = 26, $\frac{\text{number of rules}}{\text{integrand size}}=0.231$, Rules used = {3989, 3972, 482, 541, 536, 209}

$$\frac{2A\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{5/2}d}-\frac{23A\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}\,a^{5/2}d}+\frac{7A\sin(c+dx)\csc^2\left(\frac{1}{2}(c+dx)\right)}{16a^2d\sqrt{a-a\sec(c+dx)}}-\frac{A\sin(c+dx)\cos(c+dx)\csc^4\left(\frac{1}{2}(c+dx)\right)}{8a^2d\sqrt{a-a\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + A*Sec[c + d*x])/(a - a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(a^(5/2)*d) -
 (23*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])])/(
8*Sqrt[2]*a^(5/2)*d) + (7*A*Csc[(c + d*x)/2]^2*Sin[c + d*x])/(16*a^2*d*Sqrt
[a - a*Sec[c + d*x]]) - (A*Cos[c + d*x]*Csc[(c + d*x)/2]^4*Sin[c + d*x])/(8
*a^2*d*Sqrt[a - a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 482

```
Int[((e_.)*(x_))^(m_.)*((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_
))^(q_), x_Symbol] :> Simp[e^(n - 1)*(e*x)^(m - n + 1)*(a + b*x^n)^(p + 1)*
((c + d*x^n)^(q + 1)/(n*(b*c - a*d)*(p + 1))), x] - Dist[e^n/(n*(b*c - a*d)
*(p + 1)), Int[(e*x)^(m - n)*(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(m -
n + 1) + d*(m + n*(p + q + 1) + 1)*x^n, x], x], x] /; FreeQ[{a, b, c, d, e,
 q}, x] && NeQ[b*c - a*d, 0] && IGtQ[n, 0] && LtQ[p, -1] && GeQ[n, m - n +
1] && GtQ[m - n + 1, 0] && IntBinomialQ[a, b, c, d, e, m, n, p, q, x]
```

Rule 536

```
Int[((e_) + (f_.)*(x_)^(n_))/(((a_) + (b_.)*(x_)^(n_))*((c_) + (d_.)*(x_)^(
n_))), x_Symbol] :> Dist[(b*e - a*f)/(b*c - a*d), Int[1/(a + b*x^n), x], x]
 - Dist[(d*e - c*f)/(b*c - a*d), Int[1/(c + d*x^n), x], x] /; FreeQ[{a, b,
c, d, e, f, n}, x]
```

Rule 541

```
Int[((a_) + (b_.)*(x_)^(n_))^(p_)*((c_) + (d_.)*(x_)^(n_))^(q_.)*((e_) + (f
_.)*(x_)^(n_)), x_Symbol] :> Simp[(-(b*e - a*f))*x*(a + b*x^n)^(p + 1)*((c
+ d*x^n)^(q + 1)/(a*n*(b*c - a*d)*(p + 1))), x] + Dist[1/(a*n*(b*c - a*d)*(
p + 1)), Int[(a + b*x^n)^(p + 1)*(c + d*x^n)^q*Simp[c*(b*e - a*f) + e*n*(b*
c - a*d)*(p + 1) + d*(b*e - a*f)*(n*(p + q + 2) + 1)*x^n, x], x], x] /; Fre
eQ[{a, b, c, d, e, f, n, q}, x] && LtQ[p, -1]
```

Rule 3972

```
Int[cot[(c_.) + (d_.)*(x_)]^(m_.)*(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n
_.), x_Symbol] :> Dist[-2*(a^(m/2 + n + 1/2)/d), Subst[Int[x^m*((2 + a*x^2)
^(m/2 + n - 1/2)/(1 + a*x^2)), x], x, Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]]
], x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0] && IntegerQ[m/2] && In
tegerQ[n - 1/2]
```

Rule 3989

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(
d_.) + (c_))^(n_.), x_Symbol] :> Dist[((-a)*c)^m, Int[Cot[e + f*x]^(2*m)*(c
 + d*Csc[e + f*x])^(n - m), x], x] /; FreeQ[{a, b, c, d, e, f, n}, x] && Eq
Q[b*c + a*d, 0] && EqQ[a^2 - b^2, 0] && IntegerQ[m] && RationalQ[n] &&  !(I
ntegerQ[n] && GtQ[m - n, 0])
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+A\sec(c+dx)}{(a-a\sec(c+dx))^{5/2}}\,dx &= -\left((aA)\int \frac{\tan^2(c+dx)}{(a-a\sec(c+dx))^{7/2}}\,dx\right)\\
&= \frac{(2A)\text{Subst}\left(\int \frac{x^2}{(1+ax^2)(2+ax^2)^3}\,dx, x, -\frac{\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{ad}\\
&= -\frac{A\cos(c+dx)\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{8a^2d\sqrt{a-a\sec(c+dx)}} - \frac{A\text{Subst}\left(\int \frac{1-3ax^2}{(1+ax^2)(2+ax^2)^2}\,dx,\right.}{2a^2}\\
&= \frac{7A\csc^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{16a^2d\sqrt{a-a\sec(c+dx)}} - \frac{A\cos(c+dx)\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx}{8a^2d\sqrt{a-a\sec(c+dx)}}\\
&= \frac{7A\csc^2\left(\frac{1}{2}(c+dx)\right)\sin(c+dx)}{16a^2d\sqrt{a-a\sec(c+dx)}} - \frac{A\cos(c+dx)\csc^4\left(\frac{1}{2}(c+dx)\right)\sin(c+dx}{8a^2d\sqrt{a-a\sec(c+dx)}}\\
&= \frac{2A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{23A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}\,a^{5/2}d}
\end{aligned}
$$

Mathematica [**C**] Result contains complex when optimal does not.
time = 6.89, size = 421, normalized size = 2.77

$$
A\left(\frac{e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(-16idx+16\sinh^{-1}\left(e^{i(c+dx)}\right)+23\sqrt{2}\log\left(1-e^{i(c+dx)}\right)+16\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)-23\sqrt{2}\log\left(1+e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)\right)\sec^{\frac{5}{2}}(c+dx)\sin^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{2\sqrt{2}d(a-a\sec(c+dx))^{5/2}}+\frac{\sec^3(c+dx)\left(-\frac{11\cos\left(\frac{c}{2}\right)\cos\left(\frac{dx}{2}\right)}{d}+\frac{15\cot\left(\frac{c}{2}\right)\csc\left(\frac{c}{2}+\frac{dx}{2}\right)}{2d}-\frac{\cot\left(\frac{c}{2}\right)\csc^3\left(\frac{c}{2}+\frac{dx}{2}\right)}{d}-\frac{15\csc\left(\frac{c}{2}\right)\csc^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{2d}+\frac{\csc\left(\frac{c}{2}\right)\csc^4\left(\frac{c}{2}+\frac{dx}{2}\right)\sin\left(\frac{dx}{2}\right)}{d}+\frac{11\sin\left(\frac{c}{2}\right)\sin\left(\frac{dx}{2}\right)}{d}\right)\sin^5\left(\frac{c}{2}+\frac{dx}{2}\right)}{(a-a\sec(c+dx))^{5/2}}\right)
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + A*Sec[c + d*x])/(a - a*Sec[c + d*x])^(5/2),x]
```

```
[Out] A*((Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
d*x))]*((-16*I)*d*x + 16*ArcSinh[E^(I*(c + d*x))] + 23*Sqrt[2]*Log[1 - E^(I
*(c + d*x))] + 16*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - 23*Sqrt[2]*Log[1
 + E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]])*Sec[c + d*x]^(
5/2)*Sin[c/2 + (d*x)/2]^5)/(2*Sqrt[2]*d*E^((I/2)*(c + d*x))*(a - a*Sec[c +
d*x])^(5/2)) + (Sec[c + d*x]^3*((-11*Cos[c/2]*Cos[(d*x)/2])/d + (15*Cot[c/2
]*Csc[c/2 + (d*x)/2])/(2*d) - (Cot[c/2]*Csc[c/2 + (d*x)/2]^3)/d - (15*Csc[c
/2]*Csc[c/2 + (d*x)/2]^2*Sin[(d*x)/2])/(2*d) + (Csc[c/2]*Csc[c/2 + (d*x)/2]
^4*Sin[(d*x)/2])/d + (11*Sin[c/2]*Sin[(d*x)/2])/d)*Sin[c/2 + (d*x)/2]^5)/(a
 - a*Sec[c + d*x])^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 694 vs. $2(127) = 254$.
time = 4.70, size = 695, normalized size = 4.57

method	result
default	$-\frac{A(-1+\cos(dx+c))^4\left(21\sqrt{2}\,(\cos^3(dx+c))\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}+33\sqrt{2}\,(\cos^2(dx+c))\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{5}{2}}+23\sqrt{2}\,(\cos^3(dx+c))\left(-\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/12*A/d*(-1+cos(d*x+c))^4*(21*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+33*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+23*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+3*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-23*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-9*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*cos(d*x+c)^3-69*2^(1/2)*cos(d*x+c)^3*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-23*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)*cos(d*x+c)-11*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*cos(d*x+c)^2+69*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/2)*cos(d*x+c)^2+23*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)-96*cos(d*x+c)^3*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+37*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*cos(d*x+c)+69*2^(1/2)*cos(d*x+c)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+96*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^2-21*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)-69*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+96*cos(d*x+c)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-96*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)))/(a*(-1+cos(d*x+c))/cos(d*x+c))^(5/2)/sin(d*x+c)^7/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)*2^(1/2)

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] integrate((A*sec(d*x + c) + A)/(-a*sec(d*x + c) + a)^(5/2), x)

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 255 vs. $2(127) = 254$.
time = 1.29, size = 590, normalized size = 3.88

$$\left[-\frac{23\sqrt{2}\left(A\cos(dx+c)^2-2A\cos(dx+c)+A\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}+(3a\cos(dx+c)+a)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+32\left(A\cos(dx+c)^2-2A\cos(dx+c)+A\right)\sqrt{-a}\log\left(\frac{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}-(2a\cos(dx+c)+a)\sin(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)-4\left(11A\cos(dx+c)^3+4A\cos(dx+c)^2-7A\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{32\left(a^3d\cos(dx+c)^2-2a^3d\cos(dx+c)+a^3d\right)\sin(dx+c)}, \frac{23\sqrt{2}\left(A\cos(dx+c)^2-2A\cos(dx+c)+A\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)-32\left(A\cos(dx+c)^2-2A\cos(dx+c)+A\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)+2\left(11A\cos(dx+c)^3+4A\cos(dx+c)^2-7A\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{16\left(a^3d\cos(dx+c)^2-2a^3d\cos(dx+c)+a^3d\right)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] [-1/32*(23*sqrt(2)*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(-a)*log((
2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a
)/cos(d*x + c)) + (3*a*cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)*
sin(d*x + c)))*sin(d*x + c) + 32*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*
sqrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x +
c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))*
sin(d*x + c) - 4*(11*A*cos(d*x + c)^3 + 4*A*cos(d*x + c)^2 - 7*A*cos(d*x +
c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^2 - 2*a^3
*d*cos(d*x + c) + a^3*d)*sin(d*x + c)), 1/16*(23*sqrt(2)*(A*cos(d*x + c)^2
- 2*A*cos(d*x + c) + A)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/co
s(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 32*(A*cos(d
*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/
cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) + 2*(11*A*c
os(d*x + c)^3 + 4*A*cos(d*x + c)^2 - 7*A*cos(d*x + c))*sqrt((a*cos(d*x + c)
 - a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) + a^3*d)
*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$A\left(\int \frac{\sec(c+dx)}{a^2\sqrt{-a\sec(c+dx)+a}\,\sec^2(c+dx) - 2a^2\sqrt{-a\sec(c+dx)+a}\,\sec(c+dx) + a^2\sqrt{-a\sec(c+dx)+a}}\,dx + \int \frac{1}{a^2\sqrt{-a\sec(c+dx)+a}\,\sec^2(c+dx) - 2a^2\sqrt{-a\sec(c+dx)+a}\,\sec(c+dx) + a^2\sqrt{-a\sec(c+dx)+a}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(5/2),x)
```

```
[Out] A*(Integral(sec(c + d*x)/(a**2*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x)**2 -
2*a**2*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a**2*sqrt(-a*sec(c + d*x) +
 a)), x) + Integral(1/(a**2*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x)**2 - 2*a
**2*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a**2*sqrt(-a*sec(c + d*x) + a)
), x))
```

Giac [A]
time = 1.28, size = 138, normalized size = 0.91

$$\frac{\frac{23\sqrt{2}\,A\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}} - \frac{32\,A\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}}{2\sqrt{a}}\right)}{a^{\frac{5}{2}}} - \frac{\sqrt{2}\left(9\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^{\frac{3}{2}}A+7\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}\,Aa\right)}{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{16\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="giac")
```

```
[Out] 1/16*(23*sqrt(2)*A*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/a^(5/
2) - 32*A*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/a^
(5/2) - sqrt(2)*(9*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(3/2)*A + 7*sqrt(a*tan(1/
2*d*x + 1/2*c)^2 - a)*A*a)/(a^4*tan(1/2*d*x + 1/2*c)^4))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{A}{\cos(c+d\,x)}}{\left(a-\frac{a}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + A/cos(c + d*x))/(a - a/cos(c + d*x))^(5/2),x)
```

```
[Out] int((A + A/cos(c + d*x))/(a - a/cos(c + d*x))^(5/2), x)
```

3.176 $\int \frac{\cos(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=184

$$\frac{7A\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{5/2}d}-\frac{79A\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}\,a^{5/2}d}-\frac{A\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}}-$$

```
[Out] 7*A*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/a^(5/2)/d-1/2*A*sin(d
*x+c)/d/(a-a*sec(d*x+c))^(5/2)-11/8*A*sin(d*x+c)/a/d/(a-a*sec(d*x+c))^(3/2)
-79/16*A*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a-a*sec(d*x+c))^(1/2))/a^(5
/2)/d*2^(1/2)+23/8*A*sin(d*x+c)/a^2/d/(a-a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 184, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.188$, Rules used = {4105, 4107, 4005, 3859, 209, 3880}

$$\frac{7A\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{5/2}d}-\frac{79A\text{ArcTan}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{2}\,\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}\,a^{5/2}d}+\frac{23A\sin(c+dx)}{8a^2d\sqrt{a-a\sec(c+dx)}}-\frac{11A\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}}-\frac{A\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (7*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(a^(5/2)*d) -
 (79*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])])/(
8*Sqrt[2]*a^(5/2)*d) - (A*Sin[c + d*x])/(2*d*(a - a*Sec[c + d*x])^(5/2)) -
(11*A*Sin[c + d*x])/(8*a*d*(a - a*Sec[c + d*x])^(3/2)) + (23*A*Sin[c + d*x]
)/(8*a^2*d*Sqrt[a - a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
```

```
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{5/2}}\,dx = -\frac{A\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} + \frac{\int \frac{\cos(c+dx)(6aA+5aA\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{A\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{11A\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{\int \frac{\cos(c+d}{\sqrt{}}}{}$$

$$= -\frac{A\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{11A\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{23A}{8a^2d\sqrt{a}}$$

$$= -\frac{A\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{11A\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{23A}{8a^2d\sqrt{a}}$$

$$= -\frac{A\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{11A\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{23A}{8a^2d\sqrt{a}}$$

$$= \frac{7A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{79A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+}{\sqrt{2}\sqrt{a-a\sec(}}\right.}{8\sqrt{2}a^{5/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.04, size = 298, normalized size = 1.62

$$\frac{A\left(\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(-56idx+56\sinh^{-1}\left(e^{i(c+dx)}\right)+79\sqrt{2}\log\left(1-e^{i(c+dx)}\right)+56\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)-79\sqrt{2}\log\left(1+e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)\right)+\frac{1}{4}\left(-12\cos\left(\frac{1}{2}(c+dx)\right)+23\cos\left(\frac{3}{2}(c+dx)\right)-31\cos\left(\frac{5}{2}(c+dx)\right)+4\cos\left(\frac{7}{2}(c+dx)\right)\right)\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\right)\sec^{\frac{5}{2}}(c+dx)\sin^5\left(\frac{1}{2}(c+dx)\right)}{4d(a-a\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (A*((Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*((-56*I)*d*x + 56*ArcSinh[E^(I*(c + d*x))] + 79*Sqrt[2]*Log
[1 - E^(I*(c + d*x))] + 56*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - 79*Sqrt
[2]*Log[1 + E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/E^((
I/2)*(c + d*x)) + ((-12*Cos[(c + d*x)/2] + 23*Cos[(3*(c + d*x))/2] - 31*Cos
[(5*(c + d*x))/2] + 4*Cos[(7*(c + d*x))/2])*Csc[(c + d*x)/2]^4*Sqrt[Sec[c +
 d*x]])/4)*Sec[c + d*x]^(5/2)*Sin[(c + d*x)/2]^5)/(4*d*(a - a*Sec[c + d*x])
^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 787 vs. $2(155) = 310$.
time = 8.95, size = 788, normalized size = 4.28

method	result
default	$-\frac{A(-1+\cos(dx+c))^5\left(195\sqrt{2}\,(\cos^4(dx+c))\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}+450\sqrt{2}\,(\cos^3(dx+c))\left(-\frac{2\cos(dx+c)}{1+\cos(dx+c)}\right)^{\frac{7}{2}}+237\sqrt{2}\,(\cos^4(dx+c)$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/60*A/d*(-1+cos(d*x+c))^5*(195*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(1+cos
(d*x+c)))^(7/2)+450*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/
2)+237*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+180*2^(1/2
)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-210*2^(1/2)*cos(d*x+c)*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-395*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c
)/(1+cos(d*x+c)))^(3/2)-474*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(5/2)-135*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+120*2^(1/2)*cos
(d*x+c)^5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-343*2^(1/2)*cos(d*x+c)^4*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+1185*2^(1/2)*cos(d*x+c)^4*arctan(1/(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(1/2))+790*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(3/2)+237*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+736*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*cos(d*x+c)^3+1680*cos(d*x+c)^4*ar
ctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-578*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*2^(1/2)*cos(d*x+c)^2-2370*arctan(1/(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(1/2))*2^(1/2)*cos(d*x+c)^2-395*(-2*cos(d*x+c)/(1+cos(d*x+c)))^
(3/2)*2^(1/2)-280*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*cos(d*x+c)-3
360*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^2+3
45*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)+1185*2^(1/2)*arctan(1/(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(1/2))+1680*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*2^(1/2)))/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)/(a*(-1+cos(d*x+c)
)/cos(d*x+c))^(5/2)/sin(d*x+c)^9*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)/(-a*sec(d*x + c) + a)^(5/2), x)
```

Fricas [A]
time = 1.76, size = 612, normalized size = 3.33

$$\left[-\frac{79 \sqrt{2} \left(A \cos\left(dx+c\right)^{2}-2 A \cos\left(dx+c\right)+A\right) \sqrt{-a} \log\left(\frac{2 \sqrt{2} \left(\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right) \sqrt{-a} \sqrt{\frac{a \cos\left(dx+c\right)-a}{\cos\left(dx+c\right)}}+\left(3 a \cos\left(dx+c\right)+a\right) \sin\left(dx+c\right)}{\left(\cos\left(dx+c\right)-1\right) \sin\left(dx+c\right)}\right) \sin\left(dx+c\right)+112 \left(A \cos\left(dx+c\right)^{2}-2 A \cos\left(dx+c\right)+A\right) \sqrt{-a} \log\left(\frac{2 \left(\cos\left(dx+c\right)^{2}+\cos\left(dx+c\right)\right) \sqrt{-a} \sqrt{\frac{a \cos\left(dx+c\right)-a}{\cos\left(dx+c\right)}}-\left(2 a \cos\left(dx+c\right)+a\right) \sin\left(dx+c\right)}{\sin\left(dx+c\right)}\right) \sin\left(dx+c\right)+4 \left(8 A \cos\left(dx+c\right)^{4}-27 A \cos\left(dx+c\right)^{3}-12 A \cos\left(dx+c\right)^{2}+23 A \cos\left(dx+c\right)\right) \sqrt{\frac{a \cos\left(dx+c\right)-a}{\cos\left(dx+c\right)}}}{32 \left(a^{3} d \cos\left(dx+c\right)^{2}-2 a^{3} d \cos\left(dx+c\right)+a^{3} d\right) \sin\left(dx+c\right)}, \frac{79 \sqrt{2} \left(A \cos\left(dx+c\right)^{2}-2 A \cos\left(dx+c\right)+A\right) \sqrt{a} \arctan\left(\frac{\sqrt{2} \sqrt{\frac{a \cos\left(dx+c\right)-a}{\cos\left(dx+c\right)}} \cos\left(dx+c\right)}{\sqrt{a} \sin\left(dx+c\right)}\right) \sin\left(dx+c\right)-112 \left(A \cos\left(dx+c\right)^{2}-2 A \cos\left(dx+c\right)+A\right) \sqrt{a} \arctan\left(\frac{\sqrt{\frac{a \cos\left(dx+c\right)-a}{\cos\left(dx+c\right)}} \cos\left(dx+c\right)}{\sqrt{a} \sin\left(dx+c\right)}\right) \sin\left(dx+c\right)-2 \left(8 A \cos\left(dx+c\right)^{4}-27 A \cos\left(dx+c\right)^{3}-12 A \cos\left(dx+c\right)^{2}+23 A \cos\left(dx+c\right)\right) \sqrt{\frac{a \cos\left(dx+c\right)-a}{\cos\left(dx+c\right)}}}{16 \left(a^{3} d \cos\left(dx+c\right)^{2}-2 a^{3} d \cos\left(dx+c\right)+a^{3} d\right) \sin\left(dx+c\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] [-1/32*(79*sqrt(2)*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(-a)*log((
2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) - a
)/cos(d*x + c)) + (3*a*cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)*
sin(d*x + c)))*sin(d*x + c) + 112*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)
*sqrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x +
 c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c))
*sin(d*x + c) + 4*(8*A*cos(d*x + c)^4 - 27*A*cos(d*x + c)^3 - 12*A*cos(d*x
+ c)^2 + 23*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^3*
d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c)), 1/16*(79*sq
rt(2)*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(a)*arctan(sqrt(2)*sqrt
((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*si
n(d*x + c) - 112*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(a)*arctan(s
qrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))
*sin(d*x + c) - 2*(8*A*cos(d*x + c)^4 - 27*A*cos(d*x + c)^3 - 12*A*cos(d*x
+ c)^2 + 23*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^3*
d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) + a^3*d)*sin(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [A]
time = 1.24, size = 183, normalized size = 0.99

$$\frac{\frac{79 \sqrt{2} A \arctan\left(\frac{\sqrt{a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{112 A \arctan\left(\frac{\sqrt{2} \sqrt{a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-a}}{2 \sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{16 \sqrt{2} \sqrt{a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-a}\, A}{\left(a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}+a\right) a^{2}}-\frac{\sqrt{2} \left(17 \left(a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-a\right)^{\frac{3}{2}} A+15 \sqrt{a \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{2}-a}\, A a\right)}{a^{4} \tan\left(\frac{1}{2} dx+\frac{1}{2} c\right)^{4}}}{16 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] 1/16*(79*sqrt(2)*A*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/a^(5/
2) - 112*A*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/a
^(5/2) - 16*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*A/((a*tan(1/2*d*x +
1/2*c)^2 + a)*a^2) - sqrt(2)*(17*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(3/2)*A + 1
5*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*A*a)/(a^4*tan(1/2*d*x + 1/2*c)^4))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)\,\left(A+\frac{A}{\cos(c+d\,x)}\right)}{{\left(a-\frac{a}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(5/2), x)
```

3.177 $\int \frac{\cos^2(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=236

$$\frac{59A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{4a^{5/2}d}-\frac{167A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}\,a^{5/2}d}-\frac{A\cos(c+dx)\sin(c+dx}{2d(a-a\sec(c+dx))^{5/2}}$$

```
[Out] 59/4*A*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/a^(5/2)/d-1/2*A*co
s(d*x+c)*sin(d*x+c)/d/(a-a*sec(d*x+c))^(5/2)-15/8*A*cos(d*x+c)*sin(d*x+c)/a
/d/(a-a*sec(d*x+c))^(3/2)-167/16*A*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/2)/(a
-a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+49/8*A*sin(d*x+c)/a^2/d/(a-a*sec(d*
x+c))^(1/2)+23/8*A*cos(d*x+c)*sin(d*x+c)/a^2/d/(a-a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4105, 4107, 4005, 3859, 209, 3880}

$$\frac{59A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{4a^{5/2}d}-\frac{167A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}\,a^{5/2}d}+\frac{49A\sin(c+dx)}{8a^2d\sqrt{a-a\sec(c+dx)}}+\frac{23A\sin(c+dx)\cos(c+dx)}{8a^2d\sqrt{a-a\sec(c+dx)}}-\frac{15A\sin(c+dx)\cos(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}}-\frac{A\sin(c+dx)\cos(c+dx)}{2d(a-a\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (59*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(4*a^(5/2)*d
) - (167*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]])
])/(8*Sqrt[2]*a^(5/2)*d) - (A*Cos[c + d*x]*Sin[c + d*x])/(2*d*(a - a*Sec[c
+ d*x])^(5/2)) - (15*A*Cos[c + d*x]*Sin[c + d*x])/(8*a*d*(a - a*Sec[c + d*x
])^(3/2)) + (49*A*Sin[c + d*x])/(8*a^2*d*Sqrt[a - a*Sec[c + d*x]]) + (23*A*
Cos[c + d*x]*Sin[c + d*x])/(8*a^2*d*Sqrt[a - a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{5/2}}\,dx = -\frac{A\cos(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} + \frac{\int \frac{\cos^2(c+dx)(8aA+7aA\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}}\,dx}{4a^2}$$

$$= -\frac{A\cos(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{15A\cos(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{\int \text{co}}{}$$

$$= -\frac{A\cos(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{15A\cos(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{23A}{8a}$$

$$= -\frac{A\cos(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{15A\cos(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{}{8a^2}$$

$$= -\frac{A\cos(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{15A\cos(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{}{8a^2}$$

$$= -\frac{A\cos(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{15A\cos(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \frac{}{8a^2}$$

$$= \frac{59A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{4a^{5/2}d} - \frac{167A\tan^{-1}\left(\frac{\sqrt{a}\tan}{\sqrt{2}\sqrt{a-a\,\text{s}}}\right)}{8\sqrt{2}a^{5/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.08, size = 308, normalized size = 1.31

$$\frac{A\left(\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(-118idx+118\sinh^{-1}\left(e^{i(c+dx)}\right)+167\sqrt{2}\log\left(1-e^{i(c+dx)}\right)+118\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)-167\sqrt{2}\log\left(1+e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)\right)+\frac{1}{4}\left(-19\cos\left(\frac{1}{2}(c+dx)\right)+54\cos\left(\frac{3}{2}(c+dx)\right)-62\cos\left(\frac{5}{2}(c+dx)\right)+10\cos\left(\frac{7}{2}(c+dx)\right)+\cos\left(\frac{9}{2}(c+dx)\right)\right)\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\right)\sec^{\frac{5}{2}}(c+dx)\sin^5\left(\frac{1}{2}(c+dx)\right)}{4d(a-a\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(5/2),
x]
```

```
[Out] (A*((Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*((-118*I)*d*x + 118*ArcSinh[E^(I*(c + d*x))] + 167*Sqrt[2]*
Log[1 - E^(I*(c + d*x))] + 118*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - 167
*Sqrt[2]*Log[1 + E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))
/E^((I/2)*(c + d*x)) + ((-19*Cos[(c + d*x)/2] + 54*Cos[(3*(c + d*x))/2] - 6
2*Cos[(5*(c + d*x))/2] + 10*Cos[(7*(c + d*x))/2] + Cos[(9*(c + d*x))/2])*Cs
c[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]])/4)*Sec[c + d*x]^(5/2)*Sin[(c + d*x)/2]
^5)/(4*d*(a - a*Sec[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1474 vs. $2(201) = 402$.

time = 9.09, size = 1475, normalized size = 6.25

method	result	size
default	Expression too large to display	1475

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/420*A/d*(-1+cos(d*x+c))^6*(17535*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(1/2))+5145*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)+24780*arct
an(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+5010*2^(1/2)*cos(d*x+c
)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+5010*2^(1/2)*cos(d*x+c)^2*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(7/2)-7014*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(5/2)-2505*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/
2)+11690*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-35070*2^
(1/2)*cos(d*x+c)^3*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-5845*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)*cos(d*x+c)-1995*2^(1/2)*cos(d*x+c)
^5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)-1322*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*2^(1/2)*cos(d*x+c)^3-12768*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1
/2)*cos(d*x+c)^2+1015*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*cos(d*x+
c)+17535*2^(1/2)*cos(d*x+c)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-
7014*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+420*2^(1/2)*
cos(d*x+c)^7*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-2505*2^(1/2)*cos(d*x+c)^5
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+3507*2^(1/2)*cos(d*x+c)^5*(-2*cos(d*x
+c)/(1+cos(d*x+c)))^(5/2)-5845*2^(1/2)*cos(d*x+c)^5*(-2*cos(d*x+c)/(1+cos(d
*x+c)))^(3/2)+17535*2^(1/2)*cos(d*x+c)^5*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x
+c)))^(1/2))-49560*cos(d*x+c)^3*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*2^(1/2))-2505*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-11633*2^(1/
2)*cos(d*x+c)^5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+3507*2^(1/2)*cos(d*x+c
)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+15573*2^(1/2)*cos(d*x+c)^4*(-2*cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)+11690*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(3/2)-35070*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1/
2)*cos(d*x+c)^2+3507*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-49560*arc
tan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^2+3570*2^(
1/2)*cos(d*x+c)^6*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-6405*2^(1/2)*cos(d*x
+c)^4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)-5670*2^(1/2)*cos(d*x+c)^3*(-2*co
s(d*x+c)/(1+cos(d*x+c)))^(9/2)-2505*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(7/2)+1470*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))
^(9/2)+4305*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)+3507*2^
(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-5845*2^(1/2)*cos(d*
x+c)^4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)+17535*2^(1/2)*cos(d*x+c)^4*arct
```

```
an(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+1575*2^(1/2)*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(9/2)+24780*cos(d*x+c)^4*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*2^(1/2))-5845*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)+24780*
cos(d*x+c)*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+24780*c
os(d*x+c)^5*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)))/(-2*c
os(d*x+c)/(1+cos(d*x+c)))^(5/2)/(a*(-1+cos(d*x+c))/cos(d*x+c))^(5/2)/sin(d*
x+c)^11*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)^2/(-a*sec(d*x + c) + a)^(5/2),
x)
```

Fricas [A]
time = 1.61, size = 634, normalized size = 2.69

$$\left[-\frac{167\sqrt{2}\left(A\cos(dx+c)^2-2A\cos(dx+c)+A\right)\sqrt{-a}\log\left(\frac{2\sqrt{2}\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}+(3a\cos(dx+c)+a)\sin(dx+c)}{(\cos(dx+c)-1)\sin(dx+c)}\right)\sin(dx+c)+236\left(A\cos(dx+c)^2-2A\cos(dx+c)+A\right)\sqrt{-a}\log\left(\frac{2\left(\cos(dx+c)^2+\cos(dx+c)\right)\sqrt{-a}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}-(2a\cos(dx+c)+a)\sin(dx+c)}{\sin(dx+c)}\right)\sin(dx+c)+4\left(4A\cos(dx+c)^5+22A\cos(dx+c)^4-57A\cos(dx+c)^3-26A\cos(dx+c)^2+49A\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{32\left(a^3d\cos(dx+c)^2-2a^3d\cos(dx+c)+a^3d\right)\sin(dx+c)},\ \frac{167\sqrt{2}\left(A\cos(dx+c)^2-2A\cos(dx+c)+A\right)\sqrt{a}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)-236\left(A\cos(dx+c)^2-2A\cos(dx+c)+A\right)\sqrt{a}\arctan\left(\frac{\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}\cos(dx+c)}{\sqrt{a}\sin(dx+c)}\right)\sin(dx+c)-2\left(4A\cos(dx+c)^5+22A\cos(dx+c)^4-57A\cos(dx+c)^3-26A\cos(dx+c)^2+49A\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)-a}{\cos(dx+c)}}}{16\left(a^3d\cos(dx+c)^2-2a^3d\cos(dx+c)+a^3d\right)\sin(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/32*(167*sqrt(2)*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(-a)*log(
(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) -
a)/cos(d*x + c)) + (3*a*cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)
*sin(d*x + c)))*sin(d*x + c) + 236*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A
)*sqrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x
+ c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c)
)*sin(d*x + c) + 4*(4*A*cos(d*x + c)^5 + 22*A*cos(d*x + c)^4 - 57*A*cos(d*x
 + c)^3 - 26*A*cos(d*x + c)^2 + 49*A*cos(d*x + c))*sqrt((a*cos(d*x + c) - a
)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) + a^3*d)*sin
(d*x + c)), 1/16*(167*sqrt(2)*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqr
t(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)/(s
qrt(a)*sin(d*x + c)))*sin(d*x + c) - 236*(A*cos(d*x + c)^2 - 2*A*cos(d*x +
c) + A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a)/cos(d*x + c))*cos(d*x + c)
/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 2*(4*A*cos(d*x + c)^5 + 22*A*cos(d*
x + c)^4 - 57*A*cos(d*x + c)^3 - 26*A*cos(d*x + c)^2 + 49*A*cos(d*x + c))*s
qrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^2 - 2*a^3*d*co
s(d*x + c) + a^3*d)*sin(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$A\left(\int \frac{\cos^2(c+dx)}{a^2\sqrt{-a\sec(c+dx)+a}\,\sec^2(c+dx)-2a^2\sqrt{-a\sec(c+dx)+a}\,\sec(c+dx)+a^2\sqrt{-a\sec(c+dx)+a}}\,dx+\int \frac{\cos^2(c+dx)\sec(c+dx)}{a^2\sqrt{-a\sec(c+dx)+a}\,\sec^2(c+dx)-2a^2\sqrt{-a\sec(c+dx)+a}\,\sec(c+dx)+a^2\sqrt{-a\sec(c+dx)+a}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(5/2),x)
```

```
[Out] A*(Integral(cos(c + d*x)**2/(a**2*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x)**2
 - 2*a**2*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) + a**2*sqrt(-a*sec(c + d*x
) + a)), x) + Integral(cos(c + d*x)**2*sec(c + d*x)/(a**2*sqrt(-a*sec(c + d
*x) + a)*sec(c + d*x)**2 - 2*a**2*sqrt(-a*sec(c + d*x) + a)*sec(c + d*x) +
a**2*sqrt(-a*sec(c + d*x) + a)), x))
```

Giac [A]
time = 1.28, size = 224, normalized size = 0.95

$$\frac{\frac{167\sqrt{2}A\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{236A\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}}{2\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{\sqrt{2}\left(69\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^{\frac{7}{2}}A+315\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^{\frac{5}{2}}Aa+444\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^{\frac{3}{2}}Aa^2+196\sqrt{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a}\,Aa^3\right)}{\left(\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)^2+3\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a\right)a+2a^2\right)^2a^2}}{16d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] 1/16*(167*sqrt(2)*A*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/a^(5
/2) - 236*A*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/
a^(5/2) - sqrt(2)*(69*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(7/2)*A + 315*(a*tan(1
/2*d*x + 1/2*c)^2 - a)^(5/2)*A*a + 444*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(3/2)
*A*a^2 + 196*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*A*a^3)/(((a*tan(1/2*d*x + 1
/2*c)^2 - a)^2 + 3*(a*tan(1/2*d*x + 1/2*c)^2 - a)*a + 2*a^2)^2*a^2))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(A+\frac{A}{\cos\left(c+d\,x\right)}\right)}{{\left(a-\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(5/2), x)
```

3.178 $\int \frac{\cos^3(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=280

$$\frac{203A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{8a^{5/2}d}-\frac{287A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}a^{5/2}d}-\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5}}$$

```
[Out] 203/8*A*arctan(a^(1/2)*tan(d*x+c)/(a-a*sec(d*x+c))^(1/2))/a^(5/2)/d-1/2*A*c
os(d*x+c)^2*sin(d*x+c)/d/(a-a*sec(d*x+c))^(5/2)-19/8*A*cos(d*x+c)^2*sin(d*x
+c)/a/d/(a-a*sec(d*x+c))^(3/2)-287/16*A*arctan(1/2*a^(1/2)*tan(d*x+c)*2^(1/
2)/(a-a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)+21/2*A*sin(d*x+c)/a^2/d/(a-a*s
ec(d*x+c))^(1/2)+119/24*A*cos(d*x+c)*sin(d*x+c)/a^2/d/(a-a*sec(d*x+c))^(1/2
)+77/24*A*cos(d*x+c)^2*sin(d*x+c)/a^2/d/(a-a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 34, $\frac{\text{number of rules}}{\text{integrand size}} = 0.176$, Rules used = {4105, 4107, 4005, 3859, 209, 3880}

$$\frac{203A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{8a^{5/2}d}-\frac{287A\text{ArcTan}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{2}\sqrt{a-a\sec(c+dx)}}\right)}{8\sqrt{2}a^{5/2}d}+\frac{21A\sin(c+dx)}{2a^2d\sqrt{a-a\sec(c+dx)}}+\frac{77A\sin(c+dx)\cos^2(c+dx)}{24a^2d\sqrt{a-a\sec(c+dx)}}+\frac{119A\sin(c+dx)\cos(c+dx)}{24a^2d\sqrt{a-a\sec(c+dx)}}-\frac{19A\sin(c+dx)\cos^2(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}}-\frac{A\sin(c+dx)\cos^2(c+dx)}{2d(a-a\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (203*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/Sqrt[a - a*Sec[c + d*x]]])/(8*a^(5/2)*
d) - (287*A*ArcTan[(Sqrt[a]*Tan[c + d*x])/(Sqrt[2]*Sqrt[a - a*Sec[c + d*x]]
)])/(8*Sqrt[2]*a^(5/2)*d) - (A*Cos[c + d*x]^2*Sin[c + d*x])/(2*d*(a - a*Sec
[c + d*x])^(5/2)) - (19*A*Cos[c + d*x]^2*Sin[c + d*x])/(8*a*d*(a - a*Sec[c
+ d*x])^(3/2)) + (21*A*Sin[c + d*x])/(2*a^2*d*Sqrt[a - a*Sec[c + d*x]]) + (
119*A*Cos[c + d*x]*Sin[c + d*x])/(24*a^2*d*Sqrt[a - a*Sec[c + d*x]]) + (77*
A*Cos[c + d*x]^2*Sin[c + d*x])/(24*a^2*d*Sqrt[a - a*Sec[c + d*x]])
```

Rule 209

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[b, 2]))*A
rcTan[Rt[b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && PosQ[a/b] && (GtQ[a
, 0] || GtQ[b, 0])
```

Rule 3859

```
Int[Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[-2*(b/d),
 Subst[Int[1/(a + x^2), x], x, b*(Cot[c + d*x]/Sqrt[a + b*Csc[c + d*x]])],
x] /; FreeQ[{a, b, c, d}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3880

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Dist[-2/f, Subst[Int[1/(2*a + x^2), x], x, b*(Cot[e + f*x]/Sqrt[a
 + b*Csc[e + f*x]])], x] /; FreeQ[{a, b, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4005

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c/a, Int[Sqrt[a + b*Csc[e + f*x]], x], x] - D
ist[(b*c - a*d)/a, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; Fre
eQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+A\sec(c+dx))}{(a-a\sec(c+dx))^{5/2}}\,dx = -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} + \frac{\int \frac{\cos^3(c+dx)(10aA+9aA\sec(c+dx))}{(a-a\sec(c+dx))^{3/2}}dx}{4a^2}$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{19A\cos^2(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \cdots$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{19A\cos^2(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \cdots$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{19A\cos^2(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \cdots$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{19A\cos^2(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \cdots$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{19A\cos^2(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \cdots$$

$$= -\frac{A\cos^2(c+dx)\sin(c+dx)}{2d(a-a\sec(c+dx))^{5/2}} - \frac{19A\cos^2(c+dx)\sin(c+dx)}{8ad(a-a\sec(c+dx))^{3/2}} + \cdots$$

$$= \frac{203A\tan^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a-a\sec(c+dx)}}\right)}{8a^{5/2}d} - \frac{287A\tan^{-1}\left(\frac{\sqrt{a}\tan\cdots}{\sqrt{2}\sqrt{a-\cdots}}\right)}{8\sqrt{2}a^{5/2}d}\cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 2.45, size = 323, normalized size = 1.15

$$\frac{A\left(21\sqrt{2}e^{-\frac{1}{2}i(c+dx)}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\left(-29idx+29\sinh^{-1}\left(e^{i(c+dx)}\right)+41\sqrt{2}\log\left(1-e^{i(c+dx)}\right)+29\log\left(1+\sqrt{1+e^{2i(c+dx)}}\right)-41\sqrt{2}\log\left(1+e^{i(c+dx)}+\sqrt{2}\sqrt{1+e^{2i(c+dx)}}\right)\right)+\frac{1}{8}\left(-173\cos\left(\frac{1}{2}(c+dx)\right)+575\cos\left(\frac{3}{2}(c+dx)\right)-625\cos\left(\frac{5}{2}(c+dx)\right)+112\cos\left(\frac{7}{2}(c+dx)\right)+13\cos\left(\frac{9}{2}(c+dx)\right)+2\cos\left(\frac{11}{2}(c+dx)\right)\right)\csc^4\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\right)\sec^{\frac{5}{2}}(c+dx)\sin^5\left(\frac{1}{2}(c+dx)\right)}{12d(a-a\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + A*Sec[c + d*x]))/(a - a*Sec[c + d*x])^(5/2),
x]
```

```
[Out] (A*((21*Sqrt[2]*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^
((2*I)*(c + d*x))]*((-29*I)*d*x + 29*ArcSinh[E^(I*(c + d*x))] + 41*Sqrt[2]*
Log[1 - E^(I*(c + d*x))] + 29*Log[1 + Sqrt[1 + E^((2*I)*(c + d*x))]] - 41*S
qrt[2]*Log[1 + E^(I*(c + d*x)) + Sqrt[2]*Sqrt[1 + E^((2*I)*(c + d*x))]]))/E
^((I/2)*(c + d*x)) + ((-173*Cos[(c + d*x)/2] + 575*Cos[(3*(c + d*x))/2] - 6
25*Cos[(5*(c + d*x))/2] + 112*Cos[(7*(c + d*x))/2] + 13*Cos[(9*(c + d*x))/2
```

```
] + 2*Cos[(11*(c + d*x))/2])*Csc[(c + d*x)/2]^4*Sqrt[Sec[c + d*x]])/8)*Sec[
c + d*x]^(5/2)*Sin[(c + d*x)/2]^5)/(12*d*(a - a*Sec[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1963 vs. $2(241) = 482$.
time = 8.64, size = 1964, normalized size = 7.01

method	result	size
default	Expression too large to display	1964

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/180*A/d*(-1+cos(d*x+c))^7*(12915*2^(1/2)*arctan(1/(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2))+3780*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)+18270*arc
tan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+7380*2^(1/2)*cos(d*x+
c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+1845*2^(1/2)*cos(d*x+c)^2*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(7/2)-10332*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+
cos(d*x+c)))^(5/2)-3690*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(
7/2)+17220*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)-51660*
2^(1/2)*cos(d*x+c)^3*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))-8610*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)*cos(d*x+c)+2870*2^(1/2)*cos(d*x+
c)^5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)-10335*(-2*cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*2^(1/2)*cos(d*x+c)^3-8652*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^
(1/2)*cos(d*x+c)^2+4515*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2)*cos(d*
x+c)+25830*2^(1/2)*cos(d*x+c)*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
)-2583*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+4215*2^(1/
2)*cos(d*x+c)^7*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-3690*2^(1/2)*cos(d*x+c
)^5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+5166*2^(1/2)*cos(d*x+c)^5*(-2*cos(
d*x+c)/(1+cos(d*x+c)))^(5/2)-8610*2^(1/2)*cos(d*x+c)^5*(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(3/2)+25830*2^(1/2)*cos(d*x+c)^5*arctan(1/(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(1/2))-73080*cos(d*x+c)^3*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*2^(1/2))-1845*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)+14285*2^
(1/2)*cos(d*x+c)^5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+5166*2^(1/2)*cos(d*
x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+6254*2^(1/2)*cos(d*x+c)^4*(-2*cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)+4305*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+c
os(d*x+c)))^(3/2)-12915*arctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))*2^(1
/2)*cos(d*x+c)^2+2583*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-18270*ar
ctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))*cos(d*x+c)^2-945*2^(
1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)+18270*cos(d*x+c)^6*arctan(1/2*(-
2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-15112*2^(1/2)*cos(d*x+c)^6*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)-1435*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(
1+cos(d*x+c)))^(9/2)-5740*2^(1/2)*cos(d*x+c)^3*(-2*cos(d*x+c)/(1+cos(d*x+c)
))^(9/2)+1845*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(7/2)-143
```

```
5*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)+2870*2^(1/2)*co
s(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)-2583*2^(1/2)*cos(d*x+c)^4*(-2
*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)+4305*2^(1/2)*cos(d*x+c)^4*(-2*cos(d*x+c)/
(1+cos(d*x+c)))^(3/2)-12915*2^(1/2)*cos(d*x+c)^4*arctan(1/(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2))+1435*2^(1/2)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)-1827
0*cos(d*x+c)^4*arctan(1/2*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))-430
5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2)*2^(1/2)+36540*cos(d*x+c)*arctan(1/2*
(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+36540*cos(d*x+c)^5*arctan(1/2
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*2^(1/2))+12915*2^(1/2)*cos(d*x+c)^6*a
rctan(1/(-2*cos(d*x+c)/(1+cos(d*x+c)))^(1/2))+120*2^(1/2)*cos(d*x+c)^9*(-2*
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)+930*2^(1/2)*cos(d*x+c)^8*(-2*cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)+1125*2^(1/2)*cos(d*x+c)^6*(-2*cos(d*x+c)/(1+cos(d*x+c))
)^(11/2)+4680*2^(1/2)*cos(d*x+c)^5*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)+14
35*2^(1/2)*cos(d*x+c)^6*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(9/2)+6525*2^(1/2)*c
os(d*x+c)^4*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)+1800*2^(1/2)*cos(d*x+c)^3
*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/2)-1845*2^(1/2)*cos(d*x+c)^6*(-2*cos(d*
x+c)/(1+cos(d*x+c)))^(7/2)-3825*2^(1/2)*cos(d*x+c)^2*(-2*cos(d*x+c)/(1+cos(
d*x+c)))^(11/2)-3600*2^(1/2)*cos(d*x+c)*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(11/
2)+2583*2^(1/2)*cos(d*x+c)^6*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(5/2)-4305*2^(1
/2)*cos(d*x+c)^6*(-2*cos(d*x+c)/(1+cos(d*x+c)))^(3/2))/(-2*cos(d*x+c)/(1+co
s(d*x+c)))^(5/2)/(a*(-1+cos(d*x+c))/cos(d*x+c))^(5/2)/sin(d*x+c)^13*2^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

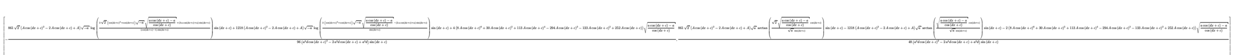
Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((A*sec(d*x + c) + A)*cos(d*x + c)^3/(-a*sec(d*x + c) + a)^(5/2),
x)
```

Fricas [A]
time = 2.37, size = 656, normalized size = 2.34



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] [-1/96*(861*sqrt(2)*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(-a)*log(
(2*sqrt(2)*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x + c) -
```

```
a)/cos(d*x + c)) + (3*a*cos(d*x + c) + a)*sin(d*x + c))/((cos(d*x + c) - 1)
*sin(d*x + c)))*sin(d*x + c) + 1218*(A*cos(d*x + c)^2 - 2*A*cos(d*x + c) +
A)*sqrt(-a)*log((2*(cos(d*x + c)^2 + cos(d*x + c))*sqrt(-a)*sqrt((a*cos(d*x
 + c) - a)/cos(d*x + c)) - (2*a*cos(d*x + c) + a)*sin(d*x + c))/sin(d*x + c
))*sin(d*x + c) + 4*(8*A*cos(d*x + c)^6 + 30*A*cos(d*x + c)^5 + 113*A*cos(d
*x + c)^4 - 294*A*cos(d*x + c)^3 - 133*A*cos(d*x + c)^2 + 252*A*cos(d*x + c
))*sqrt((a*cos(d*x + c) - a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^2 - 2*a^3*
d*cos(d*x + c) + a^3*d)*sin(d*x + c)), 1/48*(861*sqrt(2)*(A*cos(d*x + c)^2
- 2*A*cos(d*x + c) + A)*sqrt(a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) - a)/co
s(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 1218*(A*cos
(d*x + c)^2 - 2*A*cos(d*x + c) + A)*sqrt(a)*arctan(sqrt((a*cos(d*x + c) - a
)/cos(d*x + c))*cos(d*x + c)/(sqrt(a)*sin(d*x + c)))*sin(d*x + c) - 2*(8*A*
cos(d*x + c)^6 + 30*A*cos(d*x + c)^5 + 113*A*cos(d*x + c)^4 - 294*A*cos(d*x
 + c)^3 - 133*A*cos(d*x + c)^2 + 252*A*cos(d*x + c))*sqrt((a*cos(d*x + c) -
 a)/cos(d*x + c)))/((a^3*d*cos(d*x + c)^2 - 2*a^3*d*cos(d*x + c) + a^3*d)*s
in(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**A**]
time = 1.30, size = 234, normalized size = 0.84

$$\frac{\frac{861\sqrt{2}\,A\arctan\left(\frac{\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}}{\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{1218\,A\arctan\left(\frac{\sqrt{2}\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}}{2\sqrt{a}}\right)}{a^{\frac{5}{2}}}-\frac{2\sqrt{2}\left(129\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^{\frac{5}{2}}A+560\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^{\frac{3}{2}}Aa+636\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}\,Aa^2\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a\right)^3a^2}-\frac{3\sqrt{2}\left(33\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a\right)^{\frac{3}{2}}A+31\sqrt{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a}\,Aa\right)}{a^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4}}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+A*sec(d*x+c))/(a-a*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] 1/48*(861*sqrt(2)*A*arctan(sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))/a^(5
/2) - 1218*A*arctan(1/2*sqrt(2)*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)/sqrt(a))
/a^(5/2) - 2*sqrt(2)*(129*(a*tan(1/2*d*x + 1/2*c)^2 - a)^(5/2)*A + 560*(a*t
an(1/2*d*x + 1/2*c)^2 - a)^(3/2)*A*a + 636*sqrt(a*tan(1/2*d*x + 1/2*c)^2 -
a)*A*a^2)/((a*tan(1/2*d*x + 1/2*c)^2 + a)^3*a^2) - 3*sqrt(2)*(33*(a*tan(1/2
*d*x + 1/2*c)^2 - a)^(3/2)*A + 31*sqrt(a*tan(1/2*d*x + 1/2*c)^2 - a)*A*a)/(
a^4*tan(1/2*d*x + 1/2*c)^4))/d
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^3\,\left(A+\frac{A}{\cos\left(c+d\,x\right)}\right)}{{\left(a-\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(5/2),x)
[Out] int((cos(c + d*x)^3*(A + A/cos(c + d*x)))/(a - a/cos(c + d*x))^(5/2), x)
```

3.179 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=199

$$-\frac{6a(A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(7A+5B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{21d}$$

```
[Out] 2/21*a*(7*A+5*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a*(A+B)*sec(d*x+c)^(5/2)
*sin(d*x+c)/d+2/7*a*B*sec(d*x+c)^(7/2)*sin(d*x+c)/d+6/5*a*(A+B)*sin(d*x+c)*
sec(d*x+c)^(1/2)/d-6/5*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/d+2/21*a*(7*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4082, 3872, 3853, 3856, 2720, 2719}

$$\frac{2a(A+B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}+\frac{2a(7A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{6a(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(7A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{6a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aB\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-6*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(5*d) + (2*a*(7*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(21*d) + (6*a*(A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
5*d) + (2*a*(7*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*a*(A +
 B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d) + (2*a*B*Sec[c + d*x]^(7/2)*Sin[
c + d*x])/(7*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx &= \frac{2aB\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}+\frac{2}{7}\int \sec^{\frac{5}{2}}(c+\\ &= \frac{2aB\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{7d}+(a(A+B))\int\\ &= \frac{2a(7A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{21d}+\frac{2a(A\\ &= \frac{6a(A+B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d}+\frac{2a(7A\\ &= \frac{2a(7A+5B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\,\surd}{21d}\\ &= -\frac{6a(A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\,\surd}{5d}\end{aligned}$$

Mathematica [A]
time = 0.81, size = 200, normalized size = 1.01

$$\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))(A+B\sec(c+dx))\left(-63(A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(7A+5B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+63A\sin(c+dx)+63B\sin(c+dx)+35A\tan(c+dx)+25B\tan(c+dx)+21A\sec(c+dx)\tan(c+dx)+21B\sec(c+dx)\tan(c+dx)+15B\sec^2(c+dx)\tan(c+dx)\right)}{105d(B+A\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])*(A + B*Sec[c + d*x])*(-63*(A + B)*
Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(7*A + 5*B)*Sqrt[Cos[c + d
*x]]*EllipticF[(c + d*x)/2, 2] + 63*A*Sin[c + d*x] + 63*B*Sin[c + d*x] + 35
*A*Tan[c + d*x] + 25*B*Tan[c + d*x] + 21*A*Sec[c + d*x]*Tan[c + d*x] + 21*B
*Sec[c + d*x]*Tan[c + d*x] + 15*B*Sec[c + d*x]^2*Tan[c + d*x]))/(105*d*(B +
 A*Cos[c + d*x])*Sec[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 690 vs. $2(223) = 446$.
time = 3.71, size = 691, normalized size = 3.47

method	result
default	$-\frac{a\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{4\left(\frac{A}{2}+\frac{B}{2}\right)\left(24\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-12\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4/5*(1/2*A+1/
2*B)/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6
*sin(1/2*d*x+1/2*c)^2-1)*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*
d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*A*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
))+2*B*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 233, normalized size = 1.17

$$\frac{-5i\sqrt{2}(7A+5B)a\cos(dx+c)^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(7A+5B)a\cos(dx+c)^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-63i\sqrt{2}(A+B)a\cos(dx+c)^3\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+63i\sqrt{2}(A+B)a\cos(dx+c)^3\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(63(A+B)a\cos(dx+c)^3+5(7A+5B)a\cos(dx+c)^2+21(A+B)a\cos(dx+c)+15Ba\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{105d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/105*(-5*I*sqrt(2)*(7*A + 5*B)*a*cos(d*x + c)^3*weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*(7*A + 5*B)*a*cos(d*x + c)^3*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*I*sqrt(2)*(A
 + B)*a*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c))) + 63*I*sqrt(2)*(A + B)*a*cos(d*x + c)^3*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
) + 2*(63*(A + B)*a*cos(d*x + c)^3 + 5*(7*A + 5*B)*a*cos(d*x + c)^2 + 21*(A
 + B)*a*cos(d*x + c) + 15*B*a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x
+ c)^3)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2), x)
```

3.180 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=172

$$-\frac{2a(5A+3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

```
[Out] 2/3*a*(A+B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*a*B*sec(d*x+c)^(5/2)*sin(d*x+
c)/d+2/5*a*(5*A+3*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/5*a*(5*A+3*B)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4082, 3872, 3853, 3856, 2719, 2720}

$$\frac{2a(A+B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2a(5A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aB\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*a*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(5*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(3*d) + (2*a*(5*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(5*d) + (2*a*(A + B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a*B*Sec[
c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx &= \frac{2aB\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2}{5}\int \sec^{\frac{3}{2}}(c+d\\ &= \frac{2aB\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+(a(A+B))\int \mathrm{s}\\ &= \frac{2a(5A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d}+\frac{2a(A}{}\\ &= \frac{2a(5A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d}+\frac{2a(A}{}\\ &= -\frac{2a(5A+3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{5d}\end{aligned}$$

Mathematica [A]
time = 0.69, size = 168, normalized size = 0.98

$$\frac{a\sec^2\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))(A+B\sec(c+dx))\left(-3(5A+3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+15A\sin(c+dx)+9B\sin(c+dx)+5A\tan(c+dx)+5B\tan(c+dx)+3B\sec(c+dx)\tan(c+dx)\right)}{15d(B+A\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (a*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])*(A + B*Sec[c + d*x])*(-3*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 15*A*Sin[c + d*x] + 9*B*Sin[c + d*x] + 5*A*Tan[c + d*x] + 5*B*Tan[c + d*x] + 3*B*Sec[c + d*x]*Tan[c + d*x]))/(15*d*(B + A*Cos[c + d*x])*Sec[c + d*x]^(3/2))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 634 vs. $2(200) = 400$.
time = 3.23, size = 635, normalized size = 3.69

method	result
default	$-\frac{a\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2B\left(24\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-12\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\cdots}\right.}{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/5*B/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+4*(1/2*A+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.56, size = 219, normalized size = 1.27

$$\frac{-5i\sqrt{2}(A+B)a\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(A+B)a\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}(5A+3B)a\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}(5A+3B)a\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(3(5A+3B)a\cos(dx+c)^2+5(A+B)a\cos(dx+c)+3Ba\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*(A + B)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*(A + B)*a*cos(d*x + c)^2*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(5*A + 3*B)*
a*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + 3*I*sqrt(2)*(5*A + 3*B)*a*cos(d*x + c)^2*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) +
 2*(3*(5*A + 3*B)*a*cos(d*x + c)^2 + 5*(A + B)*a*cos(d*x + c) + 3*B*a)*sin(
d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2), x)
```

3.181 $\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=135

$$-\frac{2a(A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a(3A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{3d}$$

```
[Out] 2/3*a*B*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2*a*(A+B)*sin(d*x+c)*sec(d*x+c)^(1/2)
/d-2*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*a*(3*A+B)*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 135, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4082, 3872, 3856, 2720, 3853, 2719}

$$\frac{2a(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2a(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aB\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/d + (2*a*(3*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*a*(A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d + (2*a
*B*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{2aB\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2}{3}\int \sqrt{\sec(c}$$
$$= \frac{2aB\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+(a(A+B))$$
$$= \frac{2a(A+B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{2aB}{}$$
$$= \frac{2a(3A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$
$$= -\frac{2a(A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Mathematica [A]
time = 0.56, size = 94, normalized size = 0.70

$$\frac{a\sec^{\frac{3}{2}}(c+dx)\left(-6(A+B)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3A+B)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2(B+3(A+B)\cos(c+dx))\sin(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*Sec[c + d*x]^(3/2)*(-6*(A + B)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2,
 2] + 2*(3*A + B)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 2*(B + 3*(
A + B)*Cos[c + d*x])*Sin[c + d*x]))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(171) = 342$.
time = 2.28, size = 400, normalized size = 2.96

method	result
default	$-\frac{a\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \text{El}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -a*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*(
-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+4*(1/2*A+1/2*B)/sin(1/2*d*x+1/2*c)^
2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.77, size = 188, normalized size = 1.39

$$\frac{-i\sqrt{2}(3A+B)a\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}(3A+B)a\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}(A+B)a\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}(A+B)a\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2(3(A+B)a\cos(dx+c)+Ba)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(3*A + B)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c)) + I*sqrt(2)*(3*A + B)*a*cos(d*x + c)*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(A + B)*a*cos(d*x
 + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c))) + 3*I*sqrt(2)*(A + B)*a*cos(d*x + c)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*(A + B)*a*c
os(d*x + c) + B*a)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sqrt{\sec\left(c+dx\right)}\,dx+\int A\sec^{\frac{3}{2}}\left(c+dx\right)dx+\int B\sec^{\frac{3}{2}}\left(c+dx\right)dx+\int B\sec^{\frac{5}{2}}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(A*sqrt(sec(c + d*x)), x) + Integral(A*sec(c + d*x)**(3/2), x) +
 Integral(B*sec(c + d*x)**(3/2), x) + Integral(B*sec(c + d*x)**(5/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2), x)
```

3.182 $\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=106

$$\frac{2a(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}$$

```
[Out] 2*a*B*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2*a*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d+2*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 106, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4082, 3872, 3856, 2719, 2720}

$$\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aB\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/d + (2*a*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{2aB\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+2\int \frac{\frac{1}{2}a(A-B)+\frac{1}{2}a(A\cdots}{\sqrt{\sec(c+\cdots}}$$

$$= \frac{2aB\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+(a(A-B))\int \frac{1}{\sqrt{\sec(c+\cdots}}$$

$$= \frac{2aB\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+\left(a(A-B)\sqrt{\cos(c+dx\cdots}\right.$$

$$= \frac{2a(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}\cdots$$

Mathematica [A]
time = 0.33, size = 77, normalized size = 0.73

$$\frac{2a\sqrt{\sec(c+dx)}\left((A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+(A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+B\sin(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*Sqrt[Sec[c + d*x]]*((A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2] + (A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + B*Sin[c + d*x])
)/d
```

Maple [A]
time = 1.22, size = 240, normalized size = 2.26

method	result

default	$-\frac{2a\left(A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}-A\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2*a*(A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-2*B*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+B*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 141, normalized size = 1.33

$$\frac{-i\sqrt{2}\,(A+B)a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,(A+B)a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+i\sqrt{2}\,(A-B)a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\,(A-B)a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2Ba\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] (-I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
 c)) + I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c)) + I*sqrt(2)*(A - B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) - I*sqrt(2)*(A - B)*a*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*B*a
*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{A}{\sqrt{\sec(c+dx)}}\,dx + \int A\sqrt{\sec(c+dx)}\,dx + \int B\sqrt{\sec(c+dx)}\,dx + \int B\sec^{\frac{3}{2}}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(A/sqrt(sec(c + d*x)), x) + Integral(A*sqrt(sec(c + d*x)), x) +
Integral(B*sqrt(sec(c + d*x)), x) + Integral(B*sec(c + d*x)**(3/2), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/(1/cos(c + d*x))^(1/2), x)
```

3.183 $\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=110

$$\frac{2a(A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2a(A+3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{3d}$$

```
[Out] 2/3*a*A*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/d+2/3*a*(A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 110, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4081, 3872, 3856, 2719, 2720}

$$\frac{2a(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/d + (2*a*(A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{3}{2}a(A+B)-\frac{1}{2}a(A+3B)\sec(c$$
$$= \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + (a(A+B))\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx + \frac{1}{3}($$
$$= \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \Big(a(A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx}$$
$$= \frac{2a(A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A]
time = 0.33, size = 83, normalized size = 0.75

$$\frac{a\sqrt{\sec(c+dx)}\left(6(A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(A+3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+A\sin(2(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(6*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2] + 2*(A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + A*Sin[2*(c
+ d*x)]))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(148) = 296$.
time = 1.11, size = 321, normalized size = 2.92

method	result

default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(4A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*A*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2+A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+3*B*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)-3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.89, size = 142, normalized size = 1.29

$$\frac{2\,Aa\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-i\,\sqrt{2}\,(A+3\,B)a\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\,\sqrt{2}\,(A+3\,B)a\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+3i\,\sqrt{2}\,(A+B)a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-3i\,\sqrt{2}\,(A+B)a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] 1/3*(2*A*a*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*(A + 3*B)*a*weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(A + 3*B)*a*w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*(A +
 B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c))) - 3*I*sqrt(2)*(A + B)*a*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{A}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{A}{\sqrt{\sec(c+dx)}}\,dx+\int \frac{B}{\sqrt{\sec(c+dx)}}\,dx+\int B\sqrt{\sec(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] a*(Integral(A/sec(c + d*x)**(3/2), x) + Integral(A/sqrt(sec(c + d*x)), x) +
 Integral(B/sqrt(sec(c + d*x)), x) + Integral(B*sqrt(sec(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/(1/cos(c + d*x))^(3/2), x)
```

3.184 $\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=141

$$\frac{2a(3A+5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\text{se}}}{3d}$$

```
[Out] 2/5*a*A*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*a*(A+B)*sin(d*x+c)/d/sec(d*x+c)^(
1/2)+2/5*a*(3*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*a*(
A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4081, 3872, 3854, 3856, 2720, 2719}

$$\frac{2a(A+B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(3A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*a*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(5*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (
2*a*(A + B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}a(A+B)-\frac{1}{2}a(3A+5B)\sec(c}{\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + (a(A+B))\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{5}(a(3$$

$$= \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(a(A+B))$$

$$= \frac{2a(3A+5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

$$= \frac{2a(3A+5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}$$

Mathematica [A]
time = 0.60, size = 99, normalized size = 0.70

$$\frac{a\sqrt{\sec(c+dx)}\left(6(3A+5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(5(A+B)+3A\cos(c+dx))\sin(2(c+dx))\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(6*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2] + 10*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5*(A +
B) + 3*A*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(173) = 346$.
time = 1.05, size = 355, normalized size = 2.52

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(-24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(44A+20B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-24*A*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(44*A+20*B)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-16*A-10*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)-9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+5*B*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)-15*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.81, size = 169, normalized size = 1.20

$$\frac{-5i\sqrt{2}\,(A+B)a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,(A+B)a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}\,(3A+5B)a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\,(3A+5B)a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(3Aa\cos(dx+c)^2+5(A+B)a\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) + 3*I*sqrt(2)*(3*A + 5*B)*a*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(3*A +
5*B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))) + 2*(3*A*a*cos(d*x + c)^2 + 5*(A + B)*a*cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{A}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\int \frac{A}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{B}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{B}{\sqrt{\sec(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] a*(Integral(A/sec(c + d*x)**(5/2), x) + Integral(A/sec(c + d*x)**(3/2), x)
+ Integral(B/sec(c + d*x)**(3/2), x) + Integral(B/sqrt(sec(c + d*x)), x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/(1/cos(c + d*x))^(5/2), x)
```

3.185 $\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$\frac{6a(A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2a(5A+7B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}$$

```
[Out] 2/7*a*A*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*a*(A+B)*sin(d*x+c)/d/sec(d*x+c)^(
3/2)+2/21*a*(5*A+7*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5*a*(A+B)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*a*(5*A+7*B)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4081, 3872, 3854, 3856, 2719, 2720}

$$\frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(5A+7B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a(5A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (6*a*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(5*d) + (2*a*(5*A + 7*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(21*d) + (2*a*A*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) +
(2*a*(A + B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*(5*A + 7*B)*Sin[
c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

```
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}-\frac{2}{7}\int \frac{-\frac{7}{2}a(A+B)-\frac{1}{2}a(5A+7B)\sec(c\\ &= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+(a(A+B))\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\frac{1}{7}(a(5\\ &= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(5A+7B)}{21d\sqrt{\sec(}}\\ &= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a(A+B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(5A+7B)}{21d\sqrt{\sec(}}\\ &= \frac{6a(A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\end{aligned}$$

Mathematica [A]
time = 1.04, size = 113, normalized size = 0.66

$$\frac{a\sqrt{\sec(c+dx)}\left(252(A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(5A+7B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(65A+70B+42(A+B)\cos(c+dx)+15A\cos(2(c+dx)))\sin(2(c+dx))\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (a*Sqrt[Sec[c + d*x]]*(252*(A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2] + 20*(5*A + 7*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (65*A
+ 70*B + 42*(A + B)*Cos[c + d*x] + 15*A*Cos[2*(c + d*x)])*Sin[2*(c + d*x)])
)/(210*d)
```

Maple [A]
time = 1.16, size = 383, normalized size = 2.23

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(240A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-528A-168B)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*A*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-528*A-168*B)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(448*A+308*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-12
2*A-112*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)-63*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+35*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-63*B*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.91, size = 187, normalized size = 1.09

$$\frac{-5i\sqrt{2}\,(5A+7B)a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,(5A+7B)a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+63i\sqrt{2}\,(A+B)a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-63i\sqrt{2}\,(A+B)a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(15Aa\cos(dx+c)^3+21(A+B)a\cos(dx+c)^2+5(5A+7B)a\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="
fricas")
```

```
[Out] 1/105*(-5*I*sqrt(2)*(5*A + 7*B)*a*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 5*I*sqrt(2)*(5*A + 7*B)*a*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*(A + B)*a*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*
(A + B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(15*A*a*cos(d*x + c)^3 + 21*(A + B)*a*cos(d*x + c)^2 +
 5*(5*A + 7*B)*a*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int\frac{A}{\sec^{\frac{7}{2}}(c+dx)}\,dx+\int\frac{A}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\int\frac{B}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\int\frac{B}{\sec^{\frac{3}{2}}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] a*(Integral(A/sec(c + d*x)**(7/2), x) + Integral(A/sec(c + d*x)**(5/2), x)
+ Integral(B/sec(c + d*x)**(5/2), x) + Integral(B/sec(c + d*x)**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/(1/cos(c + d*x))^(7/2), x)
```

3.186 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=234

$$-\frac{4a^2(4A+3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(7A+6B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}$$

```
[Out] 4/21*a^2*(7*A+6*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/35*a^2*(7*A+9*B)*sec(d*x
+c)^(5/2)*sin(d*x+c)/d+2/7*B*sec(d*x+c)^(5/2)*(a^2+a^2*sec(d*x+c))*sin(d*x+
c)/d+4/5*a^2*(4*A+3*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-4/5*a^2*(4*A+3*B)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/21*a^2*(7*A+6*B)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.20, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4103, 4082, 3872, 3853, 3856, 2719, 2720}

$$\frac{2a^2(7A+9B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d}+\frac{4a^2(7A+6B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{4a^2(4A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(7A+6B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^2(4A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-4*a^2*(4*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^2*(7*A + 6*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^2*(4*A + 3*B)*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(5*d) + (4*a^2*(7*A + 6*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2
1*d) + (2*a^2*(7*A + 9*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*B*Se
c[c + d*x]^(5/2)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(7*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{2B\sec^{\frac{5}{2}}(c+dx)\left(a^2+a^2\sec(c+dx)\right)\sin(c+d}{7d}$$

$$= \frac{2a^2(7A+9B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2B\,\mathrm{s}}{}$$

$$= \frac{2a^2(7A+9B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2B\,\mathrm{s}}{}$$

$$= \frac{4a^2(4A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{4a^2}{}$$

$$= \frac{4a^2(4A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{4a^2}{}$$

$$= -\frac{4a^2(4A+3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 4.74, size = 463, normalized size = 1.98

$$\frac{a^2e^{-idx}\cos^3(c+dx)\csc(c)\sec^4\left(\frac{1}{2}(c+dx)\right)\left(7\sqrt{2}(4A+3B)e^{2idx}(-1+e^{2ic})\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-\frac{e^{-i(c-dx)}(-1+e^{2ic})\left(7A(-5+9e^{i(c+dx)}-5e^{2i(c+dx)}+36e^{3i(c+dx)}+5e^{4i(c+dx)}+39e^{5i(c+dx)}+5e^{6i(c+dx)}+12e^{7i(c+dx)})+3B(-10+7e^{i(c+dx)}-20e^{2i(c+dx)}+63e^{3i(c+dx)}+20e^{4i(c+dx)}+77e^{5i(c+dx)}+10e^{6i(c+dx)}+21e^{7i(c+dx)})+5i(7A+6B)(1+e^{2i(c+dx)})^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)\sqrt{\sec(c+dx)}}{(1+e^{2i(c+dx)})^3}\right)(1+\sec(c+dx))^2(A+B\sec(c+dx))}{210d(B+A\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*Cos[c + d*x]^3*Csc[c]*Sec[(c + d*x)/2]^4*(7*Sqrt[2]*(4*A + 3*B)*E^((2*
I)*d*x)*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
 + d*x))] - ((-1 + E^((2*I)*c))*(7*A*(-5 + 9*E^(I*(c + d*x)) - 5*E^((2*I)*(
c + d*x)) + 36*E^((3*I)*(c + d*x)) + 5*E^((4*I)*(c + d*x)) + 39*E^((5*I)*(c
 + d*x)) + 5*E^((6*I)*(c + d*x)) + 12*E^((7*I)*(c + d*x))) + 3*B*(-10 + 7*E
^(I*(c + d*x)) - 20*E^((2*I)*(c + d*x)) + 63*E^((3*I)*(c + d*x)) + 20*E^((4
*I)*(c + d*x)) + 77*E^((5*I)*(c + d*x)) + 10*E^((6*I)*(c + d*x)) + 21*E^((7
*I)*(c + d*x))) + (5*I)*(7*A + 6*B)*(1 + E^((2*I)*(c + d*x)))^3*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2])*Sqrt[Sec[c + d*x]])/(E^(I*(c - d*x))*(1
+ E^((2*I)*(c + d*x)))^3))*(1 + Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/(210*
d*E^(I*d*x)*(B + A*Cos[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 824 vs. $2(258) = 516$.

time = 3.98, size = 825, normalized size = 3.53

method	result	size
default	Expression too large to display	825

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(8*(1/2*A+1/
4*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*B*(-1/56*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2
-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+8/5*(1/4*A+1/2*B)/sin(1/2*
d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+
1/2*c)^2-1)*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1
/2*d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)+2*A/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*
d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.92, size = 263, normalized size = 1.12

$$-\frac{2\left(5i\sqrt{2}\,(7A+6B)a^2\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}\,(7A+6B)a^2\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+21i\sqrt{2}\,(4A+3B)a^2\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21i\sqrt{2}\,(4A+3B)a^2\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(42(4A+3B)a^2\cos(dx+c)^3+10(7A+6B)a^2\cos(dx+c)^2+21(A+2B)a^2\cos(dx+c)+15Ba^2\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{105\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] -2/105*(5*I*sqrt(2)*(7*A + 6*B)*a^2*cos(d*x + c)^3*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(7*A + 6*B)*a^2*cos(d*x + c
)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*I*sqrt(2
)*(4*A + 3*B)*a^2*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*(4*A + 3*B)*a^2*cos(
d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) - (42*(4*A + 3*B)*a^2*cos(d*x + c)^3 + 10*(7*A + 6*B)*a^2
*cos(d*x + c)^2 + 21*(A + 2*B)*a^2*cos(d*x + c) + 15*B*a^2)*sin(d*x + c)/sq
rt(cos(d*x + c)))/(d*cos(d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2), x)
```

3.187 $\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=199

$$-\frac{4a^2(5A+4B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(2A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\right.}{3d}$$

```
[Out] 2/15*a^2*(5*A+7*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*B*sec(d*x+c)^(3/2)*(a^
2+a^2*sec(d*x+c))*sin(d*x+c)/d+4/5*a^2*(5*A+4*B)*sin(d*x+c)*sec(d*x+c)^(1/2
)/d-4/5*a^2*(5*A+4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/3*a^
2*(2*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.19, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4103, 4082, 3872, 3856, 2720, 3853, 2719}

$$\frac{2a^2(5A+7B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d}+\frac{4a^2(5A+4B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(2A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2(5A+4B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2B\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-4*a^2*(5*A + 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^2*(2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^2*(5*A + 4*B)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(5*d) + (2*a^2*(5*A + 7*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d
) + (2*B*Sec[c + d*x]^(3/2)*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx &= \frac{2B\sec^{\frac{3}{2}}(c+dx)\,(a^2+a^2\sec(c+dx))\sin(c}{5d} \\
&= \frac{2a^2(5A+7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}+\frac{2}{} \\
&= \frac{2a^2(5A+7B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d}+\frac{2}{} \\
&= \frac{4a^2(5A+4B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d}+ \\
&= \frac{4a^2(2A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right.}{3d} \\
&= -\frac{4a^2(5A+4B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx\right.}{5d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.32, size = 321, normalized size = 1.61

$$
\frac{a^2e^{-ic}(-1+e^{2ic})\csc(c)\left(5A+10B-30Ae^{i(c+dx)}-18Be^{i(c+dx)}-60Ae^{3i(c+dx)}-54Be^{3i(c+dx)}-5Ae^{4i(c+dx)}-10Be^{4i(c+dx)}-30Ae^{5i(c+dx)}-24Be^{5i(c+dx)}-10i(2A+B)\left(1+e^{2i(c+dx)}\right)^2\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+2(5A+4B)e^{i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\sec^4\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^2(A+B\sec(c+dx))}{60d\left(1+e^{2i(c+dx)}\right)^2(B+A\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(-1 + E^((2*I)*c))*Csc[c]*(5*A + 10*B - 30*A*E^(I*(c + d*x)) - 18*B*E^
(I*(c + d*x)) - 60*A*E^((3*I)*(c + d*x)) - 54*B*E^((3*I)*(c + d*x)) - 5*A*E
^((4*I)*(c + d*x)) - 10*B*E^((4*I)*(c + d*x)) - 30*A*E^((5*I)*(c + d*x)) -
24*B*E^((5*I)*(c + d*x)) - (10*I)*(2*A + B)*(1 + E^((2*I)*(c + d*x)))^2*Sqr
t[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(5*A + 4*B)*E^(I*(c + d*x))*(
1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(
c + d*x))])*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/(
60*d*E^(I*c)*(1 + E^((2*I)*(c + d*x)))^2*(B + A*Cos[c + d*x])*Sec[c + d*x]^
(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 715 vs. $2(227) = 454$.
time = 3.67, size = 716, normalized size = 3.60

method	result

default	$-\frac{a^{2}\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}{\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETURNVE RBOSE)

[Out] -a^2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+8*(1/4*A+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*B/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+8*(1/2*A+1/4*B)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.42, size = 239, normalized size = 1.20

$$-\frac{2\left(5i\sqrt{2}\,(2A+B)a^2\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}\,(2A+B)a^2\cos(dx+c)^2\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}\,(5A+4B)a^2\cos(dx+c)^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\,(5A+4B)a^2\cos(dx+c)^2\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(6(5A+4B)a^2\cos(dx+c)^2+5(A+2B)a^2\cos(dx+c)+3Ba^2\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{15d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -2/15*(5*I*sqrt(2)*(2*A + B)*a^2*cos(d*x + c)^2*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(2*A + B)*a^2*cos(d*x + c)^2*w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*(5*A
 + 4*B)*a^2*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(5*A + 4*B)*a^2*cos(d*x + c
)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) - (6*(5*A + 4*B)*a^2*cos(d*x + c)^2 + 5*(A + 2*B)*a^2*cos(d*x +
c) + 3*B*a^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^2\,\sqrt{\frac{1}{\cos(c+d\,x)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2), x)
```

3.188 $\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=160

$$-\frac{4a^2B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{4a^2(3A+2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3d}$$

```
[Out] 2/3*a^2*(3*A+5*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/3*B*(a^2+a^2*sec(d*x+c))*
sin(d*x+c)*sec(d*x+c)^(1/2)/d-4*a^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d+4/3*a^2*(3*A+2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 160, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4103, 4082, 3872, 3856, 2719, 2720}

$$\frac{2a^2(3A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{4a^2(3A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2B\sin(c+dx)\sqrt{\sec(c+dx)}\,(a^2\sec(c+dx)+a^2)}{3d}-\frac{4a^2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (-4*a^2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
d + (4*a^2*(3*A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*d) + (2*a^2*(3*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3
*d) + (2*B*Sqrt[Sec[c + d*x]]*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx &= \frac{2B\sqrt{\sec(c+dx)}\,(a^2+a^2\sec(c+dx))\sin(c+dx)}{3d}+\frac{2}{3}\int \\ &= \frac{2a^2(3A+5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d}+\frac{2B\sqrt{\sec(c+}}{} \\ &= \frac{2a^2(3A+5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d}+\frac{2B\sqrt{\sec(c+}}{} \\ &= \frac{2a^2(3A+5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d}+\frac{2B\sqrt{\sec(c+}}{} \\ &= -\frac{4a^2B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{d}+\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 3.43, size = 295, normalized size = 1.84

$$\frac{a^2 \sec^4\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^2(A+B\sec(c+dx))\left(-\frac{i\sqrt{2}\cos^3(c+dx)\left(3B\sqrt{1+e^{2i(c+dx)}}+3B(-1+e^{2ic})\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)+(3A+2B)e^{i(c+dx)}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)}{d(-1+e^{2ic})\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}}+\frac{-3(-A-4B+A\cos(2c))\cos(dx)\csc(c)+6A\cos(c)\sin(dx)+2B\tan(c+dx)}{4d\sec^{\frac{5}{2}}(c+dx)}\right)}{3(B+A\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + B*Sec[c + d*x])*(((-I)*Sq
rt[2]*Cos[c + d*x]^3*(3*B*Sqrt[1 + E^((2*I)*(c + d*x))] + 3*B*(-1 + E^((2*I
)*c))*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] + (3*A + 2*B)
*E^(I*(c + d*x))*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2
*I)*(c + d*x))]))/(d*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*
(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]) + (-3*(-A - 4*B + A*Cos[2*c])*C
os[d*x]*Csc[c] + 6*A*Cos[c]*Sin[d*x] + 2*B*Tan[c + d*x])/(4*d*Sec[c + d*x]^
(5/2))))/(3*(B + A*Cos[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(194) = 388$.
time = 1.50, size = 513, normalized size = 3.21

method	result
default	$-\frac{4a^2\left(6\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,(A+2B)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\cdots}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/3*a^2*(6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A+2*B)*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(3*A+7*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+2*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2)))*sin(1/2*d*x+1/2*c)^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*B*(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2
```

```
*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*
x+1/2*c)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x
)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.67, size = 202, normalized size = 1.26

$$-\frac{2\left(i\sqrt{2}\,(3A+2B)a^2\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{2}\,(3A+2B)a^2\cos(dx+c)\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}\,Ba^2\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\,Ba^2\cos(dx+c)\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{(3(A+2B)a^2\cos(dx+c)+Ba^2)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{3d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -2/3*(I*sqrt(2)*(3*A + 2*B)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) - I*sqrt(2)*(3*A + 2*B)*a^2*cos(d*x + c)*weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*B*a^2*cos
(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c))) - 3*I*sqrt(2)*B*a^2*cos(d*x + c)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*(A + 2*B)*a^2
*cos(d*x + c) + B*a^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\frac{A}{\sqrt{\sec(c+dx)}}\,dx+\int 2A\sqrt{\sec(c+dx)}\,dx+\int A\sec^{\frac{3}{2}}(c+dx)\,dx+\int B\sqrt{\sec(c+dx)}\,dx+\int 2B\sec^{\frac{3}{2}}(c+dx)\,dx+\int B\sec^{\frac{5}{2}}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] a**2*(Integral(A/sqrt(sec(c + d*x)), x) + Integral(2*A*sqrt(sec(c + d*x)),
x) + Integral(A*sec(c + d*x)**(3/2), x) + Integral(B*sqrt(sec(c + d*x)), x)
 + Integral(2*B*sec(c + d*x)**(3/2), x) + Integral(B*sec(c + d*x)**(5/2), x
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/(1/cos(c + d*x))^(1/2), x
)
```

3.189 $\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=158

$$\frac{4a^2A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{4a^2(2A+3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}{3d}$$

```
[Out] 2/3*A*(a^2+a^2*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(1/2)-2/3*a^2*(A-3*B)*si
n(d*x+c)*sec(d*x+c)^(1/2)/d+4*a^2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^
(1/2)/d+4/3*a^2*(2*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 158, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4102, 4082, 3872, 3856, 2719, 2720}

$$-\frac{2a^2(A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{4a^2(2A+3B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2A\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{3d\sqrt{\sec(c+dx)}}+\frac{4a^2A\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (4*a^2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d
 + (4*a^2*(2*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(3*d) - (2*a^2*(A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d)
 + (2*A*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A(a^2+a^2\sec(c+dx))\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{(a+a\sec(c+ \cdots}{\cdots}$$

$$= -\frac{2a^2(A-3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2A(a^2+a^2\sec \cdots}{3d\sqrt{\cdots}}$$

$$= -\frac{2a^2(A-3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2A(a^2+a^2\sec \cdots}{3d\sqrt{\cdots}}$$

$$= -\frac{2a^2(A-3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2A(a^2+a^2\sec \cdots}{3d\sqrt{\cdots}}$$

$$= \frac{4a^2A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d} + \frac{4a^2(\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 3.09, size = 299, normalized size = 1.89

$$\frac{a^2\sec^4\left(\frac{1}{2}(c+dx)\right)(1+\sec(c+dx))^2(A+B\sec(c+dx))\left(\frac{i\sqrt{2}\cos^3(c+dx)\left(3A\sqrt{1+e^{2i(c+dx)}}+3A(-1+e^{2ic})\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\frac{3}{4};-e^{2i(c+dx)}\right)-(2A+3B)e^{i(c+dx)}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};-e^{2i(c+dx)}\right)\right)}{d(-1+e^{2ic})\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}}+\frac{-3(2A-B)\cos(dx)\csc(c)-3(2A+B)\cos(2c+dx)\csc(c)+A\sin(2(c+dx))}{4d\sec^{\frac{5}{2}}(c+dx)}\right)}{3(B+A\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),
x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]^4*(1 + Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((I*Sqrt[
2]*Cos[c + d*x]^3*(3*A*Sqrt[1 + E^((2*I)*(c + d*x))] + 3*A*(-1 + E^((2*I)*c
))*Hypergeometric2F1[-1/4, 1/2, 3/4, -E^((2*I)*(c + d*x))] - (2*A + 3*B)*E^
(I*(c + d*x))*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/4, 1/2, 5/4, -E^((2*I)
*(c + d*x))]))/(d*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c
+ d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]) + (-3*(2*A - B)*Cos[d*x]*Csc[c] -
3*(2*A + B)*Cos[2*c + d*x]*Csc[c] + A*Sin[2*(c + d*x)])/(4*d*Sec[c + d*x]^(
5/2))))/(3*(B + A*Cos[c + d*x]))
```

Maple [A]
time = 1.34, size = 245, normalized size = 1.55

method	result
default	$-\frac{4a^2\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)A+2A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/3*a^2*(2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)*A+2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-3*A*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)-3*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3*B*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maple [F]

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.81, size = 166, normalized size = 1.05

$$-\frac{2 \left(i \sqrt{2}\, (2 A+3 B) a^{2} \mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i \sin(dx+c)) - i \sqrt{2}\, (2 A+3 B) a^{2} \mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i \sin(dx+c)) - 3 i \sqrt{2}\, A a^{2} \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i \sin(dx+c))) + 3 i \sqrt{2}\, A a^{2} \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i \sin(dx+c))) - \frac{(A a^{2} \cos(dx+c)+3 B a^{2}) \sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{3 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] -2/3*(I*sqrt(2)*(2*A + 3*B)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) - I*sqrt(2)*(2*A + 3*B)*a^2*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*A*a^2*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(2)*A*a^2*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
c))) - (A*a^2*cos(d*x + c) + 3*B*a^2)*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2} \left(\int \frac{A}{\sec^{\frac{3}{2}}(c+dx)}\, dx + \int \frac{2A}{\sqrt{\sec(c+dx)}}\, dx + \int A \sqrt{\sec(c+dx)}\, dx + \int \frac{B}{\sqrt{\sec(c+dx)}}\, dx + \int 2B \sqrt{\sec(c+dx)}\, dx + \int B \sec^{\frac{3}{2}}(c+dx)\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] a**2*(Integral(A/sec(c + d*x)**(3/2), x) + Integral(2*A/sqrt(sec(c + d*x)),
 x) + Integral(A*sqrt(sec(c + d*x)), x) + Integral(B/sqrt(sec(c + d*x)), x)
 + Integral(2*B*sqrt(sec(c + d*x)), x) + Integral(B*sec(c + d*x)**(3/2), x)
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/(1/cos(c + d*x))^(3/2),x)
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/(1/cos(c + d*x))^(3/2), x
)
```

3.190 $\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=166

$$\frac{4a^2(4A+5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(A+2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{3d}$$

```
[Out] 2/5*A*(a^2+a^2*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/15*a^2*(7*A+5*B)
*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4/5*a^2*(4*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/d+4/3*a^2*(A+2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 166, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4102, 4081, 3872, 3856, 2719, 2720}

$$\frac{2a^2(7A+5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{4a^2(A+2B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2(4A+5B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2A\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (4*a^2*(4*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^2*(A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*(7*A + 5*B)*Sin[c + d*x])/(15*d*Sqrt
[Sec[c + d*x]]) + (2*A*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(5*d*Sec[c +
d*x]^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a^2+a^2\sec(c+dx))\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2}{5}\int \frac{(a+a\sec(c+$$

$$= \frac{2a^2(7A+5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2A(a^2+a^2\sec(c+dx))\sin(c}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2(7A+5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2A(a^2+a^2\sec(c+dx))\sin(c}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2(7A+5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2A(a^2+a^2\sec(c+dx))\sin(c}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{4a^2(4A+5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}{5d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 1.90, size = 153, normalized size = 0.92

$$\frac{a^2\sqrt{\sec(c+dx)}\left(20(A+2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-4i(4A+5B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(48iA+60iB+10(2A+B)\sin(c+dx)+3A\sin(2(c+dx)))\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),
x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(20*(A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2] - (4*I)*(4*A + 5*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*H
ypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*x]*((48*I
)*A + (60*I)*B + 10*(2*A + B)*Sin[c + d*x] + 3*A*Sin[2*(c + d*x)])))/(15*d)
```

Maple [A]
time = 1.19, size = 357, normalized size = 2.15

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(-12A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(32A+10B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\text{co}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*A*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(32*A+10*B)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-13*A-5*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2)-12*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+10*B*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)-15*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.77, size = 187, normalized size = 1.13

$$-\frac{2\left(5i\sqrt{2}(A+2B)a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(A+2B)a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}(4A+5B)a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}(4A+5B)a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{(3Aa^2\cos(dx+c)^2+5(2A+B)a^2\cos(dx+c))\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] -2/15*(5*I*sqrt(2)*(A + 2*B)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) - 5*I*sqrt(2)*(A + 2*B)*a^2*weierstrassPInverse(-4, 0, cos(
d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(4*A + 5*B)*a^2*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(
2)*(4*A + 5*B)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) - (3*A*a^2*cos(d*x + c)^2 + 5*(2*A + B)*a^2*cos(d
*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\frac{A}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\int\frac{2A}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{A}{\sqrt{\sec(c+dx)}}\,dx+\int\frac{B}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{2B}{\sqrt{\sec(c+dx)}}\,dx+\int B\sqrt{\sec(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] a**2*(Integral(A/sec(c + d*x)**(5/2), x) + Integral(2*A/sec(c + d*x)**(3/2)
, x) + Integral(A/sqrt(sec(c + d*x)), x) + Integral(B/sec(c + d*x)**(3/2),
x) + Integral(2*B/sqrt(sec(c + d*x)), x) + Integral(B*sqrt(sec(c + d*x)), x
))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/(1/cos(c + d*x))^(5/2),x)
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/(1/cos(c + d*x))^(5/2), x
)
```

3.191 $\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=201

$$\frac{4a^2(3A+4B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^2(6A+7B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right.$$

```
[Out] 2/35*a^2*(9*A+7*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*A*(a^2+a^2*sec(d*x+c))
*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/21*a^2*(6*A+7*B)*sin(d*x+c)/d/sec(d*x+c)^(
1/2)+4/5*a^2*(3*A+4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/21*
a^2*(6*A+7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.20, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4102, 4081, 3872, 3854, 3856, 2720, 2719}

$$\frac{2a^2(9A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^2(6A+7B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^2(6A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(3A+4B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2A\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (4*a^2*(3*A + 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^2*(6*A + 7*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(9*A + 7*B)*Sin[c + d*x])/(35*d*S
ec[c + d*x]^(3/2)) + (4*a^2*(6*A + 7*B)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*
x]]) + (2*A*(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

```
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a^2+a^2\sec(c+dx))\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+a\sec(c+$$

$$= \frac{2a^2(9A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A(a^2+a^2\sec(c+dx))\sin(c}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(9A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A(a^2+a^2\sec(c+dx))\sin(c}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(9A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^2(6A+7B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}} +$$

$$= \frac{4a^2(3A+4B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}{5d}$$

$$= \frac{4a^2(3A+4B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}{5d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.51, size = 193, normalized size = 0.96

$$\frac{a^2e^{-idx}\sqrt{\sec(c+dx)}(\cos(dx)+i\sin(dx))\left(40(6A+7B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)-56i(3A+4B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(504iA+672iB+5(51A+56B)\sin(c+dx)+42(2A+B)\sin(2(c+dx))+15A\sin(3(c+dx)))\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),
x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(40*(6*A + 7*B)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(3*A + 4*B)*E^(I*(c + d*x))*Sqrt
[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d
*x))] + Cos[c + d*x]*((504*I)*A + (672*I)*B + 5*(51*A + 56*B)*Sin[c + d*x]
+ 42*(2*A + B)*Sin[2*(c + d*x)] + 15*A*Sin[3*(c + d*x)])))/(210*d*E^(I*d*x)
)
```

Maple [A]
time = 1.10, size = 385, normalized size = 1.92

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(120A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-348A-84B)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(120*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-348*A-84*B)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(378*A+224*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-1
17*A-91*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+30*A*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)-63*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+35*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-84*B*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d
*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.02, size = 211, normalized size = 1.05

$$-\frac{2\left(5i\sqrt{2}\,(6A+7B)a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}\,(6A+7B)a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-21i\sqrt{2}\,(3A+4B)a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21i\sqrt{2}\,(3A+4B)a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(15Aa^2\cos(dx+c)^3+21(2A+B)a^2\cos(dx+c)^2+10(6A+7B)a^2\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] -2/105*(5*I*sqrt(2)*(6*A + 7*B)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) - 5*I*sqrt(2)*(6*A + 7*B)*a^2*weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(2)*(3*A + 4*B)*a^2*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*
I*sqrt(2)*(3*A + 4*B)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c))) - (15*A*a^2*cos(d*x + c)^3 + 21*(2*A + B)*
```

```
a^2*cos(d*x + c)^2 + 10*(6*A + 7*B)*a^2*cos(d*x + c))*sin(d*x + c)/sqrt(cos
(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{A}{\sec^{\frac{7}{2}}(c+dx)}\,dx + \int \frac{2A}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{A}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{B}{\sec^{\frac{5}{2}}(c+dx)}\,dx + \int \frac{2B}{\sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{B}{\sqrt{\sec(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] a**2*(Integral(A/sec(c + d*x)**(7/2), x) + Integral(2*A/sec(c + d*x)**(5/2)
, x) + Integral(A/sec(c + d*x)**(3/2), x) + Integral(B/sec(c + d*x)**(5/2),
 x) + Integral(2*B/sec(c + d*x)**(3/2), x) + Integral(B/sqrt(sec(c + d*x)),
 x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/(1/cos(c + d*x))^(7/2), x
)
```

3.192 $\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=234

$$\frac{4a^2(8A+9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{4a^2(5A+6B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}$$

```
[Out] 2/63*a^2*(11*A+9*B)*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/45*a^2*(8*A+9*B)*sin(d*
x+c)/d/sec(d*x+c)^(3/2)+2/9*A*(a^2+a^2*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^
(7/2)+4/21*a^2*(5*A+6*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4/15*a^2*(8*A+9*B)*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/21*a^2*(5*A+6*B)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.22, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4102, 4081, 3872, 3854, 3856, 2719, 2720}

$$\frac{4a^2(8A+9B)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2(11A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^2(5A+6B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^2(5A+6B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(8A+9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2A\sin(c+dx)\left(a^2\sec(c+dx)+a^2\right)}{9d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),x]
```

```
[Out] (4*a^2*(8*A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(15*d) + (4*a^2*(5*A + 6*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*(11*A + 9*B)*Sin[c + d*x])/(63*d
*Sec[c + d*x]^(5/2)) + (4*a^2*(8*A + 9*B)*Sin[c + d*x])/(45*d*Sec[c + d*x]^
(3/2)) + (4*a^2*(5*A + 6*B)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*A*
(a^2 + a^2*Sec[c + d*x])*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2A(a^2+a^2\sec(c+dx))\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+a\sec(c+d$$

$$= \frac{2a^2(11A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A(a^2+a^2\sec(c+dx))\sin(c}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^2(11A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2A(a^2+a^2\sec(c+dx))\sin(c}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^2(11A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2(8A+9B)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{2a^2(11A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^2(8A+9B)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)} +$$

$$= \frac{4a^2(8A+9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{15d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.07, size = 217, normalized size = 0.93

$$\frac{a^2e^{-idx}\sqrt{\sec(c+dx)}\,(\cos(dx)+i\sin(dx))\left(240(5A+6B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-112i(8A+9B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(2688iA+3024iB+30(46A+51B)\sin(c+dx)+14(37A+36B)\sin(2(c+dx))+180A\sin(3(c+dx))+90B\sin(3(c+dx))+35A\sin(4(c+dx)))\right)}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),
x]
```

```
[Out] (a^2*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(240*(5*A + 6*B)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2] - (112*I)*(8*A + 9*B)*E^(I*(c + d*x))*Sq
rt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
 d*x))] + Cos[c + d*x]*((2688*I)*A + (3024*I)*B + 30*(46*A + 51*B)*Sin[c +
d*x] + 14*(37*A + 36*B)*Sin[2*(c + d*x)] + 180*A*Sin[3*(c + d*x)] + 90*B*Si
n[3*(c + d*x)] + 35*A*Sin[4*(c + d*x)])))/(1260*d*E^(I*d*x))
```

Maple [A]
time = 1.26, size = 413, normalized size = 1.76

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(-560A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(1840A+360B)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-560*A*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(1840*A+360*B)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-2368*A-1044*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(1568*A+1134*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-387*A-351*B)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*A*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-168*A*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)+90*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-189*B*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(9/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.70, size = 231, normalized size = 0.99

$$-\frac{2\left(15i\sqrt{2}\,(5A+6B)a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) - 15i\sqrt{2}\,(5A+6B)a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)) - 21i\sqrt{2}\,(8A+9B)a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) + 21i\sqrt{2}\,(8A+9B)a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))) - \frac{\left(35Aa^2\cos(dx+c)^4+45(2A+B)a^2\cos(dx+c)^3+14(8A+9B)a^2\cos(dx+c)^2+30(5A+6B)a^2\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] -2/315*(15*I*sqrt(2)*(5*A + 6*B)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) - 15*I*sqrt(2)*(5*A + 6*B)*a^2*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(2)*(8*A + 9*B)*a^2*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 2
1*I*sqrt(2)*(8*A + 9*B)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
```

```
0, cos(d*x + c) - I*sin(d*x + c))) - (35*A*a^2*cos(d*x + c)^4 + 45*(2*A + B
)*a^2*cos(d*x + c)^3 + 14*(8*A + 9*B)*a^2*cos(d*x + c)^2 + 30*(5*A + 6*B)*a
^2*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sec(d*x + c)^(9/2), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^2}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/(1/cos(c + d*x))^(9/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/(1/cos(c + d*x))^(9/2), x
)
```

3.193 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=277

$$-\frac{4a^3(21A+17B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{4a^3(13A+11B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{21d}$$

```
[Out] 4/21*a^3*(13*A+11*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+4/105*a^3*(24*A+23*B)*se
c(d*x+c)^(5/2)*sin(d*x+c)/d+2/9*a*B*sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*sin
(d*x+c)/d+2/63*(9*A+13*B)*sec(d*x+c)^(5/2)*(a^3+a^3*sec(d*x+c))*sin(d*x+c)/
d+4/15*a^3*(21*A+17*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-4/15*a^3*(21*A+17*B)*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/21*a^3*(13*A+11*B)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.31, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4103, 4082, 3872, 3853, 3856, 2719, 2720}

$$\frac{4a^3(24A+23B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{105d}+\frac{4a^3(13A+11B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2(9A+13B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{63d}+\frac{4a^3(21A+17B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{4a^3(13A+11B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(21A+17B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2aB\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^2}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-4*a^3*(21*A + 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(15*d) + (4*a^3*(13*A + 11*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(21*A + 17*B)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(15*d) + (4*a^3*(13*A + 11*B)*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(21*d) + (4*a^3*(24*A + 23*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(105
*d) + (2*a*B*Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(9*d)
+ (2*(9*A + 13*B)*Sec[c + d*x]^(5/2)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])
/(63*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{2aB\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+ \ldots}{9d}$$

$$= \frac{2aB\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c+ \ldots}{9d}$$

$$= \frac{4a^3(24A+23B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d} + \ldots$$

$$= \frac{4a^3(24A+23B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{105d} + \ldots$$

$$= \frac{4a^3(21A+17B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d} + \ldots$$

$$= \frac{4a^3(21A+17B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d} + \ldots$$

$$= -\frac{4a^3(21A+17B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx \ldots\right.}{15d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.91, size = 793, normalized size = 2.86

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (7*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Cos[c + d*x]^4*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*
d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x
))])*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/(30*
Sqrt[2]*d*E^(I*d*x)*(B + A*Cos[c + d*x])) + (17*B*Sqrt[E^(I*(c + d*x))/(1 +
 E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c + d*x]^4*Csc[c]*
(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hyperg
eometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2 + (d*x)/2]^6*(a +
 a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/(90*Sqrt[2]*d*E^(I*d*x)*(B + A*Cos
[c + d*x])) + (13*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sec[c/2 +
(d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/(42*d*(B + A*Cos[c
+ d*x])*Sec[c + d*x]^(7/2)) + (11*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sec[c/2 + (d*x)/2]^6*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/(42
*d*(B + A*Cos[c + d*x])*Sec[c + d*x]^(7/2)) + (Sec[c/2 + (d*x)/2]^6*(a + a*
Sec[c + d*x])^3*(A + B*Sec[c + d*x])*(((21*A + 17*B)*Cos[d*x]*Csc[c])/(30*d
```

```
) + (B*Sec[c]*Sec[c + d*x]^4*Sin[d*x])/(36*d) + (Sec[c]*Sec[c + d*x]^3*(7*B
*Sin[c] + 9*A*Sin[d*x] + 27*B*Sin[d*x]))/(252*d) + (Sec[c]*Sec[c + d*x]^2*(
45*A*Sin[c] + 135*B*Sin[c] + 189*A*Sin[d*x] + 238*B*Sin[d*x]))/(1260*d) + (
Sec[c]*Sec[c + d*x]*(189*A*Sin[c] + 238*B*Sin[c] + 390*A*Sin[d*x] + 330*B*S
in[d*x]))/(1260*d) + ((13*A + 11*B)*Tan[c])/(42*d)))/((B + A*Cos[c + d*x])*
Sec[c + d*x]^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1152 vs. $2(297) = 594$.
time = 5.44, size = 1153, normalized size = 4.16

method	result	size
default	Expression too large to display	1153

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*(3/8*A+1
/8*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+16*(1/8*A+3/8*B)*(-1/56*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/
2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+16/5*(3/8*A+
3/8*B)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)
^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x
+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*B*(-1/144*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^5-7/
180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)
/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)+7/15*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-7/15*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
```

```
))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))+2*A/sin(1/2*d*x+1/2*c)^2/(2*sin(
1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2
*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.85, size = 283, normalized size = 1.02

$$-\frac{2\left(15i\sqrt{2}\,(13\,A+11\,B)a^3\cos\left(dx+c\right)^4 \mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-15i\sqrt{2}\,(13\,A+11\,B)a^3\cos\left(dx+c\right)^4 \mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+21i\sqrt{2}\,(21\,A+17\,B)a^3\cos\left(dx+c\right)^4 \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-21i\sqrt{2}\,(21\,A+17\,B)a^3\cos\left(dx+c\right)^4 \mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))-\frac{\left(42\,(21\,A+17\,B)a^3\cos(dx+c)^4+30\,(13\,A+11\,B)a^3\cos(dx+c)^3+7\,(27\,A+34\,B)a^3\cos(dx+c)^2+45\,(A+3\,B)a^3\cos(dx+c)+35\,Ba^3\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}\right)}{315\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] -2/315*(15*I*sqrt(2)*(13*A + 11*B)*a^3*cos(d*x + c)^4*weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c)) - 15*I*sqrt(2)*(13*A + 11*B)*a^3*cos(d
*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*I*
sqrt(2)*(21*A + 17*B)*a^3*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*(21*A + 17*B
)*a^3*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) - I*sin(d*x + c))) - (42*(21*A + 17*B)*a^3*cos(d*x + c)^4 + 30*(13
*A + 11*B)*a^3*cos(d*x + c)^3 + 7*(27*A + 34*B)*a^3*cos(d*x + c)^2 + 45*(A
+ 3*B)*a^3*cos(d*x + c) + 35*B*a^3)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos
(d*x + c)^4)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7316 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^3\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2), x)
```

3.194 $\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=244

$$-\frac{4a^3(9A+7B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3(21A+13B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+d\right.}{21d}$$

```
[Out] 4/105*a^3*(42*A+41*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*a*B*sec(d*x+c)^(3/2
)*(a+a*sec(d*x+c))^2*sin(d*x+c)/d+2/35*(7*A+11*B)*sec(d*x+c)^(3/2)*(a^3+a^3
*sec(d*x+c))*sin(d*x+c)/d+4/5*a^3*(9*A+7*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-4
/5*a^3*(9*A+7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/21*a^3*(2
1*A+13*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.28, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4103, 4082, 3872, 3856, 2720, 3853, 2719}

$$\frac{4a^3(42A+41B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{105d}+\frac{2(7A+11B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{35d}+\frac{4a^3(9A+7B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3(21A+13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^3(9A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aB\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-4*a^3*(9*A + 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(21*A + 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(9*A + 7*B)*Sqrt[Sec[c + d*x]]
*Sin[c + d*x])/(5*d) + (4*a^3*(42*A + 41*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x]
)/(105*d) + (2*a*B*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/
(7*d) + (2*(7*A + 11*B)*Sec[c + d*x]^(3/2)*(a^3 + a^3*Sec[c + d*x])*Sin[c +
 d*x])/(35*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{2aB\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c\cdots}{7d}$$
$$= \frac{2aB\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2\sin(c\cdots}{7d}$$
$$= \frac{4a^3(42A+41B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \cdots$$
$$= \frac{4a^3(42A+41B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d} + \cdots$$
$$= \frac{4a^3(9A+7B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \cdots$$
$$= \frac{4a^3(21A+13B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\cdots\right.}{21d}$$
$$= -\frac{4a^3(9A+7B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx\cdots\right.}{5d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.49, size = 465, normalized size = 1.91

$$\frac{a^3e^{-idx}\cos^4(c+dx)\csc(c)\sec^6\left(\frac{1}{2}(c+dx)\right)\left(7\sqrt{2}\,(9A+7B)e^{2idx}(-1+e^{2ic})\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-\frac{e^{-i(c-dx)}(-1+e^{2ic})\left(21A\left(-5+16e^{i(c+dx)}-5e^{2i(c+dx)}+54e^{3i(c+dx)}+5e^{4i(c+dx)}+56e^{5i(c+dx)}+5e^{6i(c+dx)}+18e^{7i(c+dx)}\right)+2B\left(-65+84e^{i(c+dx)}-95e^{2i(c+dx)}+441e^{3i(c+dx)}+95e^{4i(c+dx)}+504e^{5i(c+dx)}+65e^{6i(c+dx)}+147e^{7i(c+dx)}\right)+10i(21A+13B)\left(1+e^{2i(c+dx)}\right)^3\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)|2\right)\right)\sqrt{\sec(c+dx)}}{2\left(1+e^{2i(c+dx)}\right)^3}\right)(1+\sec(c+dx))^3(A+B\sec(c+dx))}{420d(B+A\cos(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*Cos[c + d*x]^4*Csc[c]*Sec[(c + d*x)/2]^6*(7*Sqrt[2]*(9*A + 7*B)*E^((2*
I)*d*x)*(-1 + E^((2*I)*c))*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*
Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c
 + d*x))] - ((-1 + E^((2*I)*c))*(21*A*(-5 + 16*E^(I*(c + d*x)) - 5*E^((2*I)
*(c + d*x)) + 54*E^((3*I)*(c + d*x)) + 5*E^((4*I)*(c + d*x)) + 56*E^((5*I)*
(c + d*x)) + 5*E^((6*I)*(c + d*x)) + 18*E^((7*I)*(c + d*x))) + 2*B*(-65 + 8
4*E^(I*(c + d*x)) - 95*E^((2*I)*(c + d*x)) + 441*E^((3*I)*(c + d*x)) + 95*E
^((4*I)*(c + d*x)) + 504*E^((5*I)*(c + d*x)) + 65*E^((6*I)*(c + d*x)) + 147
*E^((7*I)*(c + d*x))) + (10*I)*(21*A + 13*B)*(1 + E^((2*I)*(c + d*x)))^3*Sq
rt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2])*Sqrt[Sec[c + d*x]])/(2*E^(I*(c
- d*x))*(1 + E^((2*I)*(c + d*x)))^3))*(1 + Sec[c + d*x])^3*(A + B*Sec[c + d
*x]))/(420*d*E^(I*d*x)*(B + A*Cos[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 903 vs. $2(268) = 536$.
time = 4.14, size = 904, normalized size = 3.70

method	result	size
default	Expression too large to display	904

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+16*
(3/8*A+3/8*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*B*(-1/56*cos(1/2*d
*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x
+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+16/5*(1/8*A+3/8*B
)/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)
/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x
+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)+16*(3/8*A+1/8*B)/sin(1/2*d*x+1/2*c)^2/(2*sin(
1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2
*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 263, normalized size = 1.08

$$-\frac{2 \left(5 i \sqrt{2}\, (21 A + 13 B) a^{3} \cos\left(d x + c\right)^{3} \mathrm{weierstrassPInverse}\left(-4, 0, \cos\left(d x + c\right) + i \sin\left(d x + c\right)\right) - 5 i \sqrt{2}\, (21 A + 13 B) a^{3} \cos\left(d x + c\right)^{3} \mathrm{weierstrassPInverse}\left(-4, 0, \cos\left(d x + c\right) - i \sin\left(d x + c\right)\right) + 21 i \sqrt{2}\, (9 A + 7 B) a^{3} \cos\left(d x + c\right)^{3} \mathrm{weierstrassZeta}\left(-4, 0, \mathrm{weierstrassPInverse}\left(-4, 0, \cos\left(d x + c\right) + i \sin\left(d x + c\right)\right)\right) - 21 i \sqrt{2}\, (9 A + 7 B) a^{3} \cos\left(d x + c\right)^{3} \mathrm{weierstrassZeta}\left(-4, 0, \mathrm{weierstrassPInverse}\left(-4, 0, \cos\left(d x + c\right) - i \sin\left(d x + c\right)\right)\right) - \frac{\left(42 (9 A + 7 B) a^{3} \cos(dx+c)^{3} + 5 (21 A + 26 B) a^{3} \cos(dx+c)^{2} + 21 (A + 3 B) a^{3} \cos(dx+c) + 15 B a^{3}\right) \sin(dx+c)}{\sqrt{\cos\left(d x + c\right)}}\right)}{105 d \cos\left(d x + c\right)^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -2/105*(5*I*sqrt(2)*(21*A + 13*B)*a^3*cos(d*x + c)^3*weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(21*A + 13*B)*a^3*cos(d*x
 + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*I*sq
rt(2)*(9*A + 7*B)*a^3*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*(9*A + 7*B)*a^3*
cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c))) - (42*(9*A + 7*B)*a^3*cos(d*x + c)^3 + 5*(21*A + 26*B
)*a^3*cos(d*x + c)^2 + 21*(A + 3*B)*a^3*cos(d*x + c) + 15*B*a^3)*sin(d*x +
c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2),x)
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2), x)
```

3.195 $\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=211

$$-\frac{4a^3(5A+9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3(5A+3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\right.}{3d}$$

```
[Out] 4/15*a^3*(20*A+21*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/5*a*B*(a+a*sec(d*x+c))
^2*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/15*(5*A+9*B)*(a^3+a^3*sec(d*x+c))*sin(d*
x+c)*sec(d*x+c)^(1/2)/d-4/5*a^3*(5*A+9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/d+4/3*a^3*(5*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d
```

Rubi [A]
time = 0.27, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4103, 4082, 3872, 3856, 2719, 2720}

$$\frac{4a^3(20A+21B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(5A+9B)\sin(c+dx)\sqrt{\sec(c+dx)}\,(a^3\sec(c+dx)+a^3)}{15d}+\frac{4a^3(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^3(5A+9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aB\sin(c+dx)\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (-4*a^3*(5*A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(5*A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)
/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (4*a^3*(20*A + 21*B)*Sqrt[Sec[c + d*x]]*
Sin[c + d*x])/(15*d) + (2*a*B*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2*Sin
[c + d*x])/(5*d) + (2*(5*A + 9*B)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x
])*Sin[c + d*x])/(15*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{2aB\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2\sin(c+dx)}{5d}+\frac{2}{5}\int$$

$$= \frac{2aB\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2\sin(c+dx)}{5d}+\frac{2(5A$$

$$= \frac{4a^3(20A+21B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d}+\frac{2aB\sqrt{\sec(c}{}$$

$$= \frac{4a^3(20A+21B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d}+\frac{2aB\sqrt{\sec(c}{}$$

$$= \frac{4a^3(20A+21B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d}+\frac{2aB\sqrt{\sec(c}{}$$

$$= -\frac{4a^3(5A+9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+d}}{5d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.43, size = 244, normalized size = 1.16

$$\frac{a^3 e^{-idx} \sec^{\frac{5}{2}}(c+dx)(\cos(dx)+i\sin(dx)) \left(-90iA\cos(c+dx)-162iB\cos(c+dx)-30iA\cos(3(c+dx))-54iB\cos(3(c+dx))+40(5A+3B)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2i(5A+9B)e^{-i(c+dx)}\left(1+e^{2i(c+dx)}\right)^{5/2} {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+45A\sin(c+dx)+66B\sin(c+dx)+10A\sin(2(c+dx))+30B\sin(2(c+dx))+45A\sin(3(c+dx))+54B\sin(3(c+dx))\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (a^3*Sec[c + d*x]^(5/2)*(Cos[d*x] + I*Sin[d*x])*((-90*I)*A*Cos[c + d*x] - (
162*I)*B*Cos[c + d*x] - (30*I)*A*Cos[3*(c + d*x)] - (54*I)*B*Cos[3*(c + d*x
)] + 40*(5*A + 3*B)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + ((2*I)*(
5*A + 9*B)*(1 + E^((2*I)*(c + d*x)))^(5/2)*Hypergeometric2F1[1/2, 3/4, 7/4,
 -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) + 45*A*Sin[c + d*x] + 66*B*Sin[c +
d*x] + 10*A*Sin[2*(c + d*x)] + 30*B*Sin[2*(c + d*x)] + 45*A*Sin[3*(c + d*x)
] + 54*B*Sin[3*(c + d*x)]))/(30*d*E^(I*d*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 915 vs. $2(239) = 478$.
time = 3.38, size = 916, normalized size = 4.34

method	result	size
default	Expression too large to display	916

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/15*a^3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(8*sin(
1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*
d*x+1/2*c)^3*(180*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-100*A*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-60*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*
d*x+1/2*c)^4+216*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-60*B*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-108*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d
*x+1/2*c)^4-190*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+100*A*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+60*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*
x+1/2*c)^2-246*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+60*B*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*
```

```
c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+108*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x
+1/2*c)^2+50*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)*A-25*A*EllipticF(cos(1
/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)-15*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+72*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x
+1/2*c)^2-15*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-27*B*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2))*(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.68, size = 243, normalized size = 1.15

$$-\frac{2\left(5i\sqrt{2}\,(5A+3B)a^3\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}\,(5A+3B)a^3\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}\,(5A+9B)a^3\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}\,(5A+9B)a^3\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(9(5A+6B)a^3\cos(dx+c)^2+5(A+3B)a^3\cos(dx+c)+3Ba^3\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{15\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -2/15*(5*I*sqrt(2)*(5*A + 3*B)*a^3*cos(d*x + c)^2*weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(5*A + 3*B)*a^3*cos(d*x + c)
^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*
(5*A + 9*B)*a^3*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(5*A + 9*B)*a^3*cos(d*x
 + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c))) - (9*(5*A + 6*B)*a^3*cos(d*x + c)^2 + 5*(A + 3*B)*a^3*cos(d*
x + c) + 3*B*a^3)*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^3}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(1/2), x
)
```

3.196 $\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=199

$$\frac{4a^3(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{20a^3(A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}$$

```
[Out] 2/3*a*A*(a+a*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4/3*a^3*(A+4*B)*si
n(d*x+c)*sec(d*x+c)^(1/2)/d-2/3*(A-B)*(a^3+a^3*sec(d*x+c))*sin(d*x+c)*sec(d
*x+c)^(1/2)/d+4*a^3*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+20
/3*a^3*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.27, antiderivative size = 199, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {4102, 4103, 4082, 3872, 3856, 2719, 2720}

$$\frac{4a^3(A+4B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}-\frac{2(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}\,(a^3\sec(c+dx)+a^3)}{3d}+\frac{20a^3(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)(a\sec(c+dx)+a)^2}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (4*a^3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (20*a^3*(A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(3*d) + (4*a^3*(A + 4*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3
*d) + (2*a*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
- (2*(A - B)*Sqrt[Sec[c + d*x]]*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(3*d
)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{(a+a\sec(c+
\\
&= \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2(A-B)\sqrt{\sec(c+
\\
&= \frac{4a^3(A+4B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2aA(a+a\sec(c}{3d\sqrt{\mathrm{se}}}
\\
&= \frac{4a^3(A+4B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2aA(a+a\sec(c}{3d\sqrt{\mathrm{se}}}
\\
&= \frac{4a^3(A+4B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2aA(a+a\sec(c}{3d\sqrt{\mathrm{se}}}
\\
&= \frac{4a^3(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.00, size = 202, normalized size = 1.02

$$
\frac{a^3e^{-idx}\sec^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(12iA-12iB+12iA\cos(2(c+dx))-12iB\cos(2(c+dx))+40(A+B)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)-4i(A-B)\left(1+e^{2i(c+dx)}\right)^{3/2}{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+A\sin(c+dx)+4B\sin(c+dx)+6A\sin(2(c+dx))+18B\sin(2(c+dx))+A\sin(3(c+dx))\right)}{6d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),
x]
```

```
[Out] (a^3*Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*((12*I)*A - (12*I)*B + (12*
I)*A*Cos[2*(c + d*x)] - (12*I)*B*Cos[2*(c + d*x)] + 40*(A + B)*Cos[c + d*x]
^(3/2)*EllipticF[(c + d*x)/2, 2] - (4*I)*(A - B)*(1 + E^((2*I)*(c + d*x)))^
(3/2)*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + A*Sin[c + d*
x] + 4*B*Sin[c + d*x] + 6*A*Sin[2*(c + d*x)] + 18*B*Sin[2*(c + d*x)] + A*Si
n[3*(c + d*x)]))/(6*d*E^(I*d*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 653 vs. $2(229) = 458$.
time = 1.70, size = 654, normalized size = 3.29

method	result

default	$-\frac{4a^3\left(-4A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\mathrm{s}\cdots}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/3*a^3*(-4*A*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*(5*A+9*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A+5*B)*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(5*A*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2)))*sin(1/2*d*x+1/2*c)^2+5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-3*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*B*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d
*x+1/2*c)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.65, size = 215, normalized size = 1.08

$$-\frac{2\left(5i\sqrt{2}(A+B)a^3\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(A+B)a^3\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}(A-B)a^3\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}(A-B)a^3\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(Aa^3\cos(dx+c)^2+3(A+3B)a^3\cos(dx+c)+Ba^3\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{3d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] -2/3*(5*I*sqrt(2)*(A + B)*a^3*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(A + B)*a^3*cos(d*x + c)*weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(A - B)*a^3*c
os(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c))) + 3*I*sqrt(2)*(A - B)*a^3*cos(d*x + c)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (A*a^3*c
os(d*x + c)^2 + 3*(A + 3*B)*a^3*cos(d*x + c) + B*a^3)*sin(d*x + c)/sqrt(cos
(d*x + c)))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int \frac{A}{\sec^{\frac{3}{2}}{\left(c + dx \right)}}\, dx + \int \frac{3A}{\sqrt{\sec{\left(c + dx \right)}}}\, dx + \int 3A\sqrt{\sec{\left(c + dx \right)}}\, dx + \int A\sec^{\frac{3}{2}}{\left(c + dx \right)}\, dx + \int \frac{B}{\sqrt{\sec{\left(c + dx \right)}}}\, dx + \int 3B\sqrt{\sec{\left(c + dx \right)}}\, dx + \int 3B\sec^{\frac{3}{2}}{\left(c + dx \right)}\, dx + \int B\sec^{\frac{5}{2}}{\left(c + dx \right)}\, dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] a**3*(Integral(A/sec(c + d*x)**(3/2), x) + Integral(3*A/sqrt(sec(c + d*x)),
 x) + Integral(3*A*sqrt(sec(c + d*x)), x) + Integral(A*sec(c + d*x)**(3/2),
 x) + Integral(B/sqrt(sec(c + d*x)), x) + Integral(3*B*sqrt(sec(c + d*x)),
x) + Integral(3*B*sec(c + d*x)**(3/2), x) + Integral(B*sec(c + d*x)**(5/2),
 x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(3/2), x
)
```

3.197 $\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=211

$$\frac{4a^3(9A+5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3(3A+5B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

```
[Out] 2/5*a*A*(a+a*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/15*(9*A+5*B)*(a^
3+a^3*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(1/2)-4/15*a^3*(6*A-5*B)*sin(d*x+
c)*sec(d*x+c)^(1/2)/d+4/5*a^3*(9*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/d+4/3*a^3*(3*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
d
```

Rubi [A]
time = 0.27, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4102, 4082, 3872, 3856, 2719, 2720}

$$-\frac{4a^3(6A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(9A+5B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{15d\sqrt{\sec(c+dx)}}+\frac{4a^3(3A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^3(9A+5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aA\sin(c+dx)(a\sec(c+dx)+a)^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (4*a^3*(9*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (4*a^3*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/
2, 2]*Sqrt[Sec[c + d*x]])/(3*d) - (4*a^3*(6*A - 5*B)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(15*d) + (2*a*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c
+ d*x]^(3/2)) + (2*(9*A + 5*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(15*d
*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2}{5}\int \frac{(a+a\sec(c+$$

$$= \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(9A+5B)\left(a^3+\right.}{15d\sqrt{}}$$

$$= -\frac{4a^3(6A-5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d}+\frac{2aA(a+a\,\mathrm{s}}{5d}$$

$$= -\frac{4a^3(6A-5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d}+\frac{2aA(a+a\,\mathrm{s}}{5d}$$

$$= -\frac{4a^3(6A-5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d}+\frac{2aA(a+a\,\mathrm{s}}{5d}$$

$$= \frac{4a^3(9A+5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+d}}{5d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.80, size = 207, normalized size = 0.98

$$\frac{a^3 e^{-idx} \sqrt{\sec(c+dx)} (\cos(dx) + i\sin(dx)) \left(216iA\cos(c+dx) + 120iB\cos(c+dx) + 40(3A+5B)\sqrt{\cos(c+dx)}\, F\left(\frac{1}{2}(c+dx)\middle|2\right) - 8i(9A+5B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\, {}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right) + 3A\sin(c+dx) + 60B\sin(c+dx) + 30A\sin(2(c+dx)) + 10B\sin(2(c+dx)) + 3A\sin(3(c+dx))\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),
x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*((216*I)*A*Cos[c + d*x] + (
120*I)*B*Cos[c + d*x] + 40*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2] - (8*I)*(9*A + 5*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*
Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 3*A*Sin[c + d*x] +
 60*B*Sin[c + d*x] + 30*A*Sin[2*(c + d*x)] + 10*B*Sin[2*(c + d*x)] + 3*A*Si
n[3*(c + d*x)]))/(30*d*E^(I*d*x))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 518 vs. 2(239) = 478.
time = 1.57, size = 519, normalized size = 2.46

method	result
default	$-\frac{4a^3\left(-12A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\mathrm{si}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/15*a^3*(-12*A*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(21*A+5*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)-2*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(9*A+10*B)*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)-27*A*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+25*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-15*B*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*
```

```
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2
-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 197, normalized size = 0.93

$$-\frac{2\left(5i\sqrt{2}(3A+5B)a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(3A+5B)a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}(9A+5B)a^3\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}(9A+5B)a^3\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{(3Aa^3\cos(dx+c)^2+5(3A+B)a^3\cos(dx+c)+15Ba^3)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] -2/15*(5*I*sqrt(2)*(3*A + 5*B)*a^3*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) - 5*I*sqrt(2)*(3*A + 5*B)*a^3*weierstrassPInverse(-4, 0,
cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(9*A + 5*B)*a^3*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*s
qrt(2)*(9*A + 5*B)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) - I*sin(d*x + c))) - (3*A*a^3*cos(d*x + c)^2 + 5*(3*A + B)*a^3*c
os(d*x + c) + 15*B*a^3)*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^3\left(\int\frac{A}{\sec^{\frac{5}{2}}(c+dx)}\,dx+\int\frac{3A}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{3A}{\sqrt{\sec(c+dx)}}\,dx+\int A\sqrt{\sec(c+dx)}\,dx+\int\frac{B}{\sec^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{3B}{\sqrt{\sec(c+dx)}}\,dx+\int 3B\sqrt{\sec(c+dx)}\,dx+\int B\sec^{\frac{3}{2}}(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] a**3*(Integral(A/sec(c + d*x)**(5/2), x) + Integral(3*A/sec(c + d*x)**(3/2)
, x) + Integral(3*A/sqrt(sec(c + d*x)), x) + Integral(A*sqrt(sec(c + d*x)),
 x) + Integral(B/sec(c + d*x)**(3/2), x) + Integral(3*B/sqrt(sec(c + d*x)),
 x) + Integral(3*B*sqrt(sec(c + d*x)), x) + Integral(B*sec(c + d*x)**(3/2),
 x))
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^3}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(5/2), x
)
```

3.198 $\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=211

$$\frac{4a^3(7A+9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{4a^3(13A+21B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\right.$$

```
[Out] 2/7*a*A*(a+a*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/35*(11*A+7*B)*(a
^3+a^3*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(3/2)+4/105*a^3*(41*A+42*B)*sin(
d*x+c)/d/sec(d*x+c)^(1/2)+4/5*a^3*(7*A+9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d+4/21*a^3*(13*A+21*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/d
```

Rubi [A]

time = 0.28, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4102, 4081, 3872, 3856, 2719, 2720}

$$\frac{2(11A+7B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{4a^3(41A+42B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{4a^3(13A+21B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(7A+9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aA\sin(c+dx)(a\sec(c+dx)+a)^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (4*a^3*(7*A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (4*a^3*(13*A + 21*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x
)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(41*A + 42*B)*Sin[c + d*x])/(10
5*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*
Sec[c + d*x]^(5/2)) + (2*(11*A + 7*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x]
)/(35*d*Sec[c + d*x]^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+a\sec(c+
\\
&= \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(11A+7B)\left(a^3+
\\
&= \frac{4a^3(41A+42B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2aA(a+a\sec(c+dx))^2\sin
\\
&= \frac{4a^3(41A+42B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2aA(a+a\sec(c+dx))^2\sin
\\
&= \frac{4a^3(41A+42B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2aA(a+a\sec(c+dx))^2\sin
\\
&= \frac{4a^3(7A+9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.63, size = 194, normalized size = 0.92

$$\frac{a^3 e^{-idx}\sqrt{\sec(c+dx)}\,(\cos(dx)+i\sin(dx))\left(40(13A+21B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-56i(7A+9B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(168i(7A+9B)+5(107A+84B)\sin(c+dx)+42(3A+B)\sin(2(c+dx))+15A\sin(3(c+dx)))\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),
x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(40*(13*A + 21*B)*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2] - (56*I)*(7*A + 9*B)*E^(I*(c + d*x))*Sq
rt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c +
 d*x))] + Cos[c + d*x]*((168*I)*(7*A + 9*B) + 5*(107*A + 84*B)*Sin[c + d*x]
 + 42*(3*A + B)*Sin[2*(c + d*x)] + 15*A*Sin[3*(c + d*x)])))/(210*d*E^(I*d*x
))
```

Maple [A]
time = 1.10, size = 385, normalized size = 1.82

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^3\left(120A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-432A-84B)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(120*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-432*A-84*B)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(602*A+294*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2
08*A-126*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-147*A*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)+65*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-189*B*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)+105*B*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.81, size = 211, normalized size = 1.00

$$-\frac{2\left(5i\sqrt{2}\,(13\,A+21\,B)a^{3}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))-5i\sqrt{2}\,(13\,A+21\,B)a^{3}\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))-21i\sqrt{2}\,(7\,A+9\,B)a^{3}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+21i\sqrt{2}\,(7\,A+9\,B)a^{3}\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))-\frac{\left(15\,Aa^{3}\cos(dx+c)^{3}+21\,(3\,A+B)a^{3}\cos(dx+c)^{2}+5\,(26\,A+21\,B)a^{3}\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] -2/105*(5*I*sqrt(2)*(13*A + 21*B)*a^3*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(13*A + 21*B)*a^3*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(2)*(7*A + 9*B)*a^3*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
 21*I*sqrt(2)*(7*A + 9*B)*a^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c))) - (15*A*a^3*cos(d*x + c)^3 + 21*(3*A +
 B)*a^3*cos(d*x + c)^2 + 5*(26*A + 21*B)*a^3*cos(d*x + c))*sin(d*x + c)/sqr
t(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{3}\left(\int\frac{A}{\sec^{\frac{7}{2}}\left(c+dx\right)}\,dx+\int\frac{3A}{\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx+\int\frac{3A}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int\frac{A}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int\frac{B}{\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx+\int\frac{3B}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx+\int\frac{3B}{\sqrt{\sec\left(c+dx\right)}}\,dx+\int B\sqrt{\sec\left(c+dx\right)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] a**3*(Integral(A/sec(c + d*x)**(7/2), x) + Integral(3*A/sec(c + d*x)**(5/2)
, x) + Integral(3*A/sec(c + d*x)**(3/2), x) + Integral(A/sqrt(sec(c + d*x))
, x) + Integral(B/sec(c + d*x)**(5/2), x) + Integral(3*B/sec(c + d*x)**(3/2
), x) + Integral(3*B/sqrt(sec(c + d*x)), x) + Integral(B*sqrt(sec(c + d*x))
, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(7/2), x
)
```

3.199 $\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=244

$$\frac{4a^3(17A+21B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{4a^3(11A+13B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{21d}$$

```
[Out] 4/105*a^3*(23*A+24*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/9*a*A*(a+a*sec(d*x+c)
)^2*sin(d*x+c)/d/sec(d*x+c)^(7/2)+2/63*(13*A+9*B)*(a^3+a^3*sec(d*x+c))*sin(
d*x+c)/d/sec(d*x+c)^(5/2)+4/21*a^3*(11*A+13*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2
)+4/15*a^3*(17*A+21*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/21*
a^3*(11*A+13*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.30, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4102, 4081, 3872, 3854, 3856, 2720, 2719}

$$\frac{4a^3(23A+24B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(13A+9B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{4a^3(11A+13B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{4a^3(11A+13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^3(17A+21B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2aA\sin(c+dx)(a\sec(c+dx)+a)^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),x]
```

```
[Out] (4*a^3*(17*A + 21*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(15*d) + (4*a^3*(11*A + 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (4*a^3*(23*A + 24*B)*Sin[c + d*x])/
(105*d*Sec[c + d*x]^(3/2)) + (4*a^3*(11*A + 13*B)*Sin[c + d*x])/(21*d*Sqrt[
Sec[c + d*x]]) + (2*a*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(9*d*Sec[c + d
*x]^(7/2)) + (2*(13*A + 9*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(63*d*S
ec[c + d*x]^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx &= \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+a\sec(c+
\\
&= \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(13A+9B)\left(a^3+\right.}{63d\,\mathrm{se}}
\\
&= \frac{4a^3(23A+24B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+a\sec(c+dx))^2\sin}{9d\sec^{\frac{7}{2}}(c+dx)}
\\
&= \frac{4a^3(23A+24B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+a\sec(c+dx))^2\sin}{9d\sec^{\frac{7}{2}}(c+dx)}
\\
&= \frac{4a^3(23A+24B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)} + \frac{4a^3(11A+13B)\sin(c+dx}{21d\sqrt{\sec(c+dx)}}
\\
&= \frac{4a^3(17A+21B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}{15d}
\\
&= \frac{4a^3(17A+21B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}{15d}
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.92, size = 196, normalized size = 0.80

$$\frac{a^3\sqrt{\sec(c+dx)}\left(240(11A+13B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-112i(17A+21B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(5712iA+7056iB+30(97A+107B)\sin(c+dx)+14(73A+54B)\sin(2(c+dx))+270A\sin(3(c+dx))+90B\sin(3(c+dx))+35A\sin(4(c+dx)))\right)}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),
x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(240*(11*A + 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2] - (112*I)*(17*A + 21*B)*E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + Cos[c + d*
x]*((5712*I)*A + (7056*I)*B + 30*(97*A + 107*B)*Sin[c + d*x] + 14*(73*A + 5
4*B)*Sin[2*(c + d*x)] + 270*A*Sin[3*(c + d*x)] + 90*B*Sin[3*(c + d*x)] + 35
*A*Sin[4*(c + d*x)])))/(1260*d)
```

Maple [A]
time = 1.27, size = 413, normalized size = 1.69

method	result

default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^3 \left(-560A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(2200A+360B)\left(\sin^8\left(\frac{dx}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(-560*A*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(2200*A+360*B)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-3412*A-1296*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(2702*A+1806*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-738*A-624*B)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+165*A*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-357*A
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)+195*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-441*B*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*
d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.76, size = 231, normalized size = 0.95

$$-\frac{2\left(15i\sqrt{2}\,(11A+13B)a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-15i\sqrt{2}\,(11A+13B)a^3\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-21i\sqrt{2}\,(17A+21B)a^3\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21i\sqrt{2}\,(17A+21B)a^3\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(35Aa^3\cos(dx+c)^4+45(3A+B)a^3\cos(dx+c)^3+7(34A+27B)a^3\cos(dx+c)^2+30(11A+13B)a^3\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] -2/315*(15*I*sqrt(2)*(11*A + 13*B)*a^3*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) - 15*I*sqrt(2)*(11*A + 13*B)*a^3*weierstrassPInverse(
```

```
-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(2)*(17*A + 21*B)*a^3*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
)) + 21*I*sqrt(2)*(17*A + 21*B)*a^3*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (35*A*a^3*cos(d*x + c)^4 + 45*
(3*A + B)*a^3*cos(d*x + c)^3 + 7*(34*A + 27*B)*a^3*cos(d*x + c)^2 + 30*(11*
A + 13*B)*a^3*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(9/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(9/2), x
)
```

3.200 $\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=277

$$\frac{4a^3(15A+17B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{4a^3(105A+121B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{231d}$$

```
[Out] 20/693*a^3*(21*A+22*B)*sin(d*x+c)/d/sec(d*x+c)^(5/2)+4/45*a^3*(15*A+17*B)*s
in(d*x+c)/d/sec(d*x+c)^(3/2)+2/11*a*A*(a+a*sec(d*x+c))^2*sin(d*x+c)/d/sec(d
*x+c)^(9/2)+2/99*(15*A+11*B)*(a^3+a^3*sec(d*x+c))*sin(d*x+c)/d/sec(d*x+c)^(
7/2)+4/231*a^3*(105*A+121*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+4/15*a^3*(15*A+1
7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+4/231*a^3*(105*A+121*B)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*
c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.32, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4102, 4081, 3872, 3854, 3856, 2719, 2720}

$$\frac{4a^3(15A+17B)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{20a^3(21A+22B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)}+\frac{2(15A+11B)\sin(c+dx)\left(a^3\sec(c+dx)+a^3\right)}{99d\sec^{\frac{7}{2}}(c+dx)}+\frac{4a^3(105A+121B)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{4a^3(105A+121B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{4a^3(15A+17B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2aA\sin(c+dx)(a\sec(c+dx)+a)^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(11/2),x]
```

```
[Out] (4*a^3*(15*A + 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(15*d) + (4*a^3*(105*A + 121*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(231*d) + (20*a^3*(21*A + 22*B)*Sin[c + d*
x])/(693*d*Sec[c + d*x]^(5/2)) + (4*a^3*(15*A + 17*B)*Sin[c + d*x])/(45*d*S
ec[c + d*x]^(3/2)) + (4*a^3*(105*A + 121*B)*Sin[c + d*x])/(231*d*Sqrt[Sec[c
 + d*x]]) + (2*a*A*(a + a*Sec[c + d*x])^2*Sin[c + d*x])/(11*d*Sec[c + d*x]^
(9/2)) + (2*(15*A + 11*B)*(a^3 + a^3*Sec[c + d*x])*Sin[c + d*x])/(99*d*Sec[
c + d*x]^(7/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{11}{2}}(c+dx)}\,dx &= \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2}{11}\int \frac{(a+a\sec(c
\\
&= \frac{2aA(a+a\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2(15A+11B)\,(a^3}{99a
\\
&= \frac{20a^3(21A+22B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+a\sec(c+dx))^2 s}{11d\sec^{\frac{9}{2}}(c+dx}
\\
&= \frac{20a^3(21A+22B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+a\sec(c+dx))^2 s}{11d\sec^{\frac{9}{2}}(c+dx}
\\
&= \frac{20a^3(21A+22B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^3(15A+17B)\sin(c+}{45d\sec^{\frac{3}{2}}(c+dx)}
\\
&= \frac{20a^3(21A+22B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)} + \frac{4a^3(15A+17B)\sin(c+}{45d\sec^{\frac{3}{2}}(c+dx)}
\\
&= \frac{4a^3(15A+17B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{15d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.65, size = 239, normalized size = 0.86

$$
\frac{a^3e^{-idx}\sqrt{\sec(c+dx)}\,(\cos(dx)+i\sin(dx))\left(480(105A+121B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)-2464i(15A+17B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+\cos(c+dx)(110880iA+125664iB+30(1953A+2134B)\sin(c+dx)+308(75A+73B)\sin(2(c+dx))+8505A\sin(3(c+dx))+5940B\sin(3(c+dx))+2310A\sin(4(c+dx))+770B\sin(4(c+dx))+315A\sin(5(c+dx)))\right)}{27720d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(11/2)
,x]
```

```
[Out] (a^3*Sqrt[Sec[c + d*x]]*(Cos[d*x] + I*Sin[d*x])*(480*(105*A + 121*B)*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2] - (2464*I)*(15*A + 17*B)*E^(I*(c + d
*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*
I)*(c + d*x))] + Cos[c + d*x]*((110880*I)*A + (125664*I)*B + 30*(1953*A + 2
134*B)*Sin[c + d*x] + 308*(75*A + 73*B)*Sin[2*(c + d*x)] + 8505*A*Sin[3*(c
+ d*x)] + 5940*B*Sin[3*(c + d*x)] + 2310*A*Sin[4*(c + d*x)] + 770*B*Sin[4*(
c + d*x)] + 315*A*Sin[5*(c + d*x)])))/(27720*d*E^(I*d*x))
```

Maple [A]
time = 1.15, size = 441, normalized size = 1.59

method	result

default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^3\left(10080A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-43680A-6160B)\left(\sin^{10}\left(\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] -4/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^3*(10080*
A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12+(-43680*A-6160*B)*sin(1/2*d*x+1/
2*c)^10*cos(1/2*d*x+1/2*c)+(77280*A+24200*B)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d
*x+1/2*c)+(-72240*A-37532*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(39270
*A+29722*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-8820*A-8118*B)*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+1575*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-3465*A*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)+1815*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)-3927*B*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d
*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorith
m="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(11/2),
x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 251, normalized size = 0.91

$$-\frac{2\left(15i\sqrt{2}(105A+121B)a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-15i\sqrt{2}(105A+121B)a^3\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-231i\sqrt{2}(15A+17B)a^3\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+231i\sqrt{2}(15A+17B)a^3\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{\left(315Aa^3\cos(dx+c)^5+385(3A+B)a^3\cos(dx+c)^4+135(14A+11B)a^3\cos(dx+c)^3+154(15A+17B)a^3\cos(dx+c)^2+30(105A+121B)a^3\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}\right)}{3465d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorith
m="fricas")
```

```
[Out] -2/3465*(15*I*sqrt(2)*(105*A + 121*B)*a^3*weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c)) - 15*I*sqrt(2)*(105*A + 121*B)*a^3*weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 231*I*sqrt(2)*(15*A + 17*B)*a^
3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) + 231*I*sqrt(2)*(15*A + 17*B)*a^3*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (315*A*a^3*cos(d*x + c)
^5 + 385*(3*A + B)*a^3*cos(d*x + c)^4 + 135*(14*A + 11*B)*a^3*cos(d*x + c)^
3 + 154*(15*A + 17*B)*a^3*cos(d*x + c)^2 + 30*(105*A + 121*B)*a^3*cos(d*x +
 c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorith
m="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^3/sec(d*x + c)^(11/2),
x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^3}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{11/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(11/2),x
)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^3)/(1/cos(c + d*x))^(11/2),
x)
```

3.201 $\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=229

$$\frac{3(5A-7B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad}+\frac{5(A-B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{3ad}$$

```
[Out] 5/3*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d-1/5*(5*A-7*B)*sec(d*x+c)^(5/2)*si
n(d*x+c)/a/d+(A-B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))-3/5*(5*A-
7*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d+3/5*(5*A-7*B)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/a/d+5/3*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)/a/d
```

Rubi [A]
time = 0.16, antiderivative size = 229, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4104, 3872, 3853, 3856, 2720, 2719}

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{d(a\sec(c+dx)+a)}-\frac{(5A-7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5ad}+\frac{5(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}-\frac{3(5A-7B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5ad}+\frac{5(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(5A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*(5*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*a*d) + (5*(A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*a*d) - (3*(5*A - 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
5*a*d) + (5*(A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) - ((5*A - 7*B)
*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*a*d) + ((A - B)*Sec[c + d*x]^(7/2)*Sin
[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx = \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sec^{\frac{5}{2}}(c+dx)\left(\frac{5}{2}a(A-B)\right.$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(5A-7B)\int \sec^{\frac{7}{2}}(c+dx)\,dx}{2a}$$

$$= \frac{5(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3ad} - \frac{(5A-7B)\sec^{\frac{5}{2}}(c+dx)\sin}{5ad}$$

$$= -\frac{3(5A-7B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5ad} + \frac{5(A-B)\sec^{\frac{3}{2}}(c+dx}{3ad}$$

$$= \frac{5(A-B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{3ad} - \frac{3(5A}{}$$

$$= \frac{3(5A-7B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{5ad} + \frac{5(A}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.71, size = 814, normalized size = 3.55

$$-\frac{Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\csc\left(\frac{c}{2}\right)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\sec\left(\frac{c}{2}\right)(A+B\sec(c+dx))}{\sqrt{2}\,d(B+A\cos(c+dx))(a+a\sec(c+dx))}+\frac{7Be^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\csc\left(\frac{c}{2}\right)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\sec\left(\frac{c}{2}\right)(A+B\sec(c+dx))}{5\sqrt{2}\,d(B+A\cos(c+dx))(a+a\sec(c+dx))}+\frac{5A\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\cos(c+dx)}\csc\left(\frac{c}{2}\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)\sec\left(\frac{c}{2}\right)\sqrt{\sec(c+dx)}(A+B\sec(c+dx))\sin(c)}{3d(B+A\cos(c+dx))(a+a\sec(c+dx))}-\frac{5B\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\cos(c+dx)}\csc\left(\frac{c}{2}\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)\sec\left(\frac{c}{2}\right)\sqrt{\sec(c+dx)}(A+B\sec(c+dx))\sin(c)}{3d(B+A\cos(c+dx))(a+a\sec(c+dx))}+\frac{\cos^2\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\sec(c+dx)}(A+B\sec(c+dx))\left(\frac{3(-5A+7B)\cos(dx)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)}{5d}-\frac{(-A+B)\sec\left(\frac{c}{2}\right)\sec(c)\left(-\sin\left(\frac{c}{2}\right)+5\sin\left(\frac{3c}{2}\right)\right)}{3d}-\frac{2\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-A\sin\left(\frac{dx}{2}\right)+B\sin\left(\frac{dx}{2}\right)\right)}{d}+\frac{4B\sec(c)\sec^2(c+dx)\sin(dx)}{5d}+\frac{4\sec(c)\sec(c+dx)(3B\sin(c)+5A\sin(dx)-5B\sin(dx))}{15d}\right)}{(B+A\cos(c+dx))(a+a\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] -((A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E
^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*
(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*x]))/(Sqrt[2]*d*E^(I*d*x)*(B + A*Cos
[c + d*x])*(a + a*Sec[c + d*x]))) + (7*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)
*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*(
-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hyperge
ometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*(A + B*Sec[c + d*
x]))/(5*Sqrt[2]*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) + (5
*A*Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2,
2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos
[c + d*x])*(a + a*Sec[c + d*x])) - (5*B*Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d
*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sqrt[Sec[c + d*x]]*(A + B*
Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) + (Co
s[c/2 + (d*x)/2]^2*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x])*((3*(-5*A + 7*B)
*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) - ((-A + B)*Sec[c/2]*Sec[c]*(-Sin[c/2] +
 5*Sin[(3*c)/2]))/(3*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-(A*Sin[(d*x)/2])
 + B*Sin[(d*x)/2]))/d + (4*B*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(5*d) + (4*Sec
[c]*Sec[c + d*x]*(3*B*Sin[c] + 5*A*Sin[d*x] - 5*B*Sin[d*x]))/(15*d)))/((B +
 A*Cos[c + d*x])*(a + a*Sec[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 778 vs. $2(255) = 510$.
time = 3.76, size = 779, normalized size = 3.40

method	result	size
default	Expression too large to display	779

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*((2*A-2*B)*(-1
/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
```

```
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(A-B)*(cos(1/2*d*x+1/2*c)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)+2/5*B/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+
6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos
(1/2*d*x+1/2*c)-12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/2*c)^4-24*sin(1/2
*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*sin(1/2*d*x+1/
2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(-2*A+2*B)/sin(1/2*d
*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.56, size = 339, normalized size = 1.48

$$-\frac{25\left(\sqrt{2}\,(i\,A-i\,B)\cos\left(dx+c\right)^{3}+\sqrt{2}\,(i\,A-i\,B)\cos\left(dx+c\right)^{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+25\left(\sqrt{2}\,(-i\,A+i\,B)\cos\left(dx+c\right)^{3}+\sqrt{2}\,(-i\,A+i\,B)\cos\left(dx+c\right)^{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+9\left(\sqrt{2}\,(-5i\,A+7i\,B)\cos\left(dx+c\right)^{3}+\sqrt{2}\,(-5i\,A+7i\,B)\cos\left(dx+c\right)^{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+9\left(\sqrt{2}\,(5i\,A-7i\,B)\cos\left(dx+c\right)^{3}+\sqrt{2}\,(5i\,A-7i\,B)\cos\left(dx+c\right)^{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+\frac{2\left(9\,(5\,A-7\,B)\cos(dx+c)^{3}+2\,(10\,A-19\,B)\cos(dx+c)^{2}-2\,(5\,A-2\,B)\cos(dx+c)-6\,B\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{30\left(ad\cos\left(dx+c\right)^{3}+ad\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] -1/30*(25*(sqrt(2)*(I*A - I*B)*cos(d*x + c)^3 + sqrt(2)*(I*A - I*B)*cos(d*x
 + c)^2)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 25*(sq
rt(2)*(-I*A + I*B)*cos(d*x + c)^3 + sqrt(2)*(-I*A + I*B)*cos(d*x + c)^2)*we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 9*(sqrt(2)*(-5*I*
A + 7*I*B)*cos(d*x + c)^3 + sqrt(2)*(-5*I*A + 7*I*B)*cos(d*x + c)^2)*weiers
trassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)))
 + 9*(sqrt(2)*(5*I*A - 7*I*B)*cos(d*x + c)^3 + sqrt(2)*(5*I*A - 7*I*B)*cos(
d*x + c)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
```

```
- I*sin(d*x + c))) + 2*(9*(5*A - 7*B)*cos(d*x + c)^3 + 2*(10*A - 19*B)*cos(
d*x + c)^2 - 2*(5*A - 2*B)*cos(d*x + c) - 6*B)*sin(d*x + c)/sqrt(cos(d*x +
c)))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
giac")
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}}{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a/cos(c + d*x)),x)
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a/cos(c + d*x)), x)
```

3.202 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=192

$$-\frac{3(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}-\frac{(3A-5B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\text{se}}}{3ad}$$

```
[Out] -1/3*(3*A-5*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d+(A-B)*sec(d*x+c)^(5/2)*sin(d
*x+c)/d/(a+a*sec(d*x+c))+3*(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d-3*(A-B)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d-1/3*(3*A-5*B)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A]

time = 0.14, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4104, 3872, 3853, 3856, 2719, 2720}

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{d(a\sec(c+dx)+a)}-\frac{(3A-5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3ad}+\frac{3(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}-\frac{(3A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(a*d) - ((3*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*a*d) + (3*(A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) -
((3*A - 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*d) + ((A - B)*Sec[c + d*
x]^(5/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx = \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(A-B)-\right.}{a}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(3A-5B)\int \sec^{\frac{5}{2}}(c+dx)\,dx}{2a}$$

$$= \frac{3(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} - \frac{(3A-5B)\sec^{\frac{3}{2}}(c+dx)\sin}{3ad}$$

$$= \frac{3(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} - \frac{(3A-5B)\sec^{\frac{3}{2}}(c+dx)\sin}{3ad}$$

$$= -\frac{3(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(3A}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 3.62, size = 372, normalized size = 1.94

$$\frac{e^{-\frac{1}{2}i(c+dx)}\cos\left(\frac{1}{2}(c+dx)\right)\left(-\left((3A-5B)\left(1+e^{i(c+dx)}+e^{2i(c+dx)}+e^{3i(c+dx)}\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\right)+i\left(-3A+5B-6Ae^{i(c+dx)}+8Be^{i(c+dx)}-12Ae^{2i(c+dx)}+10Be^{2i(c+dx)}-6Ae^{3i(c+dx)}+4Be^{3i(c+dx)}-9Ae^{4i(c+dx)}+9Be^{4i(c+dx)}+3(A-B)e^{i(c+dx)}\sqrt{1+e^{2i(c+dx)}}\left(1+e^{i(c+dx)}+e^{2i(c+dx)}+e^{3i(c+dx)}\right){}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\right)\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{3ad\left(1+e^{2i(c+dx)}\right)(B+A\cos(c+dx))(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]*(-((3*A - 5*B)*(1 + E^(I*(c + d*x)) + E^((2*I)*(c + d*x))
 + E^((3*I)*(c + d*x)))*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]) + I*(
-3*A + 5*B - 6*A*E^(I*(c + d*x)) + 8*B*E^(I*(c + d*x)) - 12*A*E^((2*I)*(c +
 d*x)) + 10*B*E^((2*I)*(c + d*x)) - 6*A*E^((3*I)*(c + d*x)) + 4*B*E^((3*I)*
(c + d*x)) - 9*A*E^((4*I)*(c + d*x)) + 9*B*E^((4*I)*(c + d*x)) + 3*(A - B)*
E^(I*(c + d*x))*Sqrt[1 + E^((2*I)*(c + d*x))]*(1 + E^(I*(c + d*x)) + E^((2*
I)*(c + d*x)) + E^((3*I)*(c + d*x)))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((
2*I)*(c + d*x))]))*Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(3*a*d*E^((I/2)
*(c + d*x))*(1 + E^((2*I)*(c + d*x)))*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x
]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. 2(226) = 452.
time = 2.82, size = 466, normalized size = 2.43

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2B\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.}}{6\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(2*B*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))+(-A+B)*(cos(1/2*d*x+1/2*c)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)+(2*A-2*B)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
```

```
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.58, size = 308, normalized size = 1.60

$$\frac{\left(\sqrt{2}\,(3i\,A-5i\,B)\cos(dx+c)^2+\sqrt{2}\,(3i\,A-5i\,B)\cos(dx+c)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\left(\sqrt{2}\,(-3i\,A+5i\,B)\cos(dx+c)^2+\sqrt{2}\,(-3i\,A+5i\,B)\cos(dx+c)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-9\left(\sqrt{2}\,(i\,A-i\,B)\cos(dx+c)^2+\sqrt{2}\,(i\,A-i\,B)\cos(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-9\left(\sqrt{2}\,(-i\,A+i\,B)\cos(dx+c)^2+\sqrt{2}\,(-i\,A+i\,B)\cos(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+\frac{2\left(9\,(A-B)\cos(dx+c)^2+2\,(3\,A-2\,B)\cos(dx+c)+2\,B\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/6*((sqrt(2)*(3*I*A - 5*I*B)*cos(d*x + c)^2 + sqrt(2)*(3*I*A - 5*I*B)*cos(
d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt
(2)*(-3*I*A + 5*I*B)*cos(d*x + c)^2 + sqrt(2)*(-3*I*A + 5*I*B)*cos(d*x + c)
)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*(sqrt(2)*(I
*A - I*B)*cos(d*x + c)^2 + sqrt(2)*(I*A - I*B)*cos(d*x + c))*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*(sq
rt(2)*(-I*A + I*B)*cos(d*x + c)^2 + sqrt(2)*(-I*A + I*B)*cos(d*x + c))*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
)) + 2*(9*(A - B)*cos(d*x + c)^2 + 2*(3*A - 2*B)*cos(d*x + c) + 2*B)*sin(d*
x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{a+\frac{a}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x)),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x)), x)
```

3.203 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=153

$$\frac{(A-3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}+\frac{(A-B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}{ad}$$

```
[Out] (A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))-(A-3*B)*sin(d*x+c)*sec
(d*x+c)^(1/2)/a/d+(A-3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+
(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A]
time = 0.13, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4104, 3872, 3856, 2720, 3853, 2719}

$$\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d(a\sec(c+dx)+a)}-\frac{(A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad}+\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) + ((A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(a*d) - ((A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*d) + ((A - B
)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx = \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \sqrt{\sec(c+dx)}\,\left(\frac{1}{2}a(A-B\right.}{}$$

$$= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(A-3B)\int\sec^{\frac{3}{2}}(c+dx)\,dx}{2a}$$

$$= -\frac{(A-3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{ad} + \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{si}}{d(a+a\sec(c+d}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(A-3}{}$$

$$= \frac{(A-3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(A-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 4.67, size = 420, normalized size = 2.75

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)(A+B\sec(c+dx))\left(-2\sqrt{2}\,Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)+6\sqrt{2}\,Be^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\,\sqrt{1+e^{2i(c+dx)}}\,\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)+12A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}-12B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}-6\sqrt{\sec(c+dx)}\,\left(2(A-3B)\cos(dx)\csc(c)+2(-A+B)\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{6ad(B+A\cos(c+dx))(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(A + B*Sec[c + d*x])*((-2*Sqrt[2]*A*Sqrt[E^(I*(c + d*x)
)/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[
1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2
F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + (6*Sqrt[2]*B*Sqrt[E^(
I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c
]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hype
rgeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + 12*A*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - 12*B*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - 6*Sqrt[Sec[c + d*x
]]*(2*(A - 3*B)*Cos[d*x]*Csc[c] + 2*(-A + B)*Tan[(c + d*x)/2])))/(6*a*d*(B
+ A*Cos[c + d*x])*(1 + Sec[c + d*x]))
```

Maple [A]
time = 1.67, size = 318, normalized size = 2.08

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(-cos(1/2*d*x+
1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A-3*B)*sin(1/2*d*x+1/2*c)^4-(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A-5*B)*sin(1/2*d*x+1/2*c)^
2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.45, size = 248, normalized size = 1.62

$$\frac{\left(\sqrt{2}\,(-i\,A+i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(-i\,A+i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\left(\sqrt{2}\,(i\,A-i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(i\,A-i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+\left(\sqrt{2}\,(i\,A-3i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(i\,A-3i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+\left(\sqrt{2}\,(-i\,A+3i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(-i\,A+3i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))-\frac{2\,((A-3\,B)\cos(dx+c)-2\,B)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{2\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/2*((sqrt(2)*(-I*A + I*B)*cos(d*x + c) + sqrt(2)*(-I*A + I*B))*weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*(I*A - I*B)*cos(d
*x + c) + sqrt(2)*(I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*
sin(d*x + c)) + (sqrt(2)*(I*A - 3*I*B)*cos(d*x + c) + sqrt(2)*(I*A - 3*I*B)
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) + (sqrt(2)*(-I*A + 3*I*B)*cos(d*x + c) + sqrt(2)*(-I*A + 3*I*B))*w
eierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x +
 c))) - 2*((A - 3*B)*cos(d*x + c) - 2*B)*sin(d*x + c)/sqrt(cos(d*x + c)))/(
a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^{\frac{3}{2}}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx+\int \frac{B\sec^{\frac{5}{2}}\left(c+dx\right)}{\sec\left(c+dx\right)+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*sec(c + d*x)**(3/2)/(sec(c + d*x) + 1), x) + Integral(B*sec(c +
 d*x)**(5/2)/(sec(c + d*x) + 1), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{a+\frac{a}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x)),x)
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x)), x)
```

3.204 $\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=123

$$-\frac{(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}+\frac{(A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}{ad}$$

```
[Out] (A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))-(A-B)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d+(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 123, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4104, 3872, 3856, 2719, 2720}

$$\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}+\frac{(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] -(((A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d)) + ((A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(a*d) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c
+ d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx &= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))}+\frac{\int \frac{-\frac{1}{2}a(A-B)+\frac{1}{2}a(A+B)\sec(c+d}{\sqrt{\sec(c+dx)}}}{a^2}\\ &= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{(A-B)\int \frac{1}{\sqrt{\sec(c+dx)}}}{2a}\\ &= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))}-\frac{\Big((A-B)\sqrt{\cos(c+dx)}\,\sqrt{}}{}\\ &= -\frac{(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}+\frac{(A-}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 1.22, size = 200, normalized size = 1.63

$$\frac{e^{-\frac{1}{2}i(4c+dx)}(-1+e^{2ic})\left(-3i(A+B)\left(1+e^{i(c+dx)}\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(A-B)\left(-3\left(1+e^{2i(c+dx)}\right)+e^{i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\right)\left(\csc\left(\frac{c}{2}\right)+i\sec\left(\frac{c}{2}\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}}{24ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((-1 + E^((2*I)*c))*((-3*I)*(A + B)*(1 + E^(I*(c + d*x)))*Sqrt[Cos[c + d*x]
]*EllipticF[(c + d*x)/2, 2] + (A - B)*(-3*(1 + E^((2*I)*(c + d*x))) + E^(I*
(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeometr
```

```
ic2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))*(Csc[c/2] + I*Sec[c/2])*Sec[(c
 + d*x)/2]*Sqrt[Sec[c + d*x]])/(24*a*d*E^((I/2)*(4*c + d*x)))
```

Maple [A]
time = 1.10, size = 243, normalized size = 1.98

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(A*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+B*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+
1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.58, size = 237, normalized size = 1.93

$$\frac{2(A-B)\sqrt{\cos(dx+c)}\sin(dx+c)+\left(\sqrt{2}(-iA-iB)\cos(dx+c)+\sqrt{2}(-iA-iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(\sqrt{2}(iA+iB)\cos(dx+c)+\sqrt{2}(iA+iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+\left(\sqrt{2}(-iA+iB)\cos(dx+c)+\sqrt{2}(-iA+iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+\left(\sqrt{2}(iA-iB)\cos(dx+c)+\sqrt{2}(iA-iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/2*(2*(A - B)*sqrt(cos(d*x + c))*sin(d*x + c) + (sqrt(2)*(-I*A - I*B)*cos(
d*x + c) + sqrt(2)*(-I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) +
```

```
I*sin(d*x + c)) + (sqrt(2)*(I*A + I*B)*cos(d*x + c) + sqrt(2)*(I*A + I*B))*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (sqrt(2)*(-I*A
+ I*B)*cos(d*x + c) + sqrt(2)*(-I*A + I*B))*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + (sqrt(2)*(I*A - I*B)*c
os(d*x + c) + sqrt(2)*(I*A - I*B))*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sqrt{\sec{(c+dx)}}}{\sec{(c+dx)}+1}\,dx + \int \frac{B\sec^{\frac{3}{2}}{(c+dx)}}{\sec{(c+dx)}+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*sqrt(sec(c + d*x))/(sec(c + d*x) + 1), x) + Integral(B*sec(c +
d*x)**(3/2)/(sec(c + d*x) + 1), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{a+\frac{a}{\cos(c+d\,x)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x)),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x)), x)
```

3.205 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=128

$$\frac{(3A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}-\frac{(A-B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{ad}$$

```
[Out] -(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))+(3*A-B)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d-(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/a/d
```

Rubi [A]
time = 0.12, antiderivative size = 128, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4105, 3872, 3856, 2719, 2720}

$$-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d(a\sec(c+dx)+a)}-\frac{(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] ((3*A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a*d) - ((A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(a*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*(a + a*Sec[c +
 d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))}\,dx = -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}a(3A-B)-\frac{1}{2}a(A-B)\sec(c+d}{\sqrt{\sec(c+dx)}}}{a^2}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{(A-B)\int\sqrt{\sec(c+dx)}}{2a}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d(a+a\sec(c+dx))} - \frac{\left((A-B)\sqrt{\cos(c+dx)}\right.}{}$$

$$= \frac{(3A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{(A-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.80, size = 445, normalized size = 3.48

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left(-6\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)+2\sqrt{2}Be^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)-\frac{6\left((2A-B)\cos\left(\frac{1}{2}(c-dx)\right)+A\cos\left(\frac{1}{2}(3c+dx)\right)\right)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)}{\sqrt{\sec(c+dx)}}-12A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}+12B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}\right)(A+B\sec(c+dx))}{6ad(B+A\cos(c+dx))(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*((-6*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c +
d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x
))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^
((2*I)*(c + d*x))]))/E^(I*d*x) + (2*Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^(
```

```
(2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*
I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/
4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) - (6*((2*A - B)*Cos[(c - d*x)/2]
+ A*Cos[(3*c + d*x)/2])*Csc[c/2]*Sec[c/2]*Sec[(c + d*x)/2])/Sqrt[Sec[c + d*
x]] - 12*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
+ 12*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])*(A
+ B*Sec[c + d*x]))/(6*a*d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x]))
```

Maple [A]
time = 1.11, size = 244, normalized size = 1.91

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(A\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(s\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x
+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 241, normalized size = 1.88

$$-\frac{2(A-B)\sqrt{\cos(dx+c)}\sin(dx+c)-\left(\sqrt{2}(iA-iB)\cos(dx+c)+\sqrt{2}(iA-iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-\left(\sqrt{2}(-iA+iB)\cos(dx+c)+\sqrt{2}(-iA+iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-\left(\sqrt{2}(3iA-iB)\cos(dx+c)+\sqrt{2}(3iA-iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-\left(\sqrt{2}(-3iA+iB)\cos(dx+c)+\sqrt{2}(-3iA+iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{2(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] -1/2*(2*(A - B)*sqrt(cos(d*x + c))*sin(d*x + c) - (sqrt(2)*(I*A - I*B)*cos(
d*x + c) + sqrt(2)*(I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) - (sqrt(2)*(-I*A + I*B)*cos(d*x + c) + sqrt(2)*(-I*A + I*B))
*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - (sqrt(2)*(3*I*
A - I*B)*cos(d*x + c) + sqrt(2)*(3*I*A - I*B))*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - (sqrt(2)*(-3*I*A +
I*B)*cos(d*x + c) + sqrt(2)*(-3*I*A + I*B))*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)))))/(a*d*cos(d*x + c) + a*d
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx+\int \frac{B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] (Integral(A/(sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x) + Integral(B*sec
(c + d*x)/(sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)\,\sqrt{\frac{1}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(1/2)), x)
```

3.206 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=164

$$-\frac{3(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}+\frac{(5A-3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{3ad}$$

```
[Out] 1/3*(5*A-3*B)*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)-(A-B)*sin(d*x+c)/d/(a+a*sec(d
*x+c))/sec(d*x+c)^(1/2)-3*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/a/d+1/3*(5*A-3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A]
time = 0.14, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4105, 3872, 3854, 3856, 2720, 2719}

$$\frac{(5A-3B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)}+\frac{(5A-3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-3*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(a*d) + ((5*A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(3*a*d) + ((5*A - 3*B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]
) - ((A - B)*Sin[c + d*x])/(d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = -\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}a(5A-3B)-\frac{3}{2}a(A-B)\sec(c+dx}{\sec^{\frac{3}{2}}(c+dx)}}{a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))} + \frac{(5A-3B)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx}{2a} -$$

$$= \frac{(5A-3B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(A-B)\sin(c+dx)}{d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))} + \frac{(5A}{}$$

$$= -\frac{3(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(5A-}{3ad}$$

$$= -\frac{3(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} + \frac{(5A-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.52, size = 232, normalized size = 1.41

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)(\cos(dx)+i\sin(dx))\left(2(5A-3B)\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+3i(A-B)e^{\frac{1}{2}i(c+dx)}\left(1+e^{i(c+dx)}\right)\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+2\cos(c+dx)\left(-9i(A-B)\cos\left(\frac{1}{2}(c+dx)\right)+(5A-3B+2A\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{3ad(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(3/2)*(Cos[d*x] + I*Sin[d*x])*(2*(5*A - 3*B)
*Cos[(c + d*x)/2]*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (3*I)*(A -
 B)*E^((I/2)*(c + d*x))*(1 + E^(I*(c + d*x)))*Sqrt[1 + E^((2*I)*(c + d*x))]
*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))] + 2*Cos[c + d*x]*((
-9*I)*(A - B)*Cos[(c + d*x)/2] + (5*A - 3*B + 2*A*Cos[c + d*x])*Sin[(c + d*
x)/2])))/(3*a*d*E^(I*d*x)*(1 + Sec[c + d*x]))
```

Maple [A]
time = 1.12, size = 262, normalized size = 1.60

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{3a\cos\left(\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-3*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))-8*A*sin(1/2*d*x+1/2*c)^6+(18*A-6*B)*sin(1/2*d*x+1/2*c)^4+(-7*A
+3*B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.60, size = 260, normalized size = 1.59

$$\frac{\left(\sqrt{2}\,(-5i\,A+3i\,B)\cos(dx+c)+\sqrt{2}\,(-5i\,A+3i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\left(\sqrt{2}\,(5i\,A-3i\,B)\cos(dx+c)+\sqrt{2}\,(5i\,A-3i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-9\left(\sqrt{2}\,(i\,A-i\,B)\cos(dx+c)+\sqrt{2}\,(i\,A-i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-9\left(\sqrt{2}\,(-i\,A+i\,B)\cos(dx+c)+\sqrt{2}\,(-i\,A+i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+\frac{2\left(2\,A\cos(dx+c)^2+(5\,A-3\,B)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/6*((sqrt(2)*(-5*I*A + 3*I*B)*cos(d*x + c) + sqrt(2)*(-5*I*A + 3*I*B))*wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*(5*I*A -
3*I*B)*cos(d*x + c) + sqrt(2)*(5*I*A - 3*I*B))*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) - 9*(sqrt(2)*(I*A - I*B)*cos(d*x + c) + sqrt(
2)*(I*A - I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c))) - 9*(sqrt(2)*(-I*A + I*B)*cos(d*x + c) + sqrt(2)*(-I
*A + I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) + 2*(2*A*cos(d*x + c)^2 + (5*A - 3*B)*cos(d*x + c))*sin(d
*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec^{\frac{5}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx+\int \frac{B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)+\sec^{\frac{3}{2}}(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A/(sec(c + d*x)**(5/2) + sec(c + d*x)**(3/2)), x) + Integral(B*se
c(c + d*x)/(sec(c + d*x)**(5/2) + sec(c + d*x)**(3/2)), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(3/2)), x)
```

3.207 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=197

$$\frac{3(7A-5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad}-\frac{5(A-B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{3ad}$$

```
[Out] 1/5*(7*A-5*B)*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)-(A-B)*sin(d*x+c)/d/sec(d*x+c)
^(3/2)/(a+a*sec(d*x+c))-5/3*(A-B)*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)+3/5*(7*A-
5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d-5/3*(A-B)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d
```

Rubi [A]

time = 0.15, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4105, 3872, 3854, 3856, 2719, 2720}

$$-\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)}+\frac{(7A-5B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{5(A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{5(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(7A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (3*(7*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*a*d) - (5*(A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*a*d) + ((7*A - 5*B)*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3
/2)) - (5*(A - B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]) - ((A - B)*Sin[c
 + d*x])/(d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

```
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = -\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}+\frac{\int \frac{\frac{1}{2}a(7A-5B)-\frac{5}{2}a(A-B)\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}\,d}{a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}+\frac{(7A-5B)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx}{2a}-$$

$$= \frac{(7A-5B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{5(A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{(A-B)\sin}{d\sec^{\frac{3}{2}}(c+dx)(a}$$

$$= \frac{(7A-5B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}-\frac{5(A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}-\frac{(A-B)\sin}{d\sec^{\frac{3}{2}}(c+dx)(a}$$

$$= \frac{3(7A-5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad}-\frac{5(A-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 3.87, size = 540, normalized size = 2.74

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)(A+B\sec(c+dx))\left(-84\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)+60\sqrt{2}Be^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)-200A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}+200B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}\left(-3(51A-40B+(33A-20B)\cos(2c))\cos(dx)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)-40(A-B)\cos(2dx)\sin(2c)+12A\cos(3dx)\sin(3c)+120(A-B)\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)+12(33A-20B)\cos(c)\sin(dx)-40(A-B)\cos(2c)\sin(2dx)+12A\cos(3c)\sin(3dx)+120(A-B)\tan\left(\frac{c}{2}\right)\right)\right)}{60ad(B+A\cos(c+dx))(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(A + B*Sec[c + d*x])*((-84*Sqrt[2]*A*Sqrt[E^(I*(c + d*x
))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt
[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric
2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + (60*Sqrt[2]*B*Sqrt[E
^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc
[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hy
pergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) - 200*A*Sq
rt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + 200*B*Sqrt[
Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + Sqrt[Sec[c + d
*x]]*(-3*(51*A - 40*B + (33*A - 20*B)*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2]
- 40*(A - B)*Cos[2*d*x]*Sin[2*c] + 12*A*Cos[3*d*x]*Sin[3*c] + 120*(A - B)*S
ec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] + 12*(33*A - 20*B)*Cos[c]*Sin[d*x] -
40*(A - B)*Cos[2*c]*Sin[2*d*x] + 12*A*Cos[3*c]*Sin[3*d*x] + 120*(A - B)*Tan
[c/2])))/(60*a*d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x]))
```

Maple [A]
time = 1.18, size = 282, normalized size = 1.43

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x
+1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(25*A
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*A*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))-25*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-45*B*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))+48*A*sin(1/2*d*x+1/2*c)^8+(-56*A-40*B)*sin(1/2*d*x+1/2*c
)^6+(-30*A+90*B)*sin(1/2*d*x+1/2*c)^4+(23*A-35*B)*sin(1/2*d*x+1/2*c)^2)/a/c
os(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.63, size = 278, normalized size = 1.41

$$-\frac{25\left(\sqrt{2}\left(-iA+iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-iA+iB\right)\right)\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+25\left(\sqrt{2}\left(iA-iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(iA-iB\right)\right)\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+9\left(\sqrt{2}\left(-7iA+5iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-7iA+5iB\right)\right)\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+9\left(\sqrt{2}\left(7iA-5iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(7iA-5iB\right)\right)\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-\frac{2\left(6A\cos\left(dx+c\right)^{3}-2\left(2A-5B\right)\cos\left(dx+c\right)^{2}-25\left(A-B\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{30\left(ad\cos\left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] -1/30*(25*(sqrt(2)*(-I*A + I*B)*cos(d*x + c) + sqrt(2)*(-I*A + I*B))*weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 25*(sqrt(2)*(I*A - I*
B)*cos(d*x + c) + sqrt(2)*(I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) + 9*(sqrt(2)*(-7*I*A + 5*I*B)*cos(d*x + c) + sqrt(2)*
(-7*I*A + 5*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c))) + 9*(sqrt(2)*(7*I*A - 5*I*B)*cos(d*x + c) + sqrt(2
)*(7*I*A - 5*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) - 2*(6*A*cos(d*x + c)^3 - 2*(2*A - 5*B)*cos(d*x +
 c)^2 - 25*(A - B)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(
d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec^{\frac{7}{2}}\left(c+dx\right)+\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx+\int \frac{B\sec\left(c+dx\right)}{\sec^{\frac{7}{2}}\left(c+dx\right)+\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A/(sec(c + d*x)**(7/2) + sec(c + d*x)**(5/2)), x) + Integral(B*se
c(c + d*x)/(sec(c + d*x)**(7/2) + sec(c + d*x)**(5/2)), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(5/2)), x)
```

3.208 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=230

$$-\frac{21(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad}+\frac{5(9A-7B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{21ad}$$

```
[Out] 1/7*(9*A-7*B)*sin(d*x+c)/a/d/sec(d*x+c)^(5/2)-7/5*(A-B)*sin(d*x+c)/a/d/sec(
d*x+c)^(3/2)-(A-B)*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))+5/21*(9*A
-7*B)*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)-21/5*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/a/d+5/21*(9*A-7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/a/d
```

Rubi [A]
time = 0.18, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4105, 3872, 3854, 3856, 2720, 2719}

$$-\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)}-\frac{7(A-B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}+\frac{(9A-7B)\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)}+\frac{5(9A-7B)\sin(c+dx)}{21ad\sqrt{\sec(c+dx)}}+\frac{5(9A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21ad}-\frac{21(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-21*(A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(5*a*d) + (5*(9*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(21*a*d) + ((9*A - 7*B)*Sin[c + d*x])/(7*a*d*Sec[c + d*x]
^(5/2)) - (7*(A - B)*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) + (5*(9*A - 7
*B)*Sin[c + d*x])/(21*a*d*Sqrt[Sec[c + d*x]]) - ((A - B)*Sin[c + d*x])/(d*S
ec[c + d*x]^(5/2)*(a + a*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = -\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}a(9A-7B)-\frac{7}{2}a(A-B)\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)} d}{a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{d\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))} + \frac{(9A-7B)\int \frac{1}{\sec^{\frac{7}{2}}(c+dx)}\,dx}{2a} - $$

$$= \frac{(9A-7B)\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)} - \frac{7(A-B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{(A-B)\,\mathrm{si}}{d\sec^{\frac{5}{2}}(c+dx)(a}$$

$$= \frac{(9A-7B)\sin(c+dx)}{7ad\sec^{\frac{5}{2}}(c+dx)} - \frac{7(A-B)\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} + \frac{5(9A-7B)\sin(c}{21ad\sqrt{\sec(c+}}$$

$$= -\frac{21(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} + \frac{(9A}{7a}$$

$$= -\frac{21(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5ad} + \frac{5(9A}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 4.12, size = 568, normalized size = 2.47

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)(A+B\sec(c+dx))\left(588\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)-588\sqrt{2}Be^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)+1800A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}-1400B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}+\sqrt{\sec(c+dx)}\left(63(A-B)(17+11\cos(2c))\cos(dx)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)+20(27A-14B)\cos(2dx)\sin(2c)-84(A-B)\cos(3dx)\sin(3c)+30A\cos(4dx)\sin(4c)-840(A-B)\sec\left(\frac{c}{2}\right)\sec\left(\frac{1}{2}(c+dx)\right)\sin\left(\frac{dx}{2}\right)-2772(A-B)\cos(c)\sin(dx)+20(27A-14B)\cos(2c)\sin(2dx)-84(A-B)\cos(3c)\sin(3dx)+30A\cos(4c)\sin(4dx)-840(A-B)\tan\left(\frac{c}{2}\right)\right)\right)}{420ad(B+A\cos(c+dx))(1+\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(7/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*(A + B*Sec[c + d*x])*((588*Sqrt[2]*A*Sqrt[E^(I*(c + d*x
))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc[c]*(-3*Sqrt
[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric
2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) - (588*Sqrt[2]*B*Sqrt[
E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cs
c[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*H
ypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + 1800*A*
Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - 1400*B*Sq
rt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] + Sqrt[Sec[c
+ d*x]]*(63*(A - B)*(17 + 11*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2] + 20*(27*
A - 14*B)*Cos[2*d*x]*Sin[2*c] - 84*(A - B)*Cos[3*d*x]*Sin[3*c] + 30*A*Cos[4
*d*x]*Sin[4*c] - 840*(A - B)*Sec[c/2]*Sec[(c + d*x)/2]*Sin[(d*x)/2] - 2772*
(A - B)*Cos[c]*Sin[d*x] + 20*(27*A - 14*B)*Cos[2*c]*Sin[2*d*x] - 84*(A - B)
*Cos[3*c]*Sin[3*d*x] + 30*A*Cos[4*c]*Sin[4*d*x] - 840*(A - B)*Tan[c/2])))/(
420*a*d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x]))
```

Maple [A]
time = 1.19, size = 300, normalized size = 1.30

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(225*
A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+441*A*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))-175*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-441*B*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2)))-480*A*sin(1/2*d*x+1/2*c)^10+(864*A+336*B)*sin(1/2*d*
x+1/2*c)^8+(-888*A-392*B)*sin(1/2*d*x+1/2*c)^6+(930*A-210*B)*sin(1/2*d*x+1/
2*c)^4+(-321*A+161*B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*
x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(7/2)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.79, size = 297, normalized size = 1.29

$$-\frac{25\left(\sqrt{2}\,(9i\,A-7i\,B)\cos(dx+c)+\sqrt{2}\,(9i\,A-7i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+25\left(\sqrt{2}\,(-9i\,A+7i\,B)\cos(dx+c)+\sqrt{2}\,(-9i\,A+7i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+441\left(\sqrt{2}\,(i\,A-i\,B)\cos(dx+c)+\sqrt{2}\,(i\,A-i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+441\left(\sqrt{2}\,(-i\,A+i\,B)\cos(dx+c)+\sqrt{2}\,(-i\,A+i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{2\left(30\,A\cos(dx+c)^4-6\,(2\,A-7\,B)\cos(dx+c)^3+2\,(39\,A-14\,B)\cos(dx+c)^2+25\,(9\,A-7\,B)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{210\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] -1/210*(25*(sqrt(2)*(9*I*A - 7*I*B)*cos(d*x + c) + sqrt(2)*(9*I*A - 7*I*B))
*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 25*(sqrt(2)*(-
```

```
9*I*A + 7*I*B)*cos(d*x + c) + sqrt(2)*(-9*I*A + 7*I*B))*weierstrassPInverse
(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 441*(sqrt(2)*(I*A - I*B)*cos(d*x +
 c) + sqrt(2)*(I*A - I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) + 441*(sqrt(2)*(-I*A + I*B)*cos(d*x + c)
+ sqrt(2)*(-I*A + I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c))) - 2*(30*A*cos(d*x + c)^4 - 6*(2*A - 7*B)*cos
(d*x + c)^3 + 2*(39*A - 14*B)*cos(d*x + c)^2 + 25*(9*A - 7*B)*cos(d*x + c))
*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec^{\frac{9}{2}}(c+dx)+\sec^{\frac{7}{2}}(c+dx)}\,dx+\int \frac{B\sec(c+dx)}{\sec^{\frac{9}{2}}(c+dx)+\sec^{\frac{7}{2}}(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(7/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A/(sec(c + d*x)**(9/2) + sec(c + d*x)**(7/2)), x) + Integral(B*se
c(c + d*x)/(sec(c + d*x)**(9/2) + sec(c + d*x)**(7/2)), x))/a
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sec(d*x + c)^(7/2)), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(7/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))*(1/cos(c + d*x))^(7/2)), x)
```

3.209 $\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=237

$$-\frac{(4A-7B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}-\frac{5(A-2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{s}}}{3a^2d}$$

```
[Out] -5/3*(A-2*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d+1/3*(4*A-7*B)*sec(d*x+c)^(5/
2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))+1/3*(A-B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/
(a+a*sec(d*x+c))^2+(4*A-7*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d-(4*A-7*B)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d-5/3*(A-2*B)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.25, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4104, 3872, 3853, 3856, 2719, 2720}

$$\frac{(4A-7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}-\frac{5(A-2B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d}+\frac{(4A-7B)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}-\frac{5(A-2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(4A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(((4*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a^2*d)) - (5*(A - 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(3*a^2*d) + ((4*A - 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(a^2*d) - (5*(A - 2*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d) + ((4*A
 - 7*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*x])) + ((A
 - B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-\frac{3}{2}a(A-3B)\,\mathrm{s}}{a+a\sec(c+dx)}}{3a^2}$$

$$= \frac{(4A-7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c}{3d(a+a\sec(c+dx))^2}$$

$$= \frac{(4A-7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c}{3d(a+a\sec(c+dx))^2}$$

$$= \frac{(4A-7B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} - \frac{5(A-2B)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{si}}{3a^2d}$$

$$= \frac{(4A-7B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} - \frac{5(A-2B)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{si}}{3a^2d}$$

$$= -\frac{(4A-7B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{a^2d} - \frac{5(A}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.04, size = 865, normalized size = 3.65

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,
x]
```

```
[Out] (4*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(3*d*E^(I*
d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - (7*Sqrt[2]*B*Sqrt[E^(I*
(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2
 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-
1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Se
c[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(3*d*E^(I*d*x)*(B + A*Cos[c + d*x
])*(a + a*Sec[c + d*x])^2) - (10*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*
Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c
 + d*x])*Sin[c])/(3*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + (20*B*
Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*
Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c
+ d*x])*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^(3/2)*
(A + B*Sec[c + d*x])*((-2*(-4*A + 7*B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/d + (2*S
ec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(3*d) +
(4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-5*A*Sin[(d*x)/2] + 8*B*Sin[(d*x)/2]))/(3*d
) + (8*B*Sec[c]*Sec[c + d*x]*Sin[d*x])/(3*d) + (4*(2*B - 5*A*Cos[c] + 10*B*
Cos[c])*Sec[c]*Tan[c/2])/(3*d) + (2*(-A + B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])
/(3*d)))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(265) = 530$.
time = 3.49, size = 723, normalized size = 3.05

method	result	size
default	Expression too large to display	723

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*(4*B*(-1
/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
```

```
(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(-2*A+4*B)*(cos(1/2*d*x+1/2*c)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)+1/3*(-A+B)*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1
/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*c)^6+20*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x
+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(sin(1/2*d*x+1/2*c)^2-1)+(4*A-8*B)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*
d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.49, size = 426, normalized size = 1.80

$$-\frac{5\left(\sqrt{2}(-iA+2iB)\cos(dx+c)^3+2\sqrt{2}(-iA+2iB)\cos(dx+c)^2+\sqrt{2}(-iA+2iB)\cos(dx+c)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\left(\sqrt{2}(iA-2iB)\cos(dx+c)^3+2\sqrt{2}(iA-2iB)\cos(dx+c)^2+\sqrt{2}(iA-2iB)\cos(dx+c)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3\left(\sqrt{2}(4iA-7iB)\cos(dx+c)^3+2\sqrt{2}(4iA-7iB)\cos(dx+c)^2+\sqrt{2}(4iA-7iB)\cos(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(\sqrt{2}(-4iA+7iB)\cos(dx+c)^3+2\sqrt{2}(-4iA+7iB)\cos(dx+c)^2+\sqrt{2}(-4iA+7iB)\cos(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{2(3(4A-7B)\cos(dx+c)^3+(19A-32B)\cos(dx+c)^2+2(3A-4B)\cos(dx+c)+2B)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2+a^2d\cos(dx+c))}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] -1/6*(5*(sqrt(2)*(-I*A + 2*I*B)*cos(d*x + c)^3 + 2*sqrt(2)*(-I*A + 2*I*B)*c
os(d*x + c)^2 + sqrt(2)*(-I*A + 2*I*B)*cos(d*x + c))*weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(sqrt(2)*(I*A - 2*I*B)*cos(d*x + c)
^3 + 2*sqrt(2)*(I*A - 2*I*B)*cos(d*x + c)^2 + sqrt(2)*(I*A - 2*I*B)*cos(d*x
 + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(sqrt(
2)*(4*I*A - 7*I*B)*cos(d*x + c)^3 + 2*sqrt(2)*(4*I*A - 7*I*B)*cos(d*x + c)^
2 + sqrt(2)*(4*I*A - 7*I*B)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstras
```

```
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(sqrt(2)*(-4*I*A + 7*I
*B)*cos(d*x + c)^3 + 2*sqrt(2)*(-4*I*A + 7*I*B)*cos(d*x + c)^2 + sqrt(2)*(-
4*I*A + 7*I*B)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*(4*A - 7*B)*cos(d*x + c)^3 + (19
*A - 32*B)*cos(d*x + c)^2 + 2*(3*A - 4*B)*cos(d*x + c) + 2*B)*sin(d*x + c)/
sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*
cos(d*x + c))
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^2, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a/cos(c + d*x))^2, x
)
```

3.210 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=204

$$\frac{(A-4B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{(2A-5B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}$$

```
[Out] 1/3*(2*A-5*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(1+sec(d*x+c))+1/3*(A-B)*se
c(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-(A-4*B)*sin(d*x+c)*sec(d*x+c
)^(1/2)/a^2/d+(A-4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+1/
3*(2*A-5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.24, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4104, 3872, 3856, 2720, 3853, 2719}

$$\frac{(2A-5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3a^2d(\sec(c+dx)+1)}-\frac{(A-4B)\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d}+\frac{(2A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(A-4B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((A - 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a^2*d) + ((2*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*a^2*d) - ((A - 4*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*
d) + ((2*A - 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Sec[c + d*
x])) + ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^
2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(A-B)-\frac{1}{2}a(A-7B\right.}{a+a\sec(c+dx)}}{3a^2}$$

$$= \frac{(2A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(}{3d(a+a\sec(c+dx)}$$

$$= \frac{(2A-5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(}{3d(a+a\sec(c+dx)}$$

$$= -\frac{(A-4B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d} + \frac{(2A-5B)\sec^{\frac{3}{2}}(c+dx}{3a^2d(1+\sec(c+}$$

$$= \frac{(2A-5B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d} - \frac{(A-}{}$$

$$= \frac{(A-4B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(2A-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.05, size = 455, normalized size = 2.23

$$\frac{\cos^4\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)(A+B\sec(c+dx))\left(-2\sqrt{2}Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)+8\sqrt{2}Be^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\csc(c)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)+8A\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}-20B\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}-2\sqrt{\sec(c+dx)}\left(6(A-4B)\cos(dx)\csc(c)-(3(A-2B)+(2A-5B)\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{3a^2d(B+A\cos(c+dx))(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,
x]
```

```
[Out] (Cos[(c + d*x)/2]^4*Sec[c + d*x]*(A + B*Sec[c + d*x])*((-2*Sqrt[2]*A*Sqrt[E
^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Csc
[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hy
pergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x) + (8*Sqrt[
2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c +
 d*x))]*Csc[c]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((
2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]))/E^(I*d*x)
 + 8*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - 20
*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]] - 2*Sqrt
[Sec[c + d*x]]*(6*(A - 4*B)*Cos[d*x]*Csc[c] - (3*(A - 2*B) + (2*A - 5*B)*Co
s[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])))/(3*a^2*d*(B + A*Cos[c +
d*x])*(1 + Sec[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(236) = 472$.
time = 1.70, size = 492, normalized size = 2.41

method	result
default	$\frac{2\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(2A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/6*(2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+12*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))-5*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*EllipticE(cos(1/2*
```

```
d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(A-4*B)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*A-43*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(7*A-37*B)*sin(1/2*d*x+1/2*c)^2)/a
^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.40, size = 365, normalized size = 1.79

$$\frac{\left(\sqrt{2}\,(-2i\,A+5i\,B)\cos(dx+c)^2-2\sqrt{2}\,(2i\,A-5i\,B)\cos(dx+c)+\sqrt{2}\,(-2i\,A+5i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\left(\sqrt{2}\,(2i\,A-5i\,B)\cos(dx+c)^2-2\sqrt{2}\,(-2i\,A+5i\,B)\cos(dx+c)+\sqrt{2}\,(2i\,A-5i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-3\left(\sqrt{2}\,(-i\,A+4i\,B)\cos(dx+c)^2+2\sqrt{2}\,(-i\,A+4i\,B)\cos(dx+c)+\sqrt{2}\,(-i\,A+4i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-3\left(\sqrt{2}\,(i\,A-4i\,B)\cos(dx+c)^2+2\sqrt{2}\,(i\,A-4i\,B)\cos(dx+c)+\sqrt{2}\,(i\,A-4i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-\frac{2\left(3\,(A-4\,B)\cos(dx+c)^2+(4\,A-19\,B)\cos(dx+c)-6\,B\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] 1/6*((sqrt(2)*(-2*I*A + 5*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(2*I*A - 5*I*B)*c
os(d*x + c) + sqrt(2)*(-2*I*A + 5*I*B))*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) + (sqrt(2)*(2*I*A - 5*I*B)*cos(d*x + c)^2 - 2*sqrt(2
)*(-2*I*A + 5*I*B)*cos(d*x + c) + sqrt(2)*(2*I*A - 5*I*B))*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(sqrt(2)*(-I*A + 4*I*B)*cos(d
*x + c)^2 + 2*sqrt(2)*(-I*A + 4*I*B)*cos(d*x + c) + sqrt(2)*(-I*A + 4*I*B))
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c))) - 3*(sqrt(2)*(I*A - 4*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(I*A - 4*I*B)
*cos(d*x + c) + sqrt(2)*(I*A - 4*I*B))*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*(A - 4*B)*cos(d*x + c)
^2 + (4*A - 19*B)*cos(d*x + c) - 6*B)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2
*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3066 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x))^2, x
)
```

3.211 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=161

$$\frac{B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{(A+2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d}$$

```
[Out] 1/3*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2-B*sin(d*x+c)*sec
(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))+B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/a^2/d+1/3*(A+2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.21, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4104, 3872, 3856, 2719, 2720}

$$\frac{(A+2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{B\sin(c+dx)\sqrt{\sec(c+dx)}}{a^2d(\sec(c+dx)+1)}+\frac{B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*d)
 + ((A + 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(3*a^2*d) - (B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Sec[c + d*x
])) + ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(a + a*Sec[c + d*x])^2
)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{\sqrt{\sec(c+dx)}\,\left(\frac{1}{2}a(A-B)+\frac{1}{2}a(\right.}{a+a\sec(c+dx)}}{3a^2}$$

$$= -\frac{B\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{3d(a+a\sec(c+dx))^2}$$

$$= -\frac{B\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{3d(a+a\sec(c+dx))^2}$$

$$= -\frac{B\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a^2d(1+\sec(c+dx))} + \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx}{3d(a+a\sec(c+dx))^2}$$

$$= \frac{B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(A+2B)\sqrt{\cos}}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 2.73, size = 256, normalized size = 1.59

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(-iBe^{-i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+8(A+2B)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-i\sin\left(\frac{1}{2}(c+dx)\right)\right)+2i\cos(c+dx)(-A+7B+(A+5B)\cos(c+dx)-i(A-B)\sin(c+dx))\right)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)}{6a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,
x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(((-I)*B*(1 + E^(I*(c + d*x)))^3*Sqrt[
1 + E^((2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*
x))])/E^(I*(c + d*x)) + 8*(A + 2*B)*Cos[(c + d*x)/2]^3*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/2]) + (2*I)*Co
s[c + d*x]*(-A + 7*B + (A + 5*B)*Cos[c + d*x] - I*(A - B)*Sin[c + d*x]))*(C
os[(c + 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c +
d*x])^2)
```

Maple [**A**]
time = 1.37, size = 350, normalized size = 2.17

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*cos(1/2*d
*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^6+4*B*cos(1/2
*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+2*A*cos(1/2*d*x+1/2*c)^4+16*B*cos(1/2*d*x+1/2*c)^4-3*A*cos(
1/2*d*x+1/2*c)^2-3*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.92, size = 325, normalized size = 2.02

$$\frac{\left(\sqrt{2}\left(-iA-2iB\right)\cos\left(dx+c\right)^{2}-2\sqrt{2}\left(iA+2iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-iA-2iB\right)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+\left(\sqrt{2}\left(iA+2iB\right)\cos\left(dx+c\right)^{2}-2\sqrt{2}\left(-iA-2iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(iA+2iB\right)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-3\left(-i\sqrt{2}B\cos\left(dx+c\right)^{2}-2i\sqrt{2}B\cos\left(dx+c\right)-i\sqrt{2}B\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-3\left(i\sqrt{2}B\cos\left(dx+c\right)^{2}+2i\sqrt{2}B\cos\left(dx+c\right)+i\sqrt{2}B\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-\frac{2\left(3B\cos\left(dx+c\right)^{2}-\left(A-4B\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{6\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] 1/6*((sqrt(2)*(-I*A - 2*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(I*A + 2*I*B)*cos(d
*x + c) + sqrt(2)*(-I*A - 2*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + (sqrt(2)*(I*A + 2*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(-I*A
- 2*I*B)*cos(d*x + c) + sqrt(2)*(I*A + 2*I*B))*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) - 3*(-I*sqrt(2)*B*cos(d*x + c)^2 - 2*I*sqrt(2
)*B*cos(d*x + c) - I*sqrt(2)*B)*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(I*sqrt(2)*B*cos(d*x + c)^2 + 2*
I*sqrt(2)*B*cos(d*x + c) + I*sqrt(2)*B)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*B*cos(d*x + c)^2 - (A
 - 4*B)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^
2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sec^{\frac{3}{2}}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx+\int \frac{B\sec^{\frac{5}{2}}\left(c+dx\right)}{\sec^{2}\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*sec(c + d*x)**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)
+ Integral(B*sec(c + d*x)**(5/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x)
)/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x))^2,x)
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x))^2, x
)
```

3.212 $\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=168

$$-\frac{A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{(2A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{3a^2d}$$

```
[Out] 1/3*(2*A+B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))+1/3*(A-B)*sin(
d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^2-A*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*
sec(d*x+c)^(1/2)/a^2/d+1/3*(2*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)/a^2/d
```

Rubi [A]
time = 0.21, antiderivative size = 168, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4104, 4105, 3872, 3856, 2719, 2720}

$$\frac{(2A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}+\frac{(2A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{A\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -((A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*
d)) + ((2*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(3*a^2*d) + ((2*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*d*(1
+ Sec[c + d*x])) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Se
c[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2} + \frac{\int \frac{-\frac{1}{2}a(A-B)+\frac{3}{2}a(A+B)\sec(c}{\sqrt{\sec(c+dx)}\,(a+a\sec(c}}{3a^2}$$

$$= \frac{(2A+B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sqrt{\sec(c+dx)}}{3d(a+a\sec(c+}$$

$$= \frac{(2A+B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sqrt{\sec(c+dx)}}{3d(a+a\sec(c+}$$

$$= \frac{(2A+B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))} + \frac{(A-B)\sqrt{\sec(c+dx)}}{3d(a+a\sec(c+}$$

$$= -\frac{A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{(2A+B)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 3.40, size = 256, normalized size = 1.52

$$\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left(8(2A+B)\cos^3\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\left(\cos\left(\frac{1}{2}(c+dx)\right)-i\sin\left(\frac{1}{2}(c+dx)\right)\right)+i\left(Ae^{-i(c+dx)}\left(1+e^{i(c+dx)}\right)^3\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)-2\cos(c+dx)(5A+B+(7A-B)\cos(c+dx)-i(A-B)\sin(c+dx))\right)\right)\left(\cos\left(\frac{1}{2}(c+3dx)\right)+i\sin\left(\frac{1}{2}(c+3dx)\right)\right)}{6a^2d(1+\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,
x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(8*(2*A + B)*Cos[(c + d*x)/2]^3*Sqrt[C
os[c + d*x]]*EllipticF[(c + d*x)/2, 2]*(Cos[(c + d*x)/2] - I*Sin[(c + d*x)/
2]) + I*((A*(1 + E^(I*(c + d*x)))^3*Sqrt[1 + E^((2*I)*(c + d*x))]*Hypergeom
etric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(I*(c + d*x)) - 2*Cos[c +
d*x]*(5*A + B + (7*A - B)*Cos[c + d*x] - I*(A - B)*Sin[c + d*x])))*(Cos[(c
+ 3*d*x)/2] + I*Sin[(c + 3*d*x)/2]))/(6*a^2*d*E^(I*d*x)*(1 + Sec[c + d*x])^
2)
```

Maple [A]
time = 1.37, size = 350, normalized size = 2.08

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12A\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4A\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^6+4*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*cos(1/2
*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-20*A*cos(1/2*d*x+1/2*c)^4+2*B*cos(1/2*d*x+1/2*c)^4+9*A*cos(
1/2*d*x+1/2*c)^2-3*B*cos(1/2*d*x+1/2*c)^2-A+B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^2, x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.60, size = 324, normalized size = 1.93

$$\frac{\left(\sqrt{2}\,(-2i\,A-i\,B)\cos(dx+c)^2-2\,\sqrt{2}\,(2i\,A+i\,B)\cos(dx+c)+\sqrt{2}\,(-2i\,A-i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\left(\sqrt{2}\,(2i\,A+i\,B)\cos(dx+c)^2-2\,\sqrt{2}\,(-2i\,A-i\,B)\cos(dx+c)+\sqrt{2}\,(2i\,A+i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-3\left(i\,\sqrt{2}\,A\cos(dx+c)^2+2i\,\sqrt{2}\,A\cos(dx+c)+i\,\sqrt{2}\,A\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-3\left(-i\,\sqrt{2}\,A\cos(dx+c)^2-2i\,\sqrt{2}\,A\cos(dx+c)-i\,\sqrt{2}\,A\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+\frac{2\left(3\,A\cos(dx+c)^2+(2\,A+B)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] 1/6*((sqrt(2)*(-2*I*A - I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(2*I*A + I*B)*cos(d
*x + c) + sqrt(2)*(-2*I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + (sqrt(2)*(2*I*A + I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(-2*I*
A - I*B)*cos(d*x + c) + sqrt(2)*(2*I*A + I*B))*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) - 3*(I*sqrt(2)*A*cos(d*x + c)^2 + 2*I*sqrt(2)
*A*cos(d*x + c) + I*sqrt(2)*A)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*A*cos(d*x + c)^2 - 2*
I*sqrt(2)*A*cos(d*x + c) - I*sqrt(2)*A)*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*A*cos(d*x + c)^2 + (2
*A + B)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^
2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sqrt{\sec(c+dx)}}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx+\int \frac{B\sec^{\frac{3}{2}}(c+dx)}{\sec^2(c+dx)+2\sec(c+dx)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*sqrt(sec(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) +
 Integral(B*sec(c + d*x)**(3/2)/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x))
/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x))^2, x
)
```

3.213 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=177

$$\frac{(4A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}-\frac{(5A-2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(}}{3a^2d}$$

```
[Out] -1/3*(5*A-2*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/(1+sec(d*x+c))-1/3*(A-B)*s
in(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^2+(4*A-B)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d-1/3*(5*A-2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.22, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4105, 3872, 3856, 2719, 2720}

$$-\frac{(5A-2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d(\sec(c+dx)+1)}-\frac{(5A-2B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(4A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] ((4*A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(a^2*d) - ((5*A - 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(3*a^2*d) - ((5*A - 2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*
a^2*d*(1 + Sec[c + d*x])) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*
(a + a*Sec[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d(a+a\sec(c+dx))^2}+\frac{\int \frac{\frac{1}{2}a(7A-B)-\frac{3}{2}a(A-B)\sec(c\cdots}{\sqrt{\sec(c+dx)}\,(a+a\sec(c\cdots}}{3a^2}$$

$$= -\frac{(5A-2B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{(A-B)\sqrt{\sec(c+dx)\cdots}}{3d(a+a\sec(c+\cdots}$$

$$= -\frac{(5A-2B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{(A-B)\sqrt{\sec(c+dx)\cdots}}{3d(a+a\sec(c+\cdots}$$

$$= -\frac{(5A-2B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a^2d(1+\sec(c+dx))}-\frac{(A-B)\sqrt{\sec(c+dx)\cdots}}{3d(a+a\sec(c+\cdots}$$

$$= \frac{(4A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}-\frac{(5A\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.82, size = 854, normalized size = 4.82

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^2),
x]
```

[Out] (-4*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(3*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + (Sqrt[2]*B*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(3*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - (10*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + (4*B*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x])*((-2*(3*A - B + A*Cos[2*c])*Cos[d*x]*Csc[c/2]*Sec[c/2])/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-7*A*Sin[(d*x)/2] + 4*B*Sin[(d*x)/2]))/(3*d) + (8*A*Cos[c]*Sin[d*x])/d - (4*(-7*A + 4*B)*Tan[c/2])/(3*d) + (2*(-A + B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)

Maple [A]
time = 1.59, size = 421, normalized size = 2.38

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24A\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10A\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-\cdots}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*A*cos(1/2*d*x+1/2*c)^6+10*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+24*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^6-4*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-38*A*cos(1/2*d*x+1/2*c)^4+20*B*cos(1/2*d*x+1/2*c)^4+15*A*cos(1/2*d*x+1/2*c)^2-9*B*cos(1/2*d*x+1/2*c)^2-A+B)/cos(1/2*d*x+1/2*c)^

```
3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="maxima")
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.70, size = 364, normalized size = 2.06

$$\frac{\left(\sqrt{2}\,(5i\,A-2i\,B)\cos(dx+c)^2-2\sqrt{2}\,(-5i\,A+2i\,B)\cos(dx+c)+\sqrt{2}\,(5i\,A-2i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+\left(\sqrt{2}\,(-5i\,A+2i\,B)\cos(dx+c)^2-2\sqrt{2}\,(5i\,A-2i\,B)\cos(dx+c)+\sqrt{2}\,(-5i\,A+2i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3\left(\sqrt{2}\,(-4i\,A+i\,B)\cos(dx+c)^2+2\sqrt{2}\,(-4i\,A+i\,B)\cos(dx+c)+\sqrt{2}\,(-4i\,A+i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3\left(\sqrt{2}\,(4i\,A-i\,B)\cos(dx+c)^2+2\sqrt{2}\,(4i\,A-i\,B)\cos(dx+c)+\sqrt{2}\,(4i\,A-i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{2\left(3\,(2\,A-B)\cos(dx+c)^2+(5\,A-2\,B)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6\left(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="fricas")
[Out] 1/6*((sqrt(2)*(5*I*A - 2*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(-5*I*A + 2*I*B)*c
os(d*x + c) + sqrt(2)*(5*I*A - 2*I*B))*weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c)) + (sqrt(2)*(-5*I*A + 2*I*B)*cos(d*x + c)^2 - 2*sqrt(2
)*(5*I*A - 2*I*B)*cos(d*x + c) + sqrt(2)*(-5*I*A + 2*I*B))*weierstrassPInve
rse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(sqrt(2)*(-4*I*A + I*B)*cos(d
*x + c)^2 + 2*sqrt(2)*(-4*I*A + I*B)*cos(d*x + c) + sqrt(2)*(-4*I*A + I*B))
*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c))) - 3*(sqrt(2)*(4*I*A - I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(4*I*A - I*B)
*cos(d*x + c) + sqrt(2)*(4*I*A - I*B))*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*(2*A - B)*cos(d*x + c)
^2 + (5*A - 2*B)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(
d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec^{\frac{5}{2}}(c+dx)+2\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx+\int \frac{B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)+2\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2/sec(d*x+c)**(1/2),x)
```

```
[Out] (Integral(A/(sec(c + d*x)**(5/2) + 2*sec(c + d*x)**(3/2) + sqrt(sec(c + d*x
))), x) + Integral(B*sec(c + d*x)/(sec(c + d*x)**(5/2) + 2*sec(c + d*x)**(3
/2) + sqrt(sec(c + d*x))), x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^2\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)), x
)
```

3.214 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=211

$$-\frac{(7A-4B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{5(2A-B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\text{se}}}{3a^2d}$$

```
[Out] 5/3*(2*A-B)*sin(d*x+c)/a^2/d/sec(d*x+c)^(1/2)-1/3*(7*A-4*B)*sin(d*x+c)/a^2/
d/(1+sec(d*x+c))/sec(d*x+c)^(1/2)-1/3*(A-B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^2
/sec(d*x+c)^(1/2)-(7*A-4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^
2/d+5/3*(2*A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.24, antiderivative size = 211, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4105, 3872, 3854, 3856, 2720, 2719}

$$\frac{5(2A-B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{(7A-4B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(\sec(c+dx)+1)}+\frac{5(2A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(7A-4B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] -(((7*A - 4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a^2*d)) + (5*(2*A - B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(3*a^2*d) + (5*(2*A - B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec
[c + d*x]]) - ((7*A - 4*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]]*(1 + S
ec[c + d*x])) - ((A - B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c
 + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^2} + \frac{\int \frac{\frac{3}{2}a(3A-B)-\frac{5}{2}a(A-B)\sec(c+}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx)}}{3a^2}$$

$$= -\frac{(7A-4B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(1+\sec(c+dx))} - \frac{(A-B)\sin(c+}{3d\sqrt{\sec(c+dx)}\,(a+a\,\mathrm{s}}$$

$$= -\frac{(7A-4B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(1+\sec(c+dx))} - \frac{(A-B)\sin(c+}{3d\sqrt{\sec(c+dx)}\,(a+a\,\mathrm{s}}$$

$$= \frac{5(2A-B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{(7A-4B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}\,(1+\sec(c+dx))} -$$

$$= -\frac{(7A-4B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{5(2A}{3a}$$

$$= -\frac{(7A-4B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{5(2A}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.92, size = 899, normalized size = 4.26

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),
x]
```

```
[Out] (7*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(3*d*E^(I*
d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - (4*Sqrt[2]*B*Sqrt[E^(I*
(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2
 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-
1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Se
c[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(3*d*E^(I*d*x)*(B + A*Cos[c + d*x
])*(a + a*Sec[c + d*x])^2) + (20*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*
Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c
 + d*x])*Sin[c])/(3*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - (10*B*
Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*
```

```
Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c
+ d*x])*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^(3/2)*
(A + B*Sec[c + d*x])*((-2*(-5*A + 3*B - 2*A*Cos[2*c] + B*Cos[2*c])*Cos[d*x]
*Csc[c/2]*Sec[c/2])/d + (4*A*Cos[2*d*x]*Sin[2*c])/(3*d) - (2*Sec[c/2]*Sec[c
/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(3*d) + (4*Sec[c/2]*S
ec[c/2 + (d*x)/2]*(-10*A*Sin[(d*x)/2] + 7*B*Sin[(d*x)/2]))/(3*d) + (8*(-2*A
 + B)*Cos[c]*Sin[d*x])/d + (4*A*Cos[2*c]*Sin[2*d*x])/(3*d) + (4*(-10*A + 7*
B)*Tan[c/2])/(3*d) - (2*(-A + B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/((B
 + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [A]
time = 1.59, size = 435, normalized size = 2.06

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(16A\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+12A\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20A\left(\cos^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{\right.}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/6/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*A*cos(
1/2*d*x+1/2*c)^8+12*A*cos(1/2*d*x+1/2*c)^6+20*A*cos(1/2*d*x+1/2*c)^3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+42*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*
B*cos(1/2*d*x+1/2*c)^6-10*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-
24*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*A*cos(1/2*d*x+1/2*c)
^4+38*B*cos(1/2*d*x+1/2*c)^4+21*A*cos(1/2*d*x+1/2*c)^2-15*B*cos(1/2*d*x+1/2
*c)^2-A+B)/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.98, size = 377, normalized size = 1.79

$$-\frac{5\left(\sqrt{2}\,(2iA - iB)\cos(dx+c)^2 + 2\sqrt{2}\,(2iA - iB)\cos(dx+c) + \sqrt{2}\,(2iA - iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c) + i\sin(dx+c)) + 5\left(\sqrt{2}\,(-2iA + iB)\cos(dx+c)^2 + 2\sqrt{2}\,(-2iA + iB)\cos(dx+c) + \sqrt{2}\,(-2iA + iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c) - i\sin(dx+c)) + 3\left(\sqrt{2}\,(7iA - 4iB)\cos(dx+c)^2 + 2\sqrt{2}\,(7iA - 4iB)\cos(dx+c) + \sqrt{2}\,(7iA - 4iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c) + i\sin(dx+c))) + 3\left(\sqrt{2}\,(-7iA + 4iB)\cos(dx+c)^2 + 2\sqrt{2}\,(-7iA + 4iB)\cos(dx+c) + \sqrt{2}\,(-7iA + 4iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c) - i\sin(dx+c))) - \frac{2\left(2A\cos(dx+c)^3 + (13A - 6B)\cos(dx+c)^2 + 5(2A - B)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6\left(a^2d\cos(dx+c)^2 + 2a^2d\cos(dx+c) + a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] -1/6*(5*(sqrt(2)*(2*I*A - I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(2*I*A - I*B)*cos
(d*x + c) + sqrt(2)*(2*I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + 5*(sqrt(2)*(-2*I*A + I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(-
2*I*A + I*B)*cos(d*x + c) + sqrt(2)*(-2*I*A + I*B))*weierstrassPInverse(-4,
 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(sqrt(2)*(7*I*A - 4*I*B)*cos(d*x + c
)^2 + 2*sqrt(2)*(7*I*A - 4*I*B)*cos(d*x + c) + sqrt(2)*(7*I*A - 4*I*B))*wei
erstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c
))) + 3*(sqrt(2)*(-7*I*A + 4*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(-7*I*A + 4*I*
B)*cos(d*x + c) + sqrt(2)*(-7*I*A + 4*I*B))*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(2*A*cos(d*x + c)^3
+ (13*A - 6*B)*cos(d*x + c)^2 + 5*(2*A - B)*cos(d*x + c))*sin(d*x + c)/sqrt
(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec^{\frac{7}{2}}(c+dx) + 2\sec^{\frac{5}{2}}(c+dx) + \sec^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{B\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx) + 2\sec^{\frac{5}{2}}(c+dx) + \sec^{\frac{3}{2}}(c+dx)}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A/(sec(c + d*x)**(7/2) + 2*sec(c + d*x)**(5/2) + sec(c + d*x)**(3
/2)), x) + Integral(B*sec(c + d*x)/(sec(c + d*x)**(7/2) + 2*sec(c + d*x)**(
5/2) + sec(c + d*x)**(3/2)), x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^2\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)),x)
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)), x
)
```

3.215 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=244

$$\frac{7(8A-5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5a^2d}-\frac{5(3A-2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\text{se}}}{3a^2d}$$

```
[Out] 7/15*(8*A-5*B)*sin(d*x+c)/a^2/d/sec(d*x+c)^(3/2)-(3*A-2*B)*sin(d*x+c)/a^2/d
/sec(d*x+c)^(3/2)/(1+sec(d*x+c))-1/3*(A-B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a
+a*sec(d*x+c))^2-5/3*(3*A-2*B)*sin(d*x+c)/a^2/d/sec(d*x+c)^(1/2)+7/5*(8*A-5
*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d-5/3*(3*A-2*B)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1
/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.27, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4105, 3872, 3854, 3856, 2719, 2720}

$$-\frac{(3A-2B)\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(\sec(c+dx)+1)}+\frac{7(8A-5B)\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)}-\frac{5(3A-2B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{5(3A-2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{7(8A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{(A-B)\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (7*(8*A - 5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*a^2*d) - (5*(3*A - 2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*a^2*d) + (7*(8*A - 5*B)*Sin[c + d*x])/(15*a^2*d*Se
c[c + d*x]^(3/2)) - (5*(3*A - 2*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]
]) - ((3*A - 2*B)*Sin[c + d*x])/(a^2*d*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x]
)) - ((A - B)*Sin[c + d*x])/(3*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = -\frac{(A-B)\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(11A-5B)-\frac{7}{2}a(A-B)\sec(c+}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}}{3a^2}$$

$$= -\frac{(3A-2B)\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c}$$

$$= -\frac{(3A-2B)\sin(c+dx)}{a^2d\sec^{\frac{3}{2}}(c+dx)(1+\sec(c+dx))} - \frac{(A-B)\sin(c+dx)}{3d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c}$$

$$= \frac{7(8A-5B)\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5(3A-2B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{(3A-}{a^2d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{7(8A-5B)\sin(c+dx)}{15a^2d\sec^{\frac{3}{2}}(c+dx)} - \frac{5(3A-2B)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{(3A-}{a^2d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{7(8A-5B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5a^2d} - \frac{5(3A}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.01, size = 946, normalized size = 3.88

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),
x]
```

```
[Out] (-56*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c +
d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4,
-E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(15*d*E^
(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + (7*Sqrt[2]*B*Sqrt[E^
(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[
c/2 + (d*x)/2]^4*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)
*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])
*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x]))/(3*d*E^(I*d*x)*(B + A*Cos[c +
d*x])*(a + a*Sec[c + d*x])^2) - (10*A*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x
]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Se
c[c + d*x])*Sin[c])/(d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + (20*B
*Cos[c/2 + (d*x)/2]^4*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]
```

```
*Sec[c/2]*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c
 + d*x])*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*Sec[c + d*x]^(3/2)
*(A + B*Sec[c + d*x])*(((-151*A + 100*B - 73*A*Cos[2*c] + 40*B*Cos[2*c])*Co
s[d*x]*Csc[c/2]*Sec[c/2])/(10*d) + (4*(-2*A + B)*Cos[2*d*x]*Sin[2*c])/(3*d)
 + (2*A*Cos[3*d*x]*Sin[3*c])/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*
Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-1
3*A*Sin[(d*x)/2] + 10*B*Sin[(d*x)/2]))/(3*d) - (2*(-73*A + 40*B)*Cos[c]*Sin
[d*x])/(5*d) + (4*(-2*A + B)*Cos[2*c]*Sin[2*d*x])/(3*d) + (2*A*Cos[3*c]*Sin
[3*d*x])/(5*d) - (4*(-13*A + 10*B)*Tan[c/2])/(3*d) + (2*(-A + B)*Sec[c/2 +
(d*x)/2]^2*Tan[c/2])/(3*d)))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [A]
time = 1.61, size = 465, normalized size = 1.91

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(96A\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-352A\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+80B\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/30/a^2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*A*cos
(1/2*d*x+1/2*c)^10-352*A*cos(1/2*d*x+1/2*c)^8+80*B*cos(1/2*d*x+1/2*c)^8+120
*A*cos(1/2*d*x+1/2*c)^6-150*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-336*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+60*B*cos(1/2*d*x+1/2
*c)^6+100*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+210*B*cos(1/2*d*
x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))+266*A*cos(1/2*d*x+1/2*c)^4-240*B*cos(1
/2*d*x+1/2*c)^4-135*A*cos(1/2*d*x+1/2*c)^2+105*B*cos(1/2*d*x+1/2*c)^2+5*A-5
*B)/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2)),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.98, size = 395, normalized size = 1.62

$$-\frac{25\left(\sqrt{2}\left(-3i A+2i B\right)\cos\left(dx+c\right)^{2}+2\sqrt{2}\left(-3i A+2i B\right)\cos\left(dx+c\right)+\sqrt{2}\left(-3i A+2i B\right)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+25\left(\sqrt{2}\left(3i A-2i B\right)\cos\left(dx+c\right)^{2}+2\sqrt{2}\left(3i A-2i B\right)\cos\left(dx+c\right)+\sqrt{2}\left(3i A-2i B\right)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+21\left(\sqrt{2}\left(-8i A+5i B\right)\cos\left(dx+c\right)^{2}+2\sqrt{2}\left(-8i A+5i B\right)\cos\left(dx+c\right)+\sqrt{2}\left(-8i A+5i B\right)\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+21\left(\sqrt{2}\left(8i A-5i B\right)\cos\left(dx+c\right)^{2}+2\sqrt{2}\left(8i A-5i B\right)\cos\left(dx+c\right)+\sqrt{2}\left(8i A-5i B\right)\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-\frac{2\left(6A\cos\left(dx+c\right)^{4}-2\left(4A-5B\right)\cos\left(dx+c\right)^{3}-\left(94A-65B\right)\cos\left(dx+c\right)^{2}-25\left(3A-2B\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{30\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] -1/30*(25*(sqrt(2)*(-3*I*A + 2*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(-3*I*A + 2*
I*B)*cos(d*x + c) + sqrt(2)*(-3*I*A + 2*I*B))*weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c)) + 25*(sqrt(2)*(3*I*A - 2*I*B)*cos(d*x + c)^2 +
 2*sqrt(2)*(3*I*A - 2*I*B)*cos(d*x + c) + sqrt(2)*(3*I*A - 2*I*B))*weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*(sqrt(2)*(-8*I*A + 5
*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(-8*I*A + 5*I*B)*cos(d*x + c) + sqrt(2)*(-
8*I*A + 5*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c))) + 21*(sqrt(2)*(8*I*A - 5*I*B)*cos(d*x + c)^2 + 2*sqr
t(2)*(8*I*A - 5*I*B)*cos(d*x + c) + sqrt(2)*(8*I*A - 5*I*B))*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(6*
A*cos(d*x + c)^4 - 2*(4*A - 5*B)*cos(d*x + c)^3 - (94*A - 65*B)*cos(d*x + c
)^2 - 25*(3*A - 2*B)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*
cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec^{\frac{9}{2}}\left(c+dx\right)+2\sec^{\frac{7}{2}}\left(c+dx\right)+\sec^{\frac{5}{2}}\left(c+dx\right)}\, dx+\int \frac{B\sec\left(c+dx\right)}{\sec^{\frac{9}{2}}\left(c+dx\right)+2\sec^{\frac{7}{2}}\left(c+dx\right)+\sec^{\frac{5}{2}}\left(c+dx\right)}\, dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A/(sec(c + d*x)**(9/2) + 2*sec(c + d*x)**(7/2) + sec(c + d*x)**(5
/2)), x) + Integral(B*sec(c + d*x)/(sec(c + d*x)**(9/2) + 2*sec(c + d*x)**(
7/2) + sec(c + d*x)**(5/2)), x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*sec(d*x + c)^(5/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^2\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^2*(1/cos(c + d*x))^(5/2)), x
)
```

3.216 $\int \frac{\sec^{\frac{9}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=292

$$-\frac{7(7A-17B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(13A-33B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}$$

```
[Out] -1/6*(13*A-33*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^3/d+1/5*(A-B)*sec(d*x+c)^(9/
2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3+1/3*(A-2*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/
a/d/(a+a*sec(d*x+c))^2+7/30*(7*A-17*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a^3+a
^3*sec(d*x+c))+7/10*(7*A-17*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^3/d-7/10*(7*A-
17*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d-1/6*(13*A-33*B)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A]
time = 0.39, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4104, 3872, 3853, 3856, 2719, 2720}

$$\frac{7(7A-17B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{30d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(13A-33B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6a^3d}+\frac{7(7A-17B)\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(13A-33B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{7(7A-17B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A-B)\sin(c+dx)\sec^{\frac{9}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{(A-2B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(9/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-7*(7*A - 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(10*a^3*d) - ((13*A - 33*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + (7*(7*A - 17*B)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(10*a^3*d) - ((13*A - 33*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*
a^3*d) + ((A - B)*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(5*d*(a + a*Sec[c + d*x]
)^3) + ((A - 2*B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*a*d*(a + a*Sec[c + d*
x])^2) + (7*(7*A - 17*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(30*d*(a^3 + a^3*
Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \frac{(A-B)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^{\frac{7}{2}}(c+dx)\left(\frac{7}{2}a(A-B)-\frac{1}{2}a(3A-13B}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= \frac{(A-B)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-2B)\sec^{\frac{7}{2}}(c+dx)\sin(c+}{3ad(a+a\sec(c+dx))^2}$$

$$= \frac{(A-B)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-2B)\sec^{\frac{7}{2}}(c+dx)\sin(c+}{3ad(a+a\sec(c+dx))^2}$$

$$= \frac{(A-B)\sec^{\frac{9}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-2B)\sec^{\frac{7}{2}}(c+dx)\sin(c+}{3ad(a+a\sec(c+dx))^2}$$

$$= \frac{7(7A-17B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{10a^3d} - \frac{(13A-33B)\sec^{\frac{3}{2}}(c+}{6a^3d}$$

$$= \frac{7(7A-17B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{10a^3d} - \frac{(13A-33B)\sec^{\frac{3}{2}}(c+}{6a^3d}$$

$$= -\frac{7(7A-17B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{(}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 8.28, size = 953, normalized size = 3.26

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(9/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,
x]
```

```
[Out] (49*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*E
^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (119*Sqrt[2]*B*Sqrt
[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*C
os[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d
*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x)
)])*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*E^(I*d*x)*(B + A*Co
s[c + d*x])*(a + a*Sec[c + d*x])^3) - (26*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c
 + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A
+ B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)
```

```
 + (22*B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*
x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*Sin[c])/(d*(B + A
*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]
^(5/2)*(A + B*Sec[c + d*x])*((-14*(-7*A + 17*B)*Cos[d*x]*Csc[c/2]*Sec[c/2])
/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/
2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-8*A*Sin[(d*x)/2] + 13*B*Sin
[(d*x)/2]))/(15*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-13*A*Sin[(d*x)/2] + 2
9*B*Sin[(d*x)/2]))/(3*d) + (16*B*Sec[c]*Sec[c + d*x]*Sin[d*x])/(3*d) + (4*(
4*B - 13*A*Cos[c] + 33*B*Cos[c])*Sec[c]*Tan[c/2]))/(3*d) + (4*(-8*A + 13*B)*
Sec[c/2 + (d*x)/2]^2*Tan[c/2]))/(15*d) + (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Ta
n[c/2]))/(5*d)))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(312) = 624$.
time = 2.70, size = 876, normalized size = 3.00

method	result	size
default	Expression too large to display	876

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/60*(4*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(65*A*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-165*B*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))+357*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6-10*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-165*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+357*B*Ell
ipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)
+8*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(65*A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-165*B*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+357*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1
/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*(65*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-147*A*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))-165*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+357*B*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-168*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(7*A-17*B)*sin(1/2*d*x+1/2*c)^10+8*(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(482*A-1167*B)*sin(1/2*d*x+1/2*c
)^8-10*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(461*A-1111*B)*
sin(1/2*d*x+1/2*c)^6+14*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
```

```
)*(169*A-404*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(439*A-1029*B)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d*x+1/2*c)^
5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(3/2)/sin(1/2*d*x+1/2*c)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.76, size = 536, normalized size = 1.84

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] -1/60*(5*(sqrt(2)*(-13*I*A + 33*I*B)*cos(d*x + c)^4 + 3*sqrt(2)*(-13*I*A +
33*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-13*I*A + 33*I*B)*cos(d*x + c)^2 + sqrt
(2)*(-13*I*A + 33*I*B)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) + 5*(sqrt(2)*(13*I*A - 33*I*B)*cos(d*x + c)^4 + 3*sqrt(
2)*(13*I*A - 33*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(13*I*A - 33*I*B)*cos(d*x +
 c)^2 + sqrt(2)*(13*I*A - 33*I*B)*cos(d*x + c))*weierstrassPInverse(-4, 0,
cos(d*x + c) - I*sin(d*x + c)) + 21*(sqrt(2)*(7*I*A - 17*I*B)*cos(d*x + c)^
4 + 3*sqrt(2)*(7*I*A - 17*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(7*I*A - 17*I*B)*
cos(d*x + c)^2 + sqrt(2)*(7*I*A - 17*I*B)*cos(d*x + c))*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*(sqrt(2
)*(-7*I*A + 17*I*B)*cos(d*x + c)^4 + 3*sqrt(2)*(-7*I*A + 17*I*B)*cos(d*x +
c)^3 + 3*sqrt(2)*(-7*I*A + 17*I*B)*cos(d*x + c)^2 + sqrt(2)*(-7*I*A + 17*I*
B)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c))) - 2*(21*(7*A - 17*B)*cos(d*x + c)^4 + 2*(188*A - 45
3*B)*cos(d*x + c)^3 + 5*(59*A - 139*B)*cos(d*x + c)^2 + 60*(A - 2*B)*cos(d*
x + c) + 20*B)*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^4 + 3*a
^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(9/2)/(a*sec(d*x + c) + a)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{9/2}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(9/2))/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(9/2))/(a + a/cos(c + d*x))^3, x
)
```

3.217 $\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=261

$$\frac{(9A-49B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{(3A-13B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{6a^3d}$$

```
[Out] 1/5*(A-B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3+1/15*(3*A-8*B)*s
ec(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2+1/6*(3*A-13*B)*sec(d*x+c)
^(3/2)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))-1/10*(9*A-49*B)*sin(d*x+c)*sec(d*x
+c)^(1/2)/a^3/d+1/10*(9*A-49*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/a^3/d+1/6*(3*A-13*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elli
pticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A]
time = 0.35, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.182, Rules used = {4104, 3872, 3856, 2720, 3853, 2719}

$$\frac{(3A-13B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{6d\left(a^3\sec(c+dx)+a^3\right)}-\frac{(9A-49B)\sin(c+dx)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{(3A-13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A-49B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{(3A-8B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((9*A - 49*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(10*a^3*d) + ((3*A - 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((9*A - 49*B)*Sqrt[Sec[c + d*x]]*Sin[c + d
*x])/(10*a^3*d) + ((A - B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*Sec
[c + d*x])^3) + ((3*A - 8*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(15*a*d*(a +
a*Sec[c + d*x])^2) + ((3*A - 13*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(6*d*(a
^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-\frac{1}{2}a(A-11B)\right.}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\sec^{\frac{5}{2}}(c+dx)\sin(c}{15ad(a+a\sec(c+dx))^2}$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\sec^{\frac{5}{2}}(c+dx)\sin(c}{15ad(a+a\sec(c+dx))^2}$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A-8B)\sec^{\frac{5}{2}}(c+dx)\sin(c}{15ad(a+a\sec(c+dx))^2}$$

$$= -\frac{(9A-49B)\sqrt{\sec(c+dx)}\sin(c+dx)}{10a^3d} + \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\,s}{5d(a+a\sec(c+}$$

$$= \frac{(3A-13B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d} - \frac{(9A}{}$$

$$= \frac{(9A-49B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(3A}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.44, size = 924, normalized size = 3.54

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,
x]
```

```
[Out] (-3*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((
2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d
*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -
E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(5*d*E^
(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (49*Sqrt[2]*B*Sqrt[E
^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos
[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x
)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]
)*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*E^(I*d*x)*(B + A*Cos[
c + d*x])*(a + a*Sec[c + d*x])^3) + (2*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c +
d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B
*Sec[c + d*x])*Sin[c])/(d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (2
```

```
6*B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2,
 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Co
s[c + d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(5
/2)*(A + B*Sec[c + d*x])*((2*(-9*A + 49*B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d
) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/
(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-3*A*Sin[(d*x)/2] + 8*B*Sin[(d*x)
/2]))/(15*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-3*A*Sin[(d*x)/2] + 13*B*Sin
[(d*x)/2]))/(3*d) - (4*(-3*A + 13*B)*Tan[c/2])/(3*d) - (4*(-3*A + 8*B)*Sec[
c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c/
2])/(5*d)))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(285) = 570$.
time = 1.98, size = 685, normalized size = 2.62

method	result
default	$\frac{-2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(15A\,\mathrm{EllipticF}\left(\cos\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/60*(-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(15*A*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))-27*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(15*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*A*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-65*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(15*A*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-27*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+
1/2*c)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(9*A-49*B)*s
in(1/2*d*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
(147*A-817*B)*sin(1/2*d*x+1/2*c)^6+6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(43*A-248*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*(69*A-439*B)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d
*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.62, size = 481, normalized size = 1.84

$$-\frac{5\left(\sqrt{2}(3iA-13iB)\cos(dx+c)^3+3\sqrt{2}(3iA-13iB)\cos(dx+c)^2+3\sqrt{2}(3iA-13iB)\cos(dx+c)+\sqrt{2}(3iA-13iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\left(\sqrt{2}(-3iA+13iB)\cos(dx+c)^3+3\sqrt{2}(-3iA+13iB)\cos(dx+c)^2+3\sqrt{2}(-3iA+13iB)\cos(dx+c)+\sqrt{2}(-3iA+13iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3\left(\sqrt{2}(-9iA+49iB)\cos(dx+c)^3+3\sqrt{2}(-9iA+49iB)\cos(dx+c)^2+3\sqrt{2}(-9iA+49iB)\cos(dx+c)+\sqrt{2}(-9iA+49iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(\sqrt{2}(9iA-49iB)\cos(dx+c)^3+3\sqrt{2}(9iA-49iB)\cos(dx+c)^2+3\sqrt{2}(9iA-49iB)\cos(dx+c)+\sqrt{2}(9iA-49iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(3(9A-49B)\cos(dx+c)^3+2(33A-188B)\cos(dx+c)^2+5(9A-59B)\cos(dx+c)-60B\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{60\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] -1/60*(5*(sqrt(2)*(3*I*A - 13*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(3*I*A - 13*I
*B)*cos(d*x + c)^2 + 3*sqrt(2)*(3*I*A - 13*I*B)*cos(d*x + c) + sqrt(2)*(3*I
*A - 13*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5
*(sqrt(2)*(-3*I*A + 13*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-3*I*A + 13*I*B)*co
s(d*x + c)^2 + 3*sqrt(2)*(-3*I*A + 13*I*B)*cos(d*x + c) + sqrt(2)*(-3*I*A +
 13*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(sq
rt(2)*(-9*I*A + 49*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-9*I*A + 49*I*B)*cos(d*
x + c)^2 + 3*sqrt(2)*(-9*I*A + 49*I*B)*cos(d*x + c) + sqrt(2)*(-9*I*A + 49*
I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c))) + 3*(sqrt(2)*(9*I*A - 49*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(9*I*
A - 49*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(9*I*A - 49*I*B)*cos(d*x + c) + sqrt
(2)*(9*I*A - 49*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c))) + 2*(3*(9*A - 49*B)*cos(d*x + c)^3 + 2*(33*A -
 188*B)*cos(d*x + c)^2 + 5*(9*A - 59*B)*cos(d*x + c) - 60*B)*sin(d*x + c)/s
qrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d
*cos(d*x + c) + a^3*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a/cos(c + d*x))^3, x
)
```

3.218 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=220

$$\frac{(A+9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{(A+3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{6a^3d}$$

```
[Out] 1/5*(A-B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3+1/15*(A-6*B)*sec
(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^2-1/10*(A+9*B)*sin(d*x+c)*sec
(d*x+c)^(1/2)/d/(a^3+a^3*sec(d*x+c))+1/10*(A+9*B)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/a^3/d+1/6*(A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/a^3/d
```

Rubi [A]
time = 0.33, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4104, 3872, 3856, 2719, 2720}

$$-\frac{(A+9B)\sin(c+dx)\sqrt{\sec(c+dx)}}{10d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A+9B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}+\frac{(A-6B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((A + 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])
/(10*a^3*d) + ((A + 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[
Sec[c + d*x]])/(6*a^3*d) + ((A - B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d*(
a + a*Sec[c + d*x])^3) + ((A - 6*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*a*
d*(a + a*Sec[c + d*x])^2) - ((A + 9*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(10
*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(A-B)+\frac{1}{2}a(A+9B\right.}{(a+a\sec(c+dx))^2}}{5a^2}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-6B)\sec^{\frac{3}{2}}(c+dx)\sin(c}{15ad(a+a\sec(c+dx))}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-6B)\sec^{\frac{3}{2}}(c+dx)\sin(c}{15ad(a+a\sec(c+dx))}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-6B)\sec^{\frac{3}{2}}(c+dx)\sin(c}{15ad(a+a\sec(c+dx))}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(A-6B)\sec^{\frac{3}{2}}(c+dx)\sin(c}{15ad(a+a\sec(c+dx))}$$

$$= \frac{(A+9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(A+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 7.02, size = 919, normalized size = 4.18

$$-\frac{\sqrt{2}\,Ae^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\csc\left(\frac{c}{2}\right)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\sec\left(\frac{c}{2}\right)\sec^2(c+dx)(A+B\sec(c+dx))}{15d(B+A\cos(c+dx))(a+a\sec(c+dx))^3}-\frac{3\sqrt{2}\,Be^{-idx}\sqrt{\frac{e^{i(c+dx)}}{1+e^{2i(c+dx)}}}\sqrt{1+e^{2i(c+dx)}}\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\csc\left(\frac{c}{2}\right)\left(-3\sqrt{1+e^{2i(c+dx)}}+e^{2idx}(-1+e^{2ic})\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)\right)\sec\left(\frac{c}{2}\right)\sec^2(c+dx)(A+B\sec(c+dx))}{5d(B+A\cos(c+dx))(a+a\sec(c+dx))^3}+\frac{2A\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\cos(c+dx)}\csc\left(\frac{c}{2}\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)\sec\left(\frac{c}{2}\right)\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))\sin(c)}{3d(B+A\cos(c+dx))(a+a\sec(c+dx))^3}+\frac{2B\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{\cos(c+dx)}\csc\left(\frac{c}{2}\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)\sec\left(\frac{c}{2}\right)\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))\sin(c)}{d(B+A\cos(c+dx))(a+a\sec(c+dx))^3}+\frac{\cos^6\left(\frac{c}{2}+\frac{dx}{2}\right)\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))\left(-\frac{2(A+9B)\cos(dx)\csc\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}\right)}{5d}+\frac{2\sec\left(\frac{c}{2}\right)\sec^5\left(\frac{c}{2}+\frac{dx}{2}\right)\left(-A\sin\left(\frac{dx}{2}\right)+B\sin\left(\frac{dx}{2}\right)\right)}{5d}+\frac{4\sec\left(\frac{c}{2}\right)\sec\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\sin\left(\frac{dx}{2}\right)+3B\sin\left(\frac{dx}{2}\right)\right)}{3d}+\frac{4\sec\left(\frac{c}{2}\right)\sec^3\left(\frac{c}{2}+\frac{dx}{2}\right)\left(2A\sin\left(\frac{dx}{2}\right)+3B\sin\left(\frac{dx}{2}\right)\right)}{15d}+\frac{4(A+3B)\tan\left(\frac{c}{2}\right)}{3d}+\frac{4(2A+3B)\sec^2\left(\frac{c}{2}+\frac{dx}{2}\right)\tan\left(\frac{c}{2}\right)}{15d}+\frac{2(-A+B)\sec^4\left(\frac{c}{2}+\frac{dx}{2}\right)\tan\left(\frac{c}{2}\right)}{5d}\right)}{(B+A\cos(c+dx))(a+a\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,
x]
```

```
[Out] -1/15*(Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E
^((2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c
+ d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4
, -E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(d*E
^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (3*Sqrt[2]*B*Sqrt[E
^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos
[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x
)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))]
)*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(5*d*E^(I*d*x)*(B + A*Cos[c
 + d*x])*(a + a*Sec[c + d*x])^3) + (2*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d
*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B*
Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (
2*B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2,
 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*Sin[c])/(d*(B + A*Cos[
c + d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(5/2
)*(A + B*Sec[c + d*x])*((-2*(A + 9*B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) + (
2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d)
 + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + 3*B*Sin[(d*x)/2]))/(3*d
) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(2*A*Sin[(d*x)/2] + 3*B*Sin[(d*x)/2]))
/(15*d) + (4*(A + 3*B)*Tan[c/2])/(3*d) + (4*(2*A + 3*B)*Sec[c/2 + (d*x)/2]^
2*Tan[c/2])/(15*d) + (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/((B
 + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [A]
time = 1.49, size = 451, normalized size = 2.05

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12A\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10A\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^8-10*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
```

```
(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*cos(1/
2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+108*B*cos(1/2*d*x+1/2*c)^8-30*B*co
s(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+54*B*cos(1/2*d*x+1/2*c)^5*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-22*A*cos(1/2*d*x+1/2*c)^6-138*B*cos(1/2*d*x+1/2*c)^6+
6*A*cos(1/2*d*x+1/2*c)^4+24*B*cos(1/2*d*x+1/2*c)^4+7*A*cos(1/2*d*x+1/2*c)^2
+3*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.51, size = 474, normalized size = 2.15

$$-\frac{5\left(\sqrt{2}\,(iA+3iB)\cos(dx+c)^3+3\sqrt{2}\,(iA+3iB)\cos(dx+c)^2+3\sqrt{2}\,(iA+3iB)\cos(dx+c)+\sqrt{2}\,(iA+3iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\left(\sqrt{2}\,(-iA-3iB)\cos(dx+c)^3+3\sqrt{2}\,(-iA-3iB)\cos(dx+c)^2+3\sqrt{2}\,(-iA-3iB)\cos(dx+c)+\sqrt{2}\,(-iA-3iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3\left(\sqrt{2}\,(-iA-9iB)\cos(dx+c)^3+3\sqrt{2}\,(-iA-9iB)\cos(dx+c)^2+3\sqrt{2}\,(-iA-9iB)\cos(dx+c)+\sqrt{2}\,(-iA-9iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\left(\sqrt{2}\,(iA+9iB)\cos(dx+c)^3+3\sqrt{2}\,(iA+9iB)\cos(dx+c)^2+3\sqrt{2}\,(iA+9iB)\cos(dx+c)+\sqrt{2}\,(iA+9iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+\frac{2\left(3(A+9B)\cos(dx+c)^3+2(2A+33B)\cos(dx+c)^2-5(A-9B)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{60\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] -1/60*(5*(sqrt(2)*(I*A + 3*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(I*A + 3*I*B)*co
s(d*x + c)^2 + 3*sqrt(2)*(I*A + 3*I*B)*cos(d*x + c) + sqrt(2)*(I*A + 3*I*B)
)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(sqrt(2)*(-
I*A - 3*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-I*A - 3*I*B)*cos(d*x + c)^2 + 3*s
qrt(2)*(-I*A - 3*I*B)*cos(d*x + c) + sqrt(2)*(-I*A - 3*I*B))*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(sqrt(2)*(-I*A - 9*I*B)*cos
(d*x + c)^3 + 3*sqrt(2)*(-I*A - 9*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-I*A - 9
*I*B)*cos(d*x + c) + sqrt(2)*(-I*A - 9*I*B))*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(sqrt(2)*(I*A + 9*I
*B)*cos(d*x + c)^3 + 3*sqrt(2)*(I*A + 9*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(I*
A + 9*I*B)*cos(d*x + c) + sqrt(2)*(I*A + 9*I*B))*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*(A + 9*B)*co
s(d*x + c)^3 + 2*(2*A + 33*B)*cos(d*x + c)^2 - 5*(A - 9*B)*cos(d*x + c))*si
```

```
n(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)
^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^3, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x))^3,x)
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x))^3, x
)
```

3.219 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=216

$$-\frac{(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{(A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}{6a^3d}$$

```
[Out] 1/5*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^3-1/15*(A+4*B)*sin
(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*(A+B)*sin(d*x+c)*sec(d*
x+c)^(1/2)/d/(a^3+a^3*sec(d*x+c))-1/10*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*se
c(d*x+c)^(1/2)/a^3/d+1/6*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)
/a^3/d
```

Rubi [A]
time = 0.33, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4104, 4105, 3872, 3856, 2719, 2720}

$$\frac{(A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{5d(a\sec(c+dx)+a)^3}-\frac{(A+4B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15ad(a\sec(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/10*((A - B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(a^3*d) + ((A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(6*a^3*d) + ((A - B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*(a
 + a*Sec[c + d*x])^3) - ((A + 4*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*a*d
*(a + a*Sec[c + d*x])^2) + ((A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(6*d*(
a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{\sqrt{\sec(c+dx)}\,(\frac{1}{2}a(A-B)+\frac{1}{2}a}{(a+a\sec(c+dx)}}{5a^2}$$

$$= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(A+4B)\sqrt{\sec(c+dx)}\,\sin(}{15ad(a+a\sec(c+dx)}$$

$$= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(A+4B)\sqrt{\sec(c+dx)}\,\sin(}{15ad(a+a\sec(c+dx)}$$

$$= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(A+4B)\sqrt{\sec(c+dx)}\,\sin(}{15ad(a+a\sec(c+dx)}$$

$$= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\sec(c+dx))^3} - \frac{(A+4B)\sqrt{\sec(c+dx)}\,\sin(}{15ad(a+a\sec(c+dx)}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(A+}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.97, size = 918, normalized size = 4.25

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,
x]
```

```
[Out] (Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I
)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x)
)] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^(
(2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*E^(I
*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (Sqrt[2]*B*Sqrt[E^(I*(
c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2
+ (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-1
 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Sec
[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*E^(I*d*x)*(B + A*Cos[c + d
*x])*(a + a*Sec[c + d*x])^3) + (2*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]
*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[
c + d*x])*Sin[c])/(3*d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (2*B*
Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*
```

```
Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c
+ d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(5/2)*
(A + B*Sec[c + d*x])*((-2*(-A + B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) - (2*S
ec[c/2]*Sec[c/2 + (d*x)/2]^5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) +
(4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/(3*d) + (
4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-7*A*Sin[(d*x)/2] + 2*B*Sin[(d*x)/2]))/(15
*d) + (4*(A + B)*Tan[c/2])/(3*d) + (4*(-7*A + 2*B)*Sec[c/2 + (d*x)/2]^2*Tan
[c/2])/(15*d) - (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/((B + A*
Cos[c + d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [A]
time = 1.45, size = 451, normalized size = 2.09

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(12A\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10A\left(\cos^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^8+10*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*cos(1
/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^8+10*B*co
s(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-6*B*cos(1/2*d*x+1/2*c)^5*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)^6+22*B*cos(1/2*d*x+1/2*c)^6-24*
A*cos(1/2*d*x+1/2*c)^4-6*B*cos(1/2*d*x+1/2*c)^4+17*A*cos(1/2*d*x+1/2*c)^2-7
*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

[Out] Timed out

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.70, size = 472, normalized size = 2.19

$$-\frac{5\left(\sqrt{2}\left(iA+iB\right)\cos\left(dx+c\right)^{3}+3\sqrt{2}\left(iA+iB\right)\cos\left(dx+c\right)^{2}+3\sqrt{2}\left(iA+iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(iA+iB\right)\right){\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+5\left(\sqrt{2}\left(-iA-iB\right)\cos\left(dx+c\right)^{3}+3\sqrt{2}\left(-iA-iB\right)\cos\left(dx+c\right)^{2}+3\sqrt{2}\left(-iA-iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-iA-iB\right)\right){\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+3\left(\sqrt{2}\left(iA-iB\right)\cos\left(dx+c\right)^{3}+3\sqrt{2}\left(iA-iB\right)\cos\left(dx+c\right)^{2}+3\sqrt{2}\left(iA-iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(iA-iB\right)\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+3\left(\sqrt{2}\left(-iA+iB\right)\cos\left(dx+c\right)^{3}+3\sqrt{2}\left(-iA+iB\right)\cos\left(dx+c\right)^{2}+3\sqrt{2}\left(-iA+iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-iA+iB\right)\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-\frac{2\left(3\left(A-B\right)\cos\left(dx+c\right)^{3}+2\left(7A-2B\right)\cos\left(dx+c\right)^{2}+5\left(A+B\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{60\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] -1/60*(5*(sqrt(2)*(I*A + I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(I*A + I*B)*cos(d*
x + c)^2 + 3*sqrt(2)*(I*A + I*B)*cos(d*x + c) + sqrt(2)*(I*A + I*B))*weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(sqrt(2)*(-I*A - I*
B)*cos(d*x + c)^3 + 3*sqrt(2)*(-I*A - I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-I*A
 - I*B)*cos(d*x + c) + sqrt(2)*(-I*A - I*B))*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 3*(sqrt(2)*(I*A - I*B)*cos(d*x + c)^3 + 3*sqr
t(2)*(I*A - I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(I*A - I*B)*cos(d*x + c) + sqrt
(2)*(I*A - I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + 3*(sqrt(2)*(-I*A + I*B)*cos(d*x + c)^3 + 3*sqrt(2
)*(-I*A + I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-I*A + I*B)*cos(d*x + c) + sqrt(
2)*(-I*A + I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) - I*sin(d*x + c))) - 2*(3*(A - B)*cos(d*x + c)^3 + 2*(7*A - 2*B)*cos(d
*x + c)^2 + 5*(A + B)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d
*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x))^3,x)
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x))^3, x
)
```

3.220 $\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=222

$$-\frac{(9A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{(3A+B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec}}{6a^3d}$$

```
[Out] 1/5*(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^3+1/15*(3*A+2*B)*s
in(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2+1/6*(3*A+B)*sin(d*x+c)*se
c(d*x+c)^(1/2)/d/(a^3+a^3*sec(d*x+c))-1/10*(9*A+B)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1
/2)*sec(d*x+c)^(1/2)/a^3/d+1/6*(3*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+
c)^(1/2)/a^3/d
```

Rubi [A]
time = 0.33, antiderivative size = 222, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4104, 4105, 3872, 3856, 2719, 2720}

$$\frac{(3A+B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\left(a^3\sec(c+dx)+a^3\right)}+\frac{(3A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A+B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(3A+2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15ad(a\sec(c+dx)+a)^2}+\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] -1/10*((9*A + B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(a^3*d) + ((3*A + B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(6*a^3*d) + ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*
d*(a + a*Sec[c + d*x])^3) + ((3*A + 2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(
15*a*d*(a + a*Sec[c + d*x])^2) + ((3*A + B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{\int \frac{-\frac{1}{2}a(A-B)+\frac{5}{2}a(A+B)\sec(c}{\sqrt{\sec(c+dx)}\,(a+a\sec(c}}{5a^2}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A+2B)\sqrt{\sec(c+dx)}}{15ad(a+a\sec(c+}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A+2B)\sqrt{\sec(c+dx)}}{15ad(a+a\sec(c+}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A+2B)\sqrt{\sec(c+dx)}}{15ad(a+a\sec(c+}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3} + \frac{(3A+2B)\sqrt{\sec(c+dx)}}{15ad(a+a\sec(c+}$$

$$= -\frac{(9A+B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(3}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.07, size = 919, normalized size = 4.14

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,
x]
```

```
[Out] (3*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2
*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*
x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E
^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(5*d*E^(
I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (Sqrt[2]*B*Sqrt[E^(I*
(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*Cos[c/2
 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d*x)*(-
1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])*Se
c[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*E^(I*d*x)*(B + A*Cos[c +
d*x])*(a + a*Sec[c + d*x])^3) + (2*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]
]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Sec
[c + d*x])*Sin[c])/(d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (2*B*C
os[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*S
```

```
ec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*Sin[c])/(3*d*(B + A*Cos[c +
 d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(5/2)*(
A + B*Sec[c + d*x])*((2*(9*A + B)*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) + (4*Se
c[c/2]*Sec[c/2 + (d*x)/2]*(-9*A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/(3*d) + (2*
Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) -
 (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-12*A*Sin[(d*x)/2] + 7*B*Sin[(d*x)/2]))/
(15*d) + (4*(-9*A + B)*Tan[c/2])/(3*d) - (4*(-12*A + 7*B)*Sec[c/2 + (d*x)/2
]^2*Tan[c/2])/(15*d) + (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(
(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [A]
time = 1.50, size = 451, normalized size = 2.03

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(108A\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+30A\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*co
s(1/2*d*x+1/2*c)^8+30*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+54*A
*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*cos(1/2*d*x+1/2*c)^8+1
0*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*B*cos(1/2*d*x+1/2*c)^5
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))-198*A*cos(1/2*d*x+1/2*c)^6-2*B*cos(1/2*d*x+1/2*c
)^6+114*A*cos(1/2*d*x+1/2*c)^4-24*B*cos(1/2*d*x+1/2*c)^4-27*A*cos(1/2*d*x+1
/2*c)^2+17*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

[Out] Timed out

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 474, normalized size = 2.14

$$-\frac{5\left(\sqrt{2}\,(3i\,A+i\,B)\cos\left(dx+c\right)^{3}+3\,\sqrt{2}\,(3i\,A+i\,B)\cos\left(dx+c\right)^{2}+3\,\sqrt{2}\,(3i\,A+i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(3i\,A+i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+5\left(\sqrt{2}\,(-3i\,A-i\,B)\cos\left(dx+c\right)^{3}+3\,\sqrt{2}\,(-3i\,A-i\,B)\cos\left(dx+c\right)^{2}+3\,\sqrt{2}\,(-3i\,A-i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(-3i\,A-i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+3\left(\sqrt{2}\,(9i\,A+i\,B)\cos\left(dx+c\right)^{3}+3\,\sqrt{2}\,(9i\,A+i\,B)\cos\left(dx+c\right)^{2}+3\,\sqrt{2}\,(9i\,A+i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(9i\,A+i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))+3\left(\sqrt{2}\,(-9i\,A-i\,B)\cos\left(dx+c\right)^{3}+3\,\sqrt{2}\,(-9i\,A-i\,B)\cos\left(dx+c\right)^{2}+3\,\sqrt{2}\,(-9i\,A-i\,B)\cos\left(dx+c\right)+\sqrt{2}\,(-9i\,A-i\,B)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))-\frac{2\left(3\,(9\,A+B)\cos\left(dx+c\right)^{3}+2\,(18\,A+7\,B)\cos\left(dx+c\right)^{2}+5\,(3\,A+B)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{60\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] -1/60*(5*(sqrt(2)*(3*I*A + I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(3*I*A + I*B)*co
s(d*x + c)^2 + 3*sqrt(2)*(3*I*A + I*B)*cos(d*x + c) + sqrt(2)*(3*I*A + I*B)
)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(sqrt(2)*(-
3*I*A - I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-3*I*A - I*B)*cos(d*x + c)^2 + 3*s
qrt(2)*(-3*I*A - I*B)*cos(d*x + c) + sqrt(2)*(-3*I*A - I*B))*weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(sqrt(2)*(9*I*A + I*B)*cos(
d*x + c)^3 + 3*sqrt(2)*(9*I*A + I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(9*I*A + I*
B)*cos(d*x + c) + sqrt(2)*(9*I*A + I*B))*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(sqrt(2)*(-9*I*A - I*B)
*cos(d*x + c)^3 + 3*sqrt(2)*(-9*I*A - I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-9*I
*A - I*B)*cos(d*x + c) + sqrt(2)*(-9*I*A - I*B))*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(3*(9*A + B)*co
s(d*x + c)^3 + 2*(18*A + 7*B)*cos(d*x + c)^2 + 5*(3*A + B)*cos(d*x + c))*si
n(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)
^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x))^3, x
)
```

3.221 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=228

$$\frac{(49A-9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(13A-3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{s}}}{6a^3d}$$

```
[Out] -1/5*(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^3-1/15*(8*A-3*B)*
sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^2-1/6*(13*A-3*B)*sin(d*x+c
)*sec(d*x+c)^(1/2)/d/(a^3+a^3*sec(d*x+c))+1/10*(49*A-9*B)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d-1/6*(13*A-3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2
)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A]
time = 0.34, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4105, 3872, 3856, 2719, 2720}

$$-\frac{(13A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6d\,(a^3\sec(c+dx)+a^3)}-\frac{(13A-3B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(49A-9B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(8A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15ad(a\sec(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] ((49*A - 9*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x
]])/(10*a^3*d) - ((13*A - 3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]
*Sqrt[Sec[c + d*x]])/(6*a^3*d) - ((A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(5*d*(a + a*Sec[c + d*x])^3) - ((8*A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x]
)/(15*a*d*(a + a*Sec[c + d*x])^2) - ((13*A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(6*d*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3}+\frac{\int \frac{\frac{1}{2}a(11A-B)-\frac{5}{2}a(A-B)\sec(\cdots}{\sqrt{\sec(c+dx)}\,(a+a\sec(\cdots}}{5a^2}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{(8A-3B)\sqrt{\sec(c+dx)}\cdots}{15ad(a+a\sec(c+\cdots}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{(8A-3B)\sqrt{\sec(c+dx)}\cdots}{15ad(a+a\sec(c+\cdots}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{(8A-3B)\sqrt{\sec(c+dx)}\cdots}{15ad(a+a\sec(c+\cdots}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d(a+a\sec(c+dx))^3}-\frac{(8A-3B)\sqrt{\sec(c+dx)}\cdots}{15ad(a+a\sec(c+\cdots}$$

$$= \frac{(49A-9B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(13\cdots}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.63, size = 364, normalized size = 1.60

$$-\frac{e^{-idx}\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))(\cos(dx)+i\sin(dx))\left(160(13A-3B)\cos^5\left(\frac{1}{2}(c+dx)\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+i(49A-9B)e^{-\frac{3}{2}i(c+dx)}\left(1+e^{i(c+dx)}\right)^5\sqrt{1+e^{2i(c+dx)}}\,{}_2F_1\left(\frac{1}{2},\frac{3}{4};\frac{7}{4};-e^{2i(c+dx)}\right)+2\cos(c+dx)\left(-30i(49A-9B)\cos\left(\frac{1}{2}(c+dx)\right)-15i(49A-9B)\cos\left(\frac{3}{2}(c+dx)\right)-147iA\cos\left(\frac{5}{2}(c+dx)\right)+27iB\cos\left(\frac{5}{2}(c+dx)\right)+142A\sin\left(\frac{1}{2}(c+dx)\right)-42B\sin\left(\frac{1}{2}(c+dx)\right)+205A\sin\left(\frac{3}{2}(c+dx)\right)-45B\sin\left(\frac{3}{2}(c+dx)\right)+87A\sin\left(\frac{5}{2}(c+dx)\right)-27B\sin\left(\frac{5}{2}(c+dx)\right)\right)\right)}{120a^3d(B+A\cos(c+dx))(1+\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^3),
x]
```

```
[Out] -1/120*(Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*(Cos[d*x]
+ I*Sin[d*x])*(160*(13*A - 3*B)*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2] + (I*(49*A - 9*B)*(1 + E^(I*(c + d*x)))^5*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E
^(((3*I)/2)*(c + d*x)) + 2*Cos[c + d*x]*((-30*I)*(49*A - 9*B)*Cos[(c + d*x)
/2] - (15*I)*(49*A - 9*B)*Cos[(3*(c + d*x))/2] - (147*I)*A*Cos[(5*(c + d*x)
)/2] + (27*I)*B*Cos[(5*(c + d*x))/2] + 142*A*Sin[(c + d*x)/2] - 42*B*Sin[(c
 + d*x)/2] + 205*A*Sin[(3*(c + d*x))/2] - 45*B*Sin[(3*(c + d*x))/2] + 87*A*
Sin[(5*(c + d*x))/2] - 27*B*Sin[(5*(c + d*x))/2])))/(a^3*d*E^(I*d*x)*(B + A
*Cos[c + d*x])*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 1.68, size = 451, normalized size = 1.98

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(348A\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+130A\left(\cos^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{\cdots}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*A*cos
(1/2*d*x+1/2*c)^8+130*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+294*
A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-108*B*cos(1/2*d*x+1/2*c)^8
-30*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-54*B*cos(1/2*d*x+1/2*c
)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2))-578*A*cos(1/2*d*x+1/2*c)^6+198*B*cos(1/2*d*x+
1/2*c)^6+264*A*cos(1/2*d*x+1/2*c)^4-114*B*cos(1/2*d*x+1/2*c)^4-37*A*cos(1/2
*d*x+1/2*c)^2+27*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/cos(1/2*d*x+1/2*c)^5/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.91, size = 478, normalized size = 2.10

$$-\frac{5\left(\sqrt{2}\left(-13 i A+3 i B\right)\cos\left(dx+c\right)^{3}+3\sqrt{2}\left(-13 i A+3 i B\right)\cos\left(dx+c\right)^{2}+3\sqrt{2}\left(-13 i A+3 i B\right)\cos\left(dx+c\right)+\sqrt{2}\left(-13 i A+3 i B\right)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+5\left(\sqrt{2}\left(13 i A-3 i B\right)\cos\left(dx+c\right)^{3}+3\sqrt{2}\left(13 i A-3 i B\right)\cos\left(dx+c\right)^{2}+3\sqrt{2}\left(13 i A-3 i B\right)\cos\left(dx+c\right)+\sqrt{2}\left(13 i A-3 i B\right)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+3\left(\sqrt{2}\left(-49 i A+9 i B\right)\cos\left(dx+c\right)^{3}+3\sqrt{2}\left(-49 i A+9 i B\right)\cos\left(dx+c\right)^{2}+3\sqrt{2}\left(-49 i A+9 i B\right)\cos\left(dx+c\right)+\sqrt{2}\left(-49 i A+9 i B\right)\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+3\left(\sqrt{2}\left(49 i A-9 i B\right)\cos\left(dx+c\right)^{3}+3\sqrt{2}\left(49 i A-9 i B\right)\cos\left(dx+c\right)^{2}+3\sqrt{2}\left(49 i A-9 i B\right)\cos\left(dx+c\right)+\sqrt{2}\left(49 i A-9 i B\right)\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)+\frac{2\left(3\left(29A-9B\right)\cos\left(dx+c\right)^{3}+2\left(73A-18B\right)\cos\left(dx+c\right)^{2}+5\left(13A-3B\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{60\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/60*(5*(sqrt(2)*(-13*I*A + 3*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-13*I*A + 3
*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-13*I*A + 3*I*B)*cos(d*x + c) + sqrt(2)*(
-13*I*A + 3*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
 + 5*(sqrt(2)*(13*I*A - 3*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(13*I*A - 3*I*B)*
cos(d*x + c)^2 + 3*sqrt(2)*(13*I*A - 3*I*B)*cos(d*x + c) + sqrt(2)*(13*I*A
 - 3*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(sq
rt(2)*(-49*I*A + 9*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-49*I*A + 9*I*B)*cos(d*
x + c)^2 + 3*sqrt(2)*(-49*I*A + 9*I*B)*cos(d*x + c) + sqrt(2)*(-49*I*A + 9*
I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c))) + 3*(sqrt(2)*(49*I*A - 9*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(49*I
*A - 9*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(49*I*A - 9*I*B)*cos(d*x + c) + sqrt
(2)*(49*I*A - 9*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c))) + 2*(3*(29*A - 9*B)*cos(d*x + c)^3 + 2*(73*A -
 18*B)*cos(d*x + c)^2 + 5*(13*A - 3*B)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(
d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x
 + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sec^{\frac{7}{2}}(c+dx)+3\sec^{\frac{5}{2}}(c+dx)+3\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx+\int \frac{B\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)+3\sec^{\frac{5}{2}}(c+dx)+3\sec^{\frac{3}{2}}(c+dx)+\sqrt{\sec(c+dx)}}\,dx}{a^{3}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3/sec(d*x+c)**(1/2),x)
```

```
[Out] (Integral(A/(sec(c + d*x)**(7/2) + 3*sec(c + d*x)**(5/2) + 3*sec(c + d*x)**
(3/2) + sqrt(sec(c + d*x))), x) + Integral(B*sec(c + d*x)/(sec(c + d*x)**(7
/2) + 3*sec(c + d*x)**(5/2) + 3*sec(c + d*x)**(3/2) + sqrt(sec(c + d*x))),
x))/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^3\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)), x
)
```

3.222 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=261

$$-\frac{7(17A-7B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}+\frac{(33A-13B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}$$

```
[Out] 1/6*(33*A-13*B)*sin(d*x+c)/a^3/d/sec(d*x+c)^(1/2)-1/5*(A-B)*sin(d*x+c)/d/(a
+a*sec(d*x+c))^3/sec(d*x+c)^(1/2)-1/3*(2*A-B)*sin(d*x+c)/a/d/(a+a*sec(d*x+c
))^2/sec(d*x+c)^(1/2)-7/30*(17*A-7*B)*sin(d*x+c)/d/(a^3+a^3*sec(d*x+c))/sec
(d*x+c)^(1/2)-7/10*(17*A-7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
a^3/d+1/6*(33*A-13*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A]
time = 0.37, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4105, 3872, 3854, 3856, 2720, 2719}

$$\frac{(33A-13B)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}}-\frac{7(17A-7B)\sin(c+dx)}{30d\sqrt{\sec(c+dx)}\,(a^3\sec(c+dx)+a^3)}+\frac{(33A-13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{7(17A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(2A-B)\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (-7*(17*A - 7*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(10*a^3*d) + ((33*A - 13*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(6*a^3*d) + ((33*A - 13*B)*Sin[c + d*x])/(6*a^3*d*
Sqrt[Sec[c + d*x]]) - ((A - B)*Sin[c + d*x])/(5*d*Sqrt[Sec[c + d*x]]*(a + a
*Sec[c + d*x])^3) - ((2*A - B)*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]]*(a +
 a*Sec[c + d*x])^2) - (7*(17*A - 7*B)*Sin[c + d*x])/(30*d*Sqrt[Sec[c + d*x]
]*(a^3 + a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(13A-3B)-\frac{7}{2}a(A-B)\sec(\cdots}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx\cdots}}{5a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{(2A-B)\sin(c\cdots}{3ad\sqrt{\sec(c+dx)}\,(a+a\cdots}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{(2A-B)\sin(c\cdots}{3ad\sqrt{\sec(c+dx)}\,(a+a\cdots}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} - \frac{(2A-B)\sin(c\cdots}{3ad\sqrt{\sec(c+dx)}\,(a+a\cdots}$$

$$= \frac{(33A-13B)\sin(c+dx)}{6a^3d\sqrt{\sec(c+dx)}} - \frac{(A-B)\sin(c+dx)}{5d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^3} \cdots$$

$$= -\frac{7(17A-7B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(3\cdots}{\cdots}$$

$$= -\frac{7(17A-7B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d} + \frac{(3\cdots}{\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.01, size = 377, normalized size = 1.44

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),
x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*(Cos[d*x] + I*Sin
[d*x])*(160*(33*A - 13*B)*Cos[(c + d*x)/2]^5*Sqrt[Cos[c + d*x]]*EllipticF[(
c + d*x)/2, 2] + ((7*I)*(17*A - 7*B)*(1 + E^(I*(c + d*x)))^5*Sqrt[1 + E^((2
*I)*(c + d*x))]*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x))])/E^(
((3*I)/2)*(c + d*x)) + 2*Cos[c + d*x]*((-210*I)*(17*A - 7*B)*Cos[(c + d*x)/
2] - (105*I)*(17*A - 7*B)*Cos[(3*(c + d*x))/2] - (357*I)*A*Cos[(5*(c + d*x)
)/2] + (147*I)*B*Cos[(5*(c + d*x))/2] + 352*A*Sin[(c + d*x)/2] - 142*B*Sin[
(c + d*x)/2] + 545*A*Sin[(3*(c + d*x))/2] - 205*B*Sin[(3*(c + d*x))/2] + 22
7*A*Sin[(5*(c + d*x))/2] - 87*B*Sin[(5*(c + d*x))/2] + 10*A*Sin[(7*(c + d*x
))/2])))/(120*a^3*d*E^(I*d*x)*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x])^3)
```

Maple [A]
time = 1.65, size = 465, normalized size = 1.78

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(160A\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+468A\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+330A\left(\cos^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*A*co
s(1/2*d*x+1/2*c)^10+468*A*cos(1/2*d*x+1/2*c)^8+330*A*cos(1/2*d*x+1/2*c)^5*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))+714*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-348*B*cos(1/2*d*x+1/2*c)^8-130*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))-294*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*A*cos(1/2
*d*x+1/2*c)^6+578*B*cos(1/2*d*x+1/2*c)^6+474*A*cos(1/2*d*x+1/2*c)^4-264*B*c
os(1/2*d*x+1/2*c)^4-47*A*cos(1/2*d*x+1/2*c)^2+37*B*cos(1/2*d*x+1/2*c)^2+3*A
-3*B)/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.50, size = 489, normalized size = 1.87

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] -1/60*(5*(sqrt(2)*(33*I*A - 13*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(33*I*A - 13
*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(33*I*A - 13*I*B)*cos(d*x + c) + sqrt(2)*(
33*I*A - 13*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
 + 5*(sqrt(2)*(-33*I*A + 13*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-33*I*A + 13*I
*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-33*I*A + 13*I*B)*cos(d*x + c) + sqrt(2)*(-
33*I*A + 13*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
 + 21*(sqrt(2)*(17*I*A - 7*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(17*I*A - 7*I*B)
*cos(d*x + c)^2 + 3*sqrt(2)*(17*I*A - 7*I*B)*cos(d*x + c) + sqrt(2)*(17*I*A
 - 7*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c))) + 21*(sqrt(2)*(-17*I*A + 7*I*B)*cos(d*x + c)^3 + 3*sqrt(2
)*(-17*I*A + 7*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-17*I*A + 7*I*B)*cos(d*x +
c) + sqrt(2)*(-17*I*A + 7*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(
-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(20*A*cos(d*x + c)^4 + 3*(79*A -
 29*B)*cos(d*x + c)^3 + 2*(188*A - 73*B)*cos(d*x + c)^2 + 5*(33*A - 13*B)*c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3
*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2)),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^3\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)), x
)
```

3.223 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=294

$$\frac{7(33A-17B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{10a^3d}-\frac{(21A-11B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}$$

```
[Out] 7/30*(33*A-17*B)*sin(d*x+c)/a^3/d/sec(d*x+c)^(3/2)-1/5*(A-B)*sin(d*x+c)/d/s
ec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3-1/15*(12*A-7*B)*sin(d*x+c)/a/d/sec(d*x+c
)^(3/2)/(a+a*sec(d*x+c))^2-3/10*(21*A-11*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(
a^3+a^3*sec(d*x+c))-1/2*(21*A-11*B)*sin(d*x+c)/a^3/d/sec(d*x+c)^(1/2)+7/10*
(33*A-17*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d-1/2*(21*A-11
*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/d
```

Rubi [A]
time = 0.39, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4105, 3872, 3854, 3856, 2719, 2720}

$$-\frac{3(21A-11B)\sin(c+dx)}{10d\sec^{\frac{3}{2}}(c+dx)\left(a^3\sec(c+dx)+a^3\right)}+\frac{7(33A-17B)\sin(c+dx)}{30a^3d\sec^{\frac{3}{2}}(c+dx)}-\frac{(21A-11B)\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}}-\frac{(21A-11B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{2a^3d}+\frac{7(33A-17B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(12A-7B)\sin(c+dx)}{15ad\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] (7*(33*A - 17*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(10*a^3*d) - ((21*A - 11*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(2*a^3*d) + (7*(33*A - 17*B)*Sin[c + d*x])/(30*a^3
*d*Sec[c + d*x]^(3/2)) - ((21*A - 11*B)*Sin[c + d*x])/(2*a^3*d*Sqrt[Sec[c +
 d*x]]) - ((A - B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x
])^3) - ((12*A - 7*B)*Sin[c + d*x])/(15*a*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c
 + d*x])^2) - (3*(21*A - 11*B)*Sin[c + d*x])/(10*d*Sec[c + d*x]^(3/2)*(a^3
+ a^3*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = -\frac{(A-B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} + \frac{\int \frac{\frac{5}{2}a(3A-B)-\frac{9}{2}a(A-B)\sec(c+dx}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{(12A-7B)\sin(c+}{15ad\sec^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{se}}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{(12A-7B)\sin(c+}{15ad\sec^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{se}}$$

$$= -\frac{(A-B)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3} - \frac{(12A-7B)\sin(c+}{15ad\sec^{\frac{3}{2}}(c+dx)(a+a\,\mathrm{se}}$$

$$= \frac{7(33A-17B)\sin(c+dx)}{30a^3d\sec^{\frac{3}{2}}(c+dx)} - \frac{(21A-11B)\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{(}{5d\sec^{\frac{3}{2}}(c}$$

$$= \frac{7(33A-17B)\sin(c+dx)}{30a^3d\sec^{\frac{3}{2}}(c+dx)} - \frac{(21A-11B)\sin(c+dx)}{2a^3d\sqrt{\sec(c+dx)}} - \frac{(}{5d\sec^{\frac{3}{2}}(c}$$

$$= \frac{7(33A-17B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{10a^3d} - \frac{(2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.48, size = 1032, normalized size = 3.51

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),
x]
```

```
[Out] (-77*Sqrt[2]*A*Sqrt[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^(
(2*I)*(c + d*x))]*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c +
d*x))] + E^((2*I)*d*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4,
-E^((2*I)*(c + d*x))])*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(5*d*E
^(I*d*x)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (119*Sqrt[2]*B*Sqrt
[E^(I*(c + d*x))/(1 + E^((2*I)*(c + d*x)))]*Sqrt[1 + E^((2*I)*(c + d*x))]*C
os[c/2 + (d*x)/2]^6*Csc[c/2]*(-3*Sqrt[1 + E^((2*I)*(c + d*x))] + E^((2*I)*d
*x)*(-1 + E^((2*I)*c))*Hypergeometric2F1[1/2, 3/4, 7/4, -E^((2*I)*(c + d*x)
)])*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(15*d*E^(I*d*x)*(B + A*Co
s[c + d*x])*(a + a*Sec[c + d*x])^3) - (42*A*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c
```

```
 + d*x]]*Csc[c/2]*EllipticF[(c + d*x)/2, 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A
+ B*Sec[c + d*x])*Sin[c])/(d*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) +
 (22*B*Cos[c/2 + (d*x)/2]^6*Sqrt[Cos[c + d*x]]*Csc[c/2]*EllipticF[(c + d*x)
/2, 2]*Sec[c/2]*Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x])*Sin[c])/(d*(B + A*C
os[c + d*x])*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*Sec[c + d*x]^(
5/2)*(A + B*Sec[c + d*x])*(((-329*A + 178*B - 133*A*Cos[2*c] + 60*B*Cos[2*c
])*Cos[d*x]*Csc[c/2]*Sec[c/2])/(5*d) + (8*(-3*A + B)*Cos[2*d*x]*Sin[2*c])/(
3*d) + (4*A*Cos[3*d*x]*Sin[3*c])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(
-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]
^3*(-27*A*Sin[(d*x)/2] + 22*B*Sin[(d*x)/2]))/(15*d) - (4*Sec[c/2]*Sec[c/2 +
 (d*x)/2]*(-69*A*Sin[(d*x)/2] + 43*B*Sin[(d*x)/2]))/(3*d) - (4*(-133*A + 60
*B)*Cos[c]*Sin[d*x])/(5*d) + (8*(-3*A + B)*Cos[2*c]*Sin[2*d*x])/(3*d) + (4*
A*Cos[3*c]*Sin[3*d*x])/(5*d) - (4*(-69*A + 43*B)*Tan[c/2])/(3*d) + (4*(-27*
A + 22*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(-A + B)*Sec[c/2 + (d*
x)/2]^4*Tan[c/2])/(5*d)))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [A]
time = 1.60, size = 493, normalized size = 1.68

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(192A\left(\cos^{12}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-864A\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+160B\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/60/a^3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(192*A*co
s(1/2*d*x+1/2*c)^12-864*A*cos(1/2*d*x+1/2*c)^10+160*B*cos(1/2*d*x+1/2*c)^10
-228*A*cos(1/2*d*x+1/2*c)^8-630*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-1386*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+468*B*cos(1/2*d
*x+1/2*c)^8+330*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+714*B*cos(
1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+1590*A*cos(1/2*d*x+1/2*c)^6-1058
*B*cos(1/2*d*x+1/2*c)^6-744*A*cos(1/2*d*x+1/2*c)^4+474*B*cos(1/2*d*x+1/2*c)
^4+57*A*cos(1/2*d*x+1/2*c)^2-47*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/cos(1/2*d*x
+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+
1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.00, size = 506, normalized size = 1.72

$$-\frac{15\left(\sqrt{2}(-21iA+11iB)\cos(dx+c)^3+3\sqrt{2}(-21iA+11iB)\cos(dx+c)^2+3\sqrt{2}(-21iA+11iB)\cos(dx+c)+\sqrt{2}(-21iA+11iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+15\left(\sqrt{2}(21iA-11iB)\cos(dx+c)^3+3\sqrt{2}(21iA-11iB)\cos(dx+c)^2+3\sqrt{2}(21iA-11iB)\cos(dx+c)+\sqrt{2}(21iA-11iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+21\left(\sqrt{2}(-33iA+17iB)\cos(dx+c)^3+3\sqrt{2}(-33iA+17iB)\cos(dx+c)^2+3\sqrt{2}(-33iA+17iB)\cos(dx+c)+\sqrt{2}(-33iA+17iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21\left(\sqrt{2}(33iA-17iB)\cos(dx+c)^3+3\sqrt{2}(33iA-17iB)\cos(dx+c)^2+3\sqrt{2}(33iA-17iB)\cos(dx+c)+\sqrt{2}(33iA-17iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{2\left(12A\cos(dx+c)^5-4(6A-5B)\cos(dx+c)^4-3(147A-79B)\cos(dx+c)^3-2(357A-188B)\cos(dx+c)^2-15(21A-11B)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{60\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] -1/60*(15*(sqrt(2)*(-21*I*A + 11*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-21*I*A +
 11*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-21*I*A + 11*I*B)*cos(d*x + c) + sqrt(
2)*(-21*I*A + 11*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
+ c)) + 15*(sqrt(2)*(21*I*A - 11*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(21*I*A -
11*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(21*I*A - 11*I*B)*cos(d*x + c) + sqrt(2)
*(21*I*A - 11*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c
)) + 21*(sqrt(2)*(-33*I*A + 17*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-33*I*A + 1
7*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-33*I*A + 17*I*B)*cos(d*x + c) + sqrt(2)
*(-33*I*A + 17*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c))) + 21*(sqrt(2)*(33*I*A - 17*I*B)*cos(d*x + c)^3
+ 3*sqrt(2)*(33*I*A - 17*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(33*I*A - 17*I*B)*
cos(d*x + c) + sqrt(2)*(33*I*A - 17*I*B))*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(12*A*cos(d*x + c)^5 -
 4*(6*A - 5*B)*cos(d*x + c)^4 - 3*(147*A - 79*B)*cos(d*x + c)^3 - 2*(357*A
- 188*B)*cos(d*x + c)^2 - 15*(21*A - 11*B)*cos(d*x + c))*sin(d*x + c)/sqrt(
cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos
(d*x + c) + a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*sec(d*x + c)^(5/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^3\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^3*(1/cos(c + d*x))^(5/2)), x
)
```

3.224 $\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=176

$$\frac{\sqrt{a}\,(6A+5B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{a(6A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{a(6A+5B)\sec^{\frac{5}{2}}(c+}{12d\sqrt{a+a\,\mathrm{se}}}$$

```
[Out] 1/8*(6*A+5*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+
1/8*a*(6*A+5*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/12*a
*(6*A+5*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/3*a*B*sec
(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.19, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4101, 3888, 3886, 221}

$$\frac{a(6A+5B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{a(6A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}\,(6A+5B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{aB\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{3d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[a]*(6*A + 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(8*d) + (a*(6*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt[a + a
*Sec[c + d*x]]) + (a*(6*A + 5*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(12*d*Sqr
t[a + a*Sec[c + d*x]]) + (a*B*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*Sqrt[a
+ a*Sec[c + d*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \frac{aB\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{1}{6}(6A+5B)$$

$$= \frac{a(6A+5B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{aB\,\mathrm{s}}{3d}$$

$$= \frac{a(6A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{a(6A}{}$$

$$= \frac{a(6A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{a(6A}{}$$

$$= \frac{\sqrt{a}\,(6A+5B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right.}{8d}$$

Mathematica [A]
time = 1.48, size = 131, normalized size = 0.74

$$\frac{\sqrt{a(1+\sec(c+dx))}\left(3\sqrt{2}\,(6A+5B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\sec\left(\frac{1}{2}(c+dx)\right)+(18A+31B+4(6A+5B)\cos(c+dx)+3(6A+5B)\cos(2(c+dx)))\sec^3(c+dx)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{48d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(6*A + 5*B)*ArcTanh[Sqrt[2]*Sin[(c +
 d*x)/2]]*Sec[(c + d*x)/2] + (18*A + 31*B + 4*(6*A + 5*B)*Cos[c + d*x] + 3*
(6*A + 5*B)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*Tan[(c + d*x)/2]))/(48*d*Sqrt[
Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 407 vs. $2(150) = 300$.
time = 7.11, size = 408, normalized size = 2.32

method	result
default	$\frac{\left(18A(\cos^3(dx+c))\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}-18A(\cos^3(dx+c))\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/96/d*(18*A*cos(d*x+c)^3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c
)+sin(d*x+c))*2^(1/2))*2^(1/2)-18*A*cos(d*x+c)^3*arctan(1/4*(-2/(1+cos(d*x+
c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*2^(1/2)+15*B*cos(d*x+c)^3*arc
tan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/2
)-15*B*cos(d*x+c)^3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(
d*x+c))*2^(1/2))*2^(1/2)+36*A*cos(d*x+c)^2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(
1/2)+30*B*cos(d*x+c)^2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+24*A*cos(d*x+c)
*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+20*B*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos
(d*x+c)))^(1/2)+16*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c))*(1/cos(d*x+c))^(
5/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+
c)^2*(cos(d*x+c)^2-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3342 vs. $2(150) = 300$.
time = 0.78, size = 3342, normalized size = 18.99

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/96*(6*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sq
```

rt(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(s
qrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d
*x + c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*
x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*x
+ 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 +
sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*
c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos
(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d
*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*
c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c)
+ 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
2) - 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 +
4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2
(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x +
 c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2
)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2
*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin
(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^
2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqr
t(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arcta
n2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1
2*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c))))*A*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1
)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
+ 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*c
os(2*d*x + 2*c) + 1) + (60*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x
+ 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(11/2*arctan2(sin(d*x + c), cos(d*x
 + c))) + 20*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqr
t(2)*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(d*x + c), cos(d*x + c))) + 168*(
sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x
 + 2*c))*cos(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 168*(sqrt(2)*sin(6*
d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(5
/2*arctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*sin(6*d*x + 6*c) + 3*
sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(
d*x + c), cos(d*x + c))) - 60*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d

```
*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d
*x + c))) - 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x +
 6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 1
8*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x +
2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) + 2) + 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6
*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*
cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x
 + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*
c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x +
c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
 + 2) - 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c
) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*co
s(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(si...
```

Fricas [A]
time = 1.52, size = 448, normalized size = 2.55

$$\left[\frac{3\left((6A+5B)\cos(dx+c)^3+(6A+5B)\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(3(6A+5B)\cos(dx+c)^2+2(6A+5B)\cos(dx+c)+8B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{96\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}, \frac{3\left((6A+5B)\cos(dx+c)^3+(6A+5B)\cos(dx+c)^2\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)+\frac{2\left(3(6A+5B)\cos(dx+c)^2+2(6A+5B)\cos(dx+c)+8B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{48\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/96*(3*((6*A + 5*B)*cos(d*x + c)^3 + (6*A + 5*B)*cos(d*x + c)^2)*sqrt(a)*
log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x
+ c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos
(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*(6*A + 5*B)*cos
(d*x + c)^2 + 2*(6*A + 5*B)*cos(d*x + c) + 8*B)*sqrt((a*cos(d*x + c) + a)/c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x
 + c)^2), 1/48*(3*((6*A + 5*B)*cos(d*x + c)^3 + (6*A + 5*B)*cos(d*x + c)^2)
*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(3*
(6*A + 5*B)*cos(d*x + c)^2 + 2*(6*A + 5*B)*cos(d*x + c) + 8*B)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c
)^3 + d*cos(d*x + c)^2)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(5/2),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2),
x)
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2),
 x)
```

3.225 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=131

$$\frac{\sqrt{a}\,(4A+3B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{a(4A+3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{aB\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 1/4*(4*A+3*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+
1/4*a*(4*A+3*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2*a*
B*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4101, 3888, 3886, 221}

$$\frac{a(4A+3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}\,(4A+3B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aB\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[a]*(4*A + 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/(4*d) + (a*(4*A + 3*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a
*Sec[c + d*x]]) + (a*B*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec
[c + d*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
```

```
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \frac{aB\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}+\frac{1}{4}(4A+3B)$$

$$= \frac{a(4A+3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{aB\,\mathrm{s}}{2d}$$

$$= \frac{a(4A+3B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{aB\,\mathrm{s}}{2d}$$

$$= \frac{\sqrt{a}\,(4A+3B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right.}{4d}$$

Mathematica [A]
time = 0.52, size = 106, normalized size = 0.81

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{2}\,(4A+3B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sec(c+dx)(4A+3B+2B\sec(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(4*A + 3*B)*ArcTanh[S
qrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(4*A + 3*B + 2*B*Sec[c + d*x])*Si
n[(c + d*x)/2]))/(8*d*Sqrt[Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(111) = 222$.
time = 7.66, size = 344, normalized size = 2.63

method	result
default	$-\frac{(-1+\cos(dx+c))\left(4A(\cos^2(dx+c))\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-4A(\cos^2(dx+c))\sqrt{2}\,\text{ar}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/8/d*(-1+cos(d*x+c))*(4*A*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+
c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))-4*A*cos(d*x+c)^2*2^(1/2)*arct
an(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))+3*B*cos
(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*
x+c))*2^(1/2))-3*B*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2
)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))+8*A*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d
*x+c)))^(1/2)+6*B*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+4*B*(-2/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c))*(1/cos(d*x+c))^(3/2)*(a*(1+cos(d*x+c))/cos
(d*x+c))^(1/2)/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1927 vs. $2(111) = 222$.
time = 0.70, size = 1927, normalized size = 14.71

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/16*(4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
+ 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x +
 2*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*co
s(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d
```

```
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(
d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*s
qrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*A*s
qrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) +
 (12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*si
n(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2)*s
in(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x + c),
 cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c)
)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4
*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos
(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log
(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d
*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 3*(
2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*
d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4
*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x
 + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
- 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)
*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x +
 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*co
s(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 +
2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*
c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2)*cos
(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(sin(d
*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x
 + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 12*(sqrt(
2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2
```

```
(sin(d*x + c), cos(d*x + c))))*B*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2
 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x
 + 2*c) + 1))/d
```

Fricas [A]
time = 2.22, size = 402, normalized size = 3.07

$$\left[\frac{\left((4A+3B)\cos(dx+c)^2+(4A+3B)\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left((4A+3B)\cos(dx+c)+2B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{16\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}, \frac{\left((4A+3B)\cos(dx+c)^2+(4A+3B)\cos(dx+c)\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)+\frac{2\left((4A+3B)\cos(dx+c)+2B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{8\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(((4*A + 3*B)*cos(d*x + c)^2 + (4*A + 3*B)*cos(d*x + c))*sqrt(a)*log(
(a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c)
)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
 + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*((4*A + 3*B)*cos(d*x +
 c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*
x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/8*(((4*A + 3*B)*cos(d*x + c
)^2 + (4*A + 3*B)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 -
 a*cos(d*x + c) - 2*a)) + 2*((4*A + 3*B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^
2 + d*cos(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(3/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2),
 x)
```

3.226 $\int \sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=78

$$\frac{\sqrt{a}\,(2A+B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{aB\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] (2*A+B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+a*B*se
c(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 78, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {4101, 3886, 221}

$$\frac{\sqrt{a}\,(2A+B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[a]*(2*A + B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]
)/d + (a*B*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
```

```
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \frac{aB\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}(2A+B)$$

$$(2A+B)\mathrm{Su}$$

$$= \frac{aB\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \overline{}$$

$$= \frac{\sqrt{a}\,(2A+B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right.}{d}$$

Mathematica [A]
time = 0.28, size = 89, normalized size = 1.14

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}\,\left(\sqrt{2}\,(2A+B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos(c+dx)+2B\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (Sec[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(2
*A + B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] + 2*B*Sin[(c + d*x)/
2]))/(2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 277 vs. $2(68) = 136$.
time = 8.00, size = 278, normalized size = 3.56

method	result
default	$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(2A\cos(dx+c)\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(-1-\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)+2A\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(2*A*cos(d*x
+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))
*2^(1/2))+2*A*cos(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+co
s(d*x+c)+sin(d*x+c))*2^(1/2))+B*cos(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*
x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))+B*cos(d*x+c)*2^(1/2)*arcta
n(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))+2*B*(-2/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c))*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(c
os(d*x+c)^2-1)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 905 vs. $2(68) = 136$.
time = 0.69, size = 905, normalized size = 11.60

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*(2*A*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log
(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c
) + 2)) - (4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
 + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
+ 2*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*
```

```
sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*
sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x +
2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*B*
sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))
/d
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 144 vs. $2(68) = 136$.
time = 1.58, size = 322, normalized size = 4.13

$$\left[\frac{((2A+B)\cos(dx+c)+2A+B)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4B\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4(d\cos(dx+c)+d)}, \frac{((2A+B)\cos(dx+c)+2A+B)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)+\frac{2B\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{2(d\cos(dx+c)+d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(((2*A + B)*cos(d*x + c) + 2*A + B)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*
a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x
 + c)^3 + cos(d*x + c)^2)) + 4*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/2*(((2*A + B)*cos(d*
x + c) + 2*A + B)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c
) - 2*a)) + 2*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(c
os(d*x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)\right)\sqrt{\sec\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*sqrt(sec(c + d*x))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*sqrt(sec(d*x + c)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2),
x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2),
 x)
```

3.227 $\int \frac{\sqrt{a+a\sec(c+dx)}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=76

$$\frac{2\sqrt{a}\,B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2aA\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*B*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)/d+2*a*A*sin(
d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.10, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {4100, 3886, 221}

$$\frac{2aA\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{2\sqrt{a}\,B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a]*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a*A*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
```

```
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{2aA\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + B\int \sqrt{\sec(c+dx)}\,\sqrt{a+\cdots}$$

$$= \frac{2aA\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} - \frac{(2B)\text{Subst}\left(\int \frac{1}{\sqrt{1+\frac{x^2}{a}}}\cdots\right)}{\cdots}$$

$$= \frac{2\sqrt{a}\,B\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2aA\sqrt{\sec(c+d\cdots}}{d\sqrt{a+a\,\text{se}\cdots}}$$

Mathematica [A]
time = 0.39, size = 83, normalized size = 1.09

$$\frac{2a\left(A\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\,\sin(c+dx)-B\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\tan(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (2*a*(A*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sin[c + d*x] - B*ArcSin[S
qrt[Sec[c + d*x]]]*Tan[c + d*x]))/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec
[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 176 vs. $2(66) = 132$.
time = 6.86, size = 177, normalized size = 2.33

method	result
default	$-\frac{\left(B\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}\,\sin(dx+c)+B\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\text{a}\cdots\right.}{2d\sin(dx+c)\sqrt{\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2/d*(B*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1
+cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/2)*sin(d*x+c)+B*(-2/(1+cos(d*x+c)))^(
1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2
))*2^(1/2)*sin(d*x+c)+4*A*cos(d*x+c)-4*A)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/
2)/sin(d*x+c)/(1/cos(d*x+c))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(66) = 132$.
time = 0.62, size = 262, normalized size = 3.45

$$\frac{4\sqrt{2}A\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+B\sqrt{a}\left(\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)-\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)+\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)-\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(4*sqrt(2)*A*sqrt(a)*sin(1/2*d*x + 1/2*c) + B*sqrt(a)*(log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 136 vs. $2(66) = 132$.
time = 1.77, size = 307, normalized size = 4.04

$$\left[\frac{4A\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(B\cos\left(dx+c\right)+B\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{2\left(d\cos\left(dx+c\right)+d\right)}, \frac{2A\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(B\cos\left(dx+c\right)+B\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)}{d\cos\left(dx+c\right)+d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(4*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + (B*cos(d*x + c) + B)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x +
```

```
c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + c
os(d*x + c)^2)))/(d*cos(d*x + c) + d), (2*A*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (B*cos(d*x + c) + B)*sqrt(-a)*ar
ctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) +
d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)\right)}{\sqrt{\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))/sqrt(sec(c + d*x))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/sqrt(sec(d*x + c)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2
),x)
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2
), x)
```

3.228 $\int \frac{\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=82

$$\frac{2a(A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2A\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*a*(A+3*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/3*A*si
n(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.11, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.057$, Rules used = {4098, 3889}

$$\frac{2a(A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a*(A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x
]]) + (2*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{1}{3}(A+3B)\int \frac{\sqrt{a+}}{\sqrt{}}$$

$$= \frac{2a(A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{a+a\sec(c}}{3d\sqrt{\sec}}$$

Mathematica [A]
time = 0.23, size = 56, normalized size = 0.68

$$\frac{2(2A+3B+A\cos(c+dx))\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2
),x]
```

```
[Out] (2*(2*A + 3*B + A*Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2]
)/(3*d*Sqrt[Sec[c + d*x]])
```

Maple [A]
time = 7.65, size = 75, normalized size = 0.91

method	result	size
default	$-\frac{2(-1+\cos(dx+c))(A\cos(dx+c)+2A+3B)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(\cos^2(dx+c))\left(\frac{1}{\cos(dx+c)}\right)^{\frac{3}{2}}}{3d\sin(dx+c)}$	75

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(A*cos(d*x+c)+2*A+3*B)*(a*(1+cos(d*x+c))/cos(d*x+c))
^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)
```

Maxima [A]
time = 0.66, size = 134, normalized size = 1.63

$$\frac{\sqrt{2}\left(3\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)+2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+3\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\right)A\sqrt{a}+12\sqrt{2}B\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/6*(sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a) + 12*sqrt(2)*B*
sqrt(a)*sin(1/2*d*x + 1/2*c))/d
```

Fricas [A]
time = 1.27, size = 74, normalized size = 0.90

$$\frac{2\left(A\cos\left(dx+c\right)^{2}+\left(2\,A+3\,B\right)\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] 2/3*(A*cos(d*x + c)^2 + (2*A + 3*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)\right)}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))/sec(c + d*x)**(3/2
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(3/2),
 x)
```

Mupad [B]
time = 2.79, size = 81, normalized size = 0.99

$$\frac{\cos(c+dx)\sqrt{\frac{1}{\cos(c+dx)}}\sqrt{\frac{a(\cos(c+dx)+1)}{\cos(c+dx)}}(4A\sin(c+dx)+6B\sin(c+dx)+A\sin(2c+2dx))}{3d(\cos(c+dx)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(3/2
),x)
```

```
[Out] (cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x))^
(1/2)*(4*A*sin(c + d*x) + 6*B*sin(c + d*x) + A*sin(2*c + 2*d*x)))/(3*d*(cos
(c + d*x) + 1))
```

3.229 $\int \frac{\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=130

$$\frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{2a(4A+5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{4a(4A+5B)\sqrt{\sec(c+dx)}}{15d\sqrt{a+a\sec(c+}}$$

```
[Out] 2/5*a*A*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+2/15*a*(4*A+5*
B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+4/15*a*(4*A+5*B)*si
n(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {4100, 3890, 3889}

$$\frac{4a(4A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a(4A+5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*a*A*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*(4*A + 5*B)*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*
x]]) + (4*a*(4*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*S
ec[c + d*x]])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
```

```
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{5}(4A+5B)\int \frac{\sqrt{a}}{}$$

$$= \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a(4A+5B}{15d\sqrt{\sec(c+dx)}}$$

$$= \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a(4A+5B}{15d\sqrt{\sec(c+dx)}}$$

Mathematica [A]
time = 0.30, size = 71, normalized size = 0.55

$$\frac{a(19A+20B+2(4A+5B)\cos(c+dx)+3A\cos(2(c+dx)))\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2
),x]
```

```
[Out] (a*(19*A + 20*B + 2*(4*A + 5*B)*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)])*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 7.26, size = 96, normalized size = 0.74

method	result
default	$-\frac{2(-1+\cos(dx+c))\left(3A\left(\cos^{2}(dx+c)\right)+4A\cos(dx+c)+5B\cos(dx+c)+8A+10B\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(\cos^{3}(dx+c)\right)\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}}{15d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(3*A*cos(d*x+c)^2+4*A*cos(d*x+c)+5*B*cos(d*x+c)+8*A
+10*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2
)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 317 vs. $2(112) = 224$.
time = 0.61, size = 317, normalized size = 2.44

$$\frac{\sqrt{2}\left(30\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)-30\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\sin\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)-5\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\sin\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+6\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\sin\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+30\sin\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\right)A\sqrt{a}+10\sqrt{2}\left(3\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)+2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+3\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\right)B\sqrt{a}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/60*(sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*
d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)*si
n(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*d*x
+ 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) +
 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*A*sqrt(a)
 + 10*sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*B*sqrt(a))/d
```

Fricas [A]
time = 2.19, size = 92, normalized size = 0.71

$$\frac{2\left(3\,A\cos\left(dx+c\right)^{3}+\left(4\,A+5\,B\right)\cos\left(dx+c\right)^{2}+2\left(4\,A+5\,B\right)\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*(3*A*cos(d*x + c)^3 + (4*A + 5*B)*cos(d*x + c)^2 + 2*(4*A + 5*B)*cos(d
*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x +
 c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)\right)}{\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))/sec(c + d*x)**(5/2
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(5/2),
 x)
```

Mupad [B]
time = 3.38, size = 106, normalized size = 0.82

$$\frac{\cos(c+d\,x)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,\left(\cos(c+d\,x)+1\right)}{\cos(c+d\,x)}}\,\left(35\,A\,\sin(c+d\,x)+40\,B\,\sin(c+d\,x)+8\,A\,\sin(2\,c+2\,d\,x)+3\,A\,\sin(3\,c+3\,d\,x)+10\,B\,\sin(2\,c+2\,d\,x)\right)}{30\,d\,\left(\cos(c+d\,x)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(5/2
),x)
```

```
[Out] (cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x))^
(1/2)*(35*A*sin(c + d*x) + 40*B*sin(c + d*x) + 8*A*sin(2*c + 2*d*x) + 3*A*s
in(3*c + 3*d*x) + 10*B*sin(2*c + 2*d*x)))/(30*d*(cos(c + d*x) + 1))
```

3.230 $\int \frac{\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=175

$$\frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{2a(6A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{8a(6A+7B)\sin(c+}{105d\sqrt{\sec(c+dx)}\sqrt{a+a\,\mathrm{se}}}$$

```
[Out] 2/7*a*A*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+2/35*a*(6*A+7*
B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+8/105*a*(6*A+7*B)*s
in(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16/105*a*(6*A+7*B)*sin(
d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {4100, 3890, 3889}

$$\frac{2a(6A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{16a(6A+7B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{8a(6A+7B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*a*A*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (2
*a*(6*A + 7*B)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*
x]]) + (8*a*(6*A + 7*B)*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*
Sec[c + d*x]]) + (16*a*(6*A + 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*
Sqrt[a + a*Sec[c + d*x]])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{1}{7}(6A+7B)\int \frac{\sqrt{a}}{} \\ &= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{2a(6A+7B)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{}} \\ &= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{2a(6A+7B)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{}} \\ &= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{2a(6A+7B)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{}}\end{aligned}$$

Mathematica [A]
time = 0.33, size = 91, normalized size = 0.52

$$\frac{2a(15A+3(6A+7B)\sec(c+dx)+4(6A+7B)\sec^2(c+dx)+8(6A+7B)\sec^3(c+dx))\sin(c+dx)}{105d\sec^{\frac{5}{2}}(c+dx)\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2
),x]
```

```
[Out] (2*a*(15*A + 3*(6*A + 7*B)*Sec[c + d*x] + 4*(6*A + 7*B)*Sec[c + d*x]^2 + 8*
(6*A + 7*B)*Sec[c + d*x]^3)*Sin[c + d*x])/(105*d*Sec[c + d*x]^(5/2)*Sqrt[a*
(1 + Sec[c + d*x])])
```

Maple [A]
time = 7.54, size = 118, normalized size = 0.67

method	result
default	$-\frac{2(-1+\cos(dx+c))(15A(\cos^3(dx+c))+18A(\cos^2(dx+c))+21B(\cos^2(dx+c))+24A\cos(dx+c)+28B\cos(dx+c)+48A+56B)\sqrt{\frac{a(1+}{c}}}{105d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+18*A*cos(d*x+c)^2+21*B*cos(d*x+
c)^2+24*A*cos(d*x+c)+28*B*cos(d*x+c)+48*A+56*B)*(a*(1+cos(d*x+c))/cos(d*x+c
))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 498 vs. $2(151) = 302$.
time = 0.63, size = 498, normalized size = 2.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 1/840*(3*sqrt(2)*(105*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7
/2*c)))*sin(7/2*d*x + 7/2*c) + 35*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 7*cos(2/7*arctan2(sin(7/2*d*x +
7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 105*cos(7/2*d*x + 7/2
*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 35*cos(7
/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
) - 7*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*
x + 7/2*c))) + 10*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7/
2*c), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c))))*A*sqrt(a) + 14*sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c
), cos(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*
d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x
+ 5/2*c)*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*c
os(5/2*d*x + 5/2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2
*c))) + 6*sin(5/2*d*x + 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), co
s(5/2*d*x + 5/2*c))) + 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x
 + 5/2*c))))*B*sqrt(a))/d
```

Fricas [**A**]
time = 2.69, size = 110, normalized size = 0.63

$$\frac{2 \left(15 \, A \cos\left(dx+c\right)^{4} + 3 \, \left(6 \, A + 7 \, B\right) \cos\left(dx+c\right)^{3} + 4 \, \left(6 \, A + 7 \, B\right) \cos\left(dx+c\right)^{2} + 8 \, \left(6 \, A + 7 \, B\right) \cos\left(dx+c\right)\right) \sqrt{\frac{a \cos\left(dx+c\right) + a}{\cos\left(dx+c\right)}} \sin\left(dx+c\right)}{105 \, \left(d \cos\left(dx+c\right) + d\right) \sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*(15*A*cos(d*x + c)^4 + 3*(6*A + 7*B)*cos(d*x + c)^3 + 4*(6*A + 7*B)*c
os(d*x + c)^2 + 8*(6*A + 7*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/sec(d*x + c)^(7/2),
 x)
```

Mupad [B]
time = 4.19, size = 130, normalized size = 0.74

$$\frac{\cos(c+d\,x)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,(420\,A\,\sin(c+d\,x)+490\,B\,\sin(c+d\,x)+126\,A\,\sin(2\,c+2\,d\,x)+36\,A\,\sin(3\,c+3\,d\,x)+15\,A\,\sin(4\,c+4\,d\,x)+112\,B\,\sin(2\,c+2\,d\,x)+42\,B\,\sin(3\,c+3\,d\,x))}{420\,d\,(\cos(c+d\,x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(7/2
),x)
```

```
[Out] (cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x))^
(1/2)*(420*A*sin(c + d*x) + 490*B*sin(c + d*x) + 126*A*sin(2*c + 2*d*x) + 3
6*A*sin(3*c + 3*d*x) + 15*A*sin(4*c + 4*d*x) + 112*B*sin(2*c + 2*d*x) + 42*
B*sin(3*c + 3*d*x)))/(420*d*(cos(c + d*x) + 1))
```

3.231 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=227

$$\frac{a^{3/2}(88A+75B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d}+\frac{a^2(88A+75B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}}+\frac{a^2(88A+75B}{96d\sqrt{}}$$

```
[Out] 1/64*a^(3/2)*(88*A+75*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))
/d+1/64*a^2*(88*A+75*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2
)+1/96*a^2*(88*A+75*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)
+1/24*a^2*(8*A+9*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/
4*a*B*sec(d*x+c)^(7/2)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.33, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4103, 4101, 3888, 3886, 221}

$$\frac{a^{3/2}(88A+75B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^2(8A+9B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(88A+75B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(88A+75B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{aB\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(3/2)*(88*A + 75*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(64*d) + (a^2*(88*A + 75*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*d*Sq
rt[a + a*Sec[c + d*x]]) + (a^2*(88*A + 75*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x
])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(8*A + 9*B)*Sec[c + d*x]^(7/2)*Si
n[c + d*x])/(24*d*Sqrt[a + a*Sec[c + d*x]]) + (a*B*Sec[c + d*x]^(7/2)*Sqrt[
a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{aB\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+}{4d}$$

$$= \frac{a^2(8A+9B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}} + \frac{aB}{}$$

$$= \frac{a^2(88A+75B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} + $$

$$= \frac{a^2(88A+75B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + $$

$$= \frac{a^2(88A+75B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}} + $$

$$= \frac{a^{3/2}(88A+75B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+d}}\right.}{64d}$$

Mathematica [A]
time = 1.42, size = 153, normalized size = 0.67

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(6\sqrt{2}(88A+75B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+(352A+492B+(1048A+1155B)\cos(c+dx)+4(88A+75B)\cos(2(c+dx))+264A\cos(3(c+dx))+225B\cos(3(c+dx)))\sec^4(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)\right)}{768d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(88*A + 75*B)*Arc
Tanh[Sqrt[2]*Sin[(c + d*x)/2]] + (352*A + 492*B + (1048*A + 1155*B)*Cos[c +
 d*x] + 4*(88*A + 75*B)*Cos[2*(c + d*x)] + 264*A*Cos[3*(c + d*x)] + 225*B*C
os[3*(c + d*x)])*Sec[c + d*x]^4*Sin[(c + d*x)/2]))/(768*d*Sqrt[Sec[c + d*x]
])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 476 vs. 2(195) = 390.
time = 7.26, size = 477, normalized size = 2.10

method	result

default	$\frac{(-1+\cos(dx+c))\left(264A\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)-\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}\,(\cos^4(dx+c))-264A\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/384/d*(-1+cos(d*x+c))*(264*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(
d*x+c)-sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)^4-264*A*arctan(1/4*(-2/(1+co
s(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)^4+22
5*B*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))
*2^(1/2)*cos(d*x+c)^4-225*B*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x
+c)+sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)^4-528*A*(-2/(1+cos(d*x+c)))^(1/
2)*cos(d*x+c)^3*sin(d*x+c)-450*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^3*sin
(d*x+c)-352*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-300*B*(-2/(
1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-128*A*cos(d*x+c)*sin(d*x+c)*(-
2/(1+cos(d*x+c)))^(1/2)-240*B*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/
2)-96*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))
^(1/2)*(1/cos(d*x+c))^(5/2)/cos(d*x+c)/sin(d*x+c)^2/(-2/(1+cos(d*x+c)))^(1/
2)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 5879 vs. $2(195) = 390$.
time = 1.07, size = 5879, normalized size = 25.90

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] -1/768*(8*(132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
```

```
 2*c))) - 132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x
 + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c)
+ 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)
*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*
d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos
(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6
*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2
*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) +
 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d
*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 33*(a*cos(6*d*x + 6*c)^2 + 9
*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a
*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*
x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x
+ 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*
c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(
2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)
^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2
 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a
*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos
(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x
 + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 2) - 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3
*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*
x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(9/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2
)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a*cos(6*d*x + 6*c
) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a
```

```
)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*cos(
6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c)
+ sqrt(2)*a)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 132*(sq
rt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2
*d*x + 2*c) + sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*A*sqrt(a)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x +
6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9
*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*
x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2
*c) + 1) + 3*(300*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c
) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*...
```

Fricas [A]
time = 1.78, size = 494, normalized size = 2.18

$$\left[\frac{3\left((88A+75B)a\cos(dx+c)^4+(88A+75B)a\cos(dx+c)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(3(88A+75B)a\cos(dx+c)^3+2(88A+75B)a\cos(dx+c)^2+8(8A+15B)a\cos(dx+c)+48Ba\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{768\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}, \frac{3\left((88A+75B)a\cos(dx+c)^4+(88A+75B)a\cos(dx+c)^3\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)+\frac{2\left(3(88A+75B)a\cos(dx+c)^3+2(88A+75B)a\cos(dx+c)^2+8(8A+15B)a\cos(dx+c)+48Ba\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{384\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] [1/768*(3*((88*A + 75*B)*a*cos(d*x + c)^4 + (88*A + 75*B)*a*cos(d*x + c)^3)
*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2
*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*(88*A
+ 75*B)*a*cos(d*x + c)^3 + 2*(88*A + 75*B)*a*cos(d*x + c)^2 + 8*(8*A + 15*B
)*a*cos(d*x + c) + 48*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
+ c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1/384*(3*((
88*A + 75*B)*a*cos(d*x + c)^4 + (88*A + 75*B)*a*cos(d*x + c)^3)*sqrt(-a)*ar
ctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(3*(88*A + 75*B
)*a*cos(d*x + c)^3 + 2*(88*A + 75*B)*a*cos(d*x + c)^2 + 8*(8*A + 15*B)*a*co
s(d*x + c) + 48*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/s
qrt(cos(d*x + c)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2),
x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2),
 x)
```

3.232 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=180

$$\frac{a^{3/2}(14A+11B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{a^2(14A+11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}}+\frac{a^2(6A+7B)\,\mathrm{se}}{12d\sqrt{a\,+}}$$

```
[Out] 1/8*a^(3/2)*(14*A+11*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/
d+1/8*a^2*(14*A+11*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+
1/12*a^2*(6*A+7*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/3
*a*B*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.26, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4103, 4101, 3888, 3886, 221}

$$\frac{a^{3/2}(14A+11B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^2(6A+7B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{12d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(14A+11B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{8d\sqrt{a\sec(c+dx)+a}}+\frac{aB\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(3/2)*(14*A + 11*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(8*d) + (a^2*(14*A + 11*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(8*d*Sqrt
[a + a*Sec[c + d*x]]) + (a^2*(6*A + 7*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(
12*d*Sqrt[a + a*Sec[c + d*x]]) + (a*B*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c +
 d*x]]*Sin[c + d*x])/(3*d)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{aB\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c+}{3d}$$

$$= \frac{a^2(6A+7B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{12d\sqrt{a+a\sec(c+dx)}} + \frac{aB\,\text{s}}{}$$

$$= \frac{a^2(14A+11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{a^2}{}$$

$$= \frac{a^2(14A+11B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{8d\sqrt{a+a\sec(c+dx)}} + \frac{a^2}{}$$

$$= \frac{a^{3/2}(14A+11B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx}}\right.}{8d}$$

Mathematica [A]
time = 1.21, size = 134, normalized size = 0.74

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(3\sqrt{2}\,(14A+11B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+(7(6A+7B)+4(6A+11B)\cos(c+dx)+(42A+33B)\cos(2(c+dx)))\sec^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)\right)}{48d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(14*A + 11*B)*Arc
Tanh[Sqrt[2]*Sin[(c + d*x)/2]] + (7*(6*A + 7*B) + 4*(6*A + 11*B)*Cos[c + d*
x] + (42*A + 33*B)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*Sin[(c + d*x)/2]))/(48*
d*Sqrt[Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 416 vs. $2(154) = 308$.
time = 6.98, size = 417, normalized size = 2.32

method	result
default	$-\frac{\left(42A(\cos^3(dx+c))\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)-\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}-42A(\cos^3(dx+c))\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/96/d*(42*A*2^(1/2)*cos(d*x+c)^3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+
cos(d*x+c)-sin(d*x+c))*2^(1/2))-42*A*2^(1/2)*cos(d*x+c)^3*arctan(1/4*(-2/(1
+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))+33*B*2^(1/2)*cos(d*x
+c)^3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2
))-33*B*2^(1/2)*cos(d*x+c)^3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*
x+c)+sin(d*x+c))*2^(1/2))-84*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d
*x+c)-66*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)-24*A*cos(d*x+c
)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)-44*B*cos(d*x+c)*sin(d*x+c)*(-2/(1+co
s(d*x+c)))^(1/2)-16*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c))*(a*(1+cos(d*x+c
))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)*(-2/(1+cos(d*x+c)))^(1/2)/cos(d*x
+c)/sin(d*x+c)^2*(cos(d*x+c)^2-1)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 4606 vs. $2(154) = 308$.
time = 0.84, size = 4606, normalized size = 25.59

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] -1/96*(6*(56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*
sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt(2)*a*sin
(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*sin(
7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*sin(
5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3/2*d*x +
3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*(a
*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
```

(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(3*sqrt(2)*a*cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c),

```
cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2...
```

Fricas [A]
time = 2.05, size = 458, normalized size = 2.54

$$\left[\frac{3\left((14A+11B)a\cos(dx+c)^3+(14A+11B)a\cos(dx+c)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(3(14A+11B)a\cos(dx+c)^2+2(6A+11B)a\cos(dx+c)+8Ba\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{96\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}, \frac{3\left((14A+11B)a\cos(dx+c)^3+(14A+11B)a\cos(dx+c)^2\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)+\frac{2\left(3(14A+11B)a\cos(dx+c)^2+2(6A+11B)a\cos(dx+c)+8Ba\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{48\left(d\cos(dx+c)^3+d\cos(dx+c)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] [1/96*(3*((14*A + 11*B)*a*cos(d*x + c)^3 + (14*A + 11*B)*a*cos(d*x + c)^2)*
sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*
cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*(14*A +
 11*B)*a*cos(d*x + c)^2 + 2*(6*A + 11*B)*a*cos(d*x + c) + 8*B*a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
 c)^3 + d*cos(d*x + c)^2), 1/48*(3*((14*A + 11*B)*a*cos(d*x + c)^3 + (14*A
+ 11*B)*a*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos
(d*x + c) - 2*a)) + 2*(3*(14*A + 11*B)*a*cos(d*x + c)^2 + 2*(6*A + 11*B)*a*
cos(d*x + c) + 8*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2),
 x)
```

3.233 $\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=133

$$\frac{a^{3/2}(12A+7B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{a^2(4A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{aB\sec^{\frac{3}{2}}(c+dx)\,\sqrt{\ldots}}{\ldots}$$

```
[Out] 1/4*a^(3/2)*(12*A+7*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d
+1/4*a^2*(4*A+5*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/2
*a*B*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 133, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4103, 4101, 3886, 221}

$$\frac{a^{3/2}(12A+7B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a^2(4A+5B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{aB\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(3/2)*(12*A + 7*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]])/(4*d) + (a^2*(4*A + 5*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a
+ a*Sec[c + d*x]]) + (a*B*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c
 + d*x])/(2*d)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
```

```
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{aB\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c \ldots}{2d}$$

$$= \frac{a^2(4A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \ldots$$

$$= \frac{a^2(4A+5B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \ldots$$

$$= \frac{a^{3/2}(12A+7B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+d\ldots}}\right.\ldots}{4d}$$

Mathematica [A]
time = 0.69, size = 107, normalized size = 0.80

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{2}\,(12A+7B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sec(c+dx)(4A+7B+2B\sec(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(12*A + 7*B)*ArcTan
h[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(4*A + 7*B + 2*B*Sec[c + d*x])
*Sin[(c + d*x)/2]))/(8*d*Sqrt[Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(113) = 226$.
time = 7.64, size = 353, normalized size = 2.65

method	result
default	$\frac{(-1+\cos(dx+c))\left(12A(\cos^2(dx+c))\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)-\sin(dx+c))\sqrt{2}}{4}\right)-12A(\cos^2(dx+c))\sqrt{2}\,\ldots\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/8/d*(-1+cos(d*x+c))*(12*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x
+c)-sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-12*A*arctan(1/4*(-2/(1+cos(d*
x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)+7*B*ar
ctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*cos(d
*x+c)^2*2^(1/2)-7*B*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(
d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-8*A*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(
d*x+c)))^(1/2)-14*B*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)-4*B*(-2
/(1+cos(d*x+c)))^(1/2)*sin(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/c
os(d*x+c))^(1/2)/cos(d*x+c)/sin(d*x+c)^2/(-2/(1+cos(d*x+c)))^(1/2)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3389 vs. $2(113) = 226$.
time = 0.73, size = 3389, normalized size = 25.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/16*(4*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
```

```
x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*d*
x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2*d*x
+ 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*a*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - 4*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))*sin(
2*d*x + 2*c))*A*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1) - (56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) - 24*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt
(2)*a*sin(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(
2)*a*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(
2)*a*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(
2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3
/2*d*x + 3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(
8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c
```

```
), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
 + 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*
a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),...
```

Fricas [A]
time = 2.78, size = 410, normalized size = 3.08

$$\left[\frac{\left((12A+7B)a\cos(dx+c)^2+(12A+7B)a\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left((4A+7B)a\cos(dx+c)+2Ba\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{16\left(d\cos(dx+c)^2+d\cos(dx+c)\right)},\ \frac{\left((12A+7B)a\cos(dx+c)^2+(12A+7B)a\cos(dx+c)\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)+\frac{2\left((4A+7B)a\cos(dx+c)+2Ba\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{8\left(d\cos(dx+c)^2+d\cos(dx+c)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(((12*A + 7*B)*a*cos(d*x + c)^2 + (12*A + 7*B)*a*cos(d*x + c))*sqrt(a
)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*
x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(c
os(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*((4*A + 7*B)*a*c
os(d*x + c) + 2*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/s
qrt(cos(d*x + c)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/8*(((12*A + 7*B)*
a*cos(d*x + c)^2 + (12*A + 7*B)*a*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*
cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*((4*A + 7*B)*a*cos(d*x + c) + 2
*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c
)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2),
x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2),
 x)
```

3.234 $\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=124

$$\frac{a^{3/2}(2A+3B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a^2(2A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{aB\sqrt{\sec(c+dx)}}{}$$

```
[Out] a^(3/2)*(2*A+3*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+a^2*
(2*A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+a*B*sin(d*x+c)
*sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.20, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4103, 4100, 3886, 221}

$$\frac{a^{3/2}(2A+3B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^2(2A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}+\frac{aB\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^(3/2)*(2*A + 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/d + (a^2*(2*A - B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[a + a*Sec[c
 + d*x]]) + (a*B*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/
d
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
```

```
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{aB\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{d}+\int \frac{\sqrt{}}{}$$

$$= \frac{a^2(2A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{aB\sqrt{\sec(c+dx}}{}$$

$$= \frac{a^2(2A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}+\frac{aB\sqrt{\sec(c+dx}}{}$$

$$= \frac{a^{3/2}(2A+3B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a^2(2A-}{}$$

Mathematica [A]
time = 1.58, size = 107, normalized size = 0.86

$$\frac{a^2\left(2A\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)-3B\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)+(B+2A\cos(c+dx))\sqrt{(-1+\cos(c+dx))\sec^2(c+dx)}\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (a^2*(2*A*ArcSin[Sqrt[1 - Sec[c + d*x]]] - 3*B*ArcSin[Sqrt[Sec[c + d*x]]] +
 (B + 2*A*Cos[c + d*x])*Sqrt[(-1 + Cos[c + d*x])*Sec[c + d*x]^2])*Tan[c + d
*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 345 vs. $2(110) = 220$.
time = 8.14, size = 346, normalized size = 2.79

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(2A\sqrt{-\frac{2}{1+\cos(dx+c)}}\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}(1+\cos(dx+c)-\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}\cos(dx+c)\sin(\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(2*A*(-2/(1+cos(d*x+c)))^(1/2)*ar
ctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*2^(1/
2)*cos(d*x+c)*sin(d*x+c)-2*A*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+co
s(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)*sin(
d*x+c)+3*B*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(
1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)*sin(d*x+c)-3*B*(-2/(1+
cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d
*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)*sin(d*x+c)-8*A*cos(d*x+c)^2+8*A*cos(d*x+
c)-4*B*cos(d*x+c)+4*B)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1417 vs. $2(110) = 220$.
time = 0.65, size = 1417, normalized size = 11.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*(sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*
a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c))*A*sq
rt(a) + (3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
```

```
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*d*
x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2*d*x
+ 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*a*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - 4*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))*sin(
2*d*x + 2*c))*B*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1))/d
```

<u>Fricas</u> [**A**]
time = 3.19, size = 364, normalized size = 2.94

$$\left[\frac{\left((2A+3B)a\cos(dx+c)+(2A+3B)a\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4(2Aa\cos(dx+c)+Ba)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4(d\cos(dx+c)+d)},\ \frac{\left((2A+3B)a\cos(dx+c)+(2A+3B)a\right)\sqrt{-a}\,\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)+\frac{2(2Aa\cos(dx+c)+Ba)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{2(d\cos(dx+c)+d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(((2*A + 3*B)*a*cos(d*x + c) + (2*A + 3*B)*a)*sqrt(a)*log((a*cos(d*x +
 c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sq
```

```
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a
)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(2*A*a*cos(d*x + c) + B*a)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*
x + c) + d), 1/2*(((2*A + 3*B)*a*cos(d*x + c) + (2*A + 3*B)*a)*sqrt(-a)*arc
tan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(2*A*a*cos(d*x +
 c) + B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*
x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(a(\sec{(c+dx)}+1))^{\frac{3}{2}}\,(A+B\sec{(c+dx)})}{\sqrt{\sec{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*(A + B*sec(c + d*x))/sqrt(sec(c + d*
x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{3/2}}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(1/2
), x)
```

3.235 $\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=125

$$\frac{2a^{3/2}B\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^2(4A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}+\frac{2aA\sqrt{a+a\sec(c+dx)}}{3d\sqrt{\sec(c+dx}}$$

```
[Out] 2*a^(3/2)*B*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2/3*a^2*(4
*A+3*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/3*a*A*sin(d*
x+c)*(a+a*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 125, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4102, 4100, 3886, 221}

$$\frac{2a^{3/2}B\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2(4A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*a^(3/2)*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a^2*(4*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c +
 d*x]]) + (2*a*A*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d
*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
```

```
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{\sqrt{a+a\sec}}{}$$

$$= \frac{2a^2(4A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2aA\sqrt{a+a}}{3d}$$

$$= \frac{2a^2(4A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{2aA\sqrt{a+a}}{3d}$$

$$= \frac{2a^{3/2}B\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^2(4A+3B)}{3d\sqrt{}}$$

Mathematica [A]
time = 0.63, size = 109, normalized size = 0.87

$$\frac{2a^2\left((5A+3B+A\cos(c+dx))\sqrt{1-\sec(c+dx)}+3B\,\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)\sqrt{\sec(c+dx)}\right)\tan(c+dx)}{3d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] (2*a^2*((5*A + 3*B + A*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] + 3*B*ArcSin[Sq
rt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Tan[c + d*x])/(3*d*Sqrt[-((-1 + S
ec[c + d*x])*Sec[c + d*x])]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 7.28, size = 211, normalized size = 1.69

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3B\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}\,\sin(dx+c)+3B}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/6/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*B*(-2/(1+cos(d*x+c)))^(1/2)*a
rctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1
/2)*sin(d*x+c)+3*B*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))
^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/2)*sin(d*x+c)+4*A*cos(d*x+c
)^2+16*A*cos(d*x+c)+12*B*cos(d*x+c)-20*A-12*B)*cos(d*x+c)^2*(1/cos(d*x+c))^
(3/2)/sin(d*x+c)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(107) = 214.
time = 0.62, size = 314, normalized size = 2.51

$$\frac{3\sqrt{2}\left(\sqrt{2}\,a\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)-\sqrt{2}\,a\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)+\sqrt{2}\,a\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)-\sqrt{2}\,a\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)+8a\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)B\sqrt{a}+4\left(\sqrt{2}\,a\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+9\sqrt{2}\,a\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}}{12d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(3*sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
 2) - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(
2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c))*B
*sqrt(a) + 4*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/
2*c))*A*sqrt(a))/d
```

Fricas [A]
time = 2.16, size = 368, normalized size = 2.94

$$\left[\frac{3\left(Ba\cos\left(dx+c\right)+Ba\right)\sqrt{a}\log\left(\frac{a\cos\left(dx+c\right)^{3}-7\,a\cos\left(dx+c\right)^{2}-\frac{4\left(\cos\left(dx+c\right)^{2}-2\cos\left(dx+c\right)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}+8\,a}{\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}}\right)+\frac{4\left(Aa\cos\left(dx+c\right)^{2}+\left(5\,A+3\,B\right)a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{6\left(d\cos\left(dx+c\right)+d\right)},\ \frac{3\left(Ba\cos\left(dx+c\right)+Ba\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)-2\,a}\right)+\frac{2\left(Aa\cos\left(dx+c\right)^{2}+\left(5\,A+3\,B\right)a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{3\left(d\cos\left(dx+c\right)+d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/6*(3*(B*a*cos(d*x + c) + B*a)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*
x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3
+ cos(d*x + c)^2)) + 4*(A*a*cos(d*x + c)^2 + (5*A + 3*B)*a*cos(d*x + c))*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*c
os(d*x + c) + d), 1/3*(3*(B*a*cos(d*x + c) + B*a)*sqrt(-a)*arctan(2*sqrt(-a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(
a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(A*a*cos(d*x + c)^2 + (5*A +
3*B)*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/s
qrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\left(A+B\sec\left(c+dx\right)\right)}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*(A + B*sec(c + d*x))/sec(c + d*x)**(
3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(3/2
), x)
```

3.236 $\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=131

$$\frac{8a^2(3A+5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{2a(3A+5B)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2A(a+a\sec(c+\ldots}{5d\sec\ldots}$$

```
[Out] 2/5*A*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+8/15*a^2*(3*A+5*
B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/15*a*(3*A+5*B)*si
n(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {4098, 3894, 3889}

$$\frac{8a^2(3A+5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a(3A+5B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{15d\sqrt{\sec(c+dx)}}+\frac{2A\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (8*a^2*(3*A + 5*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c
+ d*x]]) + (2*a*(3*A + 5*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sq
rt[Sec[c + d*x]]) + (2*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d*Sec[
c + d*x]^(3/2))
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Dist[b*((2*m - 1)/(d*m)), Int[(a + b*Csc[
e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ
[2*m]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
```

```
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{1}{5}(3A+5B)\int \frac{(a}{}$$

$$= \frac{2a(3A+5B)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2A(a+a}{}$$

$$= \frac{8a^2(3A+5B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2a(3A+5B)}{}$$

Mathematica [A]
time = 0.52, size = 73, normalized size = 0.56

$$\frac{a^2(39A+50B+2(9A+5B)\cos(c+dx)+3A\cos(2(c+dx)))\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5
/2),x]
```

```
[Out] (a^2*(39*A + 50*B + 2*(9*A + 5*B)*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)])*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 7.08, size = 97, normalized size = 0.74

method	result
default	$-\frac{2(-1+\cos(dx+c))(3A(\cos^2(dx+c))+9A\cos(dx+c)+5B\cos(dx+c)+18A+25B)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(\cos^3(dx+c))\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}}a}{15d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(3*A*cos(d*x+c)^2+9*A*cos(d*x+c)+5*B*cos(d*x+c)+18*
A+25*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/
2)/sin(d*x+c)*a
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(113) = 226$.
time = 0.61, size = 250, normalized size = 1.91

$$\frac{3\sqrt{2}\left(20\,a\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,a\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)-20\,a\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\sin\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)-5\,a\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\sin\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)+2\,a\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+5\,a\sin\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)+20\,a\sin\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right),\cos\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)\right)\right)\right)A\sqrt{a}+20\left(\sqrt{2}\,a\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+9\sqrt{2}\,a\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/60*(3*sqrt(2)*(20*a*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5
/2*c)))*sin(5/2*d*x + 5/2*c) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), co
s(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4
/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x +
 5/2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*a*
sin(5/2*d*x + 5/2*c) + 5*a*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*
x + 5/2*c))) + 20*a*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2
*c))))*A*sqrt(a) + 20*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2
*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [**A**]
time = 1.15, size = 94, normalized size = 0.72

$$\frac{2\left(3\,Aa\cos\left(dx+c\right)^{3}+\left(9\,A+5\,B\right)a\cos\left(dx+c\right)^{2}+\left(18\,A+25\,B\right)a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*(3*A*a*cos(d*x + c)^3 + (9*A + 5*B)*a*cos(d*x + c)^2 + (18*A + 25*B)*a
*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos
(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\left(A+B\sec\left(c+dx\right)\right)}{\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(3/2)*(A + B*sec(c + d*x))/sec(c + d*x)**(
5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2
), x)
```

Mupad [B]
time = 3.40, size = 107, normalized size = 0.82

$$\frac{a\cos(c+d\,x)\,\sqrt{\frac{1}{\cos(c+d\,x)}}\,\sqrt{\frac{a\,(\cos(c+d\,x)+1)}{\cos(c+d\,x)}}\,(75\,A\,\sin(c+d\,x)+100\,B\,\sin(c+d\,x)+18\,A\,\sin(2\,c+2\,d\,x)+3\,A\,\sin(3\,c+3\,d\,x)+10\,B\,\sin(2\,c+2\,d\,x))}{30\,d\,(\cos(c+d\,x)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(5/2
),x)
```

```
[Out] (a*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x)
)^(1/2)*(75*A*sin(c + d*x) + 100*B*sin(c + d*x) + 18*A*sin(2*c + 2*d*x) + 3
*A*sin(3*c + 3*d*x) + 10*B*sin(2*c + 2*d*x)))/(30*d*(cos(c + d*x) + 1))
```

3.237 $\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=181

$$\frac{2a^2(8A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(52A+63B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{4a^2(52A+63B)\sqrt{\sec(c-}}{105d\sqrt{a+a\sec(}}$$

```
[Out] 2/35*a^2*(8*A+7*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+2/1
05*a^2*(52*A+63*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+4/1
05*a^2*(52*A+63*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/7
*a*A*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)
```

Rubi [A]
time = 0.28, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4102, 4100, 3890, 3889}

$$\frac{2a^2(8A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4a^2(52A+63B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(52A+63B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*a^2*(8*A + 7*B)*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c
+ d*x]]) + (2*a^2*(52*A + 63*B)*Sin[c + d*x])/(105*d*Sqrt[Sec[c + d*x]]*Sqr
t[a + a*Sec[c + d*x]]) + (4*a^2*(52*A + 63*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Sqrt[a + a*Sec[c + d*x]]*Sin[
c + d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
```

```
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{\sqrt{a+a\sec(}$$

$$= \frac{2a^2(8A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2aA\sqrt{a+a\sec(}}{7d\sec}$$

$$= \frac{2a^2(8A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(52A+}{105d\sqrt{\sec(c+d}}$$

$$= \frac{2a^2(8A+7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(52A+}{105d\sqrt{\sec(c+d}}$$

Mathematica [A]
time = 0.47, size = 92, normalized size = 0.51

$$\frac{2a^2(15A+3(13A+7B)\sec(c+dx)+(52A+63B)\sec^2(c+dx)+2(52A+63B)\sec^3(c+dx))\sin(c+dx)}{105d\sec^{\frac{5}{2}}(c+dx)\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7
/2),x]
```

```
[Out] (2*a^2*(15*A + 3*(13*A + 7*B)*Sec[c + d*x] + (52*A + 63*B)*Sec[c + d*x]^2 +
 2*(52*A + 63*B)*Sec[c + d*x]^3)*Sin[c + d*x])/(105*d*Sec[c + d*x]^(5/2)*Sq
rt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 7.12, size = 119, normalized size = 0.66

method	result
default	$-\frac{2(-1+\cos(dx+c))(15A(\cos^3(dx+c))+39A(\cos^2(dx+c))+21B(\cos^2(dx+c))+52A\cos(dx+c)+63B\cos(dx+c)+104A+126B)\sqrt{}}{105d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+39*A*cos(d*x+c)^2+21*B*cos(d*x+
c)^2+52*A*cos(d*x+c)+63*B*cos(d*x+c)+104*A+126*B)*(a*(1+cos(d*x+c))/cos(d*x
+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)*a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 514 vs. $2(157) = 314$.
time = 0.63, size = 514, normalized size = 2.84

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 1/840*(sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7
/2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c),
cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/2*
d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/2*d
*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
175*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*
x + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*
c), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*arcta
n2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(sin
(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A*sqrt(a) + 42*sqrt(2)*(20*a*cos(4/5*a
rctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) +
5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))*sin(5/2*d*
x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan2(sin(5/2*d*x + 5/2*c)
, cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x + 5/2*c)*sin(2/5*arctan2(sin(5/2
*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*a*sin(5/2*d*x + 5/2*c) + 5*a*sin(
3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 20*a*sin(1/5*arc
tan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*B*sqrt(a))/d
```

Fricas [A]
time = 2.69, size = 113, normalized size = 0.62

$$\frac{2\left(15\,Aa\cos\left(dx+c\right)^{4}+3\left(13\,A+7\,B\right)a\cos\left(dx+c\right)^{3}+\left(52\,A+63\,B\right)a\cos\left(dx+c\right)^{2}+2\left(52\,A+63\,B\right)a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*(15*A*a*cos(d*x + c)^4 + 3*(13*A + 7*B)*a*cos(d*x + c)^3 + (52*A + 63
*B)*a*cos(d*x + c)^2 + 2*(52*A + 63*B)*a*cos(d*x + c))*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2
), x)
```

Mupad [B]
time = 4.34, size = 131, normalized size = 0.72

$$\frac{a\,\cos\left(c+d\,x\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}\,\left(910\,A\,\sin\left(c+d\,x\right)+1050\,B\,\sin\left(c+d\,x\right)+238\,A\,\sin\left(2\,c+2\,d\,x\right)+78\,A\,\sin\left(3\,c+3\,d\,x\right)+15\,A\,\sin\left(4\,c+4\,d\,x\right)+252\,B\,\sin\left(2\,c+2\,d\,x\right)+42\,B\,\sin\left(3\,c+3\,d\,x\right)\right)}{420\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(7/2
),x)
```

```
[Out] (a*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x)
)^(1/2)*(910*A*sin(c + d*x) + 1050*B*sin(c + d*x) + 238*A*sin(2*c + 2*d*x)
+ 78*A*sin(3*c + 3*d*x) + 15*A*sin(4*c + 4*d*x) + 252*B*sin(2*c + 2*d*x) +
42*B*sin(3*c + 3*d*x)))/(420*d*(cos(c + d*x) + 1))
```

3.238 $\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=228

$$\frac{2a^2(10A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(34A+39B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{8a^2(34A+39B)\sin(c+dx)}{315d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2/63*a^2*(10*A+9*B)*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+2/
105*a^2*(34*A+39*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+8/
315*a^2*(34*A+39*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16
/315*a^2*(34*A+39*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2
/9*a*A*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(7/2)
```

Rubi [A]
time = 0.34, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4102, 4100, 3890, 3889}

$$\frac{2a^2(34A+39B)\sin(c+dx)}{105d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(10A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(34A+39B)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{8a^2(34A+39B)\sin(c+dx)}{315d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*a^2*(10*A + 9*B)*Sin[c + d*x])/(63*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c
 + d*x]]) + (2*a^2*(34*A + 39*B)*Sin[c + d*x])/(105*d*Sec[c + d*x]^(3/2)*Sq
rt[a + a*Sec[c + d*x]]) + (8*a^2*(34*A + 39*B)*Sin[c + d*x])/(315*d*Sqrt[Se
c[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (16*a^2*(34*A + 39*B)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Sqrt[a + a*Se
c[c + d*x]]*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{2}{9}\int \frac{\sqrt{a+a\sec(c}{}$$

$$= \frac{2a^2(10A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{2aA\sqrt{a+a\sec(}{9d\sec^{}}$$

$$= \frac{2a^2(10A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(34A+}{105d\sec^{\frac{3}{2}}(c+dx}$$

$$= \frac{2a^2(10A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(34A+}{105d\sec^{\frac{3}{2}}(c+dx}$$

$$= \frac{2a^2(10A+9B)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(34A+}{105d\sec^{\frac{3}{2}}(c+dx}$$

Mathematica [A]
time = 0.67, size = 110, normalized size = 0.48

$$\frac{2a^2(35A+5(17A+9B)\sec(c+dx)+3(34A+39B)\sec^2(c+dx)+4(34A+39B)\sec^3(c+dx)+8(34A+39B)\sec^4(c+dx))\sin(c+dx)}{315d\sec^{\frac{7}{2}}(c+dx)\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9
/2),x]
```

[Out] (2*a^2*(35*A + 5*(17*A + 9*B)*Sec[c + d*x] + 3*(34*A + 39*B)*Sec[c + d*x]^2 + 4*(34*A + 39*B)*Sec[c + d*x]^3 + 8*(34*A + 39*B)*Sec[c + d*x]^4)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(7/2)*Sqrt[a*(1 + Sec[c + d*x])])

Maple [A]
time = 7.58, size = 141, normalized size = 0.62

method	result
default	$-\frac{2(-1+\cos(dx+c))(35A(\cos^4(dx+c))+85A(\cos^3(dx+c))+45B(\cos^3(dx+c))+102A(\cos^2(dx+c))+117B(\cos^2(dx+c))+136A\cos(}{315d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)

[Out] -2/315/d*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+85*A*cos(d*x+c)^3+45*B*cos(d*x+c)^3+102*A*cos(d*x+c)^2+117*B*cos(d*x+c)^2+136*A*cos(d*x+c)+156*B*cos(d*x+c)+272*A+312*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(9/2)*cos(d*x+c)^5/sin(d*x+c)*a

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 700 vs. 2(198) = 396.
time = 0.64, size = 700, normalized size = 3.07

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="maxima")

[Out] 1/5040*(sqrt(2)*(3780*a*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 1050*a*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 378*a*cos(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 135*a*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) - 3780*a*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 1050*a*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 378*a*cos(9/2*d*x + 9/2*c)*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 135*a*cos(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*a*sin(9/2*d*x + 9/2*c) + 135*a*sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 378*a*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 1050*a*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 3780*a*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*A*sqrt(a) + 6*sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x

```
+ 7/2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c
), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7
/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/
2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))
 - 175*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2
*d*x + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7
/2*c), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*ar
ctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(
sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(sin(7/
2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*B*sqrt(a))/d
```

Fricas [A]
time = 1.97, size = 132, normalized size = 0.58

$$\frac{2\,\left(35\,Aa\cos\left(dx+c\right)^{5}+5\,(17\,A+9\,B)a\cos\left(dx+c\right)^{4}+3\,(34\,A+39\,B)a\cos\left(dx+c\right)^{3}+4\,(34\,A+39\,B)a\cos\left(dx+c\right)^{2}+8\,(34\,A+39\,B)a\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{315\,\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 2/315*(35*A*a*cos(d*x + c)^5 + 5*(17*A + 9*B)*a*cos(d*x + c)^4 + 3*(34*A +
39*B)*a*cos(d*x + c)^3 + 4*(34*A + 39*B)*a*cos(d*x + c)^2 + 8*(34*A + 39*B)
*a*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*c
os(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(9/2
), x)
```

Mupad [B]
time = 5.30, size = 155, normalized size = 0.68

$$\frac{a\cos\left(c+d\,x\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}\,\left(4830\,A\,\sin\left(c+d\,x\right)+5460\,B\,\sin\left(c+d\,x\right)+1428\,A\,\sin\left(2\,c+2\,d\,x\right)+513\,A\,\sin\left(3\,c+3\,d\,x\right)+170\,A\,\sin\left(4\,c+4\,d\,x\right)+35\,A\,\sin\left(5\,c+5\,d\,x\right)+1428\,B\,\sin\left(2\,c+2\,d\,x\right)+468\,B\,\sin\left(3\,c+3\,d\,x\right)+90\,B\,\sin\left(4\,c+4\,d\,x\right)\right)}{2520\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(9/2
),x)
```

```
[Out] (a*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*x)
)^(1/2)*(4830*A*sin(c + d*x) + 5460*B*sin(c + d*x) + 1428*A*sin(2*c + 2*d*x
) + 513*A*sin(3*c + 3*d*x) + 170*A*sin(4*c + 4*d*x) + 35*A*sin(5*c + 5*d*x)
 + 1428*B*sin(2*c + 2*d*x) + 468*B*sin(3*c + 3*d*x) + 90*B*sin(4*c + 4*d*x)
))/(2520*d*(cos(c + d*x) + 1))
```

3.239 $\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=274

$$\frac{a^{5/2}(326A+283B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{128d}+\frac{a^3(326A+283B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}}+\frac{a^3(326A+\cdots}{19\cdots}$$

```
[Out] 1/128*a^(5/2)*(326*A+283*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/
2))/d+1/5*a*B*sec(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+1/128*a^
3*(326*A+283*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/192*
a^3*(326*A+283*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/24
0*a^3*(170*A+157*B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/
40*a^2*(10*A+13*B)*sec(d*x+c)^(7/2)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.46, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4103, 4101, 3888, 3886, 221}

$$\frac{a^{5/2}(326A+283B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{128d}+\frac{a^3(170A+157B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{240d\sqrt{a\sec(c+dx)+a}}+\frac{a^3(326A+283B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{192d\sqrt{a\sec(c+dx)+a}}+\frac{a^3(326A+283B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{128d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(10A+13B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{40d}+\frac{aB\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(5/2)*(326*A + 283*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]])/(128*d) + (a^3*(326*A + 283*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(12
8*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(326*A + 283*B)*Sec[c + d*x]^(5/2)*Sin
[c + d*x])/(192*d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(170*A + 157*B)*Sec[c +
d*x]^(7/2)*Sin[c + d*x])/(240*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(10*A + 13
*B)*Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(40*d) + (a*B
*Sec[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aB\sec^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+}{5d}$$

$$= \frac{a^2(10A+13B)\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+}}{40d}$$

$$= \frac{a^3(170A+157B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{240d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{a^3(326A+283B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{192d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{a^3(326A+283B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{a^3(326A+283B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{128d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{a^{5/2}(326A+283B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+}}\right.}{128d}$$

Mathematica [**A**]
time = 2.17, size = 178, normalized size = 0.65

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(60\sqrt{2}(326A+283B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+(22030A+24863B+36(650A+781B)\cos(c+dx)+4(6730A+6509B)\cos(2(c+dx))+6520A\cos(3(c+dx))+5660B\cos(3(c+dx))+4890A\cos(4(c+dx))+4245B\cos(4(c+dx)))\sec^5(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)\right)}{15360d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(60*Sqrt[2]*(326*A + 283*B
)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + (22030*A + 24863*B + 36*(650*A + 781*
B)*Cos[c + d*x] + 4*(6730*A + 6509*B)*Cos[2*(c + d*x)] + 6520*A*Cos[3*(c +
d*x)] + 5660*B*Cos[3*(c + d*x)] + 4890*A*Cos[4*(c + d*x)] + 4245*B*Cos[4*(c
 + d*x)])*Sec[c + d*x]^5*Sin[(c + d*x)/2]))/(15360*d*Sqrt[Sec[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 542 vs. 2(236) = 472.
time = 7.93, size = 543, normalized size = 1.98

method	result
default	$\frac{\left(4890A\sqrt{2}\,(\cos^5(dx+c))\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(-1-\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)+4890A\sqrt{2}\,(\cos^5(dx+c))\arctan\left(\frac{\sqrt{}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETU
RNVERBOSE)

[Out] 1/7680/d*(4890*A*2^(1/2)*cos(d*x+c)^5*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*
(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))+4890*A*2^(1/2)*cos(d*x+c)^5*arctan(1/4*
(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))+4245*B*2^(1/2)
*cos(d*x+c)^5*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c
))*2^(1/2))+4245*B*2^(1/2)*cos(d*x+c)^5*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2
)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))+9780*A*sin(d*x+c)*cos(d*x+c)^4*(-2/(1+
cos(d*x+c)))^(1/2)+8490*B*sin(d*x+c)*cos(d*x+c)^4*(-2/(1+cos(d*x+c)))^(1/2)
+6520*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(1+cos(d*x+c)))^(1/2)+5660*B*sin(d*x+c)
*cos(d*x+c)^3*(-2/(1+cos(d*x+c)))^(1/2)+3680*A*(-2/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)^2*sin(d*x+c)+4528*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x
+c)+960*A*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+2784*B*cos(d*x+c)
*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+768*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d
x+c))(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(5/2)*(-2/(1+cos(
d*x+c)))^(1/2)/cos(d*x+c)^2/sin(d*x+c)^2*(cos(d*x+c)^2-1)*a^2

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 9242 vs. $2(236) = 472$.
time = 1.65, size = 9242, normalized size = 33.73

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")

[Out] -1/7680*(10*(1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x +
 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*co
s(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*sin(
8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4
*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6
*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*
c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a
^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*

d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2
*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*
x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt
(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*s
in(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)
*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(
2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1956*(
sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a
^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(
6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^
2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^
2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 +
8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x
 + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*
x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2
*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin
(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*
d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^
2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 +
 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c)
 + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d
*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2
*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos
(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*
sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin
(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqr
t(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 489*(a^2*c
os(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 +
 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*
c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
 + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(
6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8
*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos
(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a
^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(

```
8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d
*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16
*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*
c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*
x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x +
2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*c
os(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2
*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*
a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) +
3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c)...
```

Fricas [A]
time = 3.63, size = 558, normalized size = 2.04

$$\left[\frac{15\left((326A+283B)a^2\cos(dx+c)^5+(326A+283B)a^2\cos(dx+c)^4\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(15(326A+283B)a^2\cos(dx+c)^4+10(326A+283B)a^2\cos(dx+c)^3+8(230A+283B)a^2\cos(dx+c)^2+48(10A+29B)a^2\cos(dx+c)+384Ba^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{7680\left(d\cos(dx+c)^5+d\cos(dx+c)^4\right)},\ \frac{15\left((326A+283B)a^2\cos(dx+c)^5+(326A+283B)a^2\cos(dx+c)^4\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)+\frac{2\left(15(326A+283B)a^2\cos(dx+c)^4+10(326A+283B)a^2\cos(dx+c)^3+8(230A+283B)a^2\cos(dx+c)^2+48(10A+29B)a^2\cos(dx+c)+384Ba^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3840\left(d\cos(dx+c)^5+d\cos(dx+c)^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] [1/7680*(15*((326*A + 283*B)*a^2*cos(d*x + c)^5 + (326*A + 283*B)*a^2*cos(d
*x + c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x
+ c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4
*(15*(326*A + 283*B)*a^2*cos(d*x + c)^4 + 10*(326*A + 283*B)*a^2*cos(d*x +
c)^3 + 8*(230*A + 283*B)*a^2*cos(d*x + c)^2 + 48*(10*A + 29*B)*a^2*cos(d*x
+ c) + 384*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt
(cos(d*x + c)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4), 1/3840*(15*((326*A +
 283*B)*a^2*cos(d*x + c)^5 + (326*A + 283*B)*a^2*cos(d*x + c)^4)*sqrt(-a)*a
rctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(15*(326*A + 2
83*B)*a^2*cos(d*x + c)^4 + 10*(326*A + 283*B)*a^2*cos(d*x + c)^3 + 8*(230*A
 + 283*B)*a^2*cos(d*x + c)^2 + 48*(10*A + 29*B)*a^2*cos(d*x + c) + 384*B*a^
2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))
/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2),
x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2),
 x)
```

3.240 $\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=227

$$\frac{a^{5/2}(200A+163B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{64d}+\frac{a^3(200A+163B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}}+\frac{a^3(104A$$

```
[Out] 1/64*a^(5/2)*(200*A+163*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2
))/d+1/4*a*B*sec(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+1/64*a^3*
(200*A+163*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/96*a^3
*(104*A+95*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/24*a^2
*(8*A+11*B)*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.40, antiderivative size = 227, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4103, 4101, 3888, 3886, 221}

$$\frac{a^{5/2}(200A+163B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^3(104A+95B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{96d\sqrt{a\sec(c+dx)+a}}+\frac{a^3(200A+163B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{64d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(8A+11B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{24d}+\frac{aB\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(5/2)*(200*A + 163*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]])/(64*d) + (a^3*(200*A + 163*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(64*
d*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(104*A + 95*B)*Sec[c + d*x]^(5/2)*Sin[c
+ d*x])/(96*d*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(8*A + 11*B)*Sec[c + d*x]^(5
/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (a*B*Sec[c + d*x]^(5/2)
*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aB\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c}{4d}$$

$$= \frac{a^2(8A+11B)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+}}{24d}$$

$$= \frac{a^3(104A+95B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{96d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{a^3(200A+163B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^3(200A+163B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{64d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^{5/2}(200A+163B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+d}{\sqrt{a+a\sec(c+}}\right.}{64d}$$

Mathematica [A]
time = 1.48, size = 154, normalized size = 0.68

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(6\sqrt{2}\,(200A+163B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+(544A+844B+(2056A+2203B)\cos(c+dx)+(544A+652B)\cos(2(c+dx))+600A\cos(3(c+dx))+489B\cos(3(c+dx)))\sec^4(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)\right)}{768d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(200*A + 163*B)
*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + (544*A + 844*B + (2056*A + 2203*B)*Cos
[c + d*x] + (544*A + 652*B)*Cos[2*(c + d*x)] + 600*A*Cos[3*(c + d*x)] + 489
*B*Cos[3*(c + d*x)])*Sec[c + d*x]^4*Sin[(c + d*x)/2]))/(768*d*Sqrt[Sec[c +
d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(195) = 390$.
time = 7.63, size = 479, normalized size = 2.11

method	result

default	$-\frac{(-1+\cos(dx+c))\left(600A\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}\,(\cos^4(dx+c))+600A\sqrt{2}\,(\cos^4(dx+c$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -1/384/d*(-1+cos(d*x+c))*(600*A*2^(1/2)*cos(d*x+c)^4*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))+600*A*2^(1/2)*cos(d*x+c)^4*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))+489*B*2^(1/2)*cos(d*x+c)^4*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))+489*B*2^(1/2)*cos(d*x+c)^4*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))+1200*A*sin(d*x+c)*cos(d*x+c)^3*(-2/(1+cos(d*x+c)))^(1/2)+978*B*sin(d*x+c)*cos(d*x+c)^3*(-2/(1+cos(d*x+c)))^(1/2)+544*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+652*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+128*A*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+368*B*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+96*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(3/2)/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^2*a^2

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 7331 vs. $2(195) = 390$.
time = 1.16, size = 7331, normalized size = 32.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/768*(8*(300*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(6*d*x + 6*c) - 28*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 28*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 28*(sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - sqrt(2)*a^2*sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) - 300*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 114*sqrt(2)*a^2*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 114*sqrt(2)*a^2*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x

+ 3/2*c))) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x

```
+ 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c
) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(
a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^2*
cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(...
```

Fricas [A]
time = 3.42, size = 518, normalized size = 2.28

$$\left[\frac{3\left((200A+163B)a^2\cos(dx+c)^4+(200A+163B)a^2\cos(dx+c)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(3(200A+163B)a^2\cos(dx+c)^3+2(136A+163B)a^2\cos(dx+c)^2+8(8A+23B)a^2\cos(dx+c)+48Ba^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{768\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}, \frac{3\left((200A+163B)a^2\cos(dx+c)^4+(200A+163B)a^2\cos(dx+c)^3\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)+\frac{2\left(3(200A+163B)a^2\cos(dx+c)^3+2(136A+163B)a^2\cos(dx+c)^2+8(8A+23B)a^2\cos(dx+c)+48Ba^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{384\left(d\cos(dx+c)^4+d\cos(dx+c)^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] [1/768*(3*((200*A + 163*B)*a^2*cos(d*x + c)^4 + (200*A + 163*B)*a^2*cos(d*x
 + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x +
c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(
3*(200*A + 163*B)*a^2*cos(d*x + c)^3 + 2*(136*A + 163*B)*a^2*cos(d*x + c)^2
 + 8*(8*A + 23*B)*a^2*cos(d*x + c) + 48*B*a^2)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^4 + d*cos(d*x
+ c)^3), 1/384*(3*((200*A + 163*B)*a^2*cos(d*x + c)^4 + (200*A + 163*B)*a^2
*cos(d*x + c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c)
 - 2*a)) + 2*(3*(200*A + 163*B)*a^2*cos(d*x + c)^3 + 2*(136*A + 163*B)*a^2*
cos(d*x + c)^2 + 8*(8*A + 23*B)*a^2*cos(d*x + c) + 48*B*a^2)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^
4 + d*cos(d*x + c)^3)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2),
 x)
```

3.241 $\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=180

$$\frac{a^{5/2}(38A+25B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{8d}+\frac{a^3(54A+49B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}}+\frac{a^2(2A+3B)\,\mathrm{se}}{}$$

```
[Out] 1/8*a^(5/2)*(38*A+25*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/
d+1/3*a*B*sec(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+1/24*a^3*(54
*A+49*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+1/4*a^2*(2*A+
3*B)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.33, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4103, 4101, 3886, 221}

$$\frac{a^{5/2}(38A+25B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^3(54A+49B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{24d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(2A+3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{4d}+\frac{aB\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(5/2)*(38*A + 25*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(8*d) + (a^3*(54*A + 49*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(24*d*Sqr
t[a + a*Sec[c + d*x]]) + (a^2*(2*A + 3*B)*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec
[c + d*x]]*Sin[c + d*x])/(4*d) + (a*B*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x
])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
```

```
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aB\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(}{3d}$$

$$= \frac{a^2(2A+3B)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}}{4d}$$

$$= \frac{a^3(54A+49B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^3(54A+49B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{24d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{a^{5/2}(38A+25B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx}{\sqrt{a+a\sec(c+}}\right.}{8d}$$

Mathematica [A]
time = 1.29, size = 133, normalized size = 0.74

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(3\sqrt{2}(38A+25B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+(66A+91B+4(6A+17B)\cos(c+dx)+(66A+75B)\cos(2(c+dx)))\sec^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)\right)}{48d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

[In] Integrate[Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]

[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(38*A + 25*B)*A
rcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + (66*A + 91*B + 4*(6*A + 17*B)*Cos[c + d*
x] + (66*A + 75*B)*Cos[2*(c + d*x)])*Sec[c + d*x]^3*Sin[(c + d*x)/2]))/(48*
d*Sqrt[Sec[c + d*x]])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(154) = 308$.
time = 7.30, size = 419, normalized size = 2.33

method	result
default	$\left(114A(\cos^3(dx+c))\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}-114A\sqrt{2}\,(\cos^3(dx+c))\arctan\left(\frac{\sqrt{-\frac{}{1+}}}{}\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)

[Out] 1/96/d*(114*A*2^(1/2)*cos(d*x+c)^3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+
cos(d*x+c)+sin(d*x+c))*2^(1/2))-114*A*2^(1/2)*cos(d*x+c)^3*arctan(1/4*(-2/(
1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))+75*B*2^(1/2)*cos(d*
x+c)^3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/
2))-75*B*2^(1/2)*cos(d*x+c)^3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d
*x+c)-sin(d*x+c))*2^(1/2))+132*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin
(d*x+c)+150*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2*sin(d*x+c)+24*A*cos(d*
x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+68*B*cos(d*x+c)*sin(d*x+c)*(-2/(1
+cos(d*x+c)))^(1/2)+16*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c))*(a*(1+cos(d*
x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1/2)*(-2/(1+cos(d*x+c)))^(1/2)/cos(
d*x+c)^2/sin(d*x+c)^2*(cos(d*x+c)^2-1)*a^2

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 6297 vs. $2(154) = 308$.
time = 3.61, size = 6297, normalized size = 34.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="maxima")

[Out] -1/96*(6*(88*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 56*sqrt(2)
*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x + 3

```
/2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a
^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^2
- 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*c
os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x +
1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*(a^2*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/
2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x + 4
*c)^2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 2*(22*sqrt(2)*a^2*sin(7/2*d*x + 7/2*
c) - 14*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 14*sqrt(2)*a^2*sin(3/2*d*x + 3/2
*c) - 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
```

```
- 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*(a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*co
s(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) -
 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*
a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) +
 4*(11*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c) - 7*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c
) + 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) - 11*sqr...
```

Fricas [A]
time = 2.91, size = 478, normalized size = 2.66

$$\left[\frac{3 \, {\left({\left(38 \, A + 25 \, B\right)} a^{2} \cos\left(d x + c\right)^{3} + {\left(38 \, A + 25 \, B\right)} a^{2} \cos\left(d x + c\right)^{2}\right)} \sqrt{a} \log\left(\frac{a \cos\left(d x + c\right)^{3} - 7 \, a \cos\left(d x + c\right)^{2} - \frac{4 \, {\left(\cos\left(d x + c\right)^{2} - 2 \, \cos\left(d x + c\right)\right)} \sqrt{a} \sqrt{\frac{a \cos\left(d x + c\right) + a}{\cos\left(d x + c\right)}} \sin\left(d x + c\right)}{\sqrt{\cos\left(d x + c\right)}} + 8 \, a}{\cos\left(d x + c\right)^{3} + \cos\left(d x + c\right)^{2}}\right) + \frac{4 \, {\left(3 \, {\left(22 \, A + 25 \, B\right)} a^{2} \cos\left(d x + c\right)^{2} + 2 \, {\left(6 \, A + 17 \, B\right)} a^{2} \cos\left(d x + c\right) + 8 \, B a^{2}\right)} \sqrt{\frac{a \cos\left(d x + c\right) + a}{\cos\left(d x + c\right)}} \sin\left(d x + c\right)}{\sqrt{\cos\left(d x + c\right)}}}{96 \, {\left(d \cos\left(d x + c\right)^{3} + d \cos\left(d x + c\right)^{2}\right)}}, \frac{3 \, {\left({\left(38 \, A + 25 \, B\right)} a^{2} \cos\left(d x + c\right)^{3} + {\left(38 \, A + 25 \, B\right)} a^{2} \cos\left(d x + c\right)^{2}\right)} \sqrt{-a} \arctan\left(\frac{2 \, \sqrt{-a} \sqrt{\frac{a \cos\left(d x + c\right) + a}{\cos\left(d x + c\right)}} \sqrt{\cos\left(d x + c\right)} \sin\left(d x + c\right)}{a \cos\left(d x + c\right)^{2} - a \cos\left(d x + c\right) - 2 \, a}\right) + \frac{2 \, {\left(3 \, {\left(22 \, A + 25 \, B\right)} a^{2} \cos\left(d x + c\right)^{2} + 2 \, {\left(6 \, A + 17 \, B\right)} a^{2} \cos\left(d x + c\right) + 8 \, B a^{2}\right)} \sqrt{\frac{a \cos\left(d x + c\right) + a}{\cos\left(d x + c\right)}} \sin\left(d x + c\right)}{\sqrt{\cos\left(d x + c\right)}}}{48 \, {\left(d \cos\left(d x + c\right)^{3} + d \cos\left(d x + c\right)^{2}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/96*(3*((38*A + 25*B)*a^2*cos(d*x + c)^3 + (38*A + 25*B)*a^2*cos(d*x + c)
^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2
- 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x +
 c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(3*(22
*A + 25*B)*a^2*cos(d*x + c)^2 + 2*(6*A + 17*B)*a^2*cos(d*x + c) + 8*B*a^2)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d
*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/48*(3*((38*A + 25*B)*a^2*cos(d*x + c
)^3 + (38*A + 25*B)*a^2*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x
+ c)^2 - a*cos(d*x + c) - 2*a)) + 2*(3*(22*A + 25*B)*a^2*cos(d*x + c)^2 + 2
*(6*A + 17*B)*a^2*cos(d*x + c) + 8*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^
2)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2),
x)
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2),
 x)
```

3.242 $\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=180

$$\frac{a^{5/2}(20A+19B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{a^3(4A-9B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2(4A+7B)\sqrt{\text{s}}}{}$$

```
[Out] 1/4*a^(5/2)*(20*A+19*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/
d+1/2*a*B*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+1/4*a^3*(4*A
-9*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+1/4*a^2*(4*A+7*B
)*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.32, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4103, 4100, 3886, 221}

$$\frac{a^{5/2}(20A+19B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a^3(4A-9B)\sin(c+dx)\sqrt{\sec(c+dx)}}{4d\sqrt{a\sec(c+dx)+a}}+\frac{a^2(4A+7B)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{4d}+\frac{aB\sin(c+dx)\sqrt{\sec(c+dx)}(a\sec(c+dx)+a)^{3/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (a^(5/2)*(20*A + 19*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]])/(4*d) + (a^3*(4*A - 9*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[a
 + a*Sec[c + d*x]]) + (a^2*(4*A + 7*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c
+ d*x]]*Sin[c + d*x])/(4*d) + (a*B*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^
(3/2)*Sin[c + d*x])/(2*d)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
```

```
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{aB\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{2d}+\frac{1}{2}$$

$$= \frac{a^2(4A+7B)\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,\sin(c+}{4d}$$

$$= \frac{a^3(4A-9B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2(4A+7B)}{}$$

$$= \frac{a^3(4A-9B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}}+\frac{a^2(4A+7B)}{}$$

$$= \frac{a^{5/2}(20A+19B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4d}+\frac{a^3(4}{}$$

Mathematica [A]

time = 1.94, size = 137, normalized size = 0.76

$$\frac{a^3\left(20A\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)\tan(c+dx)-19B\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\tan(c+dx)+\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\,(8A\sin(c+dx)+(4A+11B+2B\sec(c+dx))\tan(c+dx))\right)}{4d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*
x]],x]
```

[Out] (a^3*(20*A*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 19*B*ArcSin[Sqrt[Sec[c + d*x]]]*Tan[c + d*x] + Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*(8*A*Sin[c + d*x] + (4*A + 11*B + 2*B*Sec[c + d*x])*Tan[c + d*x])))/(4*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(154) = 308$.
time = 7.79, size = 386, normalized size = 2.14

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(20A\left(\cos^2(dx+c)\right)\sqrt{2}\,\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)-\sin(dx+c)}{4}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/16/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(20*A*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))-20*A*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))+19*B*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))-19*B*cos(d*x+c)^2*2^(1/2)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))-32*A*cos(d*x+c)^3+16*A*cos(d*x+c)^2-44*B*cos(d*x+c)^2+16*A*cos(d*x+c)+36*B*cos(d*x+c)+8*B)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/cos(d*x+c)*a^2

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 14322 vs. $2(154) = 308$.
time = 3.60, size = 14322, normalized size = 79.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/16*(4*(8*a^2*cos(1/2*d*x + 1/2*c)^4*sin(1/2*d*x + 1/2*c) + 16*a^2*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^3 + 8*a^2*sin(1/2*d*x + 1/2*c)^5 + 5*(sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(

```
1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^4 + 10*(sqrt(2)*
a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*cos(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
+ 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2
 + 5*(sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqr
t(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(1/2*d*x + 1/2*c)^4 + (8*a^2
*sin(1/2*d*x + 1/2*c)^3 + (5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
 2) - 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8
*a^2*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + 5*(sqrt(2)*a^2*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*
sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^2 + (5*sqrt(2)*a^2*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) + 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2) - 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
```

```
+ 1/2*c) + 2) + 8*a^2*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 5*(sqr
t(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(1/2*d*x + 1/2*c)^2 + 2*(8*a^2*cos(1/
2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 5*(sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr...
```

Fricas [A]
time = 3.28, size = 454, normalized size = 2.52

$$\left[\frac{\left((20\,A+19\,B)a^2\cos\left(dx+c\right)^2+(20\,A+19\,B)a^2\cos\left(dx+c\right)\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\left(8\,Aa^2\cos(dx+c)^2+(4\,A+11\,B)a^2\cos(dx+c)+2\,Ba^2\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{16\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)},\ \frac{\left((20\,A+19\,B)a^2\cos\left(dx+c\right)^2+(20\,A+19\,B)a^2\cos\left(dx+c\right)\right)\sqrt{-a}\,\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2\,a}\right)+\frac{2\left(8\,Aa^2\cos(dx+c)^2+(4\,A+11\,B)a^2\cos(dx+c)+2\,Ba^2\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{8\left(d\cos\left(dx+c\right)^2+d\cos\left(dx+c\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(((20*A + 19*B)*a^2*cos(d*x + c)^2 + (20*A + 19*B)*a^2*cos(d*x + c))*
sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*
cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(8*A*a^2*c
os(d*x + c)^2 + (4*A + 11*B)*a^2*cos(d*x + c) + 2*B*a^2)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2 +
d*cos(d*x + c)), 1/8*(((20*A + 19*B)*a^2*cos(d*x + c)^2 + (20*A + 19*B)*a^2
*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) -
 2*a)) + 2*(8*A*a^2*cos(d*x + c)^2 + (4*A + 11*B)*a^2*cos(d*x + c) + 2*B*a^
2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))
/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{5/2}}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(1/2
), x)
```

3.243 $\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=177

$$\frac{a^{5/2}(2A+5B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{a^3(14A+3B)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}}-\frac{a^2(2A-3B)\sqrt{\mathrm{se}\ldots}}{}$$

```
[Out] a^(5/2)*(2*A+5*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2/3*
a*A*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+1/3*a^3*(14*A+3*B)
*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)-1/3*a^2*(2*A-3*B)*sin
(d*x+c)*sec(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.32, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4102, 4103, 4100, 3886, 221}

$$\frac{a^{5/2}(2A+5B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^3(14A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}-\frac{a^2(2A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}{3d}+\frac{2aA\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (a^(5/2)*(2*A + 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]])/d + (a^3*(14*A + 3*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*
Sec[c + d*x]]) - (a^2*(2*A - 3*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x
]]*Sin[c + d*x])/(3*d) + (2*a*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3
*d*Sqrt[Sec[c + d*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
```

```
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{(a+a\sec(}{}$$

$$= -\frac{a^2(2A-3B)\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,\sin(c}{3d}$$

$$= \frac{a^3(14A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} - \frac{a^2(2A-3B}{}$$

$$= \frac{a^3(14A+3B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} - \frac{a^2(2A-3B}{}$$

$$= \frac{a^{5/2}(2A+5B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{a^3(14A}{}$$

Mathematica [A]
time = 0.92, size = 133, normalized size = 0.75

$$\frac{a^3\left(3(2A+5B)\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)+\sqrt{1-\sec(c+dx)}\,(2A\sin(c+dx)+(16A+6B+3B\sec(c+dx))\tan(c+dx))\right)}{3d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] (a^3*(3*(2*A + 5*B)*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^(3/2)*Sin[c
 + d*x] + Sqrt[1 - Sec[c + d*x]]*(2*A*Sin[c + d*x] + (16*A + 6*B + 3*B*Sec[
c + d*x])*Tan[c + d*x])))/(3*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sq
rt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(153) = 306.
time = 7.34, size = 376, normalized size = 2.12

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(6A\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(-1-\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sqrt{2}\,\sin(dx+c)\,\mathrm{co}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/12/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(6*A*arctan(1/4*(-2/(1+cos(d*x+
c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*2^
(1/2)*sin(d*x+c)*cos(d*x+c)+6*A*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1
+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)*s
in(d*x+c)+15*B*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+
c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*2^(1/2)*sin(d*x+c)*cos(d*x+c)+15*B*(
-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)
+sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)*sin(d*x+c)+8*A*cos(d*x+c)^3+56*A*c
os(d*x+c)^2+24*B*cos(d*x+c)^2-64*A*cos(d*x+c)-12*B*cos(d*x+c)-12*B)*cos(d*x
+c)*(1/cos(d*x+c))^(3/2)/sin(d*x+c)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 12088 vs. 2(153) = 306.
time = 0.80, size = 12088, normalized size = 68.29

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(sqrt(2)*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))*sin(3/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^2*log(2*co
s(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*a^2*sin(3/2*d*
x + 3/2*c) + 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))))*A*sqrt(a) + 3*(8*a^2*cos(1/2*d*x + 1/2*c)^4*sin(1/2*d*x + 1/2*c) + 1
6*a^2*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^3 + 8*a^2*sin(1/2*d*x + 1
/2*c)^5 + 5*(sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2
) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(
2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*
cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^4 +
 10*(sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt
(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)
*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2
*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d
*x + 1/2*c)^2 + 5*(sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*
```

```
c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) +
 sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*
a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(1/2*d*x + 1/2*
c)^4 + (8*a^2*sin(1/2*d*x + 1/2*c)^3 + (5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) - 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*si
n(1/2*d*x + 1/2*c) + 2) + 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2) - 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) + 8*a^2*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + 5*(sqrt(2)
*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*co
s(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*cos(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
 + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^2 + (5*sqrt(2)*a^2*log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 5*sqrt(2)*a^2*log(2*cos(1
/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*
c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 5*sq...
```

Fricas [A]
time = 3.90, size = 424, normalized size = 2.40

$$\left[\frac{3\,\left((2\,A+5\,B)a^2\cos\left(dx+c\right)+(2\,A+5\,B)a^2\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\,\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\,\left(2\,Aa^2\cos(dx+c)^2+2\,(8\,A+3\,B)a^2\cos(dx+c)+3\,Ba^2\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{12\,\left(d\cos\left(dx+c\right)+d\right)},\ \frac{3\,\left((2\,A+5\,B)a^2\cos\left(dx+c\right)+(2\,A+5\,B)a^2\right)\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2\,a}\right)+\frac{2\,\left(2\,Aa^2\cos(dx+c)^2+2\,(8\,A+3\,B)a^2\cos(dx+c)+3\,Ba^2\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{6\,\left(d\cos\left(dx+c\right)+d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/12*(3*((2*A + 5*B)*a^2*cos(d*x + c) + (2*A + 5*B)*a^2)*sqrt(a)*log((a*co
s(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqr
t(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*(2*A*a^2*cos(d*x + c)^2 + 2
*(8*A + 3*B)*a^2*cos(d*x + c) + 3*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x
+ c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/6*(3*((2*A +
```

```
 5*B)*a^2*cos(d*x + c) + (2*A + 5*B)*a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*
x + c)^2 - a*cos(d*x + c) - 2*a)) + 2*(2*A*a^2*cos(d*x + c)^2 + 2*(8*A + 3*
B)*a^2*cos(d*x + c) + 3*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{5/2}}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(3/2
), x)
```

3.244 $\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=172

$$\frac{2a^{5/2}B\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}+\frac{2a^3(32A+35B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(8A+5B)\sqrt{a+a}}{15d\sqrt{\mathrm{se}}}$$

```
[Out] 2*a^(5/2)*B*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d+2/5*a*A*(a
+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/15*a^3*(32*A+35*B)*sin
(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/15*a^2*(8*A+5*B)*sin(d*
x+c)*(a+a*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4102, 4100, 3886, 221}

$$\frac{2a^{5/2}B\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^3(32A+35B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(8A+5B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{15d\sqrt{\sec(c+dx)}}+\frac{2aA\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*a^(5/2)*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/d +
(2*a^3*(32*A + 35*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[
c + d*x]]) + (2*a^2*(8*A + 5*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*
d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*
d*Sec[c + d*x]^(3/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
```

```
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+a\sec(}{}$$

$$= \frac{2a^2(8A+5B)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2aA(a}{}$$

$$= \frac{2a^3(32A+35B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(8A+}{}$$

$$= \frac{2a^3(32A+35B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(8A+}{}$$

$$= \frac{2a^{5/2}B\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d} + \frac{2a^3(32A+35}{15d}$$

Mathematica [A]

time = 1.92, size = 127, normalized size = 0.74

$$\frac{a^3\left((89A+80B+2(14A+5B)\cos(c+dx)+3A\cos(2(c+dx)))\sqrt{1-\sec(c+dx)}+30B\,\mathrm{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)\sqrt{\sec(c+dx)}\right)\tan(c+dx)}{15d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5
/2),x]
```

```
[Out] (a^3*((89*A + 80*B + 2*(14*A + 5*B)*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)])*Sq
rt[1 - Sec[c + d*x]] + 30*B*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x
]])*Tan[c + d*x])/(15*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sqrt[a*(1
 + Sec[c + d*x])])
```

Maple [A]
time = 7.51, size = 235, normalized size = 1.37

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(15B\sqrt{-\frac{2}{1+\cos(dx+c)}}\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}(-1-\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}\sin(dx+c)-\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/30/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(15*B*(-2/(1+cos(d*x+c)))^(1/2)
*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))*2
^(1/2)*sin(d*x+c)+15*B*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+
c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/2)*sin(d*x+c)+12*A*cos(d
*x+c)^3+44*A*cos(d*x+c)^2+20*B*cos(d*x+c)^2+116*A*cos(d*x+c)+140*B*cos(d*x+
c)-172*A-160*B)*(1/cos(d*x+c))^(5/2)*cos(d*x+c)^3/sin(d*x+c)*a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 655 vs. $2(148) = 296$.
time = 0.64, size = 655, normalized size = 3.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/60*(5*sqrt(2)*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c)))*sin(3/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan
2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arc
```

```
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^2*log(2*
cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log
(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*a^2*sin(3/2*
d*x + 3/2*c) + 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))))*B*sqrt(a) + 2*(3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2
*sin(3/2*d*x + 3/2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*A*sqrt(a))/d
```

Fricas [A]
time = 2.75, size = 424, normalized size = 2.47

$$\left[\frac{15\,(Ba^2\cos(dx+c)+Ba^2)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-7\,a\cos(dx+c)^2-\frac{4\,\left(\cos(dx+c)^2-2\,\cos(dx+c)\right)\sqrt{a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{4\,\left(3\,Aa^2\cos(dx+c)^3+(14\,A+5\,B)a^2\cos(dx+c)^2+(43\,A+40\,B)a^2\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{30\,(d\cos(dx+c)+d)},\ \frac{15\,(Ba^2\cos(dx+c)+Ba^2)\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2\,a}\right)+\frac{2\,\left(3\,Aa^2\cos(dx+c)^3+(14\,A+5\,B)a^2\cos(dx+c)^2+(43\,A+40\,B)a^2\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15\,(d\cos(dx+c)+d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/30*(15*(B*a^2*cos(d*x + c) + B*a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*
cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x +
 c)^3 + cos(d*x + c)^2)) + 4*(3*A*a^2*cos(d*x + c)^3 + (14*A + 5*B)*a^2*cos
(d*x + c)^2 + (43*A + 40*B)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d), 1/15*(15*
(B*a^2*cos(d*x + c) + B*a^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*
cos(d*x + c) - 2*a)) + 2*(3*A*a^2*cos(d*x + c)^3 + (14*A + 5*B)*a^2*cos(d*x
 + c)^2 + (43*A + 40*B)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(5/2
), x)
```

3.245 $\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=178

$$\frac{64a^3(5A+7B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}+\frac{16a^2(5A+7B)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{105d\sqrt{\sec(c+dx)}}+\frac{2a(5A+7B)}{}$$

```
[Out] 2/35*a*(5*A+7*B)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*A
*(a+a*sec(d*x+c))^(5/2)*sin(d*x+c)/d/sec(d*x+c)^(5/2)+64/105*a^3*(5*A+7*B)*
sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+16/105*a^2*(5*A+7*B)*s
in(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.21, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {4098, 3894, 3889}

$$\frac{64a^3(5A+7B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(5A+7B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{105d\sqrt{\sec(c+dx)}}+\frac{2a(5A+7B)\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2A\sin(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (64*a^3*(5*A + 7*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[
c + d*x]]) + (16*a^2*(5*A + 7*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(10
5*d*Sqrt[Sec[c + d*x]]) + (2*a*(5*A + 7*B)*(a + a*Sec[c + d*x])^(3/2)*Sin[c
 + d*x])/(35*d*Sec[c + d*x]^(3/2)) + (2*A*(a + a*Sec[c + d*x])^(5/2)*Sin[c
+ d*x])/(7*d*Sec[c + d*x]^(5/2))
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Dist[b*((2*m - 1)/(d*m)), Int[(a + b*Csc[
e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ
[2*m]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A(a+a\sec(c+dx))^{5/2}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{1}{7}(5A+7B)\int \frac{(a$$

$$= \frac{2a(5A+7B)(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2A(a+$$

$$= \frac{16a^2(5A+7B)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{105d\sqrt{\sec(c+dx)}} + \frac{2a(5A$$

$$= \frac{64a^3(5A+7B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{16a^2(5A+7$$

Mathematica [A]
time = 0.56, size = 91, normalized size = 0.51

$$\frac{2a^3(15A+3(20A+7B)\sec(c+dx)+(115A+98B)\sec^2(c+dx)+(230A+301B)\sec^3(c+dx))\sin(c+dx)}{105d\sec^{\frac{5}{2}}(c+dx)\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7
/2),x]
```

```
[Out] (2*a^3*(15*A + 3*(20*A + 7*B)*Sec[c + d*x] + (115*A + 98*B)*Sec[c + d*x]^2
+ (230*A + 301*B)*Sec[c + d*x]^3)*Sin[c + d*x])/(105*d*Sec[c + d*x]^(5/2)*S
qrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 7.45, size = 121, normalized size = 0.68

method	result
default	$-\frac{2(-1+\cos(dx+c))(15A(\cos^3(dx+c))+60A(\cos^2(dx+c))+21B(\cos^2(dx+c))+115A\cos(dx+c)+98B\cos(dx+c)+230A+301B)\sqrt{a}}{105d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+60*A*cos(d*x+c)^2+21*B*cos(d*x+
c)^2+115*A*cos(d*x+c)+98*B*cos(d*x+c)+230*A+301*B)*(a*(1+cos(d*x+c))/cos(d*
x+c))^(1/2)*cos(d*x+c)^4*(1/cos(d*x+c))^(7/2)/sin(d*x+c)*a^2
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 385 vs. $2(154) = 308$.
time = 0.63, size = 385, normalized size = 2.16

$$\frac{5\sqrt{2}\left(315\,a^2\cos\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+77\,a^2\cos\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+21\,a^2\cos\left(\frac{2}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)-315\,a^2\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\sin\left(\frac{6}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)-77\,a^2\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\sin\left(\frac{4}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)-21\,a^2\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\sin\left(\frac{2}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)+6\,a^2\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)+21\,a^2\sin\left(\frac{5}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)+77\,a^2\sin\left(\frac{3}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)+315\,a^2\sin\left(\frac{1}{7}\arctan\left(\sin\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right),\cos\left(\frac{7}{2}\,dx+\frac{7}{2}\,c\right)\right)\right)\right)A\sqrt{a}+28\left(3\sqrt{2}\,a^2\sin\left(\frac{5}{2}\,dx+\frac{5}{2}\,c\right)+25\sqrt{2}\,a^2\sin\left(\frac{3}{2}\,dx+\frac{3}{2}\,c\right)+150\sqrt{2}\,a^2\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)B\sqrt{a}}{840\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] 1/840*(5*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4/7*arctan2(sin(7/2*d*x + 7/2
*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*a^2*cos(2/7*arctan2(s
in(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 315*a^2*
cos(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c),
cos(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/
2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*sin(7/2*d*x + 7/2*c) + 21*a^
2*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 77*a^2*sin
(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 315*a^2*sin(1/7
*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A*sqrt(a) + 28*(3*sq
rt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 150*
sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*B*sqrt(a))/d
```

Fricas [**A**]
time = 1.39, size = 120, normalized size = 0.67

$$\frac{2\left(15\,Aa^2\cos\left(dx+c\right)^4+3\left(20\,A+7\,B\right)a^2\cos\left(dx+c\right)^3+\left(115\,A+98\,B\right)a^2\cos\left(dx+c\right)^2+\left(230\,A+301\,B\right)a^2\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*(15*A*a^2*cos(d*x + c)^4 + 3*(20*A + 7*B)*a^2*cos(d*x + c)^3 + (115*A
 + 98*B)*a^2*cos(d*x + c)^2 + (230*A + 301*B)*a^2*cos(d*x + c))*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*
x + c)))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2
), x)
```

Mupad [B]
time = 4.35, size = 133, normalized size = 0.75

$$\frac{a^2\cos\left(c+d\,x\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{a\,\left(\cos\left(c+d\,x\right)+1\right)}{\cos\left(c+d\,x\right)}}\,\left(1960\,A\,\sin\left(c+d\,x\right)+2450\,B\,\sin\left(c+d\,x\right)+490\,A\,\sin\left(2\,c+2\,d\,x\right)+120\,A\,\sin\left(3\,c+3\,d\,x\right)+15\,A\,\sin\left(4\,c+4\,d\,x\right)+392\,B\,\sin\left(2\,c+2\,d\,x\right)+42\,B\,\sin\left(3\,c+3\,d\,x\right)\right)}{420\,d\,\left(\cos\left(c+d\,x\right)+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(7/2
),x)
```

```
[Out] (a^2*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*
x))^(1/2)*(1960*A*sin(c + d*x) + 2450*B*sin(c + d*x) + 490*A*sin(2*c + 2*d*
x) + 120*A*sin(3*c + 3*d*x) + 15*A*sin(4*c + 4*d*x) + 392*B*sin(2*c + 2*d*x
) + 42*B*sin(3*c + 3*d*x)))/(420*d*(cos(c + d*x) + 1))
```

3.246 $\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=228

$$\frac{2a^3(124A+135B)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{2a^3(292A+345B)\sin(c+dx)}{315d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{4a^3(292A+345B)\sqrt{\sec\ldots}}{315d\sqrt{a+a\,\mathrm{s}\ldots}}$$

```
[Out] 2/9*a*A*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(7/2)+2/315*a^3*(124
*A+135*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+2/315*a^3*(2
92*A+345*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+4/315*a^3*
(292*A+345*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/21*a^2
*(4*A+3*B)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)
```

Rubi [A]
time = 0.41, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4102, 4100, 3890, 3889}

$$\frac{2a^3(124A+135B)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{4a^3(292A+345B)\sin(c+dx)\sqrt{\sec(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(292A+345B)\sin(c+dx)}{315d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(4A+3B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{21d\sec^{\frac{5}{2}}(c+dx)}+\frac{2aA\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*a^3*(124*A + 135*B)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*S
ec[c + d*x]]) + (2*a^3*(292*A + 345*B)*Sin[c + d*x])/(315*d*Sqrt[Sec[c + d*
x]]*Sqrt[a + a*Sec[c + d*x]]) + (4*a^3*(292*A + 345*B)*Sqrt[Sec[c + d*x]]*S
in[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(4*A + 3*B)*Sqrt[a +
 a*Sec[c + d*x]]*Sin[c + d*x])/(21*d*Sec[c + d*x]^(5/2)) + (2*a*A*(a + a*Se
c[c + d*x])^(3/2)*Sin[c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2}{9}\int \frac{(a+a\sec(c}{}$$

$$= \frac{2a^2(4A+3B)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2aA(a-}{}$$

$$= \frac{2a^3(124A+135B)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(4A+3B)\sqrt{}}{2}$$

$$= \frac{2a^3(124A+135B)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(292A-}{315d\sqrt{\sec(c+d}}$$

$$= \frac{2a^3(124A+135B)\sin(c+dx)}{315d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(292A-}{315d\sqrt{\sec(c+d}}$$

Mathematica [A]
time = 0.71, size = 108, normalized size = 0.47

$$\frac{2a^3(35A+5(26A+9B)\sec(c+dx)+3(73A+60B)\sec^2(c+dx)+(292A+345B)\sec^3(c+dx)+(584A+690B)\sec^4(c+dx))\sin(c+dx)}{315d\sec^{\frac{7}{2}}(c+dx)\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9
/2),x]
```

[Out] (2*a^3*(35*A + 5*(26*A + 9*B)*Sec[c + d*x] + 3*(73*A + 60*B)*Sec[c + d*x]^2 + (292*A + 345*B)*Sec[c + d*x]^3 + (584*A + 690*B)*Sec[c + d*x]^4)*Sin[c + d*x])/(315*d*Sec[c + d*x]^(7/2)*Sqrt[a*(1 + Sec[c + d*x])])

Maple [A]
time = 7.41, size = 143, normalized size = 0.63

method	result
default	$-\frac{2(-1+\cos(dx+c))(35A(\cos^4(dx+c))+130A(\cos^3(dx+c))+45B(\cos^3(dx+c))+219A(\cos^2(dx+c))+180B(\cos^2(dx+c))+292A\,\mathrm{co}}{315d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x,method=_RETURNVERBOSE)

[Out] -2/315/d*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+130*A*cos(d*x+c)^3+45*B*cos(d*x+c)^3+219*A*cos(d*x+c)^2+180*B*cos(d*x+c)^2+292*A*cos(d*x+c)+345*B*cos(d*x+c)+584*A+690*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(9/2)*cos(d*x+c)^5/sin(d*x+c)*a^2

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 746 vs. 2(198) = 396.
time = 0.65, size = 746, normalized size = 3.27

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="maxima")

[Out] 1/5040*(sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 756*a^2*cos(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 225*a^2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a^2*cos(9/2*d*x + 9/2*c)*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 225*a^2*cos(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*A*sqrt(a) + 30*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7

```
/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4
/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c
) + 21*a^2*cos(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin
(7/2*d*x + 7/2*c) - 315*a^2*cos(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*
x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*ar
ctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7
/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*
sin(7/2*d*x + 7/2*c) + 21*a^2*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2
*d*x + 7/2*c))) + 77*a^2*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
+ 7/2*c))) + 315*a^2*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c))))*B*sqrt(a))/d
```

Fricas [**A**]
time = 2.20, size = 141, normalized size = 0.62

$$\frac{2\,\left(35\,Aa^{2}\cos\left(dx+c\right)^{5}+5\,\left(26\,A+9\,B\right)a^{2}\cos\left(dx+c\right)^{4}+3\,\left(73\,A+60\,B\right)a^{2}\cos\left(dx+c\right)^{3}+\left(292\,A+345\,B\right)a^{2}\cos\left(dx+c\right)^{2}+2\,\left(292\,A+345\,B\right)a^{2}\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{315\,\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 2/315*(35*A*a^2*cos(d*x + c)^5 + 5*(26*A + 9*B)*a^2*cos(d*x + c)^4 + 3*(73*
A + 60*B)*a^2*cos(d*x + c)^3 + (292*A + 345*B)*a^2*cos(d*x + c)^2 + 2*(292*
A + 345*B)*a^2*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*
x + c)/((d*cos(d*x + c) + d)*sqrt(cos(d*x + c)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2
), x)
```

Mupad [B]
time = 5.55, size = 157, normalized size = 0.69

$$\frac{a^2\cos(c+dx)\sqrt{\frac{1}{\cos(c+dx)}}\sqrt{\frac{a(\cos(c+dx)+1)}{\cos(c+dx)}}\left(10290 A\sin(c+dx)+11760 B\sin(c+dx)+2856 A\sin(2c+2dx)+981 A\sin(3c+3dx)+260 A\sin(4c+4dx)+35 A\sin(5c+5dx)+2940 B\sin(2c+2dx)+720 B\sin(3c+3dx)+90 B\sin(4c+4dx)\right)}{2520 d(\cos(c+dx)+1)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(9/2
),x)
```

```
[Out] (a^2*cos(c + d*x)*(1/cos(c + d*x))^(1/2)*((a*(cos(c + d*x) + 1))/cos(c + d*
x))^(1/2)*(10290*A*sin(c + d*x) + 11760*B*sin(c + d*x) + 2856*A*sin(2*c + 2
*d*x) + 981*A*sin(3*c + 3*d*x) + 260*A*sin(4*c + 4*d*x) + 35*A*sin(5*c + 5*
d*x) + 2940*B*sin(2*c + 2*d*x) + 720*B*sin(3*c + 3*d*x) + 90*B*sin(4*c + 4*
d*x)))/(2520*d*(cos(c + d*x) + 1))
```

3.247 $\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=275

$$\frac{2a^3(194A+209B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{2a^3(710A+803B)\sin(c+dx)}{1155d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}+\frac{8a^3(710A+803B)\sin(c+dx)}{3465d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2/11*a*A*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(9/2)+2/693*a^3*(19
4*A+209*B)*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+2/1155*a^3*
(710*A+803*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+8/3465*a
^3*(710*A+803*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16/34
65*a^3*(710*A+803*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2
/99*a^2*(14*A+11*B)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(7/2)
```

Rubi [A]
time = 0.46, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4102, 4100, 3890, 3889}

$$\frac{2a^3(710A+803B)\sin(c+dx)}{1155d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(194A+209B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{16a^3(710A+803B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3465d\sqrt{a\sec(c+dx)+a}}+\frac{8a^3(710A+803B)\sin(c+dx)}{3465d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(14A+11B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{99d\sec^{\frac{7}{2}}(c+dx)}+\frac{2aA\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(11/2),x
]
```

```
[Out] (2*a^3*(194*A + 209*B)*Sin[c + d*x])/(693*d*Sec[c + d*x]^(5/2)*Sqrt[a + a*S
ec[c + d*x]]) + (2*a^3*(710*A + 803*B)*Sin[c + d*x])/(1155*d*Sec[c + d*x]^(
3/2)*Sqrt[a + a*Sec[c + d*x]]) + (8*a^3*(710*A + 803*B)*Sin[c + d*x])/(3465
*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (16*a^3*(710*A + 803*B)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(3465*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*
(14*A + 11*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(99*d*Sec[c + d*x]^(7/
2)) + (2*a*A*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9
/2))
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
```

```
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} + \frac{2}{11}\int \frac{(a+a\,\mathrm{sec}}{}$$

$$= \frac{2a^2(14A+11B)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2aA}{}$$

$$= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^2(14A+11}{}$$

$$= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(710A}{1155d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(710A}{1155d\sec^{\frac{3}{2}}(c+}$$

$$= \frac{2a^3(194A+209B)\sin(c+dx)}{693d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2a^3(710A}{1155d\sec^{\frac{3}{2}}(c+}$$

Mathematica [A]

time = 3.79, size = 127, normalized size = 0.46

$$\frac{2a^3(315A+35(32A+11B)\sec(c+dx)+5(355A+286B)\sec^2(c+dx)+3(710A+803B)\sec^3(c+dx)+4(710A+803B)\sec^4(c+dx)+8(710A+803B)\sec^5(c+dx))\sin(c+dx)}{3465d\sec^{\frac{9}{2}}(c+dx)\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(1
1/2),x]
```

```
[Out] (2*a^3*(315*A + 35*(32*A + 11*B)*Sec[c + d*x] + 5*(355*A + 286*B)*Sec[c + d
*x]^2 + 3*(710*A + 803*B)*Sec[c + d*x]^3 + 4*(710*A + 803*B)*Sec[c + d*x]^4
 + 8*(710*A + 803*B)*Sec[c + d*x]^5)*Sin[c + d*x])/(3465*d*Sec[c + d*x]^(9/
2)*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 7.72, size = 165, normalized size = 0.60

method	result
default	$-\frac{2(-1+\cos(dx+c))\left(315A\left(\cos^5(dx+c)\right)+1120A\left(\cos^4(dx+c)\right)+385B\left(\cos^4(dx+c)\right)+1775A\left(\cos^3(dx+c)\right)+1430B\left(\cos^3(dx+c)\right)+2130\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x,method=_RET
URNVERBOSE)
```

```
[Out] -2/3465/d*(-1+cos(d*x+c))*(315*A*cos(d*x+c)^5+1120*A*cos(d*x+c)^4+385*B*cos
(d*x+c)^4+1775*A*cos(d*x+c)^3+1430*B*cos(d*x+c)^3+2130*A*cos(d*x+c)^2+2409*
B*cos(d*x+c)^2+2840*A*cos(d*x+c)+3212*B*cos(d*x+c)+5680*A+6424*B)*(a*(1+cos
(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(11/2)*cos(d*x+c)^6/sin(d*x+c)*a^
2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 945 vs. 2(239) = 478.
time = 0.65, size = 945, normalized size = 3.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] 1/110880*(5*sqrt(2)*(31878*a^2*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), co
s(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 8778*a^2*cos(8/11*arctan2(s
in(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 34
65*a^2*cos(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*si
```

```
n(11/2*d*x + 11/2*c) + 1287*a^2*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), co
s(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 385*a^2*cos(2/11*arctan2(si
n(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) - 318
78*a^2*cos(11/2*d*x + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos
(11/2*d*x + 11/2*c))) - 8778*a^2*cos(11/2*d*x + 11/2*c)*sin(8/11*arctan2(si
n(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 3465*a^2*cos(11/2*d*x + 11
/2*c)*sin(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 1
287*a^2*cos(11/2*d*x + 11/2*c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos
(11/2*d*x + 11/2*c))) - 385*a^2*cos(11/2*d*x + 11/2*c)*sin(2/11*arctan2(sin
(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 126*a^2*sin(11/2*d*x + 11/2
*c) + 385*a^2*sin(9/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*
c))) + 1287*a^2*sin(7/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/
2*c))) + 3465*a^2*sin(5/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 1
1/2*c))) + 8778*a^2*sin(3/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x +
 11/2*c))) + 31878*a^2*sin(1/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*
x + 11/2*c))))*A*sqrt(a) + 22*sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x
 + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*a
rctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) +
756*a^2*cos(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/
2*d*x + 9/2*c) + 225*a^2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c)))*sin(9/2*d*x + 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arc
tan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x +
9/2*c)*sin(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a
^2*cos(9/2*d*x + 9/2*c)*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
 9/2*c))) - 225*a^2*cos(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*
arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arct
an2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(si
n(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*B*sqrt(a))/d
```

Fricas [A]
time = 1.70, size = 162, normalized size = 0.59

$$\frac{2\,\left(315\,Aa^{2}\cos\left(dx+c\right)^{6}+35\,\left(32\,A+11\,B\right)a^{2}\cos\left(dx+c\right)^{5}+5\,\left(355\,A+286\,B\right)a^{2}\cos\left(dx+c\right)^{4}+3\,\left(710\,A+803\,B\right)a^{2}\cos\left(dx+c\right)^{3}+4\,\left(710\,A+803\,B\right)a^{2}\cos\left(dx+c\right)^{2}+8\,\left(710\,A+803\,B\right)a^{2}\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin\left(dx+c\right)}{3465\,\left(d\cos\left(dx+c\right)+d\right)\sqrt{\cos\left(dx+c\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] 2/3465*(315*A*a^2*cos(d*x + c)^6 + 35*(32*A + 11*B)*a^2*cos(d*x + c)^5 + 5*
(355*A + 286*B)*a^2*cos(d*x + c)^4 + 3*(710*A + 803*B)*a^2*cos(d*x + c)^3 +
 4*(710*A + 803*B)*a^2*cos(d*x + c)^2 + 8*(710*A + 803*B)*a^2*cos(d*x + c))
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/((d*cos(d*x + c) + d)
*sqrt(cos(d*x + c)))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(11/
2), x)
```

Mupad [**B**]
time = 8.77, size = 392, normalized size = 1.43

$$\frac{\sqrt{a-\frac{a}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}}\left(2\sin\left(\frac{11\,c}{4}+\frac{11\,d\,x}{4}\right)^2+\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)\,1{}\mathrm{i}-1\right)\left(\frac{A\,a^2\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)\left(-2\sin\left(\frac{11\,c}{4}+\frac{11\,d\,x}{4}\right)^2+\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)\,1{}\mathrm{i}+1\right)}{88\,d}+\frac{a^2\sin\left(\frac{9\,c}{2}+\frac{9\,d\,x}{2}\right)\left(5\,A+2\,B\right)\left(-2\sin\left(\frac{11\,c}{4}+\frac{11\,d\,x}{4}\right)^2+\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)\,1{}\mathrm{i}+1\right)}{72\,d}+\frac{a^2\sin\left(\frac{7\,c}{2}+\frac{7\,d\,x}{2}\right)\left(13\,A+10\,B\right)\left(-2\sin\left(\frac{11\,c}{4}+\frac{11\,d\,x}{4}\right)^2+\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)\,1{}\mathrm{i}+1\right)}{56\,d}+\frac{a^2\sin\left(\frac{3\,c}{2}+\frac{3\,d\,x}{2}\right)\left(19\,A+20\,B\right)\left(-2\sin\left(\frac{11\,c}{4}+\frac{11\,d\,x}{4}\right)^2+\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)\,1{}\mathrm{i}+1\right)}{12\,d}+\frac{a^2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(23\,A+26\,B\right)\left(-2\sin\left(\frac{11\,c}{4}+\frac{11\,d\,x}{4}\right)^2+\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)\,1{}\mathrm{i}+1\right)}{4\,d}+\frac{a^2\sin\left(\frac{5\,c}{2}+\frac{5\,d\,x}{2}\right)\left(25\,A+24\,B\right)\left(-2\sin\left(\frac{11\,c}{4}+\frac{11\,d\,x}{4}\right)^2+\sin\left(\frac{11\,c}{2}+\frac{11\,d\,x}{2}\right)\,1{}\mathrm{i}+1\right)}{40\,d}\right)}{2\sqrt{-\frac{1}{2\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1}}\left(2\sin\left(\frac{c}{4}+\frac{d\,x}{4}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(11/
2),x)
```

```
[Out] ((a - a/(2*sin(c/2 + (d*x)/2)^2 - 1))^(1/2)*(sin((11*c)/2 + (11*d*x)/2)*1i
+ 2*sin((11*c)/4 + (11*d*x)/4)^2 - 1)*((A*a^2*sin((11*c)/2 + (11*d*x)/2)*(s
in((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(88*d)
+ (a^2*sin((9*c)/2 + (9*d*x)/2)*(5*A + 2*B)*(sin((11*c)/2 + (11*d*x)/2)*1i
- 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(72*d) + (a^2*sin((7*c)/2 + (7*d*x)/
2)*(13*A + 10*B)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)
/4)^2 + 1))/(56*d) + (a^2*sin((3*c)/2 + (3*d*x)/2)*(19*A + 20*B)*(sin((11*c
)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(12*d) + (a^2*s
in(c/2 + (d*x)/2)*(23*A + 26*B)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*
c)/4 + (11*d*x)/4)^2 + 1))/(4*d) + (a^2*sin((5*c)/2 + (5*d*x)/2)*(25*A + 24
*B)*(sin((11*c)/2 + (11*d*x)/2)*1i - 2*sin((11*c)/4 + (11*d*x)/4)^2 + 1))/(
40*d)))/(2*(-1/(2*sin(c/2 + (d*x)/2)^2 - 1))^(1/2)*(2*sin(c/4 + (d*x)/4)^2
- 1))
```

3.248 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=190

$$-\frac{(4A-7B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{a}\,d}+\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{(4A-\ldots}{4\ldots}$$

```
[Out] -1/4*(4*A-7*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)
+(A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x
+c))^(1/2))*2^(1/2)/d/a^(1/2)+1/4*(4*A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+
a*sec(d*x+c))^(1/2)+1/2*B*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1
/2)
```

Rubi [A]

time = 0.37, antiderivative size = 190, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {4106, 4108, 3893, 212, 3886, 221}

$$\frac{(4A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{(4A-7B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}\,d}+\frac{B\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -1/4*((4*A - 7*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])
/(Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c +
 d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + ((4*A - B)*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(4*d*Sqrt[a + a*Sec[c + d*x]]) + (B*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4106

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fre
eQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& GtQ[n, 1]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}dx = \frac{B\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3aB}{2}+\frac{1}{2}a(4A-B)\sec(c+dx)\right)}{\sqrt{a+a\sec(c+dx)}}d}{2a}$$

$$= \frac{(4A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{B\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(4A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{B\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(4A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d\sqrt{a+a\sec(c+dx)}} + \frac{B\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(4A-7B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4\sqrt{a}\,d} + \frac{\sqrt{2}\,(A-B)\tanh}{}$$

Mathematica [A]
time = 0.89, size = 125, normalized size = 0.66

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(8(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)-\sqrt{2}\,(4A-7B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sec(c+dx)(4A-B+2B\sec(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]
],x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(8*(A - B)*ArcTanh[Sin[(c + d*x)/2]] -
 Sqrt[2]*(4*A - 7*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(4*
A - B + 2*B*Sec[c + d*x])*Sin[(c + d*x)/2]))/(4*d*Sqrt[a*(1 + Sec[c + d*x])
])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 422 vs. $2(159) = 318$.
time = 7.67, size = 423, normalized size = 2.23

method	result

default	$-\frac{\left(4A(\cos^2(dx+c))\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-4A(\cos^2(dx+c))\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{}{1+c}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/16/d*(4*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))
*2^(1/2))*cos(d*x+c)^2*2^(1/2)-4*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+
cos(d*x+c)-sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-7*B*arctan(1/4*(-2/(1+
cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)+
7*B*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))
*cos(d*x+c)^2*2^(1/2)-16*A*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*
x+c)))^(1/2))-8*A*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+16*B*cos(
d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))+2*B*cos(d*x+c)*si
n(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)-4*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)
)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2/(
1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2524 vs. $2(159) = 318$.
time = 0.72, size = 2524, normalized size = 13.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/16*(4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
+ 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x +
 2*c) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) -
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*co
s(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(
d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*s
```

qrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2
*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
 1) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2
)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(
sin(d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*A/((cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a)) - (4*(sqrt(2)*
sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arctan2(sin(d*x + c)
, cos(d*x + c))) - 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c
))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) + 20*(sqrt(2)*sin(4*d*x + 4
*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c
))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x
 + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 +
4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x +
2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)
^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(
d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*s
qrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 7*(2*(2*cos(2*d*
x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2
+ sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x +
2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*
x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin
(d*x + c), cos(d*x + c))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x +
4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*s
in(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c
) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arc
tan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c
), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
+ 2) - 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(
2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqr

```
t(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*
x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 +
 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) + 8*(sqrt(2)*cos(4*d*x
 + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*s
qrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2
*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 +
sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 ...
```

Fricas [A]
time = 2.68, size = 617, normalized size = 3.25

$$\left[-\frac{\left((4A-7B)\cos(dx+c)^2+(4A-7B)\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{8\sqrt{2}\left((A-B)a\cos(dx+c)^2+(A-B)a\cos(dx+c)\right)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}-\frac{4\left((4A-B)\cos(dx+c)+2B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{16\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)},\ -\frac{8\sqrt{2}\left((A-B)a\cos(dx+c)^2+(A-B)a\cos(dx+c)\right)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+\left((4A-7B)\cos(dx+c)^2+(4A-7B)\cos(dx+c)\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)-\frac{2\left((4A-B)\cos(dx+c)+2B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{8\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/16*(((4*A - 7*B)*cos(d*x + c)^2 + (4*A - 7*B)*cos(d*x + c))*sqrt(a)*log
((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c
))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*
x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) + 8*sqrt(2)*((A - B)*a*co
s(d*x + c)^2 + (A - B)*a*cos(d*x + c))*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a
) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a) - 4*
((4*A - B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c)), -1/8*
(8*sqrt(2)*((A - B)*a*cos(d*x + c)^2 + (A - B)*a*cos(d*x + c))*sqrt(-1/a)*a
rctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d
*x + c))/sin(d*x + c)) + ((4*A - 7*B)*cos(d*x + c)^2 + (4*A - 7*B)*cos(d*x
+ c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) -
2*((4*A - B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*si
n(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(a*sec(d*x + c) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x))^(1/2
), x)
```

3.249 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=141

$$\frac{(2A-B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{B\sec^{\frac{3}{2}}(c+\cdots}{d\sqrt{a+a\cdots}}$$

```
[Out] (2*A-B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)-(A-B)*
arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1
/2))*2^(1/2)/d/a^(1/2)+B*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/
2)
```

Rubi [A]
time = 0.25, antiderivative size = 141, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {4106, 4108, 3893, 212, 3886, 221}

$$-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{(2A-B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{B\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] ((2*A - B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[
a]*d) - (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/
(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d) + (B*Sec[c + d*x]^(3/2)*Si
n[c + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
```

```
x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4106

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fre
eQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& GtQ[n, 1]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \frac{B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{\int \frac{\sqrt{\sec(c+dx)}\left(\frac{aB}{2}+\frac{1}{2}a(2A-B)\sec(c+d\right.}{\sqrt{a+a\sec(c+dx)}}}{a}$$

$$= \frac{B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{(2A-B)\int\sqrt{\sec(c+dx)}\sqrt{a+a\,\mathrm{se}}}{2a}$$

$$= \frac{B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{(2(A-B))\mathrm{Subst}\left(\int\frac{1}{2a-x^2}\,dx, x, -\frac{a}{}\right.}{d}$$

$$= \frac{(2A-B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} - \frac{\sqrt{2}(A-B)\tanh^{-1}\left(\right.}{}$$

Mathematica [A]
time = 0.41, size = 106, normalized size = 0.75

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sqrt{\sec(c+dx)}\left(-2(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)+\sqrt{2}(2A-B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\sec(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]
],x]
```

```
[Out] (Cos[(c + d*x)/2]*Sqrt[Sec[c + d*x]]*(-2*(A - B)*ArcTanh[Sin[(c + d*x)/2]]
+ Sqrt[2]*(2*A - B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*B*Sec[c + d*x]*Si
n[(c + d*x)/2]))/(d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. 2(120) = 240.
time = 7.73, size = 350, normalized size = 2.48

method	result
default	$\frac{(-1+\cos(dx+c))\left(2A\cos(dx+c)\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)-\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}-2A\cos(dx+c)\sqrt{2}\arctan\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/d*(-1+cos(d*x+c))*(2*A*cos(d*x+c)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*
(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*2^(1/2)-2*A*cos(d*x+c)*2^(1/2)*arctan(1/
4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))-B*cos(d*x+c)
*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*2^
(1/2)+B*cos(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+
c)+sin(d*x+c))*2^(1/2))+4*A*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x
+c)))^(1/2))-4*B*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)
)-2*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c))*cos(d*x+c)*(1/cos(d*x+c))^(3/2)
*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^2/(-2/(1+cos(d*x+c)))^(1/2)
/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1353 vs. $2(120) = 240$.
time = 0.67, size = 1353, normalized size = 9.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/4*(2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x +
c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(
2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(
2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(
2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(
2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 2))*A/sqrt(a) + (4*sqrt(2)*cos(3/2*arctan2(
sin(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))*sin(2*d*x + 2*c) + (cos(2*d*x + 2*c)^2 + sin(2*
d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqr
t(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arcta
```

```
n2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*
c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos
(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d
*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2
*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d
*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin
(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin
(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 +
 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c))) + 1) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2
+ sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(co
s(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c)
, cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) -
4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*
x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c))))*B/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1)*sqrt(a)))/d
```

Fricas [A]
time = 3.06, size = 531, normalized size = 3.77

$$\left[-\frac{((2A-B)\cos(dx+c)+2A-B)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2+\frac{4(\cos(dx+c)^2-2\cos(dx+c))\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)+\frac{2\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}-\frac{4B\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{4(ad\cos(dx+c)+ad)}, \frac{2\sqrt{2}((A-B)a\cos(dx+c)+(A-B)a)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+((2A-B)\cos(dx+c)+2A-B)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)+\frac{2B\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{2(ad\cos(dx+c)+ad)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/4*(((2*A - B)*cos(d*x + c) + 2*A - B)*sqrt(a)*log((a*cos(d*x + c)^3 - 7
*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*
x + c)^3 + cos(d*x + c)^2)) + 2*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a
)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)
^2 + 2*cos(d*x + c) + 1))/sqrt(a) - 4*B*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d), 1/2*(2*sqrt
(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c))
```

```
 + ((2*A - B)*cos(d*x + c) + 2*A - B)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x +
c)^2 - a*cos(d*x + c) - 2*a)) + 2*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(a*sec(d*x + c) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x))^(1/2
), x)
```

3.250 $\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=100

$$\frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}$$

```
[Out] 2*B*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/d/a^(1/2)+(A-B)*arct
anh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))
*2^(1/2)/d/a^(1/2)
```

Rubi [A]

time = 0.15, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4108, 3893, 212, 3886, 221}

$$\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(Sqrt[a]*d)
+ (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = (A-B)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}\,dx + \frac{B\int \sqrt{\sec(c+dx)}\,\sqrt{a-}}{a}$$

$$= -\frac{(2(A-B))\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x, -\frac{a\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{d}$$

$$= \frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{}}{\sqrt{}}\right.}{\sqrt{}}$$

Mathematica [A]
time = 0.20, size = 95, normalized size = 0.95

$$\frac{\left(-2B\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)+\sqrt{2}\,(-A+B)\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]
],x]
```

[Out] ((-2*B*ArcSin[Sqrt[Sec[c + d*x]]] + Sqrt[2]*(-A + B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]])*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. $2(83) = 166$.
time = 7.79, size = 211, normalized size = 2.11

method	result
default	$\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\cos(dx+c)\left(-B\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)-\sin(dx+c))\sqrt{2}}{4}\right)+B\sqrt{2}\,\text{a}\right.$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] 1/2/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*(-B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))+B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))+2*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-2*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)))*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2*(cos(d*x+c)^2-1)/a

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 567 vs. $2(83) = 166$.
time = 0.65, size = 567, normalized size = 5.67

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] 1/2*((sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*A/sqrt(a) - (sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +

```
2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
2) - log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
2) + log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
2))*B/sqrt(a))/d
```

Fricas [A]
time = 3.23, size = 366, normalized size = 3.66

$$\left[-\frac{\sqrt{2} (A-B)\sqrt{a} \log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-B\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{2ad}, -\frac{\sqrt{2}(A-B)a\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-B\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*(A - B)*sqrt(a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*co
s(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - B*sqrt(a)*log((a*c
os(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sq
rt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c
)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a*d), -(sqrt(2)*(A - B)*a*sq
rt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*
sqrt(cos(d*x + c))/sin(d*x + c)) - B*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c
)^2 - a*cos(d*x + c) - 2*a)))/(a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\sqrt{\sec(c+dx)}}{\sqrt{a(\sec(c+dx)+1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(sec(c + d*x))/sqrt(a*(sec(c + d*x) + 1))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(a*sec(d*x + c) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x))^(1/2
), x)
```

3.251 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=99

$$-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2A\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] -(A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x
+c))^(1/2))*2^(1/2)/d/a^(1/2)+2*A*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*
x+c))^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 99, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {4098, 3893, 212}

$$\frac{2A\sin(c+dx)\sqrt{\sec(c+dx)}}{d\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[a]*d)) + (2*A*Sqrt[Sec[c + d*x]]*Sin[c
 + d*x])/(d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
```

```
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + (-A+B)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+d}}$$

$$= \frac{2A\sqrt{\sec(c+dx)}\sin(c+dx)}{d\sqrt{a+a\sec(c+dx)}} + \frac{(2(A-B))\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x\right.}{d}$$

$$= -\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2A\sqrt{\text{se}}}{d\sqrt{}}$$

Mathematica [A]
time = 0.28, size = 114, normalized size = 1.15

$$\frac{\left(2A\sqrt{1-\sec(c+dx)}+\sqrt{2}(A-B)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sqrt{\sec(c+dx)}\right)\tan(c+dx)}{d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]
),x]
```

```
[Out] ((2*A*Sqrt[1 - Sec[c + d*x]] + Sqrt[2]*(A - B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c +
 d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Tan[c + d*x])/(d*Sqrt[-
((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 7.39, size = 150, normalized size = 1.52

method	result
default	$\frac{\left(\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,A\sin(dx+c)-\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(}}\right.}{d\sqrt{\frac{1}{\cos(dx+c)}}\,\sin(dx+c)a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*(arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(
1/2)*A*sin(d*x+c)-arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+c
os(d*x+c)))^(1/2)*B*sin(d*x+c)-2*A*cos(d*x+c)+2*A)*(a*(1+cos(d*x+c))/cos(d*
x+c))^(1/2)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)/a
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 195 vs. $2(84) = 168$.
time = 0.61, size = 195, normalized size = 1.97

$$-\frac{\frac{\left(\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-4\,\sqrt{2}\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)A}{\sqrt{a}}-\frac{\left(\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-2\,\sin\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right)\right)B}{\sqrt{a}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*((sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))*A/
sqrt(a) - (sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*B/sqrt(a))/d
```

Fricas [**A**]
time = 1.77, size = 306, normalized size = 3.09

$$\left[\frac{4\,A\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\frac{\sqrt{2}\left((A-B)a\cos(dx+c)+(A-B)a\right)\log\left(-\frac{\cos(dx+c)^{2}-\frac{2\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)}{\sqrt{a}}}{2\left(ad\cos\left(dx+c\right)+ad\right)},\ \frac{\sqrt{2}\left((A-B)a\cos\left(dx+c\right)+(A-B)a\right)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{-\frac{1}{a}}\sqrt{\cos\left(dx+c\right)}}{\sin(dx+c)}\right)+2\,A\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{ad\cos\left(dx+c\right)+ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(4*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) - sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^2
- 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
```

```
))/sqrt(a))/(a*d*cos(d*x + c) + a*d), (sqrt(2)*((A - B)*a*cos(d*x + c) + (A
 - B)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*A*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec{\left(c + dx \right)}}{\sqrt{a \left(\sec{\left(c + dx \right)} + 1\right)} \sqrt{\sec{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(sqrt(a*(sec(c + d*x) + 1))*sqrt(sec(c + d*x)
)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos\left(c+d\,x\right)}}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)
),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)
), x)
```

3.252 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=142

$$\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c+dx)}}-\frac{2(A-3B)\sqrt{s}}{3d\sqrt{a}}$$

```
[Out] (A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+
c))^(1/2))*2^(1/2)/d/a^(1/2)+2/3*A*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d
*x+c))^(1/2)-2/3*(A-3*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/
2)
```

Rubi [A]
time = 0.21, antiderivative size = 142, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4107, 4098, 3893, 212}

$$-\frac{2(A-3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\,\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])]])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(3*d*Sqrt[Sec
[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 3*B)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(A-3B)+aA\,\mathrm{s}}{\sqrt{\sec(c+dx)}\sqrt{a+}}}{3a}$$

$$= \frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2(A-3B)\sqrt{\sec(c+dx)}}{3d\sqrt{a+a\sec(c+}}$$

$$= \frac{2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}\sqrt{a+a\sec(c+dx)}} - \frac{2(A-3B)\sqrt{\sec(c+dx)}}{3d\sqrt{a+a\sec(c+}}$$

$$= \frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{}{3d\sqrt{\sec(c+}}$$

Mathematica [A]
time = 0.42, size = 132, normalized size = 0.93

$$\frac{\left(2(-A+3B+A\cos(c+dx))\sqrt{1-\sec(c+dx)}-3\sqrt{2}\,(A-B)\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sqrt{\sec(c+dx)}\right)\tan(c+dx)}{3d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]
),x]
```

```
[Out] ((2*(-A + 3*B + A*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 3*Sqrt[2]*(A - B)*
ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*
x]])*Tan[c + d*x])/(3*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*Sqrt[a*(1
 + Sec[c + d*x])])
```

Maple [A]
time = 8.05, size = 183, normalized size = 1.29

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,A\sin(dx+c)-3\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{1}{}}}{2}\right.\right.}{3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/3/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*arctan(1/2*sin(d*x+c)*(-2/(1+
cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*A*sin(d*x+c)-3*arctan(1/2*sin
(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c)+2
*A*cos(d*x+c)^2-4*A*cos(d*x+c)+6*B*cos(d*x+c)+2*A-6*B)*(1/cos(d*x+c))^(3/2)
*cos(d*x+c)^2/sin(d*x+c)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 387 vs. $2(119) = 238$.
time = 0.63, size = 387, normalized size = 2.73

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/6*((3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A/sqrt(a) + 3*(sqrt(2)
*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
```

```
c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))*B/sqrt(a))/d
```

Fricas [A]
time = 2.32, size = 354, normalized size = 2.49

$$\left[-\frac{\frac{3\,\sqrt{2}\left((A-B)a\cos(dx+c)+(A-B)a\right)\log\left(-\frac{\cos(dx+c)^2+\frac{2\,\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}-\frac{4\left(A\cos(dx+c)^2-(A-3B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{6\left(ad\cos(dx+c)+ad\right)}, -\frac{3\,\sqrt{2}\left((A-B)a\cos(dx+c)+(A-B)a\right)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-\frac{2\left(A\cos(dx+c)^2-(A-3B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3\left(ad\cos(dx+c)+ad\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/6*(3*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^2
+ 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
))/sqrt(a) - 4*(A*cos(d*x + c)^2 - (A - 3*B)*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)
+ a*d), -1/3*(3*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*arc
tan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x
 + c))/sin(d*x + c)) - 2*(A*cos(d*x + c)^2 - (A - 3*B)*cos(d*x + c))*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos
(d*x + c) + a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec(c + dx)}{\sqrt{a\left(\sec(c + dx) + 1\right)}\,\sec^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(sqrt(a*(sec(c + d*x) + 1))*sec(c + d*x)**(3/
2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)
),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)
), x)
```

3.253 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=187

$$-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}-\frac{2(A-}{15d\sqrt{\sec(c+d}}$$

```
[Out] -(A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x
+c))^(1/2))*2^(1/2)/d/a^(1/2)+2/5*A*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(
d*x+c))^(1/2)-2/15*(A-5*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(
1/2)+2/15*(13*A-5*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {4107, 4098, 3893, 212}

$$\frac{2(13A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}-\frac{2(A-5B)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}\,\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])]])/(Sqrt[a]*d)) + (2*A*Sin[c + d*x])/(5*d*Sec[c
 + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 5*B)*Sin[c + d*x])/(15*d*
Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*(13*A - 5*B)*Sqrt[Sec[c +
 d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(A-5B)+2aA\sec(c}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c}}}{5a}$$

$$= \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-5B)\sin(c}{15d\sqrt{\sec(c+dx)}\sqrt{a+}}$$

$$= \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-5B)\sin(c}{15d\sqrt{\sec(c+dx)}\sqrt{a+}}$$

$$= \frac{2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-5B)\sin(c}{15d\sqrt{\sec(c+dx)}\sqrt{a+}}$$

$$= -\frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{}{5d\sec^{\frac{3}{2}}(}$$

Mathematica [A]
time = 1.14, size = 133, normalized size = 0.71

$$\frac{(29A-10B-2(A-5B)\cos(c+dx)+3A\cos(2(c+dx)))\sqrt{\sec(c+dx)}\sin(c+dx)+\frac{15\sqrt{2}(A-B)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)}{\sqrt{1-\sec(c+dx)}}}{15d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]
),x]
```

```
[Out] ((29*A - 10*B - 2*(A - 5*B)*Cos[c + d*x] + 3*A*Cos[2*(c + d*x)])*Sqrt[Sec[c
 + d*x]]*Sin[c + d*x] + (15*Sqrt[2]*(A - B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*
x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x])/Sqrt[1 - Sec[c + d*x]])/(15*d*Sq
rt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 7.75, size = 205, normalized size = 1.10

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(15\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,A\sin(dx+c)-6A\left(\cos^{3}(dx+c)\right)-15\arctan\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/15/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(15*arctan(1/2*sin(d*x+c)*(-2/(1
+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*A*sin(d*x+c)-6*A*cos(d*x+c)^
3-15*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(
1/2)*B*sin(d*x+c)+8*A*cos(d*x+c)^2-10*B*cos(d*x+c)^2-28*A*cos(d*x+c)+20*B*c
os(d*x+c)+26*A-10*B)*(1/cos(d*x+c))^(5/2)*cos(d*x+c)^3/sin(d*x+c)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(158) = 316$.
time = 0.66, size = 640, normalized size = 3.42

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/60*(sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*
d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)*si
n(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5
*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(s
in(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d
*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d
*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c
```

```
), cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2
*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c))))*A/sqrt(a) - 10*(3*sqrt(2)*cos(2/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*
cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) + 3*
sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin
(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*B/sqrt(a))/d
```

Fricas [A]
time = 2.73, size = 388, normalized size = 2.07

$$\left[-\frac{\frac{15\sqrt{2}\left((A-B)a\cos(dx+c)+(A-B)a\right)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}-\frac{4\left(3A\cos(dx+c)^3-(A-5B)\cos(dx+c)^2+(13A-5B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{30\left(ad\cos(dx+c)+ad\right)}, \frac{15\sqrt{2}\left((A-B)a\cos(dx+c)+(A-B)a\right)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+\frac{2\left(3A\cos(dx+c)^3-(A-5B)\cos(dx+c)^2+(13A-5B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15\left(ad\cos(dx+c)+ad\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/30*(15*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^
2 - 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
 1))/sqrt(a) - 4*(3*A*cos(d*x + c)^3 - (A - 5*B)*cos(d*x + c)^2 + (13*A - 5
*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt
(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d), 1/15*(15*sqrt(2)*((A - B)*a*cos(d
*x + c) + (A - B)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*(3*A*cos(d*x +
c)^3 - (A - 5*B)*cos(d*x + c)^2 + (13*A - 5*B)*cos(d*x + c))*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c
) + a*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(5/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)
),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)
), x)
```

3.254 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=230

$$\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d}+\frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}-\frac{2(A-7\ldots}{35d\sec^{\frac{3}{2}}(c+dx\ldots}$$

```
[Out] (A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+
c))^(1/2))*2^(1/2)/d/a^(1/2)+2/7*A*sin(d*x+c)/d/sec(d*x+c)^(5/2)/(a+a*sec(d
*x+c))^(1/2)-2/35*(A-7*B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1
/2)+2/105*(31*A-7*B)*sin(d*x+c)/d/sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)-2
/105*(43*A-91*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.45, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4107, 4098, 3893, 212}

$$-\frac{2(A-7B)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{2(43A-91B)\sin(c+dx)\sqrt{\sec(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{2(31A-7B)\sin(c+dx)}{105d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])]])/(Sqrt[a]*d) + (2*A*Sin[c + d*x])/(7*d*Sec[c +
d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) - (2*(A - 7*B)*Sin[c + d*x])/(35*d*Sec
[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (2*(31*A - 7*B)*Sin[c + d*x])/(
105*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (2*(43*A - 91*B)*Sqrt[
Sec[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(A-7B)+3aA\sec(c+}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c}}}{7a}$$

$$= \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-7B)\sin(c+}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{se}}}$$

$$= \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-7B)\sin(c+}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{se}}}$$

$$= \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-7B)\sin(c+}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{se}}}$$

$$= \frac{2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \frac{2(A-7B)\sin(c+}{35d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{se}}}$$

$$= \frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{\sqrt{a}\,d} + \frac{2}{7d\sec^{\frac{5}{2}}(c+}$$

Mathematica [A]

time = 1.53, size = 152, normalized size = 0.66

$$-\frac{-\frac{2(-15A+3(A-7B)\sec(c+dx)+(-31A+7B)\sec^2(c+dx)+(43A-91B)\sec^3(c+dx))\sin(c+dx)}{\sec^{\frac{5}{2}}(c+dx)}-\frac{105\sqrt{2}(A-B)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)}{\sqrt{1-\sec(c+dx)}}}{105d\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]
),x]
```

```
[Out] ((-2*(-15*A + 3*(A - 7*B)*Sec[c + d*x] + (-31*A + 7*B)*Sec[c + d*x]^2 + (43
*A - 91*B)*Sec[c + d*x]^3)*Sin[c + d*x])/Sec[c + d*x]^(5/2) - (105*Sqrt[2]*
(A - B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c +
 d*x])/Sqrt[1 - Sec[c + d*x]])/(105*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 7.56, size = 227, normalized size = 0.99

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(30A\left(\cos^4(dx+c)\right)+105\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,A\sin(dx+c)-36A($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/105/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(30*A*cos(d*x+c)^4+105*arctan(
1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*A*sin(d
*x+c)-36*A*cos(d*x+c)^3-105*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)
)*(-2/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c)+42*B*cos(d*x+c)^3+68*A*cos(d*x+c)^
2-56*B*cos(d*x+c)^2-148*A*cos(d*x+c)+196*B*cos(d*x+c)+86*A-182*B)*(1/cos(d*
x+c))^(7/2)*cos(d*x+c)^4/sin(d*x+c)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 805 vs. 2(195) = 390.
time = 0.66, size = 805, normalized size = 3.50

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/840*(sqrt(2)*(525*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c)))*sin(7/2*d*x + 7/2*c) - 175*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*cos(2/7*arctan2(sin(7/2*d*x +
 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 525*cos(7/2*d*x + 7/
2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*cos
(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c
))) - 21*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2
*d*x + 7/2*c))) - 420*log(cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c)))^2 + sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
)^2 + 2*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 1) +
 420*log(cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))^2 + s
in(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))^2 - 2*sin(1/7*a
rctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 1) - 30*sin(7/2*d*x +
 7/2*c) + 21*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
 175*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 525*sin
(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A/sqrt(a) - 14*s
qrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))*sin
(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5
/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arctan2(sin(
5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)*sin(2/5*a
rctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*
d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(
5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x
+ 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2*d*x +
5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), c
os(5/2*d*x + 5/2*c))))*B/sqrt(a))/d
```

Fricas [A]

time = 1.70, size = 422, normalized size = 1.83

$$\left[-\frac{\frac{105\sqrt{2}\left((A-B)a\cos(dx+c)+(A-B)a\right)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}-\frac{4\left(15A\cos(dx+c)^4-3(A-7B)\cos(dx+c)^3+(31A-7B)\cos(dx+c)^2-(43A-91B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{210\left(ad\cos(dx+c)+ad\right)}, -\frac{105\sqrt{2}\left((A-B)a\cos(dx+c)+(A-B)a\right)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-\frac{2\left(15A\cos(dx+c)^4-3(A-7B)\cos(dx+c)^3+(31A-7B)\cos(dx+c)^2-(43A-91B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{105\left(ad\cos(dx+c)+ad\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/210*(105*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c
)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c)
 + 1))/sqrt(a) - 4*(15*A*cos(d*x + c)^4 - 3*(A - 7*B)*cos(d*x + c)^3 + (31*
```

```
A - 7*B)*cos(d*x + c)^2 - (43*A - 91*B)*cos(d*x + c))*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c) + a*d
), -1/105*(105*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*arct
an(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x
+ c))/sin(d*x + c)) - 2*(15*A*cos(d*x + c)^4 - 3*(A - 7*B)*cos(d*x + c)^3 +
 (31*A - 7*B)*cos(d*x + c)^2 - (43*A - 91*B)*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a*d*cos(d*x + c)
+ a*d)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6192 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*sec(d*x + c)^(7/2)
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/2)
),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(7/2)
), x)
```

3.255 $\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=247

$$-\frac{(12A-19B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{3/2}d}+\frac{(9A-13B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B}{2d}$$

```
[Out] -1/4*(12*A-19*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)
/d+1/2*(A-B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+1/4*(9*A-
13*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+
c))^(1/2))/a^(3/2)/d*2^(1/2)+1/4*(6*A-7*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/
(a+a*sec(d*x+c))^(1/2)-1/2*(A-2*B)*sec(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*sec
(d*x+c))^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4104, 4106, 4108, 3893, 212, 3886, 221}

$$\frac{(9A-13B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(12A-19B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{3/2}d}+\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{(A-2B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}+\frac{(6A-7B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] -1/4*((12*A - 19*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
])/(a^(3/2)*d) + ((9*A - 13*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c +
d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)
*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((6*A
- 7*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(4*a*d*Sqrt[a + a*Sec[c + d*x]]) -
((A - 2*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]]
)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4106

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fre
eQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& GtQ[n, 1]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-2a(A-2B)\mathrm{se}\right.}{\sqrt{a+a\sec(c+dx)}}}{2a^2}$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(A-2B)\sec^{\frac{5}{2}}(c+dx)\sin(c+}{2ad\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(6A-7B)\sec^{\frac{3}{2}}(c+dx)\sin(c}{4ad\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(6A-7B)\sec^{\frac{3}{2}}(c+dx)\sin(c}{4ad\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(6A-7B)\sec^{\frac{3}{2}}(c+dx)\sin(c}{4ad\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{(12A-19B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{4a^{3/2}d} + \frac{(9A-13B)\tanh}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 497 vs. 2(247) = 494.

time = 4.44, size = 497, normalized size = 2.01

$$\frac{4(6A-7B)\mathrm{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)+8(9A-13B)\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)+6A\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)-7B\sqrt{1-\sec(c+dx)}\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)+4A\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)-3B\sqrt{1-\sec(c+dx)}\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)+2B\sqrt{1-\sec(c+dx)}\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)-9\sqrt{2}A\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)+13\sqrt{2}B\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\tan(c+dx)-9\sqrt{2}A\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec(c+dx)\tan(c+dx)+13\sqrt{2}B\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec(c+dx)\tan(c+dx)}{4d\sqrt{1-\sec(c+dx)}(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (4*(6*A - 7*B)*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Cos[(c + d*x)/2]^3*Sec[c + d*
x]^2*Sin[(c + d*x)/2] + 8*(9*A - 13*B)*ArcSin[Sqrt[Sec[c + d*x]]]*Cos[(c +
d*x)/2]^3*Sec[c + d*x]^2*Sin[(c + d*x)/2] + 6*A*Sqrt[1 - Sec[c + d*x]]*Sec[
c + d*x]^(3/2)*Sin[c + d*x] - 7*B*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2)
*Sin[c + d*x] + 4*A*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x]
- 3*B*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 2*B*Sqrt[1 -
 Sec[c + d*x]]*Sec[c + d*x]^(7/2)*Sin[c + d*x] - 9*Sqrt[2]*A*ArcTan[(Sqrt[2
```

```
]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] + 13*Sqrt[2]*B*A
rcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d*x] - 9
*Sqrt[2]*A*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[
c + d*x]*Tan[c + d*x] + 13*Sqrt[2]*B*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sq
rt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x])/(4*d*Sqrt[1 - Sec[c + d*x]
]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 542 vs. $2(208) = 416$.
time = 7.80, size = 543, normalized size = 2.20

method	result
default	$\frac{\left(-12A\sin(dx+c)\left(\cos^2(dx+c)\right)\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-12A\sin(dx+c)\left(\cos^2(dx+c)\right)\cdots\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/16/d*(-12*A*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c))
)^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))-12*A*sin(d*x+c)*cos(d*x+c)^2*2^(
1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2
))+19*B*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2
)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))+19*B*sin(d*x+c)*cos(d*x+c)^2*2^(1/2)*a
rctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))+36*
A*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-
12*A*cos(d*x+c)^3*(-2/(1+cos(d*x+c)))^(1/2)-52*B*sin(d*x+c)*cos(d*x+c)^2*ar
ctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))+14*B*cos(d*x+c)^3*(-2/(1+cos
(d*x+c)))^(1/2)+4*A*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)-8*B*cos(d*x+c)^2
*(-2/(1+cos(d*x+c)))^(1/2)+8*A*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)-10*B*co
s(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+4*B*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos
(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(7/2)*(-2/(1+cos(d*x
+c)))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 13364 vs. $2(208) = 416$.
time = 2.08, size = 13364, normalized size = 54.11

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] -1/16*(4*(12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8*(
sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - sin(3/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(s
in(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*
d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + s
qrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4
*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4
*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2
)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt
(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(
2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 3*(s
qrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*s
qrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4
*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(
2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*
sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*c
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x
+ 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(2*d*x + 2*c) + s
```

```
qrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*si
n(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*s
qrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d
*x + 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sq
rt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*
c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
2*c) + 2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4
*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d...
```

Fricas [A]
time = 4.87, size = 761, normalized size = 3.08

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")

[Out] [-1/16*(2*sqrt(2)*((9*A - 13*B)*cos(d*x + c)^3 + 2*(9*A - 13*B)*cos(d*x + c
)^2 + (9*A - 13*B)*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)
*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + ((
12*A - 19*B)*cos(d*x + c)^3 + 2*(12*A - 19*B)*cos(d*x + c)^2 + (12*A - 19*B
)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos
(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)
) - 4*((6*A - 7*B)*cos(d*x + c)^2 + (4*A - 3*B)*cos(d*x + c) + 2*B)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*co
s(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c)), -1/8*(2*sqrt(2
)*((9*A - 13*B)*cos(d*x + c)^3 + 2*(9*A - 13*B)*cos(d*x + c)^2 + (9*A - 13*
B)*cos(d*x + c))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)
```

```
/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + ((12*A - 19*B)*cos(d*
x + c)^3 + 2*(12*A - 19*B)*cos(d*x + c)^2 + (12*A - 19*B)*cos(d*x + c))*sqr
t(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*((6*A -
 7*B)*cos(d*x + c)^2 + (4*A - 3*B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^3
+ 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^(3/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a/cos(c + d*x))^(3/2
), x)
```

3.256 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=197

$$\frac{(2A-3B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}-\frac{(5A-9B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\,\mathrm{se}}{2d(a+}$$

```
[Out] (2*A-3*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+1/2*
(A-B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)-1/4*(5*A-9*B)*ar
ctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2
))/a^(3/2)/d*2^(1/2)-1/2*(A-3*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d
*x+c))^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4104, 4106, 4108, 3893, 212, 3886, 221}

$$-\frac{(5A-9B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(2A-3B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{(A-3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2ad\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((2*A - 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(
3/2)*d) - ((5*A - 9*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sec[c +
 d*x]^(5/2)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) - ((A - 3*B)*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
```

```
x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4106

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fre
eQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& GtQ[n, 1]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(A-B)-a(A-3B)\,\mathrm{s}\right.}{\sqrt{a+a\sec(c+dx)}}}{2a^2}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(A-3B)\sec^{\frac{3}{2}}(c+dx)\sin(c}{2ad\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(A-3B)\sec^{\frac{3}{2}}(c+dx)\sin(c}{2ad\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(A-3B)\sec^{\frac{3}{2}}(c+dx)\sin(c}{2ad\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{(2A-3B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} - \frac{(5A-9B)\tanh^{-1}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.16, size = 668, normalized size = 3.39

$$\frac{\left(\frac{1}{8}+\frac{i}{8}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\left(\frac{2i\sqrt{2}\left((-3+i)+\sqrt{2}\right)\left((1+i)+\sqrt{2}\right)(2A-3B)\operatorname{ArcTan}\left(\frac{\cos\left(\frac{1}{4}(c+dx)\right)-\left(-1+\sqrt{2}\right)\sin\left(\frac{1}{4}(c+dx)\right)}{\left(1+\sqrt{2}\right)\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)}\right)}{i+\sqrt{2}} - \frac{2\sqrt{2}\left((-1+i)+\sqrt{2}\right)\left((3+i)+\sqrt{2}\right)(2A-3B)\operatorname{ArcTan}\left(\frac{\cos\left(\frac{1}{4}(c+dx)\right)-\left(1+\sqrt{2}\right)\sin\left(\frac{1}{4}(c+dx)\right)}{\left(-1+\sqrt{2}\right)\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)}\right)}{i+\sqrt{2}} + (4-4i)(5A-9B)\log\left(\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)\right) - (4-4i)(5A-9B)\log\left(\cos\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{1}{4}(c+dx)\right)\right) + \frac{(4+4i)\left(-2i+\sqrt{2}\right)(2A-3B)\log\left(\sqrt{2}+2\sin\left(\frac{1}{2}(c+dx)\right)\right)}{i+\sqrt{2}} + \frac{i\sqrt{2}\left((-1+i)+\sqrt{2}\right)\left((3+i)+\sqrt{2}\right)(2A-3B)\log\left(2-\sqrt{2}\cos\left(\frac{1}{2}(c+dx)\right)-\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)}{i+\sqrt{2}} + \frac{\sqrt{2}\left((-3+i)+\sqrt{2}\right)\left((1+i)+\sqrt{2}\right)(2A-3B)\log\left(2+\sqrt{2}\cos\left(\frac{1}{2}(c+dx)\right)-\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)}{i+\sqrt{2}} - \frac{(2-2i)(A-B)}{\left(\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)\right)^2} + \frac{(2-2i)(A-B)}{\left(\cos\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{1}{4}(c+dx)\right)\right)^2} + \frac{(8-8i)B}{\cos\left(\frac{1}{4}(c+dx)\right)-\sin\left(\frac{1}{4}(c+dx)\right)} - \frac{(8-8i)B}{\cos\left(\frac{1}{4}(c+dx)\right)+\sin\left(\frac{1}{4}(c+dx)\right)}\right)}{d(a(1+\sec(c+dx)))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3
/2),x]
```

```
[Out] ((1/8 + I/8)*Cos[(c + d*x)/2]^3*Sec[c + d*x]^(3/2)*(((2*I)*Sqrt[2]*((-3 + I
) + Sqrt[2])*((1 + I) + Sqrt[2])*(2*A - 3*B)*ArcTan[(Cos[(c + d*x)/4] - (-1
 + Sqrt[2])*Sin[(c + d*x)/4])/((1 + Sqrt[2])*Cos[(c + d*x)/4] - Sin[(c + d*
x)/4])])/(I + Sqrt[2]) - (2*Sqrt[2]*((-1 + I) + Sqrt[2])*((3 + I) + Sqrt[2]
)*(2*A - 3*B)*ArcTan[(Cos[(c + d*x)/4] - (1 + Sqrt[2])*Sin[(c + d*x)/4])/((
-1 + Sqrt[2])*Cos[(c + d*x)/4] - Sin[(c + d*x)/4])])/(I + Sqrt[2]) + (4 - 4
*I)*(5*A - 9*B)*Log[Cos[(c + d*x)/4] - Sin[(c + d*x)/4]] - (4 - 4*I)*(5*A -
 9*B)*Log[Cos[(c + d*x)/4] + Sin[(c + d*x)/4]] + ((4 + 4*I)*(-2*I + Sqrt[2]
)*(2*A - 3*B)*Log[Sqrt[2] + 2*Sin[(c + d*x)/2]])/(I + Sqrt[2]) + (I*Sqrt[2]
*((-1 + I) + Sqrt[2])*((3 + I) + Sqrt[2])*(2*A - 3*B)*Log[2 - Sqrt[2]*Cos[(
c + d*x)/2] - Sqrt[2]*Sin[(c + d*x)/2]])/(I + Sqrt[2]) + (Sqrt[2]*((-3 + I)
 + Sqrt[2])*((1 + I) + Sqrt[2])*(2*A - 3*B)*Log[2 + Sqrt[2]*Cos[(c + d*x)/2
```

```
] - Sqrt[2]*Sin[(c + d*x)/2]])/(I + Sqrt[2]) - ((2 - 2*I)*(A - B))/(Cos[(c
+ d*x)/4] - Sin[(c + d*x)/4])^2 + ((2 - 2*I)*(A - B))/(Cos[(c + d*x)/4] + S
in[(c + d*x)/4])^2 + ((8 - 8*I)*B)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) -
((8 - 8*I)*B)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])))/(d*(a*(1 + Sec[c + d*
x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(166) = 332$.
time = 7.68, size = 479, normalized size = 2.43

method	result
default	$\left(\frac{1}{\cos(dx+c)}\right)^{\frac{5}{2}} (\cos^2(dx+c)) \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}} \left(2A\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(-1-\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sin(dx+c)\sqrt{2}\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/d*(1/cos(d*x+c))^(5/2)*cos(d*x+c)^2*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)
*(2*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/
2))*sin(d*x+c)*2^(1/2)*cos(d*x+c)+2*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*
(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*sin(d*x+c)*2^(1/2)*cos(d*x+c)-3*B*arctan
(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))*sin(d*x+
c)*2^(1/2)*cos(d*x+c)-3*B*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c
)+sin(d*x+c))*2^(1/2))*sin(d*x+c)*2^(1/2)*cos(d*x+c)+A*cos(d*x+c)^2*(-2/(1+
cos(d*x+c)))^(1/2)-5*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin
(d*x+c)*cos(d*x+c)-3*B*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+9*B*arctan(1/
2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+c)-A*cos(d*x+c)*
(-2/(1+cos(d*x+c)))^(1/2)+B*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+2*B*(-2/(1
+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1
)/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 7057 vs. $2(166) = 332$.
time = 1.01, size = 7057, normalized size = 35.82

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((4*(sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*
```

cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x + 2*c

```
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 1) - 4*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(2*d*x + 2*c) - 4*(cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 8*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(2*d*x + 2*c) + 1)*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))))*A/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*
a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4
*sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(
2)*a*cos(2*d*x + 2*c) + 4*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a)) - (12*(s
in(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(3/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +...
```

Fricas [A]
time = 3.34, size = 669, normalized size = 3.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((5*A - 9*B)*cos(d*x + c)^2 + 2*(5*A - 9*B)*cos(d*x + c) + 5
*A - 9*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c
) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 2*((2*A - 3*B)*cos(d*x +
c)^2 + 2*(2*A - 3*B)*cos(d*x + c) + 2*A - 3*B)*sqrt(a)*log((a*cos(d*x + c)^
3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(c
os(d*x + c)^3 + cos(d*x + c)^2)) + 4*((A - 3*B)*cos(d*x + c) - 2*B)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*co
s(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(sqrt(2)*((5*A - 9*B)*cos
(d*x + c)^2 + 2*(5*A - 9*B)*cos(d*x + c) + 5*A - 9*B)*sqrt(-a)*arctan(sqrt(
2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*s
in(d*x + c))) + 2*((2*A - 3*B)*cos(d*x + c)^2 + 2*(2*A - 3*B)*cos(d*x + c)
+ 2*A - 3*B)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2
```

```
*a)) - 2*((A - 3*B)*cos(d*x + c) - 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d
*x + c) + a^2*d)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(3/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x))^(3/2
), x)
```

3.257 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=145

$$\frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d}+\frac{(A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)}{2d(a+a\sec(c+$$

```
[Out] 2*B*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(3/2)/d+1/2*(A-B)*
sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+1/4*(A-5*B)*arctanh(1/
2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(3/
2)/d*2^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {4104, 4108, 3893, 212, 3886, 221}

$$\frac{(A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(3/2)*d)
+ ((A - 5*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqr
t[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\sqrt{\sec(c+dx)}\,\left(\frac{1}{2}a(A-B)+2aB\right.}{\sqrt{a+a\sec(c+dx)}}}{2a^2}$$

$$= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(A-5B)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+d}}}{4a}$$

$$= \frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(A-5B)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx, x\right.}{2a}$$

$$= \frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{3/2}d} + \frac{(A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\text{se}}}{\sqrt{2}\,\sqrt{a}}\right.}{2\sqrt{2}\,a^{3/2}}$$

Mathematica [A]
time = 0.82, size = 113, normalized size = 0.78

$$\frac{\sqrt{\sec(c+dx)}\left((A-5B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)+4\sqrt{2}\,B\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)+(A-B)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{2ad\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*((A - 5*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] +
 4*Sqrt[2]*B*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + (A - B)*T
an[(c + d*x)/2]))/(2*a*d*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 315 vs. 2(120) = 240.
time = 7.74, size = 316, normalized size = 2.18

method	result
default	$-\frac{\left(-2B\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}\,\sin(dx+c)+2B\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx\right.}{4}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/d*(-2*B*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))
*2^(1/2))*2^(1/2)*sin(d*x+c)+2*B*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+co
s(d*x+c)-sin(d*x+c))*2^(1/2))*2^(1/2)*sin(d*x+c)+A*cos(d*x+c)*(-2/(1+cos(d*
x+c)))^(1/2)-A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-
B*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+5*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos
(d*x+c)))^(1/2))*sin(d*x+c)-A*(-2/(1+cos(d*x+c)))^(1/2)+B*(-2/(1+cos(d*x+c)
))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(
3/2)*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3*(cos(d*x+c)^2-1)/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 17843 vs. $2(120) = 240$.
time = 1.28, size = 17843, normalized size = 123.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((32*(cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*
d*x + 3/2*c) + cos(d*x + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin
(d*x + c))*cos(3*d*x + 3*c)^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) +
 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x
 + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(
3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(4/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(
3*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) +
1)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d
*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(2
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 32*(cos(3/2*d*x
 + 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + cos(d*
x + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(3*d*x
+ 3*c)^2 + 32*(6*cos(d*x + c) + 1)*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) +
96*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 96*sin(2*d*x + 2*c)^2*sin(3/2*
d*x + 3/2*c) + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x +
3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3
*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c)
+ 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*
cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x +
3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2
*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(2/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*(2*(3*cos(d*x + c) + 1)*cos(2
```

```
*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c
) + 3*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 2*(3*sin(3/2*d*x + 3/2*c)*s
in(d*x + c) + cos(3/2*d*x + 3/2*c))*sin(2*d*x + 2*c) + (3*cos(d*x + c)^2 +
3*sin(d*x + c)^2 + 2*cos(d*x + c))*sin(3/2*d*x + 3/2*c) + 2*cos(3/2*d*x + 3
/2*c)*sin(d*x + c))*cos(3*d*x + 3*c) - 4*(6*(sin(2*d*x + 2*c) + sin(d*x + c
))*sin(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^3 + (2*(3*cos(2*d*x + 2*c) + 3*cos
(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) +
1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 9*sin(2*d*x
 + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x
 + c) + 1)*sin(3*d*x + 3*c) + 3*(2*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1
)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1)*cos(2*d*x
+ 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 6*(sin(2*d*x + 2*c) + si
n(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c)^2 + 9*sin(2*d*x + 2*c)^2 +
18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x + c) + 1)*s
in(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*(2*(3*cos(2
*d*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6
*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x +
 c)^2 + 6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x +
3*c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*
x + c)^2 + 6*cos(d*x + c) + 1)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))))*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) - 4*(8*cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 72*cos(2*d*x + 2*c)^2*
sin(3/2*d*x + 3/2*c) - 144*cos(2*d*x + 2*c)*cos(d*x + c)*sin(3/2*d*x + 3/2*
c) - 8*sin(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 72*sin(2*d*x + 2*c)^2*sin(
3/2*d*x + 3/2*c) - 16*(3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + 3*cos(3/2*
d*x + 3/2*c)*sin(d*x + c) - sin(3/2*d*x + 3/2*c))*cos(3*d*x + 3*c) - 48*(co
s(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*
c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d
*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2
*c)*sin(d*x + c))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) - 16*(cos(3*d*x + 3*c)*cos(3/2*d*x + 3/2*c) + 3*sin(2*d*x + 2*c)*sin(3/
2*d*x + 3/2*c) + 3*sin(3/2*d*x + 3/2*c)*sin(d*x + c) + cos(3/2*d*x + 3/2*c)
)*sin(3*d*x + 3*c) - 48*(3*sin(3/2*d*x + 3/2*c)*sin(d*x + c) + cos(3/2*d*x
+ 3/2*c))*sin(2*d*x + 2*c) - 8*(9*cos(d*x + c)^2 + 9*sin(d*x + c)^2 - 1)*si
n(3/2*d*x + 3/2*c) - 48*cos(3/2*d*x + 3/2*c)*sin(d*x + c) + 3*(2*(3*cos(2*d
*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(
3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c
)^2 + 6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*
c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x
+ c)^2 + 6*cos(d*x + c) + 1)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) - 4*(8*cos(3*d*x + 3*c)^2*sin(3/2*d*x + 3/2*c) - 72*cos(2*d*x + 2*c)^2*si
n(3/2*d*x + 3/2*c) - 144*cos(2*d*x + 2*c)*cos(d...
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 265 vs.

$2(120) = 240.$
time = 2.30, size = 601, normalized size = 4.14

$$\left[-\frac{\sqrt{2}\left((A-5B)\cos(dx+c)^2+2(A-5B)\cos(dx+c)+A-5B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4(A-B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-4\left(B\cos(dx+c)^2+2B\cos(dx+c)+B\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-7a\cos(dx+c)^2-\frac{4\left(\cos(dx+c)^2-2\cos(dx+c)\right)\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}, -\frac{\sqrt{2}\left((A-5B)\cos(dx+c)^2+2(A-5B)\cos(dx+c)+A-5B\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-2(A-B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-4\left(B\cos(dx+c)^2+2B\cos(dx+c)+B\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((A - 5*B)*cos(d*x + c)^2 + 2*(A - 5*B)*cos(d*x + c) + A - 5
*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*
a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(A - B)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 4*(B*cos(d*x + c)^2 +
2*B*cos(d*x + c) + B)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 -
4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x +
 c)^2)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(
2)*((A - 5*B)*cos(d*x + c)^2 + 2*(A - 5*B)*cos(d*x + c) + A - 5*B)*sqrt(-a)
*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))/(a*sin(d*x + c))) - 2*(A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c) - 4*(B*cos(d*x + c)^2 + 2*B*cos(d*x + c
) + B)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/
(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x))^(3/2
), x)
```

3.258 $\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=107

$$\frac{(3A+B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}$$

```
[Out] -1/2*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)+1/4*(3*A+B)
*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(
1/2))/a^(3/2)/d*2^(1/2)
```

Rubi [A]
time = 0.12, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {4097, 3893, 212}

$$\frac{(3A+B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((3*A + B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sec[c + d*x]^(3/2)*
Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4097

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
```

```
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n},
 x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m
, -1]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(3A+B)\int \frac{\sqrt{\sec(c+}}{\sqrt{a+a\sec(c}}}{4a}$$

$$= -\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(3A+B)\mathrm{Subst}\left(\int \frac{1}{2a-x^2}\,d\right.}{}$$

$$= \frac{(3A+B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{(A-B)\sec^{\frac{3}{2}}}{2d(a+a}$$

Mathematica [A]
time = 0.22, size = 84, normalized size = 0.79

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\left((3A+B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^2\left(\frac{1}{2}(c+dx)\right)+(-A+B)\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(3/2)*((3*A + B)*ArcTanh[Sin[(c + d*x)/2]]*C
os[(c + d*x)/2]^2 + (-A + B)*Sin[(c + d*x)/2]))/(d*(a*(1 + Sec[c + d*x]))^(
3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 218 vs. 2(88) = 176.
time = 8.41, size = 219, normalized size = 2.05

method	result

default	$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\,\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\cos(dx+c)\left(A\cos(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}+3A\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)s\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*(
A*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+3*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos
(d*x+c)))^(1/2))*sin(d*x+c)-B*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+B*arctan
(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-A*(-2/(1+cos(d*x+c)))
^(1/2)+B*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3*
(cos(d*x+c)^2-1)/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 16752 vs. $2(88) = 176$.
time = 1.27, size = 16752, normalized size = 156.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2*c)
^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x
 + c)^2 + 3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/
2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin
(d*x + c)^2 + 2*(6*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 2
*sin(3/2*d*x + 3/2*c) + 2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 4*(3*log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
 1) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
```

+ 1/2*c) + 1) + 2*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c) + cos(3/2*d*x + 3/2*c) - cos(1/2*d*x + 1/2*c))*sin(2*d*x + 2*c) - 4*(2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) + 8*cos(3/2*d*x + 3/2*c)*sin(d*x + c) - 8*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 4*sin(1/2*d*x + 1/2*c))*A/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(d*x + c)^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(2*d*x + 2*c)*sin(d*x + c) + 4*sqrt(2)*a*sin(d*x + c)^2 + 4*sqrt(2)*a*cos(d*x + c) + 2*(2*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + sqrt(2)*a)*sqrt(a)) + (32*(cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + cos(d*x + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(3*d*x + 3*c)^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 32*(cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) + cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + cos(d*x + c)*sin(3/2*d*x + 3/2*c) + cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(3*d*x + 3*c)^2 + 32*(6*cos(d*x + c) + 1)*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 96*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 96*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 96*(cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 3*cos(3/2*d*x + 3/2*c)*sin(2*d*x + 2*c) - (3*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) - cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 3*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 32*(2*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c)*sin(3/2*d*x + 3/2*c) + 3*cos(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 3*sin(2*d*x + 2*c)^2*sin(3/2*d*x + 3/2*c) + 2*(3*sin(3/2*d*x + 3/2*c)*sin(d*x + c) + cos(3/2*d*x + 3/2*c))*sin(2*d*x + 2*c) + (3*cos(d*x + c)^2 + 3*sin(d*x + c)^2 + 2*cos(d*x + c))*sin(3/2*d*x + 3/2*c) + 2*cos(3/2*d*x + 3/2*c)*sin(d*x + c))*cos(3*d*x + 3*c) - 4*(6*(sin(2*d*x + 2*c) + sin(d*x + c))*sin(3*d*x + 3*c)^2 + sin(3*d*x + 3*c)^3 + (2*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)*cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1)*cos(2*d*x + 2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 9*sin(2*d*x + 2*c)^2 + 18*sin(2*d*x + 2*c)*sin(d*x + c) + 9*sin(d*x + c)^2 + 6*cos(d*x +

```
c) + 1)*sin(3*d*x + 3*c) + 3*(2*(3*cos(2*d*x + 2*c) + 3*cos(d*x + c) + 1)*
cos(3*d*x + 3*c) + cos(3*d*x + 3*c)^2 + 6*(3*cos(d*x + c) + 1)*cos(2*d*x +
2*c) + 9*cos(2*d*x + 2*c)^2 + 9*cos(d*x + c)^2 + 6*(sin(2*d*x + 2*c) + sin(
d*x + c))*sin(3*d*x + 3*c) + sin(3*d*x + 3*c)^2...
```

Fricas [A]
time = 2.00, size = 376, normalized size = 3.51

$$\left[\frac{\sqrt{2}\left((3A+B)\cos(dx+c)^2+2(3A+B)\cos(dx+c)+3A+B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4(A-B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}, -\frac{\sqrt{2}\left((3A+B)\cos(dx+c)^2+2(3A+B)\cos(dx+c)+3A+B\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)+2(A-B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*((3*A + B)*cos(d*x + c)^2 + 2*(3*A + B)*cos(d*x + c) + 3*A +
B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(A - B)*sqrt((a*cos(d*x + c) +
a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 +
2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)*((3*A + B)*cos(d*x + c)^2 + 2*
(3*A + B)*cos(d*x + c) + 3*A + B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(A
- B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\sqrt{\sec(c+dx)}}{(a(\sec(c+dx)+1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(sec(c + d*x))/(a*(sec(c + d*x) + 1))**(3
/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x))^(3/2
), x)
```

3.259 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=156

$$-\frac{(7A-3B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}}+\frac{(5A-B)\sqrt{\sec(c}{2ad\sqrt{a+c}$$

```
[Out] -1/4*(7*A-3*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a
*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-1/2*(A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)
/d/(a+a*sec(d*x+c))^(3/2)+1/2*(5*A-B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*
sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.24, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4105, 4098, 3893, 212}

$$-\frac{(7A-3B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(5A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\sqrt{a\sec(c+dx)+a}}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] -1/2*((7*A - 3*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2
]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[2]*a^(3/2)*d) - ((A - B)*Sqrt[Sec[c + d
*x]]*Sin[c + d*x])/(2*d*(a + a*Sec[c + d*x])^(3/2)) + ((5*A - B)*Sqrt[Sec[c
 + d*x]]*Sin[c + d*x])/(2*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(5A-B)-a(A-B}{\sqrt{\sec(c+dx)}\,\sqrt{a+}}}{2a^2}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(5A-B)\sqrt{\sec(c+dx}}{2ad\sqrt{a+a\sec(c}}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(5A-B)\sqrt{\sec(c+dx}}{2ad\sqrt{a+a\sec(c}}$$

$$= -\frac{(7A-3B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{(A-B)}{2d(}$$

Mathematica [A]
time = 1.43, size = 174, normalized size = 1.12

$$\frac{2\sqrt{2}\,(7A-3B)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)\sec^2(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)+\left((5A-B)\sqrt{1-\sec(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)+4A\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\right)\sin(c+dx)}{2d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] (2*Sqrt[2]*(7*A - 3*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c +
 d*x]]]*Cos[(c + d*x)/2]^3*Sec[c + d*x]^2*Sin[(c + d*x)/2] + ((5*A - B)*Sqr
t[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + 4*A*Sqrt[-((-1 + Sec[c + d*x])*Sec
[c + d*x])])*Sin[c + d*x])/(2*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]
))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 286 vs. $2(131) = 262$.
time = 8.29, size = 287, normalized size = 1.84

method	result
default	$-\frac{(-1+\cos(dx+c))\left(7A\sqrt{-\frac{2}{1+\cos(dx+c)}}\ \arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sin(dx+c)\cos(dx+c)-3B\sqrt{-\frac{2}{1+\cos(dx+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))*(7*A*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/2*sin(d*x+c)
*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+c)-3*B*(-2/(1+cos(d*x+c)))^(
1/2)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+c)
+7*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/
2)*A*sin(d*x+c)-3*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+c
os(d*x+c)))^(1/2)*B*sin(d*x+c)-8*A*cos(d*x+c)^2-2*A*cos(d*x+c)+2*B*cos(d*x+
c)+10*A-2*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^3/(1/cos(d*x+c)
)^(1/2)/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 8208 vs. $2(131) = 262$.
time = 0.74, size = 8208, normalized size = 52.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/4*((4*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c
)^4 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
```

sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4 + 4*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^4 + 70*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + 7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^4 - 8*sin(1/2*d*x + 1/2*c)^5 + 28*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^3 + 4*(21*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 24*sin(1/2*d*x + 1/2*c)^2 - 20)*sin(3/2*d*x + 3/2*c)^3 - 8*(10*cos(1/2*d*x + 1/2*c)^2 + 3)*sin(1/2*d*x + 1/2*c)^3 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^2 + 2)*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c)^2 + (427*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + 35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 40*sin(1/2*d*x + 1/2*c)^3 - 8*(61*cos(1/2*d*x + 1/2*c)^2 + 9)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 +

```
sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*
c))*cos(3/2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 +
(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1
/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*
x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*
x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x
+ 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c
) - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x +...
```

Fricas [A]
time = 1.43, size = 430, normalized size = 2.76

$$\left[-\frac{\sqrt{2}\left((7A-3B)\cos\left(dx+c\right)^2+2\,(7A-3B)\cos\left(dx+c\right)+7A-3B\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-\frac{4\left(4A\cos(dx+c)^2+(5A-B)\cos(dx+c)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{8\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)},\ \frac{\sqrt{2}\left((7A-3B)\cos\left(dx+c\right)^2+2\,(7A-3B)\cos\left(dx+c\right)+7A-3B\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}}{a\sin(dx+c)}\right)+\frac{2\left(4A\cos(dx+c)^2+(5A-B)\cos(dx+c)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{4\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((7*A - 3*B)*cos(d*x + c)^2 + 2*(7*A - 3*B)*cos(d*x + c) + 7
*A - 3*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c
) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(4*A*cos(d*x + c)^2 + (
5*A - B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d),
 1/4*(sqrt(2)*((7*A - 3*B)*cos(d*x + c)^2 + 2*(7*A - 3*B)*cos(d*x + c) + 7*
A - 3*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(4*A*cos(d*x + c)^2 + (5*A
- B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqr
t(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec\left(c + dx\right)}{\left(a\left(\sec\left(c + dx\right) + 1\right)\right)^{\frac{3}{2}}\sqrt{\sec\left(c + dx\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/((a*(sec(c + d*x) + 1))**(3/2)*sqrt(sec(c + d
*x))), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c
))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{3/2}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)
),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)
), x)
```

3.260 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=203

$$\frac{(11A-7B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}}+\frac{(7A-}{6ad\sqrt{\sec(c+}}$$

```
[Out] 1/4*(11*A-7*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a
*sec(d*x+c))^(1/2))/a^(3/2)/d*2^(1/2)-1/2*(A-B)*sin(d*x+c)/d/(a+a*sec(d*x+c
))^(3/2)/sec(d*x+c)^(1/2)+1/6*(7*A-3*B)*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)/(a+
a*sec(d*x+c))^(1/2)-1/6*(19*A-15*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*se
c(d*x+c))^(1/2)
```

Rubi [A]

time = 0.35, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4105, 4107, 4098, 3893, 212}

$$\frac{(11A-7B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(19A-15B)\sin(c+dx)\sqrt{\sec(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}+\frac{(7A-3B)\sin(c+dx)}{6ad\sqrt{\sec(c+dx)}\,\sqrt{a\sec(c+dx)+a}}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] ((11*A - 7*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])])/(2*Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2
*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + ((7*A - 3*B)*Sin[c + d*
x])/(6*a*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((19*A - 15*B)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(6*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(7A-3B)-2a(A-}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a}}}{2a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}} + \frac{(7A-3B)}{6ad\sqrt{\sec(c+dx)}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}} + \frac{(7A-3B)}{6ad\sqrt{\sec(c+dx)}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{3/2}} + \frac{(7A-3B)}{6ad\sqrt{\sec(c+dx)}}$$

$$= \frac{(11A-7B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d} - \frac{(}{2d\sqrt{\sec(c}}$$

Mathematica [A]
time = 1.81, size = 173, normalized size = 0.85

$$\frac{-6\sqrt{2}\,(11A-7B)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^3\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)+\sqrt{1-\sec(c+dx)}\,(12(-A+B)+(-17A+15B+2A\cos(2(c+dx)))\sec(c+dx))\tan(c+dx)}{6d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\,(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] (-6*Sqrt[2]*(11*A - 7*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c
 + d*x]]]*Cos[(c + d*x)/2]^3*Sec[c + d*x]^(5/2)*Sin[(c + d*x)/2] + Sqrt[1 -
 Sec[c + d*x]]*(12*(-A + B) + (-17*A + 15*B + 2*A*Cos[2*(c + d*x)])*Sec[c +
 d*x])*Tan[c + d*x])/(6*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*(a*(1 +
 Sec[c + d*x]))^(3/2))
```

Maple [A]
time = 7.70, size = 317, normalized size = 1.56

method	result

default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(33A\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sin(dx+c)\cos(dx+c)-21}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/12/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(33*A*(-2/(1+cos
(d*x+c)))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)
*cos(d*x+c)-21*B*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(1+cos
(d*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+c)+33*arctan(1/2*sin(d*x+c)*(-2/(1+cos(
d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*A*sin(d*x+c)+8*A*cos(d*x+c)^3-21*
arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*
B*sin(d*x+c)-32*A*cos(d*x+c)^2+24*B*cos(d*x+c)^2-14*A*cos(d*x+c)+6*B*cos(d*
x+c)+38*A-30*B)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)^3/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 41138 vs. $2(172) = 344$.
time = 1.14, size = 41138, normalized size = 202.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] -1/12*(3*(4*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/
2*c)^4 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4 + 4*(7*log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) +
1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^4 + 70*(log(cos(1/2*d*x +
 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(
1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*
cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + 7*(log(cos(1/2*d*x + 1/2*c)
^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x
 + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2
*d*x + 1/2*c)^4 - 8*sin(1/2*d*x + 1/2*c)^5 + 28*(7*(log(cos(1/2*d*x + 1/2*c
)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*
```

x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^3 + 4*(21*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 24*sin(1/2*d*x + 1/2*c)^2 - 20)*sin(3/2*d*x + 3/2*c)^3 - 8*(10*cos(1/2*d*x + 1/2*c)^2 + 3)*sin(1/2*d*x + 1/2*c)^3 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^2 + 2)*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c)^2 + (427*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + 35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 40*sin(1/2*d*x + 1/2*c)^3 - 8*(61*cos(1/2*d*x + 1/2*c)^2 + 9)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/

```
2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2
*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d
*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)
^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/
2*c) - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*...
```

Fricas [**A**]
time = 1.54, size = 464, normalized size = 2.29

$$\left[-\frac{3\sqrt{2}\left((11A-7B)\cos\left(dx+c\right)^{2}+2(11A-7B)\cos\left(dx+c\right)+11A-7B\right)\sqrt{a}\log\left(-\frac{a\cos\left(dx+c\right)^{2}+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-2a\cos\left(dx+c\right)-3a}{\cos\left(dx+c\right)^{2}+2\cos\left(dx+c\right)+1}\right)-\frac{4\left(4A\cos\left(dx+c\right)^{3}-12(A-B)\cos\left(dx+c\right)^{2}-(19A-15B)\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{24\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)},\ -\frac{3\sqrt{2}\left((11A-7B)\cos\left(dx+c\right)^{2}+2(11A-7B)\cos\left(dx+c\right)+11A-7B\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}}{a\sin\left(dx+c\right)}\right)-\frac{2\left(4A\cos\left(dx+c\right)^{3}-12(A-B)\cos\left(dx+c\right)^{2}-(19A-15B)\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{12\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/24*(3*sqrt(2)*((11*A - 7*B)*cos(d*x + c)^2 + 2*(11*A - 7*B)*cos(d*x + c
) + 11*A - 7*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d
*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(4*A*cos(d*x + c)
^3 - 12*(A - B)*cos(d*x + c)^2 - (19*A - 15*B)*cos(d*x + c))*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x +
 c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/12*(3*sqrt(2)*((11*A - 7*B)*cos(d
*x + c)^2 + 2*(11*A - 7*B)*cos(d*x + c) + 11*A - 7*B)*sqrt(-a)*arctan(sqrt(
2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*s
in(d*x + c))) - 2*(4*A*cos(d*x + c)^3 - 12*(A - B)*cos(d*x + c)^2 - (19*A -
 15*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/s
qrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec\left(c+dx\right)}{\left(a\left(\sec\left(c+dx\right)+1\right)\right)^{\frac{3}{2}}\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/((a*(sec(c + d*x) + 1))**(3/2)*sec(c + d*x)**
(3/2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{3/2}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)
),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)
), x)
```

3.261 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=250

$$-\frac{(15A-11B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}+\frac{(9A-}{10ad\sec^{\frac{3}{2}}(c+}$$

```
[Out] -1/2*(A-B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)-1/4*(15*A-1
1*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c
))^(1/2))/a^(3/2)/d*2^(1/2)+1/10*(9*A-5*B)*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)/
(a+a*sec(d*x+c))^(1/2)-1/30*(39*A-35*B)*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)/(a+
a*sec(d*x+c))^(1/2)+1/30*(147*A-95*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/d/(a+a*
sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.46, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4105, 4107, 4098, 3893, 212}

$$-\frac{(15A-11B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(9A-5B)\sin(c+dx)}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{(A-B)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}+\frac{(147A-95B)\sin(c+dx)\sqrt{\sec(c+dx)}}{30ad\sqrt{a\sec(c+dx)+a}}-\frac{(39A-35B)\sin(c+dx)}{30ad\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] -1/2*((15*A - 11*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[2]*a^(3/2)*d) - ((A - B)*Sin[c + d*x]
)/(2*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)) + ((9*A - 5*B)*Sin[c
+ d*x])/(10*a*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) - ((39*A - 35*
B)*Sin[c + d*x])/(30*a*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + ((1
47*A - 95*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(30*a*d*Sqrt[a + a*Sec[c + d*
x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
```

```
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\int \frac{\frac{1}{2}a(9A-5B)-3a(A-B)\mathrm{s}}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec}}}{2a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(9A-5B)\sin(}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(9A-5B)\sin(}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(9A-5B)\sin(}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{(9A-5B)\sin(}{10ad\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= -\frac{(15A-11B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{2\sqrt{2}a^{3/2}d} - \frac{}{2d\sec^{\frac{3}{2}}(}$$

Mathematica [A]
time = 1.43, size = 171, normalized size = 0.68

$$\frac{\sec(c+dx)\left((141A-85B+3(39A-20B)\cos(c+dx)+(-6A+10B)\cos(2(c+dx))+3A\cos(3(c+dx)))\sqrt{\sec(c+dx)}\sin(c+dx)+\frac{15\sqrt{2}(15A-11B)\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\tan(c+dx)}{\sqrt{1-\sec(c+dx)}}\right)}{30d(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] (Sec[c + d*x]*((141*A - 85*B + 3*(39*A - 20*B)*Cos[c + d*x] + (-6*A + 10*B)
*Cos[2*(c + d*x)] + 3*A*Cos[3*(c + d*x)])*Sqrt[Sec[c + d*x]]*Sin[c + d*x] +
 (15*Sqrt[2]*(15*A - 11*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec
[c + d*x]]]*Cos[(c + d*x)/2]^2*Tan[c + d*x])/Sqrt[1 - Sec[c + d*x]]))/(30*d
*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A]
time = 8.28, size = 339, normalized size = 1.36

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(24A(\cos^4(dx+c))-225A\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sin\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/60/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(24*A*cos(d*x+c)
^4-225*A*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c))
)^(1/2))*sin(d*x+c)*cos(d*x+c)+165*B*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/2*s
in(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+c)-48*A*cos(d*x+c)^
3-225*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^
(1/2)*A*sin(d*x+c)+40*B*cos(d*x+c)^3+165*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d
*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c)+240*A*cos(d*x+c)^2-16
0*B*cos(d*x+c)^2+78*A*cos(d*x+c)-70*B*cos(d*x+c)-294*A+190*B)*(1/cos(d*x+c)
)^(5/2)*cos(d*x+c)^3/sin(d*x+c)^3/a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 2.09, size = 500, normalized size = 2.00

$$\left[-\frac{15\sqrt{2}\left((15A-11B)\cos(dx+c)^2+2(15A-11B)\cos(dx+c)+15A-11B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-\frac{4\left(12A\cos(dx+c)^4-4(3A-5B)\cos(dx+c)^3+12(9A-5B)\cos(dx+c)^2+(147A-95B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{120\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}, \frac{15\sqrt{2}\left((15A-11B)\cos(dx+c)^2+2(15A-11B)\cos(dx+c)+15A-11B\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)+\frac{2\left(12A\cos(dx+c)^4-4(3A-5B)\cos(dx+c)^3+12(9A-5B)\cos(dx+c)^2+(147A-95B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{60\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/120*(15*sqrt(2)*((15*A - 11*B)*cos(d*x + c)^2 + 2*(15*A - 11*B)*cos(d*x
 + c) + 15*A - 11*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqr
```

```
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*
cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(12*A*cos(d*
x + c)^4 - 4*(3*A - 5*B)*cos(d*x + c)^3 + 12*(9*A - 5*B)*cos(d*x + c)^2 + (
147*A - 95*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^
2*d), 1/60*(15*sqrt(2)*((15*A - 11*B)*cos(d*x + c)^2 + 2*(15*A - 11*B)*cos(
d*x + c) + 15*A - 11*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(12*A*cos(d*
x + c)^4 - 4*(3*A - 5*B)*cos(d*x + c)^3 + 12*(9*A - 5*B)*cos(d*x + c)^2 + (
147*A - 95*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x
 + c)/sqrt(cos(d*x + c)))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^
2*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{3/2}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)
),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)
), x)
```

3.262 $\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=246

$$\frac{(2A-5B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d}-\frac{(43A-115B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(A-B}{4d(}$$

```
[Out] (2*A-5*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+1/4*
(A-B)*sec(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+1/16*(7*A-15*B)*
sec(d*x+c)^(5/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)-1/32*(43*A-115*B)*ar
ctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2
))/a^(5/2)/d*2^(1/2)-1/16*(11*A-35*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(a+
a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 246, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4104, 4106, 4108, 3893, 212, 3886, 221}

$$-\frac{(43A-115B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(2A-5B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-35B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(A-B)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{(7A-15B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((2*A - 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(
5/2)*d) - ((43*A - 115*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])
/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sec
[c + d*x]^(7/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((7*A - 15
*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) -
((11*A - 35*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c
+ d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4106

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fre
eQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& GtQ[n, 1]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\sec^{\frac{5}{2}}(c+dx)\left(\frac{5}{2}a(A-B)-a(A-5B)\,\mathrm{s}\right.}{(a+a\sec(c+dx))^{3/2}}}{4a^2}$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(7A-15B)\sec^{\frac{5}{2}}(c+dx)\sin}{16ad(a+a\sec(c+dx))}$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(7A-15B)\sec^{\frac{5}{2}}(c+dx)\sin}{16ad(a+a\sec(c+dx))}$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(7A-15B)\sec^{\frac{5}{2}}(c+dx)\sin}{16ad(a+a\sec(c+dx))}$$

$$= \frac{(A-B)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(7A-15B)\sec^{\frac{5}{2}}(c+dx)\sin}{16ad(a+a\sec(c+dx))}$$

$$= \frac{(2A-5B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} - \frac{(43A-115B)\tanh}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 941 vs. 2(246) = 492.
time = 6.17, size = 941, normalized size = 3.83

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5
/2),x]
```

```
[Out] -1/4*(A*Sec[c + d*x]^(9/2)*Sin[c + d*x])/(d*(a*(1 + Sec[c + d*x]))^(5/2)) -
 (B*Sec[c + d*x]^(11/2)*Sin[c + d*x])/(4*d*(a*(1 + Sec[c + d*x]))^(5/2)) +
(3*A*Sec[c + d*x]^(9/2)*(1 + Sec[c + d*x])*Sin[c + d*x])/(16*d*(a*(1 + Sec[
c + d*x]))^(5/2)) + (7*B*Sec[c + d*x]^(11/2)*(1 + Sec[c + d*x])*Sin[c + d*x
])/(16*d*(a*(1 + Sec[c + d*x]))^(5/2)) - (11*A*Sec[c + d*x]^(3/2)*(1 + Sec[
c + d*x])^2*Sin[c + d*x])/(16*d*(a*(1 + Sec[c + d*x]))^(5/2)) + (35*B*Sec[c
 + d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*(a*(1 + Sec[c + d*x]
))^(5/2)) + (7*A*Sec[c + d*x]^(5/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*
```

```
d*(a*(1 + Sec[c + d*x]))^(5/2)) - (15*B*Sec[c + d*x]^(5/2)*(1 + Sec[c + d*x
])^2*Sin[c + d*x])/(16*d*(a*(1 + Sec[c + d*x]))^(5/2)) - (3*A*Sec[c + d*x]^
(7/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*(a*(1 + Sec[c + d*x]))^(5/2)
) + (11*B*Sec[c + d*x]^(7/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*(a*(1
 + Sec[c + d*x]))^(5/2)) - (7*B*Sec[c + d*x]^(9/2)*(1 + Sec[c + d*x])^2*Sin
[c + d*x])/(16*d*(a*(1 + Sec[c + d*x]))^(5/2)) - (11*A*ArcSin[Sqrt[1 - Sec[
c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*
(a*(1 + Sec[c + d*x]))^(5/2)) + (35*B*ArcSin[Sqrt[1 - Sec[c + d*x]]]*(1 + S
ec[c + d*x])^2*Tan[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d
*x]))^(5/2)) - (43*A*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c
+ d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)) + (115*B
*ArcSin[Sqrt[Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x])/(16*d*Sqrt[1
 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)) + (43*A*ArcTan[(Sqrt[2]*Sqrt
[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d*x])^2*Tan[c + d*x])/
(16*Sqrt[2]*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)) - (115*B
*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*(1 + Sec[c + d
*x])^2*Tan[c + d*x])/(16*Sqrt[2]*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d
*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 830 vs. $2(209) = 418$.
time = 8.41, size = 831, normalized size = 3.38

method	result
default	$\frac{\left(\frac{1}{\cos(dx+c)}\right)^{\frac{7}{2}} \left(\cos^3(dx+c)\right) \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\, (-1+\cos(dx+c))^2 \left(-16A \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\, (1+\cos(dx+c)-\sin(dx+c))\sqrt{}}{4}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/16/d*(1/cos(d*x+c))^(7/2)*cos(d*x+c)^3*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2
)*(-1+cos(d*x+c))^2*(-16*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+
c)-sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)+16*A*sin(d*x+c)*cos
(d*x+c)^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*
x+c))*2^(1/2))+40*B*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(
d*x+c))*2^(1/2))*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)-40*B*sin(d*x+c)*cos(d*x+c)
^2*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2
^(1/2))+11*A*cos(d*x+c)^3*(-2/(1+cos(d*x+c)))^(1/2)-43*A*sin(d*x+c)*cos(d*x
+c)^2*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-16*A*arctan(1/4*(-2/
(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*cos(d*x+c)*sin(d*x
+c)*2^(1/2)+16*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x
```

```
+c))*2^(1/2))*sin(d*x+c)*2^(1/2)*cos(d*x+c)-35*B*cos(d*x+c)^3*(-2/(1+cos(d*
x+c)))^(1/2)+115*B*sin(d*x+c)*cos(d*x+c)^2*arctan(1/2*sin(d*x+c)*(-2/(1+cos
(d*x+c)))^(1/2))+40*B*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-si
n(d*x+c))*2^(1/2))*cos(d*x+c)*sin(d*x+c)*2^(1/2)-40*B*arctan(1/4*(-2/(1+cos
(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*sin(d*x+c)*2^(1/2)*cos(d
*x+c)+4*A*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)-43*A*arctan(1/2*sin(d*x+c)
*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+c)-20*B*cos(d*x+c)^2*(-2/(1+
cos(d*x+c)))^(1/2)+115*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*s
in(d*x+c)*cos(d*x+c)-15*A*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+39*B*cos(d*x
+c)*(-2/(1+cos(d*x+c)))^(1/2)+16*B*(-2/(1+cos(d*x+c)))^(1/2))/(-2/(1+cos(d*
x+c)))^(1/2)/sin(d*x+c)^5/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 14037 vs. $2(209) = 418$.
time = 4.93, size = 14037, normalized size = 57.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*((44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16*(19
*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 19*sin(3/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 11*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(
2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*
x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4
*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*
sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos
(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x +
4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x
```

```
+ 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*si
n(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x +
 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 2) - 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2
+ 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sq
rt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(
4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*
sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(
2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2
*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin
(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(
2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqr
t(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 16*(sqrt(2
)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sq
rt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 1
6*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(
2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) +
6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*
x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +...
```

Fricas [A]
time = 2.45, size = 803, normalized size = 3.26

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((43*A - 115*B)*cos(d*x + c)^3 + 3*(43*A - 115*B)*cos(d*x +
 c)^2 + 3*(43*A - 115*B)*cos(d*x + c) + 43*A - 115*B)*sqrt(a)*log(-(a*cos(d
*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*co
s(d*x + c) + 1)) + 16*((2*A - 5*B)*cos(d*x + c)^3 + 3*(2*A - 5*B)*cos(d*x +
 c)^2 + 3*(2*A - 5*B)*cos(d*x + c) + 2*A - 5*B)*sqrt(a)*log((a*cos(d*x + c)
^3 - 7*a*cos(d*x + c)^2 + 4*(cos(d*x + c)^2 - 2*cos(d*x + c))*sqrt(a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)) + 8*a)/(
cos(d*x + c)^3 + cos(d*x + c)^2)) + 4*((11*A - 35*B)*cos(d*x + c)^2 + 5*(3*
A - 11*B)*cos(d*x + c) - 16*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(
d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/32*(sqrt(2)*((43*A - 115*B)*cos(d*x + c
)^3 + 3*(43*A - 115*B)*cos(d*x + c)^2 + 3*(43*A - 115*B)*cos(d*x + c) + 43*
A - 115*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 16*((2*A - 5*B)*cos(d*x + c
)^3 + 3*(2*A - 5*B)*cos(d*x + c)^2 + 3*(2*A - 5*B)*cos(d*x + c) + 2*A - 5*B
)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*((
11*A - 35*B)*cos(d*x + c)^2 + 5*(3*A - 11*B)*cos(d*x + c) - 16*B)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(
d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(a*sec(d*x + c) + a)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a/cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + a/cos(c + d*x))^(5/2
), x)
```

3.263 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=194

$$\frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d}+\frac{(3A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(A-B)\sec^{\frac{5}{2}}(c+\ldots}{4d(a+a\sec(\ldots}$$

[Out] 2*B*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d+1/4*(A-B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+1/16*(3*A-11*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+1/32*(3*A-43*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)

Rubi [A]

time = 0.38, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {4104, 4108, 3893, 212, 3886, 221}

$$\frac{(3A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{(3A-11B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]])/(a^(5/2)*d)
+ ((3*A - 43*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sec[c + d*x]^
(5/2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((3*A - 11*B)*Sec[c
+ d*x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
```

```
x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(A-B)+4aB\sec(c+\right.}{(a+a\sec(c+dx))^{3/2}}}{4a^2}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3A-11B)\sec^{\frac{3}{2}}(c+dx)\sin}{16ad(a+a\sec(c+dx))}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3A-11B)\sec^{\frac{3}{2}}(c+dx)\sin}{16ad(a+a\sec(c+dx))}$$

$$= \frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(3A-11B)\sec^{\frac{3}{2}}(c+dx)\sin}{16ad(a+a\sec(c+dx))}$$

$$= \frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)}{a^{5/2}d} + \frac{(3A-43B)\tanh^{-1}\left(\frac{\sqrt{a}}{\sqrt{2}}\right.}{16\sqrt{2}\,a}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 570 vs. 2(194) = 388.
time = 5.39, size = 570, normalized size = 2.94

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5
/2),x]
```

```
[Out] (16*(3*A - 11*B)*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Cos[(c + d*x)/2]^5*Sec[c +
d*x]^3*Sin[(c + d*x)/2] + 16*(3*A - 43*B)*ArcSin[Sqrt[Sec[c + d*x]]]*Cos[(c
 + d*x)/2]^5*Sec[c + d*x]^3*Sin[(c + d*x)/2] + 6*A*Sqrt[1 - Sec[c + d*x]]*S
ec[c + d*x]^(3/2)*Sin[c + d*x] - 22*B*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(
3/2)*Sin[c + d*x] + 14*A*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c +
d*x] - 30*B*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(5/2)*Sin[c + d*x] - 3*Sqrt
[2]*A*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Tan[c + d
*x] + 43*Sqrt[2]*B*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x
]]]*Tan[c + d*x] - 6*Sqrt[2]*A*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 -
 Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x] + 86*Sqrt[2]*B*ArcTan[(Sqrt[2]*Sq
rt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]*Tan[c + d*x] - 3*Sqr
t[2]*A*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sec[c +
```

```
d*x]^2*Tan[c + d*x] + 43*Sqrt[2]*B*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt
[1 - Sec[c + d*x]]]*Sec[c + d*x]^2*Tan[c + d*x])/(32*d*Sqrt[1 - Sec[c + d*x
]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 549 vs. $2(163) = 326$.
time = 8.34, size = 550, normalized size = 2.84

method	result
default	$\frac{(-1+\cos(dx+c))^2 \left(16B \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(-1-\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sin(dx+c)\sqrt{2}\,\cos(dx+c)+16B\arctan\left(\sqrt{\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/16/d*(-1+cos(d*x+c))^2*(16*B*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos
(d*x+c)+sin(d*x+c))*2^(1/2))*sin(d*x+c)*2^(1/2)*cos(d*x+c)+16*B*arctan(1/4*
(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*sin(d*x+c)*2^(
1/2)*cos(d*x+c)+3*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*
x+c)*cos(d*x+c)-3*A*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)-43*B*arctan(1/2*
sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+c)+16*B*arctan(1/4
*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/2)*sin(
d*x+c)+16*B*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*
2^(1/2))*2^(1/2)*sin(d*x+c)+11*B*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+3*A
*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)-4*A*cos(d*x+c)
*(-2/(1+cos(d*x+c)))^(1/2)-43*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(
1/2))*sin(d*x+c)+4*B*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+7*A*(-2/(1+cos(d*
x+c)))^(1/2)-15*B*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^
(1/2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)^5/(-2/(1+cos(d*x+c)))^(1
/2)/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 89320 vs. $2(163) = 326$.
time = 23.62, size = 89320, normalized size = 460.41

Too large to display

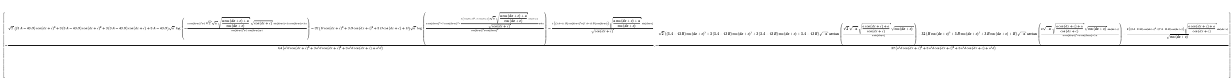
Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*((512*((2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(
5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c)
 + (2*cos(2*d*x + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(4*d*x + 4
*c)*sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(5*d
*x + 5*c)^2 + 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2
*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*
d*x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c
) + 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x
 + 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*s
in(5/2*d*x + 5/2*c))*cos(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/
2*c)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c
) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*
x + 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c)
+ 5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x +
 5/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin
(5/2*d*x + 5/2*c))*cos(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*
c)))^2 + 10240*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c)
+ cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x
+ 4*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) +
5*cos(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5
/2*c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5
/2*d*x + 5/2*c))*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
))^2 + 2560*(5*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + c
os(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4
*c) + 10*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*c
os(d*x + c) + 1)*sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*
c) - 5*cos(4*d*x + 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*
d*x + 5/2*c))*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^
2 - 512*((2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2
*d*x + 5/2*c)*sin(4*d*x + 4*c) + 2*cos(5/2*d*x + 5/2*c)*sin(3*d*x + 3*c) +
(2*cos(2*d*x + 2*c) + cos(d*x + c))*sin(5/2*d*x + 5/2*c) + cos(4*d*x + 4*c)
*sin(5/2*d*x + 5/2*c) + 2*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(5*d*x
+ 5*c)^2 + 2560*cos(4*d*x + 4*c)^2*sin(5/2*d*x + 5/2*c) + 1024*(20*cos(2*d*
x + 2*c) + 10*cos(d*x + c) + 1)*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c) + 102
40*cos(3*d*x + 3*c)^2*sin(5/2*d*x + 5/2*c) + 2560*sin(4*d*x + 4*c)^2*sin(5/
2*d*x + 5/2*c) + 10240*sin(3*d*x + 3*c)^2*sin(5/2*d*x + 5/2*c) + 2560*(5*(2
*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*
c)*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*
d*x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*
sin(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x
+ 4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin
(8/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*s
in(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)
*sin(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*
x + 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*si
```

```
n(5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x +
4*c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(6
/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 10240*(5*(2*sin
(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*s
in(5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x
+ 5/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(
5/2*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*
c)*sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(4/5
*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2560*(5*(2*sin(2*
d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(
5*d*x + 5*c) + 5*cos(5/2*d*x + 5/2*c)*sin(4*d*x + 4*c) + 10*cos(5/2*d*x + 5
/2*c)*sin(3*d*x + 3*c) - (10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/2
*d*x + 5/2*c) - cos(5*d*x + 5*c)*sin(5/2*d*x + 5/2*c) - 5*cos(4*d*x + 4*c)*
sin(5/2*d*x + 5/2*c) - 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*sin(2/5*ar
ctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 512*(5*cos(4*d*x + 4
*c)^2*sin(5/2*d*x + 5/2*c) + 4*(10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*c
os(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c) + 20*cos(3*d*x + 3*c)^2*sin(5/2*d*x +
5/2*c) + 5*sin(4*d*x + 4*c)^2*sin(5/2*d*x + 5/2*c) + 20*sin(3*d*x + 3*c)^2*
sin(5/2*d*x + 5/2*c) + 2*((10*cos(2*d*x + 2*c) + 5*cos(d*x + c) + 1)*sin(5/
2*d*x + 5/2*c) + 10*cos(3*d*x + 3*c)*sin(5/2*d*x + 5/2*c))*cos(4*d*x + 4*c)
 + 2*(2*sin(2*d*x + 2*c) + sin(d*x + c))*cos(5/2*d*x + 5/2*c) + 2*(5*(2*sin
(2*d*x + 2*c) + sin(d*x + c))*sin(5/2*d*x + 5/2...
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 339 vs. $2(163) = 326$.
time = 1.81, size = 749, normalized size = 3.86



Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((3*A - 43*B)*cos(d*x + c)^3 + 3*(3*A - 43*B)*cos(d*x + c)^
2 + 3*(3*A - 43*B)*cos(d*x + c) + 3*A - 43*B)*sqrt(a)*log(-(a*cos(d*x + c)^
2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x +
c) + 1)) - 32*(B*cos(d*x + c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c) + B
)*sqrt(a)*log((a*cos(d*x + c)^3 - 7*a*cos(d*x + c)^2 - 4*(cos(d*x + c)^2 -
2*cos(d*x + c))*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c
)/sqrt(cos(d*x + c)) + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 4*((3*A -
11*B)*cos(d*x + c)^2 + (7*A - 15*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a
^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((3*A -
 43*B)*cos(d*x + c)^3 + 3*(3*A - 43*B)*cos(d*x + c)^2 + 3*(3*A - 43*B)*cos(
```

```
d*x + c) + 3*A - 43*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 32*(B*cos(d*x +
 c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c) + B)*sqrt(-a)*arctan(2*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/
(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)) - 2*((3*A - 11*B)*cos(d*x + c)^2
 + (7*A - 15*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d
*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
+ 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(5/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + a/cos(c + d*x))^(5/2
), x)
```

3.264 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=156

$$\frac{(5A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}+\frac{(5A+3B)\sec^{\frac{3}{2}}(c+d\ldots}{16ad(a+a\sec(c\ldots}$$

```
[Out] -1/4*(A-B)*sec(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)+1/16*(5*A+3
*B)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+1/32*(5*A+3*B)*a
rctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/
2))/a^(5/2)/d*2^(1/2)
```

Rubi [A]

time = 0.18, antiderivative size = 156, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4097, 3895, 3893, 212}

$$\frac{(5A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(A-B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}+\frac{(5A+3B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((5*A + 3*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqr
t[a + a*Sec[c + d*x]])])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sec[c + d*x]^(5/
2)*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((5*A + 3*B)*Sec[c + d*
x]^(3/2)*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3895

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
```

```
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[d*((m + 1)/(b*(2*m + 1))),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 4097

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n},
 x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m
, -1]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\int \frac{\sec^{\frac{3}{2}}(c+dx)}{(a+a\sec(c+dx))^3}}{8a}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\sec^{\frac{3}{2}}(c+dx)\sin}{16ad(a+a\sec(c+dx}$$

$$= -\frac{(A-B)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\sec^{\frac{3}{2}}(c+dx)\sin}{16ad(a+a\sec(c+dx}$$

$$= \frac{(5A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}a^{5/2}d} - \frac{(A-B)\sec^{\frac{5}{2}}}{4d(a+}$$

Mathematica [A]
time = 0.69, size = 106, normalized size = 0.68

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\left((5A+3B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^4\left(\frac{1}{2}(c+dx)\right)+\frac{1}{2}(A+7B+(5A+3B)\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5
/2),x]
```

```
[Out] (Cos[(c + d*x)/2]*Sec[c + d*x]^(5/2)*((5*A + 3*B)*ArcTanh[Sin[(c + d*x)/2]]
*Cos[(c + d*x)/2]^4 + ((A + 7*B + (5*A + 3*B)*Cos[c + d*x])*Sin[(c + d*x)/2
])/2))/(4*d*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(131) = 262$.
time = 7.98, size = 350, normalized size = 2.24

method	result
default	$-\frac{(-1+\cos(dx+c))^2\left(5A(\cos^2(dx+c))\sqrt{-\frac{2}{1+\cos(dx+c)}}-5A\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sin(dx+c)\cos(dx+c)+3B(\ldots$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/16/d*(-1+cos(d*x+c))^2*(5*A*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)-5*A*a
rctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+c)+3*B*c
os(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)-3*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(
d*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+c)-4*A*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1
/2)-5*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)+4*B*cos
(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)-3*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+
c)))^(1/2))*sin(d*x+c)-A*(-2/(1+cos(d*x+c)))^(1/2)-7*B*(-2/(1+cos(d*x+c)))^
(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2
)/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^5/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 87207 vs. $2(131) = 262$.
time = 23.81, size = 87207, normalized size = 559.02

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*((4*(3*sin(3/2*d*x + 3/2*c) + 5*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) - 3*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 40*(2*si
n(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(
7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(2*sin(3*d*x
+ 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2
*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(3*sin(3/2*d*x + 3/2*
```

c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*(3*sin(3/2*d*x
+ 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*
cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 5*(16*cos(3*
d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8
*(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ 1) - 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c)))^2 + 8*(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c)))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3
*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*

```
c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 1) - 48*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 80*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3*d*x + 3*c)
+ 48*cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 4*(3*cos(3/2*d*x + 3/2*c) + 5*
cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*cos(5/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 5*cos(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 20*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) - 12*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(...
```

Fricas [A]
time = 2.25, size = 498, normalized size = 3.19

$$\left[\frac{\sqrt{2}\left((5A+3B)\cos(dx+c)^3+3(5A+3B)\cos(dx+c)^2+3(5A+3B)\cos(dx+c)+5A+3B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+\frac{4\left((5A+3B)\cos(dx+c)^2+(A+7B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}, -\frac{\sqrt{2}\left((5A+3B)\cos(dx+c)^3+3(5A+3B)\cos(dx+c)^2+3(5A+3B)\cos(dx+c)+5A+3B\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-\frac{2\left((5A+3B)\cos(dx+c)^2+(A+7B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((5*A + 3*B)*cos(d*x + c)^3 + 3*(5*A + 3*B)*cos(d*x + c)^2 +
 3*(5*A + 3*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2
*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
1)) + 4*((5*A + 3*B)*cos(d*x + c)^2 + (A + 7*B)*cos(d*x + c))*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqr
t(2)*((5*A + 3*B)*cos(d*x + c)^3 + 3*(5*A + 3*B)*cos(d*x + c)^2 + 3*(5*A +
3*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*((5*A
 + 3*B)*cos(d*x + c)^2 + (A + 7*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^
3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3010 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + a/cos(c + d*x))^(5/2
), x)
```

3.265 $\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=156

$$\frac{(19A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(A-B)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}-\frac{(9A-B)\sec^{\frac{3}{2}}(c+d}{16ad(a+a\sec(c}$$

```
[Out] -1/4*(A-B)*sec(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-1/16*(9*A-B
)*sec(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)+1/32*(19*A+5*B)*ar
ctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2
))/a^(5/2)/d*2^(1/2)
```

Rubi [A]

time = 0.37, antiderivative size = 203, normalized size of antiderivative = 1.30, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4104, 4105, 4098, 3893, 212}

$$\frac{(19A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(9A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(5A+3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{16ad(a\sec(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((19*A + 5*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sqrt[Sec[c + d*
x]]*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) + ((5*A + 3*B)*Sqrt[Sec[
c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) - ((9*A - B)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{-\frac{1}{2}a(A-B)+2a(A+B)\sec(c-}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+}}{4a^2}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\sqrt{\sec(c+dx)}}{16ad(a+a\sec(c+d}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\sqrt{\sec(c+dx)}}{16ad(a+a\sec(c+d}$$

$$= \frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\sqrt{\sec(c+dx)}}{16ad(a+a\sec(c+d}$$

$$= \frac{(19A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} + \frac{(A-B)\sqrt{}}{4d(a}$$

Mathematica [A]
time = 1.03, size = 103, normalized size = 0.66

$$\frac{\sqrt{\sec(c+dx)}\left(2(19A+5B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)+(-13A+5B+(-9A+B)\sec(c+dx))\tan\left(\frac{1}{2}(c+dx)\right)\right)}{16ad(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5
/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(2*(19*A + 5*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)
/2]^3*Sec[c + d*x] + (-13*A + 5*B + (-9*A + B)*Sec[c + d*x])*Tan[(c + d*x)/
2]))/(16*a*d*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. 2(131) = 262.
time = 7.90, size = 347, normalized size = 2.22

method	result
default	$\frac{\sqrt{\frac{1}{\cos(dx+c)}}\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\cos(dx+c)(-1+\cos(dx+c))^2\left(19A\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sin(dx+c)\cos(dx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/16/d*(1/cos(d*x+c))^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)*
(-1+cos(d*x+c))^2*(19*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*si
n(d*x+c)*cos(d*x+c)+13*A*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+5*B*arctan(
1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+c)-5*B*cos(d*x
+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+19*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c
)))^(1/2))*sin(d*x+c)-4*A*cos(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+5*B*arctan(1
/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)+4*B*cos(d*x+c)*(-2/(1+c
os(d*x+c)))^(1/2)-9*A*(-2/(1+cos(d*x+c)))^(1/2)+B*(-2/(1+cos(d*x+c)))^(1/2)
)/sin(d*x+c)^5/(-2/(1+cos(d*x+c)))^(1/2)/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 5924 vs. $2(131) = 262$.
time = 1.43, size = 5924, normalized size = 37.97

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*((19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1))*cos(4*d*x + 4*c)^2 + 304*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(
3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 304*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(d*x + c)^2 + 19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(4*d*x + 4*c)^2 + 304*(log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
1/2*c) + 1))*sin(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2
+ 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
```

```
+ 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin
(1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 2*(76*(log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x
+ 3*c) + 114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(
d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 26*sin(7/2*d*x + 7/2*c) - 10*sin(5/2*d*x
 + 5/2*c) + 10*sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(4*d*x +
4*c) + 104*(2*sin(3*d*x + 3*c) + 3*sin(2*d*x + 2*c) + 2*sin(d*x + c))*cos(7
/2*d*x + 7/2*c) + 8*(114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c
) + 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*
d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 10*sin(5/2*d*x + 5/2*c) + 10
*sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(3*d*x + 3*c) + 40*(3*s
in(2*d*x + 2*c) + 2*sin(d*x + c))*cos(5/2*d*x + 5/2*c) + 12*(76*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 10*sin(3/2*d*x + 3/2*c) + 26*
sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 8*(19*log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x
+ 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 26*sin(
1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/
2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c) +
 57*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1
/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + s
in(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c)
+ 13*cos(7/2*d*x + 7/2*c) + 5*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c)
 - 13*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) - 52*(4*cos(3*d*x + 3*c) + 6*c
os(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sin(7/2*d*x + 7/2*c) + 16*(57*(log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
 - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/
```

```
2*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin
(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + ...
```

Fricas [A]
time = 3.78, size = 502, normalized size = 3.22

$$\left[\frac{\sqrt{2}\left((19\,A+5\,B)\cos\left(dx+c\right)^{3}+3\,(19\,A+5\,B)\cos\left(dx+c\right)^{2}+3\,(19\,A+5\,B)\cos\left(dx+c\right)+19\,A+5\,B\right)\sqrt{a}\,\log\left(-\frac{a\cos(dx+c)^{2}-2\,\sqrt{2}\,\sqrt{a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-2\,a\cos(dx+c)-3\,a}{\cos(dx+c)^{2}+2\,\cos(dx+c)+1}\right)-\frac{4\left((13\,A-5\,B)\cos(dx+c)^{2}+(9\,A-B)\cos(dx+c)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{64\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)},\ -\frac{\sqrt{2}\left((19\,A+5\,B)\cos\left(dx+c\right)^{3}+3\,(19\,A+5\,B)\cos\left(dx+c\right)^{2}+3\,(19\,A+5\,B)\cos\left(dx+c\right)+19\,A+5\,B\right)\sqrt{-a}\,\arctan\left(\frac{\sqrt{2}\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}}{a\sin(dx+c)}\right)+\frac{2\left((13\,A-5\,B)\cos(dx+c)^{2}+(9\,A-B)\cos(dx+c)\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{32\left(a^{3}d\cos\left(dx+c\right)^{3}+3\,a^{3}d\cos\left(dx+c\right)^{2}+3\,a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((19*A + 5*B)*cos(d*x + c)^3 + 3*(19*A + 5*B)*cos(d*x + c)^2
 + 3*(19*A + 5*B)*cos(d*x + c) + 19*A + 5*B)*sqrt(a)*log(-(a*cos(d*x + c)^2
 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c
) + 1)) - 4*((13*A - 5*B)*cos(d*x + c)^2 + (9*A - B)*cos(d*x + c))*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos
(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32
*(sqrt(2)*((19*A + 5*B)*cos(d*x + c)^3 + 3*(19*A + 5*B)*cos(d*x + c)^2 + 3*
(19*A + 5*B)*cos(d*x + c) + 19*A + 5*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c)))
+ 2*((13*A - 5*B)*cos(d*x + c)^2 + (9*A - B)*cos(d*x + c))*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c
)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\sqrt{\sec(c+dx)}}{(a(\sec(c+dx)+1))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(sec(c + d*x))/(a*(sec(c + d*x) + 1))**(5
/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(a*sec(d*x + c) + a)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + a/cos(c + d*x))^(5/2
), x)
```

3.266 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=203

$$-\frac{(75A-19B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(A-B)\sqrt{\sec(c+dx)}\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}}-\frac{(13A-5B)\sqrt{}}{16ad(a-}$$

```
[Out] -1/32*(75*A-19*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(
a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*(A-B)*sin(d*x+c)*sec(d*x+c)^(1
/2)/d/(a+a*sec(d*x+c))^(5/2)-1/16*(13*A-5*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/
d/(a+a*sec(d*x+c))^(3/2)+1/16*(49*A-9*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/d/
(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {4105, 4098, 3893, 212}

$$-\frac{(75A-19B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(49A-9B)\sin(c+dx)\sqrt{\sec(c+dx)}}{16a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(13A-5B)\sin(c+dx)\sqrt{\sec(c+dx)}}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)\sqrt{\sec(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] -1/16*((75*A - 19*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[2]*a^(5/2)*d) - ((A - B)*Sqrt[Sec[c
+ d*x]]*Sin[c + d*x])/(4*d*(a + a*Sec[c + d*x])^(5/2)) - ((13*A - 5*B)*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(16*a*d*(a + a*Sec[c + d*x])^(3/2)) + ((49*A -
 9*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(16*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(9A-B)-2a(A-B)\mathrm{se}}{\sqrt{\sec(c+dx)}\,(a+a\,\mathrm{sec}}}{4a^2}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B)\sqrt{\sec(c+}}{16ad(a+a\sec(c}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B)\sqrt{\sec(c+}}{16ad(a+a\sec(c}$$

$$= -\frac{(A-B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B)\sqrt{\sec(c+}}{16ad(a+a\sec(c}$$

$$= -\frac{(75A-19B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{(A-}{4}$$

Mathematica [A]
time = 2.32, size = 206, normalized size = 1.01

$$\frac{4\sqrt{2}\,(75A-19B)\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^5\left(\frac{1}{2}(c+dx)\right)\sec^3(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)+\left((85A-13B)\sqrt{1-\sec(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)+(49A-9B)\sqrt{1-\sec(c+dx)}\,\sec^{\frac{5}{2}}(c+dx)+32A\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\right)\sin(c+dx)}{16d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] (4*Sqrt[2]*(75*A - 19*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c
 + d*x]]]*Cos[(c + d*x)/2]^5*Sec[c + d*x]^3*Sin[(c + d*x)/2] + ((85*A - 13*
B)*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + (49*A - 9*B)*Sqrt[1 - Sec[c
+ d*x]]*Sec[c + d*x]^(5/2) + 32*A*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]
)*Sin[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. $2(172) = 344$.
time = 7.66, size = 419, normalized size = 2.06

method	result
default	$\frac{(-1+\cos(dx+c))^2\left(75A\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)(\cos^2(dx+c))\sqrt{-\frac{2}{1+\cos(dx+c)}}\ \sin(dx+c)-19B\arctan\left(\frac{\sin(d}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/32/d*(-1+cos(d*x+c))^2*(75*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1
/2))*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-19*B*arctan(1/2*sin(
d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)*si
n(d*x+c)+150*A*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d
*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+c)-38*B*(-2/(1+cos(d*x+c)))^(1/2)*arctan(
1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+c)+75*arctan(1
/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*A*sin(d*
x+c)-64*A*cos(d*x+c)^3-19*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*
(-2/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c)-106*A*cos(d*x+c)^2+26*B*cos(d*x+c)^2
+72*A*cos(d*x+c)-8*B*cos(d*x+c)+98*A-18*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1
/2)/sin(d*x+c)^5/(1/cos(d*x+c))^(1/2)/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 261506 vs. $2(172) = 344$.
time = 3.65, size = 261506, normalized size = 1288.21

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")

[Out] -1/32*((576*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x +
 5/2*c)^6 + 14400*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(3/2
*d*x + 3/2*c)^6 + 187500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^6 + 576*(75
*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/
2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)^6 + 518
4*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^6
+ 262500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4*sin(1/2*d*x + 1/2*c)^2 +
77700*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin
(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^4 + 270
0*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2
*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^6 - 2304*sin(1/2*d*x + 1/2*c)^7 +
96*(86*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*
c) + 10275*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8768*cos(1/2*d*x + 1/2*
c)*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c)^5 + 88800*(75*(log(cos(1/2*d*
x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*cos(1/2*d*x + 1/2*c) - 64*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos
(3/2*d*x + 3/2*c)^5 + 96*(62*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + s
in(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*
c))*sin(3/2*d*x + 3/2*c) + 2625*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 22
40*sin(1/2*d*x + 1/2*c)^2 - 1996)*sin(5/2*d*x + 5/2*c)^5 + 864*(1275*(log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1
```

```
) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 1088*sin(1/2*d*x + 1/2*c)^2 - 920)*sin(3
/2*d*x + 3/2*c)^5 - 16*(4144*cos(1/2*d*x + 1/2*c)^2 + 675)*sin(1/2*d*x + 1/
2*c)^5 + 16*((75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x
+ 5/2*c)^2 + 25*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(3/2*d
*x + 3/2*c)^2 + 7500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (75*log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
- 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)^2 + 9*(75*log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1
) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
+ 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 300*(log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 256*sin(1/2*d*x + 1/2*c)^3 + 10*((75*
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2
*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 150*(l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c...
```

Fricas [A]
time = 2.71, size = 524, normalized size = 2.58

$$\left[-\frac{\sqrt{2}\left((75A-19B)\cos(dx+c)^3+3(75A-19B)\cos(dx+c)^2+3(75A-19B)\cos(dx+c)+75A-19B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-\frac{4\left(32A\cos(dx+c)^3+(85A-13B)\cos(dx+c)^2+(49A-9B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}, \frac{\sqrt{2}\left((75A-19B)\cos(dx+c)^3+3(75A-19B)\cos(dx+c)^2+3(75A-19B)\cos(dx+c)+75A-19B\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)+\frac{2\left(32A\cos(dx+c)^3+(85A-13B)\cos(dx+c)^2+(49A-9B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((75*A - 19*B)*cos(d*x + c)^3 + 3*(75*A - 19*B)*cos(d*x + c
)^2 + 3*(75*A - 19*B)*cos(d*x + c) + 75*A - 19*B)*sqrt(a)*log(-(a*cos(d*x +
 c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) - 4*(32*A*cos(d*x + c)^3 + (85*A - 13*B)*cos(d*x + c)^2 + (49*
A - 9*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)
/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d), 1/32*(sqrt(2)*((75*A - 19*B)*cos(d*x + c)^3 + 3*(
75*A - 19*B)*cos(d*x + c)^2 + 3*(75*A - 19*B)*cos(d*x + c) + 75*A - 19*B)*s
qrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqr
```

```
t(cos(d*x + c))/(a*sin(d*x + c))) + 2*(32*A*cos(d*x + c)^3 + (85*A - 13*B)*
cos(d*x + c)^2 + (49*A - 9*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*c
os(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c
))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{5/2}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)
),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)
), x)
```

3.267 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=250

$$\frac{(163A-75B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(A-B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}}-\frac{(1}{16ad\sqrt{\sec(}$$

```
[Out] 1/32*(163*A-75*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(
a+a*sec(d*x+c))^(1/2))/a^(5/2)/d*2^(1/2)-1/4*(A-B)*sin(d*x+c)/d/(a+a*sec(d*
x+c))^(5/2)/sec(d*x+c)^(1/2)-1/16*(17*A-9*B)*sin(d*x+c)/a/d/(a+a*sec(d*x+c)
)^(3/2)/sec(d*x+c)^(1/2)+1/48*(95*A-39*B)*sin(d*x+c)/a^2/d/sec(d*x+c)^(1/2)
/(a+a*sec(d*x+c))^(1/2)-1/48*(299*A-147*B)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^2/
d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$,
Rules used = {4105, 4107, 4098, 3893, 212}

$$\frac{(163A-75B)\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(299A-147B)\sin(c+dx)\sqrt{\sec(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(95A-39B)\sin(c+dx)}{48a^2d\sqrt{\sec(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{(17A-9B)\sin(c+dx)}{16ad\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] ((163*A - 75*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])]])/(16*Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])
/(4*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)) - ((17*A - 9*B)*Sin[c
+ d*x])/(16*a*d*Sqrt[Sec[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)) + ((95*A - 3
9*B)*Sin[c + d*x])/(48*a^2*d*Sqrt[Sec[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) -
 ((299*A - 147*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Sec
[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
```

```
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(11A-3B)-3a(A-B}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(}}{4a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B}{16ad\sqrt{\sec(c+dx)}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B}{16ad\sqrt{\sec(c+dx)}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B}{16ad\sqrt{\sec(c+dx)}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\sec(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B}{16ad\sqrt{\sec(c+dx)}}$$

$$= \frac{(163A-75B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{}{4d\sqrt{\sec}}$$

Mathematica [A]
time = 1.79, size = 193, normalized size = 0.77

$$\frac{-12\sqrt{2}(163A-75B)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)+2(-379A+195B+(-479A+255B)\cos(c+dx)+(-80A+48B)\cos(2(c+dx))+8A\cos(3(c+dx)))\sqrt{1-\sec(c+dx)}\sec^2(c+dx)\tan(c+dx)}{96d\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}\,(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] (-12*Sqrt[2]*(163*A - 75*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Se
c[c + d*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(7/2)*Sin[c + d*x] + 2*(-379*A
 + 195*B + (-479*A + 255*B)*Cos[c + d*x] + (-80*A + 48*B)*Cos[2*(c + d*x)]
+ 8*A*Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^2*Tan[c + d*x])
/(96*d*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])]*(a*(1 + Sec[c + d*x]))^(5/
2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 448 vs. $2(213) = 426$.
time = 7.94, size = 449, normalized size = 1.80

method	result
default	$-\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2\left(489A\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)(\cos^2(dx+c))\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(dx$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/96/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(489*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)-225*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+978*A*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+c)+64*A*cos(d*x+c)^4-450*B*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+c)+489*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*A*sin(d*x+c)-384*A*cos(d*x+c)^3-225*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c)+192*B*cos(d*x+c)^3-686*A*cos(d*x+c)^2+318*B*cos(d*x+c)^2+408*A*cos(d*x+c)-216*B*cos(d*x+c)+598*A-294*B)*(1/cos(d*x+c))^(3/2)*cos(d*x+c)^2/sin(d*x+c)^5/a^3

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 407281 vs. $2(213) = 426$.
time = 6.31, size = 407281, normalized size = 1629.12

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] -1/96*(3*(576*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c)^6 + 14400*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^6 + 187500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^6 + 576*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(

```
1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)^6 + 5
184*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
 + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^
6 + 262500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4*sin(1/2*d*x + 1/2*c)^2
+ 77700*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^4 + 2
700*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1
/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^6 - 2304*sin(1/2*d*x + 1/2*c)^7
+ 96*(86*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/
2*c) + 10275*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8768*cos(1/2*d*x + 1/
2*c)*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c)^5 + 88800*(75*(log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) +
 1))*cos(1/2*d*x + 1/2*c) - 64*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*c
os(3/2*d*x + 3/2*c)^5 + 96*(62*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 +
 sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/
2*c))*sin(3/2*d*x + 3/2*c) + 2625*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + si
n(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) -
2240*sin(1/2*d*x + 1/2*c)^2 - 1996)*sin(5/2*d*x + 5/2*c)^5 + 864*(1275*(log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 1088*sin(1/2*d*x + 1/2*c)^2 - 920)*sin
(3/2*d*x + 3/2*c)^5 - 16*(4144*cos(1/2*d*x + 1/2*c)^2 + 675)*sin(1/2*d*x +
1/2*c)^5 + 16*((75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(5/2*d*
x + 5/2*c)^2 + 25*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(3/2
*d*x + 3/2*c)^2 + 7500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (75*log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1
) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
```

```
+ 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)^2 + 9*(75*log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*
x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 300*(lo
g(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)
+ 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
+ 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 256*sin(1/2*d*x + 1/2*c)^3 + 10*((7
5*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2
*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1
/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 150*
(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 1/2...
```

Fricas [A]
time = 2.99, size = 564, normalized size = 2.26

$$\left[-\frac{3\sqrt{2}\left((163A-75B)\cos(dx+c)^3+3(163A-75B)\cos(dx+c)^2+3(163A-75B)\cos(dx+c)+163A-75B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-\frac{4\left(32A\cos(dx+c)^4-32(5A-3B)\cos(dx+c)^3-(503A-255B)\cos(dx+c)^2-(299A-147B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{192\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}, -\frac{3\sqrt{2}\left((163A-75B)\cos(dx+c)^3+3(163A-75B)\cos(dx+c)^2+3(163A-75B)\cos(dx+c)+163A-75B\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-\frac{2\left(32A\cos(dx+c)^4-32(5A-3B)\cos(dx+c)^3-(503A-255B)\cos(dx+c)^2-(299A-147B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{96\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/192*(3*sqrt(2)*((163*A - 75*B)*cos(d*x + c)^3 + 3*(163*A - 75*B)*cos(d*
x + c)^2 + 3*(163*A - 75*B)*cos(d*x + c) + 163*A - 75*B)*sqrt(a)*log(-(a*co
s(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) - 4*(32*A*cos(d*x + c)^4 - 32*(5*A - 3*B)*cos(d*x + c)^
3 - (503*A - 255*B)*cos(d*x + c)^2 - (299*A - 147*B)*cos(d*x + c))*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos
(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/96
*(3*sqrt(2)*((163*A - 75*B)*cos(d*x + c)^3 + 3*(163*A - 75*B)*cos(d*x + c)^
2 + 3*(163*A - 75*B)*cos(d*x + c) + 163*A - 75*B)*sqrt(-a)*arctan(sqrt(2)*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d
*x + c))) - 2*(32*A*cos(d*x + c)^4 - 32*(5*A - 3*B)*cos(d*x + c)^3 - (503*A
 - 255*B)*cos(d*x + c)^2 - (299*A - 147*B)*cos(d*x + c))*sqrt((a*cos(d*x +
c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^
3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{5/2}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)
),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)
), x)
```

3.268 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=297

$$-\frac{(283A-163B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(A-B)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}-\frac{(21A}{16ad\sec^{\frac{3}{2}}(c}$$

```
[Out] -1/4*(A-B)*sin(d*x+c)/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2)-1/16*(21*A-
13*B)*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)-1/32*(283*A-16
3*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c
))^(1/2))/a^(5/2)/d*2^(1/2)+1/80*(157*A-85*B)*sin(d*x+c)/a^2/d/sec(d*x+c)^(
3/2)/(a+a*sec(d*x+c))^(1/2)-1/240*(787*A-475*B)*sin(d*x+c)/a^2/d/sec(d*x+c)
^(1/2)/(a+a*sec(d*x+c))^(1/2)+1/240*(2671*A-1495*B)*sin(d*x+c)*sec(d*x+c)^(
1/2)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.61, antiderivative size = 297, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$,
Rules used = {4105, 4107, 4098, 3893, 212}

$$-\frac{(283A-163B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(157A-85B)\sin(c+dx)}{80a^2d\sec^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{(2671A-1495B)\sin(c+dx)\sqrt{\sec(c+dx)}}{240a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(787A-475B)\sin(c+dx)}{240a^2d\sqrt{\sec(c+dx)}\,\sqrt{a\sec(c+dx)+a}}-\frac{(21A-13B)\sin(c+dx)}{16ad\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] -1/16*((283*A - 163*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])])/(Sqrt[2]*a^(5/2)*d) - ((A - B)*Sin[c + d
*x])/(4*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)) - ((21*A - 13*B)*S
in[c + d*x])/(16*a*d*Sec[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)) + ((157
*A - 85*B)*Sin[c + d*x])/(80*a^2*d*Sec[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*
x]]) - ((787*A - 475*B)*Sin[c + d*x])/(240*a^2*d*Sqrt[Sec[c + d*x]]*Sqrt[a
+ a*Sec[c + d*x]]) + ((2671*A - 1495*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2
40*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
```

```
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = -\frac{(A-B)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{\int \frac{\frac{1}{2}a(13A-5B)-4a(A-B)\sec}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+d}}{4a^2}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B)\,\mathrm{si}}{16ad\sec^{\frac{3}{2}}(c+dx)(a+}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B)\,\mathrm{si}}{16ad\sec^{\frac{3}{2}}(c+dx)(a+}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B)\,\mathrm{si}}{16ad\sec^{\frac{3}{2}}(c+dx)(a+}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B)\,\mathrm{si}}{16ad\sec^{\frac{3}{2}}(c+dx)(a+}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B)\,\mathrm{si}}{16ad\sec^{\frac{3}{2}}(c+dx)(a+}$$

$$= -\frac{(283A-163B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)}{16\sqrt{2}\,a^{5/2}d} - \frac{}{4d\sec}$$

Mathematica [A]

time = 2.24, size = 196, normalized size = 0.66

$$\frac{\sec^2(c+dx)\left((3491A-1895B+5(887A-479B)\cos(c+dx)+16(52A-25B)\cos(2(c+dx))-40A\cos(3(c+dx))+40B\cos(3(c+dx))+12A\cos(4(c+dx)))\sqrt{\sec(c+dx)}\sin(c+dx)+\frac{30\sqrt{2}(283A-163B)\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\tan(c+dx)}{\sqrt{1-\sec(c+dx)}}\right)}{240d(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] (Sec[c + d*x]^2*((3491*A - 1895*B + 5*(887*A - 479*B)*Cos[c + d*x] + 16*(52
*A - 25*B)*Cos[2*(c + d*x)] - 40*A*Cos[3*(c + d*x)] + 40*B*Cos[3*(c + d*x)]
 + 12*A*Cos[4*(c + d*x)])*Sqrt[Sec[c + d*x]]*Sin[c + d*x] + (30*Sqrt[2]*(28
3*A - 163*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Co
s[(c + d*x)/2]^4*Tan[c + d*x])/Sqrt[1 - Sec[c + d*x]]))/(240*d*(a*(1 + Sec[
c + d*x]))^(5/2))
```

Maple [A]
time = 7.89, size = 471, normalized size = 1.59

method	result
default	$\frac{\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2\left(4245A\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)(\cos^2(dx+c))\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\sin(dx}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/480/d*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(4245*A*arcta
n(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2*(-2/(1+cos(d*x+c))
)^(1/2)*sin(d*x+c)-192*A*cos(d*x+c)^5-2445*B*arctan(1/2*sin(d*x+c)*(-2/(1+c
os(d*x+c)))^(1/2))*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+8490*A
*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))
*sin(d*x+c)*cos(d*x+c)+512*A*cos(d*x+c)^4-4890*B*(-2/(1+cos(d*x+c)))^(1/2)*
arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+c)-320*
B*cos(d*x+c)^4+4245*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1
+cos(d*x+c)))^(1/2)*A*sin(d*x+c)-3456*A*cos(d*x+c)^3-2445*arctan(1/2*sin(d*
x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c)+1920
*B*cos(d*x+c)^3-5974*A*cos(d*x+c)^2+3430*B*cos(d*x+c)^2+3768*A*cos(d*x+c)-2
040*B*cos(d*x+c)+5342*A-2990*B)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c
)^5/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 387118 vs. $2(254) = 508$.
time = 8.84, size = 387118, normalized size = 1303.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/480*((16*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) + 3*sin(5*d*x + 5*c)^
2*sin(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(3
/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 300*(cos(5*d*x +
5*c)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*
d*x + 5/2*c))))*cos(13/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
))^4 + 4096*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) + 3*sin(5*d*x + 5*c)
^2*sin(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(
3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 300*(cos(5*d*x +
```

```
 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c))))*cos(11/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))^4 + 20736*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) + 3*sin(5*d*x + 5*
c)^2*sin(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*si
n(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 300*(cos(5*d*x
 + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c))))*cos(9/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*
c)))^4 + 4096*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) + 3*sin(5*d*x + 5*
c)^2*sin(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*si
n(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 300*(cos(5*d*x
 + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c))))*cos(7/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*
c)))^4 + 48*sin(5/2*d*x + 5/2*c)^5 + 16*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*x +
 5/2*c) + 3*sin(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*c)^
2 + sin(5*d*x + 5*c)^2)*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x +
 5/2*c))) + 300*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan2(s
in(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sin(13/5*arctan2(sin(5/2*d*x +
 5/2*c), cos(5/2*d*x + 5/2*c)))^4 + 4096*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*x
+ 5/2*c) + 3*sin(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*c)
^2 + sin(5*d*x + 5*c)^2)*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x
+ 5/2*c))) + 300*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan2(
sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sin(11/5*arctan2(sin(5/2*d*x
+ 5/2*c), cos(5/2*d*x + 5/2*c)))^4 + 20736*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*
x + 5/2*c) + 3*sin(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*
c)^2 + sin(5*d*x + 5*c)^2)*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*
x + 5/2*c))) + 300*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sin(9/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c)))^4 + 4096*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*
x + 5/2*c) + 3*sin(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*
c)^2 + sin(5*d*x + 5*c)^2)*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*
x + 5/2*c))) + 300*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sin(7/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c)))^4 + 4*(24*cos(5*d*x + 5*c)^2*cos(5/2*d*x
+ 5/2*c)*sin(5/2*d*x + 5/2*c) + 24*cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c)^2*
sin(5/2*d*x + 5/2*c) + 24*cos(5*d*x + 5*c)*cos(5/2*d*x + 5/2*c)*sin(5/2*d*x
 + 5/2*c) + 64*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) + 3*sin(5*d*x + 5
*c)^2*sin(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*s
in(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 300*(cos(5*d*
x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(
5/2*d*x + 5/2*c))))*cos(11/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/
2*c))) + 96*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) + 3*sin(5*d*x + 5*c)
^2*sin(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(
3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 300*(cos(5*d*x +
 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c))))*cos(9/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
```

```
)) + 64*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) + 3*sin(5*d*x + 5*c)^2*s
in(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(3/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 300*(cos(5*d*x + 5*c
)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x
 + 5/2*c))))*cos(7/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) +
 (24*sin(5/2*d*x + 5/2*c)^2 - 4265)*sin(5*d*x + 5*c) - 645*(cos(5*d*x + 5*c
)^2 + sin(5*d*x + 5*c)^2)*sin(12/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*
x + 5/2*c))) - 7247*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(8/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 4803*(cos(5*d*x + 5*c)^2
+ sin(5*d*x + 5*c)^2)*sin(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5
/2*c))) - 872*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(4/5*arctan2(sin
(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 200*(cos(5*d*x + 5*c)^2*cos(5/2
*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c)^2 + cos(5*d*x + 5*c)*
cos(5/2*d*x + 5/2*c) + sin(5*d*x + 5*c)*sin(5/2...
```

Fricas [A]
time = 2.36, size = 592, normalized size = 1.99

$$\left[-\frac{15\sqrt{2}\left((283A-163B)\cos(dx+c)^3+3(283A-163B)\cos(dx+c)^2+3(283A-163B)\cos(dx+c)+283A-163B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-\frac{4\left(96A\cos(dx+c)^5-160(A-B)\cos(dx+c)^4+32(49A-25B)\cos(dx+c)^3+5(911A-503B)\cos(dx+c)^2+(2671A-1495B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{960\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)},\ \frac{15\sqrt{2}\left((283A-163B)\cos(dx+c)^3+3(283A-163B)\cos(dx+c)^2+3(283A-163B)\cos(dx+c)+283A-163B\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)+\frac{2\left(96A\cos(dx+c)^5-160(A-B)\cos(dx+c)^4+32(49A-25B)\cos(dx+c)^3+5(911A-503B)\cos(dx+c)^2+(2671A-1495B)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{480\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/960*(15*sqrt(2)*((283*A - 163*B)*cos(d*x + c)^3 + 3*(283*A - 163*B)*cos
(d*x + c)^2 + 3*(283*A - 163*B)*cos(d*x + c) + 283*A - 163*B)*sqrt(a)*log(-
(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^
2 + 2*cos(d*x + c) + 1)) - 4*(96*A*cos(d*x + c)^5 - 160*(A - B)*cos(d*x + c
)^4 + 32*(49*A - 25*B)*cos(d*x + c)^3 + 5*(911*A - 503*B)*cos(d*x + c)^2 +
(2671*A - 1495*B)*cos(d*x + c))*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sin
(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^
2 + 3*a^3*d*cos(d*x + c) + a^3*d), 1/480*(15*sqrt(2)*((283*A - 163*B)*cos(d
*x + c)^3 + 3*(283*A - 163*B)*cos(d*x + c)^2 + 3*(283*A - 163*B)*cos(d*x +
c) + 283*A - 163*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(96*A*cos(d*x +
c)^5 - 160*(A - B)*cos(d*x + c)^4 + 32*(49*A - 25*B)*cos(d*x + c)^3 + 5*(91
1*A - 503*B)*cos(d*x + c)^2 + (2671*A - 1495*B)*cos(d*x + c))*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d*cos(d*x
+ c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6192 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2)
),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + a/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2)
), x)
```

3.269 $\int (a+a\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=406

$$\frac{3\sqrt{2}\,AF_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(a+a\sec(c+dx))^{2/3}\tan(c+dx)}{7d\sqrt{1-\sec(c+dx)}}+\frac{3B(a+a\sec(}{2d(1}$$

[Out] 3/2*B*(a+a*sec(d*x+c))^(2/3)*tan(d*x+c)/d/(1+sec(d*x+c))+3/7*A*AppellF1(7/6,1,1/2,13/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*(a+a*sec(d*x+c))^(2/3)*2^(1/2)*tan(d*x+c)/d/(1-sec(d*x+c))^(1/2)-1/4*3^(3/4)*B*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(2/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(1+sec(d*x+c))/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)

Rubi [A]
time = 0.43, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4009, 3864, 3863, 141, 3913, 3912, 52, 65, 231}

$$\frac{3\sqrt{2}\,A\tan(c+dx)(a\sec(c+dx)+a)^{2/3}F_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{7d\sqrt{1-\sec(c+dx)}}-\frac{3^{3/4}B\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}(a\sec(c+dx)+a)^{2/3}F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\sqrt[3]{2}\,d(1-\sec(c+dx))(\sec(c+dx)+1)\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}+\frac{3B\tan(c+dx)(a\sec(c+dx)+a)^{2/3}}{2d(\sec(c+dx)+1)}$$

Warning: Unable to verify antiderivative.

[In] Int[(a + a*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]

[Out] (3*Sqrt[2]*A*AppellF1[7/6, 1/2, 1, 13/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(7*d*Sqrt[1 - Sec[c + d*x]]) + (3*B*(a + a*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(2*d*(1 + Sec[c + d*x])) - (3^(3/4)*B*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*2^(1/3)*d*(1

```
- Sec[c + d*x])*(1 + Sec[c + d*x])*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3
) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^
(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] && !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) && !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] && !IntegerQ[m] && !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] && !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 3863

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]])), Subst[Int[(1
+ b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] && !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 3864

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 4009

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[2*m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx = A\int (a+a\sec(c+dx))^{2/3}\,dx + B\int \sec(c+dx)(a+a\sec$$

$$= \frac{\left(A(a+a\sec(c+dx))^{2/3}\right)\int (1+\sec(c+dx))^{2/3}\,dx}{(1+\sec(c+dx))^{2/3}} + \frac{(B(}{}$$

$$= -\frac{\left(A(a+a\sec(c+dx))^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int \frac{\sqrt[6]{1+x}}{\sqrt{1-x}}\right.}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))^{7}}$$

$$= \frac{3\sqrt{2}\,AF_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(}{7d\sqrt{1-\sec(c+dx)}}$$

$$= \frac{3\sqrt{2}\,AF_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(}{7d\sqrt{1-\sec(c+dx)}}$$

$$= \frac{3\sqrt{2}\,AF_1\left(\frac{7}{6};\frac{1}{2},1;\frac{13}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(}{7d\sqrt{1-\sec(c+dx)}}$$

<u>**Mathematica**</u> [B] Leaf count is larger than twice the leaf count of optimal. 4445 vs. $2(406) = 812$.
time = 20.29, size = 4445, normalized size = 10.95

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*B*Cos[c + d*x]*((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d
*x]))^(2/3)*(A + B*Sec[c + d*x])*Tan[(c + d*x)/2])/(2*d*(B + A*Cos[c + d*x]
)*(1 + Sec[c + d*x])^(2/3)) + (Cos[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x
])^(2/3)*(a*(1 + Sec[c + d*x]))^(2/3)*(A + B*Sec[c + d*x])*((A*Cos[c + d*x]
*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])^(2/3))/2 + Sec[(c + d*x)/2]^2*((A*(1
 + Sec[c + d*x])^(2/3))/2 + (B*(1 + Sec[c + d*x])^(2/3))/4))*Tan[(c + d*x)/
2]*(2*B*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] +
2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(Cos
```

```
[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^4 + 9*AppellF1[1/2, 2/
3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(3*(4*A + B)*Cos[(c + d
*x)/2]^2 + B*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2)))/(3*2^(1
/3)*d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x])^(2/3)*(9*AppellF1[1/2, 2/3, 1
, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3,
2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1,
5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)*((Sec[(c
 + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(2*B*AppellF1[3/2, 2/3
, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-3*AppellF1[3/2, 2/3, 2
, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(Cos[c + d*x]*Sec[(c + d*x)/2
]^2)^(2/3)*Tan[(c + d*x)/2]^4 + 9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2]*(3*(4*A + B)*Cos[(c + d*x)/2]^2 + B*AppellF1[3/2
, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(
c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2)))/(6*2^(1/3)*(9*AppellF1[1/2, 2/3,
 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3
, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1
, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)) - ((C
os[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*Tan[(c + d*x)/2]*(2*B*AppellF1[3/2, 2
/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-3*AppellF1[3/2, 2/3,
 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1,
 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(Cos[c + d*x]*Sec[(c + d*x)
/2]^2)^(2/3)*Tan[(c + d*x)/2]^4 + 9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2]*(3*(4*A + B)*Cos[(c + d*x)/2]^2 + B*AppellF1[3
/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec
[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2))*(2*(-3*AppellF1[3/2, 2/3, 2, 5/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2,
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2] + 9*(-1/3*(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (2*AppellF1[3/2, 5/3, 1, 5/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2])/9) + 2*Tan[(c + d*x)/2]^2*(-3*((-6*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2
*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c
 + d*x)/2]^2*Tan[(c + d*x)/2])/5) + 2*((-3*AppellF1[5/2, 5/3, 2, 7/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5
+ AppellF1[5/2, 8/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2]))))/(3*2^(1/3)*(9*AppellF1[1/2, 2/3, 1, 3/2,
 Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2) + ((Cos[(c
+ d*x)/2]^2*Sec[c + d*x])^(2/3)*Tan[(c + d*x)/2]*(4*B*AppellF1[3/2, 2/3, 1,
 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-3*AppellF1[3/2, 2/3, 2, 5/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2,
```

```
Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*(Cos[c + d*x]*
Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^3 + 2*B*(-3*AppellF1[3/2, 2/3, 2
, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*(Cos[c + d*x]*Sec[(c + d*x)/2
]^2)^(2/3)*Tan[(c + d*x)/2]^4*((-3*AppellF1[5/2, 2/3, 2, 7/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*App
ellF1[5/2, 5/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2])/5) + (4*B*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^4*(-(Sec[(c + d*x)/2]^2*Sin[c + d
*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*
Sec[(c + d*x)/2]^2)^(1/3)) + 9*(-1/3*(AppellF1[...
```

Maple [**F**]
time = 0.29, size = 0, normalized size = 0.00

$$\int \left(a+a\sec \left(dx+c\right)\right)^{\frac{2}{3}} \left(A+B\sec \left(dx+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
[Out] int((a+a*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(2/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(a\left(\sec \left(c+dx\right)+1\right)\right)^{\frac{2}{3}} \left(A+B\sec \left(c+dx\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(2/3)*(A + B*sec(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(2/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(2/3),x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(2/3), x)
```

3.270 $\int \frac{A+B\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=354

$$\frac{3\sqrt{2}\,AF_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[3]{a+a\sec(c+dx)}}-\frac{3^{3/4}BF\left(\operatorname{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{1}}\right.\right.}{\sqrt[3]{}}$$

```
[Out] 3*A*AppellF1(1/6,1,1/2,7/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*2^(1/2)*tan(d*x
+c)/d/(a+a*sec(d*x+c))^(1/3)/(1-sec(d*x+c))^(1/2)-1/2*3^(3/4)*B*((2^(1/3)-(
1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)
))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+
c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2
)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2
^(1/2))*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/
3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2
)*tan(d*x+c)*2^(2/3)/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(1/3)/(-(1+sec(d*x+c
))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^
(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 354, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.320$, Rules used = {4009, 3864, 3863, 141, 3913, 3912, 65, 231}

$$\frac{3\sqrt{2}\,A\tan(c+dx)F_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[3]{a\sec(c+dx)+a}}-\frac{3^{3/4}B\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,F\left(\operatorname{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{\sqrt[3]{2}\,d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,\sqrt[3]{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] (3*Sqrt[2]*A*AppellF1[1/6, 1/2, 1, 7/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d
*x]]*Tan[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(1/3)) -
(3^(3/4)*B*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/
3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(
2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*
x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
 d*x])^(1/3))^2]*Tan[c + d*x])/(2^(1/3)*d*(1 - Sec[c + d*x])*(a + a*Sec[c +
```

```
d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])
^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 3863

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]])), Subst[Int[(1
+ b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 3864

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
```

```
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4009

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx = A\int \frac{1}{\sqrt[3]{a+a\sec(c+dx)}}\,dx + B\int \frac{\sec(c+dx)}{\sqrt[3]{a+a\sec(c+dx)}}\,dx$$

$$= \frac{\left(A\sqrt[3]{1+\sec(c+dx)}\right)\int \frac{1}{\sqrt[3]{1+\sec(c+dx)}}\,dx}{\sqrt[3]{a+a\sec(c+dx)}} + \frac{\left(B\sqrt[3]{1+\sec(c+dx)}\right)\int \frac{}{\sqrt[3]{}}}{\sqrt[3]{a+a\sec(c}}$$

$$= -\frac{(A\tan(c+dx))\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x(1+x)^{5/6}}\,dx, x, \sec(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[6]{1+\sec(c+dx)}\,\sqrt[3]{a+a\sec(c+dx)}} - \frac{(B\tan(c+}{d\sqrt{1-\sec(}}$$

$$= \frac{3\sqrt{2}\,AF_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[3]{a+a\sec(c+dx)}} - \frac{(6B\tan}{d\sqrt{1}}$$

$$= \frac{3\sqrt{2}\,AF_1\left(\frac{1}{6};\frac{1}{2},1;\frac{7}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt[3]{a+a\sec(c+dx)}} - \frac{3^{3/4}BF}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2709 vs. $2(354) = 708$.
time = 19.27, size = 2709, normalized size = 7.65

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(1/3),x]
```

```
[Out] (2^(2/3)*Cos[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(1 + Sec[c +
d*x])^(1/3)*(A + B*Sec[c + d*x])*((B*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])^
(2/3))/2 + (A*Cos[c + d*x]*Sec[(c + d*x)/2]^2*(1 + Sec[c + d*x])^(2/3))/2)*
Tan[(c + d*x)/2]*((-A + B)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2
]^2 + (27*(A + B)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*Cos[(c + d*x)/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(3*d*(B + A*Cos[c + d*x])*(
a*(1 + Sec[c + d*x]))^(1/3)*((Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c
+ d*x])^(2/3)*((-A + B)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2
 + (27*(A + B)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2]*Cos[(c + d*x)/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)))/(3*2^(1/3)) + (2^(2/3)*(Cos[(c
 + d*x)/2]^2*Sec[c + d*x])^(2/3)*Tan[(c + d*x)/2]*((-A + B)*AppellF1[3/2, 2
/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*(Co
s[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2] + (-A + B)*(Cos[c + d
*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2, 2/3, 2,
 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c +
d*x)/2])/5 + (2*AppellF1[5/2, 5/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5) + (2*(-A + B)*AppellF1[3/2
, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2*
(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(
c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(1/3)) - (27*(A + B)*App
ellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d
*x)/2]*Sin[(c + d*x)/2])/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) + (27*(A + B)*Cos[(c + d*x)/2]^2*(-
1/3*(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Se
c[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c +
 d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9))/(
```

```
9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(
-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*
AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c
 + d*x)/2]^2) - (27*(A + B)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2,
-Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2*(2*(-3*AppellF1[3/2, 2/3, 2, 5/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] +
 9*(-1/3*(AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^
2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/
9) + 2*Tan[(c + d*x)/2]^2*(-3*((-6*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*App
ellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2])/5) + 2*((-3*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + Ap
pellF1[5/2, 8/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2]))))/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)
/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2))/3 + (2*2^(2/3)*Tan[(c + d
*x)/2]*((-A + B)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2]*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (27*
(A + B)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*Cos[(c + d*x)/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan
[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(
c + d*x)/2]) + Cos[(c + d*x)/2]^2*Sec[c + d*x]*Tan[c + d*x]))/(9*(Cos[(c +
d*x)/2]^2*Sec[c + d*x])^(1/3))))
```

Maple [F]
time = 0.28, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec \left(dx+c\right)}{\left(a+a\sec \left(dx+c\right)\right)^{\frac{1}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/3),x)
```

```
[Out] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec{(c + dx)}}{\sqrt[3]{a\left(\sec{(c + dx)} + 1\right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/3),x)
[Out] Integral((A + B*sec(c + d*x))/(a*(sec(c + d*x) + 1))**(1/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/3),x, algorithm="giac")
[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(1/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+d\,x)}}{\left(a + \frac{a}{\cos(c+d\,x)}\right)^{1/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(1/3),x)
[Out] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(1/3), x)
```

3.271 $\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^{4/3}}\,dx$

Optimal. Leaf size=415

$$\frac{3B\tan(c+dx)}{5ad(1+\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}}-\frac{3\sqrt{2}\,AF_1\left(-\frac{5}{6};\frac{1}{2},1;\frac{1}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+}{5ad\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}}$$

```
[Out] 3/5*B*tan(d*x+c)/a/d/(1+sec(d*x+c))/(a+a*sec(d*x+c))^(1/3)-3/5*A*AppellF1(-
5/6,1,1/2,1/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*2^(1/2)*tan(d*x+c)/a/d/(1+se
c(d*x+c))/(a+a*sec(d*x+c))^(1/3)/(1-sec(d*x+c))^(1/2)-1/10*3^(3/4)*B*((2^(1
/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^
(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec
(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3
^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+
1/4*2^(1/2))*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c)
)^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)
^(1/2)*tan(d*x+c)*2^(2/3)/a/d/(1-sec(d*x+c))/(a+a*sec(d*x+c))^(1/3)/(-(1+se
c(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3
)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]
time = 0.30, antiderivative size = 415, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.360$, Rules used = {4009, 3864, 3863, 141, 3913, 3912, 53, 65, 231}

$$-\frac{3\sqrt{2}\,A\tan(c+dx)F_1\left(-\frac{5}{6};\frac{1}{2},1;\frac{1}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{5ad\sqrt{1-\sec(c+dx)}\,(\sec(c+dx)+1)\sqrt[3]{a\sec(c+dx)+a}}-\frac{3^{3/4}B\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{5\sqrt[3]{2}\,ad(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,\sqrt[3]{a\sec(c+dx)+a}}+\frac{3B\tan(c+dx)}{5ad(\sec(c+dx)+1)\sqrt[3]{a\sec(c+dx)+a}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(4/3),x]
```

```
[Out] (3*B*Tan[c + d*x])/(5*a*d*(1 + Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)) -
(3*Sqrt[2]*A*AppellF1[-5/6, 1/2, 1, 1/6, (1 + Sec[c + d*x])/2, 1 + Sec[c +
d*x]]*Tan[c + d*x])/(5*a*d*Sqrt[1 - Sec[c + d*x]]*(1 + Sec[c + d*x])*(a + a
*Sec[c + d*x])^(1/3)) - (3^(3/4)*B*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3]
)*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/
3))], (2 + Sqrt[3])/4]*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) +
 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1
```

```
 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(5*2^(1/3)*a*d*(1 -
Sec[c + d*x])*(a + a*Sec[c + d*x])^(1/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(
2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 3863

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]])), Subst[Int[(1
+ b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 3864

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] && !IntegerQ[2*n] && !GtQ[a, 0]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] && !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 4009

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^{4/3}}\,dx = A\int \frac{1}{(a+a\sec(c+dx))^{4/3}}\,dx + B\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{4/3}}\,dx$$

$$= \frac{\left(A\sqrt[3]{1+\sec(c+dx)}\right)\int \frac{1}{(1+\sec(c+dx))^{4/3}}\,dx}{a\sqrt[3]{a+a\sec(c+dx)}} + \frac{\left(B\sqrt[3]{1+\sec(c+dx)}\right)\int \frac{\text{se}}{(1+\text{se}}}{a\sqrt[3]{a+a\sec(c+dx}}$$

$$= -\frac{(A\tan(c+dx))\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x(1+x)^{11/6}}\,dx, x, \sec(c+dx)\right)}{ad\sqrt{1-\sec(c+dx)}\sqrt[6]{1+\sec(c+dx)}\sqrt[3]{a+a\sec(c+dx)}} - \frac{(B\tan(c}{ad\sqrt{1-}}$$

$$= \frac{3B\tan(c+dx)}{5ad(1+\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}} - \frac{3\sqrt{2}\,AF_1\left(-\frac{5}{6};\frac{1}{2},1;\frac{1}{6};\frac{1}{2}(1+\text{sec}\right.}{5ad\sqrt{1-\sec(c+dx)}\,(1+}$$

$$= \frac{3B\tan(c+dx)}{5ad(1+\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}} - \frac{3\sqrt{2}\,AF_1\left(-\frac{5}{6};\frac{1}{2},1;\frac{1}{6};\frac{1}{2}(1+\text{sec}\right.}{5ad\sqrt{1-\sec(c+dx)}\,(1+}$$

$$= \frac{3B\tan(c+dx)}{5ad(1+\sec(c+dx))\sqrt[3]{a+a\sec(c+dx)}} - \frac{3\sqrt{2}\,AF_1\left(-\frac{5}{6};\frac{1}{2},1;\frac{1}{6};\frac{1}{2}(1+\text{sec}\right.}{5ad\sqrt{1-\sec(c+dx)}\,(1+}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 2901 vs. $2(415) = 830$.
time = 19.25, size = 2901, normalized size = 6.99

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(4/3),x]
```

```
[Out] (Cos[c + d*x]*((1 + Cos[c + d*x])*Sec[c + d*x])^(2/3)*(1 + Sec[c + d*x])^(4
/3)*(A + B*Sec[c + d*x])*((3*Sec[(c + d*x)/2]*(-(A*Sin[(c + d*x)/2]) + B*Si
n[(c + d*x)/2]))/5 - (3*Sec[(c + d*x)/2]^3*(-(A*Sin[(c + d*x)/2]) + B*Sin[(
c + d*x)/2]))/10))/(d*(B + A*Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^(4/3)) +
(2^(2/3)*Cos[c + d*x]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(1 + Sec[c +
d*x])^(4/3)*(A + B*Sec[c + d*x])*((A*Cos[c + d*x]*Sec[(c + d*x)/2]^2*(1 + S
ec[c + d*x])^(2/3))/2 + Sec[(c + d*x)/2]^2*(-1/10*(A*(1 + Sec[c + d*x])^(2/
3)) + (B*(1 + Sec[c + d*x])^(2/3))/10))*Tan[(c + d*x)/2]*((-6*A + B)*Appell
```

F1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*x]
*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (27*(4*A + B)*AppellF1[1/2,
 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2)/
(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*
(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2
*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(
c + d*x)/2]^2)))/(15*d*(B + A*Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^(4/3)*((
Sec[(c + d*x)/2]^2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*((-6*A + B)*Appe
llF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(Cos[c + d*
x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (27*(4*A + B)*AppellF1[1/
2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]^2
)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] +
2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] +
 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan
[(c + d*x)/2]^2)))/(15*2^(1/3)) + (2^(2/3)*(Cos[(c + d*x)/2]^2*Sec[c + d*x]
)^(2/3)*Tan[(c + d*x)/2]*((-6*A + B)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*(Cos[c + d*x]*Sec[(c + d*x
)/2]^2)^(2/3)*Tan[(c + d*x)/2] + (-6*A + B)*(Cos[c + d*x]*Sec[(c + d*x)/2]^
2)^(2/3)*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2, 2/3, 2, 7/2, Tan[(c + d*x)/2
]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*Appel
lF1[5/2, 5/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x
)/2]^2*Tan[(c + d*x)/2])/5) + (2*(-6*A + B)*AppellF1[3/2, 2/3, 1, 5/2, Tan[
(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]
^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(C
os[c + d*x]*Sec[(c + d*x)/2]^2)^(1/3)) - (27*(4*A + B)*AppellF1[1/2, 2/3, 1
, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c + d*x)/2]*Sin[(c + d
*x)/2])/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2
]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
])*Tan[(c + d*x)/2]^2) + (27*(4*A + B)*Cos[(c + d*x)/2]^2*(-1/3*(AppellF1[3
/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2]) + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Ta
n[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9))/(9*AppellF1[1/2,
 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(-3*AppellF1[3/2
, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5
/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2) -
 (27*(4*A + B)*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x
)/2]^2]*Cos[(c + d*x)/2]^2*(2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] + 9*(-1/3*(App
ellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2]) + (2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) + 2*Tan[(c
 + d*x)/2]^2*(-3*((-6*AppellF1[5/2, 2/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (2*AppellF1[5/2, 5/

```
3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2])/5) + 2*((-3*AppellF1[5/2, 5/3, 2, 7/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + AppellF1[5/2, 8
/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan
[(c + d*x)/2]))))/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c +
 d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2))/15 + (2*2^(2/3)*Tan[(c + d*x)/2]*((-6*
A + B)*AppellF1[3/2, 2/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*
(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)*Tan[(c + d*x)/2]^2 + (27*(4*A + B)*
AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Cos[(c
+ d*x)/2]^2)/(9*AppellF1[1/2, 2/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2] + 2*(-3*AppellF1[3/2, 2/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d
*x)/2]^2] + 2*AppellF1[3/2, 5/3, 1, 5/2, Tan[(c...
```

Maple [**F**]
time = 0.31, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec(dx+c)}{(a+a\sec(dx+c))^{\frac{4}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(4/3),x)
[Out] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(4/3),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(4/3),x, algorithm="maxima")
[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(4/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(4/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec\left(c + dx\right)}{\left(a\left(\sec\left(c + dx\right) + 1\right)\right)^{\frac{4}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(4/3),x)
[Out] Integral((A + B*sec(c + d*x))/(a*(sec(c + d*x) + 1))**(4/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(4/3),x, algorithm="giac")
[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+d\,x)}}{{\left(a + \frac{a}{\cos(c+d\,x)}\right)}^{4/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(4/3),x)
[Out] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(4/3), x)
```

3.272 $\int (a+a\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=787

$$\frac{3aB\sqrt[3]{a+a\sec(c+dx)}\,\tan(c+dx)}{4d}+\frac{3\sqrt{2}\,aAF_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)(1+\sec}{11d\sqrt{1-\sec(c+dx)}}$$

```
[Out] 3/4*a*B*(a+a*sec(d*x+c))^(1/3)*tan(d*x+c)/d-15/4*a*B*(a+a*sec(d*x+c))^(1/3)
*(1+3^(1/2))*tan(d*x+c)/d/(1+sec(d*x+c))^(2/3)/(2^(1/3)-(1+sec(d*x+c))^(1/3
)*(1+3^(1/2)))+3/11*a*A*AppellF1(11/6,1,1/2,17/6,1+sec(d*x+c),1/2+1/2*sec(d
*x+c))*(1+sec(d*x+c))*(a+a*sec(d*x+c))^(1/3)*2^(1/2)*tan(d*x+c)/d/(1-sec(d*
x+c))^(1/2)+15/4*3^(1/4)*a*B*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/
(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))
^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticE((1
-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)
*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(1/3)*(2^(
1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*
x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)
*2^(1/3)/d/(1-sec(d*x+c))/(1+sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1
/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/
2)+5/8*3^(3/4)*a*B*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(
1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-
3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-
(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2
)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(1/3)*(2^(1/3)-(1+se
c(d*x+c))^(1/3))*(1-3^(1/2))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(
d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+
c)*2^(1/3)/d/(1-sec(d*x+c))/(1+sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^
(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(
1/2)
```

Rubi [A]

time = 0.59, antiderivative size = 787, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {4009, 3864, 3863, 141, 3913, 3912, 52, 65, 314, 231, 1895}

$$\frac{3\sqrt{2}\,aA\tan(c+dx)(\sec(c+dx)+1)\sqrt[3]{a\sec(c+dx)+a}\,F_1\left(\frac{11}{6};\frac{1}{2},1;\frac{17}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{11d\sqrt{1-\sec(c+dx)}}+\frac{5\,3^{3/4}\left(1-\sqrt{3}\right)aB\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{4\,2^{2/3}d(1-\sec(c+dx))(\sec(c+dx)+1)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}+\frac{15\sqrt[4]{3}\,aB\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2\,2^{2/3}d(1-\sec(c+dx))(\sec(c+dx)+1)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}+\frac{3aB\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}}{4d}-\frac{15\left(1+\sqrt{3}\right)aB\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}}{4d(\sec(c+dx)+1)^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + a*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*a*B*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*d) + (3*Sqrt[2]*a*A*Appe
llF1[11/6, 1/2, 1, 17/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*(1 + Sec[c
 + d*x])*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(11*d*Sqrt[1 - Sec[c + d*
x]]) - (15*(1 + Sqrt[3])*a*B*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(4*d*
(1 + Sec[c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))
) + (15*3^(1/4)*a*B*EllipticE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt
[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqr
t[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(
2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2*2^(2/
3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])
^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 +
Sec[c + d*x])^(1/3))^2)]) + (5*3^(3/4)*(1 - Sqrt[3])*a*B*EllipticF[ArcCos[(
2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*
(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^
(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x]
)^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d
*x])^(1/3))^2]*Tan[c + d*x])/(4*2^(2/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d
*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(
1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 52

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^n/(b*(m + n + 1))), x] + Dist[n*((b*c - a*d)/(
b*(m + n + 1))), Int[(a + b*x)^m*(c + d*x)^(n - 1), x], x] /; FreeQ[{a, b,
c, d}, x] && NeQ[b*c - a*d, 0] && GtQ[n, 0] && NeQ[m + n + 1, 0] &&  !(IGtQ
[m, 0] && ( !IntegerQ[n] || (GtQ[m, 0] && LtQ[m - n, 0]))) &&  !ILtQ[m + n
+ 2, 0] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
```

```
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 3863

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]])), Subst[Int[(1
+ b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 3864

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
```

```
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4009

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rubi steps

$$\int (a+a\sec(c+dx))^{4/3}(A+B\sec(c+dx))\,dx = A\int (a+a\sec(c+dx))^{4/3}\,dx + B\int \sec(c+dx)(a+a\,\mathrm{se}$$

$$= \frac{\left(aA\sqrt[3]{a+a\sec(c+dx)}\right)\int (1+\sec(c+dx))^{4/3}\,dx}{\sqrt[3]{1+\sec(c+dx)}} + \frac{(c}{}$$

$$= -\frac{\left(aA\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)\right)\mathrm{Subst}\left(\int \frac{(1+x)^5}{\sqrt{1-}}\right.}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+dx)}$$

$$= \frac{3aB\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} + \frac{3\sqrt{2}\,aAF_1\left(\frac{11}{6};\frac{1}{2}\right.}{}$$

$$= \frac{3aB\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} + \frac{3\sqrt{2}\,aAF_1\left(\frac{11}{6};\frac{1}{2}\right.}{}$$

$$= \frac{3aB\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} + \frac{3\sqrt{2}\,aAF_1\left(\frac{11}{6};\frac{1}{2}\right.}{}$$

$$= \frac{3aB\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)}{4d} + \frac{3\sqrt{2}\,aAF_1\left(\frac{11}{6};\frac{1}{2}\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 4110 vs. $2(787) = 1574$.
time = 19.30, size = 4110, normalized size = 5.22

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(4/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]*((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(a*(1 + Sec[c + d*x])
)^(4/3)*(A + B*Sec[c + d*x])*((3*(4*A + 5*B)*Sin[c + d*x])/4 + (3*B*Tan[c +
 d*x])/4))/(d*(B + A*Cos[c + d*x])*(1 + Sec[c + d*x])^(4/3)) + (Cos[c + d*x
]*(a*(1 + Sec[c + d*x]))^(4/3)*(A + B*Sec[c + d*x])*(2*A*(1 + Sec[c + d*x])
```

```
^(1/3) + (5*B*(1 + Sec[c + d*x])^(1/3))/4 + Cos[c + d*x]*(-3*A*(1 + Sec[c +
 d*x])^(1/3) - (15*B*(1 + Sec[c + d*x])^(1/3))/4))*Tan[(c + d*x)/2]*(-(((4*
A + 5*B)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)) - (9*(3*Appe
llF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-4*A + 5*B
 + 5*(4*A + 7*B)*Cos[c + d*x]) - 4*(4*A + 5*B)*(3*AppellF1[3/2, 1/3, 2, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2))/(2
*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(6*2^(2/3)*d*(B + A*Cos[c + d*x]
)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(1 + Sec[c + d*x])^(4/3)*((Sec[(c
 + d*x)/2]^2*(-(((4*A + 5*B)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2,
 -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)
^(2/3)) - (9*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2]*(-4*A + 5*B + 5*(4*A + 7*B)*Cos[c + d*x]) - 4*(4*A + 5*B)*(3*Appel
lF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3
/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan
[(c + d*x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/
2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Ta
n[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(12*2^(2/3)*
(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)) + (Tan[(c + d*x)/2]*(-(((4*A + 5*B
)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(
c + d*x)/2]^2*Tan[(c + d*x)/2])/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3)) -
((4*A + 5*B)*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2, 1/3, 2, 7/2, Tan[(c + d*
x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (App
ellF1[5/2, 4/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2])/5))/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3) + (2
*(4*A + 5*B)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/
2]^2]*Tan[(c + d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)
^(5/3)) + (9*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]*(3*AppellF1[1/2, 1/3, 1, 3
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*(-4*A + 5*B + 5*(4*A + 7*B)*Co
s[c + d*x]) - 4*(4*A + 5*B)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^
2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -T
an[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2))/(2*(-1 + Tan[(c + d*x
)/2]^2)^2*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)
/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
])*Tan[(c + d*x)/2]^2)) + (9*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*(-4*A + 5*B + 5*(4*A + 7*B)*Cos[c + d*x]) - 4*(4*A
 + 5*B)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*
```

```
Cos[c + d*x]*Tan[(c + d*x)/2]^2)*(2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c +
d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/
2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2] - 9*(-1/3*(
AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c
+ d*x)/2]^2*Tan[(c + d*x)/2]) + (AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2
]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/9) + 2*Tan[(
c + d*x)/2]^2*((3*AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 - (4*AppellF1[5/2, 7/3, 1
, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c +
 d*x)/2])/5 + 3*((-6*AppellF1[5/2, 1/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2, 4/3,
2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c
+ d*x)/2])/5))))/(2*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan
[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2)^2) - (9*(-15*(4*
A + 7*B)*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2
]*Sin[c + d*x] - 4*(4*A + 5*B)*(3*AppellF1[3/2,...
```

Maple [**F**]
time = 0.27, size = 0, normalized size = 0.00

$$\int \left(a+a \sec \left(d x+c\right)\right)^{\frac{4}{3}} \left(A+B \sec \left(d x+c\right)\right) d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
[Out] int((a+a*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(4/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(c+dx)+1))^{\frac{4}{3}} (A+B\sec(c+dx))\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(4/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((a*(sec(c + d*x) + 1))**(4/3)*(A + B*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(4/3)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(4/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{4/3} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(4/3),x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(4/3), x)
```

3.273 $\int \sqrt[3]{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=739

$$\frac{3\sqrt{2}\,AF_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\sqrt[3]{a+a\sec(c+dx)}\,\tan(c+dx)}{5d\sqrt{1-\sec(c+dx)}}-\frac{3\Big(1+}{d(1+\sec(c+}$$

```
[Out] -3*B*(a+a*sec(d*x+c))^(1/3)*(1+3^(1/2))*tan(d*x+c)/d/(1+sec(d*x+c))^(2/3)/(
2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))+3/5*A*AppellF1(5/6,1,1/2,11/6,1+s
ec(d*x+c),1/2+1/2*sec(d*x+c))*(a+a*sec(d*x+c))^(1/3)*2^(1/2)*tan(d*x+c)/d/(
1-sec(d*x+c))^(1/2)+3*2^(1/3)*3^(1/4)*B*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3
^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+
sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*E
llipticE((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*
x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))
^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3
)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
*tan(d*x+c)/d/(1-sec(d*x+c))/(1+sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2
^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^
(1/2)+1/2*3^(3/4)*B*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-
(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1
-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)
-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/
2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))*(a+a*sec(d*x+c))^(1/3)*(2^(1/3)-(1+s
ec(d*x+c))^(1/3))*(1-3^(1/2))*((2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec
(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x
+c)*2^(1/3)/d/(1-sec(d*x+c))/(1+sec(d*x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2
^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^
(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 739, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4009, 3864, 3863, 141, 3913, 3912, 65, 314, 231, 1895}

$$\frac{3\sqrt{2}A\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}\,F_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{5d\sqrt{1-\sec(c+dx)}}+\frac{3^{3/4}\left(1-\sqrt{3}\right)B\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}\,F\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2^{2/3}d(1-\sec(c+dx))(\sec(c+dx)+1)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}+\frac{3\sqrt[3]{2}\sqrt[4]{3}B\tan(c+dx)\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\sqrt[3]{a\sec(c+dx)+a}\,E\left(\text{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{d(1-\sec(c+dx))(\sec(c+dx)+1)^{2/3}\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}}-\frac{3\left(1+\sqrt{3}\right)B\tan(c+dx)\sqrt[3]{a\sec(c+dx)+a}}{d(\sec(c+dx)+1)^{2/3}\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(a + a*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*Sqrt[2]*A*AppellF1[5/6, 1/2, 1, 11/6, (1 + Sec[c + d*x])/2, 1 + Sec[c +
d*x]]*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(5*d*Sqrt[1 - Sec[c + d*x]])
 - (3*(1 + Sqrt[3])*B*(a + a*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*(1 + Sec[
c + d*x])^(2/3)*(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) + (3*2^
(1/3)*3^(1/4)*B*EllipticE[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x]
)^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])
/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2
^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1
/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(d*(1 - Sec[
c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3
) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^
(1/3))^2)]) + (3^(3/4)*(1 - Sqrt[3])*B*EllipticF[ArcCos[(2^(1/3) - (1 - Sqr
t[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])
^(1/3))], (2 + Sqrt[3])/4]*(a + a*Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c
 + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec
[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan
[c + d*x])/(2^(2/3)*d*(1 - Sec[c + d*x])*(1 + Sec[c + d*x])^(2/3)*Sqrt[-(((
1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1
 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)])
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 3863

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]])), Subst[Int[(1
+ b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 3864

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
```

```
, 0] && !IntegerQ[m] && !GtQ[a, 0]
```

Rule 4009

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[2*m]
```

Rubi steps

$$\int \sqrt[3]{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = A\int \sqrt[3]{a+a\sec(c+dx)}\,dx + B\int \sec(c+dx)\sqrt[3]{a+a\sec}$$

$$= \frac{\left(A\sqrt[3]{a+a\sec(c+dx)}\right)\int \sqrt[3]{1+\sec(c+dx)}\,dx}{\sqrt[3]{1+\sec(c+dx)}} + \frac{\left(B\sqrt[3]{\right.}{}$$

$$= -\frac{\left(A\sqrt[3]{a+a\sec(c+dx)}\tan(c+dx)\right)\mathrm{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x}\right.}{d\sqrt{1-\sec(c+dx)}\,(1+\sec(c+d}$$

$$= \frac{3\sqrt{2}\,AF_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\sqrt[3]{}}{5d\sqrt{1-\sec(c+dx)}}$$

$$= \frac{3\sqrt{2}\,AF_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\sqrt[3]{}}{5d\sqrt{1-\sec(c+dx)}}$$

$$= \frac{3\sqrt{2}\,AF_1\left(\frac{5}{6};\frac{1}{2},1;\frac{11}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\sqrt[3]{}}{5d\sqrt{1-\sec(c+dx)}}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 5094 vs. $2(739) = 1478$.
time = 20.85, size = 5094, normalized size = 6.89

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + a*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x]),x]
[Out] Result too large to show
```

Maple [**F**]
time = 0.25, size = 0, normalized size = 0.00

$$\int \left(a+a\sec \left(dx+c\right)\right)^{\frac{1}{3}} \left(A+B\sec \left(dx+c\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x)
[Out] int((a+a*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(1/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a\left(\sec \left(c+dx\right)+1\right)} \left(A+B\sec \left(c+dx\right)\right) dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(1/3)*(A+B*sec(d*x+c)),x)
[Out] Integral((a*(sec(c + d*x) + 1))**(1/3)*(A + B*sec(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(1/3), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{1/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/3),x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/3), x)
```

3.274 $\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=764

$$\frac{3B\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}}-\frac{3\sqrt{2}\,AF_1\left(-\frac{1}{6};\frac{1}{2},1;\frac{5}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(c+dx)\right)\tan(c+dx)}{d\sqrt{1-\sec(c+dx)}\,(a+a\sec(c+dx))^{2/3}}+\frac{3}{d(a+a\,\mathrm{se}}$$

```
[Out] 3*B*tan(d*x+c)/d/(a+a*sec(d*x+c))^(2/3)+3*B*(1+sec(d*x+c))^(1/3)*(1+3^(1/2)
)*tan(d*x+c)/d/(a+a*sec(d*x+c))^(2/3)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1
/2)))-3*A*AppellF1(-1/6,1,1/2,5/6,1+sec(d*x+c),1/2+1/2*sec(d*x+c))*2^(1/2)*
tan(d*x+c)/d/(a+a*sec(d*x+c))^(2/3)/(1-sec(d*x+c))^(1/2)-3*2^(1/3)*3^(1/4)*
B*((2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/
3)*(1+3^(1/2)))^2)^(1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3
)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))*EllipticE((1-(2^(1/3)-(1+sec(d*x+c))^(1
/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*
6^(1/2)+1/4*2^(1/2))*(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*((
2^(2/3)+2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(
d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)/d/(1-sec(d*x+c))/(a+a*sec(d*
x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-
(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)-1/2*3^(3/4)*B*((2^(1/3)-(1+sec(d
*x+c))^(1/3)*(1-3^(1/2)))^2/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(
1/2)/(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))*(2^(1/3)-(1+sec(d*x+c))^(1/
3)*(1+3^(1/2)))*EllipticF((1-(2^(1/3)-(1+sec(d*x+c))^(1/3)*(1-3^(1/2)))^2/(
2^(1/3)-(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2),1/4*6^(1/2)+1/4*2^(1/2))
*(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))*(1-3^(1/2))*((2^(2/3)+
2^(1/3)*(1+sec(d*x+c))^(1/3)+(1+sec(d*x+c))^(2/3))/(2^(1/3)-(1+sec(d*x+c))^
(1/3)*(1+3^(1/2)))^2)^(1/2)*tan(d*x+c)*2^(1/3)/d/(1-sec(d*x+c))/(a+a*sec(d*
x+c))^(2/3)/(-(1+sec(d*x+c))^(1/3)*(2^(1/3)-(1+sec(d*x+c))^(1/3))/(2^(1/3)-
(1+sec(d*x+c))^(1/3)*(1+3^(1/2)))^2)^(1/2)
```

Rubi [A]

time = 0.52, antiderivative size = 764, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.440$, Rules used = {4009, 3864, 3863, 141, 3913, 3912, 53, 65, 314, 231, 1895}

$$-\frac{3\sqrt{2}\,A\tan(c+dx)F_1\left(-\frac{1}{6};\frac{1}{2},1;\frac{5}{6};\frac{1}{2}(\sec(c+dx)+1),\sec(c+dx)+1\right)}{d\sqrt{1-\sec(c+dx)}\,(a\sec(c+dx)+a)^{2/3}}-\frac{3^{3/4}\left(1-\sqrt{3}\right)B\tan(c+dx)\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,F\left(\mathrm{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{2^{2/3}d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,(a\sec(c+dx)+a)^{2/3}}-\frac{3\sqrt[3]{2}\sqrt[4]{3}B\tan(c+dx)\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)\sqrt{\frac{(\sec(c+dx)+1)^{2/3}+\sqrt[3]{2}\sqrt[3]{\sec(c+dx)+1}+2^{2/3}}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,E\left(\mathrm{ArcCos}\left(\frac{\sqrt[3]{2}-\left(1-\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}{\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}}\right)|\frac{1}{4}\left(2+\sqrt{3}\right)\right)}{d(1-\sec(c+dx))\sqrt{-\frac{\sqrt[3]{\sec(c+dx)+1}\left(\sqrt[3]{2}-\sqrt[3]{\sec(c+dx)+1}\right)}{\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)^2}}\,(a\sec(c+dx)+a)^{2/3}}+\frac{3B\tan(c+dx)}{d(a\sec(c+dx)+a)^{2/3}}+\frac{3\left(1+\sqrt{3}\right)B\tan(c+dx)\sqrt[3]{\sec(c+dx)+1}}{d\left(\sqrt[3]{2}-\left(1+\sqrt{3}\right)\sqrt[3]{\sec(c+dx)+1}\right)(a\sec(c+dx)+a)^{2/3}}$$

Warning: Unable to verify antiderivative.

```
[In] Int[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (3*B*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^(2/3)) - (3*Sqrt[2]*A*AppellF1[-
1/6, 1/2, 1, 5/6, (1 + Sec[c + d*x])/2, 1 + Sec[c + d*x]]*Tan[c + d*x])/(d*
Sqrt[1 - Sec[c + d*x]]*(a + a*Sec[c + d*x])^(2/3)) + (3*(1 + Sqrt[3])*B*(1
+ Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*(a + a*Sec[c + d*x])^(2/3)*(2^(1/3)
- (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))) - (3*2^(1/3)*3^(1/4)*B*EllipticE
[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^(1/3))/(2^(1/3) - (1 +
Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4]*(1 + Sec[c + d*x])^(1
/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/3) + 2^(1/3)*(1 + Sec[c
 + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Se
c[c + d*x])^(1/3))^2]*Tan[c + d*x])/(d*(1 - Sec[c + d*x])*(a + a*Sec[c + d*
x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1
/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2)]) - (3^(3/4)*(1
 - Sqrt[3])*B*EllipticF[ArcCos[(2^(1/3) - (1 - Sqrt[3])*(1 + Sec[c + d*x])^
(1/3))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))], (2 + Sqrt[3])/4
]*(1 + Sec[c + d*x])^(1/3)*(2^(1/3) - (1 + Sec[c + d*x])^(1/3))*Sqrt[(2^(2/
3) + 2^(1/3)*(1 + Sec[c + d*x])^(1/3) + (1 + Sec[c + d*x])^(2/3))/(2^(1/3)
- (1 + Sqrt[3])*(1 + Sec[c + d*x])^(1/3))^2]*Tan[c + d*x])/(2^(2/3)*d*(1 -
Sec[c + d*x])*(a + a*Sec[c + d*x])^(2/3)*Sqrt[-(((1 + Sec[c + d*x])^(1/3)*(
2^(1/3) - (1 + Sec[c + d*x])^(1/3)))/(2^(1/3) - (1 + Sqrt[3])*(1 + Sec[c +
d*x])^(1/3))^2)])
```

Rule 53

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> Simp[
(a + b*x)^(m + 1)*((c + d*x)^(n + 1)/((b*c - a*d)*(m + 1))), x] - Dist[d*((
m + n + 2)/((b*c - a*d)*(m + 1))), Int[(a + b*x)^(m + 1)*(c + d*x)^n, x], x
] /; FreeQ[{a, b, c, d, n}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] &&  !(LtQ
[n, -1] && (EqQ[a, 0] || (NeQ[c, 0] && LtQ[m - n, 0] && IntegerQ[n]))) && I
ntLinearQ[a, b, c, d, m, n, x]
```

Rule 65

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_), x_Symbol] :> With[
{p = Denominator[m]}, Dist[p/b, Subst[Int[x^(p*(m + 1) - 1)*(c - a*(d/b) +
d*(x^p/b))^n, x], x, (a + b*x)^(1/p)], x]] /; FreeQ[{a, b, c, d}, x] && NeQ
[b*c - a*d, 0] && LtQ[-1, m, 0] && LeQ[-1, n, 0] && LeQ[Denominator[n], Den
ominator[m]] && IntLinearQ[a, b, c, d, m, n, x]
```

Rule 141

```
Int[((a_) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_))
^(p_), x_Symbol] :> Simp[(b*e - a*f)^p*((a + b*x)^(m + 1)/(b^(p + 1)*(m + 1
)*(b/(b*c - a*d))^n))*AppellF1[m + 1, -n, -p, m + 2, (-d)*((a + b*x)/(b*c -
 a*d)), (-f)*((a + b*x)/(b*e - a*f))], x] /; FreeQ[{a, b, c, d, e, f, m, n}
, x] &&  !IntegerQ[m] &&  !IntegerQ[n] && IntegerQ[p] && GtQ[b/(b*c - a*d),
```

```
 0] &&  !(GtQ[d/(d*a - c*b), 0] && SimplerQ[c + d*x, a + b*x])
```

Rule 231

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a, 3]],
s = Denom[Rt[b/a, 3]]}, Simp[x*(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/
(s + (1 + Sqrt[3])*r*x^2)^2]/(2*3^(1/4)*s*Sqrt[a + b*x^6]*Sqrt[r*x^2*((s +
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)^2)]))*EllipticF[ArcCos[(s + (1 - Sqrt[3])*
r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2 + Sqrt[3])/4], x]] /; FreeQ[{a, b}, x
]
```

Rule 314

```
Int[(x_)^4/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r = Numer[Rt[b/a,
3]], s = Denom[Rt[b/a, 3]]}, Dist[(Sqrt[3] - 1)*(s^2/(2*r^2)), Int[1/Sqrt[a
 + b*x^6], x], x] - Dist[1/(2*r^2), Int[((Sqrt[3] - 1)*s^2 - 2*r^2*x^4)/Sqr
t[a + b*x^6], x], x]] /; FreeQ[{a, b}, x]
```

Rule 1895

```
Int[((c_) + (d_.)*(x_)^4)/Sqrt[(a_) + (b_.)*(x_)^6], x_Symbol] :> With[{r =
 Numer[Rt[b/a, 3]], s = Denom[Rt[b/a, 3]]}, Simp[(1 + Sqrt[3])*d*s^3*x*(Sqr
t[a + b*x^6]/(2*a*r^2*(s + (1 + Sqrt[3])*r*x^2))), x] - Simp[3^(1/4)*d*s*x*
(s + r*x^2)*(Sqrt[(s^2 - r*s*x^2 + r^2*x^4)/(s + (1 + Sqrt[3])*r*x^2)^2]/(2
*r^2*Sqrt[(r*x^2*(s + r*x^2))/(s + (1 + Sqrt[3])*r*x^2)^2]*Sqrt[a + b*x^6])
)*EllipticE[ArcCos[(s + (1 - Sqrt[3])*r*x^2)/(s + (1 + Sqrt[3])*r*x^2)], (2
 + Sqrt[3])/4], x]] /; FreeQ[{a, b, c, d}, x] && EqQ[2*Rt[b/a, 3]^2*c - (1
- Sqrt[3])*d, 0]
```

Rule 3863

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^n*(Cot
[c + d*x]/(d*Sqrt[1 + Csc[c + d*x]]*Sqrt[1 - Csc[c + d*x]])), Subst[Int[(1
+ b*(x/a))^(n - 1/2)/(x*Sqrt[1 - b*(x/a)]), x], x, Csc[c + d*x]], x] /; Fre
eQ[{a, b, c, d, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] && GtQ[a, 0]
```

Rule 3864

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(n_), x_Symbol] :> Dist[a^IntPar
t[n]*((a + b*Csc[c + d*x])^FracPart[n]/(1 + (b/a)*Csc[c + d*x])^FracPart[n]
), Int[(1 + (b/a)*Csc[c + d*x])^n, x], x] /; FreeQ[{a, b, c, d, n}, x] && E
qQ[a^2 - b^2, 0] &&  !IntegerQ[2*n] &&  !GtQ[a, 0]
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
```

```
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4009

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Dist[c, Int[(a + b*Csc[e + f*x])^m, x], x] + Dist
[d, Int[(a + b*Csc[e + f*x])^m*Csc[e + f*x], x], x] /; FreeQ[{a, b, c, d, e
, f, m}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[2*m]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+a\sec(c+dx))^{2/3}}\,dx = A\int \frac{1}{(a+a\sec(c+dx))^{2/3}}\,dx + B\int \frac{\sec(c+dx)}{(a+a\sec(c+dx))^{2/3}}\,dx$$

$$= \frac{\left(A(1+\sec(c+dx))^{2/3}\right)\int \frac{1}{(1+\sec(c+dx))^{2/3}}\,dx}{(a+a\sec(c+dx))^{2/3}} + \frac{\left(B(1+\sec(c+dx))^{2/3}\right)\int \frac{}{(1+}}{(a+a\sec(c+dx))}$$

$$= -\frac{\left(A\sqrt[6]{1+\sec(c+dx)}\,\tan(c+dx)\right)\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,x(1+x)^{7/6}}\,dx, x, \sec(c}{d\sqrt{1-\sec(c+dx)}\,(a+a\sec(c+dx))^{2/3}}$$

$$= \frac{3B\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} - \frac{3\sqrt{2}\,AF_1\left(-\frac{1}{6};\frac{1}{2},1;\frac{5}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(}{d\sqrt{1-\sec(c+dx)}\,(a+a\sec(c+a}$$

$$= \frac{3B\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} - \frac{3\sqrt{2}\,AF_1\left(-\frac{1}{6};\frac{1}{2},1;\frac{5}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(}{d\sqrt{1-\sec(c+dx)}\,(a+a\sec(c+a}$$

$$= \frac{3B\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} - \frac{3\sqrt{2}\,AF_1\left(-\frac{1}{6};\frac{1}{2},1;\frac{5}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(}{d\sqrt{1-\sec(c+dx)}\,(a+a\sec(c+a}$$

$$= \frac{3B\tan(c+dx)}{d(a+a\sec(c+dx))^{2/3}} - \frac{3\sqrt{2}\,AF_1\left(-\frac{1}{6};\frac{1}{2},1;\frac{5}{6};\frac{1}{2}(1+\sec(c+dx)),1+\sec(}{d\sqrt{1-\sec(c+dx)}\,(a+a\sec(c+a}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 4066 vs. $2(764) = 1528$.
time = 19.15, size = 4066, normalized size = 5.32

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + a*Sec[c + d*x])^(2/3),x]
```

```
[Out] (Cos[c + d*x]*((1 + Cos[c + d*x])*Sec[c + d*x])^(1/3)*(1 + Sec[c + d*x])^(2
/3)*(A + B*Sec[c + d*x])*(3*Sec[(c + d*x)/2]*(-(A*Sin[(c + d*x)/2]) + B*Sin
[(c + d*x)/2]) - 3*(-A + B)*Sin[c + d*x]))/(d*(B + A*Cos[c + d*x])*(a*(1 +
Sec[c + d*x]))^(2/3)) - (2^(1/3)*Cos[c + d*x]*(1 + Sec[c + d*x])^(2/3)*(A +
```

```
 B*Sec[c + d*x])*(2*A*(1 + Sec[c + d*x])^(1/3) - B*(1 + Sec[c + d*x])^(1/3)
 + Cos[c + d*x]*(-3*A*(1 + Sec[c + d*x])^(1/3) + 3*B*(1 + Sec[c + d*x])^(1/
3)))*Tan[(c + d*x)/2]*(((A - B)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]
^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x)/2]
^2)^(2/3) - (9*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c +
d*x)/2]^2]*(A + B + (-5*A + 7*B)*Cos[c + d*x]) + 4*(A - B)*(3*AppellF1[3/2,
 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3,
 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*
x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(
c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(3*d*(B + A*Cos[c +
d*x])*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(2/3)*(a*(1 + Sec[c + d*x]))^(2/3)*
(-1/3*(Sec[(c + d*x)/2]^2*(((A - B)*AppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x
)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c + d*x)/2]^2)/(Cos[c + d*x]*Sec[(c + d*x
)/2]^2)^(2/3) - (9*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2]*(A + B + (-5*A + 7*B)*Cos[c + d*x]) + 4*(A - B)*(3*AppellF1[
3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2,
4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c
+ d*x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c
 + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))))/(2^(2/3)*(Cos[(c
 + d*x)/2]^2*Sec[c + d*x])^(2/3)) - (2^(1/3)*Tan[(c + d*x)/2]*(((A - B)*App
ellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d
*x)/2]^2*Tan[(c + d*x)/2])/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3) + ((A -
B)*Tan[(c + d*x)/2]^2*((-3*AppellF1[5/2, 1/3, 2, 7/2, Tan[(c + d*x)/2]^2, -
Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + (AppellF1[5/2,
 4/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2])/5))/(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(2/3) - (2*(A - B)*A
ppellF1[3/2, 1/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Tan[(c +
 d*x)/2]^2*(-(Sec[(c + d*x)/2]^2*Sin[c + d*x]) + Cos[c + d*x]*Sec[(c + d*x)
/2]^2*Tan[(c + d*x)/2]))/(3*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(5/3)) + (9*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d
*x)/2]^2, -Tan[(c + d*x)/2]^2]*(A + B + (-5*A + 7*B)*Cos[c + d*x]) + 4*(A -
 B)*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
- AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[
c + d*x]*Tan[(c + d*x)/2]^2))/(2*(-1 + Tan[(c + d*x)/2]^2)^2*(-9*AppellF1[1
/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] + 2*(3*AppellF1[3
/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4
/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Tan[(c + d*x)/2]^2))
+ (9*(3*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]
*(A + B + (-5*A + 7*B)*Cos[c + d*x]) + 4*(A - B)*(3*AppellF1[3/2, 1/3, 2, 5
/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, T
an[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Cos[c + d*x]*Tan[(c + d*x)/2]^2)*(
```

```
2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2] -
AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2])*Sec[(c
 + d*x)/2]^2*Tan[(c + d*x)/2] - 9*(-1/3*(AppellF1[3/2, 1/3, 2, 5/2, Tan[(c
+ d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]) + (A
ppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2])/9) + 2*Tan[(c + d*x)/2]^2*((3*AppellF1[5/2, 4/
3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2])/5 - (4*AppellF1[5/2, 7/3, 1, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c
 + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5 + 3*((-6*AppellF1[5/2,
 1/3, 3, 7/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2])/5 + (AppellF1[5/2, 4/3, 2, 7/2, Tan[(c + d*x)/2]^2, -Tan[(
c + d*x)/2]^2]*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])/5))))/(2*(-1 + Tan[(c +
 d*x)/2]^2)*(-9*AppellF1[1/2, 1/3, 1, 3/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2] + 2*(3*AppellF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*
x)/2]^2] - AppellF1[3/2, 4/3, 1, 5/2, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]
^2])*Tan[(c + d*x)/2]^2)^2) - (9*(-3*(-5*A + 7*B)*AppellF1[1/2, 1/3, 1, 3/2
, Tan[(c + d*x)/2]^2, -Tan[(c + d*x)/2]^2]*Sin[c + d*x] + 4*(A - B)*(3*Appe
llF1[3/2, 1/3, 2, 5/2, Tan[(c + d*x)/2]^2, -Tan...
```

Maple [F]
time = 0.25, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec\left(dx+c\right)}{\left(a+a\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(2/3),x)
```

```
[Out] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(2/3),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(2/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(2/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec(c + dx)}{(a(\sec(c + dx) + 1))^{\frac{2}{3}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(2/3),x)
[Out] Integral((A + B*sec(c + d*x))/(a*(sec(c + d*x) + 1))**(2/3), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(2/3),x, algorithm="giac")
[Out] integrate((B*sec(d*x + c) + A)/(a*sec(d*x + c) + a)^(2/3), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+d\,x)}}{\left(a + \frac{a}{\cos(c+d\,x)}\right)^{2/3}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(2/3),x)
[Out] int((A + B/cos(c + d*x))/(a + a/cos(c + d*x))^(2/3), x)
```

3.275 $\int (c\sec(e+fx))^n(a+a\sec(e+fx))^m(A+B\sec(e+fx))\,dx$

Optimal. Leaf size=197

$$-\frac{BF_1\left(n;\frac{1}{2},-\frac{1}{2}-m;1+n;\sec(e+fx),-\sec(e+fx)\right)(c\sec(e+fx))^n(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\sec(e}{fn\sqrt{1-\sec(e+fx)}}$$

```
[Out] -B*AppellF1(n,-1/2-m,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(c*sec(f*x+e))^n*(1+se
c(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/n/(1-sec(f*x+e))^(1/2)-(
A-B)*AppellF1(n,1/2-m,1/2,1+n,-sec(f*x+e),sec(f*x+e))*(c*sec(f*x+e))^n*(1+s
ec(f*x+e))^(-1/2-m)*(a+a*sec(f*x+e))^m*tan(f*x+e)/f/n/(1-sec(f*x+e))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4108, 3913, 3912, 138}

$$-\frac{(A-B)\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m(c\sec(e+fx))^nF_1\left(n;\frac{1}{2},\frac{1}{2}-m;n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}}-\frac{B\tan(e+fx)(\sec(e+fx)+1)^{-m-\frac{1}{2}}(a\sec(e+fx)+a)^m(c\sec(e+fx))^nF_1\left(n;\frac{1}{2},-m-\frac{1}{2};n+1;\sec(e+fx),-\sec(e+fx)\right)}{fn\sqrt{1-\sec(e+fx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(c*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m*(A + B*Sec[e + f*x]),x]
```

```
[Out] -((B*AppellF1[n, 1/2, -1/2 - m, 1 + n, Sec[e + f*x], -Sec[e + f*x]]*(c*Sec[
e + f*x])^n*(1 + Sec[e + f*x])^(-1/2 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*
x])/(f*n*Sqrt[1 - Sec[e + f*x]])) - ((A - B)*AppellF1[n, 1/2, 1/2 - m, 1 +
n, Sec[e + f*x], -Sec[e + f*x]]*(c*Sec[e + f*x])^n*(1 + Sec[e + f*x])^(-1/2
 - m)*(a + a*Sec[e + f*x])^m*Tan[e + f*x])/(f*n*Sqrt[1 - Sec[e + f*x]])
```

Rule 138

```
Int[((b_.)*(x_))^(m_)*((c_) + (d_.)*(x_))^(n_)*((e_) + (f_.)*(x_))^(p_), x_
Symbol] :> Simp[c^n*e^p*((b*x)^(m + 1)/(b*(m + 1)))*AppellF1[m + 1, -n, -p,
 m + 2, (-d)*(x/c), (-f)*(x/e)], x] /; FreeQ[{b, c, d, e, f, m, n, p}, x] &
&  !IntegerQ[m] &&  !IntegerQ[n] && GtQ[c, 0] && (IntegerQ[p] || GtQ[e, 0])
```

Rule 3912

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^2*d*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x
]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(d*x)^(n - 1)*((a + b*x)^(m - 1/2)
/Sqrt[a - b*x]), x], x, Csc[e + f*x]], x] /; FreeQ[{a, b, d, e, f, m, n}, x
] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (c\sec(e+fx))^n(a+a\sec(e+fx))^m(A+B\sec(e+fx))\,dx = (A-B)\int (c\sec(e+fx))^n(a+a\sec(e+f$$

$$= \left((A-B)(1+\sec(e+fx))^{-m}(a+a\sec(e\right.$$

$$= -\frac{\left((A-B)c(1+\sec(e+fx))^{-\frac{1}{2}-m}(a+a\right.}{}$$

$$= -\frac{BF_1\left(n;\frac{1}{2},-\frac{1}{2}-m;1+n;\sec(e+fx),-\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 4897 vs. $2(197) = 394$.
time = 21.84, size = 4897, normalized size = 24.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(c*Sec[e + f*x])^n*(a + a*Sec[e + f*x])^m*(A + B*Sec[e + f*x]),x]
```

```
[Out] (2^(1 + m)*(Sec[(e + f*x)/2]^2)^n*Sec[e + f*x]^(-1 - n)*(c*Sec[e + f*x])^n*
(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*(a*(1 + Sec[e + f*x]))^m*(A + B*S
ec[e + f*x])*(A*Sec[e + f*x]^n*(1 + Sec[e + f*x])^m + B*Sec[e + f*x]^(1 + n
)*(1 + Sec[e + f*x])^m)*Tan[(e + f*x)/2]*((-3*A*AppellF1[1/2, m + n, 1 - n,
 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x])/(3*AppellF1[1/
2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n
```

)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]
 + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e
 + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (B*AppellF1[1/2, 1 + m + n, -n, 3/2, T
an[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])/(AppellF1[1/2, 1 + m + n, -n, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(n*AppellF1[3/2, 1 + m + n,
1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + m + n)*AppellF1
[3/2, 2 + m + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e
+ f*x)/2]^2)/3)))/(f*(B + A*Cos[e + f*x])*(1 + Sec[e + f*x])^m*(-1 + Tan[(e
 + f*x)/2]^2)*(-((2^(1 + m)*(Sec[(e + f*x)/2]^2)^(1 + n)*(Cos[(e + f*x)/2]^
2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2]^2*((-3*A*AppellF1[1/2, m + n, 1 -
n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x])/(3*AppellF1[
1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 +
 n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (B*AppellF1[1/2, 1 + m + n, -n, 3/2,
 Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])/(AppellF1[1/2, 1 + m + n, -n, 3/
2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(n*AppellF1[3/2, 1 + m + n
, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + m + n)*Appell
F1[3/2, 2 + m + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(
e + f*x)/2]^2)/3)))/(-1 + Tan[(e + f*x)/2]^2)^2) + (2^m*(Sec[(e + f*x)/2]^2
)^(1 + n)*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*((-3*A*AppellF1[1/2, m
+ n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Cos[e + f*x])/(3*
AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
 f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]
^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (B*AppellF1[1/2, 1 + m + n,
 -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])/(AppellF1[1/2, 1 + m +
n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (2*(n*AppellF1[3/2,
1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (1 + m +
n)*AppellF1[3/2, 2 + m + n, -n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2])*Tan[(e + f*x)/2]^2)/3)))/(-1 + Tan[(e + f*x)/2]^2) + (2^(1 + m)*n*(Sec[
(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x])^(m + n)*Tan[(e + f*x)/2
]^2*((-3*A*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f
*x)/2]^2]*Cos[e + f*x])/(3*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2
]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Ta
n[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1
 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) -
(B*AppellF1[1/2, 1 + m + n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2])/(AppellF1[1/2, 1 + m + n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2
]^2] + (2*(n*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[
(e + f*x)/2]^2] + (1 + m + n)*AppellF1[3/2, 2 + m + n, -n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2)/3)))/(-1 + Tan[(e + f*x)
/2]^2) + (2^(1 + m)*(Sec[(e + f*x)/2]^2)^n*(Cos[(e + f*x)/2]^2*Sec[e + f*x]
)^(m + n)*Tan[(e + f*x)/2]*((3*A*AppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e +
f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sin[e + f*x])/(3*AppellF1[1/2, m + n, 1 - n

```
, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + 2*((-1 + n)*AppellF1[3/2,
 m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] + (m + n)*Appe
llF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2])*
Tan[(e + f*x)/2]^2) - (3*A*Cos[e + f*x]*(-1/3*((1 - n)*AppellF1[3/2, m + n,
 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Ta
n[(e + f*x)/2]) + ((m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*
x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3))/(3*A
ppellF1[1/2, m + n, 1 - n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2] +
2*((-1 + n)*AppellF1[3/2, m + n, 2 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e +
f*x)/2]^2] + (m + n)*AppellF1[3/2, 1 + m + n, 1 - n, 5/2, Tan[(e + f*x)/2]^
2, -Tan[(e + f*x)/2]^2])*Tan[(e + f*x)/2]^2) - (B*((n*AppellF1[3/2, 1 + m +
 n, 1 - n, 5/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2
*Tan[(e + f*x)/2])/3 + ((1 + m + n)*AppellF1[3/2, 2 + m + n, -n, 5/2, Tan[(
e + f*x)/2]^2, -Tan[(e + f*x)/2]^2]*Sec[(e + f*x)/2]^2*Tan[(e + f*x)/2])/3)
)/(AppellF1[1/2, 1 + m + n, -n, 3/2, Tan[(e + f*x)/2]^2, -Tan[(e + f*x)/2]^
2] + (2*(n*AppellF1[3/2, 1 + m + n, 1 - n, 5/2,...
```

Maple [F]
time = 0.41, size = 0, normalized size = 0.00

$$\int \left(c \sec \left(f x +e \right)\right)^{n} \left(a +a \sec \left(f x +e \right)\right)^{m} \left(A +B \sec \left(f x +e \right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(f*x+e))^n*(a+a*sec(f*x+e))^m*(A+B*sec(f*x+e)),x)
```

```
[Out] int((c*sec(f*x+e))^n*(a+a*sec(f*x+e))^m*(A+B*sec(f*x+e)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(f*x+e))^n*(a+a*sec(f*x+e))^m*(A+B*sec(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(f*x + e) + A)*(a*sec(f*x + e) + a)^m*(c*sec(f*x + e))^n, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(f*x+e))^n*(a+a*sec(f*x+e))^m*(A+B*sec(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(f*x + e) + A)*(a*sec(f*x + e) + a)^m*(c*sec(f*x + e))^n, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (a(\sec(e+fx)+1))^m (c\sec(e+fx))^n (A+B\sec(e+fx))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(f*x+e))**n*(a+a*sec(f*x+e))**m*(A+B*sec(f*x+e)),x)
```

```
[Out] Integral((a*(sec(e + f*x) + 1))**m*(c*sec(e + f*x))**n*(A + B*sec(e + f*x))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(f*x+e))^n*(a+a*sec(f*x+e))^m*(A+B*sec(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(f*x + e) + A)*(a*sec(f*x + e) + a)^m*(c*sec(f*x + e))^n, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos(e+f\,x)}\right)\,\left(a+\frac{a}{\cos(e+f\,x)}\right)^m\,\left(\frac{c}{\cos(e+f\,x)}\right)^n\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(e + f*x))*(a + a/cos(e + f*x))^m*(c/cos(e + f*x))^n,x)
```

```
[Out] int((A + B/cos(e + f*x))*(a + a/cos(e + f*x))^m*(c/cos(e + f*x))^n, x)
```

3.276 $\int \sec^{-1-n}(c+dx)(a+a\sec(c+dx))^n(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=164

$$\frac{A\sec^{-n}(c+dx)(a+a\sec(c+dx))^n\sin(c+dx)}{d(1+n)}+\frac{(B+An+Bn)\,{}_2F_1\left(\frac{1}{2}-n,-n;1-n;-\frac{2\sec(c+dx)}{1-\sec(c+dx)}\right)\sec^{1-n}}{dn(1+n)(1+}$$

```
[Out] A*(a+a*sec(d*x+c))^n*sin(d*x+c)/d/(1+n)/(sec(d*x+c)^n)+(A*n+B*n+B)*hypergeo
m([-n, 1/2-n],[1-n],-2*sec(d*x+c)/(1-sec(d*x+c)))*sec(d*x+c)^(1-n)*((1+sec(
d*x+c))/(1-sec(d*x+c)))^(1/2-n)*(a+a*sec(d*x+c))^n*sin(d*x+c)/d/n/(1+n)/(1+
sec(d*x+c))
```

Rubi [A]
time = 0.16, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}=0.114$, Rules used = {4098, 3913, 3910, 134}

$$\frac{(An+Bn+B)\sin(c+dx)\sec^{1-n}(c+dx)\left(\frac{\sec(c+dx)+1}{1-\sec(c+dx)}\right)^{\frac{1}{2}-n}(a\sec(c+dx)+a)^n\,{}_2F_1\left(\frac{1}{2}-n,-n;1-n;-\frac{2\sec(c+dx)}{1-\sec(c+dx)}\right)}{dn(n+1)(\sec(c+dx)+1)}+\frac{A\sin(c+dx)\sec^{-n}(c+dx)(a\sec(c+dx)+a)^n}{d(n+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(-1 - n)*(a + a*Sec[c + d*x])^n*(A + B*Sec[c + d*x]),x]
```

```
[Out] (A*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*(1 + n)*Sec[c + d*x]^n) + ((B +
A*n + B*n)*Hypergeometric2F1[1/2 - n, -n, 1 - n, (-2*Sec[c + d*x])/(1 - Sec
[c + d*x])]*Sec[c + d*x]^(1 - n)*((1 + Sec[c + d*x])/(1 - Sec[c + d*x]))^(1
/2 - n)*(a + a*Sec[c + d*x])^n*Sin[c + d*x])/(d*n*(1 + n)*(1 + Sec[c + d*x]
))
```

Rule 134

```
Int[((a_.) + (b_.)*(x_))^(m_)*((c_.) + (d_.)*(x_))^(n_)*((e_.) + (f_.)*(x_)
)^(p_), x_Symbol] :> Simp[((a + b*x)^(m + 1)*(c + d*x)^n*((e + f*x)^(p + 1)
/((b*e - a*f)*(m + 1)))*Hypergeometric2F1[m + 1, -n, m + 2, (-(d*e - c*f))*
((a + b*x)/((b*c - a*d)*(e + f*x)))])/((b*e - a*f)*((c + d*x)/((b*c - a*d)*
(e + f*x))))^n, x] /; FreeQ[{a, b, c, d, e, f, m, n, p}, x] && EqQ[m + n +
p + 2, 0] &&  !IntegerQ[n]
```

Rule 3910

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Dist[(-(a*(d/b))^n)*(Cot[e + f*x]/(a^(n - 2)*f*Sqrt
[a + b*Csc[e + f*x]]*Sqrt[a - b*Csc[e + f*x]])), Subst[Int[(a - x)^(n - 1)*
((2*a - x)^(m - 1/2)/Sqrt[x]), x], x, a - b*Csc[e + f*x]], x] /; FreeQ[{a,
```

```
b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] &&  !IntegerQ[m] && GtQ[a, 0] &&
  !IntegerQ[n] && GtQ[a*(d/b), 0]
```

Rule 3913

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))^(m_), x_Symbol] :> Dist[a^IntPart[m]*((a + b*Csc[e + f*x])^FracPart[m
]/(1 + (b/a)*Csc[e + f*x])^FracPart[m]), Int[(1 + (b/a)*Csc[e + f*x])^m*(d*
Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2
, 0] &&  !IntegerQ[m] &&  !GtQ[a, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \sec^{-1-n}(c+dx)(a+a\sec(c+dx))^n(A+B\sec(c+dx))\,dx = \frac{A\sec^{-n}(c+dx)(a+a\sec(c+dx))^n\sin(c}{d(1+n)}$$

$$= \frac{A\sec^{-n}(c+dx)(a+a\sec(c+dx))^n\sin(c}{d(1+n)}$$

$$= \frac{A\sec^{-n}(c+dx)(a+a\sec(c+dx))^n\sin(c}{d(1+n)}$$

$$= \frac{A\sec^{-n}(c+dx)(a+a\sec(c+dx))^n\sin(c}{d(1+n)}$$

Mathematica [A]
time = 1.16, size = 111, normalized size = 0.68

$$\frac{\left(A+\frac{(B+An+Bn)\left(-\cot^2\left(\frac{1}{2}(c+dx)\right)\right)^{\frac{1}{2}-n}{}_2F_1\left(\frac{1}{2}-n,-n;1-n;\csc^2\left(\frac{1}{2}(c+dx)\right)\right)}{n(1+\cos(c+dx))}\right)\sec^{-n}(c+dx)(a(1+\sec(c+dx)))^n\sin(c+dx)}{d(1+n)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(-1 - n)*(a + a*Sec[c + d*x])^n*(A + B*Sec[c + d*x])
,x]
```

```
[Out] ((A + ((B + A*n + B*n)*(-Cot[(c + d*x)/2]^2)^(1/2 - n)*Hypergeometric2F1[1/
2 - n, -n, 1 - n, Csc[(c + d*x)/2]^2])/(n*(1 + Cos[c + d*x])))*(a*(1 + Sec[
c + d*x]))^n*Sin[c + d*x])/(d*(1 + n)*Sec[c + d*x]^n)
```

Maple [F]
time = 0.52, size = 0, normalized size = 0.00

$$\int \left(\sec^{-1-n}\left(dx+c\right)\right)\left(a+a\sec\left(dx+c\right)\right)^{n}\left(A+B\sec\left(dx+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorith
m="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^n*sec(d*x + c)^(-n - 1)
, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorith
m="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^n*sec(d*x + c)^(-n - 1),
 x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(-1-n)*(a+a*sec(d*x+c))**n*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(-1-n)*(a+a*sec(d*x+c))^n*(A+B*sec(d*x+c)),x, algorith
m="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^n*sec(d*x + c)^(-n - 1)
, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^n}{\left(\frac{1}{\cos(c+d\,x)}\right)^{n+1}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^n)/(1/cos(c + d*x))^(n + 1),
x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^n)/(1/cos(c + d*x))^(n + 1),
 x)
```

3.277 $\int \sec^3(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=114

$$\frac{(4aA+3bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(Ab+aB)\tan(c+dx)}{d}+\frac{(4aA+3bB)\sec(c+dx)\tan(c+dx)}{8d}+\frac{bB\sec^3(\cdots}{}$$

```
[Out] 1/8*(4*A*a+3*B*b)*arctanh(sin(d*x+c))/d+(A*b+B*a)*tan(d*x+c)/d+1/8*(4*A*a+3
*B*b)*sec(d*x+c)*tan(d*x+c)/d+1/4*b*B*sec(d*x+c)^3*tan(d*x+c)/d+1/3*(A*b+B*
a)*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 114, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4082, 3872, 3853, 3855, 3852}

$$\frac{(aB+Ab)\tan^3(c+dx)}{3d}+\frac{(aB+Ab)\tan(c+dx)}{d}+\frac{(4aA+3bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4aA+3bB)\tan(c+dx)\sec(c+dx)}{8d}+\frac{bB\tan(c+dx)\sec^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((4*a*A + 3*b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((A*b + a*B)*Tan[c + d*x])/
d + ((4*a*A + 3*b*B)*Sec[c + d*x]*Tan[c + d*x])/(8*d) + (b*B*Sec[c + d*x]^3
*Tan[c + d*x])/(4*d) + ((A*b + a*B)*Tan[c + d*x]^3)/(3*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{bB\sec^3(c+dx)\tan(c+dx)}{4d}+\frac{1}{4}\int \sec^3(c+dx$$
$$= \frac{bB\sec^3(c+dx)\tan(c+dx)}{4d}+(Ab+aB)\int \mathrm{se}$$
$$= \frac{(4aA+3bB)\sec(c+dx)\tan(c+dx)}{8d}+\frac{bB\sec^3}{}$$
$$= \frac{(4aA+3bB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(Ab+aB)}{}$$

Mathematica [A]
time = 0.67, size = 85, normalized size = 0.75

$$\frac{3(4aA+3bB)\tanh^{-1}(\sin(c+dx))+\sec(c+dx)\left(12aA+9bB+8(Ab+aB)(2+\cos(2(c+dx)))\sec(c+dx)+6bB\sec^2(c+dx)\right)\tan(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*(4*a*A + 3*b*B)*ArcTanh[Sin[c + d*x]] + Sec[c + d*x]*(12*a*A + 9*b*B + 8
*(A*b + a*B)*(2 + Cos[2*(c + d*x)])*Sec[c + d*x] + 6*b*B*Sec[c + d*x]^2)*Ta
n[c + d*x])/(24*d)
```

Maple [A]
time = 0.38, size = 131, normalized size = 1.15

method	result

derivativedivides	$\frac{Aa\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-Ba\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)-Ab\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(}{d}$
default	$\frac{Aa\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-Ba\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)-Ab\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(}{d}$
norman	$\frac{\frac{(4Aa-8Ab-8Ba+5Bb)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{(4Aa+8Ab+8Ba+5Bb)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}-\frac{(12Aa-40Ab-40Ba-9Bb)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{12d}-\frac{(12Aa+}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^4}$
risch	$-\frac{i\left(12Aa\,e^{7i(dx+c)}+9Bb\,e^{7i(dx+c)}+12Aa\,e^{5i(dx+c)}+33Bb\,e^{5i(dx+c)}-48Ab\,e^{4i(dx+c)}-48Ba\,e^{4i(dx+c)}-12Aa\,e^{3i(dx+c)}-\right.}{12d\left(e^{2i(dx+c)}+1\right)^4}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-B*a*(-2/
3-1/3*sec(d*x+c)^2)*tan(d*x+c)-A*b*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+B*b*(
-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)
)))
```

Maxima [A]
time = 0.28, size = 163, normalized size = 1.43

$$\frac{16\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Ba+16\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Ab-3\,Bb\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)-12\,Aa\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/48*(16*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a + 16*(tan(d*x + c)^3 + 3*tan
(d*x + c))*A*b - 3*B*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)
^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) -
 1)) - 12*A*a*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1)
+ log(sin(d*x + c) - 1)))/d
```

Fricas [A]
time = 4.31, size = 136, normalized size = 1.19

$$\frac{3\left(4\,Aa+3\,Bb\right)\cos\left(dx+c\right)^4\log\left(\sin\left(dx+c\right)+1\right)-3\left(4\,Aa+3\,Bb\right)\cos\left(dx+c\right)^4\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\left(Ba+Ab\right)\cos\left(dx+c\right)^3+3\left(4\,Aa+3\,Bb\right)\cos\left(dx+c\right)^2+6\,Bb+8\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/48*(3*(4*A*a + 3*B*b)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(4*A*a + 3
*B*b)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2*(16*(B*a + A*b)*cos(d*x + c
```

```
)^3 + 3*(4*A*a + 3*B*b)*cos(d*x + c)^2 + 6*B*b + 8*(B*a + A*b)*cos(d*x + c)
)*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec{(c + dx)})(a + b\sec{(c + dx)})\sec^3{(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*sec(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 304 vs. 2(106) = 212.
time = 0.47, size = 304, normalized size = 2.67

$$\frac{3\,(4\,Aa + 3\,Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\,(4\,Aa + 3\,Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(12\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 24\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 24\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 + 15\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7 - 12\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 40\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 40\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 + 9\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5 - 12\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 40\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 40\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 9\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 + 12\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 24\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 24\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) + 15\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2 - 1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
[Out] 1/24*(3*(4*A*a + 3*B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(4*A*a + 3*B
*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(12*A*a*tan(1/2*d*x + 1/2*c)^7 -
 24*B*a*tan(1/2*d*x + 1/2*c)^7 - 24*A*b*tan(1/2*d*x + 1/2*c)^7 + 15*B*b*tan
(1/2*d*x + 1/2*c)^7 - 12*A*a*tan(1/2*d*x + 1/2*c)^5 + 40*B*a*tan(1/2*d*x +
1/2*c)^5 + 40*A*b*tan(1/2*d*x + 1/2*c)^5 + 9*B*b*tan(1/2*d*x + 1/2*c)^5 - 1
2*A*a*tan(1/2*d*x + 1/2*c)^3 - 40*B*a*tan(1/2*d*x + 1/2*c)^3 - 40*A*b*tan(1
/2*d*x + 1/2*c)^3 + 9*B*b*tan(1/2*d*x + 1/2*c)^3 + 12*A*a*tan(1/2*d*x + 1/2
*c) + 24*B*a*tan(1/2*d*x + 1/2*c) + 24*A*b*tan(1/2*d*x + 1/2*c) + 15*B*b*ta
n(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 5.73, size = 194, normalized size = 1.70

$$\frac{\left(A\,a - 2\,A\,b - 2\,B\,a + \frac{5\,B\,b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7 + \left(\frac{10\,A\,b}{3} - A\,a + \frac{10\,B\,a}{3} + \frac{3\,B\,b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5 + \left(\frac{3\,B\,b}{4} - \frac{10\,A\,b}{3} - \frac{10\,B\,a}{3} - A\,a\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3 + \left(A\,a + 2\,A\,b + 2\,B\,a + \frac{5\,B\,b}{4}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8 - 4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6 + 6\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4 - 4\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2 + 1\right)} + \frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(A\,a + \frac{3\,B\,b}{4}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/cos(c + d*x)^3,x)
[Out] (tan(c/2 + (d*x)/2)*(A*a + 2*A*b + 2*B*a + (5*B*b)/4) + tan(c/2 + (d*x)/2)^
7*(A*a - 2*A*b - 2*B*a + (5*B*b)/4) - tan(c/2 + (d*x)/2)^3*(A*a + (10*A*b)/
3 + (10*B*a)/3 - (3*B*b)/4) + tan(c/2 + (d*x)/2)^5*((10*A*b)/3 - A*a + (10*
B*a)/3 + (3*B*b)/4))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 -
4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) + (atanh(tan(c/2 + (d*x
)/2))*(A*a + (3*B*b)/4))/d
```

3.278 $\int \sec^2(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=93

$$\frac{(Ab+aB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(3aA+2bB)\tan(c+dx)}{3d}+\frac{(Ab+aB)\sec(c+dx)\tan(c+dx)}{2d}+\frac{bB\sec^2(c+\ldots}{}$$

```
[Out] 1/2*(A*b+B*a)*arctanh(sin(d*x+c))/d+1/3*(3*A*a+2*B*b)*tan(d*x+c)/d+1/2*(A*b
+B*a)*sec(d*x+c)*tan(d*x+c)/d+1/3*b*B*sec(d*x+c)^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.09, antiderivative size = 93, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4082, 3872, 3852, 8, 3853, 3855}

$$\frac{(3aA+2bB)\tan(c+dx)}{3d}+\frac{(aB+Ab)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(aB+Ab)\tan(c+dx)\sec(c+dx)}{2d}+\frac{bB\tan(c+dx)\sec^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((A*b + a*B)*ArcTanh[Sin[c + d*x]])/(2*d) + ((3*a*A + 2*b*B)*Tan[c + d*x])/
(3*d) + ((A*b + a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*d) + (b*B*Sec[c + d*x]^2
*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^2(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx &= \frac{bB\sec^2(c+dx)\tan(c+dx)}{3d}+\frac{1}{3}\int \sec^2(c+dx\\ &= \frac{bB\sec^2(c+dx)\tan(c+dx)}{3d}+(Ab+aB)\int \mathrm{se}\\ &= \frac{(Ab+aB)\sec(c+dx)\tan(c+dx)}{2d}+\frac{bB\sec^2(c}{}\\ &= \frac{(Ab+aB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(3aA+2bB}{3}\end{aligned}$$

Mathematica [A]
time = 0.31, size = 67, normalized size = 0.72

$$\frac{3(Ab+aB)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(6aA+6bB+3(Ab+aB)\sec(c+dx)+2bB\tan^2(c+dx)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*(A*b + a*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(6*a*A + 6*b*B + 3*(A*b
 + a*B)*Sec[c + d*x] + 2*b*B*Tan[c + d*x]^2))/(6*d)
```

Maple [A]
time = 0.31, size = 105, normalized size = 1.13

method	result

derivativedivides	$\frac{Aa\tan(dx+c)+Ba\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+Ab\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-}{d}$
default	$\frac{Aa\tan(dx+c)+Ba\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+Ab\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-}{d}$
norman	$\frac{\frac{4(3Aa+Bb)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{(2Aa-Ab-Ba+2Bb)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{(2Aa+Ab+Ba+2Bb)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}-\frac{(Ab+Ba)\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{2d}$
risch	$-\frac{i\left(3Ab\,e^{5i(dx+c)}+3Ba\,e^{5i(dx+c)}-6Aa\,e^{4i(dx+c)}-12Aa\,e^{2i(dx+c)}-12Bb\,e^{2i(dx+c)}-3Ab\,e^{i(dx+c)}-3Ba\,e^{i(dx+c)}-6Aa-4\right.}{3d\left(e^{2i(dx+c)}+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a*tan(d*x+c)+B*a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*
x+c)))+A*b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-B*b*(-
2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.27, size = 127, normalized size = 1.37

$$\frac{4\left(\tan(dx+c)^3+3\tan(dx+c)\right)Bb-3\,Ba\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-3\,Ab\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+12\,Aa\tan(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*b - 3*B*a*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*A*b*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 12*A*a*tan(d*x + c))/d
```

Fricas [A]
time = 4.15, size = 115, normalized size = 1.24

$$\frac{3\,(Ba+Ab)\cos(dx+c)^3\log(\sin(dx+c)+1)-3\,(Ba+Ab)\cos(dx+c)^3\log(-\sin(dx+c)+1)+2\left(2\,(3\,Aa+2\,Bb)\cos(dx+c)^2+2\,Bb+3\,(Ba+Ab)\cos(dx+c)\right)\sin(dx+c)}{12\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(B*a + A*b)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(B*a + A*b)*co
s(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*(3*A*a + 2*B*b)*cos(d*x + c)^2 +
 2*B*b + 3*(B*a + A*b)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))(a + b\sec(c + dx))\sec^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*sec(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 210 vs. 2(85) = 170.
time = 0.50, size = 210, normalized size = 2.26

$$\frac{3(Ba+Ab)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3(Ba+Ab)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(6Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3Ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-12Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6Aa\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3Ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+6Bb\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(B*a + A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(B*a + A*b)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*A*a*tan(1/2*d*x + 1/2*c)^5 - 3*B*a*tan
(1/2*d*x + 1/2*c)^5 - 3*A*b*tan(1/2*d*x + 1/2*c)^5 + 6*B*b*tan(1/2*d*x + 1/
2*c)^5 - 12*A*a*tan(1/2*d*x + 1/2*c)^3 - 4*B*b*tan(1/2*d*x + 1/2*c)^3 + 6*A
*a*tan(1/2*d*x + 1/2*c) + 3*B*a*tan(1/2*d*x + 1/2*c) + 3*A*b*tan(1/2*d*x +
1/2*c) + 6*B*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 4.39, size = 145, normalized size = 1.56

$$\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\right)(Ab+Ba)}{d}-\frac{(2Aa-Ab-Ba+2Bb)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-4Aa-\frac{4Bb}{3}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+(2Aa+Ab+Ba+2Bb)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/cos(c + d*x)^2,x)
```

```
[Out] (atanh(tan(c/2 + (d*x)/2))*(A*b + B*a))/d - (tan(c/2 + (d*x)/2)*(2*A*a + A*
b + B*a + 2*B*b) - tan(c/2 + (d*x)/2)^3*(4*A*a + (4*B*b)/3) + tan(c/2 + (d*
x)/2)^5*(2*A*a - A*b - B*a + 2*B*b))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2
 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1))
```

3.279 $\int \sec(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=61

$$\frac{(2aA+bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(Ab+aB)\tan(c+dx)}{d}+\frac{bB\sec(c+dx)\tan(c+dx)}{2d}$$

```
[Out] 1/2*(2*A*a+B*b)*arctanh(sin(d*x+c))/d+(A*b+B*a)*tan(d*x+c)/d+1/2*b*B*sec(d*
x+c)*tan(d*x+c)/d
```

Rubi [A]
time = 0.05, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.185$, Rules used = {4082, 3872, 3855, 3852, 8}

$$\frac{(aB+Ab)\tan(c+dx)}{d}+\frac{(2aA+bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{bB\tan(c+dx)\sec(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((2*a*A + b*B)*ArcTanh[Sin[c + d*x]])/(2*d) + ((A*b + a*B)*Tan[c + d*x])/d
+ (b*B*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx &= \frac{bB\sec(c+dx)\tan(c+dx)}{2d}+\frac{1}{2}\int \sec(c+dx)(2\\ &= \frac{bB\sec(c+dx)\tan(c+dx)}{2d}+(Ab+aB)\int \sec^2\\ &= \frac{(2aA+bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{bB\sec(c+d}{}\\ &= \frac{(2aA+bB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{(Ab+aB)\,\mathrm{t}}{d}\end{aligned}$$

Mathematica [A]
time = 0.04, size = 75, normalized size = 1.23

$$\frac{aA\tanh^{-1}(\sin(c+dx))}{d}+\frac{bB\tanh^{-1}(\sin(c+dx))}{2d}+\frac{Ab\tan(c+dx)}{d}+\frac{aB\tan(c+dx)}{d}+\frac{bB\sec(c+dx)\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
[Out] (a*A*ArcTanh[Sin[c + d*x]])/d + (b*B*ArcTanh[Sin[c + d*x]])/(2*d) + (A*b*Ta
n[c + d*x])/d + (a*B*Tan[c + d*x])/d + (b*B*Sec[c + d*x]*Tan[c + d*x])/(2*d
)
```

Maple [A]
time = 0.25, size = 75, normalized size = 1.23

method	result
derivativedivides	$\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))+Ba\tan(dx+c)+Ab\tan(dx+c)+Bb\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$
default	$\frac{Aa\ln(\sec(dx+c)+\tan(dx+c))+Ba\tan(dx+c)+Ab\tan(dx+c)+Bb\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)}{d}$

norman	$\frac{\frac{(2Ab+2Ba+Bb)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-\frac{(2Ab+2Ba-Bb)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{(2Aa+Bb)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2d}+\frac{(2Aa+Bb)\ln\left(\tan\left(\frac{dx}{2}\right.\right.}{2d}$
risch	$-\frac{i\left(Bb\,e^{3i(dx+c)}-2Ab\,e^{2i(dx+c)}-2Ba\,e^{2i(dx+c)}-Bb\,e^{i(dx+c)}-2Ab-2Ba\right)}{d\left(e^{2i(dx+c)}+1\right)^2}-\frac{\ln\left(e^{i(dx+c)}-i\right)Aa}{d}-\frac{\ln\left(e^{i(dx+c)}-i\right)Bb}{2d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a*ln(sec(d*x+c)+tan(d*x+c))+B*a*tan(d*x+c)+A*b*tan(d*x+c)+B*b*(1/2*s
ec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 88, normalized size = 1.44

$$-\frac{Bb\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-4\,Aa\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)-4\,Ba\tan\left(dx+c\right)-4\,Ab\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima
")
```

```
[Out] -1/4*(B*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + lo
g(sin(d*x + c) - 1)) - 4*A*a*log(sec(d*x + c) + tan(d*x + c)) - 4*B*a*tan(d
*x + c) - 4*A*b*tan(d*x + c))/d
```

Fricas [A]
time = 3.41, size = 96, normalized size = 1.57

$$\frac{(2\,Aa+Bb)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(2\,Aa+Bb)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\,(Bb+2\,(Ba+Ab)\cos\left(dx+c\right))\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/4*((2*A*a + B*b)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (2*A*a + B*b)*cos
(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(B*b + 2*(B*a + A*b)*cos(d*x + c))*s
in(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\sec\left(c+dx\right))\left(a+b\sec\left(c+dx\right)\right)\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*sec(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 153 vs. $2(57) = 114$.
time = 0.45, size = 153, normalized size = 2.51

$$\frac{(2\,Aa + Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - (2\,Aa + Bb)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*((2*A*a + B*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (2*A*a + B*b)*log(a
bs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(2*B*a*tan(1/2*d*x + 1/2*c)^3 + 2*A*b*tan
(1/2*d*x + 1/2*c)^3 - B*b*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*
c) - 2*A*b*tan(1/2*d*x + 1/2*c) - B*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x +
1/2*c)^2 - 1)^2)/d
```

Mupad [B]
time = 3.14, size = 104, normalized size = 1.70

$$\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,A\,b+2\,B\,a+B\,b\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(2\,A\,b+2\,B\,a-B\,b\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4-2\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}+\frac{\mathrm{atanh}\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)\,\left(2\,A\,a+B\,b\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/cos(c + d*x),x)
```

```
[Out] (tan(c/2 + (d*x)/2)*(2*A*b + 2*B*a + B*b) - tan(c/2 + (d*x)/2)^3*(2*A*b + 2
*B*a - B*b))/(d*(tan(c/2 + (d*x)/2)^4 - 2*tan(c/2 + (d*x)/2)^2 + 1)) + (ata
nh(tan(c/2 + (d*x)/2))*(2*A*a + B*b))/d
```

3.280 $\int (a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=35

$$aAx+\frac{(Ab+aB)\tanh^{-1}(\sin(c+dx))}{d}+\frac{bB\tan(c+dx)}{d}$$

```
[Out] a*A*x+(A*b+B*a)*arctanh(sin(d*x+c))/d+b*B*tan(d*x+c)/d
```

Rubi [A]
time = 0.02, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {3999, 3852, 8, 3855}

$$\frac{(aB+Ab)\tanh^{-1}(\sin(c+dx))}{d}+aAx+\frac{bB\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] a*A*x + ((A*b + a*B)*ArcTanh[Sin[c + d*x]])/d + (b*B*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3999

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))*(csc[(e_.) + (f_.)*(x_)]*(d_.) +
 (c_)), x_Symbol] :> Simp[a*c*x, x] + (Dist[b*d, Int[Csc[e + f*x]^2, x], x]
 + Dist[b*c + a*d, Int[Csc[e + f*x], x], x]) /; FreeQ[{a, b, c, d, e, f}, x
] && NeQ[b*c - a*d, 0] && NeQ[b*c + a*d, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))(A+B\sec(c+dx))\,dx = aAx+(bB)\int \sec^2(c+dx)\,dx+(Ab+aB)\int \sec(c+dx)\,d$$
$$= aAx+\frac{(Ab+aB)\tanh^{-1}(\sin(c+dx))}{d}-\frac{(bB)\text{Subst}(\int 1\,dx}{}$$
$$= aAx+\frac{(Ab+aB)\tanh^{-1}(\sin(c+dx))}{d}+\frac{bB\tan(c+dx)}{d}$$

Mathematica [**A**]
time = 0.02, size = 43, normalized size = 1.23

$$aAx+\frac{Ab\tanh^{-1}(\sin(c+dx))}{d}+\frac{aB\tanh^{-1}(\sin(c+dx))}{d}+\frac{bB\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] a*A*x + (A*b*ArcTanh[Sin[c + d*x]])/d + (a*B*ArcTanh[Sin[c + d*x]])/d + (b*
B*Tan[c + d*x])/d
```

Maple [**A**]
time = 0.19, size = 57, normalized size = 1.63

method	result
derivativedivides	$\frac{Aa(dx+c)+Ba\ln(\sec(dx+c)+\tan(dx+c))+Ab\ln(\sec(dx+c)+\tan(dx+c))+Bb\tan(dx+c)}{d}$
default	$\frac{Aa(dx+c)+Ba\ln(\sec(dx+c)+\tan(dx+c))+Ab\ln(\sec(dx+c)+\tan(dx+c))+Bb\tan(dx+c)}{d}$
norman	$\frac{aAx\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-aAx-\frac{2Bb\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}}{\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1}+\frac{(Ab+Ba)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{(Ab+Ba)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{d}$
risch	$aAx+\frac{2iBb}{d\left(e^{2i(dx+c)}+1\right)}+\frac{\ln\left(e^{i(dx+c)}+i\right)Ab}{d}+\frac{a\ln\left(e^{i(dx+c)}+i\right)B}{d}-\frac{\ln\left(e^{i(dx+c)}-i\right)Ab}{d}-\frac{a\ln\left(e^{i(dx+c)}-i\right)B}{d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a*(d*x+c)+B*a*ln(sec(d*x+c)+tan(d*x+c))+A*b*ln(sec(d*x+c)+tan(d*x+c)
)+B*b*tan(d*x+c))
```

Maxima [**A**]
time = 0.27, size = 56, normalized size = 1.60

$$\frac{(dx+c)Aa+Ba\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+Ab\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+Bb\tan\left(dx+c\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] ((d*x + c)*A*a + B*a*log(sec(d*x + c) + tan(d*x + c)) + A*b*log(sec(d*x + c
) + tan(d*x + c)) + B*b*tan(d*x + c))/d
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 85 vs. $2(35) = 70$.
time = 4.40, size = 85, normalized size = 2.43

$$\frac{2\,Aadx\cos\left(dx+c\right)+\left(Ba+Ab\right)\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-\left(Ba+Ab\right)\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+2\,Bb\sin\left(dx+c\right)}{2\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/2*(2*A*a*d*x*cos(d*x + c) + (B*a + A*b)*cos(d*x + c)*log(sin(d*x + c) + 1
) - (B*a + A*b)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*B*b*sin(d*x + c))/(
d*cos(d*x + c))
```

Sympy [B] Leaf count of result is larger than twice the leaf count of optimal. 71 vs. $2(32) = 64$.
time = 4.61, size = 71, normalized size = 2.03

$$\begin{cases}\frac{Aa(c+dx)+Ab\log\left(\tan\left(c+dx\right)+\sec\left(c+dx\right)\right)+Ba\log\left(\tan\left(c+dx\right)+\sec\left(c+dx\right)\right)+Bb\tan\left(c+dx\right)}{d} & \text{for } d\neq 0\\ x(A+B\sec\left(c\right))\left(a+b\sec\left(c\right)\right) & \text{otherwise}\end{cases}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Piecewise(((A*a*(c + d*x) + A*b*log(tan(c + d*x) + sec(c + d*x)) + B*a*log(
tan(c + d*x) + sec(c + d*x)) + B*b*tan(c + d*x))/d, Ne(d, 0)), (x*(A + B*se
c(c))*(a + b*sec(c)), True))
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 84 vs. $2(35) = 70$.
time = 0.47, size = 84, normalized size = 2.40

$$\frac{(dx+c)Aa+(Ba+Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(Ba+Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((d*x + c)*A*a + (B*a + A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (B*a + A*
b)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*B*b*tan(1/2*d*x + 1/2*c)/(tan(1/2
*d*x + 1/2*c)^2 - 1))/d
```

Mupad [B]
time = 2.24, size = 114, normalized size = 3.26

$$\frac{2\,A\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{B\,b\,\sin\left(c+d\,x\right)}{d\,\cos\left(c+d\,x\right)}-\frac{A\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,2{}\mathrm{i}}{d}-\frac{B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,2{}\mathrm{i}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*A*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d - (A*b*atan((sin(c/2
+ (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*2i)/d - (B*a*atan((sin(c/2 + (d*x)/2)*1i
)/cos(c/2 + (d*x)/2))*2i)/d + (B*b*sin(c + d*x))/(d*cos(c + d*x))
```

3.281 $\int \cos(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=35

$$(Ab+aB)x+\frac{bB\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\sin(c+dx)}{d}$$

```
[Out] (A*b+B*a)*x+b*B*arctanh(sin(d*x+c))/d+a*A*sin(d*x+c)/d
```

Rubi [A]
time = 0.03, antiderivative size = 35, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 2, integrand size = 27, $\frac{\text{number of rules}}{\text{integrand size}} = 0.074$, Rules used = {4081, 3855}

$$x(aB+Ab)+\frac{aA\sin(c+dx)}{d}+\frac{bB\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (A*b + a*B)*x + (b*B*ArcTanh[Sin[c + d*x]])/d + (a*A*Sin[c + d*x])/d
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx=\frac{aA\sin(c+dx)}{d}-\int(-Ab-aB-bB\sec(c+dx$$

$$=(Ab+aB)x+\frac{aA\sin(c+dx)}{d}+(bB)\int\sec(c+$$

$$=(Ab+aB)x+\frac{bB\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\sin(}{}$$

Mathematica [A]
time = 0.04, size = 46, normalized size = 1.31

$$Abx+aBx+\frac{bB\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\cos(dx)\sin(c)}{d}+\frac{aA\cos(c)\sin(dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
[Out] A*b*x + a*B*x + (b*B*ArcTanh[Sin[c + d*x]])/d + (a*A*Cos[d*x]*Sin[c])/d + (
a*A*Cos[c]*Sin[d*x])/d
```

Maple [A]
time = 0.21, size = 48, normalized size = 1.37

method	result
derivativedivides	$\frac{A\sin(dx+c)a+Ba(dx+c)+Ab(dx+c)+Bb\ln(\sec(dx+c)+\tan(dx+c))}{d}$
default	$\frac{A\sin(dx+c)a+Ba(dx+c)+Ab(dx+c)+Bb\ln(\sec(dx+c)+\tan(dx+c))}{d}$
risch	$Abx+Bax-\frac{iAa\,e^{i(dx+c)}}{2d}+\frac{iAa\,e^{-i(dx+c)}}{2d}+\frac{\ln(e^{i(dx+c)}+i)Bb}{d}-\frac{\ln(e^{i(dx+c)}-i)Bb}{d}$
norman	$\frac{(-Ab-Ba)x+(Ab+Ba)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{2Aa\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}+\frac{2Aa\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{Bb\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{d}-\frac{Bb\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
[Out] 1/d*(A*sin(d*x+c)*a+B*a*(d*x+c)+A*b*(d*x+c)+B*b*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.30, size = 58, normalized size = 1.66

$$\frac{2\,(dx+c)Ba+2\,(dx+c)Ab+Bb(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+2\,Aa\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima
")
[Out] 1/2*(2*(d*x + c)*B*a + 2*(d*x + c)*A*b + B*b*(log(sin(d*x + c) + 1) - log(s
in(d*x + c) - 1)) + 2*A*a*sin(d*x + c))/d
```

Fricas [A]
time = 3.72, size = 54, normalized size = 1.54

$$\frac{2\,(Ba+Ab)dx+Bb\log(\sin(dx+c)+1)-Bb\log(-\sin(dx+c)+1)+2\,Aa\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fricas
")
```

```
[Out] 1/2*(2*(B*a + A*b)*d*x + B*b*log(sin(d*x + c) + 1) - B*b*log(-sin(d*x + c)
+ 1) + 2*A*a*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))(a + b\sec(c + dx))\cos(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*cos(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 79 vs. 2(35) = 70.
time = 0.46, size = 79, normalized size = 2.26

$$\frac{Bb\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - Bb\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + (Ba + Ab)(dx + c) + \frac{2\,Aa\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (B*b*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - B*b*log(abs(tan(1/2*d*x + 1/2*c)
- 1)) + (B*a + A*b)*(d*x + c) + 2*A*a*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1
/2*c)^2 + 1))/d
```

Mupad [B]
time = 2.19, size = 100, normalized size = 2.86

$$\frac{A\,a\,\sin\left(c+d\,x\right)}{d}+\frac{2\,A\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,a\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x)),x)
```

```
[Out] (A*a*sin(c + d*x))/d + (2*A*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
d + (2*B*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*b*atanh(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d
```

3.282 $\int \cos^2(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=52

$$\frac{1}{2}(aA+2bB)x+\frac{(Ab+aB)\sin(c+dx)}{d}+\frac{aA\cos(c+dx)\sin(c+dx)}{2d}$$

```
[Out] 1/2*(A*a+2*B*b)*x+(A*b+B*a)*sin(d*x+c)/d+1/2*a*A*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 52, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.138, Rules used = {4081, 3872, 2717, 8}

$$\frac{(aB+Ab)\sin(c+dx)}{d}+\frac{1}{2}x(aA+2bB)+\frac{aA\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((a*A + 2*b*B)*x)/2 + ((A*b + a*B)*Sin[c + d*x])/d + (a*A*Cos[c + d*x]*Sin[
c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{aA\cos(c+dx)\sin(c+dx)}{2d} - \frac{1}{2}\int \cos(c+dx)(-$$
$$= \frac{aA\cos(c+dx)\sin(c+dx)}{2d} - (-Ab-aB)\int \cos$$
$$= \frac{1}{2}(aA+2bB)x + \frac{(Ab+aB)\sin(c+dx)}{d} + \frac{aA\,\mathrm{c}}{}$$

Mathematica [A]
time = 0.10, size = 51, normalized size = 0.98

$$\frac{2aAc+2aAdx+4bBdx+4(Ab+aB)\sin(c+dx)+aA\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*A*c + 2*a*A*d*x + 4*b*B*d*x + 4*(A*b + a*B)*Sin[c + d*x] + a*A*Sin[2*(
c + d*x)])/(4*d)
```

Maple [A]
time = 0.21, size = 57, normalized size = 1.10

method	result
risch	$\frac{aAx}{2}+xBb+\frac{\sin(dx+c)Ab}{d}+\frac{\sin(dx+c)Ba}{d}+\frac{Aa\sin(2dx+2c)}{4d}$
derivativedivides	$\frac{Aa\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ab\sin(dx+c)+Ba\sin(dx+c)+Bb(dx+c)}{d}$
default	$\frac{Aa\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ab\sin(dx+c)+Ba\sin(dx+c)+Bb(dx+c)}{d}$
norman	$\frac{\left(-\frac{Aa}{2}-Bb\right)x+\left(-\frac{Aa}{2}-Bb\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{Aa}{2}+Bb\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{Aa}{2}+Bb\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{(Aa-2Ab-2}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*b*sin(d*x+c)+B*a*sin(d
*x+c)+B*b*(d*x+c))
```

Maxima [A]
time = 0.29, size = 55, normalized size = 1.06

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa+4\,(dx+c)Bb+4\,Ba\sin(dx+c)+4\,Ab\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a + 4*(d*x + c)*B*b + 4*B*a*sin(d*x
 + c) + 4*A*b*sin(d*x + c))/d
```

Fricas [A]
time = 4.80, size = 42, normalized size = 0.81

$$\frac{(Aa+2\,Bb)dx+(Aa\cos\left(dx+c\right)+2\,Ba+2\,Ab)\sin\left(dx+c\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*((A*a + 2*B*b)*d*x + (A*a*cos(d*x + c) + 2*B*a + 2*A*b)*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(A+B\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)\cos^{2}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*cos(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 121 vs. 2(48) = 96.
time = 0.48, size = 121, normalized size = 2.33

$$\frac{(Aa+2\,Bb)(dx+c)-\frac{2\left(Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-2\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{2}}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((A*a + 2*B*b)*(d*x + c) - 2*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/
2*d*x + 1/2*c)^3 - 2*A*b*tan(1/2*d*x + 1/2*c)^3 - A*a*tan(1/2*d*x + 1/2*c)
- 2*B*a*tan(1/2*d*x + 1/2*c) - 2*A*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1
/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 2.03, size = 50, normalized size = 0.96

$$\frac{A\,a\,x}{2}+B\,b\,x+\frac{A\,b\,\sin\left(c+d\,x\right)}{d}+\frac{B\,a\,\sin\left(c+d\,x\right)}{d}+\frac{A\,a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + b/cos(c + d*x)),x)
[Out] (A*a*x)/2 + B*b*x + (A*b*sin(c + d*x))/d + (B*a*sin(c + d*x))/d + (A*a*sin(
2*c + 2*d*x))/(4*d)
```

3.283 $\int \cos^3(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=84

$$\frac{1}{2}(Ab+aB)x+\frac{(2aA+3bB)\sin(c+dx)}{3d}+\frac{(Ab+aB)\cos(c+dx)\sin(c+dx)}{2d}+\frac{aA\cos^2(c+dx)\sin(c+dx)}{3d}$$

```
[Out] 1/2*(A*b+B*a)*x+1/3*(2*A*a+3*B*b)*sin(d*x+c)/d+1/2*(A*b+B*a)*cos(d*x+c)*sin
(d*x+c)/d+1/3*a*A*cos(d*x+c)^2*sin(d*x+c)/d
```

Rubi [A]
time = 0.08, antiderivative size = 84, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4081, 3872, 2715, 8, 2717}

$$\frac{(2aA+3bB)\sin(c+dx)}{3d}+\frac{(aB+Ab)\sin(c+dx)\cos(c+dx)}{2d}+\frac{1}{2}x(aB+Ab)+\frac{aA\sin(c+dx)\cos^2(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((A*b + a*B)*x)/2 + ((2*a*A + 3*b*B)*Sin[c + d*x])/(3*d) + ((A*b + a*B)*Cos
[c + d*x]*Sin[c + d*x])/(2*d) + (a*A*Cos[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{aA\cos^2(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\int \cos^2(c+dx)($$

$$= \frac{aA\cos^2(c+dx)\sin(c+dx)}{3d} - (-Ab-aB)\int \mathrm{co}$$

$$= \frac{(2aA+3bB)\sin(c+dx)}{3d} + \frac{(Ab+aB)\cos(c+d}{2d}$$

$$= \frac{1}{2}(Ab+aB)x + \frac{(2aA+3bB)\sin(c+dx)}{3d} + \frac{(Ab}{}$$

Mathematica [A]
time = 0.19, size = 75, normalized size = 0.89

$$\frac{6Abc+6aBc+6Abdx+6aBdx+3(3aA+4bB)\sin(c+dx)+3(Ab+aB)\sin(2(c+dx))+aA\sin(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (6*A*b*c + 6*a*B*c + 6*A*b*d*x + 6*a*B*d*x + 3*(3*a*A + 4*b*B)*Sin[c + d*x]
 + 3*(A*b + a*B)*Sin[2*(c + d*x)] + a*A*Sin[3*(c + d*x)])/(12*d)
```

Maple [A]
time = 0.32, size = 85, normalized size = 1.01

method	result
derivativedivides	$\frac{\frac{Aa\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+Ab\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+B\sin(dx+c)b}{d}$
default	$\frac{\frac{Aa\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+Ab\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+Ba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+B\sin(dx+c)b}{d}$
risch	$\frac{Abx}{2}+\frac{Bax}{2}+\frac{3aA\sin(dx+c)}{4d}+\frac{\sin(dx+c)Bb}{d}+\frac{Aa\sin(3dx+3c)}{12d}+\frac{\sin(2dx+2c)Ab}{4d}+\frac{\sin(2dx+2c)Ba}{4d}$

norman	$\frac{\left(-\frac{Ab}{2}-\frac{Ba}{2}\right)x+\left(\frac{Ab}{2}+\frac{Ba}{2}\right)x\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-Ab-Ba)x\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(Ab+Ba)x\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{(2Aa-Ab-B}{}}{\left(1+\tan^{2}\left(\frac{dx}{2}\right.\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(1/3*A*a*(2+cos(d*x+c)^2)*sin(d*x+c)+A*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2
*d*x+1/2*c)+B*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*sin(d*x+c)*b)
```

Maxima [A]
time = 0.27, size = 79, normalized size = 0.94

$$-\frac{4\left(\sin\left(dx+c\right)^{3}-3\sin\left(dx+c\right)\right)Aa-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ba-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ab-12\,Bb\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a - 3*(2*d*x + 2*c + sin(2*d*x
 + 2*c))*B*a - 3*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b - 12*B*b*sin(d*x + c)
)/d
```

Fricas [A]
time = 5.04, size = 60, normalized size = 0.71

$$\frac{3\left(Ba+Ab\right)dx+\left(2\,Aa\cos\left(dx+c\right)^{2}+4\,Aa+6\,Bb+3\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/6*(3*(B*a + A*b)*d*x + (2*A*a*cos(d*x + c)^2 + 4*A*a + 6*B*b + 3*(B*a + A
*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(A+B\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)\cos^{3}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*cos(c + d*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 180 vs. 2(76) = 152.

time = 0.48, size = 180, normalized size = 2.14

$$\frac{3\,(Ba+Ab)(dx+c)+\frac{2\left(6\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+12\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+6\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(B*a + A*b)*(d*x + c) + 2*(6*A*a*tan(1/2*d*x + 1/2*c)^5 - 3*B*a*tan(
1/2*d*x + 1/2*c)^5 - 3*A*b*tan(1/2*d*x + 1/2*c)^5 + 6*B*b*tan(1/2*d*x + 1/2
*c)^5 + 4*A*a*tan(1/2*d*x + 1/2*c)^3 + 12*B*b*tan(1/2*d*x + 1/2*c)^3 + 6*A*
a*tan(1/2*d*x + 1/2*c) + 3*B*a*tan(1/2*d*x + 1/2*c) + 3*A*b*tan(1/2*d*x + 1
/2*c) + 6*B*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [**B**]

time = 2.07, size = 84, normalized size = 1.00

$$\frac{A\,b\,x}{2}+\frac{B\,a\,x}{2}+\frac{3\,A\,a\,\sin\left(c+d\,x\right)}{4\,d}+\frac{B\,b\,\sin\left(c+d\,x\right)}{d}+\frac{A\,a\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{A\,b\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + b/cos(c + d*x)),x)
```

```
[Out] (A*b*x)/2 + (B*a*x)/2 + (3*A*a*sin(c + d*x))/(4*d) + (B*b*sin(c + d*x))/d +
 (A*a*sin(3*c + 3*d*x))/(12*d) + (A*b*sin(2*c + 2*d*x))/(4*d) + (B*a*sin(2*
c + 2*d*x))/(4*d)
```

3.284 $\int \cos^4(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=105

$$\frac{1}{8}(3aA+4bB)x+\frac{(Ab+aB)\sin(c+dx)}{d}+\frac{(3aA+4bB)\cos(c+dx)\sin(c+dx)}{8d}+\frac{aA\cos^3(c+dx)\sin(c+dx)}{4d}$$

```
[Out] 1/8*(3*A*a+4*B*b)*x+(A*b+B*a)*sin(d*x+c)/d+1/8*(3*A*a+4*B*b)*cos(d*x+c)*sin
(d*x+c)/d+1/4*a*A*cos(d*x+c)^3*sin(d*x+c)/d-1/3*(A*b+B*a)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.10, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4081, 3872, 2713, 2715, 8}

$$-\frac{(aB+Ab)\sin^3(c+dx)}{3d}+\frac{(aB+Ab)\sin(c+dx)}{d}+\frac{(3aA+4bB)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(3aA+4bB)+\frac{aA\sin(c+dx)\cos^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((3*a*A + 4*b*B)*x)/8 + ((A*b + a*B)*Sin[c + d*x])/d + ((3*a*A + 4*b*B)*Cos
[c + d*x]*Sin[c + d*x])/(8*d) + (a*A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (
(A*b + a*B)*Sin[c + d*x]^3)/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{aA\cos^3(c+dx)\sin(c+dx)}{4d} - \frac{1}{4}\int \cos^3(c+dx)($$
$$= \frac{aA\cos^3(c+dx)\sin(c+dx)}{4d} - (-Ab-aB)\int \mathrm{co}$$
$$= \frac{(3aA+4bB)\cos(c+dx)\sin(c+dx)}{8d} + \frac{aA\cos^3(}{}$$
$$= \frac{1}{8}(3aA+4bB)x + \frac{(Ab+aB)\sin(c+dx)}{d} + \frac{(3a}{}$$

Mathematica [A]
time = 0.27, size = 91, normalized size = 0.87

$$\frac{36aAc+48bBc+36aAdx+48bBdx+96(Ab+aB)\sin(c+dx)-32(Ab+aB)\sin^3(c+dx)+24(aA+bB)\sin(2(c+dx))+3aA\sin(4(c+dx))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (36*a*A*c + 48*b*B*c + 36*a*A*d*x + 48*b*B*d*x + 96*(A*b + a*B)*Sin[c + d*x
] - 32*(A*b + a*B)*Sin[c + d*x]^3 + 24*(a*A + b*B)*Sin[2*(c + d*x)] + 3*a*A
*Sin[4*(c + d*x)])/(96*d)
```

Maple [A]
time = 0.41, size = 107, normalized size = 1.02

method	result
derivativedivides	$\frac{Aa\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Ab\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+\frac{Ba\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+Bb\left(\frac{\cos}{}\right.}{d}$

default	$\frac{Aa\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{Ab\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+\frac{Ba\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+Bb\left(\frac{c}{}\right.}{d}$
risch	$\frac{3aAx}{8}+\frac{xBb}{2}+\frac{3\sin(dx+c)Ab}{4d}+\frac{3\sin(dx+c)Ba}{4d}+\frac{Aa\sin(4dx+4c)}{32d}+\frac{\sin(3dx+3c)Ab}{12d}+\frac{\sin(3dx+3c)Ba}{12d}+$
norman	$\left(-\frac{3Aa}{8}-\frac{Bb}{2}\right)x+\left(-\frac{9Aa}{8}-\frac{3Bb}{2}\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{3Aa}{4}-Bb\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{3Aa}{4}+Bb\right)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+($

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3*A
*b*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*B*a*(2+cos(d*x+c)^2)*sin(d*x+c)+B*b*(1/2
*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))
```

Maxima [A]
time = 0.30, size = 101, normalized size = 0.96

$$\frac{3\,(12\,dx+12\,c+\sin\left(4\,dx+4\,c\right)+8\,\sin\left(2\,dx+2\,c\right))Aa-32\,\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Ba-32\,\left(\sin\left(dx+c\right)^{3}-3\,\sin\left(dx+c\right)\right)Ab+24\,(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right))Bb}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a - 32*(s
in(d*x + c)^3 - 3*sin(d*x + c))*B*a - 32*(sin(d*x + c)^3 - 3*sin(d*x + c))*
A*b + 24*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*b)/d
```

Fricas [A]
time = 4.31, size = 81, normalized size = 0.77

$$\frac{3\,(3\,Aa+4\,Bb)dx+\left(6\,Aa\cos\left(dx+c\right)^{3}+8\,(Ba+Ab)\cos\left(dx+c\right)^{2}+16\,Ba+16\,Ab+3\,(3\,Aa+4\,Bb)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/24*(3*(3*A*a + 4*B*b)*d*x + (6*A*a*cos(d*x + c)^3 + 8*(B*a + A*b)*cos(d*x
 + c)^2 + 16*B*a + 16*A*b + 3*(3*A*a + 4*B*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(A+B\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)\cos^{4}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*cos(c + d*x)**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 272 vs. 2(97) = 194.
time = 0.47, size = 272, normalized size = 2.59

$$\frac{3\,(3\,Aa+4\,Bb)(dx+c)-\frac{2\left(15\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-24\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-24\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}+12\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{7}-9\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-40\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-40\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+12\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+9\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-40\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-40\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-12\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-15\,Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-12\,Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{4}}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/24*(3*(3*A*a + 4*B*b)*(d*x + c) - 2*(15*A*a*tan(1/2*d*x + 1/2*c)^7 - 24*B
*a*tan(1/2*d*x + 1/2*c)^7 - 24*A*b*tan(1/2*d*x + 1/2*c)^7 + 12*B*b*tan(1/2*
d*x + 1/2*c)^7 - 9*A*a*tan(1/2*d*x + 1/2*c)^5 - 40*B*a*tan(1/2*d*x + 1/2*c)
^5 - 40*A*b*tan(1/2*d*x + 1/2*c)^5 + 12*B*b*tan(1/2*d*x + 1/2*c)^5 + 9*A*a*
tan(1/2*d*x + 1/2*c)^3 - 40*B*a*tan(1/2*d*x + 1/2*c)^3 - 40*A*b*tan(1/2*d*x
 + 1/2*c)^3 - 12*B*b*tan(1/2*d*x + 1/2*c)^3 - 15*A*a*tan(1/2*d*x + 1/2*c) -
 24*B*a*tan(1/2*d*x + 1/2*c) - 24*A*b*tan(1/2*d*x + 1/2*c) - 12*B*b*tan(1/2
*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 2.13, size = 117, normalized size = 1.11

$$\frac{3\,A\,a\,x}{8}+\frac{B\,b\,x}{2}+\frac{3\,A\,b\,\sin\left(c+d\,x\right)}{4\,d}+\frac{3\,B\,a\,\sin\left(c+d\,x\right)}{4\,d}+\frac{A\,a\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{A\,a\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{A\,b\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{B\,a\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{B\,b\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + b/cos(c + d*x)),x)
```

```
[Out] (3*A*a*x)/8 + (B*b*x)/2 + (3*A*b*sin(c + d*x))/(4*d) + (3*B*a*sin(c + d*x))
/(4*d) + (A*a*sin(2*c + 2*d*x))/(4*d) + (A*a*sin(4*c + 4*d*x))/(32*d) + (A*
b*sin(3*c + 3*d*x))/(12*d) + (B*a*sin(3*c + 3*d*x))/(12*d) + (B*b*sin(2*c +
 2*d*x))/(4*d)
```

3.285 $\int \sec^3(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=198

$$\frac{(4a^2A+3Ab^2+6abB)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4b^2B+5a(2Ab+aB))\tan(c+dx)}{5d}+\frac{(4a^2A+3Ab^2+6abB)}{8}$$

```
[Out] 1/8*(4*A*a^2+3*A*b^2+6*B*a*b)*arctanh(sin(d*x+c))/d+1/5*(4*b^2*B+5*a*(2*A*b
+B*a))*tan(d*x+c)/d+1/8*(4*A*a^2+3*A*b^2+6*B*a*b)*sec(d*x+c)*tan(d*x+c)/d+1
/20*b*(5*A*b+6*B*a)*sec(d*x+c)^3*tan(d*x+c)/d+1/5*b*B*sec(d*x+c)^3*(a+b*sec
(d*x+c))*tan(d*x+c)/d+1/15*(4*b^2*B+5*a*(2*A*b+B*a))*tan(d*x+c)^3/d
```

Rubi [A]
time = 0.20, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4111, 4132, 3852, 4131, 3853, 3855}

$$\frac{(4a^2A+6abB+3Ab^2)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4a^2A+6abB+3Ab^2)\tan(c+dx)\sec(c+dx)}{8d}+\frac{(5a(aB+2Ab)+4b^2B)\tan^3(c+dx)}{15d}+\frac{(5a(aB+2Ab)+4b^2B)\tan(c+dx)}{5d}+\frac{b(6aB+5Ab)\tan(c+dx)\sec^3(c+dx)}{20d}+\frac{bB\tan(c+dx)\sec^3(c+dx)(a+b\sec(c+dx))}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((4*a^2*A + 3*A*b^2 + 6*a*b*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*b^2*B + 5
*a*(2*A*b + a*B))*Tan[c + d*x])/(5*d) + ((4*a^2*A + 3*A*b^2 + 6*a*b*B)*Sec[
c + d*x]*Tan[c + d*x])/(8*d) + (b*(5*A*b + 6*a*B)*Sec[c + d*x]^3*Tan[c + d*
x])/(20*d) + (b*B*Sec[c + d*x]^3*(a + b*Sec[c + d*x])*Tan[c + d*x])/(5*d) +
 ((4*b^2*B + 5*a*(2*A*b + a*B))*Tan[c + d*x]^3)/(15*d)
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] && !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}
\int \sec^3(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx &= \frac{bB\sec^3(c+dx)(a+b\sec(c+dx))\tan(c+dx)}{5d} \\
&= \frac{bB\sec^3(c+dx)(a+b\sec(c+dx))\tan(c+dx)}{5d} \\
&= \frac{b(5Ab+6aB)\sec^3(c+dx)\tan(c+dx)}{20d} + \frac{bB\sec \\
&= \frac{(4b^2B+5a(2Ab+aB))\tan(c+dx)}{5d} + \frac{(4a^2A+ \\
&= \frac{(4a^2A+3Ab^2+6abB)\tanh^{-1}(\sin(c+dx))}{8d} + \frac{(
\end{aligned}$$

Mathematica [A]
time = 1.60, size = 150, normalized size = 0.76

$$\frac{15(4a^2A+3Ab^2+6abB)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(15(4a^2A+3Ab^2+6abB)\sec(c+dx)+30b(Ab+2aB)\sec^3(c+dx)+8(15(2aAb+a^2B+b^2B)+5(2aAb+a^2B+2b^2B)\tan^2(c+dx)+3b^2B\tan^4(c+dx))\right)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (15*(4*a^2*A + 3*A*b^2 + 6*a*b*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(15*
(4*a^2*A + 3*A*b^2 + 6*a*b*B)*Sec[c + d*x] + 30*b*(A*b + 2*a*B)*Sec[c + d*x
]^3 + 8*(15*(2*a*A*b + a^2*B + b^2*B) + 5*(2*a*A*b + a^2*B + 2*b^2*B)*Tan[c
 + d*x]^2 + 3*b^2*B*Tan[c + d*x]^4)))/(120*d)
```

Maple [A]
time = 0.44, size = 221, normalized size = 1.12

method	result
derivativedivides	$\frac{a^2A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a^2B\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)-2Aba\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right.}{}$
default	$\frac{a^2A\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)-a^2B\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right)\tan(dx+c)-2Aba\left(-\frac{2}{3}-\frac{\left(\sec^2(dx+c)\right)}{3}\right.}{}$
norman	$-\frac{4\left(50Aba+25a^2B+29b^2B\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{15d}+\frac{\left(4a^2A-16Aba+5A\,b^2-8a^2B+10Bab-8b^2B\right)\left(\tan^9\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}-\frac{\left(4a^2A+16Aba+5A\right.}{}$
risch	$-\frac{i\left(60a^2A\,e^{9i(dx+c)}+45A\,b^2e^{9i(dx+c)}+90Bab\,e^{9i(dx+c)}+120A\,a^2e^{7i(dx+c)}+210A\,b^2e^{7i(dx+c)}+420Bab\,e^{7i(dx+c)}-480\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-a^2*B*
(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)-2*A*b*a*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+
c)+2*B*a*b*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c
)+tan(d*x+c)))+A*b^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln
(sec(d*x+c)+tan(d*x+c)))-b^2*B*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^2)*t
an(d*x+c))
```

Maxima [A]
time = 0.28, size = 276, normalized size = 1.39

$$\frac{80\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Ba^2+160\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Aab+16\left(3\tan\left(dx+c\right)^5+10\tan\left(dx+c\right)^3+15\tan\left(dx+c\right)\right)Bb^2-30\,Bab\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)-15\,Ab^2\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)-60\,Aa^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/240*(80*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2 + 160*(tan(d*x + c)^3 + 3
*tan(d*x + c))*A*a*b + 16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan(d*
```

```
x + c))*B*b^2 - 30*B*a*b*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 15*A*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 -
2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1))
- 60*A*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + l
og(sin(d*x + c) - 1)))/d
```

Fricas [**A**]
time = 4.61, size = 208, normalized size = 1.05

$$\frac{15\left(4Aa^{2}+6Bab+3Ab^{2}\right)\cos\left(dx+c\right)^{5}\log\left(\sin\left(dx+c\right)+1\right)-15\left(4Aa^{2}+6Bab+3Ab^{2}\right)\cos\left(dx+c\right)^{5}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(16\left(5Ba^{2}+10Aab+4Bb^{2}\right)\cos\left(dx+c\right)^{4}+15\left(4Aa^{2}+6Bab+3Ab^{2}\right)\cos\left(dx+c\right)^{3}+24Bb^{2}+8\left(5Ba^{2}+10Aab+4Bb^{2}\right)\cos\left(dx+c\right)^{2}+30\left(2Bab+Ab^{2}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d\cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*cos(d*x + c)^5*log(sin(d*x + c) + 1
) - 15*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*cos(d*x + c)^5*log(-sin(d*x + c) + 1)
+ 2*(16*(5*B*a^2 + 10*A*a*b + 4*B*b^2)*cos(d*x + c)^4 + 15*(4*A*a^2 + 6*B*a
*b + 3*A*b^2)*cos(d*x + c)^3 + 24*B*b^2 + 8*(5*B*a^2 + 10*A*a*b + 4*B*b^2)*
cos(d*x + c)^2 + 30*(2*B*a*b + A*b^2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*
x + c)^5)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(A+B\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)^{2}\sec^{3}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*sec(c + d*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 528 vs. 2(186) = 372.
time = 0.54, size = 528, normalized size = 2.67

$$\frac{15\left(4Aa^{2}+6Bab+3Ab^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-15\left(4Aa^{2}+6Bab+3Ab^{2}\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)+\frac{2\left(60Aa^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-120Ba^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-240Aab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+150Bab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}+75Ab^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-120Bb^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{9}-120Aa^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+320Ba^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+640Aab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-60Bab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-30Ab^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}+160Bb^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{7}-400Ba^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-800Aab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}-464Bb^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{5}+120Aa^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+320Ba^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+640Aab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+60Bab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+30Ab^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}+160Bb^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{3}-60Aa^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-120Ba^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-240Aab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-150Bab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-75Ab^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-120Bb^{2}\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^{2}-1\right)^{5}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))
- 15*(4*A*a^2 + 6*B*a*b + 3*A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(
```

```
60*A*a^2*tan(1/2*d*x + 1/2*c)^9 - 120*B*a^2*tan(1/2*d*x + 1/2*c)^9 - 240*A*
a*b*tan(1/2*d*x + 1/2*c)^9 + 150*B*a*b*tan(1/2*d*x + 1/2*c)^9 + 75*A*b^2*ta
n(1/2*d*x + 1/2*c)^9 - 120*B*b^2*tan(1/2*d*x + 1/2*c)^9 - 120*A*a^2*tan(1/2
*d*x + 1/2*c)^7 + 320*B*a^2*tan(1/2*d*x + 1/2*c)^7 + 640*A*a*b*tan(1/2*d*x
+ 1/2*c)^7 - 60*B*a*b*tan(1/2*d*x + 1/2*c)^7 - 30*A*b^2*tan(1/2*d*x + 1/2*c
)^7 + 160*B*b^2*tan(1/2*d*x + 1/2*c)^7 - 400*B*a^2*tan(1/2*d*x + 1/2*c)^5 -
 800*A*a*b*tan(1/2*d*x + 1/2*c)^5 - 464*B*b^2*tan(1/2*d*x + 1/2*c)^5 + 120*
A*a^2*tan(1/2*d*x + 1/2*c)^3 + 320*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 640*A*a*b
*tan(1/2*d*x + 1/2*c)^3 + 60*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 30*A*b^2*tan(1/
2*d*x + 1/2*c)^3 + 160*B*b^2*tan(1/2*d*x + 1/2*c)^3 - 60*A*a^2*tan(1/2*d*x
+ 1/2*c) - 120*B*a^2*tan(1/2*d*x + 1/2*c) - 240*A*a*b*tan(1/2*d*x + 1/2*c)
- 150*B*a*b*tan(1/2*d*x + 1/2*c) - 75*A*b^2*tan(1/2*d*x + 1/2*c) - 120*B*b^
2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B]
time = 5.71, size = 359, normalized size = 1.81

$$\frac{\operatorname{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{Aa^2}{2}+\frac{3Bab}{4}+\frac{3Ab^2}{8}\right)}{2Aa^2+3Bab+\frac{3Ab^2}{2}}\right)\left(Aa^2+\frac{3Bab}{2}+\frac{3Ab^2}{4}\right)}{d}-\frac{\left(2Ba^2-\frac{5Ab^2}{4}-Aa^2+2Bb^2+4Aab-\frac{5Bab}{2}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(2Aa^2+\frac{Ab^2}{2}-\frac{16Ba^2}{3}-\frac{8Bb^2}{3}-\frac{32Aab}{3}+Bab\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(\frac{20Ba^2}{3}+\frac{40Aab}{3}+\frac{116Bb^2}{15}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-2Aa^2-\frac{Ab^2}{2}-\frac{16Ba^2}{3}-\frac{8Bb^2}{3}-\frac{32Aab}{3}-Bab\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(Aa^2+\frac{5Ab^2}{4}+2Ba^2+2Bb^2+4Aab+\frac{5Bab}{2}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/cos(c + d*x)^3,x)
```

```
[Out] (atanh((4*tan(c/2 + (d*x)/2)*((A*a^2)/2 + (3*A*b^2)/8 + (3*B*a*b)/4))/(2*A*
a^2 + (3*A*b^2)/2 + 3*B*a*b))*(A*a^2 + (3*A*b^2)/4 + (3*B*a*b)/2))/d - (tan
(c/2 + (d*x)/2)^5*((20*B*a^2)/3 + (116*B*b^2)/15 + (40*A*a*b)/3) - tan(c/2
+ (d*x)/2)^9*(A*a^2 + (5*A*b^2)/4 - 2*B*a^2 - 2*B*b^2 - 4*A*a*b + (5*B*a*b)
/2) - tan(c/2 + (d*x)/2)^3*(2*A*a^2 + (A*b^2)/2 + (16*B*a^2)/3 + (8*B*b^2)/
3 + (32*A*a*b)/3 + B*a*b) + tan(c/2 + (d*x)/2)^7*(2*A*a^2 + (A*b^2)/2 - (16
*B*a^2)/3 - (8*B*b^2)/3 - (32*A*a*b)/3 + B*a*b) + tan(c/2 + (d*x)/2)*(A*a^2
 + (5*A*b^2)/4 + 2*B*a^2 + 2*B*b^2 + 4*A*a*b + (5*B*a*b)/2))/(d*(5*tan(c/2
+ (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/
2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.286 $\int \sec^2(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=179

$$\frac{(8aAb+4a^2B+3b^2B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4a^2Ab+4Ab^3-a^3B+8ab^2B)\tan(c+dx)}{6bd}+\frac{(8aAb-2a^2B+$$

[Out] 1/8*(8*A*a*b+4*B*a^2+3*B*b^2)*arctanh(sin(d*x+c))/d+1/6*(4*A*a^2*b+4*A*b^3-B*a^3+8*B*a*b^2)*tan(d*x+c)/b/d+1/24*(8*A*a*b-2*B*a^2+9*B*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/12*(4*A*b-B*a)*(a+b*sec(d*x+c))^2*tan(d*x+c)/b/d+1/4*B*(a+b*sec(d*x+c))^3*tan(d*x+c)/b/d

Rubi [A]
time = 0.22, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4095, 4087, 4082, 3872, 3855, 3852, 8}

$$\frac{(4a^2B+8aAb+3b^2B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(-2a^2B+8aAb+9b^2B)\tan(c+dx)\sec(c+dx)}{24d}+\frac{(a^3(-B)+4a^2Ab+8ab^2B+4Ab^3)\tan(c+dx)}{6bd}+\frac{(4Ab-aB)\tan(c+dx)(a+b\sec(c+dx))^2}{12bd}+\frac{B\tan(c+dx)(a+b\sec(c+dx))^3}{4bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((8*a*A*b + 4*a^2*B + 3*b^2*B)*ArcTanh[Sin[c + d*x]])/(8*d) + ((4*a^2*A*b +
 4*A*b^3 - a^3*B + 8*a*b^2*B)*Tan[c + d*x])/(6*b*d) + ((8*a*A*b - 2*a^2*B +
 9*b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((4*A*b - a*B)*(a + b*Sec[c +
 d*x])^2*Tan[c + d*x])/(12*b*d) + (B*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(
4*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{B(a+b\sec(c+dx))^3\tan(c+dx)}{4bd} + \frac{\int \sec(c+\ldots}{}$$
$$= \frac{(4Ab-aB)(a+b\sec(c+dx))^2\tan(c+dx)}{12bd} + \ldots$$
$$= \frac{(8aAb-2a^2B+9b^2B)\sec(c+dx)\tan(c+dx)}{24d} \ldots$$
$$= \frac{(8aAb-2a^2B+9b^2B)\sec(c+dx)\tan(c+dx)}{24d} \ldots$$
$$= \frac{(8aAb+4a^2B+3b^2B)\tanh^{-1}(\sin(c+dx))}{8d} + \ldots$$
$$= \frac{(8aAb+4a^2B+3b^2B)\tanh^{-1}(\sin(c+dx))}{8d} + \ldots$$

Mathematica [A]
time = 0.78, size = 120, normalized size = 0.67

$$\frac{3(8aAb+4a^2B+3b^2B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(24(a^2A+Ab^2+2abB)+3(8aAb+4a^2B+3b^2B)\sec(c+dx)+6b^2B\sec^3(c+dx)+8b(Ab+2aB)\tan^2(c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
[Out] (3*(8*a*A*b + 4*a^2*B + 3*b^2*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(24*(
a^2*A + A*b^2 + 2*a*b*B) + 3*(8*a*A*b + 4*a^2*B + 3*b^2*B)*Sec[c + d*x] + 6
*b^2*B*Sec[c + d*x]^3 + 8*b*(A*b + 2*a*B)*Tan[c + d*x]^2))/(24*d)
```

Maple [A]
time = 0.45, size = 185, normalized size = 1.03

method	result
derivativedivides	$\frac{a^2A\tan(dx+c)+a^2B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2Aba\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c)}{2}\ldots\right.}{}$
default	$\frac{a^2A\tan(dx+c)+a^2B\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right)+2Aba\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c)}{2}\ldots\right.}{}$
norman	$\frac{-\frac{\left(8a^2A-8Aba+8A\,b^2-4a^2B+16Bab-5b^2B\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{4d}+\frac{\left(8a^2A+8Aba+8A\,b^2+4a^2B+16Bab+5b^2B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{4d}+\frac{(72a^2A\ldots}{}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\ldots\right.}$
risch	$-\frac{i\left(24Aab\,e^{7i(dx+c)}+12B\,a^2e^{7i(dx+c)}+9B\,b^2e^{7i(dx+c)}-24A\,a^2e^{6i(dx+c)}+24Aab\,e^{5i(dx+c)}+12B\,a^2e^{5i(dx+c)}+33B\,b^2e^{5i\ldots}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*tan(d*x+c)+a^2*B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+ta
n(d*x+c)))+2*A*b*a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))
)-2*B*a*b*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)-A*b^2*(-2/3-1/3*sec(d*x+c)^2)*
tan(d*x+c)+b^2*B*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec
(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 228, normalized size = 1.27

$$\frac{32\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)Bab+16\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)Ab^{2}-3\,Bb^{2}\left(\frac{2\left(3\sin(dx+c)^{3}-5\sin(dx+c)\right)}{\sin(dx+c)^{4}-2\sin(dx+c)^{2}+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)-12\,Ba^{2}\left(\frac{2\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-24\,Aab\left(\frac{2\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+48\,Aa^{2}\tan\left(dx+c\right)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/48*(32*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a*b + 16*(tan(d*x + c)^3 + 3*t
an(d*x + c))*A*b^2 - 3*B*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*
x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x
+ c) - 1)) - 12*B*a^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
c) + 1) + log(sin(d*x + c) - 1)) - 24*A*a*b*(2*sin(d*x + c)/(sin(d*x + c)^2
 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*A*a^2*tan(d*x +
 c))/d
```

Fricas [A]
time = 4.80, size = 180, normalized size = 1.01

$$\frac{3\left(4\,Ba^{2}+8\,Aab+3\,Bb^{2}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left(4\,Ba^{2}+8\,Aab+3\,Bb^{2}\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(8\left(3\,Aa^{2}+4\,Bab+2\,Ab^{2}\right)\cos\left(dx+c\right)^{3}+6\,Bb^{2}+3\left(4\,Ba^{2}+8\,Aab+3\,Bb^{2}\right)\cos\left(dx+c\right)^{2}+8\left(2\,Bab+Ab^{2}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48\,d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/48*(3*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*cos(d*x + c)^4*log(sin(d*x + c) + 1)
- 3*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*cos(d*x + c)^4*log(-sin(d*x + c) + 1) + 2
*(8*(3*A*a^2 + 4*B*a*b + 2*A*b^2)*cos(d*x + c)^3 + 6*B*b^2 + 3*(4*B*a^2 + 8
*A*a*b + 3*B*b^2)*cos(d*x + c)^2 + 8*(2*B*a*b + A*b^2)*cos(d*x + c))*sin(d*
x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + B\sec\left(c + dx\right)\right)\left(a + b\sec\left(c + dx\right)\right)^{2}\sec^{2}\left(c + dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*sec(c + d*x)**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. 2(167) = 334.
time = 0.52, size = 478, normalized size = 2.67

$$\frac{3\,(4\,Ba^2+8\,Aab+3\,Bb^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(4\,Ba^2+8\,Aab+3\,Bb^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(24\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-12\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-24\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+48\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+24\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-15\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-72\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+24\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-80\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-9\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+72\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+80\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+40\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-9\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-12\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-48\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-15\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) -
3*(4*B*a^2 + 8*A*a*b + 3*B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(24*
A*a^2*tan(1/2*d*x + 1/2*c)^7 - 12*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 24*A*a*b*t
an(1/2*d*x + 1/2*c)^7 + 48*B*a*b*tan(1/2*d*x + 1/2*c)^7 + 24*A*b^2*tan(1/2*
d*x + 1/2*c)^7 - 15*B*b^2*tan(1/2*d*x + 1/2*c)^7 - 72*A*a^2*tan(1/2*d*x + 1
/2*c)^5 + 12*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 24*A*a*b*tan(1/2*d*x + 1/2*c)^5
 - 80*B*a*b*tan(1/2*d*x + 1/2*c)^5 - 40*A*b^2*tan(1/2*d*x + 1/2*c)^5 - 9*B*
b^2*tan(1/2*d*x + 1/2*c)^5 + 72*A*a^2*tan(1/2*d*x + 1/2*c)^3 + 12*B*a^2*tan
(1/2*d*x + 1/2*c)^3 + 24*A*a*b*tan(1/2*d*x + 1/2*c)^3 + 80*B*a*b*tan(1/2*d*
x + 1/2*c)^3 + 40*A*b^2*tan(1/2*d*x + 1/2*c)^3 - 9*B*b^2*tan(1/2*d*x + 1/2*
c)^3 - 24*A*a^2*tan(1/2*d*x + 1/2*c) - 12*B*a^2*tan(1/2*d*x + 1/2*c) - 24*A
*a*b*tan(1/2*d*x + 1/2*c) - 48*B*a*b*tan(1/2*d*x + 1/2*c) - 24*A*b^2*tan(1/
2*d*x + 1/2*c) - 15*B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1
)^4)/d
```

Mupad [**B**]
time = 5.69, size = 317, normalized size = 1.77

$$\frac{\mathrm{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(\frac{B\,a^2}{2}+A\,a\,b+\frac{3\,B\,b^2}{8}\right)}{2\,B\,a^2+4\,A\,a\,b+\frac{3\,B\,b^2}{2}}\right)\left(B\,a^2+2\,A\,a\,b+\frac{3\,B\,b^2}{4}\right)}{d}-\frac{\left(2\,A\,a^2+2\,A\,b^2-B\,a^2-\frac{5\,B\,b^2}{4}-2\,A\,a\,b+4\,B\,a\,b\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(B\,a^2-\frac{10\,A\,b^2}{3}-6\,A\,a^2-\frac{3\,B\,b^2}{4}+2\,A\,a\,b-\frac{20\,B\,a\,b}{3}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(6\,A\,a^2+\frac{10\,A\,b^2}{3}+B\,a^2-\frac{3\,B\,b^2}{4}+2\,A\,a\,b+\frac{20\,B\,a\,b}{3}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(-2\,A\,a^2-2\,A\,b^2-B\,a^2-\frac{5\,B\,b^2}{4}-2\,A\,a\,b-4\,B\,a\,b\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/cos(c + d*x)^2,x)
```

```
[Out] (atanh((4*tan(c/2 + (d*x)/2)*((B*a^2)/2 + (3*B*b^2)/8 + A*a*b))/(2*B*a^2 +
(3*B*b^2)/2 + 4*A*a*b))*(B*a^2 + (3*B*b^2)/4 + 2*A*a*b))/d - (tan(c/2 + (d*
x)/2)^7*(2*A*a^2 + 2*A*b^2 - B*a^2 - (5*B*b^2)/4 - 2*A*a*b + 4*B*a*b) + tan
(c/2 + (d*x)/2)^3*(6*A*a^2 + (10*A*b^2)/3 + B*a^2 - (3*B*b^2)/4 + 2*A*a*b +
 (20*B*a*b)/3) - tan(c/2 + (d*x)/2)^5*(6*A*a^2 + (10*A*b^2)/3 - B*a^2 + (3*
B*b^2)/4 - 2*A*a*b + (20*B*a*b)/3) - tan(c/2 + (d*x)/2)*(2*A*a^2 + 2*A*b^2
+ B*a^2 + (5*B*b^2)/4 + 2*A*a*b + 4*B*a*b))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*
tan(c/2 + (d*x)/2)^2 - 4*tan(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.287 $\int \sec(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=116

$$\frac{(2a^2A+Ab^2+2abB)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{2(3aAb+a^2B+b^2B)\tan(c+dx)}{3d}+\frac{b(3Ab+2aB)\sec(c+dx)}{6d}$$

```
[Out] 1/2*(2*A*a^2+A*b^2+2*B*a*b)*arctanh(sin(d*x+c))/d+2/3*(3*A*a*b+B*a^2+B*b^2)
*tan(d*x+c)/d+1/6*b*(3*A*b+2*B*a)*sec(d*x+c)*tan(d*x+c)/d+1/3*B*(a+b*sec(d*
x+c))^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.12, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4087, 4082, 3872, 3855, 3852, 8}

$$\frac{2(a^2B+3aAb+b^2B)\tan(c+dx)}{3d}+\frac{(2a^2A+2abB+Ab^2)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b(2aB+3Ab)\tan(c+dx)\sec(c+dx)}{6d}+\frac{B\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((2*a^2*A + A*b^2 + 2*a*b*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (2*(3*a*A*b + a
^2*B + b^2*B)*Tan[c + d*x])/(3*d) + (b*(3*A*b + 2*a*B)*Sec[c + d*x]*Tan[c +
 d*x])/(6*d) + (B*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{B(a+b\sec(c+dx))^2\tan(c+dx)}{3d} + \frac{1}{3}\int \sec(c+$$
$$= \frac{b(3Ab+2aB)\sec(c+dx)\tan(c+dx)}{6d} + \frac{B(a+}{}$$
$$= \frac{b(3Ab+2aB)\sec(c+dx)\tan(c+dx)}{6d} + \frac{B(a+}{}$$
$$= \frac{(2a^2A+Ab^2+2abB)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{b(3}{}$$
$$= \frac{(2a^2A+Ab^2+2abB)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{2(3}{}$$

Mathematica [A]
time = 0.51, size = 92, normalized size = 0.79

$$\frac{3(2a^2A+Ab^2+2abB)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\,(3b(Ab+2aB)\sec(c+dx)+2(6aAb+3a^2B+3b^2B+b^2B\tan^2(c+dx)))}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*(2*a^2*A + A*b^2 + 2*a*b*B)*ArcTanh[Sin[c + d*x]] + Tan[c + d*x]*(3*b*(A
*b + 2*a*B)*Sec[c + d*x] + 2*(6*a*A*b + 3*a^2*B + 3*b^2*B + b^2*B*Tan[c + d
*x]^2)))/(6*d)
```

Maple [A]

time = 0.33, size = 143, normalized size = 1.23

method	result
derivativedivides	$\frac{a^2 A \ln(\sec(dx+c)+\tan(dx+c))+a^2 B \tan(dx+c)+2Aba\tan(dx+c)+2Bab\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right.}{d}$
default	$\frac{a^2 A \ln(\sec(dx+c)+\tan(dx+c))+a^2 B \tan(dx+c)+2Aba\tan(dx+c)+2Bab\left(\frac{\sec(dx+c)\tan(dx+c)}{2}+\frac{\ln(\sec(dx+c)+\tan(dx+c))}{2}\right.}{d}$
norman	$\frac{\frac{4(6Aba+3a^2B+b^2B)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{(4Aba-Ab^2+2a^2B-2Bab+2b^2B)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}-\frac{(4Aba+Ab^2+2a^2B+2Bab+2b^2B)\,\mathrm{ta}}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$
risch	$-\frac{i(3Ab^2 e^{5i(dx+c)}+6Bab\, e^{5i(dx+c)}-12Aab\, e^{4i(dx+c)}-6Ba^2 e^{4i(dx+c)}-24Aab\, e^{2i(dx+c)}-12Ba^2 e^{2i(dx+c)}-12Bb^2 e^{2i(}}{3d\left(e^{2i(dx+c)}+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*A*ln(sec(d*x+c)+tan(d*x+c))+a^2*B*tan(d*x+c)+2*A*b*a*tan(d*x+c)+2*
B*a*b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+A*b^2*(1/2*
sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-b^2*B*(-2/3-1/3*sec(d*
x+c)^2)*tan(d*x+c))
```

Maxima [A]

time = 0.27, size = 165, normalized size = 1.42

$$\frac{4\left(\tan\left(dx+c\right)^3+3\tan\left(dx+c\right)\right)Bb^2-6\,Bab\left(\frac{2\sin\left(dx+c\right)}{\sin\left(dx+c\right)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-3\,Ab^2\left(\frac{2\sin\left(dx+c\right)}{\sin\left(dx+c\right)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+12\,Aa^2\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+12\,Ba^2\tan\left(dx+c\right)+24\,Aab\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*b^2 - 6*B*a*b*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*A*
b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(
d*x + c) - 1)) + 12*A*a^2*log(sec(d*x + c) + tan(d*x + c)) + 12*B*a^2*tan(d
*x + c) + 24*A*a*b*tan(d*x + c))/d
```

Fricas [A]

time = 3.81, size = 150, normalized size = 1.29

$$\frac{3\left(2\,Aa^2+2\,Bab+Ab^2\right)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\left(2\,Aa^2+2\,Bab+Ab^2\right)\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(2\,Bb^2+2\left(3\,Ba^2+6\,Aab+2\,Bb^2\right)\cos\left(dx+c\right)^2+3\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(3*(2*A*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^3*log(sin(d*x + c) + 1) -
3*(2*A*a^2 + 2*B*a*b + A*b^2)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*
B*b^2 + 2*(3*B*a^2 + 6*A*a*b + 2*B*b^2)*cos(d*x + c)^2 + 3*(2*B*a*b + A*b^2
)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))(a + b\sec(c + dx))^2 \sec(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*sec(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 294 vs. 2(108) = 216.
time = 0.52, size = 294, normalized size = 2.53

$$\frac{3\,(2\,Aa^2+2\,Bab+Ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\,(2\,Aa^2+2\,Bab+Ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(2*A*a^2 + 2*B*a*b + A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(
2*A*a^2 + 2*B*a*b + A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(6*B*a^2*
tan(1/2*d*x + 1/2*c)^5 + 12*A*a*b*tan(1/2*d*x + 1/2*c)^5 - 6*B*a*b*tan(1/2*
d*x + 1/2*c)^5 - 3*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*B*b^2*tan(1/2*d*x + 1/2
*c)^5 - 12*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 24*A*a*b*tan(1/2*d*x + 1/2*c)^3 -
 4*B*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^2*tan(1/2*d*x + 1/2*c) + 12*A*a*b*t
an(1/2*d*x + 1/2*c) + 6*B*a*b*tan(1/2*d*x + 1/2*c) + 3*A*b^2*tan(1/2*d*x +
1/2*c) + 6*B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 5.44, size = 227, normalized size = 1.96

$$\frac{\mathrm{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\left(A\,a^2+B\,a\,b+\frac{A\,b^2}{2}\right)}{4\,A\,a^2+4\,B\,a\,b+2\,A\,b^2}\right)\left(2\,A\,a^2+2\,B\,a\,b+A\,b^2\right)}{d}-\frac{\left(2\,B\,a^2-A\,b^2+2\,B\,b^2+4\,A\,a\,b-2\,B\,a\,b\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(-4\,B\,a^2-8\,A\,a\,b-\frac{4\,B\,b^2}{3}\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(A\,b^2+2\,B\,a^2+2\,B\,b^2+4\,A\,a\,b+2\,B\,a\,b\right)\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^6-3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4+3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/cos(c + d*x),x)
```

```
[Out] (atanh((4*tan(c/2 + (d*x)/2)*(A*a^2 + (A*b^2)/2 + B*a*b))/(4*A*a^2 + 2*A*b^
2 + 4*B*a*b))*(2*A*a^2 + A*b^2 + 2*B*a*b))/d - (tan(c/2 + (d*x)/2)*(A*b^2 +
 2*B*a^2 + 2*B*b^2 + 4*A*a*b + 2*B*a*b) - tan(c/2 + (d*x)/2)^3*(4*B*a^2 + (
4*B*b^2)/3 + 8*A*a*b) + tan(c/2 + (d*x)/2)^5*(2*B*a^2 - A*b^2 + 2*B*b^2 + 4
*A*a*b - 2*B*a*b))/(d*(3*tan(c/2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + ta
n(c/2 + (d*x)/2)^6 - 1))
```

3.288 $\int (a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=86

$$a^2Ax+\frac{(4aAb+2a^2B+b^2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b(2Ab+3aB)\tan(c+dx)}{2d}+\frac{bB(a+b\sec(c+dx))\tan(}{2d}$$

```
[Out] a^2*A*x+1/2*(4*A*a*b+2*B*a^2+B*b^2)*arctanh(sin(d*x+c))/d+1/2*b*(2*A*b+3*B*
a)*tan(d*x+c)/d+1/2*b*B*(a+b*sec(d*x+c))*tan(d*x+c)/d
```

Rubi [A]
time = 0.06, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4003, 3855, 3852, 8}

$$\frac{(2a^2B+4aAb+b^2B)\tanh^{-1}(\sin(c+dx))}{2d}+a^2Ax+\frac{b(3aB+2Ab)\tan(c+dx)}{2d}+\frac{bB\tan(c+dx)(a+b\sec(c+dx))}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] a^2*A*x + ((4*a*A*b + 2*a^2*B + b^2*B)*ArcTanh[Sin[c + d*x]])/(2*d) + (b*(2
*A*b + 3*a*B)*Tan[c + d*x])/(2*d) + (b*B*(a + b*Sec[c + d*x])*Tan[c + d*x])
/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4003

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m
+ (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m
- 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
```

```
- a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\int (a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{bB(a+b\sec(c+dx))\tan(c+dx)}{2d} + \frac{1}{2}\int (2a^2A + (4aAb +$$

$$= a^2Ax + \frac{bB(a+b\sec(c+dx))\tan(c+dx)}{2d} + \frac{1}{2}(b(2Ab+3aB$$

$$= a^2Ax + \frac{(4aAb+2a^2B+b^2B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{bB(a-$$

$$= a^2Ax + \frac{(4aAb+2a^2B+b^2B)\tanh^{-1}(\sin(c+dx))}{2d} + \frac{b(2Ab$$

Mathematica [A]
time = 0.29, size = 67, normalized size = 0.78

$$\frac{2a^2Adx + (4aAb+2a^2B+b^2B)\tanh^{-1}(\sin(c+dx)) + b(2Ab+4aB+bB\sec(c+dx))\tan(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
[Out] (2*a^2*A*d*x + (4*a*A*b + 2*a^2*B + b^2*B)*ArcTanh[Sin[c + d*x]] + b*(2*A*b
 + 4*a*B + b*B*Sec[c + d*x])*Tan[c + d*x])/(2*d)
```

Maple [A]
time = 0.30, size = 112, normalized size = 1.30

method	result
derivativedivides	$\frac{a^2A(dx+c)+a^2B\ln(\sec(dx+c)+\tan(dx+c))+2Aba\ln(\sec(dx+c)+\tan(dx+c))+2Bab\tan(dx+c)+Ab^2\tan(dx+c)+b^2B(}{d}$
default	$\frac{a^2A(dx+c)+a^2B\ln(\sec(dx+c)+\tan(dx+c))+2Aba\ln(\sec(dx+c)+\tan(dx+c))+2Bab\tan(dx+c)+Ab^2\tan(dx+c)+b^2B(}{d}$
norman	$\frac{a^2Ax+a^2Ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{b(2Ab+4Ba+Bb)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{d}-2a^2Ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\frac{b(2Ab+4Ba-Bb)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} -$
risch	$a^2Ax - \frac{ib\left(Bb\,e^{3i(dx+c)}-2Ab\,e^{2i(dx+c)}-4Ba\,e^{2i(dx+c)}-Bb\,e^{i(dx+c)}-2Ab-4Ba\right)}{d\left(e^{2i(dx+c)}+1\right)^2} - \frac{2\ln\left(e^{i(dx+c)}-i\right)Aba}{d} - \frac{a^2\ln(e$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*A*(d*x+c)+a^2*B*ln(sec(d*x+c)+tan(d*x+c))+2*A*b*a*ln(sec(d*x+c)+ta
n(d*x+c))+2*B*a*b*tan(d*x+c)+A*b^2*tan(d*x+c)+b^2*B*(1/2*sec(d*x+c)*tan(d*x
+c)+1/2*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.27, size = 126, normalized size = 1.47

$$\frac{4\,(dx+c)Aa^2-Bb^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+4\,Ba^2\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+8\,Aab\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+8\,Bab\tan\left(dx+c\right)+4\,Ab^2\tan\left(dx+c\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/4*(4*(d*x + c)*A*a^2 - B*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(s
in(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 4*B*a^2*log(sec(d*x + c) + tan(
d*x + c)) + 8*A*a*b*log(sec(d*x + c) + tan(d*x + c)) + 8*B*a*b*tan(d*x + c)
 + 4*A*b^2*tan(d*x + c))/d
```

Fricas [A]
time = 4.05, size = 136, normalized size = 1.58

$$\frac{4\,Aa^2dx\cos\left(dx+c\right)^2+\left(2\,Ba^2+4\,Aab+Bb^2\right)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-\left(2\,Ba^2+4\,Aab+Bb^2\right)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(Bb^2+2\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/4*(4*A*a^2*d*x*cos(d*x + c)^2 + (2*B*a^2 + 4*A*a*b + B*b^2)*cos(d*x + c)^
2*log(sin(d*x + c) + 1) - (2*B*a^2 + 4*A*a*b + B*b^2)*cos(d*x + c)^2*log(-s
in(d*x + c) + 1) + 2*(B*b^2 + 2*(2*B*a*b + A*b^2)*cos(d*x + c))*sin(d*x + c
))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A+B\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)^2\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 192 vs. 2(80) = 160.
time = 0.49, size = 192, normalized size = 2.23

$$\frac{2\,(dx+c)Aa^2+\left(2\,Ba^2+4\,Aab+Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-\left(2\,Ba^2+4\,Aab+Bb^2\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(4\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/2*(2*(d*x + c)*A*a^2 + (2*B*a^2 + 4*A*a*b + B*b^2)*log(abs(tan(1/2*d*x +
1/2*c) + 1)) - (2*B*a^2 + 4*A*a*b + B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1
)) - 2*(4*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^2*tan(1/2*d*x + 1/2*c)^3 - B
*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*B*a*b*tan(1/2*d*x + 1/2*c) - 2*A*b^2*tan(1/
2*d*x + 1/2*c) - B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^2
)/d
```

Mupad [B]
time = 2.74, size = 176, normalized size = 2.05

$$\frac{2\left(A\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+B\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+\frac{B\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+2\,A\,a\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\right)}{d}+\frac{\frac{A\,b^2\,\sin(2\,c+2\,d\,x)}{2}+\frac{B\,b^2\,\sin(c+d\,x)}{2}+B\,a\,b\,\sin\left(2\,c+2\,d\,x\right)}{d\,\left(\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*(A*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + B*a^2*atanh(sin(c/2
 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (B*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 +
 (d*x)/2)))/2 + 2*A*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/d +
((A*b^2*sin(2*c + 2*d*x))/2 + (B*b^2*sin(c + d*x))/2 + B*a*b*sin(2*c + 2*d*
x))/(d*(cos(2*c + 2*d*x)/2 + 1/2))
```

3.289 $\int \cos(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=60

$$a(2Ab+aB)x+\frac{b(Ab+2aB)\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2A\sin(c+dx)}{d}+\frac{b^2B\tan(c+dx)}{d}$$

```
[Out] a*(2*A*b+B*a)*x+b*(A*b+2*B*a)*arctanh(sin(d*x+c))/d+a^2*A*sin(d*x+c)/d+b^2*
B*tan(d*x+c)/d
```

Rubi [A]
time = 0.07, antiderivative size = 60, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4109, 3855, 3852, 8}

$$\frac{a^2A\sin(c+dx)}{d}+\frac{b(2aB+Ab)\tanh^{-1}(\sin(c+dx))}{d}+ax(aB+2Ab)+\frac{b^2B\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] a*(2*A*b + a*B)*x + (b*(A*b + 2*a*B)*ArcTanh[Sin[c + d*x]])/d + (a^2*A*Sin[
c + d*x])/d + (b^2*B*Tan[c + d*x])/d
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4109

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a^2*A*Cos[
e + f*x]*((d*Csc[e + f*x])^(n + 1)/(d*f*n)), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
```

```
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{a^2A\sin(c+dx)}{d} - \int \left(-a(2Ab+aB)+\left(-Ab^2\right.\right.$$
$$= a(2Ab+aB)x + \frac{a^2A\sin(c+dx)}{d} + \left(b^2B\right)\int \sec$$
$$= a(2Ab+aB)x + \frac{b(Ab+2aB)\tanh^{-1}(\sin(c+dx}{d}$$
$$= a(2Ab+aB)x + \frac{b(Ab+2aB)\tanh^{-1}(\sin(c+dx}{d}$$

Mathematica [A]
time = 0.52, size = 109, normalized size = 1.82

$$\frac{a(2Ab+aB)(c+dx) - b(Ab+2aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + b(Ab+2aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + a^2A\sin(c+dx) + b^2B\tan(c+dx)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(2*A*b + a*B)*(c + d*x) - b*(A*b + 2*a*B)*Log[Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]] + b*(A*b + 2*a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + a^2
*A*Sin[c + d*x] + b^2*B*Tan[c + d*x])/d
```

Maple [A]
time = 0.29, size = 86, normalized size = 1.43

method	result
derivativedivides	$\frac{a^2A\sin(dx+c)+a^2B(dx+c)+2Aba(dx+c)+2Bab\ln(\sec(dx+c)+\tan(dx+c))+A\,b^2\ln(\sec(dx+c)+\tan(dx+c))+b^2B\tan(d}{d}$
default	$\frac{a^2A\sin(dx+c)+a^2B(dx+c)+2Aba(dx+c)+2Bab\ln(\sec(dx+c)+\tan(dx+c))+A\,b^2\ln(\sec(dx+c)+\tan(dx+c))+b^2B\tan(d}{d}$
risch	$2Aabx+B\,a^2x-\frac{ia^2A\,e^{i(dx+c)}}{2d}+\frac{ia^2A\,e^{-i(dx+c)}}{2d}+\frac{2ib^2B}{d\left(e^{2i(dx+c)}+1\right)}-\frac{\ln\left(e^{i(dx+c)}-i\right)A\,b^2}{d}-\frac{2\ln\left(e^{i(dx+c)}\right.}{d}$
norman	$\frac{(2Aba+a^2B)x+(-2Aba-a^2B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-2Aba-a^2B)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(2Aba+a^2B)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(a^2*A*sin(d*x+c)+a^2*B*(d*x+c)+2*A*b*a*(d*x+c)+2*B*a*b*ln(sec(d*x+c)+t
an(d*x+c))+A*b^2*ln(sec(d*x+c)+tan(d*x+c))+b^2*B*tan(d*x+c))
```

Maxima [A]
time = 0.29, size = 103, normalized size = 1.72

$$\frac{2\,(dx+c)Ba^2+4\,(dx+c)Aab+2\,Bab(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+Ab^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+2\,Aa^2\sin(dx+c)+2\,Bb^2\tan(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/2*(2*(d*x + c)*B*a^2 + 4*(d*x + c)*A*a*b + 2*B*a*b*(log(sin(d*x + c) + 1)
 - log(sin(d*x + c) - 1)) + A*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c)
 - 1)) + 2*A*a^2*sin(d*x + c) + 2*B*b^2*tan(d*x + c))/d
```

Fricas [A]
time = 3.87, size = 117, normalized size = 1.95

$$\frac{2\,(Ba^2+2\,Aab)dx\cos(dx+c)+(2\,Bab+Ab^2)\cos(dx+c)\log(\sin(dx+c)+1)-(2\,Bab+Ab^2)\cos(dx+c)\log(-\sin(dx+c)+1)+2\,(Aa^2\cos(dx+c)+Bb^2)\sin(dx+c)}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/2*(2*(B*a^2 + 2*A*a*b)*d*x*cos(d*x + c) + (2*B*a*b + A*b^2)*cos(d*x + c)*
log(sin(d*x + c) + 1) - (2*B*a*b + A*b^2)*cos(d*x + c)*log(-sin(d*x + c) +
1) + 2*(A*a^2*cos(d*x + c) + B*b^2)*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\sec(c+dx))\,(a+b\sec(c+dx))^2\cos(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*cos(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 154 vs. 2(60) = 120.
time = 0.49, size = 154, normalized size = 2.57

$$\frac{(Ba^2+2\,Aab)(dx+c)+(2\,Bab+Ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(2\,Bab+Ab^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+\frac{2\left(Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] ((B*a^2 + 2*A*a*b)*(d*x + c) + (2*B*a*b + A*b^2)*log(abs(tan(1/2*d*x + 1/2*
c) + 1)) - (2*B*a*b + A*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) + 2*(A*a^2*
tan(1/2*d*x + 1/2*c)^3 - B*b^2*tan(1/2*d*x + 1/2*c)^3 - A*a^2*tan(1/2*d*x +
 1/2*c) - B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1))/d
```

Mupad [B]
time = 2.55, size = 163, normalized size = 2.72

$$\frac{B\,b^2\tan(c+d\,x)}{d}+\frac{2\,B\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,A\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{A\,a^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d\,\cos\left(c+d\,x\right)}+\frac{4\,A\,a\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,B\,a\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (B*b^2*tan(c + d*x))/d + (2*B*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)))/d + (2*A*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*a^2*s
in(2*c + 2*d*x))/(2*d*cos(c + d*x)) + (4*A*a*b*atan(sin(c/2 + (d*x)/2)/cos(
c/2 + (d*x)/2)))/d + (4*B*a*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))
/d
```

3.290 $\int \cos^2(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=80

$$\frac{1}{2}\left(a^2A+2Ab^2+4abB\right)x+\frac{b^2B\tanh^{-1}(\sin(c+dx))}{d}+\frac{a(2Ab+aB)\sin(c+dx)}{d}+\frac{a^2A\cos(c+dx)\sin(c+dx}{2d}$$

```
[Out] 1/2*(A*a^2+2*A*b^2+4*B*a*b)*x+b^2*B*arctanh(sin(d*x+c))/d+a*(2*A*b+B*a)*sin
(d*x+c)/d+1/2*a^2*A*cos(d*x+c)*sin(d*x+c)/d
```

Rubi [A]
time = 0.13, antiderivative size = 80, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4109, 4132, 8, 4130, 3855}

$$\frac{1}{2}x\left(a^2A+4abB+2Ab^2\right)+\frac{a^2A\sin(c+dx)\cos(c+dx)}{2d}+\frac{a(aB+2Ab)\sin(c+dx)}{d}+\frac{b^2B\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((a^2*A + 2*A*b^2 + 4*a*b*B)*x)/2 + (b^2*B*ArcTanh[Sin[c + d*x]])/d + (a*(2
*A*b + a*B)*Sin[c + d*x])/d + (a^2*A*Cos[c + d*x]*Sin[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4109

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a^2*A*Cos[
e + f*x]*((d*Csc[e + f*x])^(n + 1)/(d*f*n)), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
```

```
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{a^2A\cos(c+dx)\sin(c+dx)}{2d} - \frac{1}{2}\int \cos(c+dx)$$

$$= \frac{a^2A\cos(c+dx)\sin(c+dx)}{2d} - \frac{1}{2}\int \cos(c+dx)$$

$$= \frac{1}{2}\left(a^2A+2Ab^2+4abB\right)x + \frac{a(2Ab+aB)\sin(c}{d}$$

$$= \frac{1}{2}\left(a^2A+2Ab^2+4abB\right)x + \frac{b^2B\tanh^{-1}(\sin(c+}{d}$$

Mathematica [A]
time = 0.23, size = 120, normalized size = 1.50

$$\frac{2(a^2A+2Ab^2+4abB)(c+dx)-4b^2B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4b^2B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+4a(2Ab+aB)\sin(c+dx)+a^2A\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2*A + 2*A*b^2 + 4*a*b*B)*(c + d*x) - 4*b^2*B*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 4*b^2*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 4*a*(
2*A*b + a*B)*Sin[c + d*x] + a^2*A*Sin[2*(c + d*x)])/(4*d)
```

Maple [A]
time = 0.23, size = 94, normalized size = 1.18

method	result
derivativedivides	$\frac{a^2A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2B\sin(dx+c)+2Aba\sin(dx+c)+2Bab(dx+c)+Ab^2(dx+c)+b^2B\ln(\sec(dx+c)+\tan}{d}$
default	$\frac{a^2A\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2B\sin(dx+c)+2Aba\sin(dx+c)+2Bab(dx+c)+Ab^2(dx+c)+b^2B\ln(\sec(dx+c)+\tan}{d}$

risch	$\frac{a^2Ax}{2}+xAb^2+2xBab-\frac{ie^{i(dx+c)}Aba}{d}-\frac{ie^{i(dx+c)}a^2B}{2d}+\frac{ie^{-i(dx+c)}Aba}{d}+\frac{ie^{-i(dx+c)}a^2B}{2d}+\frac{\ln(e^{i(dx+}}{}$
norman	$\frac{(\frac{1}{2}a^2A+Ab^2+2Bab)x+(\frac{1}{2}a^2A+Ab^2+2Bab)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-a^2A-2Ab^2-4Bab)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{a(Aa+4Ab+}{}}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^2*B*sin(d*x+c)+2*A*b
*a*sin(d*x+c)+2*B*a*b*(d*x+c)+A*b^2*(d*x+c)+b^2*B*ln(sec(d*x+c)+tan(d*x+c))
)
```

Maxima [A]
time = 0.28, size = 99, normalized size = 1.24

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^2+8\,(dx+c)Bab+4\,(dx+c)Ab^2+2\,Bb^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+4\,Ba^2\sin(dx+c)+8\,Aab\sin(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2 + 8*(d*x + c)*B*a*b + 4*(d*x +
c)*A*b^2 + 2*B*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*B*a^
2*sin(d*x + c) + 8*A*a*b*sin(d*x + c))/d
```

Fricas [A]
time = 4.41, size = 87, normalized size = 1.09

$$\frac{Bb^2\log(\sin(dx+c)+1)-Bb^2\log(-\sin(dx+c)+1)+(Aa^2+4\,Bab+2\,Ab^2)dx+(Aa^2\cos(dx+c)+2\,Ba^2+4\,Aab)\sin(dx+c)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2*(B*b^2*log(sin(d*x + c) + 1) - B*b^2*log(-sin(d*x + c) + 1) + (A*a^2 +
4*B*a*b + 2*A*b^2)*d*x + (A*a^2*cos(d*x + c) + 2*B*a^2 + 4*A*a*b)*sin(d*x +
 c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int(A+B\sec(c+dx))\,(a+b\sec(c+dx))^2\cos^2(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*cos(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 178 vs. $2(76) = 152$.
time = 0.52, size = 178, normalized size = 2.22

$$\frac{2\,Bb^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-2\,Bb^2\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+(Aa^2+4\,Bab+2\,Ab^2)(dx+c)-\frac{2\left(Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2*(2*B*b^2*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 2*B*b^2*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + (A*a^2 + 4*B*a*b + 2*A*b^2)*(d*x + c) - 2*(A*a^2*tan(1/
2*d*x + 1/2*c)^3 - 2*B*a^2*tan(1/2*d*x + 1/2*c)^3 - 4*A*a*b*tan(1/2*d*x + 1
/2*c)^3 - A*a^2*tan(1/2*d*x + 1/2*c) - 2*B*a^2*tan(1/2*d*x + 1/2*c) - 4*A*a
*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^2)/d
```

Mupad [B]
time = 2.38, size = 169, normalized size = 2.11

$$\frac{B\,a^2\,\sin\left(c+d\,x\right)}{d}+\frac{A\,a^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,A\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{A\,a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{2\,A\,a\,b\,\sin\left(c+d\,x\right)}{d}+\frac{4\,B\,a\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (B*a^2*sin(c + d*x))/d + (A*a^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))
)/d + (2*A*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (2*B*b^2*at
anh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*a^2*sin(2*c + 2*d*x))/(4
*d) + (2*A*a*b*sin(c + d*x))/d + (4*B*a*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 +
 (d*x)/2)))/d
```

3.291 $\int \cos^3(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=107

$$\frac{1}{2}\left(2aAb+a^2B+2b^2B\right)x+\frac{(2a^2A+3Ab^2+6abB)\sin(c+dx)}{3d}+\frac{a(2Ab+aB)\cos(c+dx)\sin(c+dx)}{2d}+\frac{a^2A$$

```
[Out] 1/2*(2*A*a*b+B*a^2+2*B*b^2)*x+1/3*(2*A*a^2+3*A*b^2+6*B*a*b)*sin(d*x+c)/d+1/
2*a*(2*A*b+B*a)*cos(d*x+c)*sin(d*x+c)/d+1/3*a^2*A*cos(d*x+c)^2*sin(d*x+c)/d
```

Rubi [A]
time = 0.15, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4109, 4132, 2717, 4130, 8}

$$\frac{(2a^2A+6abB+3Ab^2)\sin(c+dx)}{3d}+\frac{1}{2}x\left(a^2B+2aAb+2b^2B\right)+\frac{a^2A\sin(c+dx)\cos^2(c+dx)}{3d}+\frac{a(aB+2Ab)\sin(c+dx)\cos(c+dx)}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((2*a*A*b + a^2*B + 2*b^2*B)*x)/2 + ((2*a^2*A + 3*A*b^2 + 6*a*b*B)*Sin[c +
d*x])/(3*d) + (a*(2*A*b + a*B)*Cos[c + d*x]*Sin[c + d*x])/(2*d) + (a^2*A*Co
s[c + d*x]^2*Sin[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4109

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a^2*A*Cos[
e + f*x]*((d*Csc[e + f*x])^(n + 1)/(d*f*n)), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
```

```
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{a^2A\cos^2(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\int \cos^2(c+dx$$

$$= \frac{a^2A\cos^2(c+dx)\sin(c+dx)}{3d} - \frac{1}{3}\int \cos^2(c+dx$$

$$= \frac{(2a^2A+3Ab^2+6abB)\sin(c+dx)}{3d} + \frac{a(2Ab+$$

$$= \frac{1}{2}\left(2aAb+a^2B+2b^2B\right)x + \frac{(2a^2A+3Ab^2+6a}{3d}$$

Mathematica [A]
time = 0.25, size = 90, normalized size = 0.84

$$\frac{6(2aAb+a^2B+2b^2B)(c+dx)+3(3a^2A+4Ab^2+8abB)\sin(c+dx)+3a(2Ab+aB)\sin(2(c+dx))+a^2A\sin(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
[Out] (6*(2*a*A*b + a^2*B + 2*b^2*B)*(c + d*x) + 3*(3*a^2*A + 4*A*b^2 + 8*a*b*B)*
Sin[c + d*x] + 3*a*(2*A*b + a*B)*Sin[2*(c + d*x)] + a^2*A*Sin[3*(c + d*x)])
/(12*d)
```

Maple [A]
time = 0.32, size = 114, normalized size = 1.07

method	result
derivativedivides	$\frac{\frac{a^2A\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+2Aba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+A\,b^2\sin(dx+c)+2}{d}$
default	$\frac{\frac{a^2A\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+2Aba\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^2B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+A\,b^2\sin(dx+c)+2}{d}$

risch	$Aabx+\frac{B\,a^2x}{2}+x\,b^2B+\frac{3a^2A\sin(dx+c)}{4d}+\frac{\sin(dx+c)A\,b^2}{d}+\frac{2\sin(dx+c)Bab}{d}+\frac{a^2A\sin(3dx+3c)}{12d}+\sin($
norman	$\frac{(Aba+\frac{1}{2}a^2B+b^2B)x+(Aba+\frac{1}{2}a^2B+b^2B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(Aba+\frac{1}{2}a^2B+b^2B)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(Aba+\frac{1}{2}a^2B+b^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/3*a^2*A*(2+cos(d*x+c)^2)*sin(d*x+c)+2*A*b*a*(1/2*cos(d*x+c)*sin(d*x+
c)+1/2*d*x+1/2*c)+a^2*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*b^2*sin
(d*x+c)+2*B*a*b*sin(d*x+c)+b^2*B*(d*x+c))
```

Maxima [A]
time = 0.27, size = 108, normalized size = 1.01

$$-\frac{4\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)Aa^2-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ba^2-6\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aab-12\left(dx+c\right)Bb^2-24\,Bab\sin\left(dx+c\right)-12\,Ab^2\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2 - 3*(2*d*x + 2*c + sin(2*d
*x + 2*c))*B*a^2 - 6*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a*b - 12*(d*x + c)*
B*b^2 - 24*B*a*b*sin(d*x + c) - 12*A*b^2*sin(d*x + c))/d
```

Fricas [A]
time = 4.20, size = 85, normalized size = 0.79

$$\frac{3\left(Ba^2+2\,Aab+2\,Bb^2\right)dx+\left(2\,Aa^2\cos\left(dx+c\right)^2+4\,Aa^2+12\,Bab+6\,Ab^2+3\left(Ba^2+2\,Aab\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(B*a^2 + 2*A*a*b + 2*B*b^2)*d*x + (2*A*a^2*cos(d*x + c)^2 + 4*A*a^2
+ 12*B*a*b + 6*A*b^2 + 3*(B*a^2 + 2*A*a*b)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\left(A+B\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)^2\cos^3\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*cos(c + d*x)**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 254 vs. 2(99) = 198.
time = 0.46, size = 254, normalized size = 2.37

$$\frac{3\,(Ba^2+2\,Aab+2\,Bb^2)(dx+c)+\frac{2\left(6\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-6\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+24\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+12\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/6*(3*(B*a^2 + 2*A*a*b + 2*B*b^2)*(d*x + c) + 2*(6*A*a^2*tan(1/2*d*x + 1/2
*c)^5 - 3*B*a^2*tan(1/2*d*x + 1/2*c)^5 - 6*A*a*b*tan(1/2*d*x + 1/2*c)^5 + 1
2*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*A*a^2*t
an(1/2*d*x + 1/2*c)^3 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^2*tan(1/2*
d*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2*tan(1/2*d*x + 1/2*c
) + 6*A*a*b*tan(1/2*d*x + 1/2*c) + 12*B*a*b*tan(1/2*d*x + 1/2*c) + 6*A*b^2*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 2.10, size = 115, normalized size = 1.07

$$\frac{B\,a^2\,x}{2}+B\,b^2\,x+\frac{3\,A\,a^2\,\sin\left(c+d\,x\right)}{4\,d}+\frac{A\,b^2\,\sin\left(c+d\,x\right)}{d}+A\,a\,b\,x+\frac{A\,a^2\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{B\,a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{2\,B\,a\,b\,\sin\left(c+d\,x\right)}{d}+\frac{A\,a\,b\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (B*a^2*x)/2 + B*b^2*x + (3*A*a^2*sin(c + d*x))/(4*d) + (A*b^2*sin(c + d*x))
/d + A*a*b*x + (A*a^2*sin(3*c + 3*d*x))/(12*d) + (B*a^2*sin(2*c + 2*d*x))/(
4*d) + (2*B*a*b*sin(c + d*x))/d + (A*a*b*sin(2*c + 2*d*x))/(2*d)
```

3.292 $\int \cos^4(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=136

$$\frac{1}{8}\left(3a^2A+4Ab^2+8abB\right)x+\frac{(2aAb+a^2B+b^2B)\sin(c+dx)}{d}+\frac{(3a^2A+4Ab^2+8abB)\cos(c+dx)\sin(c+dx)}{8d}$$

```
[Out] 1/8*(3*A*a^2+4*A*b^2+8*B*a*b)*x+(2*A*a*b+B*a^2+B*b^2)*sin(d*x+c)/d+1/8*(3*A
*a^2+4*A*b^2+8*B*a*b)*cos(d*x+c)*sin(d*x+c)/d+1/4*a^2*A*cos(d*x+c)^3*sin(d*
x+c)/d-1/3*a*(2*A*b+B*a)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.18, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4109, 4132, 2715, 8, 4129, 3092}

$$\frac{(a^2B+2aAb+b^2B)\sin(c+dx)}{d}+\frac{(3a^2A+8abB+4Ab^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x(3a^2A+8abB+4Ab^2)+\frac{a^2A\sin(c+dx)\cos^3(c+dx)}{4d}-\frac{a(aB+2Ab)\sin^3(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((3*a^2*A + 4*A*b^2 + 8*a*b*B)*x)/8 + ((2*a*A*b + a^2*B + b^2*B)*Sin[c + d*
x])/d + ((3*a^2*A + 4*A*b^2 + 8*a*b*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (
a^2*A*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) - (a*(2*A*b + a*B)*Sin[c + d*x]^3)
/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 4109

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a^2*A*Cos[
e + f*x]*((d*Csc[e + f*x])^(n + 1)/(d*f*n)), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4129

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx &= \frac{a^2A\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{1}{4}\int \cos^3(c+dx\\ &= \frac{a^2A\cos^3(c+dx)\sin(c+dx)}{4d}-\frac{1}{4}\int \cos^3(c+dx\\ &= \frac{(3a^2A+4Ab^2+8abB)\cos(c+dx)\sin(c+dx)}{8d}\\ &= \frac{1}{8}\left(3a^2A+4Ab^2+8abB\right)x+\frac{(3a^2A+4Ab^2+8\\ &= \frac{1}{8}\left(3a^2A+4Ab^2+8abB\right)x+\frac{(2aAb+a^2B+b^2}{d}\end{aligned}$$

Mathematica [**A**]
time = 0.47, size = 118, normalized size = 0.87

$$\frac{12(3a^2A+4Ab^2+8abB)(c+dx)+24(6aAb+3a^2B+4b^2B)\sin(c+dx)+24(a^2A+Ab^2+2abB)\sin(2(c+dx))+8a(2Ab+aB)\sin(3(c+dx))+3a^2A\sin(4(c+dx))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (12*(3*a^2*A + 4*A*b^2 + 8*a*b*B)*(c + d*x) + 24*(6*a*A*b + 3*a^2*B + 4*b^2
*B)*Sin[c + d*x] + 24*(a^2*A + A*b^2 + 2*a*b*B)*Sin[2*(c + d*x)] + 8*a*(2*A
*b + a*B)*Sin[3*(c + d*x)] + 3*a^2*A*Sin[4*(c + d*x)])/(96*d)
```

Maple [A]
time = 0.44, size = 152, normalized size = 1.12

method	result
derivativedivides	$\frac{a^2A\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^2B\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+\frac{2Aba\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+2}{d}$
default	$\frac{a^2A\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+\frac{a^2B\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+\frac{2Aba\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+2}{d}$
risch	$\frac{3a^2Ax}{8}+\frac{xAb^2}{2}+xBab+\frac{3\sin(dx+c)Aba}{2d}+\frac{3\sin(dx+c)a^2B}{4d}+\frac{\sin(dx+c)b^2B}{d}+\frac{a^2A\sin(4dx+4c)}{32d}+\sin($
norman	$\frac{(\frac{3}{8}a^2A+\frac{1}{2}Ab^2+Bab)x+(-\frac{3}{2}a^2A-2Ab^2-4Bab)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-\frac{3}{8}a^2A-\frac{1}{2}Ab^2-Bab)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-\frac{3}{8}a^2}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(a^2*A*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3
*a^2*B*(2+cos(d*x+c)^2)*sin(d*x+c)+2/3*A*b*a*(2+cos(d*x+c)^2)*sin(d*x+c)+2*
B*a*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*b^2*(1/2*cos(d*x+c)*sin(d
*x+c)+1/2*d*x+1/2*c)+b^2*B*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 142, normalized size = 1.04

$$\frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Aa^2-32\,(\sin(dx+c)^3-3\,\sin(dx+c))Ba^2-64\,(\sin(dx+c)^3-3\,\sin(dx+c))Aab+48\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Bab+24\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ab^2+96\,Bb^2\sin(dx+c)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2 - 32*
(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2 - 64*(sin(d*x + c)^3 - 3*sin(d*x +
c))*A*a*b + 48*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a*b + 24*(2*d*x + 2*c + s
in(2*d*x + 2*c))*A*b^2 + 96*B*b^2*sin(d*x + c))/d
```

Fricas [A]
time = 4.29, size = 114, normalized size = 0.84

$$\frac{3\,(3\,Aa^2+8\,Bab+4\,Ab^2)dx+\left(6\,Aa^2\cos(dx+c)^3+16\,Ba^2+32\,Aab+24\,Bb^2+8\,(Ba^2+2\,Aab)\cos(dx+c)^2+3\,(3\,Aa^2+8\,Bab+4\,Ab^2)\cos(dx+c)\right)\sin(dx+c)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/24*(3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*d*x + (6*A*a^2*cos(d*x + c)^3 + 16*B*
a^2 + 32*A*a*b + 24*B*b^2 + 8*(B*a^2 + 2*A*a*b)*cos(d*x + c)^2 + 3*(3*A*a^2
 + 8*B*a*b + 4*A*b^2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 437 vs. 2(128) = 256.
time = 0.48, size = 437, normalized size = 3.21

$$\frac{3\,(3\,Aa^2+8\,Bab+4\,Ab^2)(dx+c)-\frac{2\left(15\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-24\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-48\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+24\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+12\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-24\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-9\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-80\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+24\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+12\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-72\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-80\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-12\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-72\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-48\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-12\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(3*A*a^2 + 8*B*a*b + 4*A*b^2)*(d*x + c) - 2*(15*A*a^2*tan(1/2*d*x +
 1/2*c)^7 - 24*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 48*A*a*b*tan(1/2*d*x + 1/2*c)
^7 + 24*B*a*b*tan(1/2*d*x + 1/2*c)^7 + 12*A*b^2*tan(1/2*d*x + 1/2*c)^7 - 24
*B*b^2*tan(1/2*d*x + 1/2*c)^7 - 9*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 40*B*a^2*t
an(1/2*d*x + 1/2*c)^5 - 80*A*a*b*tan(1/2*d*x + 1/2*c)^5 + 24*B*a*b*tan(1/2*
d*x + 1/2*c)^5 + 12*A*b^2*tan(1/2*d*x + 1/2*c)^5 - 72*B*b^2*tan(1/2*d*x + 1
/2*c)^5 + 9*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^2*tan(1/2*d*x + 1/2*c)^3
- 80*A*a*b*tan(1/2*d*x + 1/2*c)^3 - 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 - 12*A*
b^2*tan(1/2*d*x + 1/2*c)^3 - 72*B*b^2*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^2*tan
(1/2*d*x + 1/2*c) - 24*B*a^2*tan(1/2*d*x + 1/2*c) - 48*A*a*b*tan(1/2*d*x +
1/2*c) - 24*B*a*b*tan(1/2*d*x + 1/2*c) - 12*A*b^2*tan(1/2*d*x + 1/2*c) - 24
*B*b^2*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [**B**]
time = 2.18, size = 169, normalized size = 1.24

$$\frac{3\,A\,a^2\,x}{8}+\frac{A\,b^2\,x}{2}+\frac{3\,B\,a^2\,\sin\left(c+d\,x\right)}{4\,d}+\frac{B\,b^2\,\sin\left(c+d\,x\right)}{d}+B\,a\,b\,x+\frac{A\,a^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{A\,a^2\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{A\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,a^2\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{3\,A\,a\,b\,\sin\left(c+d\,x\right)}{2\,d}+\frac{A\,a\,b\,\sin\left(3\,c+3\,d\,x\right)}{6\,d}+\frac{B\,a\,b\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (3*A*a^2*x)/8 + (A*b^2*x)/2 + (3*B*a^2*sin(c + d*x))/(4*d) + (B*b^2*sin(c +
 d*x))/d + B*a*b*x + (A*a^2*sin(2*c + 2*d*x))/(4*d) + (A*a^2*sin(4*c + 4*d*
x))/(32*d) + (A*b^2*sin(2*c + 2*d*x))/(4*d) + (B*a^2*sin(3*c + 3*d*x))/(12*
d) + (3*A*a*b*sin(c + d*x))/(2*d) + (A*a*b*sin(3*c + 3*d*x))/(6*d) + (B*a*b
*sin(2*c + 2*d*x))/(2*d)
```

3.293 $\int \cos^5(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=180

$$\frac{1}{8}\left(6aAb+3a^2B+4b^2B\right)x+\frac{\left(4a^2A+5Ab^2+10abB\right)\sin(c+dx)}{5d}+\frac{\left(6aAb+3a^2B+4b^2B\right)\cos(c+dx)\sin(c-$$

```
[Out] 1/8*(6*A*a*b+3*B*a^2+4*B*b^2)*x+1/5*(4*A*a^2+5*A*b^2+10*B*a*b)*sin(d*x+c)/d
+1/8*(6*A*a*b+3*B*a^2+4*B*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/4*a*(2*A*b+B*a)*co
s(d*x+c)^3*sin(d*x+c)/d+1/5*a^2*A*cos(d*x+c)^4*sin(d*x+c)/d-1/15*(4*A*a^2+5
*A*b^2+10*B*a*b)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.19, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4109, 4132, 2713, 4130, 2715, 8}

$$-\frac{\left(4a^2A+10abB+5Ab^2\right)\sin^3(c+dx)}{15d}+\frac{\left(4a^2A+10abB+5Ab^2\right)\sin(c+dx)}{5d}+\frac{\left(3a^2B+6aAb+4b^2B\right)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(3a^2B+6aAb+4b^2B\right)+\frac{a^2A\sin(c+dx)\cos^4(c+dx)}{5d}+\frac{a(aB+2Ab)\sin(c+dx)\cos^3(c+dx)}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((6*a*A*b + 3*a^2*B + 4*b^2*B)*x)/8 + ((4*a^2*A + 5*A*b^2 + 10*a*b*B)*Sin[c
 + d*x])/(5*d) + ((6*a*A*b + 3*a^2*B + 4*b^2*B)*Cos[c + d*x]*Sin[c + d*x])/
(8*d) + (a*(2*A*b + a*B)*Cos[c + d*x]^3*Sin[c + d*x])/(4*d) + (a^2*A*Cos[c
+ d*x]^4*Sin[c + d*x])/(5*d) - ((4*a^2*A + 5*A*b^2 + 10*a*b*B)*Sin[c + d*x]
^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2713

```
Int[sin[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
nd[(1 - x^2)^((n - 1)/2), x], x], x, Cos[c + d*x]], x] /; FreeQ[{c, d}, x]
&& IGtQ[(n - 1)/2, 0]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 4109

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a^2*A*Cos[
e + f*x]*((d*Csc[e + f*x])^(n + 1)/(d*f*n)), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{a^2A\cos^4(c+dx)\sin(c+dx)}{5d} - \frac{1}{5}\int \cos^4(c+$$

$$= \frac{a^2A\cos^4(c+dx)\sin(c+dx)}{5d} - \frac{1}{5}\int \cos^4(c+$$

$$= \frac{a(2Ab+aB)\cos^3(c+dx)\sin(c+dx)}{4d} + \frac{a^2A}{}$$

$$= \frac{(4a^2A+5Ab^2+10abB)\sin(c+dx)}{5d} + \frac{(6aAb}{}$$

$$= \frac{1}{8}\left(6aAb+3a^2B+4b^2B\right)x + \frac{(4a^2A+5Ab^2+}{}$$

Mathematica [A]
time = 0.55, size = 146, normalized size = 0.81

$$\frac{60(6aAb+3a^2B+4b^2B)(c+dx)+60(5a^2A+6Ab^2+12abB)\sin(c+dx)+120(2aAb+a^2B+b^2B)\sin(2(c+dx))+10(5a^2A+4Ab^2+8abB)\sin(3(c+dx))+15a(2Ab+aB)\sin(4(c+dx))+6a^2A\sin(5(c+dx))}{480d}$$

Antiderivative was successfully verified.

[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]

[Out] (60*(6*a*A*b + 3*a^2*B + 4*b^2*B)*(c + d*x) + 60*(5*a^2*A + 6*A*b^2 + 12*a*b*B)*Sin[c + d*x] + 120*(2*a*A*b + a^2*B + b^2*B)*Sin[2*(c + d*x)] + 10*(5*a^2*A + 4*A*b^2 + 8*a*b*B)*Sin[3*(c + d*x)] + 15*a*(2*A*b + a*B)*Sin[4*(c + d*x)] + 6*a^2*A*Sin[5*(c + d*x)])/(480*d)

Maple [A]
time = 0.51, size = 184, normalized size = 1.02

method	result
derivativedivides	$\frac{\frac{a^2 A\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+a^2 B\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2Aba\left(\frac{\left(\cos^3(dx+\right.}{}\right.}{}$
default	$\frac{\frac{a^2 A\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+a^2 B\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+2Aba\left(\frac{\left(\cos^3(dx+\right.}{}\right.}{}$
risch	$\frac{3Aabx}{4}+\frac{3B\,a^2x}{8}+\frac{x\,b^2B}{2}+\frac{5a^2A\sin(dx+c)}{8d}+\frac{3\sin(dx+c)A\,b^2}{4d}+\frac{3\sin(dx+c)Bab}{2d}+\frac{a^2A\sin(5dx+5c)}{80d}+\sin($
norman	$\frac{(\frac{3}{4}Aba+\frac{3}{8}a^2B+\frac{1}{2}b^2B)x+(-\frac{15}{4}Aba-\frac{15}{8}a^2B-\frac{5}{2}b^2B)x\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-\frac{15}{4}Aba-\frac{15}{8}a^2B-\frac{5}{2}b^2B)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(1/5*a^2*A*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^2*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*A*b*a*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2/3*B*a*b*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*A*b^2*(2+cos(d*x+c)^2)*sin(d*x+c)+b^2*B*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c))

Maxima [A]
time = 0.28, size = 176, normalized size = 0.98

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^2+15\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(2\,dx+2\,c)\right)Ba^2+30\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(2\,dx+2\,c)\right)Aab-320\left(\sin(dx+c)^3-3\sin(dx+c)\right)Bab-160\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ab^2+120\left(2\,dx+2\,c+\sin(2\,dx+2\,c)\right)Bb^2}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^2 + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^2 + 30*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a*b - 320*(sin(d*x + c

```
)^3 - 3*sin(d*x + c))*B*a*b - 160*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*b^2 +
 120*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*b^2)/d
```

Fricas [A]
time = 2.91, size = 142, normalized size = 0.79

$$\frac{15\,(3\,Ba^2+6\,Aab+4\,Bb^2)dx+\left(24\,Aa^2\cos\left(dx+c\right)^4+30\,(Ba^2+2\,Aab)\cos\left(dx+c\right)^3+64\,Aa^2+160\,Bab+80\,Ab^2+8\,(4\,Aa^2+10\,Bab+5\,Ab^2)\cos\left(dx+c\right)^2+15\,(3\,Ba^2+6\,Aab+4\,Bb^2)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(3*B*a^2 + 6*A*a*b + 4*B*b^2)*d*x + (24*A*a^2*cos(d*x + c)^4 + 30
*(B*a^2 + 2*A*a*b)*cos(d*x + c)^3 + 64*A*a^2 + 160*B*a*b + 80*A*b^2 + 8*(4*
A*a^2 + 10*B*a*b + 5*A*b^2)*cos(d*x + c)^2 + 15*(3*B*a^2 + 6*A*a*b + 4*B*b^
2)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 487 vs. 2(168) = 336.
time = 0.50, size = 487, normalized size = 2.71

$$\frac{15\,(3\,Ba^2+6\,Aab+4\,Bb^2)(dx+c)+\frac{2\left(120\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-150\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+240\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+120\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-60\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+160\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-30\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-60\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+640\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+320\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-120\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+464\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+800\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+400\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+160\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+30\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+60\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+640\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+320\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+75\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+150\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+240\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+120\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+60\,Bb^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(3*B*a^2 + 6*A*a*b + 4*B*b^2)*(d*x + c) + 2*(120*A*a^2*tan(1/2*d*
x + 1/2*c)^9 - 75*B*a^2*tan(1/2*d*x + 1/2*c)^9 - 150*A*a*b*tan(1/2*d*x + 1/
2*c)^9 + 240*B*a*b*tan(1/2*d*x + 1/2*c)^9 + 120*A*b^2*tan(1/2*d*x + 1/2*c)^
9 - 60*B*b^2*tan(1/2*d*x + 1/2*c)^9 + 160*A*a^2*tan(1/2*d*x + 1/2*c)^7 - 30
*B*a^2*tan(1/2*d*x + 1/2*c)^7 - 60*A*a*b*tan(1/2*d*x + 1/2*c)^7 + 640*B*a*b
*tan(1/2*d*x + 1/2*c)^7 + 320*A*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*B*b^2*tan(
1/2*d*x + 1/2*c)^7 + 464*A*a^2*tan(1/2*d*x + 1/2*c)^5 + 800*B*a*b*tan(1/2*d
*x + 1/2*c)^5 + 400*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 160*A*a^2*tan(1/2*d*x +
1/2*c)^3 + 30*B*a^2*tan(1/2*d*x + 1/2*c)^3 + 60*A*a*b*tan(1/2*d*x + 1/2*c)^
```

```
3 + 640*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 320*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 1
20*B*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^2*tan(1/2*d*x + 1/2*c) + 75*B*a^2
*tan(1/2*d*x + 1/2*c) + 150*A*a*b*tan(1/2*d*x + 1/2*c) + 240*B*a*b*tan(1/2*
d*x + 1/2*c) + 120*A*b^2*tan(1/2*d*x + 1/2*c) + 60*B*b^2*tan(1/2*d*x + 1/2*
c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B]
time = 5.81, size = 307, normalized size = 1.71

$$\frac{x\left(\frac{3\,B\,a^2}{4}+\frac{3\,A\,a\,b}{2}+B\,b^2\right)}{2}+\frac{\left(2\,A\,a^2+2\,A\,b^2-\frac{5\,B\,a^2}{4}-B\,b^2-\frac{5\,A\,a\,b}{2}+4\,B\,a\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^9+\left(\frac{8\,A\,a^2}{3}+\frac{16\,A\,b^2}{3}-\frac{B\,a^2}{2}-2\,B\,b^2-A\,a\,b+\frac{32\,B\,a\,b}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^7+\left(\frac{116\,A\,a^2}{15}+\frac{40\,B\,a\,b}{3}+\frac{20\,A\,b^2}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5+\left(\frac{8\,A\,a^2}{3}+\frac{16\,A\,b^2}{3}+\frac{B\,a^2}{2}+2\,B\,b^2+A\,a\,b+\frac{32\,B\,a\,b}{3}\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3+\left(2\,A\,a^2+2\,A\,b^2+\frac{5\,B\,a^2}{4}+B\,b^2+\frac{5\,A\,a\,b}{2}+4\,B\,a\,b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^{10}+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^8+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6+10\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4+5\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (x*((3*B*a^2)/4 + B*b^2 + (3*A*a*b)/2))/2 + (tan(c/2 + (d*x)/2)^5*((116*A*a
^2)/15 + (20*A*b^2)/3 + (40*B*a*b)/3) + tan(c/2 + (d*x)/2)^9*(2*A*a^2 + 2*A
*b^2 - (5*B*a^2)/4 - B*b^2 - (5*A*a*b)/2 + 4*B*a*b) + tan(c/2 + (d*x)/2)^3*
((8*A*a^2)/3 + (16*A*b^2)/3 + (B*a^2)/2 + 2*B*b^2 + A*a*b + (32*B*a*b)/3) +
 tan(c/2 + (d*x)/2)^7*((8*A*a^2)/3 + (16*A*b^2)/3 - (B*a^2)/2 - 2*B*b^2 - A
*a*b + (32*B*a*b)/3) + tan(c/2 + (d*x)/2)*(2*A*a^2 + 2*A*b^2 + (5*B*a^2)/4
+ B*b^2 + (5*A*a*b)/2 + 4*B*a*b))/(d*(5*tan(c/2 + (d*x)/2)^2 + 10*tan(c/2 +
 (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 + 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 +
(d*x)/2)^10 + 1))
```

3.294 $\int \sec^2(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=252

$$\frac{(12a^2Ab+3Ab^3+4a^3B+9ab^2B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(15a^3Ab+60aAb^3-3a^4B+52a^2b^2B+16b^4B)\,\mathrm{ta}}{30bd}$$

```
[Out] 1/8*(12*A*a^2*b+3*A*b^3+4*B*a^3+9*B*a*b^2)*arctanh(sin(d*x+c))/d+1/30*(15*A
*a^3*b+60*A*a*b^3-3*B*a^4+52*B*a^2*b^2+16*B*b^4)*tan(d*x+c)/b/d+1/120*(30*A
*a^2*b+45*A*b^3-6*B*a^3+71*B*a*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/60*(15*A*a*b-
3*B*a^2+16*B*b^2)*(a+b*sec(d*x+c))^2*tan(d*x+c)/b/d+1/20*(5*A*b-B*a)*(a+b*s
ec(d*x+c))^3*tan(d*x+c)/b/d+1/5*B*(a+b*sec(d*x+c))^4*tan(d*x+c)/b/d
```

Rubi [A]
time = 0.32, antiderivative size = 252, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4095, 4087, 4082, 3872, 3855, 3852, 8}

$$\frac{(-3a^2B+15aAb+16b^2B)\tan(c+dx)(a+b\sec(c+dx))^2}{60bd}+\frac{(4a^3B+12a^2Ab+9ab^2B+3Ab^3)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(-6a^3B+30a^2Ab+71ab^2B+45Ab^3)\tan(c+dx)\sec(c+dx)}{120d}+\frac{(-3a^4B+15a^3Ab+52a^2b^2B+60aAb^3+16b^4B)\tan(c+dx)}{30bd}+\frac{(5Ab-aB)\tan(c+dx)(a+b\sec(c+dx))^3}{20bd}+\frac{B\tan(c+dx)(a+b\sec(c+dx))^4}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((12*a^2*A*b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*ArcTanh[Sin[c + d*x]])/(8*d)
+ ((15*a^3*A*b + 60*a*A*b^3 - 3*a^4*B + 52*a^2*b^2*B + 16*b^4*B)*Tan[c + d*
x])/(30*b*d) + ((30*a^2*A*b + 45*A*b^3 - 6*a^3*B + 71*a*b^2*B)*Sec[c + d*x]
*Tan[c + d*x])/(120*d) + ((15*a*A*b - 3*a^2*B + 16*b^2*B)*(a + b*Sec[c + d*
x])^2*Tan[c + d*x])/(60*b*d) + ((5*A*b - a*B)*(a + b*Sec[c + d*x])^3*Tan[c
+ d*x])/(20*b*d) + (B*(a + b*Sec[c + d*x])^4*Tan[c + d*x])/(5*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{B(a+b\sec(c+dx))^4\tan(c+dx)}{5bd} + \frac{\int \sec(c+ \ldots}{\ldots}$$
$$= \frac{(5Ab-aB)(a+b\sec(c+dx))^3\tan(c+dx)}{20bd} + \ldots$$
$$= \frac{(15aAb-3a^2B+16b^2B)\,(a+b\sec(c+dx))^2 \ldots}{60bd}$$
$$= \frac{(30a^2Ab+45Ab^3-6a^3B+71ab^2B)\sec(c+d \ldots}{120d}$$
$$= \frac{(30a^2Ab+45Ab^3-6a^3B+71ab^2B)\sec(c+d \ldots}{120d}$$
$$= \frac{(12a^2Ab+3Ab^3+4a^3B+9ab^2B)\tanh^{-1}(\sin(\ldots}{8d}$$
$$= \frac{(12a^2Ab+3Ab^3+4a^3B+9ab^2B)\tanh^{-1}(\sin(\ldots}{8d}$$

Mathematica [A]
time = 3.47, size = 181, normalized size = 0.72

$$\frac{15(12a^2Ab+3Ab^3+4a^3B+9ab^2B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(15(12a^2Ab+3Ab^3+4a^3B+9ab^2B)\sec(c+dx)+30b^2(Ab+3aB)\sec^3(c+dx)+8(15(a^3A+3aAb^2+3a^2bB+b^3B)+5b(3aAb+3a^2B+2b^2B)\tan^2(c+dx)+3b^3B\tan^4(c+dx))\right)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (15*(12*a^2*A*b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*ArcTanh[Sin[c + d*x]] + Ta
n[c + d*x]*(15*(12*a^2*A*b + 3*A*b^3 + 4*a^3*B + 9*a*b^2*B)*Sec[c + d*x] +
30*b^2*(A*b + 3*a*B)*Sec[c + d*x]^3 + 8*(15*(a^3*A + 3*a*A*b^2 + 3*a^2*b*B
+ b^3*B) + 5*b*(3*a*A*b + 3*a^2*B + 2*b^2*B)*Tan[c + d*x]^2 + 3*b^3*B*Tan[c
 + d*x]^4)))/(120*d)
```

Maple [A]
time = 0.53, size = 275, normalized size = 1.09 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^3*tan(d*x+c)+a^3*B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+ta
n(d*x+c)))+3*A*a^2*b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c
)))-3*B*b*a^2*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)-3*A*b^2*a*(-2/3-1/3*sec(d*
x+c)^2)*tan(d*x+c)+3*B*a*b^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c
)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+A*b^3*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))
```

```
*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-B*b^3*(-8/15-1/5*sec(d*x+c)^4-4/
15*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.27, size = 341, normalized size = 1.35

$$\frac{240 \left(\tan\left(dx+c\right)^{3}+3 \tan\left(dx+c\right)\right) Ba^{2}b+240 \left(\tan\left(dx+c\right)^{3}+3 \tan\left(dx+c\right)\right) Aab^{2}+16 \left(3 \tan\left(dx+c\right)^{5}+10 \tan\left(dx+c\right)^{3}+15 \tan\left(dx+c\right)\right) Bb^{3}-45 Bab^{2}\left(\frac{2 \left(3 \sin(dx+c)^{3}-5 \sin(dx+c)\right)}{\sin(dx+c)^{4}-2 \sin(dx+c)^{2}+1}-3 \log\left(\sin\left(dx+c\right)+1\right)+3 \log\left(\sin\left(dx+c\right)-1\right)\right)-15 Ab^{3}\left(\frac{2 \left(3 \sin(dx+c)^{3}-5 \sin(dx+c)\right)}{\sin(dx+c)^{4}-2 \sin(dx+c)^{2}+1}-3 \log\left(\sin\left(dx+c\right)+1\right)+3 \log\left(\sin\left(dx+c\right)-1\right)\right)-60 Ba^{3}\left(\frac{2 \sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-180 Aa^{2}b\left(\frac{2 \sin(dx+c)}{\sin(dx+c)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+240 Aa^{3} \tan\left(dx+c\right)}{240 d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/240*(240*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2*b + 240*(tan(d*x + c)^3
+ 3*tan(d*x + c))*A*a*b^2 + 16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*t
an(d*x + c))*B*b^3 - 45*B*a*b^2*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin
(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d
*x + c) - 1)) - 15*A*b^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x +
c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c)
 - 1)) - 60*B*a^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) +
 1) + log(sin(d*x + c) - 1)) - 180*A*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2
- 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*A*a^3*tan(d*x +
 c))/d
```

Fricas [A]
time = 2.45, size = 249, normalized size = 0.99

$$\frac{15 \left(4 Ba^{3}+12 Aa^{2}b+9 Bab^{2}+3 Ab^{3}\right) \cos\left(dx+c\right)^{5} \log\left(\sin\left(dx+c\right)+1\right)-15 \left(4 Ba^{3}+12 Aa^{2}b+9 Bab^{2}+3 Ab^{3}\right) \cos\left(dx+c\right)^{5} \log\left(-\sin\left(dx+c\right)+1\right)+2 \left(8 \left(15 Aa^{3}+30 Ba^{2}b+30 Aab^{2}+8 Bb^{3}\right) \cos\left(dx+c\right)^{4}+24 Bb^{3}+15 \left(4 Ba^{3}+12 Aa^{2}b+9 Bab^{2}+3 Ab^{3}\right) \cos\left(dx+c\right)^{3}+8 \left(15 Ba^{2}b+15 Aab^{2}+4 Bb^{3}\right) \cos\left(dx+c\right)^{2}+30 \left(3 Bab^{2}+Ab^{3}\right) \cos\left(dx+c\right)\right) \sin\left(dx+c\right)}{240 d \cos\left(dx+c\right)^{5}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*cos(d*x + c)^5*log(s
in(d*x + c) + 1) - 15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*cos(d*x
+ c)^5*log(-sin(d*x + c) + 1) + 2*(8*(15*A*a^3 + 30*B*a^2*b + 30*A*a*b^2 +
8*B*b^3)*cos(d*x + c)^4 + 24*B*b^3 + 15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 +
 3*A*b^3)*cos(d*x + c)^3 + 8*(15*B*a^2*b + 15*A*a*b^2 + 4*B*b^3)*cos(d*x +
c)^2 + 30*(3*B*a*b^2 + A*b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A+B\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)^{3}\sec^{2}\left(c+dx\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3*sec(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(239) = 478$.
time = 0.51, size = 722, normalized size = 2.87

$$\frac{15\left(4Ba^3+12Aa^2b+9Bab^2+3Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-15\left(4Ba^3+12Aa^2b+9Bab^2+3Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(120Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-60Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-180Aa^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+360Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+360Aab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-225Bab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-75Ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9+120Bb^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^9-480Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+120Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+360Aa^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-960Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-960Aab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+90Bab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+30Ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-160Bb^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+720Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+1200Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+1200Aab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+464Bb^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-480Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-120Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-360Aa^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-960Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-960Aab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-90Bab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-30Ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-160Bb^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+120Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+60Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+180Aa^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+360Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+360Aab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+225Bab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+75Ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+120Bb^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^5}}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*log(abs(tan(1/2*d*x
+ 1/2*c) + 1)) - 15*(4*B*a^3 + 12*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*log(abs(ta
n(1/2*d*x + 1/2*c) - 1)) - 2*(120*A*a^3*tan(1/2*d*x + 1/2*c)^9 - 60*B*a^3*t
an(1/2*d*x + 1/2*c)^9 - 180*A*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*B*a^2*b*ta
n(1/2*d*x + 1/2*c)^9 + 360*A*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 225*B*a*b^2*tan
(1/2*d*x + 1/2*c)^9 - 75*A*b^3*tan(1/2*d*x + 1/2*c)^9 + 120*B*b^3*tan(1/2*d
*x + 1/2*c)^9 - 480*A*a^3*tan(1/2*d*x + 1/2*c)^7 + 120*B*a^3*tan(1/2*d*x +
1/2*c)^7 + 360*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 960*B*a^2*b*tan(1/2*d*x + 1
/2*c)^7 - 960*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 90*B*a*b^2*tan(1/2*d*x + 1/2
*c)^7 + 30*A*b^3*tan(1/2*d*x + 1/2*c)^7 - 160*B*b^3*tan(1/2*d*x + 1/2*c)^7
+ 720*A*a^3*tan(1/2*d*x + 1/2*c)^5 + 1200*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 +
1200*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 464*B*b^3*tan(1/2*d*x + 1/2*c)^5 - 48
0*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 120*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 360*A*a
^2*b*tan(1/2*d*x + 1/2*c)^3 - 960*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 960*A*a*
b^2*tan(1/2*d*x + 1/2*c)^3 - 90*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 30*A*b^3*t
an(1/2*d*x + 1/2*c)^3 - 160*B*b^3*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^3*tan(1/
2*d*x + 1/2*c) + 60*B*a^3*tan(1/2*d*x + 1/2*c) + 180*A*a^2*b*tan(1/2*d*x +
1/2*c) + 360*B*a^2*b*tan(1/2*d*x + 1/2*c) + 360*A*a*b^2*tan(1/2*d*x + 1/2*c
) + 225*B*a*b^2*tan(1/2*d*x + 1/2*c) + 75*A*b^3*tan(1/2*d*x + 1/2*c) + 120*
B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^5)/d
```

Mupad [B]
time = 5.79, size = 470, normalized size = 1.87

$$\frac{\mathrm{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(\frac{Ba^3}{2}+\frac{3Aa^2b}{2}+\frac{9Bab^2}{8}+\frac{3Ab^3}{8}\right)}{2Ba^3+6Aa^2b+\frac{9Bab^2}{2}+\frac{3Ab^3}{2}}\right)\left(Ba^3+3Aa^2b+\frac{9Bab^2}{4}+\frac{3Ab^3}{4}\right)}{d}-\frac{\left(2Aa^3-\frac{5Ab^3}{4}-Ba^3+2Bb^3+6Aab^2-3Aa^2b-\frac{15Bab^2}{4}+6Ba^2b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(\frac{Ab^3}{2}-8Aa^3+2Ba^3-\frac{8Bb^3}{3}-16Aab^2+6Aa^2b+\frac{3Bab^2}{2}-16Ba^2b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(12Aa^3+20Ba^2b+20Aab^2+\frac{116Bb^3}{15}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-8Aa^3-\frac{Ab^3}{2}-2Ba^3-\frac{8Bb^3}{3}-16Aab^2-6Aa^2b-\frac{3Bab^2}{2}-16Ba^2b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(2Aa^3+\frac{5Ab^3}{4}+Ba^3+2Bb^3+6Aab^2+3Aa^2b+\frac{15Bab^2}{4}+6Ba^2b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/cos(c + d*x)^2,x)
```

```
[Out] (atanh((4*tan(c/2 + (d*x)/2)*((3*A*b^3)/8 + (B*a^3)/2 + (3*A*a^2*b)/2 + (9*
B*a*b^2)/8))/((3*A*b^3)/2 + 2*B*a^3 + 6*A*a^2*b + (9*B*a*b^2)/2))*((3*A*b^3
)/4 + B*a^3 + 3*A*a^2*b + (9*B*a*b^2)/4))/d - (tan(c/2 + (d*x)/2)*(2*A*a^3
```

```
+ (5*A*b^3)/4 + B*a^3 + 2*B*b^3 + 6*A*a*b^2 + 3*A*a^2*b + (15*B*a*b^2)/4 +
6*B*a^2*b) + tan(c/2 + (d*x)/2)^5*(12*A*a^3 + (116*B*b^3)/15 + 20*A*a*b^2 +
 20*B*a^2*b) + tan(c/2 + (d*x)/2)^9*(2*A*a^3 - (5*A*b^3)/4 - B*a^3 + 2*B*b^
3 + 6*A*a*b^2 - 3*A*a^2*b - (15*B*a*b^2)/4 + 6*B*a^2*b) - tan(c/2 + (d*x)/2
)^3*(8*A*a^3 + (A*b^3)/2 + 2*B*a^3 + (8*B*b^3)/3 + 16*A*a*b^2 + 6*A*a^2*b +
 (3*B*a*b^2)/2 + 16*B*a^2*b) - tan(c/2 + (d*x)/2)^7*(8*A*a^3 - (A*b^3)/2 -
2*B*a^3 + (8*B*b^3)/3 + 16*A*a*b^2 - 6*A*a^2*b - (3*B*a*b^2)/2 + 16*B*a^2*b
))/(d*(5*tan(c/2 + (d*x)/2)^2 - 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x
)/2)^6 - 5*tan(c/2 + (d*x)/2)^8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.295 $\int \sec(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=180

$$\frac{(8a^3A+12aAb^2+12a^2bB+3b^3B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(16a^2Ab+4Ab^3+3a^3B+12ab^2B)\tan(c+dx)}{6d}+$$

```
[Out] 1/8*(8*A*a^3+12*A*a*b^2+12*B*a^2*b+3*B*b^3)*arctanh(sin(d*x+c))/d+1/6*(16*A
*a^2*b+4*A*b^3+3*B*a^3+12*B*a*b^2)*tan(d*x+c)/d+1/24*b*(20*A*a*b+6*B*a^2+9*
B*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/12*(4*A*b+3*B*a)*(a+b*sec(d*x+c))^2*tan(d*
x+c)/d+1/4*B*(a+b*sec(d*x+c))^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.22, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4087, 4082, 3872, 3855, 3852, 8}

$$\frac{b(6a^2B+20aAb+9b^2B)\tan(c+dx)\sec(c+dx)}{24d}+\frac{(3a^3B+16a^2Ab+12ab^2B+4Ab^3)\tan(c+dx)}{6d}+\frac{(8a^3A+12a^2bB+12aAb^2+3b^3B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(3aB+4Ab)\tan(c+dx)(a+b\sec(c+dx))^2}{12d}+\frac{B\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((8*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*ArcTanh[Sin[c + d*x]])/(8*d)
 + ((16*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*Tan[c + d*x])/(6*d) + (b*
(20*a*A*b + 6*a^2*B + 9*b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(24*d) + ((4*A*b
+ 3*a*B)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(12*d) + (B*(a + b*Sec[c + d*
x])^3*Tan[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\begin{aligned}\int \sec(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx &= \frac{B(a+b\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{1}{4}\int \sec(c+\\ &= \frac{(4Ab+3aB)(a+b\sec(c+dx))^2\tan(c+dx)}{12d}+\\ &= \frac{b(20aAb+6a^2B+9b^2B)\sec(c+dx)\tan(c+dx}{24d}\\ &= \frac{b(20aAb+6a^2B+9b^2B)\sec(c+dx)\tan(c+dx}{24d}\\ &= \frac{(8a^3A+12aAb^2+12a^2bB+3b^3B)\tanh^{-1}(\sin(c}{8d}\\ &= \frac{(8a^3A+12aAb^2+12a^2bB+3b^3B)\tanh^{-1}(\sin(c}{8d}\end{aligned}$$

Mathematica [A]
time = 0.95, size = 140, normalized size = 0.78

$$\frac{3(8a^3A+12aAb^2+12a^2bB+3b^3B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(24(3a^2Ab+Ab^3+a^3B+3ab^2B)+9b(4aAb+4a^2B+b^2B)\sec(c+dx)+6b^3B\sec^3(c+dx)+8b^2(Ab+3aB)\tan^2(c+dx)\right)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (3*(8*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*ArcTanh[Sin[c + d*x]] + Ta
n[c + d*x]*(24*(3*a^2*A*b + A*b^3 + a^3*B + 3*a*b^2*B) + 9*b*(4*a*A*b + 4*a
^2*B + b^2*B)*Sec[c + d*x] + 6*b^3*B*Sec[c + d*x]^3 + 8*b^2*(A*b + 3*a*B)*T
an[c + d*x]^2))/(24*d)
```

Maple [A]
time = 0.46, size = 223, normalized size = 1.24 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a^3*ln(sec(d*x+c)+tan(d*x+c))+a^3*B*tan(d*x+c)+3*A*a^2*b*tan(d*x+c)+
3*B*b*a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+3*A*b^2
*a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-3*B*a*b^2*(-2/
3-1/3*sec(d*x+c)^2)*tan(d*x+c)-A*b^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+B*b
^3*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*
x+c))))
```

Maxima [A]
time = 0.28, size = 266, normalized size = 1.48

$$\frac{48\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)Bab^{2}+16\left(\tan\left(dx+c\right)^{3}+3\tan\left(dx+c\right)\right)Ab^{3}-3Bb^{3}\left(\frac{2\left(3\sin\left(dx+c\right)^{3}-5\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\sin\left(dx+c\right)^{2}+1}-3\log\left(\sin\left(dx+c\right)+1\right)+3\log\left(\sin\left(dx+c\right)-1\right)\right)-36Ba^{2}b\left(\frac{2\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-36Aab^{2}\left(\frac{2\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+48Aa^{3}\log\left(\sec\left(dx+c\right)+\tan\left(dx+c\right)\right)+48Ba^{3}\tan\left(dx+c\right)+144Aa^{2}b\tan\left(dx+c\right)}{48d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/48*(48*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a*b^2 + 16*(tan(d*x + c)^3 + 3
*tan(d*x + c))*A*b^3 - 3*B*b^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(
d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*
x + c) - 1)) - 36*B*a^2*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*
x + c) + 1) + log(sin(d*x + c) - 1)) - 36*A*a*b^2*(2*sin(d*x + c)/(sin(d*x
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 48*A*a^3*log
(sec(d*x + c) + tan(d*x + c)) + 48*B*a^3*tan(d*x + c) + 144*A*a^2*b*tan(d*x
 + c))/d
```

Fricas [A]
time = 4.54, size = 211, normalized size = 1.17

$$\frac{3\left(8Aa^{3}+12Ba^{2}b+12Aab^{2}+3Bb^{3}\right)\cos\left(dx+c\right)^{4}\log\left(\sin\left(dx+c\right)+1\right)-3\left(8Aa^{3}+12Ba^{2}b+12Aab^{2}+3Bb^{3}\right)\cos\left(dx+c\right)^{4}\log\left(-\sin\left(dx+c\right)+1\right)+2\left(6Bb^{3}+8\left(3Ba^{3}+9Aa^{2}b+6Bab^{2}+2Ab^{3}\right)\cos\left(dx+c\right)^{3}+9\left(4Ba^{2}b+4Aab^{2}+Bb^{3}\right)\cos\left(dx+c\right)^{2}+8\left(3Bab^{2}+Ab^{3}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{48d\cos\left(dx+c\right)^{4}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/48*(3*(8*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 3*B*b^3)*cos(d*x + c)^4*log(si
n(d*x + c) + 1) - 3*(8*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 3*B*b^3)*cos(d*x +
 c)^4*log(-sin(d*x + c) + 1) + 2*(6*B*b^3 + 8*(3*B*a^3 + 9*A*a^2*b + 6*B*a*
b^2 + 2*A*b^3)*cos(d*x + c)^3 + 9*(4*B*a^2*b + 4*A*a*b^2 + B*b^3)*cos(d*x +
 c)^2 + 8*(3*B*a*b^2 + A*b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4
)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\sec(c+dx))\,(a+b\sec(c+dx))^3\sec(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3*sec(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 586 vs. 2(170) = 340.
time = 0.53, size = 586, normalized size = 3.26

$$\frac{3\left(8Aa^3+12Ba^2b+12Aab^2+3Bb^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)-3\left(8Aa^3+12Ba^2b+12Aab^2+3Bb^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)-\frac{2\left(24Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+72Aa^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-36Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-36Aab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+72Bab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+24Ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-15Bb^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-72Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-216Aa^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+36Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+36Aab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-120Bab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-40Ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-9Bb^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+72Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+216Aa^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+36Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+36Aab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+120Bab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+40Ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-9Bb^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-24Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-72Aa^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-36Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-36Aab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-72Bab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-24Ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-15Bb^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^4}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/24*(3*(8*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 3*B*b^3)*log(abs(tan(1/2*d*x +
 1/2*c) + 1)) - 3*(8*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 3*B*b^3)*log(abs(tan
(1/2*d*x + 1/2*c) - 1)) - 2*(24*B*a^3*tan(1/2*d*x + 1/2*c)^7 + 72*A*a^2*b*t
an(1/2*d*x + 1/2*c)^7 - 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 36*A*a*b^2*tan(
1/2*d*x + 1/2*c)^7 + 72*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 + 24*A*b^3*tan(1/2*d
*x + 1/2*c)^7 - 15*B*b^3*tan(1/2*d*x + 1/2*c)^7 - 72*B*a^3*tan(1/2*d*x + 1/
2*c)^5 - 216*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^2*b*tan(1/2*d*x + 1/2*
c)^5 + 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 120*B*a*b^2*tan(1/2*d*x + 1/2*c)
^5 - 40*A*b^3*tan(1/2*d*x + 1/2*c)^5 - 9*B*b^3*tan(1/2*d*x + 1/2*c)^5 + 72*
B*a^3*tan(1/2*d*x + 1/2*c)^3 + 216*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 36*B*a^
2*b*tan(1/2*d*x + 1/2*c)^3 + 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*B*a*b^
2*tan(1/2*d*x + 1/2*c)^3 + 40*A*b^3*tan(1/2*d*x + 1/2*c)^3 - 9*B*b^3*tan(1/
2*d*x + 1/2*c)^3 - 24*B*a^3*tan(1/2*d*x + 1/2*c) - 72*A*a^2*b*tan(1/2*d*x +
 1/2*c) - 36*B*a^2*b*tan(1/2*d*x + 1/2*c) - 36*A*a*b^2*tan(1/2*d*x + 1/2*c)
 - 72*B*a*b^2*tan(1/2*d*x + 1/2*c) - 24*A*b^3*tan(1/2*d*x + 1/2*c) - 15*B*b
^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 6.01, size = 395, normalized size = 2.19

$$\frac{\mathrm{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(Aa^3+\frac{3Ba^2b}{2}+\frac{3Aab^2}{2}+\frac{3Bb^3}{8}\right)}{4Aa^3+6Ba^2b+6Aab^2+\frac{3Bb^3}{2}}\right)\left(2Aa^3+3Ba^2b+3Aab^2+\frac{3Bb^3}{4}\right)}{d}-\frac{\left(2Ab^3+2Ba^3-\frac{5Bb^3}{4}-3Aab^2+6Aa^2b+6Bab^2-3Ba^2b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(3Aab^2-6Ba^3-\frac{3Bb^3}{4}-\frac{10Ab^3}{3}-18Aa^2b-10Bab^2+3Ba^2b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(\frac{10Ab^3}{3}+6Ba^3-\frac{3Bb^3}{4}+3Aab^2+18Aa^2b+10Bab^2+3Ba^2b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(-2Ab^3-2Ba^3-\frac{5Bb^3}{4}-3Aab^2-6Aa^2b-6Bab^2-3Ba^2b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6+6\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4-4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2+1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/cos(c + d*x),x)
```

```
[Out] (atanh((4*tan(c/2 + (d*x)/2)*(A*a^3 + (3*B*b^3)/8 + (3*A*a*b^2)/2 + (3*B*a^
2*b)/2))/(4*A*a^3 + (3*B*b^3)/2 + 6*A*a*b^2 + 6*B*a^2*b))*(2*A*a^3 + (3*B*b
^3)/4 + 3*A*a*b^2 + 3*B*a^2*b))/d - (tan(c/2 + (d*x)/2)^7*(2*A*b^3 + 2*B*a^
3 - (5*B*b^3)/4 - 3*A*a*b^2 + 6*A*a^2*b + 6*B*a*b^2 - 3*B*a^2*b) + tan(c/2
+ (d*x)/2)^3*((10*A*b^3)/3 + 6*B*a^3 - (3*B*b^3)/4 + 3*A*a*b^2 + 18*A*a^2*b
 + 10*B*a*b^2 + 3*B*a^2*b) - tan(c/2 + (d*x)/2)^5*((10*A*b^3)/3 + 6*B*a^3 +
 (3*B*b^3)/4 - 3*A*a*b^2 + 18*A*a^2*b + 10*B*a*b^2 - 3*B*a^2*b) - tan(c/2 +
 (d*x)/2)*(2*A*b^3 + 2*B*a^3 + (5*B*b^3)/4 + 3*A*a*b^2 + 6*A*a^2*b + 6*B*a*
b^2 + 3*B*a^2*b))/(d*(6*tan(c/2 + (d*x)/2)^4 - 4*tan(c/2 + (d*x)/2)^2 - 4*t
an(c/2 + (d*x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1))
```

3.296 $\int (a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=137

$$a^3Ax+\frac{(6a^2Ab+Ab^3+2a^3B+3ab^2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b(9aAb+8a^2B+2b^2B)\tan(c+dx)}{3d}+\frac{b^2(3Ab\cdots}{\cdots}$$

```
[Out] a^3*A*x+1/2*(6*A*a^2*b+A*b^3+2*B*a^3+3*B*a*b^2)*arctanh(sin(d*x+c))/d+1/3*b
*(9*A*a*b+8*B*a^2+2*B*b^2)*tan(d*x+c)/d+1/6*b^2*(3*A*b+5*B*a)*sec(d*x+c)*ta
n(d*x+c)/d+1/3*b*B*(a+b*sec(d*x+c))^2*tan(d*x+c)/d
```

Rubi [A]

time = 0.12, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4003, 4133, 3855, 3852, 8}

$$a^3Ax+\frac{b(8a^2B+9aAb+2b^2B)\tan(c+dx)}{3d}+\frac{(2a^3B+6a^2Ab+3ab^2B+Ab^3)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{b^2(5aB+3Ab)\tan(c+dx)\sec(c+dx)}{6d}+\frac{bB\tan(c+dx)(a+b\sec(c+dx))^2}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] a^3*A*x + ((6*a^2*A*b + A*b^3 + 2*a^3*B + 3*a*b^2*B)*ArcTanh[Sin[c + d*x]])
/(2*d) + (b*(9*a*A*b + 8*a^2*B + 2*b^2*B)*Tan[c + d*x])/(3*d) + (b^2*(3*A*b
 + 5*a*B)*Sec[c + d*x]*Tan[c + d*x])/(6*d) + (b*B*(a + b*Sec[c + d*x])^2*Ta
n[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4003

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m
+ (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m
```

```
- 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
 - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4133

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A
 + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{bB(a+b\sec(c+dx))^2\tan(c+dx)}{3d}+\frac{1}{3}\int (a+b\sec(c+dx$$
$$= \frac{b^2(3Ab+5aB)\sec(c+dx)\tan(c+dx)}{6d}+\frac{bB(a+b\sec(c+}{3}$$
$$= a^3Ax+\frac{b^2(3Ab+5aB)\sec(c+dx)\tan(c+dx)}{6d}+\frac{bB(a+b}{}$$
$$= a^3Ax+\frac{(6a^2Ab+Ab^3+2a^3B+3ab^2B)\tanh^{-1}(\sin(c+dx}{2d}$$
$$= a^3Ax+\frac{(6a^2Ab+Ab^3+2a^3B+3ab^2B)\tanh^{-1}(\sin(c+dx}{2d}$$

Mathematica [A]
time = 0.61, size = 108, normalized size = 0.79

$$\frac{6a^3Adx+3(6a^2Ab+Ab^3+2a^3B+3ab^2B)\tanh^{-1}(\sin(c+dx))+3b(6aAb+6a^2B+2b^2B+b(Ab+3aB)\sec(c+dx))\tan(c+dx)+2b^3B\tan^3(c+dx)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (6*a^3*A*d*x + 3*(6*a^2*A*b + A*b^3 + 2*a^3*B + 3*a*b^2*B)*ArcTanh[Sin[c +
d*x]] + 3*b*(6*a*A*b + 6*a^2*B + 2*b^2*B + b*(A*b + 3*a*B)*Sec[c + d*x])*Ta
n[c + d*x] + 2*b^3*B*Tan[c + d*x]^3)/(6*d)
```

Maple [A]
time = 0.36, size = 180, normalized size = 1.31

method	result

derivativedivides	$\frac{A\,a^3(dx+c)+a^3B\ln(\sec(dx+c)+\tan(dx+c))+3A\,a^2b\ln(\sec(dx+c)+\tan(dx+c))+3Bb\,a^2\tan(dx+c)+3A\,b^2a\tan(dx+c)+\cdots}{\cdots}$
default	$\frac{A\,a^3(dx+c)+a^3B\ln(\sec(dx+c)+\tan(dx+c))+3A\,a^2b\ln(\sec(dx+c)+\tan(dx+c))+3Bb\,a^2\tan(dx+c)+3A\,b^2a\tan(dx+c)+\cdots}{\cdots}$
norman	$\frac{a^3Ax\left(\tan^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a^3Ax+3a^3Ax\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-3a^3Ax\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\frac{4b\left(9Aba+9a^2B+b^2B\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3d}-\frac{b(\cdots}{\cdots}}{\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^3}$
risch	$a^3Ax-\frac{ib\left(3A\,b^2e^{5i(dx+c)}+9Bab\,e^{5i(dx+c)}-18Aab\,e^{4i(dx+c)}-18B\,a^2e^{4i(dx+c)}-36Aab\,e^{2i(dx+c)}-36B\,a^2e^{2i(dx+c)}-1\cdots\right.}{3d\left(e^{2i(dx+c)}+1\right)^3}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a^3*(d*x+c)+a^3*B*ln(sec(d*x+c)+tan(d*x+c))+3*A*a^2*b*ln(sec(d*x+c)+
tan(d*x+c))+3*B*b*a^2*tan(d*x+c)+3*A*b^2*a*tan(d*x+c)+3*B*a*b^2*(1/2*sec(d*
x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+A*b^3*(1/2*sec(d*x+c)*tan(d*
x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-B*b^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c
))
```

Maxima [A]

time = 0.27, size = 202, normalized size = 1.47

$$\frac{12\,(dx+c)Aa^3+4\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Bb^3-9\,Bab^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-3\,Ab^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+12\,Ba^3\log(\sec(dx+c)+\tan(dx+c))+36\,Aa^2b\log(\sec(dx+c)+\tan(dx+c))+36\,Ba^2b\tan(dx+c)+36\,Aab^2\tan(dx+c)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/12*(12*(d*x + c)*A*a^3 + 4*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*b^3 - 9*B*
a*b^2*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(si
n(d*x + c) - 1)) - 3*A*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d
*x + c) + 1) + log(sin(d*x + c) - 1)) + 12*B*a^3*log(sec(d*x + c) + tan(d*x
 + c)) + 36*A*a^2*b*log(sec(d*x + c) + tan(d*x + c)) + 36*B*a^2*b*tan(d*x +
 c) + 36*A*a*b^2*tan(d*x + c))/d
```

Fricas [A]

time = 5.16, size = 189, normalized size = 1.38

$$\frac{12\,Aa^3dx\cos(dx+c)^3+3\,(2\,Ba^3+6\,Aa^2b+3\,Bab^2+Ab^3)\cos(dx+c)^3\log(\sin(dx+c)+1)-3\,(2\,Ba^3+6\,Aa^2b+3\,Bab^2+Ab^3)\cos(dx+c)^3\log(-\sin(dx+c)+1)+2\left(2\,Bb^3+2\,(9\,Ba^2b+9\,Aab^2+2\,Bb^3)\cos(dx+c)^2+3\,(3\,Bab^2+Ab^3)\cos(dx+c)\right)\sin(dx+c)}{12\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(12*A*a^3*d*x*cos(d*x + c)^3 + 3*(2*B*a^3 + 6*A*a^2*b + 3*B*a*b^2 + A*
b^3)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*B*a^3 + 6*A*a^2*b + 3*B*a*
```

```
b^2 + A*b^3)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) + 2*(2*B*b^3 + 2*(9*B*a^
2*b + 9*A*a*b^2 + 2*B*b^3)*cos(d*x + c)^2 + 3*(3*B*a*b^2 + A*b^3)*cos(d*x +
 c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A + B\sec\left(c + dx\right)\right)\left(a + b\sec\left(c + dx\right)\right)^{3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 336 vs. 2(129) = 258.
time = 0.51, size = 336, normalized size = 2.45

$$\frac{6\left(dx+c\right)Aa^{3}+3\left(2\,Ba^{3}+6\,Aa^{2}b+3\,Bab^{2}+Ab^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-3\left(2\,Ba^{3}+6\,Aa^{2}b+3\,Bab^{2}+Ab^{3}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(18\,Ba^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+18\,Aab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-9\,Bab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ab^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+6\,Bb^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-36\,Ba^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-36\,Aab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}-4\,Bb^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+18\,Ba^{2}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+18\,Aab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,Bab^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Ab^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,Bb^{3}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(6*(d*x + c)*A*a^3 + 3*(2*B*a^3 + 6*A*a^2*b + 3*B*a*b^2 + A*b^3)*log(ab
s(tan(1/2*d*x + 1/2*c) + 1)) - 3*(2*B*a^3 + 6*A*a^2*b + 3*B*a*b^2 + A*b^3)*
log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(18*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 +
 18*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 9*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 3*A
*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*B*b^3*tan(1/2*d*x + 1/2*c)^5 - 36*B*a^2*b*t
an(1/2*d*x + 1/2*c)^3 - 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 4*B*b^3*tan(1/2
*d*x + 1/2*c)^3 + 18*B*a^2*b*tan(1/2*d*x + 1/2*c) + 18*A*a*b^2*tan(1/2*d*x
+ 1/2*c) + 9*B*a*b^2*tan(1/2*d*x + 1/2*c) + 3*A*b^3*tan(1/2*d*x + 1/2*c) +
6*B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^3)/d
```

Mupad [B]
time = 4.03, size = 526, normalized size = 3.84

$$\frac{\frac{A\,b^3\,\sin\left(2\,c+2\,d\,x\right)}{4}+\frac{B\,b^3\,\sin\left(3\,c+3\,d\,x\right)}{6}+\frac{B\,b^3\,\sin\left(c+d\,x\right)}{2}+\frac{3\,A\,a\,b^2\,\sin\left(c+d\,x\right)}{4}+\frac{3\,B\,a^2\,b\,\sin\left(c+d\,x\right)}{4}+\frac{3\,A\,a^3\,\cos\left(c+d\,x\right)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}-\frac{A\,b^3\,\cos\left(c+d\,x\right)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,3{}\mathrm{i}}{4}-\frac{B\,a^3\,\cos\left(c+d\,x\right)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,3{}\mathrm{i}}{2}+\frac{3\,A\,a\,b^2\,\sin\left(3\,c+3\,d\,x\right)}{4}+\frac{3\,B\,a\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{4}+\frac{3\,B\,a^2\,b\,\sin\left(3\,c+3\,d\,x\right)}{4}+\frac{A\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos\left(3\,c+3\,d\,x\right)}{2}-\frac{A\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos\left(3\,c+3\,d\,x\right)\,1{}\mathrm{i}}{4}-\frac{B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos\left(3\,c+3\,d\,x\right)\,1{}\mathrm{i}}{2}-\frac{A\,a^2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos\left(3\,c+3\,d\,x\right)\,3{}\mathrm{i}}{2}-\frac{B\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\cos\left(3\,c+3\,d\,x\right)\,3{}\mathrm{i}}{4}-\frac{A\,a^2\,b\,\cos\left(c+d\,x\right)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,9{}\mathrm{i}}{2}-\frac{B\,a\,b^2\,\cos\left(c+d\,x\right)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,9{}\mathrm{i}}{4}}{d\,\left(\frac{3\,\cos\left(c+d\,x\right)}{4}+\frac{\cos\left(3\,c+3\,d\,x\right)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3,x)
```

```
[Out] ((A*b^3*sin(2*c + 2*d*x))/4 + (B*b^3*sin(3*c + 3*d*x))/6 + (B*b^3*sin(c + d
*x))/2 + (3*A*a*b^2*sin(c + d*x))/4 + (3*B*a^2*b*sin(c + d*x))/4 + (3*A*a^3
*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 - (A*b^3*cos(c
 + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i)/4 - (B*a^3*cos
```

```
(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i)/2 + (3*A*a*b
^2*sin(3*c + 3*d*x))/4 + (3*B*a*b^2*sin(2*c + 2*d*x))/4 + (3*B*a^2*b*sin(3*
c + 3*d*x))/4 + (A*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c
+ 3*d*x))/2 - (A*b^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3
*c + 3*d*x)*1i)/4 - (B*a^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))
*cos(3*c + 3*d*x)*1i)/2 - (A*a^2*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (
d*x)/2))*cos(3*c + 3*d*x)*3i)/2 - (B*a*b^2*atan((sin(c/2 + (d*x)/2)*1i)/cos
(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*3i)/4 - (A*a^2*b*cos(c + d*x)*atan((sin(c
/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*9i)/2 - (B*a*b^2*cos(c + d*x)*atan((s
in(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*9i)/4)/(d*((3*cos(c + d*x))/4 + c
os(3*c + 3*d*x)/4))
```

3.297 $\int \cos(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=119

$$a^2(3Ab+aB)x+\frac{b(6aAb+6a^2B+b^2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a^2(2aA-bB)\sin(c+dx)}{2d}+\frac{bB(a+b\sec(c+$$

```
[Out] a^2*(3*A*b+B*a)*x+1/2*b*(6*A*a*b+6*B*a^2+B*b^2)*arctanh(sin(d*x+c))/d+1/2*a
^2*(2*A*a-B*b)*sin(d*x+c)/d+1/2*b*B*(a+b*sec(d*x+c))^2*sin(d*x+c)/d+b^2*(A*
b+2*B*a)*tan(d*x+c)/d
```

Rubi [A]
time = 0.14, antiderivative size = 131, normalized size of antiderivative = 1.10, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4110, 4133, 3855, 3852, 8}

$$-\frac{b(2a^2A-3abB-Ab^2)\tan(c+dx)}{d}+\frac{b(6a^2B+6aAb+b^2B)\tanh^{-1}(\sin(c+dx))}{2d}+a^2x(aB+3Ab)-\frac{b^2(2aA-bB)\tan(c+dx)\sec(c+dx)}{2d}+\frac{aA\sin(c+dx)(a+b\sec(c+dx))^2}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] a^2*(3*A*b + a*B)*x + (b*(6*a*A*b + 6*a^2*B + b^2*B)*ArcTanh[Sin[c + d*x]])
/(2*d) + (a*A*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/d - (b*(2*a^2*A - A*b^2
- 3*a*b*B)*Tan[c + d*x])/d - (b^2*(2*a*A - b*B)*Sec[c + d*x]*Tan[c + d*x])/
(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
```

```
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4133

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A
 + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aA(a+b\sec(c+dx))^2\sin(c+dx)}{d} - \int (a+b\,\mathrm{se}$$
$$= \frac{aA(a+b\sec(c+dx))^2\sin(c+dx)}{d} - \frac{b^2(2aA-b}{}$$
$$= a^2(3Ab+aB)x + \frac{aA(a+b\sec(c+dx))^2\sin(c+}{d}$$
$$= a^2(3Ab+aB)x + \frac{b(6aAb+6a^2B+b^2B)\tanh^{-1}}{2d}$$
$$= a^2(3Ab+aB)x + \frac{b(6aAb+6a^2B+b^2B)\tanh^{-1}}{2d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 399 vs. 2(119) = 238.
time = 0.98, size = 399, normalized size = 3.35

$$\frac{\sec^2(c+dx)\left(6a^2Abc+2a^3Bc+6a^2Abdx+2a^3Bdx-6aAb^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-6a^2bB\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-b^3B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+6aAb^2\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+6a^2bB\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+b^3B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\cos(2(c+dx))\left(2a^2(3Ab+aB)(c+dx)-b(6aAb+6a^2B+b^2B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+b(6aAb+6a^2B+b^2B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+(a^3A+2b^3B)\sin(c+dx)+2Ab^3\sin(2(c+dx))+6ab^2B\sin(2(c+dx))+a^3A\sin(3(c+dx))\right)}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^2*(6*a^2*A*b*c + 2*a^3*B*c + 6*a^2*A*b*d*x + 2*a^3*B*d*x - 6*
a*A*b^2*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] - 6*a^2*b*B*Log[Cos[(c + d
*x)/2] - Sin[(c + d*x)/2]] - b^3*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]]
 + 6*a*A*b^2*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + 6*a^2*b*B*Log[Cos[(
```

```
c + d*x)/2] + Sin[(c + d*x)/2]] + b^3*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x
)/2]] + Cos[2*(c + d*x)]*(2*a^2*(3*A*b + a*B)*(c + d*x) - b*(6*a*A*b + 6*a^
2*B + b^2*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + b*(6*a*A*b + 6*a^2*
B + b^2*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + (a^3*A + 2*b^3*B)*Si
n[c + d*x] + 2*A*b^3*Sin[2*(c + d*x)] + 6*a*b^2*B*Sin[2*(c + d*x)] + a^3*A*
Sin[3*(c + d*x)]))/(4*d)
```

Maple [A]
time = 0.38, size = 141, normalized size = 1.18

method	result
derivativedivides	$\frac{A\,a^{3} \sin(dx+c)+a^{3} B(dx+c)+3A\,a^{2} b(dx+c)+3Bb\,a^{2} \ln(\sec(dx+c)+\tan(dx+c))+3A\,b^{2} a \ln(\sec(dx+c)+\tan(dx+c))+3}{d}$
default	$\frac{A\,a^{3} \sin(dx+c)+a^{3} B(dx+c)+3A\,a^{2} b(dx+c)+3Bb\,a^{2} \ln(\sec(dx+c)+\tan(dx+c))+3A\,b^{2} a \ln(\sec(dx+c)+\tan(dx+c))+3}{d}$
risch	$3A\,a^{2} b x+B\,a^{3} x-\frac{iA\,a^{3} \mathrm{e}^{i(dx+c)}}{2d}+\frac{iA\,a^{3} \mathrm{e}^{-i(dx+c)}}{2d}-\frac{ib^{2} \left(Bb\,\mathrm{e}^{3i(dx+c)}-2Ab\,\mathrm{e}^{2i(dx+c)}-6Ba\,\mathrm{e}^{2i(dx+c)}-Bb\,\mathrm{e}\right.}{d\left(\mathrm{e}^{2i(dx+c)}+1\right)^{2}}$
norman	$\frac{(-3A\,a^{2} b-a^{3} B)x+(-6A\,a^{2} b-2a^{3} B)x\left(\tan^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(3A\,a^{2} b+a^{3} B)x\left(\tan^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(6A\,a^{2} b+2a^{3} B)x\left(\tan^{2}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a^3*sin(d*x+c)+a^3*B*(d*x+c)+3*A*a^2*b*(d*x+c)+3*B*b*a^2*ln(sec(d*x+
c)+tan(d*x+c))+3*A*b^2*a*ln(sec(d*x+c)+tan(d*x+c))+3*B*a*b^2*tan(d*x+c)+A*b
^3*tan(d*x+c)+B*b^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)
)))
```

Maxima [A]
time = 0.28, size = 169, normalized size = 1.42

$$\frac{4\,(dx+c)Ba^{3}+12\,(dx+c)Aa^{2}b-Bb^{3}\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^{2}-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+6\,Ba^{2}b(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+6\,Aab^{2}(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+4\,Aa^{3}\sin(dx+c)+12\,Bab^{2}\tan(dx+c)+4\,Ab^{3}\tan(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/4*(4*(d*x + c)*B*a^3 + 12*(d*x + c)*A*a^2*b - B*b^3*(2*sin(d*x + c)/(sin(
d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 6*B*a^2*
b*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 6*A*a*b^2*(log(sin(d*x
+ c) + 1) - log(sin(d*x + c) - 1)) + 4*A*a^3*sin(d*x + c) + 12*B*a*b^2*tan(
d*x + c) + 4*A*b^3*tan(d*x + c))/d
```

Fricas [A]
time = 3.15, size = 167, normalized size = 1.40

$$\frac{4\,(Ba^{3}+3\,Aa^{2}b)dx\cos(dx+c)^{2}+(6\,Ba^{2}b+6\,Aab^{2}+Bb^{3})\cos(dx+c)^{2}\log(\sin(dx+c)+1)-(6\,Ba^{2}b+6\,Aab^{2}+Bb^{3})\cos(dx+c)^{2}\log(-\sin(dx+c)+1)+2\left(2\,Aa^{3}\cos(dx+c)^{2}+Bb^{3}+2\,(3\,Bab^{2}+Ab^{3})\cos(dx+c)\right)\sin(dx+c)}{4\,d\cos(dx+c)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/4*(4*(B*a^3 + 3*A*a^2*b)*d*x*cos(d*x + c)^2 + (6*B*a^2*b + 6*A*a*b^2 + B*
b^3)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (6*B*a^2*b + 6*A*a*b^2 + B*b^3)
*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(2*A*a^3*cos(d*x + c)^2 + B*b^3
+ 2*(3*B*a*b^2 + A*b^3)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))(a + b\sec(c + dx))^3 \cos(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3*cos(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 241 vs. 2(113) = 226.
time = 0.51, size = 241, normalized size = 2.03

$$\frac{\frac{4\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} + 2\,(Ba^3+3\,Aa^2b)(dx+c) + (6\,Ba^2b+6\,Aab^2+Bb^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right) - (6\,Ba^2b+6\,Aab^2+Bb^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right) - \frac{2\left(6\,Bab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Bb^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,Bab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,Ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Bb^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*(4*A*a^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 2*(B*a^3 +
 3*A*a^2*b)*(d*x + c) + (6*B*a^2*b + 6*A*a*b^2 + B*b^3)*log(abs(tan(1/2*d*x
 + 1/2*c) + 1)) - (6*B*a^2*b + 6*A*a*b^2 + B*b^3)*log(abs(tan(1/2*d*x + 1/2
*c) - 1)) - 2*(6*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^3*tan(1/2*d*x + 1/2
*c)^3 - B*b^3*tan(1/2*d*x + 1/2*c)^3 - 6*B*a*b^2*tan(1/2*d*x + 1/2*c) - 2*A
*b^3*tan(1/2*d*x + 1/2*c) - B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*
c)^2 - 1)^2)/d
```

Mupad [B]
time = 3.60, size = 249, normalized size = 2.09

$$\frac{\frac{A\,a^3\,\sin(3\,c+3\,d\,x)}{4}+\frac{A\,b^3\,\sin(2\,c+2\,d\,x)}{2}+\frac{A\,a^3\,\sin(c+d\,x)}{4}+\frac{B\,b^3\,\sin(c+d\,x)}{2}+\frac{3\,B\,a\,b^2\,\sin(2\,c+2\,d\,x)}{2}}{d\,\left(\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{1}{2}\right)} - \frac{2\left(-B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+\frac{B\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,1{}\mathrm{i}}{2}-3\,A\,a^2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+A\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,3{}\mathrm{i}+B\,a^2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,3{}\mathrm{i}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3,x)
```

```
[Out] ((A*a^3*sin(3*c + 3*d*x))/4 + (A*b^3*sin(2*c + 2*d*x))/2 + (A*a^3*sin(c + d
*x))/4 + (B*b^3*sin(c + d*x))/2 + (3*B*a*b^2*sin(2*c + 2*d*x))/2)/(d*(cos(2
*c + 2*d*x)/2 + 1/2)) - (2*((B*b^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (
d*x)/2))*1i)/2 - B*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - 3*A*a^
2*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + A*a*b^2*atan((sin(c/2 + (
d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i + B*a^2*b*atan((sin(c/2 + (d*x)/2)*1i)/c
os(c/2 + (d*x)/2))*3i))/d
```

3.298 $\int \cos^2(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=124

$$\frac{1}{2}a\left(a^2A+6Ab^2+6abB\right)x+\frac{b^2(Ab+3aB)\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2(2Ab+aB)\sin(c+dx)}{d}+\frac{aA\cos(c+dx)(}{}$$

```
[Out] 1/2*a*(A*a^2+6*A*b^2+6*B*a*b)*x+b^2*(A*b+3*B*a)*arctanh(sin(d*x+c))/d+a^2*(
2*A*b+B*a)*sin(d*x+c)/d+1/2*a*A*cos(d*x+c)*(a+b*sec(d*x+c))^2*sin(d*x+c)/d-
1/2*b^2*(A*a-2*B*b)*tan(d*x+c)/d
```

Rubi [A]
time = 0.22, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4110, 4161, 4132, 8, 4130, 3855}

$$\frac{1}{2}ax\left(a^2A+6abB+6Ab^2\right)+\frac{a^2(aB+2Ab)\sin(c+dx)}{d}-\frac{b^2(aA-2bB)\tan(c+dx)}{2d}+\frac{b^2(3aB+Ab)\tanh^{-1}(\sin(c+dx))}{d}+\frac{aA\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(a^2*A + 6*A*b^2 + 6*a*b*B)*x)/2 + (b^2*(A*b + 3*a*B)*ArcTanh[Sin[c + d*
x]])/d + (a^2*(2*A*b + a*B)*Sin[c + d*x])/d + (a*A*Cos[c + d*x]*(a + b*Sec[
c + d*x])^2*Sin[c + d*x])/(2*d) - (b^2*(a*A - 2*b*B)*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4161

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2
) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n
 + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &
&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aA\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}$$
$$= \frac{aA\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}$$
$$= \frac{aA\cos(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}$$
$$= \frac{1}{2}a\left(a^2A+6Ab^2+6abB\right)x + \frac{a^2(2Ab+aB)\,\mathrm{si}}{d}$$
$$= \frac{1}{2}a\left(a^2A+6Ab^2+6abB\right)x + \frac{b^2(Ab+3aB)\,\mathrm{ta}}{}$$

Mathematica [A]
time = 0.71, size = 217, normalized size = 1.75

$$\frac{2a(a^2A+6Ab^2+6abB)(c+dx)-4b^2(Ab+3aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+4b^2(Ab+3aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{4b^3B\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{4b^3B\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}+4a^2(3Ab+aB)\sin(c+dx)+a^3A\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*(a^2*A + 6*A*b^2 + 6*a*b*B)*(c + d*x) - 4*b^2*(A*b + 3*a*B)*Log[Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]] + 4*b^2*(A*b + 3*a*B)*Log[Cos[(c + d*x)/2] +
 Sin[(c + d*x)/2]] + (4*b^3*B*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c
+ d*x)/2]) + (4*b^3*B*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2
]) + 4*a^2*(3*A*b + a*B)*Sin[c + d*x] + a^3*A*Sin[2*(c + d*x)])/(4*d)
```

Maple [A]

time = 0.29, size = 132, normalized size = 1.06

method	result
derivativedivides	$\frac{A\,a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^3B\sin(dx+c)+3A\,a^2b\sin(dx+c)+3Bb\,a^2(dx+c)+3A\,b^2a(dx+c)+3Ba\,b^2\ln(\sec(dx\cdots}{d}$
default	$\frac{A\,a^3\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^3B\sin(dx+c)+3A\,a^2b\sin(dx+c)+3Bb\,a^2(dx+c)+3A\,b^2a(dx+c)+3Ba\,b^2\ln(\sec(dx\cdots}{d}$
risch	$\frac{a^3Ax}{2}+3Aa\,b^2x+3B\,a^2bx-\frac{iA\,a^3\mathrm{e}^{2i(dx+c)}}{8d}-\frac{3i\mathrm{e}^{i(dx+c)}A\,a^2b}{2d}-\frac{i\mathrm{e}^{i(dx+c)}a^3B}{2d}+\frac{3i\mathrm{e}^{-i(dx+c)}A\,a^2b}{2d}+\cdots$
norman	$\frac{\left(-\frac{1}{2}A\,a^3-3A\,b^2a-3Bb\,a^2\right)x+\left(-\frac{1}{2}A\,a^3-3A\,b^2a-3Bb\,a^2\right)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}A\,a^3+3A\,b^2a+3Bb\,a^2\right)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cdots}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^3*B*sin(d*x+c)+3*A*a
^2*b*sin(d*x+c)+3*B*b*a^2*(d*x+c)+3*A*b^2*a*(d*x+c)+3*B*a*b^2*ln(sec(d*x+c)
+tan(d*x+c))+A*b^3*ln(sec(d*x+c)+tan(d*x+c))+B*b^3*tan(d*x+c))
```

Maxima [A]

time = 0.29, size = 144, normalized size = 1.16

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^3+12\,(dx+c)Ba^2b+12\,(dx+c)Aab^2+6\,Bab^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+2\,Ab^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+4\,Ba^3\sin(dx+c)+12\,Aa^2b\sin(dx+c)+4\,Bb^3\tan(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3 + 12*(d*x + c)*B*a^2*b + 12*(d*
x + c)*A*a*b^2 + 6*B*a*b^2*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1))
+ 2*A*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 4*B*a^3*sin(d*x
 + c) + 12*A*a^2*b*sin(d*x + c) + 4*B*b^3*tan(d*x + c))/d
```

Fricas [A]

time = 1.45, size = 152, normalized size = 1.23

$$\frac{(Aa^3+6\,Ba^2b+6\,Aab^2)dx\cos(dx+c)+(3\,Bab^2+Ab^3)\cos(dx+c)\log(\sin(dx+c)+1)-(3\,Bab^2+Ab^3)\cos(dx+c)\log(-\sin(dx+c)+1)+\left(Aa^3\cos(dx+c)^2+2\,Bb^3+2\,(Ba^3+3\,Aa^2b)\cos(dx+c)\right)\sin(dx+c)}{2\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/2*((A*a^3 + 6*B*a^2*b + 6*A*a*b^2)*d*x*cos(d*x + c) + (3*B*a*b^2 + A*b^3)
*cos(d*x + c)*log(sin(d*x + c) + 1) - (3*B*a*b^2 + A*b^3)*cos(d*x + c)*log(
-sin(d*x + c) + 1) + (A*a^3*cos(d*x + c)^2 + 2*B*b^3 + 2*(B*a^3 + 3*A*a^2*b
)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))\,(a + b\sec(c + dx))^3\cos^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3*cos(c + d*x)**2, x)
```

Giac [A]
time = 0.55, size = 234, normalized size = 1.89

$$-\frac{\frac{4\,Bb^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1} - (Aa^3 + 6\,Ba^2b + 6\,Aab^2)(dx + c) - 2\,(3\,Bab^2 + Ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) + 2\,(3\,Bab^2 + Ab^3)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) + \frac{2\left(Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 2\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - 6\,Aa^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3 - Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 2\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right) - 6\,Aa^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*B*b^3*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - (A*a^3 +
6*B*a^2*b + 6*A*a*b^2)*(d*x + c) - 2*(3*B*a*b^2 + A*b^3)*log(abs(tan(1/2*d*
x + 1/2*c) + 1)) + 2*(3*B*a*b^2 + A*b^3)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
 + 2*(A*a^3*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^3*tan(1/2*d*x + 1/2*c)^3 - 6*A*a
^2*b*tan(1/2*d*x + 1/2*c)^3 - A*a^3*tan(1/2*d*x + 1/2*c) - 2*B*a^3*tan(1/2*
d*x + 1/2*c) - 6*A*a^2*b*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)
^2)/d
```

Mupad [B]
time = 3.33, size = 236, normalized size = 1.90

$$\frac{A\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right) - A\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,2{}\mathrm{i} + 6\,A\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right) + 6\,B\,a^2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right) - B\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,6{}\mathrm{i}}{d} + \frac{\frac{A\,a^3\,\sin(3\,c+3\,d\,x)}{8} + \frac{B\,a^3\,\sin(2\,c+2\,d\,x)}{2} + \frac{A\,a^3\,\sin(c+d\,x)}{8} + B\,b^3\,\sin\left(c+d\,x\right) + \frac{3\,A\,a^2\,b\,\sin(2\,c+2\,d\,x)}{2}}{d\,\cos\left(c+d\,x\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (A*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - A*b^3*atan((sin(c/2 +
(d*x)/2)*1i)/cos(c/2 + (d*x)/2))*2i + 6*A*a*b^2*atan(sin(c/2 + (d*x)/2)/cos
(c/2 + (d*x)/2)) + 6*B*a^2*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) -
B*a*b^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*6i)/d + ((A*a^3*si
n(3*c + 3*d*x))/8 + (B*a^3*sin(2*c + 2*d*x))/2 + (A*a^3*sin(c + d*x))/8 + B
*b^3*sin(c + d*x) + (3*A*a^2*b*sin(2*c + 2*d*x))/2)/(d*cos(c + d*x))
```

3.299 $\int \cos^3(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=145

$$\frac{1}{2}\left(3a^2Ab+2Ab^3+a^3B+6ab^2B\right)x+\frac{b^3B\tanh^{-1}(\sin(c+dx))}{d}+\frac{a(2a^2A+8Ab^2+9abB)\sin(c+dx)}{3d}+\frac{a^2(5\ldots$$

```
[Out] 1/2*(3*A*a^2*b+2*A*b^3+B*a^3+6*B*a*b^2)*x+b^3*B*arctanh(sin(d*x+c))/d+1/3*a
*(2*A*a^2+8*A*b^2+9*B*a*b)*sin(d*x+c)/d+1/6*a^2*(5*A*b+3*B*a)*cos(d*x+c)*si
n(d*x+c)/d+1/3*a*A*cos(d*x+c)^2*(a+b*sec(d*x+c))^2*sin(d*x+c)/d
```

Rubi [A]

time = 0.23, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4110, 4159, 4132, 8, 4130, 3855}

$$\frac{a(2a^2A+9abB+8Ab^2)\sin(c+dx)}{3d}+\frac{a^2(3aB+5Ab)\sin(c+dx)\cos(c+dx)}{6d}+\frac{1}{2}x\left(a^3B+3a^2Ab+6ab^2B+2Ab^3\right)+\frac{aA\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{3d}+\frac{b^3B\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((3*a^2*A*b + 2*A*b^3 + a^3*B + 6*a*b^2*B)*x)/2 + (b^3*B*ArcTanh[Sin[c + d*
x]])/d + (a*(2*a^2*A + 8*A*b^2 + 9*a*b*B)*Sin[c + d*x])/(3*d) + (a^2*(5*A*b
 + 3*a*B)*Cos[c + d*x]*Sin[c + d*x])/(6*d) + (a*A*Cos[c + d*x]^2*(a + b*Sec
[c + d*x])^2*Sin[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^3(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx &= \frac{aA\cos^2(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{3d} \\ &= \frac{a^2(5Ab+3aB)\cos(c+dx)\sin(c+dx)}{6d}+\frac{aA\,\mathrm{co}}{} \\ &= \frac{a^2(5Ab+3aB)\cos(c+dx)\sin(c+dx)}{6d}+\frac{aA\,\mathrm{co}}{} \\ &= \frac{1}{2}\left(3a^2Ab+2Ab^3+a^3B+6ab^2B\right)x+\frac{a(2a^2A-}{} \\ &= \frac{1}{2}\left(3a^2Ab+2Ab^3+a^3B+6ab^2B\right)x+\frac{b^3B\tanh}{}\end{aligned}$$

Mathematica [A]
time = 0.38, size = 159, normalized size = 1.10

$$\frac{6(3a^2Ab+2Ab^3+a^3B+6ab^2B)(c+dx)-12b^3B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12b^3B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+9a(a^2A+4Ab^2+4abB)\sin(c+dx)+3a^2(3Ab+aB)\sin(2(c+dx))+a^3A\sin(3(c+dx))}{12d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (6*(3*a^2*A*b + 2*A*b^3 + a^3*B + 6*a*b^2*B)*(c + d*x) - 12*b^3*B*Log[Cos[(
c + d*x)/2] - Sin[(c + d*x)/2]] + 12*b^3*B*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + 9*a*(a^2*A + 4*A*b^2 + 4*a*b*B)*Sin[c + d*x] + 3*a^2*(3*A*b + a*
B)*Sin[2*(c + d*x)] + a^3*A*Sin[3*(c + d*x)])/(12*d)
```

Maple [A]

time = 0.33, size = 151, normalized size = 1.04

method	result
derivativedivides	$\frac{\frac{A\,a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+a^3B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3A\,a^2b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3Bb\,a^2\sin(dx}{d}$
default	$\frac{\frac{A\,a^3\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}+a^3B\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3A\,a^2b\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+3Bb\,a^2\sin(dx}{d}$
risch	$\frac{3A\,a^2bx}{2}+xA\,b^3+\frac{B\,a^3x}{2}+3xBa\,b^2-\frac{3iA\,a^3\mathrm{e}^{i(dx+c)}}{8d}-\frac{3i\mathrm{e}^{i(dx+c)}A\,b^2a}{2d}-\frac{3i\mathrm{e}^{i(dx+c)}Bb\,a^2}{2d}+\frac{3iA\,a^3\mathrm{e}^{}}{8}$
norman	$\frac{(-\frac{3}{2}A\,a^2b-A\,b^3-\frac{1}{2}a^3B-3Ba\,b^2)x+(-\frac{9}{2}A\,a^2b-3A\,b^3-\frac{3}{2}a^3B-9Ba\,b^2)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(\frac{3}{2}A\,a^2b+A\,b^3+\frac{1}{2}a^3B+3B}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/3*A*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+a^3*B*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+3*A*a^2*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*B*b*a
^2*sin(d*x+c)+3*A*b^2*a*sin(d*x+c)+3*B*a*b^2*(d*x+c)+A*b^3*(d*x+c)+B*b^3*ln
(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]

time = 0.27, size = 152, normalized size = 1.05

$$-\frac{4\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)Aa^3-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ba^3-9\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^2b-36\left(dx+c\right)Bab^2-12\left(dx+c\right)Ab^3-6\,Bb^3\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-36\,Ba^2b\sin\left(dx+c\right)-36\,Aab^2\sin\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^3 - 3*(2*d*x + 2*c + sin(2*d
*x + 2*c))*B*a^3 - 9*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^2*b - 36*(d*x + c
)*B*a*b^2 - 12*(d*x + c)*A*b^3 - 6*B*b^3*(log(sin(d*x + c) + 1) - log(sin(d
*x + c) - 1)) - 36*B*a^2*b*sin(d*x + c) - 36*A*a*b^2*sin(d*x + c))/d
```

Fricas [A]

time = 1.49, size = 131, normalized size = 0.90

$$\frac{3\,Bb^3\log\left(\sin\left(dx+c\right)+1\right)-3\,Bb^3\log\left(-\sin\left(dx+c\right)+1\right)+3\left(Ba^3+3\,Aa^2b+6\,Bab^2+2\,Ab^3\right)dx+\left(2\,Aa^3\cos\left(dx+c\right)^2+4\,Aa^3+18\,Ba^2b+18\,Aab^2+3\left(Ba^3+3\,Aa^2b\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*B*b^3*log(sin(d*x + c) + 1) - 3*B*b^3*log(-sin(d*x + c) + 1) + 3*(B*
a^3 + 3*A*a^2*b + 6*B*a*b^2 + 2*A*b^3)*d*x + (2*A*a^3*cos(d*x + c)^2 + 4*A*
a^3 + 18*B*a^2*b + 18*A*a*b^2 + 3*(B*a^3 + 3*A*a^2*b)*cos(d*x + c))*sin(d*x
 + c))/d
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c+dx))\,(a + b\sec(c+dx))^3\cos^3(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3*cos(c + d*x)**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 314 vs. 2(137) = 274.
time = 0.49, size = 314, normalized size = 2.17

$$\frac{6\,Bb^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-6\,Bb^3\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\,(Ba^3+3\,Aa^2b+6\,Bab^2+2\,Ab^3)(dx+c)+\frac{2\left(6\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-9\,Aa^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18\,Ba^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18\,Aab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+36\,Ba^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+36\,Aab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+9\,Aa^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+18\,Ba^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+18\,Aab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/6*(6*B*b^3*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 6*B*b^3*log(abs(tan(1/2*d
*x + 1/2*c) - 1)) + 3*(B*a^3 + 3*A*a^2*b + 6*B*a*b^2 + 2*A*b^3)*(d*x + c) +
 2*(6*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^3*tan(1/2*d*x + 1/2*c)^5 - 9*A*a
^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 18*A*a*b^
2*tan(1/2*d*x + 1/2*c)^5 + 4*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 36*B*a^2*b*tan(
1/2*d*x + 1/2*c)^3 + 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^3*tan(1/2*d*
x + 1/2*c) + 3*B*a^3*tan(1/2*d*x + 1/2*c) + 9*A*a^2*b*tan(1/2*d*x + 1/2*c)
+ 18*B*a^2*b*tan(1/2*d*x + 1/2*c) + 18*A*a*b^2*tan(1/2*d*x + 1/2*c))/(tan(1
/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [**B**]
time = 3.90, size = 1924, normalized size = 13.27

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3,x)
[Out] (tan(c/2 + (d*x)/2)*(2*A*a^3 + B*a^3 + 6*A*a*b^2 + 3*A*a^2*b + 6*B*a^2*b) +
 tan(c/2 + (d*x)/2)^3*((4*A*a^3)/3 + 12*A*a*b^2 + 12*B*a^2*b) + tan(c/2 + (
d*x)/2)^5*(2*A*a^3 - B*a^3 + 6*A*a*b^2 - 3*A*a^2*b + 6*B*a^2*b))/(d*(3*tan(
c/2 + (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1)) + (a
tan((((A*b^3*1i + (B*a^3*1i)/2 + (A*a^2*b*3i)/2 + B*a*b^2*3i)*(32*A*b^3 + 1
6*B*a^3 + 32*B*b^3 + 48*A*a^2*b + 96*B*a*b^2) + tan(c/2 + (d*x)/2)*(32*A^2*
b^6 + 8*B^2*a^6 + 32*B^2*b^6 + 96*A^2*a^2*b^4 + 72*A^2*a^4*b^2 + 288*B^2*a^
2*b^4 + 96*B^2*a^4*b^2 + 192*A*B*a*b^5 + 48*A*B*a^5*b + 320*A*B*a^3*b^3))*(
A*b^3*1i + (B*a^3*1i)/2 + (A*a^2*b*3i)/2 + B*a*b^2*3i)*1i - ((A*b^3*1i + (B
*a^3*1i)/2 + (A*a^2*b*3i)/2 + B*a*b^2*3i)*(32*A*b^3 + 16*B*a^3 + 32*B*b^3 +
 48*A*a^2*b + 96*B*a*b^2) - tan(c/2 + (d*x)/2)*(32*A^2*b^6 + 8*B^2*a^6 + 32
*B^2*b^6 + 96*A^2*a^2*b^4 + 72*A^2*a^4*b^2 + 288*B^2*a^2*b^4 + 96*B^2*a^4*b
^2 + 192*A*B*a*b^5 + 48*A*B*a^5*b + 320*A*B*a^3*b^3))*(A*b^3*1i + (B*a^3*1i
)/2 + (A*a^2*b*3i)/2 + B*a*b^2*3i)*1i)/(((A*b^3*1i + (B*a^3*1i)/2 + (A*a^2*
b*3i)/2 + B*a*b^2*3i)*(32*A*b^3 + 16*B*a^3 + 32*B*b^3 + 48*A*a^2*b + 96*B*a
*b^2) + tan(c/2 + (d*x)/2)*(32*A^2*b^6 + 8*B^2*a^6 + 32*B^2*b^6 + 96*A^2*a^
2*b^4 + 72*A^2*a^4*b^2 + 288*B^2*a^2*b^4 + 96*B^2*a^4*b^2 + 192*A*B*a*b^5 +
 48*A*B*a^5*b + 320*A*B*a^3*b^3))*(A*b^3*1i + (B*a^3*1i)/2 + (A*a^2*b*3i)/2
 + B*a*b^2*3i) + ((A*b^3*1i + (B*a^3*1i)/2 + (A*a^2*b*3i)/2 + B*a*b^2*3i)*(
32*A*b^3 + 16*B*a^3 + 32*B*b^3 + 48*A*a^2*b + 96*B*a*b^2) - tan(c/2 + (d*x)
/2)*(32*A^2*b^6 + 8*B^2*a^6 + 32*B^2*b^6 + 96*A^2*a^2*b^4 + 72*A^2*a^4*b^2
+ 288*B^2*a^2*b^4 + 96*B^2*a^4*b^2 + 192*A*B*a*b^5 + 48*A*B*a^5*b + 320*A*B
*a^3*b^3))*(A*b^3*1i + (B*a^3*1i)/2 + (A*a^2*b*3i)/2 + B*a*b^2*3i) - 64*A*B
^2*b^9 + 64*A^2*B*b^9 - 192*B^3*a*b^8 + 576*B^3*a^2*b^7 - 32*B^3*a^3*b^6 +
192*B^3*a^4*b^5 + 16*B^3*a^6*b^3 + 384*A*B^2*a*b^8 - 96*A*B^2*a^2*b^7 + 640
*A*B^2*a^3*b^6 + 96*A*B^2*a^5*b^4 + 192*A^2*B*a^2*b^7 + 144*A^2*B*a^4*b^5))
*(2*A*b^3 + B*a^3 + 3*A*a^2*b + 6*B*a*b^2))/d - (B*b^3*atan((B*b^3*(tan(c/2
 + (d*x)/2)*(32*A^2*b^6 + 8*B^2*a^6 + 32*B^2*b^6 + 96*A^2*a^2*b^4 + 72*A^2*
a^4*b^2 + 288*B^2*a^2*b^4 + 96*B^2*a^4*b^2 + 192*A*B*a*b^5 + 48*A*B*a^5*b +
 320*A*B*a^3*b^3) + B*b^3*(32*A*b^3 + 16*B*a^3 + 32*B*b^3 + 48*A*a^2*b + 96
*B*a*b^2))*1i + B*b^3*(tan(c/2 + (d*x)/2)*(32*A^2*b^6 + 8*B^2*a^6 + 32*B^2*
b^6 + 96*A^2*a^2*b^4 + 72*A^2*a^4*b^2 + 288*B^2*a^2*b^4 + 96*B^2*a^4*b^2 +
192*A*B*a*b^5 + 48*A*B*a^5*b + 320*A*B*a^3*b^3) - B*b^3*(32*A*b^3 + 16*B*a^
3 + 32*B*b^3 + 48*A*a^2*b + 96*B*a*b^2))*1i)/(64*A^2*B*b^9 - 64*A*B^2*b^9 -
 192*B^3*a*b^8 + B*b^3*(tan(c/2 + (d*x)/2)*(32*A^2*b^6 + 8*B^2*a^6 + 32*B^2
*b^6 + 96*A^2*a^2*b^4 + 72*A^2*a^4*b^2 + 288*B^2*a^2*b^4 + 96*B^2*a^4*b^2 +
 192*A*B*a*b^5 + 48*A*B*a^5*b + 320*A*B*a^3*b^3) + B*b^3*(32*A*b^3 + 16*B*a
^3 + 32*B*b^3 + 48*A*a^2*b + 96*B*a*b^2)) - B*b^3*(tan(c/2 + (d*x)/2)*(32*A
^2*b^6 + 8*B^2*a^6 + 32*B^2*b^6 + 96*A^2*a^2*b^4 + 72*A^2*a^4*b^2 + 288*B^2
*a^2*b^4 + 96*B^2*a^4*b^2 + 192*A*B*a*b^5 + 48*A*B*a^5*b + 320*A*B*a^3*b^3)
 - B*b^3*(32*A*b^3 + 16*B*a^3 + 32*B*b^3 + 48*A*a^2*b + 96*B*a*b^2)) + 576*
B^3*a^2*b^7 - 32*B^3*a^3*b^6 + 192*B^3*a^4*b^5 + 16*B^3*a^6*b^3 + 384*A*B^2
*a*b^8 - 96*A*B^2*a^2*b^7 + 640*A*B^2*a^3*b^6 + 96*A*B^2*a^5*b^4 + 192*A^2*
```

```
B*a^2*b^7 + 144*A^2*B*a^4*b^5))*2i)/d
```

3.300 $\int \cos^4(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=179

$$\frac{1}{8}\left(3a^3A+12aAb^2+12a^2bB+8b^3B\right)x+\frac{(6a^2Ab+3Ab^3+2a^3B+9ab^2B)\sin(c+dx)}{3d}+\frac{a(3a^2A+10Ab^2+$$

```
[Out] 1/8*(3*A*a^3+12*A*a*b^2+12*B*a^2*b+8*B*b^3)*x+1/3*(6*A*a^2*b+3*A*b^3+2*B*a^
3+9*B*a*b^2)*sin(d*x+c)/d+1/8*a*(3*A*a^2+10*A*b^2+12*B*a*b)*cos(d*x+c)*sin(
d*x+c)/d+1/6*a^2*(3*A*b+2*B*a)*cos(d*x+c)^2*sin(d*x+c)/d+1/4*a*A*cos(d*x+c)
^3*(a+b*sec(d*x+c))^2*sin(d*x+c)/d
```

Rubi [A]
time = 0.28, antiderivative size = 179, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4110, 4159, 4132, 2717, 4130, 8}

$$\frac{a(3a^2A+12abB+10Ab^2)\sin(c+dx)\cos(c+dx)}{8d}+\frac{a^2(2aB+3Ab)\sin(c+dx)\cos^2(c+dx)}{6d}+\frac{(2a^3B+6a^2Ab+9ab^2B+3Ab^3)\sin(c+dx)}{3d}+\frac{1}{8}x(3a^3A+12a^2bB+12aAb^2+8b^3B)+\frac{aA\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((3*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 8*b^3*B)*x)/8 + ((6*a^2*A*b + 3*A*b^3
 + 2*a^3*B + 9*a*b^2*B)*Sin[c + d*x])/(3*d) + (a*(3*a^2*A + 10*A*b^2 + 12*a
*b*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(3*A*b + 2*a*B)*Cos[c + d*x]^
2*Sin[c + d*x])/(6*d) + (a*A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*Sin[c +
d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
```

```
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aA\cos^3(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{4d}$$
$$= \frac{a^2(3Ab+2aB)\cos^2(c+dx)\sin(c+dx)}{6d} + \frac{aA\,c}{}$$
$$= \frac{a^2(3Ab+2aB)\cos^2(c+dx)\sin(c+dx)}{6d} + \frac{aA\,c}{}$$
$$= \frac{(6a^2Ab+3Ab^3+2a^3B+9ab^2B)\sin(c+dx)}{3d} +$$
$$= \frac{1}{8}\left(3a^3A+12aAb^2+12a^2bB+8b^3B\right)x + \frac{(6a^2A}{}$$

Mathematica [A]
time = 0.44, size = 140, normalized size = 0.78

$$\frac{12(3a^3A+12aAb^2+12a^2bB+8b^3B)\,(c+dx)+24(9a^2Ab+4Ab^3+3a^3B+12ab^2B)\sin(c+dx)+24a(a^2A+3Ab^2+3abB)\sin(2(c+dx))+8a^2(3Ab+aB)\sin(3(c+dx))+3a^3A\sin(4(c+dx))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (12*(3*a^3*A + 12*a*A*b^2 + 12*a^2*b*B + 8*b^3*B)*(c + d*x) + 24*(9*a^2*A*b
 + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*Sin[c + d*x] + 24*a*(a^2*A + 3*A*b^2 + 3
*a*b*B)*Sin[2*(c + d*x)] + 8*a^2*(3*A*b + a*B)*Sin[3*(c + d*x)] + 3*a^3*A*S
in[4*(c + d*x)])/(96*d)
```

Maple [A]
time = 0.42, size = 180, normalized size = 1.01

method	result
derivativedivides	$\frac{A\,a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+A\,a^2b\left(2+\cos^2(dx+c)\right)\sin(dx+c)+\frac{a^3B\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{}$
default	$\frac{A\,a^3\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+A\,a^2b\left(2+\cos^2(dx+c)\right)\sin(dx+c)+\frac{a^3B\left(2+\cos^2(dx+c)\right)\sin(dx+c)}{3}}{}$
risch	$\frac{3a^3Ax}{8}+\frac{3Aa\,b^2x}{2}+\frac{3B\,a^2bx}{2}+xB\,b^3+\frac{9\sin(dx+c)A\,a^2b}{4d}+\frac{\sin(dx+c)A\,b^3}{d}+\frac{3a^3B\sin(dx+c)}{4d}+\frac{3\sin(dx-}{}$
norman	$\frac{(-\frac{3}{8}A\,a^3-\frac{3}{2}A\,b^2a-\frac{3}{2}Bb\,a^2-B\,b^3)x+(-\frac{9}{8}A\,a^3-\frac{9}{2}A\,b^2a-\frac{9}{2}Bb\,a^2-3B\,b^3)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-\frac{9}{8}A\,a^3-\frac{9}{2}A\,b^2a-\frac{9}{2}Bb}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^3*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+A*a
^2*b*(2+cos(d*x+c)^2)*sin(d*x+c)+1/3*a^3*B*(2+cos(d*x+c)^2)*sin(d*x+c)+3*A*
b^2*a*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+3*B*b*a^2*(1/2*cos(d*x+c)*s
in(d*x+c)+1/2*d*x+1/2*c)+A*b^3*sin(d*x+c)+3*B*a*b^2*sin(d*x+c)+B*b^3*(d*x+c
))
```

Maxima [A]
time = 0.28, size = 171, normalized size = 0.96

$$\frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Aa^3-32\,(\sin(dx+c)^3-3\,\sin(dx+c))Ba^3-96\,(\sin(dx+c)^3-3\,\sin(dx+c))Aa^2b+72\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^2b+72\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aab^2+96\,(dx+c)Bb^3+288\,Bab^2\sin(dx+c)+96\,Ab^3\sin(dx+c)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3 - 32*
(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3 - 96*(sin(d*x + c)^3 - 3*sin(d*x +
c))*A*a^2*b + 72*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2*b + 72*(2*d*x + 2*c
```

```
 + sin(2*d*x + 2*c))*A*a*b^2 + 96*(d*x + c)*B*b^3 + 288*B*a*b^2*sin(d*x + c
) + 96*A*b^3*sin(d*x + c))/d
```

Fricas [A]
time = 1.68, size = 136, normalized size = 0.76

$$\frac{3\,(3\,Aa^3+12\,Ba^2b+12\,Aab^2+8\,Bb^3)dx+\left(6\,Aa^3\cos\left(dx+c\right)^3+16\,Ba^3+48\,Aa^2b+72\,Bab^2+24\,Ab^3+8\,(Ba^3+3\,Aa^2b)\cos\left(dx+c\right)^2+9\,(Aa^3+4\,Ba^2b+4\,Aab^2)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/24*(3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*d*x + (6*A*a^3*cos(d*
x + c)^3 + 16*B*a^3 + 48*A*a^2*b + 72*B*a*b^2 + 24*A*b^3 + 8*(B*a^3 + 3*A*a
^2*b)*cos(d*x + c)^2 + 9*(A*a^3 + 4*B*a^2*b + 4*A*a*b^2)*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 536 vs. 2(169) = 338.
time = 0.52, size = 536, normalized size = 2.99

$$\frac{3\,(3\,Aa^3+12\,Ba^2b+12\,Aab^2+8\,Bb^3)(dx+c)-\frac{2\left(15\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-24\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-72\,Aa^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+36\,Ba^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+36\,Aab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-72\,Bab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-24\,Ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-9\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-40\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-120\,Aa^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+36\,Ba^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+36\,Aab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-216\,Bab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-72\,Ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+9\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-40\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-120\,Aa^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-36\,Ba^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-36\,Aab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-216\,Bab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-72\,Ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-15\,Aa^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,Ba^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-72\,Aa^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-36\,Ba^2b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-36\,Aab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-72\,Bab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-24\,Ab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^4}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(3*(3*A*a^3 + 12*B*a^2*b + 12*A*a*b^2 + 8*B*b^3)*(d*x + c) - 2*(15*A*a
^3*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 72*A*a^2*b*ta
n(1/2*d*x + 1/2*c)^7 + 36*B*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 36*A*a*b^2*tan(1
/2*d*x + 1/2*c)^7 - 72*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 24*A*b^3*tan(1/2*d*
x + 1/2*c)^7 - 9*A*a^3*tan(1/2*d*x + 1/2*c)^5 - 40*B*a^3*tan(1/2*d*x + 1/2*
c)^5 - 120*A*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^2*b*tan(1/2*d*x + 1/2*c)
^5 + 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^5 - 216*B*a*b^2*tan(1/2*d*x + 1/2*c)^5
 - 72*A*b^3*tan(1/2*d*x + 1/2*c)^5 + 9*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*B*
a^3*tan(1/2*d*x + 1/2*c)^3 - 120*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 36*B*a^2*
```

```
b*tan(1/2*d*x + 1/2*c)^3 - 36*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - 216*B*a*b^2*
tan(1/2*d*x + 1/2*c)^3 - 72*A*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^3*tan(1/2
*d*x + 1/2*c) - 24*B*a^3*tan(1/2*d*x + 1/2*c) - 72*A*a^2*b*tan(1/2*d*x + 1/
2*c) - 36*B*a^2*b*tan(1/2*d*x + 1/2*c) - 36*A*a*b^2*tan(1/2*d*x + 1/2*c) -
72*B*a*b^2*tan(1/2*d*x + 1/2*c) - 24*A*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d
*x + 1/2*c)^2 + 1)^4)/d
```

Mupad [B]
time = 2.49, size = 202, normalized size = 1.13

$$\frac{3\,A\,a^3\,x}{8}+B\,b^3\,x+\frac{3\,A\,a\,b^2\,x}{2}+\frac{3\,B\,a^2\,b\,x}{2}+\frac{A\,b^3\,\sin\left(c+d\,x\right)}{d}+\frac{3\,B\,a^3\,\sin\left(c+d\,x\right)}{4\,d}+\frac{A\,a^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{A\,a^3\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{B\,a^3\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{3\,A\,a\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{A\,a^2\,b\,\sin\left(3\,c+3\,d\,x\right)}{4\,d}+\frac{3\,B\,a^2\,b\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{9\,A\,a^2\,b\,\sin\left(c+d\,x\right)}{4\,d}+\frac{3\,B\,a\,b^2\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (3*A*a^3*x)/8 + B*b^3*x + (3*A*a*b^2*x)/2 + (3*B*a^2*b*x)/2 + (A*b^3*sin(c
+ d*x))/d + (3*B*a^3*sin(c + d*x))/(4*d) + (A*a^3*sin(2*c + 2*d*x))/(4*d) +
 (A*a^3*sin(4*c + 4*d*x))/(32*d) + (B*a^3*sin(3*c + 3*d*x))/(12*d) + (3*A*a
*b^2*sin(2*c + 2*d*x))/(4*d) + (A*a^2*b*sin(3*c + 3*d*x))/(4*d) + (3*B*a^2*
b*sin(2*c + 2*d*x))/(4*d) + (9*A*a^2*b*sin(c + d*x))/(4*d) + (3*B*a*b^2*sin
(c + d*x))/d
```

3.301 $\int \cos^5(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=221

$$\frac{1}{8}\left(9a^2Ab+4Ab^3+3a^3B+12ab^2B\right)x+\frac{(4a^3A+14aAb^2+15a^2bB+5b^3B)\sin(c+dx)}{5d}+\frac{(9a^2Ab+4Ab^3+3a$$

```
[Out] 1/8*(9*A*a^2*b+4*A*b^3+3*B*a^3+12*B*a*b^2)*x+1/5*(4*A*a^3+14*A*a*b^2+15*B*a
^2*b+5*B*b^3)*sin(d*x+c)/d+1/8*(9*A*a^2*b+4*A*b^3+3*B*a^3+12*B*a*b^2)*cos(d
*x+c)*sin(d*x+c)/d+1/20*a^2*(7*A*b+5*B*a)*cos(d*x+c)^3*sin(d*x+c)/d+1/5*a*A
*cos(d*x+c)^4*(a+b*sec(d*x+c))^2*sin(d*x+c)/d-1/15*a*(4*A*a^2+12*A*b^2+15*B
*a*b)*sin(d*x+c)^3/d
```

Rubi [A]
time = 0.32, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4110, 4159, 4132, 2715, 8, 4129, 3092}

$$-\frac{a(4a^2A+15abB+12Ab^2)\sin^3(c+dx)}{15d}+\frac{a^2(5aB+7Ab)\sin(c+dx)\cos^3(c+dx)}{20d}+\frac{(4a^3A+15a^2bB+14aAb^2+5b^3B)\sin(c+dx)}{5d}+\frac{(3a^3B+9a^2Ab+12ab^2B+4Ab^3)\sin(c+dx)\cos(c+dx)}{8d}+\frac{1}{8}x\left(3a^3B+9a^2Ab+12ab^2B+4Ab^3\right)+\frac{aA\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*x)/8 + ((4*a^3*A + 14*a*A*b^2
 + 15*a^2*b*B + 5*b^3*B)*Sin[c + d*x])/(5*d) + ((9*a^2*A*b + 4*A*b^3 + 3*a^
3*B + 12*a*b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(8*d) + (a^2*(7*A*b + 5*a*B)*C
os[c + d*x]^3*Sin[c + d*x])/(20*d) + (a*A*Cos[c + d*x]^4*(a + b*Sec[c + d*x
])^2*Sin[c + d*x])/(5*d) - (a*(4*a^2*A + 12*A*b^2 + 15*a*b*B)*Sin[c + d*x]^
3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
```

```
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4129

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{aA\cos^4(c+dx)(a+b\sec(c+dx))^2\sin(c+dx)}{5d}$$
$$= \frac{a^2(7Ab+5aB)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{aA\,c}{}$$
$$= \frac{a^2(7Ab+5aB)\cos^3(c+dx)\sin(c+dx)}{20d} + \frac{aA\,c}{}$$
$$= \frac{(9a^2Ab+4Ab^3+3a^3B+12ab^2B)\cos(c+dx)\,s}{8d}$$
$$= \frac{1}{8}\left(9a^2Ab+4Ab^3+3a^3B+12ab^2B\right)x + \frac{(9a^2A}{}$$
$$= \frac{1}{8}\left(9a^2Ab+4Ab^3+3a^3B+12ab^2B\right)x + \frac{(4a^3A}{}$$

Mathematica [A]
time = 0.75, size = 176, normalized size = 0.80

$$\frac{60(9a^2Ab+4Ab^3+3a^3B+12ab^2B)(c+dx)+60(5a^3A+18aAb^2+18a^2bB+8b^3B)\sin(c+dx)+120(3a^2Ab+Ab^3+a^3B+3ab^2B)\sin(2(c+dx))+10a(5a^2A+12Ab^2+12abB)\sin(3(c+dx))+15a^2(3Ab+aB)\sin(4(c+dx))+6a^3A\sin(5(c+dx))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (60*(9*a^2*A*b + 4*A*b^3 + 3*a^3*B + 12*a*b^2*B)*(c + d*x) + 60*(5*a^3*A +
18*a*A*b^2 + 18*a^2*b*B + 8*b^3*B)*Sin[c + d*x] + 120*(3*a^2*A*b + A*b^3 +
a^3*B + 3*a*b^2*B)*Sin[2*(c + d*x)] + 10*a*(5*a^2*A + 12*A*b^2 + 12*a*b*B)*
Sin[3*(c + d*x)] + 15*a^2*(3*A*b + a*B)*Sin[4*(c + d*x)] + 6*a^3*A*Sin[5*(c
 + d*x)])/(480*d)
```

Maple [A]
time = 0.54, size = 227, normalized size = 1.03 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/5*A*a^3*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+a^3*B*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+3*A*a^2*b*(1/4*(cos(d
*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+B*b*a^2*(2+cos(d*x+c)^2)*
sin(d*x+c)+A*b^2*a*(2+cos(d*x+c)^2)*sin(d*x+c)+3*B*a*b^2*(1/2*cos(d*x+c)*si
n(d*x+c)+1/2*d*x+1/2*c)+A*b^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+B*b
^3*sin(d*x+c))
```

Maxima [A]
time = 0.27, size = 217, normalized size = 0.98

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^3+15\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(2\,dx+2\,c)\right)Ba^3+45\left(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\sin(2\,dx+2\,c)\right)Aa^2b-480\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba^2b-480\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aab^2+360\left(2\,dx+2\,c+\sin(2\,dx+2\,c)\right)Bab^2+120\left(2\,dx+2\,c+\sin(2\,dx+2\,c)\right)Ab^3+480\,Bb^3\sin(dx+c)}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^3 +
15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^3 + 45*(12*d
*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2*b - 480*(sin(d*x +
 c)^3 - 3*sin(d*x + c))*B*a^2*b - 480*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a
*b^2 + 360*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a*b^2 + 120*(2*d*x + 2*c + si
n(2*d*x + 2*c))*A*b^3 + 480*B*b^3*sin(d*x + c))/d
```

Fricas [A]
time = 2.77, size = 174, normalized size = 0.79

$$\frac{15\,(3\,Ba^3+9\,Aa^2b+12\,Bab^2+4\,Ab^3)dx+\left(24\,Aa^3\cos\left(dx+c\right)^4+64\,Aa^3+240\,Ba^2b+240\,Aab^2+120\,Bb^3+30\,(Ba^3+3\,Aa^2b)\cos\left(dx+c\right)^3+8\,(4\,Aa^3+15\,Ba^2b+15\,Aab^2)\cos\left(dx+c\right)^2+15\,(3\,Ba^3+9\,Aa^2b+12\,Bab^2+4\,Ab^3)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/120*(15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*d*x + (24*A*a^3*cos(
d*x + c)^4 + 64*A*a^3 + 240*B*a^2*b + 240*A*a*b^2 + 120*B*b^3 + 30*(B*a^3 +
 3*A*a^2*b)*cos(d*x + c)^3 + 8*(4*A*a^3 + 15*B*a^2*b + 15*A*a*b^2)*cos(d*x
+ c)^2 + 15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*cos(d*x + c))*sin(
d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 672 vs. 2(209) = 418.
time = 0.51, size = 672, normalized size = 3.04

$$\frac{15\,(3\,Ba^3+9\,Aa^2b+12\,Bab^2+4\,Ab^3)(dx+c)+\frac{2\left(\text{[illegible]}\right)}{\text{[illegible]}}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(3*B*a^3 + 9*A*a^2*b + 12*B*a*b^2 + 4*A*b^3)*(d*x + c) + 2*(120*A
*a^3*tan(1/2*d*x + 1/2*c)^9 - 75*B*a^3*tan(1/2*d*x + 1/2*c)^9 - 225*A*a^2*b
*tan(1/2*d*x + 1/2*c)^9 + 360*B*a^2*b*tan(1/2*d*x + 1/2*c)^9 + 360*A*a*b^2*
tan(1/2*d*x + 1/2*c)^9 - 180*B*a*b^2*tan(1/2*d*x + 1/2*c)^9 - 60*A*b^3*tan(
1/2*d*x + 1/2*c)^9 + 120*B*b^3*tan(1/2*d*x + 1/2*c)^9 + 160*A*a^3*tan(1/2*d
*x + 1/2*c)^7 - 30*B*a^3*tan(1/2*d*x + 1/2*c)^7 - 90*A*a^2*b*tan(1/2*d*x +
1/2*c)^7 + 960*B*a^2*b*tan(1/2*d*x + 1/2*c)^7 + 960*A*a*b^2*tan(1/2*d*x + 1
/2*c)^7 - 360*B*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 120*A*b^3*tan(1/2*d*x + 1/2*
c)^7 + 480*B*b^3*tan(1/2*d*x + 1/2*c)^7 + 464*A*a^3*tan(1/2*d*x + 1/2*c)^5
+ 1200*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 1200*A*a*b^2*tan(1/2*d*x + 1/2*c)^5
 + 720*B*b^3*tan(1/2*d*x + 1/2*c)^5 + 160*A*a^3*tan(1/2*d*x + 1/2*c)^3 + 30
*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 90*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 + 960*B*a
^2*b*tan(1/2*d*x + 1/2*c)^3 + 960*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 360*B*a*
b^2*tan(1/2*d*x + 1/2*c)^3 + 120*A*b^3*tan(1/2*d*x + 1/2*c)^3 + 480*B*b^3*t
an(1/2*d*x + 1/2*c)^3 + 120*A*a^3*tan(1/2*d*x + 1/2*c) + 75*B*a^3*tan(1/2*d
*x + 1/2*c) + 225*A*a^2*b*tan(1/2*d*x + 1/2*c) + 360*B*a^2*b*tan(1/2*d*x +
1/2*c) + 360*A*a*b^2*tan(1/2*d*x + 1/2*c) + 180*B*a*b^2*tan(1/2*d*x + 1/2*c
) + 60*A*b^3*tan(1/2*d*x + 1/2*c) + 120*B*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/
2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B]
time = 2.73, size = 277, normalized size = 1.25

$$\frac{A\,b^3\,x}{2}+\frac{3\,B\,a^3\,x}{8}+\frac{9\,A\,a^2\,b\,x}{8}+\frac{3\,B\,a\,b^2\,x}{2}+\frac{5\,A\,a^3\,\sin\left(c+d\,x\right)}{8\,d}+\frac{B\,b^3\,\sin\left(c+d\,x\right)}{d}+\frac{5\,A\,a^3\,\sin\left(3\,c+3\,d\,x\right)}{48\,d}+\frac{A\,a^3\,\sin\left(5\,c+5\,d\,x\right)}{80\,d}+\frac{A\,b^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,a^3\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,a^3\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{3\,A\,a^2\,b\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{A\,a\,b^2\,\sin\left(3\,c+3\,d\,x\right)}{4\,d}+\frac{3\,A\,a^2\,b\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{3\,B\,a\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,a^2\,b\,\sin\left(3\,c+3\,d\,x\right)}{4\,d}+\frac{9\,A\,a\,b^2\,\sin\left(c+d\,x\right)}{4\,d}+\frac{9\,B\,a^2\,b\,\sin\left(c+d\,x\right)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3,x)
```

```
[Out] (A*b^3*x)/2 + (3*B*a^3*x)/8 + (9*A*a^2*b*x)/8 + (3*B*a*b^2*x)/2 + (5*A*a^3*
sin(c + d*x))/(8*d) + (B*b^3*sin(c + d*x))/d + (5*A*a^3*sin(3*c + 3*d*x))/(
48*d) + (A*a^3*sin(5*c + 5*d*x))/(80*d) + (A*b^3*sin(2*c + 2*d*x))/(4*d) +
(B*a^3*sin(2*c + 2*d*x))/(4*d) + (B*a^3*sin(4*c + 4*d*x))/(32*d) + (3*A*a^2
*b*sin(2*c + 2*d*x))/(4*d) + (A*a*b^2*sin(3*c + 3*d*x))/(4*d) + (3*A*a^2*b*
sin(4*c + 4*d*x))/(32*d) + (3*B*a*b^2*sin(2*c + 2*d*x))/(4*d) + (B*a^2*b*si
n(3*c + 3*d*x))/(4*d) + (9*A*a*b^2*sin(c + d*x))/(4*d) + (9*B*a^2*b*sin(c +
 d*x))/(4*d)
```

3.302 $\int \sec^2(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=334

$$\frac{(32a^3Ab+24aAb^3+8a^4B+36a^2b^2B+5b^4B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{(24a^4Ab+224a^2Ab^3+32Ab^5-4a^5B$$

```
[Out] 1/16*(32*A*a^3*b+24*A*a*b^3+8*B*a^4+36*B*a^2*b^2+5*B*b^4)*arctanh(sin(d*x+c
))/d+1/60*(24*A*a^4*b+224*A*a^2*b^3+32*A*b^5-4*B*a^5+121*B*a^3*b^2+128*B*a*
b^4)*tan(d*x+c)/b/d+1/240*(48*A*a^3*b+232*A*a*b^3-8*B*a^4+178*B*a^2*b^2+75*
B*b^4)*sec(d*x+c)*tan(d*x+c)/d+1/120*(24*A*a^2*b+32*A*b^3-4*B*a^3+53*B*a*b^
2)*(a+b*sec(d*x+c))^2*tan(d*x+c)/b/d+1/120*(24*A*a*b-4*B*a^2+25*B*b^2)*(a+b
*sec(d*x+c))^3*tan(d*x+c)/b/d+1/30*(6*A*b-B*a)*(a+b*sec(d*x+c))^4*tan(d*x+c
)/b/d+1/6*B*(a+b*sec(d*x+c))^5*tan(d*x+c)/b/d
```

Rubi [A]
time = 0.46, antiderivative size = 334, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4095, 4087, 4082, 3872, 3855, 3852, 8}

$$\frac{(-4a^2B+24aAb+25b^2B)\tan(c+dx)(a+b\sec(c+dx))^3}{120bd}+\frac{(-4a^3B+24a^2Ab+53ab^2B+32Ab^3)\tan(c+dx)(a+b\sec(c+dx))^2}{120bd}+\frac{(8a^4B+32a^3Ab+36a^2b^2B+24aAb^3+5b^4B)\tanh^{-1}(\sin(c+dx))}{16d}+\frac{(-8a^4B+48a^3Ab+178a^2b^2B+232aAb^3+75b^4B)\tan(c+dx)\sec(c+dx)}{240d}+\frac{(-4a^5B+24a^4Ab+121a^3b^2B+224a^2Ab^3+128ab^4B+32Ab^5)\tan(c+dx)}{60bd}+\frac{(6Ab-aB)\tan(c+dx)(a+b\sec(c+dx))^4}{30bd}+\frac{B\tan(c+dx)(a+b\sec(c+dx))^5}{6bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((32*a^3*A*b + 24*a*A*b^3 + 8*a^4*B + 36*a^2*b^2*B + 5*b^4*B)*ArcTanh[Sin[c
 + d*x]])/(16*d) + ((24*a^4*A*b + 224*a^2*A*b^3 + 32*A*b^5 - 4*a^5*B + 121*
a^3*b^2*B + 128*a*b^4*B)*Tan[c + d*x])/(60*b*d) + ((48*a^3*A*b + 232*a*A*b^
3 - 8*a^4*B + 178*a^2*b^2*B + 75*b^4*B)*Sec[c + d*x]*Tan[c + d*x])/(240*d)
+ ((24*a^2*A*b + 32*A*b^3 - 4*a^3*B + 53*a*b^2*B)*(a + b*Sec[c + d*x])^2*Ta
n[c + d*x])/(120*b*d) + ((24*a*A*b - 4*a^2*B + 25*b^2*B)*(a + b*Sec[c + d*x
])^3*Tan[c + d*x])/(120*b*d) + ((6*A*b - a*B)*(a + b*Sec[c + d*x])^4*Tan[c
+ d*x])/(30*b*d) + (B*(a + b*Sec[c + d*x])^5*Tan[c + d*x])/(6*b*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{B(a+b\sec(c+dx))^5\tan(c+dx)}{6bd} + \frac{\int \sec(c+$$
$$= \frac{(6Ab-aB)(a+b\sec(c+dx))^4\tan(c+dx)}{30bd} +$$
$$= \frac{(24aAb-4a^2B+25b^2B)\,(a+b\sec(c+dx))^3}{120bd}$$
$$= \frac{(24a^2Ab+32Ab^3-4a^3B+53ab^2B)\,(a+b\sec}{120bd}$$
$$= \frac{(48a^3Ab+232aAb^3-8a^4B+178a^2b^2B+75b}{240d}$$
$$= \frac{(48a^3Ab+232aAb^3-8a^4B+178a^2b^2B+75b}{240d}$$
$$= \frac{(32a^3Ab+24aAb^3+8a^4B+36a^2b^2B+5b^4B}{16d}$$
$$= \frac{(32a^3Ab+24aAb^3+8a^4B+36a^2b^2B+5b^4B}{16d}$$

Mathematica [A]
time = 2.91, size = 244, normalized size = 0.73

$$\frac{15(32a^3Ab+24aAb^3+8a^4B+36a^2b^2B+5b^4B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(240(a^4A+6a^2Ab^2+Ab^4+4a^3bB+4ab^3B)+15(32a^3Ab+24aAb^3+8a^4B+36a^2b^2B+5b^4B)\sec(c+dx)+10b^2(24aAb+36a^2B+5b^2B)\sec^3(c+dx)+40b^4B\sec^5(c+dx)+160b(3a^2Ab+Ab^3+2a^3B+4ab^2B)\tan^2(c+dx)+48b^3(Ab+4aB)\tan^4(c+dx)\right)}{240d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (15*(32*a^3*A*b + 24*a*A*b^3 + 8*a^4*B + 36*a^2*b^2*B + 5*b^4*B)*ArcTanh[Si
n[c + d*x]] + Tan[c + d*x]*(240*(a^4*A + 6*a^2*A*b^2 + A*b^4 + 4*a^3*b*B +
4*a*b^3*B) + 15*(32*a^3*A*b + 24*a*A*b^3 + 8*a^4*B + 36*a^2*b^2*B + 5*b^4*B
)*Sec[c + d*x] + 10*b^2*(24*a*A*b + 36*a^2*B + 5*b^2*B)*Sec[c + d*x]^3 + 40
*b^4*B*Sec[c + d*x]^5 + 160*b*(3*a^2*A*b + A*b^3 + 2*a^3*B + 4*a*b^2*B)*Tan
[c + d*x]^2 + 48*b^3*(A*b + 4*a*B)*Tan[c + d*x]^4))/(240*d)
```

Maple [A]
time = 0.61, size = 375, normalized size = 1.12 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^4*tan(d*x+c)+a^4*B*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+ta
n(d*x+c)))+4*A*a^3*b*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c
)))-4*B*b*a^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)-6*A*b^2*a^2*(-2/3-1/3*sec(
```

```
d*x+c)^2)*tan(d*x+c)+6*B*a^2*b^2*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d
*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))+4*A*a*b^3*(-(-1/4*sec(d*x+c)^3-3/8*sec
(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c)))-4*B*b^3*a*(-8/15-1/5*sec
(d*x+c)^4-4/15*sec(d*x+c)^2)*tan(d*x+c)-A*b^4*(-8/15-1/5*sec(d*x+c)^4-4/15*
sec(d*x+c)^2)*tan(d*x+c)+B*b^4*(-(-1/6*sec(d*x+c)^5-5/24*sec(d*x+c)^3-5/16*
sec(d*x+c))*tan(d*x+c)+5/16*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.31, size = 474, normalized size = 1.42

$$\frac{640\,\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Ba^{3}b+960\,\left(\tan\left(dx+c\right)^{3}+3\,\tan\left(dx+c\right)\right)Aa^{2}b^{2}+128\,\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)Bab^{3}+32\,\left(3\,\tan\left(dx+c\right)^{5}+10\,\tan\left(dx+c\right)^{3}+15\,\tan\left(dx+c\right)\right)Ab^{4}-5\,Bb^{4}\left(\frac{2\,\left(15\,\sin\left(dx+c\right)^{5}-40\,\sin\left(dx+c\right)^{3}+33\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{6}-3\,\sin\left(dx+c\right)^{4}+3\,\sin\left(dx+c\right)^{2}-1}-15\,\log\left(\sin\left(dx+c\right)+1\right)+15\,\log\left(\sin\left(dx+c\right)-1\right)\right)-180\,Ba^{2}b^{2}\left(\frac{2\,\left(3\,\sin\left(dx+c\right)^{3}-5\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-120\,Aab^{3}\left(\frac{2\,\left(3\,\sin\left(dx+c\right)^{3}-5\,\sin\left(dx+c\right)\right)}{\sin\left(dx+c\right)^{4}-2\,\sin\left(dx+c\right)^{2}+1}-3\,\log\left(\sin\left(dx+c\right)+1\right)+3\,\log\left(\sin\left(dx+c\right)-1\right)\right)-120\,Ba^{4}\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-480\,Aa^{3}b\left(\frac{2\,\sin\left(dx+c\right)}{\sin\left(dx+c\right)^{2}-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+480\,Aa^{4}\tan\left(dx+c\right)}{480\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/480*(640*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^3*b + 960*(tan(d*x + c)^3
+ 3*tan(d*x + c))*A*a^2*b^2 + 128*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 1
5*tan(d*x + c))*B*a*b^3 + 32*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15*tan
(d*x + c))*A*b^4 - 5*B*b^4*(2*(15*sin(d*x + c)^5 - 40*sin(d*x + c)^3 + 33*s
in(d*x + c))/(sin(d*x + c)^6 - 3*sin(d*x + c)^4 + 3*sin(d*x + c)^2 - 1) - 1
5*log(sin(d*x + c) + 1) + 15*log(sin(d*x + c) - 1)) - 180*B*a^2*b^2*(2*(3*s
in(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3
*log(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*A*a*b^3*(2*(3*sin(d
*x + c)^3 - 5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log
(sin(d*x + c) + 1) + 3*log(sin(d*x + c) - 1)) - 120*B*a^4*(2*sin(d*x + c)/(
sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 480*
A*a^3*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(
sin(d*x + c) - 1)) + 480*A*a^4*tan(d*x + c))/d
```

Fricas [A]
time = 2.13, size = 327, normalized size = 0.98

$$\frac{15\,\left(8\,Ba^{4}+32\,Aa^{3}b+36\,Ba^{2}b^{2}+24\,Aab^{3}+5\,Bb^{4}\right)\cos\left(dx+c\right)^{6}\log\left(\sin\left(dx+c\right)+1\right)-15\,\left(8\,Ba^{4}+32\,Aa^{3}b+36\,Ba^{2}b^{2}+24\,Aab^{3}+5\,Bb^{4}\right)\cos\left(dx+c\right)^{6}\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(16\,\left(15\,Aa^{4}+40\,Ba^{3}b+60\,Aa^{2}b^{2}+32\,Bab^{3}+8\,Ab^{4}\right)\cos\left(dx+c\right)^{5}+40\,Bb^{4}+15\,\left(8\,Ba^{4}+32\,Aa^{3}b+36\,Ba^{2}b^{2}+24\,Aab^{3}+5\,Bb^{4}\right)\cos\left(dx+c\right)^{4}+32\,\left(10\,Ba^{3}b+15\,Aa^{2}b^{2}+8\,Bab^{3}+2\,Ab^{4}\right)\cos\left(dx+c\right)^{3}+10\,\left(36\,Ba^{2}b^{2}+24\,Aab^{3}+5\,Bb^{4}\right)\cos\left(dx+c\right)^{2}+48\,\left(4\,Bab^{3}+Ab^{4}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{480\,d\cos\left(dx+c\right)^{6}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/480*(15*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2*b^2 + 24*A*a*b^3 + 5*B*b^4)*cos(
d*x + c)^6*log(sin(d*x + c) + 1) - 15*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2*b^2
+ 24*A*a*b^3 + 5*B*b^4)*cos(d*x + c)^6*log(-sin(d*x + c) + 1) + 2*(16*(15*A
*a^4 + 40*B*a^3*b + 60*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*cos(d*x + c)^5 + 4
0*B*b^4 + 15*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2*b^2 + 24*A*a*b^3 + 5*B*b^4)*c
os(d*x + c)^4 + 32*(10*B*a^3*b + 15*A*a^2*b^2 + 8*B*a*b^3 + 2*A*b^4)*cos(d*
```

```
x + c)^3 + 10*(36*B*a^2*b^2 + 24*A*a*b^3 + 5*B*b^4)*cos(d*x + c)^2 + 48*(4*
B*a*b^3 + A*b^4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^6)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))(a + b\sec(c + dx))^4 \sec^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**4*sec(c + d*x)**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1186 vs. $2(319) = 638$.
time = 0.55, size = 1186, normalized size = 3.55

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/240*(15*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2*b^2 + 24*A*a*b^3 + 5*B*b^4)*log(
abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(8*B*a^4 + 32*A*a^3*b + 36*B*a^2*b^2 +
24*A*a*b^3 + 5*B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(240*A*a^4*tan
(1/2*d*x + 1/2*c)^11 - 120*B*a^4*tan(1/2*d*x + 1/2*c)^11 - 480*A*a^3*b*tan(
1/2*d*x + 1/2*c)^11 + 960*B*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 1440*A*a^2*b^2*
tan(1/2*d*x + 1/2*c)^11 - 900*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 600*A*a*b
^3*tan(1/2*d*x + 1/2*c)^11 + 960*B*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 240*A*b^
4*tan(1/2*d*x + 1/2*c)^11 - 165*B*b^4*tan(1/2*d*x + 1/2*c)^11 - 1200*A*a^4*
tan(1/2*d*x + 1/2*c)^9 + 360*B*a^4*tan(1/2*d*x + 1/2*c)^9 + 1440*A*a^3*b*ta
n(1/2*d*x + 1/2*c)^9 - 3520*B*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 5280*A*a^2*b^2
*tan(1/2*d*x + 1/2*c)^9 + 1260*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 840*A*a*b
^3*tan(1/2*d*x + 1/2*c)^9 - 2240*B*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 560*A*b^4
*tan(1/2*d*x + 1/2*c)^9 - 25*B*b^4*tan(1/2*d*x + 1/2*c)^9 + 2400*A*a^4*tan(
1/2*d*x + 1/2*c)^7 - 240*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 960*A*a^3*b*tan(1/2
*d*x + 1/2*c)^7 + 5760*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 8640*A*a^2*b^2*tan(
1/2*d*x + 1/2*c)^7 - 360*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 240*A*a*b^3*tan
(1/2*d*x + 1/2*c)^7 + 4992*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 1248*A*b^4*tan(
1/2*d*x + 1/2*c)^7 - 450*B*b^4*tan(1/2*d*x + 1/2*c)^7 - 2400*A*a^4*tan(1/2*
d*x + 1/2*c)^5 - 240*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 960*A*a^3*b*tan(1/2*d*x
 + 1/2*c)^5 - 5760*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 8640*A*a^2*b^2*tan(1/2*
d*x + 1/2*c)^5 - 360*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 240*A*a*b^3*tan(1/2
*d*x + 1/2*c)^5 - 4992*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 1248*A*b^4*tan(1/2*
```

```
d*x + 1/2*c)^5 - 450*B*b^4*tan(1/2*d*x + 1/2*c)^5 + 1200*A*a^4*tan(1/2*d*x
+ 1/2*c)^3 + 360*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 1440*A*a^3*b*tan(1/2*d*x +
1/2*c)^3 + 3520*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 5280*A*a^2*b^2*tan(1/2*d*x
 + 1/2*c)^3 + 1260*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 840*A*a*b^3*tan(1/2*d
*x + 1/2*c)^3 + 2240*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 560*A*b^4*tan(1/2*d*x
 + 1/2*c)^3 - 25*B*b^4*tan(1/2*d*x + 1/2*c)^3 - 240*A*a^4*tan(1/2*d*x + 1/2
*c) - 120*B*a^4*tan(1/2*d*x + 1/2*c) - 480*A*a^3*b*tan(1/2*d*x + 1/2*c) - 9
60*B*a^3*b*tan(1/2*d*x + 1/2*c) - 1440*A*a^2*b^2*tan(1/2*d*x + 1/2*c) - 900
*B*a^2*b^2*tan(1/2*d*x + 1/2*c) - 600*A*a*b^3*tan(1/2*d*x + 1/2*c) - 960*B*
a*b^3*tan(1/2*d*x + 1/2*c) - 240*A*b^4*tan(1/2*d*x + 1/2*c) - 165*B*b^4*tan
(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^6)/d
```

Mupad [B]
time = 5.74, size = 709, normalized size = 2.12

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^4)/cos(c + d*x)^2,x)
```

```
[Out] (atanh((4*tan(c/2 + (d*x)/2)*((B*a^4)/2 + (5*B*b^4)/16 + (9*B*a^2*b^2)/4 +
(3*A*a*b^3)/2 + 2*A*a^3*b))/(2*B*a^4 + (5*B*b^4)/4 + 9*B*a^2*b^2 + 6*A*a*b^
3 + 8*A*a^3*b))*(B*a^4 + (5*B*b^4)/8 + (9*B*a^2*b^2)/2 + 3*A*a*b^3 + 4*A*a^
3*b))/d + (tan(c/2 + (d*x)/2)*(2*A*a^4 + 2*A*b^4 + B*a^4 + (11*B*b^4)/8 + 1
2*A*a^2*b^2 + (15*B*a^2*b^2)/2 + 5*A*a*b^3 + 4*A*a^3*b + 8*B*a*b^3 + 8*B*a^
3*b) - tan(c/2 + (d*x)/2)^11*(2*A*a^4 + 2*A*b^4 - B*a^4 - (11*B*b^4)/8 + 12
*A*a^2*b^2 - (15*B*a^2*b^2)/2 - 5*A*a*b^3 - 4*A*a^3*b + 8*B*a*b^3 + 8*B*a^3
*b) - tan(c/2 + (d*x)/2)^3*(10*A*a^4 + (14*A*b^4)/3 + 3*B*a^4 - (5*B*b^4)/2
4 + 44*A*a^2*b^2 + (21*B*a^2*b^2)/2 + 7*A*a*b^3 + 12*A*a^3*b + (56*B*a*b^3)
/3 + (88*B*a^3*b)/3) + tan(c/2 + (d*x)/2)^9*(10*A*a^4 + (14*A*b^4)/3 - 3*B*
a^4 + (5*B*b^4)/24 + 44*A*a^2*b^2 - (21*B*a^2*b^2)/2 - 7*A*a*b^3 - 12*A*a^3
*b + (56*B*a*b^3)/3 + (88*B*a^3*b)/3) + tan(c/2 + (d*x)/2)^5*(20*A*a^4 + (5
2*A*b^4)/5 + 2*B*a^4 + (15*B*b^4)/4 + 72*A*a^2*b^2 + 3*B*a^2*b^2 + 2*A*a*b^
3 + 8*A*a^3*b + (208*B*a*b^3)/5 + 48*B*a^3*b) - tan(c/2 + (d*x)/2)^7*(20*A*
a^4 + (52*A*b^4)/5 - 2*B*a^4 - (15*B*b^4)/4 + 72*A*a^2*b^2 - 3*B*a^2*b^2 -
2*A*a*b^3 - 8*A*a^3*b + (208*B*a*b^3)/5 + 48*B*a^3*b))/(d*(15*tan(c/2 + (d*
x)/2)^4 - 6*tan(c/2 + (d*x)/2)^2 - 20*tan(c/2 + (d*x)/2)^6 + 15*tan(c/2 + (
d*x)/2)^8 - 6*tan(c/2 + (d*x)/2)^10 + tan(c/2 + (d*x)/2)^12 + 1))
```

3.303 $\int \sec(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=250

$$\frac{(8a^4A+24a^2Ab^2+3Ab^4+16a^3bB+12ab^3B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(95a^3Ab+80aAb^3+12a^4B+112a^2b}{30d}$$

```
[Out] 1/8*(8*A*a^4+24*A*a^2*b^2+3*A*b^4+16*B*a^3*b+12*B*a*b^3)*arctanh(sin(d*x+c)
)/d+1/30*(95*A*a^3*b+80*A*a*b^3+12*B*a^4+112*B*a^2*b^2+16*B*b^4)*tan(d*x+c)
/d+1/120*b*(130*A*a^2*b+45*A*b^3+24*B*a^3+116*B*a*b^2)*sec(d*x+c)*tan(d*x+c
)/d+1/60*(35*A*a*b+12*B*a^2+16*B*b^2)*(a+b*sec(d*x+c))^2*tan(d*x+c)/d+1/20*
(5*A*b+4*B*a)*(a+b*sec(d*x+c))^3*tan(d*x+c)/d+1/5*B*(a+b*sec(d*x+c))^4*tan(
d*x+c)/d
```

Rubi [A]

time = 0.34, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4087, 4082, 3872, 3855, 3852, 8}

$$\frac{(12a^2B+35aAb+16b^2B)\tan(c+dx)(a+b\sec(c+dx))^2}{60d}+\frac{b(24a^3B+130a^2Ab+116ab^2B+45Ab^3)\tan(c+dx)\sec(c+dx)}{120d}+\frac{(12a^4B+95a^3Ab+112a^2b^2B+80aAb^3+16b^4B)\tan(c+dx)}{30d}+\frac{(8a^4A+16a^3bB+24a^2Ab^2+12ab^3B+3Ab^4)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{(4aB+5Ab)\tan(c+dx)(a+b\sec(c+dx))^3}{20d}+\frac{B\tan(c+dx)(a+b\sec(c+dx))^4}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((8*a^4*A + 24*a^2*A*b^2 + 3*A*b^4 + 16*a^3*b*B + 12*a*b^3*B)*ArcTanh[Sin[c
 + d*x]])/(8*d) + ((95*a^3*A*b + 80*a*A*b^3 + 12*a^4*B + 112*a^2*b^2*B + 16
*b^4*B)*Tan[c + d*x])/(30*d) + (b*(130*a^2*A*b + 45*A*b^3 + 24*a^3*B + 116*
a*b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(120*d) + ((35*a*A*b + 12*a^2*B + 16*b^
2*B)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(60*d) + ((5*A*b + 4*a*B)*(a + b*
Sec[c + d*x])^3*Tan[c + d*x])/(20*d) + (B*(a + b*Sec[c + d*x])^4*Tan[c + d*
x])/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{B(a+b\sec(c+dx))^4\tan(c+dx)}{5d}+\frac{1}{5}\int \sec(c+$$

$$= \frac{(5Ab+4aB)(a+b\sec(c+dx))^3\tan(c+dx)}{20d}+$$

$$= \frac{\left(35aAb+12a^2B+16b^2B\right)\left(a+b\sec(c+dx)\right)^2\text{ta}}{60d}$$

$$= \frac{b(130a^2Ab+45Ab^3+24a^3B+116ab^2B)\sec(c+}{120d}$$

$$= \frac{b(130a^2Ab+45Ab^3+24a^3B+116ab^2B)\sec(c+}{120d}$$

$$= \frac{(8a^4A+24a^2Ab^2+3Ab^4+16a^3bB+12ab^3B)\text{ta}}{8d}$$

$$= \frac{(8a^4A+24a^2Ab^2+3Ab^4+16a^3bB+12ab^3B)\text{ta}}{8d}$$

Mathematica [**A**]
time = 4.38, size = 198, normalized size = 0.79

$$\frac{15(8a^4A+24a^2Ab^2+3Ab^4+16a^3bB+12ab^3B)\tanh^{-1}(\sin(c+dx))+\tan(c+dx)\left(120(4a^3Ab+4aAb^3+a^4B+6a^2b^2B+b^4B)+15b(24a^2Ab+3Ab^3+16a^3B+12ab^2B)\sec(c+dx)+30b^3(Ab+4aB)\sec^3(c+dx)+80b^2(2aAb+3a^2B+b^2B)\tan^2(c+dx)+24b^4B\tan^4(c+dx)\right)}{120d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (15*(8*a^4*A + 24*a^2*A*b^2 + 3*A*b^4 + 16*a^3*b*B + 12*a*b^3*B)*ArcTanh[Si
n[c + d*x]] + Tan[c + d*x]*(120*(4*a^3*A*b + 4*a*A*b^3 + a^4*B + 6*a^2*b^2*
B + b^4*B) + 15*b*(24*a^2*A*b + 3*A*b^3 + 16*a^3*B + 12*a*b^2*B)*Sec[c + d*
x] + 30*b^3*(A*b + 4*a*B)*Sec[c + d*x]^3 + 80*b^2*(2*a*A*b + 3*a^2*B + b^2*
B)*Tan[c + d*x]^2 + 24*b^4*B*Tan[c + d*x]^4))/(120*d)
```

Maple [**A**]
time = 0.52, size = 313, normalized size = 1.25 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a^4*ln(sec(d*x+c)+tan(d*x+c))+a^4*B*tan(d*x+c)+4*A*a^3*b*tan(d*x+c)+
4*B*b*a^3*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+6*A*b^2
*a^2*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-6*B*a^2*b^2*
(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)-4*A*a*b^3*(-2/3-1/3*sec(d*x+c)^2)*tan(d*
x+c)+4*B*b^3*a*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d
*x+c)+tan(d*x+c)))+A*b^4*(-(-1/4*sec(d*x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/
8*ln(sec(d*x+c)+tan(d*x+c)))-B*b^4*(-8/15-1/5*sec(d*x+c)^4-4/15*sec(d*x+c)^
2)*tan(d*x+c))
```

Maxima [**A**]
time = 0.29, size = 379, normalized size = 1.52

$$\frac{480\left(\tan(dx+c)^3+3\tan(dx+c)\right)Ba^2b^2+320\left(\tan(dx+c)^3+3\tan(dx+c)\right)Aab^3+16\left(3\tan(dx+c)^5+10\tan(dx+c)^3+15\tan(dx+c)\right)Bb^4-60Bab^3\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-15Ab^4\left(\frac{2\left(3\sin(dx+c)^3-5\sin(dx+c)\right)}{\sin(dx+c)^4-2\sin(dx+c)^2+1}-3\log(\sin(dx+c)+1)+3\log(\sin(dx+c)-1)\right)-240Ba^3b\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)-360Aa^2b^2\left(\frac{2\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+240Aa^4\log(\sec(dx+c)+\tan(dx+c))+240Ba^4\tan(dx+c)+960Aa^3b\tan(dx+c)}{240d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/240*(480*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a^2*b^2 + 320*(tan(d*x + c)^
3 + 3*tan(d*x + c))*A*a*b^3 + 16*(3*tan(d*x + c)^5 + 10*tan(d*x + c)^3 + 15
*tan(d*x + c))*B*b^4 - 60*B*a*b^3*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(s
in(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin
(d*x + c) - 1)) - 15*A*b^4*(2*(3*sin(d*x + c)^3 - 5*sin(d*x + c))/(sin(d*x
+ c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c) + 1) + 3*log(sin(d*x +
c) - 1)) - 240*B*a^3*b*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x +
 c) + 1) + log(sin(d*x + c) - 1)) - 360*A*a^2*b^2*(2*sin(d*x + c)/(sin(d*x
```

```
+ c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 240*A*a^4*lo
g(sec(d*x + c) + tan(d*x + c)) + 240*B*a^4*tan(d*x + c) + 960*A*a^3*b*tan(d
*x + c))/d
```

Fricas [A]
time = 2.30, size = 281, normalized size = 1.12

$$\frac{15\,(8\,Aa^4+16\,Ba^3b+24\,Aa^2b^2+12\,Bab^3+3\,Ab^4)\cos\left(dx+c\right)^5\log\left(\sin\left(dx+c\right)+1\right)-15\,(8\,Aa^4+16\,Ba^3b+24\,Aa^2b^2+12\,Bab^3+3\,Ab^4)\cos\left(dx+c\right)^5\log\left(-\sin\left(dx+c\right)+1\right)+2\,\left(24\,Bb^4+8\,(15\,Ba^4+60\,Aa^3b+60\,Ba^2b^2+40\,Aab^3+8\,Bb^4)\cos\left(dx+c\right)^4+15\,(16\,Ba^3b+24\,Aa^2b^2+12\,Bab^3+3\,Ab^4)\cos\left(dx+c\right)^3+16\,(15\,Ba^2b^2+10\,Aab^3+2\,Bb^4)\cos\left(dx+c\right)^2+30\,(4\,Bab^3+Ab^4)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d\cos\left(dx+c\right)^5}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/240*(15*(8*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 12*B*a*b^3 + 3*A*b^4)*cos(
d*x + c)^5*log(sin(d*x + c) + 1) - 15*(8*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2
+ 12*B*a*b^3 + 3*A*b^4)*cos(d*x + c)^5*log(-sin(d*x + c) + 1) + 2*(24*B*b^4
 + 8*(15*B*a^4 + 60*A*a^3*b + 60*B*a^2*b^2 + 40*A*a*b^3 + 8*B*b^4)*cos(d*x
+ c)^4 + 15*(16*B*a^3*b + 24*A*a^2*b^2 + 12*B*a*b^3 + 3*A*b^4)*cos(d*x + c)
^3 + 16*(15*B*a^2*b^2 + 10*A*a*b^3 + 2*B*b^4)*cos(d*x + c)^2 + 30*(4*B*a*b^
3 + A*b^4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^5)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A+B\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)^4\sec\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**4*sec(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 850 vs. 2(238) = 476.
time = 0.55, size = 850, normalized size = 3.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/120*(15*(8*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 12*B*a*b^3 + 3*A*b^4)*log(
abs(tan(1/2*d*x + 1/2*c) + 1)) - 15*(8*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 +
12*B*a*b^3 + 3*A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(120*B*a^4*tan
(1/2*d*x + 1/2*c)^9 + 480*A*a^3*b*tan(1/2*d*x + 1/2*c)^9 - 240*B*a^3*b*tan(
```

```
1/2*d*x + 1/2*c)^9 - 360*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 + 720*B*a^2*b^2*t
an(1/2*d*x + 1/2*c)^9 + 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^9 - 300*B*a*b^3*ta
n(1/2*d*x + 1/2*c)^9 - 75*A*b^4*tan(1/2*d*x + 1/2*c)^9 + 120*B*b^4*tan(1/2*
d*x + 1/2*c)^9 - 480*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 1920*A*a^3*b*tan(1/2*d*
x + 1/2*c)^7 + 480*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 720*A*a^2*b^2*tan(1/2*d
*x + 1/2*c)^7 - 1920*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 1280*A*a*b^3*tan(1/
2*d*x + 1/2*c)^7 + 120*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 30*A*b^4*tan(1/2*d*
x + 1/2*c)^7 - 160*B*b^4*tan(1/2*d*x + 1/2*c)^7 + 720*B*a^4*tan(1/2*d*x + 1
/2*c)^5 + 2880*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 2400*B*a^2*b^2*tan(1/2*d*x
+ 1/2*c)^5 + 1600*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 + 464*B*b^4*tan(1/2*d*x +
1/2*c)^5 - 480*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 1920*A*a^3*b*tan(1/2*d*x + 1/
2*c)^3 - 480*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 720*A*a^2*b^2*tan(1/2*d*x + 1
/2*c)^3 - 1920*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 1280*A*a*b^3*tan(1/2*d*x
+ 1/2*c)^3 - 120*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 30*A*b^4*tan(1/2*d*x + 1/
2*c)^3 - 160*B*b^4*tan(1/2*d*x + 1/2*c)^3 + 120*B*a^4*tan(1/2*d*x + 1/2*c)
+ 480*A*a^3*b*tan(1/2*d*x + 1/2*c) + 240*B*a^3*b*tan(1/2*d*x + 1/2*c) + 360
*A*a^2*b^2*tan(1/2*d*x + 1/2*c) + 720*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 480*
A*a*b^3*tan(1/2*d*x + 1/2*c) + 300*B*a*b^3*tan(1/2*d*x + 1/2*c) + 75*A*b^4*
tan(1/2*d*x + 1/2*c) + 120*B*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c
)^2 - 1)^5)/d
```

Mupad [B]
time = 6.01, size = 555, normalized size = 2.22

$$\frac{\mathrm{atanh}\left(\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\left(A\,a^4+2\,B\,a^3\,b+3\,A\,a^2\,b^2+\frac{3\,B\,a\,b^3}{2}+\frac{3\,A\,b^4}{8}\right)}{4\,A\,a^4+8\,B\,a^3\,b+12\,A\,a^2\,b^2+6\,B\,a\,b^3+\frac{3\,A\,b^4}{2}}\right)\left(2\,A\,a^4+4\,B\,a^3\,b+6\,A\,a^2\,b^2+3\,B\,a\,b^3+\frac{3\,A\,b^4}{4}\right)}{d}-\frac{\left(2\,B\,a^4-\frac{5\,A\,b^4}{4}+2\,B\,b^4-6\,A\,a^2\,b^2+12\,B\,a^2\,b^2+8\,A\,a\,b^3+8\,A\,a^3\,b-5\,B\,a\,b^3-4\,B\,a^3\,b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^9+\left(\frac{A\,b^4}{2}-8\,B\,a^4-\frac{8\,B\,b^4}{3}+12\,A\,a^2\,b^2-32\,B\,a^2\,b^2-\frac{64\,A\,a\,b^3}{3}-32\,A\,a^3\,b+2\,B\,a\,b^3+8\,B\,a^3\,b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^7+\left(12\,B\,a^4+48\,A\,a^3\,b+40\,B\,a^2\,b^2+\frac{80\,A\,a\,b^3}{3}+\frac{116\,B\,b^4}{15}\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5+\left(-\frac{A\,b^4}{2}-8\,B\,a^4-\frac{8\,B\,b^4}{3}-12\,A\,a^2\,b^2-32\,B\,a^2\,b^2-\frac{64\,A\,a\,b^3}{3}-32\,A\,a^3\,b-2\,B\,a\,b^3-8\,B\,a^3\,b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3+\left(\frac{5\,A\,b^4}{4}+2\,B\,a^4+2\,B\,b^4+6\,A\,a^2\,b^2+12\,B\,a^2\,b^2+8\,A\,a\,b^3+8\,A\,a^3\,b+5\,B\,a\,b^3+4\,B\,a^3\,b\right)\tan\left(\frac{c}{2}+\frac{dx}{2}\right)}{d\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^{10}-5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^8+10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6-10\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4+5\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2-1\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^4)/cos(c + d*x),x)
```

```
[Out] (atanh((4*tan(c/2 + (d*x)/2)*(A*a^4 + (3*A*b^4)/8 + 3*A*a^2*b^2 + (3*B*a*b^
3)/2 + 2*B*a^3*b))/(4*A*a^4 + (3*A*b^4)/2 + 12*A*a^2*b^2 + 6*B*a*b^3 + 8*B*
a^3*b))*(2*A*a^4 + (3*A*b^4)/4 + 6*A*a^2*b^2 + 3*B*a*b^3 + 4*B*a^3*b))/d -
(tan(c/2 + (d*x)/2)*((5*A*b^4)/4 + 2*B*a^4 + 2*B*b^4 + 6*A*a^2*b^2 + 12*B*a
^2*b^2 + 8*A*a*b^3 + 8*A*a^3*b + 5*B*a*b^3 + 4*B*a^3*b) + tan(c/2 + (d*x)/2
)^5*(12*B*a^4 + (116*B*b^4)/15 + 40*B*a^2*b^2 + (80*A*a*b^3)/3 + 48*A*a^3*b
) + tan(c/2 + (d*x)/2)^9*(2*B*a^4 - (5*A*b^4)/4 + 2*B*b^4 - 6*A*a^2*b^2 + 1
2*B*a^2*b^2 + 8*A*a*b^3 + 8*A*a^3*b - 5*B*a*b^3 - 4*B*a^3*b) - tan(c/2 + (d
*x)/2)^3*((A*b^4)/2 + 8*B*a^4 + (8*B*b^4)/3 + 12*A*a^2*b^2 + 32*B*a^2*b^2 +
 (64*A*a*b^3)/3 + 32*A*a^3*b + 2*B*a*b^3 + 8*B*a^3*b) - tan(c/2 + (d*x)/2)^
7*(8*B*a^4 - (A*b^4)/2 + (8*B*b^4)/3 - 12*A*a^2*b^2 + 32*B*a^2*b^2 + (64*A*
a*b^3)/3 + 32*A*a^3*b - 2*B*a*b^3 - 8*B*a^3*b))/(d*(5*tan(c/2 + (d*x)/2)^2
- 10*tan(c/2 + (d*x)/2)^4 + 10*tan(c/2 + (d*x)/2)^6 - 5*tan(c/2 + (d*x)/2)^
8 + tan(c/2 + (d*x)/2)^10 - 1))
```

3.304 $\int (a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=200

$$a^4Ax+\frac{(32a^3Ab+16aAb^3+8a^4B+24a^2b^2B+3b^4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b(34a^2Ab+4Ab^3+19a^3B+16}{6d}$$

```
[Out] a^4*A*x+1/8*(32*A*a^3*b+16*A*a*b^3+8*B*a^4+24*B*a^2*b^2+3*B*b^4)*arctanh(si
n(d*x+c))/d+1/6*b*(34*A*a^2*b+4*A*b^3+19*B*a^3+16*B*a*b^2)*tan(d*x+c)/d+1/2
4*b^2*(32*A*a*b+26*B*a^2+9*B*b^2)*sec(d*x+c)*tan(d*x+c)/d+1/12*b*(4*A*b+7*B
*a)*(a+b*sec(d*x+c))^2*tan(d*x+c)/d+1/4*b*B*(a+b*sec(d*x+c))^3*tan(d*x+c)/d
```

Rubi [A]
time = 0.22, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4003, 4141, 4133, 3855, 3852, 8}

$$a^4Ax+\frac{b^2(26a^2B+32aAb+9b^2B)\tan(c+dx)\sec(c+dx)}{24d}+\frac{b(19a^3B+34a^2Ab+16ab^2B+4Ab^3)\tan(c+dx)}{6d}+\frac{(8a^4B+32a^3Ab+24a^2b^2B+16aAb^3+3b^4B)\tanh^{-1}(\sin(c+dx))}{8d}+\frac{b(7aB+4Ab)\tan(c+dx)(a+b\sec(c+dx))^2}{12d}+\frac{bB\tan(c+dx)(a+b\sec(c+dx))^3}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] a^4*A*x + ((32*a^3*A*b + 16*a*A*b^3 + 8*a^4*B + 24*a^2*b^2*B + 3*b^4*B)*Arc
Tanh[Sin[c + d*x]])/(8*d) + (b*(34*a^2*A*b + 4*A*b^3 + 19*a^3*B + 16*a*b^2*
B)*Tan[c + d*x])/(6*d) + (b^2*(32*a*A*b + 26*a^2*B + 9*b^2*B)*Sec[c + d*x]*
Tan[c + d*x])/(24*d) + (b*(4*A*b + 7*a*B)*(a + b*Sec[c + d*x])^2*Tan[c + d*
x])/(12*d) + (b*B*(a + b*Sec[c + d*x])^3*Tan[c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4003

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
```

```
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m
+ (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m
- 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
 - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4133

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A
 + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x]
```

Rule 4141

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a
 + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)
*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{bB(a+b\sec(c+dx))^3\tan(c+dx)}{4d}+\frac{1}{4}\int (a+b\sec(c+dx$$

$$= \frac{b(4Ab+7aB)(a+b\sec(c+dx))^2\tan(c+dx)}{12d}+\frac{bB(a+b}{}$$

$$= \frac{b^2(32aAb+26a^2B+9b^2B)\sec(c+dx)\tan(c+dx)}{24d}+\frac{b(4A}{}$$

$$= a^4Ax+\frac{b^2(32aAb+26a^2B+9b^2B)\sec(c+dx)\tan(c+dx}{24d}$$

$$= a^4Ax+\frac{(32a^3Ab+16aAb^3+8a^4B+24a^2b^2B+3b^4B)\tan}{8d}$$

$$= a^4Ax+\frac{(32a^3Ab+16aAb^3+8a^4B+24a^2b^2B+3b^4B)\tan}{8d}$$

Mathematica [A]
time = 1.09, size = 160, normalized size = 0.80

$$\frac{24a^4Adx+3(32a^3Ab+16aAb^3+8a^4B+24a^2b^2B+3b^4B)\tanh^{-1}(\sin(c+dx))+3b(8(6a^2Ab+Ab^3+4a^3B+4ab^2B)+b(16aAb+24a^2B+3b^2B)\sec(c+dx)+2b^3B\sec^3(c+dx))\tan(c+dx)+8b^3(Ab+4aB)\tan^3(c+dx)}{24d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (24*a^4*A*d*x + 3*(32*a^3*A*b + 16*a*A*b^3 + 8*a^4*B + 24*a^2*b^2*B + 3*b^4
*B)*ArcTanh[Sin[c + d*x]] + 3*b*(8*(6*a^2*A*b + A*b^3 + 4*a^3*B + 4*a*b^2*B
) + b*(16*a*A*b + 24*a^2*B + 3*b^2*B)*Sec[c + d*x] + 2*b^3*B*Sec[c + d*x]^3
)*Tan[c + d*x] + 8*b^3*(A*b + 4*a*B)*Tan[c + d*x]^3)/(24*d)
```

Maple [A]
time = 0.45, size = 260, normalized size = 1.30 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a^4*(d*x+c)+a^4*B*ln(sec(d*x+c)+tan(d*x+c))+4*A*a^3*b*ln(sec(d*x+c)+
tan(d*x+c))+4*B*b*a^3*tan(d*x+c)+6*A*b^2*a^2*tan(d*x+c)+6*B*a^2*b^2*(1/2*se
c(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))+4*A*a*b^3*(1/2*sec(d*x+c
)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c)))-4*B*b^3*a*(-2/3-1/3*sec(d*x+c)^
2)*tan(d*x+c)-A*b^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c)+B*b^4*(-(-1/4*sec(d*
x+c)^3-3/8*sec(d*x+c))*tan(d*x+c)+3/8*ln(sec(d*x+c)+tan(d*x+c))))
```

Maxima [A]
time = 0.28, size = 303, normalized size = 1.52

$$\frac{48\,(dx+c)Aa^4+64\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Bab^3+16\left(\tan(dx+c)^3+3\,\tan(dx+c)\right)Ab^4-3\,Bb^4\left(\frac{2\left(3\,\sin(dx+c)^3-5\,\sin(dx+c)\right)}{\sin(dx+c)^4-2\,\sin(dx+c)^2+1}-3\,\log\left(\sin(dx+c)+1\right)+3\,\log\left(\sin(dx+c)-1\right)\right)-72\,Ba^2b^2\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin(dx+c)+1\right)+\log\left(\sin(dx+c)-1\right)\right)-48\,Aab^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin(dx+c)+1\right)+\log\left(\sin(dx+c)-1\right)\right)+48\,Ba^4\log\left(\sec(dx+c)+\tan(dx+c)\right)+192\,Aa^3b\log\left(\sec(dx+c)+\tan(dx+c)\right)+192\,Ba^3b\tan(dx+c)+288\,Aa^2b^2\tan(dx+c)}{48\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] 1/48*(48*(d*x + c)*A*a^4 + 64*(tan(d*x + c)^3 + 3*tan(d*x + c))*B*a*b^3 + 1
6*(tan(d*x + c)^3 + 3*tan(d*x + c))*A*b^4 - 3*B*b^4*(2*(3*sin(d*x + c)^3 -
5*sin(d*x + c))/(sin(d*x + c)^4 - 2*sin(d*x + c)^2 + 1) - 3*log(sin(d*x + c
) + 1) + 3*log(sin(d*x + c) - 1)) - 72*B*a^2*b^2*(2*sin(d*x + c)/(sin(d*x +
 c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 48*A*a*b^3*(2
*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x +
c) - 1)) + 48*B*a^4*log(sec(d*x + c) + tan(d*x + c)) + 192*A*a^3*b*log(sec(
d*x + c) + tan(d*x + c)) + 192*B*a^3*b*tan(d*x + c) + 288*A*a^2*b^2*tan(d*x
 + c))/d
```

Fricas [A]
time = 1.75, size = 250, normalized size = 1.25

$$\frac{48\,Aa^4dx\cos(dx+c)^4+3\left(8\,Ba^4+32\,Aa^3b+24\,Ba^2b^2+16\,Aab^3+3\,Bb^4\right)\cos(dx+c)^4\log\left(\sin(dx+c)+1\right)-3\left(8\,Ba^4+32\,Aa^3b+24\,Ba^2b^2+16\,Aab^3+3\,Bb^4\right)\cos(dx+c)^4\log\left(-\sin(dx+c)+1\right)+2\left(6\,Bb^4+16\left(6\,Ba^3b+9\,Aa^2b^2+4\,Bab^3+Ab^4\right)\cos(dx+c)^3+3\left(24\,Ba^2b^2+16\,Aab^3+3\,Bb^4\right)\cos(dx+c)^2+8\left(4\,Bab^3+Ab^4\right)\cos(dx+c)\right)\sin(dx+c)}{48\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/48*(48*A*a^4*d*x*cos(d*x + c)^4 + 3*(8*B*a^4 + 32*A*a^3*b + 24*B*a^2*b^2
+ 16*A*a*b^3 + 3*B*b^4)*cos(d*x + c)^4*log(sin(d*x + c) + 1) - 3*(8*B*a^4 +
 32*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 3*B*b^4)*cos(d*x + c)^4*log(-sin(
d*x + c) + 1) + 2*(6*B*b^4 + 16*(6*B*a^3*b + 9*A*a^2*b^2 + 4*B*a*b^3 + A*b^
4)*cos(d*x + c)^3 + 3*(24*B*a^2*b^2 + 16*A*a*b^3 + 3*B*b^4)*cos(d*x + c)^2
+ 8*(4*B*a*b^3 + A*b^4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c+dx))(a + b\sec(c+dx))^4\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 635 vs. 2(190) = 380.
time = 0.52, size = 635, normalized size = 3.18

$$\frac{24(dx+c)Aa^4 + 3(8Ba^4 + 32Aa^3b + 24Ba^2b^2 + 16Aab^3 + 3Bb^4)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right) - 3(8Ba^4 + 32Aa^3b + 24Ba^2b^2 + 16Aab^3 + 3Bb^4)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right) - \frac{2\left(96Ba^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 + 144Aa^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 72Ba^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 48Aab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 + 96Bab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 + 24Ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 15Bb^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7 - 288Ba^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 432Aa^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 72Ba^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 48Aab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 160Bab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 40Ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 - 9Bb^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5 + 288Ba^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 432Aa^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 72Ba^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 48Aab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 160Bab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 40Ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 9Bb^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 96Ba^3b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 144Aa^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 72Ba^2b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 48Aab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 96Bab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 24Ab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 15Bb^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^4}}{24d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/24*(24*(d*x + c)*A*a^4 + 3*(8*B*a^4 + 32*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*
b^3 + 3*B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(8*B*a^4 + 32*A*a^3*b
 + 24*B*a^2*b^2 + 16*A*a*b^3 + 3*B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1))
- 2*(96*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 144*A*a^2*b^2*tan(1/2*d*x + 1/2*c)
^7 - 72*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 48*A*a*b^3*tan(1/2*d*x + 1/2*c)^
7 + 96*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 24*A*b^4*tan(1/2*d*x + 1/2*c)^7 - 1
5*B*b^4*tan(1/2*d*x + 1/2*c)^7 - 288*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 432*A
*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 72*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 48*
A*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 160*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 40*A*
b^4*tan(1/2*d*x + 1/2*c)^5 - 9*B*b^4*tan(1/2*d*x + 1/2*c)^5 + 288*B*a^3*b*t
an(1/2*d*x + 1/2*c)^3 + 432*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 72*B*a^2*b^2
*tan(1/2*d*x + 1/2*c)^3 + 48*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 160*B*a*b^3*t
an(1/2*d*x + 1/2*c)^3 + 40*A*b^4*tan(1/2*d*x + 1/2*c)^3 - 9*B*b^4*tan(1/2*d
*x + 1/2*c)^3 - 96*B*a^3*b*tan(1/2*d*x + 1/2*c) - 144*A*a^2*b^2*tan(1/2*d*x
 + 1/2*c) - 72*B*a^2*b^2*tan(1/2*d*x + 1/2*c) - 48*A*a*b^3*tan(1/2*d*x + 1/
2*c) - 96*B*a*b^3*tan(1/2*d*x + 1/2*c) - 24*A*b^4*tan(1/2*d*x + 1/2*c) - 15
*B*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1)^4)/d
```

Mupad [B]
time = 5.01, size = 1969, normalized size = 9.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^4,x)

[Out] (9*B*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (27*B*b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/8 + 4*A*b^4*sin(2*c + 2*d*x) + A*b^4*sin(4*c + 4*d*x) + (9*B*b^4*sin(3*c + 3*d*x))/8 + 9*A*a^4*atan((64*A^2*a^8*sin(c/2 + (d*x)/2) + 64*B^2*a^8*sin(c/2 + (d*x)/2) + 9*B^2*b^8*sin(c/2 + (d*x)/2) + 256*A^2*a^2*b^6*sin(c/2 + (d*x)/2) + 1024*A^2*a^4*b^4*sin(c/2 + (d*x)/2) + 1024*A^2*a^6*b^2*sin(c/2 + (d*x)/2) + 144*B^2*a^2*b^6*sin(c/2 + (d*x)/2) + 624*B^2*a^4*b^4*sin(c/2 + (d*x)/2) + 384*B^2*a^6*b^2*sin(c/2 + (d*x)/2) + 960*A*B*a^3*b^5*sin(c/2 + (d*x)/2) + 1792*A*B*a^5*b^3*sin(c/2 + (d*x)/2) + 96*A*B*a*b^7*sin(c/2 + (d*x)/2) + 512*A*B*a^7*b*sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2)*(64*A^2*a^8 + 64*B^2*a^8 + 9*B^2*b^8 + 256*A^2*a^2*b^6 + 1024*A^2*a^4*b^4 + 1024*A^2*a^6*b^2 + 144*B^2*a^2*b^6 + 624*B^2*a^4*b^4 + 384*B^2*a^6*b^2 + 96*A*B*a*b^7 + 512*A*B*a^7*b + 960*A*B*a^3*b^5 + 1792*A*B*a^5*b^3))) + (33*B*b^4*sin(c + d*x))/8 + 12*A*a^4*cos(2*c + 2*d*x)*atan((64*A^2*a^8*sin(c/2 + (d*x)/2) + 64*B^2*a^8*sin(c/2 + (d*x)/2) + 9*B^2*b^8*sin(c/2 + (d*x)/2) + 256*A^2*a^2*b^6*sin(c/2 + (d*x)/2) + 1024*A^2*a^4*b^4*sin(c/2 + (d*x)/2) + 1024*A^2*a^6*b^2*sin(c/2 + (d*x)/2) + 144*B^2*a^2*b^6*sin(c/2 + (d*x)/2) + 624*B^2*a^4*b^4*sin(c/2 + (d*x)/2) + 384*B^2*a^6*b^2*sin(c/2 + (d*x)/2) + 960*A*B*a^3*b^5*sin(c/2 + (d*x)/2) + 1792*A*B*a^5*b^3*sin(c/2 + (d*x)/2) + 96*A*B*a*b^7*sin(c/2 + (d*x)/2) + 512*A*B*a^7*b*sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2)*(64*A^2*a^8 + 64*B^2*a^8 + 9*B^2*b^8 + 256*A^2*a^2*b^6 + 1024*A^2*a^4*b^4 + 1024*A^2*a^6*b^2 + 144*B^2*a^2*b^6 + 624*B^2*a^4*b^4 + 384*B^2*a^6*b^2 + 96*A*B*a*b^7 + 512*A*B*a^7*b + 960*A*B*a^3*b^5 + 1792*A*B*a^5*b^3))) + 3*A*a^4*cos(4*c + 4*d*x)*atan((64*A^2*a^8*sin(c/2 + (d*x)/2) + 64*B^2*a^8*sin(c/2 + (d*x)/2) + 9*B^2*b^8*sin(c/2 + (d*x)/2) + 256*A^2*a^2*b^6*sin(c/2 + (d*x)/2) + 1024*A^2*a^4*b^4*sin(c/2 + (d*x)/2) + 1024*A^2*a^6*b^2*sin(c/2 + (d*x)/2) + 144*B^2*a^2*b^6*sin(c/2 + (d*x)/2) + 624*B^2*a^4*b^4*sin(c/2 + (d*x)/2) + 384*B^2*a^6*b^2*sin(c/2 + (d*x)/2) + 960*A*B*a^3*b^5*sin(c/2 + (d*x)/2) + 1792*A*B*a^5*b^3*sin(c/2 + (d*x)/2) + 96*A*B*a*b^7*sin(c/2 + (d*x)/2) + 512*A*B*a^7*b*sin(c/2 + (d*x)/2))/(cos(c/2 + (d*x)/2)*(64*A^2*a^8 + 64*B^2*a^8 + 9*B^2*b^8 + 256*A^2*a^2*b^6 + 1024*A^2*a^4*b^4 + 1024*A^2*a^6*b^2 + 144*B^2*a^2*b^6 + 624*B^2*a^4*b^4 + 384*B^2*a^6*b^2 + 96*A*B*a*b^7 + 512*A*B*a^7*b + 960*A*B*a^3*b^5 + 1792*A*B*a^5*b^3))) + 6*A*a*b^3*sin(c + d*x) + 18*A*a*b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 36*A*a^3*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 6*A*a*b^3*sin(3*c + 3*d*x) + 16*B*a*b^3*sin(2*c + 2*d*x) + 12*B*a^3*b*sin(2*c + 2*d*x) + 4*B*a*b^3*sin(4*c + 4*d*x) + 6*B*a^3*b*sin(4*c + 4*d*x) + 9*B*a^2*b^2*sin(c + d*x) + 12*B*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x) + 3*B*a^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(4*c + 4*d*x) + 27*B*a^2*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + (9*B*b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x))/2 + (9*B*b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(4*c + 4*d*x

```
))/8 + 18*A*a^2*b^2*sin(2*c + 2*d*x) + 9*A*a^2*b^2*sin(4*c + 4*d*x) + 9*B*a
^2*b^2*sin(3*c + 3*d*x) + 24*A*a*b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2))*cos(2*c + 2*d*x) + 48*A*a^3*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*
x)/2))*cos(2*c + 2*d*x) + 6*A*a*b^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2))*cos(4*c + 4*d*x) + 12*A*a^3*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2))*cos(4*c + 4*d*x) + 36*B*a^2*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d
*x)/2))*cos(2*c + 2*d*x) + 9*B*a^2*b^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2))*cos(4*c + 4*d*x))/(12*d*(cos(2*c + 2*d*x)/2 + cos(4*c + 4*d*x)/8 +
 3/8))
```

3.305 $\int \cos(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=195

$$a^3(4Ab+aB)x+\frac{b(12a^2Ab+Ab^3+8a^3B+4ab^2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{aA(a+b\sec(c+dx))^3\sin(c+dx)}{d}-$$

```
[Out] a^3*(4*A*b+B*a)*x+1/2*b*(12*A*a^2*b+A*b^3+8*B*a^3+4*B*a*b^2)*arctanh(sin(d*
x+c))/d+a*A*(a+b*sec(d*x+c))^3*sin(d*x+c)/d-1/3*b*(6*A*a^3-12*A*a*b^2-17*B*
a^2*b-2*B*b^3)*tan(d*x+c)/d-1/6*b^2*(6*A*a^2-3*A*b^2-8*B*a*b)*sec(d*x+c)*ta
n(d*x+c)/d-1/3*b*(3*A*a-B*b)*(a+b*sec(d*x+c))^2*tan(d*x+c)/d
```

Rubi [A]
time = 0.24, antiderivative size = 195, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4110, 4141, 4133, 3855, 3852, 8}

$$a^3x(aB+4Ab)-\frac{b^2(6a^2A-8abB-3Ab^2)\tan(c+dx)\sec(c+dx)}{6d}-\frac{b(6a^3A-17a^2bB-12aAb^2-2b^3B)\tan(c+dx)}{3d}+\frac{b(8a^3B+12a^2Ab+4ab^2B+Ab^3)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b(3aA-bB)\tan(c+dx)(a+b\sec(c+dx))^2}{3d}+\frac{aA\sin(c+dx)(a+b\sec(c+dx))^3}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] a^3*(4*A*b + a*B)*x + (b*(12*a^2*A*b + A*b^3 + 8*a^3*B + 4*a*b^2*B)*ArcTanh
[Sin[c + d*x]])/(2*d) + (a*A*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/d - (b*(6
*a^3*A - 12*a*A*b^2 - 17*a^2*b*B - 2*b^3*B)*Tan[c + d*x])/(3*d) - (b^2*(6*a
^2*A - 3*A*b^2 - 8*a*b*B)*Sec[c + d*x]*Tan[c + d*x])/(6*d) - (b*(3*a*A - b*
B)*(a + b*Sec[c + d*x])^2*Tan[c + d*x])/(3*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4133

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A
 + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x]
```

Rule 4141

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a
 + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)
*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{aA(a+b\sec(c+dx))^3\sin(c+dx)}{d} - \int (a+b\mathrm{s}$$
$$= \frac{aA(a+b\sec(c+dx))^3\sin(c+dx)}{d} - \frac{b(3aA-}{}$$
$$= \frac{aA(a+b\sec(c+dx))^3\sin(c+dx)}{d} - \frac{b^2(6a^2A}{}$$
$$= a^3(4Ab+aB)x + \frac{aA(a+b\sec(c+dx))^3\sin(c}{d}$$
$$= a^3(4Ab+aB)x + \frac{b(12a^2Ab+Ab^3+8a^3B+4a}{2d}$$
$$= a^3(4Ab+aB)x + \frac{b(12a^2Ab+Ab^3+8a^3B+4a}{2d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1051 vs. $2(195) = 390$.

time = 6.31, size = 1051, normalized size = 5.39

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^3*(4*A*b + a*B)*(c + d*x)*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*S
ec[c + d*x]))/(d*(b + a*Cos[c + d*x])^4*(B + A*Cos[c + d*x])) + ((-12*a^2*A
*b^2 - A*b^4 - 8*a^3*b*B - 4*a*b^3*B)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]]*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]))/(2*d*(b + a
*Cos[c + d*x])^4*(B + A*Cos[c + d*x])) + ((12*a^2*A*b^2 + A*b^4 + 8*a^3*b*B
 + 4*a*b^3*B)*Cos[c + d*x]^5*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]*(a +
b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]))/(2*d*(b + a*Cos[c + d*x])^4*(B + A*
Cos[c + d*x])) + ((3*A*b^4 + 12*a*b^3*B + b^4*B)*Cos[c + d*x]^5*(a + b*Sec[
c + d*x])^4*(A + B*Sec[c + d*x]))/(12*d*(b + a*Cos[c + d*x])^4*(B + A*Cos[c
 + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) + (b^4*B*Cos[c + d*x]^5*(
a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x])*Sin[(c + d*x)/2])/(6*d*(b + a*Co
s[c + d*x])^4*(B + A*Cos[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^3)
 + (b^4*B*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x])*Sin[(c
 + d*x)/2])/(6*d*(b + a*Cos[c + d*x])^4*(B + A*Cos[c + d*x])*(Cos[(c + d*x)
/2] + Sin[(c + d*x)/2])^3) + ((-3*A*b^4 - 12*a*b^3*B - b^4*B)*Cos[c + d*x]^
5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]))/(12*d*(b + a*Cos[c + d*x])^4
*(B + A*Cos[c + d*x])*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (2*Cos[c +
 d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x])*(6*a*A*b^3*Sin[(c + d*x
)/2] + 9*a^2*b^2*B*Sin[(c + d*x)/2] + b^4*B*Sin[(c + d*x)/2]))/(3*d*(b + a*
Cos[c + d*x])^4*(B + A*Cos[c + d*x])*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
 + (2*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x])*(6*a*A*b^3
*Sin[(c + d*x)/2] + 9*a^2*b^2*B*Sin[(c + d*x)/2] + b^4*B*Sin[(c + d*x)/2]))
/(3*d*(b + a*Cos[c + d*x])^4*(B + A*Cos[c + d*x])*(Cos[(c + d*x)/2] + Sin[(
c + d*x)/2])) + (a^4*A*Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c +
 d*x])*Sin[c + d*x])/(d*(b + a*Cos[c + d*x])^4*(B + A*Cos[c + d*x]))
```

Maple [A]
time = 0.46, size = 209, normalized size = 1.07

method	result
derivativedivides	$\frac{A\,a^4\sin(dx+c)+a^4B(dx+c)+4A\,a^3b(dx+c)+4Bb\,a^3\ln(\sec(dx+c)+\tan(dx+c))+6A\,b^2a^2\ln(\sec(dx+c)+\tan(dx+c))+6B}{}$
default	$\frac{A\,a^4\sin(dx+c)+a^4B(dx+c)+4A\,a^3b(dx+c)+4Bb\,a^3\ln(\sec(dx+c)+\tan(dx+c))+6A\,b^2a^2\ln(\sec(dx+c)+\tan(dx+c))+6B}{}$
risch	$4A\,a^3bx+a^4xB-\frac{iA\,a^4e^{i(dx+c)}}{2d}+\frac{iA\,a^4e^{-i(dx+c)}}{2d}-\frac{ib^2(3A\,b^2e^{5i(dx+c)}+12Bab\,e^{5i(dx+c)}-24Aab\,e^{4i(dx+c)}-}{}$

norman	$\frac{(4Aa^3b+a^4B)x+(-12Aa^3b-3a^4B)x\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-12Aa^3b-3a^4B)x\left(\tan^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(4Aa^3b+a^4B)x\left(\tan^{10}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(A*a^4*sin(d*x+c)+a^4*B*(d*x+c)+4*A*a^3*b*(d*x+c)+4*B*b*a^3*ln(sec(d*x+
c)+tan(d*x+c))+6*A*b^2*a^2*ln(sec(d*x+c)+tan(d*x+c))+6*B*a^2*b^2*tan(d*x+c)
+4*A*a*b^3*tan(d*x+c)+4*B*b^3*a*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c
)+tan(d*x+c)))+A*b^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+c
)))-B*b^4*(-2/3-1/3*sec(d*x+c)^2)*tan(d*x+c))
```

Maxima [A]
time = 0.28, size = 245, normalized size = 1.26

$$\frac{12\,(dx+c)Ba^4+48\,(dx+c)Aa^3b+4\left(\tan\left(dx+c\right)^3+3\,\tan\left(dx+c\right)\right)Bb^4-12\,Bab^3\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)-3\,Ab^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log\left(\sin\left(dx+c\right)+1\right)+\log\left(\sin\left(dx+c\right)-1\right)\right)+24\,Ba^3b(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+36\,Aa^2b^2(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right))+12\,Aa^4\sin\left(dx+c\right)+72\,Ba^2b^2\tan\left(dx+c\right)+48\,Aab^3\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] 1/12*(12*(d*x + c)*B*a^4 + 48*(d*x + c)*A*a^3*b + 4*(tan(d*x + c)^3 + 3*tan
(d*x + c))*B*b^4 - 12*B*a*b^3*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(si
n(d*x + c) + 1) + log(sin(d*x + c) - 1)) - 3*A*b^4*(2*sin(d*x + c)/(sin(d*x
 + c)^2 - 1) - log(sin(d*x + c) + 1) + log(sin(d*x + c) - 1)) + 24*B*a^3*b*
(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 36*A*a^2*b^2*(log(sin(d*x
 + c) + 1) - log(sin(d*x + c) - 1)) + 12*A*a^4*sin(d*x + c) + 72*B*a^2*b^2*
tan(d*x + c) + 48*A*a*b^3*tan(d*x + c))/d
```

Fricas [A]
time = 1.27, size = 219, normalized size = 1.12

$$\frac{12\,(Ba^4+4\,Aa^3b)dx\cos\left(dx+c\right)^3+3\,(8\,Ba^3b+12\,Aa^2b^2+4\,Bab^3+Ab^4)\cos\left(dx+c\right)^3\log\left(\sin\left(dx+c\right)+1\right)-3\,(8\,Ba^3b+12\,Aa^2b^2+4\,Bab^3+Ab^4)\cos\left(dx+c\right)^3\log\left(-\sin\left(dx+c\right)+1\right)+2\left(6\,Aa^4\cos\left(dx+c\right)^3+2\,Bb^4+4\,(9\,Ba^2b^2+6\,Aab^3+Bb^4)\cos\left(dx+c\right)^2+3\,(4\,Bab^3+Ab^4)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{12\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] 1/12*(12*(B*a^4 + 4*A*a^3*b)*d*x*cos(d*x + c)^3 + 3*(8*B*a^3*b + 12*A*a^2*b
^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(8*B*a^3*b
 + 12*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*cos(d*x + c)^3*log(-sin(d*x + c) + 1)
+ 2*(6*A*a^4*cos(d*x + c)^3 + 2*B*b^4 + 4*(9*B*a^2*b^2 + 6*A*a*b^3 + B*b^4)
*cos(d*x + c)^2 + 3*(4*B*a*b^3 + A*b^4)*cos(d*x + c))*sin(d*x + c))/(d*cos(
d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))(a + b\sec(c + dx))^4 \cos(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**4*cos(c + d*x), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 387 vs. 2(187) = 374.
time = 0.51, size = 387, normalized size = 1.98

$$\frac{\frac{12\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1} + 6\,(Ba^4 + 4\,Aa^3b)(dx + c) + 3\,(8\,Ba^3b + 12\,Aa^2b^2 + 4\,Bab^3 + Ab^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right) - 3\,(8\,Ba^3b + 12\,Aa^2b^2 + 4\,Bab^3 + Ab^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right) - \frac{2\left(36\,Ba^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+24\,Aab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12\,Bab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,Bb^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-72\,Ba^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-48\,Aab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,Bb^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+36\,Ba^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+24\,Aab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,Bab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,Bb^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(12*A*a^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 + 1) + 6*(B*a^4
+ 4*A*a^3*b)*(d*x + c) + 3*(8*B*a^3*b + 12*A*a^2*b^2 + 4*B*a*b^3 + A*b^4)*l
og(abs(tan(1/2*d*x + 1/2*c) + 1)) - 3*(8*B*a^3*b + 12*A*a^2*b^2 + 4*B*a*b^3
 + A*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(36*B*a^2*b^2*tan(1/2*d*x
+ 1/2*c)^5 + 24*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 12*B*a*b^3*tan(1/2*d*x + 1
/2*c)^5 - 3*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*B*b^4*tan(1/2*d*x + 1/2*c)^5 -
 72*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 48*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 -
4*B*b^4*tan(1/2*d*x + 1/2*c)^3 + 36*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 24*A*a
*b^3*tan(1/2*d*x + 1/2*c) + 12*B*a*b^3*tan(1/2*d*x + 1/2*c) + 3*A*b^4*tan(1
/2*d*x + 1/2*c) + 6*B*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 - 1
)^3)/d
```

Mupad [B]
time = 4.90, size = 636, normalized size = 3.26

$$\frac{\frac{A\,a^4\sin(2\,c+2\,d\,x)}{4}+\frac{A\,a^4\sin(4\,c+4\,d\,x)}{8}+\frac{A\,b^4\sin(2\,c+2\,d\,x)}{4}+\frac{B\,b^4\sin(3\,c+3\,d\,x)}{6}+\frac{B\,b^4\sin(c+d\,x)}{2}+A\,a\,b^3\sin(c+d\,x)+\frac{3\,B\,a^4\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}-\frac{A\,b^4\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,3{}\mathrm{i}}{4}+A\,a\,b^3\sin(3\,c+3\,d\,x)+B\,a\,b^3\sin(2\,c+2\,d\,x)+\frac{3\,B\,a^2\,b^2\sin(c+d\,x)}{2}+\frac{B\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(3\,c+3\,d\,x)}{2}-\frac{A\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(3\,c+3\,d\,x)\,1{}\mathrm{i}}{4}+\frac{3\,B\,a^2\,b^2\sin(3\,c+3\,d\,x)}{2}+2\,A\,a^3\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(3\,c+3\,d\,x)-A\,a^2\,b^2\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,9{}\mathrm{i}-B\,a\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(3\,c+3\,d\,x)\,1{}\mathrm{i}-B\,a^3\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(3\,c+3\,d\,x)\,2{}\mathrm{i}-A\,a^2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\cos(3\,c+3\,d\,x)\,3{}\mathrm{i}+6\,A\,a^3\,b\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)-B\,a\,b^3\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,3{}\mathrm{i}-B\,a^3\,b\cos(c+d\,x)\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,6{}\mathrm{i}}{d\,\left(\frac{3\cos(c+d\,x)}{4}+\frac{\cos(3\,c+3\,d\,x)}{4}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^4,x)
```

```
[Out] ((A*a^4*sin(2*c + 2*d*x))/4 + (A*a^4*sin(4*c + 4*d*x))/8 + (A*b^4*sin(2*c +
 2*d*x))/4 + (B*b^4*sin(3*c + 3*d*x))/6 + (B*b^4*sin(c + d*x))/2 + A*a*b^3*
sin(c + d*x) + (3*B*a^4*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x
)/2)))/2 - (A*b^4*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)
/2))*3i)/4 + A*a*b^3*sin(3*c + 3*d*x) + B*a*b^3*sin(2*c + 2*d*x) + (3*B*a^2
```

```
*b^2*sin(c + d*x))/2 + (B*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*c
os(3*c + 3*d*x))/2 - (A*b^4*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2)
)*cos(3*c + 3*d*x)*1i)/4 + (3*B*a^2*b^2*sin(3*c + 3*d*x))/2 + 2*A*a^3*b*ata
n(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x) - A*a^2*b^2*cos(c
 + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*9i - B*a*b^3*atan(
(sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*1i - B*a^3*b*a
tan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*2i - A*a^2
*b^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(3*c + 3*d*x)*3i +
 6*A*a^3*b*cos(c + d*x)*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) - B*a*b
^3*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*3i - B*a^3
*b*cos(c + d*x)*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*6i)/(d*((3
*cos(c + d*x))/4 + cos(3*c + 3*d*x)/4))
```

3.306 $\int \cos^2(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=209

$$\frac{1}{2}a^2\left(a^2A+12Ab^2+8abB\right)x+\frac{b^2(8aAb+12a^2B+b^2B)\tanh^{-1}(\sin(c+dx))}{2d}+\frac{a(5Ab+2aB)(a+b\sec(c+d$$

```
[Out] 1/2*a^2*(A*a^2+12*A*b^2+8*B*a*b)*x+1/2*b^2*(8*A*a*b+12*B*a^2+B*b^2)*arctanh
(sin(d*x+c))/d+1/2*a*(5*A*b+2*B*a)*(a+b*sec(d*x+c))^2*sin(d*x+c)/d+1/2*a*A*
cos(d*x+c)*(a+b*sec(d*x+c))^3*sin(d*x+c)/d-1/2*b*(13*A*a^2*b-2*A*b^3+4*B*a^
3-8*B*a*b^2)*tan(d*x+c)/d-1/2*b^2*(6*A*a*b+2*B*a^2-B*b^2)*sec(d*x+c)*tan(d*
x+c)/d
```

Rubi [A]
time = 0.30, antiderivative size = 209, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4110, 4179, 4133, 3855, 3852, 8}

$$\frac{b^2(12a^2B+8aAb+b^2B)\tanh^{-1}(\sin(c+dx))}{2d}-\frac{b^2(2a^2B+6aAb-b^2B)\tan(c+dx)\sec(c+dx)}{2d}+\frac{1}{2}a^2x\left(a^2A+8abB+12Ab^2\right)-\frac{b(4a^3B+13a^2Ab-8ab^2B-2Ab^3)\tan(c+dx)}{2d}+\frac{a(2aB+5Ab)\sin(c+dx)(a+b\sec(c+dx))^2}{2d}+\frac{aA\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^3}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^2*(a^2*A + 12*A*b^2 + 8*a*b*B)*x)/2 + (b^2*(8*a*A*b + 12*a^2*B + b^2*B)*
ArcTanh[Sin[c + d*x]])/(2*d) + (a*(5*A*b + 2*a*B)*(a + b*Sec[c + d*x])^2*Si
n[c + d*x])/(2*d) + (a*A*Cos[c + d*x]*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/
(2*d) - (b*(13*a^2*A*b - 2*A*b^3 + 4*a^3*B - 8*a*b^2*B)*Tan[c + d*x])/(2*d)
 - (b^2*(6*a*A*b + 2*a^2*B - b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(2*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3852

```
Int[csc[(c_.) + (d_.)*(x_)]^(n_), x_Symbol] :> Dist[-d^(-1), Subst[Int[Expa
ndIntegrand[(1 + x^2)^(n/2 - 1), x], x], x, Cot[c + d*x]], x] /; FreeQ[{c,
d}, x] && IGtQ[n/2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4133

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Simp[(-b)*C*Csc[e +
 f*x]*(Cot[e + f*x]/(2*f)), x] + Dist[1/2, Int[Simp[2*A*a + (2*B*a + b*(2*A
 + C))*Csc[e + f*x] + 2*(a*C + B*b)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B, C}, x]
```

Rule 4179

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{aA\cos(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{2d}$$

$$= \frac{a(5Ab+2aB)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}$$

$$= \frac{a(5Ab+2aB)(a+b\sec(c+dx))^2\sin(c+dx)}{2d}$$

$$= \frac{1}{2}a^2\left(a^2A+12Ab^2+8abB\right)x + \frac{a(5Ab+2aB)}{}$$

$$= \frac{1}{2}a^2\left(a^2A+12Ab^2+8abB\right)x + \frac{b^2(8aAb+12a}{}$$

$$= \frac{1}{2}a^2\left(a^2A+12Ab^2+8abB\right)x + \frac{b^2(8aAb+12a}{}$$

Mathematica [A]

time = 1.85, size = 310, normalized size = 1.48

$$\frac{2a^2(a^2A+12Ab^2+8abB)(c+dx)-2b^2(8aAb+12a^2B+b^2B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+2b^2(8aAb+12a^2B+b^2B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{b^4B}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4b^3(Ab+4aB)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}-\frac{b^4B}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2}+\frac{4b^3(Ab+4aB)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}+4a^3(4Ab+aB)\sin(c+dx)+a^4A\sin(2(c+dx))}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^2*(a^2*A + 12*A*b^2 + 8*a*b*B)*(c + d*x) - 2*b^2*(8*a*A*b + 12*a^2*B +
 b^2*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*b^2*(8*a*A*b + 12*a^2*
B + b^2*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (b^4*B)/(Cos[(c + d*x
)/2] - Sin[(c + d*x)/2])^2 + (4*b^3*(A*b + 4*a*B)*Sin[(c + d*x)/2])/(Cos[(c
 + d*x)/2] - Sin[(c + d*x)/2]) - (b^4*B)/(Cos[(c + d*x)/2] + Sin[(c + d*x)/
2])^2 + (4*b^3*(A*b + 4*a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2]) + 4*a^3*(4*A*b + a*B)*Sin[c + d*x] + a^4*A*Sin[2*(c + d*x)])/(4*d
)
```

Maple [A]

time = 0.43, size = 187, normalized size = 0.89

method	result
derivativedivides	$A\,a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^4B\sin(dx+c)+4A\,a^3b\sin(dx+c)+4Bb\,a^3(dx+c)+6A\,b^2a^2(dx+c)+6B\,a^2b^2\ln(\sec($
default	$A\,a^4\left(\frac{\cos(dx+c)\sin(dx+c)}{2}+\frac{dx}{2}+\frac{c}{2}\right)+a^4B\sin(dx+c)+4A\,a^3b\sin(dx+c)+4Bb\,a^3(dx+c)+6A\,b^2a^2(dx+c)+6B\,a^2b^2\ln(\sec($
risch	$\frac{a^4Ax}{2}+6A\,a^2b^2x+4B\,a^3bx-\frac{i\mathrm{e}^{i(dx+c)}a^4B}{2d}-\frac{iA\,a^4\mathrm{e}^{2i(dx+c)}}{8d}+\frac{i\mathrm{e}^{-i(dx+c)}a^4B}{2d}+\frac{iA\,a^4\mathrm{e}^{-2i(dx+c)}}{8d}-$
norman	$\left(\frac{1}{2}A\,a^4+6A\,b^2a^2+4Bb\,a^3\right)x+\left(-\frac{1}{2}A\,a^4-6A\,b^2a^2-4Bb\,a^3\right)x\left(\tan^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{1}{2}A\,a^4-6A\,b^2a^2-4Bb\,a^3\right)x\left(\tan^8\left(\frac{dx}{2}+\right.\right.$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+a^4*B*sin(d*x+c)+4*A*a
^3*b*sin(d*x+c)+4*B*b*a^3*(d*x+c)+6*A*b^2*a^2*(d*x+c)+6*B*a^2*b^2*ln(sec(d*
x+c)+tan(d*x+c))+4*A*a*b^3*ln(sec(d*x+c)+tan(d*x+c))+4*B*b^3*a*tan(d*x+c)+A
*b^4*tan(d*x+c)+B*b^4*(1/2*sec(d*x+c)*tan(d*x+c)+1/2*ln(sec(d*x+c)+tan(d*x+
c))))
```

Maxima [A]

time = 0.28, size = 209, normalized size = 1.00

$$\frac{(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^4+16\,(dx+c)Ba^3b+24\,(dx+c)Aa^2b^2-Bb^4\left(\frac{2\,\sin(dx+c)}{\sin(dx+c)^2-1}-\log(\sin(dx+c)+1)+\log(\sin(dx+c)-1)\right)+12\,Ba^2b^2(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+8\,Aab^3(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+4\,Ba^4\sin(dx+c)+16\,Aa^3b\sin(dx+c)+16\,Bab^3\tan(dx+c)+4\,Ab^4\tan(dx+c)}{4\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/4*((2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^4 + 16*(d*x + c)*B*a^3*b + 24*(d*
x + c)*A*a^2*b^2 - B*b^4*(2*sin(d*x + c)/(sin(d*x + c)^2 - 1) - log(sin(d*x
 + c) + 1) + log(sin(d*x + c) - 1)) + 12*B*a^2*b^2*(log(sin(d*x + c) + 1) -
 log(sin(d*x + c) - 1)) + 8*A*a*b^3*(log(sin(d*x + c) + 1) - log(sin(d*x +
c) - 1)) + 4*B*a^4*sin(d*x + c) + 16*A*a^3*b*sin(d*x + c) + 16*B*a*b^3*tan(
d*x + c) + 4*A*b^4*tan(d*x + c))/d
```

Fricas [**A**]
time = 2.33, size = 202, normalized size = 0.97

$$\frac{2\,(Aa^4+8\,Ba^3b+12\,Aa^2b^2)dx\cos\left(dx+c\right)^2+(12\,Ba^2b^2+8\,Aab^3+Bb^4)\cos\left(dx+c\right)^2\log\left(\sin\left(dx+c\right)+1\right)-(12\,Ba^2b^2+8\,Aab^3+Bb^4)\cos\left(dx+c\right)^2\log\left(-\sin\left(dx+c\right)+1\right)+2\left(Aa^4\cos\left(dx+c\right)^3+Bb^4+2\,(Ba^4+4\,Aa^3b)\cos\left(dx+c\right)^2+2\,(4\,Bab^3+Ab^4)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{4\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/4*(2*(A*a^4 + 8*B*a^3*b + 12*A*a^2*b^2)*d*x*cos(d*x + c)^2 + (12*B*a^2*b^
2 + 8*A*a*b^3 + B*b^4)*cos(d*x + c)^2*log(sin(d*x + c) + 1) - (12*B*a^2*b^2
 + 8*A*a*b^3 + B*b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) + 2*(A*a^4*cos(
d*x + c)^3 + B*b^4 + 2*(B*a^4 + 4*A*a^3*b)*cos(d*x + c)^2 + 2*(4*B*a*b^3 +
A*b^4)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 528 vs. 2(197) = 394.
time = 0.52, size = 528, normalized size = 2.53

$$\frac{(Aa^4+8\,Ba^3b+12\,Aa^2b^2)(dx+c)+(12\,Ba^2b^2+8\,Aab^3+Bb^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-(12\,Ba^2b^2+8\,Aab^3+Bb^4)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-2\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-8\,Aa^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+8\,Bab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+2\,Ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-Bb^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-3\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+2\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+8\,Aa^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+8\,Bab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+2\,Ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Bb^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+3\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+8\,Aa^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-8\,Bab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,Ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,Bb^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,Aa^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-8\,Bab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,Ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Bb^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^4-1\right)^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/2*((A*a^4 + 8*B*a^3*b + 12*A*a^2*b^2)*(d*x + c) + (12*B*a^2*b^2 + 8*A*a*b
^3 + B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - (12*B*a^2*b^2 + 8*A*a*b^3
+ B*b^4)*log(abs(tan(1/2*d*x + 1/2*c) - 1)) - 2*(A*a^4*tan(1/2*d*x + 1/2*c)
^7 - 2*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 8*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 8*
B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 2*A*b^4*tan(1/2*d*x + 1/2*c)^7 - B*b^4*tan
(1/2*d*x + 1/2*c)^7 - 3*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 2*B*a^4*tan(1/2*d*x
+ 1/2*c)^5 + 8*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 8*B*a*b^3*tan(1/2*d*x + 1/2
*c)^5 + 2*A*b^4*tan(1/2*d*x + 1/2*c)^5 - 3*B*b^4*tan(1/2*d*x + 1/2*c)^5 + 3
*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^4*tan(1/2*d*x + 1/2*c)^3 + 8*A*a^3*b*
tan(1/2*d*x + 1/2*c)^3 - 8*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*A*b^4*tan(1/2
*d*x + 1/2*c)^3 - 3*B*b^4*tan(1/2*d*x + 1/2*c)^3 - A*a^4*tan(1/2*d*x + 1/2*
c) - 2*B*a^4*tan(1/2*d*x + 1/2*c) - 8*A*a^3*b*tan(1/2*d*x + 1/2*c) - 8*B*a*
b^3*tan(1/2*d*x + 1/2*c) - 2*A*b^4*tan(1/2*d*x + 1/2*c) - B*b^4*tan(1/2*d*x
 + 1/2*c))/(tan(1/2*d*x + 1/2*c)^4 - 1)^2)/d
```

Mupad [B]
time = 4.39, size = 330, normalized size = 1.58

$$\frac{2\left(\frac{A\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+\frac{B\,b^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{2}+4\,A\,a\,b^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+4\,B\,a^3\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+6\,A\,a^2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)+6\,B\,a^2\,b^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\right)}{d}+\frac{\frac{A\,a^4\,\sin(2\,c+2\,d\,x)}{8}+\frac{A\,a^4\,\sin(4\,c+4\,d\,x)}{16}+\frac{A\,b^4\,\sin(2\,c+2\,d\,x)}{2}+\frac{B\,a^4\,\sin(3\,c+3\,d\,x)}{4}+\frac{B\,a^4\,\sin(c+d\,x)}{4}+\frac{B\,b^4\,\sin(c+d\,x)}{2}+A\,a^3\,b\,\sin\left(c+d\,x\right)+A\,a^3\,b\,\sin\left(3\,c+3\,d\,x\right)+2\,B\,a\,b^3\,\sin\left(2\,c+2\,d\,x\right)}{d\,\left(\frac{\cos(2\,c+2\,d\,x)}{2}+\frac{1}{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^4,x)
```

```
[Out] (2*((A*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + (B*b^4*atanh(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/2 + 4*A*a*b^3*atanh(sin(c/2 + (d*x)/2
)/cos(c/2 + (d*x)/2)) + 4*B*a^3*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2
)) + 6*A*a^2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)) + 6*B*a^2*b^2*
atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2))))/d + ((A*a^4*sin(2*c + 2*d*x)
)/8 + (A*a^4*sin(4*c + 4*d*x))/16 + (A*b^4*sin(2*c + 2*d*x))/2 + (B*a^4*sin
(3*c + 3*d*x))/4 + (B*a^4*sin(c + d*x))/4 + (B*b^4*sin(c + d*x))/2 + A*a^3*
b*sin(c + d*x) + A*a^3*b*sin(3*c + 3*d*x) + 2*B*a*b^3*sin(2*c + 2*d*x))/(d*
(cos(2*c + 2*d*x)/2 + 1/2))
```

3.307 $\int \cos^3(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=198

$$\frac{1}{2}a\left(4a^2Ab+8Ab^3+a^3B+12ab^2B\right)x+\frac{b^3(Ab+4aB)\tanh^{-1}(\sin(c+dx))}{d}+\frac{a^2(2a^2A+9Ab^2+9abB)\sin(c$$

```
[Out] 1/2*a*(4*A*a^2*b+8*A*b^3+B*a^3+12*B*a*b^2)*x+b^3*(A*b+4*B*a)*arctanh(sin(d*
x+c))/d+1/3*a^2*(2*A*a^2+9*A*b^2+9*B*a*b)*sin(d*x+c)/d+1/2*a*(2*A*b+B*a)*co
s(d*x+c)*(a+b*sec(d*x+c))^2*sin(d*x+c)/d+1/3*a*A*cos(d*x+c)^2*(a+b*sec(d*x+
c))^3*sin(d*x+c)/d-1/6*b^2*(8*A*a*b+3*B*a^2-6*B*b^2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.39, antiderivative size = 198, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4110, 4179, 4161, 4132, 8, 4130, 3855}

$$\frac{a^2(2a^2A+9abB+9Ab^2)\sin(c+dx)}{3d}-\frac{b^2(3a^2B+8aAb-6b^2B)\tan(c+dx)}{6d}+\frac{1}{2}ax\left(a^3B+4a^2Ab+12ab^2B+8Ab^3\right)+\frac{b^3(4aB+Ab)\tanh^{-1}(\sin(c+dx))}{d}+\frac{a(aB+2Ab)\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^2}{2d}+\frac{aA\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^3}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(4*a^2*A*b + 8*A*b^3 + a^3*B + 12*a*b^2*B)*x)/2 + (b^3*(A*b + 4*a*B)*Arc
Tanh[Sin[c + d*x]])/d + (a^2*(2*a^2*A + 9*A*b^2 + 9*a*b*B)*Sin[c + d*x])/(3
*d) + (a*(2*A*b + a*B)*Cos[c + d*x]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(2
*d) + (a*A*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^3*Sin[c + d*x])/(3*d) - (b^2
*(8*a*A*b + 3*a^2*B - 6*b^2*B)*Tan[c + d*x])/(6*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
```

```
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4161

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2
) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n
 + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &
&  !LtQ[n, -1]
```

Rule 4179

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \cos^3(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx &= \frac{aA\cos^2(c+dx)(a+b\sec(c+dx))^3\sin(c+dx}{3d} \\
&= \frac{a(2Ab+aB)\cos(c+dx)(a+b\sec(c+dx))^2\,\mathrm{s}}{2d} \\
&= \frac{a(2Ab+aB)\cos(c+dx)(a+b\sec(c+dx))^2\,\mathrm{s}}{2d} \\
&= \frac{a(2Ab+aB)\cos(c+dx)(a+b\sec(c+dx))^2\,\mathrm{s}}{2d} \\
&= \frac{1}{2}a\left(4a^2Ab+8Ab^3+a^3B+12ab^2B\right)x+\frac{a^2(2a}{} \\
&= \frac{1}{2}a\left(4a^2Ab+8Ab^3+a^3B+12ab^2B\right)x+\frac{b^3(A}{}
\end{aligned}
$$

Mathematica [A]
time = 1.09, size = 257, normalized size = 1.30

$$
\frac{6a(4a^2Ab+8Ab^3+a^3B+12ab^2B)(c+dx)-12b^3(Ab+4aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+12b^3(Ab+4aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+\frac{12b^4B\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)}+\frac{12b^4B\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}+3a^2(3a^2A+24Ab^2+16abB)\sin(c+dx)+3a^3(4Ab+aB)\sin(2(c+dx))+a^4A\sin(3(c+dx))}{12d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (6*a*(4*a^2*A*b + 8*A*b^3 + a^3*B + 12*a*b^2*B)*(c + d*x) - 12*b^3*(A*b + 4
*a*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 12*b^3*(A*b + 4*a*B)*Log[C
os[(c + d*x)/2] + Sin[(c + d*x)/2]] + (12*b^4*B*Sin[(c + d*x)/2])/(Cos[(c +
 d*x)/2] - Sin[(c + d*x)/2]) + (12*b^4*B*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2
] + Sin[(c + d*x)/2]) + 3*a^2*(3*a^2*A + 24*A*b^2 + 16*a*b*B)*Sin[c + d*x]
+ 3*a^3*(4*A*b + a*B)*Sin[2*(c + d*x)] + a^4*A*Sin[3*(c + d*x)])/(12*d)
```

Maple [A]
time = 0.40, size = 189, normalized size = 0.95 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/3*A*a^4*(2+cos(d*x+c)^2)*sin(d*x+c)+a^4*B*(1/2*cos(d*x+c)*sin(d*x+c)
+1/2*d*x+1/2*c)+4*A*a^3*b*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+4*B*b*a
^3*sin(d*x+c)+6*A*b^2*a^2*sin(d*x+c)+6*B*a^2*b^2*(d*x+c)+4*A*a*b^3*(d*x+c)+
4*B*b^3*a*ln(sec(d*x+c)+tan(d*x+c))+A*b^4*ln(sec(d*x+c)+tan(d*x+c))+B*b^4*t
an(d*x+c))
```

Maxima [A]
time = 0.28, size = 197, normalized size = 0.99

$$-\frac{4\left(\sin\left(dx+c\right)^{3}-3\sin\left(dx+c\right)\right)Aa^{4}-3\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Ba^{4}-12\left(2\,dx+2\,c+\sin\left(2\,dx+2\,c\right)\right)Aa^{3}b-72\left(dx+c\right)Ba^{2}b^{2}-48\left(dx+c\right)Aab^{3}-24\,Bab^{3}\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-6\,Ab^{4}\left(\log\left(\sin\left(dx+c\right)+1\right)-\log\left(\sin\left(dx+c\right)-1\right)\right)-48\,Ba^{3}b\sin\left(dx+c\right)-72\,Aa^{2}b^{2}\sin\left(dx+c\right)-12\,Bb^{4}\tan\left(dx+c\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/12*(4*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^4 - 3*(2*d*x + 2*c + sin(2*d
*x + 2*c))*B*a^4 - 12*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a^3*b - 72*(d*x +
c)*B*a^2*b^2 - 48*(d*x + c)*A*a*b^3 - 24*B*a*b^3*(log(sin(d*x + c) + 1) - l
og(sin(d*x + c) - 1)) - 6*A*b^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) -
 1)) - 48*B*a^3*b*sin(d*x + c) - 72*A*a^2*b^2*sin(d*x + c) - 12*B*b^4*tan(d
*x + c))/d
```

Fricas [A]
time = 1.91, size = 196, normalized size = 0.99

$$\frac{3\left(Ba^{4}+4\,Aa^{3}b+12\,Ba^{2}b^{2}+8\,Aab^{3}\right)dx\cos\left(dx+c\right)+3\left(4\,Bab^{3}+Ab^{4}\right)\cos\left(dx+c\right)\log\left(\sin\left(dx+c\right)+1\right)-3\left(4\,Bab^{3}+Ab^{4}\right)\cos\left(dx+c\right)\log\left(-\sin\left(dx+c\right)+1\right)+\left(2\,Aa^{4}\cos\left(dx+c\right)^{3}+6\,Bb^{4}+3\left(Ba^{4}+4\,Aa^{3}b\right)\cos\left(dx+c\right)^{2}+4\left(Aa^{4}+6\,Ba^{3}b+9\,Aa^{2}b^{2}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/6*(3*(B*a^4 + 4*A*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3)*d*x*cos(d*x + c) + 3*
(4*B*a*b^3 + A*b^4)*cos(d*x + c)*log(sin(d*x + c) + 1) - 3*(4*B*a*b^3 + A*b
^4)*cos(d*x + c)*log(-sin(d*x + c) + 1) + (2*A*a^4*cos(d*x + c)^3 + 6*B*b^4
 + 3*(B*a^4 + 4*A*a^3*b)*cos(d*x + c)^2 + 4*(A*a^4 + 6*B*a^3*b + 9*A*a^2*b^
2)*cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [A]
time = 0.55, size = 371, normalized size = 1.87

$$-\frac{\frac{12\,Bb^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}-1}-3\left(Ba^{4}+4\,Aa^{3}b+12\,Ba^{2}b^{2}+8\,Aab^{3}\right)\left(dx+c\right)-6\left(4\,Bab^{3}+Ab^{4}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)+6\left(4\,Bab^{3}+Ab^{4}\right)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)-\frac{2\left(6\,Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-3\,Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}-12\,Aa^{3}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+24\,Ba^{3}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+36\,Aa^{2}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{5}+4\,Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+48\,Ba^{3}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+72\,Aa^{2}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{3}+6\,Aa^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Ba^{4}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+12\,Aa^{3}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+24\,Ba^{3}b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+36\,Aa^{2}b^{2}\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^{2}+1\right)^{3}}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] -1/6*(12*B*b^4*tan(1/2*d*x + 1/2*c)/(tan(1/2*d*x + 1/2*c)^2 - 1) - 3*(B*a^4
 + 4*A*a^3*b + 12*B*a^2*b^2 + 8*A*a*b^3)*(d*x + c) - 6*(4*B*a*b^3 + A*b^4)*
log(abs(tan(1/2*d*x + 1/2*c) + 1)) + 6*(4*B*a*b^3 + A*b^4)*log(abs(tan(1/2*
d*x + 1/2*c) - 1)) - 2*(6*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^4*tan(1/2*d*
x + 1/2*c)^5 - 12*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 24*B*a^3*b*tan(1/2*d*x +
 1/2*c)^5 + 36*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*A*a^4*tan(1/2*d*x + 1/2
*c)^3 + 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 72*A*a^2*b^2*tan(1/2*d*x + 1/2*
c)^3 + 6*A*a^4*tan(1/2*d*x + 1/2*c) + 3*B*a^4*tan(1/2*d*x + 1/2*c) + 12*A*a
^3*b*tan(1/2*d*x + 1/2*c) + 24*B*a^3*b*tan(1/2*d*x + 1/2*c) + 36*A*a^2*b^2*
tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^3)/d
```

Mupad [B]
time = 4.31, size = 2523, normalized size = 12.74

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^4,x)
```

```
[Out] - (tan(c/2 + (d*x)/2)^7*(2*A*a^4 - B*a^4 - 2*B*b^4 + 12*A*a^2*b^2 - 4*A*a^3
*b + 8*B*a^3*b) - tan(c/2 + (d*x)/2)*(2*A*a^4 + B*a^4 + 2*B*b^4 + 12*A*a^2*
b^2 + 4*A*a^3*b + 8*B*a^3*b) + tan(c/2 + (d*x)/2)^3*((2*A*a^4)/3 + B*a^4 -
6*B*b^4 - 12*A*a^2*b^2 + 4*A*a^3*b - 8*B*a^3*b) + tan(c/2 + (d*x)/2)^5*(B*a
^4 - (2*A*a^4)/3 - 6*B*b^4 + 12*A*a^2*b^2 + 4*A*a^3*b + 8*B*a^3*b))/(d*(2*t
an(c/2 + (d*x)/2)^2 - 2*tan(c/2 + (d*x)/2)^6 - tan(c/2 + (d*x)/2)^8 + 1)) -
 (atan(((A*b^4 + 4*B*a*b^3)*((A*b^4 + 4*B*a*b^3)*(32*A*b^4 + 16*B*a^4 + 192
*B*a^2*b^2 + 128*A*a*b^3 + 64*A*a^3*b + 128*B*a*b^3) + tan(c/2 + (d*x)/2)*(
32*A^2*b^8 + 8*B^2*a^8 + 512*A^2*a^2*b^6 + 512*A^2*a^4*b^4 + 128*A^2*a^6*b^
2 + 512*B^2*a^2*b^6 + 1152*B^2*a^4*b^4 + 192*B^2*a^6*b^2 + 256*A*B*a*b^7 +
64*A*B*a^7*b + 1536*A*B*a^3*b^5 + 896*A*B*a^5*b^3))*1i - (A*b^4 + 4*B*a*b^3
)*((A*b^4 + 4*B*a*b^3)*(32*A*b^4 + 16*B*a^4 + 192*B*a^2*b^2 + 128*A*a*b^3 +
 64*A*a^3*b + 128*B*a*b^3) - tan(c/2 + (d*x)/2)*(32*A^2*b^8 + 8*B^2*a^8 + 5
12*A^2*a^2*b^6 + 512*A^2*a^4*b^4 + 128*A^2*a^6*b^2 + 512*B^2*a^2*b^6 + 1152
*B^2*a^4*b^4 + 192*B^2*a^6*b^2 + 256*A*B*a*b^7 + 64*A*B*a^7*b + 1536*A*B*a^
3*b^5 + 896*A*B*a^5*b^3))*1i)/((A*b^4 + 4*B*a*b^3)*((A*b^4 + 4*B*a*b^3)*(32
*A*b^4 + 16*B*a^4 + 192*B*a^2*b^2 + 128*A*a*b^3 + 64*A*a^3*b + 128*B*a*b^3)
 + tan(c/2 + (d*x)/2)*(32*A^2*b^8 + 8*B^2*a^8 + 512*A^2*a^2*b^6 + 512*A^2*a
^4*b^4 + 128*A^2*a^6*b^2 + 512*B^2*a^2*b^6 + 1152*B^2*a^4*b^4 + 192*B^2*a^6
*b^2 + 256*A*B*a*b^7 + 64*A*B*a^7*b + 1536*A*B*a^3*b^5 + 896*A*B*a^5*b^3))
+ (A*b^4 + 4*B*a*b^3)*((A*b^4 + 4*B*a*b^3)*(32*A*b^4 + 16*B*a^4 + 192*B*a^2
*b^2 + 128*A*a*b^3 + 64*A*a^3*b + 128*B*a*b^3) - tan(c/2 + (d*x)/2)*(32*A^2
```

*b^8 + 8*B^2*a^8 + 512*A^2*a^2*b^6 + 512*A^2*a^4*b^4 + 128*A^2*a^6*b^2 + 51
2*B^2*a^2*b^6 + 1152*B^2*a^4*b^4 + 192*B^2*a^6*b^2 + 256*A*B*a*b^7 + 64*A*B
*a^7*b + 1536*A*B*a^3*b^5 + 896*A*B*a^5*b^3)) - 256*A^3*a*b^11 + 1024*A^3*a
^2*b^10 - 128*A^3*a^3*b^9 + 1024*A^3*a^4*b^8 + 256*A^3*a^6*b^6 - 6144*B^3*a
^4*b^8 + 9216*B^3*a^5*b^7 - 512*B^3*a^6*b^6 + 1536*B^3*a^7*b^5 + 64*B^3*a^9
*b^3 - 7168*A*B^2*a^3*b^9 + 14592*A*B^2*a^4*b^8 - 2304*A*B^2*a^5*b^7 + 7552
*A*B^2*a^6*b^6 + 528*A*B^2*a^8*b^4 - 2432*A^2*B*a^2*b^10 + 7168*A^2*B*a^3*b
^9 - 1056*A^2*B*a^4*b^8 + 5888*A^2*B*a^5*b^7 + 1152*A^2*B*a^7*b^5))*(A*b^4*
2i + B*a*b^3*8i))/d - (a*atan(((a*(tan(c/2 + (d*x)/2)*(32*A^2*b^8 + 8*B^2*a
^8 + 512*A^2*a^2*b^6 + 512*A^2*a^4*b^4 + 128*A^2*a^6*b^2 + 512*B^2*a^2*b^6
+ 1152*B^2*a^4*b^4 + 192*B^2*a^6*b^2 + 256*A*B*a*b^7 + 64*A*B*a^7*b + 1536*
A*B*a^3*b^5 + 896*A*B*a^5*b^3) - (a*(8*A*b^3 + B*a^3 + 4*A*a^2*b + 12*B*a*b
^2)*(32*A*b^4 + 16*B*a^4 + 192*B*a^2*b^2 + 128*A*a*b^3 + 64*A*a^3*b + 128*B
*a*b^3)*1i)/2)*(8*A*b^3 + B*a^3 + 4*A*a^2*b + 12*B*a*b^2))/2 + (a*(tan(c/2
+ (d*x)/2)*(32*A^2*b^8 + 8*B^2*a^8 + 512*A^2*a^2*b^6 + 512*A^2*a^4*b^4 + 12
8*A^2*a^6*b^2 + 512*B^2*a^2*b^6 + 1152*B^2*a^4*b^4 + 192*B^2*a^6*b^2 + 256*
A*B*a*b^7 + 64*A*B*a^7*b + 1536*A*B*a^3*b^5 + 896*A*B*a^5*b^3) + (a*(8*A*b^
3 + B*a^3 + 4*A*a^2*b + 12*B*a*b^2)*(32*A*b^4 + 16*B*a^4 + 192*B*a^2*b^2 +
128*A*a*b^3 + 64*A*a^3*b + 128*B*a*b^3)*1i)/2)*(8*A*b^3 + B*a^3 + 4*A*a^2*b
 + 12*B*a*b^2))/2)/(1024*A^3*a^2*b^10 - 256*A^3*a*b^11 - 128*A^3*a^3*b^9 +
1024*A^3*a^4*b^8 + 256*A^3*a^6*b^6 - 6144*B^3*a^4*b^8 + 9216*B^3*a^5*b^7 -
512*B^3*a^6*b^6 + 1536*B^3*a^7*b^5 + 64*B^3*a^9*b^3 - (a*(tan(c/2 + (d*x)/2
)*(32*A^2*b^8 + 8*B^2*a^8 + 512*A^2*a^2*b^6 + 512*A^2*a^4*b^4 + 128*A^2*a^6
*b^2 + 512*B^2*a^2*b^6 + 1152*B^2*a^4*b^4 + 192*B^2*a^6*b^2 + 256*A*B*a*b^7
 + 64*A*B*a^7*b + 1536*A*B*a^3*b^5 + 896*A*B*a^5*b^3) - (a*(8*A*b^3 + B*a^3
 + 4*A*a^2*b + 12*B*a*b^2)*(32*A*b^4 + 16*B*a^4 + 192*B*a^2*b^2 + 128*A*a*b
^3 + 64*A*a^3*b + 128*B*a*b^3)*1i)/2)*(8*A*b^3 + B*a^3 + 4*A*a^2*b + 12*B*a
*b^2)*1i)/2 + (a*(tan(c/2 + (d*x)/2)*(32*A^2*b^8 + 8*B^2*a^8 + 512*A^2*a^2*
b^6 + 512*A^2*a^4*b^4 + 128*A^2*a^6*b^2 + 512*B^2*a^2*b^6 + 1152*B^2*a^4*b^
4 + 192*B^2*a^6*b^2 + 256*A*B*a*b^7 + 64*A*B*a^7*b + 1536*A*B*a^3*b^5 + 896
*A*B*a^5*b^3) + (a*(8*A*b^3 + B*a^3 + 4*A*a^2*b + 12*B*a*b^2)*(32*A*b^4 + 1
6*B*a^4 + 192*B*a^2*b^2 + 128*A*a*b^3 + 64*A*a^3*b + 128*B*a*b^3)*1i)/2)*(8
*A*b^3 + B*a^3 + 4*A*a^2*b + 12*B*a*b^2)*1i)/2 - 7168*A*B^2*a^3*b^9 + 14592
*A*B^2*a^4*b^8 - 2304*A*B^2*a^5*b^7 + 7552*A*B^2*a^6*b^6 + 528*A*B^2*a^8*b^
4 - 2432*A^2*B*a^2*b^10 + 7168*A^2*B*a^3*b^9 - 1056*A^2*B*a^4*b^8 + 5888*A^
2*B*a^5*b^7 + 1152*A^2*B*a^7*b^5))*(8*A*b^3 + B*a^3 + 4*A*a^2*b + 12*B*a*b^
2))/d

3.308 $\int \cos^4(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=216

$$\frac{1}{8}\left(3a^4A+24a^2Ab^2+8Ab^4+16a^3bB+32ab^3B\right)x+\frac{b^4B\tanh^{-1}(\sin(c+dx))}{d}+\frac{a(16a^2Ab+19Ab^3+4a^3B+\ldots}{6d}$$

```
[Out] 1/8*(3*A*a^4+24*A*a^2*b^2+8*A*b^4+16*B*a^3*b+32*B*a*b^3)*x+b^4*B*arctanh(si
n(d*x+c))/d+1/6*a*(16*A*a^2*b+19*A*b^3+4*B*a^3+34*B*a*b^2)*sin(d*x+c)/d+1/2
4*a^2*(9*A*a^2+26*A*b^2+32*B*a*b)*cos(d*x+c)*sin(d*x+c)/d+1/12*a*(7*A*b+4*B
*a)*cos(d*x+c)^2*(a+b*sec(d*x+c))^2*sin(d*x+c)/d+1/4*a*A*cos(d*x+c)^3*(a+b*
sec(d*x+c))^3*sin(d*x+c)/d
```

Rubi [A]
time = 0.39, antiderivative size = 216, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4110, 4179, 4159, 4132, 8, 4130, 3855}

$$\frac{a^2(9a^2A+32abB+26Ab^2)\sin(c+dx)\cos(c+dx)}{24d}+\frac{a(4a^3B+16a^2Ab+34ab^2B+19Ab^3)\sin(c+dx)}{6d}+\frac{1}{8}x\left(3a^4A+16a^3bB+24a^2Ab^2+32ab^3B+8Ab^4\right)+\frac{a(4aB+7Ab)\sin(c+dx)\cos^2(c+dx)(a+b\sec(c+dx))^2}{12d}+\frac{aA\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^3}{4d}+\frac{b^4B\tanh^{-1}(\sin(c+dx))}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((3*a^4*A + 24*a^2*A*b^2 + 8*A*b^4 + 16*a^3*b*B + 32*a*b^3*B)*x)/8 + (b^4*B
*ArcTanh[Sin[c + d*x]])/d + (a*(16*a^2*A*b + 19*A*b^3 + 4*a^3*B + 34*a*b^2*
B)*Sin[c + d*x])/(6*d) + (a^2*(9*a^2*A + 26*A*b^2 + 32*a*b*B)*Cos[c + d*x]*
Sin[c + d*x])/(24*d) + (a*(7*A*b + 4*a*B)*Cos[c + d*x]^2*(a + b*Sec[c + d*x
])^2*Sin[c + d*x])/(12*d) + (a*A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^3*Sin[
c + d*x])/(4*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
```

```
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4179

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^4(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx &= \frac{aA\cos^3(c+dx)(a+b\sec(c+dx))^3\sin(c+dx}{4d} \\ &= \frac{a(7Ab+4aB)\cos^2(c+dx)(a+b\sec(c+dx))}{12d} \\ &= \frac{a^2(9a^2A+26Ab^2+32abB)\cos(c+dx)\sin(c-}{24d} \\ &= \frac{a^2(9a^2A+26Ab^2+32abB)\cos(c+dx)\sin(c-}{24d} \\ &= \frac{1}{8}\big(3a^4A+24a^2Ab^2+8Ab^4+16a^3bB+32ab^3 \\ &= \frac{1}{8}\big(3a^4A+24a^2Ab^2+8Ab^4+16a^3bB+32ab^3\end{aligned}$$

Mathematica [A]
time = 0.61, size = 210, normalized size = 0.97

$$\frac{12(3a^4A+24a^2Ab^2+8Ab^4+16a^3bB+32ab^3B)(c+dx)-96b^4B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+96b^4B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)+24a(12a^2Ab+16Ab^3+3a^3B+24ab^2B)\sin(c+dx)+24a^2(a^2A+6Ab^2+4abB)\sin(2(c+dx))+8a^3(4Ab+aB)\sin(3(c+dx))+3a^4A\sin(4(c+dx))}{96d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (12*(3*a^4*A + 24*a^2*A*b^2 + 8*A*b^4 + 16*a^3*b*B + 32*a*b^3*B)*(c + d*x)
- 96*b^4*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 96*b^4*B*Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]] + 24*a*(12*a^2*A*b + 16*A*b^3 + 3*a^3*B + 24*a
*b^2*B)*Sin[c + d*x] + 24*a^2*(a^2*A + 6*A*b^2 + 4*a*b*B)*Sin[2*(c + d*x)]
+ 8*a^3*(4*A*b + a*B)*Sin[3*(c + d*x)] + 3*a^4*A*Sin[4*(c + d*x)])/(96*d)
```

Maple [A]
time = 0.42, size = 218, normalized size = 1.01 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^4*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+1/3
*a^4*B*(2+cos(d*x+c)^2)*sin(d*x+c)+4/3*A*a^3*b*(2+cos(d*x+c)^2)*sin(d*x+c)+
4*B*b*a^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+6*A*b^2*a^2*(1/2*cos(d*
x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+6*B*a^2*b^2*sin(d*x+c)+4*A*a*b^3*sin(d*x+c)+
4*B*b^3*a*(d*x+c)+A*b^4*(d*x+c)+B*b^4*ln(sec(d*x+c)+tan(d*x+c)))
```

Maxima [A]
time = 0.27, size = 215, normalized size = 1.00

$$\frac{3\,(12\,dx+12\,c+\sin(4\,dx+4\,c)+8\,\sin(2\,dx+2\,c))Aa^4-32\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Ba^4-128\left(\sin(dx+c)^3-3\,\sin(dx+c)\right)Aa^3b+96\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Ba^3b+144\,(2\,dx+2\,c+\sin(2\,dx+2\,c))Aa^2b^2+384\,(dx+c)Bab^3+96\,(dx+c)Ab^4+48\,Bb^4(\log(\sin(dx+c)+1)-\log(\sin(dx+c)-1))+576\,Ba^2b^2\sin(dx+c)+384\,Aab^3\sin(dx+c)}{96\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] 1/96*(3*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^4 - 32*
(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^4 - 128*(sin(d*x + c)^3 - 3*sin(d*x +
 c))*A*a^3*b + 96*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^3*b + 144*(2*d*x + 2
*c + sin(2*d*x + 2*c))*A*a^2*b^2 + 384*(d*x + c)*B*a*b^3 + 96*(d*x + c)*A*b
^4 + 48*B*b^4*(log(sin(d*x + c) + 1) - log(sin(d*x + c) - 1)) + 576*B*a^2*b
^2*sin(d*x + c) + 384*A*a*b^3*sin(d*x + c))/d
```

Fricas [A]
time = 4.43, size = 183, normalized size = 0.85

$$\frac{12\,Bb^4\log\left(\sin\left(dx+c\right)+1\right)-12\,Bb^4\log\left(-\sin\left(dx+c\right)+1\right)+3\left(3\,Aa^4+16\,Ba^3b+24\,Aa^2b^2+32\,Bab^3+8\,Ab^4\right)dx+\left(6\,Aa^4\cos\left(dx+c\right)^3+16\,Ba^4+64\,Aa^3b+144\,Ba^2b^2+96\,Aab^3+8\left(Ba^4+4\,Aa^3b\right)\cos\left(dx+c\right)^2+3\left(3\,Aa^4+16\,Ba^3b+24\,Aa^2b^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/24*(12*B*b^4*log(sin(d*x + c) + 1) - 12*B*b^4*log(-sin(d*x + c) + 1) + 3*
(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*d*x + (6*A*a^4
*cos(d*x + c)^3 + 16*B*a^4 + 64*A*a^3*b + 144*B*a^2*b^2 + 96*A*a*b^3 + 8*(B
*a^4 + 4*A*a^3*b)*cos(d*x + c)^2 + 3*(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2)*
cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 603 vs. 2(206) = 412.
time = 0.54, size = 603, normalized size = 2.79

$$\frac{24\,Bb^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)-24\,Bb^4\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)+3\left(3\,Aa^4+16\,Ba^3b+24\,Aa^2b^2+32\,Bab^3+8\,Ab^4\right)\left(dx+c\right)-\text{[illegible]}}{24\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/24*(24*B*b^4*log(abs(tan(1/2*d*x + 1/2*c) + 1)) - 24*B*b^4*log(abs(tan(1/
2*d*x + 1/2*c) - 1)) + 3*(3*A*a^4 + 16*B*a^3*b + 24*A*a^2*b^2 + 32*B*a*b^3
+ 8*A*b^4)*(d*x + c) - 2*(15*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 24*B*a^4*tan(1/
2*d*x + 1/2*c)^7 - 96*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 48*B*a^3*b*tan(1/2*d
*x + 1/2*c)^7 + 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 144*B*a^2*b^2*tan(1/2
*d*x + 1/2*c)^7 - 96*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 - 9*A*a^4*tan(1/2*d*x +
 1/2*c)^5 - 40*B*a^4*tan(1/2*d*x + 1/2*c)^5 - 160*A*a^3*b*tan(1/2*d*x + 1/2
*c)^5 + 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 72*A*a^2*b^2*tan(1/2*d*x + 1/2*
c)^5 - 432*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 288*A*a*b^3*tan(1/2*d*x + 1/2
*c)^5 + 9*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^4*tan(1/2*d*x + 1/2*c)^3 -
160*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 48*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 72
*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 432*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 -
288*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 15*A*a^4*tan(1/2*d*x + 1/2*c) - 24*B*a
^4*tan(1/2*d*x + 1/2*c) - 96*A*a^3*b*tan(1/2*d*x + 1/2*c) - 48*B*a^3*b*tan(
1/2*d*x + 1/2*c) - 72*A*a^2*b^2*tan(1/2*d*x + 1/2*c) - 144*B*a^2*b^2*tan(1/
2*d*x + 1/2*c) - 96*A*a*b^3*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 +
 1)^4)/d
```

Mupad [B]
time = 3.38, size = 369, normalized size = 1.71

$$\frac{3\,B\,a^4\,\sin\left(c+d\,x\right)}{4\,d}+\frac{3\,A\,a^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{4\,d}+\frac{2\,A\,b^4\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{2\,B\,b^4\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{A\,a^4\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{A\,a^4\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{B\,a^4\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{8\,B\,a\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{4\,B\,a^3\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{A\,a^3\,b\,\sin\left(3\,c+3\,d\,x\right)}{3\,d}+\frac{B\,a^3\,b\,\sin\left(2\,c+2\,d\,x\right)}{d}+\frac{6\,B\,a^2\,b^2\,\sin\left(c+d\,x\right)}{d}+\frac{6\,A\,a^2\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d}+\frac{3\,A\,a^2\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{4\,A\,a\,b^3\,\sin\left(c+d\,x\right)}{d}+\frac{3\,A\,a^3\,b\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^4,x)
```

```
[Out] (3*B*a^4*sin(c + d*x))/(4*d) + (3*A*a^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (
d*x)/2)))/(4*d) + (2*A*b^4*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d +
 (2*B*b^4*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*a^4*sin(2*c
+ 2*d*x))/(4*d) + (A*a^4*sin(4*c + 4*d*x))/(32*d) + (B*a^4*sin(3*c + 3*d*x)
)/(12*d) + (8*B*a*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (4*B
*a^3*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (A*a^3*b*sin(3*c +
3*d*x))/(3*d) + (B*a^3*b*sin(2*c + 2*d*x))/d + (6*B*a^2*b^2*sin(c + d*x))/d
 + (6*A*a^2*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/d + (3*A*a^2*b
^2*sin(2*c + 2*d*x))/(2*d) + (4*A*a*b^3*sin(c + d*x))/d + (3*A*a^3*b*sin(c
+ d*x))/d
```

3.309 $\int \cos^5(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=258

$$\frac{1}{8}\left(12a^3Ab+16aAb^3+3a^4B+24a^2b^2B+8b^4B\right)x+\frac{(8a^4A+60a^2Ab^2+15Ab^4+40a^3bB+60ab^3B)\sin(c+$$

```
[Out] 1/8*(12*A*a^3*b+16*A*a*b^3+3*B*a^4+24*B*a^2*b^2+8*B*b^4)*x+1/15*(8*A*a^4+60
*A*a^2*b^2+15*A*b^4+40*B*a^3*b+60*B*a*b^3)*sin(d*x+c)/d+1/40*a*(60*A*a^2*b+
56*A*b^3+15*B*a^3+110*B*a*b^2)*cos(d*x+c)*sin(d*x+c)/d+1/30*a^2*(8*A*a^2+18
*A*b^2+25*B*a*b)*cos(d*x+c)^2*sin(d*x+c)/d+1/20*a*(8*A*b+5*B*a)*cos(d*x+c)^
3*(a+b*sec(d*x+c))^2*sin(d*x+c)/d+1/5*a*A*cos(d*x+c)^4*(a+b*sec(d*x+c))^3*s
in(d*x+c)/d
```

Rubi [A]
time = 0.46, antiderivative size = 258, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4110, 4179, 4159, 4132, 2717, 4130, 8}

$$\frac{a^2(8a^2A+25abB+18Ab^2)\sin(c+dx)\cos^2(c+dx)}{30d}+\frac{a(15a^3B+60a^2Ab+110ab^2B+56Ab^3)\sin(c+dx)\cos(c+dx)}{40d}+\frac{(8a^4A+40a^3bB+60a^2Ab^2+60ab^3B+15Ab^4)\sin(c+dx)}{15d}+\frac{1}{8}x\left(3a^4B+12a^3Ab+24a^2b^2B+16aAb^3+8b^4B\right)+\frac{a(5aB+8Ab)\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^2}{20d}+\frac{aA\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^3}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((12*a^3*A*b + 16*a*A*b^3 + 3*a^4*B + 24*a^2*b^2*B + 8*b^4*B)*x)/8 + ((8*a^
4*A + 60*a^2*A*b^2 + 15*A*b^4 + 40*a^3*b*B + 60*a*b^3*B)*Sin[c + d*x])/(15*
d) + (a*(60*a^2*A*b + 56*A*b^3 + 15*a^3*B + 110*a*b^2*B)*Cos[c + d*x]*Sin[c
 + d*x])/(40*d) + (a^2*(8*a^2*A + 18*A*b^2 + 25*a*b*B)*Cos[c + d*x]^2*Sin[c
 + d*x])/(30*d) + (a*(8*A*b + 5*a*B)*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^2*
Sin[c + d*x])/(20*d) + (a*A*Cos[c + d*x]^4*(a + b*Sec[c + d*x])^3*Sin[c + d
*x])/(5*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2717

```
Int[sin[Pi/2 + (c_.) + (d_.)*(x_)], x_Symbol] :> Simp[Sin[c + d*x]/d, x] /;
 FreeQ[{c, d}, x]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
```

```
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4179

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^5(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{aA\cos^4(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{5d}$$

$$= \frac{a(8Ab+5aB)\cos^3(c+dx)(a+b\sec(c+dx))^2 \ldots}{20d}$$

$$= \frac{a^2(8a^2A+18Ab^2+25abB)\cos^2(c+dx)\sin(c+\ldots}{30d}$$

$$= \frac{a^2(8a^2A+18Ab^2+25abB)\cos^2(c+dx)\sin(c+\ldots}{30d}$$

$$= \frac{(8a^4A+60a^2Ab^2+15Ab^4+40a^3bB+60ab^3B\ldots}{15d}$$

$$= \frac{1}{8}\left(12a^3Ab+16aAb^3+3a^4B+24a^2b^2B+8b^4B\ldots\right.$$

Mathematica [A]

time = 0.64, size = 263, normalized size = 1.02

$$\frac{720a^3Abc+960aAb^3c+180a^4Bc+1440a^2b^2Bc+480b^4Bc+720a^3Abdx+960aAb^3dx+180a^4Bdx+1440a^2b^2Bdx+480b^4Bdx+60(5a^4A+36a^2Ab^2+8Ab^4+24a^3bB+32ab^3B)\sin(c+dx)+120a(4a^2Ab+4Ab^3+a^3B+6ab^2B)\sin(2(c+dx))+50a^4A\sin(3(c+dx))+240a^2Ab^2\sin(3(c+dx))+160a^3bB\sin(3(c+dx))+60a^3Ab\sin(4(c+dx))+15a^4B\sin(4(c+dx))+6a^4A\sin(5(c+dx))}{480d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^5*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (720*a^3*A*b*c + 960*a*A*b^3*c + 180*a^4*B*c + 1440*a^2*b^2*B*c + 480*b^4*B
*c + 720*a^3*A*b*d*x + 960*a*A*b^3*d*x + 180*a^4*B*d*x + 1440*a^2*b^2*B*d*x
 + 480*b^4*B*d*x + 60*(5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a
*b^3*B)*Sin[c + d*x] + 120*a*(4*a^2*A*b + 4*A*b^3 + a^3*B + 6*a*b^2*B)*Sin[
2*(c + d*x)] + 50*a^4*A*Sin[3*(c + d*x)] + 240*a^2*A*b^2*Sin[3*(c + d*x)] +
 160*a^3*b*B*Sin[3*(c + d*x)] + 60*a^3*A*b*Sin[4*(c + d*x)] + 15*a^4*B*Sin[
4*(c + d*x)] + 6*a^4*A*Sin[5*(c + d*x)])/(480*d)
```

Maple [A]

time = 0.42, size = 258, normalized size = 1.00

method	result
derivativedivides	$\frac{\frac{A\,a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+4A\,a^3b\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a^4B\left(\frac{\left(\cos^3(d\ldots\right.}{\ldots}\right.}{\ldots}$
default	$\frac{\frac{A\,a^4\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c)}{5}+4A\,a^3b\left(\frac{\left(\cos^3(dx+c)+\frac{3\cos(dx+c)}{2}\right)\sin(dx+c)}{4}+\frac{3dx}{8}+\frac{3c}{8}\right)+a^4B\left(\frac{\left(\cos^3(d\ldots\right.}{\ldots}\right.}{\ldots}$

risch	$\frac{3A\,a^3bx}{2} + 2Aa\,b^3x + \frac{3a^4xB}{8} + 3B\,a^2b^2x + xB\,b^4 + \frac{5\sin(dx+c)A\,a^4}{8d} + \frac{9\sin(dx+c)A\,b^2a^2}{2d} + \frac{\sin(dx+}{d}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^5*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(1/5*A*a^4*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*A*a^3*b*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+a^4*B*(1/4*(cos(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*A*b^2*a^2*(2+cos(d*x+c)^2)*sin(d*x+c)+4/3*B*b*a^3*(2+cos(d*x+c)^2)*sin(d*x+c)+4*A*a*b^3*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+6*B*a^2*b^2*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*b^4*sin(d*x+c)+4*B*b^3*a*sin(d*x+c)+B*b^4*(d*x+c))

Maxima [A]
time = 0.28, size = 246, normalized size = 0.95

$$\frac{32\left(3\sin(dx+c)^5-10\sin(dx+c)^3+15\sin(dx+c)\right)Aa^4+15\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Ba^4+60\left(12dx+12c+\sin(4dx+4c)+8\sin(2dx+2c)\right)Aa^3b-640\left(\sin(dx+c)^3-3\sin(dx+c)\right)Ba^3b-960\left(\sin(dx+c)^3-3\sin(dx+c)\right)Aa^2b^2+720\left(2dx+2c+\sin(2dx+2c)\right)Ba^2b^2+480\left(2dx+2c+\sin(2dx+2c)\right)Aab^3+480(dx+c)Bb^4+1920Bab^3\sin(dx+c)+480Ab^4\sin(dx+c)}{480d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/480*(32*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x + c))*A*a^4 + 15*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B*a^4 + 60*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^3*b - 640*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^3*b - 960*(sin(d*x + c)^3 - 3*sin(d*x + c))*A*a^2*b^2 + 720*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a^2*b^2 + 480*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*a*b^3 + 480*(d*x + c)*B*b^4 + 1920*B*a*b^3*sin(d*x + c) + 480*A*b^4*sin(d*x + c))/d

Fricas [A]
time = 2.31, size = 197, normalized size = 0.76

$$\frac{15\left(3Ba^4+12Aa^3b+24Ba^2b^2+16Aab^3+8Bb^4\right)dx+\left(24Aa^4\cos(dx+c)^4+64Aa^4+320Ba^3b+480Aa^2b^2+480Bab^3+120Ab^4+30\left(Ba^4+4Aa^3b\right)\cos(dx+c)^3+16\left(2Aa^4+10Ba^3b+15Aa^2b^2\right)\cos(dx+c)^2+15\left(3Ba^4+12Aa^3b+24Ba^2b^2+16Aab^3\right)\cos(dx+c)\right)\sin(dx+c)}{120d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fricas")

[Out] 1/120*(15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 8*B*b^4)*d*x + (24*A*a^4*cos(d*x + c)^4 + 64*A*a^4 + 320*B*a^3*b + 480*A*a^2*b^2 + 480*B*a*b^3 + 120*A*b^4 + 30*(B*a^4 + 4*A*a^3*b)*cos(d*x + c)^3 + 16*(2*A*a^4 + 10*B*a^3*b + 15*A*a^2*b^2)*cos(d*x + c)^2 + 15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3)*cos(d*x + c))*sin(d*x + c))/d

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**5*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4370 deep
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 791 vs. 2(246) = 492.
time = 0.53, size = 791, normalized size = 3.07

$$\frac{15\,(3\,Ba^4+12\,Aa^3b+24\,Ba^2b^2+16\,Aab^3+8\,Bb^4)(dx+c)+\frac{2\,\left(120\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-75\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-300\,Aa^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+480\,Ba^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+720\,Aa^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-360\,Ba^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9-240\,Aab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+480\,Bab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+120\,Ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^9+160\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-30\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-120\,Aa^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1280\,Ba^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1920\,Aa^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-720\,Ba^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7-480\,Aab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+1920\,Bab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+480\,Ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^7+464\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+1600\,Ba^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+2400\,Aa^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+2880\,Bab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+720\,Ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+160\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+30\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,Aa^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1280\,Ba^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1920\,Aa^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+720\,Ba^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+480\,Aab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+1920\,Bab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+480\,Ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+120\,Aa^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+75\,Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+300\,Aa^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+480\,Ba^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+720\,Aa^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+360\,Ba^2b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+240\,Aab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+480\,Bab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+120\,Ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^5}}{120\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^5*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/120*(15*(3*B*a^4 + 12*A*a^3*b + 24*B*a^2*b^2 + 16*A*a*b^3 + 8*B*b^4)*(d*x
 + c) + 2*(120*A*a^4*tan(1/2*d*x + 1/2*c)^9 - 75*B*a^4*tan(1/2*d*x + 1/2*c)
^9 - 300*A*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 480*B*a^3*b*tan(1/2*d*x + 1/2*c)^
9 + 720*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 360*B*a^2*b^2*tan(1/2*d*x + 1/2*
c)^9 - 240*A*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 480*B*a*b^3*tan(1/2*d*x + 1/2*c
)^9 + 120*A*b^4*tan(1/2*d*x + 1/2*c)^9 + 160*A*a^4*tan(1/2*d*x + 1/2*c)^7 -
 30*B*a^4*tan(1/2*d*x + 1/2*c)^7 - 120*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 128
0*B*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 1920*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 -
720*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 +
 1920*B*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 480*A*b^4*tan(1/2*d*x + 1/2*c)^7 + 4
64*A*a^4*tan(1/2*d*x + 1/2*c)^5 + 1600*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 + 240
0*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 + 2880*B*a*b^3*tan(1/2*d*x + 1/2*c)^5 +
720*A*b^4*tan(1/2*d*x + 1/2*c)^5 + 160*A*a^4*tan(1/2*d*x + 1/2*c)^3 + 30*B*
a^4*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^3*b*tan(1/2*d*x + 1/2*c)^3 + 1280*B*a^
3*b*tan(1/2*d*x + 1/2*c)^3 + 1920*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 720*B*
a^2*b^2*tan(1/2*d*x + 1/2*c)^3 + 480*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 1920*
B*a*b^3*tan(1/2*d*x + 1/2*c)^3 + 480*A*b^4*tan(1/2*d*x + 1/2*c)^3 + 120*A*a
^4*tan(1/2*d*x + 1/2*c) + 75*B*a^4*tan(1/2*d*x + 1/2*c) + 300*A*a^3*b*tan(1
/2*d*x + 1/2*c) + 480*B*a^3*b*tan(1/2*d*x + 1/2*c) + 720*A*a^2*b^2*tan(1/2*
d*x + 1/2*c) + 360*B*a^2*b^2*tan(1/2*d*x + 1/2*c) + 240*A*a*b^3*tan(1/2*d*x
 + 1/2*c) + 480*B*a*b^3*tan(1/2*d*x + 1/2*c) + 120*A*b^4*tan(1/2*d*x + 1/2*
c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^5)/d
```

Mupad [B]
time = 2.71, size = 307, normalized size = 1.19

$$\frac{3\,B\,a^4\,x}{8}+B\,b^4\,x+2\,A\,a\,b^3\,x+\frac{3\,A\,a^3\,b\,x}{2}+\frac{5\,A\,a^4\,\sin\left(c+d\,x\right)}{8\,d}+\frac{A\,b^4\,\sin\left(c+d\,x\right)}{d}+3\,B\,a^2\,b^2\,x+\frac{5\,A\,a^4\,\sin\left(3\,c+3\,d\,x\right)}{48\,d}+\frac{A\,a^4\,\sin\left(5\,c+5\,d\,x\right)}{80\,d}+\frac{B\,a^4\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{B\,a^4\,\sin\left(4\,c+4\,d\,x\right)}{32\,d}+\frac{A\,a\,b^3\,\sin\left(2\,c+2\,d\,x\right)}{d}+\frac{A\,a^3\,b\,\sin\left(2\,c+2\,d\,x\right)}{d}+\frac{A\,a^3\,b\,\sin\left(4\,c+4\,d\,x\right)}{8\,d}+\frac{9\,A\,a^2\,b^2\,\sin\left(c+d\,x\right)}{2\,d}+\frac{B\,a^3\,b\,\sin\left(3\,c+3\,d\,x\right)}{3\,d}+\frac{A\,a^2\,b^2\,\sin\left(3\,c+3\,d\,x\right)}{2\,d}+\frac{3\,B\,a^2\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{4\,B\,a\,b^3\,\sin\left(c+d\,x\right)}{d}+\frac{3\,B\,a^3\,b\,\sin\left(c+d\,x\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^5*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^4,x)
```

```
[Out] (3*B*a^4*x)/8 + B*b^4*x + 2*A*a*b^3*x + (3*A*a^3*b*x)/2 + (5*A*a^4*sin(c +
d*x))/(8*d) + (A*b^4*sin(c + d*x))/d + 3*B*a^2*b^2*x + (5*A*a^4*sin(3*c + 3
*d*x))/(48*d) + (A*a^4*sin(5*c + 5*d*x))/(80*d) + (B*a^4*sin(2*c + 2*d*x))/
(4*d) + (B*a^4*sin(4*c + 4*d*x))/(32*d) + (A*a*b^3*sin(2*c + 2*d*x))/d + (A
*a^3*b*sin(2*c + 2*d*x))/d + (A*a^3*b*sin(4*c + 4*d*x))/(8*d) + (9*A*a^2*b^
2*sin(c + d*x))/(2*d) + (B*a^3*b*sin(3*c + 3*d*x))/(3*d) + (A*a^2*b^2*sin(3
*c + 3*d*x))/(2*d) + (3*B*a^2*b^2*sin(2*c + 2*d*x))/(2*d) + (4*B*a*b^3*sin(
c + d*x))/d + (3*B*a^3*b*sin(c + d*x))/d
```

3.310 $\int \cos^6(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=309

$$\frac{1}{16}\left(5a^4A+36a^2Ab^2+8Ab^4+24a^3bB+32ab^3B\right)x+\frac{(48a^3Ab+53aAb^3+12a^4B+87a^2b^2B+15b^4B)\sin(c}{15d}$$

```
[Out] 1/16*(5*A*a^4+36*A*a^2*b^2+8*A*b^4+24*B*a^3*b+32*B*a*b^3)*x+1/15*(48*A*a^3*
b+53*A*a*b^3+12*B*a^4+87*B*a^2*b^2+15*B*b^4)*sin(d*x+c)/d+1/16*(5*A*a^4+36*
A*a^2*b^2+8*A*b^4+24*B*a^3*b+32*B*a*b^3)*cos(d*x+c)*sin(d*x+c)/d+1/120*a^2*
(25*A*a^2+48*A*b^2+72*B*a*b)*cos(d*x+c)^3*sin(d*x+c)/d+1/10*a*(3*A*b+2*B*a)
*cos(d*x+c)^4*(a+b*sec(d*x+c))^2*sin(d*x+c)/d+1/6*a*A*cos(d*x+c)^5*(a+b*sec
(d*x+c))^3*sin(d*x+c)/d-1/15*a*(16*A*a^2*b+13*A*b^3+4*B*a^3+27*B*a*b^2)*sin
(d*x+c)^3/d
```

Rubi [A]
time = 0.53, antiderivative size = 309, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4110, 4179, 4159, 4132, 2715, 8, 4129, 3092}

$$\frac{a^2(25a^2A+72abB+48Ab^2)\sin(c+dx)\cos^3(c+dx)}{120d}-\frac{a(4a^3B+16a^2Ab+27ab^2B+13Ab^3)\sin^3(c+dx)}{15d}+\frac{(12a^4B+48a^3Ab+87a^2b^2B+53aAb^3+15b^4B)\sin(c+dx)}{15d}+\frac{(5a^4A+24a^3bB+36a^2Ab^2+32ab^3B+8Ab^4)\sin(c+dx)\cos(c+dx)}{16d}+\frac{1}{16}x\left(5a^4A+24a^3bB+36a^2Ab^2+32ab^3B+8Ab^4\right)+\frac{a(2aB+3Ab)\sin(c+dx)\cos^4(c+dx)(a+b\sec(c+dx))^2}{10d}+\frac{aA\sin(c+dx)\cos^5(c+dx)(a+b\sec(c+dx))^3}{6d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^3*B)*x)/16 + ((48*
a^3*A*b + 53*a*A*b^3 + 12*a^4*B + 87*a^2*b^2*B + 15*b^4*B)*Sin[c + d*x])/(1
5*d) + ((5*a^4*A + 36*a^2*A*b^2 + 8*A*b^4 + 24*a^3*b*B + 32*a*b^3*B)*Cos[c
+ d*x]*Sin[c + d*x])/(16*d) + (a^2*(25*a^2*A + 48*A*b^2 + 72*a*b*B)*Cos[c +
 d*x]^3*Sin[c + d*x])/(120*d) + (a*(3*A*b + 2*a*B)*Cos[c + d*x]^4*(a + b*Se
c[c + d*x])^2*Sin[c + d*x])/(10*d) + (a*A*Cos[c + d*x]^5*(a + b*Sec[c + d*x
])^3*Sin[c + d*x])/(6*d) - (a*(16*a^2*A*b + 13*A*b^3 + 4*a^3*B + 27*a*b^2*B
)*Sin[c + d*x]^3)/(15*d)
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 3092

```
Int[sin[(e_.) + (f_.)*(x_)]^(m_.)*((A_) + (C_.)*sin[(e_.) + (f_.)*(x_)]^2),
 x_Symbol] :> Dist[-f^(-1), Subst[Int[(1 - x^2)^((m - 1)/2)*(A + C - C*x^2)
, x], x, Cos[e + f*x]], x] /; FreeQ[{e, f, A, C}, x] && IGtQ[(m + 1)/2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4129

```
Int[csc[(e_.) + (f_.)*(x_)]^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.) + (A_)),
 x_Symbol] :> Int[(C + A*Sin[e + f*x]^2)/Sin[e + f*x]^(m + 2), x] /; FreeQ[
{e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && ILtQ[(m + 1)/2, 0]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rule 4179

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
```

```
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^6(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx = \frac{aA\cos^5(c+dx)(a+b\sec(c+dx))^3\sin(c+dx)}{6d}$$

$$= \frac{a(3Ab+2aB)\cos^4(c+dx)(a+b\sec(c+dx))^2}{10d}$$

$$= \frac{a^2(25a^2A+48Ab^2+72abB)\cos^3(c+dx)\sin(c}{120d}$$

$$= \frac{a^2(25a^2A+48Ab^2+72abB)\cos^3(c+dx)\sin(c}{120d}$$

$$= \frac{(5a^4A+36a^2Ab^2+8Ab^4+24a^3bB+32ab^3B)}{16d}$$

$$= \frac{1}{16}\left(5a^4A+36a^2Ab^2+8Ab^4+24a^3bB+32ab^3\right.$$

$$= \frac{1}{16}\left(5a^4A+36a^2Ab^2+8Ab^4+24a^3bB+32ab^3\right.$$

Mathematica [A]
time = 1.22, size = 333, normalized size = 1.08

$$\frac{300a^4Ac+2160a^2Ab^2c+480Ab^4c+1440a^3bBc+1920ab^3Bc+300a^4Adx+2160a^2Ab^2dx+480Ab^4dx+1440a^3bBdx+1920ab^3Bdx+120(20a^3Ab+24aAb^3+5a^4B+36a^2b^2B+8b^4B)\sin(c+dx)+15(15a^4A+96a^2Ab^2+16Ab^4+64a^3bB+64ab^3B)\sin(2(c+dx))+400a^3Ab\sin(3(c+dx))+320aAb^3\sin(3(c+dx))+100a^4B\sin(3(c+dx))+480a^2b^2B\sin(3(c+dx))+45a^4A\sin(4(c+dx))+180a^2Ab^2\sin(4(c+dx))+120a^3bB\sin(4(c+dx))+48a^3Ab\sin(5(c+dx))+12a^4B\sin(5(c+dx))+5a^4A\sin(6(c+dx))}{960d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^6*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (300*a^4*A*c + 2160*a^2*A*b^2*c + 480*A*b^4*c + 1440*a^3*b*B*c + 1920*a*b^3
*B*c + 300*a^4*A*d*x + 2160*a^2*A*b^2*d*x + 480*A*b^4*d*x + 1440*a^3*b*B*d*
x + 1920*a*b^3*B*d*x + 120*(20*a^3*A*b + 24*a*A*b^3 + 5*a^4*B + 36*a^2*b^2*
B + 8*b^4*B)*Sin[c + d*x] + 15*(15*a^4*A + 96*a^2*A*b^2 + 16*A*b^4 + 64*a^3
*b*B + 64*a*b^3*B)*Sin[2*(c + d*x)] + 400*a^3*A*b*Sin[3*(c + d*x)] + 320*a*
A*b^3*Sin[3*(c + d*x)] + 100*a^4*B*Sin[3*(c + d*x)] + 480*a^2*b^2*B*Sin[3*(
c + d*x)] + 45*a^4*A*Sin[4*(c + d*x)] + 180*a^2*A*b^2*Sin[4*(c + d*x)] + 12
0*a^3*b*B*Sin[4*(c + d*x)] + 48*a^3*A*b*Sin[5*(c + d*x)] + 12*a^4*B*Sin[5*(
c + d*x)] + 5*a^4*A*Sin[6*(c + d*x)])/(960*d)
```

Maple [A]
time = 0.57, size = 316, normalized size = 1.02

method	result
derivativedivides	$\frac{A\,a^4\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{a^4B\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c}{5}}{}$
default	$\frac{A\,a^4\left(\frac{\left(\cos^5(dx+c)+\frac{5\left(\cos^3(dx+c)\right)}{4}+\frac{15\cos(dx+c)}{8}\right)\sin(dx+c)}{6}+\frac{5dx}{16}+\frac{5c}{16}\right)+\frac{a^4B\left(\frac{8}{3}+\cos^4(dx+c)+\frac{4\left(\cos^2(dx+c)\right)}{3}\right)\sin(dx+c}{5}}{}$
risch	$\frac{\sin(2dx+2c)A\,b^4}{4d}+\frac{A\,a^4\sin(6dx+6c)}{192d}+\frac{5\sin(3dx+3c)a^4B}{48d}+\frac{15\sin(2dx+2c)A\,a^4}{64d}+\frac{xA\,b^4}{2}+\frac{5a^4Ax}{16}+2xB$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^6*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(A*a^4*(1/6*(cos(d*x+c)^5+5/4*cos(d*x+c)^3+15/8*cos(d*x+c))*sin(d*x+c)+
5/16*d*x+5/16*c)+1/5*a^4*B*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4
/5*A*a^3*b*(8/3+cos(d*x+c)^4+4/3*cos(d*x+c)^2)*sin(d*x+c)+4*B*b*a^3*(1/4*(c
os(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+6*A*b^2*a^2*(1/4*(cos
(d*x+c)^3+3/2*cos(d*x+c))*sin(d*x+c)+3/8*d*x+3/8*c)+2*B*a^2*b^2*(2+cos(d*x+
c)^2)*sin(d*x+c)+4/3*A*a*b^3*(2+cos(d*x+c)^2)*sin(d*x+c)+4*B*b^3*a*(1/2*cos
(d*x+c)*sin(d*x+c)+1/2*d*x+1/2*c)+A*b^4*(1/2*cos(d*x+c)*sin(d*x+c)+1/2*d*x+
1/2*c)+B*b^4*sin(d*x+c))
```

Maxima [A]
time = 0.29, size = 307, normalized size = 0.99

$$-\frac{5\left(4\sin\left(2dx+2c\right)^3-60dx-60c-9\sin\left(4dx+4c\right)-48\sin\left(2dx+2c\right)\right)Aa^4-64\left(3\sin\left(dx+c\right)^5-10\sin\left(dx+c\right)^3+15\sin\left(dx+c\right)\right)Ba^4-256\left(3\sin\left(dx+c\right)^5-10\sin\left(dx+c\right)^3+15\sin\left(dx+c\right)\right)Aa^3b-120\left(12dx+12c+\sin\left(4dx+4c\right)+8\sin\left(2dx+2c\right)\right)Ba^3b-180\left(12dx+12c+\sin\left(4dx+4c\right)+8\sin\left(2dx+2c\right)\right)Aa^2b^2+1920\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)Ba^2b^2+1280\left(\sin\left(dx+c\right)^3-3\sin\left(dx+c\right)\right)Aab^3-960\left(2dx+2c+\sin\left(2dx+2c\right)\right)Bab^3-240\left(2dx+2c+\sin\left(2dx+2c\right)\right)Ab^4-960\,Bb^4\sin\left(dx+c\right)}{960\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] -1/960*(5*(4*sin(2*d*x + 2*c)^3 - 60*d*x - 60*c - 9*sin(4*d*x + 4*c) - 48*s
in(2*d*x + 2*c))*A*a^4 - 64*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(
d*x + c))*B*a^4 - 256*(3*sin(d*x + c)^5 - 10*sin(d*x + c)^3 + 15*sin(d*x +
c))*A*a^3*b - 120*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*B
*a^3*b - 180*(12*d*x + 12*c + sin(4*d*x + 4*c) + 8*sin(2*d*x + 2*c))*A*a^2*
b^2 + 1920*(sin(d*x + c)^3 - 3*sin(d*x + c))*B*a^2*b^2 + 1280*(sin(d*x + c)
^3 - 3*sin(d*x + c))*A*a*b^3 - 960*(2*d*x + 2*c + sin(2*d*x + 2*c))*B*a*b^3
 - 240*(2*d*x + 2*c + sin(2*d*x + 2*c))*A*b^4 - 960*B*b^4*sin(d*x + c))/d
```

Fricas [A]
time = 2.02, size = 243, normalized size = 0.79

$$\frac{15\left(5\,Aa^4+24\,Ba^3b+36\,Aa^2b^2+32\,Bab^3+8\,Ab^4\right)dx+\left(40\,Aa^4\cos\left(dx+c\right)^5+128\,Ba^4+512\,Aa^3b+960\,Ba^2b^2+640\,Aab^3+240\,Bb^4+48\left(Ba^4+4\,Aa^3b\right)\cos\left(dx+c\right)^4+10\left(5\,Aa^4+24\,Ba^3b+36\,Aa^2b^2\right)\cos\left(dx+c\right)^3+32\left(2\,Ba^4+8\,Aa^3b+15\,Ba^2b^2+10\,Aab^3\right)\cos\left(dx+c\right)^2+15\left(5\,Aa^4+24\,Ba^3b+36\,Aa^2b^2+32\,Bab^3+8\,Ab^4\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{240\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] 1/240*(15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*d*x
+ (40*A*a^4*cos(d*x + c)^5 + 128*B*a^4 + 512*A*a^3*b + 960*B*a^2*b^2 + 640*
A*a*b^3 + 240*B*b^4 + 48*(B*a^4 + 4*A*a^3*b)*cos(d*x + c)^4 + 10*(5*A*a^4 +
 24*B*a^3*b + 36*A*a^2*b^2)*cos(d*x + c)^3 + 32*(2*B*a^4 + 8*A*a^3*b + 15*B
*a^2*b^2 + 10*A*a*b^3)*cos(d*x + c)^2 + 15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2
*b^2 + 32*B*a*b^3 + 8*A*b^4)*cos(d*x + c))*sin(d*x + c))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**6*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1127 vs. 2(295) = 590.
time = 0.51, size = 1127, normalized size = 3.65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^6*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] 1/240*(15*(5*A*a^4 + 24*B*a^3*b + 36*A*a^2*b^2 + 32*B*a*b^3 + 8*A*b^4)*(d*x
 + c) - 2*(165*A*a^4*tan(1/2*d*x + 1/2*c)^11 - 240*B*a^4*tan(1/2*d*x + 1/2*
c)^11 - 960*A*a^3*b*tan(1/2*d*x + 1/2*c)^11 + 600*B*a^3*b*tan(1/2*d*x + 1/2
*c)^11 + 900*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^11 - 1440*B*a^2*b^2*tan(1/2*d*x
 + 1/2*c)^11 - 960*A*a*b^3*tan(1/2*d*x + 1/2*c)^11 + 480*B*a*b^3*tan(1/2*d*
x + 1/2*c)^11 + 120*A*b^4*tan(1/2*d*x + 1/2*c)^11 - 240*B*b^4*tan(1/2*d*x +
 1/2*c)^11 - 25*A*a^4*tan(1/2*d*x + 1/2*c)^9 - 560*B*a^4*tan(1/2*d*x + 1/2*
c)^9 - 2240*A*a^3*b*tan(1/2*d*x + 1/2*c)^9 + 840*B*a^3*b*tan(1/2*d*x + 1/2*
c)^9 + 1260*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^9 - 5280*B*a^2*b^2*tan(1/2*d*x +
 1/2*c)^9 - 3520*A*a*b^3*tan(1/2*d*x + 1/2*c)^9 + 1440*B*a*b^3*tan(1/2*d*x
+ 1/2*c)^9 + 360*A*b^4*tan(1/2*d*x + 1/2*c)^9 - 1200*B*b^4*tan(1/2*d*x + 1/
2*c)^9 + 450*A*a^4*tan(1/2*d*x + 1/2*c)^7 - 1248*B*a^4*tan(1/2*d*x + 1/2*c)
^7 - 4992*A*a^3*b*tan(1/2*d*x + 1/2*c)^7 + 240*B*a^3*b*tan(1/2*d*x + 1/2*c)
^7 + 360*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^7 - 8640*B*a^2*b^2*tan(1/2*d*x + 1/
```

```
2*c)^7 - 5760*A*a*b^3*tan(1/2*d*x + 1/2*c)^7 + 960*B*a*b^3*tan(1/2*d*x + 1/
2*c)^7 + 240*A*b^4*tan(1/2*d*x + 1/2*c)^7 - 2400*B*b^4*tan(1/2*d*x + 1/2*c)
^7 - 450*A*a^4*tan(1/2*d*x + 1/2*c)^5 - 1248*B*a^4*tan(1/2*d*x + 1/2*c)^5 -
 4992*A*a^3*b*tan(1/2*d*x + 1/2*c)^5 - 240*B*a^3*b*tan(1/2*d*x + 1/2*c)^5 -
 360*A*a^2*b^2*tan(1/2*d*x + 1/2*c)^5 - 8640*B*a^2*b^2*tan(1/2*d*x + 1/2*c)
^5 - 5760*A*a*b^3*tan(1/2*d*x + 1/2*c)^5 - 960*B*a*b^3*tan(1/2*d*x + 1/2*c)
^5 - 240*A*b^4*tan(1/2*d*x + 1/2*c)^5 - 2400*B*b^4*tan(1/2*d*x + 1/2*c)^5 +
 25*A*a^4*tan(1/2*d*x + 1/2*c)^3 - 560*B*a^4*tan(1/2*d*x + 1/2*c)^3 - 2240*
A*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 840*B*a^3*b*tan(1/2*d*x + 1/2*c)^3 - 1260*
A*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 - 5280*B*a^2*b^2*tan(1/2*d*x + 1/2*c)^3 -
3520*A*a*b^3*tan(1/2*d*x + 1/2*c)^3 - 1440*B*a*b^3*tan(1/2*d*x + 1/2*c)^3 -
 360*A*b^4*tan(1/2*d*x + 1/2*c)^3 - 1200*B*b^4*tan(1/2*d*x + 1/2*c)^3 - 165
*A*a^4*tan(1/2*d*x + 1/2*c) - 240*B*a^4*tan(1/2*d*x + 1/2*c) - 960*A*a^3*b*
tan(1/2*d*x + 1/2*c) - 600*B*a^3*b*tan(1/2*d*x + 1/2*c) - 900*A*a^2*b^2*tan
(1/2*d*x + 1/2*c) - 1440*B*a^2*b^2*tan(1/2*d*x + 1/2*c) - 960*A*a*b^3*tan(1
/2*d*x + 1/2*c) - 480*B*a*b^3*tan(1/2*d*x + 1/2*c) - 120*A*b^4*tan(1/2*d*x
+ 1/2*c) - 240*B*b^4*tan(1/2*d*x + 1/2*c))/(tan(1/2*d*x + 1/2*c)^2 + 1)^6)/
d
```

Mupad [B]
time = 3.18, size = 403, normalized size = 1.30

$$\frac{5\,A\,a^4\,x}{16}+\frac{A\,b^4\,x}{2}+2\,B\,a\,b^3\,x+\frac{3\,B\,a^3\,b\,x}{2}+\frac{5\,B\,a^4\,\sin\left(c+d\,x\right)}{8\,d}+\frac{B\,b^4\,\sin\left(c+d\,x\right)}{d}+\frac{9\,A\,a^2\,b^2\,x}{4}+\frac{15\,A\,a^4\,\sin\left(2\,c+2\,d\,x\right)}{64\,d}+\frac{3\,A\,a^4\,\sin\left(4\,c+4\,d\,x\right)}{64\,d}+\frac{A\,a^4\,\sin\left(6\,c+6\,d\,x\right)}{192\,d}+\frac{A\,b^4\,\sin\left(2\,c+2\,d\,x\right)}{4\,d}+\frac{5\,B\,a^4\,\sin\left(3\,c+3\,d\,x\right)}{48\,d}+\frac{B\,a^4\,\sin\left(5\,c+5\,d\,x\right)}{80\,d}+\frac{A\,a\,b^3\,\sin\left(3\,c+3\,d\,x\right)}{3\,d}+\frac{5\,A\,a^3\,b\,\sin\left(3\,c+3\,d\,x\right)}{12\,d}+\frac{A\,a^3\,b\,\sin\left(5\,c+5\,d\,x\right)}{20\,d}+\frac{B\,a\,b^3\,\sin\left(2\,c+2\,d\,x\right)}{d}+\frac{B\,a^3\,b\,\sin\left(2\,c+2\,d\,x\right)}{d}+\frac{B\,a^3\,b\,\sin\left(4\,c+4\,d\,x\right)}{8\,d}+\frac{9\,B\,a^2\,b^2\,\sin\left(c+d\,x\right)}{2\,d}+\frac{3\,A\,a^2\,b^2\,\sin\left(2\,c+2\,d\,x\right)}{2\,d}+\frac{3\,A\,a^2\,b^2\,\sin\left(4\,c+4\,d\,x\right)}{16\,d}+\frac{B\,a^2\,b^2\,\sin\left(3\,c+3\,d\,x\right)}{2\,d}+\frac{3\,A\,a\,b^3\,\sin\left(c+d\,x\right)}{d}+\frac{5\,A\,a^3\,b\,\sin\left(c+d\,x\right)}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^6*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^4,x)
```

```
[Out] (5*A*a^4*x)/16 + (A*b^4*x)/2 + 2*B*a*b^3*x + (3*B*a^3*b*x)/2 + (5*B*a^4*sin
(c + d*x))/(8*d) + (B*b^4*sin(c + d*x))/d + (9*A*a^2*b^2*x)/4 + (15*A*a^4*s
in(2*c + 2*d*x))/(64*d) + (3*A*a^4*sin(4*c + 4*d*x))/(64*d) + (A*a^4*sin(6*
c + 6*d*x))/(192*d) + (A*b^4*sin(2*c + 2*d*x))/(4*d) + (5*B*a^4*sin(3*c + 3
*d*x))/(48*d) + (B*a^4*sin(5*c + 5*d*x))/(80*d) + (A*a*b^3*sin(3*c + 3*d*x)
)/(3*d) + (5*A*a^3*b*sin(3*c + 3*d*x))/(12*d) + (A*a^3*b*sin(5*c + 5*d*x))/
(20*d) + (B*a*b^3*sin(2*c + 2*d*x))/d + (B*a^3*b*sin(2*c + 2*d*x))/d + (B*a
^3*b*sin(4*c + 4*d*x))/(8*d) + (9*B*a^2*b^2*sin(c + d*x))/(2*d) + (3*A*a^2*
b^2*sin(2*c + 2*d*x))/(2*d) + (3*A*a^2*b^2*sin(4*c + 4*d*x))/(16*d) + (B*a^
2*b^2*sin(3*c + 3*d*x))/(2*d) + (3*A*a*b^3*sin(c + d*x))/d + (5*A*a^3*b*sin
(c + d*x))/(2*d)
```

3.311 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=187

$$\frac{(2a^2+b^2)(Ab-aB)\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{2a^3(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^4\sqrt{a+b}\,d}-\frac{(3aAb-3a^2B-}{3b}$$

[Out] 1/2*(2*a^2+b^2)*(A*b-B*a)*arctanh(sin(d*x+c))/b^4/d-2*a^3*(A*b-B*a)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b^4/d/(a-b)^(1/2)/(a+b)^(1/2)-1/3*(3*A*a*b-3*B*a^2-2*B*b^2)*tan(d*x+c)/b^3/d+1/2*(A*b-B*a)*sec(d*x+c)*tan(d*x+c)/b^2/d+1/3*B*sec(d*x+c)^2*tan(d*x+c)/b/d

Rubi [A]
time = 0.44, antiderivative size = 187, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4118, 4177, 4167, 4083, 3855, 3916, 2738, 214}

$$-\frac{2a^3(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d\sqrt{a-b}\,\sqrt{a+b}}+\frac{(2a^2+b^2)(Ab-aB)\tanh^{-1}(\sin(c+dx))}{2b^4d}-\frac{(-3a^2B+3aAb-2b^2B)\tan(c+dx)}{3b^3d}+\frac{(Ab-aB)\tan(c+dx)\sec(c+dx)}{2b^2d}+\frac{B\tan(c+dx)\sec^2(c+dx)}{3bd}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] ((2*a^2 + b^2)*(A*b - a*B)*ArcTanh[Sin[c + d*x]])/(2*b^4*d) - (2*a^3*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^4*Sqrt[a + b]*d) - ((3*a*A*b - 3*a^2*B - 2*b^2*B)*Tan[c + d*x])/(3*b^3*d) + ((A*b - a*B)*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*d) + (B*Sec[c + d*x]^2*Tan[c + d*x])/(3*b*d)

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4118

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d^
2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 2)/(b*f*
(m + n))), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e +
 f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m +
n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n
, 0] &&  !IGtQ[m, 1]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4177

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(-C)*Csc[e + f*x]*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m +
1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e +
 f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(
m + 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \frac{B\sec^2(c+dx)\tan(c+dx)}{3bd} + \frac{\int \frac{\sec^2(c+dx)(2aB+2bB\sec(c+dx)+3(Ab-aB)\text{s}}{a+b\sec(c+dx)}}{3b}$$

$$= \frac{(Ab-aB)\sec(c+dx)\tan(c+dx)}{2b^2d} + \frac{B\sec^2(c+dx)\tan(c+dx)}{3bd} +$$

$$= -\frac{(3aAb-3a^2B-2b^2B)\tan(c+dx)}{3b^3d} + \frac{(Ab-aB)\sec(c+dx)\tan}{2b^2d}$$

$$= -\frac{(3aAb-3a^2B-2b^2B)\tan(c+dx)}{3b^3d} + \frac{(Ab-aB)\sec(c+dx)\tan}{2b^2d}$$

$$= \frac{(2a^2+b^2)(Ab-aB)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{(3aAb-3a^2B-2b^2B}{3b^3d}$$

$$= \frac{(2a^2+b^2)(Ab-aB)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{(3aAb-3a^2B-2b^2B}{3b^3d}$$

$$= \frac{(2a^2+b^2)(Ab-aB)\tanh^{-1}(\sin(c+dx))}{2b^4d} - \frac{2a^3(Ab-aB)\tanh^{-1}}{\sqrt{a-b}\,b}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 422 vs. $2(187) = 374$.
time = 2.74, size = 422, normalized size = 2.26

$$\frac{\frac{24a^3(Ab-aB)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + 6(2a^2+b^2)(-Ab+aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) - 6(2a^2+b^2)(-Ab+aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{b^2(3Ab+(-3a+b)B)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{2b^3B\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} + \frac{4b(-3aAb+3a^2B+2b^2B)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} + \frac{2b^3B\sin\left(\frac{1}{2}(c+dx)\right)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^3} - \frac{b^2(3Ab+(-3a+b)B)}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{4b(-3aAb+3a^2B+2b^2B)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}}{12b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((24*a^3*(A*b - a*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/
Sqrt[a^2 - b^2] + 6*(2*a^2 + b^2)*(-(A*b) + a*B)*Log[Cos[(c + d*x)/2] - Sin
[(c + d*x)/2]] - 6*(2*a^2 + b^2)*(-(A*b) + a*B)*Log[Cos[(c + d*x)/2] + Sin[
(c + d*x)/2]] + (b^2*(3*A*b + (-3*a + b)*B))/(Cos[(c + d*x)/2] - Sin[(c + d
*x)/2])^2 + (2*b^3*B*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]
)^3 + (4*b*(-3*a*A*b + 3*a^2*B + 2*b^2*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/
2] - Sin[(c + d*x)/2]) + (2*b^3*B*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] + Sin
[(c + d*x)/2])^3 - (b^2*(3*A*b + (-3*a + b)*B))/(Cos[(c + d*x)/2] + Sin[(c
+ d*x)/2])^2 + (4*b*(-3*a*A*b + 3*a^2*B + 2*b^2*B)*Sin[(c + d*x)/2])/(Cos[(
c + d*x)/2] + Sin[(c + d*x)/2]))/(12*b^4*d)
```

Maple [A]
time = 0.51, size = 335, normalized size = 1.79

method	result
derivativedivides	$\frac{-\frac{B}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{-Ab+Ba+Bb}{2b^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{\left(-2A\,a^{2}b-A\,b^{3}+2a^{3}B+Ba\,b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^{4}}-\frac{-2Aba-A\,b^{2}+2a^{2}B+Ba}{2b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{}$
default	$\frac{-\frac{B}{3b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{3}}-\frac{-Ab+Ba+Bb}{2b^{2}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^{2}}+\frac{\left(-2A\,a^{2}b-A\,b^{3}+2a^{3}B+Ba\,b^{2}\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^{4}}-\frac{-2Aba-A\,b^{2}+2a^{2}B+Ba}{2b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-\right.}}{}$
risch	$\frac{i\left(-3A\,b^{2}e^{5i(dx+c)}+3Bab\,e^{5i(dx+c)}-6Aab\,e^{4i(dx+c)}+6B\,a^{2}e^{4i(dx+c)}-12Aab\,e^{2i(dx+c)}+12B\,a^{2}e^{2i(dx+c)}+12B\,b^{2}e^{2i(dx\right.}{3d\,b^{3}\left(e^{2i(dx+c)}+1\right)^{3}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-1/3*B/b/(tan(1/2*d*x+1/2*c)-1)^3-1/2*(-A*b+B*a+B*b)/b^2/(tan(1/2*d*x+
1/2*c)-1)^2+1/2/b^4*(-2*A*a^2*b-A*b^3+2*B*a^3+B*a*b^2)*ln(tan(1/2*d*x+1/2*c
)-1)-1/2*(-2*A*a*b-A*b^2+2*B*a^2+B*a*b+2*B*b^2)/b^3/(tan(1/2*d*x+1/2*c)-1)-
2*a^3*(A*b-B*a)/b^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((
a+b)*(a-b))^(1/2))-1/3*B/b/(tan(1/2*d*x+1/2*c)+1)^3-1/2*(A*b-B*a-B*b)/b^2/(
tan(1/2*d*x+1/2*c)+1)^2+1/2*(2*A*a^2*b+A*b^3-2*B*a^3-B*a*b^2)/b^4*ln(tan(1/
2*d*x+1/2*c)+1)-1/2*(-2*A*a*b-A*b^2+2*B*a^2+B*a*b+2*B*b^2)/b^3/(tan(1/2*d*x
+1/2*c)+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 343 vs. $2(170) = 340$.
time = 3.32, size = 743, normalized size = 3.97

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [-1/12*(6*(B*a^4 - A*a^3*b)*sqrt(a^2 - b^2)*cos(d*x + c)^3*log((2*a*b*cos(d
*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c)
+ a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
 b^2)) + 3*(2*B*a^5 - 2*A*a^4*b - B*a^3*b^2 + A*a^2*b^3 - B*a*b^4 + A*b^5)*
cos(d*x + c)^3*log(sin(d*x + c) + 1) - 3*(2*B*a^5 - 2*A*a^4*b - B*a^3*b^2 +
 A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(d*x + c)^3*log(-sin(d*x + c) + 1) - 2*(2*
B*a^2*b^3 - 2*B*b^5 + 2*(3*B*a^4*b - 3*A*a^3*b^2 - B*a^2*b^3 + 3*A*a*b^4 -
2*B*b^5)*cos(d*x + c)^2 - 3*(B*a^3*b^2 - A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(d
*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*d*cos(d*x + c)^3), 1/12*(12*(B*a^4
- A*a^3*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/(
(a^2 - b^2)*sin(d*x + c)))*cos(d*x + c)^3 - 3*(2*B*a^5 - 2*A*a^4*b - B*a^3*
b^2 + A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(d*x + c)^3*log(sin(d*x + c) + 1) + 3
*(2*B*a^5 - 2*A*a^4*b - B*a^3*b^2 + A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(d*x +
c)^3*log(-sin(d*x + c) + 1) + 2*(2*B*a^2*b^3 - 2*B*b^5 + 2*(3*B*a^4*b - 3*A
*a^3*b^2 - B*a^2*b^3 + 3*A*a*b^4 - 2*B*b^5)*cos(d*x + c)^2 - 3*(B*a^3*b^2 -
 A*a^2*b^3 - B*a*b^4 + A*b^5)*cos(d*x + c))*sin(d*x + c))/((a^2*b^4 - b^6)*
d*cos(d*x + c)^3)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec^4(c + dx)}{a + b\sec(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. 2(170) = 340.
time = 0.50, size = 412, normalized size = 2.20

$$-\frac{\frac{3\left(2Ba^3-2Aa^2b+Bab^2-Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^4}-\frac{3\left(2Ba^3-2Aa^2b+Bab^2-Ab^3\right)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^4}-\frac{12\left(Ba^4-Aa^3b\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,b^4}+\frac{2\left(6Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6Aab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3Bab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-3Ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+6Bb^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-12Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+12Aab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4Bb^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6Ba^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-6Aab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3Bab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3Ab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+6Bb^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)^3b^3}}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] -1/6*(3*(2*B*a^3 - 2*A*a^2*b + B*a*b^2 - A*b^3)*log(abs(tan(1/2*d*x + 1/2*c
) + 1))/b^4 - 3*(2*B*a^3 - 2*A*a^2*b + B*a*b^2 - A*b^3)*log(abs(tan(1/2*d*x
```

```
 + 1/2*c) - 1))/b^4 - 12*(B*a^4 - A*a^3*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2
)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c
))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*b^4) + 2*(6*B*a^2*tan(1/2*d*x + 1/2
*c)^5 - 6*A*a*b*tan(1/2*d*x + 1/2*c)^5 + 3*B*a*b*tan(1/2*d*x + 1/2*c)^5 - 3
*A*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*B*b^2*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^2*t
an(1/2*d*x + 1/2*c)^3 + 12*A*a*b*tan(1/2*d*x + 1/2*c)^3 - 4*B*b^2*tan(1/2*d
*x + 1/2*c)^3 + 6*B*a^2*tan(1/2*d*x + 1/2*c) - 6*A*a*b*tan(1/2*d*x + 1/2*c)
 - 3*B*a*b*tan(1/2*d*x + 1/2*c) + 3*A*b^2*tan(1/2*d*x + 1/2*c) + 6*B*b^2*ta
n(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^3*b^3))/d
```

Mupad [B]
time = 6.93, size = 2500, normalized size = 13.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + b/cos(c + d*x))),x)
```

```
[Out] - ((tan(c/2 + (d*x)/2)*(A*b^2 + 2*B*a^2 + 2*B*b^2 - 2*A*a*b - B*a*b))/b^3 +
 (tan(c/2 + (d*x)/2)^5*(2*B*a^2 - A*b^2 + 2*B*b^2 - 2*A*a*b + B*a*b))/b^3 -
 (4*tan(c/2 + (d*x)/2)^3*(3*B*a^2 + B*b^2 - 3*A*a*b))/(3*b^3))/(d*(3*tan(c/
2 + (d*x)/2)^2 - 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 - 1)) - (ata
n(((((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A^2*a*b^8 + 16*B^2*a^8*
b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 - 16*A^2*a^5*b^4 + 16*A
^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*b^6 + 7*B^2*a^4*b^5 -
13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*A*B*a*b^8 + 16*A*B*a^8
*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5 - 32*A*B*a^5*b^4 + 32*
A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 - (((8*(2*A*b^13 + 2*A*a^2*b^11 - 6*A*a^
3*b^10 + 4*A*a^4*b^9 + 2*B*a^2*b^11 - 2*B*a^3*b^10 + 6*B*a^4*b^9 - 4*B*a^5*
b^8 - 2*A*a*b^12 - 2*B*a*b^12))/b^9 - (4*tan(c/2 + (d*x)/2)*(8*a*b^10 - 16*
a^2*b^9 + 8*a^3*b^8)*(A*b^3 - 2*B*a^3 + 2*A*a^2*b - B*a*b^2))/b^10)*(A*b^3
- 2*B*a^3 + 2*A*a^2*b - B*a*b^2))/(2*b^4))*(A*b^3 - 2*B*a^3 + 2*A*a^2*b - B
*a*b^2)*1i)/(2*b^4) + (((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A^2*
a*b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 - 16
*A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*b^6
 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*A*B
*a*b^8 + 16*A*B*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5 - 3
2*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 + (((8*(2*A*b^13 + 2*
A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a^4*b^9 + 2*B*a^2*b^11 - 2*B*a^3*b^10 + 6*B
*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b^12 - 2*B*a*b^12))/b^9 + (4*tan(c/2 + (d*x)
/2)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*(A*b^3 - 2*B*a^3 + 2*A*a^2*b - B*a*
b^2))/b^10)*(A*b^3 - 2*B*a^3 + 2*A*a^2*b - B*a*b^2))/(2*b^4))*(A*b^3 - 2*B*
a^3 + 2*A*a^2*b - B*a*b^2)*1i)/(2*b^4))/((16*(4*B^3*a^11 - 6*B^3*a^10*b + A
^3*a^3*b^8 - 2*A^3*a^4*b^7 + 5*A^3*a^5*b^6 - 6*A^3*a^6*b^5 + 6*A^3*a^7*b^4
- 4*A^3*a^8*b^3 - B^3*a^6*b^5 + 2*B^3*a^7*b^4 - 5*B^3*a^8*b^3 + 6*B^3*a^9*b
```

```
^2 - 12*A*B^2*a^10*b + 3*A*B^2*a^5*b^6 - 6*A*B^2*a^6*b^5 + 15*A*B^2*a^7*b^4
 - 18*A*B^2*a^8*b^3 + 18*A*B^2*a^9*b^2 - 3*A^2*B*a^4*b^7 + 6*A^2*B*a^5*b^6
- 15*A^2*B*a^6*b^5 + 18*A^2*B*a^7*b^4 - 18*A^2*B*a^8*b^3 + 12*A^2*B*a^9*b^2
))/b^9 - (((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A^2*a*b^8 + 16*B^
2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 - 16*A^2*a^5*b^4
+ 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*b^6 + 7*B^2*a^4*
b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*A*B*a*b^8 + 16*A
*B*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5 - 32*A*B*a^5*b^4
 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 - (((8*(2*A*b^13 + 2*A*a^2*b^11 -
6*A*a^3*b^10 + 4*A*a^4*b^9 + 2*B*a^2*b^11 - 2*B*a^3*b^10 + 6*B*a^4*b^9 - 4*
B*a^5*b^8 - 2*A*a*b^12 - 2*B*a*b^12))/b^9 - (4*tan(c/2 + (d*x)/2)*(8*a*b^10
 - 16*a^2*b^9 + 8*a^3*b^8)*(A*b^3 - 2*B*a^3 + 2*A*a^2*b - B*a*b^2))/b^10)*(
A*b^3 - 2*B*a^3 + 2*A*a^2*b - B*a*b^2))/(2*b^4))*(A*b^3 - 2*B*a^3 + 2*A*a^2
*b - B*a*b^2))/(2*b^4) + (((8*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A
^2*a*b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 -
 16*A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*
b^6 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*
A*B*a*b^8 + 16*A*B*a^8*b + 6*A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5
- 32*A*B*a^5*b^4 + 32*A*B*a^6*b^3 - 32*A*B*a^7*b^2))/b^6 + (((8*(2*A*b^13 +
 2*A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a^4*b^9 + 2*B*a^2*b^11 - 2*B*a^3*b^10 +
6*B*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b^12 - 2*B*a*b^12))/b^9 + (4*tan(c/2 + (d
*x)/2)*(8*a*b^10 - 16*a^2*b^9 + 8*a^3*b^8)*(A*b^3 - 2*B*a^3 + 2*A*a^2*b - B
*a*b^2))/b^10)*(A*b^3 - 2*B*a^3 + 2*A*a^2*b - B*a*b^2))/(2*b^4))*(A*b^3 - 2
*B*a^3 + 2*A*a^2*b - B*a*b^2))/(2*b^4)))*(A*b^3 - 2*B*a^3 + 2*A*a^2*b - B*a
*b^2)*1i)/(b^4*d) - (a^3*atan(((a^3*((a + b)*(a - b))^(1/2)*(A*b - B*a)*((8
*tan(c/2 + (d*x)/2)*(A^2*b^9 - 8*B^2*a^9 - 3*A^2*a*b^8 + 16*B^2*a^8*b + 7*A
^2*a^2*b^7 - 13*A^2*a^3*b^6 + 16*A^2*a^4*b^5 - 16*A^2*a^5*b^4 + 16*A^2*a^6*
b^3 - 8*A^2*a^7*b^2 + B^2*a^2*b^7 - 3*B^2*a^3*b^6 + 7*B^2*a^4*b^5 - 13*B^2*
a^5*b^4 + 16*B^2*a^6*b^3 - 16*B^2*a^7*b^2 - 2*A*B*a*b^8 + 16*A*B*a^8*b + 6*
A*B*a^2*b^7 - 14*A*B*a^3*b^6 + 26*A*B*a^4*b^5 - 32*A*B*a^5*b^4 + 32*A*B*a^6
*b^3 - 32*A*B*a^7*b^2))/b^6 + (a^3*((a + b)*(a - b))^(1/2)*((8*(2*A*b^13 +
2*A*a^2*b^11 - 6*A*a^3*b^10 + 4*A*a^4*b^9 + 2*B*a^2*b^11 - 2*B*a^3*b^10 + 6
*B*a^4*b^9 - 4*B*a^5*b^8 - 2*A*a*b^12 - 2*B*a*b^12))/b^9 + (8*a^3*tan(c/2 +
 (d*x)/2)*((a + b)*(a - b))^(1/2)*(A*b - B*a)*(8*a*b^10 - 16*a^2*b^9 + 8*a^
3*b^8))/(b^6*(b^6 - a^2*b^4)))*(A*b - B*a))/(b^6 - a^2*b^4))*1i)/(b^6 - a^2
*b^4) + (a^3*((a + b)*(a - b))^(1/2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(A^
2*b^9 - 8*B^2*a^9 - 3*A^2*a*b^8 + 16*B^2*a^8*b + 7*A^2*a^2*b^7 - 13*A^2*a^3
*b^6 + 16*A^2*a^4*b^5 - 16*A^2*a^5*b^4 + 16*A^2*a^6*b^3 - 8*A^2*a^7*b^2 + B
^2*a^2*b^7 - 3*B^2*a^3*b^6 + 7*B^2*a^4*b^5 - 13*B^2*a^5*b^4 + 16*B^2*a^6*b^
3 - 16*B^2*a^7*b^2 - 2*A*B*a*b^8 + 16*A*B*a^8*b...
```

3.312 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=143

$$-\frac{(2aAb-2a^2B-b^2B)\tanh^{-1}(\sin(c+dx))}{2b^3d}+\frac{2a^2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^3\sqrt{a+b}\,d}+\frac{(Ab-aB)\tan}{b^2d}$$

[Out] -1/2*(2*A*a*b-2*B*a^2-B*b^2)*arctanh(sin(d*x+c))/b^3/d+2*a^2*(A*b-B*a)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/b^3/d/(a-b)^(1/2)/(a+b)^(1/2)+(A*b-B*a)*tan(d*x+c)/b^2/d+1/2*B*sec(d*x+c)*tan(d*x+c)/b/d

Rubi [A]
time = 0.27, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4118, 4167, 4083, 3855, 3916, 2738, 214}

$$\frac{2a^2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d\sqrt{a-b}\sqrt{a+b}}-\frac{(-2a^2B+2aAb-b^2B)\tanh^{-1}(\sin(c+dx))}{2b^3d}+\frac{(Ab-aB)\tan(c+dx)}{b^2d}+\frac{B\tan(c+dx)\sec(c+dx)}{2bd}$$

Antiderivative was successfully verified.

[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] -1/2*((2*a*A*b - 2*a^2*B - b^2*B)*ArcTanh[Sin[c + d*x]])/(b^3*d) + (2*a^2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^3*Sqrt[a + b]*d) + ((A*b - a*B)*Tan[c + d*x])/(b^2*d) + (B*Sec[c + d*x]*Tan[c + d*x])/(2*b*d)

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x] /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4118

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d^
2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 2)/(b*f*
(m + n))), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e +
 f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m +
n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n
, 0] &&  !IGtQ[m, 1]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \frac{B\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\int \frac{\sec(c+dx)(aB+bB\sec(c+dx)+2(Ab-aB)\sec^2}{a+b\sec(c+dx)}}{2b}$$

$$= \frac{(Ab-aB)\tan(c+dx)}{b^2d} + \frac{B\sec(c+dx)\tan(c+dx)}{2bd} + \frac{\int \frac{\sec(c+dx)}{}}{}$$

$$= \frac{(Ab-aB)\tan(c+dx)}{b^2d} + \frac{B\sec(c+dx)\tan(c+dx)}{2bd} + \frac{(a^2(Ab-}{}$$

$$= -\frac{(2aAb-2a^2B-b^2B)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{(Ab-aB)\tan(c}{b^2d}$$

$$= -\frac{(2aAb-2a^2B-b^2B)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{(Ab-aB)\tan(c}{b^2d}$$

$$= -\frac{(2aAb-2a^2B-b^2B)\tanh^{-1}(\sin(c+dx))}{2b^3d} + \frac{2a^2(Ab-aB)\tan}{\sqrt{a-}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 300 vs. 2(143) = 286.
time = 1.86, size = 300, normalized size = 2.10

$$\frac{\frac{8a^2(-Ab+aB)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} - 2(-2aAb+2a^2B+b^2B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2(-2aAb+2a^2B+b^2B)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{b^2B}{\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{4b(Ab-aB)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)} - \frac{b^2B}{\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{4b(Ab-aB)\sin\left(\frac{1}{2}(c+dx)\right)}{\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)}}{4b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
[Out] ((8*a^2*(-(A*b) + a*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]
)/Sqrt[a^2 - b^2] - 2*(-2*a*A*b + 2*a^2*B + b^2*B)*Log[Cos[(c + d*x)/2] - S
in[(c + d*x)/2]] + 2*(-2*a*A*b + 2*a^2*B + b^2*B)*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] + (b^2*B)/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2 + (4*b*(A
*b - a*B)*Sin[(c + d*x)/2])/(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]) - (b^2*B)
/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2 + (4*b*(A*b - a*B)*Sin[(c + d*x)/2
])/(Cos[(c + d*x)/2] + Sin[(c + d*x)/2]))/(4*b^3*d)
```

Maple [A]
time = 0.42, size = 229, normalized size = 1.60

method	result
derivativedivides	$\frac{\frac{2a^2(Ab-Ba)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^3\sqrt{(a+b)(a-b)}} + \frac{B}{2b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2} - \frac{2Ab-2Ba-Bb}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)} + \frac{(2Aba-2a^2B-b^2B)\ln(\tan}{2b^3}}{d}$

default	$\frac{\frac{2a^2(Ab-Ba)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^3\sqrt{(a+b)(a-b)}}+\frac{B}{2b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2Ab-2Ba-Bb}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(2Aba-2a^2B-b^2B\right)\ln\left(\tan\left(\frac{d}{\cdot}\right.\right.}{2b^3}}{d}$
risch	$-\frac{i\left(Bb\,e^{3i(dx+c)}-2Ab\,e^{2i(dx+c)}+2Ba\,e^{2i(dx+c)}-Bb\,e^{i(dx+c)}-2Ab+2Ba\right)}{b^2d\left(e^{2i(dx+c)}+1\right)^2}+\frac{\ln\left(e^{i(dx+c)}-i\right)Aa}{b^2d}-\frac{\ln\left(e^{i(dx+c)}-i\right)a^2B}{b^3d}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*a^2*(A*b-B*a)/b^3/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2))+1/2*B/b/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(2*A*b-2*B*a-B
*b)/b^2/(tan(1/2*d*x+1/2*c)-1)+1/2*(2*A*a*b-2*B*a^2-B*b^2)/b^3*ln(tan(1/2*d
*x+1/2*c)-1)-1/2*B/b/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(2*A*b-2*B*a-B*b)/b^2/(ta
n(1/2*d*x+1/2*c)+1)+1/2/b^3*(-2*A*a*b+2*B*a^2+B*b^2)*ln(tan(1/2*d*x+1/2*c)+
1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(130) = 260$.
time = 11.54, size = 609, normalized size = 4.26

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [-1/4*(2*(B*a^3 - A*a^2*b)*sqrt(a^2 - b^2)*cos(d*x + c)^2*log((2*a*b*cos(d*
x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) +
 a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
```

```
b^2)) - (2*B*a^4 - 2*A*a^3*b - B*a^2*b^2 + 2*A*a*b^3 - B*b^4)*cos(d*x + c)^
2*log(sin(d*x + c) + 1) + (2*B*a^4 - 2*A*a^3*b - B*a^2*b^2 + 2*A*a*b^3 - B*
b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(B*a^2*b^2 - B*b^4 - 2*(B*a^
3*b - A*a^2*b^2 - B*a*b^3 + A*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 -
b^5)*d*cos(d*x + c)^2), -1/4*(4*(B*a^3 - A*a^2*b)*sqrt(-a^2 + b^2)*arctan(-
sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x +
 c)^2 - (2*B*a^4 - 2*A*a^3*b - B*a^2*b^2 + 2*A*a*b^3 - B*b^4)*cos(d*x + c)^
2*log(sin(d*x + c) + 1) + (2*B*a^4 - 2*A*a^3*b - B*a^2*b^2 + 2*A*a*b^3 - B*
b^4)*cos(d*x + c)^2*log(-sin(d*x + c) + 1) - 2*(B*a^2*b^2 - B*b^4 - 2*(B*a^
3*b - A*a^2*b^2 - B*a*b^3 + A*b^4)*cos(d*x + c))*sin(d*x + c))/((a^2*b^3 -
b^5)*d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec^3(c + dx)}{a + b\sec(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 269 vs. 2(130) = 260.
time = 0.50, size = 269, normalized size = 1.88

$$\frac{\frac{(2Ba^2 - 2Aab + Bb^2)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right)}{b^3} - \frac{(2Ba^2 - 2Aab + Bb^2)\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)}{b^3} - \frac{4(Ba^3 - Aa^2b)\left(\pi\left\lfloor\frac{dx+c}{2\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}(-2a + 2b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - b\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\sqrt{-a^2 + b^2}}\right)\right)}{\sqrt{-a^2 + b^2}\,b^3} + \frac{2\left(2Ba\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 2Ab\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 + Bb\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^3 - 2Ba\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 2Ab\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + Bb\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{\left(\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - 1\right)^2 b^2}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((2*B*a^2 - 2*A*a*b + B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 - (
2*B*a^2 - 2*A*a*b + B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^3 - 4*(B*a^
3 - A*a^2*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a
*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a
^2 + b^2)*b^3) + 2*(2*B*a*tan(1/2*d*x + 1/2*c)^3 - 2*A*b*tan(1/2*d*x + 1/2*
c)^3 + B*b*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c) + 2*A*b*tan(
1/2*d*x + 1/2*c) + B*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^
2*b^2))/d
```

Mupad [B]
time = 6.08, size = 2500, normalized size = 17.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + b/cos(c + d*x))),x)

[Out] (B*a*sin(2*c + 2*d*x))/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (A*b*sin(2*c + 2*d*x))/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*b*sin(c + d*x))/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (A*a*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*1i)/(d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (B*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*1i)/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (A*a^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*1i)/(b^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (B*a^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*1i)/(2*b*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*a^4*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*1i)/(b^3*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (A*a^2*sin(2*c + 2*d*x))/(2*b*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*a^3*sin(2*c + 2*d*x))/(2*b^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (A*a*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*1i)/(d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (B*b*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*1i)/(2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (B*a^2*sin(c + d*x))/(2*b*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (A*a^3*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*1i)/(b^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (B*a^2*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*1i)/(2*b*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*a^4*atan((sin(c/2 + (d*x)/2)*1i)/cos(c/2 + (d*x)/2))*cos(2*c + 2*d*x)*1i)/(b^3*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (A*a^2*atan(((8*B^2*a^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2) - 8*B^2*a^9*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + B^2*b^9*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - B^2*a*b^8*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 4*A^2*a^2*b^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 4*A^2*a^3*b^6*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 4*A^2*a^4*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 8*A^2*a^5*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2) + 12*A^2*a^5*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 8*A^2*a^7*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 2*B^2*a^2*b^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 2*B^2*a^3*b^6*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 3*B^2*a^4*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 3*B^2*a^5*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 8*B^2*a^7*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 4*A*B*a*b^8*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 16*A*B*a^6*b*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2) + 16*A*B*a^8*b*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 4*A*B*a^2*b^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 4*A*B*a^5*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 20*A*B*a^6*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))*1i)/(cos(c/2 + (d*x)/2)*(a^2*b - b^3)*(B^2*b^7 + 4*A^2*a^2*b^5 - 4*A^2*a^4*b^3 + 2*B^2*a^2*b^5 - 3*B^2*a^4*b^3 - 4*A*B*a*b^6 + 4*A*B*a^5*b^2)))*((a + b)*(a - b))^(1/2)*1i)/(b^2*d*(a^2 - b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) - (B*a^3*atan(((8*B^2*a^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2) - 8*B^2*a^9*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + B^2*b^9*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - B^2*a*b^

```
8*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 4*A^2*a^2*b^7*sin(c/2 + (d*x)/2)*(
a^2 - b^2)^(1/2) - 4*A^2*a^3*b^6*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 4*A
^2*a^4*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 8*A^2*a^5*b^2*sin(c/2 + (
d*x)/2)*(a^2 - b^2)^(3/2) + 12*A^2*a^5*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(
1/2) - 8*A^2*a^7*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 2*B^2*a^2*b^7*s
in(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 2*B^2*a^3*b^6*sin(c/2 + (d*x)/2)*(a^2
 - b^2)^(1/2) - 3*B^2*a^4*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 3*B^2*
a^5*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 8*B^2*a^7*b^2*sin(c/2 + (d*x
)/2)*(a^2 - b^2)^(1/2) - 4*A*B*a*b^8*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) -
 16*A*B*a^6*b*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2) + 16*A*B*a^8*b*sin(c/2 +
 (d*x)/2)*(a^2 - b^2)^(1/2) + 4*A*B*a^2*b^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^
(1/2) + 4*A*B*a^5*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 20*A*B*a^6*b^3
*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))*1i)/(cos(c/2 + (d*x)/2)*(a^2*b - b^3
)*(B^2*b^7 + 4*A^2*a^2*b^5 - 4*A^2*a^4*b^3 + 2*B^2*a^2*b^5 - 3*B^2*a^4*b^3
- 4*A*B*a*b^6 + 4*A*B*a^5*b^2)))*((a + b)*(a - b))^(1/2)*1i)/(b^3*d*(a^2 -
b^2)*(cos(2*c + 2*d*x)/2 + 1/2)) + (A*a^2*atan(((8*B^2*a^7*sin(c/2 + (d*x)/
2)*(a^2 - b^2)^(3/2) - 8*B^2*a^9*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + B^2
*b^9*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - B^2*a*b^8*sin(c/2 + (d*x)/2)*(a
^2 - b^2)^(1/2) + 4*A^2*a^2*b^7*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 4*A^
2*a^3*b^6*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 4*A^2*a^4*b^5*sin(c/2 + (d
*x)/2)*(a^2 - b^2)^(1/2) + 8*A^2*a^5*b^2*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/
2) + 12*A^2*a^5*b^4*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 8*A^2*a^7*b^2*si
n(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 2*B^2*a^2*b^7*sin(c/2 + (d*x)/2)*(a^2
- b^2)^(1/2) - 2*B^2*a^3*b^6*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) - 3*B^2*a
^4*b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2) + 3*B^2*a^5*b^4*sin(c/2 + (d*x)
/2)*(a^2 - b^2)^(1/2) + 8*B^2*a^7*b^2*sin(c/2 +...
```

3.313 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=98

$$\frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{b^2d}-\frac{2a(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}\,d}+\frac{B\tan(c+dx)}{bd}$$

```
[Out] (A*b-B*a)*arctanh(sin(d*x+c))/b^2/d-2*a*(A*b-B*a)*arctanh((a-b)^(1/2)*tan(1
/2*d*x+1/2*c)/(a+b)^(1/2))/b^2/d/(a-b)^(1/2)/(a+b)^(1/2)+B*tan(d*x+c)/b/d
```

Rubi [A]
time = 0.15, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4095, 12, 3874, 3855, 3916, 2738, 214}

$$\frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{b^2d}-\frac{2a(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d\sqrt{a-b}\,\sqrt{a+b}}+\frac{B\tan(c+dx)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((A*b - a*B)*ArcTanh[Sin[c + d*x]])/(b^2*d) - (2*a*(A*b - a*B)*ArcTanh[(Sqr
t[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b^2*Sqrt[a + b]*d) +
(B*Tan[c + d*x])/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3874

```
Int[csc[(e_.) + (f_.)*(x_)]^2/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Sym
bol] :> Dist[1/b, Int[Csc[e + f*x], x], x] - Dist[a/b, Int[Csc[e + f*x]/(a
+ b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, e, f}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \frac{B\tan(c+dx)}{bd} + \frac{\int \frac{(Ab-aB)\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx}{b}$$

$$= \frac{B\tan(c+dx)}{bd} + \frac{(Ab-aB)\int \frac{\sec^2(c+dx)}{a+b\sec(c+dx)}\,dx}{b}$$

$$= \frac{B\tan(c+dx)}{bd} + \frac{(Ab-aB)\int \sec(c+dx)\,dx}{b^2} - \frac{(a(Ab-aB))\int \cdots}{b^2}$$

$$= \frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{B\tan(c+dx)}{bd} - \frac{(a(Ab-aB)) \cdots}{\cdots}$$

$$= \frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{B\tan(c+dx)}{bd} - \frac{(2a(Ab-aB \cdots}{\cdots}$$

$$= \frac{(Ab-aB)\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{2a(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\cdots}{\sqrt{\cdots}}\right)}{\sqrt{a-b}\,b^2\sqrt{a+b}\,\cdots}$$

Mathematica [A]
time = 0.60, size = 130, normalized size = 1.33

$$\frac{-\frac{2a(-Ab+aB)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}-(Ab-aB)\left(\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)+bB\tan(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((-2*a*(-(A*b) + a*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])
/Sqrt[a^2 - b^2] - (A*b - a*B)*(Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] -
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]]) + b*B*Tan[c + d*x])/(b^2*d)
```

Maple [A]
time = 0.37, size = 144, normalized size = 1.47

method	result
derivativedivides	$\frac{-\frac{B}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(Ab-Ba)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{2a(Ab-Ba)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2\sqrt{(a+b)(a-b)}}-\frac{B}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(-}{}}{d}$
default	$\frac{-\frac{B}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(Ab-Ba)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^2}-\frac{2a(Ab-Ba)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2\sqrt{(a+b)(a-b)}}-\frac{B}{b\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(-}{}}{d}$
risch	$\frac{2iB}{db\left(e^{2i(dx+c)}+1\right)}+\frac{\ln\left(e^{i(dx+c)}+i\right)A}{bd}-\frac{\ln\left(e^{i(dx+c)}+i\right)Ba}{b^2d}-\frac{\ln\left(e^{i(dx+c)}-i\right)A}{bd}+\frac{\ln\left(e^{i(dx+c)}-i\right)Ba}{b^2d}+\frac{a\ln\left(e^{i(dx+}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-B/b/(tan(1/2*d*x+1/2*c)+1)+(A*b-B*a)/b^2*ln(tan(1/2*d*x+1/2*c)+1)-2*a
*(A*b-B*a)/b^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*
(a-b))^(1/2))-B/b/(tan(1/2*d*x+1/2*c)-1)+1/b^2*(-A*b+B*a)*ln(tan(1/2*d*x+1/
2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 208 vs. $2(90) = 180$.
time = 2.50, size = 472, normalized size = 4.82

$$\left[-\frac{(Ba^2-Aab)\sqrt{a^2-b^2}\cos(dx+c)\log\left(\frac{2ab\cos(dx+c)-(a^2-2b^2)\cos(dx+c)^2-2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2a^2-b^2}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right)+(Ba^3-Aa^2b-Bab^2+Ab^3)\cos(dx+c)\log(\sin(dx+c)+1)-(Ba^3-Aa^2b-Bab^2+Ab^3)\cos(dx+c)\log(-\sin(dx+c)+1)-2(Ba^2b-Bb^3)\sin(dx+c)}{2(a^2b^2-b^4)d\cos(dx+c)}, \frac{2(Ba^2-Aab)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)\cos(dx+c)-(Ba^3-Aa^2b-Bab^2+Ab^3)\cos(dx+c)\log(\sin(dx+c)+1)+(Ba^3-Aa^2b-Bab^2+Ab^3)\cos(dx+c)\log(-\sin(dx+c)+1)+2(Ba^2b-Bb^3)\sin(dx+c)}{2(a^2b^2-b^4)d\cos(dx+c)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [-1/2*((B*a^2 - A*a*b)*sqrt(a^2 - b^2)*cos(d*x + c)*log((2*a*b*cos(d*x + c)
 - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*si
n(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))
+ (B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) -
(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*cos(d*x + c)*log(-sin(d*x + c) + 1) - 2
*(B*a^2*b - B*b^3)*sin(d*x + c))/((a^2*b^2 - b^4)*d*cos(d*x + c)), 1/2*(2*(
B*a^2 - A*a*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) +
a)/((a^2 - b^2)*sin(d*x + c)))*cos(d*x + c) - (B*a^3 - A*a^2*b - B*a*b^2 +
A*b^3)*cos(d*x + c)*log(sin(d*x + c) + 1) + (B*a^3 - A*a^2*b - B*a*b^2 + A*
b^3)*cos(d*x + c)*log(-sin(d*x + c) + 1) + 2*(B*a^2*b - B*b^3)*sin(d*x + c)
)/((a^2*b^2 - b^4)*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec^2(c + dx)}{a + b\sec(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Giac [A]
time = 0.50, size = 176, normalized size = 1.80

$$-\frac{\frac{(Ba-Ab)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^2}-\frac{(Ba-Ab)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^2}+\frac{2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)b}-\frac{2(Ba^2-Aab)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] -((B*a - A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^2 - (B*a - A*b)*log(abs(
tan(1/2*d*x + 1/2*c) - 1))/b^2 + 2*B*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1
/2*c)^2 - 1)*b) - 2*(B*a^2 - A*a*b)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-
2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt
(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*b^2))/d
```

Mupad [B]
time = 2.97, size = 719, normalized size = 7.34

$$\frac{2\,B\,a\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\,\left(a^2-b^2\right)}-\frac{2\,A\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\,\left(a^2-b^2\right)}-\frac{B\,b\,\tan\left(c+d\,x\right)}{d\,\left(a^2-b^2\right)}-\frac{B\,a^2\,\ln\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\,{\left(a^2-b^2\right)}^{3/2}}+\frac{2\,A\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d\,\left(a^2-b^2\right)}-\frac{2\,B\,a^3\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^2\,d\,\left(a^2-b^2\right)}-\frac{A\,a^3\,\ln\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d\,{\left(a^2-b^2\right)}^{3/2}}+\frac{B\,a^4\,\ln\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b^2\,d\,{\left(a^2-b^2\right)}^{3/2}}+\frac{A\,a\,b\,\ln\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\,{\left(a^2-b^2\right)}^{3/2}}+\frac{B\,a^2\,\tan\left(c+d\,x\right)}{b\,d\,\left(a^2-b^2\right)}-\frac{B\,a^2\,\ln\left(\frac{b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sqrt{\left(a+b\right)\,\left(a-b\right)}}{b^2\,d\,\left(a^2-b^2\right)}+\frac{A\,a\,\ln\left(\frac{b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sqrt{\left(a+b\right)\,\left(a-b\right)}}{b\,d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + b/cos(c + d*x))),x)
```

```
[Out] (2*B*a*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)) - (2*A
*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)) - (B*b*tan
(c + d*x))/(d*(a^2 - b^2)) - (B*a^2*log((a*sin(c/2 + (d*x)/2) - b*sin(c/2 +
 (d*x)/2) + cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/cos(c/2 + (d*x)/2)))/(d*(
a^2 - b^2)^(3/2)) + (2*A*a^2*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
(b*d*(a^2 - b^2)) - (2*B*a^3*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/
(b^2*d*(a^2 - b^2)) - (A*a^3*log((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/
2) + cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/cos(c/2 + (d*x)/2)))/(b*d*(a^2 -
 b^2)^(3/2)) + (B*a^4*log((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2) + co
s(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/cos(c/2 + (d*x)/2)))/(b^2*d*(a^2 - b^2)
^(3/2)) + (A*a*b*log((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2) + cos(c/2
 + (d*x)/2)*(a^2 - b^2)^(1/2))/cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)^(3/2)) +
 (B*a^2*tan(c + d*x))/(b*d*(a^2 - b^2)) - (B*a^2*log((b*sin(c/2 + (d*x)/2)
- a*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/cos(c/2 + (d
*x)/2))*((a + b)*(a - b))^(1/2))/(b^2*d*(a^2 - b^2)) + (A*a*log((b*sin(c/2
+ (d*x)/2) - a*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/c
os(c/2 + (d*x)/2))*((a + b)*(a - b))^(1/2))/(b*d*(a^2 - b^2))
```

3.314 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=76

$$\frac{B\tanh^{-1}(\sin(c+dx))}{bd}+\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,d}$$

```
[Out] B*arctanh(sin(d*x+c))/b/d+2*(A*b-B*a)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c
)/(a+b)^(1/2))/b/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]

time = 0.08, antiderivative size = 76, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4083, 3855, 3916, 2738, 214}

$$\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{bd\sqrt{a-b}\,\sqrt{a+b}}+\frac{B\tanh^{-1}(\sin(c+dx))}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(b*d) + (2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tan[(
c + d*x)/2])/Sqrt[a + b]])/(Sqrt[a - b]*b*Sqrt[a + b]*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx &= \frac{B\int\sec(c+dx)\,dx}{b}+\frac{(Ab-aB)\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{b}\\ &= \frac{B\tanh^{-1}(\sin(c+dx))}{bd}+\frac{(Ab-aB)\int\frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b^2}\\ &= \frac{B\tanh^{-1}(\sin(c+dx))}{bd}+\frac{(2(Ab-aB))\text{Subst}\left(\int\frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\text{t}\right.}{b^2d}\\ &= \frac{B\tanh^{-1}(\sin(c+dx))}{bd}+\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{\sqrt{a-b}\,b\sqrt{a+b}\,d}\end{aligned}$$

Mathematica [A]
time = 0.19, size = 112, normalized size = 1.47

$$\frac{\frac{2(-Ab+aB)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+B\left(-\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((2*(-(A*b) + a*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sq
rt[a^2 - b^2] + B*(-Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + Log[Cos[(c +
 d*x)/2] + Sin[(c + d*x)/2]]))/(b*d)
```

Maple [A]
time = 0.35, size = 92, normalized size = 1.21

method	result

derivativedivides	$\frac{\frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}-\frac{2(-Ab+Ba)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{\frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b}-\frac{B\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b}-\frac{2(-Ab+Ba)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b\sqrt{(a+b)(a-b)}}}{d}$
risch	$\frac{\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)A}{\sqrt{a^2-b^2}\,d}-\frac{\ln\left(e^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)Ba}{\sqrt{a^2-b^2}\,db}-\frac{\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2+\ldots}{a\sqrt{\ldots}}\right.}{\sqrt{a^2-\ldots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(B/b*ln(tan(1/2*d*x+1/2*c)+1)-B/b*ln(tan(1/2*d*x+1/2*c)-1)-2/b*(-A*b+B*
a)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)
))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima
")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 3.02, size = 316, normalized size = 4.16

$$\left[-\frac{(Ba-Ab)\sqrt{a^2-b^2}\log\left(\frac{2ab\cos(dx+c)-(a^2-2b^2)\cos(dx+c)^2+2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2a^2-b^2}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right)-(Ba^2-Bb^2)\log(\sin(dx+c)+1)+(Ba^2-Bb^2)\log(-\sin(dx+c)+1)}{2(a^2b-b^3)d}, -\frac{2(Ba-Ab)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-(Ba^2-Bb^2)\log(\sin(dx+c)+1)+(Ba^2-Bb^2)\log(-\sin(dx+c)+1)}{2(a^2b-b^3)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas
")
```

```
[Out] [-1/2*((B*a - A*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*
cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^
```

```
2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - (B*a^2 - B*b^2)
*log(sin(d*x + c) + 1) + (B*a^2 - B*b^2)*log(-sin(d*x + c) + 1))/((a^2*b -
b^3)*d), -1/2*(2*(B*a - A*b)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*c
os(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (B*a^2 - B*b^2)*log(sin(d*x
+ c) + 1) + (B*a^2 - B*b^2)*log(-sin(d*x + c) + 1))/((a^2*b - b^3)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\sec{(c + dx)}}{a + b\sec{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x)), x)
```

Giac [A]
time = 0.51, size = 127, normalized size = 1.67

$$\frac{\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b}+\frac{2\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)(Ba-Ab)}{\sqrt{-a^2+b^2}\,b}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b - B*log(abs(tan(1/2*d*x + 1/2*c) -
1))/b + 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(
1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*(B*a - A*b)/(
sqrt(-a^2 + b^2)*b))/d
```

Mupad [B]
time = 3.09, size = 573, normalized size = 7.54

$$\frac{A\,a^2\,\ln\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\,{\left(a^2-b^2\right)}^{3/2}}-\frac{A\,\ln\left(\frac{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sqrt{\left(a+b\right)\,\left(a-b\right)}}{d\,\left(a^2-b^2\right)}-\frac{2\,B\,b\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\,\left(a^2-b^2\right)}-\frac{A\,b^2\,\ln\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\,{\left(a^2-b^2\right)}^{3/2}}+\frac{2\,B\,a^2\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d\,\left(a^2-b^2\right)}-\frac{B\,a^3\,\ln\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{b\,d\,{\left(a^2-b^2\right)}^{3/2}}+\frac{B\,a\,b\,\ln\left(\frac{a\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)-b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\,{\left(a^2-b^2\right)}^{3/2}}+\frac{B\,a\,\ln\left(\frac{a\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)+b\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)-\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)\,\sqrt{\left(a+b\right)\,\left(a-b\right)}}{b\,d\,\left(a^2-b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + b/cos(c + d*x))),x)
```

```
[Out] (A*a^2*log((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2
)*(a^2 - b^2)^(1/2))/cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)^(3/2)) - (A*log((a
*cos(c/2 + (d*x)/2) + b*cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2)*(a^2 - b^2)
^(1/2))/cos(c/2 + (d*x)/2))*((a + b)*(a - b))^(1/2))/(d*(a^2 - b^2)) - (2*B
*b*atanh(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)) - (A*b^2*l
og((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)*(a^2 -
```

```
 b^2)^(1/2))/cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)^(3/2)) + (2*B*a^2*atanh(si
n(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(b*d*(a^2 - b^2)) - (B*a^3*log((a*sin
(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/
2))/cos(c/2 + (d*x)/2)))/(b*d*(a^2 - b^2)^(3/2)) + (B*a*b*log((a*sin(c/2 +
(d*x)/2) - b*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/cos
(c/2 + (d*x)/2)))/(d*(a^2 - b^2)^(3/2)) + (B*a*log((a*cos(c/2 + (d*x)/2) +
b*cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/cos(c/2 + (d*x
)/2))*((a + b)*(a - b))^(1/2))/(b*d*(a^2 - b^2))
```

3.315 $\int \frac{A+B\sec(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=67

$$\frac{Ax}{a}-\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,\sqrt{a+b}\,d}$$

```
[Out] A*x/a-2*(A*b-B*a)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a/d/(
a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]
time = 0.07, antiderivative size = 67, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.174$, Rules used = {4004, 3916, 2738, 214}

$$\frac{Ax}{a}-\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{ad\sqrt{a-b}\,\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x]),x]
```

```
[Out] (A*x)/a - (2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]
])/(a*Sqrt[a - b]*Sqrt[a + b]*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sec(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{Ax}{a}-\frac{(Ab-aB)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a}\\
&= \frac{Ax}{a}-\frac{(Ab-aB)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{ab}\\
&= \frac{Ax}{a}-\frac{(2(Ab-aB))\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{abd}\\
&= \frac{Ax}{a}-\frac{2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a\sqrt{a-b}\,\sqrt{a+b}\,d}
\end{aligned}$$

Mathematica [A]
time = 0.14, size = 68, normalized size = 1.01

$$\frac{A(c+dx)+\frac{2(Ab-aB)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}}{ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x]),x]
```

```
[Out] (A*(c + d*x) + (2*(A*b - a*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]])/Sqrt[a^2 - b^2])/(a*d)
```

Maple [A]
time = 0.31, size = 73, normalized size = 1.09

method	result
derivativedivides	$\frac{\frac{2A\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2(Ab-Ba)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}}{d}$

default	$\frac{\frac{2A\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}-\frac{2(Ab-Ba)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}}{d}$
risch	$\frac{Ax}{a}+\frac{\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)Ab}{\sqrt{a^2-b^2}\,da}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)B}{\sqrt{a^2-b^2}\,d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-}{}\right.}{\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*A/a*arctan(tan(1/2*d*x+1/2*c))-2*(A*b-B*a)/a/((a+b)*(a-b))^(1/2)*arc
tanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.59, size = 250, normalized size = 3.73

$$\left[\frac{2\,(Aa^2-Ab^2)dx-(Ba-Ab)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)}{2\,(a^3-ab^2)d},\ \frac{(Aa^2-Ab^2)dx+(Ba-Ab)\sqrt{-a^2+b^2}\,\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)}{(a^3-ab^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*(A*a^2 - A*b^2)*d*x - (B*a - A*b)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*
x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) +
 a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) +
b^2)))/((a^3 - a*b^2)*d), ((A*a^2 - A*b^2)*d*x + (B*a - A*b)*sqrt(-a^2 + b^
2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))
))/((a^3 - a*b^2)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{A+B\sec(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 274 vs. 2(58) = 116.
time = 0.49, size = 274, normalized size = 4.09

$$\frac{\frac{\left(\sqrt{-a^2+b^2}A(a-2b)|-a+b|+\sqrt{-a^2+b^2}Ba|-a+b|-\sqrt{-a^2+b^2}A|a||-a+b|+\sqrt{-a^2+b^2}B|a||-a+b|\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-\frac{b+\sqrt{(a+b)(a-b)+b^2}}{a-b}}}\right)\right)}{(a^2-2ab+b^2)a^2+(a^2b-2ab^2+b^3)|a|}+\frac{(Aa+Ba-2Ab+A|a|-B|a|)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-\frac{b-\sqrt{(a+b)(a-b)+b^2}}{a-b}}}\right)\right)}{a^2-b|a|}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((sqrt(-a^2 + b^2)*A*(a - 2*b)*abs(-a + b) + sqrt(-a^2 + b^2)*B*a*abs(-a +
b) - sqrt(-a^2 + b^2)*A*abs(a)*abs(-a + b) + sqrt(-a^2 + b^2)*B*abs(a)*abs(
-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sq
rt(-(b + sqrt((a + b)*(a - b) + b^2))/(a - b))))/((a^2 - 2*a*b + b^2)*a^2 +
 (a^2*b - 2*a*b^2 + b^3)*abs(a)) + (A*a + B*a - 2*A*b + A*abs(a) - B*abs(a)
)*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(b
- sqrt((a + b)*(a - b) + b^2))/(a - b))))/(a^2 - b*abs(a)))/d
```

Mupad [B]
time = 3.20, size = 573, normalized size = 8.55

$$\frac{2Aa\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d\,(a^2-b^2)}-\frac{B\ln\left(\frac{a\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sqrt{(a+b)\,(a-b)}}{d\,(a^2-b^2)}+\frac{Ba^2\ln\left(\frac{a\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d\,(a^2-b^2)^{3/2}}-\frac{Bb^2\ln\left(\frac{a\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d\,(a^2-b^2)^{3/2}}-\frac{2Ab^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{dx}{2}\right)}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{a\,d\,(a^2-b^2)}+\frac{Ab^3\ln\left(\frac{a\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{a\,d\,(a^2-b^2)^{3/2}}-\frac{A\,a\,b\ln\left(\frac{a\sin\left(\frac{c}{2}+\frac{dx}{2}\right)-b\sin\left(\frac{c}{2}+\frac{dx}{2}\right)+\cos\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)}{d\,(a^2-b^2)^{3/2}}+\frac{A\,b\ln\left(\frac{a\cos\left(\frac{c}{2}+\frac{dx}{2}\right)+b\cos\left(\frac{c}{2}+\frac{dx}{2}\right)-\sin\left(\frac{c}{2}+\frac{dx}{2}\right)\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{dx}{2}\right)}\right)\sqrt{(a+b)\,(a-b)}}{a\,d\,(a^2-b^2)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x)),x)
```

```
[Out] (2*A*a*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)) - (B*lo
g((a*cos(c/2 + (d*x)/2) + b*cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2)*(a^2 -
b^2)^(1/2))/cos(c/2 + (d*x)/2))*((a + b)*(a - b))^(1/2))/(d*(a^2 - b^2)) +
(B*a^2*log((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2
)*(a^2 - b^2)^(1/2))/cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)^(3/2)) - (B*b^2*lo
g((a*sin(c/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)*(a^2 -
b^2)^(1/2))/cos(c/2 + (d*x)/2)))/(d*(a^2 - b^2)^(3/2)) - (2*A*b^2*atan(sin(
c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a*d*(a^2 - b^2)) + (A*b^3*log((a*sin(c
/2 + (d*x)/2) - b*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)
)/cos(c/2 + (d*x)/2)))/(a*d*(a^2 - b^2)^(3/2)) - (A*a*b*log((a*sin(c/2 + (d
*x)/2) - b*sin(c/2 + (d*x)/2) + cos(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/cos(c
/2 + (d*x)/2)))/(d*(a^2 - b^2)^(3/2)) + (A*b*log((a*cos(c/2 + (d*x)/2) + b*
cos(c/2 + (d*x)/2) - sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/cos(c/2 + (d*x)/
2))*((a + b)*(a - b))^(1/2))/(a*d*(a^2 - b^2))
```

3.316 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=90

$$-\frac{(Ab-aB)x}{a^2}+\frac{2b(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}\,d}+\frac{A\sin(c+dx)}{ad}$$

```
[Out] -(A*b-B*a)*x/a^2+A*sin(d*x+c)/a/d+2*b*(A*b-B*a)*arctanh((a-b)^(1/2)*tan(1/2
*d*x+1/2*c)/(a+b)^(1/2))/a^2/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]

time = 0.10, antiderivative size = 90, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.172, Rules used = {4119, 12, 3868, 2738, 214}

$$\frac{2b(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d\sqrt{a-b}\sqrt{a+b}}-\frac{x(Ab-aB)}{a^2}+\frac{A\sin(c+dx)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] -(((A*b - a*B)*x)/a^2) + (2*b*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x
)/2])/Sqrt[a + b]])/(a^2*Sqrt[a - b]*Sqrt[a + b]*d) + (A*Sin[c + d*x])/(a*d
)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3868

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_))^(-1), x_Symbol] :> Simp[x/a, x]
- Dist[1/a, Int[1/(1 + (a/b)*Sin[c + d*x]), x], x] /; FreeQ[{a, b, c, d}, x
] && NeQ[a^2 - b^2, 0]
```

Rule 4119

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*n)), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx &= \frac{A\sin(c+dx)}{ad}-\frac{\int \frac{Ab-aB}{a+b\sec(c+dx)}\,dx}{a}\\
&= \frac{A\sin(c+dx)}{ad}-\frac{(Ab-aB)\int \frac{1}{a+b\sec(c+dx)}\,dx}{a}\\
&= -\frac{(Ab-aB)x}{a^2}+\frac{A\sin(c+dx)}{ad}+\frac{(Ab-aB)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^2}\\
&= -\frac{(Ab-aB)x}{a^2}+\frac{A\sin(c+dx)}{ad}+\frac{(2(Ab-aB))\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)\right.}\right.}{a^2d}\\
&= -\frac{(Ab-aB)x}{a^2}+\frac{2b(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2\sqrt{a-b}\sqrt{a+b}\,d}+\frac{A}{}
\end{aligned}
$$

Mathematica [A]
time = 0.23, size = 85, normalized size = 0.94

$$
\frac{(-Ab+aB)(c+dx)-\frac{2b(Ab-aB)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+aA\sin(c+dx)}{a^2d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
[Out] ((-(A*b) + a*B)*(c + d*x) - (2*b*(A*b - a*B)*ArcTanh[((-a + b)*Tan[(c + d*x
)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + a*A*Sin[c + d*x])/(a^2*d)
```

Maple [A]
time = 0.41, size = 111, normalized size = 1.23

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{Aa\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+(Ab-Ba)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^2}+\frac{2b(Ab-Ba)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^2\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{-\frac{2\left(-\frac{Aa\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+(Ab-Ba)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^2}+\frac{2b(Ab-Ba)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^2\sqrt{(a+b)(a-b)}}}{d}$
risch	$-\frac{xAb}{a^2}+\frac{xB}{a}-\frac{iA\,\mathrm{e}^{i(dx+c)}}{2ad}+\frac{iA\,\mathrm{e}^{-i(dx+c)}}{2ad}+\frac{b^2\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)A}{\sqrt{a^2-b^2}\,d\,a^2}-\frac{b\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-}{}\right.}{\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/d*(-2/a^2*(-A*a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+(A*b-B*a)*arc tan(tan(1/2*d*x+1/2*c)))+2*b*(A*b-B*a)/a^2/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima ")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a dditional constraints; using the 'assume' command before evaluation *may* h elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more de

Fricas [A]
time = 2.98, size = 328, normalized size = 3.64

$$\left[\frac{2\,(Ba^3-Aa^2b-Bab^2+Ab^3)dx-(Bab-Ab^2)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+2\,(Aa^3-Aab^2)\sin(dx+c)}{2\,(a^4-a^2b^2)d},\ \frac{(Ba^3-Aa^2b-Bab^2+Ab^3)dx-(Bab-Ab^2)\sqrt{-a^2+b^2}\,\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)+(Aa^3-Aab^2)\sin(dx+c)}{(a^4-a^2b^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas ")

```
[Out] [1/2*(2*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*d*x - (B*a*b - A*b^2)*sqrt(a^2
- b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2
- b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2
 + 2*a*b*cos(d*x + c) + b^2)) + 2*(A*a^3 - A*a*b^2)*sin(d*x + c))/((a^4 - a
^2*b^2)*d), ((B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*d*x - (B*a*b - A*b^2)*sqrt
(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin
(d*x + c))) + (A*a^3 - A*a*b^2)*sin(d*x + c))/((a^4 - a^2*b^2)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\cos(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)/(a + b*sec(c + d*x)), x)
```

Giac [A]
time = 0.51, size = 141, normalized size = 1.57

$$\frac{\frac{(Ba-Ab)(dx+c)}{a^2}+\frac{2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a}-\frac{2\,(Bab-Ab^2)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] ((B*a - A*b)*(d*x + c)/a^2 + 2*A*tan(1/2*d*x + 1/2*c)/((tan(1/2*d*x + 1/2*c
)^2 + 1)*a) - 2*(B*a*b - A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a
+ 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^
2 + b^2)))/(sqrt(-a^2 + b^2)*a^2))/d
```

Mupad [B]
time = 3.36, size = 740, normalized size = 8.22

$$\frac{2\,A\,b^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\,\left(a^4-a^2\,b^2\right)}+\frac{2\,B\,a^3\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\,\left(a^4-a^2\,b^2\right)}+\frac{A\,a^3\,\sin\left(c+d\,x\right)}{d\,\left(a^4-a^2\,b^2\right)}-\frac{A\,a\,b^2\,\sin\left(c+d\,x\right)}{d\,\left(a^4-a^2\,b^2\right)}+\frac{A\,b^2\,\mathrm{atan}\left(\frac{-a^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}\,1{}\mathrm{i}+b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(a^2-b^2\right)}^{3/2}\,2{}\mathrm{i}+b^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}\,2{}\mathrm{i}-a^2\,b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}\,3{}\mathrm{i}+a^3\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}\,1{}\mathrm{i}+a^4\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^6-2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4\,b^2+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^4}\right)\,\sqrt{a^2-b^2}\,2{}\mathrm{i}}{d\,\left(a^4-a^2\,b^2\right)}-\frac{2\,A\,a^2\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\,\left(a^4-a^2\,b^2\right)}-\frac{2\,B\,a\,b^2\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{d\,\left(a^4-a^2\,b^2\right)}-\frac{B\,a\,b\,\mathrm{atan}\left(\frac{-a^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}\,1{}\mathrm{i}+b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,{\left(a^2-b^2\right)}^{3/2}\,2{}\mathrm{i}+b^5\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}\,2{}\mathrm{i}-a^2\,b^3\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}\,3{}\mathrm{i}+a^3\,b^2\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}\,1{}\mathrm{i}+a^4\,b\,\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}\,1{}\mathrm{i}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^6-2\,\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^4\,b^2+\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,a^2\,b^4}\right)\,\sqrt{a^2-b^2}\,2{}\mathrm{i}}{d\,\left(a^4-a^2\,b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x)),x)
```

```
[Out] (2*A*b^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*(a^4 - a^2*b^2)) +
 (2*B*a^3*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*(a^4 - a^2*b^2))
+ (A*a^3*sin(c + d*x))/(d*(a^4 - a^2*b^2)) - (A*a*b^2*sin(c + d*x))/(d*(a^4
 - a^2*b^2)) + (A*b^2*atan((b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2)*2i - a
```

```
^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*1i + b^5*sin(c/2 + (d*x)/2)*(a^2 -
b^2)^(1/2)*2i - a^2*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*3i + a^3*b^2*s
in(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*1i + a^4*b*sin(c/2 + (d*x)/2)*(a^2 - b^
2)^(1/2)*1i)/(a^6*cos(c/2 + (d*x)/2) + a^2*b^4*cos(c/2 + (d*x)/2) - 2*a^4*b
^2*cos(c/2 + (d*x)/2)))*(a^2 - b^2)^(1/2)*2i)/(d*(a^4 - a^2*b^2)) - (2*A*a^
2*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*(a^4 - a^2*b^2)) - (2*B
*a*b^2*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(d*(a^4 - a^2*b^2)) - (
B*a*b*atan((b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(3/2)*2i - a^5*sin(c/2 + (d*
x)/2)*(a^2 - b^2)^(1/2)*1i + b^5*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*2i -
a^2*b^3*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*3i + a^3*b^2*sin(c/2 + (d*x)/2
)*(a^2 - b^2)^(1/2)*1i + a^4*b*sin(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2)*1i)/(a^
6*cos(c/2 + (d*x)/2) + a^2*b^4*cos(c/2 + (d*x)/2) - 2*a^4*b^2*cos(c/2 + (d*
x)/2)))*(a^2 - b^2)^(1/2)*2i)/(d*(a^4 - a^2*b^2))
```

3.317 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=134

$$\frac{(a^2A+2Ab^2-2abB)\,x}{2a^3}-\frac{2b^2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3\sqrt{a-b}\,\sqrt{a+b}\,d}-\frac{(Ab-aB)\sin(c+dx)}{a^2d}+\frac{A\cos(c+$$

[Out] 1/2*(A*a^2+2*A*b^2-2*B*a*b)*x/a^3-(A*b-B*a)*sin(d*x+c)/a^2/d+1/2*A*cos(d*x+c)*sin(d*x+c)/a/d-2*b^2*(A*b-B*a)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^3/d/(a-b)^(1/2)/(a+b)^(1/2)

Rubi [A]
time = 0.26, antiderivative size = 134, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4119, 4189, 4004, 3916, 2738, 214}

$$-\frac{2b^2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d\sqrt{a-b}\,\sqrt{a+b}}-\frac{(Ab-aB)\sin(c+dx)}{a^2d}+\frac{x(a^2A-2abB+2Ab^2)}{2a^3}+\frac{A\sin(c+dx)\cos(c+dx)}{2ad}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]

[Out] ((a^2*A + 2*A*b^2 - 2*a*b*B)*x)/(2*a^3) - (2*b^2*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*Sqrt[a - b]*Sqrt[a + b]*d) - ((A*b - a*B)*Sin[c + d*x])/(a^2*d) + (A*Cos[c + d*x]*Sin[c + d*x])/(2*a*d)

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4119

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*n)), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \frac{A\cos(c+dx)\sin(c+dx)}{2ad} - \frac{\int \frac{\cos(c+dx)(2(Ab-aB)-aA\sec(c+dx)-Ab\sec^2}{a+b\sec(c+dx)}}{2a}$$

$$= -\frac{(Ab-aB)\sin(c+dx)}{a^2d} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad} + \frac{\int \frac{a^2A+2A}{}}{}$$

$$= \frac{(a^2A+2Ab^2-2abB)\,x}{2a^3} - \frac{(Ab-aB)\sin(c+dx)}{a^2d} + \frac{A\cos(c+dx}{2a}$$

$$= \frac{(a^2A+2Ab^2-2abB)\,x}{2a^3} - \frac{(Ab-aB)\sin(c+dx)}{a^2d} + \frac{A\cos(c+dx}{2a}$$

$$= \frac{(a^2A+2Ab^2-2abB)\,x}{2a^3} - \frac{(Ab-aB)\sin(c+dx)}{a^2d} + \frac{A\cos(c+dx}{2a}$$

$$= \frac{(a^2A+2Ab^2-2abB)\,x}{2a^3} - \frac{2b^2(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}(c}{\sqrt{a+b}}\right.}{a^3\sqrt{a-b}\sqrt{a+b}\,d}$$

Mathematica [A]
time = 0.36, size = 121, normalized size = 0.90

$$\frac{2(a^2A+2Ab^2-2abB)(c+dx)+\frac{8b^2(Ab-aB)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+4a(-Ab+aB)\sin(c+dx)+a^2A\sin(2(c+dx))}{4a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2*A + 2*A*b^2 - 2*a*b*B)*(c + d*x) + (8*b^2*(A*b - a*B)*ArcTanh[((-a
+ b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 4*a*(-(A*b) + a*
B)*Sin[c + d*x] + a^2*A*Sin[2*(c + d*x)])/(4*a^3*d)
```

Maple [A]
time = 0.42, size = 168, normalized size = 1.25

method	result
derivativedivides	$\frac{\frac{\frac{2\left(\left(-\frac{1}{2}a^2A-Aba+a^2B\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^2A-Aba+a^2B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\left(a^2A+2Ab^2-2Bab\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{2b^2(A}{}}{d}$

default	$\frac{\frac{\frac{2\left(\left(-\frac{1}{2}a^2A-Aba+a^2B\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^2A-Aba+a^2B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\left(a^2A+2A\,b^2-2Bab\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^3}-\frac{2b^2(Ab-}{}}{d}$
risch	$\frac{Ax}{2a}+\frac{xA\,b^2}{a^3}-\frac{bBx}{a^2}+\frac{i\mathrm{e}^{i(dx+c)}Ab}{2a^2d}-\frac{i\mathrm{e}^{i(dx+c)}B}{2ad}-\frac{i\mathrm{e}^{-i(dx+c)}Ab}{2a^2d}+\frac{i\mathrm{e}^{-i(dx+c)}B}{2ad}+\frac{b^3\ln\left(\mathrm{e}^{i(dx+c)}-\frac{ia^2-ib^2-b}{a\sqrt{a}}\right.}{\sqrt{a^2-b^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^3*(((-1/2*a^2*A-A*b*a+a^2*B)*tan(1/2*d*x+1/2*c)^3+(1/2*a^2*A-A*b*a
+a^2*B)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(A*a^2+2*A*b^2-2
*B*a*b)*arctan(tan(1/2*d*x+1/2*c)))-2*b^2*(A*b-B*a)/a^3/((a+b)*(a-b))^(1/2)
*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.71, size = 427, normalized size = 3.19

$$\left[\frac{(Aa^4-2\,Ba^3b+Aa^2b^2+2\,Bab^3-2\,Ab^4)dx-(Bab^2-Ab^3)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2-2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+(2\,Ba^4-2\,Aa^3b-2\,Ba^2b^2+2\,Aab^3+(Aa^4-Aa^2b^2)\cos(dx+c))\sin(dx+c)}{2\,(a^5-a^3b^2)d},\ \frac{(Aa^4-2\,Ba^3b+Aa^2b^2+2\,Bab^3-2\,Ab^4)dx+2\,(Bab^2-Ab^3)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)+(2\,Ba^4-2\,Aa^3b-2\,Ba^2b^2+2\,Aab^3+(Aa^4-Aa^2b^2)\cos(dx+c))\sin(dx+c)}{2\,(a^5-a^3b^2)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/2*((A*a^4 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4)*d*x - (B*a*b^2
- A*b^3)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x +
c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(
a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (2*B*a^4 - 2*A*a^3*b - 2*
B*a^2*b^2 + 2*A*a*b^3 + (A*a^4 - A*a^2*b^2)*cos(d*x + c))*sin(d*x + c))/((a
^5 - a^3*b^2)*d), 1/2*((A*a^4 - 2*B*a^3*b + A*a^2*b^2 + 2*B*a*b^3 - 2*A*b^4
```

```
)*d*x + 2*(B*a*b^2 - A*b^3)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*co
s(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (2*B*a^4 - 2*A*a^3*b - 2*B*a^
2*b^2 + 2*A*a*b^3 + (A*a^4 - A*a^2*b^2)*cos(d*x + c))*sin(d*x + c))/((a^5 -
 a^3*b^2)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\cos^2(c + dx)}{a + b\sec(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/(a + b*sec(c + d*x)), x)
```

Giac [A]
time = 0.49, size = 227, normalized size = 1.69

$$\frac{\frac{(Aa^2-2\,Bab+2\,Ab^2)(dx+c)}{a^3}+\frac{4\,(Bab^2-Ab^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^3}-\frac{2\left(Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^2}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/2*((A*a^2 - 2*B*a*b + 2*A*b^2)*(d*x + c)/a^3 + 4*(B*a*b^2 - A*b^3)*(pi*fl
oor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*
c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 + b^2)*a^3) - 2*
(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c)^3 + 2*A*b*tan(1/2*
d*x + 1/2*c)^3 - A*a*tan(1/2*d*x + 1/2*c) - 2*B*a*tan(1/2*d*x + 1/2*c) + 2*
A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^2))/d
```

Mupad [B]
time = 6.02, size = 2500, normalized size = 18.66

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x)),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(A*a - 2*A*b + 2*B*a))/a^2 - (tan(c/2 + (d*x)/2)^3*(A*
a + 2*A*b - 2*B*a))/a^2)/(d*(2*tan(c/2 + (d*x)/2)^2 + tan(c/2 + (d*x)/2)^4
+ 1)) - (atan((((((((8*(2*A*a^10 + 4*A*a^6*b^4 - 6*A*a^7*b^3 + 2*A*a^8*b^2 -
 4*B*a^7*b^3 + 8*B*a^8*b^2 - 2*A*a^9*b - 4*B*a^9*b))/a^6 - (4*tan(c/2 + (d*
x)/2)*(A*a^2*1i + A*b^2*2i - B*a*b*2i)*(8*a^8*b + 8*a^6*b^3 - 16*a^7*b^2))/
```

a^7)*(A*a^2*1i + A*b^2*2i - B*a*b*2i))/(2*a^3) + (8*tan(c/2 + (d*x)/2)*(A^2*a^7 - 8*A^2*b^7 + 16*A^2*a*b^6 - 3*A^2*a^6*b - 16*A^2*a^2*b^5 + 16*A^2*a^3*b^4 - 13*A^2*a^4*b^3 + 7*A^2*a^5*b^2 - 8*B^2*a^2*b^5 + 16*B^2*a^3*b^4 - 12*B^2*a^4*b^3 + 4*B^2*a^5*b^2 + 16*A*B*a*b^6 - 4*A*B*a^6*b - 32*A*B*a^2*b^5 + 28*A*B*a^3*b^4 - 20*A*B*a^4*b^3 + 12*A*B*a^5*b^2))/a^4)*(A*a^2*1i + A*b^2*2i - B*a*b*2i)*1i)/(2*a^3) - (((((8*(2*A*a^10 + 4*A*a^6*b^4 - 6*A*a^7*b^3 + 2*A*a^8*b^2 - 4*B*a^7*b^3 + 8*B*a^8*b^2 - 2*A*a^9*b - 4*B*a^9*b))/a^6 + (4*tan(c/2 + (d*x)/2)*(A*a^2*1i + A*b^2*2i - B*a*b*2i)*(8*a^8*b + 8*a^6*b^3 - 16*a^7*b^2))/a^7)*(A*a^2*1i + A*b^2*2i - B*a*b*2i))/(2*a^3) - (8*tan(c/2 + (d*x)/2)*(A^2*a^7 - 8*A^2*b^7 + 16*A^2*a*b^6 - 3*A^2*a^6*b - 16*A^2*a^2*b^5 + 16*A^2*a^3*b^4 - 13*A^2*a^4*b^3 + 7*A^2*a^5*b^2 - 8*B^2*a^2*b^5 + 16*B^2*a^3*b^4 - 12*B^2*a^4*b^3 + 4*B^2*a^5*b^2 + 16*A*B*a*b^6 - 4*A*B*a^6*b - 32*A*B*a^2*b^5 + 28*A*B*a^3*b^4 - 20*A*B*a^4*b^3 + 12*A*B*a^5*b^2))/a^4)*(A*a^2*1i + A*b^2*2i - B*a*b*2i)*1i)/(2*a^3))/((16*(4*A^3*b^8 - 6*A^3*a*b^7 + 6*A^3*a^2*b^6 - 5*A^3*a^3*b^5 + 2*A^3*a^4*b^4 - A^3*a^5*b^3 - 4*B^3*a^3*b^5 + 4*B^3*a^4*b^4 - 12*A^2*B*a*b^7 + 12*A*B^2*a^2*b^6 - 14*A*B^2*a^3*b^5 + 6*A*B^2*a^4*b^4 - 4*A*B^2*a^5*b^3 + 16*A^2*B*a^2*b^6 - 12*A^2*B*a^3*b^5 + 9*A^2*B*a^4*b^4 - 2*A^2*B*a^5*b^3 + A^2*B*a^6*b^2))/a^6 + (((((8*(2*A*a^10 + 4*A*a^6*b^4 - 6*A*a^7*b^3 + 2*A*a^8*b^2 - 4*B*a^7*b^3 + 8*B*a^8*b^2 - 2*A*a^9*b - 4*B*a^9*b))/a^6 - (4*tan(c/2 + (d*x)/2)*(A*a^2*1i + A*b^2*2i - B*a*b*2i)*(8*a^8*b + 8*a^6*b^3 - 16*a^7*b^2))/a^7)*(A*a^2*1i + A*b^2*2i - B*a*b*2i))/(2*a^3) + (8*tan(c/2 + (d*x)/2)*(A^2*a^7 - 8*A^2*b^7 + 16*A^2*a*b^6 - 3*A^2*a^6*b - 16*A^2*a^2*b^5 + 16*A^2*a^3*b^4 - 13*A^2*a^4*b^3 + 7*A^2*a^5*b^2 - 8*B^2*a^2*b^5 + 16*B^2*a^3*b^4 - 12*B^2*a^4*b^3 + 4*B^2*a^5*b^2 + 16*A*B*a*b^6 - 4*A*B*a^6*b - 32*A*B*a^2*b^5 + 28*A*B*a^3*b^4 - 20*A*B*a^4*b^3 + 12*A*B*a^5*b^2))/a^4)*(A*a^2*1i + A*b^2*2i - B*a*b*2i))/(2*a^3) + (((((8*(2*A*a^10 + 4*A*a^6*b^4 - 6*A*a^7*b^3 + 2*A*a^8*b^2 - 4*B*a^7*b^3 + 8*B*a^8*b^2 - 2*A*a^9*b - 4*B*a^9*b))/a^6 + (4*tan(c/2 + (d*x)/2)*(A*a^2*1i + A*b^2*2i - B*a*b*2i)*(8*a^8*b + 8*a^6*b^3 - 16*a^7*b^2))/a^7)*(A*a^2*1i + A*b^2*2i - B*a*b*2i))/(2*a^3) - (8*tan(c/2 + (d*x)/2)*(A^2*a^7 - 8*A^2*b^7 + 16*A^2*a*b^6 - 3*A^2*a^6*b - 16*A^2*a^2*b^5 + 16*A^2*a^3*b^4 - 13*A^2*a^4*b^3 + 7*A^2*a^5*b^2 - 8*B^2*a^2*b^5 + 16*B^2*a^3*b^4 - 12*B^2*a^4*b^3 + 4*B^2*a^5*b^2 + 16*A*B*a*b^6 - 4*A*B*a^6*b - 32*A*B*a^2*b^5 + 28*A*B*a^3*b^4 - 20*A*B*a^4*b^3 + 12*A*B*a^5*b^2))/a^4)*(A*a^2*1i + A*b^2*2i - B*a*b*2i))/(2*a^3)))*(A*a^2*1i + A*b^2*2i - B*a*b*2i)*1i)/(a^3*d) - (b^2*atan(((b^2*((a + b)*(a - b))^(1/2)*((8*tan(c/2 + (d*x)/2)*(A^2*a^7 - 8*A^2*b^7 + 16*A^2*a*b^6 - 3*A^2*a^6*b - 16*A^2*a^2*b^5 + 16*A^2*a^3*b^4 - 13*A^2*a^4*b^3 + 7*A^2*a^5*b^2 - 8*B^2*a^2*b^5 + 16*B^2*a^3*b^4 - 12*B^2*a^4*b^3 + 4*B^2*a^5*b^2 + 16*A*B*a*b^6 - 4*A*B*a^6*b - 32*A*B*a^2*b^5 + 28*A*B*a^3*b^4 - 20*A*B*a^4*b^3 + 12*A*B*a^5*b^2))/a^4 + (b^2*((a + b)*(a - b))^(1/2)*(A*b - B*a)*((8*(2*A*a^10 + 4*A*a^6*b^4 - 6*A*a^7*b^3 + 2*A*a^8*b^2 - 4*B*a^7*b^3 + 8*B*a^8*b^2 - 2*A*a^9*b - 4*B*a^9*b))/a^6 - (8*b^2*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^(1/2)*(A*b - B*a)*(8*a^8*b + 8*a^6*b^3 - 16*a^7*b^2))/(a^4*(a^5 - a^3*b^2))))/(a^5 - a^3*b^2))*(A*b - B*a)*1i)/(a^5 - a^3*b^2) + (b^2*((a + b)*(a - b))^(1/2)*((8*tan(c/2 + (d*x)/2)*(A^2*a^7 - 8*A^2*b^7 + 16*A^2*a*b^6 - 3

```
*A^2*a^6*b - 16*A^2*a^2*b^5 + 16*A^2*a^3*b^4 - 13*A^2*a^4*b^3 + 7*A^2*a^5*b
^2 - 8*B^2*a^2*b^5 + 16*B^2*a^3*b^4 - 12*B^2*a^4*b^3 + 4*B^2*a^5*b^2 + 16*A
*B*a*b^6 - 4*A*B*a^6*b - 32*A*B*a^2*b^5 + 28*A*B*a^3*b^4 - 20*A*B*a^4*b^3 +
 12*A*B*a^5*b^2))/a^4 - (b^2*((a + b)*(a - b))^(1/2)*(A*b - B*a)*((8*(2*A*a
^10 + 4*A*a^6*b^4 - 6*A*a^7*b^3 + 2*A*a^8*b^2 - 4*B*a^7*b^3 + 8*B*a^8*b^2 -
 2*A*a^9*b - 4*B*a^9*b))/a^6 + (8*b^2*tan(c/2 + (d*x)/2)*((a + b)*(a - b))^
(1/2)*(A*b - B*a)*(8*a^8*b + 8*a^6*b^3 - 16*a^7*b^2))/(a^4*(a^5 - a^3*b^2))
))/(a^5 - a^3*b^2))*(A*b - B*a)*1i)/(a^5 - a^3*b^2))/((16*(4*A^3*b^8 - 6*A^
3*a*b^7 + 6*A^3*a^2*b^6 - 5*A^3*a^3*b^5 + 2*A^3*a^4*b^4 - A^3*a^5*b^3 - 4*B
^3*a^3*b^5 + 4*B^3*a^4*b^4 - 12*A^2*B*a*b^7 + 12*A*B^2*a^2*b^6 - 14*A*B^2*a
^3*b^5 + 6*A*B^2*a^4*b^4 - 4*A*B^2*a^5*b^3 + 16*A^2*B*a^2*b^6 - 12*A^2*B*a^
3*b^5 + 9*A^2*B*a^4*b^4 - 2*A^2*B*a^5*b^3 + A^2*B*a^6*b^2))/a^6 + (b^2*((a
+ b)*(a - b))^(1/2)*((8*tan(c/2 + (d*x)/2)*(A^2*a^7 - 8*A^2*b^7 + 16*A^2*a*
b^6 - 3*A^2*a^6*b - 16*A^2*a^2*b^5 + 16*A^2*a^3*b^4 - 13*A^2*a^4*b^3 + 7*A^
2*a^5*b^2 - 8*B^2*a^2*b^5 + 16*B^2*a^3*b^4 - 12*B^2*a^4*b^3 + 4*B^2*a^5*b^2
 + 16*A*B*a*b^6 - 4*A*B*a^6*b - 32*A*B*a^2*b^5 + 28*A*B*a^3*b^4 - 20*A*B*a^
4*b^3 + 12*A*B*a^5*b^2))/a^4 + (b^2*((a + b)*(a...
```

3.318 $\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=178

$$-\frac{(a^2+2b^2)(Ab-aB)x}{2a^4}+\frac{2b^3(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4\sqrt{a-b}\sqrt{a+b}\,d}+\frac{(2a^2A+3Ab^2-3abB)\sin(c+dx)}{3a^3d}$$

```
[Out] -1/2*(a^2+2*b^2)*(A*b-B*a)*x/a^4+1/3*(2*A*a^2+3*A*b^2-3*B*a*b)*sin(d*x+c)/a
^3/d-1/2*(A*b-B*a)*cos(d*x+c)*sin(d*x+c)/a^2/d+1/3*A*cos(d*x+c)^2*sin(d*x+c
)/a/d+2*b^3*(A*b-B*a)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a
^4/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]

time = 0.42, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {4119, 4189, 4004, 3916, 2738, 214}

$$\frac{2b^3(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d\sqrt{a-b}\sqrt{a+b}}-\frac{(Ab-aB)\sin(c+dx)\cos(c+dx)}{2a^2d}-\frac{x(a^2+2b^2)(Ab-aB)}{2a^4}+\frac{(2a^2A-3abB+3Ab^2)\sin(c+dx)}{3a^3d}+\frac{A\sin(c+dx)\cos^2(c+dx)}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/2*((a^2 + 2*b^2)*(A*b - a*B)*x)/a^4 + (2*b^3*(A*b - a*B)*ArcTanh[(Sqrt[a
 - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*Sqrt[a - b]*Sqrt[a + b]*d) + ((2
*a^2*A + 3*A*b^2 - 3*a*b*B)*Sin[c + d*x])/(3*a^3*d) - ((A*b - a*B)*Cos[c +
d*x]*Sin[c + d*x])/(2*a^2*d) + (A*Cos[c + d*x]^2*Sin[c + d*x])/(3*a*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4119

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*n)), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \frac{A\cos^2(c+dx)\sin(c+dx)}{3ad} - \frac{\int \frac{\cos^2(c+dx)(3(Ab-aB)-2aA\sec(c+dx)-2Ab\,\mathrm{se}}{a+b\sec(c+dx)}}{3a}$$

$$= -\frac{(Ab-aB)\cos(c+dx)\sin(c+dx)}{2a^2d} + \frac{A\cos^2(c+dx)\sin(c+dx)}{3ad}$$

$$= \frac{(2a^2A+3Ab^2-3abB)\sin(c+dx)}{3a^3d} - \frac{(Ab-aB)\cos(c+dx)\sin(c}{2a^2d}$$

$$= -\frac{(a^2+2b^2)(Ab-aB)x}{2a^4} + \frac{(2a^2A+3Ab^2-3abB)\sin(c+dx)}{3a^3d} - ($$

$$= -\frac{(a^2+2b^2)(Ab-aB)x}{2a^4} + \frac{(2a^2A+3Ab^2-3abB)\sin(c+dx)}{3a^3d} - ($$

$$= -\frac{(a^2+2b^2)(Ab-aB)x}{2a^4} + \frac{(2a^2A+3Ab^2-3abB)\sin(c+dx)}{3a^3d} - ($$

$$= -\frac{(a^2+2b^2)(Ab-aB)x}{2a^4} + \frac{2b^3(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan(\frac{1}{2}(c+}{\sqrt{a+b}}\right.}{a^4\sqrt{a-b}\,\sqrt{a+b}\,d}$$

Mathematica [A]

time = 0.52, size = 152, normalized size = 0.85

$$\frac{6(a^2+2b^2)(-Ab+aB)(c+dx) - \frac{24b^3(Ab-aB)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + 3a(3a^2A+4Ab^2-4abB)\sin(c+dx) + 3a^2(-Ab+aB)\sin(2(c+dx)) + a^3A\sin(3(c+dx))}{12a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (6*(a^2 + 2*b^2)*(-(A*b) + a*B)*(c + d*x) - (24*b^3*(A*b - a*B)*ArcTanh[((-
a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + 3*a*(3*a^2*A +
 4*A*b^2 - 4*a*b*B)*Sin[c + d*x] + 3*a^2*(-(A*b) + a*B)*Sin[2*(c + d*x)] +
a^3*A*Sin[3*(c + d*x)])/(12*a^4*d)
```

Maple [A]

time = 0.47, size = 242, normalized size = 1.36

method	result
derivativedivides	$-\frac{2\left(\frac{\left(-A\,a^3-\frac{1}{2}A\,a^2b-A\,b^2a+\frac{1}{2}a^3B+Bb\,a^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{2}{3}A\,a^3-2A\,b^2a+2Bb\,a^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-A\,a^3-A\,b^2a+Bb\,a^2\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}\right.}{a^4}$

default	$-\frac{2\left(\frac{\left(-A\,a^3-\frac{1}{2}A\,a^2b-A\,b^2a+\frac{1}{2}a^3B+Bb\,a^2\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-\frac{2}{3}A\,a^3-2A\,b^2a+2Bb\,a^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(-A\,a^3-A\,b^2a+Bb\,a\right.}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^3}}{a^4}$
risch	$-\frac{xAb}{2a^2}-\frac{xA\,b^3}{a^4}+\frac{xB}{2a}+\frac{xB\,b^2}{a^3}-\frac{3iA\,e^{i(dx+c)}}{8ad}-\frac{ie^{i(dx+c)}A\,b^2}{2a^3d}-\frac{ie^{-i(dx+c)}Bb}{2a^2d}+\frac{3iA\,e^{-i(dx+c)}}{8ad}+\frac{ie^{-i(dx}}{2a}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a^4*(((-A*a^3-1/2*A*a^2*b-A*b^2*a+1/2*a^3*B+B*b*a^2)*tan(1/2*d*x+1/
2*c)^5+(-2/3*A*a^3-2*A*b^2*a+2*B*b*a^2)*tan(1/2*d*x+1/2*c)^3+(-A*a^3-A*b^2*
a+B*b*a^2+1/2*A*a^2*b-1/2*a^3*B)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^
2)^3+1/2*(A*a^2*b+2*A*b^3-B*a^3-2*B*a*b^2)*arctan(tan(1/2*d*x+1/2*c)))+2*b^
3*(A*b-B*a)/a^4/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)
*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.75, size = 547, normalized size = 3.07

$$\left[\frac{3\left(Ba^5-Aa^4b+Ba^3b^2-Aa^2b^3-2\,Bab^4+2\,Ab^5\right)dx-3\left(Bab^3-Ab^4\right)\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+\left(4\,Aa^5-6\,Ba^4b+2\,Aa^3b^2+6\,Ba^2b^3-6\,Aab^4+2\left(Aa^5-Aa^3b^2\right)\cos\left(dx+c\right)^2+3\left(Ba^5-Aa^4b-Ba^3b^2+Aa^2b^3\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\left(a^6-a^4b^2\right)d}, \frac{3\left(Ba^5-Aa^4b+Ba^3b^2-Aa^2b^3-2\,Bab^4+2\,Ab^5\right)dx-6\left(Bab^3-Ab^4\right)\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)+\left(4\,Aa^5-6\,Ba^4b+2\,Aa^3b^2+6\,Ba^2b^3-6\,Aab^4+2\left(Aa^5-Aa^3b^2\right)\cos\left(dx+c\right)^2+3\left(Ba^5-Aa^4b-Ba^3b^2+Aa^2b^3\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{6\left(a^6-a^4b^2\right)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/6*(3*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*a^2*b^3 - 2*B*a*b^4 + 2*A*b^5)*d*x
 - 3*(B*a*b^3 - A*b^4)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b
^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) +
2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (4*A*a^5 -
```

```
6*B*a^4*b + 2*A*a^3*b^2 + 6*B*a^2*b^3 - 6*A*a*b^4 + 2*(A*a^5 - A*a^3*b^2)*c
os(d*x + c)^2 + 3*(B*a^5 - A*a^4*b - B*a^3*b^2 + A*a^2*b^3)*cos(d*x + c))*s
in(d*x + c))/((a^6 - a^4*b^2)*d), 1/6*(3*(B*a^5 - A*a^4*b + B*a^3*b^2 - A*a
^2*b^3 - 2*B*a*b^4 + 2*A*b^5)*d*x - 6*(B*a*b^3 - A*b^4)*sqrt(-a^2 + b^2)*ar
ctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (
4*A*a^5 - 6*B*a^4*b + 2*A*a^3*b^2 + 6*B*a^2*b^3 - 6*A*a*b^4 + 2*(A*a^5 - A*
a^3*b^2)*cos(d*x + c)^2 + 3*(B*a^5 - A*a^4*b - B*a^3*b^2 + A*a^2*b^3)*cos(d
*x + c))*sin(d*x + c))/((a^6 - a^4*b^2)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\cos^3(c + dx)}{a + b\sec(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**3/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 360 vs. 2(161) = 322.
time = 0.46, size = 360, normalized size = 2.02

$$\frac{\frac{3\,(Ba^3 - Aa^2b + 2\,Bab^2 - 2\,Ab^3)(dx + c)}{a^4} - \frac{12\,(Bab^3 - Ab^4)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a + 2\,b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\sqrt{-a^2 + b^2}}\right)\right)}{\sqrt{-a^2 + b^2}\,a^4} + \frac{2\left(6\,Aa^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 - 3\,Ba^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 + 3\,Aab\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 - 6\,Bab\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 + 6\,Ab^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^5 + 4\,Aa^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - 12\,Bab\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 12\,Ab^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + 6\,Aa^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 3\,Ba^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 3\,Aab\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 6\,Bab\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 6\,Ab^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + 1\right)^3 a^3}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/6*(3*(B*a^3 - A*a^2*b + 2*B*a*b^2 - 2*A*b^3)*(d*x + c)/a^4 - 12*(B*a*b^3
- A*b^4)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan
(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/(sqrt(-a^2 +
 b^2)*a^4) + 2*(6*A*a^2*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^2*tan(1/2*d*x + 1/2*
c)^5 + 3*A*a*b*tan(1/2*d*x + 1/2*c)^5 - 6*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 6*
A*b^2*tan(1/2*d*x + 1/2*c)^5 + 4*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 12*B*a*b*ta
n(1/2*d*x + 1/2*c)^3 + 12*A*b^2*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*
x + 1/2*c) + 3*B*a^2*tan(1/2*d*x + 1/2*c) - 3*A*a*b*tan(1/2*d*x + 1/2*c) -
6*B*a*b*tan(1/2*d*x + 1/2*c) + 6*A*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x
+ 1/2*c)^2 + 1)^3*a^3))/d
```

Mupad [B]
time = 6.90, size = 2500, normalized size = 14.04

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x)),x)

[Out] ((tan(c/2 + (d*x)/2)*(2*A*a^2 + 2*A*b^2 + B*a^2 - A*a*b - 2*B*a*b))/a^3 + (
tan(c/2 + (d*x)/2)^5*(2*A*a^2 + 2*A*b^2 - B*a^2 + A*a*b - 2*B*a*b))/a^3 + (
4*tan(c/2 + (d*x)/2)^3*(A*a^2 + 3*A*b^2 - 3*B*a*b))/(3*a^3))/(d*(3*tan(c/2
+ (d*x)/2)^2 + 3*tan(c/2 + (d*x)/2)^4 + tan(c/2 + (d*x)/2)^6 + 1)) - (atan(
(((a^2 + 2*b^2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 - 1
6*A^2*a*b^8 + 3*B^2*a^8*b + 16*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*b^
5 - 7*A^2*a^5*b^4 + 3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*a^
3*b^6 + 16*B^2*a^4*b^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2 -
16*A*B*a*b^8 + 2*A*B*a^8*b + 32*A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4*b
^5 - 26*A*B*a^5*b^4 + 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2))/a^6 - ((a^2 + 2*b^2)
*(A*b - B*a)*((8*(2*B*a^13 - 4*A*a^8*b^5 + 6*A*a^9*b^4 - 2*A*a^10*b^3 + 2*A
*a^11*b^2 + 4*B*a^9*b^4 - 6*B*a^10*b^3 + 2*B*a^11*b^2 - 2*A*a^12*b - 2*B*a^
12*b))/a^9 - (tan(c/2 + (d*x)/2)*(a^2 + 2*b^2)*(A*b - B*a)*(8*a^10*b + 8*a^
8*b^3 - 16*a^9*b^2)*4i)/a^10)*1i)/(2*a^4)))/(2*a^4) + ((a^2 + 2*b^2)*(A*b -
 B*a)*((8*tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 - 16*A^2*a*b^8 + 3*B^2*a^
8*b + 16*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*b^5 - 7*A^2*a^5*b^4 + 3*
A^2*a^6*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*a^3*b^6 + 16*B^2*a^4*b^5
 - 16*B^2*a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2 - 16*A*B*a*b^8 + 2*A*B*a
^8*b + 32*A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4*b^5 - 26*A*B*a^5*b^4 +
14*A*B*a^6*b^3 - 6*A*B*a^7*b^2))/a^6 + ((a^2 + 2*b^2)*(A*b - B*a)*((8*(2*B*
a^13 - 4*A*a^8*b^5 + 6*A*a^9*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2 + 4*B*a^9*b^
4 - 6*B*a^10*b^3 + 2*B*a^11*b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^9 + (tan(c/2
+ (d*x)/2)*(a^2 + 2*b^2)*(A*b - B*a)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2)*4i
)/a^10)*1i)/(2*a^4)))/(2*a^4))/((16*(4*A^3*b^11 - 6*A^3*a*b^10 + 6*A^3*a^2*
b^9 - 5*A^3*a^3*b^8 + 2*A^3*a^4*b^7 - A^3*a^5*b^6 - 4*B^3*a^3*b^8 + 6*B^3*a
^4*b^7 - 6*B^3*a^5*b^6 + 5*B^3*a^6*b^5 - 2*B^3*a^7*b^4 + B^3*a^8*b^3 - 12*A
^2*B*a*b^10 + 12*A*B^2*a^2*b^9 - 18*A*B^2*a^3*b^8 + 18*A*B^2*a^4*b^7 - 15*A
*B^2*a^5*b^6 + 6*A*B^2*a^6*b^5 - 3*A*B^2*a^7*b^4 + 18*A^2*B*a^2*b^9 - 18*A^
2*B*a^3*b^8 + 15*A^2*B*a^4*b^7 - 6*A^2*B*a^5*b^6 + 3*A^2*B*a^6*b^5))/a^9 +
((a^2 + 2*b^2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 - 16
*A^2*a*b^8 + 3*B^2*a^8*b + 16*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*b^5
 - 7*A^2*a^5*b^4 + 3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*a^3
*b^6 + 16*B^2*a^4*b^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2 - 1
6*A*B*a*b^8 + 2*A*B*a^8*b + 32*A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4*b^
5 - 26*A*B*a^5*b^4 + 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2))/a^6 - ((a^2 + 2*b^2)*
(A*b - B*a)*((8*(2*B*a^13 - 4*A*a^8*b^5 + 6*A*a^9*b^4 - 2*A*a^10*b^3 + 2*A*
a^11*b^2 + 4*B*a^9*b^4 - 6*B*a^10*b^3 + 2*B*a^11*b^2 - 2*A*a^12*b - 2*B*a^1
2*b))/a^9 - (tan(c/2 + (d*x)/2)*(a^2 + 2*b^2)*(A*b - B*a)*(8*a^10*b + 8*a^8
*b^3 - 16*a^9*b^2)*4i)/a^10)*1i)/(2*a^4))*1i)/(2*a^4) - ((a^2 + 2*b^2)*(A*b
 - B*a)*((8*tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 - 16*A^2*a*b^8 + 3*B^2*
a^8*b + 16*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*b^5 - 7*A^2*a^5*b^4 +
3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*a^3*b^6 + 16*B^2*a^4*b
^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2 - 16*A*B*a*b^8 + 2*A*B
```

```
*a^8*b + 32*A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4*b^5 - 26*A*B*a^5*b^4
+ 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2))/a^6 + ((a^2 + 2*b^2)*(A*b - B*a)*((8*(2*
B*a^13 - 4*A*a^8*b^5 + 6*A*a^9*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2 + 4*B*a^9*
b^4 - 6*B*a^10*b^3 + 2*B*a^11*b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^9 + (tan(c/
2 + (d*x)/2)*(a^2 + 2*b^2)*(A*b - B*a)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2)*
4i)/a^10)*1i)/(2*a^4))*1i)/(2*a^4)))*(a^2 + 2*b^2)*(A*b - B*a))/(a^4*d) - (
b^3*atan(((b^3*((a + b)*(a - b))^(1/2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(
8*A^2*b^9 - B^2*a^9 - 16*A^2*a*b^8 + 3*B^2*a^8*b + 16*A^2*a^2*b^7 - 16*A^2*
a^3*b^6 + 13*A^2*a^4*b^5 - 7*A^2*a^5*b^4 + 3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*
B^2*a^2*b^7 - 16*B^2*a^3*b^6 + 16*B^2*a^4*b^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6
*b^3 - 7*B^2*a^7*b^2 - 16*A*B*a*b^8 + 2*A*B*a^8*b + 32*A*B*a^2*b^7 - 32*A*B
*a^3*b^6 + 32*A*B*a^4*b^5 - 26*A*B*a^5*b^4 + 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2
))/a^6 + (b^3*((a + b)*(a - b))^(1/2)*((8*(2*B*a^13 - 4*A*a^8*b^5 + 6*A*a^9
*b^4 - 2*A*a^10*b^3 + 2*A*a^11*b^2 + 4*B*a^9*b^4 - 6*B*a^10*b^3 + 2*B*a^11*
b^2 - 2*A*a^12*b - 2*B*a^12*b))/a^9 + (8*b^3*tan(c/2 + (d*x)/2)*((a + b)*(a
 - b))^(1/2)*(A*b - B*a)*(8*a^10*b + 8*a^8*b^3 - 16*a^9*b^2))/(a^6*(a^6 - a
^4*b^2)))*(A*b - B*a))/(a^6 - a^4*b^2))*1i)/(a^6 - a^4*b^2) + (b^3*((a + b)
*(a - b))^(1/2)*(A*b - B*a)*((8*tan(c/2 + (d*x)/2)*(8*A^2*b^9 - B^2*a^9 - 1
6*A^2*a*b^8 + 3*B^2*a^8*b + 16*A^2*a^2*b^7 - 16*A^2*a^3*b^6 + 13*A^2*a^4*b^
5 - 7*A^2*a^5*b^4 + 3*A^2*a^6*b^3 - A^2*a^7*b^2 + 8*B^2*a^2*b^7 - 16*B^2*a^
3*b^6 + 16*B^2*a^4*b^5 - 16*B^2*a^5*b^4 + 13*B^2*a^6*b^3 - 7*B^2*a^7*b^2 -
16*A*B*a*b^8 + 2*A*B*a^8*b + 32*A*B*a^2*b^7 - 32*A*B*a^3*b^6 + 32*A*B*a^4*b
^5 - 26*A*B*a^5*b^4 + 14*A*B*a^6*b^3 - 6*A*B*a^7*b^2))/a^6 - (b^3*((a + b)*
(a - b))^(1/2)*((8*(2*B*a^13 - 4*A*a^8*b^5 + 6*...
```

3.319 $\int \frac{\cos^4(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=240

$$\frac{(3a^4A+4a^2Ab^2+8Ab^4-4a^3bB-8ab^3B)\,x}{8a^5}-\frac{2b^4(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^5\sqrt{a-b}\,\sqrt{a+b}\,d}-\frac{(2a^2+3b^2)\,(\ldots}{}$$

```
[Out] 1/8*(3*A*a^4+4*A*a^2*b^2+8*A*b^4-4*B*a^3*b-8*B*a*b^3)*x/a^5-1/3*(2*a^2+3*b^
2)*(A*b-B*a)*sin(d*x+c)/a^4/d+1/8*(3*A*a^2+4*A*b^2-4*B*a*b)*cos(d*x+c)*sin(
d*x+c)/a^3/d-1/3*(A*b-B*a)*cos(d*x+c)^2*sin(d*x+c)/a^2/d+1/4*A*cos(d*x+c)^3
*sin(d*x+c)/a/d-2*b^4*(A*b-B*a)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b
)^(1/2))/a^5/d/(a-b)^(1/2)/(a+b)^(1/2)
```

Rubi [A]

time = 0.65, antiderivative size = 240, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4119, 4189, 4004, 3916, 2738, 214}

$$-\frac{2b^4(Ab-aB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^5d\sqrt{a-b}\,\sqrt{a+b}}-\frac{(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{3a^2d}-\frac{(2a^2+3b^2)\,(Ab-aB)\sin(c+dx)}{3a^4d}+\frac{(3a^2A-4abB+4Ab^2)\sin(c+dx)\cos(c+dx)}{8a^3d}+\frac{x(3a^4A-4a^3bB+4a^2Ab^2-8ab^3B+8Ab^4)}{8a^5}+\frac{A\sin(c+dx)\cos^3(c+dx)}{4ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((3*a^4*A + 4*a^2*A*b^2 + 8*A*b^4 - 4*a^3*b*B - 8*a*b^3*B)*x)/(8*a^5) - (2*
b^4*(A*b - a*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*S
qrt[a - b]*Sqrt[a + b]*d) - ((2*a^2 + 3*b^2)*(A*b - a*B)*Sin[c + d*x])/(3*a
^4*d) + ((3*a^2*A + 4*A*b^2 - 4*a*b*B)*Cos[c + d*x]*Sin[c + d*x])/(8*a^3*d)
 - ((A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x])/(3*a^2*d) + (A*Cos[c + d*x]^3*
Sin[c + d*x])/(4*a*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4119

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*n)), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^4(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \frac{A\cos^3(c+dx)\sin(c+dx)}{4ad} - \frac{\int \frac{\cos^3(c+dx)(4(Ab-aB)-3aA\sec(c+dx)-3Ab}{a+b\sec(c+dx)}}{4a}$$

$$= -\frac{(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{3a^2d} + \frac{A\cos^3(c+dx)\sin(c+d}{4ad}$$

$$= \frac{(3a^2A+4Ab^2-4abB)\cos(c+dx)\sin(c+dx)}{8a^3d} - \frac{(Ab-aB)\cos^2}{}$$

$$= -\frac{(2a^2+3b^2)(Ab-aB)\sin(c+dx)}{3a^4d} + \frac{(3a^2A+4Ab^2-4abB)\cos}{8a^3d}$$

$$= \frac{(3a^4A+4a^2Ab^2+8Ab^4-4a^3bB-8ab^3B)\,x}{8a^5} - \frac{(2a^2+3b^2)(Ab-}{3a}$$

$$= \frac{(3a^4A+4a^2Ab^2+8Ab^4-4a^3bB-8ab^3B)\,x}{8a^5} - \frac{(2a^2+3b^2)(Ab-}{3a}$$

$$= \frac{(3a^4A+4a^2Ab^2+8Ab^4-4a^3bB-8ab^3B)\,x}{8a^5} - \frac{(2a^2+3b^2)(Ab-}{3a}$$

$$= \frac{(3a^4A+4a^2Ab^2+8Ab^4-4a^3bB-8ab^3B)\,x}{8a^5} - \frac{2b^4(Ab-aB)\tan}{a^5\sqrt{}}$$

Mathematica [A]
time = 0.67, size = 202, normalized size = 0.84

$$\frac{12(3a^4A+4a^2Ab^2+8Ab^4-4a^3bB-8ab^3B)(c+dx)+\frac{192b^4(Ab-aB)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+24a(3a^2+4b^2)(-Ab+aB)\sin(c+dx)+24a^2(a^2A+Ab^2-abB)\sin(2(c+dx))+8a^3(-Ab+aB)\sin(3(c+dx))+3a^4A\sin(4(c+dx))}{96a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (12*(3*a^4*A + 4*a^2*A*b^2 + 8*A*b^4 - 4*a^3*b*B - 8*a*b^3*B)*(c + d*x) + (
192*b^4*(A*b - a*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/S
qrt[a^2 - b^2] + 24*a*(3*a^2 + 4*b^2)*(-(A*b) + a*B)*Sin[c + d*x] + 24*a^2*
(a^2*A + A*b^2 - a*b*B)*Sin[2*(c + d*x)] + 8*a^3*(-(A*b) + a*B)*Sin[3*(c +
d*x)] + 3*a^4*A*Sin[4*(c + d*x)])/(96*a^5*d)
```

Maple [A]
time = 0.48, size = 381, normalized size = 1.59

method	result

derivativedivides	$\frac{-\frac{2b^4(Ab-Ba)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^5\sqrt{(a+b)(a-b)}}+\frac{2\left(\left(-\frac{5}{8}A\,a^4-A\,a^3b-\frac{1}{2}A\,b^2a^2-Aa\,b^3+a^4B+\frac{1}{2}Bb\,a^3+B\,a^2b^2\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}}{}$
default	$\frac{-\frac{2b^4(Ab-Ba)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a^5\sqrt{(a+b)(a-b)}}+\frac{2\left(\left(-\frac{5}{8}A\,a^4-A\,a^3b-\frac{1}{2}A\,b^2a^2-Aa\,b^3+a^4B+\frac{1}{2}Bb\,a^3+B\,a^2b^2\right)\left(\tan^7\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}}{}$
risch	$\frac{3Ax}{8a}+\frac{xA\,b^2}{2a^3}+\frac{xA\,b^4}{a^5}-\frac{bBx}{2a^2}-\frac{xB\,b^3}{a^4}-\frac{i\mathrm{e}^{-i(dx+c)}A\,b^3}{2a^4d}-\frac{3i\mathrm{e}^{i(dx+c)}B}{8ad}+\frac{i\mathrm{e}^{i(dx+c)}A\,b^3}{2a^4d}-\frac{i\mathrm{e}^{i(dx+c)}B\,b^2}{2a^3d}+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b^4*(A*b-B*a)/a^5/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2
*c)/((a+b)*(a-b))^(1/2))+2/a^5*(((-5/8*A*a^4-A*a^3*b-1/2*A*b^2*a^2-A*a*b^3+
a^4*B+1/2*B*b*a^3+B*a^2*b^2)*tan(1/2*d*x+1/2*c)^7+(3/8*A*a^4-5/3*A*a^3*b-3*
A*a*b^3+5/3*a^4*B+3*B*a^2*b^2-1/2*A*b^2*a^2+1/2*B*b*a^3)*tan(1/2*d*x+1/2*c)
^5+(-3/8*A*a^4+1/2*A*b^2*a^2-1/2*B*b*a^3-5/3*A*a^3*b-3*A*a*b^3+5/3*a^4*B+3*
B*a^2*b^2)*tan(1/2*d*x+1/2*c)^3+(5/8*A*a^4+1/2*A*b^2*a^2-1/2*B*b*a^3-A*a^3*
b-A*a*b^3+a^4*B+B*a^2*b^2)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^4+1
/8*(3*A*a^4+4*A*a^2*b^2+8*A*b^4-4*B*a^3*b-8*B*a*b^3)*arctan(tan(1/2*d*x+1/2
*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 5.91, size = 685, normalized size = 2.85

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] [1/24*(3*(3*A*a^6 - 4*B*a^5*b + A*a^4*b^2 - 4*B*a^3*b^3 + 4*A*a^2*b^4 + 8*B
*a*b^5 - 8*A*b^6)*d*x - 12*(B*a*b^4 - A*b^5)*sqrt(a^2 - b^2)*log((2*a*b*cos
(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c
) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c)
 + b^2)) + (16*B*a^6 - 16*A*a^5*b + 8*B*a^4*b^2 - 8*A*a^3*b^3 - 24*B*a^2*b^
4 + 24*A*a*b^5 + 6*(A*a^6 - A*a^4*b^2)*cos(d*x + c)^3 + 8*(B*a^6 - A*a^5*b
- B*a^4*b^2 + A*a^3*b^3)*cos(d*x + c)^2 + 3*(3*A*a^6 - 4*B*a^5*b + A*a^4*b^
2 + 4*B*a^3*b^3 - 4*A*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^7 - a^5*b^2)
*d), 1/24*(3*(3*A*a^6 - 4*B*a^5*b + A*a^4*b^2 - 4*B*a^3*b^3 + 4*A*a^2*b^4 +
 8*B*a*b^5 - 8*A*b^6)*d*x + 24*(B*a*b^4 - A*b^5)*sqrt(-a^2 + b^2)*arctan(-s
qrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (16*B*a^
6 - 16*A*a^5*b + 8*B*a^4*b^2 - 8*A*a^3*b^3 - 24*B*a^2*b^4 + 24*A*a*b^5 + 6*
(A*a^6 - A*a^4*b^2)*cos(d*x + c)^3 + 8*(B*a^6 - A*a^5*b - B*a^4*b^2 + A*a^3
*b^3)*cos(d*x + c)^2 + 3*(3*A*a^6 - 4*B*a^5*b + A*a^4*b^2 + 4*B*a^3*b^3 - 4
*A*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^7 - a^5*b^2)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\cos^4(c+dx)}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**4/(a + b*sec(c + d*x)), x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 642 vs. 2(221) = 442.
time = 0.50, size = 642, normalized size = 2.68

$$\frac{\frac{3\left(3Aa^4-4Ba^3b+4Aa^2b^2-8Bab^3+8Ab^4\right)(dx+c)}{a^5}+\frac{48\left(Bab^4-Ab^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\sqrt{-a^2+b^2}\,a^5}-\frac{2\left(15Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-24Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+24Aa^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-12Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7+12Aab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^7-24Bab^2\tan\ldots\right)}{\ldots}}{24d}$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] 1/24*(3*(3*A*a^4 - 4*B*a^3*b + 4*A*a^2*b^2 - 8*B*a*b^3 + 8*A*b^4)*(d*x + c)
/a^5 + 48*(B*a*b^4 - A*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*
b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 +
b^2)))/(sqrt(-a^2 + b^2)*a^5) - 2*(15*A*a^3*tan(1/2*d*x + 1/2*c)^7 - 24*B*a
^3*tan(1/2*d*x + 1/2*c)^7 + 24*A*a^2*b*tan(1/2*d*x + 1/2*c)^7 - 12*B*a^2*b*
tan(1/2*d*x + 1/2*c)^7 + 12*A*a*b^2*tan(1/2*d*x + 1/2*c)^7 - 24*B*a*b^2*tan
```

```
(1/2*d*x + 1/2*c)^7 + 24*A*b^3*tan(1/2*d*x + 1/2*c)^7 - 9*A*a^3*tan(1/2*d*x
 + 1/2*c)^5 - 40*B*a^3*tan(1/2*d*x + 1/2*c)^5 + 40*A*a^2*b*tan(1/2*d*x + 1/
2*c)^5 - 12*B*a^2*b*tan(1/2*d*x + 1/2*c)^5 + 12*A*a*b^2*tan(1/2*d*x + 1/2*c
)^5 - 72*B*a*b^2*tan(1/2*d*x + 1/2*c)^5 + 72*A*b^3*tan(1/2*d*x + 1/2*c)^5 +
 9*A*a^3*tan(1/2*d*x + 1/2*c)^3 - 40*B*a^3*tan(1/2*d*x + 1/2*c)^3 + 40*A*a^
2*b*tan(1/2*d*x + 1/2*c)^3 + 12*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 - 12*A*a*b^2
*tan(1/2*d*x + 1/2*c)^3 - 72*B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 72*A*b^3*tan(
1/2*d*x + 1/2*c)^3 - 15*A*a^3*tan(1/2*d*x + 1/2*c) - 24*B*a^3*tan(1/2*d*x +
 1/2*c) + 24*A*a^2*b*tan(1/2*d*x + 1/2*c) + 12*B*a^2*b*tan(1/2*d*x + 1/2*c)
 - 12*A*a*b^2*tan(1/2*d*x + 1/2*c) - 24*B*a*b^2*tan(1/2*d*x + 1/2*c) + 24*A
*b^3*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^4*a^4))/d
```

Mupad [B]
time = 8.64, size = 2500, normalized size = 10.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^4*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x)),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(5*A*a^3 - 8*A*b^3 + 8*B*a^3 + 4*A*a*b^2 - 8*A*a^2*b +
 8*B*a*b^2 - 4*B*a^2*b))/(4*a^4) - (tan(c/2 + (d*x)/2)^7*(5*A*a^3 + 8*A*b^3
 - 8*B*a^3 + 4*A*a*b^2 + 8*A*a^2*b - 8*B*a*b^2 - 4*B*a^2*b))/(4*a^4) - (tan
(c/2 + (d*x)/2)^3*(9*A*a^3 + 72*A*b^3 - 40*B*a^3 - 12*A*a*b^2 + 40*A*a^2*b
- 72*B*a*b^2 + 12*B*a^2*b))/(12*a^4) + (tan(c/2 + (d*x)/2)^5*(9*A*a^3 - 72*
A*b^3 + 40*B*a^3 - 12*A*a*b^2 - 40*A*a^2*b + 72*B*a*b^2 + 12*B*a^2*b))/(12*
a^4))/(d*(4*tan(c/2 + (d*x)/2)^2 + 6*tan(c/2 + (d*x)/2)^4 + 4*tan(c/2 + (d*
x)/2)^6 + tan(c/2 + (d*x)/2)^8 + 1)) - (atan(((((((12*A*a^16 + 32*A*a^10*b^
6 - 48*A*a^11*b^5 + 16*A*a^12*b^4 - 4*A*a^13*b^3 + 4*A*a^14*b^2 - 32*B*a^11
*b^5 + 48*B*a^12*b^4 - 16*B*a^13*b^3 + 16*B*a^14*b^2 - 12*A*a^15*b - 16*B*a
^15*b)/a^12 - (tan(c/2 + (d*x)/2)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^2
)*(A*a^4*3i + A*b^4*8i + A*a^2*b^2*4i - B*a*b^3*8i - B*a^3*b*4i))/(16*a^13)
)*(A*a^4*3i + A*b^4*8i + A*a^2*b^2*4i - B*a*b^3*8i - B*a^3*b*4i))/(8*a^5) +
 (tan(c/2 + (d*x)/2)*(9*A^2*a^11 - 128*A^2*b^11 + 256*A^2*a*b^10 - 27*A^2*a
^10*b - 256*A^2*a^2*b^9 + 256*A^2*a^3*b^8 - 256*A^2*a^4*b^7 + 256*A^2*a^5*b
^6 - 216*A^2*a^6*b^5 + 136*A^2*a^7*b^4 - 81*A^2*a^8*b^3 + 51*A^2*a^9*b^2 -
128*B^2*a^2*b^9 + 256*B^2*a^3*b^8 - 256*B^2*a^4*b^7 + 256*B^2*a^5*b^6 - 208
*B^2*a^6*b^5 + 112*B^2*a^7*b^4 - 48*B^2*a^8*b^3 + 16*B^2*a^9*b^2 + 256*A*B*
a*b^10 - 24*A*B*a^10*b - 512*A*B*a^2*b^9 + 512*A*B*a^3*b^8 - 512*A*B*a^4*b^
7 + 464*A*B*a^5*b^6 - 368*A*B*a^6*b^5 + 264*A*B*a^7*b^4 - 152*A*B*a^8*b^3 +
 72*A*B*a^9*b^2))/(2*a^8))*(A*a^4*3i + A*b^4*8i + A*a^2*b^2*4i - B*a*b^3*8i
 - B*a^3*b*4i)*1i)/(8*a^5) - (((((12*A*a^16 + 32*A*a^10*b^6 - 48*A*a^11*b^5
 + 16*A*a^12*b^4 - 4*A*a^13*b^3 + 4*A*a^14*b^2 - 32*B*a^11*b^5 + 48*B*a^12*
b^4 - 16*B*a^13*b^3 + 16*B*a^14*b^2 - 12*A*a^15*b - 16*B*a^15*b)/a^12 + (ta
n(c/2 + (d*x)/2)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^2)*(A*a^4*3i + A*b
```

^4*8i + A*a^2*b^2*4i - B*a*b^3*8i - B*a^3*b*4i))/(16*a^13))*(A*a^4*3i + A*b^4*8i + A*a^2*b^2*4i - B*a*b^3*8i - B*a^3*b*4i))/(8*a^5) - (tan(c/2 + (d*x)/2)*(9*A^2*a^11 - 128*A^2*b^11 + 256*A^2*a*b^10 - 27*A^2*a^10*b - 256*A^2*a^2*b^9 + 256*A^2*a^3*b^8 - 256*A^2*a^4*b^7 + 256*A^2*a^5*b^6 - 216*A^2*a^6*b^5 + 136*A^2*a^7*b^4 - 81*A^2*a^8*b^3 + 51*A^2*a^9*b^2 - 128*B^2*a^2*b^9 + 256*B^2*a^3*b^8 - 256*B^2*a^4*b^7 + 256*B^2*a^5*b^6 - 208*B^2*a^6*b^5 + 112*B^2*a^7*b^4 - 48*B^2*a^8*b^3 + 16*B^2*a^9*b^2 + 256*A*B*a*b^10 - 24*A*B*a^10*b - 512*A*B*a^2*b^9 + 512*A*B*a^3*b^8 - 512*A*B*a^4*b^7 + 464*A*B*a^5*b^6 - 368*A*B*a^6*b^5 + 264*A*B*a^7*b^4 - 152*A*B*a^8*b^3 + 72*A*B*a^9*b^2))/(2*a^8))*(A*a^4*3i + A*b^4*8i + A*a^2*b^2*4i - B*a*b^3*8i - B*a^3*b*4i)*1i)/(8*a^5))/((64*A^3*b^14 - 96*A^3*a*b^13 + 96*A^3*a^2*b^12 - 104*A^3*a^3*b^11 + 104*A^3*a^4*b^10 - 88*A^3*a^5*b^9 + 48*A^3*a^6*b^8 - 33*A^3*a^7*b^7 + 18*A^3*a^8*b^6 - 9*A^3*a^9*b^5 - 64*B^3*a^3*b^11 + 96*B^3*a^4*b^10 - 96*B^3*a^5*b^9 + 80*B^3*a^6*b^8 - 32*B^3*a^7*b^7 + 16*B^3*a^8*b^6 - 192*A^2*B*a*b^13 + 192*A*B^2*a^2*b^12 - 288*A*B^2*a^3*b^11 + 288*A*B^2*a^4*b^10 - 264*A*B^2*a^5*b^9 + 168*A*B^2*a^6*b^8 - 120*A*B^2*a^7*b^7 + 48*A*B^2*a^8*b^6 - 24*A*B^2*a^9*b^5 + 288*A^2*B*a^2*b^12 - 288*A^2*B*a^3*b^11 + 288*A^2*B*a^4*b^10 - 240*A^2*B*a^5*b^9 + 192*A^2*B*a^6*b^8 - 96*A^2*B*a^7*b^7 + 57*A^2*B*a^8*b^6 - 18*A^2*B*a^9*b^5 + 9*A^2*B*a^10*b^4)/a^12 + (((((12*A*a^16 + 32*A*a^10*b^6 - 48*A*a^11*b^5 + 16*A*a^12*b^4 - 4*A*a^13*b^3 + 4*A*a^14*b^2 - 32*B*a^11*b^5 + 48*B*a^12*b^4 - 16*B*a^13*b^3 + 16*B*a^14*b^2 - 12*A*a^15*b - 16*B*a^15*b)/a^12 - (tan(c/2 + (d*x)/2)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^2)*(A*a^4*3i + A*b^4*8i + A*a^2*b^2*4i - B*a*b^3*8i - B*a^3*b*4i))/(16*a^13))*(A*a^4*3i + A*b^4*8i + A*a^2*b^2*4i - B*a*b^3*8i - B*a^3*b*4i))/(8*a^5) + (tan(c/2 + (d*x)/2)*(9*A^2*a^11 - 128*A^2*b^11 + 256*A^2*a*b^10 - 27*A^2*a^10*b - 256*A^2*a^2*b^9 + 256*A^2*a^3*b^8 - 256*A^2*a^4*b^7 + 256*A^2*a^5*b^6 - 216*A^2*a^6*b^5 + 136*A^2*a^7*b^4 - 81*A^2*a^8*b^3 + 51*A^2*a^9*b^2 - 128*B^2*a^2*b^9 + 256*B^2*a^3*b^8 - 256*B^2*a^4*b^7 + 256*B^2*a^5*b^6 - 208*B^2*a^6*b^5 + 112*B^2*a^7*b^4 - 48*B^2*a^8*b^3 + 16*B^2*a^9*b^2 + 256*A*B*a*b^10 - 24*A*B*a^10*b - 512*A*B*a^2*b^9 + 512*A*B*a^3*b^8 - 512*A*B*a^4*b^7 + 464*A*B*a^5*b^6 - 368*A*B*a^6*b^5 + 264*A*B*a^7*b^4 - 152*A*B*a^8*b^3 + 72*A*B*a^9*b^2))/(2*a^8))*(A*a^4*3i + A*b^4*8i + A*a^2*b^2*4i - B*a*b^3*8i - B*a^3*b*4i))/(8*a^5) + (((((12*A*a^16 + 32*A*a^10*b^6 - 48*A*a^11*b^5 + 16*A*a^12*b^4 - 4*A*a^13*b^3 + 4*A*a^14*b^2 - 32*B*a^11*b^5 + 48*B*a^12*b^4 - 16*B*a^13*b^3 + 16*B*a^14*b^2 - 12*A*a^15*b - 16*B*a^15*b)/a^12 + (tan(c/2 + (d*x)/2)*(128*a^12*b + 128*a^10*b^3 - 256*a^11*b^2)*(A*a^4*3i + A*b^4*8i + A*a^2*b^2*4i - B*a*b^3*8i - B*a^3*b*4i))/(16*a^13))*(A*a^4*3i + A*b^4*8i + A*a^2*b^2*4i - B*a*b^3*8i - B*a^3*b*4i))/(8*a^5) - (tan(c/2 + (d*x)/2)*(9*A^2*a^11 - 128*A^2*b^11 + 256*A^2*a*b^10 - 27*A^2*a^10*b - 256*A^2*a^2*b^9 + 256*A^2*a^3*b^8 - 256*A^2*a^4*b^7 + 256*A^2*a^5*b^6 - 216*A^2*a^6*b^5 + 136*A^2*a^7*b^4 - 81*A^2*a^8*b^3 + 51*A^2*a^9*b^2 - 128*B^2*a^2*b^9 + 256*B^2*a^3*b^8 - 256*B^2*a^4*b^7 + 256*B^2...

3.320 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=272

$$-\frac{(4aAb-6a^2B-b^2B)\tanh^{-1}(\sin(c+dx))}{2b^4d}+\frac{2a^2(2a^2Ab-3Ab^3-3a^3B+4ab^2B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}(c+$$

```
[Out] -1/2*(4*A*a*b-6*B*a^2-B*b^2)*arctanh(sin(d*x+c))/b^4/d+2*a^2*(2*A*a^2*b-3*A
*b^3-3*B*a^3+4*B*a*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))
/(a-b)^(3/2)/b^4/(a+b)^(3/2)/d+(2*A*a^2*b-A*b^3-3*B*a^3+2*B*a*b^2)*tan(d*x+
c)/b^3/(a^2-b^2)/d-1/2*(2*A*a*b-3*B*a^2+B*b^2)*sec(d*x+c)*tan(d*x+c)/b^2/(a
^2-b^2)/d+a*(A*b-B*a)*sec(d*x+c)^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c)
)
```

Rubi [A]
time = 0.58, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4114, 4177, 4167, 4083, 3855, 3916, 2738, 214}

$$\frac{a(Ab-aB)\tan(c+dx)\sec^2(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{(-3a^2B+2aAb+b^2B)\tan(c+dx)\sec(c+dx)}{2b^2d(a^2-b^2)}-\frac{(-6a^2B+4aAb-b^2B)\tanh^{-1}(\sin(c+dx))}{2b^4d}+\frac{(-3a^3B+2a^2Ab+2ab^2B-Ab^3)\tan(c+dx)}{b^3d(a^2-b^2)}+\frac{2a^2(-3a^3B+2a^2Ab+4ab^2B-3Ab^3)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{b^4d(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/2*((4*a*A*b - 6*a^2*B - b^2*B)*ArcTanh[Sin[c + d*x]])/(b^4*d) + (2*a^2*(
2*a^2*A*b - 3*A*b^3 - 3*a^3*B + 4*a*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^4*(a + b)^(3/2)*d) + ((2*a^2*A*b - A*
b^3 - 3*a^3*B + 2*a*b^2*B)*Tan[c + d*x])/(b^3*(a^2 - b^2)*d) - ((2*a*A*b -
3*a^2*B + b^2*B)*Sec[c + d*x]*Tan[c + d*x])/(2*b^2*(a^2 - b^2)*d) + (a*(A*b
 - a*B)*Sec[c + d*x]^2*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4177

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(-C)*Csc[e + f*x]*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m +
1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e +
 f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(
m + 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
```

```
NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = \frac{a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \sec^2(c+dx)(2a(Ab-aB)-b(Ab-a$$

$$= -\frac{\left(2aAb-3a^2B+b^2B\right)\sec(c+dx)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\sec}{b\left(a^2-b^2\right)d(}$$

$$= \frac{\left(2a^2Ab-Ab^3-3a^3B+2ab^2B\right)\tan(c+dx)}{b^3\left(a^2-b^2\right)d} - \frac{(2aAb-3a^2B+b^2}{2b^2}$$

$$= \frac{\left(2a^2Ab-Ab^3-3a^3B+2ab^2B\right)\tan(c+dx)}{b^3\left(a^2-b^2\right)d} - \frac{(2aAb-3a^2B+b^2}{2b^2}$$

$$= -\frac{\left(4aAb-6a^2B-b^2B\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{(2a^2Ab-Ab^3-3a}{b^3(}$$

$$= -\frac{\left(4aAb-6a^2B-b^2B\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{(2a^2Ab-Ab^3-3a}{b^3(}$$

$$= -\frac{\left(4aAb-6a^2B-b^2B\right)\tanh^{-1}(\sin(c+dx))}{2b^4d} + \frac{2a^2(2a^2Ab-3Ab^3}{}$$

Mathematica [A]
time = 6.30, size = 438, normalized size = 1.61

$$-\frac{2a^2\left(-2a^2Ab+3Ab^3+3a^3B-4ab^2B\right)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}\left(-a^2+b^2\right)d} + \frac{\left(4aAb-6a^2B-b^2B\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2b^4d} + \frac{\left(-4aAb+6a^2B+b^2B\right)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2b^4d} + \frac{B}{4b^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} - \frac{B}{4b^2d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)^2} + \frac{Ab\sin\left(\frac{1}{2}(c+dx)\right)-2aB\sin\left(\frac{1}{2}(c+dx)\right)}{b^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{Ab\sin\left(\frac{1}{2}(c+dx)\right)-2aB\sin\left(\frac{1}{2}(c+dx)\right)}{b^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{-a^3Ab\sin(c+dx)+a^4B\sin(c+dx)}{b^3(-a+b)(a+b)d(b+a\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (-2*a^2*(-2*a^2*A*b + 3*A*b^3 + 3*a^3*B - 4*a*b^2*B)*ArcTanh[((-a + b)*Tan[
(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^4*Sqrt[a^2 - b^2]*(-a^2 + b^2)*d) + ((4*
a*A*b - 6*a^2*B - b^2*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]])/(2*b^4*d
) + ((-4*a*A*b + 6*a^2*B + b^2*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])
/(2*b^4*d) + B/(4*b^2*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])^2) - B/(4*b^2
*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])^2) + (A*b*Sin[(c + d*x)/2] - 2*a*B
*Sin[(c + d*x)/2])/(b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2])) + (A*b*Sin
[(c + d*x)/2] - 2*a*B*Sin[(c + d*x)/2])/(b^3*d*(Cos[(c + d*x)/2] + Sin[(c +
 d*x)/2])) + (-(a^3*A*b*Sin[c + d*x]) + a^4*B*Sin[c + d*x])/(b^3*(-a + b)*(
a + b)*d*(b + a*Cos[c + d*x]))
```

Maple [A]
time = 0.71, size = 330, normalized size = 1.21

method	result
derivativedivides	$\frac{\frac{B}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2Ab-4Ba-Bb}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(4Aba-6a^2B-b^2B\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^4}-\frac{B}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{2Ab-4B\cdots}{2b^3\left(\tan\left(\frac{dx}{2}\cdots\right.\right.}\cdots}{}$
default	$\frac{\frac{B}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2Ab-4Ba-Bb}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(4Aba-6a^2B-b^2B\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{2b^4}-\frac{B}{2b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{2Ab-4B\cdots}{2b^3\left(\tan\left(\frac{dx}{2}\cdots\right.\right.}\cdots}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(1/2*B/b^2/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(2*A*b-4*B*a-B*b)/b^3/(tan(1/2*
d*x+1/2*c)-1)+1/2*(4*A*a*b-6*B*a^2-B*b^2)/b^4*ln(tan(1/2*d*x+1/2*c)-1)-1/2*
B/b^2/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(2*A*b-4*B*a-B*b)/b^3/(tan(1/2*d*x+1/2*c
)+1)+1/2/b^4*(-4*A*a*b+6*B*a^2+B*b^2)*ln(tan(1/2*d*x+1/2*c)+1)-2*a^2/b^4*(b
*a*(A*b-B*a)/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2
*d*x+1/2*c)^2-a-b)-(2*A*a^2*b-3*A*b^3-3*B*a^3+4*B*a*b^2)/(a+b)/(a-b)/((a+b)
*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 643 vs. $2(260) = 520$.

time = 27.31, size = 1343, normalized size = 4.94

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/4*(2*((3*B*a^6 - 2*A*a^5*b - 4*B*a^4*b^2 + 3*A*a^3*b^3)*cos(d*x + c)^3 +
 (3*B*a^5*b - 2*A*a^4*b^2 - 4*B*a^3*b^3 + 3*A*a^2*b^4)*cos(d*x + c)^2)*sqrt
(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt
(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x +
 c)^2 + 2*a*b*cos(d*x + c) + b^2)) + ((6*B*a^7 - 4*A*a^6*b - 11*B*a^5*b^2 +
 8*A*a^4*b^3 + 4*B*a^3*b^4 - 4*A*a^2*b^5 + B*a*b^6)*cos(d*x + c)^3 + (6*B*a
^6*b - 4*A*a^5*b^2 - 11*B*a^4*b^3 + 8*A*a^3*b^4 + 4*B*a^2*b^5 - 4*A*a*b^6 +
 B*b^7)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) - ((6*B*a^7 - 4*A*a^6*b - 11*
B*a^5*b^2 + 8*A*a^4*b^3 + 4*B*a^3*b^4 - 4*A*a^2*b^5 + B*a*b^6)*cos(d*x + c)
^3 + (6*B*a^6*b - 4*A*a^5*b^2 - 11*B*a^4*b^3 + 8*A*a^3*b^4 + 4*B*a^2*b^5 -
4*A*a*b^6 + B*b^7)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) + 2*(B*a^4*b^3 -
2*B*a^2*b^5 + B*b^7 - 2*(3*B*a^6*b - 2*A*a^5*b^2 - 5*B*a^4*b^3 + 3*A*a^3*b^
4 + 2*B*a^2*b^5 - A*a*b^6)*cos(d*x + c)^2 - (3*B*a^5*b^2 - 2*A*a^4*b^3 - 6*
B*a^3*b^4 + 4*A*a^2*b^5 + 3*B*a*b^6 - 2*A*b^7)*cos(d*x + c))*sin(d*x + c))/
((a^5*b^4 - 2*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 + (a^4*b^5 - 2*a^2*b^7 + b^
9)*d*cos(d*x + c)^2), -1/4*(4*((3*B*a^6 - 2*A*a^5*b - 4*B*a^4*b^2 + 3*A*a^3
*b^3)*cos(d*x + c)^3 + (3*B*a^5*b - 2*A*a^4*b^2 - 4*B*a^3*b^3 + 3*A*a^2*b^4
)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c)
 + a)/((a^2 - b^2)*sin(d*x + c))) - ((6*B*a^7 - 4*A*a^6*b - 11*B*a^5*b^2 +
8*A*a^4*b^3 + 4*B*a^3*b^4 - 4*A*a^2*b^5 + B*a*b^6)*cos(d*x + c)^3 + (6*B*a^
6*b - 4*A*a^5*b^2 - 11*B*a^4*b^3 + 8*A*a^3*b^4 + 4*B*a^2*b^5 - 4*A*a*b^6 +
B*b^7)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((6*B*a^7 - 4*A*a^6*b - 11*B
*a^5*b^2 + 8*A*a^4*b^3 + 4*B*a^3*b^4 - 4*A*a^2*b^5 + B*a*b^6)*cos(d*x + c)^
3 + (6*B*a^6*b - 4*A*a^5*b^2 - 11*B*a^4*b^3 + 8*A*a^3*b^4 + 4*B*a^2*b^5 - 4
*A*a*b^6 + B*b^7)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(B*a^4*b^3 - 2
*B*a^2*b^5 + B*b^7 - 2*(3*B*a^6*b - 2*A*a^5*b^2 - 5*B*a^4*b^3 + 3*A*a^3*b^4
 + 2*B*a^2*b^5 - A*a*b^6)*cos(d*x + c)^2 - (3*B*a^5*b^2 - 2*A*a^4*b^3 - 6*B
*a^3*b^4 + 4*A*a^2*b^5 + 3*B*a*b^6 - 2*A*b^7)*cos(d*x + c))*sin(d*x + c))/(
(a^5*b^4 - 2*a^3*b^6 + a*b^8)*d*cos(d*x + c)^3 + (a^4*b^5 - 2*a^2*b^7 + b^9
)*d*cos(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\sec^4{(c + dx)}}{(a + b\sec{(c + dx)})^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/(a + b*sec(c + d*x))**2, x)
```

Giac [A]
time = 0.52, size = 384, normalized size = 1.41

$$-\frac{\frac{4\,(3\,Ba^5-2\,Aa^4b-4\,Ba^3b^2+3\,Aa^2b^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^2b^4-b^6)\sqrt{-a^2+b^2}}-\frac{4\,(Ba^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Aa^3b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right))}{(a^2b^3-b^5)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}-\frac{(6\,Ba^2-4\,Aab+Bb^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+1\right|\right)}{b^4}+\frac{(6\,Ba^2-4\,Aab+Bb^2)\log\left(\left|\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-1\right|\right)}{b^4}-\frac{2\left(4\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Bb\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-1\right)^2b^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/2*(4*(3*B*a^5 - 2*A*a^4*b - 4*B*a^3*b^2 + 3*A*a^2*b^3)*(pi*floor(1/2*(d*
x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(
1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^4 - b^6)*sqrt(-a^2 + b^2)) - 4
*(B*a^4*tan(1/2*d*x + 1/2*c) - A*a^3*b*tan(1/2*d*x + 1/2*c))/((a^2*b^3 - b^
5)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - (6*B*a^
2 - 4*A*a*b + B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 + (6*B*a^2 - 4*
A*a*b + B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 - 2*(4*B*a*tan(1/2*d*
x + 1/2*c)^3 - 2*A*b*tan(1/2*d*x + 1/2*c)^3 + B*b*tan(1/2*d*x + 1/2*c)^3 -
4*B*a*tan(1/2*d*x + 1/2*c) + 2*A*b*tan(1/2*d*x + 1/2*c) + B*b*tan(1/2*d*x +
 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 - 1)^2*b^3))/d
```

Mupad [B]
time = 11.17, size = 2500, normalized size = 9.19

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + b/cos(c + d*x))^2),x)
```

```
[Out] (atan(-((((8*tan(c/2 + (d*x)/2)*(72*B^2*a^10 + B^2*b^10 - 2*B^2*a*b^9 - 72*
B^2*a^9*b + 16*A^2*a^2*b^8 - 32*A^2*a^3*b^7 + 20*A^2*a^4*b^6 + 64*A^2*a^5*b
^5 - 64*A^2*a^6*b^4 - 32*A^2*a^7*b^3 + 32*A^2*a^8*b^2 + 11*B^2*a^2*b^8 - 20
*B^2*a^3*b^7 + 23*B^2*a^4*b^6 - 26*B^2*a^5*b^5 + 17*B^2*a^6*b^4 + 120*B^2*a
^7*b^3 - 120*B^2*a^8*b^2 - 8*A*B*a*b^9 - 96*A*B*a^9*b + 16*A*B*a^2*b^8 - 40
*A*B*a^3*b^7 + 64*A*B*a^4*b^6 - 40*A*B*a^5*b^5 - 176*A*B*a^6*b^4 + 176*A*B*
a^7*b^3 + 96*A*B*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (((8*(2*B*b^
15 + 12*A*a^2*b^13 + 12*A*a^3*b^12 - 20*A*a^4*b^11 - 4*A*a^5*b^10 + 8*A*a^6
*b^9 + 6*B*a^2*b^13 - 16*B*a^3*b^12 - 14*B*a^4*b^11 + 28*B*a^5*b^10 + 6*B*a
^6*b^9 - 12*B*a^7*b^8 - 8*A*a*b^14))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) -
 (4*tan(c/2 + (d*x)/2)*(6*B*a^2 + B*b^2 - 4*A*a*b)*(8*a*b^13 - 8*a^2*b^12 -
 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^
2*b^7 - a^3*b^6)))*(6*B*a^2 + B*b^2 - 4*A*a*b))/(2*b^4))*(6*B*a^2 + B*b^2 -
 4*A*a*b)*1i)/(2*b^4) + (((8*tan(c/2 + (d*x)/2)*(72*B^2*a^10 + B^2*b^10 - 2
```

*B^2*a*b^9 - 72*B^2*a^9*b + 16*A^2*a^2*b^8 - 32*A^2*a^3*b^7 + 20*A^2*a^4*b^6 + 64*A^2*a^5*b^5 - 64*A^2*a^6*b^4 - 32*A^2*a^7*b^3 + 32*A^2*a^8*b^2 + 11*B^2*a^2*b^8 - 20*B^2*a^3*b^7 + 23*B^2*a^4*b^6 - 26*B^2*a^5*b^5 + 17*B^2*a^6*b^4 + 120*B^2*a^7*b^3 - 120*B^2*a^8*b^2 - 8*A*B*a*b^9 - 96*A*B*a^9*b + 16*A*B*a^2*b^8 - 40*A*B*a^3*b^7 + 64*A*B*a^4*b^6 - 40*A*B*a^5*b^5 - 176*A*B*a^6*b^4 + 176*A*B*a^7*b^3 + 96*A*B*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (((8*(2*B*b^15 + 12*A*a^2*b^13 + 12*A*a^3*b^12 - 20*A*a^4*b^11 - 4*A*a^5*b^10 + 8*A*a^6*b^9 + 6*B*a^2*b^13 - 16*B*a^3*b^12 - 14*B*a^4*b^11 + 28*B*a^5*b^10 + 6*B*a^6*b^9 - 12*B*a^7*b^8 - 8*A*a*b^14))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + (4*tan(c/2 + (d*x)/2)*(6*B*a^2 + B*b^2 - 4*A*a*b)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)))*(6*B*a^2 + B*b^2 - 4*A*a*b))/(2*b^4))*(6*B*a^2 + B*b^2 - 4*A*a*b)*1i)/(2*b^4))/((16*(108*B^3*a^11 - 54*B^3*a^10*b - 48*A^3*a^4*b^7 - 24*A^3*a^5*b^6 + 80*A^3*a^6*b^5 + 16*A^3*a^7*b^4 - 32*A^3*a^8*b^3 + 4*B^3*a^3*b^8 - 4*B^3*a^4*b^7 + 41*B^3*a^5*b^6 - 9*B^3*a^6*b^5 + 63*B^3*a^7*b^4 + 81*B^3*a^8*b^3 - 216*B^3*a^9*b^2 - 216*A*B^2*a^10*b - 3*A*B^2*a^2*b^9 + 3*A*B^2*a^3*b^8 - 63*A*B^2*a^4*b^7 + 15*A*B^2*a^5*b^6 - 186*A*B^2*a^6*b^5 - 162*A*B^2*a^7*b^4 + 468*A*B^2*a^8*b^3 + 108*A*B^2*a^9*b^2 + 24*A^2*B*a^3*b^8 - 6*A^2*B*a^4*b^7 + 168*A^2*B*a^5*b^6 + 108*A^2*B*a^6*b^5 - 336*A^2*B*a^7*b^4 - 72*A^2*B*a^8*b^3 + 144*A^2*B*a^9*b^2))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (((8*tan(c/2 + (d*x)/2)*(72*B^2*a^10 + B^2*b^10 - 2*B^2*a*b^9 - 72*B^2*a^9*b + 16*A^2*a^2*b^8 - 32*A^2*a^3*b^7 + 20*A^2*a^4*b^6 + 64*A^2*a^5*b^5 - 64*A^2*a^6*b^4 - 32*A^2*a^7*b^3 + 32*A^2*a^8*b^2 + 11*B^2*a^2*b^8 - 20*B^2*a^3*b^7 + 23*B^2*a^4*b^6 - 26*B^2*a^5*b^5 + 17*B^2*a^6*b^4 + 120*B^2*a^7*b^3 - 120*B^2*a^8*b^2 - 8*A*B*a*b^9 - 96*A*B*a^9*b + 16*A*B*a^2*b^8 - 40*A*B*a^3*b^7 + 64*A*B*a^4*b^6 - 40*A*B*a^5*b^5 - 176*A*B*a^6*b^4 + 176*A*B*a^7*b^3 + 96*A*B*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (((8*(2*B*b^15 + 12*A*a^2*b^13 + 12*A*a^3*b^12 - 20*A*a^4*b^11 - 4*A*a^5*b^10 + 8*A*a^6*b^9 + 6*B*a^2*b^13 - 16*B*a^3*b^12 - 14*B*a^4*b^11 + 28*B*a^5*b^10 + 6*B*a^6*b^9 - 12*B*a^7*b^8 - 8*A*a*b^14))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) - (4*tan(c/2 + (d*x)/2)*(6*B*a^2 + B*b^2 - 4*A*a*b)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)))*(6*B*a^2 + B*b^2 - 4*A*a*b))/(2*b^4))*(6*B*a^2 + B*b^2 - 4*A*a*b))/(2*b^4) + (((8*tan(c/2 + (d*x)/2)*(72*B^2*a^10 + B^2*b^10 - 2*B^2*a*b^9 - 72*B^2*a^9*b + 16*A^2*a^2*b^8 - 32*A^2*a^3*b^7 + 20*A^2*a^4*b^6 + 64*A^2*a^5*b^5 - 64*A^2*a^6*b^4 - 32*A^2*a^7*b^3 + 32*A^2*a^8*b^2 + 11*B^2*a^2*b^8 - 20*B^2*a^3*b^7 + 23*B^2*a^4*b^6 - 26*B^2*a^5*b^5 + 17*B^2*a^6*b^4 + 120*B^2*a^7*b^3 - 120*B^2*a^8*b^2 - 8*A*B*a*b^9 - 96*A*B*a^9*b + 16*A*B*a^2*b^8 - 40*A*B*a^3*b^7 + 64*A*B*a^4*b^6 - 40*A*B*a^5*b^5 - 176*A*B*a^6*b^4 + 176*A*B*a^7*b^3 + 96*A*B*a^8*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (((8*(2*B*b^15 + 12*A*a^2*b^13 + 12*A*a^3*b^12 - 20*A*a^4*b^11 - 4*A*a^5*b^10 + 8*A*a^6*b^9 + 6*B*a^2*b^13 - 16*B*a^3*b^12 - 14*B*a^4*b^11 + 28*B*a^5*b^10 + 6*B*a^6*b^9 - 12*B*a^7*b^8 - 8*A*a*b^14))/(a*b^11 + b^12 - a^2*b^10 - a^3*b^9) + (4*tan(c/2 + (d*x)/2)*(6*B*a^2 + B*b^2 - 4*A*a*b)*(8*a*b^13 - 8*a^2*b^12 - 16*a^3*b^11 + 16*a^4*b^10 + 8*a^5*b^9 - 8*a

```
^6*b^8))/(b^4*(a*b^8 + b^9 - a^2*b^7 - a^3*b^6)))*(6*B*a^2 + B*b^2 - 4*A*a*
b))/(2*b^4))*(6*B*a^2 + B*b^2 - 4*A*a*b))/(2*b^4)))*(6*B*a^2 + B*b^2 - 4*A*
a*b)*1i)/(b^4*d) - ((tan(c/2 + (d*x)/2)^5*(6*B*a^4 - 2*A*b^4 + B*b^4 + 2*A*
a^2*b^2 - 5*B*a^2*b^2 + 2*A*a*b^3 - 4*A*a^3*b + 3*B*a*b^3 - 3*B*a^3*b))/((a
*b^3 - b^4)*(a + b)) + (2*tan(c/2 + (d*x)/2)^3*(B*b^4 - 6*B*a^4 + 3*B*a^2*b
^2 - 2*A*a*b^3 + 4*A*a^3*b))/(b*(a*b^2 - b^3)*(a + b)) + (tan(c/2 + (d*x)/2
)*(2*A*b^4 + 6*B*a^4 + B*b^4 - 2*A*a^2*b^2 - 5*B*a^2*b^2 + 2*A*a*b^3 - 4*A*
a^3*b - 3*B*a*b^3 + 3*B*a^3*b))/(b^3*(a + b)*(a...
```

3.321 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=164

$$\frac{(Ab-2aB)\tanh^{-1}(\sin(c+dx))}{b^3d}-\frac{2a(a^2Ab-2Ab^3-2a^3B+3ab^2B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^3(a+b)^{3/2}d}+\frac{B\tan(}{b}$$

```
[Out] (A*b-2*B*a)*arctanh(sin(d*x+c))/b^3/d-2*a*(A*a^2*b-2*A*b^3-2*B*a^3+3*B*a*b^
2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^3/(a+b
)^(3/2)/d+B*tan(d*x+c)/b^2/d-a^2*(A*b-B*a)*tan(d*x+c)/b^2/(a^2-b^2)/d/(a+b*
sec(d*x+c))
```

Rubi [A]
time = 0.38, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4113, 4167, 4083, 3855, 3916, 2738, 214}

$$-\frac{a^2(Ab-aB)\tan(c+dx)}{b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{2a(-2a^3B+a^2Ab+3ab^2B-2Ab^3)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{3/2}(a+b)^{3/2}}+\frac{(Ab-2aB)\tanh^{-1}(\sin(c+dx))}{b^3d}+\frac{B\tan(c+dx)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((A*b - 2*a*B)*ArcTanh[Sin[c + d*x]])/(b^3*d) - (2*a*(a^2*A*b - 2*A*b^3 - 2
*a^3*B + 3*a*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((
a - b)^(3/2)*b^3*(a + b)^(3/2)*d) + (B*Tan[c + d*x])/(b^2*d) - (a^2*(A*b -
a*B)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4113

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-a^2)*(A*b - a*B)
*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x
] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])
^(m + 1)*Simp[a*b*(A*b - a*B)*(m + 1) - (A*b - a*B)*(a^2 + b^2*(m + 1))*Csc
[e + f*x] + b*B*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = -\frac{a^2(Ab-aB)\tan(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)\left(-ab(Ab-aB)-\left(a^2-b^2\right)(A\cdots\right.}{a+b\cdots}}{b^2\,(\cdots}$$

$$= \frac{B\tan(c+dx)}{b^2d} - \frac{a^2(Ab-aB)\tan(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\sec(c+dx)\left(-ab^2\cdots\right.}{\cdots}}{\cdots}$$

$$= \frac{B\tan(c+dx)}{b^2d} - \frac{a^2(Ab-aB)\tan(c+dx)}{b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{(Ab-2aB)\int\cdots}{b\cdots}$$

$$= \frac{(Ab-2aB)\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{B\tan(c+dx)}{b^2d} - \frac{a^2(Ab-a\cdots}{b^2\left(a^2-b^2\right)d(\cdots}$$

$$= \frac{(Ab-2aB)\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{B\tan(c+dx)}{b^2d} - \frac{a^2(Ab-a\cdots}{b^2\left(a^2-b^2\right)d(\cdots}$$

$$= \frac{(Ab-2aB)\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{2a(a^2Ab-2Ab^3-2a^3B+3ab\cdots}{(a-b)^{3/2}\cdots}$$

Mathematica [A]
time = 2.17, size = 240, normalized size = 1.46

$$\frac{-\frac{2a\left(-a^2Ab+2Ab^3+2a^3B-3ab^2B\right)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} - Ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2aB\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + Ab\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) - 2aB\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{a^2b(-Ab+aB)\sin(c+dx)}{(a-b)(a+b)(b+a\cos(c+dx))} + bB\tan(c+dx)}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((-2*a*(-(a^2*A*b) + 2*A*b^3 + 2*a^3*B - 3*a*b^2*B)*ArcTanh[((-a + b)*Tan[(
c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - A*b*Log[Cos[(c + d*x)/2]
 - Sin[(c + d*x)/2]] + 2*a*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + A*b
*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - 2*a*B*Log[Cos[(c + d*x)/2] + Si
n[(c + d*x)/2]] + (a^2*b*(-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(b +
 a*Cos[c + d*x])) + b*B*Tan[c + d*x])/(b^3*d)
```

Maple [A]
time = 0.58, size = 245, normalized size = 1.49

method	result

derivativedivides	$\frac{-\frac{B}{b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(-Ab+2Ba)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3}-\frac{B}{b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(Ab-2Ba)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^3}+\frac{2a\left(\frac{\cdots}{(a^2-b^2)(\cdots}\right.}{\cdots}}{d}$
default	$\frac{-\frac{B}{b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{(-Ab+2Ba)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^3}-\frac{B}{b^2\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{(Ab-2Ba)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^3}+\frac{2a\left(\frac{\cdots}{(a^2-b^2)(\cdots}\right.}{\cdots}}{d}$
risch	$\frac{2i\left(Aab^2e^{3i(dx+c)}-Ba^2b\,e^{3i(dx+c)}+Aa^2b\,e^{2i(dx+c)}-2Ba^3e^{2i(dx+c)}+Bab^2e^{2i(dx+c)}+Aab^2e^{i(dx+c)}-3Ba^2b\,e^{i(dx+c)}\cdots\right.}{d\,b^2\left(e^{2i(dx+c)}+1\right)\left(-a^2+b^2\right)\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-B/b^2/(tan(1/2*d*x+1/2*c)-1)+1/b^3*(-A*b+2*B*a)*ln(tan(1/2*d*x+1/2*c)
-1)-B/b^2/(tan(1/2*d*x+1/2*c)+1)+(A*b-2*B*a)/b^3*ln(tan(1/2*d*x+1/2*c)+1)+2
/b^3*a*(b*a*(A*b-B*a)/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-
b*tan(1/2*d*x+1/2*c)^2-a-b)-(A*a^2*b-2*A*b^3-2*B*a^3+3*B*a*b^2)/(a+b)/(a-b)
/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 529 vs. $2(157) = 314$.
time = 24.72, size = 1114, normalized size = 6.79

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*(((2*B*a^5 - A*a^4*b - 3*B*a^3*b^2 + 2*A*a^2*b^3)*cos(d*x + c)^2 + (2*
B*a^4*b - A*a^3*b^2 - 3*B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(a^2 - b^2
)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2
)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*
a*b*cos(d*x + c) + b^2)) - ((2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3
+ 2*B*a^2*b^4 - A*a*b^5)*cos(d*x + c)^2 + (2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*
b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*b^6)*cos(d*x + c))*log(sin(d*x + c) + 1)
+ ((2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3 + 2*B*a^2*b^4 - A*a*b^5)*
cos(d*x + c)^2 + (2*B*a^5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*a
*b^5 - A*b^6)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(B*a^4*b^2 - 2*B*a^2
*b^4 + B*b^6 + (2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b^3 + A*a^2*b^4 + B*a*b^5)*
cos(d*x + c))*sin(d*x + c))/((a^5*b^3 - 2*a^3*b^5 + a*b^7)*d*cos(d*x + c)^2
 + (a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c)), 1/2*(2*((2*B*a^5 - A*a^4*b
- 3*B*a^3*b^2 + 2*A*a^2*b^3)*cos(d*x + c)^2 + (2*B*a^4*b - A*a^3*b^2 - 3*B*
a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2
)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - ((2*B*a^6 - A*a^5*b -
4*B*a^4*b^2 + 2*A*a^3*b^3 + 2*B*a^2*b^4 - A*a*b^5)*cos(d*x + c)^2 + (2*B*a^
5*b - A*a^4*b^2 - 4*B*a^3*b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*b^6)*cos(d*x +
c))*log(sin(d*x + c) + 1) + ((2*B*a^6 - A*a^5*b - 4*B*a^4*b^2 + 2*A*a^3*b^3
 + 2*B*a^2*b^4 - A*a*b^5)*cos(d*x + c)^2 + (2*B*a^5*b - A*a^4*b^2 - 4*B*a^3
*b^3 + 2*A*a^2*b^4 + 2*B*a*b^5 - A*b^6)*cos(d*x + c))*log(-sin(d*x + c) + 1
) + 2*(B*a^4*b^2 - 2*B*a^2*b^4 + B*b^6 + (2*B*a^5*b - A*a^4*b^2 - 3*B*a^3*b
^3 + A*a^2*b^4 + B*a*b^5)*cos(d*x + c))*sin(d*x + c))/((a^5*b^3 - 2*a^3*b^5
 + a*b^7)*d*cos(d*x + c)^2 + (a^4*b^4 - 2*a^2*b^6 + b^8)*d*cos(d*x + c))]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\sec^3{(c + dx)}}{(a + b\sec{(c + dx)})^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 404 vs. 2(157) = 314.
time = 0.54, size = 404, normalized size = 2.46

$$\frac{\frac{2\,(2\,Ba^4 - Aa^3b - 3\,Ba^2b^2 + 2\,Aab^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a + 2\,b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\sqrt{-a^2 + b^2}}\right)\right)}{(a^2b^3 - b^5)\sqrt{-a^2 + b^2}} - \frac{2\left(2\,Ba^3\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - Aa^2b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - Ba^2b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - Bab^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 + Bb^3\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^3 - 2\,Ba^3\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + Aa^2b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - Ba^2b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + Bab^2\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + Bb^3\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)\right)}{\left(a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^4 - b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^4 - 2\,a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^2 + a + b\right)(a^2b^2 - b^4)} - \frac{(2\,Ba - Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 1\right|\right)}{b^3} + \frac{(2\,Ba - Ab)\log\left(\left|\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 1\right|\right)}{b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] (2*(2*B*a^4 - A*a^3*b - 3*B*a^2*b^2 + 2*A*a*b^3)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(-a^2 + b^2)))/((a^2*b^3 - b^5)*sqrt(-a^2 + b^2)) - 2*(2*B*a^3
*tan(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*
d*x + 1/2*c)^3 - B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + B*b^3*tan(1/2*d*x + 1/2*c
)^3 - 2*B*a^3*tan(1/2*d*x + 1/2*c) + A*a^2*b*tan(1/2*d*x + 1/2*c) - B*a^2*b
*tan(1/2*d*x + 1/2*c) + B*a*b^2*tan(1/2*d*x + 1/2*c) + B*b^3*tan(1/2*d*x +
1/2*c))/((a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*a*tan(1/2
*d*x + 1/2*c)^2 + a + b)*(a^2*b^2 - b^4)) - (2*B*a - A*b)*log(abs(tan(1/2*d
*x + 1/2*c) + 1))/b^3 + (2*B*a - A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^
3)/d
```

Mupad [B]
time = 10.26, size = 2500, normalized size = 15.24

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + b/cos(c + d*x))^2),x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^3*(A*a^2*b - B*b^3 - 2*B*a^3 + B*a*b^2 + B*a^2*b))/(
b^2*(a + b)*(a - b)) - (2*tan(c/2 + (d*x)/2)*(B*b^3 - 2*B*a^3 + A*a^2*b + B
*a*b^2 - B*a^2*b))/(b^2*(a + b)*(a - b)))/(d*(a + b + tan(c/2 + (d*x)/2)^4*
(a - b) - 2*a*tan(c/2 + (d*x)/2)^2)) + (atan(((((32*tan(c/2 + (d*x)/2)*(A^2
*b^8 + 8*B^2*a^8 - 2*A^2*a*b^7 - 8*B^2*a^7*b + 3*A^2*a^2*b^6 + 4*A^2*a^3*b^
5 - 5*A^2*a^4*b^4 - 2*A^2*a^5*b^3 + 2*A^2*a^6*b^2 + 4*B^2*a^2*b^6 - 8*B^2*a
^3*b^5 + 5*B^2*a^4*b^4 + 16*B^2*a^5*b^3 - 16*B^2*a^6*b^2 - 4*A*B*a*b^7 - 8*
A*B*a^7*b + 8*A*B*a^2*b^6 - 8*A*B*a^3*b^5 - 16*A*B*a^4*b^4 + 18*A*B*a^5*b^3
 + 8*A*B*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (((32*(A*a^2*b^10 -
A*b^12 - 3*A*a^3*b^9 + A*a^5*b^7 - 3*B*a^2*b^10 - 3*B*a^3*b^9 + 5*B*a^4*b^8
 + B*a^5*b^7 - 2*B*a^6*b^6 + 2*A*a*b^11 + 2*B*a*b^11))/(a*b^8 + b^9 - a^2*b
^7 - a^3*b^6) + (32*tan(c/2 + (d*x)/2)*(A*b - 2*B*a)*(2*a*b^11 - 2*a^2*b^10
 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3*(a*b^6 + b^7 - a^2*
b^5 - a^3*b^4)))*(A*b - 2*B*a))/b^3)*(A*b - 2*B*a)*1i)/b^3 + (((32*tan(c/2
+ (d*x)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^7 - 8*B^2*a^7*b + 3*A^2*a^2*b^6
 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5*b^3 + 2*A^2*a^6*b^2 + 4*B^2*a^
2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B^2*a^5*b^3 - 16*B^2*a^6*b^2 - 4
*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^2*b^6 - 8*A*B*a^3*b^5 - 16*A*B*a^4*b^4 +
 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) - (((32
*(A*a^2*b^10 - A*b^12 - 3*A*a^3*b^9 + A*a^5*b^7 - 3*B*a^2*b^10 - 3*B*a^3*b^
9 + 5*B*a^4*b^8 + B*a^5*b^7 - 2*B*a^6*b^6 + 2*A*a*b^11 + 2*B*a*b^11))/(a*b^
```

8 + b^9 - a^2*b^7 - a^3*b^6) - (32*tan(c/2 + (d*x)/2)*(A*b - 2*B*a)*(2*a*b^
11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/(b^3*(a*b
^6 + b^7 - a^2*b^5 - a^3*b^4)))*(A*b - 2*B*a))/b^3)*(A*b - 2*B*a)*1i)/b^3)/
((64*(8*B^3*a^8 - 2*A^3*a*b^7 - 4*B^3*a^7*b - 2*A^3*a^2*b^6 + 3*A^3*a^3*b^5
 + A^3*a^4*b^4 - A^3*a^5*b^3 + 12*B^3*a^4*b^4 + 6*B^3*a^5*b^3 - 20*B^3*a^6*
b^2 - 12*A*B^2*a^7*b - 20*A*B^2*a^3*b^5 - 13*A*B^2*a^4*b^4 + 32*A*B^2*a^5*b
^3 + 8*A*B^2*a^6*b^2 + 11*A^2*B*a^2*b^6 + 9*A^2*B*a^3*b^5 - 17*A^2*B*a^4*b^
4 - 5*A^2*B*a^5*b^3 + 6*A^2*B*a^6*b^2))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) +
 (((32*tan(c/2 + (d*x)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^7 - 8*B^2*a^7*b
+ 3*A^2*a^2*b^6 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5*b^3 + 2*A^2*a^6
*b^2 + 4*B^2*a^2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B^2*a^5*b^3 - 16*
B^2*a^6*b^2 - 4*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^2*b^6 - 8*A*B*a^3*b^5 - 1
6*A*B*a^4*b^4 + 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a
^3*b^4) + (((32*(A*a^2*b^10 - A*b^12 - 3*A*a^3*b^9 + A*a^5*b^7 - 3*B*a^2*b^
10 - 3*B*a^3*b^9 + 5*B*a^4*b^8 + B*a^5*b^7 - 2*B*a^6*b^6 + 2*A*a*b^11 + 2*B
*a*b^11))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*tan(c/2 + (d*x)/2)*(A*b -
 2*B*a)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*
b^6))/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)))*(A*b - 2*B*a))/b^3)*(A*b - 2
*B*a))/b^3 - (((32*tan(c/2 + (d*x)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^7 -
8*B^2*a^7*b + 3*A^2*a^2*b^6 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5*b^3
 + 2*A^2*a^6*b^2 + 4*B^2*a^2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B^2*a
^5*b^3 - 16*B^2*a^6*b^2 - 4*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^2*b^6 - 8*A*B
*a^3*b^5 - 16*A*B*a^4*b^4 + 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a*b^6 + b^7 -
 a^2*b^5 - a^3*b^4) - (((32*(A*a^2*b^10 - A*b^12 - 3*A*a^3*b^9 + A*a^5*b^7
- 3*B*a^2*b^10 - 3*B*a^3*b^9 + 5*B*a^4*b^8 + B*a^5*b^7 - 2*B*a^6*b^6 + 2*A*
a*b^11 + 2*B*a*b^11))/(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) - (32*tan(c/2 + (d*
x)/2)*(A*b - 2*B*a)*(2*a*b^11 - 2*a^2*b^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*
b^7 - 2*a^6*b^6))/(b^3*(a*b^6 + b^7 - a^2*b^5 - a^3*b^4)))*(A*b - 2*B*a))/b
^3)*(A*b - 2*B*a))/b^3))*(A*b - 2*B*a)*2i)/(b^3*d) + (a*atan(((a*((32*tan(c
/2 + (d*x)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^7 - 8*B^2*a^7*b + 3*A^2*a^2*
b^6 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5*b^3 + 2*A^2*a^6*b^2 + 4*B^2
*a^2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B^2*a^5*b^3 - 16*B^2*a^6*b^2
- 4*A*B*a*b^7 - 8*A*B*a^7*b + 8*A*B*a^2*b^6 - 8*A*B*a^3*b^5 - 16*A*B*a^4*b^
4 + 18*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a*b^6 + b^7 - a^2*b^5 - a^3*b^4) + (a
((32(A*a^2*b^10 - A*b^12 - 3*A*a^3*b^9 + A*a^5*b^7 - 3*B*a^2*b^10 - 3*B*a
^3*b^9 + 5*B*a^4*b^8 + B*a^5*b^7 - 2*B*a^6*b^6 + 2*A*a*b^11 + 2*B*a*b^11))/
(a*b^8 + b^9 - a^2*b^7 - a^3*b^6) + (32*a*tan(c/2 + (d*x)/2)*((a + b)^3*(a
- b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2)*(2*a*b^11 - 2*a^2*b
^10 - 4*a^3*b^9 + 4*a^4*b^8 + 2*a^5*b^7 - 2*a^6*b^6))/((a*b^6 + b^7 - a^2*b
^5 - a^3*b^4)*(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3)))*((a + b)^3*(a - b)^
3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b - 3*B*a*b^2))/(b^9 - 3*a^2*b^7 + 3*a^
4*b^5 - a^6*b^3))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - A*a^2*b
- 3*B*a*b^2)*1i)/(b^9 - 3*a^2*b^7 + 3*a^4*b^5 - a^6*b^3) + (a*((32*tan(c/2
+ (d*x)/2)*(A^2*b^8 + 8*B^2*a^8 - 2*A^2*a*b^7 - 8*B^2*a^7*b + 3*A^2*a^2*b^6
 + 4*A^2*a^3*b^5 - 5*A^2*a^4*b^4 - 2*A^2*a^5*b^3 + 2*A^2*a^6*b^2 + 4*B^2*a^

```
2*b^6 - 8*B^2*a^3*b^5 + 5*B^2*a^4*b^4 + 16*B^2*...
```

3.322 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=131

$$\frac{B\tanh^{-1}(\sin(c+dx))}{b^2d}-\frac{2(Ab^3+a^3B-2ab^2B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}b^2(a+b)^{3/2}d}+\frac{a(Ab-aB)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))}$$

```
[Out] B*arctanh(sin(d*x+c))/b^2/d-2*(A*b^3+B*a^3-2*B*a*b^2)*arctanh((a-b)^(1/2)*t
an(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)/b^2/(a+b)^(3/2)/d+a*(A*b-B*a)*ta
n(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.20, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}=0.194$, Rules used = {4094, 4083, 3855, 3916, 2738, 214}

$$-\frac{2(a^3B-2ab^2B+Ab^3)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d(a-b)^{3/2}(a+b)^{3/2}}+\frac{a(Ab-aB)\tan(c+dx)}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{B\tanh^{-1}(\sin(c+dx))}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(b^2*d) - (2*(A*b^3 + a^3*B - 2*a*b^2*B)*ArcTanh[
(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(3/2)*b^2*(a + b)^(3/
2)*d) + (a*(A*b - a*B)*Tan[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4094

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*(A*b - a*B)*Cot[
e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dis
t[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
Simp[b*(A*b - a*B)*(m + 1) - (a*A*b*(m + 2) - B*(a^2 + b^2*(m + 1)))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = \frac{a(Ab-aB)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{\sec(c+dx)\left(-b(Ab-aB)+\left(a^2-b^2\right)B\sec(\ldots\right.}{a+b\sec(c+dx)}}{b\left(a^2-b^2\right)}$$

$$= \frac{a(Ab-aB)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{B\int \sec(c+dx)\,dx}{b^2} - \frac{\left(Ab^3+a\ldots\right.}{\ldots}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{a(Ab-aB)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(Ab^3\ldots\right.}{\ldots}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{b^2d} + \frac{a(Ab-aB)\tan(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\left(2(Ab\ldots\right.}{\ldots}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{b^2d} - \frac{2\left(Ab^3+a^3B-2ab^2B\right)\tanh^{-1}\left(\frac{\sqrt{a-\ldots}}{\sqrt{\ldots}}\right.}{(a-b)^{3/2}b^2(a+b)^{3/2}d}$$

Mathematica [A]
time = 0.73, size = 191, normalized size = 1.46

$$\frac{\cos(c+dx)(A+B\sec(c+dx))\left(\frac{2\left(Ab^3+a\left(a^2-2b^2\right)B\right)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} - B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{ab(-Ab+aB)\sin(c+dx)}{(-a+b)(a+b)(b+a\cos(c+dx))}\right)}{b^2d(B+A\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (Cos[c + d*x]*(A + B*Sec[c + d*x])*((2*(A*b^3 + a*(a^2 - 2*b^2)*B)*ArcTanh[
((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(3/2) - B*Log[Cos
[(c + d*x)/2] - Sin[(c + d*x)/2]] + B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]] + (a*b*(-(A*b) + a*B)*Sin[c + d*x])/((-a + b)*(a + b)*(b + a*Cos[c + d*
x]))))/(b^2*d*(B + A*Cos[c + d*x]))
```

Maple [A]
time = 0.49, size = 185, normalized size = 1.41

method	result
derivativedivides	$\frac{\frac{-\frac{2ba(Ab-Ba)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}-\frac{2\left(A\,b^3+a^3B-2Ba\,b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{b^2}+\frac{B\ln\left(\tan\left(\right.\right.}{}}{d}$
default	$\frac{\frac{-\frac{2ba(Ab-Ba)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}-\frac{2\left(A\,b^3+a^3B-2Ba\,b^2\right)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{b^2}+\frac{B\ln\left(\tan\left(\right.\right.}{}}{d}$
risch	$-\frac{2i(-Ab+Ba)\left(b\,e^{i(dx+c)}+a\right)}{(a^2-b^2)db\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}+\frac{b\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)A}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}+\frac{a^3\ln\left(e^{i(dx+c)}-\frac{ia^2-ib^2-}{a\sqrt{}}\right.}{\sqrt{a^2-b^2}\,(a+b}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(2/b^2*(-b*a*(A*b-B*a)/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*
c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)-(A*b^3+B*a^3-2*B*a*b^2)/(a+b)/(a-b)/((a+b)
*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))+B/b^2*
ln(tan(1/2*d*x+1/2*c)+1)-B/b^2*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 319 vs. $2(123) = 246$.
time = 9.44, size = 694, normalized size = 5.30

$$\left[\frac{(Ba^3b - 2Bab^3 + Ab^4 + (Ba^4 - 2Ba^2b^2 + Aab^3)\cos(dx + c))\sqrt{a^2 - b^2}\log\left(\frac{2ab\cos(dx+c) - (a^2 - 2b^2)\cos(dx+c)^2 - 2\sqrt{a^2 - b^2}(b\cos(dx+c) + a)\sin(dx+c) + 2a^2 - b^2}{a^2\cos(dx+c)^2 + 2ab\cos(dx+c) + b^2}\right) + (Ba^4b - 2Ba^2b^3 + Bb^5 + (Ba^5 - 2Ba^3b^2 + Bab^4)\cos(dx + c))\log(\sin(dx + c) + 1) - (Ba^4b - 2Ba^2b^3 + Bb^5 + (Ba^5 - 2Ba^3b^2 + Bab^4)\cos(dx + c))\log(-\sin(dx + c) + 1) - 2(Ba^4b - Aa^3b^2 - Ba^2b^3 + Aab^4)\sin(dx + c)}{2((a^5b^2 - 2a^3b^4 + ab^6)d\cos(dx + c) + (a^4b^3 - 2a^2b^5 + b^7)d)}, -\frac{2(Ba^3b - 2Bab^3 + Ab^4 + (Ba^4 - 2Ba^2b^2 + Aab^3)\cos(dx + c))\sqrt{-a^2 + b^2}\arctan\left(-\frac{\sqrt{-a^2 + b^2}(b\cos(dx+c) + a)}{(a^2 - b^2)\sin(dx+c)}\right) - (Ba^4b - 2Ba^2b^3 + Bb^5 + (Ba^5 - 2Ba^3b^2 + Bab^4)\cos(dx + c))\log(\sin(dx + c) + 1) + (Ba^4b - 2Ba^2b^3 + Bb^5 + (Ba^5 - 2Ba^3b^2 + Bab^4)\cos(dx + c))\log(-\sin(dx + c) + 1) + 2(Ba^4b - Aa^3b^2 - Ba^2b^3 + Aab^4)\sin(dx + c)}{2((a^5b^2 - 2a^3b^4 + ab^6)d\cos(dx + c) + (a^4b^3 - 2a^2b^5 + b^7)d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*((B*a^3*b - 2*B*a*b^3 + A*b^4 + (B*a^4 - 2*B*a^2*b^2 + A*a*b^3)*cos(d*
x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c
)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a
^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (B*a^4*b - 2*B*a^2*b^3 + B
*b^5 + (B*a^5 - 2*B*a^3*b^2 + B*a*b^4)*cos(d*x + c))*log(sin(d*x + c) + 1)
- (B*a^4*b - 2*B*a^2*b^3 + B*b^5 + (B*a^5 - 2*B*a^3*b^2 + B*a*b^4)*cos(d*x
+ c))*log(-sin(d*x + c) + 1) - 2*(B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4
)*sin(d*x + c))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^4*b^3 -
2*a^2*b^5 + b^7)*d), -1/2*(2*(B*a^3*b - 2*B*a*b^3 + A*b^4 + (B*a^4 - 2*B*a^
2*b^2 + A*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b
*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (B*a^4*b - 2*B*a^2*b^3 + B
*b^5 + (B*a^5 - 2*B*a^3*b^2 + B*a*b^4)*cos(d*x + c))*log(sin(d*x + c) + 1)
+ (B*a^4*b - 2*B*a^2*b^3 + B*b^5 + (B*a^5 - 2*B*a^3*b^2 + B*a*b^4)*cos(d*x
+ c))*log(-sin(d*x + c) + 1) + 2*(B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4
)*sin(d*x + c))/((a^5*b^2 - 2*a^3*b^4 + a*b^6)*d*cos(d*x + c) + (a^4*b^3 -
2*a^2*b^5 + b^7)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec^2(c + dx)}{(a + b\sec(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Giac [A]
time = 0.51, size = 231, normalized size = 1.76

$$-\frac{\frac{2(Ba^3 - 2Bab^2 + Ab^3)\left(\pi\left\lfloor\frac{dx+c}{2\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}(-2a + 2b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - b\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)}{\sqrt{-a^2 + b^2}}\right)\right)}{(a^2b^2 - b^4)\sqrt{-a^2 + b^2}} - \frac{B\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) + 1\right|\right)}{b^2} + \frac{B\log\left(\left|\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - 1\right|\right)}{b^2} - \frac{2\left(Ba^2\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right) - Aab\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)\right)}{(a^2b - b^3)\left(a\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - b\tan\left(\frac{1}{2}dx + \frac{1}{2}c\right)^2 - a - b\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -(2*(B*a^3 - 2*B*a*b^2 + A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a
+ 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^
2 + b^2)))/((a^2*b^2 - b^4)*sqrt(-a^2 + b^2)) - B*log(abs(tan(1/2*d*x + 1/2
*c) + 1))/b^2 + B*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^2 - 2*(B*a^2*tan(1/2
*d*x + 1/2*c) - A*a*b*tan(1/2*d*x + 1/2*c))/((a^2*b - b^3)*(a*tan(1/2*d*x +
 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)))/d
```

Mupad [B]
time = 9.65, size = 2500, normalized size = 19.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + b/cos(c + d*x))^2),x)
```

```
[Out] - (B*atan(((B*((B*((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^7 -
 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a^2*b^4 - a
^3*b^3) - (32*B*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4
*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))))/b^
2 - (32*tan(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2
*B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 +
2*A*B*a^3*b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))*1i)/b^2 - (B*((B*((32*(A
*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 + B*a^5*b^4
+ A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (32*B*tan(c/2 +
 (d*x)/2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*
b^4))/(b^2*(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))))/b^2 + (32*tan(c/2 + (d*x)/2
)*(A^2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^
4 + 4*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 +
b^5 - a^2*b^3 - a^3*b^2))*1i)/b^2)/((B*((B*((32*(A*a^2*b^7 - B*b^9 - A*b^9
- A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(
a*b^5 + b^6 - a^2*b^4 - a^3*b^3) - (32*B*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2*a^
2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^5 -
 a^2*b^3 - a^3*b^2))))/b^2 - (32*tan(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6 +
B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5*B^2
*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2))
)/b^2 - (64*(B^3*a^5 - A*B^2*b^5 + A^2*B*b^5 + 2*B^3*a*b^4 - B^3*a^4*b + 2*
B^3*a^2*b^3 - 3*B^3*a^3*b^2 - 3*A*B^2*a*b^4 + A*B^2*a^2*b^3 + A*B^2*a^3*b^2
))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (B*((B*((32*(A*a^2*b^7 - B*b^9 - A*b
^9 - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8)
)/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (32*B*tan(c/2 + (d*x)/2)*(2*a*b^9 - 2
*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/(b^2*(a*b^4 + b^
```

```
5 - a^2*b^3 - a^3*b^2))))/b^2 + (32*tan(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6
 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5*
B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^
2)))/b^2))*2i)/(b^2*d) - (atan(((((32*tan(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a
^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 -
5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*
b^2) + (((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b
^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) +
(32*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b
^2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/
((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))
)*((a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2))/(b^8 - 3*a^2*b^6
 + 3*a^4*b^4 - a^6*b^2))*((a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a
*b^2)*1i)/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2) + (((32*tan(c/2 + (d*x)/2
)*(A^2*b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^
4 + 4*B^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 +
b^5 - a^2*b^3 - a^3*b^2) - (((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B
*a^2*b^7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a
^2*b^4 - a^3*b^3) - (32*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(A*b
^3 + B*a^3 - 2*B*a*b^2)*(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^
5*b^5 - 2*a^6*b^4))/((a*b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3
*a^4*b^4 - a^6*b^2)))*((a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^
2))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))*((a + b)^3*(a - b)^3)^(1/2)*(A
*b^3 + B*a^3 - 2*B*a*b^2)*1i)/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2))/((64
*(B^3*a^5 - A*B^2*b^5 + A^2*B*b^5 + 2*B^3*a*b^4 - B^3*a^4*b + 2*B^3*a^2*b^3
 - 3*B^3*a^3*b^2 - 3*A*B^2*a*b^4 + A*B^2*a^2*b^3 + A*B^2*a^3*b^2))/(a*b^5 +
 b^6 - a^2*b^4 - a^3*b^3) - (((32*tan(c/2 + (d*x)/2)*(A^2*b^6 + 2*B^2*a^6 +
 B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B^2*a^3*b^3 - 5*B^
2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 - a^2*b^3 - a^3*b^2)
 + (((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^7 - 3*B*a^3*b^6 +
 B*a^5*b^4 + A*a*b^8 + 2*B*a*b^8))/(a*b^5 + b^6 - a^2*b^4 - a^3*b^3) + (32*
tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2)*
(2*a*b^9 - 2*a^2*b^8 - 4*a^3*b^7 + 4*a^4*b^6 + 2*a^5*b^5 - 2*a^6*b^4))/((a*
b^4 + b^5 - a^2*b^3 - a^3*b^2)*(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2)))*((
a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2))/(b^8 - 3*a^2*b^6 + 3
*a^4*b^4 - a^6*b^2))*((a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*B*a*b^2
))/(b^8 - 3*a^2*b^6 + 3*a^4*b^4 - a^6*b^2) + (((32*tan(c/2 + (d*x)/2)*(A^2*
b^6 + 2*B^2*a^6 + B^2*b^6 - 2*B^2*a*b^5 - 2*B^2*a^5*b + 3*B^2*a^2*b^4 + 4*B
^2*a^3*b^3 - 5*B^2*a^4*b^2 - 4*A*B*a*b^5 + 2*A*B*a^3*b^3))/(a*b^4 + b^5 - a
^2*b^3 - a^3*b^2) - (((32*(A*a^2*b^7 - B*b^9 - A*b^9 - A*a^3*b^6 + B*a^2*b^
7 - 3*B*a^3*b^6 + B*a^5*b^4 + A*a*b^8 + 2*B*a*b...
```

3.323 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=100

$$\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}-\frac{(Ab-aB)\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))}$$

```
[Out] 2*(A*a-B*b)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(3/2)
/(a+b)^(3/2)/d-(A*b-B*a)*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.10, antiderivative size = 100, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4088, 12, 3916, 2738, 214}

$$\frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{3/2}(a+b)^{3/2}}-\frac{(Ab-aB)\tan(c+dx)}{d\,(a^2-b^2)\,(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(a*A - b*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b
)^(3/2)*(a + b)^(3/2)*d) - ((A*b - a*B)*Tan[c + d*x])/((a^2 - b^2)*d*(a + b
*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\begin{aligned}
\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx &= -\frac{(Ab-aB)\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))}+\frac{\int \frac{(-aA+bB)\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{-a^2+b^2}\\
&= -\frac{(Ab-aB)\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))}+\frac{(aA-bB)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a^2-b^2}\\
&= -\frac{(Ab-aB)\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))}+\frac{(aA-bB)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{b\,(a^2-b^2)}\\
&= -\frac{(Ab-aB)\tan(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))}+\frac{(2(aA-bB))\text{Subst}\left(\int \frac{1}{1+\frac{a}{b}+(1-\frac{a}{b}}\right.}{b\,(a^2-b}\\
&= \frac{2(aA-bB)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{3/2}(a+b)^{3/2}d}-\frac{(Ab-aB)\tan(c}{(a^2-b^2)\,d(a+b\sec}
\end{aligned}$$

Mathematica [A]
time = 0.38, size = 97, normalized size = 0.97

$$\frac{-\frac{2(aA-bB)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{3/2}}+\frac{(-Ab+aB)\sin(c+dx)}{(a-b)(a+b)(b+a\cos(c+dx))}}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((-2*(a*A - b*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2
 - b^2)^(3/2) + ((-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c
 + d*x])))/d
```

Maple [A]
time = 0.34, size = 132, normalized size = 1.32

method	result
derivativedivides	$\frac{\frac{2(Ab-Ba)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}+\frac{2(Aa-Bb)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{d}$
default	$\frac{\frac{2(Ab-Ba)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}+\frac{2(Aa-Bb)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{(a+b)(a-b)\sqrt{(a+b)(a-b)}}}{d}$
risch	$\frac{2i(-Ab+Ba)\left(b\,\mathrm{e}^{i(dx+c)}+a\right)}{a(a^2-b^2)d\left(a\,\mathrm{e}^{2i(dx+c)}+2b\,\mathrm{e}^{i(dx+c)}+a\right)}+\frac{a\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)A}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}-\frac{\ln\left(\mathrm{e}^{i(dx+c)}+\frac{ia^2-ib^2+b\sqrt{a}}{a\sqrt{a^2-}}\right.}{\sqrt{a^2-b^2}\,(a+b)(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2*(A*b-B*a)/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan
(1/2*d*x+1/2*c)^2-a-b)+2*(A*a-B*b)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh(
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 4.12, size = 389, normalized size = 3.89

$$\left[\frac{\left(Aab-Bb^2+\left(Aa^2-Bab\right)\cos\left(dx+c\right)\right)\sqrt{a^2-b^2}\,\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)+2\left(Ba^3-Aa^2b-Bab^2+Ab^3\right)\sin\left(dx+c\right)}{2\left(\left(a^5-2\,a^3b^2+ab^4\right)d\cos\left(dx+c\right)+\left(a^4b-2\,a^2b^3+b^5\right)d\right)},\ \frac{\left(Aab-Bb^2+\left(Aa^2-Bab\right)\cos\left(dx+c\right)\right)\sqrt{-a^2+b^2}\,\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)+\left(Ba^3-Aa^2b-Bab^2+Ab^3\right)\sin\left(dx+c\right)}{\left(a^5-2\,a^3b^2+ab^4\right)d\cos\left(dx+c\right)+\left(a^4b-2\,a^2b^3+b^5\right)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*((A*a*b - B*b^2 + (A*a^2 - B*a*b)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2
*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos
(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(
d*x + c) + b^2)) + 2*(B*a^3 - A*a^2*b - B*a*b^2 + A*b^3)*sin(d*x + c))/((a^
5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x + c) + (a^4*b - 2*a^2*b^3 + b^5)*d), ((A*a
*b - B*b^2 + (A*a^2 - B*a*b)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a
^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (B*a^3 - A*a^2
*b - B*a*b^2 + A*b^3)*sin(d*x + c))/((a^5 - 2*a^3*b^2 + a*b^4)*d*cos(d*x +
c) + (a^4*b - 2*a^2*b^3 + b^5)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec(c + dx)}{(a + b\sec(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Giac [A]
time = 0.48, size = 172, normalized size = 1.72

$$-\frac{2\left(\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)(Aa-Bb)}{(a^2-b^2)\sqrt{-a^2+b^2}}+\frac{Ba\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-Ab\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)(a^2-b^2)}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] -2*((pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*
x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))*(A*a - B*b)/((a^2 -
 b^2)*sqrt(-a^2 + b^2)) + (B*a*tan(1/2*d*x + 1/2*c) - A*b*tan(1/2*d*x + 1/2
*c))/((a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)*(a^2 -
b^2)))/d
```

Mupad [B]
time = 2.42, size = 106, normalized size = 1.06

$$\frac{2\,\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a-b}}{\sqrt{a+b}}\right)\,\left(A\,a-B\,b\right)}{d\,{\left(a+b\right)}^{3/2}\,{\left(a-b\right)}^{3/2}}-\frac{2\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A\,b-B\,a\right)}{d\,\left(a+b\right)\,\left(a-b\right)\,\left(\left(b-a\right)\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + b/cos(c + d*x))^2),x)
```

```
[Out] (2*atanh((tan(c/2 + (d*x)/2)*(a - b)^(1/2))/(a + b)^(1/2))*(A*a - B*b))/(d*
(a + b)^(3/2)*(a - b)^(3/2)) - (2*tan(c/2 + (d*x)/2)*(A*b - B*a))/(d*(a + b
)*(a - b)*(a + b - tan(c/2 + (d*x)/2)^2*(a - b)))
```

3.324 $\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=124

$$\frac{Ax}{a^2}-\frac{2(2a^2Ab-Ab^3-a^3B)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d}+\frac{b(Ab-aB)\tan(c+dx)}{a\,(a^2-b^2)\,d(a+b\sec(c+dx))}$$

```
[Out] A*x/a^2-2*(2*A*a^2*b-A*b^3-B*a^3)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a
+b)^(1/2))/a^2/(a-b)^(3/2)/(a+b)^(3/2)/d+b*(A*b-B*a)*tan(d*x+c)/a/(a^2-b^2)
/d/(a+b*sec(d*x+c))
```

Rubi [A]

time = 0.14, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4008, 4004, 3916, 2738, 214}

$$\frac{b(Ab-aB)\tan(c+dx)}{ad\,(a^2-b^2)\,(a+b\sec(c+dx))}+\frac{Ax}{a^2}-\frac{2(a^3(-B)+2a^2Ab-Ab^3)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (A*x)/a^2 - (2*(2*a^2*A*b - A*b^3 - a^3*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/(a^2*(a - b)^(3/2)*(a + b)^(3/2)*d) + (b*(A*b - a*B)*T
an[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4008

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$
\begin{aligned}
\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^2}\,dx &= \frac{b(Ab-aB)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))}-\frac{\int \frac{-A\left(a^2-b^2\right)+a(Ab-aB)\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a\left(a^2-b^2\right)}\\
&= \frac{Ax}{a^2}+\frac{b(Ab-aB)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))}-\frac{\left(2a^2Ab-Ab^3-a^3B\right)\int \frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a^2\left(a^2-b^2\right)}\\
&= \frac{Ax}{a^2}+\frac{b(Ab-aB)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))}-\frac{\left(2a^2Ab-Ab^3-a^3B\right)\int \frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{a^2b\left(a^2-b^2\right)}\\
&= \frac{Ax}{a^2}+\frac{b(Ab-aB)\tan(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))}-\frac{\left(2\left(2a^2Ab-Ab^3-a^3B\right)\right)\text{Subst}\left(\int \frac{}{1+\frac{a}{b}-} \right.}{a^2b\left(a^2-b\right.}\\
&= \frac{Ax}{a^2}-\frac{2\left(2a^2Ab-Ab^3-a^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2(a-b)^{3/2}(a+b)^{3/2}d}+\frac{b(Ab-aB)\,\text{t}}{a\left(a^2-b^2\right)d(a+}
\end{aligned}
$$

Mathematica [A]
time = 0.70, size = 155, normalized size = 1.25

$$
\frac{-\frac{2\left(-2a^2Ab+Ab^3+a^3B\right)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}}+\frac{Ab\left(a^2-b^2\right)(c+dx)+aA\left(a^2-b^2\right)(c+dx)\cos(c+dx)-ab(-Ab+aB)\sin(c+dx)}{b+a\cos(c+dx)}}{a^2(a-b)(a+b)d}
$$

Antiderivative was successfully verified.

[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^2,x]

[Out] ((-2*(-2*a^2*A*b + A*b^3 + a^3*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (A*b*(a^2 - b^2)*(c + d*x) + a*A*(a^2 - b^2)*(c + d*x)*Cos[c + d*x] - a*b*(-(A*b) + a*B)*Sin[c + d*x])/(b + a*Cos[c + d*x]))/(a^2*(a - b)*(a + b)*d)

Maple [A]
time = 0.35, size = 168, normalized size = 1.35

method	result
derivativedivides	$\frac{\frac{2A\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{-\frac{2ba(Ab-Ba)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}-\frac{2\left(2Aa^2b-Ab^3-a^3B\right)\operatorname{arctanh}\left(\frac{(a-b)\operatorname{ta}}{\sqrt{(a+}}\right.}{(a+b)(a-b)\sqrt{(a+b)(a-}}}{a^2}}{d}$
default	$\frac{\frac{2A\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^2}+\frac{-\frac{2ba(Ab-Ba)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\left(a^2-b^2\right)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}-\frac{2\left(2Aa^2b-Ab^3-a^3B\right)\operatorname{arctanh}\left(\frac{(a-b)\operatorname{ta}}{\sqrt{(a+}}\right.}{(a+b)(a-b)\sqrt{(a+b)(a-}}}{a^2}}{d}$
risch	$\frac{Ax}{a^2}-\frac{2ib(-Ab+Ba)\left(b\,e^{i(dx+c)}+a\right)}{a^2(a^2-b^2)d\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}+\frac{2\ln\left(e^{i(dx+c)}+\frac{-ia^2+ib^2+b\sqrt{a^2-b^2}}{a\sqrt{a^2-b^2}}\right)Ab}{\sqrt{a^2-b^2}\,(a+b)(a-b)d}-\frac{\ln\left(e^{i(dx+c)}+\right.}{\sqrt{a^2}}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(2*A/a^2*arctan(tan(1/2*d*x+1/2*c))+2/a^2*(-b*a*(A*b-B*a)/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)-(2*A*a^2*b-A*b^3-B*a^3)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 250 vs. $2(114) = 228$.
time = 5.28, size = 561, normalized size = 4.52

$$\left[\frac{2\,(Aa^5-2\,Aa^3b^2+Aab^4)dx\cos(dx+c)+2\,(Aa^4b-2\,Aa^2b^3+Ab^5)dx-(Ba^3b-2\,Aa^2b^2+Ab^4+(Ba^4-2\,Aa^3b+Aab^3)\cos(dx+c))\sqrt{a^2-b^2}\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2b^2)\cos(dx+c)^2-2\sqrt{a^2-b^2}(b\cos(dx+c)+a)\sin(dx+c)+2a^2-b^2}{a^2\cos(dx+c)^2+2ab\cos(dx+c)+b^2}\right)-2\,(Ba^4b-Aa^3b^2-Ba^2b^3+Aab^4)\sin(dx+c)}{2\,((a^7-2\,a^5b^2+a^3b^4)d\cos(dx+c)+(a^6b-2\,a^4b^3+a^2b^5)d)},\ \frac{(Aa^5-2\,Aa^3b^2+Aab^4)dx\cos(dx+c)+(Aa^4b-2\,Aa^2b^3+Ab^5)dx+(Ba^3b-2\,Aa^2b^2+Ab^4+(Ba^4-2\,Aa^3b+Aab^3)\cos(dx+c))\sqrt{-a^2+b^2}\arctan\left(-\frac{\sqrt{-a^2+b^2}(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)-(Ba^4b-Aa^3b^2-Ba^2b^3+Aab^4)\sin(dx+c)}{(a^7-2\,a^5b^2+a^3b^4)d\cos(dx+c)+(a^6b-2\,a^4b^3+a^2b^5)d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*(A*a^5 - 2*A*a^3*b^2 + A*a*b^4)*d*x*cos(d*x + c) + 2*(A*a^4*b - 2*A
*a^2*b^3 + A*b^5)*d*x - (B*a^3*b - 2*A*a^2*b^2 + A*b^4 + (B*a^4 - 2*A*a^3*b
 + A*a*b^3)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 -
2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c)
 + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(B*a^4
*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*sin(d*x + c))/((a^7 - 2*a^5*b^2 + a^3
*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)*d), ((A*a^5 - 2*A*a^3*
b^2 + A*a*b^4)*d*x*cos(d*x + c) + (A*a^4*b - 2*A*a^2*b^3 + A*b^5)*d*x + (B*
a^3*b - 2*A*a^2*b^2 + A*b^4 + (B*a^4 - 2*A*a^3*b + A*a*b^3)*cos(d*x + c))*s
qrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*
sin(d*x + c))) - (B*a^4*b - A*a^3*b^2 - B*a^2*b^3 + A*a*b^4)*sin(d*x + c))/
((a^7 - 2*a^5*b^2 + a^3*b^4)*d*cos(d*x + c) + (a^6*b - 2*a^4*b^3 + a^2*b^5)
*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec{\left(c + dx \right)}}{\left(a + b\sec{\left(c + dx \right)}\right)^{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**2, x)
```

Giac [A]
time = 0.47, size = 201, normalized size = 1.62

$$\frac{\frac{2\,(Ba^3-2\,Aa^2b+Ab^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4-a^2b^2)\sqrt{-a^2+b^2}}+\frac{(dx+c)A}{a^2}+\frac{2\,(Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right))}{(a^3-ab^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] (2*(B*a^3 - 2*A*a^2*b + A*b^3)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a +
 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2
```

```
 + b^2)))/((a^4 - a^2*b^2)*sqrt(-a^2 + b^2)) + (d*x + c)*A/a^2 + 2*(B*a*b*t
an(1/2*d*x + 1/2*c) - A*b^2*tan(1/2*d*x + 1/2*c))/((a^3 - a*b^2)*(a*tan(1/2
*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)))/d
```

Mupad [B]
time = 9.66, size = 2500, normalized size = 20.16

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*A*atan(((A*((A*((32*(A*a^4*b^5 - B*a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2
 - B*a^6*b^3 + B*a^7*b^2 + 2*A*a^8*b + B*a^8*b))/(a^5*b + a^6 - a^3*b^3 - a
^4*b^2) - (A*tan(c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^
4 - 4*a^7*b^3 - 2*a^8*b^2)*32i)/(a^2*(a^4*b + a^5 - a^2*b^3 - a^3*b^2)))*1i
)/a^2 + (32*tan(c/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^5
 - 2*A^2*a^5*b - 5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^5*
b + 2*A*B*a^3*b^3))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2)))/a^2 - (A*((A*((32*(
A*a^4*b^5 - B*a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2
 + 2*A*a^8*b + B*a^8*b))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (A*tan(c/2 + (
d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^
2)*32i)/(a^2*(a^4*b + a^5 - a^2*b^3 - a^3*b^2)))*1i)/a^2 - (32*tan(c/2 + (d
*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^5 - 2*A^2*a^5*b - 5*A^2*a
^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^5*b + 2*A*B*a^3*b^3))/(a^4
*b + a^5 - a^2*b^3 - a^3*b^2)))/a^2)/((64*(A^3*b^5 + A*B^2*a^5 - A^2*B*a^5
 - A^3*a*b^4 + 2*A^3*a^4*b - 3*A^3*a^2*b^3 + 2*A^3*a^3*b^2 - 3*A^2*B*a^4*b +
 A^2*B*a^2*b^3 + A^2*B*a^3*b^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (A*((A
*((32*(A*a^4*b^5 - B*a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*
a^7*b^2 + 2*A*a^8*b + B*a^8*b))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) - (A*tan(
c/2 + (d*x)/2)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2
*a^8*b^2)*32i)/(a^2*(a^4*b + a^5 - a^2*b^3 - a^3*b^2)))*1i)/a^2 + (32*tan(c
/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^5 - 2*A^2*a^5*b -
5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^5*b + 2*A*B*a^3*b^3
))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2))*1i)/a^2 + (A*((A*((32*(A*a^4*b^5 - B*
a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2 + 2*A*a^8*b +
 B*a^8*b))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (A*tan(c/2 + (d*x)/2)*(2*a^9
*b - 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2)*32i)/(a^2*(
a^4*b + a^5 - a^2*b^3 - a^3*b^2)))*1i)/a^2 - (32*tan(c/2 + (d*x)/2)*(A^2*a^
6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^5 - 2*A^2*a^5*b - 5*A^2*a^2*b^4 + 4*A^2
*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^5*b + 2*A*B*a^3*b^3))/(a^4*b + a^5 - a^2
*b^3 - a^3*b^2))*1i)/a^2)))/(a^2*d) + (atan(((((a + b)^3*(a - b)^3)^(1/2)*(
(32*tan(c/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^5 - 2*A^2
*a^5*b - 5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^5*b + 2*A*
B*a^3*b^3))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) + (((32*(A*a^4*b^5 - B*a^9 -
```

```
A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2 + 2*A*a^8*b + B*a^8
*b))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) - (32*tan(c/2 + (d*x)/2)*((a + b)^3*
(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*A*a^2*b)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b
^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b^3 - a^3*b^2)
*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*((a + b)^3*(a - b)^3)^(1/2)*(A*b
^3 + B*a^3 - 2*A*a^2*b))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*(A*b^3 +
B*a^3 - 2*A*a^2*b)*1i)/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2) + (((a + b)^
3*(a - b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 -
 2*A^2*a*b^5 - 2*A^2*a^5*b - 5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2
- 4*A*B*a^5*b + 2*A*B*a^3*b^3))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) - (((32*(
A*a^4*b^5 - B*a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2
 + 2*A*a^8*b + B*a^8*b))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (32*tan(c/2 +
(d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*A*a^2*b)*(2*a^9*b -
 2*a^4*b^6 + 2*a^5*b^5 + 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5
- a^2*b^3 - a^3*b^2)*(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*((a + b)^3*(
a - b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*A*a^2*b))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3
*a^6*b^2))*(A*b^3 + B*a^3 - 2*A*a^2*b)*1i)/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a
^6*b^2))/((64*(A^3*b^5 + A*B^2*a^5 - A^2*B*a^5 - A^3*a*b^4 + 2*A^3*a^4*b -
3*A^3*a^2*b^3 + 2*A^3*a^3*b^2 - 3*A^2*B*a^4*b + A^2*B*a^2*b^3 + A^2*B*a^3*b
^2))/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) + (((a + b)^3*(a - b)^3)^(1/2)*((32*
tan(c/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*a*b^5 - 2*A^2*a^5
*b - 5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B*a^5*b + 2*A*B*a^
3*b^3))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) + (((32*(A*a^4*b^5 - B*a^9 - A*a^
9 - 3*A*a^6*b^3 + A*a^7*b^2 - B*a^6*b^3 + B*a^7*b^2 + 2*A*a^8*b + B*a^8*b))
/(a^5*b + a^6 - a^3*b^3 - a^4*b^2) - (32*tan(c/2 + (d*x)/2)*((a + b)^3*(a -
 b)^3)^(1/2)*(A*b^3 + B*a^3 - 2*A*a^2*b)*(2*a^9*b - 2*a^4*b^6 + 2*a^5*b^5 +
 4*a^6*b^4 - 4*a^7*b^3 - 2*a^8*b^2))/((a^4*b + a^5 - a^2*b^3 - a^3*b^2)*(a^
8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2)))*((a + b)^3*(a - b)^3)^(1/2)*(A*b^3 +
 B*a^3 - 2*A*a^2*b))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2))*(A*b^3 + B*a^
3 - 2*A*a^2*b))/(a^8 - a^2*b^6 + 3*a^4*b^4 - 3*a^6*b^2) - (((a + b)^3*(a -
b)^3)^(1/2)*((32*tan(c/2 + (d*x)/2)*(A^2*a^6 + 2*A^2*b^6 + B^2*a^6 - 2*A^2*
a*b^5 - 2*A^2*a^5*b - 5*A^2*a^2*b^4 + 4*A^2*a^3*b^3 + 3*A^2*a^4*b^2 - 4*A*B
*a^5*b + 2*A*B*a^3*b^3))/(a^4*b + a^5 - a^2*b^3 - a^3*b^2) - (((32*(A*a^4*b
^5 - B*a^9 - A*a^9 - 3*A*a^6*b^3 + A*a^7*b^2 - ...
```

3.325 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=180

$$-\frac{(2Ab-aB)x}{a^3}+\frac{2b(3a^2Ab-2Ab^3-2a^3B+ab^2B)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{3/2}(a+b)^{3/2}d}+\frac{(a^2A-2Ab^2+abB)\,\text{si}}{a^2\left(a^2-b^2\right)d}$$

```
[Out] -(2*A*b-B*a)*x/a^3+2*b*(3*A*a^2*b-2*A*b^3-2*B*a^3+B*a*b^2)*arctanh((a-b)^(1
/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^3/(a-b)^(3/2)/(a+b)^(3/2)/d+(A*a^2-2*
A*b^2+B*a*b)*sin(d*x+c)/a^2/(a^2-b^2)/d+b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/
d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.38, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.207$, Rules used = {4115, 4189, 4004, 3916, 2738, 214}

$$-\frac{x(2Ab-aB)}{a^3}+\frac{(a^2A+abB-2Ab^2)\sin(c+dx)}{a^2d\,(a^2-b^2)}+\frac{b(Ab-aB)\sin(c+dx)}{ad\,(a^2-b^2)\,(a+b\sec(c+dx))}+\frac{2b(-2a^3B+3a^2Ab+ab^2B-2Ab^3)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((2*A*b - a*B)*x)/a^3) + (2*b*(3*a^2*A*b - 2*A*b^3 - 2*a^3*B + a*b^2*B)*A
rcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(3/2)*(a +
 b)^(3/2)*d) + ((a^2*A - 2*A*b^2 + a*b*B)*Sin[c + d*x])/(a^2*(a^2 - b^2)*d)
 + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{a(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{\int \frac{\cos(c+dx)\left(-a^2A+2Ab^2-abB+a(Ab-a\ldots\right.}{a+b\sec(\ldots}}{a(a^2\ldots}$$

$$= \frac{(a^2A-2Ab^2+abB)\sin(c+dx)}{a^2(a^2-b^2)\,d} + \frac{b(Ab-aB)\sin(c+dx)}{a(a^2-b^2)\,d(a+b\sec(c+dx))}$$

$$= -\frac{(2Ab-aB)x}{a^3} + \frac{(a^2A-2Ab^2+abB)\sin(c+dx)}{a^2(a^2-b^2)\,d} + \frac{b(Ab-\ldots}{a(a^2-b^2)\,a\ldots}$$

$$= -\frac{(2Ab-aB)x}{a^3} + \frac{(a^2A-2Ab^2+abB)\sin(c+dx)}{a^2(a^2-b^2)\,d} + \frac{b(Ab-\ldots}{a(a^2-b^2)\,a\ldots}$$

$$= -\frac{(2Ab-aB)x}{a^3} + \frac{(a^2A-2Ab^2+abB)\sin(c+dx)}{a^2(a^2-b^2)\,d} + \frac{b(Ab-\ldots}{a(a^2-b^2)\,a\ldots}$$

$$= -\frac{(2Ab-aB)x}{a^3} + \frac{2b(3a^2Ab-2Ab^3-2a^3B+ab^2B)\tanh^{-1}\left(\frac{\sqrt{a\ldots}}{\ldots}\right.}{a^3(a-b)^{3/2}(a+b)^{3/2}d}$$

Mathematica [A]

time = 1.14, size = 221, normalized size = 1.23

$$\frac{(b+a\cos(c+dx))(A+B\sec(c+dx))\left((-2Ab+aB)(c+dx)(b+a\cos(c+dx))\sec(c+dx)+\frac{2b(-3a^2Ab+2Ab^3+2a^3B-ab^2B)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))\sec(c+dx)}{(a^2-b^2)^{3/2}}+\frac{ab^2(-Ab+aB)\tan(c+dx)}{(a-b)(a+b)}+aA(b+a\cos(c+dx))\tan(c+dx)\right)}{a^3d(B+A\cos(c+dx))(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((b + a*Cos[c + d*x])*(A + B*Sec[c + d*x])*((-2*A*b + a*B)*(c + d*x)*(b + a
*Cos[c + d*x])*Sec[c + d*x] + (2*b*(-3*a^2*A*b + 2*A*b^3 + 2*a^3*B - a*b^2*
B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x]
)*Sec[c + d*x])/(a^2 - b^2)^(3/2) + (a*b^2*(-(A*b) + a*B)*Tan[c + d*x])/((a
 - b)*(a + b)) + a*A*(b + a*Cos[c + d*x])*Tan[c + d*x]))/(a^3*d*(B + A*Cos[
c + d*x])*(a + b*Sec[c + d*x])^2)
```

Maple [A]

time = 0.48, size = 213, normalized size = 1.18

method	result

derivativedivides	$-\frac{\frac{2\left(-\frac{Aa\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+(2Ab-Ba)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^3}-\frac{2b\left(-\frac{ba(Ab-Ba)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}-\frac{(3A\,a^2b}{}\right.}{a^3}}{d}$
default	$-\frac{\frac{2\left(-\frac{Aa\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+(2Ab-Ba)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^3}-\frac{2b\left(-\frac{ba(Ab-Ba)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a^2-b^2)\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)}-\frac{(3A\,a^2b}{}\right.}{a^3}}{d}$
risch	$-\frac{2xAb}{a^3}+\frac{xB}{a^2}-\frac{iA\,e^{i(dx+c)}}{2a^2d}+\frac{iA\,e^{-i(dx+c)}}{2a^2d}+\frac{2ib^2(-Ab+Ba)\left(b\,e^{i(dx+c)}+a\right)}{a^3(a^2-b^2)\,d\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)}+\frac{3b^2\ln\left(e^{i(dx+c)}+\frac{ia^2-}{}\right.}{\sqrt{a^2-b^2}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a^3*(-A*a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+(2*A*b-B*a)*a
rctan(tan(1/2*d*x+1/2*c)))-2*b/a^3*(-b*a*(A*b-B*a)/(a^2-b^2)*tan(1/2*d*x+1/
2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)-(3*A*a^2*b-2*A*b^3
-2*B*a^3+B*a*b^2)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x
+1/2*c)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 364 vs. $2(171) = 342$.
time = 5.20, size = 788, normalized size = 4.38

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fric
as")
```

```
[Out] [1/2*(2*(B*a^6 - 2*A*a^5*b - 2*B*a^4*b^2 + 4*A*a^3*b^3 + B*a^2*b^4 - 2*A*a*
b^5)*d*x*cos(d*x + c) + 2*(B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a^2*b^
4 + B*a*b^5 - 2*A*b^6)*d*x + (2*B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4 + 2*A*b^5
 + (2*B*a^4*b - 3*A*a^3*b^2 - B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt(a^2
 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2
 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^
2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(A*a^5*b + B*a^4*b^2 - 3*A*a^3*b^3 - B*a
^2*b^4 + 2*A*a*b^5 + (A*a^6 - 2*A*a^4*b^2 + A*a^2*b^4)*cos(d*x + c))*sin(d*
x + c))/((a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c) + (a^7*b - 2*a^5*b^3 +
a^3*b^5)*d), ((B*a^6 - 2*A*a^5*b - 2*B*a^4*b^2 + 4*A*a^3*b^3 + B*a^2*b^4 -
2*A*a*b^5)*d*x*cos(d*x + c) + (B*a^5*b - 2*A*a^4*b^2 - 2*B*a^3*b^3 + 4*A*a^
2*b^4 + B*a*b^5 - 2*A*b^6)*d*x - (2*B*a^3*b^2 - 3*A*a^2*b^3 - B*a*b^4 + 2*A
*b^5 + (2*B*a^4*b - 3*A*a^3*b^2 - B*a^2*b^3 + 2*A*a*b^4)*cos(d*x + c))*sqrt
(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin
(d*x + c))) + (A*a^5*b + B*a^4*b^2 - 3*A*a^3*b^3 - B*a^2*b^4 + 2*A*a*b^5 +
(A*a^6 - 2*A*a^4*b^2 + A*a^2*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 2*a^6
*b^2 + a^4*b^4)*d*cos(d*x + c) + (a^7*b - 2*a^5*b^3 + a^3*b^5)*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\cos(c + dx)}{(a + b\sec(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)/(a + b*sec(c + d*x))**2, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1107 vs. $2(171) = 342$.
time = 0.58, size = 1107, normalized size = 6.15

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="giac
")
```

```
[Out] ((B*a^8 - 2*A*a^7*b - 3*B*a^7*b + 5*A*a^6*b^2 - 2*B*a^6*b^2 + 4*A*a^5*b^3 +
 5*B*a^5*b^3 - 9*A*a^4*b^4 + B*a^4*b^4 - 2*A*a^3*b^5 - 2*B*a^3*b^5 + 4*A*a^
2*b^6 - B*a^3*abs(-a^5 + a^3*b^2) + 2*A*a^2*b*abs(-a^5 + a^3*b^2) - B*a^2*b
*abs(-a^5 + a^3*b^2) + A*a*b^2*abs(-a^5 + a^3*b^2) + B*a*b^2*abs(-a^5 + a^3
```

```
*b^2) - 2*A*b^3*abs(-a^5 + a^3*b^2))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + ar
ctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^4*b - a^2*b^3 + sqrt((a^5 + a^4*b - a^3*
b^2 - a^2*b^3)*(a^5 - a^4*b - a^3*b^2 + a^2*b^3) + (a^4*b - a^2*b^3)^2))/(a
^5 - a^4*b - a^3*b^2 + a^2*b^3))))/(a^4*b*abs(-a^5 + a^3*b^2) - a^2*b^3*abs
(-a^5 + a^3*b^2) + (a^5 - a^3*b^2)^2) - ((2*a^2*b + a*b^2 - 2*b^3)*sqrt(-a^
2 + b^2)*A*abs(-a^5 + a^3*b^2)*abs(-a + b) - (a^3 + a^2*b - a*b^2)*sqrt(-a^
2 + b^2)*B*abs(-a^5 + a^3*b^2)*abs(-a + b) + (2*a^7*b - 5*a^6*b^2 - 4*a^5*b
^3 + 9*a^4*b^4 + 2*a^3*b^5 - 4*a^2*b^6)*sqrt(-a^2 + b^2)*A*abs(-a + b) - (a
^8 - 3*a^7*b - 2*a^6*b^2 + 5*a^5*b^3 + a^4*b^4 - 2*a^3*b^5)*sqrt(-a^2 + b^2
)*B*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1
/2*c)/sqrt(-(a^4*b - a^2*b^3 - sqrt((a^5 + a^4*b - a^3*b^2 - a^2*b^3)*(a^5
- a^4*b - a^3*b^2 + a^2*b^3) + (a^4*b - a^2*b^3)^2))/(a^5 - a^4*b - a^3*b^2
 + a^2*b^3))))/((a^5 - a^3*b^2)^2*(a^2 - 2*a*b + b^2) - (a^6*b - 2*a^5*b^2
+ 2*a^3*b^4 - a^2*b^5)*abs(-a^5 + a^3*b^2)) + 2*(A*a^3*tan(1/2*d*x + 1/2*c)
^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - A*a*b^2*tan(1/2*d*x + 1/2*c)^3 - B*a*
b^2*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^3*tan(1/2*d*x + 1/2*c)^3 - A*a^3*tan(1/2
*d*x + 1/2*c) - A*a^2*b*tan(1/2*d*x + 1/2*c) + A*a*b^2*tan(1/2*d*x + 1/2*c)
 - B*a*b^2*tan(1/2*d*x + 1/2*c) + 2*A*b^3*tan(1/2*d*x + 1/2*c))/((a*tan(1/2
*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*b*tan(1/2*d*x + 1/2*c)^2 - a
 - b)*(a^4 - a^2*b^2)))/d
```

Mupad [B]
time = 7.07, size = 2500, normalized size = 13.89

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^2,x)
```

```
[Out] ((2*tan(c/2 + (d*x)/2)^3*(A*a*b^2 - 2*A*b^3 - A*a^3 + A*a^2*b + B*a*b^2))/(
a^2*(a + b)*(a - b)) + (2*tan(c/2 + (d*x)/2)*(A*a^3 - 2*A*b^3 - A*a*b^2 + A
*a^2*b + B*a*b^2))/(a^2*(a + b)*(a - b)))/(d*(a + b - tan(c/2 + (d*x)/2)^4*
(a - b) + 2*b*tan(c/2 + (d*x)/2)^2)) + (log(tan(c/2 + (d*x)/2) - 1i)*(2*A*b
 - B*a)*1i)/(a^3*d) - (log(tan(c/2 + (d*x)/2) + 1i)*(A*b*2i - B*a*1i))/(a^3
*d) - (b*atan(((b*((32*tan(c/2 + (d*x)/2)*(8*A^2*b^8 + B^2*a^8 - 8*A^2*a*b^
7 - 2*B^2*a^7*b - 16*A^2*a^2*b^6 + 16*A^2*a^3*b^5 + 5*A^2*a^4*b^4 - 8*A^2*a
^5*b^3 + 4*A^2*a^6*b^2 + 2*B^2*a^2*b^6 - 2*B^2*a^3*b^5 - 5*B^2*a^4*b^4 + 4*
B^2*a^5*b^3 + 3*B^2*a^6*b^2 - 8*A*B*a*b^7 - 4*A*B*a^7*b + 8*A*B*a^2*b^6 + 1
8*A*B*a^3*b^5 - 16*A*B*a^4*b^4 - 8*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a^6*b + a
^7 - a^4*b^3 - a^5*b^2) + (b*((32*(A*a^7*b^5 - 2*A*a^6*b^6 - B*a^12 + 5*A*a
^8*b^4 - 3*A*a^9*b^3 - 3*A*a^10*b^2 + B*a^7*b^5 - 3*B*a^9*b^3 + B*a^10*b^2
+ 2*A*a^11*b + 2*B*a^11*b))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (32*b*tan(c
/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b -
B*a*b^2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10
*b^2))/((a^6*b + a^7 - a^4*b^3 - a^5*b^2)*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^
```

7*b^2)))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2)*1i)/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2) + (b*((32*tan(c/2 + (d*x)/2)*(8*A^2*b^8 + B^2*a^8 - 8*A^2*a*b^7 - 2*B^2*a^7*b - 16*A^2*a^2*b^6 + 16*A^2*a^3*b^5 + 5*A^2*a^4*b^4 - 8*A^2*a^5*b^3 + 4*A^2*a^6*b^2 + 2*B^2*a^2*b^6 - 2*B^2*a^3*b^5 - 5*B^2*a^4*b^4 + 4*B^2*a^5*b^3 + 3*B^2*a^6*b^2 - 8*A*B*a*b^7 - 4*A*B*a^7*b + 8*A*B*a^2*b^6 + 18*A*B*a^3*b^5 - 16*A*B*a^4*b^4 - 8*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) - (b*((32*(A*a^7*b^5 - 2*A*a^6*b^6 - B*a^12 + 5*A*a^8*b^4 - 3*A*a^9*b^3 - 3*A*a^10*b^2 + B*a^7*b^5 - 3*B*a^9*b^3 + B*a^10*b^2 + 2*A*a^11*b + 2*B*a^11*b))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (32*b*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/((a^6*b + a^7 - a^4*b^3 - a^5*b^2)*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2)*1i)/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))/((64*(8*A^3*b^8 - 4*A^3*a*b^7 - 2*B^3*a^7*b - 20*A^3*a^2*b^6 + 6*A^3*a^3*b^5 + 12*A^3*a^4*b^4 - B^3*a^3*b^5 + B^3*a^4*b^4 + 3*B^3*a^5*b^3 - 2*B^3*a^6*b^2 - 12*A^2*B*a*b^7 + 6*A*B^2*a^2*b^6 - 5*A*B^2*a^3*b^5 - 17*A*B^2*a^4*b^4 + 9*A*B^2*a^5*b^3 + 11*A*B^2*a^6*b^2 + 8*A^2*B*a^2*b^6 + 32*A^2*B*a^3*b^5 - 13*A^2*B*a^4*b^4 - 20*A^2*B*a^5*b^3))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b*((32*tan(c/2 + (d*x)/2)*(8*A^2*b^8 + B^2*a^8 - 8*A^2*a*b^7 - 2*B^2*a^7*b - 16*A^2*a^2*b^6 + 16*A^2*a^3*b^5 + 5*A^2*a^4*b^4 - 8*A^2*a^5*b^3 + 4*A^2*a^6*b^2 + 2*B^2*a^2*b^6 - 2*B^2*a^3*b^5 - 5*B^2*a^4*b^4 + 4*B^2*a^5*b^3 + 3*B^2*a^6*b^2 - 8*A*B*a*b^7 - 4*A*B*a^7*b + 8*A*B*a^2*b^6 + 18*A*B*a^3*b^5 - 16*A*B*a^4*b^4 - 8*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) + (b*((32*(A*a^7*b^5 - 2*A*a^6*b^6 - B*a^12 + 5*A*a^8*b^4 - 3*A*a^9*b^3 - 3*A*a^10*b^2 + B*a^7*b^5 - 3*B*a^9*b^3 + B*a^10*b^2 + 2*A*a^11*b + 2*B*a^11*b))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (32*b*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2)*(2*a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/((a^6*b + a^7 - a^4*b^3 - a^5*b^2)*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2) + (b*((32*tan(c/2 + (d*x)/2)*(8*A^2*b^8 + B^2*a^8 - 8*A^2*a*b^7 - 2*B^2*a^7*b - 16*A^2*a^2*b^6 + 16*A^2*a^3*b^5 + 5*A^2*a^4*b^4 - 8*A^2*a^5*b^3 + 4*A^2*a^6*b^2 + 2*B^2*a^2*b^6 - 2*B^2*a^3*b^5 - 5*B^2*a^4*b^4 + 4*B^2*a^5*b^3 + 3*B^2*a^6*b^2 - 8*A*B*a*b^7 - 4*A*B*a^7*b + 8*A*B*a^2*b^6 + 18*A*B*a^3*b^5 - 16*A*B*a^4*b^4 - 8*A*B*a^5*b^3 + 8*A*B*a^6*b^2))/(a^6*b + a^7 - a^4*b^3 - a^5*b^2) - (b*((32*(A*a^7*b^5 - 2*A*a^6*b^6 - B*a^12 + 5*A*a^8*b^4 - 3*A*a^9*b^3 - 3*A*a^10*b^2 + B*a^7*b^5 - 3*B*a^9*b^3 + B*a^10*b^2 + 2*A*a^11*b + 2*B*a^11*b))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (32*b*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2)*(2*

```
a^11*b - 2*a^6*b^6 + 2*a^7*b^5 + 4*a^8*b^4 - 4*a^9*b^3 - 2*a^10*b^2))/((a^6
*b + a^7 - a^4*b^3 - a^5*b^2)*(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*((a
 + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3*A*a^2*b - B*a*b^2))/(a^9 -
a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2))*((a + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*
B*a^3 - 3*A*a^2*b - B*a*b^2))/(a^9 - a^3*b^6 + 3*a^5*b^4 - 3*a^7*b^2)))*((a
 + b)^3*(a - b)^3)^(1/2)*(2*A*b^3 + 2*B*a^3 - 3...
```

3.326 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=261

$$\frac{(a^2A+6Ab^2-4abB)\,x}{2a^4}-\frac{2b^2(4a^2Ab-3Ab^3-3a^3B+2ab^2B)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{3/2}(a+b)^{3/2}d}-\frac{(2a^2Ab-3A$$

```
[Out] 1/2*(A*a^2+6*A*b^2-4*B*a*b)*x/a^4-2*b^2*(4*A*a^2*b-3*A*b^3-3*B*a^3+2*B*a*b^
2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/(a-b)^(3/2)/(a+b
)^(3/2)/d-(2*A*a^2*b-3*A*b^3-B*a^3+2*B*a*b^2)*sin(d*x+c)/a^3/(a^2-b^2)/d+1/
2*(A*a^2-3*A*b^2+2*B*a*b)*cos(d*x+c)*sin(d*x+c)/a^2/(a^2-b^2)/d+b*(A*b-B*a)
*cos(d*x+c)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.59, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4115, 4189, 4004, 3916, 2738, 214}

$$\frac{(a^2A+2abB-3Ab^2)\sin(c+dx)\cos(c+dx)}{2a^2d\,(a^2-b^2)}+\frac{b(Ab-aB)\sin(c+dx)\cos(c+dx)}{ad\,(a^2-b^2)\,(a+b\sec(c+dx))}+\frac{x(a^2A-4abB+6Ab^2)}{2a^4}-\frac{(a^3(-B)+2a^2Ab+2ab^2B-3Ab^3)\sin(c+dx)}{a^3d\,(a^2-b^2)}-\frac{2b^2(-3a^3B+4a^2Ab+2ab^2B-3Ab^3)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{3/2}(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((a^2*A + 6*A*b^2 - 4*a*b*B)*x)/(2*a^4) - (2*b^2*(4*a^2*A*b - 3*A*b^3 - 3*a
^3*B + 2*a*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4
*(a - b)^(3/2)*(a + b)^(3/2)*d) - ((2*a^2*A*b - 3*A*b^3 - a^3*B + 2*a*b^2*B
)*Sin[c + d*x])/(a^3*(a^2 - b^2)*d) + ((a^2*A - 3*A*b^2 + 2*a*b*B)*Cos[c +
d*x]*Sin[c + d*x])/(2*a^2*(a^2 - b^2)*d) + (b*(A*b - a*B)*Cos[c + d*x]*Sin[
c + d*x])/(a*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = \frac{b(Ab-aB)\cos(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{\int \frac{\cos^2(c+dx)\left(-a^2A+3Ab^2-2abB-\right.}{}}{}$$

$$= \frac{\left(a^2A-3Ab^2+2abB\right)\cos(c+dx)\sin(c+dx)}{2a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\cos(}{a\left(a^2-b^2\right)d(a}$$

$$= -\frac{\left(2a^2Ab-3Ab^3-a^3B+2ab^2B\right)\sin(c+dx)}{a^3\left(a^2-b^2\right)d} + \frac{\left(a^2A-3Ab^2+2\right.}{2}$$

$$= \frac{\left(a^2A+6Ab^2-4abB\right)x}{2a^4} - \frac{\left(2a^2Ab-3Ab^3-a^3B+2ab^2B\right)\sin(c}{a^3\left(a^2-b^2\right)d}$$

$$= \frac{\left(a^2A+6Ab^2-4abB\right)x}{2a^4} - \frac{\left(2a^2Ab-3Ab^3-a^3B+2ab^2B\right)\sin(c}{a^3\left(a^2-b^2\right)d}$$

$$= \frac{\left(a^2A+6Ab^2-4abB\right)x}{2a^4} - \frac{\left(2a^2Ab-3Ab^3-a^3B+2ab^2B\right)\sin(c}{a^3\left(a^2-b^2\right)d}$$

$$= \frac{\left(a^2A+6Ab^2-4abB\right)x}{2a^4} - \frac{2b^2\left(4a^2Ab-3Ab^3-3a^3B+2ab^2B\right)\mathrm{ta}}{a^4(a-b)^{3/2}(a+}$$

Mathematica [A]
time = 1.11, size = 184, normalized size = 0.70

$$\frac{2\left(a^2A+6Ab^2-4abB\right)(c+dx) - \frac{8b^2\left(-4a^2Ab+3Ab^3+3a^3B-2ab^2B\right)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}} + 4a(-2Ab+aB)\sin(c+dx) - \frac{4ab^3(-Ab+aB)\sin(c+dx)}{(a-b)(a+b)(b+a\cos(c+dx))} + a^2A\sin(2(c+dx))}{4a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (2*(a^2*A + 6*A*b^2 - 4*a*b*B)*(c + d*x) - (8*b^2*(-4*a^2*A*b + 3*A*b^3 + 3
*a^3*B - 2*a*b^2*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(
a^2 - b^2)^(3/2) + 4*a*(-2*A*b + a*B)*Sin[c + d*x] - (4*a*b^3*(-(A*b) + a*B
)*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Cos[c + d*x])) + a^2*A*Sin[2*(c + d
*x)])/(4*a^4*d)
```

Maple [A]
time = 0.54, size = 270, normalized size = 1.03

method	result

derivativedivides	$\frac{\frac{2\left(\left(-\frac{1}{2}a^2A-2Aba+a^2B\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^2A-2Aba+a^2B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\left(a^2A+6A\,b^2-4Bab\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^4}+\frac{2b^2\left(\cdots\right.}{d}$
default	$\frac{\frac{2\left(\left(-\frac{1}{2}a^2A-2Aba+a^2B\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^2A-2Aba+a^2B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\left(a^2A+6A\,b^2-4Bab\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^4}+\frac{2b^2\left(\cdots\right.}{d}$
risch	$\frac{Ax}{2a^2}+\frac{3xA\,b^2}{a^4}-\frac{2xBb}{a^3}-\frac{i\mathrm{e}^{-i(dx+c)}Ab}{a^3d}+\frac{i\mathrm{e}^{i(dx+c)}Ab}{a^3d}+\frac{i\mathrm{e}^{-i(dx+c)}B}{2a^2d}-\frac{i\mathrm{e}^{i(dx+c)}B}{2a^2d}+\frac{iA\,\mathrm{e}^{-2i(dx+c)}}{8a^2d}-\frac{\cdots}{a^4(a^{\cdots}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(2/a^4*(((-1/2*a^2*A-2*A*b*a+a^2*B)*tan(1/2*d*x+1/2*c)^3+(1/2*a^2*A-2*A
*b*a+a^2*B)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(A*a^2+6*A*b
^2-4*B*a*b)*arctan(tan(1/2*d*x+1/2*c)))+2*b^2/a^4*(-b*a*(A*b-B*a)/(a^2-b^2)
*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)-(4*
A*a^2*b-3*A*b^3-3*B*a^3+2*B*a*b^2)/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh(
(a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 5.37, size = 970, normalized size = 3.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/2*((A*a^7 - 4*B*a^6*b + 4*A*a^5*b^2 + 8*B*a^4*b^3 - 11*A*a^3*b^4 - 4*B*a
^2*b^5 + 6*A*a*b^6)*d*x*cos(d*x + c) + (A*a^6*b - 4*B*a^5*b^2 + 4*A*a^4*b^3
 + 8*B*a^3*b^4 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*d*x + (3*B*a^3*b^3 - 4
*A*a^2*b^4 - 2*B*a*b^5 + 3*A*b^6 + (3*B*a^4*b^2 - 4*A*a^3*b^3 - 2*B*a^2*b^4
 + 3*A*a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2
- 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + (2*B*a
^6*b - 4*A*a^5*b^2 - 6*B*a^4*b^3 + 10*A*a^3*b^4 + 4*B*a^2*b^5 - 6*A*a*b^6 +
 (A*a^7 - 2*A*a^5*b^2 + A*a^3*b^4)*cos(d*x + c)^2 + (2*B*a^7 - 3*A*a^6*b -
4*B*a^5*b^2 + 6*A*a^4*b^3 + 2*B*a^3*b^4 - 3*A*a^2*b^5)*cos(d*x + c))*sin(d*
x + c))/((a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c) + (a^8*b - 2*a^6*b^3 +
a^4*b^5)*d), 1/2*((A*a^7 - 4*B*a^6*b + 4*A*a^5*b^2 + 8*B*a^4*b^3 - 11*A*a^3
*b^4 - 4*B*a^2*b^5 + 6*A*a*b^6)*d*x*cos(d*x + c) + (A*a^6*b - 4*B*a^5*b^2 +
 4*A*a^4*b^3 + 8*B*a^3*b^4 - 11*A*a^2*b^5 - 4*B*a*b^6 + 6*A*b^7)*d*x + 2*(3
*B*a^3*b^3 - 4*A*a^2*b^4 - 2*B*a*b^5 + 3*A*b^6 + (3*B*a^4*b^2 - 4*A*a^3*b^3
 - 2*B*a^2*b^4 + 3*A*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^
2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (2*B*a^6*b - 4*
A*a^5*b^2 - 6*B*a^4*b^3 + 10*A*a^3*b^4 + 4*B*a^2*b^5 - 6*A*a*b^6 + (A*a^7 -
 2*A*a^5*b^2 + A*a^3*b^4)*cos(d*x + c)^2 + (2*B*a^7 - 3*A*a^6*b - 4*B*a^5*b
^2 + 6*A*a^4*b^3 + 2*B*a^3*b^4 - 3*A*a^2*b^5)*cos(d*x + c))*sin(d*x + c))/(
(a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c) + (a^8*b - 2*a^6*b^3 + a^4*b^5)*
d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\cos^2(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/(a + b*sec(c + d*x))**2, x)
```

Giac [A]
time = 0.50, size = 340, normalized size = 1.30

$$\frac{\frac{4\,(3\,Ba^3b^2-4\,Aa^2b^3-2\,Bab^4+3\,Ab^5)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-a^4b^2)\sqrt{-a^2+b^2}}+\frac{4\,\left(Bab^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Ab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{(a^5-a^3b^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}+\frac{(Aa^2-4\,Bab+6\,Ab^2)(dx+c)}{a^4}-\frac{2\left(Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+4\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Aa\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,Ba\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+4\,Ab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^2a^3}}{2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] 1/2*(4*(3*B*a^3*b^2 - 4*A*a^2*b^3 - 2*B*a*b^4 + 3*A*b^5)*(pi*floor(1/2*(d*x
 + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1
/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6 - a^4*b^2)*sqrt(-a^2 + b^2)) + 4*
(B*a*b^3*tan(1/2*d*x + 1/2*c) - A*b^4*tan(1/2*d*x + 1/2*c))/((a^5 - a^3*b^2
)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) + (A*a^2 -
 4*B*a*b + 6*A*b^2)*(d*x + c)/a^4 - 2*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*t
an(1/2*d*x + 1/2*c)^3 + 4*A*b*tan(1/2*d*x + 1/2*c)^3 - A*a*tan(1/2*d*x + 1/
2*c) - 2*B*a*tan(1/2*d*x + 1/2*c) + 4*A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d
*x + 1/2*c)^2 + 1)^2*a^3))/d
```

Mupad [B]
time = 11.11, size = 2500, normalized size = 9.58

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^2,x)
```

```
[Out] (b^2*atan(((b^2*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(A^2*a^1
0 + 72*A^2*b^10 - 72*A^2*a*b^9 - 2*A^2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^
3*b^7 + 17*A^2*a^4*b^6 - 26*A^2*a^5*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 +
 11*A^2*a^8*b^2 + 32*B^2*a^2*b^8 - 32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2
*a^5*b^5 + 20*B^2*a^6*b^4 - 32*B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9
- 8*A*B*a^9*b + 96*A*B*a^2*b^8 + 176*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B
*a^5*b^5 + 64*A*B*a^6*b^4 - 40*A*B*a^7*b^3 + 16*A*B*a^8*b^2))/(a^8*b + a^9
- a^6*b^3 - a^7*b^2) + (b^2*((8*(2*A*a^15 - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28
*A*a^10*b^5 - 14*A*a^11*b^4 - 16*A*a^12*b^3 + 6*A*a^13*b^2 + 8*B*a^9*b^6 -
4*B*a^10*b^5 - 20*B*a^11*b^4 + 12*B*a^12*b^3 + 12*B*a^13*b^2 - 8*B*a^14*b))
/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (8*b^2*tan(c/2 + (d*x)/2)*((a + b)^
3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B*a*b^2)*(8*a^13*b -
8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/((a^8*b +
a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*((a + b
)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B*a*b^2))/(a^10 - a
^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*(3*A*b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B*a*b^2
)*1i)/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2) + (b^2*((a + b)^3*(a - b)^3)
^(1/2)*((8*tan(c/2 + (d*x)/2)*(A^2*a^10 + 72*A^2*b^10 - 72*A^2*a*b^9 - 2*A^
2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^7 + 17*A^2*a^4*b^6 - 26*A^2*a^5*b
^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11*A^2*a^8*b^2 + 32*B^2*a^2*b^8 - 32
*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a^5*b^5 + 20*B^2*a^6*b^4 - 32*B^2*a^
7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 - 8*A*B*a^9*b + 96*A*B*a^2*b^8 + 176*
A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B*a^5*b^5 + 64*A*B*a^6*b^4 - 40*A*B*a^
7*b^3 + 16*A*B*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b^2*((8*(2*A*
a^15 - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28*A*a^10*b^5 - 14*A*a^11*b^4 - 16*A*a^
```

12*b^3 + 6*A*a^13*b^2 + 8*B*a^9*b^6 - 4*B*a^10*b^5 - 20*B*a^11*b^4 + 12*B*a^12*b^3 + 12*B*a^13*b^2 - 8*B*a^14*b))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) + (8*b^2*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B*a*b^2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*((a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B*a*b^2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*(3*A*b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B*a*b^2)*1i)/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))/((16*(108*A^3*b^11 - 54*A^3*a*b^10 - 216*A^3*a^2*b^9 + 81*A^3*a^3*b^8 + 63*A^3*a^4*b^7 - 9*A^3*a^5*b^6 + 41*A^3*a^6*b^5 - 4*A^3*a^7*b^4 + 4*A^3*a^8*b^3 - 32*B^3*a^3*b^8 + 16*B^3*a^4*b^7 + 80*B^3*a^5*b^6 - 24*B^3*a^6*b^5 - 48*B^3*a^7*b^4 - 216*A^2*B*a*b^10 + 144*A*B^2*a^2*b^9 - 72*A*B^2*a^3*b^8 - 336*A*B^2*a^4*b^7 + 108*A*B^2*a^5*b^6 + 168*A*B^2*a^6*b^5 - 6*A*B^2*a^7*b^4 + 24*A*B^2*a^8*b^3 + 108*A^2*B*a^2*b^9 + 468*A^2*B*a^3*b^8 - 162*A^2*B*a^4*b^7 - 186*A^2*B*a^5*b^6 + 15*A^2*B*a^6*b^5 - 63*A^2*B*a^7*b^4 + 3*A^2*B*a^8*b^3 - 3*A^2*B*a^9*b^2))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (b^2*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(A^2*a^10 + 72*A^2*b^10 - 72*A^2*a*b^9 - 2*A^2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^7 + 17*A^2*a^4*b^6 - 26*A^2*a^5*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11*A^2*a^8*b^2 + 32*B^2*a^2*b^8 - 32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a^5*b^5 + 20*B^2*a^6*b^4 - 32*B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 - 8*A*B*a^9*b + 96*A*B*a^2*b^8 + 176*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B*a^5*b^5 + 64*A*B*a^6*b^4 - 40*A*B*a^7*b^3 + 16*A*B*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) + (b^2*((8*(2*A*a^15 - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28*A*a^10*b^5 - 14*A*a^11*b^4 - 16*A*a^12*b^3 + 6*A*a^13*b^2 + 8*B*a^9*b^6 - 4*B*a^10*b^5 - 20*B*a^11*b^4 + 12*B*a^12*b^3 + 12*B*a^13*b^2 - 8*B*a^14*b))/(a^11*b + a^12 - a^9*b^3 - a^10*b^2) - (8*b^2*tan(c/2 + (d*x)/2)*((a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B*a*b^2)*(8*a^13*b - 8*a^8*b^6 + 8*a^9*b^5 + 16*a^10*b^4 - 16*a^11*b^3 - 8*a^12*b^2))/((a^8*b + a^9 - a^6*b^3 - a^7*b^2)*(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2)))*((a + b)^3*(a - b)^3)^(1/2)*(3*A*b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B*a*b^2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2))*(3*A*b^3 + 3*B*a^3 - 4*A*a^2*b - 2*B*a*b^2))/(a^10 - a^4*b^6 + 3*a^6*b^4 - 3*a^8*b^2) + (b^2*((a + b)^3*(a - b)^3)^(1/2)*((8*tan(c/2 + (d*x)/2)*(A^2*a^10 + 72*A^2*b^10 - 72*A^2*a*b^9 - 2*A^2*a^9*b - 120*A^2*a^2*b^8 + 120*A^2*a^3*b^7 + 17*A^2*a^4*b^6 - 26*A^2*a^5*b^5 + 23*A^2*a^6*b^4 - 20*A^2*a^7*b^3 + 11*A^2*a^8*b^2 + 32*B^2*a^2*b^8 - 32*B^2*a^3*b^7 - 64*B^2*a^4*b^6 + 64*B^2*a^5*b^5 + 20*B^2*a^6*b^4 - 32*B^2*a^7*b^3 + 16*B^2*a^8*b^2 - 96*A*B*a*b^9 - 8*A*B*a^9*b + 96*A*B*a^2*b^8 + 176*A*B*a^3*b^7 - 176*A*B*a^4*b^6 - 40*A*B*a^5*b^5 + 64*A*B*a^6*b^4 - 40*A*B*a^7*b^3 + 16*A*B*a^8*b^2))/(a^8*b + a^9 - a^6*b^3 - a^7*b^2) - (b^2*((8*(2*A*a^15 - 12*A*a^8*b^7 + 6*A*a^9*b^6 + 28*A*a^10*b^5 - 14*A*a^11*b^4 - 16*A*a^12*b^3 + 6*A*a^13*b^2 + 8*B*a^9*b^6 - 4*B*a^10*b^5 - 20*B*a^11*...

3.327 $\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=346

$$-\frac{(2a^2Ab+8Ab^3-a^3B-6ab^2B)\,x}{2a^5}+\frac{2b^3(5a^2Ab-4Ab^3-4a^3B+3ab^2B)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^5(a-b)^{3/2}(a+b)^{3/2}d}+\frac{(2$$

```
[Out] -1/2*(2*A*a^2*b+8*A*b^3-B*a^3-6*B*a*b^2)*x/a^5+2*b^3*(5*A*a^2*b-4*A*b^3-4*B
*a^3+3*B*a*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^5/(a-
b)^(3/2)/(a+b)^(3/2)/d+1/3*(2*A*a^4+7*A*a^2*b^2-12*A*b^4-6*B*a^3*b+9*B*a*b^
3)*sin(d*x+c)/a^4/(a^2-b^2)/d-1/2*(2*A*a^2*b-4*A*b^3-B*a^3+3*B*a*b^2)*cos(d
*x+c)*sin(d*x+c)/a^3/(a^2-b^2)/d+1/3*(A*a^2-4*A*b^2+3*B*a*b)*cos(d*x+c)^2*s
in(d*x+c)/a^2/(a^2-b^2)/d+b*(A*b-B*a)*cos(d*x+c)^2*sin(d*x+c)/a/(a^2-b^2)/d
/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.84, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4115, 4189, 4004, 3916, 2738, 214}

$$\frac{(a^2A+3abB-4Ab^2)\sin(c+dx)\cos^2(c+dx)}{3a^2d\,(a^2-b^2)}+\frac{b(Ab-aB)\sin(c+dx)\cos^2(c+dx)}{ad\,(a^2-b^2)\,(a+b\sec(c+dx))}-\frac{(a^3(-B)+2a^2Ab+3ab^2B-4Ab^3)\sin(c+dx)\cos(c+dx)}{2a^3d\,(a^2-b^2)}+\frac{2b^3(-4a^3B+5a^2Ab+3ab^2B-4Ab^3)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^5d(a-b)^{3/2}(a+b)^{3/2}}-\frac{x(a^3(-B)+2a^2Ab-6ab^2B+8Ab^3)}{2a^5}+\frac{(2a^4A-6a^3bB+7a^2Ab^2+9ab^3B-12Ab^4)\sin(c+dx)}{3a^4d\,(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -1/2*((2*a^2*A*b + 8*A*b^3 - a^3*B - 6*a*b^2*B)*x)/a^5 + (2*b^3*(5*a^2*A*b
- 4*A*b^3 - 4*a^3*B + 3*a*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqr
t[a + b]])/(a^5*(a - b)^(3/2)*(a + b)^(3/2)*d) + ((2*a^4*A + 7*a^2*A*b^2 -
12*A*b^4 - 6*a^3*b*B + 9*a*b^3*B)*Sin[c + d*x])/(3*a^4*(a^2 - b^2)*d) - ((2
*a^2*A*b - 4*A*b^3 - a^3*B + 3*a*b^2*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*(
a^2 - b^2)*d) + ((a^2*A - 4*A*b^2 + 3*a*b*B)*Cos[c + d*x]^2*Sin[c + d*x])/(
3*a^2*(a^2 - b^2)*d) + (b*(A*b - a*B)*Cos[c + d*x]^2*Sin[c + d*x])/(a*(a^2
- b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
```

```
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = \frac{b(Ab-aB)\cos^2(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \int \frac{\cos^3(c+dx)\left(-a^2A+4Ab^2-3abB+\cdots\right.}{\cdots}$$

$$= \frac{\left(a^2A-4Ab^2+3abB\right)\cos^2(c+dx)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\cos^2(\cdots}{a\left(a^2-b^2\right)d(a\cdots}$$

$$= -\frac{\left(2a^2Ab-4Ab^3-a^3B+3ab^2B\right)\cos(c+dx)\sin(c+dx)}{2a^3\left(a^2-b^2\right)d} + \frac{\left(a^2A\cdots\right.}{\cdots}$$

$$= \frac{\left(2a^4A+7a^2Ab^2-12Ab^4-6a^3bB+9ab^3B\right)\sin(c+dx)}{3a^4\left(a^2-b^2\right)d} - \frac{\left(2a^2Ab\cdots\right.}{\cdots}$$

$$= -\frac{\left(2a^2Ab+8Ab^3-a^3B-6ab^2B\right)x}{2a^5} + \frac{\left(2a^4A+7a^2Ab^2-12Ab^4-\cdots\right.}{3a^4\left(a^2\cdots\right.}$$

$$= -\frac{\left(2a^2Ab+8Ab^3-a^3B-6ab^2B\right)x}{2a^5} + \frac{\left(2a^4A+7a^2Ab^2-12Ab^4-\cdots\right.}{3a^4\left(a^2\cdots\right.}$$

$$= -\frac{\left(2a^2Ab+8Ab^3-a^3B-6ab^2B\right)x}{2a^5} + \frac{\left(2a^4A+7a^2Ab^2-12Ab^4-\cdots\right.}{3a^4\left(a^2\cdots\right.}$$

$$= -\frac{\left(2a^2Ab+8Ab^3-a^3B-6ab^2B\right)x}{2a^5} + \frac{2b^3\left(5a^2Ab-4Ab^3-4a^3B-\cdots\right.}{a^5(a\cdots}$$

Mathematica [A]
time = 1.36, size = 224, normalized size = 0.65

$$\frac{6\left(-2a^2Ab-8Ab^3+a^3B+6ab^2B\right)(c+dx)+\frac{24b^3\left(-5a^2Ab+4Ab^3+4a^3B-3ab^2B\right)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{3/2}}+3a\left(3a^2A+12Ab^2-8abB\right)\sin(c+dx)+\frac{12ab^4(-Ab+aB)\sin(c+dx)}{(a-b)(a+b)(b+a\cos(c+dx))}+3a^2(-2Ab+aB)\sin(2(c+dx))+a^3A\sin(3(c+dx))}{12a^5d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] (6*(-2*a^2*A*b - 8*A*b^3 + a^3*B + 6*a*b^2*B)*(c + d*x) + (24*b^3*(-5*a^2*A
*b + 4*A*b^3 + 4*a^3*B - 3*a*b^2*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqr
t[a^2 - b^2]])/(a^2 - b^2)^(3/2) + 3*a*(3*a^2*A + 12*A*b^2 - 8*a*b*B)*Sin[c
 + d*x] + (12*a*b^4*(-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a + b)*(b + a*Co
s[c + d*x])) + 3*a^2*(-2*A*b + a*B)*Sin[2*(c + d*x)] + a^3*A*Sin[3*(c + d*x
)])/(12*a^5*d)
```

Maple [A]
time = 0.56, size = 346, normalized size = 1.00 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-2*b^3/a^5*(-b*a*(A*b-B*a)/(a^2-b^2)*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x
+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)-(5*A*a^2*b-4*A*b^3-4*B*a^3+3*B*a*b^2)
/(a+b)/(a-b)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a
-b))^(1/2)))-2/a^5*(((-A*a^3-A*a^2*b-3*A*b^2*a+1/2*a^3*B+2*B*b*a^2)*tan(1/2
*d*x+1/2*c)^5+(-2/3*A*a^3-6*A*b^2*a+4*B*b*a^2)*tan(1/2*d*x+1/2*c)^3+(-A*a^3
-3*A*b^2*a+2*B*b*a^2+A*a^2*b-1/2*a^3*B)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+
1/2*c)^2)^3+1/2*(2*A*a^2*b+8*A*b^3-B*a^3-6*B*a*b^2)*arctan(tan(1/2*d*x+1/2*
c))))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**A**]
time = 4.08, size = 1167, normalized size = 3.37

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="fr
icas")
```

```
[Out] [1/6*(3*(B*a^8 - 2*A*a^7*b + 4*B*a^6*b^2 - 4*A*a^5*b^3 - 11*B*a^4*b^4 + 14*
A*a^3*b^5 + 6*B*a^2*b^6 - 8*A*a*b^7)*d*x*cos(d*x + c) + 3*(B*a^7*b - 2*A*a^
6*b^2 + 4*B*a^5*b^3 - 4*A*a^4*b^4 - 11*B*a^3*b^5 + 14*A*a^2*b^6 + 6*B*a*b^7
 - 8*A*b^8)*d*x + 3*(4*B*a^3*b^4 - 5*A*a^2*b^5 - 3*B*a*b^6 + 4*A*b^7 + (4*B
*a^4*b^3 - 5*A*a^3*b^4 - 3*B*a^2*b^5 + 4*A*a*b^6)*cos(d*x + c))*sqrt(a^2 -
b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 -
b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 +
 2*a*b*cos(d*x + c) + b^2)) + (4*A*a^7*b - 12*B*a^6*b^2 + 10*A*a^5*b^3 + 30
*B*a^4*b^4 - 38*A*a^3*b^5 - 18*B*a^2*b^6 + 24*A*a*b^7 + 2*(A*a^8 - 2*A*a^6*
b^2 + A*a^4*b^4)*cos(d*x + c)^3 + (3*B*a^8 - 4*A*a^7*b - 6*B*a^6*b^2 + 8*A*
a^5*b^3 + 3*B*a^4*b^4 - 4*A*a^3*b^5)*cos(d*x + c)^2 + (4*A*a^8 - 9*B*a^7*b
```

```
+ 4*A*a^6*b^2 + 18*B*a^5*b^3 - 20*A*a^4*b^4 - 9*B*a^3*b^5 + 12*A*a^2*b^6)*c
os(d*x + c))*sin(d*x + c))/((a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*x + c) + (
a^9*b - 2*a^7*b^3 + a^5*b^5)*d), 1/6*(3*(B*a^8 - 2*A*a^7*b + 4*B*a^6*b^2 -
4*A*a^5*b^3 - 11*B*a^4*b^4 + 14*A*a^3*b^5 + 6*B*a^2*b^6 - 8*A*a*b^7)*d*x*co
s(d*x + c) + 3*(B*a^7*b - 2*A*a^6*b^2 + 4*B*a^5*b^3 - 4*A*a^4*b^4 - 11*B*a^
3*b^5 + 14*A*a^2*b^6 + 6*B*a*b^7 - 8*A*b^8)*d*x - 6*(4*B*a^3*b^4 - 5*A*a^2*
b^5 - 3*B*a*b^6 + 4*A*b^7 + (4*B*a^4*b^3 - 5*A*a^3*b^4 - 3*B*a^2*b^5 + 4*A*
a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x +
 c) + a)/((a^2 - b^2)*sin(d*x + c))) + (4*A*a^7*b - 12*B*a^6*b^2 + 10*A*a^5
*b^3 + 30*B*a^4*b^4 - 38*A*a^3*b^5 - 18*B*a^2*b^6 + 24*A*a*b^7 + 2*(A*a^8 -
 2*A*a^6*b^2 + A*a^4*b^4)*cos(d*x + c)^3 + (3*B*a^8 - 4*A*a^7*b - 6*B*a^6*b
^2 + 8*A*a^5*b^3 + 3*B*a^4*b^4 - 4*A*a^3*b^5)*cos(d*x + c)^2 + (4*A*a^8 - 9
*B*a^7*b + 4*A*a^6*b^2 + 18*B*a^5*b^3 - 20*A*a^4*b^4 - 9*B*a^3*b^5 + 12*A*a
^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*x
 + c) + (a^9*b - 2*a^7*b^3 + a^5*b^5)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\cos^3(c + dx)}{(a + b\sec(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**3/(a + b*sec(c + d*x))**2, x)
```

Giac [A]
time = 0.49, size = 473, normalized size = 1.37

$$-\frac{\frac{12\,(4\,Ba^3b^3-5\,Aa^2b^4-3\,Bab^5+4\,Ab^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^7-a^5b^2)\sqrt{-a^2+b^2}}+\frac{12\,\left(Bab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-Ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{(a^6-a^4b^2)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)}-\frac{3\,(Ba^3-2\,Aa^2b+6\,Bab^2-8\,Ab^3)(dx+c)}{a^5}-\frac{2\left(6\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-3\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+6\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5-12\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+18\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^5+4\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-24\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+36\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+6\,Aa^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Ba^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-6\,Aab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-12\,Bab\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+18\,Ab^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)^3a^4}}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm="gi
ac")
```

```
[Out] -1/6*(12*(4*B*a^3*b^3 - 5*A*a^2*b^4 - 3*B*a*b^5 + 4*A*b^6)*(pi*floor(1/2*(d
*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan
(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^7 - a^5*b^2)*sqrt(-a^2 + b^2)) +
12*(B*a*b^4*tan(1/2*d*x + 1/2*c) - A*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - a^4*
b^2)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)) - 3*(B*
a^3 - 2*A*a^2*b + 6*B*a*b^2 - 8*A*b^3)*(d*x + c)/a^5 - 2*(6*A*a^2*tan(1/2*d
*x + 1/2*c)^5 - 3*B*a^2*tan(1/2*d*x + 1/2*c)^5 + 6*A*a*b*tan(1/2*d*x + 1/2*
c)^5 - 12*B*a*b*tan(1/2*d*x + 1/2*c)^5 + 18*A*b^2*tan(1/2*d*x + 1/2*c)^5 +
4*A*a^2*tan(1/2*d*x + 1/2*c)^3 - 24*B*a*b*tan(1/2*d*x + 1/2*c)^3 + 36*A*b^2
*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^2*tan(1/2*d*x + 1/2*c) + 3*B*a^2*tan(1/2*d*
```

```
x + 1/2*c) - 6*A*a*b*tan(1/2*d*x + 1/2*c) - 12*B*a*b*tan(1/2*d*x + 1/2*c) +
 18*A*b^2*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^3*a^4))/d
```

Mupad [B]
time = 11.66, size = 2500, normalized size = 7.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^2,x)
```

```
[Out] - ((tan(c/2 + (d*x)/2)^7*(2*A*a^5 + 8*A*b^5 - B*a^5 - 6*A*a^2*b^3 + 2*A*a^3
*b^2 + 3*B*a^2*b^3 + 5*B*a^3*b^2 - 4*A*a*b^4 - 6*B*a*b^4 - 3*B*a^4*b))/(a^4
*(a + b)*(a - b)) - (tan(c/2 + (d*x)/2)*(2*A*a^5 - 8*A*b^5 + B*a^5 + 6*A*a^
2*b^3 + 2*A*a^3*b^2 + 3*B*a^2*b^3 - 5*B*a^3*b^2 - 4*A*a*b^4 + 6*B*a*b^4 - 3
*B*a^4*b))/(a^4*(a + b)*(a - b)) + (tan(c/2 + (d*x)/2)^3*(2*A*a^5 + 72*A*b^
5 + 3*B*a^5 - 38*A*a^2*b^3 - 14*A*a^3*b^2 - 9*B*a^2*b^3 + 33*B*a^3*b^2 + 12
*A*a*b^4 - 16*A*a^4*b - 54*B*a*b^4 + 9*B*a^4*b))/(3*a^4*(a + b)*(a - b)) -
(tan(c/2 + (d*x)/2)^5*(2*A*a^5 - 72*A*b^5 - 3*B*a^5 + 38*A*a^2*b^3 - 14*A*a
^3*b^2 - 9*B*a^2*b^3 - 33*B*a^3*b^2 + 12*A*a*b^4 + 16*A*a^4*b + 54*B*a*b^4
+ 9*B*a^4*b))/(3*a^4*(a + b)*(a - b)))/(d*(a + b - tan(c/2 + (d*x)/2)^8*(a
- b) + tan(c/2 + (d*x)/2)^2*(2*a + 4*b) - tan(c/2 + (d*x)/2)^6*(2*a - 4*b)
+ 6*b*tan(c/2 + (d*x)/2)^4)) - (atan((((((8*(2*B*a^18 + 16*A*a^10*b^8 - 8*
A*a^11*b^7 - 36*A*a^12*b^6 + 16*A*a^13*b^5 + 20*A*a^14*b^4 - 4*A*a^15*b^3 -
 12*B*a^11*b^7 + 6*B*a^12*b^6 + 28*B*a^13*b^5 - 14*B*a^14*b^4 - 16*B*a^15*b
^3 + 6*B*a^16*b^2 - 4*A*a^17*b))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) - (8
*tan(c/2 + (d*x)/2)*(A*b^3*4i - (B*a^3*1i)/2 + A*a^2*b*1i - B*a*b^2*3i)*(8*
a^15*b - 8*a^10*b^6 + 8*a^11*b^5 + 16*a^12*b^4 - 16*a^13*b^3 - 8*a^14*b^2))
/(a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2)))*(A*b^3*4i - (B*a^3*1i)/2 + A*a^
2*b*1i - B*a*b^2*3i))/a^5 + (8*tan(c/2 + (d*x)/2)*(128*A^2*b^12 + B^2*a^12
- 128*A^2*a*b^11 - 2*B^2*a^11*b - 192*A^2*a^2*b^10 + 192*A^2*a^3*b^9 + 8*A^
2*a^4*b^8 - 8*A^2*a^5*b^7 + 28*A^2*a^6*b^6 - 48*A^2*a^7*b^5 + 28*A^2*a^8*b^
4 - 8*A^2*a^9*b^3 + 4*A^2*a^10*b^2 + 72*B^2*a^2*b^10 - 72*B^2*a^3*b^9 - 120
*B^2*a^4*b^8 + 120*B^2*a^5*b^7 + 17*B^2*a^6*b^6 - 26*B^2*a^7*b^5 + 23*B^2*a
^8*b^4 - 20*B^2*a^9*b^3 + 11*B^2*a^10*b^2 - 192*A*B*a*b^11 - 4*A*B*a^11*b +
 192*A*B*a^2*b^10 + 304*A*B*a^3*b^9 - 304*A*B*a^4*b^8 - 28*A*B*a^5*b^7 + 40
*A*B*a^6*b^6 - 52*A*B*a^7*b^5 + 64*A*B*a^8*b^4 - 36*A*B*a^9*b^3 + 8*A*B*a^1
0*b^2))/(a^10*b + a^11 - a^8*b^3 - a^9*b^2))*(A*b^3*4i - (B*a^3*1i)/2 + A*a
^2*b*1i - B*a*b^2*3i)*1i)/a^5 - (((((8*(2*B*a^18 + 16*A*a^10*b^8 - 8*A*a^11
*b^7 - 36*A*a^12*b^6 + 16*A*a^13*b^5 + 20*A*a^14*b^4 - 4*A*a^15*b^3 - 12*B*
a^11*b^7 + 6*B*a^12*b^6 + 28*B*a^13*b^5 - 14*B*a^14*b^4 - 16*B*a^15*b^3 + 6
*B*a^16*b^2 - 4*A*a^17*b))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (8*tan(c
/2 + (d*x)/2)*(A*b^3*4i - (B*a^3*1i)/2 + A*a^2*b*1i - B*a*b^2*3i)*(8*a^15*b
 - 8*a^10*b^6 + 8*a^11*b^5 + 16*a^12*b^4 - 16*a^13*b^3 - 8*a^14*b^2))/(a^5*
(a^10*b + a^11 - a^8*b^3 - a^9*b^2)))*(A*b^3*4i - (B*a^3*1i)/2 + A*a^2*b*1i
```

```
 - B*a*b^2*3i))/a^5 - (8*tan(c/2 + (d*x)/2)*(128*A^2*b^12 + B^2*a^12 - 128*
A^2*a*b^11 - 2*B^2*a^11*b - 192*A^2*a^2*b^10 + 192*A^2*a^3*b^9 + 8*A^2*a^4*
b^8 - 8*A^2*a^5*b^7 + 28*A^2*a^6*b^6 - 48*A^2*a^7*b^5 + 28*A^2*a^8*b^4 - 8*
A^2*a^9*b^3 + 4*A^2*a^10*b^2 + 72*B^2*a^2*b^10 - 72*B^2*a^3*b^9 - 120*B^2*a
^4*b^8 + 120*B^2*a^5*b^7 + 17*B^2*a^6*b^6 - 26*B^2*a^7*b^5 + 23*B^2*a^8*b^4
 - 20*B^2*a^9*b^3 + 11*B^2*a^10*b^2 - 192*A*B*a*b^11 - 4*A*B*a^11*b + 192*A
*B*a^2*b^10 + 304*A*B*a^3*b^9 - 304*A*B*a^4*b^8 - 28*A*B*a^5*b^7 + 40*A*B*a
^6*b^6 - 52*A*B*a^7*b^5 + 64*A*B*a^8*b^4 - 36*A*B*a^9*b^3 + 8*A*B*a^10*b^2)
)/(a^10*b + a^11 - a^8*b^3 - a^9*b^2))*(A*b^3*4i - (B*a^3*1i)/2 + A*a^2*b*1
i - B*a*b^2*3i)*1i)/a^5)/((16*(256*A^3*b^14 - 128*A^3*a*b^13 - 448*A^3*a^2*
b^12 + 192*A^3*a^3*b^11 + 48*A^3*a^4*b^10 - 24*A^3*a^5*b^9 + 124*A^3*a^6*b^
8 - 20*A^3*a^7*b^7 + 20*A^3*a^8*b^6 - 108*B^3*a^3*b^11 + 54*B^3*a^4*b^10 +
216*B^3*a^5*b^9 - 81*B^3*a^6*b^8 - 63*B^3*a^7*b^7 + 9*B^3*a^8*b^6 - 41*B^3*
a^9*b^5 + 4*B^3*a^10*b^4 - 4*B^3*a^11*b^3 - 576*A^2*B*a*b^13 + 432*A*B^2*a^
2*b^12 - 216*A*B^2*a^3*b^11 - 828*A*B^2*a^4*b^10 + 324*A*B^2*a^5*b^9 + 192*
A*B^2*a^6*b^8 - 39*A*B^2*a^7*b^7 + 183*A*B^2*a^8*b^6 - 21*A*B^2*a^9*b^5 + 2
1*A*B^2*a^10*b^4 + 288*A^2*B*a^2*b^12 + 1056*A^2*B*a^3*b^11 - 432*A^2*B*a^4
*b^10 - 180*A^2*B*a^5*b^9 + 54*A^2*B*a^6*b^8 - 264*A^2*B*a^7*b^7 + 36*A^2*B
*a^8*b^6 - 36*A^2*B*a^9*b^5))/(a^14*b + a^15 - a^12*b^3 - a^13*b^2) + (((((
8*(2*B*a^18 + 16*A*a^10*b^8 - 8*A*a^11*b^7 - 36*A*a^12*b^6 + 16*A*a^13*b^5
+ 20*A*a^14*b^4 - 4*A*a^15*b^3 - 12*B*a^11*b^7 + 6*B*a^12*b^6 + 28*B*a^13*b
^5 - 14*B*a^14*b^4 - 16*B*a^15*b^3 + 6*B*a^16*b^2 - 4*A*a^17*b))/(a^14*b +
a^15 - a^12*b^3 - a^13*b^2) - (8*tan(c/2 + (d*x)/2)*(A*b^3*4i - (B*a^3*1i)/
2 + A*a^2*b*1i - B*a*b^2*3i)*(8*a^15*b - 8*a^10*b^6 + 8*a^11*b^5 + 16*a^12*
b^4 - 16*a^13*b^3 - 8*a^14*b^2))/(a^5*(a^10*b + a^11 - a^8*b^3 - a^9*b^2)))
*(A*b^3*4i - (B*a^3*1i)/2 + A*a^2*b*1i - B*a*b^2*3i))/a^5 + (8*tan(c/2 + (d
*x)/2)*(128*A^2*b^12 + B^2*a^12 - 128*A^2*a*b^11 - 2*B^2*a^11*b - 192*A^2*a
^2*b^10 + 192*A^2*a^3*b^9 + 8*A^2*a^4*b^8 - 8*A^2*a^5*b^7 + 28*A^2*a^6*b^6
- 48*A^2*a^7*b^5 + 28*A^2*a^8*b^4 - 8*A^2*a^9*b^3 + 4*A^2*a^10*b^2 + 72*B^2
*a^2*b^10 - 72*B^2*a^3*b^9 - 120*B^2*a^4*b^8 + 120*B^2*a^5*b^7 + 17*B^2*a^6
*b^6 - 26*B^2*a^7*b^5 + 23*B^2*a^8*b^4 - 20*B^2*a^9*b^3 + 11*B^2*a^10*b^2 -
 192*A*B*a*b^11 - 4*A*B*a^11*b + 192*A*B*a^2*b^10 + 304*A*B*a^3*b^9 - 304*A
*B*a^4*b^8 - 28*A*B*a^5*b^7 + 40*A*B*a^6*b^6 - ...
```

3.328 $\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=407

$$-\frac{(6aAb-12a^2B-b^2B)\tanh^{-1}(\sin(c+dx))}{2b^5d}+\frac{a^2(6a^4Ab-15a^2Ab^3+12Ab^5-12a^5B+29a^3b^2B-20ab^4}{(a-b)^{5/2}b^5(a+b)^{5/2}d}$$

```
[Out] -1/2*(6*A*a*b-12*B*a^2-B*b^2)*arctanh(sin(d*x+c))/b^5/d+a^2*(6*A*a^4*b-15*A
*a^2*b^3+12*A*b^5-12*B*a^5+29*B*a^3*b^2-20*B*a*b^4)*arctanh((a-b)^(1/2)*tan
(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^5/(a+b)^(5/2)/d+1/2*(6*A*a^4*b-1
1*A*a^2*b^3+2*A*b^5-12*B*a^5+21*B*a^3*b^2-6*B*a*b^4)*tan(d*x+c)/b^4/(a^2-b^
2)^2/d-1/2*(3*A*a^3*b-6*A*a*b^3-6*B*a^4+10*B*a^2*b^2-B*b^4)*sec(d*x+c)*tan(
d*x+c)/b^3/(a^2-b^2)^2/d+1/2*a*(A*b-B*a)*sec(d*x+c)^3*tan(d*x+c)/b/(a^2-b^2
)/d/(a+b*sec(d*x+c))^2+1/2*a*(2*A*a^2*b-5*A*b^3-4*B*a^3+7*B*a*b^2)*sec(d*x+
c)^2*tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 1.32, antiderivative size = 407, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {4114, 4183, 4177, 4167, 4083, 3855, 3916, 2738, 214}

$$\frac{a(Ab-aB)\tan(c+dx)\sec^3(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{(-12a^2B+6aAb-b^2B)\tanh^{-1}(\sin(c+dx))}{2b^5d}+\frac{a(-4a^3B+2a^2Ab+7ab^2B-5Ab^3)\tan(c+dx)\sec^2(c+dx)}{2b^2d(a^2-b^2)^2(a+b\sec(c+dx))}-\frac{(-6a^4B+3a^3Ab+10a^2b^2B-6aAb^3-b^4B)\tan(c+dx)\sec(c+dx)}{2b^3d(a^2-b^2)^2}+\frac{(-12a^5B+6a^4Ab+21a^3b^2B-11a^2Ab^3-6ab^4B+2Ab^5)\tan(c+dx)}{2b^4d(a^2-b^2)^2}+\frac{a^2(-12a^5B+6a^4Ab+29a^3b^2B-15a^2Ab^3-20ab^4B+12Ab^5)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{b^5d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/2*((6*a*A*b - 12*a^2*B - b^2*B)*ArcTanh[Sin[c + d*x]])/(b^5*d) + (a^2*(6
*a^4*A*b - 15*a^2*A*b^3 + 12*A*b^5 - 12*a^5*B + 29*a^3*b^2*B - 20*a*b^4*B)*
ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^5*(a
+ b)^(5/2)*d) + ((6*a^4*A*b - 11*a^2*A*b^3 + 2*A*b^5 - 12*a^5*B + 21*a^3*b^
2*B - 6*a*b^4*B)*Tan[c + d*x])/(2*b^4*(a^2 - b^2)^2*d) - ((3*a^3*A*b - 6*a*
A*b^3 - 6*a^4*B + 10*a^2*b^2*B - b^4*B)*Sec[c + d*x]*Tan[c + d*x])/(2*b^3*(
a^2 - b^2)^2*d) + (a*(A*b - a*B)*Sec[c + d*x]^3*Tan[c + d*x])/(2*b*(a^2 - b
^2)*d*(a + b*Sec[c + d*x])^2) + (a*(2*a^2*A*b - 5*A*b^3 - 4*a^3*B + 7*a*b^2
*B)*Sec[c + d*x]^2*Tan[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]
))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4177

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(-C)*Csc[e + f*x]*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m +
1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e +
 f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(
m + 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4183

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
- b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n +
 b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}
, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \frac{a(Ab-aB)\sec^3(c+dx)\tan(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec^3(c+dx)(3a(Ab-aB)-2b(Ab-}{}}{}$$

$$= \frac{a(Ab-aB)\sec^3(c+dx)\tan(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{a(2a^2Ab-5Ab^3-4a^3B+}{2b^2\,(a^2-b^2)^2}$$

$$= -\frac{(3a^3Ab-6aAb^3-6a^4B+10a^2b^2B-b^4B)\sec(c+dx)\tan(c+d}{2b^3\,(a^2-b^2)^2\,d}$$

$$= \frac{(6a^4Ab-11a^2Ab^3+2Ab^5-12a^5B+21a^3b^2B-6ab^4B)\tan(c+d}{2b^4\,(a^2-b^2)^2\,d}$$

$$= \frac{(6a^4Ab-11a^2Ab^3+2Ab^5-12a^5B+21a^3b^2B-6ab^4B)\tan(c+d}{2b^4\,(a^2-b^2)^2\,d}$$

$$= -\frac{(6aAb-12a^2B-b^2B)\tanh^{-1}(\sin(c+dx))}{2b^5d} + \frac{(6a^4Ab-11a^2Ab^3}{}$$

$$= -\frac{(6aAb-12a^2B-b^2B)\tanh^{-1}(\sin(c+dx))}{2b^5d} + \frac{(6a^4Ab-11a^2Ab^3}{}$$

$$= -\frac{(6aAb-12a^2B-b^2B)\tanh^{-1}(\sin(c+dx))}{2b^5d} + \frac{a^2(6a^4Ab-15a^2A}{}$$

Mathematica [A]
time = 2.99, size = 507, normalized size = 1.25

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((16*a^2*(-6*a^4*A*b + 15*a^2*A*b^3 - 12*A*b^5 + 12*a^5*B - 29*a^3*b^2*B +
20*a*b^4*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^
2)^(5/2) - 8*(-6*a*A*b + 12*a^2*B + b^2*B)*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] + 8*(-6*a*A*b + 12*a^2*B + b^2*B)*Log[Cos[(c + d*x)/2] + Sin[(c +
d*x)/2]] + (2*b*(18*a^5*A*b^2 - 32*a^3*A*b^4 + 8*a*A*b^6 - 36*a^6*b*B + 68*
a^4*b^3*B - 30*a^2*b^5*B + 4*b^7*B + (18*a^6*A*b - 25*a^4*A*b^3 - 10*a^2*A*
b^5 + 8*A*b^7 - 36*a^7*B + 47*a^5*b^2*B + 14*a^3*b^4*B - 16*a*b^6*B)*Cos[c
+ d*x] - 2*a*b*(-9*a^4*A*b + 16*a^2*A*b^3 - 4*A*b^5 + 18*a^5*B - 32*a^3*b^2
*B + 11*a*b^4*B)*Cos[2*(c + d*x)] + 6*a^6*A*b*Cos[3*(c + d*x)] - 11*a^4*A*b
```

```
^3*Cos[3*(c + d*x)] + 2*a^2*A*b^5*Cos[3*(c + d*x)] - 12*a^7*B*Cos[3*(c + d*
x)] + 21*a^5*b^2*B*Cos[3*(c + d*x)] - 6*a^3*b^4*B*Cos[3*(c + d*x)])*Sec[c +
 d*x]*Tan[c + d*x])/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2))/(16*b^5*d)
```

Maple [A]
time = 1.14, size = 464, normalized size = 1.14

method	result
derivativedivides	$\frac{-\frac{B}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{2Ab-6Ba-Bb}{2b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(-6Aba+12a^2B+b^2B\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b^5}+\frac{B}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2Ab}{2b^4\left(\tan\right.}}{}$
default	$\frac{-\frac{B}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)^2}-\frac{2Ab-6Ba-Bb}{2b^4\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(-6Aba+12a^2B+b^2B\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{2b^5}+\frac{B}{2b^3\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)^2}-\frac{2Ab}{2b^4\left(\tan\right.}}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-1/2*B/b^3/(tan(1/2*d*x+1/2*c)+1)^2-1/2*(2*A*b-6*B*a-B*b)/b^4/(tan(1/2
*d*x+1/2*c)+1)+1/2/b^5*(-6*A*a*b+12*B*a^2+B*b^2)*ln(tan(1/2*d*x+1/2*c)+1)+1
/2*B/b^3/(tan(1/2*d*x+1/2*c)-1)^2-1/2*(2*A*b-6*B*a-B*b)/b^4/(tan(1/2*d*x+1/
2*c)-1)+1/2*(6*A*a*b-12*B*a^2-B*b^2)/b^5*ln(tan(1/2*d*x+1/2*c)-1)-2*a^2/b^5
*((1/2*(4*A*a^2*b-A*a*b^2-8*A*b^3-6*B*a^3+B*a^2*b+10*B*a*b^2)*b*a/(a-b)/(a^
2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*a*b*(4*A*a^2*b+A*a*b^2-8*A*b^3-6*B*a^
3-B*a^2*b+10*B*a*b^2)/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*
c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)^2-1/2*(6*A*a^4*b-15*A*a^2*b^3+12*A*b^5-12*
B*a^5+29*B*a^3*b^2-20*B*a*b^4)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arct
anh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1194 vs. 2(387) = 774.
time = 113.82, size = 2444, normalized size = 6.00

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [-1/4*(((12*B*a^9 - 6*A*a^8*b - 29*B*a^7*b^2 + 15*A*a^6*b^3 + 20*B*a^5*b^4
- 12*A*a^4*b^5)*cos(d*x + c)^4 + 2*(12*B*a^8*b - 6*A*a^7*b^2 - 29*B*a^6*b^3
 + 15*A*a^5*b^4 + 20*B*a^4*b^5 - 12*A*a^3*b^6)*cos(d*x + c)^3 + (12*B*a^7*b
^2 - 6*A*a^6*b^3 - 29*B*a^5*b^4 + 15*A*a^4*b^5 + 20*B*a^3*b^6 - 12*A*a^2*b^
7)*cos(d*x + c)^2)*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*
cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^
2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - ((12*B*a^10 - 6
*A*a^9*b - 35*B*a^8*b^2 + 18*A*a^7*b^3 + 33*B*a^6*b^4 - 18*A*a^5*b^5 - 9*B*
a^4*b^6 + 6*A*a^3*b^7 - B*a^2*b^8)*cos(d*x + c)^4 + 2*(12*B*a^9*b - 6*A*a^8
*b^2 - 35*B*a^7*b^3 + 18*A*a^6*b^4 + 33*B*a^5*b^5 - 18*A*a^4*b^6 - 9*B*a^3*
b^7 + 6*A*a^2*b^8 - B*a*b^9)*cos(d*x + c)^3 + (12*B*a^8*b^2 - 6*A*a^7*b^3 -
 35*B*a^6*b^4 + 18*A*a^5*b^5 + 33*B*a^4*b^6 - 18*A*a^3*b^7 - 9*B*a^2*b^8 +
6*A*a*b^9 - B*b^10)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((12*B*a^10 - 6
*A*a^9*b - 35*B*a^8*b^2 + 18*A*a^7*b^3 + 33*B*a^6*b^4 - 18*A*a^5*b^5 - 9*B*
a^4*b^6 + 6*A*a^3*b^7 - B*a^2*b^8)*cos(d*x + c)^4 + 2*(12*B*a^9*b - 6*A*a^8
*b^2 - 35*B*a^7*b^3 + 18*A*a^6*b^4 + 33*B*a^5*b^5 - 18*A*a^4*b^6 - 9*B*a^3*
b^7 + 6*A*a^2*b^8 - B*a*b^9)*cos(d*x + c)^3 + (12*B*a^8*b^2 - 6*A*a^7*b^3 -
 35*B*a^6*b^4 + 18*A*a^5*b^5 + 33*B*a^4*b^6 - 18*A*a^3*b^7 - 9*B*a^2*b^8 +
6*A*a*b^9 - B*b^10)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(B*a^6*b^4 -
 3*B*a^4*b^6 + 3*B*a^2*b^8 - B*b^10 - (12*B*a^9*b - 6*A*a^8*b^2 - 33*B*a^7*
b^3 + 17*A*a^6*b^4 + 27*B*a^5*b^5 - 13*A*a^4*b^6 - 6*B*a^3*b^7 + 2*A*a^2*b^
8)*cos(d*x + c)^3 - (18*B*a^8*b^2 - 9*A*a^7*b^3 - 50*B*a^6*b^4 + 25*A*a^5*b
^5 + 43*B*a^4*b^6 - 20*A*a^3*b^7 - 11*B*a^2*b^8 + 4*A*a*b^9)*cos(d*x + c)^2
 - 2*(2*B*a^7*b^3 - A*a^6*b^4 - 6*B*a^5*b^5 + 3*A*a^4*b^6 + 6*B*a^3*b^7 - 3
*A*a^2*b^8 - 2*B*a*b^9 + A*b^10)*cos(d*x + c))*sin(d*x + c))/((a^8*b^5 - 3*
a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*d*cos(d*x + c)^4 + 2*(a^7*b^6 - 3*a^5*b^8 +
 3*a^3*b^10 - a*b^12)*d*cos(d*x + c)^3 + (a^6*b^7 - 3*a^4*b^9 + 3*a^2*b^11
- b^13)*d*cos(d*x + c)^2), -1/4*(2*((12*B*a^9 - 6*A*a^8*b - 29*B*a^7*b^2 +
15*A*a^6*b^3 + 20*B*a^5*b^4 - 12*A*a^4*b^5)*cos(d*x + c)^4 + 2*(12*B*a^8*b
- 6*A*a^7*b^2 - 29*B*a^6*b^3 + 15*A*a^5*b^4 + 20*B*a^4*b^5 - 12*A*a^3*b^6)*
```

```
cos(d*x + c)^3 + (12*B*a^7*b^2 - 6*A*a^6*b^3 - 29*B*a^5*b^4 + 15*A*a^4*b^5
+ 20*B*a^3*b^6 - 12*A*a^2*b^7)*cos(d*x + c)^2)*sqrt(-a^2 + b^2)*arctan(-sqr
t(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - ((12*B*a^1
0 - 6*A*a^9*b - 35*B*a^8*b^2 + 18*A*a^7*b^3 + 33*B*a^6*b^4 - 18*A*a^5*b^5 -
 9*B*a^4*b^6 + 6*A*a^3*b^7 - B*a^2*b^8)*cos(d*x + c)^4 + 2*(12*B*a^9*b - 6*
A*a^8*b^2 - 35*B*a^7*b^3 + 18*A*a^6*b^4 + 33*B*a^5*b^5 - 18*A*a^4*b^6 - 9*B
*a^3*b^7 + 6*A*a^2*b^8 - B*a*b^9)*cos(d*x + c)^3 + (12*B*a^8*b^2 - 6*A*a^7*
b^3 - 35*B*a^6*b^4 + 18*A*a^5*b^5 + 33*B*a^4*b^6 - 18*A*a^3*b^7 - 9*B*a^2*b
^8 + 6*A*a*b^9 - B*b^10)*cos(d*x + c)^2)*log(sin(d*x + c) + 1) + ((12*B*a^1
0 - 6*A*a^9*b - 35*B*a^8*b^2 + 18*A*a^7*b^3 + 33*B*a^6*b^4 - 18*A*a^5*b^5 -
 9*B*a^4*b^6 + 6*A*a^3*b^7 - B*a^2*b^8)*cos(d*x + c)^4 + 2*(12*B*a^9*b - 6*
A*a^8*b^2 - 35*B*a^7*b^3 + 18*A*a^6*b^4 + 33*B*a^5*b^5 - 18*A*a^4*b^6 - 9*B
*a^3*b^7 + 6*A*a^2*b^8 - B*a*b^9)*cos(d*x + c)^3 + (12*B*a^8*b^2 - 6*A*a^7*
b^3 - 35*B*a^6*b^4 + 18*A*a^5*b^5 + 33*B*a^4*b^6 - 18*A*a^3*b^7 - 9*B*a^2*b
^8 + 6*A*a*b^9 - B*b^10)*cos(d*x + c)^2)*log(-sin(d*x + c) + 1) - 2*(B*a^6*
b^4 - 3*B*a^4*b^6 + 3*B*a^2*b^8 - B*b^10 - (12*B*a^9*b - 6*A*a^8*b^2 - 33*B
*a^7*b^3 + 17*A*a^6*b^4 + 27*B*a^5*b^5 - 13*A*a^4*b^6 - 6*B*a^3*b^7 + 2*A*a
^2*b^8)*cos(d*x + c)^3 - (18*B*a^8*b^2 - 9*A*a^7*b^3 - 50*B*a^6*b^4 + 25*A*
a^5*b^5 + 43*B*a^4*b^6 - 20*A*a^3*b^7 - 11*B*a^2*b^8 + 4*A*a*b^9)*cos(d*x +
 c)^2 - 2*(2*B*a^7*b^3 - A*a^6*b^4 - 6*B*a^5*b^5 + 3*A*a^4*b^6 + 6*B*a^3*b^
7 - 3*A*a^2*b^8 - 2*B*a*b^9 + A*b^10)*cos(d*x + c))*sin(d*x + c))/((a^8*b^5
 - 3*a^6*b^7 + 3*a^4*b^9 - a^2*b^11)*d*cos(d*x + c)^4 + 2*(a^7*b^6 - 3*a^5*
b^8 + 3*a^3*b^10 - a*b^12)*d*cos(d*x + c)^3 + (a^6*b^7 - 3*a^4*b^9 + 3*a^2*
b^11 - b^13)*d*cos(d*x + c)^2)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec^5(c + dx)}{(a + b\sec(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**5/(a + b*sec(c + d*x))**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1391 vs. $2(387) = 774$.
time = 0.60, size = 1391, normalized size = 3.42

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/2*(2*(12*B*a^7 - 6*A*a^6*b - 29*B*a^5*b^2 + 15*A*a^4*b^3 + 20*B*a^3*b^4
- 12*A*a^2*b^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(
-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4
*b^5 - 2*a^2*b^7 + b^9)*sqrt(-a^2 + b^2)) - 2*(12*B*a^7*tan(1/2*d*x + 1/2*c
)^7 - 6*A*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 18*B*a^6*b*tan(1/2*d*x + 1/2*c)^7
+ 9*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 - 17*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^7
+ 9*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^7
- 16*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 2*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^7
+ 2*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 13*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^7
+ 4*A*a*b^6*tan(1/2*d*x + 1/2*c)^7 + 4*B*a*b^6*tan(1/2*d*x + 1/2*c)^7 - 2*A
*b^7*tan(1/2*d*x + 1/2*c)^7 + B*b^7*tan(1/2*d*x + 1/2*c)^7 - 36*B*a^7*tan(1
/2*d*x + 1/2*c)^5 + 18*A*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 18*B*a^6*b*tan(1/2*
d*x + 1/2*c)^5 - 9*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 + 67*B*a^5*b^2*tan(1/2*
d*x + 1/2*c)^5 - 35*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 29*B*a^4*b^3*tan(1/2
*d*x + 1/2*c)^5 + 16*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 - 26*B*a^3*b^4*tan(1/
2*d*x + 1/2*c)^5 + 10*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 5*B*a^2*b^5*tan(1/
2*d*x + 1/2*c)^5 - 4*A*a*b^6*tan(1/2*d*x + 1/2*c)^5 + 4*B*a*b^6*tan(1/2*d*x
 + 1/2*c)^5 - 2*A*b^7*tan(1/2*d*x + 1/2*c)^5 + 3*B*b^7*tan(1/2*d*x + 1/2*c)
^5 + 36*B*a^7*tan(1/2*d*x + 1/2*c)^3 - 18*A*a^6*b*tan(1/2*d*x + 1/2*c)^3 +
18*B*a^6*b*tan(1/2*d*x + 1/2*c)^3 - 9*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 67
*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 35*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 2
9*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 + 16*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 +
26*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 10*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 +
 5*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*A*a*b^6*tan(1/2*d*x + 1/2*c)^3 - 4*
B*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 2*A*b^7*tan(1/2*d*x + 1/2*c)^3 + 3*B*b^7*t
an(1/2*d*x + 1/2*c)^3 - 12*B*a^7*tan(1/2*d*x + 1/2*c) + 6*A*a^6*b*tan(1/2*d
*x + 1/2*c) - 18*B*a^6*b*tan(1/2*d*x + 1/2*c) + 9*A*a^5*b^2*tan(1/2*d*x + 1
/2*c) + 17*B*a^5*b^2*tan(1/2*d*x + 1/2*c) - 9*A*a^4*b^3*tan(1/2*d*x + 1/2*c
) + 33*B*a^4*b^3*tan(1/2*d*x + 1/2*c) - 16*A*a^3*b^4*tan(1/2*d*x + 1/2*c) +
 2*B*a^3*b^4*tan(1/2*d*x + 1/2*c) - 2*A*a^2*b^5*tan(1/2*d*x + 1/2*c) - 13*B
*a^2*b^5*tan(1/2*d*x + 1/2*c) + 4*A*a*b^6*tan(1/2*d*x + 1/2*c) - 4*B*a*b^6*
tan(1/2*d*x + 1/2*c) + 2*A*b^7*tan(1/2*d*x + 1/2*c) + B*b^7*tan(1/2*d*x + 1
/2*c))/((a^4*b^4 - 2*a^2*b^6 + b^8)*(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d
*x + 1/2*c)^4 - 2*a*tan(1/2*d*x + 1/2*c)^2 + a + b)^2) - (12*B*a^2 - 6*A*a*
b + B*b^2)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^5 + (12*B*a^2 - 6*A*a*b + B
*b^2)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^5)/d
```

Mupad [B]
time = 14.23, size = 2500, normalized size = 6.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^5*(a + b/cos(c + d*x))^3),x)
```

[Out] ((tan(c/2 + (d*x)/2)^3*(2*A*b^7 + 36*B*a^7 + 3*B*b^7 - 10*A*a^2*b^5 + 16*A*a^3*b^4 + 35*A*a^4*b^3 - 9*A*a^5*b^2 + 5*B*a^2*b^5 + 26*B*a^3*b^4 - 29*B*a^4*b^3 - 67*B*a^5*b^2 - 4*A*a*b^6 - 18*A*a^6*b - 4*B*a*b^6 + 18*B*a^6*b))/((a + b)^2*(b^6 - 2*a*b^5 + a^2*b^4)) + (tan(c/2 + (d*x)/2)^5*(3*B*b^7 - 36*B*a^7 - 2*A*b^7 + 10*A*a^2*b^5 + 16*A*a^3*b^4 - 35*A*a^4*b^3 - 9*A*a^5*b^2 + 5*B*a^2*b^5 - 26*B*a^3*b^4 - 29*B*a^4*b^3 + 67*B*a^5*b^2 - 4*A*a*b^6 + 18*A*a^6*b + 4*B*a*b^6 + 18*B*a^6*b))/((a + b)^2*(b^6 - 2*a*b^5 + a^2*b^4)) - (tan(c/2 + (d*x)/2)^7*(B*b^6 - 12*B*a^6 - 2*A*b^6 + 4*A*a^2*b^4 - 12*A*a^3*b^3 - 3*A*a^4*b^2 - 8*B*a^2*b^4 - 10*B*a^3*b^3 + 23*B*a^4*b^2 + 2*A*a*b^5 + 6*A*a^5*b + 5*B*a*b^5 + 6*B*a^5*b))/((a*b^4 - b^5)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(2*A*b^6 - 12*B*a^6 + B*b^6 - 4*A*a^2*b^4 - 12*A*a^3*b^3 + 3*A*a^4*b^2 - 8*B*a^2*b^4 + 10*B*a^3*b^3 + 23*B*a^4*b^2 + 2*A*a*b^5 + 6*A*a^5*b - 5*B*a*b^5 - 6*B*a^5*b))/((a + b)*(b^6 - 2*a*b^5 + a^2*b^4)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^4*(6*a^2 - 2*b^2) - tan(c/2 + (d*x)/2)^2*(4*a*b + 4*a^2) + tan(c/2 + (d*x)/2)^6*(4*a*b - 4*a^2) + tan(c/2 + (d*x)/2)^8*(a^2 - 2*a*b + b^2) + a^2 + b^2)) - (atan(((((8*tan(c/2 + (d*x)/2)*(288*B^2*a^14 + B^2*b^14 - 2*B^2*a*b^13 - 288*B^2*a^13*b + 36*A^2*a^2*b^12 - 72*A^2*a^3*b^11 + 36*A^2*a^4*b^10 + 288*A^2*a^5*b^9 - 288*A^2*a^6*b^8 - 432*A^2*a^7*b^7 + 441*A^2*a^8*b^6 + 288*A^2*a^9*b^5 - 288*A^2*a^10*b^4 - 72*A^2*a^11*b^3 + 72*A^2*a^12*b^2 + 21*B^2*a^2*b^12 - 40*B^2*a^3*b^11 + 74*B^2*a^4*b^10 - 108*B^2*a^5*b^9 + 18*B^2*a^6*b^8 + 872*B^2*a^7*b^7 - 827*B^2*a^8*b^6 - 1538*B^2*a^9*b^5 + 1538*B^2*a^10*b^4 + 1104*B^2*a^11*b^3 - 1104*B^2*a^12*b^2 - 12*A*B*a*b^13 - 288*A*B*a^13*b + 24*A*B*a^2*b^12 - 108*A*B*a^3*b^11 + 192*A*B*a^4*b^10 - 72*A*B*a^5*b^9 - 1008*A*B*a^6*b^8 + 984*A*B*a^7*b^7 + 1632*A*B*a^8*b^6 - 1650*A*B*a^9*b^5 - 1128*A*B*a^10*b^4 + 1128*A*B*a^11*b^3 + 288*A*B*a^12*b^2))/(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8) - (((4*(4*B*b^21 + 48*A*a^2*b^19 + 72*A*a^3*b^18 - 156*A*a^4*b^17 - 84*A*a^5*b^16 + 192*A*a^6*b^15 + 48*A*a^7*b^14 - 108*A*a^8*b^13 - 12*A*a^9*b^12 + 24*A*a^10*b^11 + 28*B*a^2*b^19 - 80*B*a^3*b^18 - 120*B*a^4*b^17 + 276*B*a^5*b^16 + 164*B*a^6*b^15 - 360*B*a^7*b^14 - 100*B*a^8*b^13 + 212*B*a^9*b^12 + 24*B*a^10*b^11 - 48*B*a^11*b^10 - 24*A*a*b^20))/(a*b^18 + b^19 - 3*a^2*b^17 - 3*a^3*b^16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^12) - (4*tan(c/2 + (d*x)/2)*(12*B*a^2 + B*b^2 - 6*A*a*b)*(8*a*b^19 - 8*a^2*b^18 - 32*a^3*b^17 + 32*a^4*b^16 + 48*a^5*b^15 - 48*a^6*b^14 - 32*a^7*b^13 + 32*a^8*b^12 + 8*a^9*b^11 - 8*a^10*b^10))/(b^5*(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8)))*(12*B*a^2 + B*b^2 - 6*A*a*b))/(2*b^5))*(12*B*a^2 + B*b^2 - 6*A*a*b)*1i)/(2*b^5) + (((8*tan(c/2 + (d*x)/2)*(288*B^2*a^14 + B^2*b^14 - 2*B^2*a*b^13 - 288*B^2*a^13*b + 36*A^2*a^2*b^12 - 72*A^2*a^3*b^11 + 36*A^2*a^4*b^10 + 288*A^2*a^5*b^9 - 288*A^2*a^6*b^8 - 432*A^2*a^7*b^7 + 441*A^2*a^8*b^6 + 288*A^2*a^9*b^5 - 288*A^2*a^10*b^4 - 72*A^2*a^11*b^3 + 72*A^2*a^12*b^2 + 21*B^2*a^2*b^12 - 40*B^2*a^3*b^11 + 74*B^2*a^4*b^10 - 108*B^2*a^5*b^9 + 18*B^2*a^6*b^8 + 872*B^2*a^7*b^7 - 827*B^2*a^8*b^6 - 1538*B^2*a^9*b^5 + 1538*B^2*a^10*b^4 + 1104*B^2*a^11*b^3 - 1104*B^2*a^12*b^2 - 12*A*B*a*b^13 - 288*A*B*a^13*b + 24*A*B*a^2*b^12 - 108*A*B*a^3*b^11 + 192*A*B*a^4*b^10 - 72*A*B*a^5*b^9 - 1008*A*B*

```
a^6*b^8 + 984*A*B*a^7*b^7 + 1632*A*B*a^8*b^6 - 1650*A*B*a^9*b^5 - 1128*A*B*
a^10*b^4 + 1128*A*B*a^11*b^3 + 288*A*B*a^12*b^2))/(a*b^14 + b^15 - 3*a^2*b^
13 - 3*a^3*b^12 + 3*a^4*b^11 + 3*a^5*b^10 - a^6*b^9 - a^7*b^8) + (((4*(4*B*
b^21 + 48*A*a^2*b^19 + 72*A*a^3*b^18 - 156*A*a^4*b^17 - 84*A*a^5*b^16 + 192
*A*a^6*b^15 + 48*A*a^7*b^14 - 108*A*a^8*b^13 - 12*A*a^9*b^12 + 24*A*a^10*b^
11 + 28*B*a^2*b^19 - 80*B*a^3*b^18 - 120*B*a^4*b^17 + 276*B*a^5*b^16 + 164*
B*a^6*b^15 - 360*B*a^7*b^14 - 100*B*a^8*b^13 + 212*B*a^9*b^12 + 24*B*a^10*b
^11 - 48*B*a^11*b^10 - 24*A*a*b^20))/(a*b^18 + b^19 - 3*a^2*b^17 - 3*a^3*b^
16 + 3*a^4*b^15 + 3*a^5*b^14 - a^6*b^13 - a^7*b^12) + (4*tan(c/2 + (d*x)/2)
*(12*B*a^2 + B*b^2 - 6*A*a*b)*(8*a*b^19 - 8*a^2*b^18 - 32*a^3*b^17 + 32*a^4
*b^16 + 48*a^5*b^15 - 48*a^6*b^14 - 32*a^7*b^13 + 32*a^8*b^12 + 8*a^9*b^11
- 8*a^10*b^10))/(b^5*(a*b^14 + b^15 - 3*a^2*b^13 - 3*a^3*b^12 + 3*a^4*b^11
+ 3*a^5*b^10 - a^6*b^9 - a^7*b^8)))*(12*B*a^2 + B*b^2 - 6*A*a*b))/(2*b^5))*
(12*B*a^2 + B*b^2 - 6*A*a*b)*1i)/(2*b^5))/((8*(1728*B^3*a^15 - 864*B^3*a^14
*b - 432*A^3*a^4*b^11 - 432*A^3*a^5*b^10 + 1404*A^3*a^6*b^9 + 756*A^3*a^7*b
^8 - 1728*A^3*a^8*b^7 - 486*A^3*a^9*b^6 + 972*A^3*a^10*b^5 + 108*A^3*a^11*b
^4 - 216*A^3*a^12*b^3 + 20*B^3*a^3*b^12 - 20*B^3*a^4*b^11 + 411*B^3*a^5*b^1
0 - 11*B^3*a^6*b^9 + 1314*B^3*a^7*b^8 + 2326*B^3*a^8*b^7 - 7829*B^3*a^9*b^6
 - 4770*B^3*a^10*b^5 + 11700*B^3*a^11*b^4 + 3456*B^3*a^12*b^3 - 7344*B^3*a^
13*b^2 - 2592*A*B^2*a^14*b - 12*A*B^2*a^2*b^13 + 12*A*B^2*a^3*b^12 - 489*A*
B^2*a^4*b^11 + 9*A*B^2*a^5*b^10 - 2892*A*B^2*a^6*b^9 - 3972*A*B^2*a^7*b^8 +
 13347*A*B^2*a^8*b^7 + 7767*A*B^2*a^9*b^6 - 185...
```

3.329 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=289

$$\frac{(Ab-3aB)\tanh^{-1}(\sin(c+dx))}{b^4d}-\frac{a(2a^4Ab-5a^2Ab^3+6Ab^5-6a^5B+15a^3b^2B-12ab^4B)\tanh^{-1}\left(\frac{\sqrt{a-}}{}\right.}{(a-b)^{5/2}b^4(a+b)^{5/2}d}$$

```
[Out] (A*b-3*B*a)*arctanh(sin(d*x+c))/b^4/d-a*(2*A*a^4*b-5*A*a^2*b^3+6*A*b^5-6*B*
a^5+15*B*a^3*b^2-12*B*a*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(
1/2))/(a-b)^(5/2)/b^4/(a+b)^(5/2)/d-1/2*(A*a*b-3*B*a^2+2*B*b^2)*tan(d*x+c)/
b^3/(a^2-b^2)/d+1/2*a*(A*b-B*a)*sec(d*x+c)^2*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*
sec(d*x+c))^2-1/2*a^2*(A*a^2*b-4*A*b^3-3*B*a^3+6*B*a*b^2)*tan(d*x+c)/b^3/(a
^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Rubi [A]

time = 0.97, antiderivative size = 289, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4114, 4175, 4167, 4083, 3855, 3916, 2738, 214}

$$\frac{a(Ab-aB)\tan(c+dx)\sec^2(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{\left(-3a^2B+aAb+2b^2B\right)\tan(c+dx)}{2b^3d\left(a^2-b^2\right)}-\frac{a^2\left(-3a^3B+a^2Ab+6ab^2B-4Ab^3\right)\tan(c+dx)}{2b^3d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{a\left(-6a^5B+2a^4Ab+15a^3b^2B-5a^2Ab^3-12ab^4B+6Ab^5\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{5/2}(a+b)^{5/2}}+\frac{(Ab-3aB)\tanh^{-1}(\sin(c+dx))}{b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((A*b - 3*a*B)*ArcTanh[Sin[c + d*x]])/(b^4*d) - (a*(2*a^4*A*b - 5*a^2*A*b^3
 + 6*A*b^5 - 6*a^5*B + 15*a^3*b^2*B - 12*a*b^4*B)*ArcTanh[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(5/2)*b^4*(a + b)^(5/2)*d) - ((a*A*b -
 3*a^2*B + 2*b^2*B)*Tan[c + d*x])/(2*b^3*(a^2 - b^2)*d) + (a*(A*b - a*B)*Se
c[c + d*x]^2*Tan[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) - (a^
2*(a^2*A*b - 4*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Tan[c + d*x])/(2*b^3*(a^2 - b^2
)^2*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4175

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x
])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*((-a)*(b*
```

```
B - a*C) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \frac{a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\int \sec^2(c+dx)\left(2a(Ab-aB)-2b(A\right.}{}$$

$$= \frac{a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a^2\left(a^2Ab-4Ab^3-3a^3B\right.}{2b^3\left(a^2-b^2\right)^2 d(a+}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\sec^2(c+dx)\,\mathrm{t}}{2b\left(a^2-b^2\right)d(a+b\sec(}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)\tan(c+dx)}{2b^3\left(a^2-b^2\right)d} + \frac{a(Ab-aB)\sec^2(c+dx)\,\mathrm{t}}{2b\left(a^2-b^2\right)d(a+b\sec(}$$

$$= \frac{(Ab-3aB)\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{\left(aAb-3a^2B+2b^2B\right)\tan(c+}{2b^3\left(a^2-b^2\right)d}$$

$$= \frac{(Ab-3aB)\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{\left(aAb-3a^2B+2b^2B\right)\tan(c+}{2b^3\left(a^2-b^2\right)d}$$

$$= \frac{(Ab-3aB)\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{a\left(2a^4Ab-5a^2Ab^3+6Ab^5-\right.}{}$$

Mathematica [A]
time = 6.50, size = 418, normalized size = 1.45

$$\frac{a\left(2a^4Ab-5a^2Ab^3+6Ab^5-6a^5B+15a^3b^2B-12ab^4B\right)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{b^4\sqrt{a^2-b^2}\left(-a^2+b^2\right)^2d} + \frac{(-Ab+3aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b^4d} + \frac{(Ab-3aB)\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)}{b^4d} + \frac{B\sin\left(\frac{1}{2}(c+dx)\right)}{b^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{B\sin\left(\frac{1}{2}(c+dx)\right)}{b^3d\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)} + \frac{a^2Ab\sin(c+dx)-a^3B\sin(c+dx)}{2b^2(-a+b)(a+b)d(b+a\cos(c+dx))^2} + \frac{-2a^4Ab\sin(c+dx)+5a^2Ab^3\sin(c+dx)+4a^5B\sin(c+dx)-7a^3b^2B\sin(c+dx)}{2b^3(-a+b)^2(a+b)^2d(b+a\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (a*(2*a^4*A*b - 5*a^2*A*b^3 + 6*A*b^5 - 6*a^5*B + 15*a^3*b^2*B - 12*a*b^4*B
)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(b^4*Sqrt[a^2 - b^2
]*(-a^2 + b^2)^2*d) + ((-(A*b) + 3*a*B)*Log[Cos[(c + d*x)/2] - Sin[(c + d*x
)/2]])/(b^4*d) + ((A*b - 3*a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]])/(
b^4*d) + (B*Sin[(c + d*x)/2])/(b^3*d*(Cos[(c + d*x)/2] - Sin[(c + d*x)/2]))
 + (B*Sin[(c + d*x)/2])/(b^3*d*(Cos[(c + d*x)/2] + Sin[(c + d*x)/2])) + (a^
2*A*b*Sin[c + d*x] - a^3*B*Sin[c + d*x])/(2*b^2*(-a + b)*(a + b)*d*(b + a*C
```

```
os[c + d*x])^2) + (-2*a^4*A*b*Sin[c + d*x] + 5*a^2*A*b^3*Sin[c + d*x] + 4*a
^5*B*Sin[c + d*x] - 7*a^3*b^2*B*Sin[c + d*x])/(2*b^3*(-a + b)^2*(a + b)^2*d
*(b + a*Cos[c + d*x]))
```

Maple [A]
time = 1.03, size = 380, normalized size = 1.31

method	result
derivativedivides	$-\frac{B}{b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(Ab-3Ba\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^{4}}-\frac{B}{b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-Ab+3Ba\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^{4}}+\frac{2a\left(\frac{\left(2A\,a^{2}b-A\right.}{}\right.}{}$
default	$-\frac{B}{b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}+\frac{\left(Ab-3Ba\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)+1\right)}{b^{4}}-\frac{B}{b^{3}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}+\frac{\left(-Ab+3Ba\right)\ln\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)-1\right)}{b^{4}}+\frac{2a\left(\frac{\left(2A\,a^{2}b-A\right.}{}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-B/b^3/(tan(1/2*d*x+1/2*c)+1)+(A*b-3*B*a)/b^4*ln(tan(1/2*d*x+1/2*c)+1)
-B/b^3/(tan(1/2*d*x+1/2*c)-1)+1/b^4*(-A*b+3*B*a)*ln(tan(1/2*d*x+1/2*c)-1)+2
*a/b^4*((1/2*(2*A*a^2*b-A*a*b^2-6*A*b^3-4*B*a^3+B*a^2*b+8*B*a*b^2)*b*a/(a-b
)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*a*b*(2*A*a^2*b+A*a*b^2-6*A*b^3-4
*B*a^3-B*a^2*b+8*B*a*b^2)/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+
1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)^2-1/2*(2*A*a^4*b-5*A*a^2*b^3+6*A*b^5-6
*B*a^5+15*B*a^3*b^2-12*B*a*b^4)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arc
tanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1026 vs. $2(277) = 554$.
time = 116.02, size = 2111, normalized size = 7.30

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [-1/4*(((6*B*a^8 - 2*A*a^7*b - 15*B*a^6*b^2 + 5*A*a^5*b^3 + 12*B*a^4*b^4 -
6*A*a^3*b^5)*cos(d*x + c)^3 + 2*(6*B*a^7*b - 2*A*a^6*b^2 - 15*B*a^5*b^3 + 5
*A*a^4*b^4 + 12*B*a^3*b^5 - 6*A*a^2*b^6)*cos(d*x + c)^2 + (6*B*a^6*b^2 - 2*
A*a^5*b^3 - 15*B*a^4*b^4 + 5*A*a^3*b^5 + 12*B*a^2*b^6 - 6*A*a*b^7)*cos(d*x
+ c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^
2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2
*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*((3*B*a^9 - A*a^8*b - 9*B*
a^7*b^2 + 3*A*a^6*b^3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 - 3*B*a^3*b^6 + A*a^2*b^7
)*cos(d*x + c)^3 + 2*(3*B*a^8*b - A*a^7*b^2 - 9*B*a^6*b^3 + 3*A*a^5*b^4 + 9
*B*a^4*b^5 - 3*A*a^3*b^6 - 3*B*a^2*b^7 + A*a*b^8)*cos(d*x + c)^2 + (3*B*a^7
*b^2 - A*a^6*b^3 - 9*B*a^5*b^4 + 3*A*a^4*b^5 + 9*B*a^3*b^6 - 3*A*a^2*b^7 -
3*B*a*b^8 + A*b^9)*cos(d*x + c))*log(sin(d*x + c) + 1) - 2*((3*B*a^9 - A*a^
8*b - 9*B*a^7*b^2 + 3*A*a^6*b^3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 - 3*B*a^3*b^6 +
 A*a^2*b^7)*cos(d*x + c)^3 + 2*(3*B*a^8*b - A*a^7*b^2 - 9*B*a^6*b^3 + 3*A*a
^5*b^4 + 9*B*a^4*b^5 - 3*A*a^3*b^6 - 3*B*a^2*b^7 + A*a*b^8)*cos(d*x + c)^2
+ (3*B*a^7*b^2 - A*a^6*b^3 - 9*B*a^5*b^4 + 3*A*a^4*b^5 + 9*B*a^3*b^6 - 3*A*
a^2*b^7 - 3*B*a*b^8 + A*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(2*B*
a^6*b^3 - 6*B*a^4*b^5 + 6*B*a^2*b^7 - 2*B*b^9 + (6*B*a^8*b - 2*A*a^7*b^2 -
17*B*a^6*b^3 + 7*A*a^5*b^4 + 13*B*a^4*b^5 - 5*A*a^3*b^6 - 2*B*a^2*b^7)*cos(
d*x + c)^2 + (9*B*a^7*b^2 - 3*A*a^6*b^3 - 25*B*a^5*b^4 + 9*A*a^4*b^5 + 20*B
*a^3*b^6 - 6*A*a^2*b^7 - 4*B*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^8*b^4 -
 3*a^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^3 + 2*(a^7*b^5 - 3*a^5*b^
7 + 3*a^3*b^9 - a*b^11)*d*cos(d*x + c)^2 + (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^1
0 - b^12)*d*cos(d*x + c)), 1/2*(((6*B*a^8 - 2*A*a^7*b - 15*B*a^6*b^2 + 5*A*
a^5*b^3 + 12*B*a^4*b^4 - 6*A*a^3*b^5)*cos(d*x + c)^3 + 2*(6*B*a^7*b - 2*A*a
^6*b^2 - 15*B*a^5*b^3 + 5*A*a^4*b^4 + 12*B*a^3*b^5 - 6*A*a^2*b^6)*cos(d*x +
 c)^2 + (6*B*a^6*b^2 - 2*A*a^5*b^3 - 15*B*a^4*b^4 + 5*A*a^3*b^5 + 12*B*a^2*
b^6 - 6*A*a*b^7)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b
*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - ((3*B*a^9 - A*a^8*b - 9*B*
a^7*b^2 + 3*A*a^6*b^3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 - 3*B*a^3*b^6 + A*a^2*b^7
)*cos(d*x + c)^3 + 2*(3*B*a^8*b - A*a^7*b^2 - 9*B*a^6*b^3 + 3*A*a^5*b^4 + 9
*B*a^4*b^5 - 3*A*a^3*b^6 - 3*B*a^2*b^7 + A*a*b^8)*cos(d*x + c)^2 + (3*B*a^7
```

```
*b^2 - A*a^6*b^3 - 9*B*a^5*b^4 + 3*A*a^4*b^5 + 9*B*a^3*b^6 - 3*A*a^2*b^7 -
3*B*a*b^8 + A*b^9)*cos(d*x + c))*log(sin(d*x + c) + 1) + ((3*B*a^9 - A*a^8*
b - 9*B*a^7*b^2 + 3*A*a^6*b^3 + 9*B*a^5*b^4 - 3*A*a^4*b^5 - 3*B*a^3*b^6 + A
*a^2*b^7)*cos(d*x + c)^3 + 2*(3*B*a^8*b - A*a^7*b^2 - 9*B*a^6*b^3 + 3*A*a^5
*b^4 + 9*B*a^4*b^5 - 3*A*a^3*b^6 - 3*B*a^2*b^7 + A*a*b^8)*cos(d*x + c)^2 +
(3*B*a^7*b^2 - A*a^6*b^3 - 9*B*a^5*b^4 + 3*A*a^4*b^5 + 9*B*a^3*b^6 - 3*A*a^
2*b^7 - 3*B*a*b^8 + A*b^9)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (2*B*a^6*
b^3 - 6*B*a^4*b^5 + 6*B*a^2*b^7 - 2*B*b^9 + (6*B*a^8*b - 2*A*a^7*b^2 - 17*B
*a^6*b^3 + 7*A*a^5*b^4 + 13*B*a^4*b^5 - 5*A*a^3*b^6 - 2*B*a^2*b^7)*cos(d*x
+ c)^2 + (9*B*a^7*b^2 - 3*A*a^6*b^3 - 25*B*a^5*b^4 + 9*A*a^4*b^5 + 20*B*a^3
*b^6 - 6*A*a^2*b^7 - 4*B*a*b^8)*cos(d*x + c))*sin(d*x + c))/((a^8*b^4 - 3*a
^6*b^6 + 3*a^4*b^8 - a^2*b^10)*d*cos(d*x + c)^3 + 2*(a^7*b^5 - 3*a^5*b^7 +
3*a^3*b^9 - a*b^11)*d*cos(d*x + c)^2 + (a^6*b^6 - 3*a^4*b^8 + 3*a^2*b^10 -
b^12)*d*cos(d*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec^4(c + dx)}{(a + b\sec(c + dx))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/(a + b*sec(c + d*x))**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 581 vs. 2(277) = 554.
time = 0.55, size = 581, normalized size = 2.01

$$\frac{\frac{(6Ba^6 - 2Aa^5b - 15Ba^4b^2 + 5Aa^3b^3 + 12Ba^2b^4 - 6Aab^5)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4b^4 - 2a^2b^6 + b^8)\sqrt{-a^2+b^2}} - \frac{4Ba^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 2Aa^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 5Ba^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 3Aa^4b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 7Ba^4b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 5Aa^3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 + 8Ba^3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 6Aa^2b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3 - 4Ba^6\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 2Aa^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 5Ba^5b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 3Aa^4b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 7Ba^4b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 5Aa^3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) + 8Ba^3b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right) - 6Aa^2b^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{(a^4b^3 - 2a^2b^5 + b^7)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2 - a - b\right)^2} - \frac{(3Ba - Ab)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^4} + \frac{(3Ba - Ab)\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^4} - \frac{2B\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\left(\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-1\right)b^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] ((6*B*a^6 - 2*A*a^5*b - 15*B*a^4*b^2 + 5*A*a^3*b^3 + 12*B*a^2*b^4 - 6*A*a*b
^5)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*
d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4*b^4 - 2*a^2
*b^6 + b^8)*sqrt(-a^2 + b^2)) - (4*B*a^6*tan(1/2*d*x + 1/2*c)^3 - 2*A*a^5*b
*tan(1/2*d*x + 1/2*c)^3 - 5*B*a^5*b*tan(1/2*d*x + 1/2*c)^3 + 3*A*a^4*b^2*ta
n(1/2*d*x + 1/2*c)^3 - 7*B*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^3*b^3*tan
(1/2*d*x + 1/2*c)^3 + 8*B*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 6*A*a^2*b^4*tan(
1/2*d*x + 1/2*c)^3 - 4*B*a^6*tan(1/2*d*x + 1/2*c) + 2*A*a^5*b*tan(1/2*d*x +
 1/2*c) - 5*B*a^5*b*tan(1/2*d*x + 1/2*c) + 3*A*a^4*b^2*tan(1/2*d*x + 1/2*c)
 + 7*B*a^4*b^2*tan(1/2*d*x + 1/2*c) - 5*A*a^3*b^3*tan(1/2*d*x + 1/2*c) + 8*
```

```
B*a^3*b^3*tan(1/2*d*x + 1/2*c) - 6*A*a^2*b^4*tan(1/2*d*x + 1/2*c))/((a^4*b^
3 - 2*a^2*b^5 + b^7)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 -
 a - b)^2) - (3*B*a - A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^4 + (3*B*a
- A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 - 2*B*tan(1/2*d*x + 1/2*c)/((
tan(1/2*d*x + 1/2*c)^2 - 1)*b^3))/d
```

Mupad [B]
time = 14.54, size = 2500, normalized size = 8.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + b/cos(c + d*x))^3),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(6*B*a^5 - 2*B*b^5 + 6*A*a^2*b^3 + A*a^3*b^2 + 4*B*a
^2*b^3 - 12*B*a^3*b^2 - 2*A*a^4*b + 2*B*a*b^4 - 3*B*a^4*b))/((a*b^3 - b^4)*
(a + b)^2) + (tan(c/2 + (d*x)/2)*(6*B*a^5 + 2*B*b^5 + 6*A*a^2*b^3 - A*a^3*b
^2 - 4*B*a^2*b^3 - 12*B*a^3*b^2 - 2*A*a^4*b + 2*B*a*b^4 + 3*B*a^4*b))/((a +
 b)*(b^5 - 2*a*b^4 + a^2*b^3)) - (2*tan(c/2 + (d*x)/2)^3*(6*B*a^6 - 2*B*b^6
 + 5*A*a^3*b^3 + 6*B*a^2*b^4 - 13*B*a^4*b^2 - 2*A*a^5*b))/(b*(a*b^2 - b^3)*
(a + b)^2*(a - b)))/(d*(2*a*b - tan(c/2 + (d*x)/2)^2*(2*a*b + 3*a^2 - b^2)
- tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^2 - tan(c/2 + (d*x)/2)
^4*(2*a*b - 3*a^2 + b^2))) + (atan(((((8*tan(c/2 + (d*x)/2)*(4*A^2*b^12 + 7
2*B^2*a^12 - 8*A^2*a*b^11 - 72*B^2*a^11*b + 24*A^2*a^2*b^10 + 32*A^2*a^3*b^
9 - 52*A^2*a^4*b^8 - 48*A^2*a^5*b^7 + 57*A^2*a^6*b^6 + 32*A^2*a^7*b^5 - 32*
A^2*a^8*b^4 - 8*A^2*a^9*b^3 + 8*A^2*a^10*b^2 + 36*B^2*a^2*b^10 - 72*B^2*a^3
*b^9 + 36*B^2*a^4*b^8 + 288*B^2*a^5*b^7 - 288*B^2*a^6*b^6 - 432*B^2*a^7*b^5
 + 441*B^2*a^8*b^4 + 288*B^2*a^9*b^3 - 288*B^2*a^10*b^2 - 24*A*B*a*b^11 - 4
8*A*B*a^11*b + 48*A*B*a^2*b^10 - 72*A*B*a^3*b^9 - 192*A*B*a^4*b^8 + 252*A*B
*a^5*b^7 + 288*A*B*a^6*b^6 - 318*A*B*a^7*b^5 - 192*A*B*a^8*b^4 + 192*A*B*a^
9*b^3 + 48*A*B*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*
b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (((8*(4*A*b^18 - 8*A*a^2*b^16 + 34*A
*a^3*b^15 + 6*A*a^4*b^14 - 36*A*a^5*b^13 - 4*A*a^6*b^12 + 18*A*a^7*b^11 + 2
*A*a^8*b^10 - 4*A*a^9*b^9 + 24*B*a^2*b^16 + 36*B*a^3*b^15 - 78*B*a^4*b^14 -
 42*B*a^5*b^13 + 96*B*a^6*b^12 + 24*B*a^7*b^11 - 54*B*a^8*b^10 - 6*B*a^9*b^
9 + 12*B*a^10*b^8 - 12*A*a*b^17 - 12*B*a*b^17))/(a*b^15 + b^16 - 3*a^2*b^14
 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) + (8*tan(c/2
+ (d*x)/2)*(A*b - 3*B*a)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14
 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^
10*b^8))/(b^4*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*
b^8 - a^6*b^7 - a^7*b^6)))*(A*b - 3*B*a))/b^4)*(A*b - 3*B*a)*1i)/b^4 + (((8
*tan(c/2 + (d*x)/2)*(4*A^2*b^12 + 72*B^2*a^12 - 8*A^2*a*b^11 - 72*B^2*a^11*
b + 24*A^2*a^2*b^10 + 32*A^2*a^3*b^9 - 52*A^2*a^4*b^8 - 48*A^2*a^5*b^7 + 57
*A^2*a^6*b^6 + 32*A^2*a^7*b^5 - 32*A^2*a^8*b^4 - 8*A^2*a^9*b^3 + 8*A^2*a^10
*b^2 + 36*B^2*a^2*b^10 - 72*B^2*a^3*b^9 + 36*B^2*a^4*b^8 + 288*B^2*a^5*b^7
```

- 288*B^2*a^6*b^6 - 432*B^2*a^7*b^5 + 441*B^2*a^8*b^4 + 288*B^2*a^9*b^3 - 288*B^2*a^10*b^2 - 24*A*B*a*b^11 - 48*A*B*a^11*b + 48*A*B*a^2*b^10 - 72*A*B*a^3*b^9 - 192*A*B*a^4*b^8 + 252*A*B*a^5*b^7 + 288*A*B*a^6*b^6 - 318*A*B*a^7*b^5 - 192*A*B*a^8*b^4 + 192*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (((8*(4*A*b^18 - 8*A*a^2*b^16 + 34*A*a^3*b^15 + 6*A*a^4*b^14 - 36*A*a^5*b^13 - 4*A*a^6*b^12 + 18*A*a^7*b^11 + 2*A*a^8*b^10 - 4*A*a^9*b^9 + 24*B*a^2*b^16 + 36*B*a^3*b^15 - 78*B*a^4*b^14 - 42*B*a^5*b^13 + 96*B*a^6*b^12 + 24*B*a^7*b^11 - 54*B*a^8*b^10 - 6*B*a^9*b^9 + 12*B*a^10*b^8 - 12*A*a*b^17 - 12*B*a*b^17))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (8*tan(c/2 + (d*x)/2)*(A*b - 3*B*a)*(8*a*b^17 - 8*a^2*b^16 - 32*a^3*b^15 + 32*a^4*b^14 + 48*a^5*b^13 - 48*a^6*b^12 - 32*a^7*b^11 + 32*a^8*b^10 + 8*a^9*b^9 - 8*a^10*b^8))/(b^4*(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6)))*(A*b - 3*B*a))/b^4)*(A*b - 3*B*a)*1i)/b^4)/((16*(108*B^3*a^12 - 12*A^3*a*b^11 - 54*B^3*a^11*b - 24*A^3*a^2*b^10 + 34*A^3*a^3*b^9 + 26*A^3*a^4*b^8 - 36*A^3*a^5*b^7 - 13*A^3*a^6*b^6 + 18*A^3*a^7*b^5 + 2*A^3*a^8*b^4 - 4*A^3*a^9*b^3 + 216*B^3*a^4*b^8 + 216*B^3*a^5*b^7 - 702*B^3*a^6*b^6 - 378*B^3*a^7*b^5 + 864*B^3*a^8*b^4 + 243*B^3*a^9*b^3 - 486*B^3*a^10*b^2 - 108*A*B^2*a^11*b - 252*A*B^2*a^3*b^9 - 324*A*B^2*a^4*b^8 + 774*A*B^2*a^5*b^7 + 486*A*B^2*a^6*b^6 - 900*A*B^2*a^7*b^5 - 279*A*B^2*a^8*b^4 + 486*A*B^2*a^9*b^3 + 54*A*B^2*a^10*b^2 + 96*A^2*B*a^2*b^10 + 156*A^2*B*a^3*b^9 - 282*A^2*B*a^4*b^8 - 198*A^2*B*a^5*b^7 + 312*A^2*B*a^6*b^6 + 105*A^2*B*a^7*b^5 - 162*A^2*B*a^8*b^4 - 18*A^2*B*a^9*b^3 + 36*A^2*B*a^10*b^2))/(a*b^15 + b^16 - 3*a^2*b^14 - 3*a^3*b^13 + 3*a^4*b^12 + 3*a^5*b^11 - a^6*b^10 - a^7*b^9) - (((8*tan(c/2 + (d*x)/2)*(4*A^2*b^12 + 72*B^2*a^12 - 8*A^2*a*b^11 - 72*B^2*a^11*b + 24*A^2*a^2*b^10 + 32*A^2*a^3*b^9 - 52*A^2*a^4*b^8 - 48*A^2*a^5*b^7 + 57*A^2*a^6*b^6 + 32*A^2*a^7*b^5 - 32*A^2*a^8*b^4 - 8*A^2*a^9*b^3 + 8*A^2*a^10*b^2 + 36*B^2*a^2*b^10 - 72*B^2*a^3*b^9 + 36*B^2*a^4*b^8 + 288*B^2*a^5*b^7 - 288*B^2*a^6*b^6 - 432*B^2*a^7*b^5 + 441*B^2*a^8*b^4 + 288*B^2*a^9*b^3 - 288*B^2*a^10*b^2 - 24*A*B*a*b^11 - 48*A*B*a^11*b + 48*A*B*a^2*b^10 - 72*A*B*a^3*b^9 - 192*A*B*a^4*b^8 + 252*A*B*a^5*b^7 + 288*A*B*a^6*b^6 - 318*A*B*a^7*b^5 - 192*A*B*a^8*b^4 + 192*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (((8*(4*A*b^18 - 8*A*a^2*b^16 + 34*A*a^3*b^15 + 6*A*a^4*b^14 - 36*A*a^5*b^13 - 4*A*a^6*b^12 + 18*A*a^7*b^11 + 2*A*a^8*b^10 - 4*A*a^9*b^9 + 24*B*a^2*b^...

3.330 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=220

$$\frac{B\tanh^{-1}(\sin(c+dx))}{b^3d}+\frac{(a^2Ab^3+2Ab^5-2a^5B+5a^3b^2B-6ab^4B)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}b^3(a+b)^{5/2}d}-\frac{a^2 \cdots}{2b^2\,(a \cdots}$$

```
[Out] B*arctanh(sin(d*x+c))/b^3/d+(A*a^2*b^3+2*A*b^5-2*B*a^5+5*B*a^3*b^2-6*B*a*b^
4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(5/2)/b^3/(a+b
)^(5/2)/d-1/2*a^2*(A*b-B*a)*tan(d*x+c)/b^2/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1
/2*a*(A*a^2*b-4*A*b^3-3*B*a^3+6*B*a*b^2)*tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*
sec(d*x+c))
```

Rubi [A]

time = 0.47, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4113, 4165, 4083, 3855, 3916, 2738, 214}

$$-\frac{a^2(Ab-aB)\tan(c+dx)}{2b^2d\,(a^2-b^2)\,(a+b\sec(c+dx))^2}+\frac{a(-3a^3B+a^2Ab+6ab^2B-4Ab^3)\tan(c+dx)}{2b^2d\,(a^2-b^2)^2\,(a+b\sec(c+dx))}+\frac{(-2a^5B+5a^3b^2B+a^2Ab^3-6ab^4B+2Ab^5)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^3d(a-b)^{5/2}(a+b)^{5/2}}+\frac{B\tanh^{-1}(\sin(c+dx))}{b^3d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(b^3*d) + ((a^2*A*b^3 + 2*A*b^5 - 2*a^5*B + 5*a^3
*b^2*B - 6*a*b^4*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((
a - b)^(5/2)*b^3*(a + b)^(5/2)*d) - (a^2*(A*b - a*B)*Tan[c + d*x])/(2*b^2*(
a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (a*(a^2*A*b - 4*A*b^3 - 3*a^3*B + 6*
a*b^2*B)*Tan[c + d*x])/(2*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4113

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-a^2)*(A*b - a*B)
*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x
] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])
^(m + 1)*Simp[a*b*(A*b - a*B)*(m + 1) - (A*b - a*B)*(a^2 + b^2*(m + 1))*Csc
[e + f*x] + b*B*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4165

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cot[e + f*x]*((a + b*Csc[e + f*x]
)^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)),
Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m +
 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2
, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx &= -\frac{a^2(Ab-aB)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \sec(c+dx)(-2ab(Ab-aB)-(a^2-\cdots}{\cdots} \\
&= -\frac{a^2(Ab-aB)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a(a^2Ab-4Ab^3-3a^3B+\cdots}{2b^2\left(a^2-b^2\right)^2d(a+\cdots} \\
&= -\frac{a^2(Ab-aB)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a(a^2Ab-4Ab^3-3a^3B+\cdots}{2b^2\left(a^2-b^2\right)^2d(a+\cdots} \\
&= \frac{B\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{a^2(Ab-aB)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a(\cdots}{\cdots} \\
&= \frac{B\tanh^{-1}(\sin(c+dx))}{b^3d} - \frac{a^2(Ab-aB)\tan(c+dx)}{2b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{a(\cdots}{\cdots} \\
&= \frac{B\tanh^{-1}(\sin(c+dx))}{b^3d} + \frac{(a^2Ab^3+2Ab^5-2a^5B+5a^3b^2B-6ab\cdots}{(a-b)^{5/2}b^3(a\cdots}
\end{aligned}
$$

Mathematica [A]
time = 1.75, size = 270, normalized size = 1.23

$$
\frac{\cos(c+dx)(A+B\sec(c+dx))\left(\frac{2(-a^2Ab^3-2Ab^5+2a^5B-5a^3b^2B+6ab^4B)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} - 2B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right) + 2B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right) + \frac{ab^2(-Ab+aB)\sin(c+dx)}{(-a+b)(a+b)(b+a\cos(c+dx))^2} + \frac{ab(-3Ab^3-2a^3B+5ab^2B)\sin(c+dx)}{(a-b)^2(a+b)^2(b+a\cos(c+dx))}\right)}{2b^3d(B+A\cos(c+dx))}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (Cos[c + d*x]*(A + B*Sec[c + d*x])*((2*(-(a^2*A*b^3) - 2*A*b^5 + 2*a^5*B -
5*a^3*b^2*B + 6*a*b^4*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2
]])/(a^2 - b^2)^(5/2) - 2*B*Log[Cos[(c + d*x)/2] - Sin[(c + d*x)/2]] + 2*B*
Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] + (a*b^2*(-(A*b) + a*B)*Sin[c + d*
x])/((-a + b)*(a + b)*(b + a*Cos[c + d*x])^2) + (a*b*(-3*A*b^3 - 2*a^3*B +
5*a*b^2*B)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*Cos[c + d*x]))))/(2*b^
3*d*(B + A*Cos[c + d*x]))
```

Maple [A]
time = 0.81, size = 305, normalized size = 1.39 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-2/b^3*((-1/2*(A*a*b^2+4*A*b^3+2*B*a^3-B*a^2*b-6*B*a*b^2)*b*a/(a-b)/(a
^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*a*b*(A*a*b^2-4*A*b^3-2*B*a^3-B*a^2*b
+6*B*a*b^2)/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan
(1/2*d*x+1/2*c)^2-a-b)^2-1/2*(A*a^2*b^3+2*A*b^5-2*B*a^5+5*B*a^3*b^2-6*B*a*b
^4)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c
)/((a+b)*(a-b))^(1/2)))-B/b^3*ln(tan(1/2*d*x+1/2*c)-1)+B/b^3*ln(tan(1/2*d*x
+1/2*c)+1))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 680 vs. 2(210) = 420.
time = 28.62, size = 1419, normalized size = 6.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [-1/4*((2*B*a^5*b^2 - 5*B*a^3*b^4 - A*a^2*b^5 + 6*B*a*b^6 - 2*A*b^7 + (2*B*
a^7 - 5*B*a^5*b^2 - A*a^4*b^3 + 6*B*a^3*b^4 - 2*A*a^2*b^5)*cos(d*x + c)^2 +
 2*(2*B*a^6*b - 5*B*a^4*b^3 - A*a^3*b^4 + 6*B*a^2*b^5 - 2*A*a*b^6)*cos(d*x
+ c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^
2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2
*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 2*(B*a^6*b^2 - 3*B*a^4*b^4 +
 3*B*a^2*b^6 - B*b^8 + (B*a^8 - 3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^6)*cos(
d*x + c)^2 + 2*(B*a^7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*cos(d*x + c)
)*log(sin(d*x + c) + 1) + 2*(B*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8
+ (B*a^8 - 3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^6)*cos(d*x + c)^2 + 2*(B*a^7
*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*cos(d*x + c))*log(-sin(d*x + c) +
 1) + 2*(3*B*a^6*b^2 - A*a^5*b^3 - 9*B*a^4*b^4 + 5*A*a^3*b^5 + 6*B*a^2*b^6
- 4*A*a*b^7 + (2*B*a^7*b - 7*B*a^5*b^3 + 3*A*a^4*b^4 + 5*B*a^3*b^5 - 3*A*a^
2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 3*a^6*b^5 + 3*a^4*b^7 - a^2*
```

```
b^9)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 + 3*a^3*b^8 - a*b^10)*d*cos(
d*x + c) + (a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*d), -1/2*((2*B*a^5*b^2
- 5*B*a^3*b^4 - A*a^2*b^5 + 6*B*a*b^6 - 2*A*b^7 + (2*B*a^7 - 5*B*a^5*b^2 -
A*a^4*b^3 + 6*B*a^3*b^4 - 2*A*a^2*b^5)*cos(d*x + c)^2 + 2*(2*B*a^6*b - 5*B*
a^4*b^3 - A*a^3*b^4 + 6*B*a^2*b^5 - 2*A*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^
2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))
) - (B*a^6*b^2 - 3*B*a^4*b^4 + 3*B*a^2*b^6 - B*b^8 + (B*a^8 - 3*B*a^6*b^2 +
 3*B*a^4*b^4 - B*a^2*b^6)*cos(d*x + c)^2 + 2*(B*a^7*b - 3*B*a^5*b^3 + 3*B*a
^3*b^5 - B*a*b^7)*cos(d*x + c))*log(sin(d*x + c) + 1) + (B*a^6*b^2 - 3*B*a^
4*b^4 + 3*B*a^2*b^6 - B*b^8 + (B*a^8 - 3*B*a^6*b^2 + 3*B*a^4*b^4 - B*a^2*b^
6)*cos(d*x + c)^2 + 2*(B*a^7*b - 3*B*a^5*b^3 + 3*B*a^3*b^5 - B*a*b^7)*cos(d
*x + c))*log(-sin(d*x + c) + 1) + (3*B*a^6*b^2 - A*a^5*b^3 - 9*B*a^4*b^4 +
5*A*a^3*b^5 + 6*B*a^2*b^6 - 4*A*a*b^7 + (2*B*a^7*b - 7*B*a^5*b^3 + 3*A*a^4*
b^4 + 5*B*a^3*b^5 - 3*A*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^8*b^3 - 3*
a^6*b^5 + 3*a^4*b^7 - a^2*b^9)*d*cos(d*x + c)^2 + 2*(a^7*b^4 - 3*a^5*b^6 +
3*a^3*b^8 - a*b^10)*d*cos(d*x + c) + (a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^1
1)*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec^3(c + dx)}{(a + b\sec(c + dx))^3}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x))**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 486 vs. 2(210) = 420.
time = 0.57, size = 486, normalized size = 2.21

$$-\frac{\frac{(2Ba^5-5Ba^3b^2-Aa^2b^3+6Bab^4-2Ab^5)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4b^3-2a^2b^5+b^7)\sqrt{-a^2+b^2}}-\frac{B\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right|\right)}{b^3}+\frac{B\log\left(\left|\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-1\right|\right)}{b^3}-\frac{2Ba^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3Ba^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+Aa^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-5Ba^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+3Aa^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6Ba^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4Aab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2Ba^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3Ba^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+Aa^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+5Ba^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3Aa^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+6Ba^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-4Aab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{(a^4b^2-2a^2b^4+b^6)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -((2*B*a^5 - 5*B*a^3*b^2 - A*a^2*b^3 + 6*B*a*b^4 - 2*A*b^5)*(pi*floor(1/2*(
d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*ta
n(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^4*b^3 - 2*a^2*b^5 + b^7)*sqrt(-a
^2 + b^2)) - B*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^3 + B*log(abs(tan(1/2*d
*x + 1/2*c) - 1))/b^3 - (2*B*a^5*tan(1/2*d*x + 1/2*c)^3 - 3*B*a^4*b*tan(1/2
*d*x + 1/2*c)^3 + A*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 5*B*a^3*b^2*tan(1/2*d*
x + 1/2*c)^3 + 3*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 6*B*a^2*b^3*tan(1/2*d*x
```

```
 + 1/2*c)^3 - 4*A*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^5*tan(1/2*d*x + 1/2*
c) - 3*B*a^4*b*tan(1/2*d*x + 1/2*c) + A*a^3*b^2*tan(1/2*d*x + 1/2*c) + 5*B*
a^3*b^2*tan(1/2*d*x + 1/2*c) - 3*A*a^2*b^3*tan(1/2*d*x + 1/2*c) + 6*B*a^2*b
^3*tan(1/2*d*x + 1/2*c) - 4*A*a*b^4*tan(1/2*d*x + 1/2*c))/((a^4*b^2 - 2*a^2
*b^4 + b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2
))/d
```

Mupad [B]
time = 11.53, size = 2500, normalized size = 11.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + b/cos(c + d*x))^3),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^3*(2*B*a^4 + A*a^2*b^2 - 6*B*a^2*b^2 + 4*A*a*b^3 - B*a
^3*b))/((a*b^2 - b^3)*(a + b)^2) - (tan(c/2 + (d*x)/2)*(2*B*a^4 - A*a^2*b^2
 - 6*B*a^2*b^2 + 4*A*a*b^3 + B*a^3*b))/((a + b)*(b^4 - 2*a*b^3 + a^2*b^2)))
/(d*(2*a*b - tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a
^2 - 2*a*b + b^2) + a^2 + b^2)) - (B*atan(-((B*((B*((8*(4*A*b^15 + 4*B*b^15
 - 6*A*a^2*b^13 + 6*A*a^3*b^12 + 2*A*a^6*b^9 - 2*A*a^7*b^8 - 8*B*a^2*b^13 +
 34*B*a^3*b^12 + 6*B*a^4*b^11 - 36*B*a^5*b^10 - 4*B*a^6*b^9 + 18*B*a^7*b^8
+ 2*B*a^8*b^7 - 4*B*a^9*b^6 - 4*A*a*b^14 - 12*B*a*b^14))/(a*b^12 + b^13 - 3
*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (8*B*
tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*
a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))
/(b^3*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*
b^5 - a^7*b^4))))/b^3 - (8*tan(c/2 + (d*x)/2)*(4*A^2*b^10 + 8*B^2*a^10 + 4*
B^2*b^10 - 8*B^2*a*b^9 - 8*B^2*a^9*b + 4*A^2*a^2*b^8 + A^2*a^4*b^6 + 24*B^2
*a^2*b^8 + 32*B^2*a^3*b^7 - 52*B^2*a^4*b^6 - 48*B^2*a^5*b^5 + 57*B^2*a^6*b^
4 + 32*B^2*a^7*b^3 - 32*B^2*a^8*b^2 - 24*A*B*a*b^9 + 8*A*B*a^3*b^7 + 2*A*B*
a^5*b^5 - 4*A*B*a^7*b^3))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^
7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))*1i)/b^3 - (B*((B*((8*(4*A*b^15 + 4*B*b^
15 - 6*A*a^2*b^13 + 6*A*a^3*b^12 + 2*A*a^6*b^9 - 2*A*a^7*b^8 - 8*B*a^2*b^13
 + 34*B*a^3*b^12 + 6*B*a^4*b^11 - 36*B*a^5*b^10 - 4*B*a^6*b^9 + 18*B*a^7*b^
8 + 2*B*a^8*b^7 - 4*B*a^9*b^6 - 4*A*a*b^14 - 12*B*a*b^14))/(a*b^12 + b^13 -
 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (8*
B*tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 4
8*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6
))/(b^3*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^
6*b^5 - a^7*b^4))))/b^3 + (8*tan(c/2 + (d*x)/2)*(4*A^2*b^10 + 8*B^2*a^10 +
4*B^2*b^10 - 8*B^2*a*b^9 - 8*B^2*a^9*b + 4*A^2*a^2*b^8 + A^2*a^4*b^6 + 24*B
^2*a^2*b^8 + 32*B^2*a^3*b^7 - 52*B^2*a^4*b^6 - 48*B^2*a^5*b^5 + 57*B^2*a^6*
b^4 + 32*B^2*a^7*b^3 - 32*B^2*a^8*b^2 - 24*A*B*a*b^9 + 8*A*B*a^3*b^7 + 2*A*
B*a^5*b^5 - 4*A*B*a^7*b^3))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*
```

b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))*1i)/b^3)/((16*(4*B^3*a^9 - 4*A*B^2*b^9 + 4*A^2*B*b^9 + 12*B^3*a*b^8 - 2*B^3*a^8*b + 24*B^3*a^2*b^7 - 34*B^3*a^3*b^6 - 26*B^3*a^4*b^5 + 36*B^3*a^5*b^4 + 13*B^3*a^6*b^3 - 18*B^3*a^7*b^2 - 20*A*B^2*a*b^8 + 6*A*B^2*a^2*b^7 + 2*A*B^2*a^3*b^6 + 2*A*B^2*a^5*b^4 - 2*A*B^2*a^6*b^3 - 2*A*B^2*a^7*b^2 + 4*A^2*B*a^2*b^7 + A^2*B*a^4*b^5))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (B*((B*((8*(4*A*b^15 + 4*B*b^15 - 6*A*a^2*b^13 + 6*A*a^3*b^12 + 2*A*a^6*b^9 - 2*A*a^7*b^8 - 8*B*a^2*b^13 + 34*B*a^3*b^12 + 6*B*a^4*b^11 - 36*B*a^5*b^10 - 4*B*a^6*b^9 + 18*B*a^7*b^8 + 2*B*a^8*b^7 - 4*B*a^9*b^6 - 4*A*a*b^14 - 12*B*a*b^14))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) - (8*B*tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))/(b^3*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))))/b^3 - (8*tan(c/2 + (d*x)/2)*(4*A^2*b^10 + 8*B^2*a^10 + 4*B^2*b^10 - 8*B^2*a*b^9 - 8*B^2*a^9*b + 4*A^2*a^2*b^8 + A^2*a^4*b^6 + 24*B^2*a^2*b^8 + 32*B^2*a^3*b^7 - 52*B^2*a^4*b^6 - 48*B^2*a^5*b^5 + 57*B^2*a^6*b^4 + 32*B^2*a^7*b^3 - 32*B^2*a^8*b^2 - 24*A*B*a*b^9 + 8*A*B*a^3*b^7 + 2*A*B*a^5*b^5 - 4*A*B*a^7*b^3))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))/b^3 + (B*((B*((8*(4*A*b^15 + 4*B*b^15 - 6*A*a^2*b^13 + 6*A*a^3*b^12 + 2*A*a^6*b^9 - 2*A*a^7*b^8 - 8*B*a^2*b^13 + 34*B*a^3*b^12 + 6*B*a^4*b^11 - 36*B*a^5*b^10 - 4*B*a^6*b^9 + 18*B*a^7*b^8 + 2*B*a^8*b^7 - 4*B*a^9*b^6 - 4*A*a*b^14 - 12*B*a*b^14))/(a*b^12 + b^13 - 3*a^2*b^11 - 3*a^3*b^10 + 3*a^4*b^9 + 3*a^5*b^8 - a^6*b^7 - a^7*b^6) + (8*B*tan(c/2 + (d*x)/2)*(8*a*b^15 - 8*a^2*b^14 - 32*a^3*b^13 + 32*a^4*b^12 + 48*a^5*b^11 - 48*a^6*b^10 - 32*a^7*b^9 + 32*a^8*b^8 + 8*a^9*b^7 - 8*a^10*b^6))/(b^3*(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4))))/b^3 + (8*tan(c/2 + (d*x)/2)*(4*A^2*b^10 + 8*B^2*a^10 + 4*B^2*b^10 - 8*B^2*a*b^9 - 8*B^2*a^9*b + 4*A^2*a^2*b^8 + A^2*a^4*b^6 + 24*B^2*a^2*b^8 + 32*B^2*a^3*b^7 - 52*B^2*a^4*b^6 - 48*B^2*a^5*b^5 + 57*B^2*a^6*b^4 + 32*B^2*a^7*b^3 - 32*B^2*a^8*b^2 - 24*A*B*a*b^9 + 8*A*B*a^3*b^7 + 2*A*B*a^5*b^5 - 4*A*B*a^7*b^3))/(a*b^10 + b^11 - 3*a^2*b^9 - 3*a^3*b^8 + 3*a^4*b^7 + 3*a^5*b^6 - a^6*b^5 - a^7*b^4)))/b^3))*2i)/(b^3*d) - (atan(((((a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^10 + 8*B^2*a^10 + 4*B^2*b^10 - 8*B^2*a*b^9 - 8*B^2*a^9*b + 4*A^2*a^2*b^8 + A^2*a^4*b^6 + 24*B^2*a^2*b^8 + 32*B^2*a^3*b^7 - 52*B^2*a^4*b^6 - 48*B^2*a^5*b^5 + 57*B^2*a^6*b^4 + 32*B^2*a^7*b^3 - 32*B^2*a^8*b^2 - 24*A*B*a*b^9 + 8*A*B*a^3*b^7 + 2*A*B*a^5*b^5 - 4*A*B*a^7...

3.331 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=180

$$-\frac{(3aAb-a^2B-2b^2B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d}+\frac{a(Ab-aB)\tan(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}+\frac{(a^2Ab+2Ab^3+}{2b\left(a^2-b^2\right)}$$

```
[Out] -(3*A*a*b-B*a^2-2*B*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2)
)/(a-b)^(5/2)/(a+b)^(5/2)/d+1/2*a*(A*b-B*a)*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*s
ec(d*x+c))^2+1/2*(A*a^2*b+2*A*b^3+B*a^3-4*B*a*b^2)*tan(d*x+c)/b/(a^2-b^2)^2
/d/(a+b*sec(d*x+c))
```

Rubi [A]

time = 0.24, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4094, 4088, 12, 3916, 2738, 214}

$$-\frac{(a^2(-B)+3aAb-2b^2B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}+\frac{a(Ab-aB)\tan(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{\left(a^3B+a^2Ab-4ab^2B+2Ab^3\right)\tan(c+dx)}{2bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(((3*a*A*b - a^2*B - 2*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[
a + b]])/((a - b)^(5/2)*(a + b)^(5/2)*d)) + (a*(A*b - a*B)*Tan[c + d*x])/(2
*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + ((a^2*A*b + 2*A*b^3 + a^3*B - 4*
a*b^2*B)*Tan[c + d*x])/(2*b*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rule 4094

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*(A*b - a*B)*Cot[
e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dis
t[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
Simp[b*(A*b - a*B)*(m + 1) - (a*A*b*(m + 2) - B*(a^2 + b^2*(m + 1)))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \frac{a(Ab-aB)\tan(c+dx)}{2b(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{\int \frac{\sec(c+dx)(-2b(Ab-aB)+(aAb+a^2B-}{(a+b\sec(c+dx))^2}}{2b(a^2-b^2)}$$

$$= \frac{a(Ab-aB)\tan(c+dx)}{2b(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{(a^2Ab+2Ab^3+a^3B-4ab^2B)}{2b(a^2-b^2)^2\,d(a+b\sec(c}$$

$$= \frac{a(Ab-aB)\tan(c+dx)}{2b(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{(a^2Ab+2Ab^3+a^3B-4ab^2B)}{2b(a^2-b^2)^2\,d(a+b\sec(c}$$

$$= \frac{a(Ab-aB)\tan(c+dx)}{2b(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{(a^2Ab+2Ab^3+a^3B-4ab^2B)}{2b(a^2-b^2)^2\,d(a+b\sec(c}$$

$$= \frac{a(Ab-aB)\tan(c+dx)}{2b(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{(a^2Ab+2Ab^3+a^3B-4ab^2B)}{2b(a^2-b^2)^2\,d(a+b\sec(c}$$

$$= -\frac{(3aAb-a^2B-2b^2B)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} + \frac{a(Al}{2b(a^2-}$$

Mathematica [A]
time = 0.75, size = 157, normalized size = 0.87

$$-\frac{\frac{2(-3aAb+a^2B+2b^2B)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{5/2}} + \frac{(-Ab+aB)\sin(c+dx)}{(a-b)(a+b)(b+a\cos(c+dx))^2} + \frac{(2a^2A+Ab^2-3abB)\sin(c+dx)}{(a-b)^2(a+b)^2(b+a\cos(c+dx))}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-2*(-3*a*A*b + a^2*B + 2*b^2*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[
a^2 - b^2]])/(a^2 - b^2)^(5/2) + ((-(A*b) + a*B)*Sin[c + d*x])/((a - b)*(a
+ b)*(b + a*Cos[c + d*x])^2) + ((2*a^2*A + A*b^2 - 3*a*b*B)*Sin[c + d*x])/(
(a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])))/(2*d)
```

Maple [A]
time = 0.48, size = 238, normalized size = 1.32

method	result
derivativedivides	$\frac{\frac{-\frac{\left(2a^2A+Aba+2Ab^2-a^2B-4Bab\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)\left(a^2+2ba+b^2\right)}+\frac{\left(2a^2A-Aba+2Ab^2+a^2B-4Bab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)\left(a^2-2ba+b^2\right)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}-\frac{\left(3Aba-a^2B-2b^2B\right)\operatorname{arctanh}}{\left(a^4-2b^2a^2+b^4\right)\sqrt{}}}{d}$

default	$\frac{\frac{-\frac{\left(2a^2A+Aba+2A\,b^2-a^2B-4Bab\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)\left(a^2+2ba+b^2\right)}+\frac{\left(2a^2A-Aba+2A\,b^2+a^2B-4Bab\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{(a+b)\left(a^2-2ba+b^2\right)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}-\frac{\left(3Aba-a^2B-2b^2B\right)\arctan}{\left(a^4-2b^2a^2+b^4\right)\sqrt{}}}{d}$
risch	$\frac{i\left(3A\,a^3b\,e^{3i(dx+c)}-B\,a^4e^{3i(dx+c)}-2B\,a^2b^2e^{3i(dx+c)}+2A\,a^4e^{2i(dx+c)}+5A\,a^2b^2e^{2i(dx+c)}+2A\,b^4e^{2i(dx+c)}-3B\,a^3b\,e^{2i(d}\right.}{a\left(-a^2+b^2\right)^2d\left(a\,e^{2i(dx-}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(2*(-1/2*(2*A*a^2+A*a*b+2*A*b^2-B*a^2-4*B*a*b)/(a-b)/(a^2+2*a*b+b^2)*ta
n(1/2*d*x+1/2*c)^3+1/2*(2*A*a^2-A*a*b+2*A*b^2+B*a^2-4*B*a*b)/(a+b)/(a^2-2*a
*b+b^2)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-
a-b)^2-(3*A*a*b-B*a^2-2*B*b^2)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arct
anh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 346 vs. $2(165) = 330$.
time = 4.28, size = 750, normalized size = 4.17

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*((B*a^2*b^2 - 3*A*a*b^3 + 2*B*b^4 + (B*a^4 - 3*A*a^3*b + 2*B*a^2*b^2)*
cos(d*x + c)^2 + 2*(B*a^3*b - 3*A*a^2*b^2 + 2*B*a*b^3)*cos(d*x + c))*sqrt(a
^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a
```

```
^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(B*a^5 + A*a^4*b - 5*B*a^3*b^2 + A*a^2
*b^3 + 4*B*a*b^4 - 2*A*b^5 + (2*A*a^5 - 3*B*a^4*b - A*a^3*b^2 + 3*B*a^2*b^3
 - A*a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2
*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*
x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d), 1/2*((B*a^2*b^2 - 3*A*
a*b^3 + 2*B*b^4 + (B*a^4 - 3*A*a^3*b + 2*B*a^2*b^2)*cos(d*x + c)^2 + 2*(B*a
^3*b - 3*A*a^2*b^2 + 2*B*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt
(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (B*a^5 + A*
a^4*b - 5*B*a^3*b^2 + A*a^2*b^3 + 4*B*a*b^4 - 2*A*b^5 + (2*A*a^5 - 3*B*a^4*
b - A*a^3*b^2 + 3*B*a^2*b^3 - A*a*b^4)*cos(d*x + c))*sin(d*x + c))/((a^8 -
3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 +
3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)
*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\sec^2(c+dx)}{(a+b\sec(c+dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. 2(165) = 330.
time = 0.52, size = 400, normalized size = 2.22

$$\frac{\frac{(Ba^2-3Aab+2Bb^2)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2a+2b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^4-2a^2b^2+b^4)\sqrt{-a^2+b^2}}-\frac{2Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-Aa^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-3Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+Aab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+4Bab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2Ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-2Aa^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-Ba^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-Aa^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3Ba^2b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-Aab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+4Bab^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2Ab^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{(a^4-2a^2b^2+b^4)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] ((B*a^2 - 3*A*a*b + 2*B*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2
*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 +
 b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) - (2*A*a^3*tan(1/2*d*x +
 1/2*c)^3 - B*a^3*tan(1/2*d*x + 1/2*c)^3 - A*a^2*b*tan(1/2*d*x + 1/2*c)^3 -
 3*B*a^2*b*tan(1/2*d*x + 1/2*c)^3 + A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*B*a*
b^2*tan(1/2*d*x + 1/2*c)^3 - 2*A*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*A*a^3*tan(1
/2*d*x + 1/2*c) - B*a^3*tan(1/2*d*x + 1/2*c) - A*a^2*b*tan(1/2*d*x + 1/2*c)
 + 3*B*a^2*b*tan(1/2*d*x + 1/2*c) - A*a*b^2*tan(1/2*d*x + 1/2*c) + 4*B*a*b^
2*tan(1/2*d*x + 1/2*c) - 2*A*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 +
b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

Mupad [B]
time = 5.42, size = 251, normalized size = 1.39

$$\frac{\operatorname{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,a-2\,b)\,(a^2-2\,a\,b+b^2)}{2\,\sqrt{a+b}\,(a-b)^{5/2}}\right)\,(B\,a^2-3\,A\,a\,b+2\,B\,b^2)}{d\,(a+b)^{5/2}\,(a-b)^{5/2}}-\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,(2\,A\,a^2+2\,A\,b^2-B\,a^2+A\,a\,b-4\,B\,a\,b)}{(a+b)^2\,(a-b)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(2\,A\,a^2+2\,A\,b^2+B\,a^2-A\,a\,b-4\,B\,a\,b)}{(a+b)\,(a^2-2\,a\,b+b^2)}}{d\,\left(2\,a\,b-\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2\,(2\,a^2-2\,b^2)+\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^4\,(a^2-2\,a\,b+b^2)+a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + b/cos(c + d*x))^3),x)
```

```
[Out] (atanh((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2*(a + b)^(1/2
)*(a - b)^(5/2)))*(B*a^2 + 2*B*b^2 - 3*A*a*b))/(d*(a + b)^(5/2)*(a - b)^(5/
2)) - ((tan(c/2 + (d*x)/2)^3*(2*A*a^2 + 2*A*b^2 - B*a^2 + A*a*b - 4*B*a*b))
/((a + b)^2*(a - b)) - (tan(c/2 + (d*x)/2)*(2*A*a^2 + 2*A*b^2 + B*a^2 - A*a
*b - 4*B*a*b))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b - tan(c/2 + (d*x)/2
)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2)
)
```

3.332 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=164

$$\frac{(2a^2A+Ab^2-3abB)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d}-\frac{(Ab-aB)\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}-\frac{(3aAb-a^2B-2b^2\dots}{2\left(a^2-b^2\right)^2 d(a+\dots}$$

```
[Out] (2*A*a^2+A*b^2-3*B*a*b)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))
/(a-b)^(5/2)/(a+b)^(5/2)/d-1/2*(A*b-B*a)*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*
x+c))^2-1/2*(3*A*a*b-B*a^2-2*B*b^2)*tan(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c
))
```

Rubi [A]
time = 0.19, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4088, 12, 3916, 2738, 214}

$$\frac{(2a^2A-3abB+Ab^2)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{5/2}(a+b)^{5/2}}-\frac{(a^2(-B)+3aAb-2b^2B)\tan(c+dx)}{2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{(Ab-aB)\tan(c+dx)}{2d\left(a^2-b^2\right)(a+b\sec(c+dx))^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((2*a^2*A + A*b^2 - 3*a*b*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/((a - b)^(5/2)*(a + b)^(5/2)*d) - ((A*b - a*B)*Tan[c + d*x])/(2*(a^2
 - b^2)*d*(a + b*Sec[c + d*x])^2) - ((3*a*A*b - a^2*B - 2*b^2*B)*Tan[c + d*
x])/(2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = -\frac{(Ab-aB)\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\sec(c+dx)(-2(aA-bB)+(Ab-aB)\,\mathrm{se}}{(a+b\sec(c+dx))^2}}{2\left(a^2-b^2\right)}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(3aAb-a^2B-2b^2B\right)\tan(c-}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(3aAb-a^2B-2b^2B\right)\tan(c-}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(3aAb-a^2B-2b^2B\right)\tan(c-}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{2\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\left(3aAb-a^2B-2b^2B\right)\tan(c-}{2\left(a^2-b^2\right)^2 d(a+b\sec(c+}$$

$$= \frac{\left(2a^2A+Ab^2-3abB\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{5/2}(a+b)^{5/2}d} - \frac{(Ab-}{2\left(a^2-b^2\right)}$$

Mathematica [A]
time = 0.94, size = 172, normalized size = 1.05

$$\frac{-\frac{2\left(2a^2A+Ab^2-3abB\right)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{5/2}} + \frac{b(Ab-aB)\sin(c+dx)}{a(a-b)(a+b)(b+a\cos(c+dx))^2} + \frac{\left(-4a^2Ab+Ab^3+2a^3B+ab^2B\right)\sin(c+dx)}{a(a-b)^2(a+b)^2(b+a\cos(c+dx))}}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((-2*(2*a^2*A + A*b^2 - 3*a*b*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a
^2 - b^2]])/(a^2 - b^2)^(5/2) + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a - b)*(a
+ b)*(b + a*Cos[c + d*x])^2) + ((-4*a^2*A*b + A*b^3 + 2*a^3*B + a*b^2*B)*Si
n[c + d*x])/(a*(a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])))/(2*d)
```

Maple [A]
time = 0.45, size = 236, normalized size = 1.44

method	result
derivativedivides	$\frac{-\frac{2\left(-\frac{\left(4Aba+A\,b^2-2a^2B-Bab-2b^2B\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ba+b^2\right)}+\frac{\left(4Aba-A\,b^2-2a^2B+Bab-2b^2B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ba+b^2\right)}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}+\frac{\left(2a^2A+A\,b^2-3Bab\right)a\cdots}{\left(a^4-2b^2a^2+b\cdots\right.}}{d}$
default	$\frac{-\frac{2\left(-\frac{\left(4Aba+A\,b^2-2a^2B-Bab-2b^2B\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ba+b^2\right)}+\frac{\left(4Aba-A\,b^2-2a^2B+Bab-2b^2B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{2(a+b)\left(a^2-2ba+b^2\right)}\right)}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}+\frac{\left(2a^2A+A\,b^2-3Bab\right)a\cdots}{\left(a^4-2b^2a^2+b\cdots\right.}}{d}$
risch	$\frac{i\left(-5A\,a^3b^2\mathrm{e}^{3i(dx+c)}+2Aa\,b^4\mathrm{e}^{3i(dx+c)}+3B\,a^4b\,\mathrm{e}^{3i(dx+c)}-4A\,a^4b\,\mathrm{e}^{2i(dx+c)}-7A\,a^2b^3\mathrm{e}^{2i(dx+c)}+2A\,b^5\mathrm{e}^{2i(dx+c)}+2B\,a^5\mathrm{e}^{2i\cdots}\right.}{a^2\left(-a^2\cdots\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*(4*A*a*b+A*b^2-2*B*a^2-B*a*b-2*B*b^2)/(a-b)/(a^2+2*a*b+b^2)*t
an(1/2*d*x+1/2*c)^3+1/2*(4*A*a*b-A*b^2-2*B*a^2+B*a*b-2*B*b^2)/(a+b)/(a^2-2*
a*b+b^2)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2
-a-b)^2+(2*A*a^2+A*b^2-3*B*a*b)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arc
tanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
```

```
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see 'assume?' for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 347 vs. $2(150) = 300$.

time = 3.87, size = 752, normalized size = 4.59

$$\left[\frac{\left(2\,Aa^{2}b^{2} - 3\,Bab^{3} + Ab^{4} + \left(2\,Aa^{4} - 3\,Ba^{3}b + Aa^{2}b^{2}\right)\cos\left(dx + c\right)^{2} + 2\,\left(2\,Aa^{3}b - 3\,Ba^{2}b^{2} + Aab^{3}\right)\cos\left(dx + c\right)\right)\sqrt{a^{2} - b^{2}}\log\left(\frac{2\,ab\cos\left(dx+c\right) - \left(a^{2} - 2\,b^{2}\right)\cos\left(dx+c\right)^{2} + 2\,\sqrt{a^{2} - b^{2}}\left(b\cos\left(dx+c\right) + a\right)\sin\left(dx+c\right) + 2\,a^{2} - b^{2}}{a^{2}\cos\left(dx+c\right)^{2} + 2\,ab\cos\left(dx+c\right) + b^{2}}\right) + 2\,\left(Ba^{4}b - 3\,Aa^{3}b^{2} + Ba^{2}b^{3} + 3\,Aab^{4} - 2\,Bb^{5} + \left(2\,Ba^{5} - 4\,Aa^{4}b - Ba^{3}b^{2} + 5\,Aa^{2}b^{3} - Bab^{4} - Ab^{5}\right)\cos\left(dx + c\right)\right)\sin\left(dx + c\right)}{4\,\left(\left(a^{8} - 3\,a^{6}b^{2} + 3\,a^{4}b^{4} - a^{2}b^{6}\right)d\cos\left(dx + c\right)^{2} + 2\,\left(a^{7}b - 3\,a^{5}b^{3} + 3\,a^{3}b^{5} - ab^{7}\right)d\cos\left(dx + c\right) + \left(a^{6}b^{2} - 3\,a^{4}b^{4} + 3\,a^{2}b^{6} - b^{8}\right)d\right)}, \frac{\left(2\,Aa^{2}b^{2} - 3\,Bab^{3} + Ab^{4} + \left(2\,Aa^{4} - 3\,Ba^{3}b + Aa^{2}b^{2}\right)\cos\left(dx + c\right)^{2} + 2\,\left(2\,Aa^{3}b - 3\,Ba^{2}b^{2} + Aab^{3}\right)\cos\left(dx + c\right)\right)\sqrt{-a^{2} + b^{2}}\arctan\left(-\frac{\sqrt{-a^{2} + b^{2}}\left(b\cos\left(dx+c\right) + a\right)}{\left(a^{2} - b^{2}\right)\sin\left(dx+c\right)}\right) + \left(Ba^{4}b - 3\,Aa^{3}b^{2} + Ba^{2}b^{3} + 3\,Aab^{4} - 2\,Bb^{5} + \left(2\,Ba^{5} - 4\,Aa^{4}b - Ba^{3}b^{2} + 5\,Aa^{2}b^{3} - Bab^{4} - Ab^{5}\right)\cos\left(dx + c\right)\right)\sin\left(dx + c\right)}{2\,\left(\left(a^{8} - 3\,a^{6}b^{2} + 3\,a^{4}b^{4} - a^{2}b^{6}\right)d\cos\left(dx + c\right)^{2} + 2\,\left(a^{7}b - 3\,a^{5}b^{3} + 3\,a^{3}b^{5} - ab^{7}\right)d\cos\left(dx + c\right) + \left(a^{6}b^{2} - 3\,a^{4}b^{4} + 3\,a^{2}b^{6} - b^{8}\right)d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] [1/4*((2*A*a^2*b^2 - 3*B*a*b^3 + A*b^4 + (2*A*a^4 - 3*B*a^3*b + A*a^2*b^2)*
cos(d*x + c)^2 + 2*(2*A*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sqrt(a
^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a
^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c
)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(B*a^4*b - 3*A*a^3*b^2 + B*a^2*b^3 + 3
*A*a*b^4 - 2*B*b^5 + (2*B*a^5 - 4*A*a^4*b - B*a^3*b^2 + 5*A*a^2*b^3 - B*a*b
^4 - A*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8 - 3*a^6*b^2 + 3*a^4*b^4 - a^2
*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 + 3*a^3*b^5 - a*b^7)*d*cos(d*
x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)*d), 1/2*((2*A*a^2*b^2 - 3*
B*a*b^3 + A*b^4 + (2*A*a^4 - 3*B*a^3*b + A*a^2*b^2)*cos(d*x + c)^2 + 2*(2*A
*a^3*b - 3*B*a^2*b^2 + A*a*b^3)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt
(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (B*a^4*b -
3*A*a^3*b^2 + B*a^2*b^3 + 3*A*a*b^4 - 2*B*b^5 + (2*B*a^5 - 4*A*a^4*b - B*a^
3*b^2 + 5*A*a^2*b^3 - B*a*b^4 - A*b^5)*cos(d*x + c))*sin(d*x + c))/((a^8 -
3*a^6*b^2 + 3*a^4*b^4 - a^2*b^6)*d*cos(d*x + c)^2 + 2*(a^7*b - 3*a^5*b^3 +
3*a^3*b^5 - a*b^7)*d*cos(d*x + c) + (a^6*b^2 - 3*a^4*b^4 + 3*a^2*b^6 - b^8)
*d)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B\sec\left(c + dx\right)\right)\sec\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 399 vs. $2(150) = 300$.

time = 0.54, size = 399, normalized size = 2.43

$$\frac{\frac{\left(2\,Aa^{2} - 3\,Bab + Ab^{2}\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi} + \frac{1}{2}\right\rfloor\mathrm{sgn}\left(-2\,a + 2\,b\right) + \arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\sqrt{-a^{2} + b^{2}}}\right)\right)}{\left(a^{4} - 2\,a^{2}b^{2} + b^{4}\right)\sqrt{-a^{2} + b^{2}}} - \frac{2\,Ba^{3}\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{3} - 4\,Aa^{2}b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{3} - Ba^{2}b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{3} + 3\,Aab^{2}\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{3} + Bab^{2}\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{3} + Ab^{3}\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{3} - 2\,Bb^{3}\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{3} - 2\,Ba^{3}\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 4\,Aa^{2}b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - Ba^{2}b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) + 3\,Aab^{2}\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - Bab^{2}\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - Ab^{3}\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right) - 2\,Bb^{3}\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)}{\left(a^{4} - 2\,a^{2}b^{2} + b^{4}\right)\left(a\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{2} - b\tan\left(\frac{1}{2}\,dx + \frac{1}{2}\,c\right)^{2} - a - b\right)^{2}}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] ((2*A*a^2 - 3*B*a*b + A*b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2
*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 +
 b^2)))/((a^4 - 2*a^2*b^2 + b^4)*sqrt(-a^2 + b^2)) - (2*B*a^3*tan(1/2*d*x +
 1/2*c)^3 - 4*A*a^2*b*tan(1/2*d*x + 1/2*c)^3 - B*a^2*b*tan(1/2*d*x + 1/2*c)
^3 + 3*A*a*b^2*tan(1/2*d*x + 1/2*c)^3 + B*a*b^2*tan(1/2*d*x + 1/2*c)^3 + A*
b^3*tan(1/2*d*x + 1/2*c)^3 - 2*B*b^3*tan(1/2*d*x + 1/2*c)^3 - 2*B*a^3*tan(1
/2*d*x + 1/2*c) + 4*A*a^2*b*tan(1/2*d*x + 1/2*c) - B*a^2*b*tan(1/2*d*x + 1/
2*c) + 3*A*a*b^2*tan(1/2*d*x + 1/2*c) - B*a*b^2*tan(1/2*d*x + 1/2*c) - A*b^
3*tan(1/2*d*x + 1/2*c) - 2*B*b^3*tan(1/2*d*x + 1/2*c))/((a^4 - 2*a^2*b^2 +
b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

Mupad [B]
time = 5.35, size = 251, normalized size = 1.53

$$\frac{\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a-2\,b\right)\,\left(a^2-2\,a\,b+b^2\right)}{2\,\sqrt{a+b}\,\,{\left(a-b\right)}^{5/2}}\right)\,\left(2\,A\,a^2-3\,B\,a\,b+A\,b^2\right)}{d\,{\left(a+b\right)}^{5/2}\,{\left(a-b\right)}^{5/2}}-\frac{\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(2\,B\,a^2-A\,b^2+2\,B\,b^2-4\,A\,a\,b+B\,a\,b\right)}{{\left(a+b\right)}^2\,\left(a-b\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A\,b^2+2\,B\,a^2+2\,B\,b^2-4\,A\,a\,b-B\,a\,b\right)}{\left(a+b\right)\,\left(a^2-2\,a\,b+b^2\right)}}{d\,\left(2\,a\,b-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(2\,a^2-2\,b^2\right)+{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(a^2-2\,a\,b+b^2\right)+a^2+b^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + b/cos(c + d*x))^3),x)
```

```
[Out] (atanh((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(a^2 - 2*a*b + b^2))/(2*(a + b)^(1/2
)*(a - b)^(5/2)))*(2*A*a^2 + A*b^2 - 3*B*a*b))/(d*(a + b)^(5/2)*(a - b)^(5/
2)) - ((tan(c/2 + (d*x)/2)^3*(2*B*a^2 - A*b^2 + 2*B*b^2 - 4*A*a*b + B*a*b))
/((a + b)^2*(a - b)) - (tan(c/2 + (d*x)/2)*(A*b^2 + 2*B*a^2 + 2*B*b^2 - 4*A
*a*b - B*a*b))/((a + b)*(a^2 - 2*a*b + b^2)))/(d*(2*a*b - tan(c/2 + (d*x)/2
)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 - 2*a*b + b^2) + a^2 + b^2)
)
```

3.333 $\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=205

$$\frac{Ax}{a^3}-\frac{(6a^4Ab-5a^2Ab^3+2Ab^5-2a^5B-a^3b^2B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3(a-b)^{5/2}(a+b)^{5/2}d}+\frac{b(Ab-aB)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2}$$

```
[Out] A*x/a^3-(6*A*a^4*b-5*A*a^2*b^3+2*A*b^5-2*B*a^5-B*a^3*b^2)*arctanh((a-b)^(1/
2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^3/(a-b)^(5/2)/(a+b)^(5/2)/d+1/2*b*(A*b
-B*a)*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/2*b*(5*A*a^2*b-2*A*b^3-
3*B*a^3)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.37, antiderivative size = 205, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4008, 4145, 4004, 3916, 2738, 214}

$$\frac{Ax}{a^3}+\frac{b(Ab-aB)\tan(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{b(-3a^3B+5a^2Ab-2Ab^3)\tan(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{(-2a^5B+6a^4Ab-a^3b^2B-5a^2Ab^3+2Ab^5)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^3d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] (A*x)/a^3 - ((6*a^4*A*b - 5*a^2*A*b^3 + 2*A*b^5 - 2*a^5*B - a^3*b^2*B)*ArcT
anh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^3*(a - b)^(5/2)*(a + b)
^(5/2)*d) + (b*(A*b - a*B)*Tan[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c +
d*x])^2) + (b*(5*a^2*A*b - 2*A*b^3 - 3*a^3*B)*Tan[c + d*x])/(2*a^2*(a^2 - b
^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
```

```
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4008

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^3}\,dx = \frac{b(Ab-aB)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{-2A\left(a^2-b^2\right)+2a(Ab-aB)\sec(c+dx)-b(Ab-aB)\sec^2}{(a+b\sec(c+dx))^2}}{2a\left(a^2-b^2\right)}$$

$$= \frac{b(Ab-aB)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b\left(5a^2Ab-2Ab^3-3a^3B\right)\tan(c+dx)}{2a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx))} + \frac{\int}{}$$

$$= \frac{Ax}{a^3} + \frac{b(Ab-aB)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b\left(5a^2Ab-2Ab^3-3a^3B\right)\tan(c+d}{2a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx)}$$

$$= \frac{Ax}{a^3} + \frac{b(Ab-aB)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b\left(5a^2Ab-2Ab^3-3a^3B\right)\tan(c+d}{2a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx)}$$

$$= \frac{Ax}{a^3} + \frac{b(Ab-aB)\tan(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b\left(5a^2Ab-2Ab^3-3a^3B\right)\tan(c+d}{2a^2\left(a^2-b^2\right)^2d(a+b\sec(c+dx)}$$

$$= \frac{Ax}{a^3} - \frac{\left(6a^4Ab-5a^2Ab^3+2Ab^5-2a^5B-a^3b^2B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right.}{a^3(a-b)^{5/2}(a+b)^{5/2}d}$$

Mathematica [A]
time = 1.49, size = 267, normalized size = 1.30

$$\frac{(b+a\cos(c+dx))\sec^2(c+dx)(A+B\sec(c+dx))\left(2A(c+dx)(b+a\cos(c+dx))^2-\frac{2\left(-6a^4Ab+5a^2Ab^3-2Ab^5+2a^5B+a^3b^2B\right)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))^2}{\left(a^2-b^2\right)^{5/2}}+\frac{ab^2(-Ab+aB)\sin(c+dx)}{(a-b)(a+b)}-\frac{ab\left(-6a^2Ab+3Ab^3+4a^3B-ab^2B\right)(b+a\cos(c+dx))\sin(c+dx)}{(a-b)^2(a+b)^2}\right)}{2a^3d(B+A\cos(c+dx))(a+b\sec(c+dx))^3}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*(2*A*(c + d*x)*(b
 + a*Cos[c + d*x])^2 - (2*(-6*a^4*A*b + 5*a^2*A*b^3 - 2*A*b^5 + 2*a^5*B + a
^3*b^2*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c
 + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^2*(-(A*b) + a*B)*Sin[c + d*x])/((a - b
)*(a + b)) - (a*b*(-6*a^2*A*b + 3*A*b^3 + 4*a^3*B - a*b^2*B)*(b + a*Cos[c +
 d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2)))/(2*a^3*d*(B + A*Cos[c + d*x])*
(a + b*Sec[c + d*x])^3)
```

Maple [A]
time = 0.48, size = 287, normalized size = 1.40 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^3*((-1/2*(6*A*a^2*b+A*a*b^2-2*A*b^3-4*B*a^3-B*a^2*b)*b*a/(a-b)/(a^
2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*a*b*(6*A*a^2*b-A*a*b^2-2*A*b^3-4*B*a^
```

```
3+B*a^2*b)/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(
1/2*d*x+1/2*c)^2-a-b)^2-1/2*(6*A*a^4*b-5*A*a^2*b^3+2*A*b^5-2*B*a^5-B*a^3*b^
2)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)
/((a+b)*(a-b))^(1/2)))+2*A/a^3*arctan(tan(1/2*d*x+1/2*c)))
```

Maxima [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(190) = 380$.
time = 3.64, size = 1152, normalized size = 5.62

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fricas")
```

```
[Out] [1/4*(4*(A*a^8 - 3*A*a^6*b^2 + 3*A*a^4*b^4 - A*a^2*b^6)*d*x*cos(d*x + c)^2
+ 8*(A*a^7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*d*x*cos(d*x + c) + 4*(A
*a^6*b^2 - 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*d*x - (2*B*a^5*b^2 - 6*A*a^4*
b^3 + B*a^3*b^4 + 5*A*a^2*b^5 - 2*A*b^7 + (2*B*a^7 - 6*A*a^6*b + B*a^5*b^2
+ 5*A*a^4*b^3 - 2*A*a^2*b^5)*cos(d*x + c)^2 + 2*(2*B*a^6*b - 6*A*a^5*b^2 +
B*a^4*b^3 + 5*A*a^3*b^4 - 2*A*a*b^6)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a
*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d
*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*
x + c) + b^2)) - 2*(3*B*a^6*b^2 - 5*A*a^5*b^3 - 3*B*a^4*b^4 + 7*A*a^3*b^5 -
 2*A*a*b^7 + (4*B*a^7*b - 6*A*a^6*b^2 - 5*B*a^5*b^3 + 9*A*a^4*b^4 + B*a^3*b
^5 - 3*A*a^2*b^6)*cos(d*x + c))*sin(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^
4 - a^5*b^6)*d*cos(d*x + c)^2 + 2*(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7
)*d*cos(d*x + c) + (a^9*b^2 - 3*a^7*b^4 + 3*a^5*b^6 - a^3*b^8)*d), 1/2*(2*(
A*a^8 - 3*A*a^6*b^2 + 3*A*a^4*b^4 - A*a^2*b^6)*d*x*cos(d*x + c)^2 + 4*(A*a^
7*b - 3*A*a^5*b^3 + 3*A*a^3*b^5 - A*a*b^7)*d*x*cos(d*x + c) + 2*(A*a^6*b^2
- 3*A*a^4*b^4 + 3*A*a^2*b^6 - A*b^8)*d*x + (2*B*a^5*b^2 - 6*A*a^4*b^3 + B*a
^3*b^4 + 5*A*a^2*b^5 - 2*A*b^7 + (2*B*a^7 - 6*A*a^6*b + B*a^5*b^2 + 5*A*a^4
*b^3 - 2*A*a^2*b^5)*cos(d*x + c)^2 + 2*(2*B*a^6*b - 6*A*a^5*b^2 + B*a^4*b^3
 + 5*A*a^3*b^4 - 2*A*a*b^6)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^
```

```
2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (3*B*a^6*b^2 -
5*A*a^5*b^3 - 3*B*a^4*b^4 + 7*A*a^3*b^5 - 2*A*a*b^7 + (4*B*a^7*b - 6*A*a^6*
b^2 - 5*B*a^5*b^3 + 9*A*a^4*b^4 + B*a^3*b^5 - 3*A*a^2*b^6)*cos(d*x + c))*si
n(d*x + c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*d*cos(d*x + c)^2 + 2*
(a^10*b - 3*a^8*b^3 + 3*a^6*b^5 - a^4*b^7)*d*cos(d*x + c) + (a^9*b^2 - 3*a^
7*b^4 + 3*a^5*b^6 - a^3*b^8)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**3, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 457 vs. 2(190) = 380.
time = 0.54, size = 457, normalized size = 2.23

$$\frac{\frac{\left(2\,Ba^5-6\,Aa^4b+Ba^3b^2+5\,Aa^2b^3-2\,Ab^5\right)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}\left(-2\,a+2\,b\right)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{\left(a^7-2\,a^5b^2+a^3b^4\right)\sqrt{-a^2+b^2}}+\frac{(dx+c)A}{a^3}+\frac{4\,Ba^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,Aa^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,Ba^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,Aa^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-Ba^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+3\,Aab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-2\,Ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,Ba^4b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+6\,Aa^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,Ba^3b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+5\,Aa^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+Ba^2b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-3\,Aab^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-2\,Ab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a^6-2\,a^4b^2+a^2b^4\right)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="giac")
```

```
[Out] ((2*B*a^5 - 6*A*a^4*b + B*a^3*b^2 + 5*A*a^2*b^3 - 2*A*b^5)*(pi*floor(1/2*(d
*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan
(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^7 - 2*a^5*b^2 + a^3*b^4)*sqrt(-a^
2 + b^2)) + (d*x + c)*A/a^3 + (4*B*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 6*A*a^3*b
^2*tan(1/2*d*x + 1/2*c)^3 - 3*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^2*b^
3*tan(1/2*d*x + 1/2*c)^3 - B*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 3*A*a*b^4*tan
(1/2*d*x + 1/2*c)^3 - 2*A*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*B*a^4*b*tan(1/2*d*
x + 1/2*c) + 6*A*a^3*b^2*tan(1/2*d*x + 1/2*c) - 3*B*a^3*b^2*tan(1/2*d*x + 1
/2*c) + 5*A*a^2*b^3*tan(1/2*d*x + 1/2*c) + B*a^2*b^3*tan(1/2*d*x + 1/2*c) -
 3*A*a*b^4*tan(1/2*d*x + 1/2*c) - 2*A*b^5*tan(1/2*d*x + 1/2*c))/((a^6 - 2*a
^4*b^2 + a^2*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a
- b)^2))/d
```

Mupad [B]
time = 11.79, size = 2500, normalized size = 12.20

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^3,x)

[Out] (2*A*atan(((A*((8*tan(c/2 + (d*x)/2)*(4*A^2*a^10 + 8*A^2*b^10 + 4*B^2*a^10 - 8*A^2*a*b^9 - 8*A^2*a^9*b - 32*A^2*a^2*b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*A^2*a^5*b^5 - 52*A^2*a^6*b^4 + 32*A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 24*A*B*a^9*b - 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + 8*A*B*a^7*b^3))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) + (A*((8*(4*A*a^15 + 4*B*a^15 - 4*A*a^6*b^9 + 2*A*a^7*b^8 + 18*A*a^8*b^7 - 4*A*a^9*b^6 - 36*A*a^10*b^5 + 6*A*a^11*b^4 + 34*A*a^12*b^3 - 8*A*a^13*b^2 - 2*B*a^8*b^7 + 2*B*a^9*b^6 + 6*B*a^12*b^3 - 6*B*a^13*b^2 - 12*A*a^14*b - 4*B*a^14*b))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (A*tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2)*8i)/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))*1i)/a^3))/a^3 + (A*((8*tan(c/2 + (d*x)/2)*(4*A^2*a^10 + 8*A^2*b^10 + 4*B^2*a^10 - 8*A^2*a*b^9 - 8*A^2*a^9*b - 32*A^2*a^2*b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*A^2*a^5*b^5 - 52*A^2*a^6*b^4 + 32*A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 24*A*B*a^9*b - 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + 8*A*B*a^7*b^3))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) - (A*((8*(4*A*a^15 + 4*B*a^15 - 4*A*a^6*b^9 + 2*A*a^7*b^8 + 18*A*a^8*b^7 - 4*A*a^9*b^6 - 36*A*a^10*b^5 + 6*A*a^11*b^4 + 34*A*a^12*b^3 - 8*A*a^13*b^2 - 2*B*a^8*b^7 + 2*B*a^9*b^6 + 6*B*a^12*b^3 - 6*B*a^13*b^2 - 12*A*a^14*b - 4*B*a^14*b))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) + (A*tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3 - 8*a^14*b^2)*8i)/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))*1i)/a^3))/a^3)/((16*(4*A^3*b^9 + 4*A*B^2*a^9 - 4*A^2*B*a^9 - 2*A^3*a*b^8 + 12*A^3*a^8*b - 18*A^3*a^2*b^7 + 13*A^3*a^3*b^6 + 36*A^3*a^4*b^5 - 26*A^3*a^5*b^4 - 34*A^3*a^6*b^3 + 24*A^3*a^7*b^2 - 20*A^2*B*a^8*b + A*B^2*a^5*b^4 + 4*A*B^2*a^7*b^2 - 2*A^2*B*a^2*b^7 - 2*A^2*B*a^3*b^6 + 2*A^2*B*a^4*b^5 + 2*A^2*B*a^6*b^3 + 6*A^2*B*a^7*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (A*((8*tan(c/2 + (d*x)/2)*(4*A^2*a^10 + 8*A^2*b^10 + 4*B^2*a^10 - 8*A^2*a*b^9 - 8*A^2*a^9*b - 32*A^2*a^2*b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*A^2*a^5*b^5 - 52*A^2*a^6*b^4 + 32*A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 24*A*B*a^9*b - 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + 8*A*B*a^7*b^3))/(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) + (A*((8*(4*A*a^15 + 4*B*a^15 - 4*A*a^6*b^9 + 2*A*a^7*b^8 + 18*A*a^8*b^7 - 4*A*a^9*b^6 - 36*A*a^10*b^5 + 6*A*a^11*b^4 + 34*A*a^12*b^3 - 8*A*a^13*b^2 - 2*B*a^8*b^7 + 2*B*a^9*b^6 + 6*B*a^12*b^3 - 6*B*a^13*b^2 - 12*A*a^14*b - 4*B*a^14*b))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (A*tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 + 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^13*b^3

```
- 8*a^14*b^2)*8i)/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a
^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))*1i)/a^3)*1i)/a^3 + (A*((8*tan(c/2 + (d*x)
/2)*(4*A^2*a^10 + 8*A^2*b^10 + 4*B^2*a^10 - 8*A^2*a*b^9 - 8*A^2*a^9*b - 32*
A^2*a^2*b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*A^2*a^5*b^5 - 52*A^2*a^6
*b^4 + 32*A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B^2*a^6*b^4 + 4*B^2*a^8*b^2 - 24*A
*B*a^9*b - 4*A*B*a^3*b^7 + 2*A*B*a^5*b^5 + 8*A*B*a^7*b^3))/(a^10*b + a^11 -
 a^4*b^7 - a^5*b^6 + 3*a^6*b^5 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2) - (A*((
8*(4*A*a^15 + 4*B*a^15 - 4*A*a^6*b^9 + 2*A*a^7*b^8 + 18*A*a^8*b^7 - 4*A*a^9
*b^6 - 36*A*a^10*b^5 + 6*A*a^11*b^4 + 34*A*a^12*b^3 - 8*A*a^13*b^2 - 2*B*a^
8*b^7 + 2*B*a^9*b^6 + 6*B*a^12*b^3 - 6*B*a^13*b^2 - 12*A*a^14*b - 4*B*a^14*
b))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3
 - 3*a^11*b^2) + (A*tan(c/2 + (d*x)/2)*(8*a^15*b - 8*a^6*b^10 + 8*a^7*b^9 +
 32*a^8*b^8 - 32*a^9*b^7 - 48*a^10*b^6 + 48*a^11*b^5 + 32*a^12*b^4 - 32*a^1
3*b^3 - 8*a^14*b^2)*8i)/(a^3*(a^10*b + a^11 - a^4*b^7 - a^5*b^6 + 3*a^6*b^5
 + 3*a^7*b^4 - 3*a^8*b^3 - 3*a^9*b^2)))*1i)/a^3)*1i)/a^3)))/(a^3*d) - ((tan
(c/2 + (d*x)/2)^3*(2*A*b^4 - 6*A*a^2*b^2 + B*a^2*b^2 - A*a*b^3 + 4*B*a^3*b)
)/((a^2*b - a^3)*(a + b)^2) + (tan(c/2 + (d*x)/2)*(2*A*b^4 - 6*A*a^2*b^2 -
B*a^2*b^2 + A*a*b^3 + 4*B*a^3*b))/((a + b)*(a^4 - 2*a^3*b + a^2*b^2)))/(d*(
2*a*b - tan(c/2 + (d*x)/2)^2*(2*a^2 - 2*b^2) + tan(c/2 + (d*x)/2)^4*(a^2 -
2*a*b + b^2) + a^2 + b^2)) + (atan(((((a + b)^5*(a - b)^5)^(1/2)*((8*tan(c/
2 + (d*x)/2)*(4*A^2*a^10 + 8*A^2*b^10 + 4*B^2*a^10 - 8*A^2*a*b^9 - 8*A^2*a^
9*b - 32*A^2*a^2*b^8 + 32*A^2*a^3*b^7 + 57*A^2*a^4*b^6 - 48*A^2*a^5*b^5 - 5
2*A^2*a^6*b^4 + 32*A^2*a^7*b^3 + 24*A^2*a^8*b^2 + B^2*a^6*b^4 + 4*B^2*a^8*b
^2 - 24*A*B*a^9*b - 4*A*B*a^3*b^7 + 2*A*B*a^5*b...
```

3.334 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=290

$$-\frac{(3Ab-aB)x}{a^4}+\frac{b(12a^4Ab-15a^2Ab^3+6Ab^5-6a^5B+5a^3b^2B-2ab^4B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{5/2}(a+b)^{5/2}d}+\frac{(2\cdots}{}$$

```
[Out] -(3*A*b-B*a)*x/a^4+b*(12*A*a^4*b-15*A*a^2*b^3+6*A*b^5-6*B*a^5+5*B*a^3*b^2-2
*B*a*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/(a-b)^(5/
2)/(a+b)^(5/2)/d+1/2*(2*A*a^4-11*A*a^2*b^2+6*A*b^4+5*B*a^3*b-2*B*a*b^3)*sin
(d*x+c)/a^3/(a^2-b^2)^2/d+1/2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec
(d*x+c))^2+1/2*b*(6*A*a^2*b-3*A*b^3-4*B*a^3+B*a*b^2)*sin(d*x+c)/a^2/(a^2-b^
2)^2/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 1.03, antiderivative size = 290, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$, Rules used = {4115, 4185, 4189, 4004, 3916, 2738, 214}

$$-\frac{x(3Ab-aB)}{a^4}+\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{b\left(-4a^3B+6a^2Ab+ab^2B-3Ab^3\right)\sin(c+dx)}{2a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{\left(2a^4A+5a^3bB-11a^2Ab^2-2ab^3B+6Ab^4\right)\sin(c+dx)}{2a^3d\left(a^2-b^2\right)^2}+\frac{b\left(-6a^5B+12a^4Ab+5a^3b^2B-15a^2Ab^3-2ab^4B+6Ab^5\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -(((3*A*b - a*B)*x)/a^4) + (b*(12*a^4*A*b - 15*a^2*A*b^3 + 6*A*b^5 - 6*a^5*
B + 5*a^3*b^2*B - 2*a*b^4*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a
+ b]])/(a^4*(a - b)^(5/2)*(a + b)^(5/2)*d) + ((2*a^4*A - 11*a^2*A*b^2 + 6*A
*b^4 + 5*a^3*b*B - 2*a*b^3*B)*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + (b*(A
*b - a*B)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (b*(6*
a^2*A*b - 3*A*b^3 - 4*a^3*B + a*b^2*B)*Sin[c + d*x])/(2*a^2*(a^2 - b^2)^2*d
*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx &= \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{\int \frac{\cos(c+dx)\left(-2a^2A+3Ab^2-abB+2a(Ab\right.}{(a+b\,\mathrm{se}}}{2a\,(a} \\
&= \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{b(6a^2Ab-3Ab^3-4a^3B+ab^2}{2a^2\left(a^2-b^2\right)^2d(a+b\sec(} \\
&= \frac{\left(2a^4A-11a^2Ab^2+6Ab^4+5a^3bB-2ab^3B\right)\sin(c+dx)}{2a^3\left(a^2-b^2\right)^2d} + \frac{b(A}{2a\,(a^2-} \\
&= -\frac{(3Ab-aB)x}{a^4} + \frac{\left(2a^4A-11a^2Ab^2+6Ab^4+5a^3bB-2ab^3B\right)\sin(}{2a^3\left(a^2-b^2\right)^2d} \\
&= -\frac{(3Ab-aB)x}{a^4} + \frac{\left(2a^4A-11a^2Ab^2+6Ab^4+5a^3bB-2ab^3B\right)\sin(}{2a^3\left(a^2-b^2\right)^2d} \\
&= -\frac{(3Ab-aB)x}{a^4} + \frac{\left(2a^4A-11a^2Ab^2+6Ab^4+5a^3bB-2ab^3B\right)\sin(}{2a^3\left(a^2-b^2\right)^2d} \\
&= -\frac{(3Ab-aB)x}{a^4} + \frac{b(12a^4Ab-15a^2Ab^3+6Ab^5-6a^5B+5a^3b^2B-}{a^4(a-b)^{5/2}(a+}
\end{aligned}
$$

Mathematica [A]
time = 2.07, size = 306, normalized size = 1.06

$$
\frac{(b+a\cos(c+dx))\sec^2(c+dx)(A+B\sec(c+dx))\left(2(-3Ab+aB)(c+dx)(b+a\cos(c+dx))^2-\frac{2b\left(12a^4Ab-15a^2Ab^3+6Ab^5-6a^5B+5a^3b^2B-2ab^4B\right)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))^2}{\left(a^2-b^2\right)^{5/2}}+\frac{ab^3(Ab-aB)\sin(c+dx)}{(a-b)(a+b)}+\frac{ab^2\left(-8a^2Ab+5Ab^3+6a^3B-3ab^2B\right)(b+a\cos(c+dx))\sin(c+dx)}{(a-b)^2(a+b)^2}+2aA(b+a\cos(c+dx))^2\sin(c+dx)\right)}{2a^4d(B+A\cos(c+dx))(a+b\sec(c+dx))^3}
$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*(2*(-3*A*b + a*B)
*(c + d*x)*(b + a*Cos[c + d*x])^2 - (2*b*(12*a^4*A*b - 15*a^2*A*b^3 + 6*A*b
^5 - 6*a^5*B + 5*a^3*b^2*B - 2*a*b^4*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])
/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^2)/(a^2 - b^2)^(5/2) + (a*b^3*(A*b -
 a*B)*Sin[c + d*x])/((a - b)*(a + b)) + (a*b^2*(-8*a^2*A*b + 5*A*b^3 + 6*a^
3*B - 3*a*b^2*B)*(b + a*Cos[c + d*x])*Sin[c + d*x])/((a - b)^2*(a + b)^2) +
 2*a*A*(b + a*Cos[c + d*x])^2*Sin[c + d*x]))/(2*a^4*d*(B + A*Cos[c + d*x])*
(a + b*Sec[c + d*x])^3)
```

Maple [A]
time = 0.62, size = 347, normalized size = 1.20

method	result
derivativedivides	$-\frac{2b\left(\frac{-\frac{\left(8A\,a^2b+A\,b^2a-4A\,b^3-6a^3B-Bb\,a^2+2Ba\,b^2\right)ba\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ba+b^2\right)}+\frac{ab\left(8A\,a^2b-A\,b^2a-4A\,b^3-6a^3B+Bb\,a^2+2Ba\,b^2\right)\tan(}{2(a+b)(a-b)^2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}\right.}{a^4}$
default	$-\frac{2b\left(\frac{-\frac{\left(8A\,a^2b+A\,b^2a-4A\,b^3-6a^3B-Bb\,a^2+2Ba\,b^2\right)ba\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^2+2ba+b^2\right)}+\frac{ab\left(8A\,a^2b-A\,b^2a-4A\,b^3-6a^3B+Bb\,a^2+2Ba\,b^2\right)\tan(}{2(a+b)(a-b)^2}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^2}\right.}{a^4}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*b/a^4*((-1/2*(8*A*a^2*b+A*a*b^2-4*A*b^3-6*B*a^3-B*a^2*b+2*B*a*b^2)*
b*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1/2*a*b*(8*A*a^2*b-A*a*b^2-4
*A*b^3-6*B*a^3+B*a^2*b+2*B*a*b^2)/(a+b)/(a-b)^2*tan(1/2*d*x+1/2*c))/(a*tan(
1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)^2-1/2*(12*A*a^4*b-15*A*a^2*b^3
+6*A*b^5-6*B*a^5+5*B*a^3*b^2-2*B*a*b^4)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(
1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))-2/a^4*(-A*a*tan
(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+(3*A*b-B*a)*arctan(tan(1/2*d*x+1/2
*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 755 vs. $2(275) = 550$.
time = 3.92, size = 1568, normalized size = 5.41

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fric
as")
```

```
[Out] [1/4*(4*(B*a^9 - 3*A*a^8*b - 3*B*a^7*b^2 + 9*A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*
a^4*b^5 - B*a^3*b^6 + 3*A*a^2*b^7)*d*x*cos(d*x + c)^2 + 8*(B*a^8*b - 3*A*a^
7*b^2 - 3*B*a^6*b^3 + 9*A*a^5*b^4 + 3*B*a^4*b^5 - 9*A*a^3*b^6 - B*a^2*b^7 +
 3*A*a*b^8)*d*x*cos(d*x + c) + 4*(B*a^7*b^2 - 3*A*a^6*b^3 - 3*B*a^5*b^4 + 9
*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7 - B*a*b^8 + 3*A*b^9)*d*x - (6*B*a^5*
b^3 - 12*A*a^4*b^4 - 5*B*a^3*b^5 + 15*A*a^2*b^6 + 2*B*a*b^7 - 6*A*b^8 + (6*
B*a^7*b - 12*A*a^6*b^2 - 5*B*a^5*b^3 + 15*A*a^4*b^4 + 2*B*a^3*b^5 - 6*A*a^2
*b^6)*cos(d*x + c)^2 + 2*(6*B*a^6*b^2 - 12*A*a^5*b^3 - 5*B*a^4*b^4 + 15*A*a
^3*b^5 + 2*B*a^2*b^6 - 6*A*a*b^7)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*
cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x
+ c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x +
 c) + b^2)) + 2*(2*A*a^7*b^2 + 5*B*a^6*b^3 - 13*A*a^5*b^4 - 7*B*a^4*b^5 + 1
7*A*a^3*b^6 + 2*B*a^2*b^7 - 6*A*a*b^8 + 2*(A*a^9 - 3*A*a^7*b^2 + 3*A*a^5*b^
4 - A*a^3*b^6)*cos(d*x + c)^2 + (4*A*a^8*b + 6*B*a^7*b^2 - 20*A*a^6*b^3 - 9
*B*a^5*b^4 + 25*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7)*cos(d*x + c))*sin(d*
x + c))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*d*cos(d*x + c)^2 + 2*(a^
11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*cos(d*x + c) + (a^10*b^2 - 3*a^8*
b^4 + 3*a^6*b^6 - a^4*b^8)*d), 1/2*(2*(B*a^9 - 3*A*a^8*b - 3*B*a^7*b^2 + 9*
A*a^6*b^3 + 3*B*a^5*b^4 - 9*A*a^4*b^5 - B*a^3*b^6 + 3*A*a^2*b^7)*d*x*cos(d*
x + c)^2 + 4*(B*a^8*b - 3*A*a^7*b^2 - 3*B*a^6*b^3 + 9*A*a^5*b^4 + 3*B*a^4*b
^5 - 9*A*a^3*b^6 - B*a^2*b^7 + 3*A*a*b^8)*d*x*cos(d*x + c) + 2*(B*a^7*b^2 -
 3*A*a^6*b^3 - 3*B*a^5*b^4 + 9*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7 - B*a*
b^8 + 3*A*b^9)*d*x - (6*B*a^5*b^3 - 12*A*a^4*b^4 - 5*B*a^3*b^5 + 15*A*a^2*b
^6 + 2*B*a*b^7 - 6*A*b^8 + (6*B*a^7*b - 12*A*a^6*b^2 - 5*B*a^5*b^3 + 15*A*a
^4*b^4 + 2*B*a^3*b^5 - 6*A*a^2*b^6)*cos(d*x + c)^2 + 2*(6*B*a^6*b^2 - 12*A*
a^5*b^3 - 5*B*a^4*b^4 + 15*A*a^3*b^5 + 2*B*a^2*b^6 - 6*A*a*b^7)*cos(d*x + c
))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b
^2)*sin(d*x + c))) + (2*A*a^7*b^2 + 5*B*a^6*b^3 - 13*A*a^5*b^4 - 7*B*a^4*b^
5 + 17*A*a^3*b^6 + 2*B*a^2*b^7 - 6*A*a*b^8 + 2*(A*a^9 - 3*A*a^7*b^2 + 3*A*a
^5*b^4 - A*a^3*b^6)*cos(d*x + c)^2 + (4*A*a^8*b + 6*B*a^7*b^2 - 20*A*a^6*b^
3 - 9*B*a^5*b^4 + 25*A*a^4*b^5 + 3*B*a^3*b^6 - 9*A*a^2*b^7)*cos(d*x + c))*s
in(d*x + c))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*d*cos(d*x + c)^2 +
2*(a^11*b - 3*a^9*b^3 + 3*a^7*b^5 - a^5*b^7)*d*cos(d*x + c) + (a^10*b^2 - 3
*a^8*b^4 + 3*a^6*b^6 - a^4*b^8)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\cos(c + dx)}{(a + b\sec(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)/(a + b*sec(c + d*x))**3, x)
```

Giac [A]
time = 0.55, size = 546, normalized size = 1.88

$$-\frac{\frac{(6\,Ba^5b-12\,Aa^4b^2-5\,Ba^3b^3+15\,Aa^2b^4+2\,Bab^5-6\,Ab^6)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^8-2\,a^6b^2+a^4b^4)\sqrt{-a^2+b^2}}+\frac{6\,Ba^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-8\,Aa^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-5\,Ba^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+7\,Aa^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-3\,Ba^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+5\,Aab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3+2\,Bab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-4\,Ab^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^3-6\,Ba^4b^2\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+8\,Aa^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,Ba^3b^3\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+7\,Aa^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+3\,Ba^2b^4\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-5\,Aab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)+2\,Bab^5\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-4\,Ab^6\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{(a^7-2\,a^5b^2+a^3b^4)\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-a-b\right)^2}-\frac{(Ba-3\,Ab)(dx+c)}{a^4}-\frac{2\,A\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+1\right)a^3}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="giac
")
```

```
[Out] -((6*B*a^5*b - 12*A*a^4*b^2 - 5*B*a^3*b^3 + 15*A*a^2*b^4 + 2*B*a*b^5 - 6*A*
b^6)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2
*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^8 - 2*a^6*b^
2 + a^4*b^4)*sqrt(-a^2 + b^2)) + (6*B*a^4*b^2*tan(1/2*d*x + 1/2*c)^3 - 8*A*
a^3*b^3*tan(1/2*d*x + 1/2*c)^3 - 5*B*a^3*b^3*tan(1/2*d*x + 1/2*c)^3 + 7*A*a
^2*b^4*tan(1/2*d*x + 1/2*c)^3 - 3*B*a^2*b^4*tan(1/2*d*x + 1/2*c)^3 + 5*A*a*
b^5*tan(1/2*d*x + 1/2*c)^3 + 2*B*a*b^5*tan(1/2*d*x + 1/2*c)^3 - 4*A*b^6*tan
(1/2*d*x + 1/2*c)^3 - 6*B*a^4*b^2*tan(1/2*d*x + 1/2*c) + 8*A*a^3*b^3*tan(1/
2*d*x + 1/2*c) - 5*B*a^3*b^3*tan(1/2*d*x + 1/2*c) + 7*A*a^2*b^4*tan(1/2*d*x
 + 1/2*c) + 3*B*a^2*b^4*tan(1/2*d*x + 1/2*c) - 5*A*a*b^5*tan(1/2*d*x + 1/2*
c) + 2*B*a*b^5*tan(1/2*d*x + 1/2*c) - 4*A*b^6*tan(1/2*d*x + 1/2*c))/((a^7 -
 2*a^5*b^2 + a^3*b^4)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2
- a - b)^2) - (B*a - 3*A*b)*(d*x + c)/a^4 - 2*A*tan(1/2*d*x + 1/2*c)/((tan(
1/2*d*x + 1/2*c)^2 + 1)*a^3))/d
```

Mupad [B]
time = 9.73, size = 2500, normalized size = 8.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(6*A*b^5 - 2*A*a^5 - 12*A*a^2*b^3 + 4*A*a^3*b^2 + B*
a^2*b^3 + 6*B*a^3*b^2 - 3*A*a*b^4 + 2*A*a^4*b - 2*B*a*b^4))/((a^3*b - a^4)*
(a + b)^2) + (tan(c/2 + (d*x)/2)*(2*A*a^5 + 6*A*b^5 - 12*A*a^2*b^3 - 4*A*a^
3*b^2 - B*a^2*b^3 + 6*B*a^3*b^2 + 3*A*a*b^4 + 2*A*a^4*b - 2*B*a*b^4))/((a +
 b)*(a^5 - 2*a^4*b + a^3*b^2)) + (2*tan(c/2 + (d*x)/2)^3*(2*A*a^6 - 6*A*b^6
 + 13*A*a^2*b^4 - 6*A*a^4*b^2 - 5*B*a^3*b^3 + 2*B*a*b^5))/(a*(a^2*b - a^3)*
(a + b)^2*(a - b)))/(d*(2*a*b + tan(c/2 + (d*x)/2)^2*(2*a*b - a^2 + 3*b^2)
+ tan(c/2 + (d*x)/2)^6*(a^2 - 2*a*b + b^2) + a^2 + b^2 - tan(c/2 + (d*x)/2)
^4*(2*a*b + a^2 - 3*b^2))) + (log(tan(c/2 + (d*x)/2) - 1i)*(3*A*b - B*a)*1i
)/(a^4*d) - (log(tan(c/2 + (d*x)/2) + 1i)*(A*b*3i - B*a*1i))/(a^4*d) - (b*a
```

tan(((b*((8*tan(c/2 + (d*x)/2)*(72*A^2*b^12 + 4*B^2*a^12 - 72*A^2*a*b^11 - 8*B^2*a^11*b - 288*A^2*a^2*b^10 + 288*A^2*a^3*b^9 + 441*A^2*a^4*b^8 - 432*A ^2*a^5*b^7 - 288*A^2*a^6*b^6 + 288*A^2*a^7*b^5 + 36*A^2*a^8*b^4 - 72*A^2*a^ 9*b^3 + 36*A^2*a^10*b^2 + 8*B^2*a^2*b^10 - 8*B^2*a^3*b^9 - 32*B^2*a^4*b^8 + 32*B^2*a^5*b^7 + 57*B^2*a^6*b^6 - 48*B^2*a^7*b^5 - 52*B^2*a^8*b^4 + 32*B^2 *a^9*b^3 + 24*B^2*a^10*b^2 - 48*A*B*a*b^11 - 24*A*B*a^11*b + 48*A*B*a^2*b^1 0 + 192*A*B*a^3*b^9 - 192*A*B*a^4*b^8 - 318*A*B*a^5*b^7 + 288*A*B*a^6*b^6 + 252*A*B*a^7*b^5 - 192*A*B*a^8*b^4 - 72*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a^ 12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^ 11*b^2) + (b*((8*(4*B*a^18 + 12*A*a^8*b^10 - 6*A*a^9*b^9 - 54*A*a^10*b^8 + 24*A*a^11*b^7 + 96*A*a^12*b^6 - 42*A*a^13*b^5 - 78*A*a^14*b^4 + 36*A*a^15*b ^3 + 24*A*a^16*b^2 - 4*B*a^9*b^9 + 2*B*a^10*b^8 + 18*B*a^11*b^7 - 4*B*a^12* b^6 - 36*B*a^13*b^5 + 6*B*a^14*b^4 + 34*B*a^15*b^3 - 8*B*a^16*b^2 - 12*A*a^ 17*b - 12*B*a^17*b))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a ^12*b^4 - 3*a^13*b^3 - 3*a^14*b^2) - (4*b*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 15*A*a^2*b^3 + 5*B*a^3*b^2 + 12*A*a^4*b - 2*B*a*b^4)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/((a ^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)*(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b ^2)))*((a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 15*A*a^2*b^3 + 5*B*a ^3*b^2 + 12*A*a^4*b - 2*B*a*b^4))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8* b^6 + 10*a^10*b^4 - 5*a^12*b^2)))*((a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6* B*a^5 - 15*A*a^2*b^3 + 5*B*a^3*b^2 + 12*A*a^4*b - 2*B*a*b^4)*1i)/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)) + (b*((8*ta n(c/2 + (d*x)/2)*(72*A^2*b^12 + 4*B^2*a^12 - 72*A^2*a*b^11 - 8*B^2*a^11*b - 288*A^2*a^2*b^10 + 288*A^2*a^3*b^9 + 441*A^2*a^4*b^8 - 432*A^2*a^5*b^7 - 2 88*A^2*a^6*b^6 + 288*A^2*a^7*b^5 + 36*A^2*a^8*b^4 - 72*A^2*a^9*b^3 + 36*A^2 *a^10*b^2 + 8*B^2*a^2*b^10 - 8*B^2*a^3*b^9 - 32*B^2*a^4*b^8 + 32*B^2*a^5*b^ 7 + 57*B^2*a^6*b^6 - 48*B^2*a^7*b^5 - 52*B^2*a^8*b^4 + 32*B^2*a^9*b^3 + 24* B^2*a^10*b^2 - 48*A*B*a*b^11 - 24*A*B*a^11*b + 48*A*B*a^2*b^10 + 192*A*B*a^ 3*b^9 - 192*A*B*a^4*b^8 - 318*A*B*a^5*b^7 + 288*A*B*a^6*b^6 + 252*A*B*a^7*b ^5 - 192*A*B*a^8*b^4 - 72*A*B*a^9*b^3 + 48*A*B*a^10*b^2))/(a^12*b + a^13 - a^6*b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2) - (b*((8*(4*B*a^18 + 12*A*a^8*b^10 - 6*A*a^9*b^9 - 54*A*a^10*b^8 + 24*A*a^11*b^7 + 96*A*a^12*b^6 - 42*A*a^13*b^5 - 78*A*a^14*b^4 + 36*A*a^15*b^3 + 24*A*a^16 *b^2 - 4*B*a^9*b^9 + 2*B*a^10*b^8 + 18*B*a^11*b^7 - 4*B*a^12*b^6 - 36*B*a^1 3*b^5 + 6*B*a^14*b^4 + 34*B*a^15*b^3 - 8*B*a^16*b^2 - 12*A*a^17*b - 12*B*a^ 17*b))/(a^15*b + a^16 - a^9*b^7 - a^10*b^6 + 3*a^11*b^5 + 3*a^12*b^4 - 3*a^ 13*b^3 - 3*a^14*b^2) + (4*b*tan(c/2 + (d*x)/2)*((a + b)^5*(a - b)^5)^(1/2)* (6*A*b^5 - 6*B*a^5 - 15*A*a^2*b^3 + 5*B*a^3*b^2 + 12*A*a^4*b - 2*B*a*b^4)*(8*a^17*b - 8*a^8*b^10 + 8*a^9*b^9 + 32*a^10*b^8 - 32*a^11*b^7 - 48*a^12*b^6 + 48*a^13*b^5 + 32*a^14*b^4 - 32*a^15*b^3 - 8*a^16*b^2))/((a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)*(a^12*b + a^13 - a^6* b^7 - a^7*b^6 + 3*a^8*b^5 + 3*a^9*b^4 - 3*a^10*b^3 - 3*a^11*b^2)))*((a + b)

```
^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 15*A*a^2*b^3 + 5*B*a^3*b^2 + 12*A*
a^4*b - 2*B*a*b^4))/(2*(a^14 - a^4*b^10 + 5*a^6*b^8 - 10*a^8*b^6 + 10*a^10*
b^4 - 5*a^12*b^2)))*((a + b)^5*(a - b)^5)^(1/2)*(6*A*b^5 - 6*B*a^5 - 15*A*a
^2*b^3 + 5*B*a^3*b^2 + 12*A*a^4*b - 2*B*a*b^4)*1i)/(2*(a^14 - a^4*b^10 + 5*
a^6*b^8 - 10*a^8*b^6 + 10*a^10*b^4 - 5*a^12*b^2)))/((16*(108*A^3*b^12 - 54*
A^3*a*b^11 - 12*B^3*a^11*b - 486*A^3*a^2*b^10 + 243*A^3*a^3*b^9 + 864*A^3*a
^4*b^8 - 378*A^3*a^5*b^7 - 702*A^3*a^6*b^6 + 216*A^3*a^7*b^5 + 216*A^3*a^8*
b^4 - 4*B^3*a^3*b^9 + 2*B^3*a^4*b^8 + 18*B^3*a^5*b^7 - 13*B^3*a^6*b^6 - 36*
B^3*a^7*b^5 + 26*B^3*a^8*b^4 + 34*B^3*a^9*b^3 - 24*B^3*a^10*b^2 - 108*A^2*B
*a*b^11 + 36*A*B^2*a^2*b^10 - 18*A*B^2*a^3*b^9 - 162*A*B^2*a^4*b^8 + 105*A*
B^2*a^5*b^7 + 312*A*B^2*a^6*b^6 - 198*A*B^2*a^7*b^5 - 282*A*B^2*a^8*b^4 + 1
56*A*B^2*a^9*b^3 + 96*A*B^2*a^10*b^2 + 54*A^2*B...
```

3.335 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=393

$$\frac{(a^2A+12Ab^2-6abB)\,x}{2a^5}-\frac{b^2(20a^4Ab-29a^2Ab^3+12Ab^5-12a^5B+15a^3b^2B-6ab^4B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5(a-b)^{5/2}(a+b)^{5/2}d}$$

```
[Out] 1/2*(A*a^2+12*A*b^2-6*B*a*b)*x/a^5-b^2*(20*A*a^4*b-29*A*a^2*b^3+12*A*b^5-12
*B*a^5+15*B*a^3*b^2-6*B*a*b^4)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)
^(1/2))/a^5/(a-b)^(5/2)/(a+b)^(5/2)/d-1/2*(6*A*a^4*b-21*A*a^2*b^3+12*A*b^5-
2*B*a^5+11*B*a^3*b^2-6*B*a*b^4)*sin(d*x+c)/a^4/(a^2-b^2)^2/d+1/2*(A*a^4-10*
A*a^2*b^2+6*A*b^4+6*B*a^3*b-3*B*a*b^3)*cos(d*x+c)*sin(d*x+c)/a^3/(a^2-b^2)^
2/d+1/2*b*(A*b-B*a)*cos(d*x+c)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+
1/2*b*(7*A*a^2*b-4*A*b^3-5*B*a^3+2*B*a*b^2)*cos(d*x+c)*sin(d*x+c)/a^2/(a^2-
b^2)^2/d/(a+b*sec(d*x+c))
```

Rubi [A]

time = 1.37, antiderivative size = 393, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4115, 4185, 4189, 4004, 3916, 2738, 214}

$$\frac{b(Ab-aB)\sin(c+dx)\cos(c+dx)}{2ad\,(a^2-b^2)\,(a+b\sec(c+dx))^2}+\frac{x(a^2A-6abB+12Ab^2)}{2a^5}+\frac{b(-5a^3B+7a^2Ab+2ab^2B-4Ab^3)\sin(c+dx)\cos(c+dx)}{2a^2d\,(a^2-b^2)^2\,(a+b\sec(c+dx))}+\frac{(a^4A+6a^3bB-10a^2Ab^2-3ab^3B+6Ab^4)\sin(c+dx)\cos(c+dx)}{2a^3d\,(a^2-b^2)^2}-\frac{(-2a^5B+6a^4Ab+11a^3b^2B-21a^2Ab^3-6ab^4B+12Ab^5)\sin(c+dx)}{2a^4d\,(a^2-b^2)^2}-\frac{b^2(-12a^5B+20a^4Ab+15a^3b^2B-29a^2Ab^3-6ab^4B+12Ab^5)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{5/2}(a+b)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((a^2*A + 12*A*b^2 - 6*a*b*B)*x)/(2*a^5) - (b^2*(20*a^4*A*b - 29*a^2*A*b^3
+ 12*A*b^5 - 12*a^5*B + 15*a^3*b^2*B - 6*a*b^4*B)*ArcTanh[(Sqrt[a - b]*Tan[
(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(5/2)*(a + b)^(5/2)*d) - ((6*a^4*A
*b - 21*a^2*A*b^3 + 12*A*b^5 - 2*a^5*B + 11*a^3*b^2*B - 6*a*b^4*B)*Sin[c +
d*x])/(2*a^4*(a^2 - b^2)^2*d) + ((a^4*A - 10*a^2*A*b^2 + 6*A*b^4 + 6*a^3*b*
B - 3*a*b^3*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^2*d) + (b*(A*b
 - a*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^
2) + (b*(7*a^2*A*b - 4*A*b^3 - 5*a^3*B + 2*a*b^2*B)*Cos[c + d*x]*Sin[c + d*
x])/(2*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
```

```
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \frac{b(Ab-aB)\cos(c+dx)\sin(c+dx)}{2a\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} - \frac{\int \frac{\cos^2(c+dx)\left(-2\left(a^2A-2Ab^2+abB\right)\ldots\right.}{(\ldots}}{\ldots}$$

$$= \frac{b(Ab-aB)\cos(c+dx)\sin(c+dx)}{2a\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{b(7a^2Ab-4Ab^3-5a^3B+2\ldots}{2a^2\,(a^2-b^2)^2\,d(\ldots}$$

$$= \frac{(a^4A-10a^2Ab^2+6Ab^4+6a^3bB-3ab^3B)\cos(c+dx)\sin(c+dx)\ldots}{2a^3\,(a^2-b^2)^2\,d}$$

$$= -\frac{(6a^4Ab-21a^2Ab^3+12Ab^5-2a^5B+11a^3b^2B-6ab^4B)\sin(c+\ldots}{2a^4\,(a^2-b^2)^2\,d}$$

$$= \frac{(a^2A+12Ab^2-6abB)\,x}{2a^5} - \frac{(6a^4Ab-21a^2Ab^3+12Ab^5-2a^5B+\ldots}{2a^4\,(a^2-b^2\ldots}$$

$$= \frac{(a^2A+12Ab^2-6abB)\,x}{2a^5} - \frac{(6a^4Ab-21a^2Ab^3+12Ab^5-2a^5B+\ldots}{2a^4\,(a^2-b^2\ldots}$$

$$= \frac{(a^2A+12Ab^2-6abB)\,x}{2a^5} - \frac{(6a^4Ab-21a^2Ab^3+12Ab^5-2a^5B+\ldots}{2a^4\,(a^2-b^2\ldots}$$

$$= \frac{(a^2A+12Ab^2-6abB)\,x}{2a^5} - \frac{b^2(20a^4Ab-29a^2Ab^3+12Ab^5-12a^5\ldots}{a^5(\ldots}$$

Mathematica [A]
time = 4.68, size = 734, normalized size = 1.87

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((16*b^2*(20*a^4*A*b - 29*a^2*A*b^3 + 12*A*b^5 - 12*a^5*B + 15*a^3*b^2*B -
6*a*b^4*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2
)^(5/2) + (4*a^8*A*c + 48*a^6*A*b^2*c - 12*a^4*A*b^4*c - 136*a^2*A*b^6*c +
96*A*b^8*c - 24*a^7*b*B*c + 72*a^3*b^5*B*c - 48*a*b^7*B*c + 4*a^8*A*d*x + 4
```

```
8*a^6*A*b^2*d*x - 12*a^4*A*b^4*d*x - 136*a^2*A*b^6*d*x + 96*A*b^8*d*x - 24*
a^7*b*B*d*x + 72*a^3*b^5*B*d*x - 48*a*b^7*B*d*x + 16*a*b*(a^2 - b^2)^2*(a^2
*A + 12*A*b^2 - 6*a*b*B)*(c + d*x)*Cos[c + d*x] + 4*(a^3 - a*b^2)^2*(a^2*A
+ 12*A*b^2 - 6*a*b*B)*(c + d*x)*Cos[2*(c + d*x)] - 8*a^7*A*b*Sin[c + d*x] -
 32*a^5*A*b^3*Sin[c + d*x] + 160*a^3*A*b^5*Sin[c + d*x] - 96*a*A*b^7*Sin[c
+ d*x] + 4*a^8*B*Sin[c + d*x] + 8*a^6*b^2*B*Sin[c + d*x] - 84*a^4*b^4*B*Sin
[c + d*x] + 48*a^2*b^6*B*Sin[c + d*x] + 2*a^8*A*Sin[2*(c + d*x)] - 48*a^6*A
*b^2*Sin[2*(c + d*x)] + 130*a^4*A*b^4*Sin[2*(c + d*x)] - 72*a^2*A*b^6*Sin[2
*(c + d*x)] + 16*a^7*b*B*Sin[2*(c + d*x)] - 64*a^5*b^3*B*Sin[2*(c + d*x)] +
 36*a^3*b^5*B*Sin[2*(c + d*x)] - 8*a^7*A*b*Sin[3*(c + d*x)] + 16*a^5*A*b^3*
Sin[3*(c + d*x)] - 8*a^3*A*b^5*Sin[3*(c + d*x)] + 4*a^8*B*Sin[3*(c + d*x)]
- 8*a^6*b^2*B*Sin[3*(c + d*x)] + 4*a^4*b^4*B*Sin[3*(c + d*x)] + a^8*A*Sin[4
*(c + d*x)] - 2*a^6*A*b^2*Sin[4*(c + d*x)] + a^4*A*b^4*Sin[4*(c + d*x)])/((
a^2 - b^2)^2*(b + a*Cos[c + d*x])^2))/(16*a^5*d)
```

Maple [A]
time = 0.74, size = 403, normalized size = 1.03

method	result
derivativedivides	$\frac{\frac{2\left(\left(-\frac{1}{2}a^2A-3Aba+a^2B\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^2A-3Aba+a^2B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\left(a^2A+12Ab^2-6Bab\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^5}+\frac{2b^2\cdots}{\cdots}$
default	$\frac{\frac{2\left(\left(-\frac{1}{2}a^2A-3Aba+a^2B\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\left(\frac{1}{2}a^2A-3Aba+a^2B\right)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{\left(1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)^2}+\left(a^2A+12Ab^2-6Bab\right)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a^5}+\frac{2b^2\cdots}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(2/a^5*(((-1/2*a^2*A-3*A*b*a+a^2*B)*tan(1/2*d*x+1/2*c)^3+(1/2*a^2*A-3*A
*b*a+a^2*B)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(A*a^2+12*A*
b^2-6*B*a*b)*arctan(tan(1/2*d*x+1/2*c)))+2*b^2/a^5*((-1/2*(10*A*a^2*b+A*a*b
^2-6*A*b^3-8*B*a^3-B*a^2*b+4*B*a*b^2)*b*a/(a-b)/(a^2+2*a*b+b^2)*tan(1/2*d*x
+1/2*c)^3+1/2*a*b*(10*A*a^2*b-A*a*b^2-6*A*b^3-8*B*a^3+B*a^2*b+4*B*a*b^2)/(a
+b)/(a-b)^2*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c
)^2-a-b)^2-1/2*(20*A*a^4*b-29*A*a^2*b^3+12*A*b^5-12*B*a^5+15*B*a^3*b^2-6*B*
a*b^4)/(a^4-2*a^2*b^2+b^4)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/
2*c)/((a+b)*(a-b))^(1/2))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 875 vs. $2(373) = 746$.
time = 3.73, size = 1811, normalized size = 4.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="fr
icas")
```

```
[Out] [1/4*(2*(A*a^10 - 6*B*a^9*b + 9*A*a^8*b^2 + 18*B*a^7*b^3 - 33*A*a^6*b^4 - 1
8*B*a^5*b^5 + 35*A*a^4*b^6 + 6*B*a^3*b^7 - 12*A*a^2*b^8)*d*x*cos(d*x + c)^2
 + 4*(A*a^9*b - 6*B*a^8*b^2 + 9*A*a^7*b^3 + 18*B*a^6*b^4 - 33*A*a^5*b^5 - 1
8*B*a^4*b^6 + 35*A*a^3*b^7 + 6*B*a^2*b^8 - 12*A*a*b^9)*d*x*cos(d*x + c) + 2
*(A*a^8*b^2 - 6*B*a^7*b^3 + 9*A*a^6*b^4 + 18*B*a^5*b^5 - 33*A*a^4*b^6 - 18*
B*a^3*b^7 + 35*A*a^2*b^8 + 6*B*a*b^9 - 12*A*b^10)*d*x - (12*B*a^5*b^4 - 20*
A*a^4*b^5 - 15*B*a^3*b^6 + 29*A*a^2*b^7 + 6*B*a*b^8 - 12*A*b^9 + (12*B*a^7*
b^2 - 20*A*a^6*b^3 - 15*B*a^5*b^4 + 29*A*a^4*b^5 + 6*B*a^3*b^6 - 12*A*a^2*b
^7)*cos(d*x + c)^2 + 2*(12*B*a^6*b^3 - 20*A*a^5*b^4 - 15*B*a^4*b^5 + 29*A*a
^3*b^6 + 6*B*a^2*b^7 - 12*A*a*b^8)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b
*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x
 + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x
+ c) + b^2)) + 2*(2*B*a^8*b^2 - 6*A*a^7*b^3 - 13*B*a^6*b^4 + 27*A*a^5*b^5 +
 17*B*a^4*b^6 - 33*A*a^3*b^7 - 6*B*a^2*b^8 + 12*A*a*b^9 + (A*a^10 - 3*A*a^8
*b^2 + 3*A*a^6*b^4 - A*a^4*b^6)*cos(d*x + c)^3 + 2*(B*a^10 - 2*A*a^9*b - 3*
B*a^8*b^2 + 6*A*a^7*b^3 + 3*B*a^6*b^4 - 6*A*a^5*b^5 - B*a^4*b^6 + 2*A*a^3*b
^7)*cos(d*x + c)^2 + (4*B*a^9*b - 11*A*a^8*b^2 - 20*B*a^7*b^3 + 43*A*a^6*b^
4 + 25*B*a^5*b^5 - 50*A*a^4*b^6 - 9*B*a^3*b^7 + 18*A*a^2*b^8)*cos(d*x + c))
*sin(d*x + c))/((a^13 - 3*a^11*b^2 + 3*a^9*b^4 - a^7*b^6)*d*cos(d*x + c)^2
+ 2*(a^12*b - 3*a^10*b^3 + 3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c) + (a^11*b^2
- 3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*d), 1/2*((A*a^10 - 6*B*a^9*b + 9*A*a^8*b
```

```
^2 + 18*B*a^7*b^3 - 33*A*a^6*b^4 - 18*B*a^5*b^5 + 35*A*a^4*b^6 + 6*B*a^3*b^
7 - 12*A*a^2*b^8)*d*x*cos(d*x + c)^2 + 2*(A*a^9*b - 6*B*a^8*b^2 + 9*A*a^7*b
^3 + 18*B*a^6*b^4 - 33*A*a^5*b^5 - 18*B*a^4*b^6 + 35*A*a^3*b^7 + 6*B*a^2*b^
8 - 12*A*a*b^9)*d*x*cos(d*x + c) + (A*a^8*b^2 - 6*B*a^7*b^3 + 9*A*a^6*b^4 +
 18*B*a^5*b^5 - 33*A*a^4*b^6 - 18*B*a^3*b^7 + 35*A*a^2*b^8 + 6*B*a*b^9 - 12
*A*b^10)*d*x + (12*B*a^5*b^4 - 20*A*a^4*b^5 - 15*B*a^3*b^6 + 29*A*a^2*b^7 +
 6*B*a*b^8 - 12*A*b^9 + (12*B*a^7*b^2 - 20*A*a^6*b^3 - 15*B*a^5*b^4 + 29*A*
a^4*b^5 + 6*B*a^3*b^6 - 12*A*a^2*b^7)*cos(d*x + c)^2 + 2*(12*B*a^6*b^3 - 20
*A*a^5*b^4 - 15*B*a^4*b^5 + 29*A*a^3*b^6 + 6*B*a^2*b^7 - 12*A*a*b^8)*cos(d*
x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^
2 - b^2)*sin(d*x + c))) + (2*B*a^8*b^2 - 6*A*a^7*b^3 - 13*B*a^6*b^4 + 27*A*
a^5*b^5 + 17*B*a^4*b^6 - 33*A*a^3*b^7 - 6*B*a^2*b^8 + 12*A*a*b^9 + (A*a^10
- 3*A*a^8*b^2 + 3*A*a^6*b^4 - A*a^4*b^6)*cos(d*x + c)^3 + 2*(B*a^10 - 2*A*a
^9*b - 3*B*a^8*b^2 + 6*A*a^7*b^3 + 3*B*a^6*b^4 - 6*A*a^5*b^5 - B*a^4*b^6 +
2*A*a^3*b^7)*cos(d*x + c)^2 + (4*B*a^9*b - 11*A*a^8*b^2 - 20*B*a^7*b^3 + 43
*A*a^6*b^4 + 25*B*a^5*b^5 - 50*A*a^4*b^6 - 9*B*a^3*b^7 + 18*A*a^2*b^8)*cos(
d*x + c))*sin(d*x + c))/((a^13 - 3*a^11*b^2 + 3*a^9*b^4 - a^7*b^6)*d*cos(d*
x + c)^2 + 2*(a^12*b - 3*a^10*b^3 + 3*a^8*b^5 - a^6*b^7)*d*cos(d*x + c) + (
a^11*b^2 - 3*a^9*b^4 + 3*a^7*b^6 - a^5*b^8)*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\cos^2(c + dx)}{(a + b\sec(c + dx))^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/(a + b*sec(c + d*x))**3, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2700 vs. 2(373) = 746.
time = 0.81, size = 2700, normalized size = 6.87

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm="gi
ac")
```

```
[Out] -1/2*(((a^6 - a^5*b + 10*a^4*b^2 + 10*a^3*b^3 - 23*a^2*b^4 - 6*a*b^5 + 12*b
^6)*sqrt(-a^2 + b^2)*A*abs(a^9 - 2*a^7*b^2 + a^5*b^4)*abs(-a + b) - 3*(2*a^
5*b + 2*a^4*b^2 - 4*a^3*b^3 - a^2*b^4 + 2*a*b^5)*sqrt(-a^2 + b^2)*B*abs(a^9
 - 2*a^7*b^2 + a^5*b^4)*abs(-a + b) - (a^15 - a^14*b + 8*a^13*b^2 - 28*a^12
*b^3 - 42*a^11*b^4 + 111*a^10*b^5 + 68*a^9*b^6 - 158*a^8*b^7 - 47*a^7*b^8 +
```

100*a^6*b^9 + 12*a^5*b^10 - 24*a^4*b^11)*sqrt(-a^2 + b^2)*A*abs(-a + b) + 3*(2*a^14*b - 6*a^13*b^2 - 8*a^12*b^3 + 21*a^11*b^4 + 12*a^10*b^5 - 28*a^9*b^6 - 8*a^8*b^7 + 17*a^7*b^8 + 2*a^6*b^9 - 4*a^5*b^10)*sqrt(-a^2 + b^2)*B*abs(-a + b))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^8*b - 2*a^6*b^3 + a^4*b^5 + sqrt((a^9 + a^8*b - 2*a^7*b^2 - 2*a^6*b^3 + a^5*b^4 + a^4*b^5)*(a^9 - a^8*b - 2*a^7*b^2 + 2*a^6*b^3 + a^5*b^4 - a^4*b^5) + (a^8*b - 2*a^6*b^3 + a^4*b^5)^2))/(a^9 - a^8*b - 2*a^7*b^2 + 2*a^6*b^3 + a^5*b^4 - a^4*b^5))))/((a^9 - 2*a^7*b^2 + a^5*b^4)^2*(a^2 - 2*a*b + b^2) + (a^10*b - 2*a^9*b^2 - a^8*b^3 + 4*a^7*b^4 - a^6*b^5 - 2*a^5*b^6 + a^4*b^7)*abs(a^9 - 2*a^7*b^2 + a^5*b^4)) + (A*a^15 - A*a^14*b - 6*B*a^14*b + 8*A*a^13*b^2 + 18*B*a^13*b^2 - 28*A*a^12*b^3 + 24*B*a^12*b^3 - 42*A*a^11*b^4 - 63*B*a^11*b^4 + 111*A*a^10*b^5 - 36*B*a^10*b^5 + 68*A*a^9*b^6 + 84*B*a^9*b^6 - 158*A*a^8*b^7 + 24*B*a^8*b^7 - 47*A*a^7*b^8 - 51*B*a^7*b^8 + 100*A*a^6*b^9 - 6*B*a^6*b^9 + 12*A*a^5*b^10 + 12*B*a^5*b^10 - 24*A*a^4*b^11 + A*a^6*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - A*a^5*b*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - 6*B*a^5*b*abs(a^9 - 2*a^7*b^2 + a^5*b^4) + 10*A*a^4*b^2*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - 6*B*a^4*b^2*abs(a^9 - 2*a^7*b^2 + a^5*b^4) + 10*A*a^3*b^3*abs(a^9 - 2*a^7*b^2 + a^5*b^4) + 12*B*a^3*b^3*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - 23*A*a^2*b^4*abs(a^9 - 2*a^7*b^2 + a^5*b^4) + 3*B*a^2*b^4*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - 6*A*a*b^5*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - 6*B*a*b^5*abs(a^9 - 2*a^7*b^2 + a^5*b^4) + 12*A*b^6*abs(a^9 - 2*a^7*b^2 + a^5*b^4))*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x + 1/2*c)/sqrt(-(a^8*b - 2*a^6*b^3 + a^4*b^5 - sqrt((a^9 + a^8*b - 2*a^7*b^2 - 2*a^6*b^3 + a^5*b^4 + a^4*b^5)*(a^9 - a^8*b - 2*a^7*b^2 + 2*a^6*b^3 + a^5*b^4 - a^4*b^5) + (a^8*b - 2*a^6*b^3 + a^4*b^5)^2))/(a^9 - a^8*b - 2*a^7*b^2 + 2*a^6*b^3 + a^5*b^4 - a^4*b^5))))/(a^8*b*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - 2*a^6*b^3*abs(a^9 - 2*a^7*b^2 + a^5*b^4) + a^4*b^5*abs(a^9 - 2*a^7*b^2 + a^5*b^4) - (a^9 - 2*a^7*b^2 + a^5*b^4)^2) + 2*(A*a^7*tan(1/2*d*x + 1/2*c)^7 - 2*B*a^7*tan(1/2*d*x + 1/2*c)^7 + 4*A*a^6*b*tan(1/2*d*x + 1/2*c)^7 + 4*B*a^6*b*tan(1/2*d*x + 1/2*c)^7 - 13*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 + 2*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^7 - 2*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 - 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^7 + 33*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 + 9*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^7 - 17*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 + 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^7 - 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^7 - 6*B*a*b^6*tan(1/2*d*x + 1/2*c)^7 + 12*A*b^7*tan(1/2*d*x + 1/2*c)^7 - 3*A*a^7*tan(1/2*d*x + 1/2*c)^5 + 2*B*a^7*tan(1/2*d*x + 1/2*c)^5 - 4*A*a^6*b*tan(1/2*d*x + 1/2*c)^5 + 4*B*a^6*b*tan(1/2*d*x + 1/2*c)^5 - 5*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 - 10*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^5 + 26*A*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 - 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^5 + 29*A*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 + 35*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^5 - 67*A*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 + 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^5 - 18*A*a*b^6*tan(1/2*d*x + 1/2*c)^5 - 18*B*a*b^6*tan(1/2*d*x + 1/2*c)^5 + 36*A*b^7*tan(1/2*d*x + 1/2*c)^5 + 3*A*a^7*tan(1/2*d*x + 1/2*c)^3 + 2*B*a^7*tan(1/2*d*x + 1/2*c)^3 - 4*A*a^6*b*tan(1/2*d*x + 1/2*c)^3 - 4*B*a^6*b*tan(1/2*d*x + 1/2*c)^3 + 5*A*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 - 10*B*a^5*b^2*tan(1/2*d*x + 1/2*c)^3 + 26*A*a^4*b^3*tan(1/2*d*x + 1/

```
2*c)^3 + 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c)^3 - 29*A*a^3*b^4*tan(1/2*d*x + 1
/2*c)^3 + 35*B*a^3*b^4*tan(1/2*d*x + 1/2*c)^3 - 67*A*a^2*b^5*tan(1/2*d*x +
1/2*c)^3 - 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c)^3 + 18*A*a*b^6*tan(1/2*d*x + 1/
2*c)^3 - 18*B*a*b^6*tan(1/2*d*x + 1/2*c)^3 + 36*A*b^7*tan(1/2*d*x + 1/2*c)^
3 - A*a^7*tan(1/2*d*x + 1/2*c) - 2*B*a^7*tan(1/2*d*x + 1/2*c) + 4*A*a^6*b*t
an(1/2*d*x + 1/2*c) - 4*B*a^6*b*tan(1/2*d*x + 1/2*c) + 13*A*a^5*b^2*tan(1/2
*d*x + 1/2*c) + 2*B*a^5*b^2*tan(1/2*d*x + 1/2*c) - 2*A*a^4*b^3*tan(1/2*d*x
+ 1/2*c) + 16*B*a^4*b^3*tan(1/2*d*x + 1/2*c) - 33*A*a^3*b^4*tan(1/2*d*x + 1
/2*c) + 9*B*a^3*b^4*tan(1/2*d*x + 1/2*c) - 17*A*a^2*b^5*tan(1/2*d*x + 1/2*c
) - 9*B*a^2*b^5*tan(1/2*d*x + 1/2*c) + 18*A*a*b^6*tan(1/2*d*x + 1/2*c) - 6*
B*a*b^6*tan(1/2*d*x + 1/2*c) + 12*A*b^7*tan(1/2*d*x + 1/2*c))/((a^8 - 2*a^6
*b^2 + a^4*b^4)*(a*tan(1/2*d*x + 1/2*c)^4 - b*tan(1/2*d*x + 1/2*c)^4 - 2*b*
tan(1/2*d*x + 1/2*c)^2 - a - b)^2))/d
```

Mupad [B]
time = 14.17, size = 2500, normalized size = 6.36

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^3,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(3*A*a^7 - 36*A*b^7 - 2*B*a^7 + 67*A*a^2*b^5 - 29*A*
a^3*b^4 - 26*A*a^4*b^3 + 5*A*a^5*b^2 - 9*B*a^2*b^5 - 35*B*a^3*b^4 + 16*B*a^
4*b^3 + 10*B*a^5*b^2 + 18*A*a*b^6 + 4*A*a^6*b + 18*B*a*b^6 - 4*B*a^6*b))/((
a + b)^2*(a^6 - 2*a^5*b + a^4*b^2)) - (tan(c/2 + (d*x)/2)^3*(3*A*a^7 + 36*A
*b^7 + 2*B*a^7 - 67*A*a^2*b^5 - 29*A*a^3*b^4 + 26*A*a^4*b^3 + 5*A*a^5*b^2 -
 9*B*a^2*b^5 + 35*B*a^3*b^4 + 16*B*a^4*b^3 - 10*B*a^5*b^2 + 18*A*a*b^6 - 4*
A*a^6*b - 18*B*a*b^6 - 4*B*a^6*b))/((a + b)^2*(a^6 - 2*a^5*b + a^4*b^2)) +
(tan(c/2 + (d*x)/2)^7*(A*a^6 - 12*A*b^6 - 2*B*a^6 + 23*A*a^2*b^4 - 10*A*a^3
*b^3 - 8*A*a^4*b^2 - 3*B*a^2*b^4 - 12*B*a^3*b^3 + 4*B*a^4*b^2 + 6*A*a*b^5 +
 5*A*a^5*b + 6*B*a*b^5 + 2*B*a^5*b))/((a^4*b - a^5)*(a + b)^2) + (tan(c/2 +
 (d*x)/2)*(A*a^6 - 12*A*b^6 + 2*B*a^6 + 23*A*a^2*b^4 + 10*A*a^3*b^3 - 8*A*a
^4*b^2 + 3*B*a^2*b^4 - 12*B*a^3*b^3 - 4*B*a^4*b^2 - 6*A*a*b^5 - 5*A*a^5*b +
 6*B*a*b^5 + 2*B*a^5*b))/((a + b)*(a^6 - 2*a^5*b + a^4*b^2)))/(d*(2*a*b - t
an(c/2 + (d*x)/2)^4*(2*a^2 - 6*b^2) + tan(c/2 + (d*x)/2)^2*(4*a*b + 4*b^2)
- tan(c/2 + (d*x)/2)^6*(4*a*b - 4*b^2) + tan(c/2 + (d*x)/2)^8*(a^2 - 2*a*b
+ b^2) + a^2 + b^2)) + (atan(((((8*tan(c/2 + (d*x)/2)*(A^2*a^14 + 288*A^2*b
^14 - 288*A^2*a*b^13 - 2*A^2*a^13*b - 1104*A^2*a^2*b^12 + 1104*A^2*a^3*b^11
 + 1538*A^2*a^4*b^10 - 1538*A^2*a^5*b^9 - 827*A^2*a^6*b^8 + 872*A^2*a^7*b^7
 + 18*A^2*a^8*b^6 - 108*A^2*a^9*b^5 + 74*A^2*a^10*b^4 - 40*A^2*a^11*b^3 + 2
1*A^2*a^12*b^2 + 72*B^2*a^2*b^12 - 72*B^2*a^3*b^11 - 288*B^2*a^4*b^10 + 288
*B^2*a^5*b^9 + 441*B^2*a^6*b^8 - 432*B^2*a^7*b^7 - 288*B^2*a^8*b^6 + 288*B^
2*a^9*b^5 + 36*B^2*a^10*b^4 - 72*B^2*a^11*b^3 + 36*B^2*a^12*b^2 - 288*A*B*a
*b^13 - 12*A*B*a^13*b + 288*A*B*a^2*b^12 + 1128*A*B*a^3*b^11 - 1128*A*B*a^4
```

*b^10 - 1650*A*B*a^5*b^9 + 1632*A*B*a^6*b^8 + 984*A*B*a^7*b^7 - 1008*A*B*a^8*b^6 - 72*A*B*a^9*b^5 + 192*A*B*a^10*b^4 - 108*A*B*a^11*b^3 + 24*A*B*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) + (((4*(4*A*a^21 - 48*A*a^10*b^11 + 24*A*a^11*b^10 + 212*A*a^12*b^9 - 100*A*a^13*b^8 - 360*A*a^14*b^7 + 164*A*a^15*b^6 + 276*A*a^16*b^5 - 120*A*a^17*b^4 - 80*A*a^18*b^3 + 28*A*a^19*b^2 + 24*B*a^11*b^10 - 12*B*a^12*b^9 - 108*B*a^13*b^8 + 48*B*a^14*b^7 + 192*B*a^15*b^6 - 84*B*a^16*b^5 - 156*B*a^17*b^4 + 72*B*a^18*b^3 + 48*B*a^19*b^2 - 24*B*a^20*b))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) - (4*tan(c/2 + (d*x)/2)*(A*a^2*1i + A*b^2*12i - B*a*b*6i)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2)))*(A*a^2*1i + A*b^2*12i - B*a*b*6i))/(2*a^5))*(A*a^2*1i + A*b^2*12i - B*a*b*6i)*1i)/(2*a^5) + (((8*tan(c/2 + (d*x)/2)*(A^2*a^14 + 288*A^2*b^14 - 288*A^2*a*b^13 - 2*A^2*a^13*b - 1104*A^2*a^2*b^12 + 1104*A^2*a^3*b^11 + 1538*A^2*a^4*b^10 - 1538*A^2*a^5*b^9 - 827*A^2*a^6*b^8 + 872*A^2*a^7*b^7 + 18*A^2*a^8*b^6 - 108*A^2*a^9*b^5 + 74*A^2*a^10*b^4 - 40*A^2*a^11*b^3 + 21*A^2*a^12*b^2 + 72*B^2*a^2*b^12 - 72*B^2*a^3*b^11 - 288*B^2*a^4*b^10 + 288*B^2*a^5*b^9 + 441*B^2*a^6*b^8 - 432*B^2*a^7*b^7 - 288*B^2*a^8*b^6 + 288*B^2*a^9*b^5 + 36*B^2*a^10*b^4 - 72*B^2*a^11*b^3 + 36*B^2*a^12*b^2 - 288*A*B*a*b^13 - 12*A*B*a^13*b + 288*A*B*a^2*b^12 + 1128*A*B*a^3*b^11 - 1128*A*B*a^4*b^10 - 1650*A*B*a^5*b^9 + 1632*A*B*a^6*b^8 + 984*A*B*a^7*b^7 - 1008*A*B*a^8*b^6 - 72*A*B*a^9*b^5 + 192*A*B*a^10*b^4 - 108*A*B*a^11*b^3 + 24*A*B*a^12*b^2))/(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2) - (((4*(4*A*a^21 - 48*A*a^10*b^11 + 24*A*a^11*b^10 + 212*A*a^12*b^9 - 100*A*a^13*b^8 - 360*A*a^14*b^7 + 164*A*a^15*b^6 + 276*A*a^16*b^5 - 120*A*a^17*b^4 - 80*A*a^18*b^3 + 28*A*a^19*b^2 + 24*B*a^11*b^10 - 12*B*a^12*b^9 - 108*B*a^13*b^8 + 48*B*a^14*b^7 + 192*B*a^15*b^6 - 84*B*a^16*b^5 - 156*B*a^17*b^4 + 72*B*a^18*b^3 + 48*B*a^19*b^2 - 24*B*a^20*b))/(a^18*b + a^19 - a^12*b^7 - a^13*b^6 + 3*a^14*b^5 + 3*a^15*b^4 - 3*a^16*b^3 - 3*a^17*b^2) + (4*tan(c/2 + (d*x)/2)*(A*a^2*1i + A*b^2*12i - B*a*b*6i)*(8*a^19*b - 8*a^10*b^10 + 8*a^11*b^9 + 32*a^12*b^8 - 32*a^13*b^7 - 48*a^14*b^6 + 48*a^15*b^5 + 32*a^16*b^4 - 32*a^17*b^3 - 8*a^18*b^2))/(a^5*(a^14*b + a^15 - a^8*b^7 - a^9*b^6 + 3*a^10*b^5 + 3*a^11*b^4 - 3*a^12*b^3 - 3*a^13*b^2)))*(A*a^2*1i + A*b^2*12i - B*a*b*6i))/(2*a^5))*(A*a^2*1i + A*b^2*12i - B*a*b*6i)*1i)/(2*a^5))/((8*(1728*A^3*b^15 - 864*A^3*a*b^14 - 7344*A^3*a^2*b^13 + 3456*A^3*a^3*b^12 + 11700*A^3*a^4*b^11 - 4770*A^3*a^5*b^10 - 7829*A^3*a^6*b^9 + 2326*A^3*a^7*b^8 + 1314*A^3*a^8*b^7 - 11*A^3*a^9*b^6 + 411*A^3*a^10*b^5 - 20*A^3*a^11*b^4 + 20*A^3*a^12*b^3 - 216*B^3*a^3*b^12 + 108*B^3*a^4*b^11 + 972*B^3*a^5*b^10 - 486*B^3*a^6*b^9 - 1728*B^3*a^7*b^8 + 756*B^3*a^8*b^7 + 1404*B^3*a^9*b^6 - 432*B^3*a^10*b^5 - 432*B^3*a^11*b^4 - 2592*A^2*B*a*b^14 + 1296*A*B^2*a^2*b^13 - 648*A*B^2*a^3*b^12 - 5724*A*B^2*a^4*b^11 + 2808*A*B^2*a^5*b^10 + 9828*A*B^2*a^6*b^9 - 4203*A*B^2*a^7*b^8 - 7524*A*B^...

3.336 $\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=418

$$\frac{(Ab-4aB)\tanh^{-1}(\sin(c+dx))}{b^5d}-\frac{a(2a^6Ab-7a^4Ab^3+8a^2Ab^5-8Ab^7-8a^7B+28a^5b^2B-35a^3b^4B+20}{(a-b)^{7/2}b^5(a+b)^{7/2}d}$$

```
[Out] (A*b-4*B*a)*arctanh(sin(d*x+c))/b^5/d-a*(2*A*a^6*b-7*A*a^4*b^3+8*A*a^2*b^5-
8*A*b^7-8*B*a^7+28*B*a^5*b^2-35*B*a^3*b^4+20*B*a*b^6)*arctanh((a-b)^(1/2)*t
an(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^(7/2)/b^5/(a+b)^(7/2)/d-1/6*(3*A*a^3*b
-8*A*a*b^3-12*B*a^4+23*B*a^2*b^2-6*B*b^4)*tan(d*x+c)/b^4/(a^2-b^2)^2/d+1/3*
a*(A*b-B*a)*sec(d*x+c)^3*tan(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*a*
(A*a^2*b-6*A*b^3-4*B*a^3+9*B*a*b^2)*sec(d*x+c)^2*tan(d*x+c)/b^2/(a^2-b^2)^2
/d/(a+b*sec(d*x+c))^2-1/2*a^2*(A*a^4*b-2*A*a^2*b^3+6*A*b^5-4*B*a^5+11*B*a^3
*b^2-12*B*a*b^4)*tan(d*x+c)/b^4/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 3.52, antiderivative size = 418, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.290$, Rules used = {4114, 4183, 4175, 4167, 4083, 3855, 3916, 2738, 214}

$$\frac{a(Ab-aB)\tan(c+dx)\sec^3(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^3}+\frac{a(-4a^3B+a^2Ab+9ab^2B-6Ab^3)\tan(c+dx)\sec^2(c+dx)}{6b^2d(a^2-b^2)^2(a+b\sec(c+dx))^2}-\frac{(-12a^4B+3a^3Ab+23a^2b^2B-8aAb^3-6b^4B)\tan(c+dx)}{6b^4d(a^2-b^2)^2}-\frac{a^2(-4a^5B+a^4Ab+11a^3b^2B-2a^2Ab^3-12ab^4B+6Ab^5)\tan(c+dx)}{2b^4d(a^2-b^2)^3(a+b\sec(c+dx))}-\frac{a(-8a^7B+2a^6Ab+28a^5b^2B-7a^4Ab^3-35a^3b^4B+8a^2Ab^5+20ab^6B-8Ab^7)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{b^5d(a-b)^{7/2}(a+b)^{7/2}}+\frac{(Ab-4aB)\tanh^{-1}(\sin(c+dx))}{b^5d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((A*b - 4*a*B)*ArcTanh[Sin[c + d*x]])/(b^5*d) - (a*(2*a^6*A*b - 7*a^4*A*b^3
 + 8*a^2*A*b^5 - 8*A*b^7 - 8*a^7*B + 28*a^5*b^2*B - 35*a^3*b^4*B + 20*a*b^6
*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*b^5
*(a + b)^(7/2)*d) - ((3*a^3*A*b - 8*a*A*b^3 - 12*a^4*B + 23*a^2*b^2*B - 6*b
^4*B)*Tan[c + d*x])/(6*b^4*(a^2 - b^2)^2*d) + (a*(A*b - a*B)*Sec[c + d*x]^3
*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (a*(a^2*A*b - 6
*A*b^3 - 4*a^3*B + 9*a*b^2*B)*Sec[c + d*x]^2*Tan[c + d*x])/(6*b^2*(a^2 - b^
2)^2*d*(a + b*Sec[c + d*x])^2) - (a^2*(a^4*A*b - 2*a^2*A*b^3 + 6*A*b^5 - 4*
a^5*B + 11*a^3*b^2*B - 12*a*b^4*B)*Tan[c + d*x])/(2*b^4*(a^2 - b^2)^3*d*(a
+ b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4175

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x
])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*((-a)*(b*
B - a*C) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4183

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
- b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n +
 b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}
, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sec^5(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx = \frac{a(Ab-aB)\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\int \frac{\sec^3(c+dx)(3a(Ab-aB)-3b(Ab-}{(}}{}$$

$$= \frac{a(Ab-aB)\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+9}{6b^2\left(a^2-b^2\right)^2 d}$$

$$= \frac{a(Ab-aB)\sec^3(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+9}{6b^2\left(a^2-b^2\right)^2 d}$$

$$= -\frac{(3a^3Ab-8aAb^3-12a^4B+23a^2b^2B-6b^4B)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2 d} + \frac{a(A}{3b}$$

$$= -\frac{(3a^3Ab-8aAb^3-12a^4B+23a^2b^2B-6b^4B)\tan(c+dx)}{6b^4\left(a^2-b^2\right)^2 d} + \frac{a(A}{3b}$$

$$= \frac{(Ab-4aB)\tanh^{-1}(\sin(c+dx))}{b^5d} - \frac{(3a^3Ab-8aAb^3-12a^4B+23}{6b^4\left(a^2-\right.}$$

$$= \frac{(Ab-4aB)\tanh^{-1}(\sin(c+dx))}{b^5d} - \frac{(3a^3Ab-8aAb^3-12a^4B+23}{6b^4\left(a^2-\right.}$$

$$= \frac{(Ab-4aB)\tanh^{-1}(\sin(c+dx))}{b^5d} - \frac{a(2a^6Ab-7a^4Ab^3+8a^2Ab^5-}{}$$

Mathematica [A]
time = 3.12, size = 548, normalized size = 1.31

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^5*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((-48*a*(-2*a^6*A*b + 7*a^4*A*b^3 - 8*a^2*A*b^5 + 8*A*b^7 + 8*a^7*B - 28*a^
5*b^2*B + 35*a^3*b^4*B - 20*a*b^6*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sq
rt[a^2 - b^2]])/(a^2 - b^2)^(7/2) - 48*(A*b - 4*a*B)*Log[Cos[(c + d*x)/2] -
 Sin[(c + d*x)/2]] + 48*(A*b - 4*a*B)*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/
2]] + (2*b*(30*a^7*A*b^2 - 90*a^5*A*b^4 + 120*a^3*A*b^6 - 120*a^8*b*B + 318
*a^6*b^3*B - 246*a^4*b^5*B - 36*a^2*b^7*B + 24*b^9*B + a*(18*a^7*A*b - 7*a^
5*A*b^3 - 50*a^3*A*b^5 + 144*a*A*b^7 - 72*a^8*B + 28*a^6*b^2*B + 305*a^4*b^
4*B - 438*a^2*b^6*B + 72*b^8*B)*Cos[c + d*x] - 6*a^2*b*(-5*a^5*A*b + 15*a^3
*A*b^3 - 20*a*A*b^5 + 20*a^6*B - 57*a^4*b^2*B + 53*a^2*b^4*B - 6*b^6*B)*Cos
```

```
[2*(c + d*x)] + 6*a^8*A*b*Cos[3*(c + d*x)] - 17*a^6*A*b^3*Cos[3*(c + d*x)]
+ 26*a^4*A*b^5*Cos[3*(c + d*x)] - 24*a^9*B*Cos[3*(c + d*x)] + 68*a^7*b^2*B*
Cos[3*(c + d*x)] - 65*a^5*b^4*B*Cos[3*(c + d*x)] + 6*a^3*b^6*B*Cos[3*(c + d
*x)])*Tan[c + d*x])/((-a^2 + b^2)^3*(b + a*Cos[c + d*x])^3))/(48*b^5*d)
```

Maple [A]
time = 1.08, size = 591, normalized size = 1.41

method	result
derivativedivides	$\frac{2a\left(\frac{\left(2Aa^4b-Aa^3b^2-6Aa^2b^3+4Aab^4+12Ab^5-6Ba^5+2Ba^4b+18Ba^3b^2-5Ba^2b^3-20Bab^4\right)ba\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^3+3ba^2+3b^2a+b^3\right)}-\frac{2\left(3Aa^4b-11A\right.}{}\right.}{}$
default	$\frac{2a\left(\frac{\left(2Aa^4b-Aa^3b^2-6Aa^2b^3+4Aab^4+12Ab^5-6Ba^5+2Ba^4b+18Ba^3b^2-5Ba^2b^3-20Bab^4\right)ba\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^3+3ba^2+3b^2a+b^3\right)}-\frac{2\left(3Aa^4b-11A\right.}{}\right.}{}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(2*a/b^5*((1/2*(2*A*a^4*b-A*a^3*b^2-6*A*a^2*b^3+4*A*a*b^4+12*A*b^5-6*B*
a^5+2*B*a^4*b+18*B*a^3*b^2-5*B*a^2*b^3-20*B*a*b^4)*b*a/(a-b)/(a^3+3*a^2*b+3
*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5-2/3*(3*A*a^4*b-11*A*a^2*b^3+18*A*b^5-9*B*a
^5+29*B*a^3*b^2-30*B*a*b^4)*b*a/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x
+1/2*c)^3+1/2*(2*A*a^4*b+A*a^3*b^2-6*A*a^2*b^3-4*A*a*b^4+12*A*b^5-6*B*a^5-2
*B*a^4*b+18*B*a^3*b^2+5*B*a^2*b^3-20*B*a*b^4)*b*a/(a+b)/(a^3-3*a^2*b+3*a*b^
2-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2-a
-b)^3-1/2*(2*A*a^6*b-7*A*a^4*b^3+8*A*a^2*b^5-8*A*b^7-8*B*a^7+28*B*a^5*b^2-3
5*B*a^3*b^4+20*B*a*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))-B/b^4/(tan(1/2*d*x+1/
2*c)-1)+1/b^5*(-A*b+4*B*a)*ln(tan(1/2*d*x+1/2*c)-1)-B/b^4/(tan(1/2*d*x+1/2*
c)+1)+(A*b-4*B*a)/b^5*ln(tan(1/2*d*x+1/2*c)+1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1688 vs. $2(404) = 808$.
time = 185.94, size = 3434, normalized size = 8.22

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [-1/12*(3*((8*B*a^11 - 2*A*a^10*b - 28*B*a^9*b^2 + 7*A*a^8*b^3 + 35*B*a^7*b
^4 - 8*A*a^6*b^5 - 20*B*a^5*b^6 + 8*A*a^4*b^7)*cos(d*x + c)^4 + 3*(8*B*a^10
*b - 2*A*a^9*b^2 - 28*B*a^8*b^3 + 7*A*a^7*b^4 + 35*B*a^6*b^5 - 8*A*a^5*b^6
- 20*B*a^4*b^7 + 8*A*a^3*b^8)*cos(d*x + c)^3 + 3*(8*B*a^9*b^2 - 2*A*a^8*b^3
 - 28*B*a^7*b^4 + 7*A*a^6*b^5 + 35*B*a^5*b^6 - 8*A*a^4*b^7 - 20*B*a^3*b^8 +
 8*A*a^2*b^9)*cos(d*x + c)^2 + (8*B*a^8*b^3 - 2*A*a^7*b^4 - 28*B*a^6*b^5 +
7*A*a^5*b^6 + 35*B*a^4*b^7 - 8*A*a^3*b^8 - 20*B*a^2*b^9 + 8*A*a*b^10)*cos(d
*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x +
c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(
a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 6*((4*B*a^12 - A*a^11*b -
 16*B*a^10*b^2 + 4*A*a^9*b^3 + 24*B*a^8*b^4 - 6*A*a^7*b^5 - 16*B*a^6*b^6 +
4*A*a^5*b^7 + 4*B*a^4*b^8 - A*a^3*b^9)*cos(d*x + c)^4 + 3*(4*B*a^11*b - A*a
^10*b^2 - 16*B*a^9*b^3 + 4*A*a^8*b^4 + 24*B*a^7*b^5 - 6*A*a^6*b^6 - 16*B*a^
5*b^7 + 4*A*a^4*b^8 + 4*B*a^3*b^9 - A*a^2*b^10)*cos(d*x + c)^3 + 3*(4*B*a^1
0*b^2 - A*a^9*b^3 - 16*B*a^8*b^4 + 4*A*a^7*b^5 + 24*B*a^6*b^6 - 6*A*a^5*b^7
 - 16*B*a^4*b^8 + 4*A*a^3*b^9 + 4*B*a^2*b^10 - A*a*b^11)*cos(d*x + c)^2 + (
4*B*a^9*b^3 - A*a^8*b^4 - 16*B*a^7*b^5 + 4*A*a^6*b^6 + 24*B*a^5*b^7 - 6*A*a
^4*b^8 - 16*B*a^3*b^9 + 4*A*a^2*b^10 + 4*B*a*b^11 - A*b^12)*cos(d*x + c))*l
og(sin(d*x + c) + 1) - 6*((4*B*a^12 - A*a^11*b - 16*B*a^10*b^2 + 4*A*a^9*b^
3 + 24*B*a^8*b^4 - 6*A*a^7*b^5 - 16*B*a^6*b^6 + 4*A*a^5*b^7 + 4*B*a^4*b^8 -
 A*a^3*b^9)*cos(d*x + c)^4 + 3*(4*B*a^11*b - A*a^10*b^2 - 16*B*a^9*b^3 + 4*
A*a^8*b^4 + 24*B*a^7*b^5 - 6*A*a^6*b^6 - 16*B*a^5*b^7 + 4*A*a^4*b^8 + 4*B*a
^3*b^9 - A*a^2*b^10)*cos(d*x + c)^3 + 3*(4*B*a^10*b^2 - A*a^9*b^3 - 16*B*a^
8*b^4 + 4*A*a^7*b^5 + 24*B*a^6*b^6 - 6*A*a^5*b^7 - 16*B*a^4*b^8 + 4*A*a^3*b
^9 + 4*B*a^2*b^10 - A*a*b^11)*cos(d*x + c)^2 + (4*B*a^9*b^3 - A*a^8*b^4 - 1
6*B*a^7*b^5 + 4*A*a^6*b^6 + 24*B*a^5*b^7 - 6*A*a^4*b^8 - 16*B*a^3*b^9 + 4*A
*a^2*b^10 + 4*B*a*b^11 - A*b^12)*cos(d*x + c))*log(-sin(d*x + c) + 1) - 2*(
6*B*a^8*b^4 - 24*B*a^6*b^6 + 36*B*a^4*b^8 - 24*B*a^2*b^10 + 6*B*b^12 + (24*
```

```
B*a^11*b - 6*A*a^10*b^2 - 92*B*a^9*b^3 + 23*A*a^8*b^4 + 133*B*a^7*b^5 - 43*
A*a^6*b^6 - 71*B*a^5*b^7 + 26*A*a^4*b^8 + 6*B*a^3*b^9)*cos(d*x + c)^3 + 3*(
20*B*a^10*b^2 - 5*A*a^9*b^3 - 77*B*a^8*b^4 + 20*A*a^7*b^5 + 110*B*a^6*b^6 -
 35*A*a^5*b^7 - 59*B*a^4*b^8 + 20*A*a^3*b^9 + 6*B*a^2*b^10)*cos(d*x + c)^2
+ (44*B*a^9*b^3 - 11*A*a^8*b^4 - 169*B*a^7*b^5 + 43*A*a^6*b^6 + 239*B*a^5*b
^7 - 68*A*a^4*b^8 - 132*B*a^3*b^9 + 36*A*a^2*b^10 + 18*B*a*b^11)*cos(d*x +
c))*sin(d*x + c))/((a^11*b^5 - 4*a^9*b^7 + 6*a^7*b^9 - 4*a^5*b^11 + a^3*b^1
3)*d*cos(d*x + c)^4 + 3*(a^10*b^6 - 4*a^8*b^8 + 6*a^6*b^10 - 4*a^4*b^12 + a
^2*b^14)*d*cos(d*x + c)^3 + 3*(a^9*b^7 - 4*a^7*b^9 + 6*a^5*b^11 - 4*a^3*b^1
3 + a*b^15)*d*cos(d*x + c)^2 + (a^8*b^8 - 4*a^6*b^10 + 6*a^4*b^12 - 4*a^2*b
^14 + b^16)*d*cos(d*x + c)), 1/6*(3*((8*B*a^11 - 2*A*a^10*b - 28*B*a^9*b^2
+ 7*A*a^8*b^3 + 35*B*a^7*b^4 - 8*A*a^6*b^5 - 20*B*a^5*b^6 + 8*A*a^4*b^7)*co
s(d*x + c)^4 + 3*(8*B*a^10*b - 2*A*a^9*b^2 - 28*B*a^8*b^3 + 7*A*a^7*b^4 + 3
5*B*a^6*b^5 - 8*A*a^5*b^6 - 20*B*a^4*b^7 + 8*A*a^3*b^8)*cos(d*x + c)^3 + 3*
(8*B*a^9*b^2 - 2*A*a^8*b^3 - 28*B*a^7*b^4 + 7*A*a^6*b^5 + 35*B*a^5*b^6 - 8*
A*a^4*b^7 - 20*B*a^3*b^8 + 8*A*a^2*b^9)*cos(d*x + c)^2 + (8*B*a^8*b^3 - 2*A
*a^7*b^4 - 28*B*a^6*b^5 + 7*A*a^5*b^6 + 35*B*a^4*b^7 - 8*A*a^3*b^8 - 20*B*a
^2*b^9 + 8*A*a*b^10)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2
)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - 3*((4*B*a^12 - A*a^11*
b - 16*B*a^10*b^2 + 4*A*a^9*b^3 + 24*B*a^8*b^4 - 6*A*a^7*b^5 - 16*B*a^6*b^6
 + 4*A*a^5*b^7 + 4*B*a^4*b^8 - A*a^3*b^9)*cos(d*x + c)^4 + 3*(4*B*a^11*b -
A*a^10*b^2 - 16*B*a^9*b^3 + 4*A*a^8*b^4 + 24*B*a^7*b^5 - 6*A*a^6*b^6 - 16*B
*a^5*b^7 + 4*A*a^4*b^8 + 4*B*a^3*b^9 - A*a^2*b^10)*cos(d*x + c)^3 + 3*(4*B*
a^10*b^2 - A*a^9*b^3 - 16*B*a^8*b^4 + 4*A*a^7*b^5 + 24*B*a^6*b^6 - 6*A*a^5*
b^7 - 16*B*a^4*b^8 + 4*A*a^3*b^9 + 4*B*a^2*b^10 - A*a*b^11)*cos(d*x + c)^2
+ (4*B*a^9*b^3 - A*a^8*b^4 - 16*B*a^7*b^5 + 4*A*a^6*b^6 + 24*B*a^5*b^7 - 6*
A*a^4*b^8 - 16*B*a^3*b^9 + 4*A*a^2*b^10 + 4*B*a*b^11 - A*b^12)*cos(d*x + c)
)*log(sin(d*x + c) + 1) + 3*((4*B*a^12 - A*a^11*b - 16*B*a^10*b^2 + 4*A*a^9
*b^3 + 24*B*a^8*b^4 - 6*A*a^7*b^5 - 16*B*a^6*b^6 + 4*A*a^5*b^7 + 4*B*a^4*b^
8 - A*a^3*b^9)*cos(d*x + c)^4 + 3*(4*B*a^11*b - A*a^10*b^2 - 16*B*a^9*b^3 +
 4*A*a^8*b^4 + 24*B*a^7*b^5 - 6*A*a^6*b^6 - 16*B*a^5*b^7 + 4*A*a^4*b^8 + 4*
B*a^3*b^9 - A*a^2*b^10)*cos(d*x + c)^3 + 3*(4*B*a^10*b^2 - A*a^9*b^3 - 16*B
*a^8*b^4 + 4*A*a^7*b^5 + 24*B*a^6*b^6 - 6*A*a^5*b^7 - 16*B*a^4*b^8 + 4*A*a^
3*b^9 + 4*B*a^2*b^10 - A*a*b^11)*cos(d*x + c)^2 + (4*B*a^9*b^3 - A*a^8*b^4
- 16*B*a^7*b^5 + 4*A*a^6*b^6 + 24*B*a^5*b^7 - 6*A*a^4*b^8 - 16*B*a^3*b^9 +
4*A*a^2*b^10 + 4*B*a*b^11 - A*b^12)*cos(d*x + c))*log(-sin(d*x + c) + 1) +
(6*B*a^8*b^4 - 24*B*a^6*b^6 + 36*B*a^4*b^8 - 24*B*a^2*b^10 + 6*B*b^12 + (24
*B*a^11*b - 6*A*a^10*b^2 - 92*B*a^9*b^3 + 23*A*a^8*b^4 + 133*B*a^7*b^5 - 43
*A*a^6*b^6 - 71*B*a^5*b^7 + 26*A*a^4*b^8 + 6*B*...
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\sec^5{(c + dx)}}{(a + b\sec{(c + dx)})^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**5/(a + b*sec(c + d*x))**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1005 vs. $2(404) = 808$.
time = 0.58, size = 1005, normalized size = 2.40

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^5*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(8*B*a^8 - 2*A*a^7*b - 28*B*a^6*b^2 + 7*A*a^5*b^3 + 35*B*a^4*b^4 - 8
*A*a^3*b^5 - 20*B*a^2*b^6 + 8*A*a*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sg
n(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/s
qrt(-a^2 + b^2)))/((a^6*b^5 - 3*a^4*b^7 + 3*a^2*b^9 - b^11)*sqrt(-a^2 + b^2
)) - (18*B*a^9*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^8*b*tan(1/2*d*x + 1/2*c)^5 -
42*B*a^8*b*tan(1/2*d*x + 1/2*c)^5 + 15*A*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 - 2
4*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 + 1
17*B*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 - 45*A*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 -
 24*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 -
 105*B*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 + 60*A*a^3*b^6*tan(1/2*d*x + 1/2*c)^5
 + 60*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 - 36*A*a^2*b^7*tan(1/2*d*x + 1/2*c)^
5 - 36*B*a^9*tan(1/2*d*x + 1/2*c)^3 + 12*A*a^8*b*tan(1/2*d*x + 1/2*c)^3 + 1
52*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 - 56*A*a^6*b^3*tan(1/2*d*x + 1/2*c)^3 -
 236*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 + 116*A*a^4*b^5*tan(1/2*d*x + 1/2*c)^
3 + 120*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 - 72*A*a^2*b^7*tan(1/2*d*x + 1/2*c
)^3 + 18*B*a^9*tan(1/2*d*x + 1/2*c) - 6*A*a^8*b*tan(1/2*d*x + 1/2*c) + 42*B
*a^8*b*tan(1/2*d*x + 1/2*c) - 15*A*a^7*b^2*tan(1/2*d*x + 1/2*c) - 24*B*a^7*
b^2*tan(1/2*d*x + 1/2*c) + 6*A*a^6*b^3*tan(1/2*d*x + 1/2*c) - 117*B*a^6*b^3
*tan(1/2*d*x + 1/2*c) + 45*A*a^5*b^4*tan(1/2*d*x + 1/2*c) - 24*B*a^5*b^4*ta
n(1/2*d*x + 1/2*c) + 6*A*a^4*b^5*tan(1/2*d*x + 1/2*c) + 105*B*a^4*b^5*tan(1
/2*d*x + 1/2*c) - 60*A*a^3*b^6*tan(1/2*d*x + 1/2*c) + 60*B*a^3*b^6*tan(1/2*
d*x + 1/2*c) - 36*A*a^2*b^7*tan(1/2*d*x + 1/2*c))/((a^6*b^4 - 3*a^4*b^6 + 3
*a^2*b^8 - b^10)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a -
 b)^3) - 3*(4*B*a - A*b)*log(abs(tan(1/2*d*x + 1/2*c) + 1))/b^5 + 3*(4*B*a
- A*b)*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^5 - 6*B*tan(1/2*d*x + 1/2*c)/((
tan(1/2*d*x + 1/2*c)^2 - 1)*b^4))/d
```

Mupad [B]
time = 19.96, size = 2500, normalized size = 5.98

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^5*(a + b/cos(c + d*x))^4),x)

[Out] ((tan(c/2 + (d*x)/2)^7*(12*A*a^2*b^5 - 2*B*b^7 - 8*B*a^7 + 4*A*a^3*b^4 - 6*A*a^4*b^3 - A*a^5*b^2 + 6*B*a^2*b^5 - 26*B*a^3*b^4 - 11*B*a^4*b^3 + 24*B*a^5*b^2 + 2*A*a^6*b + 2*B*a*b^6 + 4*B*a^6*b))/(b^4*(a + b)^3*(a - b)) - (tan(c/2 + (d*x)/2)^3*(72*B*a^8 + 18*B*b^8 + 36*A*a^2*b^6 - 96*A*a^3*b^5 - 14*A*a^4*b^4 + 59*A*a^5*b^3 + 3*A*a^6*b^2 - 72*B*a^2*b^6 - 60*B*a^3*b^5 + 273*B*a^4*b^4 + 47*B*a^5*b^3 - 236*B*a^6*b^2 - 18*A*a^7*b - 12*B*a^7*b))/(3*b^4*(a + b)^2*(a - b)^3) + (tan(c/2 + (d*x)/2)^5*(72*B*a^8 + 18*B*b^8 - 36*A*a^2*b^6 - 96*A*a^3*b^5 + 14*A*a^4*b^4 + 59*A*a^5*b^3 - 3*A*a^6*b^2 - 72*B*a^2*b^6 + 60*B*a^3*b^5 + 273*B*a^4*b^4 - 47*B*a^5*b^3 - 236*B*a^6*b^2 - 18*A*a^7*b + 12*B*a^7*b))/(3*b^4*(a + b)^3*(a - b)^2) - (tan(c/2 + (d*x)/2)*(2*B*b^7 - 8*B*a^7 + 12*A*a^2*b^5 - 4*A*a^3*b^4 - 6*A*a^4*b^3 + A*a^5*b^2 - 6*B*a^2*b^5 - 26*B*a^3*b^4 + 11*B*a^4*b^3 + 24*B*a^5*b^2 + 2*A*a^6*b + 2*B*a*b^6 - 4*B*a^6*b))/(b^4*(a + b)*(a - b)^3))/(d*(3*a*b^2 + 3*a^2*b - tan(c/2 + (d*x)/2)^4*(6*a*b^2 - 6*a^3) - tan(c/2 + (d*x)/2)^2*(6*a^2*b + 4*a^3 - 2*b^3) - tan(c/2 + (d*x)/2)^6*(4*a^3 - 6*a^2*b + 2*b^3) + a^3 + b^3 + tan(c/2 + (d*x)/2)^8*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) + (atan((((((A*b - 4*B*a)*((8*(4*A*b^24 - 12*A*a^2*b^22 + 64*A*a^3*b^21 + 20*A*a^4*b^20 - 110*A*a^5*b^19 - 30*A*a^6*b^18 + 110*A*a^7*b^17 + 30*A*a^8*b^16 - 70*A*a^9*b^15 - 14*A*a^10*b^14 + 26*A*a^11*b^13 + 2*A*a^12*b^12 - 4*A*a^13*b^11 + 40*B*a^2*b^22 + 72*B*a^3*b^21 - 190*B*a^4*b^20 - 146*B*a^5*b^19 + 386*B*a^6*b^18 + 174*B*a^7*b^17 - 434*B*a^8*b^16 - 126*B*a^9*b^15 + 286*B*a^10*b^14 + 50*B*a^11*b^13 - 104*B*a^12*b^12 - 8*B*a^13*b^11 + 16*B*a^14*b^10 - 16*A*a*b^23 - 16*B*a*b^23))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19 + 10*a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*b^13 - a^11*b^12) - (8*tan(c/2 + (d*x)/2)*(A*b - 4*B*a)*(8*a*b^23 - 8*a^2*b^22 - 48*a^3*b^21 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160*a^7*b^17 + 160*a^8*b^16 + 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48*a^12*b^12 + 8*a^13*b^11 - 8*a^14*b^10))/(b^5*(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8))))/b^5 - (8*tan(c/2 + (d*x)/2)*(4*A^2*b^16 + 128*B^2*a^16 - 8*A^2*a*b^15 - 128*B^2*a^15*b + 44*A^2*a^2*b^14 + 48*A^2*a^3*b^13 - 92*A^2*a^4*b^12 - 120*A^2*a^5*b^11 + 156*A^2*a^6*b^10 + 160*A^2*a^7*b^9 - 164*A^2*a^8*b^8 - 120*A^2*a^9*b^7 + 117*A^2*a^10*b^6 + 48*A^2*a^11*b^5 - 48*A^2*a^12*b^4 - 8*A^2*a^13*b^3 + 8*A^2*a^14*b^2 + 64*B^2*a^2*b^14 - 128*B^2*a^3*b^13 + 80*B^2*a^4*b^12 + 768*B^2*a^5*b^11 - 824*B^2*a^6*b^10 - 1920*B^2*a^7*b^9 + 2025*B^2*a^8*b^8 + 2560*B^2*a^9*b^7 - 2600*B^2*a^10*b^6 - 1920*B^2*a^11*b^5 + 1920*B^2*a^12*b^4 + 768*B^2*a^13*b^3 - 768*B^2*a^14*b^2 - 32*A*B*a*b^15 - 64*A*B*a^15*b + 64*A*B*a^2*b^14 - 160*A*B*a^3*b^13 - 384*A*B*a^4*b^12 + 592*A*B*a^5*b^11 + 960*A*B*a^6*b^10 - 1128*A*B*a^7*b^9 - 1280*A*B*a^8*b^8 + 1306*A*B*a^9*b^7 + 960*A*B*a^10*b^6 - 948*A*B*a^11*b^5 - 384*A*B*a^12*b^4 + 384*A*B*a^13*b^3 + 64*A*B*a^14*b^2))/(a*b^18 + b^19 - 5*a^2*b^17

```
- 5*a^3*b^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^
8*b^11 + 5*a^9*b^10 - a^10*b^9 - a^11*b^8))*(A*b - 4*B*a)*1i)/b^5 - ((((A*b
 - 4*B*a)*((8*(4*A*b^24 - 12*A*a^2*b^22 + 64*A*a^3*b^21 + 20*A*a^4*b^20 - 1
10*A*a^5*b^19 - 30*A*a^6*b^18 + 110*A*a^7*b^17 + 30*A*a^8*b^16 - 70*A*a^9*b
^15 - 14*A*a^10*b^14 + 26*A*a^11*b^13 + 2*A*a^12*b^12 - 4*A*a^13*b^11 + 40*
B*a^2*b^22 + 72*B*a^3*b^21 - 190*B*a^4*b^20 - 146*B*a^5*b^19 + 386*B*a^6*b^
18 + 174*B*a^7*b^17 - 434*B*a^8*b^16 - 126*B*a^9*b^15 + 286*B*a^10*b^14 + 5
0*B*a^11*b^13 - 104*B*a^12*b^12 - 8*B*a^13*b^11 + 16*B*a^14*b^10 - 16*A*a*b
^23 - 16*B*a*b^23))/(a*b^22 + b^23 - 5*a^2*b^21 - 5*a^3*b^20 + 10*a^4*b^19
+ 10*a^5*b^18 - 10*a^6*b^17 - 10*a^7*b^16 + 5*a^8*b^15 + 5*a^9*b^14 - a^10*
b^13 - a^11*b^12) + (8*tan(c/2 + (d*x)/2)*(A*b - 4*B*a)*(8*a*b^23 - 8*a^2*b
^22 - 48*a^3*b^21 + 48*a^4*b^20 + 120*a^5*b^19 - 120*a^6*b^18 - 160*a^7*b^1
7 + 160*a^8*b^16 + 120*a^9*b^15 - 120*a^10*b^14 - 48*a^11*b^13 + 48*a^12*b^
12 + 8*a^13*b^11 - 8*a^14*b^10))/(b^5*(a*b^18 + b^19 - 5*a^2*b^17 - 5*a^3*b
^16 + 10*a^4*b^15 + 10*a^5*b^14 - 10*a^6*b^13 - 10*a^7*b^12 + 5*a^8*b^11 +
5*a^9*b^10 - a^10*b^9 - a^11*b^8))))/b^5 + (8*tan(c/2 + (d*x)/2)*(4*A^2*b^1
6 + 128*B^2*a^16 - 8*A^2*a*b^15 - 128*B^2*a^15*b + 44*A^2*a^2*b^14 + 48*A^2
*a^3*b^13 - 92*A^2*a^4*b^12 - 120*A^2*a^5*b^11 + 156*A^2*a^6*b^10 + 160*A^2
*a^7*b^9 - 164*A^2*a^8*b^8 - 120*A^2*a^9*b^7 + 117*A^2*a^10*b^6 + 48*A^2*a^
11*b^5 - 48*A^2*a^12*b^4 - 8*A^2*a^13*b^3 + 8*A^2*a^14*b^2 + 64*B^2*a^2*b^1
4 - 128*B^2*a^3*b^13 + 80*B^2*a^4*b^12 + 768*B^2*a^5*b^11 - 824*B^2*a^6*b^1
0 - 1920*B^2*a^7*b^9 + 2025*B^2*a^8*b^8 + 2560*B^2*a^9*b^7 - 2600*B^2*a^10*
b^6 - 1920*B^2*a^11*b^5 + 1920*B^2*a^12*b^4 + 768*B^2*a^13*b^3 - 768*B^2*a^
14*b^2 - 32*A*B*a*b^15 - 64*A*B*a^15*b + 64*A*B*a^2*b^14 - 160*A*B*a^3*b^13
 - 384*A*B*a^4*b^12 + 592*A*B*a^5*b^11 + 960*A*B*a^6*b^10 - 1128*A*B*a^7*b^
9 - 1280*A*B*a^8*b^8 + 1306*A*B*a^9*b^7 + 960*A...
```

3.337 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=310

$$\frac{B\tanh^{-1}(\sin(c+dx))}{b^4d}-\frac{(3a^2Ab^5+2Ab^7+2a^7B-7a^5b^2B+8a^3b^4B-8ab^6B)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right.}{\sqrt{a+b}}\right.}{(a-b)^{7/2}b^4(a+b)^{7/2}d}$$

```
[Out] B*arctanh(sin(d*x+c))/b^4/d-(3*A*a^2*b^5+2*A*b^7+2*B*a^7-7*B*a^5*b^2+8*B*a^
3*b^4-8*B*a*b^6)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/(a-b)^
(7/2)/b^4/(a+b)^(7/2)/d+1/3*a*(A*b-B*a)*sec(d*x+c)^2*tan(d*x+c)/b/(a^2-b^2)
/d/(a+b*sec(d*x+c))^3+1/6*a^2*(5*A*b^3+3*B*a^3-8*B*a*b^2)*tan(d*x+c)/b^3/(a
^2-b^2)^2/d/(a+b*sec(d*x+c))^2-1/6*a*(A*a^2*b^3-16*A*b^5+9*B*a^5-28*B*a^3*b
^2+34*B*a*b^4)*tan(d*x+c)/b^3/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A]

time = 0.93, antiderivative size = 310, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4114, 4175, 4165, 4083, 3855, 3916, 2738, 214}

$$\frac{a(Ab-aB)\tan(c+dx)\sec^2(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{a^2\left(3a^3B-8ab^2B+5Ab^3\right)\tan(c+dx)}{6b^3d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}-\frac{a\left(9a^5B-28a^3b^2B+a^2Ab^3+34ab^4B-16Ab^5\right)\tan(c+dx)}{6b^3d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}-\frac{\left(2a^7B-7a^5b^2B+8a^3b^4B+3a^2Ab^5-8ab^6B+2Ab^7\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^4d(a-b)^{7/2}(a+b)^{7/2}}+\frac{B\tanh^{-1}(\sin(c+dx))}{b^4d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (B*ArcTanh[Sin[c + d*x]])/(b^4*d) - ((3*a^2*A*b^5 + 2*A*b^7 + 2*a^7*B - 7*a
^5*b^2*B + 8*a^3*b^4*B - 8*a*b^6*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/
Sqrt[a + b]])/((a - b)^(7/2)*b^4*(a + b)^(7/2)*d) + (a*(A*b - a*B)*Sec[c +
d*x]^2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (a^2*(5*A
*b^3 + 3*a^3*B - 8*a*b^2*B)*Tan[c + d*x])/(6*b^3*(a^2 - b^2)^2*d*(a + b*Sec
[c + d*x])^2) - (a*(a^2*A*b^3 - 16*A*b^5 + 9*a^5*B - 28*a^3*b^2*B + 34*a*b^
4*B)*Tan[c + d*x])/(6*b^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3855

```
Int[csc[(c_.) + (d_.)*(x_)], x_Symbol] :> Simp[-ArcTanh[Cos[c + d*x]]/d, x]
 /; FreeQ[{c, d}, x]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4083

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/(csc[(
e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[B/b, Int[Csc[e + f*x],
x], x] + Dist[(A*b - a*B)/b, Int[Csc[e + f*x]/(a + b*Csc[e + f*x]), x], x]
/; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4165

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cot[e + f*x]*((a + b*Csc[e + f*x]
)^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)),
Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m +
 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2
, 0]
```

Rule 4175

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x
```

```
])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*((-a)*(b*
B - a*C) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx = \frac{a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{\int \frac{\sec^2(c+dx)(2a(Ab-aB)-3b(A\cdots}{(a\cdots}}{3\cdots}$$

$$= \frac{a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^2\left(5Ab^3+3a^3B-8ab^2B\cdots\right.}{6b^3\left(a^2-b^2\right)^2 d(a+b\,\mathrm{se}\cdots}$$

$$= \frac{a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^2\left(5Ab^3+3a^3B-8ab^2B\cdots\right.}{6b^3\left(a^2-b^2\right)^2 d(a+b\,\mathrm{se}\cdots}$$

$$= \frac{a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{a^2\left(5Ab^3+3a^3B-8ab^2B\cdots\right.}{6b^3\left(a^2-b^2\right)^2 d(a+b\,\mathrm{se}\cdots}$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \cdots$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{b^4d} + \frac{a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \cdots$$

$$= \frac{B\tanh^{-1}(\sin(c+dx))}{b^4d} - \frac{\left(3a^2Ab^5+2Ab^7+2a^7B-7a^5b^2B+8a\cdots\right.}{(a-b)^{7\cdots}}$$

Mathematica [A]
time = 1.90, size = 369, normalized size = 1.19

$$\frac{\cos(c+dx)(A+B\sec(c+dx))\left(\frac{24\left(3a^2Ab^5+2Ab^7+2a^7B-7a^5b^2B+8a^3b^4B-8ab^6B\right)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\left(a^2-b^2\right)^{7/2}}-24B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sin\left(\frac{1}{2}(c+dx)\right)\right)+24B\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sin\left(\frac{1}{2}(c+dx)\right)\right)-\frac{2ab\left(8a^4Ab^3+a^2Ab^5+36Ab^7-6a^7B-5a^5b^2B+38a^3b^4B-72ab^6B-6ab\left(-a^2Ab^3-9Ab^5+5a^5B-15a^3b^2B+20ab^4B\right)\cos(c+dx)+a^2\left(4a^2Ab^3+11Ab^5-6a^5B+17a^3b^2B-26ab^4B\right)\cos(2(c+dx))\right)\sin(c+dx)}{\left(-a^2+b^2\right)^3(b+a\cos(c+dx))^3}\right)}{24b^4d(B+A\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (Cos[c + d*x]*(A + B*Sec[c + d*x])*((24*(3*a^2*A*b^5 + 2*A*b^7 + 2*a^7*B -
7*a^5*b^2*B + 8*a^3*b^4*B - 8*a*b^6*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/
Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) - 24*B*Log[Cos[(c + d*x)/2] - Sin[(c +
d*x)/2]] + 24*B*Log[Cos[(c + d*x)/2] + Sin[(c + d*x)/2]] - (2*a*b*(8*a^4*A*
```

```
b^3 + a^2*A*b^5 + 36*A*b^7 - 6*a^7*B - 5*a^5*b^2*B + 38*a^3*b^4*B - 72*a*b^
6*B - 6*a*b*(-(a^2*A*b^3) - 9*A*b^5 + 5*a^5*B - 15*a^3*b^2*B + 20*a*b^4*B)*
Cos[c + d*x] + a^2*(4*a^2*A*b^3 + 11*A*b^5 - 6*a^5*B + 17*a^3*b^2*B - 26*a*
b^4*B)*Cos[2*(c + d*x)])*Sin[c + d*x])/((-a^2 + b^2)^3*(b + a*Cos[c + d*x])
^3)))/(24*b^4*d*(B + A*Cos[c + d*x]))
```

Maple [A]
time = 1.26, size = 482, normalized size = 1.55

method	result
derivativedivides	$\frac{2\left(-\frac{\left(2A\,a^{2}b^{3}+3Aa\,b^{4}+6A\,b^{5}-2B\,a^{5}+B\,a^{4}b+6B\,a^{3}b^{2}-4B\,a^{2}b^{3}-12Ba\,b^{4}\right)ba\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^{3}+3b\,a^{2}+3b^{2}a+b^{3}\right)}+\frac{2\left(A\,a^{2}b^{3}+9A\,b^{5}-3B\,a^{5}+11B\,a^{3}b^{2}-\cdots}{3\left(a^{2}-2ba+b^{2}\right)\left(a^{2}\cdots}}{\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^{2}\left(\cdots\right.\right.\right.}$
default	$\frac{2\left(-\frac{\left(2A\,a^{2}b^{3}+3Aa\,b^{4}+6A\,b^{5}-2B\,a^{5}+B\,a^{4}b+6B\,a^{3}b^{2}-4B\,a^{2}b^{3}-12Ba\,b^{4}\right)ba\left(\tan^{5}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^{3}+3b\,a^{2}+3b^{2}a+b^{3}\right)}+\frac{2\left(A\,a^{2}b^{3}+9A\,b^{5}-3B\,a^{5}+11B\,a^{3}b^{2}-\cdots}{3\left(a^{2}-2ba+b^{2}\right)\left(a^{2}\cdots}}{\left(a\left(\tan^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^{2}\left(\cdots\right.\right.\right.}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(2/b^4*((-1/2*(2*A*a^2*b^3+3*A*a*b^4+6*A*b^5-2*B*a^5+B*a^4*b+6*B*a^3*b^
2-4*B*a^2*b^3-12*B*a*b^4)*b*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1
/2*c)^5+2/3*(A*a^2*b^3+9*A*b^5-3*B*a^5+11*B*a^3*b^2-18*B*a*b^4)*b*a/(a^2-2*
a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(2*A*a^2*b^3-3*A*a*b^4+6*
A*b^5-2*B*a^5-B*a^4*b+6*B*a^3*b^2+4*B*a^2*b^3-12*B*a*b^4)*b*a/(a+b)/(a^3-3*
a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*
x+1/2*c)^2-a-b)^3-1/2*(3*A*a^2*b^5+2*A*b^7+2*B*a^7-7*B*a^5*b^2+8*B*a^3*b^4-
8*B*a*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*
tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))+B/b^4*ln(tan(1/2*d*x+1/2*c)+1)-B/b
^4*ln(tan(1/2*d*x+1/2*c)-1))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1110 vs. $2(295) = 590$.
time = 70.96, size = 2278, normalized size = 7.35

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [-1/12*(3*(2*B*a^7*b^3 - 7*B*a^5*b^5 + 8*B*a^3*b^7 + 3*A*a^2*b^8 - 8*B*a*b^
9 + 2*A*b^10 + (2*B*a^10 - 7*B*a^8*b^2 + 8*B*a^6*b^4 + 3*A*a^5*b^5 - 8*B*a^
4*b^6 + 2*A*a^3*b^7)*cos(d*x + c)^3 + 3*(2*B*a^9*b - 7*B*a^7*b^3 + 8*B*a^5*
b^5 + 3*A*a^4*b^6 - 8*B*a^3*b^7 + 2*A*a^2*b^8)*cos(d*x + c)^2 + 3*(2*B*a^8*
b^2 - 7*B*a^6*b^4 + 8*B*a^4*b^6 + 3*A*a^3*b^7 - 8*B*a^2*b^8 + 2*A*a*b^9)*co
s(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x
 + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2
)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) - 6*(B*a^8*b^3 - 4*B*a^6
*b^5 + 6*B*a^4*b^7 - 4*B*a^2*b^9 + B*b^11 + (B*a^11 - 4*B*a^9*b^2 + 6*B*a^7
*b^4 - 4*B*a^5*b^6 + B*a^3*b^8)*cos(d*x + c)^3 + 3*(B*a^10*b - 4*B*a^8*b^3
+ 6*B*a^6*b^5 - 4*B*a^4*b^7 + B*a^2*b^9)*cos(d*x + c)^2 + 3*(B*a^9*b^2 - 4*
B*a^7*b^4 + 6*B*a^5*b^6 - 4*B*a^3*b^8 + B*a*b^10)*cos(d*x + c))*log(sin(d*x
 + c) + 1) + 6*(B*a^8*b^3 - 4*B*a^6*b^5 + 6*B*a^4*b^7 - 4*B*a^2*b^9 + B*b^1
1 + (B*a^11 - 4*B*a^9*b^2 + 6*B*a^7*b^4 - 4*B*a^5*b^6 + B*a^3*b^8)*cos(d*x
+ c)^3 + 3*(B*a^10*b - 4*B*a^8*b^3 + 6*B*a^6*b^5 - 4*B*a^4*b^7 + B*a^2*b^9)
*cos(d*x + c)^2 + 3*(B*a^9*b^2 - 4*B*a^7*b^4 + 6*B*a^5*b^6 - 4*B*a^3*b^8 +
B*a*b^10)*cos(d*x + c))*log(-sin(d*x + c) + 1) + 2*(11*B*a^8*b^3 - 2*A*a^7*
b^4 - 43*B*a^6*b^5 + 7*A*a^5*b^6 + 68*B*a^4*b^7 - 23*A*a^3*b^8 - 36*B*a^2*b
^9 + 18*A*a*b^10 + (6*B*a^10*b - 23*B*a^8*b^3 - 4*A*a^7*b^4 + 43*B*a^6*b^5
- 7*A*a^5*b^6 - 26*B*a^4*b^7 + 11*A*a^3*b^8)*cos(d*x + c)^2 + 3*(5*B*a^9*b^
2 - 20*B*a^7*b^4 - A*a^6*b^5 + 35*B*a^5*b^6 - 8*A*a^4*b^7 - 20*B*a^3*b^8 +
9*A*a^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^11*b^4 - 4*a^9*b^6 + 6*a^7*b^8
 - 4*a^5*b^10 + a^3*b^12)*d*cos(d*x + c)^3 + 3*(a^10*b^5 - 4*a^8*b^7 + 6*a^
6*b^9 - 4*a^4*b^11 + a^2*b^13)*d*cos(d*x + c)^2 + 3*(a^9*b^6 - 4*a^7*b^8 +
6*a^5*b^10 - 4*a^3*b^12 + a*b^14)*d*cos(d*x + c) + (a^8*b^7 - 4*a^6*b^9 + 6
*a^4*b^11 - 4*a^2*b^13 + b^15)*d), -1/6*(3*(2*B*a^7*b^3 - 7*B*a^5*b^5 + 8*B
*a^3*b^7 + 3*A*a^2*b^8 - 8*B*a*b^9 + 2*A*b^10 + (2*B*a^10 - 7*B*a^8*b^2 + 8
*B*a^6*b^4 + 3*A*a^5*b^5 - 8*B*a^4*b^6 + 2*A*a^3*b^7)*cos(d*x + c)^3 + 3*(2
*B*a^9*b - 7*B*a^7*b^3 + 8*B*a^5*b^5 + 3*A*a^4*b^6 - 8*B*a^3*b^7 + 2*A*a^2*
b^8)*cos(d*x + c)^2 + 3*(2*B*a^8*b^2 - 7*B*a^6*b^4 + 8*B*a^4*b^6 + 3*A*a^3*
```

```
b^7 - 8*B*a^2*b^8 + 2*A*a*b^9)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(
-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - 3*(B*a^8*b^3
 - 4*B*a^6*b^5 + 6*B*a^4*b^7 - 4*B*a^2*b^9 + B*b^11 + (B*a^11 - 4*B*a^9*b^2
 + 6*B*a^7*b^4 - 4*B*a^5*b^6 + B*a^3*b^8)*cos(d*x + c)^3 + 3*(B*a^10*b - 4*
B*a^8*b^3 + 6*B*a^6*b^5 - 4*B*a^4*b^7 + B*a^2*b^9)*cos(d*x + c)^2 + 3*(B*a^
9*b^2 - 4*B*a^7*b^4 + 6*B*a^5*b^6 - 4*B*a^3*b^8 + B*a*b^10)*cos(d*x + c))*l
og(sin(d*x + c) + 1) + 3*(B*a^8*b^3 - 4*B*a^6*b^5 + 6*B*a^4*b^7 - 4*B*a^2*b
^9 + B*b^11 + (B*a^11 - 4*B*a^9*b^2 + 6*B*a^7*b^4 - 4*B*a^5*b^6 + B*a^3*b^8
)*cos(d*x + c)^3 + 3*(B*a^10*b - 4*B*a^8*b^3 + 6*B*a^6*b^5 - 4*B*a^4*b^7 +
B*a^2*b^9)*cos(d*x + c)^2 + 3*(B*a^9*b^2 - 4*B*a^7*b^4 + 6*B*a^5*b^6 - 4*B*
a^3*b^8 + B*a*b^10)*cos(d*x + c))*log(-sin(d*x + c) + 1) + (11*B*a^8*b^3 -
2*A*a^7*b^4 - 43*B*a^6*b^5 + 7*A*a^5*b^6 + 68*B*a^4*b^7 - 23*A*a^3*b^8 - 36
*B*a^2*b^9 + 18*A*a*b^10 + (6*B*a^10*b - 23*B*a^8*b^3 - 4*A*a^7*b^4 + 43*B*
a^6*b^5 - 7*A*a^5*b^6 - 26*B*a^4*b^7 + 11*A*a^3*b^8)*cos(d*x + c)^2 + 3*(5*
B*a^9*b^2 - 20*B*a^7*b^4 - A*a^6*b^5 + 35*B*a^5*b^6 - 8*A*a^4*b^7 - 20*B*a^
3*b^8 + 9*A*a^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^11*b^4 - 4*a^9*b^6 + 6
*a^7*b^8 - 4*a^5*b^10 + a^3*b^12)*d*cos(d*x + c)^3 + 3*(a^10*b^5 - 4*a^8*b^
7 + 6*a^6*b^9 - 4*a^4*b^11 + a^2*b^13)*d*cos(d*x + c)^2 + 3*(a^9*b^6 - 4*a^
7*b^8 + 6*a^5*b^10 - 4*a^3*b^12 + a*b^14)*d*cos(d*x + c) + (a^8*b^7 - 4*a^6
*b^9 + 6*a^4*b^11 - 4*a^2*b^13 + b^15)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec^4(c + dx)}{(a + b\sec(c + dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/(a + b*sec(c + d*x))**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 844 vs. 2(295) = 590.
time = 0.57, size = 844, normalized size = 2.72

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] -1/3*(3*(2*B*a^7 - 7*B*a^5*b^2 + 8*B*a^3*b^4 + 3*A*a^2*b^5 - 8*B*a*b^6 + 2*
A*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1
/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^6*b^4 - 3*
a^4*b^6 + 3*a^2*b^8 - b^10)*sqrt(-a^2 + b^2)) - 3*B*log(abs(tan(1/2*d*x + 1
```

```
/2*c) + 1))/b^4 + 3*B*log(abs(tan(1/2*d*x + 1/2*c) - 1))/b^4 - (6*B*a^8*tan
(1/2*d*x + 1/2*c)^5 - 15*B*a^7*b*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^6*b^2*tan(1
/2*d*x + 1/2*c)^5 - 6*A*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 + 45*B*a^5*b^3*tan(1
/2*d*x + 1/2*c)^5 + 3*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^4*b^4*tan(1/
2*d*x + 1/2*c)^5 - 6*A*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 - 60*B*a^3*b^5*tan(1/
2*d*x + 1/2*c)^5 + 27*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 36*B*a^2*b^6*tan(1
/2*d*x + 1/2*c)^5 - 18*A*a*b^7*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^8*tan(1/2*d*
x + 1/2*c)^3 + 56*B*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 4*A*a^5*b^3*tan(1/2*d*
x + 1/2*c)^3 - 116*B*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 + 32*A*a^3*b^5*tan(1/2*
d*x + 1/2*c)^3 + 72*B*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 - 36*A*a*b^7*tan(1/2*d
*x + 1/2*c)^3 + 6*B*a^8*tan(1/2*d*x + 1/2*c) + 15*B*a^7*b*tan(1/2*d*x + 1/2
*c) - 6*B*a^6*b^2*tan(1/2*d*x + 1/2*c) - 6*A*a^5*b^3*tan(1/2*d*x + 1/2*c) -
 45*B*a^5*b^3*tan(1/2*d*x + 1/2*c) - 3*A*a^4*b^4*tan(1/2*d*x + 1/2*c) - 6*B
*a^4*b^4*tan(1/2*d*x + 1/2*c) - 6*A*a^3*b^5*tan(1/2*d*x + 1/2*c) + 60*B*a^3
*b^5*tan(1/2*d*x + 1/2*c) - 27*A*a^2*b^6*tan(1/2*d*x + 1/2*c) + 36*B*a^2*b^
6*tan(1/2*d*x + 1/2*c) - 18*A*a*b^7*tan(1/2*d*x + 1/2*c))/((a^6*b^3 - 3*a^4
*b^5 + 3*a^2*b^7 - b^9)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^
2 - a - b)^3))/d
```

Mupad [B]
time = 14.14, size = 2500, normalized size = 8.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + b/cos(c + d*x))^4),x)
```

```
[Out] - ((tan(c/2 + (d*x)/2)*(2*B*a^6 + 3*A*a^2*b^4 - 2*A*a^3*b^3 + 12*B*a^2*b^4
- 4*B*a^3*b^3 - 6*B*a^4*b^2 - 6*A*a*b^5 + B*a^5*b))/((a + b)*(3*a*b^5 - b^6
 - 3*a^2*b^4 + a^3*b^3)) - (tan(c/2 + (d*x)/2)^5*(3*A*a^2*b^4 - 2*B*a^6 + 2
*A*a^3*b^3 - 12*B*a^2*b^4 - 4*B*a^3*b^3 + 6*B*a^4*b^2 + 6*A*a*b^5 + B*a^5*b
))/((a*b^3 - b^4)*(a + b)^3) + (4*tan(c/2 + (d*x)/2)^3*(A*a^3*b^3 - 3*B*a^6
 - 18*B*a^2*b^4 + 11*B*a^4*b^2 + 9*A*a*b^5))/(3*(a + b)^2*(b^5 - 2*a*b^4 +
a^2*b^3)))/(d*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) - t
an(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) + 3*a*b^2 + 3*a^2*b
 + a^3 + b^3 - tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (B*
atan(((B*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^14 + 8*B^2*a^14 + 4*B^2*b^14 - 8*B
^2*a*b^13 - 8*B^2*a^13*b + 12*A^2*a^2*b^12 + 9*A^2*a^4*b^10 + 44*B^2*a^2*b^
12 + 48*B^2*a^3*b^11 - 92*B^2*a^4*b^10 - 120*B^2*a^5*b^9 + 156*B^2*a^6*b^8
+ 160*B^2*a^7*b^7 - 164*B^2*a^8*b^6 - 120*B^2*a^9*b^5 + 117*B^2*a^10*b^4 +
48*B^2*a^11*b^3 - 48*B^2*a^12*b^2 - 32*A*B*a*b^13 - 16*A*B*a^3*b^11 + 20*A*
B*a^5*b^9 - 34*A*B*a^7*b^7 + 12*A*B*a^9*b^5))/(a*b^16 + b^17 - 5*a^2*b^15 -
 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8
*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6) + (B*((8*(4*A*b^21 + 4*B*b^21 - 6*A
*a^2*b^19 + 6*A*a^3*b^18 - 6*A*a^4*b^17 + 6*A*a^5*b^16 + 14*A*a^6*b^15 - 14
```

```
*A*a^7*b^14 - 6*A*a^8*b^13 + 6*A*a^9*b^12 - 12*B*a^2*b^19 + 64*B*a^3*b^18 +
 20*B*a^4*b^17 - 110*B*a^5*b^16 - 30*B*a^6*b^15 + 110*B*a^7*b^14 + 30*B*a^8
*b^13 - 70*B*a^9*b^12 - 14*B*a^10*b^11 + 26*B*a^11*b^10 + 2*B*a^12*b^9 - 4*
B*a^13*b^8 - 4*A*a*b^20 - 16*B*a*b^20))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3
*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12
+ 5*a^9*b^11 - a^10*b^10 - a^11*b^9) + (8*B*tan(c/2 + (d*x)/2)*(8*a*b^21 -
8*a^2*b^20 - 48*a^3*b^19 + 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - 160*
a^7*b^15 + 160*a^8*b^14 + 120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + 48*
a^12*b^10 + 8*a^13*b^9 - 8*a^14*b^8))/(b^4*(a*b^16 + b^17 - 5*a^2*b^15 - 5*
a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^
9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6))))/b^4)*1i)/b^4 + (B*((8*tan(c/2 + (d*
x)/2)*(4*A^2*b^14 + 8*B^2*a^14 + 4*B^2*b^14 - 8*B^2*a*b^13 - 8*B^2*a^13*b +
 12*A^2*a^2*b^12 + 9*A^2*a^4*b^10 + 44*B^2*a^2*b^12 + 48*B^2*a^3*b^11 - 92*
B^2*a^4*b^10 - 120*B^2*a^5*b^9 + 156*B^2*a^6*b^8 + 160*B^2*a^7*b^7 - 164*B^
2*a^8*b^6 - 120*B^2*a^9*b^5 + 117*B^2*a^10*b^4 + 48*B^2*a^11*b^3 - 48*B^2*a
^12*b^2 - 32*A*B*a*b^13 - 16*A*B*a^3*b^11 + 20*A*B*a^5*b^9 - 34*A*B*a^7*b^7
 + 12*A*B*a^9*b^5))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13
+ 10*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^
7 - a^11*b^6) - (B*((8*(4*A*b^21 + 4*B*b^21 - 6*A*a^2*b^19 + 6*A*a^3*b^18 -
 6*A*a^4*b^17 + 6*A*a^5*b^16 + 14*A*a^6*b^15 - 14*A*a^7*b^14 - 6*A*a^8*b^13
 + 6*A*a^9*b^12 - 12*B*a^2*b^19 + 64*B*a^3*b^18 + 20*B*a^4*b^17 - 110*B*a^5
*b^16 - 30*B*a^6*b^15 + 110*B*a^7*b^14 + 30*B*a^8*b^13 - 70*B*a^9*b^12 - 14
*B*a^10*b^11 + 26*B*a^11*b^10 + 2*B*a^12*b^9 - 4*B*a^13*b^8 - 4*A*a*b^20 -
16*B*a*b^20))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3*b^17 + 10*a^4*b^16 + 10*a
^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12 + 5*a^9*b^11 - a^10*b^10 -
 a^11*b^9) - (8*B*tan(c/2 + (d*x)/2)*(8*a*b^21 - 8*a^2*b^20 - 48*a^3*b^19 +
 48*a^4*b^18 + 120*a^5*b^17 - 120*a^6*b^16 - 160*a^7*b^15 + 160*a^8*b^14 +
120*a^9*b^13 - 120*a^10*b^12 - 48*a^11*b^11 + 48*a^12*b^10 + 8*a^13*b^9 - 8
*a^14*b^8))/(b^4*(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 1
0*a^5*b^12 - 10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 -
 a^11*b^6))))/b^4)*1i)/b^4)/((16*(4*B^3*a^13 - 4*A*B^2*b^13 + 4*A^2*B*b^13
+ 16*B^3*a*b^12 - 2*B^3*a^12*b + 48*B^3*a^2*b^11 - 64*B^3*a^3*b^10 - 64*B^3
*a^4*b^9 + 110*B^3*a^5*b^8 + 66*B^3*a^6*b^7 - 110*B^3*a^7*b^6 - 34*B^3*a^8*
b^5 + 70*B^3*a^9*b^4 + 11*B^3*a^10*b^3 - 26*B^3*a^11*b^2 - 28*A*B^2*a*b^12
+ 6*A*B^2*a^2*b^11 - 22*A*B^2*a^3*b^10 + 6*A*B^2*a^4*b^9 + 14*A*B^2*a^5*b^8
 - 14*A*B^2*a^6*b^7 - 20*A*B^2*a^7*b^6 + 6*A*B^2*a^8*b^5 + 6*A*B^2*a^9*b^4
+ 12*A^2*B*a^2*b^11 + 9*A^2*B*a^4*b^9))/(a*b^19 + b^20 - 5*a^2*b^18 - 5*a^3
*b^17 + 10*a^4*b^16 + 10*a^5*b^15 - 10*a^6*b^14 - 10*a^7*b^13 + 5*a^8*b^12
+ 5*a^9*b^11 - a^10*b^10 - a^11*b^9) + (B*((8*tan(c/2 + (d*x)/2)*(4*A^2*b^1
4 + 8*B^2*a^14 + 4*B^2*b^14 - 8*B^2*a*b^13 - 8*B^2*a^13*b + 12*A^2*a^2*b^12
 + 9*A^2*a^4*b^10 + 44*B^2*a^2*b^12 + 48*B^2*a^3*b^11 - 92*B^2*a^4*b^10 - 1
20*B^2*a^5*b^9 + 156*B^2*a^6*b^8 + 160*B^2*a^7*b^7 - 164*B^2*a^8*b^6 - 120*
B^2*a^9*b^5 + 117*B^2*a^10*b^4 + 48*B^2*a^11*b^3 - 48*B^2*a^12*b^2 - 32*A*B
*a*b^13 - 16*A*B*a^3*b^11 + 20*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 12*A*B*a^9*b^
5))/(a*b^16 + b^17 - 5*a^2*b^15 - 5*a^3*b^14 + 10*a^4*b^13 + 10*a^5*b^12 -
```

```
10*a^6*b^11 - 10*a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - a^10*b^7 - a^11*b^6) +
(B*((8*(4*A*b^21 + 4*B*b^21 - 6*A*a^2*b^19 + 6*A*a^3*b^18 - 6*A*a^4*b^17 +
6*A*a^5*b^16 + 14*A*a^6*b^15 - 14*A*a^7*b^14 - 6*A*a^8*b^13 + 6*A*a^9*b^12
- 12*B*a^2*b^19 + 64*B*a^3*b^18 + 20*B*a^4*b^17...
```

3.338 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=274

$$\frac{(a^3A+4aAb^2-3a^2bB-2b^3B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}-\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2\left(a^2-b^2\right)d(a+b\sec(c+dx))^3}+\frac{a(a^2Ab}{6b}$$

```
[Out] (A*a^3+4*A*a*b^2-3*B*a^2*b-2*B*b^3)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/
(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-1/3*a^2*(A*b-B*a)*tan(d*x+c)/b^2/(a^
2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*a*(A*a^2*b-6*A*b^3-4*B*a^3+9*B*a*b^2)*tan(d
*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/6*(A*a^4*b-10*A*a^2*b^3-6*A*b^
5+2*B*a^5-5*B*a^3*b^2+18*B*a*b^4)*tan(d*x+c)/b^2/(a^2-b^2)^3/d/(a+b*sec(d*x
+c))
```

Rubi [A]
time = 0.48, antiderivative size = 274, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4113, 4165, 4088, 12, 3916, 2738, 214}

$$-\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{(a^3A-3a^2bB+4aAb^2-2b^3B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{a(-4a^3B+a^2Ab+9ab^2B-6Ab^3)\tan(c+dx)}{6b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}+\frac{(2a^5B+a^4Ab-5a^3b^2B-10a^2Ab^3+18ab^4B-6Ab^5)\tan(c+dx)}{6b^2d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((a^3*A + 4*a*A*b^2 - 3*a^2*b*B - 2*b^3*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - (a^2*(A*b - a*B)*Tan
[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (a*(a^2*A*b - 6*A
*b^3 - 4*a^3*B + 9*a*b^2*B)*Tan[c + d*x])/(6*b^2*(a^2 - b^2)^2*d*(a + b*Sec
[c + d*x])^2) + ((a^4*A*b - 10*a^2*A*b^3 - 6*A*b^5 + 2*a^5*B - 5*a^3*b^2*B
+ 18*a*b^4*B)*Tan[c + d*x])/(6*b^2*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rule 4113

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-a^2)*(A*b - a*B)
*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x
] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])
^(m + 1)*Simp[a*b*(A*b - a*B)*(m + 1) - (A*b - a*B)*(a^2 + b^2*(m + 1))*Csc
[e + f*x] + b*B*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4165

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cot[e + f*x]*((a + b*Csc[e + f*x]
)^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)),
Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m +
 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2
, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx = -\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)(-3ab(Ab-aB)-(a^2-3b\cdots}{(\cdots}}{3\cdots}$$

$$= -\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+9\cdots}{6b^2(a^2-b^2)^2\,d(a+b\,\mathrm{s}\cdots}$$

$$= -\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+9\cdots}{6b^2(a^2-b^2)^2\,d(a+b\,\mathrm{s}\cdots}$$

$$= -\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+9\cdots}{6b^2(a^2-b^2)^2\,d(a+b\,\mathrm{s}\cdots}$$

$$= -\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+9\cdots}{6b^2(a^2-b^2)^2\,d(a+b\,\mathrm{s}\cdots}$$

$$= -\frac{a^2(Ab-aB)\tan(c+dx)}{3b^2(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{a(a^2Ab-6Ab^3-4a^3B+9\cdots}{6b^2(a^2-b^2)^2\,d(a+b\,\mathrm{s}\cdots}$$

$$= \frac{(a^3A+4aAb^2-3a^2bB-2b^3B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} - \cdots$$

Mathematica [A]
time = 2.33, size = 226, normalized size = 0.82

$$\frac{-\frac{6(a^3A+4aAb^2-3a^2bB-2b^3B)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{(a^2-b^2)^{7/2}} + \frac{2(-Ab+aB)\sin(c+dx)}{(a-b)(a+b)(b+a\cos(c+dx))^3} + \frac{(3a^2A+2Ab^2-5abB)\sin(c+dx)}{(a-b)^2(a+b)^2(b+a\cos(c+dx))^2} + \frac{(-13a^2Ab-2Ab^3+4a^3B+11ab^2B)\sin(c+dx)}{(a-b)^3(a+b)^3(b+a\cos(c+dx))}}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((-6*(a^3*A + 4*a*A*b^2 - 3*a^2*b*B - 2*b^3*B)*ArcTanh[((-a + b)*Tan[(c + d
*x)/2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) + (2*(-(A*b) + a*B)*Sin[c + d*x
])/((a - b)*(a + b)*(b + a*Cos[c + d*x])^3) + ((3*a^2*A + 2*A*b^2 - 5*a*b*B
)*Sin[c + d*x])/((a - b)^2*(a + b)^2*(b + a*Cos[c + d*x])^2) + ((-13*a^2*A*
b - 2*A*b^3 + 4*a^3*B + 11*a*b^2*B)*Sin[c + d*x])/((a - b)^3*(a + b)^3*(b +
 a*Cos[c + d*x])))/(6*d)
```

Maple [A]
time = 0.70, size = 375, normalized size = 1.37 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(-2*(-1/2*(A*a^3+6*A*a^2*b+2*A*a*b^2+2*A*b^3-2*B*a^3-3*B*a^2*b-6*B*a*b^
2)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(7*A*a^2*b+3*A*
b^3-B*a^3-9*B*a*b^2)/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3+1
/2*(A*a^3-6*A*a^2*b+2*A*a*b^2-2*A*b^3+2*B*a^3-3*B*a^2*b+6*B*a*b^2)/(a+b)/(a
^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1
/2*d*x+1/2*c)^2-a-b)^3+(A*a^3+4*A*a*b^2-3*B*a^2*b-2*B*b^3)/(a^6-3*a^4*b^2+3
*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(
a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 586 vs. $2(260) = 520$.
time = 7.04, size = 1230, normalized size = 4.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(3*(A*a^3*b^3 - 3*B*a^2*b^4 + 4*A*a*b^5 - 2*B*b^6 + (A*a^6 - 3*B*a^5*
b + 4*A*a^4*b^2 - 2*B*a^3*b^3)*cos(d*x + c)^3 + 3*(A*a^5*b - 3*B*a^4*b^2 +
4*A*a^3*b^3 - 2*B*a^2*b^4)*cos(d*x + c)^2 + 3*(A*a^4*b^2 - 3*B*a^3*b^3 + 4*
A*a^2*b^4 - 2*B*a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c
) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*s
in(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))
 + 2*(2*B*a^7 + A*a^6*b - 7*B*a^5*b^2 - 11*A*a^4*b^3 + 23*B*a^3*b^4 + 4*A*a
^2*b^5 - 18*B*a*b^6 + 6*A*b^7 + (4*B*a^7 - 13*A*a^6*b + 7*B*a^5*b^2 + 11*A*
a^4*b^3 - 11*B*a^3*b^4 + 2*A*a^2*b^5)*cos(d*x + c)^2 + 3*(A*a^7 + B*a^6*b -
 10*A*a^5*b^2 + 8*B*a^4*b^3 + 7*A*a^3*b^4 - 9*B*a^2*b^5 + 2*A*a*b^6)*cos(d*
x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)
*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9
)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^1
```

```
0)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)
, 1/6*(3*(A*a^3*b^3 - 3*B*a^2*b^4 + 4*A*a*b^5 - 2*B*b^6 + (A*a^6 - 3*B*a^5*
b + 4*A*a^4*b^2 - 2*B*a^3*b^3)*cos(d*x + c)^3 + 3*(A*a^5*b - 3*B*a^4*b^2 +
4*A*a^3*b^3 - 2*B*a^2*b^4)*cos(d*x + c)^2 + 3*(A*a^4*b^2 - 3*B*a^3*b^3 + 4*
A*a^2*b^4 - 2*B*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b
^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (2*B*a^7 + A*a^6*b -
 7*B*a^5*b^2 - 11*A*a^4*b^3 + 23*B*a^3*b^4 + 4*A*a^2*b^5 - 18*B*a*b^6 + 6*A
*b^7 + (4*B*a^7 - 13*A*a^6*b + 7*B*a^5*b^2 + 11*A*a^4*b^3 - 11*B*a^3*b^4 +
2*A*a^2*b^5)*cos(d*x + c)^2 + 3*(A*a^7 + B*a^6*b - 10*A*a^5*b^2 + 8*B*a^4*b
^3 + 7*A*a^3*b^4 - 9*B*a^2*b^5 + 2*A*a*b^6)*cos(d*x + c))*sin(d*x + c))/((a
^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^
10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a
^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*
b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c+dx))\sec^3(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x))**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 693 vs. 2(260) = 520.
time = 0.54, size = 693, normalized size = 2.53

$$\frac{\frac{3\,(Aa^3-3\,Ba^2b+4\,Aab^2-2\,Bb^3)\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(-2\,a+2\,b)+\arctan\left(-\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-3\,a^4b^2+3\,a^2b^4-b^6)\sqrt{-a^2+b^2}}+\cdots}{3\,d}$$

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(A*a^3 - 3*B*a^2*b + 4*A*a*b^2 - 2*B*b^3)*(pi*floor(1/2*(d*x + c)/pi
 + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
 1/2*c))/sqrt(-a^2 + b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2
+ b^2)) + (3*A*a^5*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^5*tan(1/2*d*x + 1/2*c)^5
+ 12*A*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 3*B*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 27
*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 12
*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 27*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 6
*A*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 18*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*A*b
^5*tan(1/2*d*x + 1/2*c)^5 + 4*B*a^5*tan(1/2*d*x + 1/2*c)^3 - 28*A*a^4*b*tan
(1/2*d*x + 1/2*c)^3 + 32*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 + 16*A*a^2*b^3*ta
```

```
n(1/2*d*x + 1/2*c)^3 - 36*B*a*b^4*tan(1/2*d*x + 1/2*c)^3 + 12*A*b^5*tan(1/2
*d*x + 1/2*c)^3 - 3*A*a^5*tan(1/2*d*x + 1/2*c) - 6*B*a^5*tan(1/2*d*x + 1/2*
c) + 12*A*a^4*b*tan(1/2*d*x + 1/2*c) - 3*B*a^4*b*tan(1/2*d*x + 1/2*c) + 27*
A*a^3*b^2*tan(1/2*d*x + 1/2*c) - 6*B*a^3*b^2*tan(1/2*d*x + 1/2*c) + 12*A*a^
2*b^3*tan(1/2*d*x + 1/2*c) - 27*B*a^2*b^3*tan(1/2*d*x + 1/2*c) + 6*A*a*b^4*
tan(1/2*d*x + 1/2*c) - 18*B*a*b^4*tan(1/2*d*x + 1/2*c) + 6*A*b^5*tan(1/2*d*
x + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2
- b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

Mupad [B]
time = 6.80, size = 439, normalized size = 1.60

$$\frac{\frac{4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^3\,\left(-B\,a^3+7\,A\,a^2\,b-9\,B\,a\,b^2+3\,A\,b^3\right)}{3\,{\left(a+b\right)}^2\,\left(a^2-2\,a\,b+b^2\right)}-\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^5\,\left(A\,a^3+2\,A\,b^3-2\,B\,a^3+2\,A\,a\,b^2+6\,A\,a^2\,b-6\,B\,a\,b^2-3\,B\,a^2\,b\right)}{{\left(a+b\right)}^3\,\left(a-b\right)}+\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(A\,a^3-2\,A\,b^3+2\,B\,a^3+2\,A\,a\,b^2-6\,A\,a^2\,b+6\,B\,a\,b^2-3\,B\,a^2\,b\right)}{\left(a+b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)+3\,a\,b^2+3\,a^2\,b+a^3+b^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)\right)}+\frac{\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a-2\,b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,\sqrt{a+b}\,{\left(a-b\right)}^{7/2}}\right)\,\left(A\,a^3-3\,B\,a^2\,b+4\,A\,a\,b^2-2\,B\,b^3\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + b/cos(c + d*x))^4),x)
```

```
[Out] ((4*tan(c/2 + (d*x)/2)^3*(3*A*b^3 - B*a^3 + 7*A*a^2*b - 9*B*a*b^2))/(3*(a +
 b)^2*(a^2 - 2*a*b + b^2)) - (tan(c/2 + (d*x)/2)^5*(A*a^3 + 2*A*b^3 - 2*B*a
^3 + 2*A*a*b^2 + 6*A*a^2*b - 6*B*a*b^2 - 3*B*a^2*b))/((a + b)^3*(a - b)) +
(tan(c/2 + (d*x)/2)*(A*a^3 - 2*A*b^3 + 2*B*a^3 + 2*A*a*b^2 - 6*A*a^2*b + 6*
B*a*b^2 - 3*B*a^2*b))/((a + b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(tan(c/
2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) - tan(c/2 + (d*x)/2)^4*(
3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/
2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) + (atanh((tan(c/2 + (d*x)/
2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(a + b)^(1/2)*(a - b)^(7
/2)))*(A*a^3 - 2*B*b^3 + 4*A*a*b^2 - 3*B*a^2*b))/(d*(a + b)^(7/2)*(a - b)^(
7/2))
```

3.339 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4} \, dx$

Optimal. Leaf size=263

$$-\frac{(4a^2Ab+Ab^3-a^3B-4ab^2B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}+\frac{a(Ab-aB)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^3}+\frac{(2a^2Ab}{6b}$$

```
[Out] -(4*A*a^2*b+A*b^3-B*a^3-4*B*a*b^2)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(
a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d+1/3*a*(A*b-B*a)*tan(d*x+c)/b/(a^2-b^2
)/d/(a+b*sec(d*x+c))^3+1/6*(2*A*a^2*b+3*A*b^3+B*a^3-6*B*a*b^2)*tan(d*x+c)/b
/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/6*(2*A*a^3*b+13*A*a*b^3+B*a^4-10*B*a^2*
b^2-6*B*b^4)*tan(d*x+c)/b/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.42, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4094, 4088, 12, 3916, 2738, 214}

$$\frac{a(Ab-aB)\tan(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^3}-\frac{\left(a^3(-B)+4a^2Ab-4ab^2B+Ab^3\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}+\frac{\left(a^3B+2a^2Ab-6ab^2B+3Ab^3\right)\tan(c+dx)}{6bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}+\frac{\left(a^4B+2a^3Ab-10a^2b^2B+13aAb^3-6b^4B\right)\tan(c+dx)}{6bd\left(a^2-b^2\right)^3(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] -(((4*a^2*A*b + A*b^3 - a^3*B - 4*a*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d)) + (a*(A*b - a*B)*Tan[
c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + ((2*a^2*A*b + 3*A*b^
3 + a^3*B - 6*a*b^2*B)*Tan[c + d*x])/(6*b*(a^2 - b^2)^2*d*(a + b*Sec[c + d*
x])^2) + ((2*a^3*A*b + 13*a*A*b^3 + a^4*B - 10*a^2*b^2*B - 6*b^4*B)*Tan[c +
 d*x])/(6*b*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rule 4094

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*(A*b - a*B)*Cot[
e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dis
t[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
Simp[b*(A*b - a*B)*(m + 1) - (a*A*b*(m + 2) - B*(a^2 + b^2*(m + 1)))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx = \frac{a(Ab-aB)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{\int \frac{\sec(c+dx)(-3b(Ab-aB)+(2aAb+a^2B}{(a+b\sec(c+dx))^3}}{3b\,(a^2-b^2)}$$

$$= \frac{a(Ab-aB)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{(2a^2Ab+3Ab^3+a^3B-6ab^2B}{6b\,(a^2-b^2)^2\,d(a+b\sec(c}$$

$$= \frac{a(Ab-aB)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{(2a^2Ab+3Ab^3+a^3B-6ab^2B}{6b\,(a^2-b^2)^2\,d(a+b\sec(c}$$

$$= \frac{a(Ab-aB)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{(2a^2Ab+3Ab^3+a^3B-6ab^2B}{6b\,(a^2-b^2)^2\,d(a+b\sec(c}$$

$$= \frac{a(Ab-aB)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{(2a^2Ab+3Ab^3+a^3B-6ab^2B}{6b\,(a^2-b^2)^2\,d(a+b\sec(c}$$

$$= \frac{a(Ab-aB)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{(2a^2Ab+3Ab^3+a^3B-6ab^2B}{6b\,(a^2-b^2)^2\,d(a+b\sec(c}$$

$$= -\frac{(4a^2Ab+Ab^3-a^3B-4ab^2B)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} +$$

Mathematica [A]
time = 1.27, size = 252, normalized size = 0.96

$$\frac{\frac{24(-4a^2Ab-Ab^3+a^3B+4ab^2B)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)}{\sqrt{a^2-b^2}} + \frac{2(-6a^5A-14a^3Ab^2-25aAb^4+11a^4bB+22a^2b^3B+12b^5B-6(2a^4Ab+9a^2Ab^3-Ab^5+a^5B-9a^3b^2B-2ab^4B)\cos(c+dx)+a(-6a^4A-10a^2Ab^2+Ab^4+13a^3bB+2ab^3B)\cos(2(c+dx)))\sin(c+dx)}{(b+a\cos(c+dx))^3}}{24\,(-a^2+b^2)^3\,d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((24*(-4*a^2*A*b - A*b^3 + a^3*B + 4*a*b^2*B)*ArcTanh[((-a + b)*Tan[(c + d*
x)/2])/Sqrt[a^2 - b^2]])/Sqrt[a^2 - b^2] + (2*(-6*a^5*A - 14*a^3*A*b^2 - 25
*a*A*b^4 + 11*a^4*b*B + 22*a^2*b^3*B + 12*b^5*B - 6*(2*a^4*A*b + 9*a^2*A*b^
3 - A*b^5 + a^5*B - 9*a^3*b^2*B - 2*a*b^4*B)*Cos[c + d*x] + a*(-6*a^4*A - 1
0*a^2*A*b^2 + A*b^4 + 13*a^3*b*B + 2*a*b^3*B)*Cos[2*(c + d*x)])*Sin[c + d*x
])/(b + a*Cos[c + d*x])^3)/(24*(-a^2 + b^2)^3*d)
```

Maple [A]
time = 0.68, size = 388, normalized size = 1.48

method	result

derivativedivides	$\frac{-\frac{\left(2Aa^3+2Aa^2b+6Ab^2a+Ab^3-a^3B-6Bba^2-2Bab^2-2Bb^3\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)\left(a^3+3ba^2+3b^2a+b^3\right)}+\frac{4\left(3Aa^3+7Ab^2a-7Bba^2-3Bb^3\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^2+2ba+b^2\right)\left(a^2-2ba+b^2\right)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^3}$
default	$\frac{-\frac{\left(2Aa^3+2Aa^2b+6Ab^2a+Ab^3-a^3B-6Bba^2-2Bab^2-2Bb^3\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{(a-b)\left(a^3+3ba^2+3b^2a+b^3\right)}+\frac{4\left(3Aa^3+7Ab^2a-7Bba^2-3Bb^3\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^2+2ba+b^2\right)\left(a^2-2ba+b^2\right)}}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^3}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(2*(-1/2*(2*A*a^3+2*A*a^2*b+6*A*a*b^2+A*b^3-B*a^3-6*B*a^2*b-2*B*a*b^2-2
*B*b^3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(3*A*a^3+7
*A*a*b^2-7*B*a^2*b-3*B*b^3)/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2
*c)^3-1/2*(2*A*a^3-2*A*a^2*b+6*A*a*b^2-A*b^3+B*a^3-6*B*a^2*b+2*B*a*b^2-2*B*
b^3)/(a+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2
*c)^2-b*tan(1/2*d*x+1/2*c)^2-a-b)^3-(4*A*a^2*b+A*b^3-B*a^3-4*B*a*b^2)/(a^6-
3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*
c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="ma
xima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 592 vs. $2(247) = 494$.
time = 3.39, size = 1242, normalized size = 4.72

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(3*(B*a^3*b^3 - 4*A*a^2*b^4 + 4*B*a*b^5 - A*b^6 + (B*a^6 - 4*A*a^5*b
+ 4*B*a^4*b^2 - A*a^3*b^3)*cos(d*x + c)^3 + 3*(B*a^5*b - 4*A*a^4*b^2 + 4*B*
a^3*b^3 - A*a^2*b^4)*cos(d*x + c)^2 + 3*(B*a^4*b^2 - 4*A*a^3*b^3 + 4*B*a^2*
b^4 - A*a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2
 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x +
 c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(B*
a^6*b + 2*A*a^5*b^2 - 11*B*a^4*b^3 + 11*A*a^3*b^4 + 4*B*a^2*b^5 - 13*A*a*b^
6 + 6*B*b^7 + (6*A*a^7 - 13*B*a^6*b + 4*A*a^5*b^2 + 11*B*a^4*b^3 - 11*A*a^3
*b^4 + 2*B*a^2*b^5 + A*a*b^6)*cos(d*x + c)^2 + 3*(B*a^7 + 2*A*a^6*b - 10*B*
a^5*b^2 + 7*A*a^4*b^3 + 7*B*a^3*b^4 - 10*A*a^2*b^5 + 2*B*a*b^6 + A*b^7)*cos
(d*x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b
^8)*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*
b^9)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*
b^10)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)
*d), 1/6*(3*(B*a^3*b^3 - 4*A*a^2*b^4 + 4*B*a*b^5 - A*b^6 + (B*a^6 - 4*A*a^5
*b + 4*B*a^4*b^2 - A*a^3*b^3)*cos(d*x + c)^3 + 3*(B*a^5*b - 4*A*a^4*b^2 + 4
*B*a^3*b^3 - A*a^2*b^4)*cos(d*x + c)^2 + 3*(B*a^4*b^2 - 4*A*a^3*b^3 + 4*B*a
^2*b^4 - A*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(
b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (B*a^6*b + 2*A*a^5*b^2 -
11*B*a^4*b^3 + 11*A*a^3*b^4 + 4*B*a^2*b^5 - 13*A*a*b^6 + 6*B*b^7 + (6*A*a^7
 - 13*B*a^6*b + 4*A*a^5*b^2 + 11*B*a^4*b^3 - 11*A*a^3*b^4 + 2*B*a^2*b^5 + A
*a*b^6)*cos(d*x + c)^2 + 3*(B*a^7 + 2*A*a^6*b - 10*B*a^5*b^2 + 7*A*a^4*b^3
+ 7*B*a^3*b^4 - 10*A*a^2*b^5 + 2*B*a*b^6 + A*b^7)*cos(d*x + c))*sin(d*x + c
))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 +
 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2
+ 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) +
 (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec^2(c + dx)}{(a + b\sec(c + dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x))**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 726 vs. 2(247) = 494.
time = 0.59, size = 726, normalized size = 2.76

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="gi
ac")
```

```
[Out] 1/3*(3*(B*a^3 - 4*A*a^2*b + 4*B*a*b^2 - A*b^3)*(pi*floor(1/2*(d*x + c)/pi +
 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1
/2*c))/sqrt(-a^2 + b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 +
b^2)) - (6*A*a^5*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^5*tan(1/2*d*x + 1/2*c)^5 -
6*A*a^4*b*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 12*A
*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 + 27*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*
A*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 12
*A*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 6*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 + 3*A*b^
5*tan(1/2*d*x + 1/2*c)^5 - 6*B*b^5*tan(1/2*d*x + 1/2*c)^5 - 12*A*a^5*tan(1/
2*d*x + 1/2*c)^3 + 28*B*a^4*b*tan(1/2*d*x + 1/2*c)^3 - 16*A*a^3*b^2*tan(1/2
*d*x + 1/2*c)^3 - 16*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^3 + 28*A*a*b^4*tan(1/2*
d*x + 1/2*c)^3 - 12*B*b^5*tan(1/2*d*x + 1/2*c)^3 + 6*A*a^5*tan(1/2*d*x + 1/
2*c) + 3*B*a^5*tan(1/2*d*x + 1/2*c) + 6*A*a^4*b*tan(1/2*d*x + 1/2*c) - 12*B
*a^4*b*tan(1/2*d*x + 1/2*c) + 12*A*a^3*b^2*tan(1/2*d*x + 1/2*c) - 27*B*a^3*
b^2*tan(1/2*d*x + 1/2*c) + 27*A*a^2*b^3*tan(1/2*d*x + 1/2*c) - 12*B*a^2*b^3
*tan(1/2*d*x + 1/2*c) + 12*A*a*b^4*tan(1/2*d*x + 1/2*c) - 6*B*a*b^4*tan(1/2
*d*x + 1/2*c) - 3*A*b^5*tan(1/2*d*x + 1/2*c) - 6*B*b^5*tan(1/2*d*x + 1/2*c)
)/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/
2*d*x + 1/2*c)^2 - a - b)^3))/d
```

Mupad [B]
time = 6.65, size = 451, normalized size = 1.71

$$\frac{\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,A\,a^3-A\,b^3+B\,a^3-2\,B\,b^3+6\,A\,a\,b^2-2\,A\,a^2\,b+2\,B\,a\,b^2-6\,B\,a^2\,b\right)}{\left(a+b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}-\frac{4\,{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^3\,\left(3\,A\,a^3-7\,B\,a^2\,b+7\,A\,a\,b^2-3\,B\,b^3\right)}{3\,{\left(a+b\right)}^2\,\left(a^2-2\,a\,b+b^2\right)}+\frac{{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^5\,\left(2\,A\,a^3+A\,b^3-B\,a^3-2\,B\,b^3+6\,A\,a\,b^2+2\,A\,a^2\,b-2\,B\,a\,b^2-6\,B\,a^2\,b\right)}{{\left(a+b\right)}^3\,\left(a-b\right)}}{d\,\left({\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)+3\,a\,b^2+3\,a^2\,b+a^3+b^3-{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}^6\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)\right)}-\frac{\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\left(2\,a-2\,b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,\sqrt{a+b}\,{\left(a-b\right)}^{7/2}}\right)\,\left(-B\,a^3+4\,A\,a^2\,b-4\,B\,a\,b^2+A\,b^3\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + b/cos(c + d*x))^4),x)
```

```
[Out] ((tan(c/2 + (d*x)/2)*(2*A*a^3 - A*b^3 + B*a^3 - 2*B*b^3 + 6*A*a*b^2 - 2*A*a
^2*b + 2*B*a*b^2 - 6*B*a^2*b))/((a + b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)) -
(4*tan(c/2 + (d*x)/2)^3*(3*A*a^3 - 3*B*b^3 + 7*A*a*b^2 - 7*B*a^2*b))/(3*(a
+ b)^2*(a^2 - 2*a*b + b^2)) + (tan(c/2 + (d*x)/2)^5*(2*A*a^3 + A*b^3 - B*a^
3 - 2*B*b^3 + 6*A*a*b^2 + 2*A*a^2*b - 2*B*a*b^2 - 6*B*a^2*b))/((a + b)^3*(a
 - b)))/(d*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) - tan(
c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) + 3*a*b^2 + 3*a^2*b +
a^3 + b^3 - tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (atanh
((tan(c/2 + (d*x)/2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(a + b
)^(1/2)*(a - b)^(7/2)))*(A*b^3 - B*a^3 + 4*A*a^2*b - 4*B*a*b^2))/(d*(a + b)
^(7/2)*(a - b)^(7/2))
```

3.340 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=237

$$\frac{(2a^3A+3aAb^2-4a^2bB-b^3B)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d}-\frac{(Ab-aB)\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^3}-\frac{(5aAb-\cdots}{6\left(a^2-\cdots\right.}$$

```
[Out] (2*A*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/
(a+b)^(1/2))/(a-b)^(7/2)/(a+b)^(7/2)/d-1/3*(A*b-B*a)*tan(d*x+c)/(a^2-b^2)/d
/(a+b*sec(d*x+c))^3-1/6*(5*A*a*b-2*B*a^2-3*B*b^2)*tan(d*x+c)/(a^2-b^2)^2/d/
(a+b*sec(d*x+c))^2-1/6*(11*A*a^2*b+4*A*b^3-2*B*a^3-13*B*a*b^2)*tan(d*x+c)/(
a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 0.35, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.172$, Rules used = {4088, 12, 3916, 2738, 214}

$$-\frac{\left(-2a^2B+5aAb-3b^2B\right)\tan(c+dx)}{6d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}-\frac{(Ab-aB)\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{\left(2a^3A-4a^2bB+3aAb^2-b^3B\right)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{d(a-b)^{7/2}(a+b)^{7/2}}-\frac{\left(-2a^3B+11a^2Ab-13ab^2B+4Ab^3\right)\tan(c+dx)}{6d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((2*a^3*A + 3*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/((a - b)^(7/2)*(a + b)^(7/2)*d) - ((A*b - a*B)*Tan[c +
 d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) - ((5*a*A*b - 2*a^2*B - 3*b
^2*B)*Tan[c + d*x])/(6*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) - ((11*a^2*A
*b + 4*A*b^3 - 2*a^3*B - 13*a*b^2*B)*Tan[c + d*x])/(6*(a^2 - b^2)^3*d*(a +
b*Sec[c + d*x]))
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx = -\frac{(Ab-aB)\tan(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{\int \frac{\sec(c+dx)(-3(aA-bB)+2(Ab-aB)\,\mathrm{s}}{(a+b\sec(c+dx))^3}}{3(a^2-b^2)}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{(5aAb-2a^2B-3b^2B)\tan(c}{6(a^2-b^2)^2\,d(a+b\sec(c+}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{(5aAb-2a^2B-3b^2B)\tan(c}{6(a^2-b^2)^2\,d(a+b\sec(c+}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{(5aAb-2a^2B-3b^2B)\tan(c}{6(a^2-b^2)^2\,d(a+b\sec(c+}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{(5aAb-2a^2B-3b^2B)\tan(c}{6(a^2-b^2)^2\,d(a+b\sec(c+}$$

$$= -\frac{(Ab-aB)\tan(c+dx)}{3(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{(5aAb-2a^2B-3b^2B)\tan(c}{6(a^2-b^2)^2\,d(a+b\sec(c+}$$

$$= \frac{(2a^3A+3aAb^2-4a^2bB-b^3B)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{(a-b)^{7/2}(a+b)^{7/2}d} -$$

Mathematica [A]

time = 1.12, size = 404, normalized size = 1.70

$$\frac{(b+a\cos(c+dx))\sec^3(c+dx)(A+B\sec(c+dx))\left(\frac{24(2a^3A+3aAb^2-4a^2bB-b^3B)\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)(b+a\cos(c+dx))^3}{\sqrt{a^2-b^2}}+18a^4Ab\sin(c+dx)+39a^2Ab^3\sin(c+dx)+18Ab^5\sin(c+dx)-6a^5B\sin(c+dx)-18a^3b^2B\sin(c+dx)-51ab^4B\sin(c+dx)+54a^3Ab^2\sin(2(c+dx))+6aAb^4\sin(2(c+dx))-12a^4bB\sin(2(c+dx))-54a^2b^3B\sin(2(c+dx))+6b^5B\sin(2(c+dx))+18a^4Ab\sin(3(c+dx))-5a^2Ab^3\sin(3(c+dx))+2Ab^5\sin(3(c+dx))-6a^5B\sin(3(c+dx))-10a^3b^2B\sin(3(c+dx))+ab^4B\sin(3(c+dx))\right)}{24(-a^2+b^2)^3d(B+A\cos(c+dx))(a+b\sec(c+dx))^4}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(A + B*Sec[c + d*x])*((24*(2*a^3*A + 3
*a*A*b^2 - 4*a^2*b*B - b^3*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2
- b^2]]*(b + a*Cos[c + d*x])^3)/Sqrt[a^2 - b^2] + 18*a^4*A*b*Sin[c + d*x] +
 39*a^2*A*b^3*Sin[c + d*x] + 18*A*b^5*Sin[c + d*x] - 6*a^5*B*Sin[c + d*x] -
 18*a^3*b^2*B*Sin[c + d*x] - 51*a*b^4*B*Sin[c + d*x] + 54*a^3*A*b^2*Sin[2*(
c + d*x)] + 6*a*A*b^4*Sin[2*(c + d*x)] - 12*a^4*b*B*Sin[2*(c + d*x)] - 54*a
^2*b^3*B*Sin[2*(c + d*x)] + 6*b^5*B*Sin[2*(c + d*x)] + 18*a^4*A*b*Sin[3*(c
+ d*x)] - 5*a^2*A*b^3*Sin[3*(c + d*x)] + 2*A*b^5*Sin[3*(c + d*x)] - 6*a^5*B
*Sin[3*(c + d*x)] - 10*a^3*b^2*B*Sin[3*(c + d*x)] + a*b^4*B*Sin[3*(c + d*x)
]))/(24*(-a^2 + b^2)^3*d*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^4)
```

Maple [A]
time = 0.60, size = 376, normalized size = 1.59

method	result
derivativedivides	$-\frac{2\left(-\frac{\left(6A\,a^2b+3A\,b^2a+2A\,b^3-2a^3B-2Bb\,a^2-6Ba\,b^2-B\,b^3\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^3+3b\,a^2+3b^2a+b^3\right)}+\frac{2\left(9A\,a^2b+A\,b^3-3a^3B-7Ba\,b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^2+2ba+b^2\right)\left(a^2-2ba+b^2\right)}-\right.}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^3}$
default	$-\frac{2\left(-\frac{\left(6A\,a^2b+3A\,b^2a+2A\,b^3-2a^3B-2Bb\,a^2-6Ba\,b^2-B\,b^3\right)\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^3+3b\,a^2+3b^2a+b^3\right)}+\frac{2\left(9A\,a^2b+A\,b^3-3a^3B-7Ba\,b^2\right)\left(\tan^3\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{3\left(a^2+2ba+b^2\right)\left(a^2-2ba+b^2\right)}-\right.}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-a-b\right)^3}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2*(-1/2*(6*A*a^2*b+3*A*a*b^2+2*A*b^3-2*B*a^3-2*B*a^2*b-6*B*a*b^2-B*b^
3)/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(9*A*a^2*b+A*b^
3-3*B*a^3-7*B*a*b^2)/(a^2+2*a*b+b^2)/(a^2-2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1
/2*(6*A*a^2*b-3*A*a*b^2+2*A*b^3-2*B*a^3+2*B*a^2*b-6*B*a*b^2+B*b^3)/(a+b)/(a
^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1
/2*d*x+1/2*c)^2-a-b)^3+(2*A*a^3+3*A*a*b^2-4*B*a^2*b-B*b^3)/(a^6-3*a^4*b^2+3
*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(
a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 590 vs. 2(222) = 444.
time = 5.37, size = 1238, normalized size = 5.22

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fric
as")
```

```
[Out] [1/12*(3*(2*A*a^3*b^3 - 4*B*a^2*b^4 + 3*A*a*b^5 - B*b^6 + (2*A*a^6 - 4*B*a^
5*b + 3*A*a^4*b^2 - B*a^3*b^3)*cos(d*x + c)^3 + 3*(2*A*a^5*b - 4*B*a^4*b^2
+ 3*A*a^3*b^3 - B*a^2*b^4)*cos(d*x + c)^2 + 3*(2*A*a^4*b^2 - 4*B*a^3*b^3 +
3*A*a^2*b^4 - B*a*b^5)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c
) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*s
in(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2))
 + 2*(2*B*a^5*b^2 - 11*A*a^4*b^3 + 11*B*a^3*b^4 + 7*A*a^2*b^5 - 13*B*a*b^6
+ 4*A*b^7 + (6*B*a^7 - 18*A*a^6*b + 4*B*a^5*b^2 + 23*A*a^4*b^3 - 11*B*a^3*b
^4 - 7*A*a^2*b^5 + B*a*b^6 + 2*A*b^7)*cos(d*x + c)^2 + 3*(2*B*a^6*b - 9*A*a
^5*b^2 + 7*B*a^4*b^3 + 8*A*a^3*b^4 - 10*B*a^2*b^5 + A*a*b^6 + B*b^7)*cos(d*
x + c))*sin(d*x + c))/((a^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)
*d*cos(d*x + c)^3 + 3*(a^10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9
)*d*cos(d*x + c)^2 + 3*(a^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^1
0)*d*cos(d*x + c) + (a^8*b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)
, 1/6*(3*(2*A*a^3*b^3 - 4*B*a^2*b^4 + 3*A*a*b^5 - B*b^6 + (2*A*a^6 - 4*B*a^
5*b + 3*A*a^4*b^2 - B*a^3*b^3)*cos(d*x + c)^3 + 3*(2*A*a^5*b - 4*B*a^4*b^2
+ 3*A*a^3*b^3 - B*a^2*b^4)*cos(d*x + c)^2 + 3*(2*A*a^4*b^2 - 4*B*a^3*b^3 +
3*A*a^2*b^4 - B*a*b^5)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b
^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (2*B*a^5*b^2 - 11*A*
a^4*b^3 + 11*B*a^3*b^4 + 7*A*a^2*b^5 - 13*B*a*b^6 + 4*A*b^7 + (6*B*a^7 - 18
*A*a^6*b + 4*B*a^5*b^2 + 23*A*a^4*b^3 - 11*B*a^3*b^4 - 7*A*a^2*b^5 + B*a*b^
```

```
6 + 2*A*b^7)*cos(d*x + c)^2 + 3*(2*B*a^6*b - 9*A*a^5*b^2 + 7*B*a^4*b^3 + 8*
A*a^3*b^4 - 10*B*a^2*b^5 + A*a*b^6 + B*b^7)*cos(d*x + c))*sin(d*x + c))/((a
^11 - 4*a^9*b^2 + 6*a^7*b^4 - 4*a^5*b^6 + a^3*b^8)*d*cos(d*x + c)^3 + 3*(a^
10*b - 4*a^8*b^3 + 6*a^6*b^5 - 4*a^4*b^7 + a^2*b^9)*d*cos(d*x + c)^2 + 3*(a
^9*b^2 - 4*a^7*b^4 + 6*a^5*b^6 - 4*a^3*b^8 + a*b^10)*d*cos(d*x + c) + (a^8*
b^3 - 4*a^6*b^5 + 6*a^4*b^7 - 4*a^2*b^9 + b^11)*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec(c + dx)}{(a + b\sec(c + dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x))**4, x)
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 693 vs. 2(222) = 444.
time = 0.55, size = 693, normalized size = 2.92

$$-\frac{\frac{3\,(2Aa^3-4Ba^2b+3Aab^2-Bb^3)\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor\operatorname{sgn}(2a-2b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-a^2+b^2}}\right)\right)}{(a^6-3a^4b^2+3a^2b^4-b^6)\sqrt{-a^2+b^2}}+\frac{6Ba^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-18Aa^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6Ba^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+27Aa^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+12Ba^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6Aa^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-27Ba^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3Aab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+12Bab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-6Ab^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5+3Bb^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^5-12Ba^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+36Aa^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-16Ba^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-32Aa^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+28Bab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3-4Ab^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^3+6Ba^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-18Aa^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+6Ba^4b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-27Aa^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+12Ba^3b^2\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-6Aa^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+27Ba^2b^3\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3Aab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)+12Bab^4\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-6Ab^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)-3Bb^5\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{(a^6-3a^4b^2+3a^2b^4-b^6)\left(a\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-b\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-a-b\right)^3}}{3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="giac
")
```

```
[Out] -1/3*(3*(2*A*a^3 - 4*B*a^2*b + 3*A*a*b^2 - B*b^3)*(pi*floor(1/2*(d*x + c)/p
i + 1/2)*sgn(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x +
1/2*c))/sqrt(-a^2 + b^2)))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*sqrt(-a^2 +
 b^2)) + (6*B*a^5*tan(1/2*d*x + 1/2*c)^5 - 18*A*a^4*b*tan(1/2*d*x + 1/2*c)^
5 - 6*B*a^4*b*tan(1/2*d*x + 1/2*c)^5 + 27*A*a^3*b^2*tan(1/2*d*x + 1/2*c)^5
+ 12*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^5 - 6*A*a^2*b^3*tan(1/2*d*x + 1/2*c)^5
- 27*B*a^2*b^3*tan(1/2*d*x + 1/2*c)^5 + 3*A*a*b^4*tan(1/2*d*x + 1/2*c)^5 +
12*B*a*b^4*tan(1/2*d*x + 1/2*c)^5 - 6*A*b^5*tan(1/2*d*x + 1/2*c)^5 + 3*B*b^
5*tan(1/2*d*x + 1/2*c)^5 - 12*B*a^5*tan(1/2*d*x + 1/2*c)^3 + 36*A*a^4*b*tan
(1/2*d*x + 1/2*c)^3 - 16*B*a^3*b^2*tan(1/2*d*x + 1/2*c)^3 - 32*A*a^2*b^3*ta
n(1/2*d*x + 1/2*c)^3 + 28*B*a*b^4*tan(1/2*d*x + 1/2*c)^3 - 4*A*b^5*tan(1/2*
d*x + 1/2*c)^3 + 6*B*a^5*tan(1/2*d*x + 1/2*c) - 18*A*a^4*b*tan(1/2*d*x + 1/
2*c) + 6*B*a^4*b*tan(1/2*d*x + 1/2*c) - 27*A*a^3*b^2*tan(1/2*d*x + 1/2*c) +
 12*B*a^3*b^2*tan(1/2*d*x + 1/2*c) - 6*A*a^2*b^3*tan(1/2*d*x + 1/2*c) + 27*
B*a^2*b^3*tan(1/2*d*x + 1/2*c) - 3*A*a*b^4*tan(1/2*d*x + 1/2*c) + 12*B*a*b^
4*tan(1/2*d*x + 1/2*c) - 6*A*b^5*tan(1/2*d*x + 1/2*c) - 3*B*b^5*tan(1/2*d*x
 + 1/2*c))/((a^6 - 3*a^4*b^2 + 3*a^2*b^4 - b^6)*(a*tan(1/2*d*x + 1/2*c)^2 -
 b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3))/d
```

Mupad [B]
time = 6.67, size = 439, normalized size = 1.85

$$\frac{\frac{4\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^3\,\left(-3\,B\,a^3+9\,A\,a^2\,b-7\,B\,a\,b^2+A\,b^3\right)}{3\,{\left(a+b\right)}^2\,\left(a^2-2\,a\,b+b^2\right)}+\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^5\,\left(2\,B\,a^3-2\,A\,b^3+B\,b^3-3\,A\,a\,b^2-6\,A\,a^2\,b+6\,B\,a\,b^2+2\,B\,a^2\,b\right)}{{\left(a+b\right)}^3\,\left(a-b\right)}-\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(2\,A\,b^3-2\,B\,a^3+B\,b^3-3\,A\,a\,b^2+6\,A\,a^2\,b-6\,B\,a\,b^2+2\,B\,a^2\,b\right)}{\left(a+b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}}{d\,\left(\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^2\,\left(-3\,a^3-3\,a^2\,b+3\,a\,b^2+3\,b^3\right)-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^4\,\left(-3\,a^3+3\,a^2\,b+3\,a\,b^2-3\,b^3\right)+3\,a\,b^2+3\,a^2\,b+a^3+b^3-\tan\left(\frac{c}{2}+\frac{dx}{2}\right)^6\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)\right)}+\frac{\mathrm{atanh}\left(\frac{\tan\left(\frac{c}{2}+\frac{dx}{2}\right)\,\left(2\,a-2\,b\right)\,\left(a^3-3\,a^2\,b+3\,a\,b^2-b^3\right)}{2\,\sqrt{a+b}\,{\left(a-b\right)}^{7/2}}\right)\,\left(2\,A\,a^3-4\,B\,a^2\,b+3\,A\,a\,b^2-B\,b^3\right)}{d\,{\left(a+b\right)}^{7/2}\,{\left(a-b\right)}^{7/2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + b/cos(c + d*x))^4),x)
```

```
[Out] ((4*tan(c/2 + (d*x)/2)^3*(A*b^3 - 3*B*a^3 + 9*A*a^2*b - 7*B*a*b^2))/(3*(a +
 b)^2*(a^2 - 2*a*b + b^2)) + (tan(c/2 + (d*x)/2)^5*(2*B*a^3 - 2*A*b^3 + B*b
^3 - 3*A*a*b^2 - 6*A*a^2*b + 6*B*a*b^2 + 2*B*a^2*b))/((a + b)^3*(a - b)) -
(tan(c/2 + (d*x)/2)*(2*A*b^3 - 2*B*a^3 + B*b^3 - 3*A*a*b^2 + 6*A*a^2*b - 6*
B*a*b^2 + 2*B*a^2*b))/((a + b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3)))/(d*(tan(c/
2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) - tan(c/2 + (d*x)/2)^4*(
3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/
2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) + (atanh((tan(c/2 + (d*x)/
2)*(2*a - 2*b)*(3*a*b^2 - 3*a^2*b + a^3 - b^3))/(2*(a + b)^(1/2)*(a - b)^(7
/2)))*(2*A*a^3 - B*b^3 + 3*A*a*b^2 - 4*B*a^2*b))/(d*(a + b)^(7/2)*(a - b)^(
7/2))
```

3.341 $\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=292

$$\frac{Ax}{a^4}-\frac{(8a^6Ab-8a^4Ab^3+7a^2Ab^5-2Ab^7-2a^7B-3a^5b^2B)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4(a-b)^{7/2}(a+b)^{7/2}d}+\frac{b(Ab-aB)}{3a\left(a^2-b^2\right)d(a}$$

```
[Out] A*x/a^4-(8*A*a^6*b-8*A*a^4*b^3+7*A*a^2*b^5-2*A*b^7-2*B*a^7-3*B*a^5*b^2)*arc
tanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^4/(a-b)^(7/2)/(a+b)^(7/2
)/d+1/3*b*(A*b-B*a)*tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*b*(8*A*
a^2*b-3*A*b^3-5*B*a^3)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/6*
b*(26*A*a^4*b-17*A*a^2*b^3+6*A*b^5-11*B*a^5-4*B*a^3*b^2)*tan(d*x+c)/a^3/(a^
2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A]

time = 0.74, antiderivative size = 292, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.261$, Rules used = {4008, 4145, 4004, 3916, 2738, 214}

$$\frac{Ax}{a^4}+\frac{b(Ab-aB)\tan(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{b(-5a^3B+8a^2Ab-3Ab^3)\tan(c+dx)}{6a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}+\frac{b(-11a^5B+26a^4Ab-4a^3b^2B-17a^2Ab^3+6Ab^5)\tan(c+dx)}{6a^3d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}-\frac{(-2a^7B+8a^6Ab-3a^5b^2B-8a^4Ab^3+7a^2Ab^5-2Ab^7)\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^4d(a-b)^{7/2}(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] (A*x)/a^4 - ((8*a^6*A*b - 8*a^4*A*b^3 + 7*a^2*A*b^5 - 2*A*b^7 - 2*a^7*B - 3
*a^5*b^2*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^4*(a -
b)^(7/2)*(a + b)^(7/2)*d) + (b*(A*b - a*B)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d
*(a + b*Sec[c + d*x])^3) + (b*(8*a^2*A*b - 3*A*b^3 - 5*a^3*B)*Tan[c + d*x])
/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) + (b*(26*a^4*A*b - 17*a^2*A
*b^3 + 6*A*b^5 - 11*a^5*B - 4*a^3*b^2*B)*Tan[c + d*x])/(6*a^3*(a^2 - b^2)^3
*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4008

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^4}\,dx = \frac{b(Ab-aB)\tan(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} - \frac{\int \frac{-3A(a^2-b^2)+3a(Ab-aB)\sec(c+dx)-2b(Ab-aB)\sec^2(}{(a+b\sec(c+dx))^3}}{3a\,(a^2-b^2)}$$

$$= \frac{b(Ab-aB)\tan(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{b(8a^2Ab-3Ab^3-5a^3B)\tan(c+dx)}{6a^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + \int$$

$$= \frac{b(Ab-aB)\tan(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{b(8a^2Ab-3Ab^3-5a^3B)\tan(c+dx)}{6a^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2} + b($$

$$= \frac{Ax}{a^4} + \frac{b(Ab-aB)\tan(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{b(8a^2Ab-3Ab^3-5a^3B)\tan(c+dx}{6a^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2}$$

$$= \frac{Ax}{a^4} + \frac{b(Ab-aB)\tan(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{b(8a^2Ab-3Ab^3-5a^3B)\tan(c+dx}{6a^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2}$$

$$= \frac{Ax}{a^4} + \frac{b(Ab-aB)\tan(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^3} + \frac{b(8a^2Ab-3Ab^3-5a^3B)\tan(c+dx}{6a^2\,(a^2-b^2)^2\,d(a+b\sec(c+dx))^2}$$

$$= \frac{Ax}{a^4} - \frac{(8a^6Ab-8a^4Ab^3+7a^2Ab^5-2Ab^7-2a^7B-3a^5b^2B)\tanh^{-1}\left(\frac{\sqrt{a-b}\,t}{\sqrt{a}}\right.}{a^4(a-b)^{7/2}(a+b)^{7/2}d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 769 vs. 2(292) = 584.

time = 3.50, size = 769, normalized size = 2.63

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(A + B*Sec[c + d*x])*((-24*(-8*a^6*A*b
 + 8*a^4*A*b^3 - 7*a^2*A*b^5 + 2*A*b^7 + 2*a^7*B + 3*a^5*b^2*B)*ArcTanh[((-
a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b + a*Cos[c + d*x])^3)/(a^2 - b^
2)^(7/2) + (36*a^8*A*b*c - 84*a^6*A*b^3*c + 36*a^4*A*b^5*c + 36*a^2*A*b^7*c
 - 24*A*b^9*c + 36*a^8*A*b*d*x - 84*a^6*A*b^3*d*x + 36*a^4*A*b^5*d*x + 36*a
^2*A*b^7*d*x - 24*A*b^9*d*x + 18*a*A*(a^2 - b^2)^3*(a^2 + 4*b^2)*(c + d*x)*
Cos[c + d*x] + 36*a^2*A*b*(a^2 - b^2)^3*(c + d*x)*Cos[2*(c + d*x)] + 6*a^9*
A*c*Cos[3*(c + d*x)] - 18*a^7*A*b^2*c*Cos[3*(c + d*x)] + 18*a^5*A*b^4*c*Cos
[3*(c + d*x)] - 6*a^3*A*b^6*c*Cos[3*(c + d*x)] + 6*a^9*A*d*x*Cos[3*(c + d*x
)] - 18*a^7*A*b^2*d*x*Cos[3*(c + d*x)] + 18*a^5*A*b^4*d*x*Cos[3*(c + d*x)]
- 6*a^3*A*b^6*d*x*Cos[3*(c + d*x)] + 36*a^7*A*b^2*Sin[c + d*x] + 72*a^5*A*b
```

```
^4*Sin[c + d*x] - 57*a^3*A*b^6*Sin[c + d*x] + 24*a*A*b^8*Sin[c + d*x] - 18*
a^8*b*B*Sin[c + d*x] - 39*a^6*b^3*B*Sin[c + d*x] - 18*a^4*b^5*B*Sin[c + d*x
] + 120*a^6*A*b^3*Sin[2*(c + d*x)] - 90*a^4*A*b^5*Sin[2*(c + d*x)] + 30*a^2
*A*b^7*Sin[2*(c + d*x)] - 54*a^7*b^2*B*Sin[2*(c + d*x)] - 6*a^5*b^4*B*Sin[2
*(c + d*x)] + 36*a^7*A*b^2*Sin[3*(c + d*x)] - 32*a^5*A*b^4*Sin[3*(c + d*x)]
 + 11*a^3*A*b^6*Sin[3*(c + d*x)] - 18*a^8*b*B*Sin[3*(c + d*x)] + 5*a^6*b^3*
B*Sin[3*(c + d*x)] - 2*a^4*b^5*B*Sin[3*(c + d*x)])/(a^2 - b^2)^3))/(24*a^4*
d*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^4)
```

Maple [A]
time = 0.62, size = 464, normalized size = 1.59

method	result
derivativedivides	$\frac{2\left(-\frac{\left(12A\,a^4b+4A\,a^3b^2-6A\,a^2b^3-Aa\,b^4+2A\,b^5-6B\,a^5-3B\,a^4b-2B\,a^3b^2\right)ba\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^3+3b\,a^2+3b^2a+b^3\right)}+\frac{2\left(18A\,a^4b-11A\,a^2b^3+3A\,b^5-9B\,\cdots\right.}{3\left(a^2-2ba+b^2\right)\left(\cdots\right.}\cdots}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\cdots\right.\right.\right.}$
default	$\frac{2\left(-\frac{\left(12A\,a^4b+4A\,a^3b^2-6A\,a^2b^3-Aa\,b^4+2A\,b^5-6B\,a^5-3B\,a^4b-2B\,a^3b^2\right)ba\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)\left(a^3+3b\,a^2+3b^2a+b^3\right)}+\frac{2\left(18A\,a^4b-11A\,a^2b^3+3A\,b^5-9B\,\cdots\right.}{3\left(a^2-2ba+b^2\right)\left(\cdots\right.}\cdots}{\left(a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\cdots\right.\right.\right.}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(2/a^4*((-1/2*(12*A*a^4*b+4*A*a^3*b^2-6*A*a^2*b^3-A*a*b^4+2*A*b^5-6*B*a
^5-3*B*a^4*b-2*B*a^3*b^2)*b*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1
/2*c)^5+2/3*(18*A*a^4*b-11*A*a^2*b^3+3*A*b^5-9*B*a^5-B*a^3*b^2)*b*a/(a^2-2*
a*b+b^2)/(a^2+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(12*A*a^4*b-4*A*a^3*b^2-6
*A*a^2*b^3+A*a*b^4+2*A*b^5-6*B*a^5+3*B*a^4*b-2*B*a^3*b^2)*b*a/(a+b)/(a^3-3*
a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*
x+1/2*c)^2-a-b)^3-1/2*(8*A*a^6*b-8*A*a^4*b^3+7*A*a^2*b^5-2*A*b^7-2*B*a^7-3*
B*a^5*b^2)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*
tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))+2*A/a^4*arctan(tan(1/2*d*x+1/2*c))
)
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 905 vs. 2(276) = 552.
time = 5.84, size = 1867, normalized size = 6.39

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fricas")
```

```
[Out] [1/12*(12*(A*a^11 - 4*A*a^9*b^2 + 6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b^8)*d*
x*cos(d*x + c)^3 + 36*(A*a^10*b - 4*A*a^8*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7 +
 A*a^2*b^9)*d*x*cos(d*x + c)^2 + 36*(A*a^9*b^2 - 4*A*a^7*b^4 + 6*A*a^5*b^6
- 4*A*a^3*b^8 + A*a*b^10)*d*x*cos(d*x + c) + 12*(A*a^8*b^3 - 4*A*a^6*b^5 +
6*A*a^4*b^7 - 4*A*a^2*b^9 + A*b^11)*d*x - 3*(2*B*a^7*b^3 - 8*A*a^6*b^4 + 3*
B*a^5*b^5 + 8*A*a^4*b^6 - 7*A*a^2*b^8 + 2*A*b^10 + (2*B*a^10 - 8*A*a^9*b +
3*B*a^8*b^2 + 8*A*a^7*b^3 - 7*A*a^5*b^5 + 2*A*a^3*b^7)*cos(d*x + c)^3 + 3*(
2*B*a^9*b - 8*A*a^8*b^2 + 3*B*a^7*b^3 + 8*A*a^6*b^4 - 7*A*a^4*b^6 + 2*A*a^2
*b^8)*cos(d*x + c)^2 + 3*(2*B*a^8*b^2 - 8*A*a^7*b^3 + 3*B*a^6*b^4 + 8*A*a^5
*b^5 - 7*A*a^3*b^7 + 2*A*a*b^9)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*co
s(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^2)*(b*cos(d*x +
c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c
) + b^2)) - 2*(11*B*a^8*b^3 - 26*A*a^7*b^4 - 7*B*a^6*b^5 + 43*A*a^5*b^6 - 4
*B*a^4*b^7 - 23*A*a^3*b^8 + 6*A*a*b^10 + (18*B*a^10*b - 36*A*a^9*b^2 - 23*B
*a^8*b^3 + 68*A*a^7*b^4 + 7*B*a^6*b^5 - 43*A*a^5*b^6 - 2*B*a^4*b^7 + 11*A*a
^3*b^8)*cos(d*x + c)^2 + 3*(9*B*a^9*b^2 - 20*A*a^8*b^3 - 8*B*a^7*b^4 + 35*A
*a^6*b^5 - B*a^5*b^6 - 20*A*a^4*b^7 + 5*A*a^2*b^9)*cos(d*x + c))*sin(d*x +
c))/((a^15 - 4*a^13*b^2 + 6*a^11*b^4 - 4*a^9*b^6 + a^7*b^8)*d*cos(d*x + c)^
3 + 3*(a^14*b - 4*a^12*b^3 + 6*a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x +
c)^2 + 3*(a^13*b^2 - 4*a^11*b^4 + 6*a^9*b^6 - 4*a^7*b^8 + a^5*b^10)*d*cos(d
*x + c) + (a^12*b^3 - 4*a^10*b^5 + 6*a^8*b^7 - 4*a^6*b^9 + a^4*b^11)*d), 1/
6*(6*(A*a^11 - 4*A*a^9*b^2 + 6*A*a^7*b^4 - 4*A*a^5*b^6 + A*a^3*b^8)*d*x*cos
(d*x + c)^3 + 18*(A*a^10*b - 4*A*a^8*b^3 + 6*A*a^6*b^5 - 4*A*a^4*b^7 + A*a^
2*b^9)*d*x*cos(d*x + c)^2 + 18*(A*a^9*b^2 - 4*A*a^7*b^4 + 6*A*a^5*b^6 - 4*A
*a^3*b^8 + A*a*b^10)*d*x*cos(d*x + c) + 6*(A*a^8*b^3 - 4*A*a^6*b^5 + 6*A*a^
4*b^7 - 4*A*a^2*b^9 + A*b^11)*d*x + 3*(2*B*a^7*b^3 - 8*A*a^6*b^4 + 3*B*a^5*
b^5 + 8*A*a^4*b^6 - 7*A*a^2*b^8 + 2*A*b^10 + (2*B*a^10 - 8*A*a^9*b + 3*B*a^
8*b^2 + 8*A*a^7*b^3 - 7*A*a^5*b^5 + 2*A*a^3*b^7)*cos(d*x + c)^3 + 3*(2*B*a^
9*b - 8*A*a^8*b^2 + 3*B*a^7*b^3 + 8*A*a^6*b^4 - 7*A*a^4*b^6 + 2*A*a^2*b^8)*
cos(d*x + c)^2 + 3*(2*B*a^8*b^2 - 8*A*a^7*b^3 + 3*B*a^6*b^4 + 8*A*a^5*b^5 -
 7*A*a^3*b^7 + 2*A*a*b^9)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2
```

```
+ b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) - (11*B*a^8*b^3 - 2
6*A*a^7*b^4 - 7*B*a^6*b^5 + 43*A*a^5*b^6 - 4*B*a^4*b^7 - 23*A*a^3*b^8 + 6*A
*a*b^10 + (18*B*a^10*b - 36*A*a^9*b^2 - 23*B*a^8*b^3 + 68*A*a^7*b^4 + 7*B*a
^6*b^5 - 43*A*a^5*b^6 - 2*B*a^4*b^7 + 11*A*a^3*b^8)*cos(d*x + c)^2 + 3*(9*B
*a^9*b^2 - 20*A*a^8*b^3 - 8*B*a^7*b^4 + 35*A*a^6*b^5 - B*a^5*b^6 - 20*A*a^4
*b^7 + 5*A*a^2*b^9)*cos(d*x + c))*sin(d*x + c))/((a^15 - 4*a^13*b^2 + 6*a^1
1*b^4 - 4*a^9*b^6 + a^7*b^8)*d*cos(d*x + c)^3 + 3*(a^14*b - 4*a^12*b^3 + 6*
a^10*b^5 - 4*a^8*b^7 + a^6*b^9)*d*cos(d*x + c)^2 + 3*(a^13*b^2 - 4*a^11*b^4
 + 6*a^9*b^6 - 4*a^7*b^8 + a^5*b^10)*d*cos(d*x + c) + (a^12*b^3 - 4*a^10*b^
5 + 6*a^8*b^7 - 4*a^6*b^9 + a^4*b^11)*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^{4}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 814 vs. 2(276) = 552.
time = 0.56, size = 814, normalized size = 2.79

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="giac")
```

```
[Out] 1/3*(3*(2*B*a^7 - 8*A*a^6*b + 3*B*a^5*b^2 + 8*A*a^4*b^3 - 7*A*a^2*b^5 + 2*A
*b^7)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/
2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^10 - 3*a^8*
b^2 + 3*a^6*b^4 - a^4*b^6)*sqrt(-a^2 + b^2)) + 3*(d*x + c)*A/a^4 + (18*B*a^
7*b*tan(1/2*d*x + 1/2*c)^5 - 36*A*a^6*b^2*tan(1/2*d*x + 1/2*c)^5 - 27*B*a^6
*b^2*tan(1/2*d*x + 1/2*c)^5 + 60*A*a^5*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^5
*b^3*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^5 - 3*B*a^4*
b^4*tan(1/2*d*x + 1/2*c)^5 - 45*A*a^3*b^5*tan(1/2*d*x + 1/2*c)^5 + 6*B*a^3*
b^5*tan(1/2*d*x + 1/2*c)^5 + 6*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^5 + 15*A*a*b^
7*tan(1/2*d*x + 1/2*c)^5 - 6*A*b^8*tan(1/2*d*x + 1/2*c)^5 - 36*B*a^7*b*tan(
1/2*d*x + 1/2*c)^3 + 72*A*a^6*b^2*tan(1/2*d*x + 1/2*c)^3 + 32*B*a^5*b^3*tan
(1/2*d*x + 1/2*c)^3 - 116*A*a^4*b^4*tan(1/2*d*x + 1/2*c)^3 + 4*B*a^3*b^5*ta
n(1/2*d*x + 1/2*c)^3 + 56*A*a^2*b^6*tan(1/2*d*x + 1/2*c)^3 - 12*A*b^8*tan(1
/2*d*x + 1/2*c)^3 + 18*B*a^7*b*tan(1/2*d*x + 1/2*c) - 36*A*a^6*b^2*tan(1/2*
d*x + 1/2*c) + 27*B*a^6*b^2*tan(1/2*d*x + 1/2*c) - 60*A*a^5*b^3*tan(1/2*d*x
```

```
 + 1/2*c) + 6*B*a^5*b^3*tan(1/2*d*x + 1/2*c) + 6*A*a^4*b^4*tan(1/2*d*x + 1/
2*c) + 3*B*a^4*b^4*tan(1/2*d*x + 1/2*c) + 45*A*a^3*b^5*tan(1/2*d*x + 1/2*c)
 + 6*B*a^3*b^5*tan(1/2*d*x + 1/2*c) + 6*A*a^2*b^6*tan(1/2*d*x + 1/2*c) - 15
*A*a*b^7*tan(1/2*d*x + 1/2*c) - 6*A*b^8*tan(1/2*d*x + 1/2*c))/((a^9 - 3*a^7
*b^2 + 3*a^5*b^4 - a^3*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2
*c)^2 - a - b)^3))/d
```

Mupad [B]
time = 14.41, size = 2500, normalized size = 8.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^4,x)
```

```
[Out] ((tan(c/2 + (d*x)/2)^5*(6*A*a^2*b^4 - 2*A*b^6 - 4*A*a^3*b^3 - 12*A*a^4*b^2
+ 2*B*a^3*b^3 + 3*B*a^4*b^2 + A*a*b^5 + 6*B*a^5*b))/((a^3*b - a^4)*(a + b)^
3) - (tan(c/2 + (d*x)/2)*(2*A*b^6 - 6*A*a^2*b^4 - 4*A*a^3*b^3 + 12*A*a^4*b^
2 - 2*B*a^3*b^3 + 3*B*a^4*b^2 + A*a*b^5 - 6*B*a^5*b))/((a + b)*(3*a^5*b - a
^6 + a^3*b^3 - 3*a^4*b^2)) + (4*tan(c/2 + (d*x)/2)^3*(11*A*a^2*b^4 - 3*A*b^
6 - 18*A*a^4*b^2 + B*a^3*b^3 + 9*B*a^5*b))/(3*(a + b)^2*(a^5 - 2*a^4*b + a^
3*b^2)))/(d*(tan(c/2 + (d*x)/2)^2*(3*a*b^2 - 3*a^2*b - 3*a^3 + 3*b^3) - tan
(c/2 + (d*x)/2)^4*(3*a*b^2 + 3*a^2*b - 3*a^3 - 3*b^3) + 3*a*b^2 + 3*a^2*b +
 a^3 + b^3 - tan(c/2 + (d*x)/2)^6*(3*a*b^2 - 3*a^2*b + a^3 - b^3))) - (2*A*
atan(-((A*((8*tan(c/2 + (d*x)/2)*(4*A^2*a^14 + 8*A^2*b^14 + 4*B^2*a^14 - 8*
A^2*a*b^13 - 8*A^2*a^13*b - 48*A^2*a^2*b^12 + 48*A^2*a^3*b^11 + 117*A^2*a^4
*b^10 - 120*A^2*a^5*b^9 - 164*A^2*a^6*b^8 + 160*A^2*a^7*b^7 + 156*A^2*a^8*b
^6 - 120*A^2*a^9*b^5 - 92*A^2*a^10*b^4 + 48*A^2*a^11*b^3 + 44*A^2*a^12*b^2
+ 9*B^2*a^10*b^4 + 12*B^2*a^12*b^2 - 32*A*B*a^13*b + 12*A*B*a^5*b^9 - 34*A*
B*a^7*b^7 + 20*A*B*a^9*b^5 - 16*A*B*a^11*b^3))/(a^16*b + a^17 - a^6*b^11 -
a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5
+ 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2) + (A*((8*(4*A*a^21 + 4*B*a^21 - 4*
A*a^8*b^13 + 2*A*a^9*b^12 + 26*A*a^10*b^11 - 14*A*a^11*b^10 - 70*A*a^12*b^9
 + 30*A*a^13*b^8 + 110*A*a^14*b^7 - 30*A*a^15*b^6 - 110*A*a^16*b^5 + 20*A*a
^17*b^4 + 64*A*a^18*b^3 - 12*A*a^19*b^2 + 6*B*a^12*b^9 - 6*B*a^13*b^8 - 14*
B*a^14*b^7 + 14*B*a^15*b^6 + 6*B*a^16*b^5 - 6*B*a^17*b^4 + 6*B*a^18*b^3 - 6
*B*a^19*b^2 - 16*A*a^20*b - 4*B*a^20*b))/(a^19*b + a^20 - a^9*b^11 - a^10*b
^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 1
0*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) - (A*tan(c/2 + (d*x)/2)*(8*a^21*b - 8
*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*
a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a
^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2)*8i)/(a^4*(a^16*b + a^17 - a^6*b^11 - a^
7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 +
10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)))*1i)/a^4))/a^4 + (A*((8*tan(c/2 + (
d*x)/2)*(4*A^2*a^14 + 8*A^2*b^14 + 4*B^2*a^14 - 8*A^2*a*b^13 - 8*A^2*a^13*b
```

- 48*A^2*a^2*b^12 + 48*A^2*a^3*b^11 + 117*A^2*a^4*b^10 - 120*A^2*a^5*b^9 - 164*A^2*a^6*b^8 + 160*A^2*a^7*b^7 + 156*A^2*a^8*b^6 - 120*A^2*a^9*b^5 - 92*A^2*a^10*b^4 + 48*A^2*a^11*b^3 + 44*A^2*a^12*b^2 + 9*B^2*a^10*b^4 + 12*B^2*a^12*b^2 - 32*A*B*a^13*b + 12*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 20*A*B*a^9*b^5 - 16*A*B*a^11*b^3))/(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2) - (A*((8*(4*A*a^21 + 4*B*a^21 - 4*A*a^8*b^13 + 2*A*a^9*b^12 + 26*A*a^10*b^11 - 14*A*a^11*b^10 - 70*A*a^12*b^9 + 30*A*a^13*b^8 + 110*A*a^14*b^7 - 30*A*a^15*b^6 - 110*A*a^16*b^5 + 20*A*a^17*b^4 + 64*A*a^18*b^3 - 12*A*a^19*b^2 + 6*B*a^12*b^9 - 6*B*a^13*b^8 - 14*B*a^14*b^7 + 14*B*a^15*b^6 + 6*B*a^16*b^5 - 6*B*a^17*b^4 + 6*B*a^18*b^3 - 6*B*a^19*b^2 - 16*A*a^20*b - 4*B*a^20*b))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) + (A*tan(c/2 + (d*x)/2)*(8*a^21*b - 8*a^8*b^14 + 8*a^9*b^13 + 48*a^10*b^12 - 48*a^11*b^11 - 120*a^12*b^10 + 120*a^13*b^9 + 160*a^14*b^8 - 160*a^15*b^7 - 120*a^16*b^6 + 120*a^17*b^5 + 48*a^18*b^4 - 48*a^19*b^3 - 8*a^20*b^2)*8i)/(a^4*(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2)))*1i)/a^4))/a^4)/((16*(4*A^3*b^13 + 4*A*B^2*a^13 - 4*A^2*B*a^13 - 2*A^3*a*b^12 + 16*A^3*a^12*b - 26*A^3*a^2*b^11 + 11*A^3*a^3*b^10 + 70*A^3*a^4*b^9 - 34*A^3*a^5*b^8 - 110*A^3*a^6*b^7 + 66*A^3*a^7*b^6 + 110*A^3*a^8*b^5 - 64*A^3*a^9*b^4 - 64*A^3*a^10*b^3 + 48*A^3*a^11*b^2 - 28*A^2*B*a^12*b + 9*A*B^2*a^9*b^4 + 12*A*B^2*a^11*b^2 + 6*A^2*B*a^4*b^9 + 6*A^2*B*a^5*b^8 - 20*A^2*B*a^6*b^7 - 14*A^2*B*a^7*b^6 + 14*A^2*B*a^8*b^5 + 6*A^2*B*a^9*b^4 - 22*A^2*B*a^10*b^3 + 6*A^2*B*a^11*b^2))/(a^19*b + a^20 - a^9*b^11 - a^10*b^10 + 5*a^11*b^9 + 5*a^12*b^8 - 10*a^13*b^7 - 10*a^14*b^6 + 10*a^15*b^5 + 10*a^16*b^4 - 5*a^17*b^3 - 5*a^18*b^2) - (A*((8*tan(c/2 + (d*x)/2)*(4*A^2*a^14 + 8*A^2*b^14 + 4*B^2*a^14 - 8*A^2*a*b^13 - 8*A^2*a^13*b - 48*A^2*a^2*b^12 + 48*A^2*a^3*b^11 + 117*A^2*a^4*b^10 - 120*A^2*a^5*b^9 - 164*A^2*a^6*b^8 + 160*A^2*a^7*b^7 + 156*A^2*a^8*b^6 - 120*A^2*a^9*b^5 - 92*A^2*a^10*b^4 + 48*A^2*a^11*b^3 + 44*A^2*a^12*b^2 + 9*B^2*a^10*b^4 + 12*B^2*a^12*b^2 - 32*A*B*a^13*b + 12*A*B*a^5*b^9 - 34*A*B*a^7*b^7 + 20*A*B*a^9*b^5 - 16*A*B*a^11*b^3))/(a^16*b + a^17 - a^6*b^11 - a^7*b^10 + 5*a^8*b^9 + 5*a^9*b^8 - 10*a^10*b^7 - 10*a^11*b^6 + 10*a^12*b^5 + 10*a^13*b^4 - 5*a^14*b^3 - 5*a^15*b^2) + (A*((8*(4*A*a^21 + 4*B*a^21 - 4*A*a^8*b^13 + 2*A*a^9*b^12 + 26*A*a^10*b^11 - 14*A*a^11*b^10 - 70*A*a^12*b^9 + 30*A*a^13*b^8 + 110*A*a^14*b^7 - 30*A*a^15*b^6 - 110*A*a^16*b^5 + 20*A*a^17*b^4 + 64...

3.342 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=411

$$-\frac{(4Ab-aB)x}{a^5}+\frac{b(20a^6Ab-35a^4Ab^3+28a^2Ab^5-8Ab^7-8a^7B+8a^5b^2B-7a^3b^4B+2ab^6B)\tanh^{-1}\left(\frac{\sqrt{a}}{}\right.}{a^5(a-b)^{7/2}(a+b)^{7/2}d}$$

```
[Out] -(4*A*b-B*a)*x/a^5+b*(20*A*a^6*b-35*A*a^4*b^3+28*A*a^2*b^5-8*A*b^7-8*B*a^7+
8*B*a^5*b^2-7*B*a^3*b^4+2*B*a*b^6)*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(
a+b)^(1/2))/a^5/(a-b)^(7/2)/(a+b)^(7/2)/d+1/6*(6*A*a^6-65*A*a^4*b^2+68*A*a^
2*b^4-24*A*b^6+26*B*a^5*b-17*B*a^3*b^3+6*B*a*b^5)*sin(d*x+c)/a^4/(a^2-b^2)^
3/d+1/3*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^3+1/6*b*(9*A*
a^2*b-4*A*b^3-6*B*a^3+B*a*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c)
)^2+1/2*b*(12*A*a^4*b-11*A*a^2*b^3+4*A*b^5-6*B*a^5+2*B*a^3*b^2-B*a*b^4)*sin
(d*x+c)/a^3/(a^2-b^2)^3/d/(a+b*sec(d*x+c))
```

Rubi [A]
time = 3.62, antiderivative size = 411, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.241$,
Rules used = {4115, 4185, 4189, 4004, 3916, 2738, 214}

$$-\frac{x(4Ab-aB)}{a^5}+\frac{b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^3}+\frac{b(-6a^3B+9a^2Ab+ab^2B-4Ab^3)\sin(c+dx)}{6a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))^2}+\frac{b(-6a^5B+12a^4Ab+2a^3b^2B-11a^2Ab^3-ab^4B+4Ab^5)\sin(c+dx)}{2a^3d\left(a^2-b^2\right)^3(a+b\sec(c+dx))}+\frac{(6a^6A+26a^5bB-65a^4Ab^2-17a^3b^3B+68a^2Ab^4+6ab^5B-24Ab^6)\sin(c+dx)}{6a^4d\left(a^2-b^2\right)^3}+\frac{b(-8a^7B+20a^6Ab+8a^5b^2B-35a^4Ab^3-7a^3b^4B+28a^2Ab^5+2ab^6B-8Ab^7)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^5d(a-b)^{7/2}(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] -(((4*A*b - a*B)*x)/a^5) + (b*(20*a^6*A*b - 35*a^4*A*b^3 + 28*a^2*A*b^5 - 8
*A*b^7 - 8*a^7*B + 8*a^5*b^2*B - 7*a^3*b^4*B + 2*a*b^6*B)*ArcTanh[(Sqrt[a -
 b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^5*(a - b)^(7/2)*(a + b)^(7/2)*d) + (
(6*a^6*A - 65*a^4*A*b^2 + 68*a^2*A*b^4 - 24*A*b^6 + 26*a^5*b*B - 17*a^3*b^3
*B + 6*a*b^5*B)*Sin[c + d*x])/(6*a^4*(a^2 - b^2)^3*d) + (b*(A*b - a*B)*Sin[
c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^3) + (b*(9*a^2*A*b - 4*A*
b^3 - 6*a^3*B + a*b^2*B)*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c
+ d*x])^2) + (b*(12*a^4*A*b - 11*a^2*A*b^3 + 4*A*b^5 - 6*a^5*B + 2*a^3*b^2*
B - a*b^4*B)*Sin[c + d*x])/(2*a^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x]))
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
```

```
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
```

```
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\cos(c+dx)\left(-3a^2A+4Ab^2-abB+3a(Ab}{(a+b\operatorname{se}}}{3a\,(a}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b(9a^2Ab-4Ab^3-6a^3B+ab^2}{6a^2\left(a^2-b^2\right)^2d(a+b\sec(}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b(9a^2Ab-4Ab^3-6a^3B+ab^2}{6a^2\left(a^2-b^2\right)^2d(a+b\sec(}$$

$$= \frac{(6a^6A-65a^4Ab^2+68a^2Ab^4-24Ab^6+26a^5bB-17a^3b^3B+6ab^5}{6a^4\left(a^2-b^2\right)^3d}$$

$$= -\frac{(4Ab-aB)x}{a^5} + \frac{(6a^6A-65a^4Ab^2+68a^2Ab^4-24Ab^6+26a^5bB}{6a^4\left(a^2-b^2\right)^3d}$$

$$= -\frac{(4Ab-aB)x}{a^5} + \frac{(6a^6A-65a^4Ab^2+68a^2Ab^4-24Ab^6+26a^5bB}{6a^4\left(a^2-b^2\right)^3d}$$

$$= -\frac{(4Ab-aB)x}{a^5} + \frac{(6a^6A-65a^4Ab^2+68a^2Ab^4-24Ab^6+26a^5bB}{6a^4\left(a^2-b^2\right)^3d}$$

$$= -\frac{(4Ab-aB)x}{a^5} + \frac{b(20a^6Ab-35a^4Ab^3+28a^2Ab^5-8Ab^7-8a^7B}{a^5(a-}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1205 vs. $2(411) = 822$.
time = 6.15, size = 1205, normalized size = 2.93

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^3*(A + B*Sec[c + d*x])*((24*b*(-20*a^6*A
*b + 35*a^4*A*b^3 - 28*a^2*A*b^5 + 8*A*b^7 + 8*a^7*B - 8*a^5*b^2*B + 7*a^3*
b^4*B - 2*a*b^6*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[a^2 - b^2]]*(b
```

```
+ a*Cos[c + d*x])^3)/(a^2 - b^2)^(7/2) + (-144*a^8*A*b^2*c + 336*a^6*A*b^4*
c - 144*a^4*A*b^6*c - 144*a^2*A*b^8*c + 96*A*b^10*c + 36*a^9*b*B*c - 84*a^7
*b^3*B*c + 36*a^5*b^5*B*c + 36*a^3*b^7*B*c - 24*a*b^9*B*c - 144*a^8*A*b^2*d
*x + 336*a^6*A*b^4*d*x - 144*a^4*A*b^6*d*x - 144*a^2*A*b^8*d*x + 96*A*b^10*
d*x + 36*a^9*b*B*d*x - 84*a^7*b^3*B*d*x + 36*a^5*b^5*B*d*x + 36*a^3*b^7*B*d
*x - 24*a*b^9*B*d*x + 18*a*(a^2 - b^2)^3*(a^2 + 4*b^2)*(-4*A*b + a*B)*(c +
d*x)*Cos[c + d*x] + 36*a^2*b*(a^2 - b^2)^3*(-4*A*b + a*B)*(c + d*x)*Cos[2*(
c + d*x)] - 24*a^9*A*b*c*Cos[3*(c + d*x)] + 72*a^7*A*b^3*c*Cos[3*(c + d*x)]
 - 72*a^5*A*b^5*c*Cos[3*(c + d*x)] + 24*a^3*A*b^7*c*Cos[3*(c + d*x)] + 6*a^
10*B*c*Cos[3*(c + d*x)] - 18*a^8*b^2*B*c*Cos[3*(c + d*x)] + 18*a^6*b^4*B*c*
Cos[3*(c + d*x)] - 6*a^4*b^6*B*c*Cos[3*(c + d*x)] - 24*a^9*A*b*d*x*Cos[3*(c
 + d*x)] + 72*a^7*A*b^3*d*x*Cos[3*(c + d*x)] - 72*a^5*A*b^5*d*x*Cos[3*(c +
d*x)] + 24*a^3*A*b^7*d*x*Cos[3*(c + d*x)] + 6*a^10*B*d*x*Cos[3*(c + d*x)] -
 18*a^8*b^2*B*d*x*Cos[3*(c + d*x)] + 18*a^6*b^4*B*d*x*Cos[3*(c + d*x)] - 6*
a^4*b^6*B*d*x*Cos[3*(c + d*x)] + 18*a^9*A*b*Sin[c + d*x] - 90*a^7*A*b^3*Sin
[c + d*x] - 135*a^5*A*b^5*Sin[c + d*x] + 228*a^3*A*b^7*Sin[c + d*x] - 96*a*
A*b^9*Sin[c + d*x] + 36*a^8*b^2*B*Sin[c + d*x] + 72*a^6*b^4*B*Sin[c + d*x]
- 57*a^4*b^6*B*Sin[c + d*x] + 24*a^2*b^8*B*Sin[c + d*x] + 6*a^10*A*Sin[2*(c
 + d*x)] + 18*a^8*A*b^2*Sin[2*(c + d*x)] - 300*a^6*A*b^4*Sin[2*(c + d*x)] +
 336*a^4*A*b^6*Sin[2*(c + d*x)] - 120*a^2*A*b^8*Sin[2*(c + d*x)] + 120*a^7*
b^3*B*Sin[2*(c + d*x)] - 90*a^5*b^5*B*Sin[2*(c + d*x)] + 30*a^3*b^7*B*Sin[2
*(c + d*x)] + 18*a^9*A*b*Sin[3*(c + d*x)] - 114*a^7*A*b^3*Sin[3*(c + d*x)]
+ 125*a^5*A*b^5*Sin[3*(c + d*x)] - 44*a^3*A*b^7*Sin[3*(c + d*x)] + 36*a^8*b
^2*B*Sin[3*(c + d*x)] - 32*a^6*b^4*B*Sin[3*(c + d*x)] + 11*a^4*b^6*B*Sin[3*
(c + d*x)] + 3*a^10*A*Sin[4*(c + d*x)] - 9*a^8*A*b^2*Sin[4*(c + d*x)] + 9*a
^6*A*b^4*Sin[4*(c + d*x)] - 3*a^4*A*b^6*Sin[4*(c + d*x)])/(a^2 - b^2)^3))/(
24*a^5*d*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^4)
```

Maple [A]

time = 0.77, size = 558, normalized size = 1.36

method	result
derivativedivides	$-\frac{2\left(-\frac{A a \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+(4Ab-Ba)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^5}-\frac{2b\left(-\frac{\left(20A a^4 b+5A a^3 b^2-18A a^2 b^3-2A a b^4+6A b^5-12B a^5-4B a^4 \cdots\right.}{2(a-b)\left(a^3+3b a^2+3b^{\cdots}\right.}\cdots\right.}{\cdots}$
default	$-\frac{2\left(-\frac{A a \tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{1+\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)}+(4Ab-Ba)\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right)}{a^5}-\frac{2b\left(-\frac{\left(20A a^4 b+5A a^3 b^2-18A a^2 b^3-2A a b^4+6A b^5-12B a^5-4B a^4 \cdots\right.}{2(a-b)\left(a^3+3b a^2+3b^{\cdots}\right.}\cdots\right.}{\cdots}$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(-2/a^5*(-A*a*tan(1/2*d*x+1/2*c)/(1+tan(1/2*d*x+1/2*c)^2)+(4*A*b-B*a)*a
rctan(tan(1/2*d*x+1/2*c)))-2*b/a^5*((-1/2*(20*A*a^4*b+5*A*a^3*b^2-18*A*a^2*
b^3-2*A*a*b^4+6*A*b^5-12*B*a^5-4*B*a^4*b+6*B*a^3*b^2+B*a^2*b^3-2*B*a*b^4)*b
*a/(a-b)/(a^3+3*a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(30*A*a^4*b-29*
A*a^2*b^3+9*A*b^5-18*B*a^5+11*B*a^3*b^2-3*B*a*b^4)*b*a/(a^2-2*a*b+b^2)/(a^2
+2*a*b+b^2)*tan(1/2*d*x+1/2*c)^3-1/2*(20*A*a^4*b-5*A*a^3*b^2-18*A*a^2*b^3+2
*A*a*b^4+6*A*b^5-12*B*a^5+4*B*a^4*b+6*B*a^3*b^2-B*a^2*b^3-2*B*a*b^4)*b*a/(a
+b)/(a^3-3*a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b
*tan(1/2*d*x+1/2*c)^2-a-b)^3-1/2*(20*A*a^6*b-35*A*a^4*b^3+28*A*a^2*b^5-8*A*
b^7-8*B*a^7+8*B*a^5*b^2-7*B*a^3*b^4+2*B*a*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6
)/((a+b)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2))
))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="maxi
ma")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1251 vs. $2(393) = 786$.
time = 6.02, size = 2560, normalized size = 6.23

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fric
as")
```

```
[Out] [1/12*(12*(B*a^12 - 4*A*a^11*b - 4*B*a^10*b^2 + 16*A*a^9*b^3 + 6*B*a^8*b^4
- 24*A*a^7*b^5 - 4*B*a^6*b^6 + 16*A*a^5*b^7 + B*a^4*b^8 - 4*A*a^3*b^9)*d*x*
cos(d*x + c)^3 + 36*(B*a^11*b - 4*A*a^10*b^2 - 4*B*a^9*b^3 + 16*A*a^8*b^4 +
 6*B*a^7*b^5 - 24*A*a^6*b^6 - 4*B*a^5*b^7 + 16*A*a^4*b^8 + B*a^3*b^9 - 4*A*
a^2*b^10)*d*x*cos(d*x + c)^2 + 36*(B*a^10*b^2 - 4*A*a^9*b^3 - 4*B*a^8*b^4 +
 16*A*a^7*b^5 + 6*B*a^6*b^6 - 24*A*a^5*b^7 - 4*B*a^4*b^8 + 16*A*a^3*b^9 + B
*a^2*b^10 - 4*A*a*b^11)*d*x*cos(d*x + c) + 12*(B*a^9*b^3 - 4*A*a^8*b^4 - 4*
```

B*a^7*b^5 + 16*A*a^6*b^6 + 6*B*a^5*b^7 - 24*A*a^4*b^8 - 4*B*a^3*b^9 + 16*A*a^2*b^10 + B*a*b^11 - 4*A*b^12)*d*x - 3*(8*B*a^7*b^4 - 20*A*a^6*b^5 - 8*B*a^5*b^6 + 35*A*a^4*b^7 + 7*B*a^3*b^8 - 28*A*a^2*b^9 - 2*B*a*b^10 + 8*A*b^11 + (8*B*a^10*b - 20*A*a^9*b^2 - 8*B*a^8*b^3 + 35*A*a^7*b^4 + 7*B*a^6*b^5 - 28*A*a^5*b^6 - 2*B*a^4*b^7 + 8*A*a^3*b^8)*cos(d*x + c)^3 + 3*(8*B*a^9*b^2 - 20*A*a^8*b^3 - 8*B*a^7*b^4 + 35*A*a^6*b^5 + 7*B*a^5*b^6 - 28*A*a^4*b^7 - 2*B*a^3*b^8 + 8*A*a^2*b^9)*cos(d*x + c)^2 + 3*(8*B*a^8*b^3 - 20*A*a^7*b^4 - 8*B*a^6*b^5 + 35*A*a^5*b^6 + 7*B*a^4*b^7 - 28*A*a^3*b^8 - 2*B*a^2*b^9 + 8*A*a*b^10)*cos(d*x + c))*sqrt(a^2 - b^2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)) + 2*(6*A*a^9*b^3 + 26*B*a^8*b^4 - 71*A*a^7*b^5 - 43*B*a^6*b^6 + 133*A*a^5*b^7 + 23*B*a^4*b^8 - 92*A*a^3*b^9 - 6*B*a^2*b^10 + 24*A*a*b^11 + 6*(A*a^12 - 4*A*a^10*b^2 + 6*A*a^8*b^4 - 4*A*a^6*b^6 + A*a^4*b^8)*cos(d*x + c)^3 + (18*A*a^11*b + 36*B*a^10*b^2 - 132*A*a^9*b^3 - 68*B*a^8*b^4 + 239*A*a^7*b^5 + 43*B*a^6*b^6 - 169*A*a^5*b^7 - 11*B*a^4*b^8 + 44*A*a^3*b^9)*cos(d*x + c)^2 + 3*(6*A*a^10*b^2 + 20*B*a^9*b^3 - 59*A*a^8*b^4 - 35*B*a^7*b^5 + 110*A*a^6*b^6 + 20*B*a^5*b^7 - 77*A*a^4*b^8 - 5*B*a^3*b^9 + 20*A*a^2*b^10)*cos(d*x + c))*sin(d*x + c))/((a^16 - 4*a^14*b^2 + 6*a^12*b^4 - 4*a^10*b^6 + a^8*b^8)*d*cos(d*x + c)^3 + 3*(a^15*b - 4*a^13*b^3 + 6*a^11*b^5 - 4*a^9*b^7 + a^7*b^9)*d*cos(d*x + c)^2 + 3*(a^14*b^2 - 4*a^12*b^4 + 6*a^10*b^6 - 4*a^8*b^8 + a^6*b^10)*d*cos(d*x + c) + (a^13*b^3 - 4*a^11*b^5 + 6*a^9*b^7 - 4*a^7*b^9 + a^5*b^11)*d),
1/6*(6*(B*a^12 - 4*A*a^11*b - 4*B*a^10*b^2 + 16*A*a^9*b^3 + 6*B*a^8*b^4 - 24*A*a^7*b^5 - 4*B*a^6*b^6 + 16*A*a^5*b^7 + B*a^4*b^8 - 4*A*a^3*b^9)*d*x*cos(d*x + c)^3 + 18*(B*a^11*b - 4*A*a^10*b^2 - 4*B*a^9*b^3 + 16*A*a^8*b^4 + 6*B*a^7*b^5 - 24*A*a^6*b^6 - 4*B*a^5*b^7 + 16*A*a^4*b^8 + B*a^3*b^9 - 4*A*a^2*b^10)*d*x*cos(d*x + c)^2 + 18*(B*a^10*b^2 - 4*A*a^9*b^3 - 4*B*a^8*b^4 + 16*A*a^7*b^5 + 6*B*a^6*b^6 - 24*A*a^5*b^7 - 4*B*a^4*b^8 + 16*A*a^3*b^9 + B*a^2*b^10 - 4*A*a*b^11)*d*x*cos(d*x + c) + 6*(B*a^9*b^3 - 4*A*a^8*b^4 - 4*B*a^7*b^5 + 16*A*a^6*b^6 + 6*B*a^5*b^7 - 24*A*a^4*b^8 - 4*B*a^3*b^9 + 16*A*a^2*b^10 + B*a*b^11 - 4*A*b^12)*d*x - 3*(8*B*a^7*b^4 - 20*A*a^6*b^5 - 8*B*a^5*b^6 + 35*A*a^4*b^7 + 7*B*a^3*b^8 - 28*A*a^2*b^9 - 2*B*a*b^10 + 8*A*b^11 + (8*B*a^10*b - 20*A*a^9*b^2 - 8*B*a^8*b^3 + 35*A*a^7*b^4 + 7*B*a^6*b^5 - 28*A*a^5*b^6 - 2*B*a^4*b^7 + 8*A*a^3*b^8)*cos(d*x + c)^3 + 3*(8*B*a^9*b^2 - 20*A*a^8*b^3 - 8*B*a^7*b^4 + 35*A*a^6*b^5 + 7*B*a^5*b^6 - 28*A*a^4*b^7 - 2*B*a^3*b^8 + 8*A*a^2*b^9)*cos(d*x + c)^2 + 3*(8*B*a^8*b^3 - 20*A*a^7*b^4 - 8*B*a^6*b^5 + 35*A*a^5*b^6 + 7*B*a^4*b^7 - 28*A*a^3*b^8 - 2*B*a^2*b^9 + 8*A*a*b^10)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (6*A*a^9*b^3 + 26*B*a^8*b^4 - 71*A*a^7*b^5 - 43*B*a^6*b^6 + 133*A*a^5*b^7 + 23*B*a^4*b^8 - 92*A*a^3*b^9 - 6*B*a^2*b^10 + 24*A*a*b^11 + 6*(A*a^12 - 4*A*a^10*b^2 + 6*A*a^8*b^4 - 4*A*a^6*b^6 + A*a^4*b^8)*cos(d*x + c)^3 + (18*A*a^11*b + 36*B*a^10*b^2 - 132*A*a^9*b^3 - 68*B*a^8*b^4 + 239*A*a^7*b^5 + 43*B*a^6*b^6 - 169*A*a^5*b^7 - 11*B*a^4*b^8 + 44*A*a^3*b^9)*cos(d*x + c)^2 + 3*(6*A*a^10*b^2 + 20*B*a^9*b^3 - 59*A*a^8*b^4 - 35*B*a^7*b^5 + 110*A*a^6*b^6 + 20*B*a^5*b^7 - 77*A*a^4*b^8 - 5*B*a^3

```
*b^9 + 20*A*a^2*b^10)*cos(d*x + c))*sin(d*x + c))/((a^16 - 4*a^14*b^2 + 6*a
^12*b^4 - 4*a^10*b^6 + a^8*b^8)*d*cos(d*x + c)^3 + 3*(a^15*b - 4*a^13*b^3 +
 6*a^11*b^5 - 4*a^9*b^7 + a^7*b^9)*d*cos(d*x + c)^2 + 3*(a^14*b^2 - 4*a^12*
b^4 + 6*a^10*b^6 - 4*a^8*b^8 + a^6*b^10)*d*cos(d*x + c) + (a^13*b^3 - 4*a^1
1*b^5 + 6*a^9*b^7 - 4*a^7*b^9 + a^5*b^11)*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\cos(c + dx)}{(a + b\sec(c + dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)/(a + b*sec(c + d*x))**4, x)
```

Giac [**B**] Leaf count of result is larger than twice the leaf count of optimal. 966 vs. 2(393) = 786.
time = 0.57, size = 966, normalized size = 2.35

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="giac
")
```

```
[Out] -1/3*(3*(8*B*a^7*b - 20*A*a^6*b^2 - 8*B*a^5*b^3 + 35*A*a^4*b^4 + 7*B*a^3*b^
5 - 28*A*a^2*b^6 - 2*B*a*b^7 + 8*A*b^8)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*s
gn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/
sqrt(-a^2 + b^2)))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*sqrt(-a^2 + b^
2)) + (36*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^5 - 60*A*a^6*b^3*tan(1/2*d*x + 1/2
*c)^5 - 60*B*a^6*b^3*tan(1/2*d*x + 1/2*c)^5 + 105*A*a^5*b^4*tan(1/2*d*x + 1
/2*c)^5 - 6*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^5 + 24*A*a^4*b^5*tan(1/2*d*x + 1
/2*c)^5 + 45*B*a^4*b^5*tan(1/2*d*x + 1/2*c)^5 - 117*A*a^3*b^6*tan(1/2*d*x +
 1/2*c)^5 - 6*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^5 + 24*A*a^2*b^7*tan(1/2*d*x +
 1/2*c)^5 - 15*B*a^2*b^7*tan(1/2*d*x + 1/2*c)^5 + 42*A*a*b^8*tan(1/2*d*x +
1/2*c)^5 + 6*B*a*b^8*tan(1/2*d*x + 1/2*c)^5 - 18*A*b^9*tan(1/2*d*x + 1/2*c)
^5 - 72*B*a^7*b^2*tan(1/2*d*x + 1/2*c)^3 + 120*A*a^6*b^3*tan(1/2*d*x + 1/2*
c)^3 + 116*B*a^5*b^4*tan(1/2*d*x + 1/2*c)^3 - 236*A*a^4*b^5*tan(1/2*d*x + 1
/2*c)^3 - 56*B*a^3*b^6*tan(1/2*d*x + 1/2*c)^3 + 152*A*a^2*b^7*tan(1/2*d*x +
 1/2*c)^3 + 12*B*a*b^8*tan(1/2*d*x + 1/2*c)^3 - 36*A*b^9*tan(1/2*d*x + 1/2*
c)^3 + 36*B*a^7*b^2*tan(1/2*d*x + 1/2*c) - 60*A*a^6*b^3*tan(1/2*d*x + 1/2*c
) + 60*B*a^6*b^3*tan(1/2*d*x + 1/2*c) - 105*A*a^5*b^4*tan(1/2*d*x + 1/2*c)
- 6*B*a^5*b^4*tan(1/2*d*x + 1/2*c) + 24*A*a^4*b^5*tan(1/2*d*x + 1/2*c) - 45
*B*a^4*b^5*tan(1/2*d*x + 1/2*c) + 117*A*a^3*b^6*tan(1/2*d*x + 1/2*c) - 6*B*
```

```
a^3*b^6*tan(1/2*d*x + 1/2*c) + 24*A*a^2*b^7*tan(1/2*d*x + 1/2*c) + 15*B*a^2
*b^7*tan(1/2*d*x + 1/2*c) - 42*A*a*b^8*tan(1/2*d*x + 1/2*c) + 6*B*a*b^8*tan
(1/2*d*x + 1/2*c) - 18*A*b^9*tan(1/2*d*x + 1/2*c))/((a^10 - 3*a^8*b^2 + 3*a
^6*b^4 - a^4*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a
- b)^3) - 3*(B*a - 4*A*b)*(d*x + c)/a^5 - 6*A*tan(1/2*d*x + 1/2*c)/((tan(1/
2*d*x + 1/2*c)^2 + 1)*a^4))/d
```

Mupad [B]
time = 14.20, size = 2500, normalized size = 6.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^4,x)
```

```
[Out] (log(tan(c/2 + (d*x)/2) - 1i)*(4*A*b - B*a)*1i)/(a^5*d) - ((tan(c/2 + (d*x)
/2)*(2*A*a^7 - 8*A*b^7 + 24*A*a^2*b^5 + 11*A*a^3*b^4 - 26*A*a^4*b^3 - 6*A*a
^5*b^2 + B*a^2*b^5 - 6*B*a^3*b^4 - 4*B*a^4*b^3 + 12*B*a^5*b^2 - 4*A*a*b^6 +
 2*A*a^6*b + 2*B*a*b^6))/((a + b)*(3*a^6*b - a^7 + a^4*b^3 - 3*a^5*b^2)) -
(tan(c/2 + (d*x)/2)^3*(18*A*a^8 + 72*A*b^8 - 236*A*a^2*b^6 + 47*A*a^3*b^5 +
 273*A*a^4*b^4 - 60*A*a^5*b^3 - 72*A*a^6*b^2 + 3*B*a^2*b^6 + 59*B*a^3*b^5 -
 14*B*a^4*b^4 - 96*B*a^5*b^3 + 36*B*a^6*b^2 - 12*A*a*b^7 - 18*B*a*b^7))/(3*
(a + b)^2*(3*a^6*b - a^7 + a^4*b^3 - 3*a^5*b^2)) + (tan(c/2 + (d*x)/2)^7*(2
4*A*a^2*b^5 - 8*A*b^7 - 2*A*a^7 - 11*A*a^3*b^4 - 26*A*a^4*b^3 + 6*A*a^5*b^2
 - B*a^2*b^5 - 6*B*a^3*b^4 + 4*B*a^4*b^3 + 12*B*a^5*b^2 + 4*A*a*b^6 + 2*A*a
^6*b + 2*B*a*b^6))/((a^4*b - a^5)*(a + b)^3) + (tan(c/2 + (d*x)/2)^5*(18*A*
a^8 + 72*A*b^8 - 236*A*a^2*b^6 - 47*A*a^3*b^5 + 273*A*a^4*b^4 + 60*A*a^5*b^
3 - 72*A*a^6*b^2 - 3*B*a^2*b^6 + 59*B*a^3*b^5 + 14*B*a^4*b^4 - 96*B*a^5*b^3
 - 36*B*a^6*b^2 + 12*A*a*b^7 - 18*B*a*b^7))/(3*(a^4*b - a^5)*(a + b)^3*(a -
 b)))/(d*(3*a*b^2 + 3*a^2*b - tan(c/2 + (d*x)/2)^4*(6*a^2*b - 6*b^3) + tan(
c/2 + (d*x)/2)^2*(6*a*b^2 - 2*a^3 + 4*b^3) + tan(c/2 + (d*x)/2)^6*(2*a^3 -
6*a*b^2 + 4*b^3) + a^3 + b^3 - tan(c/2 + (d*x)/2)^8*(3*a*b^2 - 3*a^2*b + a^
3 - b^3))) - (log(tan(c/2 + (d*x)/2) + 1i)*(A*b*4i - B*a*1i))/(a^5*d) - (b*
atan(((b*((8*tan(c/2 + (d*x)/2)*(128*A^2*b^16 + 4*B^2*a^16 - 128*A^2*a*b^15
 - 8*B^2*a^15*b - 768*A^2*a^2*b^14 + 768*A^2*a^3*b^13 + 1920*A^2*a^4*b^12 -
 1920*A^2*a^5*b^11 - 2600*A^2*a^6*b^10 + 2560*A^2*a^7*b^9 + 2025*A^2*a^8*b^
8 - 1920*A^2*a^9*b^7 - 824*A^2*a^10*b^6 + 768*A^2*a^11*b^5 + 80*A^2*a^12*b^
4 - 128*A^2*a^13*b^3 + 64*A^2*a^14*b^2 + 8*B^2*a^2*b^14 - 8*B^2*a^3*b^13 -
48*B^2*a^4*b^12 + 48*B^2*a^5*b^11 + 117*B^2*a^6*b^10 - 120*B^2*a^7*b^9 - 16
4*B^2*a^8*b^8 + 160*B^2*a^9*b^7 + 156*B^2*a^10*b^6 - 120*B^2*a^11*b^5 - 92*
B^2*a^12*b^4 + 48*B^2*a^13*b^3 + 44*B^2*a^14*b^2 - 64*A*B*a*b^15 - 32*A*B*a
^15*b + 64*A*B*a^2*b^14 + 384*A*B*a^3*b^13 - 384*A*B*a^4*b^12 - 948*A*B*a^5
*b^11 + 960*A*B*a^6*b^10 + 1306*A*B*a^7*b^9 - 1280*A*B*a^8*b^8 - 1128*A*B*a
^9*b^7 + 960*A*B*a^10*b^6 + 592*A*B*a^11*b^5 - 384*A*B*a^12*b^4 - 160*A*B*a
^13*b^3 + 64*A*B*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b
```

^9 + 5*a^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5
*a^16*b^3 - 5*a^17*b^2) + (b*((a + b)^7*(a - b)^7)^(1/2)*((8*(4*B*a^24 + 16
*A*a^10*b^14 - 8*A*a^11*b^13 - 104*A*a^12*b^12 + 50*A*a^13*b^11 + 286*A*a^1
4*b^10 - 126*A*a^15*b^9 - 434*A*a^16*b^8 + 174*A*a^17*b^7 + 386*A*a^18*b^6
- 146*A*a^19*b^5 - 190*A*a^20*b^4 + 72*A*a^21*b^3 + 40*A*a^22*b^2 - 4*B*a^1
1*b^13 + 2*B*a^12*b^12 + 26*B*a^13*b^11 - 14*B*a^14*b^10 - 70*B*a^15*b^9 +
30*B*a^16*b^8 + 110*B*a^17*b^7 - 30*B*a^18*b^6 - 110*B*a^19*b^5 + 20*B*a^20
*b^4 + 64*B*a^21*b^3 - 12*B*a^22*b^2 - 16*A*a^23*b - 16*B*a^23*b))/(a^22*b
+ a^23 - a^12*b^11 - a^13*b^10 + 5*a^14*b^9 + 5*a^15*b^8 - 10*a^16*b^7 - 10
*a^17*b^6 + 10*a^18*b^5 + 10*a^19*b^4 - 5*a^20*b^3 - 5*a^21*b^2) - (4*b*tan
(c/2 + (d*x)/2)*((a + b)^7*(a - b)^7)^(1/2)*(8*A*b^7 + 8*B*a^7 - 28*A*a^2*b
^5 + 35*A*a^4*b^3 + 7*B*a^3*b^4 - 8*B*a^5*b^2 - 20*A*a^6*b - 2*B*a*b^6)*(8*
a^23*b - 8*a^10*b^14 + 8*a^11*b^13 + 48*a^12*b^12 - 48*a^13*b^11 - 120*a^14
*b^10 + 120*a^15*b^9 + 160*a^16*b^8 - 160*a^17*b^7 - 120*a^18*b^6 + 120*a^1
9*b^5 + 48*a^20*b^4 - 48*a^21*b^3 - 8*a^22*b^2))/((a^19 - a^5*b^14 + 7*a^7*
b^12 - 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)*
(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a^11*b^8 - 10*a^12*b^
7 - 10*a^13*b^6 + 10*a^14*b^5 + 10*a^15*b^4 - 5*a^16*b^3 - 5*a^17*b^2)))*(8
*A*b^7 + 8*B*a^7 - 28*A*a^2*b^5 + 35*A*a^4*b^3 + 7*B*a^3*b^4 - 8*B*a^5*b^2
- 20*A*a^6*b - 2*B*a*b^6))/(2*(a^19 - a^5*b^14 + 7*a^7*b^12 - 21*a^9*b^10 +
35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)))*((a + b)^7*(a - b)
^7)^(1/2)*(8*A*b^7 + 8*B*a^7 - 28*A*a^2*b^5 + 35*A*a^4*b^3 + 7*B*a^3*b^4 -
8*B*a^5*b^2 - 20*A*a^6*b - 2*B*a*b^6)*1i)/(2*(a^19 - a^5*b^14 + 7*a^7*b^12
- 21*a^9*b^10 + 35*a^11*b^8 - 35*a^13*b^6 + 21*a^15*b^4 - 7*a^17*b^2)) + (b
*((8*tan(c/2 + (d*x)/2)*(128*A^2*b^16 + 4*B^2*a^16 - 128*A^2*a*b^15 - 8*B^2
*a^15*b - 768*A^2*a^2*b^14 + 768*A^2*a^3*b^13 + 1920*A^2*a^4*b^12 - 1920*A^
2*a^5*b^11 - 2600*A^2*a^6*b^10 + 2560*A^2*a^7*b^9 + 2025*A^2*a^8*b^8 - 1920
*A^2*a^9*b^7 - 824*A^2*a^10*b^6 + 768*A^2*a^11*b^5 + 80*A^2*a^12*b^4 - 128*
A^2*a^13*b^3 + 64*A^2*a^14*b^2 + 8*B^2*a^2*b^14 - 8*B^2*a^3*b^13 - 48*B^2*a
^4*b^12 + 48*B^2*a^5*b^11 + 117*B^2*a^6*b^10 - 120*B^2*a^7*b^9 - 164*B^2*a^
8*b^8 + 160*B^2*a^9*b^7 + 156*B^2*a^10*b^6 - 120*B^2*a^11*b^5 - 92*B^2*a^12
*b^4 + 48*B^2*a^13*b^3 + 44*B^2*a^14*b^2 - 64*A*B*a*b^15 - 32*A*B*a^15*b +
64*A*B*a^2*b^14 + 384*A*B*a^3*b^13 - 384*A*B*a^4*b^12 - 948*A*B*a^5*b^11 +
960*A*B*a^6*b^10 + 1306*A*B*a^7*b^9 - 1280*A*B*a^8*b^8 - 1128*A*B*a^9*b^7 +
960*A*B*a^10*b^6 + 592*A*B*a^11*b^5 - 384*A*B*a^12*b^4 - 160*A*B*a^13*b^3
+ 64*A*B*a^14*b^2))/(a^18*b + a^19 - a^8*b^11 - a^9*b^10 + 5*a^10*b^9 + 5*a
^11*b^8 - 10*a^12*b^7 - 10*a^13*b^6 + 10*a^14*b...

3.343 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx$

Optimal. Leaf size=538

$$\frac{(a^2A+20Ab^2-8abB)\,x}{2a^6}-\frac{b^2(40a^6Ab-84a^4Ab^3+69a^2Ab^5-20Ab^7-20a^7B+35a^5b^2B-28a^3b^4B+8ab}{a^6(a-b)^{7/2}(a+b)^{7/2}d}$$

```
[Out] 1/2*(A*a^2+20*A*b^2-8*B*a*b)*x/a^6-b^2*(40*A*a^6*b-84*A*a^4*b^3+69*A*a^2*b^
5-20*A*b^7-20*B*a^7+35*B*a^5*b^2-28*B*a^3*b^4+8*B*a*b^6)*arctanh((a-b)^(1/2
)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))/a^6/(a-b)^(7/2)/(a+b)^(7/2)/d-1/6*(24*A*a
^6*b-146*A*a^4*b^3+167*A*a^2*b^5-60*A*b^7-6*B*a^7+65*B*a^5*b^2-68*B*a^3*b^4
+24*B*a*b^6)*sin(d*x+c)/a^5/(a^2-b^2)^3/d+1/2*(A*a^6-23*A*a^4*b^2+27*A*a^2*
b^4-10*A*b^6+12*B*a^5*b-11*B*a^3*b^3+4*B*a*b^5)*cos(d*x+c)*sin(d*x+c)/a^4/(
a^2-b^2)^3/d+1/3*b*(A*b-B*a)*cos(d*x+c)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d
*x+c))^3+1/6*b*(10*A*a^2*b-5*A*b^3-7*B*a^3+2*B*a*b^2)*cos(d*x+c)*sin(d*x+c)
/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^2+1/6*b*(48*A*a^4*b-53*A*a^2*b^3+20*A*b
^5-27*B*a^5+20*B*a^3*b^2-8*B*a*b^4)*cos(d*x+c)*sin(d*x+c)/a^3/(a^2-b^2)^3/d
/(a+b*sec(d*x+c))
```

Rubi [A]
time = 4.38, antiderivative size = 538, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4115, 4185, 4189, 4004, 3916, 2738, 214}

$$\frac{b(Ab-aB)\sin(c+dx)\cos(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^3}+\frac{x(a^2A-8abB+20Ab^2)}{2a^6}+\frac{b(-7a^3B+10a^2Ab+2ab^2B-5Ab^3)\sin(c+dx)\cos(c+dx)}{6a^2d(a^2-b^2)^2(a+b\sec(c+dx))^2}+\frac{b(-27a^5B+48a^4Ab+20a^3b^2B-53a^2Ab^3-8ab^4B+20Ab^5)\sin(c+dx)\cos(c+dx)}{6a^3d(a^2-b^2)^3(a+b\sec(c+dx))}+\frac{(a^6A+12a^5bB-23a^4Ab^2-11a^3b^3B+27a^2Ab^4+4ab^5B-10Ab^6)\sin(c+dx)\cos(c+dx)}{2a^4d(a^2-b^2)^3}-\frac{(-6a^7B+24a^6Ab+65a^5b^2B-146a^4Ab^3-68a^3b^4B+167a^2Ab^5+24ab^6B-60Ab^7)\sin(c+dx)}{6a^5d(a^2-b^2)^3}-\frac{b^2(-20a^7B+40a^6Ab+35a^5b^2B-84a^4Ab^3-28a^3b^4B+69a^2Ab^5+8ab^6B-20Ab^7)\tanh^{-1}\left(\frac{\sqrt{a-b}\tan(\frac{1}{2}(c+dx))}{\sqrt{a+b}}\right)}{a^6d(a-b)^{7/2}(a+b)^{7/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((a^2*A + 20*A*b^2 - 8*a*b*B)*x)/(2*a^6) - (b^2*(40*a^6*A*b - 84*a^4*A*b^3
+ 69*a^2*A*b^5 - 20*A*b^7 - 20*a^7*B + 35*a^5*b^2*B - 28*a^3*b^4*B + 8*a*b^
6*B)*ArcTanh[(Sqrt[a - b]*Tan[(c + d*x)/2])/Sqrt[a + b]])/(a^6*(a - b)^(7/2
)*(a + b)^(7/2)*d) - ((24*a^6*A*b - 146*a^4*A*b^3 + 167*a^2*A*b^5 - 60*A*b^
7 - 6*a^7*B + 65*a^5*b^2*B - 68*a^3*b^4*B + 24*a*b^6*B)*Sin[c + d*x])/(6*a^
5*(a^2 - b^2)^3*d) + ((a^6*A - 23*a^4*A*b^2 + 27*a^2*A*b^4 - 10*A*b^6 + 12*
a^5*b*B - 11*a^3*b^3*B + 4*a*b^5*B)*Cos[c + d*x]*Sin[c + d*x])/(2*a^4*(a^2
- b^2)^3*d) + (b*(A*b - a*B)*Cos[c + d*x]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*
(a + b*Sec[c + d*x])^3) + (b*(10*a^2*A*b - 5*A*b^3 - 7*a^3*B + 2*a*b^2*B)*C
os[c + d*x]*Sin[c + d*x])/(6*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^2) +
(b*(48*a^4*A*b - 53*a^2*A*b^3 + 20*A*b^5 - 27*a^5*B + 20*a^3*b^2*B - 8*a*b^
4*B)*Cos[c + d*x]*Sin[c + d*x])/(6*a^3*(a^2 - b^2)^3*d*(a + b*Sec[c + d*x])
)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
```

```
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^4}\,dx = \frac{b(Ab-aB)\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} - \frac{\int \frac{\cos^2(c+dx)\left(-3a^2A+5Ab^2-2abB\right.}{}}{}$$

$$= \frac{b(Ab-aB)\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b(10a^2Ab-5Ab^3-7a^3B}{6a^2\left(a^2-b^2\right)^2}$$

$$= \frac{b(Ab-aB)\cos(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^3} + \frac{b(10a^2Ab-5Ab^3-7a^3B}{6a^2\left(a^2-b^2\right)^2}$$

$$= \frac{(a^6A-23a^4Ab^2+27a^2Ab^4-10Ab^6+12a^5bB-11a^3b^3B+4ab^5}{2a^4\left(a^2-b^2\right)^3d}$$

$$= -\frac{(24a^6Ab-146a^4Ab^3+167a^2Ab^5-60Ab^7-6a^7B+65a^5b^2B}{6a^5\left(a^2-b^2\right)^3d}$$

$$= \frac{\left(a^2A+20Ab^2-8abB\right)x}{2a^6} - \frac{(24a^6Ab-146a^4Ab^3+167a^2Ab^5-6}{}$$

$$= \frac{\left(a^2A+20Ab^2-8abB\right)x}{2a^6} - \frac{(24a^6Ab-146a^4Ab^3+167a^2Ab^5-6}{}$$

$$= \frac{\left(a^2A+20Ab^2-8abB\right)x}{2a^6} - \frac{(24a^6Ab-146a^4Ab^3+167a^2Ab^5-6}{}$$

$$= \frac{\left(a^2A+20Ab^2-8abB\right)x}{2a^6} - \frac{b^2(40a^6Ab-84a^4Ab^3+69a^2Ab^5-2}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1452 vs. $2(538) = 1076$.

time = 5.99, size = 1452, normalized size = 2.70

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^4,x]
```

```
[Out] ((-96*b^2*(-40*a^6*A*b + 84*a^4*A*b^3 - 69*a^2*A*b^5 + 20*A*b^7 + 20*a^7*B
- 35*a^5*b^2*B + 28*a^3*b^4*B - 8*a*b^6*B)*ArcTanh[((-a + b)*Tan[(c + d*x)/
2])/Sqrt[a^2 - b^2]])/(a^2 - b^2)^(7/2) + (72*a^10*A*b*c + 1272*a^8*A*b^3*c
 - 3288*a^6*A*b^5*c + 1512*a^4*A*b^7*c + 1392*a^2*A*b^9*c - 960*A*b^11*c -
576*a^9*b^2*B*c + 1344*a^7*b^4*B*c - 576*a^5*b^6*B*c - 576*a^3*b^8*B*c + 38
4*a*b^10*B*c + 72*a^10*A*b*d*x + 1272*a^8*A*b^3*d*x - 3288*a^6*A*b^5*d*x +
1512*a^4*A*b^7*d*x + 1392*a^2*A*b^9*d*x - 960*A*b^11*d*x - 576*a^9*b^2*B*d*
x + 1344*a^7*b^4*B*d*x - 576*a^5*b^6*B*d*x - 576*a^3*b^8*B*d*x + 384*a*b^10
*B*d*x + 36*a*(a^2 - b^2)^3*(a^2 + 4*b^2)*(a^2*A + 20*A*b^2 - 8*a*b*B)*(c +
 d*x)*Cos[c + d*x] + 72*a^2*b*(a^2 - b^2)^3*(a^2*A + 20*A*b^2 - 8*a*b*B)*(c
 + d*x)*Cos[2*(c + d*x)] + 12*a^11*A*c*Cos[3*(c + d*x)] + 204*a^9*A*b^2*c*C
os[3*(c + d*x)] - 684*a^7*A*b^4*c*Cos[3*(c + d*x)] + 708*a^5*A*b^6*c*Cos[3*
(c + d*x)] - 240*a^3*A*b^8*c*Cos[3*(c + d*x)] - 96*a^10*b*B*c*Cos[3*(c + d*
x)] + 288*a^8*b^3*B*c*Cos[3*(c + d*x)] - 288*a^6*b^5*B*c*Cos[3*(c + d*x)] +
 96*a^4*b^7*B*c*Cos[3*(c + d*x)] + 12*a^11*A*d*x*Cos[3*(c + d*x)] + 204*a^9
*A*b^2*d*x*Cos[3*(c + d*x)] - 684*a^7*A*b^4*d*x*Cos[3*(c + d*x)] + 708*a^5*
A*b^6*d*x*Cos[3*(c + d*x)] - 240*a^3*A*b^8*d*x*Cos[3*(c + d*x)] - 96*a^10*b
*B*d*x*Cos[3*(c + d*x)] + 288*a^8*b^3*B*d*x*Cos[3*(c + d*x)] - 288*a^6*b^5*
B*d*x*Cos[3*(c + d*x)] + 96*a^4*b^7*B*d*x*Cos[3*(c + d*x)] + 6*a^11*A*Sin[c
 + d*x] - 270*a^9*A*b^2*Sin[c + d*x] + 750*a^7*A*b^4*Sin[c + d*x] + 1086*a^
5*A*b^6*Sin[c + d*x] - 2232*a^3*A*b^8*Sin[c + d*x] + 960*a*A*b^10*Sin[c + d
*x] + 72*a^10*b*B*Sin[c + d*x] - 360*a^8*b^3*B*Sin[c + d*x] - 540*a^6*b^5*B
*Sin[c + d*x] + 912*a^4*b^7*B*Sin[c + d*x] - 384*a^2*b^9*B*Sin[c + d*x] - 6
0*a^10*A*b*Sin[2*(c + d*x)] - 372*a^8*A*b^3*Sin[2*(c + d*x)] + 2772*a^6*A*b
^5*Sin[2*(c + d*x)] - 3300*a^4*A*b^7*Sin[2*(c + d*x)] + 1200*a^2*A*b^9*Sin[
2*(c + d*x)] + 24*a^11*B*Sin[2*(c + d*x)] + 72*a^9*b^2*B*Sin[2*(c + d*x)] -
 1200*a^7*b^4*B*Sin[2*(c + d*x)] + 1344*a^5*b^6*B*Sin[2*(c + d*x)] - 480*a^
3*b^8*B*Sin[2*(c + d*x)] + 9*a^11*A*Sin[3*(c + d*x)] - 279*a^9*A*b^2*Sin[3*
(c + d*x)] + 1143*a^7*A*b^4*Sin[3*(c + d*x)] - 1253*a^5*A*b^6*Sin[3*(c + d*
x)] + 440*a^3*A*b^8*Sin[3*(c + d*x)] + 72*a^10*b*B*Sin[3*(c + d*x)] - 456*a
^8*b^3*B*Sin[3*(c + d*x)] + 500*a^6*b^5*B*Sin[3*(c + d*x)] - 176*a^4*b^7*B*
Sin[3*(c + d*x)] - 30*a^10*A*b*Sin[4*(c + d*x)] + 90*a^8*A*b^3*Sin[4*(c + d
*x)] - 90*a^6*A*b^5*Sin[4*(c + d*x)] + 30*a^4*A*b^7*Sin[4*(c + d*x)] + 12*a
^11*B*Sin[4*(c + d*x)] - 36*a^9*b^2*B*Sin[4*(c + d*x)] + 36*a^7*b^4*B*Sin[4
*(c + d*x)] - 12*a^5*b^6*B*Sin[4*(c + d*x)] + 3*a^11*A*Sin[5*(c + d*x)] - 9
*a^9*A*b^2*Sin[5*(c + d*x)] + 9*a^7*A*b^4*Sin[5*(c + d*x)] - 3*a^5*A*b^6*Si
n[5*(c + d*x)])/((a^2 - b^2)^3*(b + a*Cos[c + d*x])^3))/(96*a^6*d)
```

Maple [A]
time = 0.90, size = 615, normalized size = 1.14

method	result
derivativedivides	$2b^2\left(-\frac{(30A\,a^4b+6A\,a^3b^2-34A\,a^2b^3-3Aa\,b^4+12A\,b^5-20B\,a^5-5B\,a^4b+18B\,a^3b^2+2B\,a^2b^3-6Ba\,b^4)ba\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)(a^3+3b\,a^2+3b^2a+b^3)}+\frac{2(45A\,a\cdots}{\cdots}\right.$
default	$2b^2\left(-\frac{(30A\,a^4b+6A\,a^3b^2-34A\,a^2b^3-3Aa\,b^4+12A\,b^5-20B\,a^5-5B\,a^4b+18B\,a^3b^2+2B\,a^2b^3-6Ba\,b^4)ba\left(\tan^5\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{2(a-b)(a^3+3b\,a^2+3b^2a+b^3)}+\frac{2(45A\,a\cdots}{\cdots}\right.$
risch	Expression too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x,method=_RETURNVERBOS
E)
```

```
[Out] 1/d*(2*b^2/a^6*((-1/2*(30*A*a^4*b+6*A*a^3*b^2-34*A*a^2*b^3-3*A*a*b^4+12*A*b
^5-20*B*a^5-5*B*a^4*b+18*B*a^3*b^2+2*B*a^2*b^3-6*B*a*b^4)*b*a/(a-b)/(a^3+3*
a^2*b+3*a*b^2+b^3)*tan(1/2*d*x+1/2*c)^5+2/3*(45*A*a^4*b-53*A*a^2*b^3+18*A*b
^5-30*B*a^5+29*B*a^3*b^2-9*B*a*b^4)*b*a/(a^2-2*a*b+b^2)/(a^2+2*a*b+b^2)*tan
(1/2*d*x+1/2*c)^3-1/2*(30*A*a^4*b-6*A*a^3*b^2-34*A*a^2*b^3+3*A*a*b^4+12*A*b
^5-20*B*a^5+5*B*a^4*b+18*B*a^3*b^2-2*B*a^2*b^3-6*B*a*b^4)*b*a/(a+b)/(a^3-3*
a^2*b+3*a*b^2-b^3)*tan(1/2*d*x+1/2*c))/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*
x+1/2*c)^2-a-b)^3-1/2*(40*A*a^6*b-84*A*a^4*b^3+69*A*a^2*b^5-20*A*b^7-20*B*a
^7+35*B*a^5*b^2-28*B*a^3*b^4+8*B*a*b^6)/(a^6-3*a^4*b^2+3*a^2*b^4-b^6)/((a+b
)*(a-b))^(1/2)*arctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))+2/a^6
*(((-1/2*a^2*A-4*A*b*a+a^2*B)*tan(1/2*d*x+1/2*c)^3+(1/2*a^2*A-4*A*b*a+a^2*B
)*tan(1/2*d*x+1/2*c))/(1+tan(1/2*d*x+1/2*c)^2)^2+1/2*(A*a^2+20*A*b^2-8*B*a*
b)*arctan(tan(1/2*d*x+1/2*c))))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="ma
xima")
```

[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de

Fricas [B] Leaf count of result is larger than twice the leaf count of optimal. 1416 vs. $2(516) = 1032$.
time = 2.98, size = 2890, normalized size = 5.37

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="fr
icas")
```

```
[Out] [1/12*(6*(A*a^13 - 8*B*a^12*b + 16*A*a^11*b^2 + 32*B*a^10*b^3 - 74*A*a^9*b^
4 - 48*B*a^8*b^5 + 116*A*a^7*b^6 + 32*B*a^6*b^7 - 79*A*a^5*b^8 - 8*B*a^4*b^
9 + 20*A*a^3*b^10)*d*x*cos(d*x + c)^3 + 18*(A*a^12*b - 8*B*a^11*b^2 + 16*A*
a^10*b^3 + 32*B*a^9*b^4 - 74*A*a^8*b^5 - 48*B*a^7*b^6 + 116*A*a^6*b^7 + 32*
B*a^5*b^8 - 79*A*a^4*b^9 - 8*B*a^3*b^10 + 20*A*a^2*b^11)*d*x*cos(d*x + c)^2
 + 18*(A*a^11*b^2 - 8*B*a^10*b^3 + 16*A*a^9*b^4 + 32*B*a^8*b^5 - 74*A*a^7*b
^6 - 48*B*a^6*b^7 + 116*A*a^5*b^8 + 32*B*a^4*b^9 - 79*A*a^3*b^10 - 8*B*a^2*
b^11 + 20*A*a*b^12)*d*x*cos(d*x + c) + 6*(A*a^10*b^3 - 8*B*a^9*b^4 + 16*A*a
^8*b^5 + 32*B*a^7*b^6 - 74*A*a^6*b^7 - 48*B*a^5*b^8 + 116*A*a^4*b^9 + 32*B*
a^3*b^10 - 79*A*a^2*b^11 - 8*B*a*b^12 + 20*A*b^13)*d*x - 3*(20*B*a^7*b^5 -
40*A*a^6*b^6 - 35*B*a^5*b^7 + 84*A*a^4*b^8 + 28*B*a^3*b^9 - 69*A*a^2*b^10 -
 8*B*a*b^11 + 20*A*b^12 + (20*B*a^10*b^2 - 40*A*a^9*b^3 - 35*B*a^8*b^4 + 84
*A*a^7*b^5 + 28*B*a^6*b^6 - 69*A*a^5*b^7 - 8*B*a^4*b^8 + 20*A*a^3*b^9)*cos(
d*x + c)^3 + 3*(20*B*a^9*b^3 - 40*A*a^8*b^4 - 35*B*a^7*b^5 + 84*A*a^6*b^6 +
 28*B*a^5*b^7 - 69*A*a^4*b^8 - 8*B*a^3*b^9 + 20*A*a^2*b^10)*cos(d*x + c)^2
+ 3*(20*B*a^8*b^4 - 40*A*a^7*b^5 - 35*B*a^6*b^6 + 84*A*a^5*b^7 + 28*B*a^4*b
^8 - 69*A*a^3*b^9 - 8*B*a^2*b^10 + 20*A*a*b^11)*cos(d*x + c))*sqrt(a^2 - b^
2)*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)*cos(d*x + c)^2 - 2*sqrt(a^2 - b^
2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a^2 - b^2)/(a^2*cos(d*x + c)^2 + 2
*a*b*cos(d*x + c) + b^2)) + 2*(6*B*a^10*b^3 - 24*A*a^9*b^4 - 71*B*a^8*b^5 +
 170*A*a^7*b^6 + 133*B*a^6*b^7 - 313*A*a^5*b^8 - 92*B*a^4*b^9 + 227*A*a^3*b
^10 + 24*B*a^2*b^11 - 60*A*a*b^12 + 3*(A*a^13 - 4*A*a^11*b^2 + 6*A*a^9*b^4
- 4*A*a^7*b^6 + A*a^5*b^8)*cos(d*x + c)^4 + 3*(2*B*a^13 - 5*A*a^12*b - 8*B*
a^11*b^2 + 20*A*a^10*b^3 + 12*B*a^9*b^4 - 30*A*a^8*b^5 - 8*B*a^7*b^6 + 20*A
*a^6*b^7 + 2*B*a^5*b^8 - 5*A*a^4*b^9)*cos(d*x + c)^3 + (18*B*a^12*b - 63*A*
a^11*b^2 - 132*B*a^10*b^3 + 342*A*a^9*b^4 + 239*B*a^8*b^5 - 590*A*a^7*b^6 -
 169*B*a^6*b^7 + 421*A*a^5*b^8 + 44*B*a^4*b^9 - 110*A*a^3*b^10)*cos(d*x + c
)^2 + 3*(6*B*a^11*b^2 - 23*A*a^10*b^3 - 59*B*a^9*b^4 + 146*A*a^8*b^5 + 110*
B*a^7*b^6 - 263*A*a^6*b^7 - 77*B*a^5*b^8 + 190*A*a^4*b^9 + 20*B*a^3*b^10 -
50*A*a^2*b^11)*cos(d*x + c))*sin(d*x + c))/((a^17 - 4*a^15*b^2 + 6*a^13*b^4
```

```
 - 4*a^11*b^6 + a^9*b^8)*d*cos(d*x + c)^3 + 3*(a^16*b - 4*a^14*b^3 + 6*a^12
*b^5 - 4*a^10*b^7 + a^8*b^9)*d*cos(d*x + c)^2 + 3*(a^15*b^2 - 4*a^13*b^4 +
6*a^11*b^6 - 4*a^9*b^8 + a^7*b^10)*d*cos(d*x + c) + (a^14*b^3 - 4*a^12*b^5
+ 6*a^10*b^7 - 4*a^8*b^9 + a^6*b^11)*d), 1/6*(3*(A*a^13 - 8*B*a^12*b + 16*A
*a^11*b^2 + 32*B*a^10*b^3 - 74*A*a^9*b^4 - 48*B*a^8*b^5 + 116*A*a^7*b^6 + 3
2*B*a^6*b^7 - 79*A*a^5*b^8 - 8*B*a^4*b^9 + 20*A*a^3*b^10)*d*x*cos(d*x + c)^
3 + 9*(A*a^12*b - 8*B*a^11*b^2 + 16*A*a^10*b^3 + 32*B*a^9*b^4 - 74*A*a^8*b^
5 - 48*B*a^7*b^6 + 116*A*a^6*b^7 + 32*B*a^5*b^8 - 79*A*a^4*b^9 - 8*B*a^3*b^
10 + 20*A*a^2*b^11)*d*x*cos(d*x + c)^2 + 9*(A*a^11*b^2 - 8*B*a^10*b^3 + 16*
A*a^9*b^4 + 32*B*a^8*b^5 - 74*A*a^7*b^6 - 48*B*a^6*b^7 + 116*A*a^5*b^8 + 32
*B*a^4*b^9 - 79*A*a^3*b^10 - 8*B*a^2*b^11 + 20*A*a*b^12)*d*x*cos(d*x + c) +
 3*(A*a^10*b^3 - 8*B*a^9*b^4 + 16*A*a^8*b^5 + 32*B*a^7*b^6 - 74*A*a^6*b^7 -
 48*B*a^5*b^8 + 116*A*a^4*b^9 + 32*B*a^3*b^10 - 79*A*a^2*b^11 - 8*B*a*b^12
+ 20*A*b^13)*d*x + 3*(20*B*a^7*b^5 - 40*A*a^6*b^6 - 35*B*a^5*b^7 + 84*A*a^4
*b^8 + 28*B*a^3*b^9 - 69*A*a^2*b^10 - 8*B*a*b^11 + 20*A*b^12 + (20*B*a^10*b
^2 - 40*A*a^9*b^3 - 35*B*a^8*b^4 + 84*A*a^7*b^5 + 28*B*a^6*b^6 - 69*A*a^5*b
^7 - 8*B*a^4*b^8 + 20*A*a^3*b^9)*cos(d*x + c)^3 + 3*(20*B*a^9*b^3 - 40*A*a^
8*b^4 - 35*B*a^7*b^5 + 84*A*a^6*b^6 + 28*B*a^5*b^7 - 69*A*a^4*b^8 - 8*B*a^3
*b^9 + 20*A*a^2*b^10)*cos(d*x + c)^2 + 3*(20*B*a^8*b^4 - 40*A*a^7*b^5 - 35*
B*a^6*b^6 + 84*A*a^5*b^7 + 28*B*a^4*b^8 - 69*A*a^3*b^9 - 8*B*a^2*b^10 + 20*
A*a*b^11)*cos(d*x + c))*sqrt(-a^2 + b^2)*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*
x + c) + a)/((a^2 - b^2)*sin(d*x + c))) + (6*B*a^10*b^3 - 24*A*a^9*b^4 - 71
*B*a^8*b^5 + 170*A*a^7*b^6 + 133*B*a^6*b^7 - 313*A*a^5*b^8 - 92*B*a^4*b^9 +
 227*A*a^3*b^10 + 24*B*a^2*b^11 - 60*A*a*b^12 + 3*(A*a^13 - 4*A*a^11*b^2 +
6*A*a^9*b^4 - 4*A*a^7*b^6 + A*a^5*b^8)*cos(d*x + c)^4 + 3*(2*B*a^13 - 5*A*a
^12*b - 8*B*a^11*b^2 + 20*A*a^10*b^3 + 12*B*a^9*b^4 - 30*A*a^8*b^5 - 8*B*a^
7*b^6 + 20*A*a^6*b^7 + 2*B*a^5*b^8 - 5*A*a^4*b^9)*cos(d*x + c)^3 + (18*B*a^
12*b - 63*A*a^11*b^2 - 132*B*a^10*b^3 + 342*A*a^9*b^4 + 239*B*a^8*b^5 - 590
*A*a^7*b^6 - 169*B*a^6*b^7 + 421*A*a^5*b^8 + 44*B*a^4*b^9 - 110*A*a^3*b^10)
*cos(d*x + c)^2 + 3*(6*B*a^11*b^2 - 23*A*a^10*b^3 - 59*B*a^9*b^4 + 146*A*a^
8*b^5 + 110*B*a^7*b^6 - 263*A*a^6*b^7 - 77*B*a^5*b^8 + 190*A*a^4*b^9 + 20*B
*a^3*b^10 - 50*A*a^2*b^11)*cos(d*x + c))*sin(d*x + c))/((a^17 - 4*a^15*b^2
+ 6*a^13*b^4 - 4*a^11*b^6 + a^9*b^8)*d*cos(d*x + c)^3 + 3*(a^16*b - 4*a^14*
b^3 + 6*a^12*b^5 - 4*a^10*b^7 + a^8*b^9)*d*cos(d*x + c)^2 + 3*(a^15*b^2 - 4
*a^13*b^4 + 6*a^11*b^6 - 4*a^9*b^8 + a^7*b^10)*d*cos(d*x + c) + (a^14*b^3 -
 4*a^12*b^5 + 6*a^10*b^7 - 4*a^8*b^9 + a^6*b^11)*d)]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c+dx))\cos^2(c+dx)}{(a+b\sec(c+dx))^4}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**4,x)
```

[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/(a + b*sec(c + d*x))**4, x)

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 1052 vs. $2(516) = 1032$.
time = 0.57, size = 1052, normalized size = 1.96

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^4,x, algorithm="giac")

[Out] 1/6*(6*(20*B*a^7*b^2 - 40*A*a^6*b^3 - 35*B*a^5*b^4 + 84*A*a^4*b^5 + 28*B*a^3*b^6 - 69*A*a^2*b^7 - 8*B*a*b^8 + 20*A*b^9)*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sgn(-2*a + 2*b) + arctan(-(a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqrt(-a^2 + b^2)))/((a^12 - 3*a^10*b^2 + 3*a^8*b^4 - a^6*b^6)*sqrt(-a^2 + b^2)) + 2*(60*B*a^7*b^3*tan(1/2*d*x + 1/2*c)^5 - 90*A*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 - 105*B*a^6*b^4*tan(1/2*d*x + 1/2*c)^5 + 162*A*a^5*b^5*tan(1/2*d*x + 1/2*c)^5 - 24*B*a^5*b^5*tan(1/2*d*x + 1/2*c)^5 + 48*A*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 + 117*B*a^4*b^6*tan(1/2*d*x + 1/2*c)^5 - 213*A*a^3*b^7*tan(1/2*d*x + 1/2*c)^5 - 24*B*a^3*b^7*tan(1/2*d*x + 1/2*c)^5 + 48*A*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 - 42*B*a^2*b^8*tan(1/2*d*x + 1/2*c)^5 + 81*A*a*b^9*tan(1/2*d*x + 1/2*c)^5 + 18*B*a*b^9*tan(1/2*d*x + 1/2*c)^5 - 36*A*b^10*tan(1/2*d*x + 1/2*c)^5 - 120*B*a^7*b^3*tan(1/2*d*x + 1/2*c)^3 + 180*A*a^6*b^4*tan(1/2*d*x + 1/2*c)^3 + 236*B*a^5*b^5*tan(1/2*d*x + 1/2*c)^3 - 392*A*a^4*b^6*tan(1/2*d*x + 1/2*c)^3 - 152*B*a^3*b^7*tan(1/2*d*x + 1/2*c)^3 + 284*A*a^2*b^8*tan(1/2*d*x + 1/2*c)^3 + 36*B*a*b^9*tan(1/2*d*x + 1/2*c)^3 - 72*A*b^10*tan(1/2*d*x + 1/2*c)^3 + 60*B*a^7*b^3*tan(1/2*d*x + 1/2*c) - 90*A*a^6*b^4*tan(1/2*d*x + 1/2*c) + 105*B*a^6*b^4*tan(1/2*d*x + 1/2*c) - 162*A*a^5*b^5*tan(1/2*d*x + 1/2*c) - 24*B*a^5*b^5*tan(1/2*d*x + 1/2*c) + 48*A*a^4*b^6*tan(1/2*d*x + 1/2*c) - 117*B*a^4*b^6*tan(1/2*d*x + 1/2*c) + 213*A*a^3*b^7*tan(1/2*d*x + 1/2*c) - 24*B*a^3*b^7*tan(1/2*d*x + 1/2*c) + 48*A*a^2*b^8*tan(1/2*d*x + 1/2*c) + 42*B*a^2*b^8*tan(1/2*d*x + 1/2*c) - 81*A*a*b^9*tan(1/2*d*x + 1/2*c) + 18*B*a*b^9*tan(1/2*d*x + 1/2*c) - 36*A*b^10*tan(1/2*d*x + 1/2*c))/((a^11 - 3*a^9*b^2 + 3*a^7*b^4 - a^5*b^6)*(a*tan(1/2*d*x + 1/2*c)^2 - b*tan(1/2*d*x + 1/2*c)^2 - a - b)^3) + 3*(A*a^2 - 8*B*a*b + 20*A*b^2)*(d*x + c)/a^6 - 6*(A*a*tan(1/2*d*x + 1/2*c)^3 - 2*B*a*tan(1/2*d*x + 1/2*c)^3 + 8*A*b*tan(1/2*d*x + 1/2*c)^3 - A*a*tan(1/2*d*x + 1/2*c) - 2*B*a*tan(1/2*d*x + 1/2*c) + 8*A*b*tan(1/2*d*x + 1/2*c))/((tan(1/2*d*x + 1/2*c)^2 + 1)^2*a^5))/d

Mupad [B]
time = 15.82, size = 2500, normalized size = 4.65

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^4,x)
[Out] ((tan(c/2 + (d*x)/2)*(A*a^8 + 20*A*b^8 + 2*B*a^8 - 59*A*a^2*b^6 - 27*A*a^3*
b^5 + 57*A*a^4*b^4 + 21*A*a^5*b^3 - 11*A*a^6*b^2 - 4*B*a^2*b^6 + 24*B*a^3*b
^5 + 11*B*a^4*b^4 - 26*B*a^5*b^3 - 6*B*a^6*b^2 + 10*A*a*b^7 - 7*A*a^7*b - 8
*B*a*b^7 + 2*B*a^7*b))/(a^5*(a + b)*(a - b)^3) + (2*tan(c/2 + (d*x)/2)^5*(9
*A*a^10 + 180*A*b^10 - 611*A*a^2*b^8 + 740*A*a^4*b^6 - 324*A*a^6*b^4 + 36*A
*a^8*b^2 + 248*B*a^3*b^7 - 320*B*a^5*b^5 + 132*B*a^7*b^3 - 72*B*a*b^9 - 18*
B*a^9*b))/(3*a^5*(a + b)^3*(a - b)^3) + (tan(c/2 + (d*x)/2)^9*(A*a^8 + 20*A
*b^8 - 2*B*a^8 - 59*A*a^2*b^6 + 27*A*a^3*b^5 + 57*A*a^4*b^4 - 21*A*a^5*b^3
- 11*A*a^6*b^2 + 4*B*a^2*b^6 + 24*B*a^3*b^5 - 11*B*a^4*b^4 - 26*B*a^5*b^3 +
 6*B*a^6*b^2 - 10*A*a*b^7 + 7*A*a^7*b - 8*B*a*b^7 + 2*B*a^7*b))/(a^5*(a + b
)^3*(a - b)) - (2*tan(c/2 + (d*x)/2)^3*(6*A*a^9 - 120*A*b^9 + 6*B*a^9 + 364
*A*a^2*b^7 + 71*A*a^3*b^6 - 369*A*a^4*b^5 - 45*A*a^5*b^4 + 111*A*a^6*b^3 +
3*A*a^7*b^2 + 12*B*a^2*b^7 - 148*B*a^3*b^6 - 29*B*a^4*b^5 + 159*B*a^5*b^4 +
 18*B*a^6*b^3 - 30*B*a^7*b^2 - 30*A*a*b^8 - 21*A*a^8*b + 48*B*a*b^8 - 6*B*a
^8*b))/(3*a^5*(a + b)^2*(a - b)^3) - (2*tan(c/2 + (d*x)/2)^7*(6*A*a^9 + 120
*A*b^9 - 6*B*a^9 - 364*A*a^2*b^7 + 71*A*a^3*b^6 + 369*A*a^4*b^5 - 45*A*a^5*
b^4 - 111*A*a^6*b^3 + 3*A*a^7*b^2 + 12*B*a^2*b^7 + 148*B*a^3*b^6 - 29*B*a^4
*b^5 - 159*B*a^5*b^4 + 18*B*a^6*b^3 + 30*B*a^7*b^2 - 30*A*a*b^8 + 21*A*a^8*
b - 48*B*a*b^8 - 6*B*a^8*b))/(3*a^5*(a + b)^3*(a - b)^2))/(d*(tan(c/2 + (d*
x)/2)^2*(9*a*b^2 + 3*a^2*b - a^3 + 5*b^3) + tan(c/2 + (d*x)/2)^4*(6*a*b^2 -
 6*a^2*b - 2*a^3 + 10*b^3) - tan(c/2 + (d*x)/2)^6*(6*a*b^2 + 6*a^2*b - 2*a^
3 - 10*b^3) + 3*a*b^2 + 3*a^2*b + a^3 + b^3 - tan(c/2 + (d*x)/2)^10*(3*a*b^
2 - 3*a^2*b + a^3 - b^3) + tan(c/2 + (d*x)/2)^8*(3*a^2*b - 9*a*b^2 + a^3 +
5*b^3))) - (atan(((((8*tan(c/2 + (d*x)/2)*(800*A^2*a*b^17 - 800*A^2*b^18 -
A^2*a^18 + 2*A^2*a^17*b + 4720*A^2*a^2*b^16 - 4720*A^2*a^3*b^15 - 11522*A^2
*a^4*b^14 + 11522*A^2*a^5*b^13 + 14837*A^2*a^6*b^12 - 14812*A^2*a^7*b^11 -
10385*A^2*a^8*b^10 + 10430*A^2*a^9*b^9 + 3325*A^2*a^10*b^8 - 3640*A^2*a^11*
b^7 + 45*A^2*a^12*b^6 + 350*A^2*a^13*b^5 - 209*A^2*a^14*b^4 + 68*A^2*a^15*b
^3 - 35*A^2*a^16*b^2 - 128*B^2*a^2*b^16 + 128*B^2*a^3*b^15 + 768*B^2*a^4*b^
14 - 768*B^2*a^5*b^13 - 1920*B^2*a^6*b^12 + 1920*B^2*a^7*b^11 + 2600*B^2*a^
8*b^10 - 2560*B^2*a^9*b^9 - 2025*B^2*a^10*b^8 + 1920*B^2*a^11*b^7 + 824*B^2
*a^12*b^6 - 768*B^2*a^13*b^5 - 80*B^2*a^14*b^4 + 128*B^2*a^15*b^3 - 64*B^2*
a^16*b^2 + 640*A*B*a*b^17 + 16*A*B*a^17*b - 640*A*B*a^2*b^16 - 3808*A*B*a^3
*b^15 + 3808*A*B*a^4*b^14 + 9408*A*B*a^5*b^13 - 9408*A*B*a^6*b^12 - 12430*A
*B*a^7*b^11 + 12320*A*B*a^8*b^10 + 9200*A*B*a^9*b^9 - 8960*A*B*a^10*b^8 - 3
360*A*B*a^11*b^7 + 3360*A*B*a^12*b^6 + 144*A*B*a^13*b^5 - 448*A*B*a^14*b^4
+ 240*A*B*a^15*b^3 - 32*A*B*a^16*b^2))/(a^20*b + a^21 - a^10*b^11 - a^11*b^
10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10
*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2) - (((4*(4*A*a^27 - 80*A*a^12*b^15 + 40
*A*a^13*b^14 + 516*A*a^14*b^13 - 248*A*a^15*b^12 - 1404*A*a^16*b^11 + 640*A
*a^17*b^10 + 2076*A*a^18*b^9 - 896*A*a^19*b^8 - 1764*A*a^20*b^7 + 724*A*a^2
1*b^6 + 816*A*a^22*b^5 - 316*A*a^23*b^4 - 160*A*a^24*b^3 + 52*A*a^25*b^2 +
32*B*a^13*b^14 - 16*B*a^14*b^13 - 208*B*a^15*b^12 + 100*B*a^16*b^11 + 572*B
```

*a^17*b^10 - 252*B*a^18*b^9 - 868*B*a^19*b^8 + 348*B*a^20*b^7 + 772*B*a^21*b^6 - 292*B*a^22*b^5 - 380*B*a^23*b^4 + 144*B*a^24*b^3 + 80*B*a^25*b^2 - 32*B*a^26*b))/(a^25*b + a^26 - a^15*b^11 - a^16*b^10 + 5*a^17*b^9 + 5*a^18*b^8 - 10*a^19*b^7 - 10*a^20*b^6 + 10*a^21*b^5 + 10*a^22*b^4 - 5*a^23*b^3 - 5*a^24*b^2) - (4*tan(c/2 + (d*x)/2)*(A*a^2*1i + A*b^2*20i - B*a*b*8i)*(8*a^25*b - 8*a^12*b^14 + 8*a^13*b^13 + 48*a^14*b^12 - 48*a^15*b^11 - 120*a^16*b^10 + 120*a^17*b^9 + 160*a^18*b^8 - 160*a^19*b^7 - 120*a^20*b^6 + 120*a^21*b^5 + 48*a^22*b^4 - 48*a^23*b^3 - 8*a^24*b^2))/(a^6*(a^20*b + a^21 - a^10*b^11 - a^11*b^10 + 5*a^12*b^9 + 5*a^13*b^8 - 10*a^14*b^7 - 10*a^15*b^6 + 10*a^16*b^5 + 10*a^17*b^4 - 5*a^18*b^3 - 5*a^19*b^2)))*(A*a^2*1i + A*b^2*20i - B*a*b*8i))/(2*a^6))*(A*a^2*1i + A*b^2*20i - B*a*b*8i)*1i)/(2*a^6) + (((8*tan(c/2 + (d*x)/2)*(800*A^2*a*b^17 - 800*A^2*b^18 - A^2*a^18 + 2*A^2*a^17*b + 4720*A^2*a^2*b^16 - 4720*A^2*a^3*b^15 - 11522*A^2*a^4*b^14 + 11522*A^2*a^5*b^13 + 14837*A^2*a^6*b^12 - 14812*A^2*a^7*b^11 - 10385*A^2*a^8*b^10 + 10430*A^2*a^9*b^9 + 3325*A^2*a^10*b^8 - 3640*A^2*a^11*b^7 + 45*A^2*a^12*b^6 + 350*A^2*a^13*b^5 - 209*A^2*a^14*b^4 + 68*A^2*a^15*b^3 - 35*A^2*a^16*b^2 - 128*B^2*a^2*b^16 + 128*B^2*a^3*b^15 + 768*B^2*a^4*b^14 - 768*B^2*a^5*b^13 - 1920*B^2*a^6*b^12 + 1920*B^2*a^7*b^11 + 2600*B^2*a^8*b^10 - 2560*B^2*a^9*b^9 - 2025*B^2*a^10*b^8 + 1920*B^2*a^11*b^7 + 824*B^2*a^12*b^6 - 768*B^2*a^13*b^5 - 80*B^2*a^14*b^4 + 128*B^2*a^15*b^3 - 64*B^2*a^16*b^2 + 640*A*B*a*b^17 + 16*A*B*a^17*b - 640*A*B*a^2*b^16 - 3808*A*B*a^3*b^15 + 3808*A*B*a^4*b^14 + 9408*A*B*a^5*b^13 - 9408*A*B*a^6*b^12 - 12430*A*B*a^7*b^11 + 12320*A*B*a^8*b^10 + 9200*A*B*a^9*b^9 - 8960*A*B*a^10*b^8 - 3360*A*B*a^11*b^7 + 3360*A*B*a^12*b^6 + 144*A*B*a^13*b^5 - 448*A*B*a^14*b^...

3.344 $\int \frac{\frac{bB}{a}+B\sec(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=61

$$\frac{bBx}{a^2}+\frac{2\sqrt{a-b}\,\sqrt{a+b}\,B\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d}$$

```
[Out] b*B*x/a^2+2*B*arctanh((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))*(a-b)^(1/
2)*(a+b)^(1/2)/a^2/d
```

Rubi [A]
time = 0.08, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {4004, 3916, 2738, 214}

$$\frac{2B\sqrt{a-b}\,\sqrt{a+b}\,\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d}+\frac{bBx}{a^2}$$

Antiderivative was successfully verified.

```
[In] Int[((b*B)/a + B*Sec[c + d*x])/(a + b*Sec[c + d*x]),x]
```

```
[Out] (b*B*x)/a^2 + (2*Sqrt[a - b]*Sqrt[a + b]*B*ArcTanh[(Sqrt[a - b]*Tan[(c + d*
x)/2])/Sqrt[a + b]])/(a^2*d)
```

Rule 214

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[-a/b, 2]/a)*ArcTanh[x
/Rt[-a/b, 2]], x] /; FreeQ[{a, b}, x] && NegQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{\frac{bB}{a}+B\sec(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{bBx}{a^2}-\frac{\left(-aB+\frac{b^2B}{a}\right)\int\frac{\sec(c+dx)}{a+b\sec(c+dx)}\,dx}{a}\\
&= \frac{bBx}{a^2}-\frac{\left(-aB+\frac{b^2B}{a}\right)\int\frac{1}{1+\frac{a\cos(c+dx)}{b}}\,dx}{ab}\\
&= \frac{bBx}{a^2}-\frac{\left(2\left(-aB+\frac{b^2B}{a}\right)\right)\text{Subst}\left(\int\frac{1}{1+\frac{a}{b}+\left(1-\frac{a}{b}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{abd}\\
&= \frac{bBx}{a^2}+\frac{2\sqrt{a-b}\,\sqrt{a+b}\,B\tanh^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{a^2d}
\end{aligned}$$

Mathematica [A]
time = 0.15, size = 61, normalized size = 1.00

$$\frac{B\left(b(c+dx)-2\sqrt{a^2-b^2}\,\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a^2-b^2}}\right)\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((b*B)/a + B*Sec[c + d*x])/(a + b*Sec[c + d*x]),x]
```

```
[Out] (B*(b*(c + d*x) - 2*Sqrt[a^2 - b^2]*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqr
t[a^2 - b^2]]))/(a^2*d)
```

Maple [A]
time = 0.35, size = 76, normalized size = 1.25

method	result	size
derivativedivides	$\frac{2B\left(\frac{b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{(a+b)(a-b)\operatorname{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}\right)}{da}$	76

default	$\frac{2B\left(\frac{b\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{a}+\frac{(a+b)(a-b)\,\mathrm{arctanh}\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{a\sqrt{(a+b)(a-b)}}\right)}{da}$	7
risch	$\frac{bBx}{a^2}+\frac{\sqrt{a^2-b^2}\,B\ln\left(\mathrm{e}^{i(dx+c)}+\frac{i\sqrt{a^2-b^2}+b}{a}\right)}{d\,a^2}-\frac{\sqrt{a^2-b^2}\,B\ln\left(\mathrm{e}^{i(dx+c)}-\frac{i\sqrt{a^2-b^2}-b}{a}\right)}{d\,a^2}$	1

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((b*B/a+B*sec(d*x+c))/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*B/a*(b/a*arctan(tan(1/2*d*x+1/2*c))+(a+b)*(a-b)/a/((a+b)*(a-b))^(1/2)*a
rctanh((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.58, size = 197, normalized size = 3.23

$$\left[\frac{2\,Bbdx+\sqrt{a^2-b^2}\,B\log\left(\frac{2\,ab\cos(dx+c)-(a^2-2\,b^2)\cos(dx+c)^2+2\,\sqrt{a^2-b^2}\,(b\cos(dx+c)+a)\sin(dx+c)+2\,a^2-b^2}{a^2\cos(dx+c)^2+2\,ab\cos(dx+c)+b^2}\right)}{2\,a^2d},\ \frac{Bbdx+\sqrt{-a^2+b^2}\,B\arctan\left(-\frac{\sqrt{-a^2+b^2}\,(b\cos(dx+c)+a)}{(a^2-b^2)\sin(dx+c)}\right)}{a^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] [1/2*(2*B*b*d*x + sqrt(a^2 - b^2)*B*log((2*a*b*cos(d*x + c) - (a^2 - 2*b^2)
*cos(d*x + c)^2 + 2*sqrt(a^2 - b^2)*(b*cos(d*x + c) + a)*sin(d*x + c) + 2*a
^2 - b^2)/(a^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + b^2)))/(a^2*d), (B*b*d
*x + sqrt(-a^2 + b^2)*B*arctan(-sqrt(-a^2 + b^2)*(b*cos(d*x + c) + a)/((a^2
 - b^2)*sin(d*x + c))))/(a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{B\left(\int \frac{b}{a+b\sec{(c+dx)}}\,dx+\int \frac{a\sec{(c+dx)}}{a+b\sec{(c+dx)}}\,dx\right)}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] B*(Integral(b/(a + b*sec(c + d*x)), x) + Integral(a*sec(c + d*x)/(a + b*sec
(c + d*x)), x))/a
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 187 vs. 2(52) = 104.
time = 0.49, size = 187, normalized size = 3.07

$$\frac{2\left(\frac{\sqrt{-a^2+b^2}\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-\frac{ab+\sqrt{a^2b^2+(a^2+ab)(a^2-ab)}}{a^2-ab}}}\right)\right)B|-a+b|}{a^3-a^2b}+\frac{\left(\pi\left\lfloor\frac{dx+c}{2\pi}+\frac{1}{2}\right\rfloor+\arctan\left(\frac{\tan\left(\frac{1}{2}dx+\frac{1}{2}c\right)}{\sqrt{-\frac{ab-\sqrt{a^2b^2+(a^2+ab)(a^2-ab)}}{a^2-ab}}}\right)\right)Bb}{a^2}\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((b*B/a+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 2*(sqrt(-a^2 + b^2)*(pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan(tan(1/2*d*x
+ 1/2*c)/sqrt(-(a*b + sqrt(a^2*b^2 + (a^2 + a*b)*(a^2 - a*b)))/(a^2 - a*b))
))*B*abs(-a + b)/(a^3 - a^2*b) + (pi*floor(1/2*(d*x + c)/pi + 1/2) + arctan
(tan(1/2*d*x + 1/2*c)/sqrt(-(a*b - sqrt(a^2*b^2 + (a^2 + a*b)*(a^2 - a*b)))
/(a^2 - a*b))))*B*b/a^2)/d
```

Mupad [B]
time = 2.56, size = 91, normalized size = 1.49

$$\frac{2\,B\,b\,\mathrm{atan}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)}\right)}{a^2\,d}+\frac{2\,B\,\mathrm{atanh}\left(\frac{\sin\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,\sqrt{a^2-b^2}}{\cos\left(\frac{c}{2}+\frac{d\,x}{2}\right)\,(a+b)}\right)\,\sqrt{a^2-b^2}}{a^2\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B/cos(c + d*x) + (B*b)/a)/(a + b/cos(c + d*x)),x)
```

```
[Out] (2*B*b*atan(sin(c/2 + (d*x)/2)/cos(c/2 + (d*x)/2)))/(a^2*d) + (2*B*atanh((s
in(c/2 + (d*x)/2)*(a^2 - b^2)^(1/2))/(cos(c/2 + (d*x)/2)*(a + b)))*(a^2 - b
^2)^(1/2))/(a^2*d)
```

3.345 $\int \frac{\frac{aB}{b}+B\sec(c+dx)}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=6

$$\frac{Bx}{b}$$

```
[Out] B*x/b
```

Rubi [A]
time = 0.00, antiderivative size = 6, normalized size of antiderivative = 1.00, number of steps used = 2, number of rules used = 2, integrand size = 28, $\frac{\text{number of rules}}{\text{integrand size}} = 0.071$, Rules used = {21, 8}

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Int[((a*B)/b + B*Sec[c + d*x])/(a + b*Sec[c + d*x]),x]
```

```
[Out] (B*x)/b
```

Rule 8

```
Int[a_, x_Symbol] :> Simp[a*x, x] /; FreeQ[a, x]
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rubi steps

$$\begin{aligned} \int \frac{\frac{aB}{b}+B\sec(c+dx)}{a+b\sec(c+dx)}\,dx &= \frac{B\int 1\,dx}{b}\\ &= \frac{Bx}{b} \end{aligned}$$

Mathematica [A]
time = 0.00, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a*B)/b + B*Sec[c + d*x])/(a + b*Sec[c + d*x]),x]
```

```
[Out] (B*x)/b
```

Maple [A]
time = 0.12, size = 7, normalized size = 1.17

method	result	size
default	$\frac{Bx}{b}$	7
norman	$\frac{Bx}{b}$	7
risch	$\frac{Bx}{b}$	7
derivativedivides	$\frac{B(dx+c)}{db}$	14

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a*B/b+B*sec(d*x+c))/(a+b*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] B*x/b
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] Exception raised: ValueError >> Computation failed since Maxima requested a
dditional constraints; using the 'assume' command before evaluation *may* h
elp (example of legal syntax is 'assume(4*a^2-4*b^2>0)', see `assume?` for
more de
```

Fricas [A]
time = 2.04, size = 6, normalized size = 1.00

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] B*x/b
```

Sympy [A]
time = 3.27, size = 3, normalized size = 0.50

$$\frac{Bx}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] B*x/b
```

Giac [B] Leaf count of result is larger than twice the leaf count of optimal. 13 vs. 2(6) = 12.
time = 0.51, size = 13, normalized size = 2.17

$$\frac{(dx+c)B}{bd}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a*B/b+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] (d*x + c)*B/(b*d)
```

Mupad [B]
time = 2.23, size = 6, normalized size = 1.00

$$\frac{B\,x}{b}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((B/cos(c + d*x) + (B*a)/b)/(a + b/cos(c + d*x)),x)
```

```
[Out] (B*x)/b
```

3.346 $\int \frac{a+b\sec(c+dx)}{(b+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=86

$$\frac{ax}{b^2}-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d}-\frac{a\tan(c+dx)}{bd(b+a\sec(c+dx))}$$

```
[Out] a*x/b^2-2*arctan((a-b)^(1/2)*tan(1/2*d*x+1/2*c)/(a+b)^(1/2))*(a-b)^(1/2)*(a
+b)^(1/2)/b^2/d-a*tan(d*x+c)/b/d/(b+a*sec(d*x+c))
```

Rubi [A]
time = 0.12, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 23, $\frac{\text{number of rules}}{\text{integrand size}} = 0.217$, Rules used = {4008, 4004, 3916, 2738, 211}

$$-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\text{ArcTan}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d}+\frac{ax}{b^2}-\frac{a\tan(c+dx)}{bd(a\sec(c+dx)+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])/(b + a*Sec[c + d*x])^2,x]
```

```
[Out] (a*x)/b^2 - (2*Sqrt[a - b]*Sqrt[a + b]*ArcTan[(Sqrt[a - b]*Tan[(c + d*x)/2]
)/Sqrt[a + b]])/(b^2*d) - (a*Tan[c + d*x])/(b*d*(b + a*Sec[c + d*x]))
```

Rule 211

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(Rt[a/b, 2]/a)*ArcTan[x/R
t[a/b, 2]], x] /; FreeQ[{a, b}, x] && PosQ[a/b]
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rule 4008

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rubi steps

$$\begin{aligned}\int \frac{a+b\sec(c+dx)}{(b+a\sec(c+dx))^2}\,dx &= -\frac{a\tan(c+dx)}{bd(b+a\sec(c+dx))}+\frac{\int \frac{a(a^2-b^2)+b(a^2-b^2)\sec(c+dx)}{b+a\sec(c+dx)}\,dx}{b(a^2-b^2)}\\ &= \frac{ax}{b^2}-\frac{a\tan(c+dx)}{bd(b+a\sec(c+dx))}-\frac{(a^2-b^2)\int \frac{\sec(c+dx)}{b+a\sec(c+dx)}\,dx}{b^2}\\ &= \frac{ax}{b^2}-\frac{a\tan(c+dx)}{bd(b+a\sec(c+dx))}-\frac{(a^2-b^2)\int \frac{1}{1+\frac{b\cos(c+dx)}{a}}\,dx}{ab^2}\\ &= \frac{ax}{b^2}-\frac{a\tan(c+dx)}{bd(b+a\sec(c+dx))}-\frac{(2(a^2-b^2))\,\mathrm{Subst}\left(\int \frac{1}{1+\frac{b}{a}+\left(1-\frac{b}{a}\right)x^2}\,dx,x,\tan\left(\frac{1}{2}(\right.\right.}{ab^2d}\\ &= \frac{ax}{b^2}-\frac{2\sqrt{a-b}\,\sqrt{a+b}\,\tan^{-1}\left(\frac{\sqrt{a-b}\,\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{a+b}}\right)}{b^2d}-\frac{a\tan(c+dx)}{bd(b+a\sec(c+dx))}\end{aligned}$$

Mathematica [A]
time = 0.39, size = 97, normalized size = 1.13

$$\frac{2\sqrt{-a^2+b^2}\,\tanh^{-1}\left(\frac{(-a+b)\tan\left(\frac{1}{2}(c+dx)\right)}{\sqrt{-a^2+b^2}}\right)+\frac{a(ac+adx+b(c+dx)\cos(c+dx)-b\sin(c+dx))}{a+b\cos(c+dx)}}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(a + b*Sec[c + d*x])/(b + a*Sec[c + d*x])^2,x]
```

[Out] (2*Sqrt[-a^2 + b^2]*ArcTanh[((-a + b)*Tan[(c + d*x)/2])/Sqrt[-a^2 + b^2]] + (a*(a*c + a*d*x + b*(c + d*x)*Cos[c + d*x] - b*Sin[c + d*x]))/(a + b*Cos[c + d*x]))/(b^2*d)

Maple [A]
time = 0.33, size = 120, normalized size = 1.40

method	result
derivativedivides	$\frac{-\frac{2\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{\left(a^2-b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2}+\frac{2a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}}{d}$
default	$\frac{-\frac{2\left(\frac{ab\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{a\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-b\left(\tan^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+a+b}+\frac{\left(a^2-b^2\right)\arctan\left(\frac{(a-b)\tan\left(\frac{dx}{2}+\frac{c}{2}\right)}{\sqrt{(a+b)(a-b)}}\right)}{\sqrt{(a+b)(a-b)}}\right)}{b^2}+\frac{2a\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{b^2}}{d}$
risch	$\frac{ax}{b^2}-\frac{2ia\left(e^{i(dx+c)}a+b\right)}{b^2d\left(b\,e^{2i(dx+c)}+2\,e^{i(dx+c)}a+b\right)}+\frac{\sqrt{-a^2+b^2}\,\ln\left(e^{i(dx+c)}+\frac{i\sqrt{-a^2+b^2}+a}{b}\right)}{d\,b^2}-\frac{\sqrt{-a^2+b^2}\,\ln}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))/(b+a*sec(d*x+c))^2,x,method=_RETURNVERBOSE)

[Out] 1/d*(-2/b^2*(a*b*tan(1/2*d*x+1/2*c)/(a*tan(1/2*d*x+1/2*c)^2-b*tan(1/2*d*x+1/2*c)^2+a+b)+(a^2-b^2)/((a+b)*(a-b))^(1/2)*arctan((a-b)*tan(1/2*d*x+1/2*c)/((a+b)*(a-b))^(1/2)))+2/b^2*a*arctan(tan(1/2*d*x+1/2*c)))

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: ValueError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))/(b+a*sec(d*x+c))^2,x, algorithm="maxima")

[Out] Exception raised: ValueError >> Computation failed since Maxima requested additional constraints; using the 'assume' command before evaluation *may* help (example of legal syntax is 'assume(4*b^2-4*a^2>0)', see `assume?` for more de

Fricas [A]
time = 4.00, size = 279, normalized size = 3.24

$$\left[\frac{2\,abdx\cos\left(dx+c\right)+2\,a^2dx-2\,ab\sin\left(dx+c\right)+\sqrt{-a^2+b^2}\left(b\cos\left(dx+c\right)+a\right)\log\left(\frac{2\,ab\cos(dx+c)+(2\,a^2-b^2)\cos(dx+c)^2+2\sqrt{-a^2+b^2}\,(a\cos(dx+c)+b)\sin(dx+c)-a^2+2\,b^2}{b^2\cos(dx+c)^2+2\,ab\cos(dx+c)+a^2}\right)}{2\left(b^3d\cos\left(dx+c\right)+ab^2d\right)},\ \frac{abdx\cos\left(dx+c\right)+a^2dx-ab\sin\left(dx+c\right)-\sqrt{a^2-b^2}\left(b\cos\left(dx+c\right)+a\right)\arctan\left(-\frac{a\cos(dx+c)+b}{\sqrt{a^2-b^2}\,\sin(dx+c)}\right)}{b^3d\cos\left(dx+c\right)+ab^2d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/(b+a*sec(d*x+c))^2,x, algorithm="fricas")
```

```
[Out] [1/2*(2*a*b*d*x*cos(d*x + c) + 2*a^2*d*x - 2*a*b*sin(d*x + c) + sqrt(-a^2 +
 b^2)*(b*cos(d*x + c) + a)*log((2*a*b*cos(d*x + c) + (2*a^2 - b^2)*cos(d*x
+ c)^2 + 2*sqrt(-a^2 + b^2)*(a*cos(d*x + c) + b)*sin(d*x + c) - a^2 + 2*b^2
)/(b^2*cos(d*x + c)^2 + 2*a*b*cos(d*x + c) + a^2)))/(b^3*d*cos(d*x + c) + a
*b^2*d), (a*b*d*x*cos(d*x + c) + a^2*d*x - a*b*sin(d*x + c) - sqrt(a^2 - b^
2)*(b*cos(d*x + c) + a)*arctan(-(a*cos(d*x + c) + b)/(sqrt(a^2 - b^2)*sin(d
*x + c))))/(b^3*d*cos(d*x + c) + a*b^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{a + b\sec{(c + dx)}}{\left(a\sec{(c + dx)} + b\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/(b+a*sec(d*x+c))**2,x)
```

```
[Out] Integral((a + b*sec(c + d*x))/(a*sec(c + d*x) + b)**2, x)
```

Giac [A]
time = 0.49, size = 139, normalized size = 1.62

$$\frac{\frac{(dx+c)a}{b^2} - \frac{2\,a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\left(a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)^2+a+b\right)b} - \frac{2\left(\pi\left\lfloor\frac{dx+c}{2\,\pi}+\frac{1}{2}\right\rfloor\mathrm{sgn}(2\,a-2\,b)+\arctan\left(\frac{a\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)-b\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)}{\sqrt{a^2-b^2}}\right)\right)\sqrt{a^2-b^2}}{b^2}}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))/(b+a*sec(d*x+c))^2,x, algorithm="giac")
```

```
[Out] ((d*x + c)*a/b^2 - 2*a*tan(1/2*d*x + 1/2*c)/((a*tan(1/2*d*x + 1/2*c)^2 - b*
tan(1/2*d*x + 1/2*c)^2 + a + b)*b) - 2*(pi*floor(1/2*(d*x + c)/pi + 1/2)*sg
n(2*a - 2*b) + arctan((a*tan(1/2*d*x + 1/2*c) - b*tan(1/2*d*x + 1/2*c))/sqr
t(a^2 - b^2)))*sqrt(a^2 - b^2)/b^2)/d
```

Mupad [B]
time = 2.57, size = 444, normalized size = 5.16

$$\frac{2\,\mathrm{atanh}\left(\frac{64\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}}{64\,a^4-128\,a^3\,b+128\,a\,b^3-64\,b^4}-\frac{192\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}}{128\,a\,b^2-128\,a^3-64\,b^3+\frac{64\,a^4}{b}}+\frac{192\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}}{128\,a\,b-64\,b^2-\frac{128\,a^3}{b}+\frac{64\,a^4}{b^2}}-\frac{64\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\sqrt{b^2-a^2}}{128\,a\,b-64\,b^2-\frac{128\,a^3}{b}+\frac{64\,a^4}{b^2}}\right)\sqrt{b^2-a^2}}{b^2\,d}-\frac{2\,a\,\mathrm{atan}\left(\frac{64\,a^2\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a\,b-64\,a^2-\frac{64\,a^3}{b}+\frac{64\,a^4}{b^2}}+\frac{64\,a^3\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a\,b^2-64\,a^2\,b-64\,a^3+\frac{64\,a^4}{b}}-\frac{64\,a^4\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a^4-64\,a^3\,b-64\,a^2\,b^2+64\,a\,b^3}-\frac{64\,a\,b\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{64\,a\,b-64\,a^2-\frac{64\,a^3}{b}+\frac{64\,a^4}{b^2}}\right)}{b^2\,d}-\frac{2\,a\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)}{b\,d\,\left(\left(a-b\right)\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)^2+a+b\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a + b/cos(c + d*x))/(b + a/cos(c + d*x))^2,x)
```

```
[Out] (2*atanh((64*a^3*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(128*a*b^3 - 128*a^3
*b + 64*a^4 - 64*b^4) - (192*a^2*tan(c/2 + (d*x)/2)*(b^2 - a^2)^(1/2))/(128
*a*b^2 - 128*a^3 - 64*b^3 + (64*a^4)/b) + (192*a*tan(c/2 + (d*x)/2)*(b^2 -
a^2)^(1/2))/(128*a*b - 64*b^2 - (128*a^3)/b + (64*a^4)/b^2) - (64*b*tan(c/2
 + (d*x)/2)*(b^2 - a^2)^(1/2))/(128*a*b - 64*b^2 - (128*a^3)/b + (64*a^4)/b
^2))*(b^2 - a^2)^(1/2))/(b^2*d) - (2*a*atan((64*a^2*tan(c/2 + (d*x)/2))/(64
*a*b - 64*a^2 - (64*a^3)/b + (64*a^4)/b^2) + (64*a^3*tan(c/2 + (d*x)/2))/(6
4*a*b^2 - 64*a^2*b - 64*a^3 + (64*a^4)/b) - (64*a^4*tan(c/2 + (d*x)/2))/(64
*a*b^3 - 64*a^3*b + 64*a^4 - 64*a^2*b^2) - (64*a*b*tan(c/2 + (d*x)/2))/(64*
a*b - 64*a^2 - (64*a^3)/b + (64*a^4)/b^2)))/(b^2*d) - (2*a*tan(c/2 + (d*x)/
2))/(b*d*(a + b + tan(c/2 + (d*x)/2)^2*(a - b)))
```

3.347 $\int \frac{3+\sec(c+dx)}{2-\sec(c+dx)}\,dx$

Optimal. Leaf size=87

$$\frac{3x}{2}-\frac{5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sqrt{3}\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2\sqrt{3}\,d}+\frac{5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sqrt{3}\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2\sqrt{3}\,d}$$

```
[Out] 3/2*x-5/6*ln(cos(1/2*d*x+1/2*c)-sin(1/2*d*x+1/2*c)*3^(1/2))/d*3^(1/2)+5/6*l
n(cos(1/2*d*x+1/2*c)+sin(1/2*d*x+1/2*c)*3^(1/2))/d*3^(1/2)
```

Rubi [A]
time = 0.05, antiderivative size = 87, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 21, $\frac{\text{number of rules}}{\text{integrand size}} = 0.190$, Rules used = {4004, 3916, 2738, 213}

$$-\frac{5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sqrt{3}\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2\sqrt{3}\,d}+\frac{5\log\left(\sqrt{3}\sin\left(\frac{1}{2}(c+dx)\right)+\cos\left(\frac{1}{2}(c+dx)\right)\right)}{2\sqrt{3}\,d}+\frac{3x}{2}$$

Antiderivative was successfully verified.

```
[In] Int[(3 + Sec[c + d*x])/(2 - Sec[c + d*x]),x]
```

```
[Out] (3*x)/2 - (5*Log[Cos[(c + d*x)/2] - Sqrt[3]*Sin[(c + d*x)/2]])/(2*Sqrt[3]*d
) + (5*Log[Cos[(c + d*x)/2] + Sqrt[3]*Sin[(c + d*x)/2]])/(2*Sqrt[3]*d)
```

Rule 213

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(-(Rt[-a, 2]*Rt[b, 2])^(-
1))*ArcTanh[Rt[b, 2]*(x/Rt[-a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] &&
 (LtQ[a, 0] || GtQ[b, 0])
```

Rule 2738

```
Int[((a_) + (b_.)*sin[Pi/2 + (c_.) + (d_.)*(x_)])^(-1), x_Symbol] :> With[{
e = FreeFactors[Tan[(c + d*x)/2], x]}, Dist[2*(e/d), Subst[Int[1/(a + b + (
a - b)*e^2*x^2), x], x, Tan[(c + d*x)/2]/e], x]] /; FreeQ[{a, b, c, d}, x]
&& NeQ[a^2 - b^2, 0]
```

Rule 3916

```
Int[csc[(e_.) + (f_.)*(x_)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbo
l] :> Dist[1/b, Int[1/(1 + (a/b)*Sin[e + f*x]), x], x] /; FreeQ[{a, b, e, f
}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4004

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
 (a_)), x_Symbol] :> Simp[c*(x/a), x] - Dist[(b*c - a*d)/a, Int[Csc[e + f*x
]/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c -
 a*d, 0]
```

Rubi steps

$$\begin{aligned}\int \frac{3+\sec(c+dx)}{2-\sec(c+dx)}\,dx &= \frac{3x}{2}+\frac{5}{2}\int \frac{\sec(c+dx)}{2-\sec(c+dx)}\,dx\\ &= \frac{3x}{2}-\frac{5}{2}\int \frac{1}{1-2\cos(c+dx)}\,dx\\ &= \frac{3x}{2}-\frac{5\mathrm{Subst}\left(\int \frac{1}{-1+3x^2}\,dx,x,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{d}\\ &= \frac{3x}{2}-\frac{5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)-\sqrt{3}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2\sqrt{3}\,d}+\frac{5\log\left(\cos\left(\frac{1}{2}(c+dx)\right)+\sqrt{3}\right.}{2\sqrt{3}\,d}\end{aligned}$$

Mathematica [A]
time = 0.08, size = 39, normalized size = 0.45

$$\frac{9(c+dx)+10\sqrt{3}\,\tanh^{-1}\left(\sqrt{3}\,\tan\left(\frac{1}{2}(c+dx)\right)\right)}{6d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(3 + Sec[c + d*x])/(2 - Sec[c + d*x]),x]
[Out] (9*(c + d*x) + 10*Sqrt[3]*ArcTanh[Sqrt[3]*Tan[(c + d*x)/2]])/(6*d)
```

Maple [A]
time = 0.30, size = 37, normalized size = 0.43

method	result	size
derivativedivides	$\frac{\frac{5\sqrt{3}\,\mathrm{arctanh}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{3}\right)}{3}+3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$	37
default	$\frac{\frac{5\sqrt{3}\,\mathrm{arctanh}\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{3}\right)}{3}+3\arctan\left(\tan\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}{d}$	37
risch	$\frac{3x}{2}+\frac{5\sqrt{3}\,\ln\left(\mathrm{e}^{i(dx+c)}+\frac{i\sqrt{3}}{2}-\frac{1}{2}\right)}{6d}-\frac{5\sqrt{3}\,\ln\left(\mathrm{e}^{i(dx+c)}-\frac{i\sqrt{3}}{2}-\frac{1}{2}\right)}{6d}$	57

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((3+sec(d*x+c))/(2-sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*(5/3*3^(1/2)*arctanh(tan(1/2*d*x+1/2*c)*3^(1/2))+3*arctan(tan(1/2*d*x+1
/2*c)))
```

Maxima [A]
time = 0.49, size = 80, normalized size = 0.92

$$-\frac{5\sqrt{3}\log\left(-\frac{\sqrt{3}-\frac{3\sin(dx+c)}{\cos(dx+c)+1}}{\sqrt{3}+\frac{3\sin(dx+c)}{\cos(dx+c)+1}}\right)-18\arctan\left(\frac{\sin(dx+c)}{\cos(dx+c)+1}\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sec(d*x+c))/(2-sec(d*x+c)),x, algorithm="maxima")
```

```
[Out] -1/6*(5*sqrt(3)*log(-(sqrt(3) - 3*sin(d*x + c)/(cos(d*x + c) + 1))/(sqrt(3)
 + 3*sin(d*x + c)/(cos(d*x + c) + 1))) - 18*arctan(sin(d*x + c)/(cos(d*x +
c) + 1)))/d
```

Fricas [A]
time = 3.71, size = 84, normalized size = 0.97

$$\frac{18\,dx+5\sqrt{3}\log\left(-\frac{2\cos(dx+c)^{2}+2\left(\sqrt{3}\cos(dx+c)-2\sqrt{3}\right)\sin(dx+c)+4\cos(dx+c)-7}{4\cos(dx+c)^{2}-4\cos(dx+c)+1}\right)}{12\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sec(d*x+c))/(2-sec(d*x+c)),x, algorithm="fricas")
```

```
[Out] 1/12*(18*d*x + 5*sqrt(3)*log(-(2*cos(d*x + c)^2 + 2*(sqrt(3)*cos(d*x + c) -
 2*sqrt(3))*sin(d*x + c) + 4*cos(d*x + c) - 7)/(4*cos(d*x + c)^2 - 4*cos(d*
x + c) + 1)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-\int\frac{\sec(c+dx)}{\sec(c+dx)-2}\,dx-\int\frac{3}{\sec(c+dx)-2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sec(d*x+c))/(2-sec(d*x+c)),x)
```

```
[Out] -Integral(sec(c + d*x)/(sec(c + d*x) - 2), x) - Integral(3/(sec(c + d*x) -
2), x)
```

Giac [A]
time = 0.50, size = 58, normalized size = 0.67

$$\frac{9\,dx-5\,\sqrt{3}\,\log\left(\frac{\left|-2\,\sqrt{3}+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}{\left|2\,\sqrt{3}+6\,\tan\left(\frac{1}{2}\,dx+\frac{1}{2}\,c\right)\right|}\right)+9\,c}{6\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((3+sec(d*x+c))/(2-sec(d*x+c)),x, algorithm="giac")
```

```
[Out] 1/6*(9*d*x - 5*sqrt(3)*log(abs(-2*sqrt(3) + 6*tan(1/2*d*x + 1/2*c))/abs(2*s
qrt(3) + 6*tan(1/2*d*x + 1/2*c))) + 9*c)/d
```

Mupad [B]
time = 2.30, size = 26, normalized size = 0.30

$$\frac{3\,x}{2}+\frac{5\,\sqrt{3}\,\mathrm{atanh}\left(\sqrt{3}\,\tan\left(\frac{c}{2}+\frac{d\,x}{2}\right)\right)}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(-(1/cos(c + d*x) + 3)/(1/cos(c + d*x) - 2),x)
```

```
[Out] (3*x)/2 + (5*3^(1/2)*atanh(3^(1/2)*tan(c/2 + (d*x)/2)))/(3*d)
```

3.348 $\int \sec^4(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=485

$$-\frac{2(a-b)\sqrt{a+b}\,(24a^3Ab+57aAb^3-16a^4B-24a^2b^2B+147b^4B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right.\right.}{315b^5d}$$

```
[Out] -2/315*(a-b)*(24*A*a^3*b+57*A*a*b^3-16*B*a^4-24*B*a^2*b^2+147*B*b^4)*cot(d*
x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b
)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^5/
d-2/315*(a-b)*(3*b^3*(25*A-49*B)+18*a*b^2*(A-2*B)+12*a^2*b*(2*A-B)-16*a^3*B
)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1
/2)/b^4/d-2/315*(12*A*a^2*b-75*A*b^3-8*B*a^3-13*B*a*b^2)*(a+b*sec(d*x+c))^(
1/2)*tan(d*x+c)/b^3/d+2/315*(9*A*a*b-6*B*a^2+49*B*b^2)*sec(d*x+c)*(a+b*sec(
d*x+c))^(1/2)*tan(d*x+c)/b^2/d+2/63*(9*A*b+B*a)*sec(d*x+c)^2*(a+b*sec(d*x+c
))^(1/2)*tan(d*x+c)/b/d+2/9*B*sec(d*x+c)^3*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c
)/d
```

Rubi [A]
time = 0.93, antiderivative size = 485, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4116, 4187, 4177, 4167, 4090, 3917, 4089}

$$\frac{2(-6a^2B+9aAb+49b^2B)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{315b^2d}-\frac{2(a-b)\sqrt{a+b}\,(-16a^3B+12a^2b(2A-B)+18ab^2(A-2B)+3b^3(25A-49B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{315b^4d}-\frac{2(-8a^3B+12a^2Ab-13ab^2B-75Ab^3)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{315b^3d}-\frac{2(a-b)\sqrt{a+b}\,(-16a^4B+24a^3Ab-24a^2b^2B+57aAb^3+147b^4B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{315b^5d}+\frac{2(aB+9Ab)\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{63bd}+\frac{2B\tan(c+dx)\sec^3(c+dx)\sqrt{a+b\sec(c+dx)}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^4*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(24*a^3*A*b + 57*a*A*b^3 - 16*a^4*B - 24*a^2*b^2*B
+ 147*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/(315*b^5*d) - (2*(a - b)*Sqrt[a + b]*(3*b^3*(25*
A - 49*B) + 18*a*b^2*(A - 2*B) + 12*a^2*b*(2*A - B) - 16*a^3*B)*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(315*b^4*d) - (2*(12*a^2*A*b - 75*A*b^3 - 8*a^3*B - 13*a*b^2*B)*Sqrt[a +
 b*Sec[c + d*x]]*Tan[c + d*x])/(315*b^3*d) + (2*(9*a*A*b - 6*a^2*B + 49*b^2
*B)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(315*b^2*d) + (2*(9
*A*b + a*B)*Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(63*b*d)
+ (2*B*Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(9*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4116

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n
- 1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B
*m + A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}
, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4177

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
```

```
_Symbol] :> Simp[(-C)*Csc[e + f*x]*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m +
1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e +
 f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(
m + 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rubi steps

$$\int \sec^4(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \frac{2B\sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,\tan(c+d}{9d}$$

$$= \frac{2(9Ab+aB)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{63bd}$$

$$= \frac{2(9aAb-6a^2B+49b^2B)\sec(c+dx)\sqrt{a+b}}{315b^2d}$$

$$= -\frac{2(12a^2Ab-75Ab^3-8a^3B-13ab^2B)\sqrt{a-}}{315b^3d}$$

$$= -\frac{2(12a^2Ab-75Ab^3-8a^3B-13ab^2B)\sqrt{a-}}{315b^3d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(24a^3Ab+57aAb^3-16a^4B}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3734 vs. $2(485) = 970$.
time = 25.79, size = 3734, normalized size = 7.70

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^4*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((2*(24*a^3*A*b + 57*a*A*b^3 - 16*a^4*B - 24*a^2*
b^2*B + 147*b^4*B)*Sin[c + d*x])/(315*b^4) + (2*Sec[c + d*x]^3*(9*A*b*Sin[c
 + d*x] + a*B*Sin[c + d*x]))/(63*b) + (2*Sec[c + d*x]^2*(9*a*A*b*Sin[c + d*
x] - 6*a^2*B*Sin[c + d*x] + 49*b^2*B*Sin[c + d*x]))/(315*b^2) + (2*Sec[c +
d*x]*(-12*a^2*A*b*Sin[c + d*x] + 75*A*b^3*Sin[c + d*x] + 8*a^3*B*Sin[c + d*
x] + 13*a*b^2*B*Sin[c + d*x]))/(315*b^3) + (2*B*Sec[c + d*x]^3*Tan[c + d*x]
)/9))/d + (2*((-19*a*A)/(105*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) -
 (8*a^3*A)/(105*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*a^4*
B)/(315*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (8*a^2*B)/(105*b
*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (7*b*B)/(15*Sqrt[b + a*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^4*A*Sqrt[Sec[c + d*x]])/(105*b^3*Sqrt[
b + a*Cos[c + d*x]]) - (17*a^2*A*Sqrt[Sec[c + d*x]])/(105*b*Sqrt[b + a*Cos[
c + d*x]]) + (5*A*b*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) - (4*
a*B*Sqrt[Sec[c + d*x]])/(35*Sqrt[b + a*Cos[c + d*x]]) + (16*a^5*B*Sqrt[Sec[
c + d*x]])/(315*b^4*Sqrt[b + a*Cos[c + d*x]]) + (4*a^3*B*Sqrt[Sec[c + d*x]]
)/(63*b^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^4*A*Cos[2*(c + d*x)]*Sqrt[Sec[c
+ d*x]])/(105*b^3*Sqrt[b + a*Cos[c + d*x]]) - (19*a^2*A*Cos[2*(c + d*x)]*Sq
rt[Sec[c + d*x]])/(105*b*Sqrt[b + a*Cos[c + d*x]]) - (7*a*B*Cos[2*(c + d*x)
]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) + (16*a^5*B*Cos[2*(c +
d*x)]*Sqrt[Sec[c + d*x]])/(315*b^4*Sqrt[b + a*Cos[c + d*x]]) + (8*a^3*B*Cos
[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*b^2*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[
Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(2*(a + b)*(-24*a
^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*
a^3*B + 12*a^2*b*(2*A + B) - 18*a*b^2*(A + 2*B) + 3*b^3*(25*A + 49*B))*Sqrt
[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + C
os[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-24*
a^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*Cos[c + d*x]*(b
 + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*b^4*d*(b + a*
Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]*((a*Sqrt[Cos[(c +
 d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-24*a^3*A*b - 57*a*A*b^3
+ 16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*a^3*B + 12*a^2*b*(2*
A + B) - 18*a*b^2*(A + 2*B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-24*a^3*A*b - 57*a*A*b^3
 + 16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*S
ec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*b^4*(b + a*Cos[c + d*x])^(3/2)*Sq
rt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d
```

```
*x)/2]*(2*(a + b)*(-24*a^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 147
*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)] + 2*b*(a + b)*(-16*a^3*B + 12*a^2*b*(2*A + B) - 18*a*b^2*(A + 2*B) + 3
*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)] + (-24*a^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 14
7*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)
/2]))/(315*b^4*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt
[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-24*a^3*A*b - 57*a*A*b^3 + 16*a^4*B +
24*a^2*b^2*B - 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2
]^4)/2 + ((a + b)*(-24*a^3*A*b - 57*a*A*b^3 + 16*a^4*B + 24*a^2*b^2*B - 147
*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 +
 Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])] + (b*(a + b)*(-16*a^3*B + 12*a^2*b*(2*A + B) - 18*a*b^2*(A
+ 2*B) + 3*b^3*(25*A + 49*B))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d
*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/
Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(-24*a^3*A*b - 57*a*A*b^3
+ 16*a^4*B + 24*a^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b
)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))] + (b*(a + b)*(-16*a^3*B + 12*a^2*b*(2*A + B) - 18*a*b^2*(A + 2*B) +
3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + Cos...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4394 vs. $2(447) = 894$.
time = 11.52, size = 4395, normalized size = 9.06

method	result	size
default	Expression too large to display	4395

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/315/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(-24*A*cos(d*x+c)^6*a^4*b+12*A*cos(d*x+c)^6*a^3*b^2-57*A*cos(d*x+c)^6*a
^2*b^3-75*A*cos(d*x+c)^6*a*b^4-8*B*cos(d*x+c)^6*a^4*b+24*B*cos(d*x+c)^6*a^3
*b^2-13*B*cos(d*x+c)^6*a^2*b^3-147*B*cos(d*x+c)^6*a*b^4+24*A*cos(d*x+c)^5*a
^4*b-24*A*cos(d*x+c)^5*a^3*b^2+60*A*cos(d*x+c)^5*a^2*b^3-57*A*cos(d*x+c)^5*
a*b^4+16*B*cos(d*x+c)^5*a^4*b-26*B*cos(d*x+c)^5*a^3*b^2+24*B*cos(d*x+c)^5*a
^2*b^3+85*B*cos(d*x+c)^5*a*b^4+12*A*cos(d*x+c)^4*a^3*b^2+57*A*sin(d*x+c)*co
```

```
s(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
*b^4-75*A*cos(d*x+c)^5*b^5+30*A*cos(d*x+c)^3*b^5+45*A*cos(d*x+c)*b^5+16*B*c
os(d*x+c)^6*a^5-16*B*cos(d*x+c)^5*a^5-147*B*cos(d*x+c)^5*b^5+98*B*cos(d*x+c
)^4*b^5+14*B*cos(d*x+c)^2*b^5+78*A*cos(d*x+c)^4*a*b^4-8*B*cos(d*x+c)^4*a^4*
b-10*B*cos(d*x+c)^4*a^2*b^3-3*A*cos(d*x+c)^3*a^2*b^3+2*B*cos(d*x+c)^3*a^3*b
^2+22*B*cos(d*x+c)^3*a*b^4+54*A*cos(d*x+c)^2*a*b^4-B*cos(d*x+c)^2*a^2*b^3+4
0*B*cos(d*x+c)*a*b^4+35*B*b^5-24*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+4*B*sin(d*x+c)*cos(d
*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*
b^2+24*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^2*b^3-111*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+24*A*sin(d*x+c)*cos(d*
x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b
+24*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^3*b^2+57*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-6*A*sin(d*x+c)*cos(d*x+c
)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-
57*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a*b^4-16*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-24*B*sin(d*x+c)*cos(d*x+c)^4*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-24*B
*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^2*b^3+147*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+16*B*sin(d*x+c)*cos(d*x+c)^4*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+4*B*sin
(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^3*b^2+24*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-111*B*sin(d*x+c)*cos(d*x+c)^4*(co
```

```
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+24*A*sin(
d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a^3*b^2+57*A*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+57*A*sin(d*x+c)*cos(d*x+c)^5*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-24*B*sin(d*
x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^3*b^2-24*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+147*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^4, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^5 + A*sec(d*x + c)^4)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))\sqrt{a + b\sec(c + dx)}\sec^4(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sec(c + d*x)**4, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^4, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{{\cos\left(c+d\,x\right)}^4} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x)^4,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x)^4, x)
```

3.349 $\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=397

$$\frac{2(a-b)\sqrt{a+b}\,(14a^2Ab-63Ab^3-8a^3B-19ab^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{105b^4d}$$

```
[Out] 2/105*(a-b)*(14*A*a^2*b-63*A*b^3-8*B*a^3-19*B*a*b^2)*cot(d*x+c)*EllipticE((
a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-se
c(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d+2/105*(a-b)*(b
^2*(63*A-25*B)+2*a*b*(7*A-3*B)-8*a^2*B)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c
))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+
b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/105*(7*A*a*b-4*B*a^2+25*B
*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d+2/35*(7*A*b+B*a)*sec(d*x+c)*(
a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b/d+2/7*B*sec(d*x+c)^2*(a+b*sec(d*x+c))^(1
/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.60, antiderivative size = 397, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4116, 4177, 4167, 4090, 3917, 4089}

$$\frac{2(a-b)\sqrt{a+b}\,(-8a^2B+2ab(7A-3B)+b^2(63A-25B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{105b^3d}+\frac{2(-4a^2B+7aAb+25b^2B)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{105b^2d}+\frac{2(a-b)\sqrt{a+b}\,(-8a^3B+14a^2Ab-19ab^2B-63Ab^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{105b^4d}+\frac{2(aB+7Ab)\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{35bd}+\frac{2B\tan(c+dx)\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(14*a^2*A*b - 63*A*b^3 - 8*a^3*B - 19*a*b^2*B)*Cot[c
 + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(105*b^4*d) + (2*(a - b)*Sqrt[a + b]*(b^2*(63*A - 25*B) + 2*a*b*(7
*A - 3*B) - 8*a^2*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^3*d) + (2*(7*a*A*b - 4*a^2*B + 25
*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b^2*d) + (2*(7*A*b + a*
B)*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(35*b*d) + (2*B*Sec[
c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(7*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
```

```
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4116

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n
- 1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B
*m + A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}
, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4177

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
_Symbol] :> Simp[(-C)*Csc[e + f*x]*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m +
1)/(b*f*(m + 3))), x] + Dist[1/(b*(m + 3)), Int[Csc[e + f*x]*(a + b*Csc[e +
 f*x])^m*Simp[a*C + b*(C*(m + 2) + A*(m + 3))*Csc[e + f*x] - (2*a*C - b*B*(
```

```
m + 3))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x] &&
 NeQ[a^2 - b^2, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^3(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \frac{2B\sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,\tan(c+d$$

$$= \frac{2(7Ab+aB)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{35bd}$$

$$= \frac{2(7aAb-4a^2B+25b^2B)\sqrt{a+b\sec(c+dx)}}{105b^2d}$$

$$= \frac{2(7aAb-4a^2B+25b^2B)\sqrt{a+b\sec(c+dx)}}{105b^2d}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(14a^2Ab-63Ab^3-8a^3B-1}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3330 vs. $2(397) = 794$.
time = 23.74, size = 3330, normalized size = 8.39

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((2*(-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*
B)*Sin[c + d*x])/(105*b^3) + (2*Sec[c + d*x]^2*(7*A*b*Sin[c + d*x] + a*B*Si
n[c + d*x]))/(35*b) + (2*Sec[c + d*x]*(7*a*A*b*Sin[c + d*x] - 4*a^2*B*Sin[c
 + d*x] + 25*b^2*B*Sin[c + d*x]))/(105*b^2) + (2*B*Sec[c + d*x]^2*Tan[c + d
*x])/7))/d - (2*((2*a^2*A)/(15*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]
]) - (3*A*b)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (19*a*B)/(10
5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^3*B)/(105*b^2*Sqrt[b
+ a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a*A*Sqrt[Sec[c + d*x]])/(15*Sqrt
[b + a*Cos[c + d*x]]) + (2*a^3*A*Sqrt[Sec[c + d*x]])/(15*b^2*Sqrt[b + a*Cos
[c + d*x]]) - (8*a^4*B*Sqrt[Sec[c + d*x]])/(105*b^3*Sqrt[b + a*Cos[c + d*x]
]) - (17*a^2*B*Sqrt[Sec[c + d*x]])/(105*b*Sqrt[b + a*Cos[c + d*x]]) + (5*b*
B*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) - (3*a*A*Cos[2*(c + d*x
```

```
)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]) + (2*a^3*A*Cos[2*(c + d
*x)]*Sqrt[Sec[c + d*x]])/(15*b^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^4*B*Cos[2
*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*b^3*Sqrt[b + a*Cos[c + d*x]]) - (19*a^
2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*b*Sqrt[b + a*Cos[c + d*x]]))*
Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(2*(a + b)*(
-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c
 + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(8*a^2*B - 2*a*b*
(7*A + 3*B) + b^2*(63*A + 25*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt
[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)] + (-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^
2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2])
)/(105*b^3*d*(b + a*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x
]]*(-1/105*(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)
*(-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(8*a^2*B - 2*a*
b*(7*A + 3*B) + b^2*(63*A + 25*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sq
rt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[
(c + d*x)/2]], (a - b)/(a + b)] + (-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*
b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2
]))/(b^3*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[Cos[(
c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-14*a^2*A*b + 63*A
*b^3 + 8*a^3*B + 19*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(8*a^2*B - 2*a*b*(7*A + 3*B) + b^2*
(63*A + 25*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)] + (-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)*Cos[c + d*x]*
(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(105*b^3*Sqrt[b
+ a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Se
c[c + d*x]]*(((-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)*Cos[c + d*x]*
(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-14*a^2*A*b + 63*A*b
^3 + 8*a^3*B + 19*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]
*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])] - (b*(a + b)*(8*a^2*B - 2*a*b*(7*A + 3*B
) + b^2*(63*A + 25*B))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin
[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(-14*a^2*A*b + 63*A*b^3 + 8*a^3*B
 + 19*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2))
)/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (b*(a + b)*(8*a
```

```
^2*B - 2*a*b*(7*A + 3*B) + b^2*(63*A + 25*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/
((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))] - a*(-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)*Cos[c + d
*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-14*a^2*A*b + 63*A*
b^3 + 8*a^3*B + 19*a*b^2*B)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c +
 d*x]*Tan[(c + d*x)/2] + (-14*a^2*A*b + 63*A*b^3 + 8*a^3*B + 19*a*b^2*B)*Co
s[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - (b*
(a + b)*(8*a^2*B - 2*a*b*(7*A + 3*B) + b^2*(63*...
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3437 vs. $2(363) = 726$.
time = 10.82, size = 3438, normalized size = 8.66

method	result	size
default	Expression too large to display	3438

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/105/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c
))^2*(-15*B*b^4-19*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-35*A*cos(d*x+c)^4*a*b^3+8*B*cos(d*
x+c)^4*a^3*b-20*B*cos(d*x+c)^4*a^2*b^2+19*B*cos(d*x+c)^4*a*b^3+7*A*cos(d*x+
c)^3*a^2*b^2-4*B*cos(d*x+c)^3*a^3*b-26*B*cos(d*x+c)^3*a*b^3-28*A*cos(d*x+c)
^2*a*b^3+B*cos(d*x+c)^2*a^2*b^2-18*B*cos(d*x+c)*a*b^3-14*A*cos(d*x+c)^5*a^3
*b+7*A*cos(d*x+c)^5*a^2*b^2+63*A*cos(d*x+c)^5*a*b^3-4*B*cos(d*x+c)^5*a^3*b+
19*B*cos(d*x+c)^5*a^2*b^2+25*B*cos(d*x+c)^5*a*b^3+14*A*cos(d*x+c)^4*a^3*b-1
4*A*cos(d*x+c)^4*a^2*b^2+14*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-63*A*sin(d*x+c)*cos(d*x+c
)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-14
*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^2*b^2+49*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-8*B*sin(d*x+c)*cos(d*x+c)^4*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+14*A*si
n(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
```

```
))^(1/2))*a^3*b-10*B*cos(d*x+c)^2*b^4+8*B*cos(d*x+c)^5*a^4+63*A*cos(d*x+c)^
4*b^4-8*B*cos(d*x+c)^4*a^4-42*A*cos(d*x+c)^3*b^4-21*A*cos(d*x+c)*b^4-14*A*s
in(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^2*b^2+49*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-8*B*sin(d*x+c)*cos(d*x+c)^3*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-19*B*sin(d*
x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^2*b^2-19*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+8*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+2*B*sin(d*x+c)*c
os(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*b^2+19*B*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a*b^3+14*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+25*B*cos(d*x+c)^4*b^4
+14*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^2*b^2-63*A*sin(d*x+c)*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-19*B*sin(d*x+c)*cos(d*x+c)
^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+8*B
*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^3*b+2*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+19*B*sin(d*x+c)*cos(d*x+c)^4*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-63*A*sin(
d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*b^4+63*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*b^4-8*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^4 + A*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))\sqrt{a + b\sec(c + dx)}\sec^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sec(c + d*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^3, x)
```

Mupad [**F**]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{{\cos\left(c+d\,x\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x)^3,x)
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x)^3, x)
```

3.350 $\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=314

$$-\frac{2(a-b)\sqrt{a+b}\,(5aAb-2a^2B+9b^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(}{a+}}}{15b^3d}$$

```
[Out] -2/15*(a-b)*(5*A*a*b-2*B*a^2+9*B*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))
^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b)
)^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d-2/15*(a-b)*(5*A*b-2*B*a-9*B*b
)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1
/2)/b^2/d+2/5*B*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/b/d+2/15*(5*A*b-2*B*a)*(a
+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b/d
```

Rubi [A]

time = 0.36, antiderivative size = 314, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4095, 4087, 4090, 3917, 4089}

$$-\frac{2(a-b)\sqrt{a+b}\,(-2a^2B+5aAb+9b^2B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^3d}-\frac{2(a-b)\sqrt{a+b}\,(-2aB+5Ab-9bB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^2d}+\frac{2(5Ab-2aB)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{15bd}+\frac{2B\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(5*a*A*b - 2*a^2*B + 9*b^2*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^3
*d) - (2*(a - b)*Sqrt[a + b]*(5*A*b - 2*a*B - 9*b*B)*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^2*
d) + (2*(5*A*b - 2*a*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*b*d) + (
2*B*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*b*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^2(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \frac{2B(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5bd} + \frac{2\int \mathrm{se}}{}$$

$$= \frac{2(5Ab-2aB)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx}{15bd}$$

$$= \frac{2(5Ab-2aB)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx}{15bd}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(5aAb-2a^2B+9b^2B)\cot(}{}$$

Mathematica [A]
time = 17.41, size = 434, normalized size = 1.38

$$\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\left(2(a+b)\left(-5aAb+2a^2B-9b^2B\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)+2b(a+b)(5Ab-2aB+9bB)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)+\left(-5aAb+2a^2B-9b^2B\right)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{15b^2d(b+a\cos(c+dx))\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\,\sqrt{\sec(c+dx)}}+\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{2(5aAb-2a^2B+9b^2B)\sin(c+dx)}{15b^2}+\frac{2\sec(c+dx)(5Ab\sin(c+dx)+aB\sin(c+dx))}{15b}+\frac{2}{5}B\sec(c+dx)\tan(c+dx)\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(2*(a + b
)*(-5*a*A*b + 2*a^2*B - 9*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt
[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(5*A*b - 2*a*B + 9*b*B)*Sqrt[Co
s[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[
c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-5*a*A*
b + 2*a^2*B - 9*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2
*Tan[(c + d*x)/2]))/(15*b^2*d*(b + a*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]
*Sqrt[Sec[c + d*x]]) + (Sqrt[a + b*Sec[c + d*x]]*((2*(5*a*A*b - 2*a^2*B + 9
*b^2*B)*Sin[c + d*x])/(15*b^2) + (2*Sec[c + d*x]*(5*A*b*Sin[c + d*x] + a*B*
Sin[c + d*x]))/(15*b) + (2*B*Sec[c + d*x]*Tan[c + d*x])/5))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2497 vs. $2(284) = 568$.
time = 10.71, size = 2498, normalized size = 7.96

method	result	size
default	Expression too large to display	2498

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/15/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(-5*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^2*b-3*B*b^3-5*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+5*A*cos(d*x+c)^3*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
*b^2+2*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^2*b-9*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-2*B*cos(d*x+c)^3*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+7*B*
cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a*b^2-5*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-5*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+5*A*cos(d*x+c
)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a*b^2+2*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*a^2*b-9*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-2*B*cos(d*x+c)^2*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+
7*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a*b^2+5*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+2*B*cos(d*x+c)^3*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-9*B*cos(d*x+c
)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*b^3+9*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*b^3+5*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+2*B*cos(d*x+c)^2*sin(d*x+c)*
```

```
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-9*B*cos(
d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*b^3+9*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*b^3-5*A*cos(d*x+c)*b^3-2*B*cos(d*x+c)^4*a^3+2*B
*cos(d*x+c)^3*a^3+9*B*cos(d*x+c)^3*b^3+5*A*cos(d*x+c)^4*a^2*b+5*A*cos(d*x+c
)^4*a*b^2+B*cos(d*x+c)^4*a^2*b+9*B*cos(d*x+c)^4*a*b^2-5*A*cos(d*x+c)^3*a^2*
b+5*A*cos(d*x+c)^3*a*b^2-2*B*cos(d*x+c)^3*a^2*b-5*B*cos(d*x+c)^3*a*b^2-10*A
*cos(d*x+c)^2*a*b^2+B*cos(d*x+c)^2*a^2*b-4*B*cos(d*x+c)*a*b^2-6*B*cos(d*x+c
)^2*b^3+5*A*cos(d*x+c)^3*b^3)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5/b^
2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec{(c + dx)})\sqrt{a + b\sec{(c + dx)}}\sec^2{(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sec(c + d*x)**2, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^2, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{{\cos\left(c+d\,x\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x)^2,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x)^2, x)
```

3.351 $\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=256

$$-\frac{2(a-b)\sqrt{a+b}\,(3Ab+aB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\,\sqrt{}}{3b^2d}$$

[Out] -2/3*(a-b)*(3*A*b+B*a)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+2/3*(a-b)*(3*A-B)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/3*B*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/d

Rubi [A]
time = 0.21, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4087, 4090, 3917, 4089}

$$-\frac{2(a-b)\sqrt{a+b}\,(aB+3Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}+\frac{2(a-b)\sqrt{a+b}\,(3A-B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bd}+\frac{2B\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]

[Out] (-2*(a - b)*Sqrt[a + b]*(3*A*b + a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) + (2*(a - b)*Sqrt[a + b]*(3*A - B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) + (2*B*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
```

```
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \frac{2B\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3d} + \frac{2}{3}\int \frac{\sec(\ldots}{\ldots}$$

$$= \frac{2B\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3d} + \frac{1}{3}((a-b)\ldots$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(3Ab+aB)\cot(c+dx)E\Big(\sin\ldots}{\ldots}$$

Mathematica [A]
time = 14.38, size = 408, normalized size = 1.59

$$\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\left(-2(a+b)(3Ab+aB)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\,\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+2b(a+b)(3A+B)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\,\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-(3Ab+aB)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3bd(b+a\cos(c+dx))(B+A\cos(c+dx))\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\,\sec^{\frac{3}{2}}(c+dx)}+\frac{\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\left(\frac{2(3Ab+aB)\sin(c+dx)}{3b}+\frac{2}{3}B\tan(c+dx)\right)}{d(B+A\cos(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Se
c[c + d*x])*(-2*(a + b)*(3*A*b + a*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]
```

```
*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(3*A + B)*Sqrt[Cos[c + d*x
]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])
)]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - (3*A*b + a*B)*Cos
[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b*d
*(b + a*Cos[c + d*x])*(B + A*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c +
 d*x]^(3/2)) + (Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x])*
((2*(3*A*b + a*B)*Sin[c + d*x])/(3*b) + (2*B*Tan[c + d*x])/3))/(d*(B + A*Co
s[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1751 vs. $2(230) = 460$.
time = 9.64, size = 1752, normalized size = 6.84

method	result	size
default	Expression too large to display	1752

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))^2*(3*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-B*cos(d*x+c)^2*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+B*cos(d*x+c
)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+
c)*a*b-3*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*sin(d*x+c)*a*b+3*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-B*cos(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+B*cos(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-
3*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*sin(d*x+c)*a*b-3*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+3*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-B*cos(d*x+c)^2*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
```

```
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a
^2+B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*sin(d*x+c)*b^2-3*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+3*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-B*cos(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+B*
cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*s
in(d*x+c)*b^2-b^2*B+B*cos(d*x+c)^2*b^2+B*cos(d*x+c)^3*a^2+3*A*cos(d*x+c)^2*
b^2-B*cos(d*x+c)^2*a^2-3*A*cos(d*x+c)*b^2+3*A*cos(d*x+c)^3*a*b+B*cos(d*x+c)
^3*a*b-3*A*cos(d*x+c)^2*a*b+B*cos(d*x+c)^2*a*b-2*B*cos(d*x+c)*a*b)*((b+a*co
s(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2/(b+a*cos(d*x+c))/cos(d*x+c)/si
n(d*x+c)^5/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\sec (c+dx))\sqrt {a+b\sec (c+dx)}\sec (c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{\cos\left(c+d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x), x)
```

3.352 $\int \sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=320

$$-\frac{2(a-b)\sqrt{a+b}\,B\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a}}}{bd}$$

```
[Out] -2*(a-b)*B*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(
a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/
(a-b))^(1/2)/b/d+2*(A*b+(a-b)*B)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2
)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/
2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d-2*A*cot(d*x+c)*EllipticPi((a+b*sec(d
*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec
(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d
```

Rubi [A]
time = 0.18, antiderivative size = 320, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4001, 3869, 4090, 3917, 4089}

$$\frac{2\sqrt{a+b}\,(B(a-b)+Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{2A\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}-\frac{2B(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (2*Sqrt[a + b]*(A*b + (a -
b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
(a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
+ d*x]))/(a - b))])/(b*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b
)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b
*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
```

```
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4001

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(d_
.) + (c_)), x_Symbol] :> Dist[a*c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] +
 Int[Csc[e + f*x]*((b*c + a*d + b*d*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]])
, x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = (aA)\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx + \int \frac{\sec(c+dx)(Ab+aB}{\sqrt{a+b\sec}}$$

$$= -\frac{2A\sqrt{a+b}\,\cot(c+dx)\Pi\left(\frac{a+b}{a};\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+d}}{\sqrt{a+b}}\right.\right.}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c}}{\sqrt{a+b}}\right.\right.}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.72, size = 913, normalized size = 2.85

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*B*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x])*Sin[c + d*x
])/(d*(B + A*Cos[c + d*x])) + (2*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*
x])*(a*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + b*Sqrt[(-a + b)/(a + b)]
*B*Tan[(c + d*x)/2] - 2*a*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^3 + a*S
qrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - b*Sqrt[(-a + b)/(a + b)]*B*Tan
[(c + d*x)/2]^5 + (2*I)*a*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(
-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
+ (2*I)*a*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]
*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d
*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b
)] - I*(a - b)*B*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a
- b)*(A - B)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
(a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt
[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(Sqrt[(-a
 + b)/(a + b)]*d*Sqrt[b + a*Cos[c + d*x]]*(B + A*Cos[c + d*x])*Sec[c + d*x]
^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c + d*x)/2]^2)*(1 + T
an[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1371 vs. $2(293) = 586$.
time = 10.29, size = 1372, normalized size = 4.29

method	result	size
default	Expression too large to display	1372

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] 2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2*
(A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a-A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*b-2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a-B*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+
c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-B*cos(d*x+c)
```

```
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
b+B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a+B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*sin(d*x+c)-B*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c)-B*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)+B*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c)+B*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-B*cos(d*x+c)^2*a+B
*cos(d*x+c)*a-B*cos(d*x+c)*b+B*b)/sin(d*x+c)^5/(b+a*cos(d*x+c))
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))\sqrt{a + b\sec(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2), x)
```

3.353 $\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=344

$$\frac{A(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(}{a-}}}{bd}$$

```
[Out] A*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b
))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/b/d+(A+2*B)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+s
ec(d*x+c))/(a-b))^(1/2)/d-(A*b+2*B*a)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c)
)^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+
c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+A*sin(d*x+c)*(a+b*sec(
d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.24, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4117, 4143, 4006, 3869, 3917, 4089}

$$\frac{\sqrt{a+b}\,(A+2B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}-\frac{\sqrt{a+b}\,(2aB+Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}+\frac{A(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}+\frac{A\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(A + 2*B)*Cot[c +
 d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/d - (Sqrt[a + b]*(A*b + 2*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*d) + (A*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4117

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \frac{A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \int \frac{1}{2}(Ab -$$

$$= \frac{A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{1}{2}(Ab)\int$$

$$= \frac{A(a-b)\sqrt{a+b}\,\cot(c+dx)E\Big(\sin^{-1}\Big(\frac{\sqrt{a+}}{}}{}$$

$$= \frac{A(a-b)\sqrt{a+b}\,\cot(c+dx)E\Big(\sin^{-1}\Big(\frac{\sqrt{a+}}{}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.71, size = 1107, normalized size = 3.22

[illegible]

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(a*A*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*
x)/2] + A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 2*a*A*Sqrt[(-a + b)/(
a + b)]*Tan[(c + d*x)/2]^3 + a*A*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5
- A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - (2*I)*A*b*EllipticPi[-((a
 + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)
/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(a + b)] - (4*I)*a*B*EllipticPi[-((a + b)/(a - b)),
I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)
/2]^2)/(a + b)] - (2*I)*A*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(
-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqr
t[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(a + b)] - (4*I)*a*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqr
t[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*
Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
+ d*x)/2]^2)/(a + b)] - I*A*(a - b)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + T
an[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] + (2*I)*(a - b)*B*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c
```

```
+ d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
+ b)]))/(Sqrt[(-a + b)/(a + b)]*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(b - b*Tan[(c +
d*x)/2]^4 + a*(-1 + Tan[(c + d*x)/2]^2)^2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1385 vs. $2(315) = 630$.
time = 7.46, size = 1386, normalized size = 4.03

method	result	size
default	Expression too large to display	1386

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/d*(-1+cos(d*x+c))^2*(-2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+A*cos(d*x+c)*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+2*A*co
s(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a
+b))^(1/2))*b-2*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a+2*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+4*B*cos(d*x+c)*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a-2*A*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)+A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c)+A
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c
)+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b*
sin(d*x+c)-2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a*sin(d*x+c)+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*b*sin(d*x+c)+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
```

```
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-
b)/(a+b))^(1/2))*a*sin(d*x+c)+A*cos(d*x+c)^3*a-A*a*cos(d*x+c)^2+A*cos(d*x+c
)^2*b-A*b*cos(d*x+c))*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/
(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))\sqrt{a + b\sec(c + dx)}\cos(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*cos(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+dx)\left(A+\frac{B}{\cos(c+dx)}\right)\sqrt{a+\frac{b}{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2), x)
```

3.354 $\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=429

$$\frac{(a-b)\sqrt{a+b}\,(Ab+4aB)\cot(c+dx)E\left(\mathrm{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{4abd}$$

```
[Out] 1/4*(a-b)*(A*b+4*B*a)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+s
ec(d*x+c))/(a-b))^(1/2)/a/b/d+1/4*(A*b+2*a*(A+2*B))*cot(d*x+c)*EllipticF((a
+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec
(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-1/4*(4*A*a^2-A*b^
2+4*B*a*b)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a
,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec
(d*x+c))/(a-b))^(1/2)/a^2/d+1/4*(A*b+4*B*a)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/
2)/a/d+1/2*A*cos(d*x+c)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.47, antiderivative size = 429, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4117, 4189, 4143, 4006, 3869, 3917, 4089}

$$-\frac{\sqrt{a+b}\,(4a^2A+4abB-Ab^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\mathrm{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^2d}+\frac{\sqrt{a+b}\,(2a(A+2B)+Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\mathrm{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4ad}+\frac{(a-b)\sqrt{a+b}\,(4aB+Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\mathrm{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4abd}+\frac{(4aB+Ab)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4ad}+\frac{A\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(A*b + 4*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*b*d) + (Sqrt[a + b]*(
A*b + 2*a*(A + 2*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]
/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-
((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) - (Sqrt[a + b]*(4*a^2*A - A*b^2
+ 4*a*b*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*d) + ((A*b + 4*a*B)*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(4*a*d) + (A*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(2*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
```

```
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4117

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \frac{A\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d}$$

$$= \frac{(Ab+4aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4ad}$$

$$= \frac{(Ab+4aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4ad}$$

$$= \frac{(a-b)\sqrt{a+b}\,(Ab+4aB)\cot(c+dx)E\Big(\sin}{}$$

$$= \frac{(a-b)\sqrt{a+b}\,(Ab+4aB)\cot(c+dx)E\Big(\sin}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1149 vs. $2(429) = 858$.
time = 18.02, size = 1149, normalized size = 2.68

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (A*Sqrt[a + b*Sec[c + d*x]]*Sin[2*(c + d*x)])/(4*d) + (Sqrt[a + b*Sec[c + d
*x]]*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(a*A*b*Tan[(c + d*x)/2] + A*b^2*Ta
n[(c + d*x)/2] + 4*a^2*B*Tan[(c + d*x)/2] + 4*a*b*B*Tan[(c + d*x)/2] - 2*a*
A*b*Tan[(c + d*x)/2]^3 - 8*a^2*B*Tan[(c + d*x)/2]^3 + a*A*b*Tan[(c + d*x)/2
]^5 - A*b^2*Tan[(c + d*x)/2]^5 + 4*a^2*B*Tan[(c + d*x)/2]^5 - 4*a*b*B*Tan[(
c + d*x)/2]^5 + 8*a^2*A*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
```

```
 + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(a + b)] - 2*A*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
 d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*a*b*B*EllipticPi[-1, ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
 b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*a^2*A*Ellipt
icPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt
[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(a + b)] - 2*A*b^2*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*
Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 8*a*b*B*EllipticPi[-1
, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Ta
n[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2
)/(a + b)] + (a + b)*(A*b + 4*a*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a
+ b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*(2*a*A -
A*b + 4*b*B)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*a*d*Sqrt[b + a*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x
)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2064 vs. $2(388) = 776$.
time = 7.55, size = 2065, normalized size = 4.81

method	result	size
default	Expression too large to display	2065

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^2*(A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+2*A*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+4*B*cos(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a
*b-8*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*sin(d*x+c)*a*b+8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1
```

```
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+A*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+4*B*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+
c)*a^2+2*A*cos(d*x+c)^4*a^2-2*A*cos(d*x+c)^2*a^2+8*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-2*A*cos(d*x+c)*
a*b-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*
b^2*sin(d*x+c)-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a^2*sin(d*x+c)+4*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*sin(d*x+c)*a^2+A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*sin(d*x+c)*a*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+8*B*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-8*B*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+8*A*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*si
n(d*x+c)*a^2-2*A*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/
2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*sin(d*x+c)*b^2-4*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2+4*B*cos(d*x+c)^3*a^2+A
*cos(d*x+c)^2*b^2-4*B*cos(d*x+c)^2*a^2-A*cos(d*x+c)*b^2+3*A*cos(d*x+c)^3*a*
b-A*cos(d*x+c)^2*a*b+4*B*cos(d*x+c)^2*a*b-4*B*cos(d*x+c)*a*b)*(1+cos(d*x+c)
)^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*sqrt(b*sec(d*x
+ c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))\sqrt{a + b\sec(c + dx)}\cos^2(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*cos(c + d*x)**2, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^2, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + d\,x)^2\left(A + \frac{B}{\cos(c + d\,x)}\right)\sqrt{a + \frac{b}{\cos(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2), x)
```

3.355 $\int \cos^3(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=509

$$\frac{(a-b)\sqrt{a+b}\,(16a^2A-3Ab^2+6abB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c}{a+b}}}{24a^2bd}$$

```
[Out] 1/24*(a-b)*(16*A*a^2-3*A*b^2+6*B*a*b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))
^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b)
)^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/b/d+1/24*(2*a+b)*(8*A*a-3*A*b+6
*B*a)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))
^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b)
)^(1/2)/a^2/d-1/8*(4*A*a^2*b+A*b^3+8*B*a^3-2*B*a*b^2)*cot(d*x+c)*EllipticPi
((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2
)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d+1/24
*(16*A*a^2-3*A*b^2+6*B*a*b)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^2/d+1/12*(A
*b+6*B*a)*cos(d*x+c)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d+1/3*A*cos(d*x+c)
^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.70, antiderivative size = 509, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4117, 4189, 4143, 4006, 3869, 3917, 4089}

$$\frac{(a-b)\sqrt{a+b}\,(16a^2A+6abB-3Ab^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{24a^2bd}+\frac{\sqrt{a+b}\,(2a+b)(8aA+6aB-3Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{24a^2d}+\frac{(16a^2A+6abB-3Ab^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24a^2d}-\frac{\sqrt{a+b}\,(8a^3B+4a^2Ab-2ab^2B+Ab^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{8a^3d}+\frac{(6aB+Ab)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{12ad}+\frac{A\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(16*a^2*A - 3*A*b^2 + 6*a*b*B)*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a^2*b
*d) + (Sqrt[a + b]*(2*a + b)*(8*a*A - 3*A*b + 6*a*B)*Cot[c + d*x]*EllipticF
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a^2*
d) - (Sqrt[a + b]*(4*a^2*A*b + A*b^3 + 8*a^3*B - 2*a*b^2*B)*Cot[c + d*x]*El
lipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(8*a^3*d) + ((16*a^2*A - 3*A*b^2 + 6*a*b*B)*Sqrt[a + b*Sec[c + d
*x]]*Sin[c + d*x])/(24*a^2*d) + ((A*b + 6*a*B)*Cos[c + d*x]*Sqrt[a + b*Sec[
c + d*x]]*Sin[c + d*x])/(12*a*d) + (A*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/(3*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4117

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
```

```
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^3(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \frac{A\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx}{3d}$$

$$= \frac{(Ab+6aB)\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\mathrm{s}}{12ad}$$

$$= \frac{(16a^2A-3Ab^2+6abB)\sqrt{a+b\sec(c+dx)}}{24a^2d}$$

$$= \frac{(16a^2A-3Ab^2+6abB)\sqrt{a+b\sec(c+dx)}}{24a^2d}$$

$$= \frac{(a-b)\sqrt{a+b}\,(16a^2A-3Ab^2+6abB)\cot(c}{}$$

$$= \frac{(a-b)\sqrt{a+b}\,(16a^2A-3Ab^2+6abB)\cot(c}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1548 vs. $2(509) = 1018$.
time = 19.53, size = 1548, normalized size = 3.04

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((A*Sin[c + d*x])/12 + ((A*b + 6*a*B)*Sin[2*(c +
d*x)])/(24*a) + (A*Sin[3*(c + d*x)])/12))/d - (Sqrt[a + b*Sec[c + d*x]]*Sqr
t[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan
[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(16*a^3*A*Tan[(c + d*x)/2] + 16*
a^2*A*b*Tan[(c + d*x)/2] - 3*a*A*b^2*Tan[(c + d*x)/2] - 3*A*b^3*Tan[(c + d*
x)/2] + 6*a^2*b*B*Tan[(c + d*x)/2] + 6*a*b^2*B*Tan[(c + d*x)/2] - 32*a^3*A*
Tan[(c + d*x)/2]^3 + 6*a*A*b^2*Tan[(c + d*x)/2]^3 - 12*a^2*b*B*Tan[(c + d*x
)/2]^3 + 16*a^3*A*Tan[(c + d*x)/2]^5 - 16*a^2*A*b*Tan[(c + d*x)/2]^5 - 3*a*
A*b^2*Tan[(c + d*x)/2]^5 + 3*A*b^3*Tan[(c + d*x)/2]^5 + 6*a^2*b*B*Tan[(c +
d*x)/2]^5 - 6*a*b^2*B*Tan[(c + d*x)/2]^5 + 24*a^2*A*b*EllipticPi[-1, ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
 b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*A*b^3*Ellipt
icPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/
2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] +
 48*a^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)] - 12*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(a + b)] + 24*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2
]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 6*A
*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x
)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(a + b)] + 48*a^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt
[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 12*a*b^2*
B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2
]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan
[(c + d*x)/2]^2)/(a + b)] + (a + b)*(16*a^2*A - 3*A*b^2 + 6*a*b*B)*Elliptic
E[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(
1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x
)/2]^2)/(a + b)] - 2*a*(-(A*b^2) + 2*a*b*(7*A - 3*B) + 12*a^2*B)*EllipticF[
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1
+ Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)]))/(24*a^2*d*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt
[1 + Tan[(c + d*x)/2]^2]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x
)/2]^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2953 vs. $2(464) = 928$.
time = 8.25, size = 2954, normalized size = 5.80

method	result	size

default	Expression too large to display	2954

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVE RBOSE)

[Out] -1/24/d*(-1+cos(d*x+c))^2*(-2*A*cos(d*x+c)*a*b^2-12*B*cos(d*x+c)*a^2*b+6*A*cos(d*x+c)^2*a^2*b+6*B*cos(d*x+c)^2*a*b^2-16*A*cos(d*x+c)*a^2*b+16*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+16*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+6*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3+48*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3-24*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+16*A*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-3*A*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+24*A*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b-28*A*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+6*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-12*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+12*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+8*A*cos(d*x+c)^5*a^3+3*A*cos(d*x+c)*b^3+12*B*cos(d*x+c)^4*a^3-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),

```
((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin
(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+48*B*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-24*B*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+8*A*cos(
d*x+c)^3*a^3-16*A*cos(d*x+c)^2*a^3-3*A*cos(d*x+c)^2*b^3-12*B*cos(d*x+c)^2*a
^3+12*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^2*b+16*A*cos(d*x+c)*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-3*A*cos(d*x+c)*b^2*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*
x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+24*A*cos(d
*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b)
)^(1/2))*a^2*b-28*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+6*B*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b
+6*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a*b^2-12*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^2+10*A*cos(d*x+c)^4*a^2*b-A*cos
(d*x+c)^3*a*b^2+18*B*cos(d*x+c)^3*a^2*b+3*A*cos...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^3, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3*sec(d*x + c) + A*cos(d*x + c)^3)*sqrt(b*sec(d*x
+ c) + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))\sqrt{a + b\sec(c + dx)}\cos^3(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*cos(c + d*x)**3, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^3, x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c + d\,x)^3\left(A + \frac{B}{\cos(c + d\,x)}\right)\sqrt{a + \frac{b}{\cos(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2), x)
```

3.356 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=475

$$\frac{2(a-b)\sqrt{a+b}\,(18a^3Ab-246aAb^3-8a^4B-33a^2b^2B-147b^4B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+d}}{\sqrt{a+b}}\right.\right.}{315b^4d}$$

[Out] 2/315*(a-b)*(18*A*a^3*b-246*A*a*b^3-8*B*a^4-33*B*a^2*b^2-147*B*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d-2/315*(a-b)*(3*b^3*(25*A-49*B)-3*a*b^2*(57*A-13*B)-6*a^2*b*(3*A-B)+8*a^3*B)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d-2/315*(18*A*a*b-8*B*a^2-49*B*b^2)*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/b^2/d+2/63*(9*A*b-4*B*a)*(a+b*sec(d*x+c))^(5/2)*tan(d*x+c)/b^2/d+2/9*B*sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*tan(d*x+c)/b/d-2/315*(18*A*a^2*b-75*A*b^3-8*B*a^3-39*B*a*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d

Rubi [A]
time = 0.79, antiderivative size = 475, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4118, 4167, 4087, 4090, 3917, 4089}

$$-\frac{2(-8a^2B+18aAb-49b^2B)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{315b^2d}-\frac{2(a-b)\sqrt{a+b}\,(8a^3B-6a^2b(3A-B)-3ab^2(57A-13B)+3b^3(25A-49B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{315b^3d}-\frac{2(-8a^3B+18a^2Ab-39ab^2B-75Ab^3)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{315b^2d}+\frac{2(a-b)\sqrt{a+b}\,(-8a^4B+18a^3Ab-33a^2b^2B-246aAb^3-147b^4B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{315b^4d}+\frac{2(9Ab-4aB)\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{63b^2d}+\frac{2B\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{5/2}}{9bd}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] (2*(a - b)*Sqrt[a + b]*(18*a^3*A*b - 246*a*A*b^3 - 8*a^4*B - 33*a^2*b^2*B - 147*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b^4*d) - (2*(a - b)*Sqrt[a + b]*(3*b^3*(25*A - 49*B) - 3*a*b^2*(57*A - 13*B) - 6*a^2*b*(3*A - B) + 8*a^3*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(315*b^3*d) - (2*(18*a^2*A*b - 75*A*b^3 - 8*a^3*B - 39*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(315*b^2*d) - (2*(18*a*A*b - 8*a^2*B - 49*b^2*B)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(315*b^2*d) + (2*(9*A*b - 4*a*B)*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*b^2*d) + (2*B*Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(9*b*d)

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4118

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d^
2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 2)/(b*f*
(m + n))), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e +
 f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m +
n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n
, 0] &&  !IGtQ[m, 1]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
```

```
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{2B\sec(c+dx)(a+b\sec(c+dx))^{5/2}\tan(c+d}{9bd}$$

$$= \frac{2(9Ab-4aB)(a+b\sec(c+dx))^{5/2}\tan(c+d}{63b^2d}$$

$$= -\frac{2(18aAb-8a^2B-49b^2B)\,(a+b\sec(c+dx}{315b^2d}$$

$$= -\frac{2(18a^2Ab-75Ab^3-8a^3B-39ab^2B)\,\sqrt{a+}}{315b^2d}$$

$$= -\frac{2(18a^2Ab-75Ab^3-8a^3B-39ab^2B)\,\sqrt{a+}}{315b^2d}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(18a^3Ab-246aAb^3-8a^4B-}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3766 vs. 2(475) = 950.
time = 26.17, size = 3766, normalized size = 7.93

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((2*(-18*a^3*A*b + 246*a*A*b^3 + 8
*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*Sin[c + d*x])/(315*b^3) + (2*Sec[c + d*x
]^3*(9*A*b*Sin[c + d*x] + 10*a*B*Sin[c + d*x]))/63 + (2*Sec[c + d*x]^2*(72*
a*A*b*Sin[c + d*x] + 3*a^2*B*Sin[c + d*x] + 49*b^2*B*Sin[c + d*x]))/(315*b)
 + (2*Sec[c + d*x]*(9*a^2*A*b*Sin[c + d*x] + 75*A*b^3*Sin[c + d*x] - 4*a^3*
B*Sin[c + d*x] + 88*a*b^2*B*Sin[c + d*x]))/(315*b^2) + (2*b*B*Sec[c + d*x]^
3*Tan[c + d*x])/9))/(d*(b + a*Cos[c + d*x])) - (2*((2*a^3*A)/(35*b*Sqrt[b +
 a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (82*a*A*b)/(105*Sqrt[b + a*Cos[c + d
*x]]*Sqrt[Sec[c + d*x]]) - (11*a^2*B)/(105*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Se
```

c[c + d*x]]) - (8*a^4*B)/(315*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x
]]) - (7*b^2*B)/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (31*a^2*
A*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) + (2*a^4*A*Sqrt[Sec[c
+ d*x]])/(35*b^2*Sqrt[b + a*Cos[c + d*x]]) + (5*A*b^2*Sqrt[Sec[c + d*x]])/(
21*Sqrt[b + a*Cos[c + d*x]]) - (8*a^5*B*Sqrt[Sec[c + d*x]])/(315*b^3*Sqrt[b
 + a*Cos[c + d*x]]) - (31*a^3*B*Sqrt[Sec[c + d*x]])/(315*b*Sqrt[b + a*Cos[c
 + d*x]]) + (13*a*b*B*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]]) -
(82*a^2*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]
]) + (2*a^4*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*b^2*Sqrt[b + a*Cos[c
 + d*x]]) - (8*a^5*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(315*b^3*Sqrt[b +
 a*Cos[c + d*x]]) - (11*a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*b*S
qrt[b + a*Cos[c + d*x]]) - (7*a*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(1
5*Sqrt[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*S
ec[c + d*x])^(3/2)*(2*(a + b)*(-18*a^3*A*b + 246*a*A*b^3 + 8*a^4*B + 33*a^2
*b^2*B + 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)] - 2*b*(a + b)*(8*a^3*B - 6*a^2*b*(3*A + B) + 3*a*b^2*(57*A
 + 13*B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[
(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + (-18*a^3*A*b + 246*a*A*b^3 + 8*a^4*B + 33*a^
2*b^2*B + 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*T
an[(c + d*x)/2]))/(315*b^3*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c + d*x)/2]^2
]*Sec[c + d*x]^(3/2)*(-1/315*(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c
 + d*x]*(2*(a + b)*(-18*a^3*A*b + 246*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 14
7*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((
a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)] - 2*b*(a + b)*(8*a^3*B - 6*a^2*b*(3*A + B) + 3*a*b^2*(57*A + 13*B) +
3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)] + (-18*a^3*A*b + 246*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 1
47*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x
)/2]))/(b^3*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[Co
s[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-18*a^3*A*b + 2
46*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos
[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellipti
cE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(8*a^3*B - 6*a^
2*b*(3*A + B) + 3*a*b^2*(57*A + 13*B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-18*a^3*A*b +
246*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*b^3*Sqrt[b + a*Cos[c +
d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*
(((-18*a^3*A*b + 246*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*Cos[c +
d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-18*a^3*A*b + 2
46*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/

```
((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1
 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (b*(a + b)*(8*a^
3*B - 6*a^2*b*(3*A + B) + 3*a*b^2*(57*A + 13*B) + 3*b^3*(25*A + 49*B))*Sqrt
[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x
])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x
])] + ((a + b)*(-18*a^3*A*b + 246*a*A*b^3 + 8*a^4*B + 33*a^2*b^2*B + 147*b^
4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((
b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (b*(a + b)*(8*a^3*B - 6*a
^2*b*(3*A + B) + 3*a*b^2*(57*A + 13*B) + 3*b^3*...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4393 vs. $2(437) = 874$.
time = 11.22, size = 4394, normalized size = 9.25

method	result	size
default	Expression too large to display	4394

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/315/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c
))^2*(-18*A*cos(d*x+c)^6*a^4*b+9*A*cos(d*x+c)^6*a^3*b^2+246*A*cos(d*x+c)^6*
a^2*b^3+75*A*cos(d*x+c)^6*a*b^4-4*B*cos(d*x+c)^6*a^4*b+33*B*cos(d*x+c)^6*a^
3*b^2+88*B*cos(d*x+c)^6*a^2*b^3+147*B*cos(d*x+c)^6*a*b^4+18*A*cos(d*x+c)^5*
a^4*b-18*A*cos(d*x+c)^5*a^3*b^2-165*A*cos(d*x+c)^5*a^2*b^3+246*A*cos(d*x+c)
^5*a*b^4+8*B*cos(d*x+c)^5*a^4*b-34*B*cos(d*x+c)^5*a^3*b^2+33*B*cos(d*x+c)^5
*a^2*b^3-10*B*cos(d*x+c)^5*a*b^4+9*A*cos(d*x+c)^4*a^3*b^2-246*A*sin(d*x+c)*
cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a*b^4+75*A*cos(d*x+c)^5*b^5-30*A*cos(d*x+c)^3*b^5-45*A*cos(d*x+c)*b^5+8*B*
cos(d*x+c)^6*a^5-8*B*cos(d*x+c)^5*a^5+147*B*cos(d*x+c)^5*b^5-98*B*cos(d*x+c
)^4*b^5-14*B*cos(d*x+c)^2*b^5-204*A*cos(d*x+c)^4*a*b^4-4*B*cos(d*x+c)^4*a^4
*b-68*B*cos(d*x+c)^4*a^2*b^3-81*A*cos(d*x+c)^3*a^2*b^3+B*cos(d*x+c)^3*a^3*b
^2-52*B*cos(d*x+c)^3*a*b^4-117*A*cos(d*x+c)^2*a*b^4-53*B*cos(d*x+c)^2*a^2*b
^3-85*B*cos(d*x+c)*a*b^4-35*B*b^5-18*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+2*B*sin(d*x+c)*c
os(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^3*b^2+33*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
```

a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+186*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+18*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+18*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-246*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+153*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+246*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-8*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-33*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-33*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-147*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+8*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b+2*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+33*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+186*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+18*A*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-246*A*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-246*A*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-33*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-33*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d

```
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-147*B*sin(d*x+c)*cos(d*x+c)^
5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^5 + A*a*sec(d*x + c)^3 + (B*a + A*b)*sec(d*x + c
)^4)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))(a + b\sec(c + dx))^{\frac{3}{2}} \sec^3(c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**3,
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/cos(c + d*x)^3,x)
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/cos(c + d*x)^3, x)
```

3.357 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=388

$$-\frac{2(a-b)\sqrt{a+b}\,(21a^2Ab+63Ab^3-6a^3B+82ab^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{105b^3d}$$

```
[Out] -2/105*(a-b)*(21*A*a^2*b+63*A*b^3-6*B*a^3+82*B*a*b^2)*cot(d*x+c)*EllipticE(
(a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-s
ec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d+2/105*(a-b)*(
b^2*(63*A-25*B)+6*a^2*B-a*(21*A*b-57*B*b))*cot(d*x+c)*EllipticF((a+b*sec(d*
x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/
(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+2/35*(7*A*b-2*B*a)*(a+b*
sec(d*x+c))^(3/2)*tan(d*x+c)/b/d+2/7*B*(a+b*sec(d*x+c))^(5/2)*tan(d*x+c)/b/
d+2/105*(21*A*a*b-6*B*a^2+25*B*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b/d
```

Rubi [A]

time = 0.53, antiderivative size = 388, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4095, 4087, 4090, 3917, 4089}

$$\frac{2(a-b)\sqrt{a+b}\,(6a^2B-a(21Ab-57bB)+b^2(63A-25B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{105b^2d}+\frac{2(-6a^2B+21aAb+25b^2B)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{105bd}-\frac{2(a-b)\sqrt{a+b}\,(-6a^3B+21a^2Ab+82ab^2B+63Ab^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{105b^3d}+\frac{2(7Ab-2aB)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{35bd}+\frac{2B\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(21*a^2*A*b + 63*A*b^3 - 6*a^3*B + 82*a*b^2*B)*Cot[
c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
 - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(
a - b))])/(105*b^3*d) + (2*(a - b)*Sqrt[a + b]*(b^2*(63*A - 25*B) + 6*a^2*B
 - a*(21*A*b - 57*b*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*
x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqr
t[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b^2*d) + (2*(21*a*A*b - 6*a^2*B
+ 25*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*b*d) + (2*(7*A*b -
2*a*B)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*b*d) + (2*B*(a + b*Sec[
c + d*x])^(5/2)*Tan[c + d*x])/(7*b*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{2B(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7bd} + \frac{2\int \mathrm{se}}{}$$

$$= \frac{2(7Ab-2aB)(a+b\sec(c+dx))^{3/2}\tan(c+d}{35bd}$$

$$= \frac{2(21aAb-6a^2B+25b^2B)\sqrt{a+b\sec(c+dx}}{105bd}$$

$$= \frac{2(21aAb-6a^2B+25b^2B)\sqrt{a+b\sec(c+dx}}{105bd}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(21a^2Ab+63Ab^3-6a^3B+}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3342 vs. $2(388) = 776$.

time = 24.23, size = 3342, normalized size = 8.61

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((-2*(-21*a^2*A*b - 63*A*b^3 + 6*a
^3*B - 82*a*b^2*B)*Sin[c + d*x])/(105*b^2) + (2*Sec[c + d*x]^2*(7*A*b*Sin[c
 + d*x] + 8*a*B*Sin[c + d*x]))/35 + (2*Sec[c + d*x]*(42*a*A*b*Sin[c + d*x]
+ 3*a^2*B*Sin[c + d*x] + 25*b^2*B*Sin[c + d*x]))/(105*b) + (2*b*B*Sec[c + d
*x]^2*Tan[c + d*x])/7))/(d*(b + a*Cos[c + d*x])) + (2*(-1/5*(a^2*A)/(Sqrt[b
 + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (3*A*b^2)/(5*Sqrt[b + a*Cos[c + d*
x]]*Sqrt[Sec[c + d*x]]) + (2*a^3*B)/(35*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec
[c + d*x]]) - (82*a*b*B)/(105*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]])
- (a^3*A*Sqrt[Sec[c + d*x]])/(5*b*Sqrt[b + a*Cos[c + d*x]]) + (a*A*b*Sqrt[S
ec[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]) - (31*a^2*B*Sqrt[Sec[c + d*x]])/
(105*Sqrt[b + a*Cos[c + d*x]]) + (2*a^4*B*Sqrt[Sec[c + d*x]])/(35*b^2*Sqrt[
b + a*Cos[c + d*x]]) + (5*b^2*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c +
d*x]]) - (a^3*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*b*Sqrt[b + a*Cos[c
+ d*x]]) - (3*a*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*Cos[
c + d*x]]) - (82*a^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a
*Cos[c + d*x]]) + (2*a^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*b^2*Sqr
t[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c
+ d*x])^(3/2)*(2*(a + b)*(-21*a^2*A*b - 63*A*b^3 + 6*a^3*B - 82*a*b^2*B)*Sq
```

```
rt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
 Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b
*(a + b)*(-6*a^2*B + 3*a*b*(7*A + 19*B) + b^2*(63*A + 25*B))*Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x
]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-21*a^2*A*b -
63*A*b^3 + 6*a^3*B - 82*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2]))/(105*b^2*d*(b + a*Cos[c + d*x])^2*Sqrt[Sec[(c
 + d*x)/2]^2]*Sec[c + d*x]^(3/2)*((a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*
Sin[c + d*x]*(2*(a + b)*(-21*a^2*A*b - 63*A*b^3 + 6*a^3*B - 82*a*b^2*B)*Sqr
t[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*
(a + b)*(-6*a^2*B + 3*a*b*(7*A + 19*B) + b^2*(63*A + 25*B))*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-21*a^2*A*b - 6
3*A*b^3 + 6*a^3*B - 82*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c +
d*x)/2]^2*Tan[(c + d*x)/2]))/(105*b^2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(
c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(
2*(a + b)*(-21*a^2*A*b - 63*A*b^3 + 6*a^3*B - 82*a*b^2*B)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-6*a^
2*B + 3*a*b*(7*A + 19*B) + b^2*(63*A + 25*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-21*a^2*A*b - 63*A*b^3 + 6*a^
3*B - 82*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[
(c + d*x)/2]))/(105*b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2])
+ (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-21*a^2*A*b - 63*A*b^3 + 6*a^
3*B - 82*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 +
 ((a + b)*(-21*a^2*A*b - 63*A*b^3 + 6*a^3*B - 82*a*b^2*B)*Sqrt[(b + a*Cos[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]],
(a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c
+ d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(a +
 b)*(-6*a^2*B + 3*a*b*(7*A + 19*B) + b^2*(63*A + 25*B))*Sqrt[(b + a*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c +
d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*
(-21*a^2*A*b - 63*A*b^3 + 6*a^3*B - 82*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[
c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c
 + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x]
)/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 +
Cos[c + d*x]))] + (b*(a + b)*(-6*a^2*B + 3*a*b*(7*A + 19*B) + b^2*(63*A + 2
5*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + (
(b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b
 + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(-21*a^2*A*b - 63*A*b^
3 + 6*a^3*B - 82*a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[
```

```
(c + d*x)/2] - (-21*a^2*A*b - 63*A*b^3 + 6*a^3*B - 82*a*b^2*B)*(b + a*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-21*a^2*A*b -
63*A*b^3 + 6*a^3*B - 82*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c +
 d*x)/2]^2*Tan[(c + d*x)/2]^2 + (b*(a + b)*(-6*...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3423 vs. $2(354) = 708$.
time = 11.40, size = 3424, normalized size = 8.82

method	result	size
default	Expression too large to display	3424

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/105/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c
))^2*(-15*B*b^4+21*A*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-39*B*cos(d*x+c)*a*b^3+21*A*cos(d*x
+c)^4*a^2*b^2-6*B*cos(d*x+c)^4*a^3*b-55*B*cos(d*x+c)^4*a^2*b^2+82*B*cos(d*x
+c)^4*a*b^3-63*A*cos(d*x+c)^3*a^2*b^2+3*B*cos(d*x+c)^3*a^3*b-68*B*cos(d*x+c
)^3*a*b^3-63*A*cos(d*x+c)^2*a*b^3-27*B*cos(d*x+c)^2*a^2*b^2+21*A*cos(d*x+c)
^5*a^3*b+42*A*cos(d*x+c)^5*a^2*b^2+63*A*cos(d*x+c)^5*a*b^3+3*B*cos(d*x+c)^5
*a^3*b+82*B*cos(d*x+c)^5*a^2*b^2+25*B*cos(d*x+c)^5*a*b^3-21*A*cos(d*x+c)^4*
a^3*b+25*B*cos(d*x+c)^4*b^4-21*A*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-63*A*cos(d*x+c)^4*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3
-6*B*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*sin(d*x+c)*a^3*b+51*B*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+84*A*cos(d*x+c)^4*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-21*A
*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*sin(d*x+c)*a^3*b+82*B*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+6*B*cos(d*x+c)^4*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b-82*B*cos(d*
x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
```

```
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d
*x+c)*a^2*b^2-82*B*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+21*A*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2+84*A*cos(d*x+
c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*a*b^3-21*A*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*a^3*b-10*B*cos(d*x+c)^2*b^4-21*A*cos(d*x+c)*b^4-
6*B*cos(d*x+c)^5*a^4+63*A*cos(d*x+c)^4*b^4+6*B*cos(d*x+c)^4*a^4-42*A*cos(d*
x+c)^3*b^4-21*A*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b^2-63*A*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3-6*B*cos(d*x+c)^3
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*
a^3*b+51*B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*sin(d*x+c)*a^2*b^2+82*B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+6*B*cos(d*x+c)^3*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3*b
-82*B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*sin(d*x+c)*a^2*b^2-82*B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^3+63*A*cos(d*x+c)^4*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4-63*A*
cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*sin(d*x+c)*b^4+25*B*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^4+6*B*cos(d*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^4 + A*a*sec(d*x + c)^2 + (B*a + A*b)*sec(d*x + c
)^3)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec (c + dx)) (a + b\sec (c + dx))^{\frac{3}{2}} \sec^2 (c + dx)\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**2,
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+d\,x)}\right)\,\left(a + \frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos\left(c + d\,x\right)^2}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/cos(c + d*x)^2,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/cos(c + d*x)^2, x)
```

3.358 $\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=312

$$-\frac{2(a-b)\sqrt{a+b}\,(20aAb+3a^2B+9b^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec}{a}}}{15b^2d}$$

```
[Out] -2/15*(a-b)*(20*A*a*b+3*B*a^2+9*B*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c)
)^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b
))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+2/15*(a-b)*(15*A*a-5*A*b-3*B
*a+9*B*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(
a-b))^(1/2)/b/d+2/5*B*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/15*(5*A*b+3*B*a
)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.36, antiderivative size = 312, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4087, 4090, 3917, 4089}

$$-\frac{2(a-b)\sqrt{a+b}\,(3a^2B+20aAb+9b^2B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^2d}+\frac{2(a-b)\sqrt{a+b}\,(15aA-3aB-5Ab+9bB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15bd}+\frac{2(3aB+5Ab)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{15d}+\frac{2B\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(20*a*A*b + 3*a^2*B + 9*b^2*B)*Cot[c + d*x]*Ellipti
cE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(
1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^
2*d) + (2*(a - b)*Sqrt[a + b]*(15*a*A - 5*A*b - 3*a*B + 9*b*B)*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(15*b*d) + (2*(5*A*b + 3*a*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(15*
d) + (2*B*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{2B(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{2}{5}\int \text{se}$$

$$= \frac{2(5Ab+3aB)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{15d}$$

$$= \frac{2(5Ab+3aB)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{15d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(20aAb+3a^2B+9b^2B)\cot(}{}$$

Mathematica [A]
time = 18.37, size = 502, normalized size = 1.61

$$-\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\left(2(a+b)\left(20aAb+3a^2B+9b^2B\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)-2b(a+b)(3a(5A+B)+b(5A+9B))\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)+\left(20aAb+3a^2B+9b^2B\right)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{15bd(b+a\cos(c+dx))^2(B+A\cos(c+dx))\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sec^{\frac{5}{2}}(c+dx)}+\frac{\cos^2(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\left(\frac{2\left(20aAb+3a^2B+9b^2B\right)\sin(c+dx)}{15b}+\frac{2}{15}\sec(c+dx)(5Ab\sin(c+dx)+6aB\sin(c+dx))+\frac{2}{5}bB\sec(c+dx)\tan(c+dx)\right)}{d(b+a\cos(c+dx))(B+A\cos(c+dx))}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + B
*Sec[c + d*x])*(2*(a + b)*(20*a*A*b + 3*a^2*B + 9*b^2*B)*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(3*a*(5
*A + B) + b*(5*A + 9*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*
Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/
2]], (a - b)/(a + b)] + (20*a*A*b + 3*a^2*B + 9*b^2*B)*Cos[c + d*x]*(b + a*
Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b*d*(b + a*Cos[c +
d*x])^2*(B + A*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2)) +
 (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x])*((2*(20*a*
A*b + 3*a^2*B + 9*b^2*B)*Sin[c + d*x])/(15*b) + (2*Sec[c + d*x]*(5*A*b*Sin[
c + d*x] + 6*a*B*Sin[c + d*x]))/15 + (2*b*B*Sec[c + d*x]*Tan[c + d*x])/5))/
(d*(b + a*Cos[c + d*x])*(B + A*Cos[c + d*x]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 2682 vs. $2(282) = 564$.
time = 10.98, size = 2683, normalized size = 8.60

method	result	size
default	Expression too large to display	2683

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/15/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(5*A*cos(d*x+c)^3*b^3-3*B*cos(d*x+c)^3*a^3+9*B*cos(d*x+c)^3*b^3-6*B*cos
(d*x+c)^2*b^3+3*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-3*B*b^3+12*B*cos(d*x+c)^2*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-3*B*cos(
d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a^2*b-9*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+15*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+20*A*cos(d*x+c)^
3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a*b^2-20*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^2*b-20*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+c
```

```
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+3*B*cos(d*x+c)^3*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+
12*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a*b^2-3*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-9*B*cos(d*x+c)^3*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+15*A*cos
(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^2*b+20*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-20*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-20*A*cos(d*x+c
)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a*b^2-5*A*cos(d*x+c)*b^3+3*B*cos(d*x+c)^4*a^3+5*A*cos(d*x+c)^3*sin(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+9*B*co
s(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*b^3-3*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a^3-9*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+5*A*cos(d*x+c)^2*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+9
*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*b^3-3*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3-9*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+5*A*cos(d*x+c)^4
*a*b^2+6*B*cos(d*x+c)^4*a^2*b+9*B*cos(d*x+c)^4*a*b^2-20*A*cos(d*x+c)^3*a^2*
b+20*A*cos(d*x+c)^4*a^2*b+20*A*cos(d*x+c)^3*a*b^2+3*B*cos(d*x+c)^3*a^2*b-25
*A*cos(d*x+c)^2*a*b^2-9*B*cos(d*x+c)^2*a^2*b-9*B*cos(d*x+c)*a*b^2)/(b+a*cos
(d*x+c))/cos(d*x+c)^2/sin(d*x+c)^5/b
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^3 + A*a*sec(d*x + c) + (B*a + A*b)*sec(d*x + c)^
2)*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))\,(a + b\sec(c + dx))^{\frac{3}{2}}\sec(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(3/2)*sec(c + d*x), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+d\,x)}\right)\,\left(a + \frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\cos(c + d\,x)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/cos(c + d*x),x)
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/cos(c + d*x), x)
```

3.359 $\int (a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=381

$$-\frac{2(a-b)\sqrt{a+b}\,(3Ab+4aB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{3bd}$$

```
[Out] -2/3*(a-b)*(3*A*b+4*B*a)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^
(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(
1+sec(d*x+c))/(a-b))^(1/2)/b/d-2/3*(b^2*(3*A-B)-3*a^2*B-a*(6*A*b-4*B*b))*co
t(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*
(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/
b/d-2*a*A*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,
((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(
d*x+c))/(a-b))^(1/2)/d+2/3*b*B*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.30, antiderivative size = 381, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4003, 4143, 4006, 3869, 3917, 4089}

$$-\frac{2\sqrt{a+b}\,\left(-3a^2B-a(6Ab-4bB)+b^2(3A-B)\right)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bd}-\frac{2(a-b)\sqrt{a+b}\,(4aB+3Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bd}-\frac{2aA\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}+\frac{2bB\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*A*b + 4*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) - (2*Sqrt[a +
 b]*(b^2*(3*A - B) - 3*a^2*B - a*(6*A*b - 4*b*B))*Cot[c + d*x]*EllipticF[Ar
cSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - S
ec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) - (
2*a*A*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d + (2*b*B*Sqrt[a + b*Sec[c + d
*x]]*Tan[c + d*x])/(3*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4003

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m
+ (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m
- 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
 - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{2bB\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{3a^2A}{2}+\frac{1}{2}(6a\ldots}{}$$

$$= \frac{2bB\sqrt{a+b\sec(c+dx)}\tan(c+dx)}{3d} + \frac{2}{3}\int \frac{\frac{3a^2A}{2}+\left(-\frac{1}{2}b\ldots\right.}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(3Ab+4aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{\ldots}}{}\right.\right.}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(3Ab+4aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{\ldots}}{}\right.\right.}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 6063 vs. 2(381) = 762.
time = 24.39, size = 6063, normalized size = 15.91

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2336 vs. 2(346) = 692.
time = 10.27, size = 2337, normalized size = 6.13

method	result	size
default	Expression too large to display	2337

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))^2*(6*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-4*B*cos(d*x+c)^2*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+4*B*cos(d
*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(
d*x+c)*a*b-3*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
```

```
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*sin(d*x+c)*a*b+6*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-4*B*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+4*B*cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x
+c)*a*b-3*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*sin(d*x+c)*a*b-3*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+3*A*cos(d*x+c)^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-4*B*cos
(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*si
n(d*x+c)*a^2+B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*sin(d*x+c)*b^2-3*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+3*A*cos(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-4*B*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d
*x+c)*a^2+B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*sin(d*x+c)*b^2-b^2*B+6*A*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-
1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2-3*A*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2+B*c
os(d*x+c)^2*b^2+4*B*cos(d*x+c)^3*a^2+3*A*cos(d*x+c)^2*b^2-4*B*cos(d*x+c)^2*
a^2-3*A*cos(d*x+c)*b^2+3*A*cos(d*x+c)^3*a*b+B*cos(d*x+c)^3*a*b-3*A*cos(d*x+
c)^2*a*b+4*B*cos(d*x+c)^2*a*b-5*B*cos(d*x+c)*a*b-3*A*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2+6*A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*cos(d*x+c)^2*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2
))*sin(d*x+c)*a^2+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2)*((b+a*cos(d*x+c))/
```

```
cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))(a + b\sec(c + dx))^{\frac{3}{2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2),x)
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2), x)
```

3.360 $\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=361

$$\frac{(a-b)\sqrt{a+b}\,(aA-2bB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b}{}}}{bd}$$

```
[Out] (a-b)*(A*a-2*B*b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(
(a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d
*x+c))/(a-b))^(1/2)/b/d+(2*b*(A-B)+a*(A+4*B))*cot(d*x+c)*EllipticF((a+b*sec
(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c
))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-(3*A*b+2*B*a)*cot(d*x+c)*
EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*
(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/
d+a*A*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.29, antiderivative size = 361, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4110, 4143, 4006, 3869, 3917, 4089}

$$\frac{\sqrt{a+b}\,(a(A+4B)+2b(A-B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}+\frac{(a-b)\sqrt{a+b}\,(aA-2bB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{\sqrt{a+b}\,(2aB+3Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}+\frac{aA\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(a*A - 2*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b*d) + (Sqrt[a + b]*(2*b*
(A - B) + a*(A + 4*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/d - (Sqrt[a + b]*(3*A*b + 2*a*B)*Cot[c
 + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]],
 (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c
 + d*x]))/(a - b))])/d + (a*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} - \int \frac{-\frac{1}{2}a(\cdots}{\cdots}$$

$$= \frac{aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{1}{2}(b(aA \cdots$$

$$= \frac{(a-b)\sqrt{a+b}\,(aA-2bB)\cot(c+dx)E\Big(\sin^{-}\cdots}{\cdots}$$

$$= \frac{(a-b)\sqrt{a+b}\,(aA-2bB)\cot(c+dx)E\Big(\sin^{-}\cdots}{\cdots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 971 vs. 2(361) = 722.
time = 16.18, size = 971, normalized size = 2.69

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*b*B*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(d*(b + a*Cos[
c + d*x])) + ((a + b*Sec[c + d*x])^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)
]*(a^2*A*Tan[(c + d*x)/2] + a*A*b*Tan[(c + d*x)/2] - 2*a*b*B*Tan[(c + d*x)/
2] - 2*b^2*B*Tan[(c + d*x)/2] - 2*a^2*A*Tan[(c + d*x)/2]^3 + 4*a*b*B*Tan[(c
 + d*x)/2]^3 + a^2*A*Tan[(c + d*x)/2]^5 - a*A*b*Tan[(c + d*x)/2]^5 - 2*a*b*
B*Tan[(c + d*x)/2]^5 + 2*b^2*B*Tan[(c + d*x)/2]^5 + 6*a*A*b*EllipticPi[-1,
ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 4*a^2*B*
EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
 b)] + 6*a*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Ta
n[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)
/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 4*a^2*B*EllipticPi[-1, ArcSin[Tan[
(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] +
(a + b)*(a*A - 2*b*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*
Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*(2*a*b*(A - B) + a^2*B -
 b^2*(A + B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 -
 Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)
```

```
/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(d*(b + a*Cos[c + d*x])^(3/2)*Sec[
c + d*x]^(3/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)
/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2195 vs. $2(332) = 664$.
time = 10.02, size = 2196, normalized size = 6.08

method	result	size
default	Expression too large to display	2196

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/d*(-1+cos(d*x+c))^2*(A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-4*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-2*B*cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b
+4*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*sin(d*x+c)*a*b+A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*sin(d*x+c)+6*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b+2*A*cos(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+2*B*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+
c)*b^2-A*cos(d*x+c)^2*a^2+A*cos(d*x+c)^3*a^2+2*B*cos(d*x+c)*b^2-2*b^2*B-A*c
os(d*x+c)*a*b+A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^2-2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2+4*B*cos(d*x+c)*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2+6*A*E
llipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c
)*a*b+A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(
```

```
d*x+c)*a*b-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a*b*sin(d*x+c)-2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*a*b*sin(d*x+c)+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a*b*sin(d*x+c)+A*cos(d*x+c)^2*a*b+2*B*cos(d*x+c)^2*a*b-2*B
*cos(d*x+c)*a*b+2*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*sin(d*x+c)-2*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b
))^(1/2))*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*sin(d*x+c)-2*B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)+2*B*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)+4*B*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-2*B*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2)*(
1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*
x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm="
giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2), x)
```

3.361 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=428

$$\frac{(a-b)\sqrt{a+b}\,(5Ab+4aB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-}}{4bd}$$

```
[Out] 1/4*(a-b)*(5*A*b+4*B*a)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(
1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1
+sec(d*x+c))/(a-b))^(1/2)/b/d+1/4*(2*A*a+5*A*b+4*B*a+8*B*b)*cot(d*x+c)*Elli
pticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(
b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/4*(4*A*a^
2+3*A*b^2+12*B*a*b)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2
),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(
-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+1/4*(5*A*b+4*B*a)*sin(d*x+c)*(a+b*sec(d*
x+c))^(1/2)/d+1/2*a*A*cos(d*x+c)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.51, antiderivative size = 428, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4110, 4189, 4143, 4006, 3869, 3917, 4089}

$$-\frac{\sqrt{a+b}\,(4a^2A+12abB+3Ab^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4ad}+\frac{\sqrt{a+b}\,(2aA+4aB+5Ab+8bB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4d}+\frac{(a-b)\sqrt{a+b}\,(4aB+5Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4bd}+\frac{(4aB+5Ab)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}+\frac{aA\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(5*A*b + 4*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a +
 b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))
/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*b*d) + (Sqrt[a + b]*(
2*a*A + 5*A*b + 4*a*B + 8*b*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec
[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*d) - (Sqrt[a + b]*(4*a^2*A
+ 3*A*b^2 + 12*a*b*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*
Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
 + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a*d) + ((5*A*b + 4*a*B)*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*A*Cos[c + d*x]*Sqrt[a + b
*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
```

```
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
```

```
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx}{2d}$$

$$= \frac{(5Ab+4aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d}$$

$$= \frac{(5Ab+4aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d}$$

$$= \frac{(a-b)\sqrt{a+b}\,(5Ab+4aB)\cot(c+dx)E\Big(\mathrm{si}}{}$$

$$= \frac{(a-b)\sqrt{a+b}\,(5Ab+4aB)\cot(c+dx)E\Big(\mathrm{si}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 19.21, size = 1598, normalized size = 3.73

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*A*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[2*(c + d*x)])/(4*d*(b + a*
Cos[c + d*x])) - ((a + b*Sec[c + d*x])^(3/2)*(5*a*A*b*Sqrt[(-a + b)/(a + b)
]*Tan[(c + d*x)/2] + 5*A*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 4*a^
```

```
2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + 4*a*b*Sqrt[(-a + b)/(a + b)]*
B*Tan[(c + d*x)/2] - 10*a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 - 8
*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^3 + 5*a*A*b*Sqrt[(-a + b)/(a
 + b)]*Tan[(c + d*x)/2]^5 - 5*A*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]
^5 + 4*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - 4*a*b*Sqrt[(-a + b
)/(a + b)]*B*Tan[(c + d*x)/2]^5 - (8*I)*a^2*A*EllipticPi[-((a + b)/(a - b))
, I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(a + b)] - (6*I)*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sq
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
b)] - (24*I)*a*b*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(
a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (8*I)*a
^2*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c
 + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^
2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (6
*I)*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*T
an[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
 - (24*I)*a*b*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a +
 b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
 + b)] - I*(a - b)*(5*A*b + 4*a*B)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b
)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Ta
n[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2
)/(a + b)] + (2*I)*(a - b)*(2*a*A + b*(A + 4*B))*EllipticF[I*ArcSinh[Sqrt[(
-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)
/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)]))/(4*Sqrt[(-a + b)/(a + b)]*d*(b + a*Cos[c + d*x])
^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(-1 + Tan[(c
+ d*x)/2]^2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2
]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2438 vs. $2(387) = 774$.

time = 8.33, size = 2439, normalized size = 5.70

method	result	size
default	Expression too large to display	2439

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^2*(5*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+2*A*cos(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+4*B*cos(d*x+c
)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)
*a*b-16*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*sin(d*x+c)*a*b+24*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(
d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b+5*A*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+5*A*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-8*A
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
sin(d*x+c)*b^2+4*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+8*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+2*A*cos(d*x+c)^4*a^2-2*A
*cos(d*x+c)^2*a^2+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/
(a+b))^(1/2))*a^2*sin(d*x+c)-2*A*cos(d*x+c)*a*b+6*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-4*A*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+4*B*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2+5*
A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)
*a*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b
*sin(d*x+c)+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a*b*sin(d*x+c)+24*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b
)/(a+b))^(1/2))*a*b*sin(d*x+c)-16*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+8*A*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2+6*A*EllipticPi
```

```
((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x
+c)*b^2-4*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*sin(d*x+c)*a^2+4*B*cos(d*x+c)^3*a^2+5*A*cos(d*x+c)^2*b^2-4*B*cos
(d*x+c)^2*a^2-5*A*cos(d*x+c)*b^2+7*A*cos(d*x+c)^3*a*b-5*A*cos(d*x+c)^2*a*b+
4*B*cos(d*x+c)^2*a*b-4*B*cos(d*x+c)*a*b-8*A*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)+8*B*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c))*(1+cos(d*x+c))^2*((b
+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^2*sec(d*x + c)^2 + A*a*cos(d*x + c)^2 + (B*a + A
*b)*cos(d*x + c)^2*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^2\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2), x)
```

3.362 $\int \cos^3(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=520

$$\frac{(a-b)\sqrt{a+b}\,(16a^2A+3Ab^2+30abB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c}{a+}}}{24abd}$$

```
[Out] 1/24*(a-b)*(16*A*a^2+3*A*b^2+30*B*a*b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c)
)^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b
))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d+1/24*(16*A*a^2+14*A*a*b+3*A*
b^2+12*B*a^2+30*B*a*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1
/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+
sec(d*x+c))/(a-b))^(1/2)/a/d-1/8*(12*A*a^2*b-A*b^3+8*B*a^3+6*B*a*b^2)*cot(d
*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(
1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^
(1/2)/a^2/d+1/24*(16*A*a^2+3*A*b^2+30*B*a*b)*sin(d*x+c)*(a+b*sec(d*x+c))^(1
/2)/a/d+1/12*(7*A*b+6*B*a)*cos(d*x+c)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+1
/3*a*A*cos(d*x+c)^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.81, antiderivative size = 520, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4110, 4189, 4143, 4006, 3869, 3917, 4089}

$$\frac{\sqrt{a+b}\,(16a^2A+12a^2B+14aAb+30abB+3Ab^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{24ad}+\frac{(a-b)\sqrt{a+b}\,(16a^2A+30abB+3Ab^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{24abd}+\frac{(16a^2A+30abB+3Ab^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24ad}-\frac{\sqrt{a+b}\,(8a^3B+12a^2Ab+6ab^2B-Ab^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{8a^2d}+\frac{(6aB+7Ab)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{12d}+\frac{aA\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(16*a^2*A + 3*A*b^2 + 30*a*b*B)*Cot[c + d*x]*EllipticE
[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
- Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a*b*
d) + (Sqrt[a + b]*(16*a^2*A + 14*a*A*b + 3*A*b^2 + 12*a^2*B + 30*a*b*B)*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(24*a*d) - (Sqrt[a + b]*(12*a^2*A*b - A*b^3 + 8*a^3*B + 6*a*b^2*
B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(8*a^2*d) + ((16*a^2*A + 3*A*b^2 + 30*a*b*B)*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*a*d) + ((7*A*b + 6*a*B)*Cos[c + d
*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*d) + (a*A*Cos[c + d*x]^2*Sqr
t[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
```

```
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+}{3d}$$

$$= \frac{(7Ab+6aB)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{12d}$$

$$= \frac{(16a^2A+3Ab^2+30abB)\,\sqrt{a+b\sec(c+dx}}{24ad}$$

$$= \frac{(16a^2A+3Ab^2+30abB)\,\sqrt{a+b\sec(c+dx}}{24ad}$$

$$= \frac{(a-b)\sqrt{a+b}\,(16a^2A+3Ab^2+30abB)\cot}{}$$

$$= \frac{(a-b)\sqrt{a+b}\,(16a^2A+3Ab^2+30abB)\cot}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 1535 vs. $2(520) = 1040$.
time = 19.04, size = 1535, normalized size = 2.95

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] (Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*((a*A*Sin[c + d*x])/12 + ((7*A*b + 6*a*B)*Sin[2*(c + d*x)])/24 + (a*A*Sin[3*(c + d*x)])/12))/(d*(b + a*Cos[c + d*x])) + ((a + b*Sec[c + d*x])^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(16*a^3*A*Tan[(c + d*x)/2] + 16*a^2*A*b*Tan[(c + d*x)/2] + 3*a*A*b^2*Tan[(c + d*x)/2] + 3*A*b^3*Tan[(c + d*x)/2] + 30*a^2*b*B*Tan[(c + d*x)/2] + 30*a*b^2*B*Tan[(c + d*x)/2] - 32*a^3*A*Tan[(c + d*x)/2]^3 - 6*a*A*b^2*Tan[(c + d*x)/2]^3 - 60*a^2*b*B*Tan[(c + d*x)/2]^3 + 16*a^3*A*Tan[(c + d*x)/2]^5 - 16*a^2*A*b*Tan[(c + d*x)/2]^5 + 3*a*A*b^2*Tan[(c + d*x)/2]^5 - 3*A*b^3*Tan[(c + d*x)/2]^5 + 30*a^2*b*B*Tan[(c + d*x)/2]^5 - 30*a*b^2*B*Tan[(c + d*x)/2]^5 + 72*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*a^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 36*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 72*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*a^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 36*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(16*a^2*A + 3*A*b^2 + 30*a*b*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*(12*a^2*B + b^2*(-7*A + 24*B) + a*(26*A*b - 6*b*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(24*a*d*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3141 vs. $2(475) = 950$.
time = 9.02, size = 3142, normalized size = 6.04

method	result	size

default	Expression too large to display	3142

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/24/d*(-1+cos(d*x+c))^2*(-48*B*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a-14*A*cos(d*x+c)*a*b^2-12*B
*cos(d*x+c)*a^2*b-6*A*cos(d*x+c)^2*a^2*b+30*B*cos(d*x+c)^2*a*b^2-16*A*cos(d
*x+c)*a^2*b-48*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a*b^2*sin(d*x+c)+16*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^3*sin(d*x+c)+16*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+3*A*cos(d*x+c)*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-6*A*cos(
d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b
))^(1/2))*b^3+48*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/si
n(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3-24*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+16*A*a^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d
*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+3*A*b^2*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+
72*A*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a
+b))^(1/2))*b-52*A*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*b+14*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+30*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+30*B*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+36*B*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi(
(-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+12*B*(c
```

```
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+
c)+8*A*cos(d*x+c)^5*a^3-3*A*cos(d*x+c)*b^3+12*B*cos(d*x+c)^4*a^3+3*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-6*A
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*sin
(d*x+c)+48*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1
/2))*a^3*sin(d*x+c)-24*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*a^3*sin(d*x+c)+8*A*cos(d*x+c)^3*a^3-16*A*cos(d*x+c)^2*a^3+3*A*
cos(d*x+c)^2*b^3-12*B*cos(d*x+c)^2*a^3+12*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+16*A*cos(d*x+c)
*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*b+3*A*cos(d*x+c)*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a+72*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+co
s(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b-52*A*cos(d*x+c)*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+14*
A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*a*b^2+30*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+30*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b*cos(d*x + c)^3*sec(d*x + c)^2 + A*a*cos(d*x + c)^3 + (B*a + A
*b)*cos(d*x + c)^3*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 8010 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^3\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2), x)
```

3.363 $\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=566

$$\frac{2(a-b)\sqrt{a+b}\,(110a^4Ab-3069a^2Ab^3-1617Ab^5-40a^5B-255a^3b^2B-3705ab^4B)\cot(c+dx)E\Big(\text{ArcSin}}{3465b^4d}$$

[Out] 2/3465*(a-b)*(110*A*a^4*b-3069*A*a^2*b^3-1617*A*b^5-40*B*a^5-255*B*a^3*b^2-3705*B*a*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d-2/3465*(a-b)*(6*a*b^3*(209*A-505*B)-3*b^4*(539*A-225*B)-15*a^2*b^2*(121*A-19*B)+40*a^4*B-a^3*(110*A*b-30*B*b))*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/d-2/3465*(110*A*a^2*b-539*A*b^3-40*B*a^3-335*B*a*b^2)*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/b^2/d-2/693*(22*A*a*b-8*B*a^2-81*B*b^2)*(a+b*sec(d*x+c))^(5/2)*tan(d*x+c)/b^2/d+2/99*(11*A*b-4*B*a)*(a+b*sec(d*x+c))^(7/2)*tan(d*x+c)/b^2/d+2/11*B*sec(d*x+c)*(a+b*sec(d*x+c))^(7/2)*tan(d*x+c)/b/d-2/3465*(110*A*a^3*b-1254*A*a*b^3-40*B*a^4-285*B*a^2*b^2-675*B*b^4)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d

Rubi [A]
time = 1.15, antiderivative size = 566, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4118, 4167, 4087, 4090, 3917, 4089}

$$-\frac{2(-8a^2B+22aAb-81b^2B)\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{693b^2d}-\frac{2(-40a^3B+110a^2Ab-335ab^2B-539Ab^3)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{3465b^2d}-\frac{2(a-b)\sqrt{a+b}\,(40a^4B-a^3(110Ab-30bB)-15a^2b^2(121A-19B)+6ab^3(209A-505B)-3b^4(539A-225B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3465b^3d}-\frac{2(-40a^4B+110a^3Ab-285a^2b^2B-1254aAb^3-675b^4B)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3465b^2d}+\frac{2(a-b)\sqrt{a+b}\,(-40a^5B+110a^4Ab-255a^3b^2B-3069a^2Ab^3-3705ab^4B-1617Ab^5)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3465b^4d}+\frac{2(11Ab-4aB)\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{99b^2d}+\frac{2B\tan(c+dx)\sec(c+dx)(a+b\sec(c+dx))^{7/2}}{11bd}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] (2*(a - b)*Sqrt[a + b]*(110*a^4*A*b - 3069*a^2*A*b^3 - 1617*A*b^5 - 40*a^5*B - 255*a^3*b^2*B - 3705*a*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3465*b^4*d) - (2*(a - b)*Sqrt[a + b]*(6*a*b^3*(209*A - 505*B) - 3*b^4*(539*A - 225*B) - 15*a^2*b^2*(121*A - 19*B) + 40*a^4*B - a^3*(110*A*b - 30*b*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3465*b^3*d) - (2*(110*a^3*A*b - 1254*a*A*b^3 - 40*a^4*B - 285*a^2*b^2*B - 675*b^4*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3465*b^2*d) - (2*(110*a^2*A*b - 539*A*b^3 - 40*a^3*B - 335*a*b^2*B)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(

```
3465*b^2*d) - (2*(22*a*A*b - 8*a^2*B - 81*b^2*B)*(a + b*Sec[c + d*x])^(5/2)
*Tan[c + d*x])/(693*b^2*d) + (2*(11*A*b - 4*a*B)*(a + b*Sec[c + d*x])^(7/2)
*Tan[c + d*x])/(99*b^2*d) + (2*B*Sec[c + d*x]*(a + b*Sec[c + d*x])^(7/2)*Ta
n[c + d*x])/(11*b*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4118

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d^
2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 2)/(b*f*
(m + n))), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e +
 f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m +
n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
```

```
 m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n
, 0] &&  !IGtQ[m, 1]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^3(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{2B\sec(c+dx)(a+b\sec(c+dx))^{7/2}\tan(c+d}{11bd}$$

$$= \frac{2(11Ab-4aB)(a+b\sec(c+dx))^{7/2}\tan(c+}{99b^2d}$$

$$= -\frac{2(22aAb-8a^2B-81b^2B)\left(a+b\sec(c+dx\right.}{693b^2d}$$

$$= -\frac{2(110a^2Ab-539Ab^3-40a^3B-335ab^2B)\,(}{3465b^2d}$$

$$= -\frac{2(110a^3Ab-1254aAb^3-40a^4B-285a^2b^2}{346}$$

$$= -\frac{2(110a^3Ab-1254aAb^3-40a^4B-285a^2b^2}{346}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(110a^4Ab-3069a^2Ab^3-161}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 4227 vs. 2(566) = 1132.
time = 26.89, size = 4227, normalized size = 7.47

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((2*(-110*a^4*A*b + 3069*a^2*A*b
^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*Sin[c + d*x])/(3
465*b^3) + (2*Sec[c + d*x]^4*(11*A*b^2*Sin[c + d*x] + 23*a*b*B*Sin[c + d*x]
))/99 + (2*Sec[c + d*x]^3*(209*a*A*b*Sin[c + d*x] + 113*a^2*B*Sin[c + d*x]
+ 81*b^2*B*Sin[c + d*x]))/693 + (2*Sec[c + d*x]^2*(825*a^2*A*b*Sin[c + d*x]
 + 539*A*b^3*Sin[c + d*x] + 15*a^3*B*Sin[c + d*x] + 1145*a*b^2*B*Sin[c + d*
x]))/(3465*b) + (2*Sec[c + d*x]*(55*a^3*A*b*Sin[c + d*x] + 1793*a*A*b^3*Sin
[c + d*x] - 20*a^4*B*Sin[c + d*x] + 1025*a^2*b^2*B*Sin[c + d*x] + 675*b^4*B
*Sin[c + d*x]))/(3465*b^2) + (2*b^2*B*Sec[c + d*x]^4*Tan[c + d*x])/11))/(d*
(b + a*Cos[c + d*x])^2) - (2*((2*a^4*A)/(63*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt
[Sec[c + d*x]]) - (31*a^2*A*b)/(35*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*
x]]) - (7*A*b^3)/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (17*a^3
*B)/(231*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^5*B)/(693*b^2*
Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (247*a*b^2*B)/(231*Sqrt[b +
a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (124*a^3*A*Sqrt[Sec[c + d*x]])/(315*S
qrt[b + a*Cos[c + d*x]]) + (2*a^5*A*Sqrt[Sec[c + d*x]])/(63*b^2*Sqrt[b + a*
Cos[c + d*x]]) + (38*a*A*b^2*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*
x]]) - (8*a^6*B*Sqrt[Sec[c + d*x]])/(693*b^3*Sqrt[b + a*Cos[c + d*x]]) - (7
*a^4*B*Sqrt[Sec[c + d*x]])/(99*b*Sqrt[b + a*Cos[c + d*x]]) - (26*a^2*b*B*Sq
rt[Sec[c + d*x]])/(231*Sqrt[b + a*Cos[c + d*x]]) + (15*b^3*B*Sqrt[Sec[c + d
*x]])/(77*Sqrt[b + a*Cos[c + d*x]]) - (31*a^3*A*Cos[2*(c + d*x)]*Sqrt[Sec[c
 + d*x]])/(35*Sqrt[b + a*Cos[c + d*x]]) + (2*a^5*A*Cos[2*(c + d*x)]*Sqrt[Se
c[c + d*x]])/(63*b^2*Sqrt[b + a*Cos[c + d*x]]) - (7*a*A*b^2*Cos[2*(c + d*x)
]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) - (8*a^6*B*Cos[2*(c + d
*x)]*Sqrt[Sec[c + d*x]])/(693*b^3*Sqrt[b + a*Cos[c + d*x]]) - (17*a^4*B*Cos
[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(231*b*Sqrt[b + a*Cos[c + d*x]]) - (247*a
^2*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(231*Sqrt[b + a*Cos[c + d*x]]))
*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(2*(a + b
)*(-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B +
3705*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)] - 2*b*(a + b)*(40*a^4*B - 10*a^3*b*(11*A + 3*B) + 15*a^2*b^2*(12
1*A + 19*B) + 3*b^4*(539*A + 225*B) + 6*a*b^3*(209*A + 505*B))*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-110*a^4*A*b
 + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*C
os[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(346
5*b^3*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x)/2]^2]*Sec[c + d*x]^(5/2)*
(-1/3465*(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(
-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 370
5*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)] - 2*b*(a + b)*(40*a^4*B - 10*a^3*b*(11*A + 3*B) + 15*a^2*b^2*(121*A
 + 19*B) + 3*b^4*(539*A + 225*B) + 6*a*b^3*(209*A + 505*B))*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
```

```
))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-110*a^4*A*b +
3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*Cos[
c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(b^3*(b
 + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[Cos[(c + d*x)/2]
^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-110*a^4*A*b + 3069*a^2*A*b^3
 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(40*a^4
*B - 10*a^3*b*(11*A + 3*B) + 15*a^2*b^2*(121*A + 19*B) + 3*b^4*(539*A + 225
*B) + 6*a*b^3*(209*A + 505*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
 d*x)/2]], (a - b)/(a + b)] + (-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 +
 40*a^5*B + 255*a^3*b^2*B + 3705*a*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])
*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3465*b^3*Sqrt[b + a*Cos[c + d*x]]*S
qrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-110
*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 3705*a*
b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(
-110*a^4*A*b + 3069*a^2*A*b^3 + 1617*A*b^5 + 40*a^5*B + 255*a^3*b^2*B + 370
5*a*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Elliptic
E[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(
1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(
1 + Cos[c + d*x])] - (b*(a + b)*(40*a^4*B - 10*...
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5367 vs. $2(524) = 1048$.
time = 11.27, size = 5368, normalized size = 9.48

method	result	size
default	Expression too large to display	5368

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^6 + A*a^2*sec(d*x + c)^3 + (2*B*a*b + A*b^2)*s
ec(d*x + c)^5 + (B*a^2 + 2*A*a*b)*sec(d*x + c)^4)*sqrt(b*sec(d*x + c) + a),
 x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3062 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/cos(c + d*x)^3,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/cos(c + d*x)^3, x)
```

3.364 $\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=469

$$-\frac{2(a-b)\sqrt{a+b}\,(45a^3Ab+435aAb^3-10a^4B+279a^2b^2B+147b^4B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c}{\sqrt{a+b}}\right.\right.}{315b^3d}$$

```
[Out] -2/315*(a-b)*(45*A*a^3*b+435*A*a*b^3-10*B*a^4+279*B*a^2*b^2+147*B*b^4)*cot(
d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a
+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^
3/d-2/315*(a-b)*(3*b^3*(25*A-49*B)-6*a*b^2*(60*A-19*B)+15*a^2*b*(3*A-11*B)-
10*a^3*B)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a
-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(
a-b))^(1/2)/b^2/d+2/315*(45*A*a*b-10*B*a^2+49*B*b^2)*(a+b*sec(d*x+c))^(3/2)
*tan(d*x+c)/b/d+2/63*(9*A*b-2*B*a)*(a+b*sec(d*x+c))^(5/2)*tan(d*x+c)/b/d+2/
9*B*(a+b*sec(d*x+c))^(7/2)*tan(d*x+c)/b/d+2/315*(45*A*a^2*b+75*A*b^3-10*B*a
^3+114*B*a*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b/d
```

Rubi [A]
time = 0.76, antiderivative size = 469, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4095, 4087, 4090, 3917, 4089}

$$\frac{2(-10a^2B+45aAb+49b^2B)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{315bd}-\frac{2(a-b)\sqrt{a+b}\,(-10a^3B+15a^2b(3A-11B)-6ab^2(60A-19B)+3b^3(25A-49B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{315b^2d}+\frac{2(-10a^3B+45a^2Ab+114ab^2B+75Ab^3)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{315bd}-\frac{2(a-b)\sqrt{a+b}\,(-10a^4B+45a^3Ab+279a^2b^2B+435aAb^3+147b^4B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{315b^3d}+\frac{2(9Ab-2aB)\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{63bd}+\frac{2B\tan(c+dx)(a+b\sec(c+dx))^{7/2}}{9bd}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(45*a^3*A*b + 435*a*A*b^3 - 10*a^4*B + 279*a^2*b^2*
B + 147*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(315*b^3*d) - (2*(a - b)*Sqrt[a + b]*(3*b^3*(2
5*A - 49*B) - 6*a*b^2*(60*A - 19*B) + 15*a^2*b*(3*A - 11*B) - 10*a^3*B)*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(315*b^2*d) + (2*(45*a^2*A*b + 75*A*b^3 - 10*a^3*B + 114*a*b^2*B
)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(315*b*d) + (2*(45*a*A*b - 10*a^2*
B + 49*b^2*B)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(315*b*d) + (2*(9*A*
b - 2*a*B)*(a + b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(63*b*d) + (2*B*(a + b*
Sec[c + d*x])^(7/2)*Tan[c + d*x])/(9*b*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \sec^2(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{2B(a+b\sec(c+dx))^{7/2}\tan(c+dx)}{9bd} + \frac{2\int \mathrm{se}}{}$$

$$= \frac{2(9Ab-2aB)(a+b\sec(c+dx))^{5/2}\tan(c+d}{63bd}$$

$$= \frac{2(45aAb-10a^2B+49b^2B)\,(a+b\sec(c+dx)}{315bd}$$

$$= \frac{2(45a^2Ab+75Ab^3-10a^3B+114ab^2B)\,\sqrt{a-}}{315bd}$$

$$= \frac{2(45a^2Ab+75Ab^3-10a^3B+114ab^2B)\,\sqrt{a-}}{315bd}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(45a^3Ab+435aAb^3-10a^4}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3781 vs. $2(469) = 938$.
time = 26.07, size = 3781, normalized size = 8.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((2*(45*a^3*A*b + 435*a*A*b^3 -
10*a^4*B + 279*a^2*b^2*B + 147*b^4*B)*Sin[c + d*x])/(315*b^2) + (2*Sec[c +
d*x]^3*(9*A*b^2*Sin[c + d*x] + 19*a*b*B*Sin[c + d*x]))/63 + (2*Sec[c + d*x]
^2*(135*a*A*b*Sin[c + d*x] + 75*a^2*B*Sin[c + d*x] + 49*b^2*B*Sin[c + d*x])
)/315 + (2*Sec[c + d*x]*(135*a^2*A*b*Sin[c + d*x] + 75*A*b^3*Sin[c + d*x] +
 5*a^3*B*Sin[c + d*x] + 163*a*b^2*B*Sin[c + d*x]))/(315*b) + (2*b^2*B*Sec[c
 + d*x]^3*Tan[c + d*x])/9))/(d*(b + a*Cos[c + d*x])^2) + (2*(-1/7*(a^3*A)/(
Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (29*a*A*b^2)/(21*Sqrt[b + a*
Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^4*B)/(63*b*Sqrt[b + a*Cos[c + d*x]
]*Sqrt[Sec[c + d*x]]) - (31*a^2*b*B)/(35*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[
c + d*x]]) - (7*b^3*B)/(15*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (
a^4*A*Sqrt[Sec[c + d*x]])/(7*b*Sqrt[b + a*Cos[c + d*x]]) - (2*a^2*A*b*Sqrt[
Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) + (5*A*b^3*Sqrt[Sec[c + d*x]])
/(21*Sqrt[b + a*Cos[c + d*x]]) - (124*a^3*B*Sqrt[Sec[c + d*x]])/(315*Sqrt[b
 + a*Cos[c + d*x]]) + (2*a^5*B*Sqrt[Sec[c + d*x]])/(63*b^2*Sqrt[b + a*Cos[c
 + d*x]]) + (38*a*b^2*B*Sqrt[Sec[c + d*x]])/(105*Sqrt[b + a*Cos[c + d*x]])
```

```
- (a^4*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(7*b*Sqrt[b + a*Cos[c + d*x]]
) - (29*a^2*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c +
 d*x]]) - (31*a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(35*Sqrt[b + a*Cos
[c + d*x]]) + (2*a^5*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(63*b^2*Sqrt[b
+ a*Cos[c + d*x]]) - (7*a*b^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sq
rt[b + a*Cos[c + d*x]]))*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c
 + d*x])^(5/2)*(2*(a + b)*(-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b
^2*B - 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c +
 d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)] + 2*b*(a + b)*(-10*a^3*B + 15*a^2*b*(3*A + 11*B) + 6*a*b^2*(
60*A + 19*B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*S
qrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)] + (-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B -
279*a^2*b^2*B - 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2]))/(315*b^2*d*(b + a*Cos[c + d*x])^3*Sqrt[Sec[(c + d*x
)/2]^2]*Sec[c + d*x]^(5/2)*((a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c
+ d*x]*(2*(a + b)*(-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 1
47*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
 + b)] + 2*b*(a + b)*(-10*a^3*B + 15*a^2*b*(3*A + 11*B) + 6*a*b^2*(60*A + 1
9*B) + 3*b^3*(25*A + 49*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)] + (-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*
b^2*B - 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan
[(c + d*x)/2]))/(315*b^2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2
]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-4
5*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c
+ d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*
(-10*a^3*B + 15*a^2*b*(3*A + 11*B) + 6*a*b^2*(60*A + 19*B) + 3*b^3*(25*A +
49*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)] + (-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Co
s[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(315*
b^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d
*x)/2]^2*Sec[c + d*x]]*(((-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^
2*B - 147*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 +
((a + b)*(-45*a^3*A*b - 435*a*A*b^3 + 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)
*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c
+ d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c
+ d*x])] + (b*(a + b)*(-10*a^3*B + 15*a^2*b*(3*A + 11*B) + 6*a*b^2*(60*A +
19*B) + 3*b^3*(25*A + 49*B))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
+ d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*
x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/S
```

```
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(-45*a^3*A*b - 435*a*A*b^3
+ 10*a^4*B - 279*a^2*b^2*B - 147*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])
/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a +
b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c +
d*x]))] + (b*(a + b)*(-10*a^3*B + 15*a^2*b*(3*A...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4394 vs. $2(431) = 862$.
time = 11.70, size = 4395, normalized size = 9.37

method	result	size
default	Expression too large to display	4395

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/315/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c
))^2*(45*A*cos(d*x+c)^6*a^4*b+135*A*cos(d*x+c)^6*a^3*b^2+435*A*cos(d*x+c)^6
*a^2*b^3+75*A*cos(d*x+c)^6*a*b^4+5*B*cos(d*x+c)^6*a^4*b+279*B*cos(d*x+c)^6*
a^3*b^2+163*B*cos(d*x+c)^6*a^2*b^3+147*B*cos(d*x+c)^6*a*b^4-45*A*cos(d*x+c)
^5*a^4*b+45*A*cos(d*x+c)^5*a^3*b^2-165*A*cos(d*x+c)^5*a^2*b^3+435*A*cos(d*x
+c)^5*a*b^4-10*B*cos(d*x+c)^5*a^4*b-199*B*cos(d*x+c)^5*a^3*b^2+279*B*cos(d*
x+c)^5*a^2*b^3+65*B*cos(d*x+c)^5*a*b^4-180*A*cos(d*x+c)^4*a^3*b^2-435*A*sin
(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a*b^4+75*A*cos(d*x+c)^5*b^5-30*A*cos(d*x+c)^3*b^5-45*A*cos(d*x+c)*
b^5-10*B*cos(d*x+c)^6*a^5+10*B*cos(d*x+c)^5*a^5+147*B*cos(d*x+c)^5*b^5-98*B
*cos(d*x+c)^4*b^5-14*B*cos(d*x+c)^2*b^5-330*A*cos(d*x+c)^4*a*b^4+5*B*cos(d*
x+c)^4*a^4*b-272*B*cos(d*x+c)^4*a^2*b^3-270*A*cos(d*x+c)^3*a^2*b^3-80*B*cos
(d*x+c)^3*a^3*b^2-82*B*cos(d*x+c)^3*a*b^4-180*A*cos(d*x+c)^2*a*b^4-170*B*co
s(d*x+c)^2*a^2*b^3-130*B*cos(d*x+c)*a*b^4-35*B*b^5+45*A*sin(d*x+c)*cos(d*x+
c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2
+155*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a^3*b^2+279*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+261*B*sin(d*x+c)*cos(d
*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^
4-45*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a^4*b-45*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*
```

```
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-435*A*sin(d*x+c)*cos(d*x+
c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3
+405*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a^2*b^3+435*A*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4+10*B*sin(d*x+c)*cos(d*x+
c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*b-2
79*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a^3*b^2-279*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-147*B*sin(d*x+c)*cos(d*x
+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^4-
10*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a^4*b+155*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2+279*B*sin(d*x+c)*cos(d*x+c
)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3+
261*B*sin(d*x+c)*cos(d*x+c)^4*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a*b^4-45*A*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-435*A*sin(d*x+c)*cos(d*x+c
)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3-
435*A*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a*b^4-279*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b^2-279*B*sin(d*x+c)*cos(d*x+
c)^5*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^3
-147*B*sin(d*x+c)*cos(d*x+c)^5*(cos(d*x+c)/(1+c...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^5 + A*a^2*sec(d*x + c)^2 + (2*B*a*b + A*b^2)*s
ec(d*x + c)^4 + (B*a^2 + 2*A*a*b)*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a),
 x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/cos(c + d*x)^2,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/cos(c + d*x)^2, x)
```

3.365 $\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=384

$$-\frac{2(a-b)\sqrt{a+b}\,(161a^2Ab+63Ab^3+15a^3B+145ab^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a}{a}\right.}{105b^2d}$$

```
[Out] -2/105*(a-b)*(161*A*a^2*b+63*A*b^3+15*B*a^3+145*B*a*b^2)*cot(d*x+c)*Ellipti
cE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(
1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+2/105*(a-b
)*(b^2*(63*A-25*B)-8*a*b*(7*A-15*B)+15*a^2*(7*A-B))*cot(d*x+c)*EllipticF((a
+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec
(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/35*(7*A*b+5*B*a
)*(a+b*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/7*B*(a+b*sec(d*x+c))^(5/2)*tan(d*x+
c)/d+2/105*(56*A*a*b+15*B*a^2+25*B*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]
time = 0.51, antiderivative size = 384, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4087, 4090, 3917, 4089}

$$\frac{2(a-b)\sqrt{a+b}\,(15a^2(7A-B)-8ab(7A-15B)+b^2(63A-25B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{105bd}+\frac{2(15a^2B+56aAb+25b^2B)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{105d}-\frac{2(a-b)\sqrt{a+b}\,(15a^3B+161a^2Ab+145ab^2B+63Ab^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{105b^2d}+\frac{2(5aB+7Ab)\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{35d}+\frac{2B\tan(c+dx)(a+b\sec(c+dx))^{5/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*C
ot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)
/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x])
)/(a - b))])/(105*b^2*d) + (2*(a - b)*Sqrt[a + b]*(b^2*(63*A - 25*B) - 8*a*
b*(7*A - 15*B) + 15*a^2*(7*A - B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(105*b*d) + (2*(56*a*A*b +
 15*a^2*B + 25*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(105*d) + (2*(
7*A*b + 5*a*B)*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(35*d) + (2*B*(a +
b*Sec[c + d*x])^(5/2)*Tan[c + d*x])/(7*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4087

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*((
a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[Csc[e + f*x]*(
a + b*Csc[e + f*x])^(m - 1)*Simp[b*B*m + a*A*(m + 1) + (a*B*m + A*b*(m + 1)
)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B
, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \sec(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{2B(a+b\sec(c+dx))^{5/2}\tan(c+dx)}{7d}+\frac{2}{7}\int \mathrm{se}$$

$$= \frac{2(7Ab+5aB)(a+b\sec(c+dx))^{3/2}\tan(c+dx}{35d}$$

$$= \frac{2(56aAb+15a^2B+25b^2B)\sqrt{a+b\sec(c+dx}}{105d}$$

$$= \frac{2(56aAb+15a^2B+25b^2B)\sqrt{a+b\sec(c+dx}}{105d}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(161a^2Ab+63Ab^3+15a^3B}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2957 vs. $2(384) = 768$.

time = 22.73, size = 2957, normalized size = 7.70

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x])*((2*(161*a^
2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sin[c + d*x])/(105*b) + (2*Sec[c
 + d*x]^2*(7*A*b^2*Sin[c + d*x] + 15*a*b*B*Sin[c + d*x]))/35 + (2*Sec[c + d
*x]*(77*a*A*b*Sin[c + d*x] + 45*a^2*B*Sin[c + d*x] + 25*b^2*B*Sin[c + d*x])
)/105 + (2*b^2*B*Sec[c + d*x]^2*Tan[c + d*x])/7))/(d*(b + a*Cos[c + d*x])^2
*(B + A*Cos[c + d*x])) + (2*((-23*a^2*A*b)/(15*Sqrt[b + a*Cos[c + d*x]]*Sqr
t[Sec[c + d*x]]) - (3*A*b^3)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]
) - (a^3*B)/(7*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (29*a*b^2*B)/
(21*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^3*A*Sqrt[Sec[c + d*
x]])/(15*Sqrt[b + a*Cos[c + d*x]]) + (8*a*A*b^2*Sqrt[Sec[c + d*x]])/(15*Sqr
t[b + a*Cos[c + d*x]]) - (a^4*B*Sqrt[Sec[c + d*x]])/(7*b*Sqrt[b + a*Cos[c +
 d*x]]) - (2*a^2*b*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) + (5
*b^3*B*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]) - (23*a^3*A*Cos[2*
(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*Sqrt[b + a*Cos[c + d*x]]) - (3*a*A*b^2*C
os[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*Sqrt[b + a*Cos[c + d*x]]) - (a^4*B*C
os[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(7*b*Sqrt[b + a*Cos[c + d*x]]) - (29*a^
2*b*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(21*Sqrt[b + a*Cos[c + d*x]]))*S
qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[
c + d*x])*((-2*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*((161*a^2*A*b + 63*A
*b^3 + 15*a^3*B + 145*a*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/
(a + b)] - b*(15*a^2*(7*A + B) + 8*a*b*(7*A + 15*B) + b^2*(63*A + 25*B))*El
lipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[c + d*x])/Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (161*a^2*A*b + 63*A*b^3 +
15*a^3*B + 145*a*b^2*B)*Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2)))/(105*b
*d*(b + a*Cos[c + d*x])^2*(B + A*Cos[c + d*x])*Sqrt[Sec[(c + d*x)/2]^2]*Sec
[c + d*x]^(7/2)*(-1/105*(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*
x]*((-2*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*((161*a^2*A*b + 63*A*b^3 +
15*a^3*B + 145*a*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
] - b*(15*a^2*(7*A + B) + 8*a*b*(7*A + 15*B) + b^2*(63*A + 25*B))*EllipticF
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[c + d*x])/Sqrt[(b + a*Cos[
c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (161*a^2*A*b + 63*A*b^3 + 15*a^3*
B + 145*a*b^2*B)*Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2)))/(b*Sqrt[b + a
*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[b + a*Cos[c + d*x]]*Sqrt[C
os[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*((-2*(Cos[c + d*x]/(1 + Co
s[c + d*x]))^(3/2)*((161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Ellip
ticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - b*(15*a^2*(7*A + B) + 8*a
```

```
*b*(7*A + 15*B) + b^2*(63*A + 25*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a
 - b)/(a + b)])*Sec[c + d*x])/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c
 + d*x]))] + (161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Tan[(c + d*x
)/2]*(-1 + Tan[(c + d*x)/2]^2)))/(105*b*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[b
 + a*Cos[c + d*x]]*((-2*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3/2)*((161*a^2*A
*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)] - b*(15*a^2*(7*A + B) + 8*a*b*(7*A + 15*B) + b^2*(63*A +
25*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[c + d*x])/
Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + (161*a^2*A*b + 63
*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Tan[(c + d*x)/2]*(-1 + Tan[(c + d*x)/2]^2)
)*(-(Cos[(c + d*x)/2]*Sec[c + d*x]*Sin[(c + d*x)/2]) + Cos[(c + d*x)/2]^2*S
ec[c + d*x]*Tan[c + d*x]))/(105*b*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Cos[(c + d*
x)/2]^2*Sec[c + d*x]]) + (2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^
2*Sec[c + d*x]]*((-3*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*((161*a^2*A*b +
63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)] - b*(15*a^2*(7*A + B) + 8*a*b*(7*A + 15*B) + b^2*(63*A + 25*B)
)*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)])*Sec[c + d*x]*((Cos[
c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x
])))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] + ((Cos[c + d*
x]/(1 + Cos[c + d*x]))^(3/2)*((161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^
2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - b*(15*a^2*(7*A
+ B) + 8*a*b*(7*A + 15*B) + b^2*(63*A + 25*B))*EllipticF[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)])*Sec[c + d*x]*(-((a*Sin[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d
*x])^2)))/((b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x])))^(3/2) + (161*
a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sec[(c + d*x)/2]^2*Tan[(c + d*
x)/2]^2 + ((161*a^2*A*b + 63*A*b^3 + 15*a^3*B + 145*a*b^2*B)*Sec[(c + d*x)/
2]^2*(-1 + Tan[(c + d*x)/2]^2))/2 - (2*(Cos[c + d*x]/(1 + Cos[c + d*x]))^(3
/2)*Sec[c + d*x]*(-1/2*(b*(15*a^2*(7*A + B) + 8*a*b*(7*A + 15*B) + b^2*(63*
A + 25*B))*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a -
 b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((161*a^2*A...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3636 vs. $2(350) = 700$.
time = 11.26, size = 3637, normalized size = 9.47

method	result	size
default	Expression too large to display	3637

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/105/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c
))^2*(-15*B*b^4+105*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
```

1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-10*B*cos(d*x+c)^2*b^4+15*B*cos(d*x+
c)^5*a^4+63*A*cos(d*x+c)^4*b^4-15*B*cos(d*x+c)^4*a^4-42*A*cos(d*x+c)^3*b^4-
21*A*cos(d*x+c)*b^4+25*B*cos(d*x+c)^4*b^4-161*A*cos(d*x+c)^3*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-161*A*cos
(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^2*b^2-63*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+15*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+135*B*cos(d*
x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^2*b^2+145*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-15*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-145*B*cos(d*x+
c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a^2*b^2-145*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+161*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+119*A*cos(d*x
+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a*b^3-161*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-161*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-63*A*cos(d*x+c
)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a*b^3+15*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*a^3*b+135*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+145*B*cos(d*x+c)^
4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a*b^3-15*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+

```
c),((a-b)/(a+b))^(1/2))*a^3*b-145*B*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-145*B*cos(d*x+c)^4*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
*b^3+105*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^3*b+161*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+119*A*cos(d*x+c)^3*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*
b^3-60*B*cos(d*x+c)*a*b^3-55*B*cos(d*x+c)^4*a^2*b^2+145*B*cos(d*x+c)^4*a*b^
3-238*A*cos(d*x+c)^3*a^2*b^2-60*B*cos(d*x+c)^3*a^3*b-110*B*cos(d*x+c)^3*a*b
^3-98*A*cos(d*x+c)^2*a*b^3-90*B*cos(d*x+c)^2*a^2*b^2+161*A*cos(d*x+c)^5*a^3
*b+77*A*cos(d*x+c)^5*a^2*b^2+63*A*cos(d*x+c)^5*a*b^3+45*B*cos(d*x+c)^5*a^3*
b+145*B*cos(d*x+c)^5*a^2*b^2+25*B*cos(d*x+c)^5*a*b^3-161*A*cos(d*x+c)^4*a^3
*b+161*A*cos(d*x+c)^4*a^2*b^2+35*A*cos(d*x+c)^4*a*b^3+15*B*cos(d*x+c)^4*a^3
*b+63*A*cos(d*x+c)^4*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^4 + A*a^2*sec(d*x + c) + (2*B*a*b + A*b^2)*sec
(d*x + c)^3 + (B*a^2 + 2*A*a*b)*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a), x
)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))(a + b\sec(c + dx))^{\frac{5}{2}}\sec(c + dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(5/2)*sec(c + d*x), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sec(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+d\,x)}\right)\,\left(a + \frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c + d\,x\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/cos(c + d*x),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/cos(c + d*x), x)
```

3.366 $\int (a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=442

$$-\frac{2(a-b)\sqrt{a+b}\,(35aAb+23a^2B+9b^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec}{a}}}{15bd}$$

```
[Out] -2/15*(a-b)*(35*A*a*b+23*B*a^2+9*B*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c
))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+
b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+2/15*(a^2*b*(45*A-23*B)-a*b^2
*(35*A-17*B)+b^3*(5*A-9*B)+15*a^3*B)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^
(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))
^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d-2*a^2*A*cot(d*x+c)*EllipticPi((a
+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(
b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+2/5*b*B*(a+
b*sec(d*x+c))^(3/2)*tan(d*x+c)/d+2/15*b*(5*A*b+8*B*a)*(a+b*sec(d*x+c))^(1/2
)*tan(d*x+c)/d
```

Rubi [A]
time = 0.42, antiderivative size = 442, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4003, 4141, 4143, 4006, 3869, 3917, 4089}

$$-\frac{2(a-b)\sqrt{a+b}\,(23a^2B+35aAb+9b^2B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15bd}-\frac{2a^2A\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}+\frac{2\sqrt{a+b}\,(15a^3B+a^2b(45A-23B)-ab^2(35A-17B)+b^3(5A-9B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15bd}+\frac{2b(8aB+5Ab)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{15d}+\frac{2bB\tan(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(35*a*A*b + 23*a^2*B + 9*b^2*B)*Cot[c + d*x]*Ellipt
icE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b
*d) + (2*Sqrt[a + b]*(a^2*b*(45*A - 23*B) - a*b^2*(35*A - 17*B) + b^3*(5*A
- 9*B) + 15*a^3*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/S
qrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((
b*(1 + Sec[c + d*x]))/(a - b))])/(15*b*d) - (2*a^2*A*Sqrt[a + b]*Cot[c + d*
x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a +
 b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*
x]))/(a - b))])/d + (2*b*(5*A*b + 8*a*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d
*x])/(15*d) + (2*b*B*(a + b*Sec[c + d*x])^(3/2)*Tan[c + d*x])/(5*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
```

```
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4003

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[(-b)*d*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m
 - 1)/(f*m)), x] + Dist[1/m, Int[(a + b*Csc[e + f*x])^(m - 2)*Simp[a^2*c*m
+ (b^2*d*(m - 1) + 2*a*b*c*m + a^2*d*m)*Csc[e + f*x] + b*(b*c*m + a*d*(2*m
- 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c
 - a*d, 0] && GtQ[m, 1] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4141

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a
 + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)
*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{2bB(a+b\sec(c+dx))^{3/2}\tan(c+dx)}{5d} + \frac{2}{5}\int \sqrt{a+b\sec(c}$$

$$= \frac{2b(5Ab+8aB)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{15d} + \frac{2bB(a}{}$$

$$= \frac{2b(5Ab+8aB)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{15d} + \frac{2bB(a}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(35aAb+23a^2B+9b^2B)\cot(c+dx)E\Big(}{}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(35aAb+23a^2B+9b^2B)\cot(c+dx)E\Big(}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 7138 vs. $2(442) = 884$.
time = 25.45, size = 7138, normalized size = 16.15

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3284 vs. $2(403) = 806$.
time = 11.08, size = 3285, normalized size = 7.43

method	result	size
default	Expression too large to display	3285

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 2/15/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(-5*A*cos(d*x+c)^3*b^3+23*B*cos(d*x+c)^3*a^3-9*B*cos(d*x+c)^3*b^3+6*B*co
s(d*x+c)^2*b^3-23*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+3*B*b^3+15*A*cos(d*x+c)^2*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-30*A*cos
(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2)
)*sin(d*x+c)*a^3-15*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+15*A*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3-30*A*cos(d*x+c)^
3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*sin(d*
x+c)*a^3-15*B*cos(d*x+c)^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*sin(d*x+c)*a^3-5*B*cos(d*x+c)^3*a*b^2-17*B*cos(d*x+c)^2*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2
+23*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*a^2*b+9*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-45*A*cos(d*x+c)^3*sin(d*x+c)*
(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-35*A*c
os(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a*b^2+35*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+35*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-23*B*cos(d*x
+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a^2*b-17*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+23*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+9*B*cos(d*x+c)^3*s
in(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
```

```
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*
b^2-45*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^2*b-35*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+35*A*cos(d*x+c)^2*sin(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+35
*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a*b^2+5*A*cos(d*x+c)*b^3-23*B*cos(d*x+c)^4*a^3-5*A*cos(d*x+c
)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*b^3-9*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*b^3+23*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+9*B*cos(d*x+c)^3*sin(d*x+c)
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-5*A*cos
(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*b^3-9*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="fricas")
[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec (c + dx)) (a + b\sec (c + dx))^{\frac{5}{2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos \left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos \left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2),x)
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)
```

3.367 $\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=433

$$\frac{(a-b)\sqrt{a+b}\,(3a^2A-6Ab^2-14abB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+}{a+b}}}{3bd}$$

```
[Out] a*A*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+1/3*(a-b)*(3*A*a^2-6*A*b^2-14*B*a*b
)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/
2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1
/2)/b/d+1/3*(2*a*b*(9*A-7*B)-2*b^2*(3*A-B)+3*a^2*(A+6*B))*cot(d*x+c)*Ellipt
icF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*
(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-a*(5*A*b+2*B*
a)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/
(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/d-1/3*b*(3*A*a-2*B*b)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/d
```

Rubi [A]

time = 0.44, antiderivative size = 433, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4110, 4141, 4143, 4006, 3869, 3917, 4089}

$$\frac{\sqrt{a+b}\,(3a^2(A+6B)+2ab(9A-7B)-2b^2(3A-B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3d}+\frac{(a-b)\sqrt{a+b}\,(3a^2A-14abB-6Ab^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bd}-\frac{a\sqrt{a+b}\,(2aB+5Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{d}-\frac{b(3aA-2bB)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}+\frac{aA\sin(c+dx)(a+b\sec(c+dx))^{3/2}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(3*a^2*A - 6*A*b^2 - 14*a*b*B)*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b*d) +
 (Sqrt[a + b]*(2*a*b*(9*A - 7*B) - 2*b^2*(3*A - B) + 3*a^2*(A + 6*B))*Cot[c
 + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
- b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
 - b))])/(3*d) - (a*Sqrt[a + b]*(5*A*b + 2*a*B)*Cot[c + d*x]*EllipticPi[(a
+ b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt
[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/d
 + (a*A*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/d - (b*(3*a*A - 2*b*B)*Sqr
t[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
```

```
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4141

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_.), x_Symbol] :> Simp[(-C)*Cot
[e + f*x]*((a + b*Csc[e + f*x])^m/(f*(m + 1))), x] + Dist[1/(m + 1), Int[(a
 + b*Csc[e + f*x])^(m - 1)*Simp[a*A*(m + 1) + ((A*b + a*B)*(m + 1) + b*C*m)
*Csc[e + f*x] + (b*B*(m + 1) + a*C*m)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
```

```
a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && IGtQ[2*m, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aA(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{d} - \int \sqrt{a\cdots}$$

$$= \frac{aA(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{b(3aA\cdots}{\cdots}$$

$$= \frac{aA(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{d} - \frac{b(3aA\cdots}{\cdots}$$

$$= \frac{(a-b)\sqrt{a+b}\,(3a^2A-6Ab^2-14abB)\cot(c+\cdots}{\cdots}$$

$$= \frac{(a-b)\sqrt{a+b}\,(3a^2A-6Ab^2-14abB)\cot(c+\cdots}{\cdots}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 7745 vs. $2(433) = 866$.
time = 25.67, size = 7745, normalized size = 17.89

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] Result too large to show
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3214 vs. $2(396) = 792$.
time = 11.12, size = 3215, normalized size = 7.42

method	result	size
default	Expression too large to display	3215

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/3/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
^2*(2*B*cos(d*x+c)^2*b^3+18*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+3*A*cos(d*x+c)^4*a^3-2*B*b^
3+14*B*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*sin(d*x+c)*a-3*A*cos(d*x+c)^2*a^2*b+14*B*cos(d*x+c)^2*a*b^2+3*A*c
os(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^3-6*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*b^3+12*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3+6*A*cos(d*x+c)*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b
^3+2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*b^3-6*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^3+2*B*cos(d*x+c)^3*a*b^2+3*A*cos(d*x
+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^3-6*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*b^3+12*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos
(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3-6*B*cos(d*x+c)*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+30*A*cos(
d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a
+b))^(1/2))*a^2*b+14*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-14*B*cos(d*x+c)^2*sin(d*x+c)*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
```

```
EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-14*B*cos(d*
x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a*b^2-18*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+18*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+3*A*cos(d*x+c)^2*
sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^2*b-6*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a*b^2-6*A*cos(d*x+c)*b^3+6*A*cos(d*x+c)^2*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+2*B*cos(d
*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*b^3+3*A*cos(d*x+c)^3*a^2*b-3*A*cos(d*x+c)^3*a^3+6*A*cos(d*x+c)^2*b^3
+18*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a^2*b+3*A*cos(d*x+c)*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-6*A*cos(d*x+c)*b^2*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+30*A*cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(
1/2))*a^2*b-18*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^2*b+18*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)*sec(d*x + c)^3 + A*a^2*cos(d*x + c) + (2*B*a*b
 + A*b^2)*cos(d*x + c)*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c)*sec(
d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)
```

3.368 $\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=450

$$\frac{(a-b)\sqrt{a+b}\,(9aAb+4a^2B-8b^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{4bd}$$

```
[Out] 1/2*a*A*cos(d*x+c)*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+1/4*(a-b)*(9*A*a*b+4
*B*a^2-8*B*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a
+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x
+c))/(a-b))^(1/2)/b/d+1/4*(8*b^2*(A-B)+2*a^2*(A+2*B)+3*a*b*(3*A+8*B))*cot(d
*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+
b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-1
/4*(4*A*a^2+15*A*b^2+20*B*a*b)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)
/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+
b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d+1/4*a*(7*A*b+4*B*a)*sin(d*x+c)*
(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.52, antiderivative size = 450, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4110, 4179, 4143, 4006, 3869, 3917, 4089}

$$\frac{\sqrt{a+b}\,(2a^2(A+2B)+3ab(3A+8B)+8b^2(A-B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4d}+\frac{(a-b)\sqrt{a+b}\,(4a^2B+9aAb-8b^2B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4bd}-\frac{\sqrt{a+b}\,(4a^2A+20abB+15Ab^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4d}+\frac{a(4aB+7Ab)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}+\frac{aA\sin(c+dx)\cos(c+dx)(a+b\sec(c+dx))^{3/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((a - b)*Sqrt[a + b]*(9*a*A*b + 4*a^2*B - 8*b^2*B)*Cot[c + d*x]*EllipticE[A
rcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*b*d) +
(Sqrt[a + b]*(8*b^2*(A - B) + 2*a^2*(A + 2*B) + 3*a*b*(3*A + 8*B))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(4*d) - (Sqrt[a + b]*(4*a^2*A + 15*A*b^2 + 20*a*b*B)*Cot[c + d*x]*Ell
ipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a
 - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(
a - b))])/(4*d) + (a*(7*A*b + 4*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(4*d) + (a*A*Cos[c + d*x]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
```

```
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
```

```
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4179

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^2(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c+d}{2d}$$

$$= \frac{a(7Ab+4aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx}{4d}$$

$$= \frac{a(7Ab+4aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx}{4d}$$

$$= \frac{(a-b)\sqrt{a+b}\,(9aAb+4a^2B-8b^2B)\cot(c+}{}$$

$$= \frac{(a-b)\sqrt{a+b}\,(9aAb+4a^2B-8b^2B)\cot(c+}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1326 vs. $2(450) = 900$.
time = 19.25, size = 1326, normalized size = 2.95

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*(2*b^2*B*Sin[c + d*x] + (a^2*A*S
in[2*(c + d*x)])/4))/(d*(b + a*Cos[c + d*x])^2) + ((a + b*Sec[c + d*x])^(5/
2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*(9*a^2*A*b*Tan[(c + d*x)/2] + 9*a*A*
b^2*Tan[(c + d*x)/2] + 4*a^3*B*Tan[(c + d*x)/2] + 4*a^2*b*B*Tan[(c + d*x)/2
```

```
] - 8*a*b^2*B*Tan[(c + d*x)/2] - 8*b^3*B*Tan[(c + d*x)/2] - 18*a^2*A*b*Tan[
(c + d*x)/2]^3 - 8*a^3*B*Tan[(c + d*x)/2]^3 + 16*a*b^2*B*Tan[(c + d*x)/2]^3
 + 9*a^2*A*b*Tan[(c + d*x)/2]^5 - 9*a*A*b^2*Tan[(c + d*x)/2]^5 + 4*a^3*B*Ta
n[(c + d*x)/2]^5 - 4*a^2*b*B*Tan[(c + d*x)/2]^5 - 8*a*b^2*B*Tan[(c + d*x)/2
]^5 + 8*b^3*B*Tan[(c + d*x)/2]^5 + 8*a^3*A*EllipticPi[-1, ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a*A*b^2*EllipticPi[-1,
 ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sq
rt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 40*a^2*
b*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[
(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/
(a + b)] + 8*a^3*A*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*a*A*b^2*EllipticPi[-1, ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
 b)] + 40*a^2*b*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d
*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(9*a*A*b + 4*a^2*B - 8*
b^2*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c
 + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
 b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*(2*a^3*A - a^2*b*(A - 12*B) + 12*a*b^2*
(A - B) - 4*b^3*(A + B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Ta
n[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*d*(b + a*Cos[c + d*x
])^(5/2)*Sec[c + d*x]^(5/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*
Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3270 vs. $2(409) = 818$.
time = 10.90, size = 3271, normalized size = 7.27

method	result	size
default	Expression too large to display	3271

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^2*(4*B*cos(d*x+c)^3*a^3+2*A*cos(d*x+c)^4*a^3-8*B*b^3
+30*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*
b^2*sin(d*x+c)+40*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a
+b))^(1/2))*a^2*b*sin(d*x+c)+24*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
```

os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+24*B*cos(d*x+c)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a-9*A*cos(d*x+c)*
a*b^2-4*B*cos(d*x+c)*a^2*b-9*A*cos(d*x+c)^2*a^2*b+8*B*cos(d*x+c)^2*a*b^2-2*
A*cos(d*x+c)*a^2*b+8*B*cos(d*x+c)*b^3+8*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)
/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+8*B*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+9
*A*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*b+9*A*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a+2*A*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*b-24*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-8*B*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-24*B*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+8*A*Ellipti
cPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(
d*x+c)*a^3-4*A*cos(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*sin(d*x+c)*a^3+4*B*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d
x+c),((a-b)/(a+b))^(1/2))(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^3-8*B*cos(d*x+c)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*b^3+30*A*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*si
n(d*x+c)*a*b^2+40*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+8*A*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+4*B*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-8*B*(cos(d*x+c

```
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+8*B*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+
8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*
sin(d*x+c)+11*A*cos(d*x+c)^3*a^2*b-2*A*cos(d*x+c)^2*a^3-4*B*cos(d*x+c)^2*a^
3-24*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a^2*b+9*A*cos(d*x+c)*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+9*A*cos(d*x+c)*b^2*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+
c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+2*A*cos(d*x+
c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^2*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^2 + (2*B
*a*b + A*b^2)*cos(d*x + c)^2*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c
)^2*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^2\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^2*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)
```

3.369 $\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=518

$$\frac{(a-b)\sqrt{a+b}\,(16a^2A+33Ab^2+54abB)\cot(c+dx)E\left(\operatorname{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec}{a-}}}{24bd}$$

[Out] 1/3*a*A*cos(d*x+c)^2*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+1/24*(a-b)*(16*A*a^2+33*A*b^2+54*B*a*b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b/d+1/24*(16*A*a^2+26*A*a*b+33*A*b^2+12*B*a^2+54*B*a*b+48*B*b^2)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/d-1/8*(20*A*a^2*b+5*A*b^3+8*B*a^3+30*B*a*b^2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d+1/24*(16*A*a^2+33*A*b^2+54*B*a*b)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+1/4*a*(3*A*b+2*B*a)*cos(d*x+c)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d

Rubi [A]
time = 0.87, antiderivative size = 518, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4110, 4179, 4189, 4143, 4006, 3869, 3917, 4089}

[illegible]

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] ((a - b)*Sqrt[a + b]*(16*a^2*A + 33*A*b^2 + 54*a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*b*d) + (Sqrt[a + b]*(16*a^2*A + 26*a*A*b + 33*A*b^2 + 12*a^2*B + 54*a*b*B + 48*b^2*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*d) - (Sqrt[a + b]*(20*a^2*A*b + 5*A*b^3 + 8*a^3*B + 30*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(8*a*d) + ((16*a^2*A + 33*A*b^2 + 54*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (a*(3*A*b + 2*a*B)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (a*A*Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
```

```
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4179

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^3(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos^2(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c+}{3d}$$

$$= \frac{a(3Ab+2aB)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}$$

$$= \frac{(16a^2A+33Ab^2+54abB)\sqrt{a+b\sec(c+dx}}{24d}$$

$$= \frac{(16a^2A+33Ab^2+54abB)\sqrt{a+b\sec(c+dx}}{24d}$$

$$= \frac{(a-b)\sqrt{a+b}\,(16a^2A+33Ab^2+54abB)\cot(}{}$$

$$= \frac{(a-b)\sqrt{a+b}\,(16a^2A+33Ab^2+54abB)\cot(}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1551 vs. $2(518) = 1036$.

time = 19.38, size = 1551, normalized size = 2.99

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^2*(a + b*Sec[c + d*x])^(5/2)*((a^2*A*Sin[c + d*x])/12 + (a*(1
3*A*b + 6*a*B)*Sin[2*(c + d*x)])/24 + (a^2*A*Sin[3*(c + d*x)])/12))/(d*(b +
 a*Cos[c + d*x])^2) + ((a + b*Sec[c + d*x])^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2
]^2)^(-1)]*(16*a^3*A*Tan[(c + d*x)/2] + 16*a^2*A*b*Tan[(c + d*x)/2] + 33*a*
A*b^2*Tan[(c + d*x)/2] + 33*A*b^3*Tan[(c + d*x)/2] + 54*a^2*b*B*Tan[(c + d*
x)/2] + 54*a*b^2*B*Tan[(c + d*x)/2] - 32*a^3*A*Tan[(c + d*x)/2]^3 - 66*a*A*
b^2*Tan[(c + d*x)/2]^3 - 108*a^2*b*B*Tan[(c + d*x)/2]^3 + 16*a^3*A*Tan[(c +
 d*x)/2]^5 - 16*a^2*A*b*Tan[(c + d*x)/2]^5 + 33*a*A*b^2*Tan[(c + d*x)/2]^5
- 33*A*b^3*Tan[(c + d*x)/2]^5 + 54*a^2*b*B*Tan[(c + d*x)/2]^5 - 54*a*b^2*B*
Tan[(c + d*x)/2]^5 + 120*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (
a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*A*b^3*EllipticPi[-1, ArcSin[Tan[
(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b -
a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*a^3*B*EllipticPi
```

```
[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2
]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 180
*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 -
 Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2
]^2)/(a + b)] + 120*a^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*
Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 30*A*b^3*EllipticPi[-
1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] + 48*a^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a
+ b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(
c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 180*a*b^2*B*EllipticPi[-1,
 ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan
[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)
/(a + b)] + (a + b)*(16*a^2*A + 33*A*b^2 + 54*a*b*B)*EllipticE[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d
*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b
)] - 2*(24*b^3*(A - B) + 12*a^3*B + a*b^2*(-13*A + 72*B) + a^2*(38*A*b - 6*
b*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
b*Tan[(c + d*x)/2]^2)/(a + b)]))/(24*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d
*x]^(5/2)*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3510 vs. $2(473) = 946$.
time = 8.86, size = 3511, normalized size = 6.78

method	result	size
default	Expression too large to display	3511

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/24/d*(-1+cos(d*x+c))^2*(-144*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-144*B*cos(d*x+c)*EllipticF((-1+cos(d
*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a-26*A*cos(d*x+c
)*a*b^2-12*B*cos(d*x+c)*a^2*b-18*A*cos(d*x+c)^2*a^2*b+54*B*cos(d*x+c)^2*a*b
^2-16*A*cos(d*x+c)*a^2*b-48*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+48*B*cos(d*x+c)*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
```

*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+16*A*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+16*A
*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^3+33*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*b^3+30*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticP
i((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3+48*B*cos(d*x+c)*si
n(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))
*a^3-24*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^3+16*A*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*b+33*A*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+120*A*a^2*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*Ellip
ticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b-76*A*a^2*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*si
n(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+26*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x
+c)+54*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2
*b*sin(d*x+c)+54*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a*b^2*sin(d*x+c)+180*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1
,((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+12*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+8*A*cos(d*x+c)^5*a^3-33*A*
cos(d*x+c)*b^3+12*B*cos(d*x+c)^4*a^3-48*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+48*B*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+33*A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+30*A*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+48
B(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*s

```
in(d*x+c)-24*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*a^3*sin(d*x+c)+8*A*cos(d*x+c)^3*a^3-16*A*cos(d*x+c)^2*a^3+33*A*cos(d*x+c
)^2*b^3-12*B*cos(d*x+c)^2*a^3+12*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+16*A*cos(d*x+c)*a^2*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+33*
A*cos(d*x+c)*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a+120*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x
)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^3*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^3 + (2*B
*a*b + A*b^2)*cos(d*x + c)^3*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c
)^3*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^3\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^3*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)
```

3.370 $\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=617

$$\frac{(a-b)\sqrt{a+b}\,(284a^2Ab+15Ab^3+128a^3B+264ab^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\cdots}{192abd}$$

[Out] 1/4*a*A*cos(d*x+c)^3*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+1/192*(a-b)*(284*A*a^2*b+15*A*b^3+128*B*a^3+264*B*a*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/b/d+1/192*(15*A*b^3+8*a^3*(9*A+16*B)+4*a^2*b*(71*A+52*B)+2*a*b^2*(59*A+132*B))*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a/d-1/64*(48*A*a^4+120*A*a^2*b^2-5*A*b^4+160*B*a^3*b+40*B*a*b^3)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+1/192*(284*A*a^2*b+15*A*b^3+128*B*a^3+264*B*a*b^2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d+1/96*(36*A*a^2+59*A*b^2+104*B*a*b)*cos(d*x+c)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+1/24*a*(11*A*b+8*B*a)*cos(d*x+c)^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d

Rubi [A]
time = 1.15, antiderivative size = 617, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4110, 4179, 4189, 4143, 4006, 3869, 3917, 4089}

$$\frac{(36a^2A+104abB+59Ab^2)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{96d}+\frac{\sqrt{a+b}\,(8a^3(9A+16B)+4a^2b(71A+52B)+2ab^2(59A+132B)+15Ab^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{192ad}+\frac{(a-b)\sqrt{a+b}\,(128a^3B+284a^2Ab+264ab^2B+15Ab^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{192abd}+\frac{(128a^3B+284a^2Ab+264ab^2B+15Ab^3)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{192ad}-\frac{\sqrt{a+b}\,(48a^4A+160a^3bB+120a^2Ab^2+40ab^3B-5Ab^4)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{64a^2d}+\frac{a(8aB+11Ab)\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{24d}+\frac{aA\sin(c+dx)\cos^3(c+dx)(a+b\sec(c+dx))^{3/2}}{4d}$$

Antiderivative was successfully verified.

[In] Int[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]

[Out] ((a - b)*Sqrt[a + b]*(284*a^2*A*b + 15*A*b^3 + 128*a^3*B + 264*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(192*a*b*d) + (Sqrt[a + b]*(15*A*b^3 + 8*a^3*(9*A + 16*B) + 4*a^2*b*(71*A + 52*B) + 2*a*b^2*(59*A + 132*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(192*a*d) - (Sqrt[a + b]*(48*a^4*A + 120*a^2*A*b^2 - 5*A*b^4 + 160*a^3*b*B + 40*a*b^3*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(64*a^2*d) + ((284*a^2*A*b + 15*A*b^3 + 128*a^3*B +

```
264*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192*a*d) + ((36*a^2*A
+ 59*A*b^2 + 104*a*b*B)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(96*d) + (a*(11*A*b + 8*a*B)*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c
 + d*x])/(24*d) + (a*A*Cos[c + d*x]^3*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*
x])/(4*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
```

```
LeQ[n, -1]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4179

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^4(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{aA\cos^3(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c+}{4d}$$
$$= \frac{a(11Ab+8aB)\cos^2(c+dx)\sqrt{a+b\sec(c+d}}{24d}$$
$$= \frac{(36a^2A+59Ab^2+104abB)\cos(c+dx)\sqrt{a+}}{96d}$$
$$= \frac{(284a^2Ab+15Ab^3+128a^3B+264ab^2B)\sqrt{a}}{192ad}$$
$$= \frac{(284a^2Ab+15Ab^3+128a^3B+264ab^2B)\sqrt{a}}{192ad}$$
$$= \frac{(a-b)\sqrt{a+b}\,(284a^2Ab+15Ab^3+128a^3B+}{}$$
$$= \frac{(a-b)\sqrt{a+b}\,(284a^2Ab+15Ab^3+128a^3B+}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 5172 vs. $2(617) = 1234$.
time = 24.09, size = 5172, normalized size = 8.38

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^4*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] Result too large to show
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4230 vs. $2(568) = 1136$.
time = 8.13, size = 4231, normalized size = 6.86

method	result	size
default	Expression too large to display	4231

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

[Out] -1/192/d*(-1+cos(d*x+c))^2*(64*B*cos(d*x+c)^5*a^4-15*A*cos(d*x+c)*b^4-264*B
*cos(d*x+c)*a*b^3+284*A*cos(d*x+c)*a^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+284*A*sin(d*x+c)*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+15*A*sin(d*
x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*a*b^3+720*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*b^2+72*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-644*A*sin(d*x+c)
*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a^2*b^2+118*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a*b^3+128*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+264*B*sin(d*x+c)*cos(d*
x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2
+264*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a*b^3+960*B*cos(d*x+c)*a^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticPi((-1+
cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b+240*B*cos(d*x+c)*b^3*(cos(
d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a-
608*B*cos(d*x+c)*a^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*b+208*B*cos(d*x+c)*a^2*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-384*B*sin(d*x+c)*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-15*A*co
s(d*x+c)^2*a*b^3-264*B*cos(d*x+c)^2*a^2*b^2+184*A*cos(d*x+c)^5*a^3*b+254*A*
cos(d*x+c)^4*a^2*b^2+272*B*cos(d*x+c)^4*a^3*b-144*A*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)+172*A*cos(d*x+c)^3*a
^3*b+133*A*cos(d*x+c)^3*a*b^3+472*B*cos(d*x+c)^3*a^2*b^2-284*A*cos(d*x+c)^2
*a^3*b+30*A*cos(d*x+c)^2*a^2*b^2-144*B*cos(d*x+c)^2*a^3*b+264*B*cos(d*x+c)^
2*a*b^3-72*A*cos(d*x+c)*a^3*b-284*A*cos(d*x+c)*a^2*b^2-118*A*cos(d*x+c)*a*b
^3-128*B*cos(d*x+c)*a^3*b-208*B*cos(d*x+c)*a^2*b^2+64*B*cos(d*x+c)^3*a^4+15
*A*cos(d*x+c)^2*b^4-128*B*cos(d*x+c)^2*a^4-72*A*cos(d*x+c)^2*a^4+48*A*cos(d

```
*x+c)^6*a^4+24*A*a^4*cos(d*x+c)^4+128*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)+15*A*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)+288*A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-
1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4*sin(d*x+c)-30*A*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^4*sin(d*x+c)
+15*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*b^4+288*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x
+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^4-30*A*sin(d*x+c)*cos(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^4-144*A*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a^4+128*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^4+284*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^4*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^4 + (2*B
*a*b + A*b^2)*cos(d*x + c)^4*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c
)^4*sec(d*x + c))*sqrt(b*sec(d*x + c) + a), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**4*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^4*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm
="giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^4, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^4\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^4*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)
```

3.371 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=329

$$\frac{2(a-b)\sqrt{a+b}\,(10aAb-8a^2B-9b^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c}{a+b}}}{15b^4d}$$

```
[Out] 2/15*(a-b)*(10*A*a*b-8*B*a^2-9*B*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))
^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b)
)^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d+2/15*(b^2*(5*A-9*B)-8*a^2*B+2
*a*b*(5*A+B))*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b
)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c
))/(a-b))^(1/2)/b^3/d+2/15*(5*A*b-4*B*a)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/
b^2/d+2/5*B*sec(d*x+c)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b/d
```

Rubi [A]

time = 0.39, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4118, 4167, 4090, 3917, 4089}

$$\frac{2(a-b)\sqrt{a+b}\,(-8a^2B+10aAb-9b^2B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^4d}+\frac{2\sqrt{a+b}\,(-8a^2B+2ab(5A+B)+b^2(5A-9B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{15b^3d}+\frac{2(5Ab-4aB)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{15b^2d}+\frac{2B\tan(c+dx)\sec(c+dx)\sqrt{a+b\sec(c+dx)}}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*(a - b)*Sqrt[a + b]*(10*a*A*b - 8*a^2*B - 9*b^2*B)*Cot[c + d*x]*Elliptic
E[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1
 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(15*b^4
*d) + (2*Sqrt[a + b]*(b^2*(5*A - 9*B) - 8*a^2*B + 2*a*b*(5*A + B))*Cot[c +
d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b
)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
b))])/(15*b^3*d) + (2*(5*A*b - 4*a*B)*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x]
)/(15*b^2*d) + (2*B*Sec[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(5*
b*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4118

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d^
2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 2)/(b*f*
(m + n))), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e +
 f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m +
n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n
, 0] &&  !IGtQ[m, 1]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2B\sec(c+dx)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{5bd} + \frac{2\int \frac{\sec(c+dx)(aB+}{}}{}$$

$$= \frac{2(5Ab-4aB)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{15b^2d} + \frac{2B\sec(c+dx)\sqrt{}}{}$$

$$= \frac{2(5Ab-4aB)\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{15b^2d} + \frac{2B\sec(c+dx)\sqrt{}}{}$$

$$= \frac{2(a-b)\sqrt{a+b}\,(10aAb-8a^2B-9b^2B)\cot(c+dx)E\Big(\sin^{-1}\Big(\frac{\sqrt{}}{}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3000 vs. $2(329) = 658$.

time = 21.85, size = 3000, normalized size = 9.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((2*(-10*a*A*b + 8*a^2*B + 9*b^2*B)*Sin[
c + d*x])/(15*b^3) + (2*Sec[c + d*x]*(5*A*b*Sin[c + d*x] - 4*a*B*Sin[c + d*
x]))/(15*b^2) + (2*B*Sec[c + d*x]*Tan[c + d*x])/(5*b)))/(d*Sqrt[a + b*Sec[c
 + d*x]]) - (2*((2*a*A)/(3*b*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) -
 (3*B)/(5*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^2*B)/(15*b^2*
Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (A*Sqrt[Sec[c + d*x]])/(3*Sq
rt[b + a*Cos[c + d*x]]) + (2*a^2*A*Sqrt[Sec[c + d*x]])/(3*b^2*Sqrt[b + a*Co
s[c + d*x]]) - (8*a^3*B*Sqrt[Sec[c + d*x]])/(15*b^3*Sqrt[b + a*Cos[c + d*x]
]) - (7*a*B*Sqrt[Sec[c + d*x]])/(15*b*Sqrt[b + a*Cos[c + d*x]]) + (2*a^2*A*
Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^2*Sqrt[b + a*Cos[c + d*x]]) - (8*
a^3*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(15*b^3*Sqrt[b + a*Cos[c + d*x]]
) - (3*a*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(5*b*Sqrt[b + a*Cos[c + d*x
]]))*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*(-
10*a*A*b + 8*a^2*B + 9*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
 + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^2*B + 2*a*b*(-5*A + B) + b^2*(5*A + 9
*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
] + (-10*a*A*b + 8*a^2*B + 9*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(
```

c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b^3*d*Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[a + b*Sec[c + d*x]]*(-1/15*(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-10*a*A*b + 8*a^2*B + 9*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^2*B + 2*a*b*(-5*A + B) + b^2*(5*A + 9*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-10*a*A*b + 8*a^2*B + 9*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(b^3*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-10*a*A*b + 8*a^2*B + 9*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(8*a^2*B + 2*a*b*(-5*A + B) + b^2*(5*A + 9*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-10*a*A*b + 8*a^2*B + 9*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(15*b^3*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-10*a*A*b + 8*a^2*B + 9*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-10*a*A*b + 8*a^2*B + 9*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (b*(8*a^2*B + 2*a*b*(-5*A + B) + b^2*(5*A + 9*B))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(-10*a*A*b + 8*a^2*B + 9*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - (b*(8*a^2*B + 2*a*b*(-5*A + B) + b^2*(5*A + 9*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(-10*a*A*b + 8*a^2*B + 9*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (-10*a*A*b + 8*a^2*B + 9*b^2*B)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-10*a*A*b + 8*a^2*B + 9*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]^2 - (b*(8*a^2*B + 2*a*b*(-5*A + B) + b^2*(5*A + 9*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2)/(Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[1 - ((a - b)*Tan[(c + d*x)/2]^2)/(a + b)]) + ((a + b)*(-10*a*A*b + 8*a^2*B + 9*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Sec[(c + d*x)/2]^2*Sqrt[1 - ((a - b)*Tan

```
[(c + d*x)/2]^2)/(a + b)])/Sqrt[1 - Tan[(c + d*x)/2]^2]))/(15*b^3*Sqrt[b +
a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - ((2*(a + b)*(-10*a*A*b + 8*a^2*
B + 9*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2498 vs. $2(299) = 598$.
time = 11.08, size = 2499, normalized size = 7.60

method	result	size
default	Expression too large to display	2499

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/15/d*(1+cos(d*x+c))^2*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(5*A*cos(d*x+c)^3*b^3-8*B*cos(d*x+c)^3*a^3+9*B*cos(d*x+c)^3*b^3-6*B*cos
(d*x+c)^2*b^3+8*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-3*B*b^3-10*B*cos(d*x+c)^3*a*b^2+2*B*cos
(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a*b^2-8*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-9*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-10*A*cos(d*x+c)^
3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))
*a*b^2+10*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a^2*b+10*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+8*B*cos(d*x+c)^3*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+
2*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a*b^2-8*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-9*B*cos(d*x+c)^3*sin(d*x+c)*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-10*A*cos(
d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
```

```
^(1/2))*a*b^2+10*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+10*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-5*A*cos(d*x+c)*
b^3+8*B*cos(d*x+c)^4*a^3+5*A*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+9*B*cos(d*x+c)^3*sin(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-8*B*cos(d*
x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*a^3-9*B*cos(d*x+c)^3*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*b^3+5*A*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3+9*B*cos(d*x+c)^2*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-8*B*c
os(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*a^3-9*B*cos(d*x+c)^2*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*b^3+5*A*cos(d*x+c)^4*a*b^2-4*B*cos(d*x+c)^4*
a^2*b+9*B*cos(d*x+c)^4*a*b^2+10*A*cos(d*x+c)^3*a^2*b-10*A*cos(d*x+c)^4*a^2*
b-10*A*cos(d*x+c)^3*a*b^2+8*B*cos(d*x+c)^3*a^2*b+5*A*cos(d*x+c)^2*a*b^2-4*B
*cos(d*x+c)^2*a^2*b+B*cos(d*x+c)*a*b^2)/(b+a*cos(d*x+c))/cos(d*x+c)^2/sin(d
*x+c)^5/b^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^4 + A*sec(d*x + c)^3)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\sec^3(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos(c+d\,x)^3\,\sqrt{a+\frac{b}{\cos(c+d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(1/2)), x)
```

3.372 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=261

$$-\frac{2(a-b)\sqrt{a+b}\,(3Ab-2aB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{3b^3d}$$

```
[Out] -2/3*(a-b)*(3*A*b-2*B*a)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^
(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(
1+sec(d*x+c))/(a-b))^(1/2)/b^3/d-2/3*(3*A*b-(2*a+b)*B)*cot(d*x+c)*EllipticF
((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-
sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d+2/3*B*(a+b*s
ec(d*x+c))^(1/2)*tan(d*x+c)/b/d
```

Rubi [A]
time = 0.24, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4095, 4090, 3917, 4089}

$$-\frac{2(a-b)\sqrt{a+b}\,(3Ab-2aB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^3d}-\frac{2\sqrt{a+b}\,(3Ab-B(2a+b))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d}+\frac{2B\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*(3*A*b - 2*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[
a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x
]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*d) - (2*Sqrt[a
 + b]*(3*A*b - (2*a + b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
+ d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]
*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^2*d) + (2*B*Sqrt[a + b*Sec[c
 + d*x]]*Tan[c + d*x])/(3*b*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
```

```
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4095

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*Cot[e + f*x]*
((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m + 2)), Int[
Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b*B*(m + 1) + (A*b*(m + 2) - a*B)*
Csc[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, m}, x] && NeQ[A*b - a*
B, 0] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2B\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3bd} + \frac{2\int \frac{\sec(c+dx)\left(\frac{bB}{2}+\frac{1}{2}(3Ab-2aB)\sec\right.}{\sqrt{a+b\sec(c+dx}}}{3b}$$

$$= \frac{2B\sqrt{a+b\sec(c+dx)}\,\tan(c+dx)}{3bd} + \frac{(3Ab-2aB)\int \frac{\sec(c+dx)(1+\sec}{\sqrt{a+b\sec(}}}{3b}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,(3Ab-2aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(}}{\sqrt{a+}}\right.\right.}{3b^3}$$

Mathematica [A]
time = 15.63, size = 372, normalized size = 1.43

$$\frac{2\sqrt{\sec(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2(a+b)(-3Ab+2aB)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+2b(3Ab+(-2a+b)B)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-(3Ab-2aB)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3b^2d\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{a+b\sec(c+dx)}}+\frac{(b+a\cos(c+dx))\sec(c+dx)\left(\frac{2(3Ab-2aB)\sin(c+dx)}{3b^2}+\frac{2B\tan(c+dx)}{3b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*(-3*
A*b + 2*a*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x]
)/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)
/(a + b)] + 2*b*(3*A*b + (-2*a + b)*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)] - (3*A*b - 2*a*B)*Cos[c + d*x]*(b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^2*d*Sqrt[Sec[(c + d
*x)/2]^2]*Sqrt[a + b*Sec[c + d*x]]) + ((b + a*Cos[c + d*x])*Sec[c + d*x]*((
2*(3*A*b - 2*a*B)*Sin[c + d*x])/(3*b^2) + (2*B*Tan[c + d*x])/(3*b)))/(d*Sqr
t[a + b*Sec[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1562 vs. $2(235) = 470$.
time = 11.15, size = 1563, normalized size = 5.99

method	result	size
default	Expression too large to display	1563

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))^2*(2*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-2*B*cos(d*x+c)^2*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-3*A*cos(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*
x+c)*a*b+2*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*sin(d*x+c)*a*b-2*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/si
n(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-3*A*cos(d*x+c)^2*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-3*A*cos(d*
x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d
*x+c)*b^2+3*A*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*sin(d*x+c)*b^2+2*B*cos(d*x+c)^2*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+B*cos(d*x+c)^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-3*A*cos
```

```
(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(
d*x+c)*b^2+3*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*sin(d*x+c)*b^2+2*B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+B*cos(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-b^2*B+B*cos(
d*x+c)^2*b^2-2*B*cos(d*x+c)^3*a^2+3*A*cos(d*x+c)^2*b^2+2*B*cos(d*x+c)^2*a^2
-3*A*cos(d*x+c)*b^2+3*A*cos(d*x+c)^3*a*b+B*cos(d*x+c)^3*a*b-3*A*cos(d*x+c)^
2*a*b-2*B*cos(d*x+c)^2*a*b+B*cos(d*x+c)*a*b)*((b+a*cos(d*x+c))/cos(d*x+c))^
(1/2)*(1+cos(d*x+c))^2/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)^5/b^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec{(c + dx)}) \sec^2{(c + dx)}}{\sqrt{a + b \sec{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\cos\left(c+d\,x\right)}^2\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(1/2)), x)
```

3.373 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=210

$$-\frac{2(a-b)\sqrt{a+b}\,B\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{se}}{a}}}{b^2d}$$

```
[Out] -2*(a-b)*B*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(
a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/
(a-b))^(1/2)/b^2/d+2*(A-B)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b
)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b
*(1+sec(d*x+c))/(a-b))^(1/2)/b/d
```

Rubi [A]
time = 0.13, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.097$, Rules used = {4090, 3917, 4089}

$$\frac{2\sqrt{a+b}\,(A-B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{2B(a-b)\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*(a - b)*Sqrt[a + b]*B*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c +
d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^2*d) + (2*Sqrt[a + b]*(A - B)*Co
t[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/
(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))
/(a - b))])/(b*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
```

```
f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = (A-B)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx + B\int \frac{\sec(c+dx)(1+\sec(c+\cdots}{\sqrt{a+b\sec(c+dx\cdots}}$$

$$= -\frac{2(a-b)\sqrt{a+b}\,B\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\cdots\right.}{b^2d}$$

Mathematica [A]
time = 13.52, size = 356, normalized size = 1.70

$$\frac{2B(b+a\cos(c+dx))(A+B\sec(c+dx))\sin(c+dx)}{bd(B+A\cos(c+dx))\sqrt{a+b\sec(c+dx)}}-\frac{2\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\,(A+B\sec(c+dx))\left(2(a+b)B\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-2b(A+B)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\,F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+B\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{bd(B+A\cos(c+dx))\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*B*(b + a*Cos[c + d*x])*(A + B*Sec[c + d*x])*Sin[c + d*x])/(b*d*(B + A*Co
s[c + d*x])*Sqrt[a + b*Sec[c + d*x]]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
d*x]]*(A + B*Sec[c + d*x])*(2*(a + b)*B*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x]
)]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(A + B)*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + B*Cos[c + d*x]*(b + a*Cos
[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(b*d*(B + A*Cos[c + d*x])*
Sqrt[Sec[(c + d*x)/2]^2]*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 828 vs. $2(192) = 384$.
time = 10.92, size = 829, normalized size = 3.95

method	result
default	$-\frac{2\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\,(1+\cos(dx+c))^2(-1+\cos(dx+c))^2\left(A\cos(dx+c)\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\,\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\sin(dx+c)\,\mathrm{Elli}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+c))^2
*(A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*b+B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*b-B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-B*cos(d*x+c)*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+B*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x
+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-B*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*sin(d*x+c)-B*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b*sin(d*x+c)+B*cos
(d*x+c)^2*a-B*cos(d*x+c)*a+B*cos(d*x+c)*b-B*b)/sin(d*x+c)^5/(b+a*cos(d*x+c)
)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\sec{(c + dx)}}{\sqrt{a + b\sec{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+d\,x)}}{\cos\left(c + d\,x\right)\,\sqrt{a + \frac{b}{\cos\left(c + d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + b/cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + b/cos(c + d*x))^(1/2)), x)
```

3.374 $\int \frac{A+B\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=208

$$\frac{2\sqrt{a+b}\,B\cot(c+dx)F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx))}{a-b}}}{bd}$$

```
[Out] 2*B*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(
1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^
(1/2)/b/d-2*A*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b
)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+
sec(d*x+c))/(a-b))^(1/2)/a/d
```

Rubi [A]
time = 0.08, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.120$, Rules used = {4006, 3869, 3917}

$$\frac{2B\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd}-\frac{2A\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[a + b]*B*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(b*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(a*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx = A\int \frac{1}{\sqrt{a+b\sec(c+dx)}}\,dx + B\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{2\sqrt{a+b}\,B\cot(c+dx)F\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c}{a+b}}}{bd}$$

Mathematica [A]
time = 2.39, size = 145, normalized size = 0.70

$$\frac{4\cos^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}\left((-A+B)F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+2A\Pi\left(-1;\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)\right)\sec(c+dx)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/Sqrt[a + b*Sec[c + d*x]],x]
[Out] (4*Cos[(c + d*x)/2]^2*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos
[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*((-A + B)*EllipticF[ArcSin[Tan[(c
+ d*x)/2]], (a - b)/(a + b)] + 2*A*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]],
 (a - b)/(a + b)])*Sec[c + d*x])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [A]
time = 7.96, size = 215, normalized size = 1.03

method	result
default	$-\frac{2\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\sqrt{\frac{\cos(dx+c)}{1+\cos(dx+c)}}\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}(1+\cos(dx+c))^2(-1+\cos(dx+c))\left(A\,\text{EllipticF}\left(\frac{-1+\cos(dx+c)}{\sin(dx+c)}\right.\right.}{d(b+a\cos(dx+c))\sin(dx+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1+cos(d*x+c))^2*(-1+cos(d*x+
c))*(A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-2*A*Ellipt
icPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))-B*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)))/(b+a*cos(d*x+c))/sin(d*x+c)^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="maxima")
[Out] integrate((B*sec(d*x + c) + A)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="fricas")
[Out] integral((B*sec(d*x + c) + A)/sqrt(b*sec(d*x + c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B \sec{\left(c + dx \right)}}{\sqrt{a + b \sec{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
[Out] Integral((A + B*sec(c + d*x))/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="giac")
[Out] integrate((B*sec(d*x + c) + A)/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+d\,x)}}{\sqrt{a + \frac{b}{\cos\left(c + d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^(1/2),x)
[Out] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^(1/2), x)
```

3.375 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=348

$$\frac{A(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c}{a-b}}}{abd}$$

```
[Out] A*(a-b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b
))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/a/b/d+A*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(
(a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d
*x+c))/(a-b))^(1/2)/a/d+(A*b-2*B*a)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^
(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c)
)/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+A*sin(d*x+c)*(a+b*sec(
d*x+c))^(1/2)/a/d
```

Rubi [A]
time = 0.27, antiderivative size = 348, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4119, 4144, 4006, 3869, 3917, 4089}

$$\frac{\sqrt{a+b}\,(Ab-2aB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{A\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{ad}+\frac{A(a-b)\sqrt{a+b}\,\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{abd}+\frac{A\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (A*(a - b)*Sqrt[a + b]*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x
]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt
[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a*b*d) + (A*Sqrt[a + b]*Cot[c + d*x]*
EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(a*d) + (Sqrt[a + b]*(A*b - 2*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*d) + (A*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(a*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4119

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*n)), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4144

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/Sqrt[csc[(e_.) + (f_.)*(x_)]*
(b_.) + (a_)], x_Symbol] :> Int[(A - C*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x
]], x] + Dist[C, Int[Csc[e + f*x]*((1 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*
x]]), x], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{ad} - \frac{\int \frac{\frac{1}{2}(Ab-2aB)+\frac{1}{2}Ab\sec^2(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a}$$

$$= \frac{A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{ad} - \frac{\int \frac{\frac{1}{2}(Ab-2aB)-\frac{1}{2}Ab\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx}{a} - $$

$$= \frac{A(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{abd}$$

$$= \frac{A(a-b)\sqrt{a+b}\,\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{abd}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.32, size = 1027, normalized size = 2.95

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 - Tan[(c + d*x)/2]^2)^
(-1)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(a*A*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x
)/2]*Sqrt[1 - Tan[(c + d*x)/2]^2] + A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x
)/2]*Sqrt[1 - Tan[(c + d*x)/2]^2] - a*A*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x
)/2]^3*Sqrt[1 - Tan[(c + d*x)/2]^2] + A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d
*x)/2]^3*Sqrt[1 - Tan[(c + d*x)/2]^2] + (2*I)*A*b*EllipticPi[-((a + b)/(a -
 b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (4*I)
*a*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c
 + d*x)/2]], (a + b)/(a - b)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
 + d*x)/2]^2)/(a + b)] + (2*I)*A*b*EllipticPi[-((a + b)/(a - b)), I*ArcSinh
[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2
]^2*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (
4*I)*a*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[(a + b - a*Tan[(c
 + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - I*A*(a - b)*EllipticE[I*Arc
Sinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*(1 + Tan[(c
 + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
 + b)] - (2*I)*(A*b - a*B)*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(
c + d*x)/2]], (a + b)/(a - b)]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan
```

```
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(a*Sqrt[(-a + b)/(a + b)
]*d*Sqrt[a + b*Sec[c + d*x]]*(1 + Tan[(c + d*x)/2]^2)^(3/2)*Sqrt[(a + b - a
*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1024 vs. $2(319) = 638$.
time = 9.35, size = 1025, normalized size = 2.95

method	result	size
default	Expression too large to display	1025

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/d*(-1+cos(d*x+c))^2*(A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b-2*A*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1
+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b+4*B
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x
+c)*sin(d*x+c)*a-2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*cos(d*x+c)*sin(d*x+c)*a+A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a+A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*b-2*A*EllipticPi((-1+cos(d*x+c))/sin(d
*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*b+4*B*EllipticPi((-1+cos(d*x+
c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a-2*B*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a+A*cos(d*x+c)^
3*a-A*a*cos(d*x+c)^2+A*cos(d*x+c)^2*b-A*b*cos(d*x+c))*(1+cos(d*x+c))^2*((b+
a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))/sqrt(b*sec(d*x + c)
 + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\cos{(c + dx)}}{\sqrt{a + b\sec{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(1/2), x)
```

3.376 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=435

$$-\frac{(a-b)\sqrt{a+b}\,(3Ab-4aB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\,\sqrt{}}{4a^2bd}$$

```
[Out] -1/4*(a-b)*(3*A*b-4*B*a)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^
(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(
1+sec(d*x+c))/(a-b))^(1/2)/a^2/b/d-1/4*(3*A*b-2*a*(A+2*B))*cot(d*x+c)*Ellip
ticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b
*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d-1/4*(4*A
*a^2+3*A*b^2-4*B*a*b)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1
/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)
*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d-1/4*(3*A*b-4*B*a)*sin(d*x+c)*(a+b*se
c(d*x+c))^(1/2)/a^2/d+1/2*A*cos(d*x+c)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/
d
```

Rubi [A]
time = 0.47, antiderivative size = 435, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4119, 4189, 4143, 4006, 3869, 3917, 4089}

$$-\frac{\sqrt{a+b}\,(3Ab-2a(A+2B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^2d}-\frac{(a-b)\sqrt{a+b}\,(3Ab-4aB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^2bd}-\frac{(3Ab-4aB)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4a^2d}-\frac{\sqrt{a+b}\,(4a^2A-4abB+3Ab^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^3d}+\frac{A\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{2ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] -1/4*((a - b)*Sqrt[a + b]*(3*A*b - 4*a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqr
t[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d
*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*b*d) - (Sqrt[a
 + b]*(3*A*b - 2*a*(A + 2*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[
c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b
)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^2*d) - (Sqrt[a + b]*(4*a^2
*A + 3*A*b^2 - 4*a*b*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a +
b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^3*d) - ((3*A*b - 4*a
*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*a^2*d) + (A*Cos[c + d*x]*Sqrt
[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*a*d)
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
```

```
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4119

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*n)), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{A\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2ad} - \frac{\int \frac{\cos(c+dx)\left(\frac{1}{2}(3Ab-\right.}{\sqrt{}}}{}$$

$$= -\frac{(3Ab-4aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4a^2d} + \frac{A\cos(c+dx)\sqrt{}}{}$$

$$= -\frac{(3Ab-4aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4a^2d} + \frac{A\cos(c+dx)\sqrt{}}{}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(3Ab-4aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(}}{\sqrt{a+}}\right.\right.}{4a^2}$$

$$= -\frac{(a-b)\sqrt{a+b}\,(3Ab-4aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(}}{\sqrt{a+}}\right.\right.}{4a^2}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.98, size = 1639, normalized size = 3.77

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (A*(b + a*Cos[c + d*x])*Sec[c + d*x]*Sin[2*(c + d*x)])/(4*a*d*Sqrt[a + b*Se
c[c + d*x]]) + (Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(a + b - a
*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(-3*a
*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 3*A*b^2*Sqrt[(-a + b)/(a + b
)]*Tan[(c + d*x)/2] + 4*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + 4*a
```

```
*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + 6*a*A*b*Sqrt[(-a + b)/(a + b
)]*Tan[(c + d*x)/2]^3 - 8*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^3 -
 3*a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 3*A*b^2*Sqrt[(-a + b)/
(a + b)]*Tan[(c + d*x)/2]^5 + 4*a^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/
2]^5 - 4*a*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - (8*I)*a^2*A*Elli
pticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2
]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (6*I)*A*b^2*EllipticPi[-((a +
b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a
 - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(a + b)] + (8*I)*a*b*B*EllipticPi[-((a + b)/(a - b)), I
*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)] - (8*I)*a^2*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[
(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sq
rt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
d*x)/2]^2)/(a + b)] - (6*I)*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[
Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]
^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[
(c + d*x)/2]^2)/(a + b)] + (8*I)*a*b*B*EllipticPi[-((a + b)/(a - b)), I*Arc
Sinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*
x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b
*Tan[(c + d*x)/2]^2)/(a + b)] - I*(a - b)*(-3*A*b + 4*a*B)*EllipticE[I*ArcS
inh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan
[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^
2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(2*a^2*A + 3*A*b^2 - a*b*(A + 4*
B))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(
a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b -
 a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(4*a^2*Sqrt[(-a +
b)/(a + b)]*d*Sqrt[a + b*Sec[c + d*x]]*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 +
Tan[(c + d*x)/2]^2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^2)
- b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1885 vs. $2(394) = 788$.

time = 8.96, size = 1886, normalized size = 4.34

method	result	size
default	Expression too large to display	1886

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))^2*(-3*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si
```

```
n(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+2*A*cos(d*x+c)*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+4*B*cos(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c
)*a*b-8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,((a-b)/(a+b))^(1/2))*a*b-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-3*A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+4*B*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a^2+2*A*
cos(d*x+c)^4*a^2-2*A*cos(d*x+c)^2*a^2+8*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/s
in(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)-2*A*cos(d*x+c)*a*b+6*A*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*sin(d*x
+c)-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*
sin(d*x+c)+4*B*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*sin(d*x+c)*a^2-3*A*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*sin(d*x+c)*a*b+2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*a*b*sin(d*x+c)+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-8*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/
sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+8*A*EllipticPi((-1+cos(d*
x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2+6*A
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*sin(d*x+c)*b^2-4*A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(
1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2+4*B*cos(d*x+c)^3*a^2-3*A*cos(d*x+c)^2*
b^2-4*B*cos(d*x+c)^2*a^2+3*A*cos(d*x+c)*b^2-A*cos(d*x+c)^3*a*b+3*A*cos(d*x+
c)^2*a*b+4*B*cos(d*x+c)^2*a*b-4*B*cos(d*x+c)*a*b)*(1+cos(d*x+c))^2*((b+a*co
s(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^5/a^2
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)/sqrt(b*sec(d*x
+ c) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec(c + dx)) \cos^2(c + dx)}{\sqrt{a + b \sec(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(1/2), x)
```

3.377 $\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=525

$$\frac{(a-b)\sqrt{a+b}\,(16a^2A+15Ab^2-18abB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c}{a+}}}{24a^3bd}$$

[Out] 1/24*(a-b)*(16*A*a^2+15*A*b^2-18*B*a*b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/b/d+1/24*(16*A*a^2-10*A*a*b+15*A*b^2+12*B*a^2-18*B*a*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d+1/8*(4*A*a^2*b+5*A*b^3-8*B*a^3-6*B*a*b^2)*cot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d+1/24*(16*A*a^2+15*A*b^2-18*B*a*b)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^3/d-1/12*(5*A*b-6*B*a)*cos(d*x+c)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^2/d+1/3*A*cos(d*x+c)^2*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d

Rubi [A]
time = 0.77, antiderivative size = 525, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4119, 4189, 4143, 4006, 3869, 3917, 4089}

$$-\frac{(5Ab-6aB)\sin(c+dx)\cos(c+dx)\sqrt{a+b\sec(c+dx)}}{12a^2d}+\frac{\sqrt{a+b}\,(16a^2A+12a^2B-10aAb-18abB+15Ab^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{24a^3d}+\frac{(a-b)\sqrt{a+b}\,(16a^2A-18abB+15Ab^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{24a^3bd}+\frac{(16a^2A-18abB+15Ab^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24a^3d}+\frac{\sqrt{a+b}\,(-8a^3B+4a^2Ab-6ab^2B+5Ab^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{8a^4d}+\frac{A\sin(c+dx)\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]

[Out] ((a - b)*Sqrt[a + b]*(16*a^2*A + 15*A*b^2 - 18*a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a^3*b*d) + (Sqrt[a + b]*(16*a^2*A - 10*a*A*b + 15*A*b^2 + 12*a^2*B - 18*a*b*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a^3*d) + (Sqrt[a + b]*(4*a^2*A*b + 5*A*b^3 - 8*a^3*B - 6*a*b^2*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(8*a^4*d) + ((16*a^2*A + 15*A*b^2 - 18*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*a^3*d) - ((5*A*b - 6*a*B)*Cos[c + d*x]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*a^2*d) + (A*Cos[c + d*x]^2*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d)

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4119

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*n)), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
```

```
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{A\cos^2(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3ad} - \frac{\int \frac{\cos^2(c+dx)\left(\frac{1}{2}(5Ab\cdots\right.}{\sqrt{\cdots}}}{\cdots}$$

$$= -\frac{(5Ab-6aB)\cos(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{12a^2d} + \frac{A\cos\cdots}{\cdots}$$

$$= \frac{(16a^2A+15Ab^2-18abB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24a^3d} - \frac{(5A\cdots}{\cdots}$$

$$= \frac{(16a^2A+15Ab^2-18abB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24a^3d} - \frac{(5A\cdots}{\cdots}$$

$$= \frac{(a-b)\sqrt{a+b}\,(16a^2A+15Ab^2-18abB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{\cdots}}{\cdots}\right.\right.}{\cdots}$$

$$= \frac{(a-b)\sqrt{a+b}\,(16a^2A+15Ab^2-18abB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{\cdots}}{\cdots}\right.\right.}{\cdots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1569 vs. $2(525) = 1050$.
time = 19.53, size = 1569, normalized size = 2.99

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]*((A*Sin[c + d*x])/(12*a) + ((-5*A*b + 6*
a*B)*Sin[2*(c + d*x)])/(24*a^2) + (A*Sin[3*(c + d*x)])/(12*a)))/(d*Sqrt[a +
 b*Sec[c + d*x]]) - (Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*Sqrt[(1 -
Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d
*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(16*a^3*A*Tan[(c + d*x)/2] + 16*a^2*A*b
*Tan[(c + d*x)/2] + 15*a*A*b^2*Tan[(c + d*x)/2] + 15*A*b^3*Tan[(c + d*x)/2]
 - 18*a^2*b*B*Tan[(c + d*x)/2] - 18*a*b^2*B*Tan[(c + d*x)/2] - 32*a^3*A*Tan
[(c + d*x)/2]^3 - 30*a*A*b^2*Tan[(c + d*x)/2]^3 + 36*a^2*b*B*Tan[(c + d*x)/
2]^3 + 16*a^3*A*Tan[(c + d*x)/2]^5 - 16*a^2*A*b*Tan[(c + d*x)/2]^5 + 15*a*A
*b^2*Tan[(c + d*x)/2]^5 - 15*A*b^3*Tan[(c + d*x)/2]^5 - 18*a^2*b*B*Tan[(c +
 d*x)/2]^5 + 18*a*b^2*B*Tan[(c + d*x)/2]^5 - 24*a^2*A*b*EllipticPi[-1, ArcS
in[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*A*b^3*Ell
ipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] + 48*a^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(a + b)] + 36*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]
^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 24*a^2*A*b*EllipticPi[-1, ArcSin[Tan[
(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2
]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] -
30*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c +
 d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
+ b*Tan[(c + d*x)/2]^2)/(a + b)] + 48*a^3*B*EllipticPi[-1, ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 36*a*
b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*
x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b
*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(16*a^2*A + 15*A*b^2 - 18*a*b*B)*El
lipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2
]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
 + d*x)/2]^2)/(a + b)] - 2*a*(5*A*b^2 + 2*a*b*(A - 3*B) + 12*a^2*B)*Ellipti
cF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*
x)/2]^2)/(a + b)]))/(24*a^3*d*Sqrt[a + b*Sec[c + d*x]]*Sqrt[1 + Tan[(c + d*
x)/2]^2]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2953 vs. $2(480) = 960$.
time = 8.77, size = 2954, normalized size = 5.63

method	result	size

default	Expression too large to display	2954

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/24/d*(-1+cos(d*x+c))^2*(10*A*cos(d*x+c)*a*b^2-12*B*cos(d*x+c)*a^2*b+18*A*cos(d*x+c)^2*a^2*b-18*B*cos(d*x+c)^2*a*b^2-16*A*cos(d*x+c)*a^2*b+16*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+16*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+15*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-30*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3+48*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3-24*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3+16*A*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+15*A*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-24*A*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b-4*A*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-10*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-18*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+36*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+12*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+8*A*cos(d*x+c)^5*a^3-15*A*cos(d*x+c)*b^3+12*B*cos(d*x+c)^4*a^3+15*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/si

```
n(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-30*A*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d
*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+48*B*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)-24*B*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c
)+8*A*cos(d*x+c)^3*a^3-16*A*cos(d*x+c)^2*a^3+15*A*cos(d*x+c)^2*b^3-12*B*cos
(d*x+c)^2*a^3+12*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b+16*A*cos(d*x+c)*a^2*(cos(d*x+c)/(1+cos(d
*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+15*A*cos(d*x+c)*b^2
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a-24*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,
((a-b)/(a+b))^(1/2))*a^2*b-4*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-10*A*cos(d*x+c)*sin(d*x+c)*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-18*B*co
s(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a^2*b-18*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+36*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
Pi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^2-2*A*cos(d*x+c)^
4*a^2*b+5*A*cos(d*x+c)^3*a*b^2-6*B*cos(d*x+c)^3...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\cos^3(c + dx)}{\sqrt{a + b\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**3/sqrt(a + b*sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^3/sqrt(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + d\,x)^3\,\left(A + \frac{B}{\cos(c+d\,x)}\right)}{\sqrt{a + \frac{b}{\cos(c + d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(1/2), x)
```

3.378 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=329

$$-\frac{2(6a^2Ab-3Ab^3-8a^3B+5ab^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{3b^4\sqrt{a+b}\,d}$$

```
[Out] -2/3*(6*A*a^2*b-3*A*b^3-8*B*a^3+5*B*a*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*
x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)
*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^4/d/(a+b)^(1/2)-2/3*(2*a+b)*(3*A*b-(4*a+
b)*B)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))
^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/
d/(a+b)^(1/2)-2*a^2*(A*b-B*a)*tan(d*x+c)/b^2/(a^2-b^2)/d/(a+b*sec(d*x+c))^(
1/2)+2/3*B*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^2/d
```

Rubi [A]
time = 0.46, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4113, 4167, 4090, 3917, 4089}

$$-\frac{2a^2(Ab-aB)\tan(c+dx)}{b^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2(-8a^3B+6a^2Ab+5ab^2B-3Ab^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^4d\sqrt{a+b}}-\frac{2(2a+b)(3Ab-B(4a+b))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^3d\sqrt{a+b}}+\frac{2B\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3b^2d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(6*a^2*A*b - 3*A*b^3 - 8*a^3*B + 5*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcS
in[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec
[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^4*Sqrt[a
 + b]*d) - (2*(2*a + b)*(3*A*b - (4*a + b)*B)*Cot[c + d*x]*EllipticF[ArcSin
[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
 + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a +
 b]*d) - (2*a^2*(A*b - a*B)*Tan[c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[a + b*Sec
[c + d*x]]) + (2*B*Sqrt[a + b*Sec[c + d*x]]*Tan[c + d*x])/(3*b^2*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4113

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-a^2)*(A*b - a*B)
*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x
] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])
^(m + 1)*Simp[a*b*(A*b - a*B)*(m + 1) - (A*b - a*B)*(a^2 + b^2*(m + 1))*Csc
[e + f*x] + b*B*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2a^2(Ab-aB)\tan(c+dx)}{b^2\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{1}{2}ab(Ab-aB)-\frac{1}{2}(\cdots\right.}{\sqrt{\cdots}}}{\cdots}$$

$$= -\frac{2a^2(Ab-aB)\tan(c+dx)}{b^2\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2B\sqrt{a+b\sec(c+dx)}\,\tan\cdots}{3b^2d}$$

$$= -\frac{2a^2(Ab-aB)\tan(c+dx)}{b^2\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2B\sqrt{a+b\sec(c+dx)}\,\tan\cdots}{3b^2d}$$

$$= -\frac{2(6a^2Ab-3Ab^3-8a^3B+5ab^2B)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+\cdots}}{\cdots}\right.\right.}{3b^4\cdots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3460 vs. $2(329) = 658$.
time = 24.40, size = 3460, normalized size = 10.52

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),
x]

[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(-6*a^2*A*b + 3*A*b^3 + 8*a^3*B
- 5*a*b^2*B)*Sin[c + d*x])/(3*b^3*(-a^2 + b^2)) + (2*(a^2*A*b*Sin[c + d*x]
- a^3*B*Sin[c + d*x]))/(b^2*(-a^2 + b^2)*(b + a*Cos[c + d*x])) + (2*B*Tan[c
 + d*x])/(3*b^2)))/(d*(a + b*Sec[c + d*x])^(3/2)) - (2*(b + a*Cos[c + d*x])
*((2*a^2*A)/(b*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) -
(A*b)/((-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*a*B)/
(3*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^3*B)/(3
*b^2*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a*A*Sqr
t[Sec[c + d*x]])/((-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) + (2*a^3*A*Sqrt[Se
c[c + d*x]])/(b^2*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) - (8*a^4*B*Sqrt[Se
c[c + d*x]])/(3*b^3*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) + (7*a^2*B*Sqrt[
Sec[c + d*x]])/(3*b*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) + (b*B*Sqrt[Sec[
c + d*x]])/(3*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) - (a*A*Cos[2*(c + d*x)
]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]) + (2*a^3*A*Co
s[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(b^2*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x
]]) - (8*a^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^3*(-a^2 + b^2)*Sqr
t[b + a*Cos[c + d*x]]) + (5*a^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b
```

```
*(-a^2 + b^2)*Sqrt[b + a*Cos[c + d*x]]))*Sec[c + d*x]^(3/2)*Sqrt[Cos[(c + d
*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*(-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2
*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b
)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
 - 2*b*(-2*a^2 - a*b + b^2)*(3*A*b + (-4*a + b)*B)*Sqrt[Cos[c + d*x]/(1 + C
os[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ellip
ticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-6*a^2*A*b + 3*A*b^3 + 8
*a^3*B - 5*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Ta
n[(c + d*x)/2]))/(3*b^3*(-a^2 + b^2)*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[
c + d*x])^(3/2)*(-1/3*(a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]
*(2*(a + b)*(-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*Sqrt[Cos[c + d*x]/
(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]
*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(-2*a^2 - a*b +
 b^2)*(3*A*b + (-4*a + b)*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
+ a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d
*x)/2]], (a - b)/(a + b)] + (-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*Co
s[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(b^3*
(-a^2 + b^2)*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) + (Sqrt[C
os[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-6*a^2*A*b + 3
*A*b^3 + 8*a^3*B - 5*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b
 + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c +
d*x)/2]], (a - b)/(a + b)] - 2*b*(-2*a^2 - a*b + b^2)*(3*A*b + (-4*a + b)*B
)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*
(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
 (-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(-a^2 + b^2)*Sqrt[b + a*C
os[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
 d*x]]*(((-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*Cos[c + d*x]*(b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-6*a^2*A*b + 3*A*b^3 + 8*a^3
*B - 5*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*
x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d
*x]/(1 + Cos[c + d*x])] - (b*(-2*a^2 - a*b + b^2)*(3*A*b + (-4*a + b)*B)*Sq
rt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[
(c + d*x)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d
*x])^2 - Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d
*x])] + ((a + b)*(-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x]
)*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))] - (b*(-2*a^2 - a*b + b^2)*(3*A*b + (-4*a + b)
*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b
 + a*Cos[c + d*x])*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))] - a*(-6*a^2*A*b + 3*A*b^3 +
```

```
8*a^3*B - 5*a*b^2*B)*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c +
d*x)/2] - (-6*a^2*A*b + 3*A*b^3 + 8*a^3*B - 5*a*b^2*B)*(b + a*Cos[c + d*x])
*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] + (-6*a^2*A*b + 3*A*b^3 +
 8*a^3*B - 5*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*
Tan[(c + d*x)/2]^2 - (b*(-2*a^2 - a*b + b^2)*(3...
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3328 vs. $2(301) = 602$.
time = 17.64, size = 3329, normalized size = 10.12

method	result	size
default	Expression too large to display	3329

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/3/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(B*a^2*b^2-B*b^4+B*cos(d*
x+c)^2*b^4-3*A*cos(d*x+c)*b^4-5*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^3+4*B*cos(d*x+c)*a*b^3+6*
A*cos(d*x+c)*a^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*b+6*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-3*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+
c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-6*A*sin(d*x+c)*
cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a
^2*b^2-3*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a*b^3-8*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b+5*B*sin(d*x+c)*cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+5*B*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a*b^3+8*B*cos(d*x+c)*a^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/
sin(d*x+c),((a-b)/(a+b))^(1/2))*b+2*B*cos(d*x+c)*a^2*b^2*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-5*B*sin(d*x+c)*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
```

```
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3
+3*A*cos(d*x+c)^3*a^2*b^2-4*B*cos(d*x+c)^3*a^3*b+B*cos(d*x+c)^3*a*b^3-3*A*c
os(d*x+c)^2*a*b^3+4*B*cos(d*x+c)^2*a^2*b^2+5*B*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^3+8*B*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d
*x+c)*a^3*b+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/
2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^2-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^4+3*A*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^4
-8*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x
+c)^2*sin(d*x+c)*a^4+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*b^4-6*A*cos(d*x+c)^3*a^3*b+3*A*cos(d*x+c
)^3*a*b^3-5*B*cos(d*x+c)^3*a^2*b^2+6*A*cos(d*x+c)^2*a^3*b-6*A*cos(d*x+c)^2*
a^2*b^2+8*B*cos(d*x+c)^2*a^3*b-5*B*cos(d*x+c)^2*a*b^3+3*A*cos(d*x+c)*a^2*b^
2-4*B*cos(d*x+c)*a^3*b+8*B*cos(d*x+c)^3*a^4+3*A*cos(d*x+c)^2*b^4-8*B*cos(d*
x+c)^2*a^4+6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*cos(d*x+c)^2*sin(d*x+c)*a^3*b+6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x
+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^2-3*A*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*
b^3-6*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(
d*x+c)^2*sin(d*x+c)*a^2*b^2-3*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a*b^3-8*B*(cos(d*x+c)/(1+cos(d*
x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b+5*B
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^
2*sin(d*x+c)*a^2*b^2+3*A*(cos(d*x+c)/(1+cos(d*x...
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^4 + A*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a)/(b^
2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\sec^{3}{(c + dx)}}{(a + b\sec{(c + dx)})^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x))**(3/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^3/(b*sec(d*x + c) + a)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\cos\left(c+d\,x\right)}^3\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(3/2)), x)
```

3.379 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=275

$$\frac{2(aAb-2a^2B+b^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\text{s}}{}}}{b^3\sqrt{a+b}\,d}$$

```
[Out] 2*(A*a*b-2*B*a^2+B*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(
1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/b^3/d/(a+b)^(1/2)+2*(A*b-(2*a+b)*B)*cot(d*x+c)*EllipticF((a+b
*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b)
)^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^2/d/(a+b)^(1/2)+2*a*(A*b-B*a)*tan
(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.29, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.121$, Rules used = {4094, 4090, 3917, 4089}

$$\frac{2(-2a^2B+aAb+b^2B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^3d\sqrt{a+b}}+\frac{2a(Ab-aB)\tan(c+dx)}{bd(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{2(Ab-B(2a+b))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(a*A*b - 2*a^2*B + b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(b^3*Sqrt[a + b]*d) + (2*(A*b -
(2*a + b)*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d) + (2*a*(A*b - a*B)*Tan[c + d
*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
```

```
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4094

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*(A*b - a*B)*Cot[
e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dis
t[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
Simp[b*(A*b - a*B)*(m + 1) - (a*A*b*(m + 2) - B*(a^2 + b^2*(m + 1)))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2a(Ab-aB)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{\sec(c+dx)\left(-\frac{1}{2}b(Ab-aB)-\frac{1}{2}(aAb- \cdots}{\sqrt{a+b\sec(c \cdots}}}{b\left(a^2-b^2 \cdots}$$

$$= \frac{2a(Ab-aB)\tan(c+dx)}{b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left(aAb-2a^2B+b^2B\right)\int \frac{\sec(c+ \cdots}{\sqrt{a+ \cdots}}}{b\left(a^2-b^2\right) \cdots}$$

$$= \frac{2\left(aAb-2a^2B+b^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}} \cdots\right.\right.}{b^3\sqrt{a+b}\, \cdots}$$

Mathematica [A]
time = 17.62, size = 467, normalized size = 1.70

$$\frac{(b+a\cos(c+dx))^2\sec^2(c+dx)\left(\frac{2\left(aAb-2a^2B+b^2B\right)\sin(c+dx)}{b^2\left(-a^2+b^2\right)}-\frac{2\left(aAb\sin(c+dx)-a^2B\sin(c+dx)\right)}{b\left(-a^2+b^2\right)(b+a\cos(c+dx))}\right)}{d(a+b\sec(c+dx))^{3/2}}+\frac{2(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\left(2(a+b)\left(-aAb+2a^2B-b^2B\right)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+2b(a+b)(Ab+(-2a+b)B)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+\left(-aAb+2a^2B-b^2B\right)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{b^2\left(-a^2+b^2\right)d\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}(a+b\sec(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),
x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*(a*A*b - 2*a^2*B + b^2*B)*Sin[c
+ d*x])/(b^2*(-a^2 + b^2)) - (2*(a*A*b*Sin[c + d*x] - a^2*B*Sin[c + d*x]))/
(b*(-a^2 + b^2)*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(3/2)) + (2
*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]]*(2*(a + b)*(-(a*A*b) + 2*a^2*B - b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c +
d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[Ar
cSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(A*b + (-2*a + b)*B)
*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(
1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] +
(-(a*A*b) + 2*a^2*B - b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x
)/2]^2*Tan[(c + d*x)/2]))/(b^2*(-a^2 + b^2)*d*Sqrt[Sec[(c + d*x)/2]^2]*(a +
 b*Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2274 vs. $2(255) = 510$.
time = 17.28, size = 2275, normalized size = 8.27

method	result	size
default	Expression too large to display	2275

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-B*b*a^2+B*b^3+B*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+B*c
os(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*sin(d*x+c)*a-A*cos(d*x+c)*a*b^2+2*B*cos(d*x+c)*a^2*b-A*cos(d*x+c)^2*a^2*b
-B*cos(d*x+c)^2*a*b^2+A*cos(d*x+c)*a^2*b-B*cos(d*x+c)*b^3-A*cos(d*x+c)*sin(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-B
*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+
b))^(1/2))*b^3+A*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*b+A*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)-2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(
```

```
d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)+B*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+2*B*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-2*B*cos(d*x+c)
*a^3-2*B*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2
))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*sin(d*x+c)*a^3+B*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((
a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*b^3-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-2*B*(cos(d*x+c)/(1+cos(d*x+c)))
^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*sin(d*x+c)+B*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-B*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)+2*B*cos(d*x+c)
^2*a^3+2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*a^2*b+A*cos(d*x+c)*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/
2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+A*cos(d*x+c)*b^2*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+
c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a-A*cos(d*x+c)
*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*
a*b^2-2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),(
(a-b)/(a+b))^(1/2))*a^2*b+B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2+A*cos(d*x+c)^2*a*b^2-B*cos(d*x+
c)^2*a^2*b+B*cos(d*x+c)*a*b^2)/sin(d*x+c)/(b+a*cos(d*x+c))/b^2/(a+b)/(a-b)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)/(b^
2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec^2(c + dx)}{(a + b\sec(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x))**(3/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+d\,x)}}{\cos\left(c + d\,x\right)^2\,\left(a + \frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(3/2)), x)
```

3.380 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=254

$$-\frac{2(Ab-aB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+\cdots}{a-b}}}{b^2\sqrt{a+b}\,d}$$

```
[Out] -2*(A*b-B*a)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)
/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/
2)/b^2/d/(a+b)^(1/2)+2*(A+B)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a
+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*
x+c))/(a-b))^(1/2)/b/d/(a+b)^(1/2)-2*(A*b-B*a)*tan(d*x+c)/(a^2-b^2)/d/(a+b*
sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.23, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4088, 4090, 3917, 4089}

$$-\frac{2(Ab-aB)\tan(c+dx)}{d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{2(Ab-aB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2d\sqrt{a+b}}+\frac{2(A+B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{bd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(A*b - a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt
[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(
1 + Sec[c + d*x]))/(a - b))])/(b^2*Sqrt[a + b]*d) + (2*(A + B)*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(b*Sqrt[a + b]*d) - (2*(A*b - a*B)*Tan[c + d*x])/((a^2 - b^2)*d*Sqrt[a +
b*Sec[c + d*x]])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
```

```
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2(Ab-aB)\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\sec(c+dx)\left(\frac{1}{2}(-aA+bB)-\frac{1}{2}(Ab-aB)s}{\sqrt{a+b\sec(c+dx)}}}{a^2-b^2}$$

$$= -\frac{2(Ab-aB)\tan(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{(A+B)\int \frac{\sec(c+dx)}{\sqrt{a+b\sec(c+dx)}}}{a+b}$$

$$= -\frac{2(Ab-aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{}}{b^2\sqrt{a+b}\,d}$$

Mathematica [A]
time = 14.94, size = 468, normalized size = 1.84

$$\frac{(b+a\cos(c+dx))^2\sec(c+dx)(A+B\sec(c+dx))\left(-\frac{2(Ab-aB)\sin(c+dx)}{b(-a^2+b^2)}+\frac{2(Ab\sin(c+dx)-aB\sin(c+dx))}{(-a^2+b^2)(b+a\cos(c+dx))}\right)}{d(B+A\cos(c+dx))(a+b\sec(c+dx))^{3/2}} - \frac{2(b+a\cos(c+dx))\sqrt{\sec(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\,(A+B\sec(c+dx))\left(2(a+b)(-Ab+aB)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)+2b(a+b)(A-B)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{a-b}{a+b}\right)-(Ab-aB)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{(-a^2b+b^3)\,d(B+A\cos(c+dx))\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\,(a+b\sec(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]*(A + B*Sec[c + d*x])*((-2*(A*b - a*B)*
Sin[c + d*x])/(b*(-a^2 + b^2)) + (2*(A*b*Sin[c + d*x] - a*B*Sin[c + d*x]))/
((-a^2 + b^2)*(b + a*Cos[c + d*x]))))/(d*(B + A*Cos[c + d*x])*(a + b*Sec[c
+ d*x])^(3/2)) - (2*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sqrt[Cos[(c + d
*x)/2]^2*Sec[c + d*x]]*(A + B*Sec[c + d*x])*(2*(a + b)*(-(A*b) + a*B)*Sqrt[
Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Co
s[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a
 + b)*(A - B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x
])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b
)/(a + b)] - (A*b - a*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]
^2*Tan[(c + d*x)/2]))/((-(a^2*b) + b^3)*d*(B + A*Cos[c + d*x])*Sqrt[Sec[(c
+ d*x)/2]^2]*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1633 vs. $2(234) = 468$.

time = 9.28, size = 1634, normalized size = 6.43

method	result	size
default	Expression too large to display	1634

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(A*cos(d*x+c)*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b+A*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+
c)*b^2-A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*sin(d*x+c)*a*b-A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2-B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-B*cos(d*x+c)*(cos(d*
x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2+B*co
s(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin
(d*x+c)*a^2+B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(
a+b))^(1/2))*sin(d*x+c)*a*b+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
```

```
-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+A*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a
-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-A*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-B*(cos(d*x+c)/(1+cos(d*x+c)
))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)-B*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)+B*EllipticE((-1+c
os(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2+B*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)+A*cos(d
*x+c)^2*a*b-A*cos(d*x+c)^2*b^2-B*cos(d*x+c)^2*a^2+B*cos(d*x+c)^2*a*b-A*cos(
d*x+c)*a*b+A*cos(d*x+c)*b^2+B*cos(d*x+c)*a^2-B*cos(d*x+c)*a*b)/(b+a*cos(d*x
+c))/sin(d*x+c)/b/(a+b)/(a-b)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/(b^2*
sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec(c + dx)}{(a + b\sec(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + b/cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + b/cos(c + d*x))^(3/2)), x)
```

3.381 $\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=376

$$\frac{2(Ab-aB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+\sec(c+dx}{a-b}}}{ab\sqrt{a+b}\,d}$$

```
[Out] 2*(A*b-B*a)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/
(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2
)/a/b/d/(a+b)^(1/2)-2*(A*b-B*a)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)
/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec
(d*x+c))/(a-b))^(1/2)/a/b/d/(a+b)^(1/2)-2*A*cot(d*x+c)*EllipticPi((a+b*sec(
d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-se
c(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d+2*b*(A*b-B*a)*
tan(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.28, antiderivative size = 376, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.240$, Rules used = {4008, 4143, 4006, 3869, 3917, 4089}

$$-\frac{2A\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d}+\frac{2b(Ab-aB)\tan(c+dx)}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2(Ab-aB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{abd\sqrt{a+b}}+\frac{2(Ab-aB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{abd\sqrt{a+b}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(A*b - a*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(a*b*Sqrt[a + b]*d) - (2*(A*b - a*B)*Cot[c + d
*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)
]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b
))])/(a*b*Sqrt[a + b]*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*EllipticPi[(a + b)
/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*
(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*
d) + (2*b*(A*b - a*B)*Tan[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x
]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4008

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b(Ab-aB)\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}A(a^2-b^2)+\frac{1}{2}a(Ab-aB)\sec(c+dx)+\frac{1}{2}b(Ab-aB)}{\sqrt{a+b\sec(c+dx)}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\tan(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{-\frac{1}{2}A(a^2-b^2)+\left(\frac{1}{2}a(Ab-aB)-\frac{1}{2}b(Ab-aB)\right)\sec(c+a}{\sqrt{a+b\sec(c+dx)}}}{a\left(a^2-b^2\right)}$$

$$= \frac{2(Ab-aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c}{a+b}}}{ab\sqrt{a+b}\,d}$$

$$= \frac{2(Ab-aB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c}{a+b}}}{ab\sqrt{a+b}\,d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.29, size = 1491, normalized size = 3.97

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*(-(A*b) + a*B
)*Sin[c + d*x])/(a*(a^2 - b^2)) - (2*(-(A*b^2*Sin[c + d*x]) + a*b*B*Sin[c +
 d*x]))/(a*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(B + A*Cos[c + d*x])*(a +
 b*Sec[c + d*x])^(3/2)) + (2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[c + d*x]]*
(A + B*Sec[c + d*x])*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2
]^2)/(1 + Tan[(c + d*x)/2]^2)]*(a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/
2] + A*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - a^2*Sqrt[(-a + b)/(a +
 b)]*B*Tan[(c + d*x)/2] - a*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] - 2
*a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 2*a^2*Sqrt[(-a + b)/(a +
 b)]*B*Tan[(c + d*x)/2]^3 + a*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5
 - A*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - a^2*Sqrt[(-a + b)/(a +
 b)]*B*Tan[(c + d*x)/2]^5 + a*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5
 - (2*I)*a^2*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a +
b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*A*b^2
*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d
*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[
(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (2*I)*a^2*A*EllipticPi[-(
(a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a +
```

```
b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a
*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*A*b^2*Elliptic
Pi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]],
(a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a - b)*(-(A*
b) + a*B)*EllipticE[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a
+ b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a
 + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a - b)*(2
*A*b + a*(A - B))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/
2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)
*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(a*S
qrt[(-a + b)/(a + b)]*(a^2 - b^2)*d*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x
])^(3/2)*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/(1 - Tan[(
c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2008 vs. $2(347) = 694$.

time = 9.45, size = 2009, normalized size = 5.34

method	result	size
default	Expression too large to display	2009

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERBOSE)
```

```
[Out] 1/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(B*cos(d*x+c)*a^2-A*cos(d*x
+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+
c)*a*b+A*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*sin(d*x+c)*a*b+B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+
c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+
c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b-A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)-A*cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^2
+B*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*sin(d*x+c)*a^2-2*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(
a+b))^(1/2))*a^2*sin(d*x+c)-A*cos(d*x+c)*a*b+2*A*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*
```

```
x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^2*sin(d*x+c)+A*(cos(d*x+c)/(1+co
s(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*sin(d*x+c)+B*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2-A*Ellipti
cE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a*b+A*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d*x+c)
+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*sin(d
*x+c)-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b*
sin(d*x+c)-2*A*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2)
)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*sin(d*x+c)*a^2+2*A*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),
-1,((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*b^2+A*EllipticF((-1+cos(
d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c
)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*a^2-A*co
s(d*x+c)^2*b^2-B*cos(d*x+c)^2*a^2+A*cos(d*x+c)*b^2+A*cos(d*x+c)^2*a*b+B*cos
(d*x+c)^2*a*b-B*cos(d*x+c)*a*b-B*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*
x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2)/(b+a*cos(d*x+c))/sin(d
*x+c)/a/(a+b)/(a-b)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="maxima")
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/(b^2*sec(d*x + c)^2
+ 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec\left(c + dx\right)}{\left(a + b\sec\left(c + dx\right)\right)^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+d\,x)}}{{\left(a + \frac{b}{\cos(c+d\,x)}\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^(3/2), x)
```

3.382 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=427

$$\frac{(a^2A-3Ab^2+2abB)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1+s}{}}}{a^2b\sqrt{a+b}\,d}$$

```
[Out] (A*a^2-3*A*b^2+2*B*a*b)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(
1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/a^2/b/d/(a+b)^(1/2)+(3*A*b+a*(A-2*B))*cot(d*x+c)*EllipticF((a
+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+
b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^2/d/(a+b)^(1/2)+(3*A*b-2*B*a)*c
ot(d*x+c)*EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b
))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-
b))^(1/2)/a^3/d+A*sin(d*x+c)/a/d/(a+b*sec(d*x+c))^(1/2)+b*(A*a^2-3*A*b^2+2*
B*a*b)*tan(d*x+c)/a^2/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4119, 4146, 4143, 4006, 3869, 3917, 4089}

$$\frac{\sqrt{a+b}\,(3Ab-2aB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^3d}+\frac{(a^2A+2abB-3Ab^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2bd\sqrt{a+b}}+\frac{(a(A-2B)+3Ab)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^2d\sqrt{a+b}}+\frac{b(a^2A+2abB-3Ab^2)\tan(c+dx)}{a^2d\,(a^2-b^2)\,\sqrt{a+b\sec(c+dx)}}+\frac{A\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((a^2*A - 3*A*b^2 + 2*a*b*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^2*b*Sqrt[a + b]*d) + ((3*A*b
+ a*(A - 2*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[
a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
 + Sec[c + d*x]))/(a - b))])/(a^2*Sqrt[a + b]*d) + (Sqrt[a + b]*(3*A*b - 2*
a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqr
t[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*
(1 + Sec[c + d*x]))/(a - b))])/(a^3*d) + (A*Sin[c + d*x])/(a*d*Sqrt[a + b*S
ec[c + d*x]]) + (b*(a^2*A - 3*A*b^2 + 2*a*b*B)*Tan[c + d*x])/(a^2*(a^2 - b^
2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
```

```
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4119

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*n)), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4146

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 + a^2*C)*Cot[e + f*x]*((a + b*Csc[
e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{A\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{\frac{1}{2}(3Ab-2aB)-\frac{1}{2}Ab\sec^2(c+dx)}{(a+b\sec(c+dx))^{3/2}}\,dx}{a}$$

$$= \frac{A\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} + \frac{b(a^2A-3Ab^2+2abB)\tan(c+dx)}{a^2(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2\int}{}$$

$$= \frac{A\sin(c+dx)}{ad\sqrt{a+b\sec(c+dx)}} + \frac{b(a^2A-3Ab^2+2abB)\tan(c+dx)}{a^2(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2\int}{}$$

$$= \frac{(a^2A-3Ab^2+2abB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right)}{a^2b\sqrt{a+b}\,d}$$

$$= \frac{(a^2A-3Ab^2+2abB)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)\right)}{a^2b\sqrt{a+b}\,d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1597 vs. $2(427) = 854$.
time = 19.23, size = 1597, normalized size = 3.74

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((-2*b*(A*b - a*B)*Sin[c + d*x])/(a^
2*(-a^2 + b^2)) + (2*(-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x]))/(a^2*(
a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^(3/2)) - ((b + a
*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]
*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d
```

```
*x)/2]^2)]*(a^3*A*Tan[(c + d*x)/2] + a^2*A*b*Tan[(c + d*x)/2] - 3*a*A*b^2*T
an[(c + d*x)/2] - 3*A*b^3*Tan[(c + d*x)/2] + 2*a^2*b*B*Tan[(c + d*x)/2] + 2
*a*b^2*B*Tan[(c + d*x)/2] - 2*a^3*A*Tan[(c + d*x)/2]^3 + 6*a*A*b^2*Tan[(c +
 d*x)/2]^3 - 4*a^2*b*B*Tan[(c + d*x)/2]^3 + a^3*A*Tan[(c + d*x)/2]^5 - a^2*
A*b*Tan[(c + d*x)/2]^5 - 3*a*A*b^2*Tan[(c + d*x)/2]^5 + 3*A*b^3*Tan[(c + d*
x)/2]^5 + 2*a^2*b*B*Tan[(c + d*x)/2]^5 - 2*a*b^2*B*Tan[(c + d*x)/2]^5 - 6*a
^2*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - T
an[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^
2)/(a + b)] + 6*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Ta
n[(c + d*x)/2]^2)/(a + b)] + 4*a^3*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]
], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d
*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 4*a*b^2*B*EllipticPi[-1, ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a +
 b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 6*a^2*A*b*Elli
pticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sq
rt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
d*x)/2]^2)/(a + b)] + 6*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b -
a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 4*a^3*B*EllipticPi[
-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)] - 4*a*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(
a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan
[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(a^2*A - 3*A*b^2
 + 2*a*b*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - T
an[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2
]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*(a + b)*(-(A*b) + a*B)*EllipticF
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1
 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)
/2]^2)/(a + b)]))/(a^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)*Sqrt[1 + Ta
n[(c + d*x)/2]^2]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2873 vs. $2(396) = 792$.
time = 8.94, size = 2874, normalized size = 6.73

method	result	size
default	Expression too large to display	2874

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] 1/2/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(2*A*cos(d*x+c)*a*b^2-2*B
*cos(d*x+c)*a^2*b-A*cos(d*x+c)^2*a^2*b-2*B*cos(d*x+c)^2*a*b^2+A*cos(d*x+c)*
a^2*b-A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*
sin(d*x+c)-A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c
),((a-b)/(a+b))^(1/2))*a^3+3*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x
+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3-6*A*cos(d*x+c)*sin(d*x+c)*(cos
(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3-4*B*cos(
d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b
))^(1/2))*a^3+2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^3-A*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+3*A*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^
(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-
1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+6*A*a^2*(cos(d*x+c)/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*Ell
ipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b-2*A*a^2*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*s
in(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-2*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2*sin(d*x
+c)-2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*
b*sin(d*x+c)-2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1
/2))*a*b^2*sin(d*x+c)+4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a
-b)/(a+b))^(1/2))*a*b^2*sin(d*x+c)+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d
*x+c),((a-b)/(a+b))^(1/2))*a^2*b*sin(d*x+c)-3*A*cos(d*x+c)*b^3+3*A*(cos(d*x
+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b^3*sin(d*x+c)-6*A*(
cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*b^3*sin(d
*x+c)-4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2)
)*a^3*sin(d*x+c)+2*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*a^3*sin(d*x+c)-A*cos(d*x+c)^3*a^3+A*cos(d*x+c)^2*a^3+3*A*cos(d*x+c
)^2*b^3+2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
```

```
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a^2*b-A*cos(d*x+c)*a^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1
/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+
cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+3*A*cos(d*x+c)*b^2*(cos(d*x+c
)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d
*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a+6*A*cos(d
*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b)
)^(1/2))*a^2*b-2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-2*A*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1
+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^2-2*B*cos(d*x+c)*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b-
2*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/
(a+b))^(1/2))*a*b^2+4*B*cos(d*x+c)*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c
))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*a*b^2+A*cos(d*x+c)^3*a*b^2-3*A*cos(d*
x+c)^2*a*b^2+2*B*cos(d*x+c)^2*a^2*b+2*B*cos(d*x+c)*a*b^2)/(b+a*cos(d*x+c))/
sin(d*x+c)/a^2/(a+b)/(a-b)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(b*sec(d*x + c)
 + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\cos(c + dx)}{(a + b\sec(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)/(a + b*sec(c + d*x))**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/(b*sec(d*x + c) + a)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos(c + d\,x)\,\left(A + \frac{B}{\cos(c+d\,x)}\right)}{{\left(a + \frac{b}{\cos(c+d\,x)}\right)}^{3/2}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2), x)
```

3.383 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=531

$$-\frac{(7a^2Ab-15Ab^3-4a^3B+12ab^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+}{a+b}}}{4a^3b\sqrt{a+b}\,d}$$

```
[Out] -1/4*(7*A*a^2*b-15*A*b^3-4*B*a^3+12*B*a*b^2)*cot(d*x+c)*EllipticE((a+b*sec(
d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/
2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/b/d/(a+b)^(1/2)-1/4*(15*A*b^2+a*b*(5
*A-12*B)-2*a^2*(A+2*B))*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(
1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))
/(a-b))^(1/2)/a^3/d/(a+b)^(1/2)-1/4*(4*A*a^2+15*A*b^2-12*B*a*b)*cot(d*x+c)*
EllipticPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*
(a+b)^(1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/
a^4/d-1/4*(5*A*b-4*B*a)*sin(d*x+c)/a^2/d/(a+b*sec(d*x+c))^(1/2)+1/2*A*cos(d
*x+c)*sin(d*x+c)/a/d/(a+b*sec(d*x+c))^(1/2)-1/4*b*(7*A*a^2*b-15*A*b^3-4*B*a
^3+12*B*a*b^2)*tan(d*x+c)/a^3/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.72, antiderivative size = 531, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4119, 4189, 4145, 4143, 4006, 3869, 3917, 4089}

$$-\frac{(5Ab-4aB)\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}}-\frac{\sqrt{a+b}\,(4a^2A-12abB+15Ab^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^4d}-\frac{(-2a^2(A+2B)+ab(5A-12B)+15Ab^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^3d\sqrt{a+b}}-\frac{(-4a^3B+7a^2Ab+12ab^2B-15Ab^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^3bd\sqrt{a+b}}-\frac{b(-4a^3B+7a^2Ab+12ab^2B-15Ab^3)\tan(c+dx)}{4a^3d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{A\sin(c+dx)\cos(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] -1/4*((7*a^2*A*b - 15*A*b^3 - 4*a^3*B + 12*a*b^2*B)*Cot[c + d*x]*EllipticE[
ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 -
 Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^3*b*Sq
rt[a + b]*d) - ((15*A*b^2 + a*b*(5*A - 12*B) - 2*a^2*(A + 2*B))*Cot[c + d*x
]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*
Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))
])/(4*a^3*Sqrt[a + b]*d) - (Sqrt[a + b]*(4*a^2*A + 15*A*b^2 - 12*a*b*B)*Cot
[c + d*x]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]
], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec
[c + d*x]))/(a - b))])/(4*a^4*d) - ((5*A*b - 4*a*B)*Sin[c + d*x])/(4*a^2*d*
Sqrt[a + b*Sec[c + d*x]]) + (A*Cos[c + d*x]*Sin[c + d*x])/(2*a*d*Sqrt[a + b
*Sec[c + d*x]]) - (b*(7*a^2*A*b - 15*A*b^3 - 4*a^3*B + 12*a*b^2*B)*Tan[c +
d*x])/(4*a^3*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4119

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*n)), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
```

```
B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{\cos(c+dx)\left(\frac{1}{2}(5Ab-4aB)-aA\sec(c+dx)-\frac{3}{2}Ab}{(a+b\sec(c+dx))^{3/2}}}{2a}$$

$$= -\frac{(5Ab-4aB)\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} + \frac{\int \frac{1}{4}(4a}{}$$

$$= -\frac{(5Ab-4aB)\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \frac{b(7a^2A}{}$$

$$= -\frac{(5Ab-4aB)\sin(c+dx)}{4a^2d\sqrt{a+b\sec(c+dx)}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad\sqrt{a+b\sec(c+dx)}} - \frac{b(7a^2A}{}$$

$$= -\frac{(7a^2Ab-15Ab^3-4a^3B+12ab^2B)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a}}{}\right.\right.}{4a^3}$$

$$= -\frac{(7a^2Ab-15Ab^3-4a^3B+12ab^2B)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a}}{}\right.\right.}{4a^3}$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.63, size = 2667, normalized size = 5.02

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),
x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*((2*b^2*(A*b - a*B)*Sin[c + d*x])/(a
^3*(-a^2 + b^2)) + (2*(A*b^4*Sin[c + d*x] - a*b^3*B*Sin[c + d*x]))/(a^3*(a^
2 - b^2)*(b + a*Cos[c + d*x])) + (A*Sin[2*(c + d*x)])/(4*a^2)))/(d*(a + b*S
ec[c + d*x])^(3/2)) + ((b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^
2)]*(-7*a^3*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 7*a^2*A*b^2*Sqrt[
(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 15*a*A*b^3*Sqrt[(-a + b)/(a + b)]*Tan[
(c + d*x)/2] + 15*A*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 4*a^4*Sqr
t[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + 4*a^3*b*Sqrt[(-a + b)/(a + b)]*B*T
an[(c + d*x)/2] - 12*a^2*b^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] - 12
*a*b^3*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] + 14*a^3*A*b*Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]^3 - 30*a*A*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d
*x)/2]^3 - 8*a^4*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^3 + 24*a^2*b^2*S
qrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^3 - 7*a^3*A*b*Sqrt[(-a + b)/(a + b
)]*Tan[(c + d*x)/2]^5 + 7*a^2*A*b^2*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]
^5 + 15*a*A*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 15*A*b^4*Sqrt[(
-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 + 4*a^4*Sqrt[(-a + b)/(a + b)]*B*Tan[(c
 + d*x)/2]^5 - 4*a^3*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 - 12*a^2
*b^2*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 + 12*a*b^3*Sqrt[(-a + b)/(
a + b)]*B*Tan[(c + d*x)/2]^5 - (8*I)*a^4*A*EllipticPi[-((a + b)/(a - b)), I
*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1
- Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)] - (22*I)*a^2*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[
Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a
+ b)] + (30*I)*A*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)
/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*
Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (24*I
)*a^3*b*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*T
an[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b
 - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (24*I)*a*b^3*B*E
llipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x
)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
 + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (8*I)*a^4*A*EllipticPi[-((a
 + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)
/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*T
```

```
an[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (22*I)*a^2*A*b^2*Ellip
ticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]
], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a
 + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (30*I)*A*b^4
*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d
*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*S
qrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (24*I)
*a^3*b*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)
/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
- (24*I)*a*b^3*B*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a
+ b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(
c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(
a + b)] - I*(a - b)*(-7*a^2*A*b + 15*A*b^3 + 4*a^3*B - 12*a*b^2*B)*Elliptic
E[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt
[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (2*I)*(a - b)*(2*a^3*A + 15*A*
b^3 + a^2*b*(A - 8*B) + 2*a*b^2*(5*A - 6*B))*EllipticF[I*ArcSinh[Sqrt[(-a +
 b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^
2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c +
 d*x)/2]^2)/(a + b)]))/(4*a^3*Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)*d*(a + b*S
ec[c + d*x])^(3/2)*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^2)/
(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*
x)/2]^2)))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3979 vs. $2(486) = 972$.
time = 9.36, size = 3980, normalized size = 7.50

method	result	size
default	Expression too large to display	3980

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/8/d*4^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-15*A*cos(d*x+c)*b^4-2*
A*cos(d*x+c)^4*a^2*b^2+2*A*cos(d*x+c)^4*a^4-2*A*cos(d*x+c)^2*a^4+12*B*cos(d
*x+c)*a*b^3+8*A*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/
(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*a^4-7*A*cos(d*x+c)*a^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1+cos
(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-7*A*sin(d*x+c)*cos(d*x+c)*(cos(d
*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+15*A*sin(d
```

*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-4*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-10*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b-12*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-12*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+8*B*cos(d*x+c)*a^3*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b+8*B*cos(d*x+c)*a^2*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))-15*A*cos(d*x+c)^2*a*b^3+12*B*cos(d*x+c)^2*a^2*b^2-5*A*cos(d*x+c)^3*a^3*b+5*A*cos(d*x+c)^3*a*b^3-4*B*cos(d*x+c)^3*a^2*b^2+7*A*cos(d*x+c)^2*a^3*b-5*A*cos(d*x+c)^2*a^2*b^2+4*B*cos(d*x+c)^2*a^3*b-12*B*cos(d*x+c)^2*a*b^3-2*A*cos(d*x+c)*a^3*b+7*A*cos(d*x+c)*a^2*b^2+10*A*cos(d*x+c)*a*b^3-4*B*cos(d*x+c)*a^3*b-8*B*cos(d*x+c)*a^2*b^2+4*B*cos(d*x+c)^3*a^4+15*A*cos(d*x+c)^2*b^4-4*B*cos(d*x+c)^2*a^4-24*B*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b+24*B*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^3+22*A*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2*b^2+2*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^3*b-30*A*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^4+15*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^4-4*A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4+4*B*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4+8*A*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4-30*A*cos(d*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)

```
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^4-4*A*cos(d*x+c)*sin(d*x+c
)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^4+22*A*s
in(d*x+c)*EllipticPi((-1+cos(d*x+c))/sin(d*x+c),-1,((a-b)/(a+b))^(1/2))*(co
s(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*a^2*b^2-7*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*a^3*b-7*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*a^2*b^2+15*A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))/sin
(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b^3+2*A*sin(d*x+c)*EllipticF((-1+cos(d*
x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*(cos(d*x+...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*sqrt(b*sec(d*x
+ c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\cos^2(c + dx)}{(a + b\sec(c + dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/(a + b*sec(c + d*x))**(3/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/(b*sec(d*x + c) + a)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^2\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2), x)
```

3.384 $\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=630

$$\frac{(16a^4A+41a^2Ab^2-105Ab^4-42a^3bB+90ab^3B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b}{}}}{24a^4b\sqrt{a+b}\,d}$$

```
[Out] 1/24*(16*A*a^4+41*A*a^2*b^2-105*A*b^4-42*B*a^3*b+90*B*a*b^3)*cot(d*x+c)*Ell
ipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*
x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/b/d/(a+b)^(1/2)+1/24
*(105*A*b^3+5*a*b^2*(7*A-18*B)+4*a^3*(4*A+3*B)-6*a^2*b*(A+5*B))*cot(d*x+c)*
EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec
(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d/(a+b)^(1/2)+1/8
*(12*A*a^2*b+35*A*b^3-8*B*a^3-30*B*a*b^2)*cot(d*x+c)*EllipticPi((a+b*sec(d*
x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(
d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^5/d+1/24*(16*A*a^2+3
5*A*b^2-30*B*a*b)*sin(d*x+c)/a^3/d/(a+b*sec(d*x+c))^(1/2)-1/12*(7*A*b-6*B*a
)*cos(d*x+c)*sin(d*x+c)/a^2/d/(a+b*sec(d*x+c))^(1/2)+1/3*A*cos(d*x+c)^2*sin
(d*x+c)/a/d/(a+b*sec(d*x+c))^(1/2)+1/24*b*(16*A*a^4+41*A*a^2*b^2-105*A*b^4-
42*B*a^3*b+90*B*a*b^3)*tan(d*x+c)/a^4/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 1.08, antiderivative size = 630, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4119, 4189, 4145, 4143, 4006, 3869, 3917, 4089}

$$-\frac{(7Ab-6aB)\sin(c+dx)\cos(c+dx)}{12a^2d\sqrt{a+b\sec(c+dx)}}+\frac{(16a^2A-30abB+35Ab^2)\sin(c+dx)}{24a^3d\sqrt{a+b\sec(c+dx)}}+\frac{\sqrt{a+b}\,(-8a^3B+12a^2Ab-30ab^2B+35Ab^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{8a^5d}+\frac{(4a^3(4A+3B)-6a^2b(A+5B)+5ab^2(7A-18B)+105Ab^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{24a^4d\sqrt{a+b}}+\frac{(16a^4A-42a^3bB+41a^2Ab^2+90ab^3B-105Ab^4)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{24a^4bd\sqrt{a+b}}+\frac{b(16a^4A-42a^3bB+41a^2Ab^2+90ab^3B-105Ab^4)\tan(c+dx)}{24a^4d(a^2-b^2)\sqrt{a+b\sec(c+dx)}}+\frac{A\sin(c+dx)\cos^2(c+dx)}{3ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((16*a^4*A + 41*a^2*A*b^2 - 105*A*b^4 - 42*a^3*b*B + 90*a*b^3*B)*Cot[c + d*
x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(24*a^4*b*Sqrt[a + b]*d) + ((105*A*b^3 + 5*a*b^2*(7*A - 18*B) + 4*a^3*(
4*A + 3*B) - 6*a^2*b*(A + 5*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Se
c[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(24*a^4*Sqrt[a + b]*d) + (Sqr
t[a + b]*(12*a^2*A*b + 35*A*b^3 - 8*a^3*B - 30*a*b^2*B)*Cot[c + d*x]*Ellipt
icPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a -
 b))])/(8*a^5*d) + ((16*a^2*A + 35*A*b^2 - 30*a*b*B)*Sin[c + d*x])/(24*a^3*
d*Sqrt[a + b*Sec[c + d*x]]) - ((7*A*b - 6*a*B)*Cos[c + d*x]*Sin[c + d*x])/(
```

```
12*a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (A*Cos[c + d*x]^2*Sin[c + d*x])/(3*a*d
*Sqrt[a + b*Sec[c + d*x]]) + (b*(16*a^4*A + 41*a^2*A*b^2 - 105*A*b^4 - 42*a
^3*b*B + 90*a*b^3*B)*Tan[c + d*x])/(24*a^4*(a^2 - b^2)*d*Sqrt[a + b*Sec[c +
 d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4119

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*n)), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{A\cos^2(c+dx)\sin(c+dx)}{3ad\sqrt{a+b\sec(c+dx)}} - \frac{\int \frac{\cos^2(c+dx)\left(\frac{1}{2}(7Ab-6aB)-2aA\sec(c+dx)-\frac{5}{2}A\right.}{(a+b\sec(c+dx))^{3/2}}}{3a}$$

$$= -\frac{(7Ab-6aB)\cos(c+dx)\sin(c+dx)}{12a^2d\sqrt{a+b\sec(c+dx)}} + \frac{A\cos^2(c+dx)\sin(c+dx}{3ad\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{(16a^2A+35Ab^2-30abB)\sin(c+dx)}{24a^3d\sqrt{a+b\sec(c+dx)}} - \frac{(7Ab-6aB)\cos(c+dx)}{12a^2d\sqrt{a+b\sec(c}}$$

$$= \frac{(16a^2A+35Ab^2-30abB)\sin(c+dx)}{24a^3d\sqrt{a+b\sec(c+dx)}} - \frac{(7Ab-6aB)\cos(c+dx)}{12a^2d\sqrt{a+b\sec(c}}$$

$$= \frac{(16a^2A+35Ab^2-30abB)\sin(c+dx)}{24a^3d\sqrt{a+b\sec(c+dx)}} - \frac{(7Ab-6aB)\cos(c+dx)}{12a^2d\sqrt{a+b\sec(c}}$$

$$= \frac{(16a^4A+41a^2Ab^2-105Ab^4-42a^3bB+90ab^3B)\cot(c+dx)E\Big(\mathrm{s}}{}$$

$$= \frac{(16a^4A+41a^2Ab^2-105Ab^4-42a^3bB+90ab^3B)\cot(c+dx)E\Big(\mathrm{s}}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2319 vs. $2(630) = 1260$.
time = 21.88, size = 2319, normalized size = 3.68

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),
x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^2*(-1/12*((a^4*A - a^2*A*b^2 + 24*A*b^
4 - 24*a*b^3*B)*Sin[c + d*x])/(a^4*(-a^2 + b^2)) - (2*(A*b^5*Sin[c + d*x] -
 a*b^4*B*Sin[c + d*x]))/(a^4*(a^2 - b^2)*(b + a*Cos[c + d*x])) + ((-11*A*b
+ 6*a*B)*Sin[2*(c + d*x)])/(24*a^3) + (A*Sin[3*(c + d*x)])/(12*a^2)))/(d*(a
 + b*Sec[c + d*x])^(3/2)) - ((b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)*
Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*
Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(16*a^5*A*Tan[(c + d*x)/2] +
16*a^4*A*b*Tan[(c + d*x)/2] + 41*a^3*A*b^2*Tan[(c + d*x)/2] + 41*a^2*A*b^3*
```

Tan[(c + d*x)/2] - 105*a*A*b^4*Tan[(c + d*x)/2] - 105*A*b^5*Tan[(c + d*x)/2
] - 42*a^4*b*B*Tan[(c + d*x)/2] - 42*a^3*b^2*B*Tan[(c + d*x)/2] + 90*a^2*b^
3*B*Tan[(c + d*x)/2] + 90*a*b^4*B*Tan[(c + d*x)/2] - 32*a^5*A*Tan[(c + d*x)
/2]^3 - 82*a^3*A*b^2*Tan[(c + d*x)/2]^3 + 210*a*A*b^4*Tan[(c + d*x)/2]^3 +
84*a^4*b*B*Tan[(c + d*x)/2]^3 - 180*a^2*b^3*B*Tan[(c + d*x)/2]^3 + 16*a^5*A
*Tan[(c + d*x)/2]^5 - 16*a^4*A*b*Tan[(c + d*x)/2]^5 + 41*a^3*A*b^2*Tan[(c +
 d*x)/2]^5 - 41*a^2*A*b^3*Tan[(c + d*x)/2]^5 - 105*a*A*b^4*Tan[(c + d*x)/2]
^5 + 105*A*b^5*Tan[(c + d*x)/2]^5 - 42*a^4*b*B*Tan[(c + d*x)/2]^5 + 42*a^3*
b^2*B*Tan[(c + d*x)/2]^5 + 90*a^2*b^3*B*Tan[(c + d*x)/2]^5 - 90*a*b^4*B*Tan
[(c + d*x)/2]^5 - 72*a^4*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 138*a^2*A*b^3*EllipticPi[-1, ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b -
 a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 210*A*b^5*Elliptic
Pi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]
^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 4
8*a^5*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)] + 132*a^3*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2
 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 180*a*b^4*B*EllipticPi[-1, ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a
*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 72*a^4*A*b*EllipticP
i[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1
- Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/
2]^2)/(a + b)] - 138*a^2*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a
- b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b -
 a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 210*A*b^5*Elliptic
Pi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)
/2]^2)/(a + b)] + 48*a^5*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)
/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*T
an[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 132*a^3*b^2*B*Elliptic
Pi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1
 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)
/2]^2)/(a + b)] - 180*a*b^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b -
a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(16*a^4*A +
 41*a^2*A*b^2 - 105*A*b^4 - 42*a^3*b*B + 90*a*b^3*B)*EllipticE[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d
*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b
)] - 2*a*(a + b)*(-35*A*b^3 + 12*a^3*B - 2*a^2*b*(5*A + 9*B) + 3*a*b^2*(7*A
 + 10*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan
[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^
2 + b*Tan[(c + d*x)/2]^2)/(a + b)]))/(24*a^4*(a^2 - b^2)*d*(a + b*Sec[c + d

```
*x])^(3/2)*Sqrt[1 + Tan[(c + d*x)/2]^2]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1
 + Tan[(c + d*x)/2]^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5085 vs. $2(581) = 1162$.
time = 9.31, size = 5086, normalized size = 8.07

method	result	size
default	Expression too large to display	5086

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^3/(b*sec(d*x + c) + a)^(3/2), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^3*sec(d*x + c) + A*cos(d*x + c)^3)*sqrt(b*sec(d*x
+ c) + a)/(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\cos^3{(c + dx)}}{(a + b\sec{(c + dx)})^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**3/(a + b*sec(c + d*x))**(3/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^3/(b*sec(d*x + c) + a)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^3\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^3*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2), x)
```

3.385 $\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=510

$$-\frac{2(8a^4Ab-15a^2Ab^3+3Ab^5-16a^5B+28a^3b^2B-8ab^4B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\ldots}{3(a-b)b^5(a+b)^{3/2}d}$$

```
[Out] -2/3*(8*A*a^4*b-15*A*a^2*b^3+3*A*b^5-16*B*a^5+28*B*a^3*b^2-8*B*a*b^4)*cot(d
*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*
(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b^5/(a+b)
^(3/2)/d+2/3*(9*a*b^3*(A-B)+b^4*(3*A-B)+16*a^4*B-2*a^2*b^2*(3*A+8*B)-a^3*(8
*A*b-12*B*b))*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b
)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1
/2)/b^4/(a^2-b^2)/d/(a+b)^(1/2)+2/3*a*(A*b-B*a)*sec(d*x+c)^2*tan(d*x+c)/b/(
a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)-2/3*a^2*(3*A*a^2*b-7*A*b^3-6*B*a^3+10*B*a
*b^2)*tan(d*x+c)/b^3/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)-2/3*(A*a*b-2*B*a^
2+B*b^2)*(a+b*sec(d*x+c))^(1/2)*tan(d*x+c)/b^3/(a^2-b^2)/d
```

Rubi [A]

time = 0.98, antiderivative size = 510, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4114, 4175, 4167, 4090, 3917, 4089}

$$\frac{2a(Ab-aB)\tan(c+dx)\sec^2(c+dx)}{3bd(a^2-b^2)(a+b\sec(c+dx))^{3/2}}-\frac{2(-2a^2B+aAb+b^2B)\tan(c+dx)\sqrt{a+b\sec(c+dx)}}{3b^3d(a^2-b^2)}-\frac{2a^2(-6a^3B+3a^2Ab+10ab^2B-7Ab^3)\tan(c+dx)}{3b^3d(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}+\frac{2(16a^4B-a^3(8Ab-12bB)-2a^2b^2(3A+8B)+9ab^3(A-B)+b^4(3A-B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^4d\sqrt{a+b}(a^2-b^2)}-\frac{2(-16a^5B+8a^4Ab+28a^3b^2B-15a^2Ab^3-8ab^4B+3Ab^5)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^5d(a-b)(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*(8*a^4*A*b - 15*a^2*A*b^3 + 3*A*b^5 - 16*a^5*B + 28*a^3*b^2*B - 8*a*b^4
*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a
 + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c +
d*x]))/(a - b))])/(3*(a - b)*b^5*(a + b)^(3/2)*d) + (2*(9*a*b^3*(A - B) + b
^4*(3*A - B) + 16*a^4*B - 2*a^2*b^2*(3*A + 8*B) - a^3*(8*A*b - 12*b*B))*Cot
[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(
a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/
(a - b))])/(3*b^4*Sqrt[a + b]*(a^2 - b^2)*d) + (2*a*(A*b - a*B)*Sec[c + d*x
]^2*Tan[c + d*x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*a^2*(
3*a^2*A*b - 7*A*b^3 - 6*a^3*B + 10*a*b^2*B)*Tan[c + d*x])/(3*b^3*(a^2 - b^2
)^2*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(a*A*b - 2*a^2*B + b^2*B)*Sqrt[a + b*S
ec[c + d*x]]*Tan[c + d*x])/(3*b^3*(a^2 - b^2)*d)
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
```

```
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4167

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 2)
)), x] + Dist[1/(b*(m + 2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^m*Simp[b
*A*(m + 2) + b*C*(m + 1) + (b*B*(m + 2) - a*C)*Csc[e + f*x], x], x], x] /;
FreeQ[{a, b, e, f, A, B, C, m}, x] &&  !LtQ[m, -1]
```

Rule 4175

```
Int[csc[(e_.) + (f_.)*(x_)]^2*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[
(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x
```

```
_Symbol] :> Simp[a*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x
])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x] - Dist[1/(b^2*(m + 1)*(a^2 - b^
2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(m + 1)*((-a)*(b*
B - a*C) + A*b^2) + (b*B*(a^2 + b^2*(m + 1)) - a*(A*b^2*(m + 2) + C*(a^2 +
b^2*(m + 1))))*Csc[e + f*x] - b*C*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^4(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{\sec^2(c+dx)\left(2a(Ab-aB)-\frac{3}{2}b(\right.}{}}{}$$

$$= \frac{2a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2(3a^2Ab-7Ab^3-6a^3}{3b^3\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= \frac{2a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2(3a^2Ab-7Ab^3-6a^3}{3b^3\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= \frac{2a(Ab-aB)\sec^2(c+dx)\tan(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a^2(3a^2Ab-7Ab^3-6a^3}{3b^3\left(a^2-b^2\right)^2 d\sqrt{}}$$

$$= -\frac{2(8a^4Ab-15a^2Ab^3+3Ab^5-16a^5B+28a^3b^2B-8ab^4B)\cot(c}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 4342 vs. 2(510) = 1020.
time = 27.10, size = 4342, normalized size = 8.51

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^4*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),
x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*(8*a^4*A*b - 15*a^2*A*b^3 + 3*A*
b^5 - 16*a^5*B + 28*a^3*b^2*B - 8*a*b^4*B)*Sin[c + d*x])/(3*b^4*(-a^2 + b^2
)^2) + (2*(a^2*A*b*Sin[c + d*x] - a^3*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)*
(b + a*Cos[c + d*x])^2) + (2*(-4*a^4*A*b*Sin[c + d*x] + 8*a^2*A*b^3*Sin[c +
 d*x] + 7*a^5*B*Sin[c + d*x] - 11*a^3*b^2*B*Sin[c + d*x]))/(3*b^3*(-a^2 + b
^2)^2*(b + a*Cos[c + d*x])) + (2*B*Tan[c + d*x])/(3*b^3)))/(d*(a + b*Sec[c
```

```
+ d*x])^(5/2)) + (2*(b + a*Cos[c + d*x])^2*((5*a^2*A)/((-a^2 + b^2)^2*Sqrt[
b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^4*A)/(3*b^2*(-a^2 + b^2)^2*S
qrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (A*b^2)/((-a^2 + b^2)^2*Sqrt[
b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (16*a^5*B)/(3*b^3*(-a^2 + b^2)^2*
Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (28*a^3*B)/(3*b*(-a^2 + b^2)
^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (8*a*b*B)/(3*(-a^2 + b^2)
^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^5*A*Sqrt[Sec[c + d*x
]])/(3*b^3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (17*a^3*A*Sqrt[Sec[c
+ d*x]])/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (3*a*A*b*Sqrt[Sec[
c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (5*a^2*B*Sqrt[Sec[c
+ d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (16*a^6*B*Sqrt[Sec[c +
 d*x]])/(3*b^4*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (32*a^4*B*Sqrt[Se
c[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (b^2*B*Sqrt[
Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^5*A*Cos[2
*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*
x]]) + (5*a^3*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(b*(-a^2 + b^2)^2*Sqrt
[b + a*Cos[c + d*x]]) - (a*A*b*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2
+ b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (8*a^2*B*Cos[2*(c + d*x)]*Sqrt[Sec[c +
 d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (16*a^6*B*Cos[2*(c +
d*x)]*Sqrt[Sec[c + d*x]])/(3*b^4*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) -
 (28*a^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[
b + a*Cos[c + d*x]]))*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*
x]]*(2*(a + b)*(-8*a^4*A*b + 15*a^2*A*b^3 - 3*A*b^5 + 16*a^5*B - 28*a^3*b^2
*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d
*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a -
 b)/(a + b)] + 2*b*(a + b)*(-16*a^4*B - 9*a*b^3*(A + B) + b^4*(3*A + B) + 4
*a^3*b*(2*A + 3*B) + 2*a^2*b^2*(-3*A + 8*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c +
 d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[A
rcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-8*a^4*A*b + 15*a^2*A*b^3 - 3*
A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*
x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^4*(a^2 - b^2)^2*d*Sqrt[Sec[(
c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(5/2)*((a*Sqrt[Cos[(c + d*x)/2]^2*Sec[c
 + d*x]]*Sin[c + d*x]*(2*(a + b)*(-8*a^4*A*b + 15*a^2*A*b^3 - 3*A*b^5 + 16*
a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqr
t[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(
c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*a^4*B - 9*a*b^3*(A + B) +
 b^4*(3*A + B) + 4*a^3*b*(2*A + 3*B) + 2*a^2*b^2*(-3*A + 8*B))*Sqrt[Cos[c +
 d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-8*a^4*A*b +
 15*a^2*A*b^3 - 3*A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*Cos[c + d*x]
*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^4*(a^2 - b
^2)^2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c +
 d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-8*a^4*A*b + 15*a^2*A
*b^3 - 3*A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*Sqrt[Cos[c + d*x]/(1
+ Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*El
```

```
lipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(-16*a^4*B
 - 9*a*b^3*(A + B) + b^4*(3*A + B) + 4*a^3*b*(2*A + 3*B) + 2*a^2*b^2*(-3*A
+ 8*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a
 + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
 b)] + (-8*a^4*A*b + 15*a^2*A*b^3 - 3*A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a
*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/
2]))/(3*b^4*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]
) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((-8*a^4*A*b + 15*a^2*A*b^3 -
 3*A*b^5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*Cos[c + d*x]*(b + a*Cos[c +
 d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-8*a^4*A*b + 15*a^2*A*b^3 - 3*A*b^
5 + 16*a^5*B - 28*a^3*b^2*B + 8*a*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/((a + b)
*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*
((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + Cos[c
 + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + (b*(a + b)*(-16*a^4*B -
9*a*b^3*(A + B) + b^4*(3*A + B) + 4*a^3*b*(2*A + 3*B) + 2*a^2*b^2*(-3*A + 8
*B))*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Co...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 8043 vs. $2(476) = 952$.
time = 18.33, size = 8044, normalized size = 15.77

method	result	size
default	Expression too large to display	8044

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^5 + A*sec(d*x + c)^4)*sqrt(b*sec(d*x + c) + a)/(b^
3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec^4(c + dx)}{(a + b\sec(c + dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**4/(a + b*sec(c + d*x))**(5/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^4*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^4/(b*sec(d*x + c) + a)^(5/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+d\,x)}}{\cos(c + d\,x)^4\,\left(a + \frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^4*(a + b/cos(c + d*x))^(5/2)), x)
```

3.386 $\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=417

$$\frac{2(2a^3Ab-6aAb^3-8a^4B+15a^2b^2B-3b^4B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-s}{}}}{3(a-b)b^4(a+b)^{3/2}d}$$

```
[Out] 2/3*(2*A*a^3*b-6*A*a*b^3-8*B*a^4+15*B*a^2*b^2-3*B*b^4)*cot(d*x+c)*EllipticE
((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/
(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b^4/(a+b)^(3/2)/d+2/3*(2
*a^2*b*(A-3*B)-3*b^3*(A-B)-8*a^3*B+3*a*b^2*(A+3*B))*cot(d*x+c)*EllipticF((a
+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+
b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/b^3/(a^2-b^2)/d/(a+b)^(1/2)-2/3*a
^2*(A*b-B*a)*tan(d*x+c)/b^2/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+2/3*a*(2*A*a
^2*b-6*A*b^3-5*B*a^3+9*B*a*b^2)*tan(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c
))^(1/2)
```

Rubi [A]

time = 0.60, antiderivative size = 417, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4113, 4165, 4090, 3917, 4089}

$$-\frac{2a^2(Ab-aB)\tan(c+dx)}{3b^2d\,(a^2-b^2)\,(a+b\sec(c+dx))^{3/2}}+\frac{2(-8a^3B+2a^2b(A-3B)+3ab^2(A+3B)-3b^3(A-B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^3d\sqrt{a+b}\,(a^2-b^2)}+\frac{2a(-5a^3B+2a^2Ab+9ab^2B-6Ab^3)\tan(c+dx)}{3b^2d\,(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}+\frac{2(-8a^4B+2a^3Ab+15a^2b^2B-6aAb^3-3b^4B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^4d(a-b)(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(2*a^3*A*b - 6*a*A*b^3 - 8*a^4*B + 15*a^2*b^2*B - 3*b^4*B)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(3*(a - b)*b^4*(a + b)^(3/2)*d) + (2*(2*a^2*b*(A - 3*B) - 3*b^3*(A - B) -
8*a^3*B + 3*a*b^2*(A + 3*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c
 + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)
]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*b^3*Sqrt[a + b]*(a^2 - b^2)*d
) - (2*a^2*(A*b - a*B)*Tan[c + d*x])/(3*b^2*(a^2 - b^2)*d*(a + b*Sec[c + d*
x])^(3/2)) + (2*a*(2*a^2*A*b - 6*A*b^3 - 5*a^3*B + 9*a*b^2*B)*Tan[c + d*x])
/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
```

```
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4113

```
Int[csc[(e_.) + (f_.)*(x_)]^3*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-a^2)*(A*b - a*B)
*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b^2*f*(m + 1)*(a^2 - b^2))), x
] + Dist[1/(b^2*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])
^(m + 1)*Simp[a*b*(A*b - a*B)*(m + 1) - (A*b - a*B)*(a^2 + b^2*(m + 1))*Csc
[e + f*x] + b*B*(m + 1)*(a^2 - b^2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4165

```
Int[csc[(e_.) + (f_.)*(x_)]*((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e
_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_S
ymbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cot[e + f*x]*((a + b*Csc[e + f*x]
)^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)),
Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m +
 1) - (A*b^2 - a*b*B + a^2*C + b*(A*b - a*B + b*C)*(m + 1))*Csc[e + f*x], x
], x], x] /; FreeQ[{a, b, e, f, A, B, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2
, 0]
```

Rubi steps

$$\int \frac{\sec^3(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2a^2(Ab-aB)\tan(c+dx)}{3b^2(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{3}{2}ab(Ab-aB)-\frac{1}{2}\right.}{}}{}$$

$$= -\frac{2a^2(Ab-aB)\tan(c+dx)}{3b^2(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2a(2a^2Ab-6Ab^3-5a^3B}{3b^2(a^2-b^2)^2\,d\sqrt{a}}$$

$$= -\frac{2a^2(Ab-aB)\tan(c+dx)}{3b^2(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2a(2a^2Ab-6Ab^3-5a^3B}{3b^2(a^2-b^2)^2\,d\sqrt{a}}$$

$$= \frac{2(2a^3Ab-6aAb^3-8a^4B+15a^2b^2B-3b^4B)\cot(c+dx)E\left(\sin^{-1}\right.}{3(a-}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3920 vs. $2(417) = 834$.

time = 26.21, size = 3920, normalized size = 9.40

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^3*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),
x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*(-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*
B - 15*a^2*b^2*B + 3*b^4*B)*Sin[c + d*x])/(3*b^3*(-a^2 + b^2)^2) - (2*(a*A*
b*Sin[c + d*x] - a^2*B*Sin[c + d*x]))/(3*b*(-a^2 + b^2)*(b + a*Cos[c + d*x]
)^2) - (2*(-(a^3*A*b*Sin[c + d*x]) + 5*a*A*b^3*Sin[c + d*x] + 4*a^4*B*Sin[c
 + d*x] - 8*a^2*b^2*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)^2*(b + a*Cos[c + d
*x]))))/(d*(a + b*Sec[c + d*x])^(5/2)) - (2*(b + a*Cos[c + d*x])^2*((2*a^3*
A)/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a*
A*b)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (5*a^2*
B)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*a^4*B)
/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (b^2*
B)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (5*a^2*A*
Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (2*a^4*A*
Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (A*b^
2*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^5*B*
Sqrt[Sec[c + d*x]])/(3*b^3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (17*a
^3*B*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (3
*a*b*B*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (2*a
^2*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c
```

```
+ d*x]]) + (2*a^4*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2
)^2*Sqrt[b + a*Cos[c + d*x]]) - (8*a^5*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]
])/(3*b^3*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (5*a^3*B*Cos[2*(c + d*
x)]*Sqrt[Sec[c + d*x]])/(b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (a*b*
B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d
*x]]))*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(2*(a + b)*
(-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(3*a
*b^2*(A - 3*B) + 8*a^3*B + 3*b^3*(A + B) - 2*a^2*b*(A + 3*B))*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*
x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-2*a^3*A*b +
6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Sec[
(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(5/2)*(-1/3*(a*Sqrt[Cos[(c + d*x)/2]^2
*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B -
15*a^2*b^2*B + 3*b^4*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*C
os[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2
]], (a - b)/(a + b)] - 2*b*(a + b)*(3*a*b^2*(A - 3*B) + 8*a^3*B + 3*b^3*(A
+ B) - 2*a^2*b*(A + 3*B))*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a
*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)
/2]], (a - b)/(a + b)] + (-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B +
 3*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x
)/2]))/(b^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^
2]) + (Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(-
2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(3*a*b
^2*(A - 3*B) + 8*a^3*B + 3*b^3*(A + B) - 2*a^2*b*(A + 3*B))*Sqrt[Cos[c + d*
x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]
))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-2*a^3*A*b + 6*
a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*Cos[c + d*x]*(b + a*Cos[c + d*x
])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*b^3*(a^2 - b^2)^2*Sqrt[b + a*Co
s[c + d*x]]*Sqrt[Sec[(c + d*x)/2]^2]) - (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c +
d*x]]*(((-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*Cos[c +
 d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^4)/2 + ((a + b)*(-2*a^3*A*b + 6
*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/((a
+ b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x]/(1 + C
os[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] - (b*(a + b)*(3*a*b^2*
(A - 3*B) + 8*a^3*B + 3*b^3*(A + B) - 2*a^2*b*(A + 3*B))*Sqrt[(b + a*Cos[c
+ d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (
a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c +
 d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)
*(-2*a^3*A*b + 6*a*A*b^3 + 8*a^4*B - 15*a^2*b^2*B + 3*b^4*B)*Sqrt[Cos[c + d
```

```
*x]/(1 + Cos[c + d*x])]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])
*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/(
(a + b)*(1 + Cos[c + d*x]))] - (b*(a + b)*(3*a*...
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6454 vs. $2(387) = 774$.
time = 17.88, size = 6455, normalized size = 15.48

method	result	size
default	Expression too large to display	6455

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^4 + A*sec(d*x + c)^3)*sqrt(b*sec(d*x + c) + a)/(b^
3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\sec^3{(c + dx)}}{(a + b\sec{(c + dx)})^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**3/(a + b*sec(c + d*x))**(5/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^3*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^3/(b*sec(d*x + c) + a)^(5/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\cos\left(c+d\,x\right)}^3\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^3*(a + b/cos(c + d*x))^(5/2)), x)
```

3.387 $\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=387

$$\frac{2(a^2Ab+3Ab^3+2a^3B-6ab^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}}{3(a-b)b^3(a+b)^{3/2}d}$$

```
[Out] 2/3*(A*a^2*b+3*A*b^3+2*B*a^3-6*B*a*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c
))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-
b*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b^3/(a+b)^(3/2)/d+2/3*(2*a^2*B-3*b^2*(A
+B)+a*b*(A+3*B))*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((
a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))
^(1/2)/b^2/(a^2-b^2)/d/(a+b)^(1/2)+2/3*a*(A*b-B*a)*tan(d*x+c)/b/(a^2-b^2)/d
/(a+b*sec(d*x+c))^(3/2)+2/3*(A*a^2*b+3*A*b^3+2*B*a^3-6*B*a*b^2)*tan(d*x+c)/
b/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.44, antiderivative size = 387, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4094, 4088, 4090, 3917, 4089}

$$\frac{2(2a^2B+ab(A+3B)-3b^2(A+B))\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d\sqrt{a+b}\,(a^2-b^2)}+\frac{2a(Ab-aB)\tan(c+dx)}{3bd\,(a^2-b^2)\,(a+b\sec(c+dx))^{3/2}}+\frac{2(2a^3B+a^2Ab-6ab^2B+3Ab^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^3d(a-b)(a+b)^{3/2}}+\frac{2(2a^3B+a^2Ab-6ab^2B+3Ab^3)\tan(c+dx)}{3bd\,(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[
Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c
+ d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^3*(
a + b)^(3/2)*d) + (2*(2*a^2*B - 3*b^2*(A + B) + a*b*(A + 3*B))*Cot[c + d*x]
*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*S
qrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))]
)/(3*b^2*Sqrt[a + b]*(a^2 - b^2)*d) + (2*a*(A*b - a*B)*Tan[c + d*x])/(3*b*(
a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(a^2*A*b + 3*A*b^3 + 2*a^3*B
- 6*a*b^2*B)*Tan[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rule 4094

```
Int[csc[(e_.) + (f_.)*(x_)]^2*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(
csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*(A*b - a*B)*Cot[
e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dis
t[1/(b*(m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*
Simp[b*(A*b - a*B)*(m + 1) - (a*A*b*(m + 2) - B*(a^2 + b^2*(m + 1)))*Csc[e
+ f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\sec^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2a(Ab-aB)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{\sec(c+dx)\left(-\frac{3}{2}b(Ab-aB)+\frac{1}{2}(aAb\right.}{(a+b\sec(c+d}}{3b\,(a^2-}$$

$$= \frac{2a(Ab-aB)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2(a^2Ab+3Ab^3+2a^3B-6a}{3b\,(a^2-b^2)^2\,d\sqrt{a+b\,\mathrm{s}}}$$

$$= \frac{2a(Ab-aB)\tan(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2(a^2Ab+3Ab^3+2a^3B-6a}{3b\,(a^2-b^2)^2\,d\sqrt{a+b\,\mathrm{s}}}$$

$$= \frac{2(a^2Ab+3Ab^3+2a^3B-6ab^2B)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec}}{\sqrt{a}}\right.\right.}{3(a-b)b^3(}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 3514 vs. $2(387) = 774$.

time = 23.78, size = 3514, normalized size = 9.08

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),
x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-2*(a^2*A*b + 3*A*b^3 + 2*a^3*B -
6*a*b^2*B)*Sin[c + d*x])/(3*b^2*(-a^2 + b^2)^2) + (2*(A*b*Sin[c + d*x] - a*
B*Sin[c + d*x]))/(3*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) + (2*(2*a^2*A*b*Si
n[c + d*x] + 2*A*b^3*Sin[c + d*x] + a^3*B*Sin[c + d*x] - 5*a*b^2*B*Sin[c +
d*x]))/(3*b*(-a^2 + b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b*Sec[c + d*x])^
(5/2)) + (2*(b + a*Cos[c + d*x])^2*((a^2*A)/(3*(-a^2 + b^2)^2*Sqrt[b + a*Co
s[c + d*x]]*Sqrt[Sec[c + d*x]]) + (A*b^2)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]) + (2*a^3*B)/(3*b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[
c + d*x]]*Sqrt[Sec[c + d*x]]) - (2*a*b*B)/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c
+ d*x]]*Sqrt[Sec[c + d*x]]) + (a^3*A*Sqrt[Sec[c + d*x]])/(3*b*(-a^2 + b^2)^
2*Sqrt[b + a*Cos[c + d*x]]) - (a*A*b*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*
Sqrt[b + a*Cos[c + d*x]]) - (5*a^2*B*Sqrt[Sec[c + d*x]])/(3*(-a^2 + b^2)^2*
Sqrt[b + a*Cos[c + d*x]]) + (2*a^4*B*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2
)^2*Sqrt[b + a*Cos[c + d*x]]) + (b^2*B*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*
Sqrt[b + a*Cos[c + d*x]]) + (a^3*A*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*
b*(-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (a*A*b*Cos[2*(c + d*x)]*Sqrt[S
ec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (2*a^2*B*Cos[2*(c
 + d*x)]*Sqrt[Sec[c + d*x]])/((-a^2 + b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (2
```

```
*a^4*B*Cos[2*(c + d*x)]*Sqrt[Sec[c + d*x]])/(3*b^2*(-a^2 + b^2)^2*Sqrt[b +
a*Cos[c + d*x]]))*Sec[c + d*x]^(5/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*
(2*(a + b)*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 +
 Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*Ell
ipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(a*b*(A - 3
*B) + 3*b^2*(A - B) + 2*a^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(
b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c +
 d*x)/2]], (a - b)/(a + b)] + (a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Cos
[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(-(
a^2*b) + b^3)^2*d*Sqrt[Sec[(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(5/2)*((a*S
qrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*Sin[c + d*x]*(2*(a + b)*(a^2*A*b + 3*A
*b^3 + 2*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b +
 a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*
x)/2]], (a - b)/(a + b)] - 2*b*(a + b)*(a*b*(A - 3*B) + 3*b^2*(A - B) + 2*a
^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
 b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)] + (a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c +
d*x])*Sec[(c + d*x)/2]^2*Tan[(c + d*x)/2]))/(3*(-(a^2*b) + b^3)^2*(b + a*Co
s[c + d*x])^(3/2)*Sqrt[Sec[(c + d*x)/2]^2]) - (Sqrt[Cos[(c + d*x)/2]^2*Sec[
c + d*x]]*Tan[(c + d*x)/2]*(2*(a + b)*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^
2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a +
b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)
] - 2*b*(a + b)*(a*b*(A - 3*B) + 3*b^2*(A - B) + 2*a^2*B)*Sqrt[Cos[c + d*x]
/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))
]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (a^2*A*b + 3*A*b^3
 + 2*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^
2*Tan[(c + d*x)/2]))/(3*(-(a^2*b) + b^3)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Se
c[(c + d*x)/2]^2]) + (2*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(((a^2*A*b +
3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d
*x)/2]^4)/2 + ((a + b)*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Sqrt[(b +
a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 -
 Sin[c + d*x]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] -
(b*(a + b)*(a*b*(A - 3*B) + 3*b^2*(A - B) + 2*a^2*B)*Sqrt[(b + a*Cos[c + d*
x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)]*((Cos[c + d*x]*Sin[c + d*x])/(1 + Cos[c + d*x])^2 - Sin[c + d*x
]/(1 + Cos[c + d*x])))/Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])] + ((a + b)*(a^
2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])
]*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*(-((a*Sin[c + d*x])/
((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x])*Sin[c + d*x])/((a + b
)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d
*x]))] - (b*(a + b)*(a*b*(A - 3*B) + 3*b^2*(A - B) + 2*a^2*B)*Sqrt[Cos[c +
d*x]/(1 + Cos[c + d*x])]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b
)]*(-((a*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x]))) + ((b + a*Cos[c + d*x]
)*Sin[c + d*x])/((a + b)*(1 + Cos[c + d*x])^2)))/Sqrt[(b + a*Cos[c + d*x])/
```

```
((a + b)*(1 + Cos[c + d*x]))] - a*(a^2*A*b + 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)
*Cos[c + d*x]*Sec[(c + d*x)/2]^2*Sin[c + d*x]*Tan[(c + d*x)/2] - (a^2*A*b +
 3*A*b^3 + 2*a^3*B - 6*a*b^2*B)*(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2*Sin
[c + d*x]*Tan[(c + d*x)/2] + (a^2*A*b + 3*A*b^3...
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5169 vs. $2(357) = 714$.
time = 9.60, size = 5170, normalized size = 13.36

method	result	size
default	Expression too large to display	5170

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c)^3 + A*sec(d*x + c)^2)*sqrt(b*sec(d*x + c) + a)/(b^
3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\sec^2{(c + dx)}}{(a + b\sec{(c + dx)})^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**2/(a + b*sec(c + d*x))**(5/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^2\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^2*(a + b/cos(c + d*x))^(5/2)), x)
```

3.388 $\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=353

$$-\frac{2(4aAb-a^2B-3b^2B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1}{}}}{3(a-b)b^2(a+b)^{3/2}d}$$

```
[Out] -2/3*(4*A*a*b-B*a^2-3*B*b^2)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a
+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*
x+c))/(a-b))^(1/2)/(a-b)/b^2/(a+b)^(3/2)/d+2/3*(3*A*a-A*b+B*a-3*B*b)*cot(d*
x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(
1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/(a-b)/b/(a+b)^(3
/2)/d-2/3*(A*b-B*a)*tan(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)-2/3*(4*A*
a*b-B*a^2-3*B*b^2)*tan(d*x+c)/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.39, antiderivative size = 353, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.129$, Rules used = {4088, 4090, 3917, 4089}

$$-\frac{2(a^2(-B)+4aAb-3b^2B)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3b^2d(a-b)(a+b)^{3/2}}-\frac{2(a^2(-B)+4aAb-3b^2B)\tan(c+dx)}{3d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2(Ab-aB)\tan(c+dx)}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2(3aA+aB-Ab-3bB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3bd(a-b)(a+b)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*(4*a*A*b - a^2*B - 3*b^2*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Se
c[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a +
 b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b^2*(a + b)^(3/2)*
d) + (2*(3*a*A - A*b + a*B - 3*b*B)*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a +
b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/
(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*(a - b)*b*(a + b)^(3/2
)*d) - (2*(A*b - a*B)*Tan[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(
3/2)) - (2*(4*a*A*b - a^2*B - 3*b^2*B)*Tan[c + d*x])/(3*(a^2 - b^2)^2*d*Sqr
t[a + b*Sec[c + d*x]])
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4088

```
Int[csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(cs
c[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b - a*B))*Cot[e
 + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1
/((m + 1)*(a^2 - b^2)), Int[Csc[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*Simp[
(a*A - b*B)*(m + 1) - (A*b - a*B)*(m + 2)*Csc[e + f*x], x], x], x] /; FreeQ
[{a, b, A, B, e, f}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m,
 -1]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4090

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A - B, Int[Csc[e +
f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B, Int[Csc[e + f*x]*((1 + Csc[
e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A, B}, x]
 && NeQ[a^2 - b^2, 0] && NeQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2(Ab-aB)\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\sec(c+dx)\left(-\frac{3}{2}(aA-bB)+\frac{1}{2}(Ab-\right.}{(a+b\sec(c+dx))^{3}}}{3\left(a^2-b^2\right)}$$

$$= -\frac{2(Ab-aB)\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\left(4aAb-a^2B-3b^2B\right)\tan(}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c}}$$

$$= -\frac{2(Ab-aB)\tan(c+dx)}{3\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\left(4aAb-a^2B-3b^2B\right)\tan(}{3\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c}}$$

$$= -\frac{2\left(4aAb-a^2B-3b^2B\right)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx}}{\sqrt{a+b}}\right.\right.}{3(a-b)b^2(a+}$$

Mathematica [A]

time = 18.15, size = 603, normalized size = 1.71

$$\frac{(b+a\cos(c+dx))^3\sec^2(c+dx)(A+B\sec(c+dx))\left(-\frac{2(-4aAb+a^2B+3b^2B)\sin(c+dx)}{3b(-a^2+b^2)^2}+\frac{2(Ab^2\sin(c+dx)-abB\sin(c+dx))}{3a(a^2-b^2)(b+a\cos(c+dx))^2}+\frac{2(-5a^2Ab\sin(c+dx)+Ab^3\sin(c+dx)+2a^3B\sin(c+dx)+2ab^2B\sin(c+dx))}{3a(a^2-b^2)^2(b+a\cos(c+dx))}\right)}{d(B+A\cos(c+dx))(a+b\sec(c+dx))^{5/2}}+\frac{2(b+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\,(A+B\sec(c+dx))\left(2(a+b)(-4aAb+a^2B+3b^2B)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}E\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)+2b(a+b)(3aA+Ab-aB-3bB)\sqrt{\frac{\cos(c+dx)}{1+\cos(c+dx)}}\sqrt{\frac{b+a\cos(c+dx)}{(a+b)(1+\cos(c+dx))}}F\left(\mathrm{ArcSin}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{a-b}{a+b}\right)+(-4aAb+a^2B+3b^2B)\cos(c+dx)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3b(a^2-b^2)^2d(B+A\cos(c+dx))\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}(a+b\sec(c+dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((-2*(-4*a*A*b
+ a^2*B + 3*b^2*B)*Sin[c + d*x])/(3*b*(-a^2 + b^2)^2) + (2*(A*b^2*Sin[c + d
*x] - a*b*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (2*(-
5*a^2*A*b*Sin[c + d*x] + A*b^3*Sin[c + d*x] + 2*a^3*B*Sin[c + d*x] + 2*a*b^
2*B*Sin[c + d*x]))/(3*a*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*(B + A*Cos
[c + d*x])*(a + b*Sec[c + d*x])^(5/2)) + (2*(b + a*Cos[c + d*x])^2*Sec[c +
d*x]^(3/2)*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(A + B*Sec[c + d*x])*(2*(a
 + b)*(-4*a*A*b + a^2*B + 3*b^2*B)*Sqrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sq
rt[(b + a*Cos[c + d*x])/((a + b)*(1 + Cos[c + d*x]))]*EllipticE[ArcSin[Tan[
(c + d*x)/2]], (a - b)/(a + b)] + 2*b*(a + b)*(3*a*A + A*b - a*B - 3*b*B)*S
qrt[Cos[c + d*x]/(1 + Cos[c + d*x])]*Sqrt[(b + a*Cos[c + d*x])/((a + b)*(1
+ Cos[c + d*x]))]*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] + (-
4*a*A*b + a^2*B + 3*b^2*B)*Cos[c + d*x]*(b + a*Cos[c + d*x])*Sec[(c + d*x)/
2]^2*Tan[(c + d*x)/2]))/(3*b*(a^2 - b^2)^2*d*(B + A*Cos[c + d*x])*Sqrt[Sec[
(c + d*x)/2]^2]*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4212 vs. 2(323) = 646.
time = 9.35, size = 4213, normalized size = 11.93

method	result	size
default	Expression too large to display	4213

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/3/d*4^(1/2)*(A*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-
b)/(a+b))^(1/2))*b^4+3*B*cos(d*x+c)^2*b^4-A*cos(d*x+c)*b^4-3*B*(cos(d*x+c)/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*
a*b^3+4*B*cos(d*x+c)*a*b^3-4*A*cos(d*x+c)*a^3*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*EllipticE((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-8*A*sin(d*x+c)*cos(d*x+c)*(c
os(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2-4*A*si
n(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+
```

```
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))*a*b^3+7*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(
d*x+c),((a-b)/(a+b))^(1/2))*a^2*b^2+5*A*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(
1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3+2*B*sin(d*x+c)*cos(
d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a^3*b
+4*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)
/(a+b))^(1/2))*a^2*b^2+6*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c))
)^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*a*b^3-B*cos(d*x+c)*a^3*(cos(d*x+c)/(1+
cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)
*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*b-5*B*cos(d*x+c)
*a^2*b^2*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))
^(1/2))-7*B*sin(d*x+c)*cos(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c)
,((a-b)/(a+b))^(1/2))*a*b^3-5*A*cos(d*x+c)^3*a^2*b^2+2*B*cos(d*x+c)^3*a^3*b
+2*B*cos(d*x+c)^3*a*b^3-4*A*cos(d*x+c)^2*a*b^3+4*B*cos(d*x+c)^2*a^2*b^2+3*B
*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^
2*sin(d*x+c)*a*b^3-B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b)
)^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b-4*B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/s
in(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^2+3*B*sin(d*x+
c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2)
)*b^4+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(
d*x+c)^2*sin(d*x+c)*a^4+4*A*cos(d*x+c)^3*a^3*b-3*B*cos(d*x+c)^3*a^2*b^2-4*A
*cos(d*x+c)^2*a^3*b+8*A*cos(d*x+c)^2*a^2*b^2-2*B*cos(d*x+c)^2*a^3*b-6*B*cos
(d*x+c)^2*a*b^3-3*A*cos(d*x+c)*a^2*b^2+4*A*cos(d*x+c)*a*b^3-B*cos(d*x+c)*a^
2*b^2+A*cos(d*x+c)^3*b^4-3*B*cos(d*x+c)*b^4-B*cos(d*x+c)^3*a^4+B*cos(d*x+c)
^2*a^4-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*c
os(d*x+c)^2*sin(d*x+c)*a^3*b-4*A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),
((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^2+4*A*(cos(d*x+c)/(1+cos
(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^
2+A*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+
```

```
c)^2*sin(d*x+c)*a*b^3+B*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a
+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3*b+3*B*(cos(d*x+c)/(1+cos(d*x+c)))^(
1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)/sin(d*x+c),((a-b)/(a+b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b^2+3*A*sin(d
*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*cos(d*x+c)^2*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b)/(a+b))^
(1/2))*a^3*b-3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))/sin(d*x+c),((a-b
)/(a+b))^(1/2))*b^4+3*B*sin(d*x+c)*(cos(d*x+c)/(1+cos(d*x+c)))^(1/2)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti...
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] Timed out
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c)^2 + A*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/(b^3*
sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sec(c + dx)}{(a + b\sec(c + dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)/(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="
giac")
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + b/cos(c + d*x))^(5/2)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)*(a + b/cos(c + d*x))^(5/2)), x)
```

3.389 $\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=495

$$\frac{2(7a^2Ab-3Ab^3-4a^3B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(1}{}}}{3a^2(a-b)b(a+b)^{3/2}d}$$

```
[Out] 2/3*(7*A*a^2*b-3*A*b^3-4*B*a^3)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)
/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec
(d*x+c))/(a-b))^(1/2)/a^2/(a-b)/b/(a+b)^(3/2)/d-2/3*(6*A*a^2*b-A*a*b^2-3*A*
b^3-3*B*a^3+B*a^2*b)*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2
),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a
-b))^(1/2)/a^2/(a-b)/b/(a+b)^(3/2)/d-2*A*cot(d*x+c)*EllipticPi((a+b*sec(d*x
+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(d
*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^3/d+2/3*b*(A*b-B*a)*t
an(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)+2/3*b*(7*A*a^2*b-3*A*b^3-4*B
*a^3)*tan(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.51, antiderivative size = 495, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 25, $\frac{\text{number of rules}}{\text{integrand size}} = 0.280$, Rules used = {4008, 4145, 4143, 4006, 3869, 3917, 4089}

$$-\frac{2A\sqrt{a+b}\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^3d}+\frac{2b(Ab-aB)\tan(c+dx)}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}+\frac{2(-4a^3B+7a^2Ab-3Ab^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3a^2bd(a-b)(a+b)^{3/2}}-\frac{2(-3a^3B+6a^2Ab+a^2bB-aAb^2-3Ab^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3a^2bd(a-b)(a+b)^{3/2}}+\frac{2b(-4a^3B+7a^2Ab-3Ab^3)\tan(c+dx)}{3a^2d(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(7*a^2*A*b - 3*A*b^3 - 4*a^3*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b
*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(
a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^2*(a - b)*b*(a + b)^(
3/2)*d) - (2*(6*a^2*A*b - a*A*b^2 - 3*A*b^3 - 3*a^3*B + a^2*b*B)*Cot[c + d*
x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]
*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b)
)])/(3*a^2*(a - b)*b*(a + b)^(3/2)*d) - (2*A*Sqrt[a + b]*Cot[c + d*x]*Ellip
ticPi[(a + b)/a, ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a -
 b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a
- b))])/(a^3*d) + (2*b*(A*b - a*B)*Tan[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*
Sec[c + d*x])^(3/2)) + (2*b*(7*a^2*A*b - 3*A*b^3 - 4*a^3*B)*Tan[c + d*x])/(
3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
```

```
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4008

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(d
_.) + (c_)), x_Symbol] :> Simp[b*(b*c - a*d)*Cot[e + f*x]*((a + b*Csc[e + f
*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)
), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[c*(a^2 - b^2)*(m + 1) - (a*(b*c -
a*d)*(m + 1))*Csc[e + f*x] + b*(b*c - a*d)*(m + 2)*Csc[e + f*x]^2, x], x],
x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && LtQ[m, -1] && Ne
Q[a^2 - b^2, 0] && IntegerQ[2*m]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}A\left(a^2-b^2\right)+\frac{3}{2}a(Ab-aB)\sec(c+dx)-\frac{1}{2}b(Ab-a}{(a+b\sec(c+dx))^{3/2}}}{3a\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b(7a^2Ab-3Ab^3-4a^3B)\tan(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2b(Ab-aB)\tan(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b(7a^2Ab-3Ab^3-4a^3B)\tan(c+dx)}{3a^2\left(a^2-b^2\right)^2 d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2(7a^2Ab-3Ab^3-4a^3B)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3a^2(a-b)b(a+b)^{3/2}d}$$

$$= \frac{2(7a^2Ab-3Ab^3-4a^3B)\cot(c+dx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3a^2(a-b)b(a+b)^{3/2}d}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.48, size = 2083, normalized size = 4.21

Result too large to show

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((2*(-7*a^2*A*b
 + 3*A*b^3 + 4*a^3*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2) - (2*(A*b^3*Sin[c
 + d*x] - a*b^2*B*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)*(b + a*Cos[c + d*x])^2)
 - (2*(-8*a^2*A*b^2*Sin[c + d*x] + 4*A*b^4*Sin[c + d*x] + 5*a^3*b*B*Sin[c +
 d*x] - a*b^3*B*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*(b + a*Cos[c + d*x])))
```

```
/(d*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^(5/2)) + (2*(b + a*Cos[c + d*
x])^(5/2)*Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x])*Sqrt[(a + b - a*Tan[(c +
d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(7*a^3*A*b*Sqrt
[(-a + b)/(a + b)]*Tan[(c + d*x)/2] + 7*a^2*A*b^2*Sqrt[(-a + b)/(a + b)]*Ta
n[(c + d*x)/2] - 3*a*A*b^3*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 3*A*b^
4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2] - 4*a^4*Sqrt[(-a + b)/(a + b)]*B*
Tan[(c + d*x)/2] - 4*a^3*b*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2] - 14*a
^3*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^3 + 6*a*A*b^3*Sqrt[(-a + b)/
(a + b)]*Tan[(c + d*x)/2]^3 + 8*a^4*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/
2]^3 + 7*a^3*A*b*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 7*a^2*A*b^2*Sq
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 3*a*A*b^3*Sqrt[(-a + b)/(a + b)]*
Tan[(c + d*x)/2]^5 + 3*A*b^4*Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]^5 - 4*
a^4*Sqrt[(-a + b)/(a + b)]*B*Tan[(c + d*x)/2]^5 + 4*a^3*b*Sqrt[(-a + b)/(a
+ b)]*B*Tan[(c + d*x)/2]^5 - (6*I)*a^4*A*EllipticPi[-((a + b)/(a - b)), I*A
rcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 -
Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]
^2)/(a + b)] + (12*I)*a^2*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sq
rt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c +
d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
b)] - (6*I)*A*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a
 + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqr
t[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - (6*I)*a^
4*A*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c
+ d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2
]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (12
*I)*a^2*A*b^2*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(a + b
)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c +
 d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a +
 b)] - (6*I)*A*b^4*EllipticPi[-((a + b)/(a - b)), I*ArcSinh[Sqrt[(-a + b)/(
a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan
[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)
/(a + b)] + I*(a - b)*(-7*a^2*A*b + 3*A*b^3 + 4*a^3*B)*EllipticE[I*ArcSinh[
Sqrt[(-a + b)/(a + b)]*Tan[(c + d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c
+ d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 +
b*Tan[(c + d*x)/2]^2)/(a + b)] + I*(a - b)*(-4*a*A*b^2 - 6*A*b^3 + 3*a^3*(A
 - B) + a^2*b*(9*A + B))*EllipticF[I*ArcSinh[Sqrt[(-a + b)/(a + b)]*Tan[(c
+ d*x)/2]], (a + b)/(a - b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x
)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
))/(3*a^2*Sqrt[(-a + b)/(a + b)]*(a^2 - b^2)^2*d*(B + A*Cos[c + d*x])*(a +
b*Sec[c + d*x])^(5/2)*(-1 + Tan[(c + d*x)/2]^2)*Sqrt[(1 + Tan[(c + d*x)/2]^
2)/(1 - Tan[(c + d*x)/2]^2)]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c +
 d*x)/2]^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5711 vs. $2(456) = 912$.

time = 9.98, size = 5712, normalized size = 11.54

method	result	size
default	Expression too large to display	5712

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/(b^3*sec(d*x + c)^3
+ 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec(c + dx)}{(a + b\sec(c + dx))^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="giac")
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^(5/2),x)
[Out] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^(5/2), x)
```

3.390 $\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=582

$$\frac{(3a^4A-26a^2Ab^2+15Ab^4+14a^3bB-6ab^3B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-}{}}}{3a^3(a-b)b(a+b)^{3/2}d}$$

```
[Out] A*sin(d*x+c)/a/d/(a+b*sec(d*x+c))^(3/2)+1/3*(3*A*a^4-26*A*a^2*b^2+15*A*b^4+
14*B*a^3*b-6*B*a*b^3)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/
2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(
a-b))^(1/2)/a^3/(a-b)/b/(a+b)^(3/2)/d-1/3*(15*A*b^3+a*b^2*(5*A-6*B)-3*a^3*(
A-4*B)-a^2*b*(21*A+2*B))*cot(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^
(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c)
)/(a-b))^(1/2)/a^3/(a^2-b^2)/d/(a+b)^(1/2)+(5*A*b-2*B*a)*cot(d*x+c)*Ellipti
cPi((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(
1/2)*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/d+1
/3*b*(3*A*a^2-5*A*b^2+2*B*a*b)*tan(d*x+c)/a^2/(a^2-b^2)/d/(a+b*sec(d*x+c))^
(3/2)+1/3*b*(3*A*a^4-26*A*a^2*b^2+15*A*b^4+14*B*a^3*b-6*B*a*b^3)*tan(d*x+c)
/a^3/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.77, antiderivative size = 582, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.258$, Rules used = {4119, 4146, 4145, 4143, 4006, 3869, 3917, 4089}

$$\frac{\sqrt{a+b}\,(5Ab-2aB)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{a^4d}+\frac{b(3a^2A+2abB-5Ab^2)\tan(c+dx)}{3a^2d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}-\frac{(-3a^3(A-4B)-a^2b(21A+2B)+ab^2(5A-6B)+15Ab^3)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3a^3d\sqrt{a+b}\,(a^2-b^2)}+\frac{(3a^4A+14a^3bB-26a^2Ab^2-6ab^3B+15Ab^4)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{3a^3bd(a-b)(a+b)^{3/2}}+\frac{b(3a^4A+14a^3bB-26a^2Ab^2-6ab^3B+15Ab^4)\tan(c+dx)}{3a^3d(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}+\frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((3*a^4*A - 26*a^2*A*b^2 + 15*A*b^4 + 14*a^3*b*B - 6*a*b^3*B)*Cot[c + d*x]*
EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sq
rt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])
/(3*a^3*(a - b)*b*(a + b)^(3/2)*d) - ((15*A*b^3 + a*b^2*(5*A - 6*B) - 3*a^3
*(A - 4*B) - a^2*b*(21*A + 2*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*S
ec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a
+ b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(3*a^3*Sqrt[a + b]*(a^2 - b^
2)*d) + (Sqrt[a + b]*(5*A*b - 2*a*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, Arc
Sin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Se
c[c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(a^4*d) + (A
*Sin[c + d*x])/(a*d*(a + b*Sec[c + d*x])^(3/2)) + (b*(3*a^2*A - 5*A*b^2 + 2
*a*b*B)*Tan[c + d*x])/(3*a^2*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (b
*(3*a^4*A - 26*a^2*A*b^2 + 15*A*b^4 + 14*a^3*b*B - 6*a*b^3*B)*Tan[c + d*x])
/(3*a^3*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4119

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*n)), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
```

```
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4146

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))*(csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 + a^2*C)*Cot[e + f*x]*((a + b*Csc[
e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 -
 b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*b*(
A + C)*(m + 1)*Csc[e + f*x] + (A*b^2 + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x
], x] /; FreeQ[{a, b, e, f, A, C}, x] && NeQ[a^2 - b^2, 0] && IntegerQ[2*m]
 && LtQ[m, -1]
```

Rubi steps

$$\int \frac{\cos(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} - \frac{\int \frac{\frac{1}{2}(5Ab-2aB)-\frac{3}{2}Ab\sec^2(c+dx)}{(a+b\sec(c+dx))^{5/2}}dx}{a}$$

$$= \frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{b(3a^2A-5Ab^2+2abB)\tan(c+dx)}{3a^2(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} +$$

$$= \frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{b(3a^2A-5Ab^2+2abB)\tan(c+dx)}{3a^2(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} +$$

$$= \frac{A\sin(c+dx)}{ad(a+b\sec(c+dx))^{3/2}} + \frac{b(3a^2A-5Ab^2+2abB)\tan(c+dx)}{3a^2(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} +$$

$$= \frac{(3a^4A-26a^2Ab^2+15Ab^4+14a^3bB-6ab^3B)\cot(c+dx)E\Big(\sin^{-}}{3a^3(}$$

$$= \frac{(3a^4A-26a^2Ab^2+15Ab^4+14a^3bB-6ab^3B)\cot(c+dx)E\Big(\sin^{-}}{3a^3(}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 2366 vs. $2(582) = 1164$.
time = 21.49, size = 2366, normalized size = 4.07

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((-2*b*(-10*a^2*A*b + 6*A*b^3 + 7*a^
3*B - 3*a*b^2*B)*Sin[c + d*x])/(3*a^3*(-a^2 + b^2)^2) + (2*(A*b^4*Sin[c + d
*x] - a*b^3*B*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (
2*(-11*a^2*A*b^3*Sin[c + d*x] + 7*A*b^5*Sin[c + d*x] + 8*a^3*b^2*B*Sin[c +
d*x] - 4*a*b^4*B*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x])))
)/(d*(a + b*Sec[c + d*x])^(5/2)) - ((b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]
^(5/2)*Sqrt[(1 - Tan[(c + d*x)/2]^2)^(-1)]*Sqrt[(a + b - a*Tan[(c + d*x)/2]
^2 + b*Tan[(c + d*x)/2]^2)/(1 + Tan[(c + d*x)/2]^2)]*(3*a^5*A*Tan[(c + d*x)
/2] + 3*a^4*A*b*Tan[(c + d*x)/2] - 26*a^3*A*b^2*Tan[(c + d*x)/2] - 26*a^2*A
*b^3*Tan[(c + d*x)/2] + 15*a*A*b^4*Tan[(c + d*x)/2] + 15*A*b^5*Tan[(c + d*x
)/2] + 14*a^4*b*B*Tan[(c + d*x)/2] + 14*a^3*b^2*B*Tan[(c + d*x)/2] - 6*a^2*
b^3*B*Tan[(c + d*x)/2] - 6*a*b^4*B*Tan[(c + d*x)/2] - 6*a^5*A*Tan[(c + d*x)
```

```
/2]^3 + 52*a^3*A*b^2*Tan[(c + d*x)/2]^3 - 30*a*A*b^4*Tan[(c + d*x)/2]^3 - 2
8*a^4*b*B*Tan[(c + d*x)/2]^3 + 12*a^2*b^3*B*Tan[(c + d*x)/2]^3 + 3*a^5*A*Ta
n[(c + d*x)/2]^5 - 3*a^4*A*b*Tan[(c + d*x)/2]^5 - 26*a^3*A*b^2*Tan[(c + d*x
)/2]^5 + 26*a^2*A*b^3*Tan[(c + d*x)/2]^5 + 15*a*A*b^4*Tan[(c + d*x)/2]^5 -
15*A*b^5*Tan[(c + d*x)/2]^5 + 14*a^4*b*B*Tan[(c + d*x)/2]^5 - 14*a^3*b^2*B*
Tan[(c + d*x)/2]^5 - 6*a^2*b^3*B*Tan[(c + d*x)/2]^5 + 6*a*b^4*B*Tan[(c + d*
x)/2]^5 - 30*a^4*A*b*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a +
b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan
[(c + d*x)/2]^2)/(a + b)] + 60*a^2*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x
)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c
 + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*A*b^5*EllipticPi[-1, Arc
Sin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(
a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + 12*a^5*B*El
lipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d
*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b
)] - 24*a^3*b^2*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c
 + d*x)/2]^2)/(a + b)] + 12*a*b^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]]
, (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*
x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*a^4*A*b*EllipticPi[-1, ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d
*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b
)] + 60*a^2*A*b^3*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d
*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 30*A*b^5*EllipticPi[-1, ArcSin[
Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*
x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)
] + 12*a^5*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan[
(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2
]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 24*a^3*b^2*B*EllipticPi[-1, ArcSin[T
an[(c + d*x)/2]], (a - b)/(a + b)]*Tan[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x
)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)]
 + 12*a*b^4*B*EllipticPi[-1, ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Tan
[(c + d*x)/2]^2*Sqrt[1 - Tan[(c + d*x)/2]^2]*Sqrt[(a + b - a*Tan[(c + d*x)/
2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] + (a + b)*(3*a^4*A - 26*a^2*A*b^2 + 1
5*A*b^4 + 14*a^3*b*B - 6*a*b^3*B)*EllipticE[ArcSin[Tan[(c + d*x)/2]], (a -
b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[(c + d*x)/2]^2)*Sqrt[(a +
 b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/(a + b)] - 2*a*(a + b)*(5
*A*b^3 + 3*a^3*B + 3*a^2*b*(-2*A + B) - a*b^2*(3*A + 2*B))*EllipticF[ArcSin
[Tan[(c + d*x)/2]], (a - b)/(a + b)]*Sqrt[1 - Tan[(c + d*x)/2]^2]*(1 + Tan[
(c + d*x)/2]^2)*Sqrt[(a + b - a*Tan[(c + d*x)/2]^2 + b*Tan[(c + d*x)/2]^2)/
(a + b)]))/(3*a*(a^3 - a*b^2)^2*d*(a + b*Sec[c + d*x])^(5/2)*Sqrt[1 + Tan[(
c + d*x)/2]^2]*(a*(-1 + Tan[(c + d*x)/2]^2) - b*(1 + Tan[(c + d*x)/2]^2)))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 8544 vs.

$2(541) = 1082$.
time = 9.48, size = 8545, normalized size = 14.68

method	result	size
default	Expression too large to display	8545

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="
fricas")
```

```
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(b*sec(d*x + c)
 + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) +
 a^3), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)/(b*sec(d*x + c) + a)^(5/2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(5/2), x)
```

3.391 $\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=686

$$-\frac{(33a^4Ab-170a^2Ab^3+105Ab^5-12a^5B+104a^3b^2B-60ab^4B)\cot(c+dx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+d}}{\sqrt{a+b}}\right.\right.}{12a^4(a-b)b(a+b)^{3/2}d}$$

```
[Out] -1/4*(7*A*b-4*B*a)*sin(d*x+c)/a^2/d/(a+b*sec(d*x+c))^(3/2)+1/2*A*cos(d*x+c)
*sin(d*x+c)/a/d/(a+b*sec(d*x+c))^(3/2)-1/12*(33*A*a^4*b-170*A*a^2*b^3+105*A
*b^5-12*B*a^5+104*B*a^3*b^2-60*B*a*b^4)*cot(d*x+c)*EllipticE((a+b*sec(d*x+c
))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(b*(1-sec(d*x+c))/(a+b))^(1/2)*(-
b*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a-b)/b/(a+b)^(3/2)/d+1/12*(105*A*b^4+5*a
*b^3*(7*A-12*B)+6*a^4*(A+2*B)-5*a^2*b^2*(27*A+4*B)-a^3*(27*A*b-84*B*b))*cot
(d*x+c)*EllipticF((a+b*sec(d*x+c))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(
b*(1-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^4/(a^2-b^2)
/d/(a+b)^(1/2)-1/4*(4*A*a^2+35*A*b^2-20*B*a*b)*cot(d*x+c)*EllipticPi((a+b*s
ec(d*x+c))^(1/2)/(a+b)^(1/2),(a+b)/a,((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1
-sec(d*x+c))/(a+b))^(1/2)*(-b*(1+sec(d*x+c))/(a-b))^(1/2)/a^5/d-1/12*b*(27*
A*a^2*b-35*A*b^3-12*B*a^3+20*B*a*b^2)*tan(d*x+c)/a^3/(a^2-b^2)/d/(a+b*sec(d
*x+c))^(3/2)-1/12*b*(33*A*a^4*b-170*A*a^2*b^3+105*A*b^5-12*B*a^5+104*B*a^3*
b^2-60*B*a*b^4)*tan(d*x+c)/a^4/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 1.28, antiderivative size = 686, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4119, 4189, 4145, 4143, 4006, 3869, 3917, 4089}

$$-\frac{(7Ab-4aB)\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}}-\frac{\sqrt{a+b}\,(4a^2A-20abB+35Ab^2)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}\,\Pi\left(\frac{a+b}{a};\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{4a^5d}-\frac{b(-12a^3B+27a^2Ab+20ab^2B-35Ab^3)\tan(c+dx)}{12a^3d(a^2-b^2)(a+b\sec(c+dx))^{3/2}}+\frac{(6a^4(A+2B)-a^3(27Ab-84bB)-5a^2b^2(27A+4B)+5ab^3(7A-12B)+105Ab^4)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}F\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{12a^4d\sqrt{a+b}\,(a^2-b^2)}-\frac{(-12a^5B+33a^4Ab+104a^3b^2B-170a^2Ab^3-60ab^4B+105Ab^5)\cot(c+dx)\sqrt{\frac{b(1-\sec(c+dx))}{a+b}}\sqrt{-\frac{b(\sec(c+dx)+1)}{a-b}}E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{12a^4bd(a-b)(a+b)^{3/2}}-\frac{b(-12a^5B+33a^4Ab+104a^3b^2B-170a^2Ab^3-60ab^4B+105Ab^5)\tan(c+dx)}{12a^4d(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}+\frac{A\sin(c+dx)\cos(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] -1/12*((33*a^4*A*b - 170*a^2*A*b^3 + 105*A*b^5 - 12*a^5*B + 104*a^3*b^2*B -
 60*a*b^4*B)*Cot[c + d*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
+ b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1 +
 Sec[c + d*x]))/(a - b))])/(a^4*(a - b)*b*(a + b)^(3/2)*d) + ((105*A*b^4 +
5*a*b^3*(7*A - 12*B) + 6*a^4*(A + 2*B) - 5*a^2*b^2*(27*A + 4*B) - a^3*(27*A
*b - 84*b*B))*Cot[c + d*x]*EllipticF[ArcSin[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a
 + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[c + d*x]))/(a + b)]*Sqrt[-((b*(1
+ Sec[c + d*x]))/(a - b))])/(12*a^4*Sqrt[a + b]*(a^2 - b^2)*d) - (Sqrt[a +
b]*(4*a^2*A + 35*A*b^2 - 20*a*b*B)*Cot[c + d*x]*EllipticPi[(a + b)/a, ArcSi
n[Sqrt[a + b*Sec[c + d*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[
c + d*x]))/(a + b)]*Sqrt[-((b*(1 + Sec[c + d*x]))/(a - b))])/(4*a^5*d) - ((
```

```
7*A*b - 4*a*B)*Sin[c + d*x])/(4*a^2*d*(a + b*Sec[c + d*x])^(3/2)) + (A*Cos[
c + d*x]*Sin[c + d*x])/(2*a*d*(a + b*Sec[c + d*x])^(3/2)) - (b*(27*a^2*A*b
- 35*A*b^3 - 12*a^3*B + 20*a*b^2*B)*Tan[c + d*x])/(12*a^3*(a^2 - b^2)*d*(a
+ b*Sec[c + d*x])^(3/2)) - (b*(33*a^4*A*b - 170*a^2*A*b^3 + 105*A*b^5 - 12*
a^5*B + 104*a^3*b^2*B - 60*a*b^4*B)*Tan[c + d*x])/(12*a^4*(a^2 - b^2)^2*d*S
qrt[a + b*Sec[c + d*x]])
```

Rule 3869

```
Int[1/Sqrt[csc[(c_.) + (d_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[2*(Rt[a
 + b, 2]/(a*d*Cot[c + d*x]))*Sqrt[b*((1 - Csc[c + d*x])/(a + b))]*Sqrt[(-b)
*((1 + Csc[c + d*x])/(a - b))]*EllipticPi[(a + b)/a, ArcSin[Sqrt[a + b*Csc[
c + d*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, c, d}, x] &&
NeQ[a^2 - b^2, 0]
```

Rule 3917

```
Int[csc[(e_.) + (f_.)*(x_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_S
ymbol] :> Simp[-2*(Rt[a + b, 2]/(b*f*Cot[e + f*x]))*Sqrt[(b*(1 - Csc[e + f*
x]))/(a + b)]*Sqrt[(-b)*((1 + Csc[e + f*x])/(a - b))]*EllipticF[ArcSin[Sqrt
[a + b*Csc[e + f*x]]/Rt[a + b, 2]], (a + b)/(a - b)], x] /; FreeQ[{a, b, e,
 f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4006

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)], x_Symbol] :> Dist[c, Int[1/Sqrt[a + b*Csc[e + f*x]], x], x] + D
ist[d, Int[Csc[e + f*x]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, c,
 d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rule 4119

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*n)), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
```

```
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4143

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Int[(A + (B - C
)*Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]], x] + Dist[C, Int[Csc[e + f*x]*((1
 + Csc[e + f*x])/Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b, e, f, A,
 B, C}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4145

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_))^(m_), x_Symbol] :> Simp[(A*b^2 -
a*b*B + a^2*C)*Cot[e + f*x]*((a + b*Csc[e + f*x])^(m + 1)/(a*f*(m + 1)*(a^2
 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m
 + 1)*Simp[A*(a^2 - b^2)*(m + 1) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x]
 + (A*b^2 - a*b*B + a^2*C)*(m + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
b, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^2(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{\int \frac{\cos(c+dx)\left(\frac{1}{2}(7Ab-4aB)-aA\sec(c+dx)-\frac{5}{2}Ab\,\mathrm{se}}{(a+b\sec(c+dx))^{5/2}}}{2a}$$

$$= -\frac{(7Ab-4aB)\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} + \frac{\int \frac{1}{4}(4a^2}{}$$

$$= -\frac{(7Ab-4aB)\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{b(27a^2}{}$$

$$= -\frac{(7Ab-4aB)\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{b(27a^2}{}$$

$$= -\frac{(7Ab-4aB)\sin(c+dx)}{4a^2d(a+b\sec(c+dx))^{3/2}} + \frac{A\cos(c+dx)\sin(c+dx)}{2ad(a+b\sec(c+dx))^{3/2}} - \frac{b(27a^2}{}$$

$$= -\frac{(33a^4Ab-170a^2Ab^3+105Ab^5-12a^5B+104a^3b^2B-60ab^4B)}{}$$

$$= -\frac{(33a^4Ab-170a^2Ab^3+105Ab^5-12a^5B+104a^3b^2B-60ab^4B)}{}$$

Mathematica [A]
time = 15.01, size = 821, normalized size = 1.20

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^2*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),
x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^3*((2*b^2*(-13*a^2*A*b + 9*A*b^3 + 10*
a^3*B - 6*a*b^2*B)*Sin[c + d*x])/(3*a^4*(-a^2 + b^2)^2) - (2*(A*b^5*Sin[c +
 d*x] - a*b^4*B*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) -
 (2*(-14*a^2*A*b^4*Sin[c + d*x] + 10*A*b^6*Sin[c + d*x] + 11*a^3*b^3*B*Sin[
c + d*x] - 7*a*b^5*B*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)^2*(b + a*Cos[c + d*x
])) + (A*Sin[2*(c + d*x)])/(4*a^3)))/(d*(a + b*Sec[c + d*x])^(5/2)) - ((b +
 a*Cos[c + d*x])^2*Sec[c + d*x]*(-(a*(a + b)*(-33*a^4*A*b + 170*a^2*A*b^3 -
 105*A*b^5 + 12*a^5*B - 104*a^3*b^2*B + 60*a*b^4*B)*EllipticE[ArcSin[Tan[(c
 + d*x)/2]], (a - b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])
```

```
*Sec[(c + d*x)/2]^2)/(a + b)]) + b*(a + b)*(105*A*b^5 + 6*a^5*(A + 2*B) - 3
0*a*b^4*(7*A + 2*B) + 4*a^3*b^2*(57*A + 10*B) - 3*a^4*b*(13*A + 48*B) + 2*a
^2*b^3*(-29*A + 60*B))*EllipticF[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)]
*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
 + 3*(a - b)^2*(a + b)^2*(4*a^2*A + 35*A*b^2 - 20*a*b*B)*((a - b)*EllipticF
[ArcSin[Tan[(c + d*x)/2]], (a - b)/(a + b)] - 2*a*EllipticPi[-1, ArcSin[Tan
[(c + d*x)/2]], (a - b)/(a + b)])*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2)/(a + b)] - a*(-33*a^4*A*b + 170*a^2*A*b^3 - 105*A*
b^5 + 12*a^5*B - 104*a^3*b^2*B + 60*a*b^4*B)*(b + a*Cos[c + d*x])*(Cos[c +
d*x]*Sec[(c + d*x)/2]^2)^(3/2)*Sec[c + d*x]*Tan[(c + d*x)/2]))/(12*a^5*(a^2
 - b^2)^2*d*(Cos[c + d*x]*Sec[(c + d*x)/2]^2)^(3/2)*(a + b*Sec[c + d*x])^(5
/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 10321 vs. $2(637) = 1274$.
time = 9.60, size = 10322, normalized size = 15.05

method	result	size
default	Expression too large to display	10322

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x
)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)^2*sec(d*x + c) + A*cos(d*x + c)^2)*sqrt(b*sec(d*x
+ c) + a)/(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x +
c) + a^3), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\cos^2{(c + dx)}}{(a + b\sec{(c + dx)})^{\frac{5}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*cos(c + d*x)**2/(a + b*sec(c + d*x))**(5/2),
x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^2*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^2/(b*sec(d*x + c) + a)^(5/2), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^2\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^2*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(5/2), x)
```

3.392 $\int \frac{\sec(e+fx)(A+A\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal. Leaf size=105

$$-\frac{2A(a-b)\sqrt{a+b}\,\cot(e+fx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+}{}}}{b^2f}$$

[Out] -2*A*(a-b)*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a+b)^(1/2),((a+b)/(a-b))^(1/2))*(a+b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(a-b))^(1/2)/b^2/f

Rubi [A]

time = 0.05, antiderivative size = 105, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.032$, Rules used = $\{4089\}$

$$-\frac{2A(a-b)\sqrt{a+b}\,\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)|\frac{a+b}{a-b}\right)}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(A + A*Sec[e + f*x]))/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (-2*A*(a - b)*Sqrt[a + b]*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e +
f*x]]/Sqrt[a + b]], (a + b)/(a - b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*S
qrt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b^2*f)
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(A+A\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}\,dx = -\frac{2A(a-b)\sqrt{a+b}\,\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a+b}}\right)\right.}{b^2f}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 248 vs. 2(105) = 210.

time = 9.85, size = 248, normalized size = 2.36

$$\frac{A(1+\sec(e+fx))\left(2(b+a\cos(e+fx))\tan\left(\frac{1}{2}(e+fx)\right)+\frac{\sqrt{\sec^2\left(\frac{1}{2}(e+fx)\right)}\sqrt{\cos^2\left(\frac{1}{2}(e+fx)\right)\sec(e+fx)}\left(\frac{\sqrt{\frac{a-b}{a+b}}(a+b)\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}E\left(\text{ArcSin}\left(\sqrt{\frac{a-b}{a+b}}\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a+b}{a-b}\right)}{\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}}+(b+a\cos(e+fx))\tan\left(\frac{1}{2}(e+fx)\right)\right)\left(-1+\tan^2\left(\frac{1}{2}(e+fx)\right)\right)}{\sqrt{\sec(e+fx)}}\right)}{bf\sqrt{a+b\sec(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[e + f*x]*(A + A*Sec[e + f*x]))/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (A*(1 + Sec[e + f*x])*(2*(b + a*Cos[e + f*x])*Tan[(e + f*x)/2] + (Sqrt[Sec[
(e + f*x)/2]^2]*Sqrt[Cos[(e + f*x)/2]^2*Sec[e + f*x]]*((Sqrt[(a - b)/(a + b
)]*(a + b)*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*Elliptic
E[ArcSin[Sqrt[(a - b)/(a + b)]*Tan[(e + f*x)/2]], (a + b)/(a - b)])/Sqrt[Co
s[e + f*x]/(1 + Cos[e + f*x])] + (b + a*Cos[e + f*x])*Tan[(e + f*x)/2])*(-1
 + Tan[(e + f*x)/2]^2))/Sqrt[Sec[e + f*x]]))/(b*f*Sqrt[a + b*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 641 vs. 2(96) = 192.

time = 11.81, size = 642, normalized size = 6.11

method	result
default	$-\frac{2A\sqrt{\frac{a\cos(fx+e)+b}{\cos(fx+e)}}\,(\cos(fx+e)+1)^2(-1+\cos(fx+e))^2\left(2\cos(fx+e)\,\text{EllipticF}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)},\sqrt{\frac{a-b}{a+b}}\right)\sqrt{\frac{\cos(fx+e)}{\cos(fx+e)+1}}\ \sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(A+A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2*A/f*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))
^2*(2*cos(f*x+e)*EllipticF((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*
(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(
1/2)*sin(f*x+e)*b-cos(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a
+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/
(cos(f*x+e)+1)/(a+b))^(1/2)*a-cos(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+
e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*co
s(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*b+2*(cos(f*x+e)/(cos(f*x+e)+1))^(1/
2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*EllipticF((-1+cos(f*x+e))/
sin(f*x+e),((a-b)/(a+b))^(1/2))*b*sin(f*x+e)-EllipticE((-1+cos(f*x+e))/sin(
f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((
a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*a-EllipticE((-1+cos(f*x+e))/sin
```

```
(f*x+e),((a-b)/(a+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*(
(a*cos(f*x+e)+b)/(cos(f*x+e)+1)/(a+b))^(1/2)*b+cos(f*x+e)^2*a-a*cos(f*x+e)+
cos(f*x+e)*b-b)/sin(f*x+e)^5/(a*cos(f*x+e)+b)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(A+A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((A*sec(f*x + e) + A)*sec(f*x + e)/sqrt(b*sec(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(A+A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral((A*sec(f*x + e)^2 + A*sec(f*x + e))/sqrt(b*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$A\left(\int \frac{\sec\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}}\,dx+\int \frac{\sec^{2}\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(A+A*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] A*(Integral(sec(e + f*x)/sqrt(a + b*sec(e + f*x)), x) + Integral(sec(e + f*
x)**2/sqrt(a + b*sec(e + f*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(A+A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((A*sec(f*x + e) + A)*sec(f*x + e)/sqrt(b*sec(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{A}{\cos(e+f\,x)}}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + A/cos(e + f*x))/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)),x)
[Out] int((A + A/cos(e + f*x))/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)), x)
```

3.393 $\int \frac{\sec(e+fx)(A-A\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}\,dx$

Optimal. Leaf size=107

$$\frac{2A\sqrt{a-b}\,(a+b)\cot(e+fx)E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a-b}}\right)|\frac{a-b}{a+b}\right)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(1+\text{se}}{a}}}{b^2f}$$

```
[Out] 2*A*(a+b)*cot(f*x+e)*EllipticE((a+b*sec(f*x+e))^(1/2)/(a-b)^(1/2),((a-b)/(a
+b))^(1/2))*(a-b)^(1/2)*(b*(1-sec(f*x+e))/(a+b))^(1/2)*(-b*(1+sec(f*x+e))/(
a-b))^(1/2)/b^2/f
```

Rubi [A]
time = 0.06, antiderivative size = 107, normalized size of antiderivative = 1.00, number of steps used = 1, number of rules used = 1, integrand size = 32, $\frac{\text{number of rules}}{\text{integrand size}} = 0.031$, Rules used = $\{4089\}$

$$\frac{2A\sqrt{a-b}\,(a+b)\cot(e+fx)\sqrt{\frac{b(1-\sec(e+fx))}{a+b}}\sqrt{-\frac{b(\sec(e+fx)+1)}{a-b}}\,E\left(\text{ArcSin}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a-b}}\right)|\frac{a-b}{a+b}\right)}{b^2f}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[e + f*x]*(A - A*Sec[e + f*x]))/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (2*A*Sqrt[a - b]*(a + b)*Cot[e + f*x]*EllipticE[ArcSin[Sqrt[a + b*Sec[e + f
*x]]/Sqrt[a - b]], (a - b)/(a + b)]*Sqrt[(b*(1 - Sec[e + f*x]))/(a + b)]*Sq
rt[-((b*(1 + Sec[e + f*x]))/(a - b))])/(b^2*f)
```

Rule 4089

```
Int[(csc[(e_.) + (f_.)*(x_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)))/Sqrt[c
sc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Simp[-2*(A*b - a*B)*Rt[a
 + b*(B/A), 2]*Sqrt[b*((1 - Csc[e + f*x])/(a + b))]*(Sqrt[(-b)*((1 + Csc[e
+ f*x])/(a - b))]/(b^2*f*Cot[e + f*x]))*EllipticE[ArcSin[Sqrt[a + b*Csc[e +
 f*x]]/Rt[a + b*(B/A), 2]], (a*A + b*B)/(a*A - b*B)], x] /; FreeQ[{a, b, e,
 f, A, B}, x] && NeQ[a^2 - b^2, 0] && EqQ[A^2 - B^2, 0]
```

Rubi steps

$$\int \frac{\sec(e+fx)(A-A\sec(e+fx))}{\sqrt{a+b\sec(e+fx)}}\,dx = \frac{2A\sqrt{a-b}\,(a+b)\cot(e+fx)E\left(\sin^{-1}\left(\frac{\sqrt{a+b\sec(e+fx)}}{\sqrt{a-b}}\right)|\right.}{b^2f}$$

Mathematica [A]
time = 7.40, size = 211, normalized size = 1.97

$$\frac{A(a+b)\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\sec^2\left(\frac{1}{2}(e+fx)\right)\sqrt{\sec(e+fx)}\left(\sqrt{\frac{\cos(e+fx)}{1+\cos(e+fx)}}E\left(\text{ArcSin}\left(\tan\left(\frac{1}{2}(e+fx)\right)\right)|\frac{a-b}{a+b}\right)\sqrt{1+\sec(e+fx)}-\sqrt{\frac{1}{1+\cos(e+fx)}}\sqrt{\frac{b+a\cos(e+fx)}{(a+b)(1+\cos(e+fx))}}\sqrt{\sec(e+fx)}\sin(e+fx)\right)}{bf\left(\frac{1}{1+\cos(e+fx)}\right)^{3/2}\sqrt{a+b\sec(e+fx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[e + f*x]*(A - A*Sec[e + f*x]))/Sqrt[a + b*Sec[e + f*x]],x]
```

```
[Out] (A*(a + b)*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e + f*x]))]*Sec[(e +
 f*x)/2]^2*Sqrt[Sec[e + f*x]]*(Sqrt[Cos[e + f*x]/(1 + Cos[e + f*x])]*Ellipt
icE[ArcSin[Tan[(e + f*x)/2]], (a - b)/(a + b)]*Sqrt[1 + Sec[e + f*x]] - Sqr
t[(1 + Cos[e + f*x])^(-1)]*Sqrt[(b + a*Cos[e + f*x])/((a + b)*(1 + Cos[e +
f*x]))]*Sqrt[Sec[e + f*x]]*Sin[e + f*x]))/(b*f*((1 + Cos[e + f*x])^(-1))^(3
/2)*Sqrt[a + b*Sec[e + f*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 456 vs. 2(98) = 196.
time = 11.47, size = 457, normalized size = 4.27

method	result
default	$-\frac{2A\sqrt{\frac{a\cos(fx+e)+b}{\cos(fx+e)}}\,(\cos(fx+e)+1)^2(-1+\cos(fx+e))^2\left(\cos(fx+e)\,\text{EllipticE}\left(\frac{-1+\cos(fx+e)}{\sin(fx+e)},\sqrt{\frac{a-b}{a+b}}\right)\sin(fx+e)\sqrt{\frac{\cos(fx}{\cos(fx+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(f*x+e)*(A-A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2*A/f*((a*cos(f*x+e)+b)/cos(f*x+e))^(1/2)*(cos(f*x+e)+1)^2*(-1+cos(f*x+e))
^2*(cos(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b))^(1/2))*si
n(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(cos(f*x+e)+1)
/(a+b))^(1/2)*a+cos(f*x+e)*EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+b
))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(c
os(f*x+e)+1)/(a+b))^(1/2)*b+EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a+
b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/(
cos(f*x+e)+1)/(a+b))^(1/2)*a+EllipticE((-1+cos(f*x+e))/sin(f*x+e),((a-b)/(a
+b))^(1/2))*sin(f*x+e)*(cos(f*x+e)/(cos(f*x+e)+1))^(1/2)*((a*cos(f*x+e)+b)/
(cos(f*x+e)+1)/(a+b))^(1/2)*b-cos(f*x+e)^2*a+a*cos(f*x+e)-cos(f*x+e)*b+b)/s
in(f*x+e)^5/(a*cos(f*x+e)+b)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(A-A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
maxima")
```

```
[Out] -integrate((A*sec(f*x + e) - A)*sec(f*x + e)/sqrt(b*sec(f*x + e) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(A-A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
fricas")
```

```
[Out] integral(-(A*sec(f*x + e)^2 - A*sec(f*x + e))/sqrt(b*sec(f*x + e) + a), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$-A\left(\int\left(-\frac{\sec\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}}\right)dx+\int\frac{\sec^{2}\left(e+fx\right)}{\sqrt{a+b\sec\left(e+fx\right)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(A-A*sec(f*x+e))/(a+b*sec(f*x+e))**(1/2),x)
```

```
[Out] -A*(Integral(-sec(e + f*x)/sqrt(a + b*sec(e + f*x)), x) + Integral(sec(e +
f*x)**2/sqrt(a + b*sec(e + f*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(f*x+e)*(A-A*sec(f*x+e))/(a+b*sec(f*x+e))^(1/2),x, algorithm="
giac")
```

```
[Out] integrate(-(A*sec(f*x + e) - A)*sec(f*x + e)/sqrt(b*sec(f*x + e) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int\frac{A-\frac{A}{\cos(e+f\,x)}}{\cos\left(e+f\,x\right)\,\sqrt{a+\frac{b}{\cos\left(e+f\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A - A/cos(e + f*x))/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)),x)
[Out] int((A - A/cos(e + f*x))/(cos(e + f*x)*(a + b/cos(e + f*x))^(1/2)), x)
```

3.394 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=180

$$-\frac{2(5aA+3bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(Ab+aB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

```
[Out] 2/3*(A*b+B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*b*B*sec(d*x+c)^(5/2)*sin(d*
x+c)/d+2/5*(5*A*a+3*B*b)*sin(d*x+c)*sec(d*x+c)^(1/2)/d-2/5*(5*A*a+3*B*b)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(A*b+B*a)*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d
*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.12, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4082, 3872, 3853, 3856, 2719, 2720}

$$\frac{2(aB+Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2(5aA+3bB)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(5aA+3bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2bB\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*(5*a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
 + d*x]])/(5*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(5*a*A + 3*b*B)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(5*d) + (2*(A*b + a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*
b*B*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
```

```
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$
\begin{aligned}
\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx &= \frac{2bB\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2}{5}\int \sec^{\frac{3}{2}}(c+dx \\
&= \frac{2bB\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d}+(Ab+aB)\int \mathrm{sec} \\
&= \frac{2(5aA+3bB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d}+\frac{2(A}{} \\
&= \frac{2(5aA+3bB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d}+\frac{2(A}{} \\
&= -\frac{2(5aA+3bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}
\end{aligned}
$$

Mathematica [A]
time = 1.92, size = 132, normalized size = 0.73

$$
\frac{\sec^{\frac{5}{2}}(c+dx)\left(-12(5aA+3bB)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+20(Ab+aB)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+2(15(aA+bB)+10(Ab+aB)\cos(c+dx)+3(5aA+3bB)\cos(2(c+dx)))\sin(c+dx)\right)}{30d}
$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-12*(5*a*A + 3*b*B)*Cos[c + d*x]^(5/2)*EllipticE[(c +
d*x)/2, 2] + 20*(A*b + a*B)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] +
2*(15*(a*A + b*B) + 10*(A*b + a*B)*Cos[c + d*x] + 3*(5*a*A + 3*b*B)*Cos[2*(
c + d*x)])*Sin[c + d*x]))/(30*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 635 vs. $2(208) = 416$.
time = 4.52, size = 636, normalized size = 3.53

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2(Ab+Ba)\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin}}{6\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b+B*a)*(-1
/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*B*b/(8*sin(1/2*d*x+1/2*c)^6-12*si
n(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1
/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+
1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2
*A*a/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.66, size = 235, normalized size = 1.31

$$-\frac{5\sqrt{2}\left(iBa+iAb\right)\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-iBa-iAb\right)\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+3\sqrt{2}\left(5iAa+3iBb\right)\cos\left(dx+c\right)^{2}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+3\sqrt{2}\left(-5iAa-3iBb\right)\cos\left(dx+c\right)^{2}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-\frac{2\left(3\left(5Aa+3Bb\right)\cos\left(dx+c\right)^{2}+3Bb+5\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{15d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] -1/15*(5*sqrt(2)*(I*B*a + I*A*b)*cos(d*x + c)^2*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*B*a - I*A*b)*cos(d*x + c)^2*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(5*I*
A*a + 3*I*B*b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(-5*I*A*a - 3*I*B*b)*cos(d
*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(3*(5*A*a + 3*B*b)*cos(d*x + c)^2 + 3*B*b + 5*(B*a + A
*b)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^2)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2),x)
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2), x)
```

3.395 $\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=143

$$-\frac{2(Ab+aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2(3aA+bB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{3d}$$

```
[Out] 2/3*b*B*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2*(A*b+B*a)*sin(d*x+c)*sec(d*x+c)^(1/
2)/d-2*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(3*A*a+
B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.10, antiderivative size = 143, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4082, 3872, 3856, 2720, 3853, 2719}

$$\frac{2(aB+Ab)\sin(c+dx)\sqrt{\sec(c+dx)}}{d}+\frac{2(3aA+bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bB\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/d + (2*(3*a*A + b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(3*d) + (2*(A*b + a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
+ (2*b*B*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{2bB\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+\frac{2}{3}\int \sqrt{\sec(c$$

$$= \frac{2bB\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d}+(Ab+aB)\int$$

$$= \frac{2(Ab+aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+\frac{2bI}{}$$

$$= \frac{2(3aA+bB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}$$

$$= -\frac{2(Ab+aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}$$

Mathematica [A]
time = 0.92, size = 104, normalized size = 0.73

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\left(-3(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+(3aA+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{(bB+3(Ab+aB)\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(-3*(A*b + a*B)*EllipticE[(c + d*x
)/2, 2] + (3*a*A + b*B)*EllipticF[(c + d*x)/2, 2] + ((b*B + 3*(A*b + a*B)*C
os[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(179) = 358$.
time = 3.30, size = 401, normalized size = 2.80

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2Aa\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{El}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*b
*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b+B*a)/sin(1/2*d*x+1/2*c)^2/
(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.48, size = 205, normalized size = 1.43

$$\frac{\sqrt{2}\left(-3i\,Aa-i\,Bb\right)\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\sqrt{2}\left(3i\,Aa+i\,Bb\right)\cos\left(dx+c\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-3\sqrt{2}\left(i\,Ba+i\,Ab\right)\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-3\sqrt{2}\left(-i\,Ba-i\,Ab\right)\cos\left(dx+c\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+\frac{2\left(Bb+3\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*A*a - I*B*b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*A*a + I*B*b)*cos(d*x + c)*weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(I*B*a + I*
A*b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
 + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-I*B*a - I*A*b)*cos(d*x + c)*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) +
 2*(B*b + 3*(B*a + A*b)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*c
os(d*x + c))
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))(a + b\sec(c + dx))\sqrt{\sec(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*sqrt(sec(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + d\,x)}\right)\left(a + \frac{b}{\cos(c + d\,x)}\right)\sqrt{\frac{1}{\cos(c + d\,x)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2), x)
```

3.396 $\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=111

$$\frac{2(aA-bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2(Ab+aB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{d}$$

```
[Out] 2*b*B*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2*(A*a-B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/d+2*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d
```

Rubi [A]
time = 0.09, antiderivative size = 111, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4082, 3872, 3856, 2719, 2720}

$$\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(aA-bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bB\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(a*A - b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/d + (2*b*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{2bB\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+2\int \frac{\frac{1}{2}(aA-bB)+\frac{1}{2}(Ab}{\sqrt{\sec(c-}}$$

$$=\frac{2bB\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+(Ab+aB)\int \sqrt{\sec(c+d}$$

$$=\frac{2bB\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}+\Big((Ab+aB)\sqrt{\cos(c+dx}$$

$$=\frac{2(aA-bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}$$

Mathematica [A]
time = 0.30, size = 84, normalized size = 0.76

$$\frac{2\sqrt{\sec(c+dx)}\left((aA-bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+(Ab+aB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+bB\sin(c+dx)\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*((a*A - b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2] + (A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + b*B*Sin[c
 + d*x]))/d
```

Maple [A]
time = 1.55, size = 244, normalized size = 2.20

method	result

default	$-\frac{2\left(A\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,b-A\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2*(A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b-A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a-2*B*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a+B*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.82, size = 153, normalized size = 1.38

$$\frac{\frac{2\,Bb\sin(dx+c)}{\sqrt{\cos(dx+c)}}+\sqrt{2}\,(-i\,Ba-i\,Ab)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(i\,Ba+i\,Ab)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+\sqrt{2}\,(i\,Aa-i\,Bb)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+\sqrt{2}\,(-i\,Aa+i\,Bb)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] (2*B*b*sin(d*x + c)/sqrt(cos(d*x + c)) + sqrt(2)*(-I*B*a - I*A*b)*weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*B*a + I*A*b)*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + sqrt(2)*(I*A*a
- I*B*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c))) + sqrt(2)*(-I*A*a + I*B*b)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\,(a+b\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))/sqrt(sec(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm="
giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/(1/cos(c + d*x))^(1/2),x)
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/(1/cos(c + d*x))^(1/2), x)
```

3.397 $\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=115

$$\frac{2(Ab+aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2(aA+3bB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{3d}$$

```
[Out] 2/3*a*A*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+2/3*(A*a+3*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d
```

Rubi [A]

time = 0.10, antiderivative size = 115, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4081, 3872, 3856, 2719, 2720}

$$\frac{2(aA+3bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/d + (2*(a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{2}{3}\int \frac{-\frac{3}{2}(Ab+aB)-\frac{1}{2}(aA+3bB)\sec}{\sqrt{\sec(c+dx)}}\\ &= \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-(-Ab-aB)\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx-\frac{1}{3}\\ &= \frac{2aA\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\Big(\big(-Ab-aB\big)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+}\\ &= \frac{2(Ab+aB)\sqrt{\cos(c+dx)}\,E\big(\frac{1}{2}(c+dx)\big|\,2\big)\sqrt{\sec(c+dx)}}{d}\end{aligned}$$

Mathematica [A]
time = 0.27, size = 90, normalized size = 0.78

$$\frac{\sqrt{\sec(c+dx)}\left(6(Ab+aB)\sqrt{\cos(c+dx)}\,E\big(\frac{1}{2}(c+dx)\big|\,2\big)+2(aA+3bB)\sqrt{\cos(c+dx)}\,F\big(\frac{1}{2}(c+dx)\big|\,2\big)+aA\sin(2(c+dx))\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2
, 2] + 2*(a*A + 3*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*A*S
in[2*(c + d*x)]))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(153) = 306$.
time = 1.34, size = 326, normalized size = 2.83

method	result

default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2*a+A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b+3*B*b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.70, size = 156, normalized size = 1.36

$$\frac{2\,Aa\sqrt{\cos(dx+c)}\,\sin(dx+c)+\sqrt{2}\,(-i\,Aa-3i\,Bb)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(i\,Aa+3i\,Bb)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-3\sqrt{2}\,(-i\,Ba-i\,Ab)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-3\sqrt{2}\,(i\,Ba+i\,Ab)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] 1/3*(2*A*a*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-I*A*a - 3*I*B*b)*wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A*a + 3
*I*B*b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(
2)*(-I*B*a - I*A*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
```

```
*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(I*B*a + I*A*b)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))(a+b\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))/sec(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm="
giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/(1/cos(c + d*x))^(3/2),x)
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/(1/cos(c + d*x))^(3/2), x)
```

3.398 $\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=148

$$\frac{2(3aA+5bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(Ab+aB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{3d}$$

```
[Out] 2/5*a*A*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*(A*b+B*a)*sin(d*x+c)/d/sec(d*x+c)
^(1/2)+2/5*(3*A*a+5*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3
*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/
2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.11, antiderivative size = 148, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4081, 3872, 3854, 3856, 2720, 2719}

$$\frac{2(aB+Ab)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2(aB+Ab)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(3aA+5bB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*(3*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c
+ d*x]])/(5*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a*A*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
 + (2*(A*b + a*B)*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx &= \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}-\frac{2}{5}\int \frac{-\frac{5}{2}(Ab+aB)-\frac{1}{2}(3aA+5bB)\sec\\ &= \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}-(-Ab-aB)\int \frac{1}{\sec^{\frac{3}{2}}(c+dx)}\,dx-\frac{1}{5}(-3\\ &= \frac{2aA\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(Ab+aB)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{1}{3}(-Ab-aB\\ &= \frac{2(3aA+5bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{5d}\\ &= \frac{2(3aA+5bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx}}{5d}\end{aligned}$$

Mathematica [A]

time = 0.62, size = 108, normalized size = 0.73

$$\frac{\sqrt{\sec(c+dx)}\left(6(3aA+5bB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(Ab+aB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(5Ab+5aB+3aA\cos(c+dx))\sin(2(c+dx))\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(3*a*A + 5*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*
x)/2, 2] + 10*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (5
*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(180) = 360$.
time = 1.37, size = 371, normalized size = 2.51

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+(24Aa+20Ab+20Ba)\left(\sin^4\left(\frac{dx}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*a+(24*A*a+20*A*b+20*B*a)*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+(-6*A*a-10*A*b-10*B*a)*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)+5*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b-9*A*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*a+5
*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*a-15*B*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*b)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.57, size = 184, normalized size = 1.24

$$-\frac{5\sqrt{2}\,(i\,Ba+i\,Ab)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}\,(-i\,Ba-i\,Ab)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+3\sqrt{2}\,(-3i\,Aa-5i\,Bb)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+3\sqrt{2}\,(3i\,Aa+5i\,Bb)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-\frac{2\left(3\,Aa\cos(dx+c)^2+5\,(Ba+Ab)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
fricas")
```

```
[Out] -1/15*(5*sqrt(2)*(I*B*a + I*A*b)*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + 5*sqrt(2)*(-I*B*a - I*A*b)*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(-3*I*A*a - 5*I*B*b)*weierstrassZet
a(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqr
t(2)*(3*I*A*a + 5*I*B*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
cos(d*x + c) - I*sin(d*x + c))) - 2*(3*A*a*cos(d*x + c)^2 + 5*(B*a + A*b)*c
os(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))(a + b\sec(c + dx))}{\sec^{\frac{5}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))/sec(c + d*x)**(5/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+d\,x)}\right)\,\left(a + \frac{b}{\cos(c+d\,x)}\right)}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/(1/cos(c + d*x))^(5/2), x)
```

3.399 $\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=180

$$\frac{6(Ab+aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(5aA+7bB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}$$

```
[Out] 2/7*a*A*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*(A*b+B*a)*sin(d*x+c)/d/sec(d*x+c)
^(3/2)+2/21*(5*A*a+7*B*b)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+6/5*(A*b+B*a)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(5*A*a+7*B*b)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4081, 3872, 3854, 3856, 2719, 2720}

$$\frac{2(aB+Ab)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(5aA+7bB)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2(5aA+7bB)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6(aB+Ab)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (6*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(5*d) + (2*(5*a*A + 7*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2
]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a*A*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)
) + (2*(A*b + a*B)*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*(5*a*A + 7*b
*B)*Sin[c + d*x])/(21*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
```

```
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4081

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*a*Cot[e +
f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Dist[1/(d*n), Int[(d*Csc[e + f*x])^(n
 + 1)*Simp[n*(B*a + A*b) + (B*b*n + A*a*(n + 1))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx &= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}-\frac{2}{7}\int \frac{-\frac{7}{2}(Ab+aB)-\frac{1}{2}(5aA+7bB)\sec}{\sec^{\frac{5}{2}}(c+dx)}\\ &= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}-(-Ab-aB)\int \frac{1}{\sec^{\frac{5}{2}}(c+dx)}\,dx-\frac{1}{7}(-5\\ &= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2(Ab+aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(5aA+7bB}{21d\sqrt{\sec}}\\ &= \frac{2aA\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2(Ab+aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(5aA+7bB}{21d\sqrt{\sec}}\\ &= \frac{6(Ab+aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}\end{aligned}$$

Mathematica [A]
time = 1.10, size = 125, normalized size = 0.69

$$\frac{\sqrt{\sec(c+dx)}\left(252(Ab+aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(5aA+7bB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(65aA+70bB+42(Ab+aB)\cos(c+dx)+15aA\cos(2(c+dx)))\sin(2(c+dx))\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(252*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, 2] + 20*(5*a*A + 7*b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] +
(65*a*A + 70*b*B + 42*(A*b + a*B)*Cos[c + d*x] + 15*a*A*Cos[2*(c + d*x)])*S
in[2*(c + d*x)]))/(210*d)
```

Maple [A]
time = 1.59, size = 413, normalized size = 2.29

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+(-360Aa-168Ab-168Ba)\left(\sin^6\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*A*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*a+(-360*A*a-168*A*b-168*B*a)*sin(1/2*d*x
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a+168*A*b+168*B*a+140*B*b)*sin(1/2*d*x+
1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*A*a-42*A*b-42*B*a-70*B*b)*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+25*A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b+35*B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.00, size = 203, normalized size = 1.13

$$-\frac{5\sqrt{2}\,(5i\,Aa+7i\,Bb)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}\,(-5i\,Aa-7i\,Bb)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+63\sqrt{2}\,(-i\,Ba-i\,Ab)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+63\sqrt{2}\,(i\,Ba+i\,Ab)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-\frac{2\left(15\,Aa\cos(dx+c)^3+21\,(Ba+Ab)\cos(dx+c)^2+5\,(5\,Aa+7\,Bb)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="
fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(5*I*A*a + 7*I*B*b)*weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*A*a - 7*I*B*b)*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) + 63*sqrt(2)*(-I*B*a - I*A*b)*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) +
 63*sqrt(2)*(I*B*a + I*A*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*A*a*cos(d*x + c)^3 + 21*(B*a + A
*b)*cos(d*x + c)^2 + 5*(5*A*a + 7*B*b)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(
d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\,(a+b\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))/sec(c + d*x)**(7/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/(1/cos(c + d*x))^(7/2), x)
```

3.400 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=263

$$-\frac{2(5a^2A+3Ab^2+6abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(5b^2B+7a(2Ab+aB))\sqrt{\cos(c}}{}$$

```
[Out] 2/21*(5*b^2*B+7*a*(2*A*b+B*a))*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/35*b*(7*A*b+
9*B*a)*sec(d*x+c)^(5/2)*sin(d*x+c)/d+2/7*b*B*sec(d*x+c)^(5/2)*(a+b*sec(d*x+
c))*sin(d*x+c)/d+2/5*(5*A*a^2+3*A*b^2+6*B*a*b)*sin(d*x+c)*sec(d*x+c)^(1/2)/
d-2/5*(5*A*a^2+3*A*b^2+6*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d+2/21*(5*b^2*B+7*a*(2*A*b+B*a))*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(
1/2)/d
```

Rubi [A]
time = 0.25, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4111, 4132, 3853, 3856, 2720, 4131, 2719}

$$\frac{2(5a^2A+6abB+3Ab^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{2(5a^2A+6abB+3Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(7a(aB+2Ab)+5b^2B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2(7a(aB+2Ab)+5b^2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b(9aB+7Ab)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d}+\frac{2bB\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(5*b^2*B + 7*a*(2*A*b + a*B))*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*(5*a^2*A
 + 3*A*b^2 + 6*a*b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*(5*b^2*B
+ 7*a*(2*A*b + a*B))*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*b*(7*A*b
+ 9*a*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(35*d) + (2*b*B*Sec[c + d*x]^(5/2
)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(7*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{2bB\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d}$$
$$= \frac{2bB\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{7d}$$
$$= \frac{2(5b^2B+7a(2Ab+aB))\sec^{\frac{3}{2}}(c+dx)\sin(c+}{21d}$$
$$= \frac{2(5a^2A+3Ab^2+6abB)\sqrt{\sec(c+dx)}\,\sin(c+}{5d}$$
$$= \frac{2(5b^2B+7a(2Ab+aB))\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{21d}$$
$$= -\frac{2(5a^2A+3Ab^2+6abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(\right.}{5d}$$

Mathematica [A]
time = 4.37, size = 221, normalized size = 0.84

$$\frac{\sec^{\frac{7}{2}}(c+dx)\left(-168(5a^2A+3Ab^2+6abB)\cos^{\frac{7}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+40(14aAb+7a^2B+5b^2B)\cos^{\frac{7}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2(140aAb+70a^2B+110b^2B+21(15a^2A+13Ab^2+26abB)\cos(c+dx)+10(14aAb+7a^2B+5b^2B)\cos(2(c+dx))+105a^2A\cos(3(c+dx))+63Ab^2\cos(3(c+dx))+126abB\cos(3(c+dx)))\sin(c+dx)\right)}{420d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^(7/2)*(-168*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*Cos[c + d*x]^(7/2)*
EllipticE[(c + d*x)/2, 2] + 40*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Cos[c + d*x]^
(7/2)*EllipticF[(c + d*x)/2, 2] + 2*(140*a*A*b + 70*a^2*B + 110*b^2*B + 21*
(15*a^2*A + 13*A*b^2 + 26*a*b*B)*Cos[c + d*x] + 10*(14*a*A*b + 7*a^2*B + 5*
b^2*B)*Cos[2*(c + d*x)] + 105*a^2*A*Cos[3*(c + d*x)] + 63*A*b^2*Cos[3*(c +
d*x)] + 126*a*b*B*Cos[3*(c + d*x)])*Sin[c + d*x]))/(420*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 831 vs. $2(287) = 574$.
time = 6.47, size = 832, normalized size = 3.16

method	result	size
default	Expression too large to display	832

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*(2*A*b+B*a)
*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*b*(A*b+2*B*a)/(8*sin(1/2*d*x+
1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*
c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)+2*b^2*B*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/2*
c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)
^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d
*x+1/2*c),2^(1/2)))+2*a^2*A/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

<u>Maxima</u> [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

<u>Fricas</u> [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 314, normalized size = 1.19

$$-\frac{5\sqrt{2}\,(7i\,Ba^2+14i\,Aab+5i\,Bb^2)\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}\,(-7i\,Ba^2-14i\,Aab-5i\,Bb^2)\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+21\sqrt{2}\,(5i\,Aa^2+6i\,Bab+3i\,Ab^2)\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+21\sqrt{2}\,(-5i\,Aa^2-6i\,Bab-3i\,Ab^2)\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-\frac{2\left(21\,(5\,Aa^2+6\,Bab+3\,Ab^2)\cos(dx+c)^3+15\,Bb^2+5\,(7\,Ba^2+14\,Aab+5\,Bb^2)\cos(dx+c)^2+21\,(2\,Bab+Ab^2)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{105\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(7*I*B*a^2 + 14*I*A*a*b + 5*I*B*b^2)*cos(d*x + c)^3*weier
strassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-7*I*B*a^
```

```
2 - 14*I*A*a*b - 5*I*B*b^2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(5*I*A*a^2 + 6*I*B*a*b + 3*I*A*b^2)*
cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) + 21*sqrt(2)*(-5*I*A*a^2 - 6*I*B*a*b - 3*I*A*b^2)*cos
(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))) - 2*(21*(5*A*a^2 + 6*B*a*b + 3*A*b^2)*cos(d*x + c)^3 + 1
5*B*b^2 + 5*(7*B*a^2 + 14*A*a*b + 5*B*b^2)*cos(d*x + c)^2 + 21*(2*B*a*b + A
*b^2)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^2\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2),x)
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2), x)
```

3.401 $\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=221

$$-\frac{2(3b^2B+5a(2Ab+aB))\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(3a^2A+Ab^2+2abB)\sqrt{\cos(c}{}$$

```
[Out] 2/15*b*(5*A*b+7*B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*b*B*sec(d*x+c)^(3/2)
*(a+b*sec(d*x+c))*sin(d*x+c)/d+2/5*(3*b^2*B+5*a*(2*A*b+B*a))*sin(d*x+c)*sec
(d*x+c)^(1/2)/d-2/5*(3*b^2*B+5*a*(2*A*b+B*a))*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d+2/3*(3*A*a^2+A*b^2+2*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.21, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4111, 4132, 3853, 3856, 2719, 4131, 2720}

$$\frac{2(3a^2A+2abB+Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(5a(aB+2Ab)+3b^2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}-\frac{2(5a(aB+2Ab)+3b^2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b(7aB+5Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d}+\frac{2bB\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*(3*b^2*B + 5*a*(2*A*b + a*B))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(3*a^2*A + A*b^2 + 2*a*b*B)*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*(3*b^2*B +
5*a*(2*A*b + a*B))*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*b*(5*A*b + 7
*a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(15*d) + (2*b*B*Sec[c + d*x]^(3/2)*(
a + b*Sec[c + d*x])*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
```

```
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$
\begin{aligned}
\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx &= \frac{2bB\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+ \\ {5d} \\
&= \frac{2bB\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))\sin(c+ \\ {5d} \\
&= \frac{2(3b^2B+5a(2Ab+aB))\sqrt{\sec(c+dx)}\,\sin}{5d} \\
&= \frac{2(3b^2B+5a(2Ab+aB))\sqrt{\sec(c+dx)}\,\sin}{5d} \\
&= -\frac{2(3b^2B+5a(2Ab+aB))\sqrt{\cos(c+dx)}\,E}{5d}
\end{aligned}
$$

Mathematica [**A**]
time = 2.66, size = 171, normalized size = 0.77

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(-12(10aAb+5a^2B+3b^2B)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(3a^2A+Ab^2+2abB)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2(15(2aAb+a^2B+b^2B)+10b(Ab+2aB)\cos(c+dx)+3(10aAb+5a^2B+3b^2B)\cos(2(c+dx)))\sin(c+dx)\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(-12*(10*a*A*b + 5*a^2*B + 3*b^2*B)*Cos[c + d*x]^(5/2)*
EllipticE[(c + d*x)/2, 2] + 20*(3*a^2*A + A*b^2 + 2*a*b*B)*Cos[c + d*x]^(5/
2)*EllipticF[(c + d*x)/2, 2] + 2*(15*(2*a*A*b + a^2*B + b^2*B) + 10*b*(A*b
+ 2*a*B)*Cos[c + d*x] + 3*(10*a*A*b + 5*a^2*B + 3*b^2*B)*Cos[2*(c + d*x)])*
Sin[c + d*x]))/(30*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(249) = 498$.
time = 5.27, size = 723, normalized size = 3.27

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\frac{2a^2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{d}{2}\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*b
*(A*b+2*B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*b^2*B/(8*sin(1/2*
d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+
1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*
c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)+2*a*(2*A*b+B*a)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)
^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.67, size = 286, normalized size = 1.29

$$-\frac{5\sqrt{2}\left(3i\,Aa^2+2i\,Bab+i\,Ab^2\right)\cos\left(dx+c\right)^2\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-3i\,Aa^2-2i\,Bab-i\,Ab^2\right)\cos\left(dx+c\right)^2\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+3\sqrt{2}\left(5i\,Ba^2+10i\,Aab+3i\,Bb^2\right)\cos\left(dx+c\right)^2\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+3\sqrt{2}\left(-5i\,Ba^2-10i\,Aab-3i\,Bb^2\right)\cos\left(dx+c\right)^2\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-\frac{2\left(3\,Bb^2+3\left(5\,Ba^2+10\,Aab+3\,Bb^2\right)\cos\left(dx+c\right)^2+5\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{15\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] -1/15*(5*sqrt(2)*(3*I*A*a^2 + 2*I*B*a*b + I*A*b^2)*cos(d*x + c)^2*weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-3*I*A*a^2 -
2*I*B*a*b - I*A*b^2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) + 3*sqrt(2)*(5*I*B*a^2 + 10*I*A*a*b + 3*I*B*b^2)*cos(d*x
```

```
 + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) + 3*sqrt(2)*(-5*I*B*a^2 - 10*I*A*a*b - 3*I*B*b^2)*cos(d*x +
c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c))) - 2*(3*B*b^2 + 3*(5*B*a^2 + 10*A*a*b + 3*B*b^2)*cos(d*x + c)^2
+ 5*(2*B*a*b + A*b^2)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos
(d*x + c)^2)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c)), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^2\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2),x)
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2), x)
```

3.402 $\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=177

$$\frac{2(a^2A-Ab^2-2abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2(6aAb+3a^2B+b^2B)\sqrt{\cos(c+dx)}}{3d}$$

```
[Out] 2/3*b*(3*A*b+5*B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/3*b*B*(a+b*sec(d*x+c))*
sin(d*x+c)*sec(d*x+c)^(1/2)/d+2*(A*a^2-A*b^2-2*B*a*b)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(6*A*a*b+3*B*a^2+B*b^2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.18, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4111, 4132, 3856, 2720, 4131, 2719}

$$\frac{2(3a^2B+6aAb+b^2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(a^2A-2abB-Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b(5aB+3Ab)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2bB\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(a^2*A - A*b^2 - 2*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/d + (2*(6*a*A*b + 3*a^2*B + b^2*B)*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*b*(3*A*b + 5*a*B)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*b*B*Sqrt[Sec[c + d*x]]*(a + b*Sec
[c + d*x])*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{2bB\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))\sin(c+dx)}{3d} + \frac{2}{3}\int$$

$$= \frac{2bB\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))\sin(c+dx)}{3d} + \frac{2}{3}\int$$

$$= \frac{2b(3Ab+5aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3d} + \frac{2bB\sqrt{\sec(c-}}{}$$

$$= \frac{2(6aAb+3a^2B+b^2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{3d}$$

$$= \frac{2(a^2A-Ab^2-2abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{d}$$

Mathematica [A]
time = 1.26, size = 125, normalized size = 0.71

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\left(3(a^2A-Ab^2-2abB)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+(6aAb+3a^2B+b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{b(bB+3(Ab+2aB)\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]], x]

[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(3*(a^2*A - A*b^2 - 2*a*b*B)*EllipticE[(c + d*x)/2, 2] + (6*a*A*b + 3*a^2*B + b^2*B)*EllipticF[(c + d*x)/2, 2] + (b*(b*B + 3*(A*b + 2*a*B)*Cos[c + d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(211) = 422$.
time = 3.35, size = 650, normalized size = 3.67

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2a^2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,(}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*A*b*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*b^2*B*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b*(A*b+2*B*a)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.41, size = 247, normalized size = 1.40

$$\frac{\sqrt{2}\,(-3i\,Ba^2-6i\,Aab-i\,Bb^2)\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(3i\,Ba^2+6i\,Aab+i\,Bb^2)\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-3\sqrt{2}\,(-i\,Aa^2+2i\,Bab+i\,Ab^2)\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-3\sqrt{2}\,(i\,Aa^2-2i\,Bab-i\,Ab^2)\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+\frac{2\,(Bb^2+3\,(2\,Bab+Ab^2)\cos(dx+c))\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3\,d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*B*a^2 - 6*I*A*a*b - I*B*b^2)*cos(d*x + c)*weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*B*a^2 + 6*I*A*a*
b + I*B*b^2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c)) - 3*sqrt(2)*(-I*A*a^2 + 2*I*B*a*b + I*A*b^2)*cos(d*x + c)*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) -
 3*sqrt(2)*(I*A*a^2 - 2*I*B*a*b - I*A*b^2)*cos(d*x + c)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(B*b^2 +
 3*(2*B*a*b + A*b^2)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(
d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\,(a+b\sec(c+dx))^2}{\sqrt{\sec(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2/sqrt(sec(c + d*x)), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sqrt(sec(d*x + c)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/(1/cos(c + d*x))^(1/2), x
)
```

3.403 $\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=161

$$-\frac{2(b^2B-a(2Ab+aB))\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2(a^2A+3Ab^2+6abB)\sqrt{\cos(c+$$

```
[Out] 2/3*a^2*A*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2*b^2*B*sin(d*x+c)*sec(d*x+c)^(1/2)
/d-2*(b^2*B-a*(2*A*b+B*a))*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2
/3*(A*a^2+3*A*b^2+6*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.17, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4109, 4132, 3856, 2720, 4131, 2719}

$$\frac{2(a^2A+6abB+3Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{2(b^2B-a(aB+2Ab))\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b^2B\sin(c+dx)\sqrt{\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (-2*(b^2*B - a*(2*A*b + a*B))*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*
Sqrt[Sec[c + d*x]])/d + (2*(a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*E
llipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*A*Sin[c + d*x])
/(3*d*Sqrt[Sec[c + d*x]]) + (2*b^2*B*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4109

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a^2*A*Cos[
e + f*x]*((d*Csc[e + f*x])^(n + 1)/(d*f*n)), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\begin{aligned}\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx &= \frac{2a^2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{2}{3}\int \frac{-\frac{3}{2}a(2Ab+aB)+\Big(A\Big(-\frac{a^2}{2}-\frac{3b^2}{2}\\ &= \frac{2a^2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}-\frac{2}{3}\int \frac{-\frac{3}{2}a(2Ab+aB)-\frac{3}{2}b^2B\sec^2(c+}{\sqrt{\sec(c+dx)}}\\ &= \frac{2a^2A\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}+\frac{2b^2B\sqrt{\sec(c+dx)}\,\sin(c+dx)}{d}-(b^2B\\ &= \frac{2(a^2A+3Ab^2+6abB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\mathrm{se}}}{3d}\\ &= -\frac{2(b^2B-a(2Ab+aB))\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{d}\end{aligned}$$

Mathematica [A]
time = 0.77, size = 124, normalized size = 0.77

$$\frac{\sqrt{\sec(c+dx)}\left(6(2aAb+a^2B-b^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(a^2A+3Ab^2+6abB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+2(3b^2B+a^2A\cos(c+dx))\sin(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(6*(2*a*A*b + a^2*B - b^2*B)*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2] + 2*(a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[Cos[c + d*x]]*Ellip
ticF[(c + d*x)/2, 2] + 2*(3*b^2*B + a^2*A*Cos[c + d*x])*Sin[c + d*x]))/(3*d
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. $2(197) = 394$.
time = 1.82, size = 404, normalized size = 2.51

method	result
default	$-\frac{2\left(4A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2+a^2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3*(4*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^2-2*A*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2*a^2+a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*b^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-6*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-6*B*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^2*b^2+6*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*a^2+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 208, normalized size = 1.29

$$\frac{\sqrt{2}\,(-i\,Aa^2 - 6i\,Bab - 3i\,Ab^2)\mathrm{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\,\sin(dx+c)) + \sqrt{2}\,(i\,Aa^2 + 6i\,Bab + 3i\,Ab^2)\mathrm{weierstrassPInverse}(-4, 0, \cos(dx+c) - i\,\sin(dx+c)) - 3\,\sqrt{2}\,(-i\,Ba^2 - 2i\,Aab + i\,Bb^2)\mathrm{weierstrassZeta}(-4, 0, \mathrm{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\,\sin(dx+c))) - 3\,\sqrt{2}\,(i\,Ba^2 + 2i\,Aab - i\,Bb^2)\mathrm{weierstrassZeta}(-4, 0, \mathrm{weierstrassPInverse}(-4, 0, \cos(dx+c) - i\,\sin(dx+c))) + \frac{2\,(Aa^2\cos(dx+c) + 3\,Bb^2)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A*a^2 - 6*I*B*a*b - 3*I*A*b^2)*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A*a^2 + 6*I*B*a*b + 3*I*A*b^2)*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(-I*B
*a^2 - 2*I*A*a*b + I*B*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(I*B*a^2 + 2*I*A*a*b - I*B*b
^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) + 2*(A*a^2*cos(d*x + c) + 3*B*b^2)*sin(d*x + c)/sqrt(cos(d*x + c
)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c+dx))\,(a + b\sec(c+dx))^2}{\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2/sec(c + d*x)**(3/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sec(d*x + c)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A + \frac{B}{\cos(c+d\,x)}\right)\,\left(a + \frac{b}{\cos(c+d\,x)}\right)^2}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/(1/cos(c + d*x))^(3/2),x)
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/(1/cos(c + d*x))^(3/2), x
)
```

3.404 $\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=171

$$\frac{2(3a^2A+5Ab^2+10abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(2aAb+a^2B+3b^2B)\sqrt{\cos(c+\ldots}}{\ldots}$$

```
[Out] 2/5*a^2*A*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/3*a*(2*A*b+B*a)*sin(d*x+c)/d/sec(
d*x+c)^(1/2)+2/5*(3*A*a^2+5*A*b^2+10*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d+2/3*(2*A*a*b+B*a^2+3*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.18, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {4109, 4132, 3856, 2719, 4130, 2720}

$$\frac{2(a^2B+2aAb+3b^2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(3a^2A+10abB+5Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a(aB+2Ab)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(2*a*A*b + a^2*B + 3*b^2*B)*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d) + (2*a^2*A*Sin[c
 + d*x])/(5*d*Sec[c + d*x]^(3/2)) + (2*a*(2*A*b + a*B)*Sin[c + d*x])/(3*d*S
qrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4109

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a^2*A*Cos[
e + f*x]*((d*Csc[e + f*x])^(n + 1)/(d*f*n)), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2a^2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}a(2Ab+aB)+\Big(A\Big(-\frac{3a^2}{2}-\frac{5}{}$$

$$= \frac{2a^2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}a(2Ab+aB)-\frac{5}{2}b^2B\sec^2(c+}{\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2A\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}(-2aA$$

$$= \frac{2(3a^2A+5Ab^2+10abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

$$= \frac{2(3a^2A+5Ab^2+10abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}$$

Mathematica [A]

time = 0.98, size = 128, normalized size = 0.75

$$\frac{\sqrt{\sec(c+dx)}\left(6(3a^2A+5Ab^2+10abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(2aAb+a^2B+3b^2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+a(10Ab+5aB+3aA\cos(c+dx))\sin(2(c+dx))\right)}{15d}$$

Antiderivative was successfully verified.

[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2), x]

[Out] (Sqrt[Sec[c + d*x]]*(6*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 10*(2*a*A*b + a^2*B + 3*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*(10*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[2*(c + d*x)]))/(15*d)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(203) = 406$.
time = 1.79, size = 487, normalized size = 2.85

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2+\left(24a^2A+40Aba+20a^2B\right)\left(\sin^4\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETURNVERBOSE)

[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*a^2+(24*A*a^2+40*A*a*b+20*B*a^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6*A*a^2-20*A*a*b-10*B*a^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+10*A*b*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+5*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+15*b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-30*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm="maxima")

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.53, size = 226, normalized size = 1.32

$$-\frac{5\sqrt{2}\left(i\,Ba^{2}+2i\,Aab+3i\,Bb^{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-i\,Ba^{2}-2i\,Aab-3i\,Bb^{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+3\sqrt{2}\left(-3i\,Aa^{2}-10i\,Bab-5i\,Ab^{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+3\sqrt{2}\left(3i\,Aa^{2}+10i\,Bab+5i\,Ab^{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-\frac{2\left(3\,Aa^{2}\cos\left(dx+c\right)^{2}+5\left(Ba^{2}+2\,Aab\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/15*(5*sqrt(2)*(I*B*a^2 + 2*I*A*a*b + 3*I*B*b^2)*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*B*a^2 - 2*I*A*a*b - 3*I*B
*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)
*(-3*I*A*a^2 - 10*I*B*a*b - 5*I*A*b^2)*weierstrassZeta(-4, 0, weierstrassPI
nverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(3*I*A*a^2 + 10*I
*B*a*b + 5*I*A*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c))) - 2*(3*A*a^2*cos(d*x + c)^2 + 5*(B*a^2 + 2*A*a*b
)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)^{2}}{\sec^{\frac{5}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2/sec(c + d*x)**(5/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sec(d*x + c)^(5/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^2}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/(1/cos(c + d*x))^(5/2),x)
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/(1/cos(c + d*x))^(5/2), x
)
```

3.405 $\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=213

$$\frac{2(6aAb+3a^2B+5b^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(5a^2A+7Ab^2+14abB)\sqrt{\cos(c}{}$$

```
[Out] 2/7*a^2*A*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/5*a*(2*A*b+B*a)*sin(d*x+c)/d/sec(
d*x+c)^(3/2)+2/21*(5*A*a^2+7*A*b^2+14*B*a*b)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+
2/5*(6*A*a*b+3*B*a^2+5*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
d+2/21*(5*A*a^2+7*A*b^2+14*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/d
```

Rubi [A]
time = 0.20, antiderivative size = 213, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4109, 4132, 3854, 3856, 2720, 4130, 2719}

$$\frac{2(5a^2A+14abB+7Ab^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2(5a^2A+14abB+7Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(3a^2B+6aAb+5b^2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a(aB+2Ab)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(6*a*A*b + 3*a^2*B + 5*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(5*a^2*A + 7*A*b^2 + 14*a*b*B)*Sqrt[Cos[c
 + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*A*Si
n[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*a*(2*A*b + a*B)*Sin[c + d*x])/(5*
d*Sec[c + d*x]^(3/2)) + (2*(5*a^2*A + 7*A*b^2 + 14*a*b*B)*Sin[c + d*x])/(21
*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
```

```
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4109

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a^2*A*Cos[
e + f*x]*((d*Csc[e + f*x])^(n + 1)/(d*f*n)), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2a^2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}a(2Ab+aB) + \Big(A\Big(-\frac{5a^2}{2} - 7$$

$$= \frac{2a^2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}a(2Ab+aB) - \frac{7}{2}b^2B\sec^2(c+}{\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(5a^2A}{}$$

$$= \frac{2a^2A\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(5a^2A}{}$$

$$= \frac{2(6aAb+3a^2B+5b^2B)\sqrt{\cos(c+dx)}\,E\big(\frac{1}{2}(c+dx)\big|2\big)\,\sqrt{}}{5d}$$

Mathematica [A]
time = 1.49, size = 161, normalized size = 0.76

$$\frac{\sqrt{\sec(c+dx)}\Big(84(6aAb+3a^2B+5b^2B)\sqrt{\cos(c+dx)}\,E\big(\frac{1}{2}(c+dx)\big|2\big)+20(5a^2A+7Ab^2+14abB)\sqrt{\cos(c+dx)}\,F\big(\frac{1}{2}(c+dx)\big|2\big)+(42a(2Ab+aB)\cos(c+dx)+5(13a^2A+14Ab^2+28abB+3a^2A\cos(2(c+dx))))\sin(2(c+dx))\Big)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(84*(6*a*A*b + 3*a^2*B + 5*b^2*B)*Sqrt[Cos[c + d*x]]*El
lipticE[(c + d*x)/2, 2] + 20*(5*a^2*A + 7*A*b^2 + 14*a*b*B)*Sqrt[Cos[c + d*
x]]*EllipticF[(c + d*x)/2, 2] + (42*a*(2*A*b + a*B)*Cos[c + d*x] + 5*(13*a^
2*A + 14*A*b^2 + 28*a*b*B + 3*a^2*A*Cos[2*(c + d*x)]))*Sin[2*(c + d*x)]))/(
210*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(241) = 482$.
time = 1.61, size = 548, normalized size = 2.57

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\Big(240A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2+(-360a^2A-336Aba-168a^2B}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*A*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*a^2+(-360*A*a^2-336*A*a*b-168*B*a^2)*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a^2+336*A*a*b+140*A*b^2+168*B*a
^2+280*B*a*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*A*a^2-84*A*a*b-7
0*A*b^2-42*B*a^2-140*B*a*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*a^2*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+35*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-126*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*a*b+70*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*a^2-105*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.71, size = 254, normalized size = 1.19

$$-\frac{5\sqrt{2}(5iAa^2+14iBab+7iAb^2)\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{2}(-5iAa^2-14iBab-7iAb^2)\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+21\sqrt{2}(-3iBa^2-6iAab-5iBb^2)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21\sqrt{2}(3iBa^2+6iAab+5iBb^2)\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\frac{2(15Aa^2\cos(dx+c)^3+21(Ba^2+2Aab)\cos(dx+c)^2+5(5Aa^2+14Bab+7Ab^2)\cos(dx+c))\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(5*I*A*a^2 + 14*I*B*a*b + 7*I*A*b^2)*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*A*a^2 - 14*I*B*a*b
- 7*I*A*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21
*sqrt(2)*(-3*I*B*a^2 - 6*I*A*a*b - 5*I*B*b^2)*weierstrassZeta(-4, 0, weiers
trassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(3*I*B*a^
2 + 6*I*A*a*b + 5*I*B*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*A*a^2*cos(d*x + c)^3 + 21*(B*a^2
```

```
+ 2*A*a*b)*cos(d*x + c)^2 + 5*(5*A*a^2 + 14*B*a*b + 7*A*b^2)*cos(d*x + c))*
sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\,(a+b\sec(c+dx))^2}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2/sec(c + d*x)**(7/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sec(d*x + c)^(7/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^2}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/(1/cos(c + d*x))^(7/2),x)
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/(1/cos(c + d*x))^(7/2), x
)
```

3.406 $\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=254

$$\frac{2(7a^2A+9Ab^2+18abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{2(7b^2B+5a(2Ab+aB))\sqrt{\cos(c+}}{}$$

```
[Out] 2/9*a^2*A*sin(d*x+c)/d/sec(d*x+c)^(7/2)+2/7*a*(2*A*b+B*a)*sin(d*x+c)/d/sec(
d*x+c)^(5/2)+2/45*(7*A*a^2+9*A*b^2+18*B*a*b)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+
2/21*(7*b^2*B+5*a*(2*A*b+B*a))*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/15*(7*A*a^2+
9*A*b^2+18*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(7*b^
2*B+5*a*(2*A*b+B*a))*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.22, antiderivative size = 254, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4109, 4132, 3854, 3856, 2719, 4130, 2720}

$$\frac{2(7a^2A+18abB+9Ab^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(7a^2A+18abB+9Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2A\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)}+\frac{2(5a(aB+2Ab)+7b^2B)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2(5a(aB+2Ab)+7b^2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(aB+2Ab)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*(7*a^2*A + 9*A*b^2 + 18*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(7*b^2*B + 5*a*(2*A*b + a*B))*Sqrt[Cos[
c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(21*d) + (2*a^2*A*S
in[c + d*x])/(9*d*Sec[c + d*x]^(7/2)) + (2*a*(2*A*b + a*B)*Sin[c + d*x])/(7
*d*Sec[c + d*x]^(5/2)) + (2*(7*a^2*A + 9*A*b^2 + 18*a*b*B)*Sin[c + d*x])/(4
5*d*Sec[c + d*x]^(3/2)) + (2*(7*b^2*B + 5*a*(2*A*b + a*B))*Sin[c + d*x])/(2
1*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4109

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^2*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a^2*A*Cos[
e + f*x]*((d*Csc[e + f*x])^(n + 1)/(d*f*n)), x] + Dist[1/(d*n), Int[(d*Csc[
e + f*x])^(n + 1)*(a*(2*A*b + a*B)*n + (2*a*b*B*n + A*(b^2*n + a^2*(n + 1))
)*Csc[e + f*x] + b^2*B*n*Csc[e + f*x]^2), x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2a^2A\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{2}{9}\int \frac{-\frac{9}{2}a(2Ab+aB)+\Big(A\Big(-\frac{7a^2}{2}-\frac{9b^2}{2}$$

$$= \frac{2a^2A\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{2}{9}\int \frac{-\frac{9}{2}a(2Ab+aB)-\frac{9}{2}b^2B\sec^2(c+}{\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^2A\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(7a^2A+}{}$$

$$= \frac{2a^2A\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a(2Ab+aB)\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(7a^2A+}{}$$

$$= \frac{2(7a^2A+9Ab^2+18abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{15d}$$

$$= \frac{2(7a^2A+9Ab^2+18abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{15d}$$

Mathematica [A]
time = 1.89, size = 189, normalized size = 0.74

$$\frac{\sqrt{\sec(c+dx)}\left(168(7a^2A+9Ab^2+18abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+120(10aAb+5a^2B+7b^2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(7(43a^2A+36Ab^2+72abB)\cos(c+dx)+5(156aAb+78a^2B+84b^2B+18a(2Ab+aB)\cos(2(c+dx))+7a^2A\cos(3(c+dx))))\sin(2(c+dx))\right)}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*(7*a^2*A + 9*A*b^2 + 18*a*b*B)*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, 2] + 120*(10*a*A*b + 5*a^2*B + 7*b^2*B)*Sqrt[Cos[c +
 d*x]]*EllipticF[(c + d*x)/2, 2] + (7*(43*a^2*A + 36*A*b^2 + 72*a*b*B)*Cos[
c + d*x] + 5*(156*a*A*b + 78*a^2*B + 84*b^2*B + 18*a*(2*A*b + a*B)*Cos[2*(c
 + d*x)] + 7*a^2*A*Cos[3*(c + d*x)]))*Sin[2*(c + d*x)]))/(1260*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 609 vs. $2(278) = 556$.
time = 1.82, size = 610, normalized size = 2.40

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-1120A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2+\left(2240a^2A+1440Aba+720a\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*A*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*a^2+(2240*A*a^2+1440*A*a*b+720*B*a^2)
*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*A*a^2-2160*A*a*b-504*A*b^2-
1080*B*a^2-1008*B*a*b)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(952*A*a^2+1
680*A*a*b+504*A*b^2+840*B*a^2+1008*B*a*b+420*B*b^2)*sin(1/2*d*x+1/2*c)^4*co
s(1/2*d*x+1/2*c)+(-168*A*a^2-480*A*a*b-126*A*b^2-240*B*a^2-252*B*a*b-210*B*
b^2)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+150*A*b*a*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))-147*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+
75*a^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-378*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sec(d*x + c)^(9/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.92, size = 282, normalized size = 1.11

$$-\frac{15\sqrt{2}\,(5i\,Ba^2+10i\,Aab+7i\,Bb^2)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+15\sqrt{2}\,(-5i\,Ba^2-10i\,Aab-7i\,Bb^2)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+21\sqrt{2}\,(-7i\,Aa^2-18i\,Bab-9i\,Ab^2)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+21\sqrt{2}\,(7i\,Aa^2+18i\,Bab+9i\,Ab^2)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-\frac{2\left(35\,Aa^2\cos(dx+c)^4+45\,(Ba^2+2\,Aab)\cos(dx+c)^3+7\,(7\,Aa^2+18\,Bab+9\,Ab^2)\cos(dx+c)^2+15\,(5\,Ba^2+10\,Aab+7\,Bb^2)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] -1/315*(15*sqrt(2)*(5*I*B*a^2 + 10*I*A*a*b + 7*I*B*b^2)*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 15*sqrt(2)*(-5*I*B*a^2 - 10*I*A*a*
b - 7*I*B*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) +
21*sqrt(2)*(-7*I*A*a^2 - 18*I*B*a*b - 9*I*A*b^2)*weierstrassZeta(-4, 0, wei
```

```
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(7*I*A
*a^2 + 18*I*B*a*b + 9*I*A*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(35*A*a^2*cos(d*x + c)^4 + 45*(B*
a^2 + 2*A*a*b)*cos(d*x + c)^3 + 7*(7*A*a^2 + 18*B*a*b + 9*A*b^2)*cos(d*x +
c)^2 + 15*(5*B*a^2 + 10*A*a*b + 7*B*b^2)*cos(d*x + c))*sin(d*x + c)/sqrt(co
s(d*x + c)))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sec(d*x + c)^(9/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^2}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/(1/cos(c + d*x))^(9/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/(1/cos(c + d*x))^(9/2), x
)
```

3.407 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=345

$$-\frac{2(15a^3A+27aAb^2+27a^2bB+7b^3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{2(21a^2Ab+5Ab^3 \cdots}{}$$

```
[Out] 2/21*(21*A*a^2*b+5*A*b^3+7*B*a^3+15*B*a*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+
2/45*b*(27*A*a*b+22*B*a^2+7*B*b^2)*sec(d*x+c)^(5/2)*sin(d*x+c)/d+2/63*b^2*(
9*A*b+13*B*a)*sec(d*x+c)^(7/2)*sin(d*x+c)/d+2/9*b*B*sec(d*x+c)^(5/2)*(a+b*s
ec(d*x+c))^2*sin(d*x+c)/d+2/15*(15*A*a^3+27*A*a*b^2+27*B*a^2*b+7*B*b^3)*sin
(d*x+c)*sec(d*x+c)^(1/2)/d-2/15*(15*A*a^3+27*A*a*b^2+27*B*a^2*b+7*B*b^3)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),
2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(21*A*a^2*b+5*A*b^3+7*B*a
^3+15*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(si
n(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.37, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4111, 4161, 4132, 3853, 3856, 2720, 4131, 2719}

$$\frac{2b(22a^2B+27aAb+7b^2B)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{45d}+\frac{2(7a^3B+21a^2Ab+15ab^2B+5Ab^3)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2(15a^3A+27a^2bB+27aAb^2+7b^3B)\sin(c+dx)\sqrt{\sec(c+dx)}}{15d}+\frac{2(7a^3B+21a^2Ab+15ab^2B+5Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2(15a^3A+27a^2bB+27aAb^2+7b^3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2b^2(13aB+9Ab)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{63d}+\frac{2bB\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*Sqrt[Cos[c + d*x]]*Ellip
ticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(21*a^2*A*b + 5*A*b^3
+ 7*a^3*B + 15*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[S
ec[c + d*x]])/(21*d) + (2*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*(21*a^2*A*b + 5*A*b^3 + 7*a^3*B
+ 15*a*b^2*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*b*(27*a*A*b + 22
*a^2*B + 7*b^2*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(45*d) + (2*b^2*(9*A*b +
 13*a*B)*Sec[c + d*x]^(7/2)*Sin[c + d*x])/(63*d) + (2*b*B*Sec[c + d*x]^(5/2
)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(9*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4161

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
```

```
n/(f*(n + 2))), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2
) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n
 + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &
&  !LtQ[n, -1]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{2bB\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2\sin(c+d}{9d}$$

$$= \frac{2b^2(9Ab+13aB)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d} + $$

$$= \frac{2b^2(9Ab+13aB)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{63d} + $$

$$= \frac{2(21a^2Ab+5Ab^3+7a^3B+15ab^2B)\sec^{\frac{3}{2}}(c+}{21d}$$

$$= \frac{2(15a^3A+27aAb^2+27a^2bB+7b^3B)\sqrt{\sec(c}}{15d}$$

$$= \frac{2(21a^2Ab+5Ab^3+7a^3B+15ab^2B)\sqrt{\cos(c}}{21d}$$

$$= -\frac{2(15a^3A+27aAb^2+27a^2bB+7b^3B)\sqrt{\cos}}{15a}$$

Mathematica [A]
time = 6.56, size = 452, normalized size = 1.31

$$\frac{\cos^4(c+dx)\left(\frac{2(-105a^3A-189aAb^2-189a^2bB-49b^3B)E\left(\frac{1}{2}(c+dx)|2\right)}{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}+2(105a^2Ab+25Ab^3+35a^3B+75ab^2B)\sqrt{\cos(c+dx)}F\left(\frac{1}{2}(c+dx)|2\right)\sqrt{\sec(c+dx)}\right)(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{105d(b+a\cos(c+dx))^3(B+A\cos(c+dx))}+\frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))\left(\frac{2}{15}(15a^3A+27aAb^2+27a^2bB+7b^3B)\sin(c+dx)+\frac{2}{7}\sec^3(c+dx)(Ab^3\sin(c+dx)+3ab^2B\sin(c+dx))+\frac{2}{21}\sec(c+dx)(21a^2Ab\sin(c+dx)+5Ab^3\sin(c+dx)+7a^3B\sin(c+dx)+15ab^2B\sin(c+dx))+\frac{2}{45}\sec^2(c+dx)(27aAb^2\sin(c+dx)+27a^2bB\sin(c+dx)+7b^3B\sin(c+dx))+\frac{2}{9}b^3B\sec^3(c+dx)\tan(c+dx)\right)}{d(b+a\cos(c+dx))^3(B+A\cos(c+dx))\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^4*((2*(-105*a^3*A - 189*a*A*b^2 - 189*a^2*b*B - 49*b^3*B)*Ell
ipticE[(c + d*x)/2, 2])/(Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + 2*(105*a^
2*A*b + 25*A*b^3 + 35*a^3*B + 75*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x])
)/(105*d*(b + a*Cos[c + d*x])^3*(B + A*Cos[c + d*x])) + ((a + b*Sec[c + d*x
])^3*(A + B*Sec[c + d*x])*((2*(15*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 7*b^3*B
)*Sin[c + d*x])/15 + (2*Sec[c + d*x]^3*(A*b^3*Sin[c + d*x] + 3*a*b^2*B*Sin[
c + d*x]))/7 + (2*Sec[c + d*x]*(21*a^2*A*b*Sin[c + d*x] + 5*A*b^3*Sin[c + d
*x] + 7*a^3*B*Sin[c + d*x] + 15*a*b^2*B*Sin[c + d*x]))/21 + (2*Sec[c + d*x]
```

```
^2*(27*a*A*b^2*Sin[c + d*x] + 27*a^2*b*B*Sin[c + d*x] + 7*b^3*B*Sin[c + d*x
]))/45 + (2*b^3*B*Sec[c + d*x]^3*Tan[c + d*x])/9))/(d*(b + a*Cos[c + d*x])^
3*(B + A*Cos[c + d*x])*Sec[c + d*x]^(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1165 vs. $2(365) = 730$.

time = 9.21, size = 1166, normalized size = 3.38

method	result	size
default	Expression too large to display	1166

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*(3*A*b+B*
a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+6/5*b*a*(A*b+B*a)/(8*sin(1/2*d*
x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/
2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*
x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)+2*b^2*(A*b+3*B*a)*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/
2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*A*a^3/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x
+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*B*b^3*(-1/144
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(c
os(1/2*d*x+1/2*c)^2-1/2)^5-7/180*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^3-14/15*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)/(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2
*c)^2)^(1/2)+7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))-7/15*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(Ellipt
```

icF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))))/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.20, size = 401, normalized size = 1.16

$$-\frac{15\sqrt{2}\left(7i\,Ba^3+21i\,Aa^2b+15i\,Bab^2+5i\,Ab^3\right)\cos\left(dx+c\right)^4 \mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+15\sqrt{2}\left(-7i\,Ba^3-21i\,Aa^2b-15i\,Bab^2-5i\,Ab^3\right)\cos\left(dx+c\right)^4 \mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+21\sqrt{2}\left(15i\,Aa^3+27i\,Ba^2b+27i\,Aab^2+7i\,Bb^3\right)\cos\left(dx+c\right)^4 \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+21\sqrt{2}\left(-15i\,Aa^3-27i\,Ba^2b-27i\,Aab^2-7i\,Bb^3\right)\cos\left(dx+c\right)^4 \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-\frac{2\left(21\left(15Aa^3+27Ba^2b+27Aab^2+7Bb^3\right)\cos\left(dx+c\right)^4+35Bb^3+15\left(7Ba^3+21Aa^2b+15Bab^2+5Ab^3\right)\cos\left(dx+c\right)^3+7\left(27Ba^2b+27Aab^2+7Bb^3\right)\cos\left(dx+c\right)^2+45\left(3Bab^2+Ab^3\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{315\,d\cos\left(dx+c\right)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] -1/315*(15*sqrt(2)*(7*I*B*a^3 + 21*I*A*a^2*b + 15*I*B*a*b^2 + 5*I*A*b^3)*co
s(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 15
*sqrt(2)*(-7*I*B*a^3 - 21*I*A*a^2*b - 15*I*B*a*b^2 - 5*I*A*b^3)*cos(d*x + c
)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*
(15*I*A*a^3 + 27*I*B*a^2*b + 27*I*A*a*b^2 + 7*I*B*b^3)*cos(d*x + c)^4*weier
strassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
) + 21*sqrt(2)*(-15*I*A*a^3 - 27*I*B*a^2*b - 27*I*A*a*b^2 - 7*I*B*b^3)*cos(
d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) - 2*(21*(15*A*a^3 + 27*B*a^2*b + 27*A*a*b^2 + 7*B*b^3)*co
s(d*x + c)^4 + 35*B*b^3 + 15*(7*B*a^3 + 21*A*a^2*b + 15*B*a*b^2 + 5*A*b^3)*
cos(d*x + c)^3 + 7*(27*B*a^2*b + 27*A*a*b^2 + 7*B*b^3)*cos(d*x + c)^2 + 45*
(3*B*a*b^2 + A*b^3)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d
*x + c)^4)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 7316 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^3\,\left(\frac{1}{\cos\left(c+d\,x\right)}\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2),x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2), x)
```

3.408 $\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=295

$$-\frac{2(15a^2Ab+3Ab^3+5a^3B+9ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(21a^3A+21aAb^2+$$

```
[Out] 2/21*b*(21*A*a*b+18*B*a^2+5*B*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/35*b^2*(
7*A*b+11*B*a)*sec(d*x+c)^(5/2)*sin(d*x+c)/d+2/7*b*B*sec(d*x+c)^(3/2)*(a+b*s
ec(d*x+c))^2*sin(d*x+c)/d+2/5*(15*A*a^2*b+3*A*b^3+5*B*a^3+9*B*a*b^2)*sin(d*
x+c)*sec(d*x+c)^(1/2)/d-2/5*(15*A*a^2*b+3*A*b^3+5*B*a^3+9*B*a*b^2)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(21*A*a^3+21*A*a*b^2+21*B*a^2*b
+5*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.33, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4111, 4161, 4132, 3853, 3856, 2719, 4131, 2720}

$$\frac{2b(18a^2B+21aAb+5b^2B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{21d}+\frac{2(5a^3B+15a^2Ab+9ab^2B+3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2(21a^3A+21a^2bB+21aAb^2+5b^3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2(5a^3B+15a^2Ab+9ab^2B+3Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b^2(11aB+7Ab)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{35d}+\frac{2bB\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(21*a^3*A + 21*a*A*b^2 +
21*a^2*b*B + 5*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec
[c + d*x]])/(21*d) + (2*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*b*(21*a*A*b + 18*a^2*B + 5*b^2*B)*Sec
[c + d*x]^(3/2)*Sin[c + d*x])/(21*d) + (2*b^2*(7*A*b + 11*a*B)*Sec[c + d*x]
^(5/2)*Sin[c + d*x])/(35*d) + (2*b*B*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x]
)^2*Sin[c + d*x])/(7*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3853

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Csc[c + d*x])^(n - 1)/(d*(n - 1))), x] + Dist[b^2*((n - 2)/(n - 1)),
 Int[(b*Csc[c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] &
& IntegerQ[2*n]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4161

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2
) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n
 + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &
```

```
&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{2bB\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2\sin(c+$$

$$= \frac{2b^2(7Ab+11aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d}$$

$$= \frac{2b^2(7Ab+11aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{35d}$$

$$= \frac{2(15a^2Ab+3Ab^3+5a^3B+9ab^2B)\sqrt{\sec(c}}{5d}$$

$$= \frac{2(15a^2Ab+3Ab^3+5a^3B+9ab^2B)\sqrt{\sec(c}}{5d}$$

$$= -\frac{2(15a^2Ab+3Ab^3+5a^3B+9ab^2B)\sqrt{\cos}}{5d}$$

Mathematica [A]
time = 3.88, size = 225, normalized size = 0.76

$$\frac{2\sqrt{\sec(c+dx)}\left(-21(15a^2Ab+3Ab^3+5a^3B+9ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(21a^3A+21aAb^2+21a^2bB+5b^3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+21(15a^2Ab+3Ab^3+5a^3B+9ab^2B)\sin(c+dx)+5b(21aAb+21a^2B+5b^2B)\tan(c+dx)+21b^2(Ab+3aB)\sec(c+dx)\tan(c+dx)+15b^3B\sec^2(c+dx)\tan(c+dx)\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-21*(15*a^2*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 5*(21*a^3*A + 21*a*A*b^2 + 21*a
^2*b*B + 5*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 21*(15*a^2
*A*b + 3*A*b^3 + 5*a^3*B + 9*a*b^2*B)*Sin[c + d*x] + 5*b*(21*a*A*b + 21*a^2
*B + 5*b^2*B)*Tan[c + d*x] + 21*b^2*(A*b + 3*a*B)*Sec[c + d*x]*Tan[c + d*x]
 + 15*b^3*B*Sec[c + d*x]^2*Tan[c + d*x]))/(105*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 916 vs. 2(319) = 638.
time = 6.65, size = 917, normalized size = 3.11

method	result	size
default	Expression too large to display	917

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*b
*a*(A*b+B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*b^2*(A*b+3*B*a)/(
8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/si
n(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/
2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^
2*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)+2*B*b^3*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(
1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a^2*(3*A*b+B*a)/sin(1/2*d*x+1/2*c)^2/(2
*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/si
n(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.97, size = 364, normalized size = 1.23

$$-\frac{5\sqrt{2}\left(21i\,Aa^3+21i\,Ba^2b+21i\,Aab^2+5i\,Bb^3\right)\cos\left(dx+c\right)^3\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-21i\,Aa^3-21i\,Ba^2b-21i\,Aab^2-5i\,Bb^3\right)\cos\left(dx+c\right)^3\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+21\sqrt{2}\left(5i\,Ba^3+15i\,Aa^2b+9i\,Bab^2+3i\,Ab^3\right)\cos\left(dx+c\right)^3\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+21\sqrt{2}\left(-5i\,Ba^3-15i\,Aa^2b-9i\,Bab^2-3i\,Ab^3\right)\cos\left(dx+c\right)^3\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-\frac{2\left(15\,Bb^3+21\left(5\,Ba^3+15\,Aa^2b+9\,Bab^2+3\,Ab^3\right)\cos\left(dx+c\right)^3+5\left(21\,Ba^2b+21\,Aab^2+5\,Bb^3\right)\cos\left(dx+c\right)^2+21\left(3\,Bab^2+Ab^3\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{105\,d\cos\left(dx+c\right)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm ="fricas")

[Out] -1/105*(5*sqrt(2)*(21*I*A*a^3 + 21*I*B*a^2*b + 21*I*A*a*b^2 + 5*I*B*b^3)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-21*I*A*a^3 - 21*I*B*a^2*b - 21*I*A*a*b^2 - 5*I*B*b^3)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(5*I*B*a^3 + 15*I*A*a^2*b + 9*I*B*a*b^2 + 3*I*A*b^3)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(-5*I*B*a^3 - 15*I*A*a^2*b - 9*I*B*a*b^2 - 3*I*A*b^3)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(15*B*b^3 + 21*(5*B*a^3 + 15*A*a^2*b + 9*B*a*b^2 + 3*A*b^3)*cos(d*x + c)^3 + 5*(21*B*a^2*b + 21*A*a*b^2 + 5*B*b^3)*cos(d*x + c)^2 + 21*(3*B*a*b^2 + A*b^3)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x + c)^3)

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm ="giac")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c)), x)

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^3\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2),x)

[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2), x)

3.409 $\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=244

$$\frac{2(5a^3A-15aAb^2-15a^2bB-3b^3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(9a^2Ab+Ab^3+3a^3B$$

```
[Out] 2/15*b^2*(5*A*b+9*B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/5*b*(15*A*a*b+14*B*a
^2+3*B*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+2/5*b*B*(a+b*sec(d*x+c))^2*sin(d*
x+c)*sec(d*x+c)^(1/2)/d+2/5*(5*A*a^3-15*A*a*b^2-15*B*a^2*b-3*B*b^3)*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(9*A*a^2*b+A*b^3+3*B*a^3+3*B*a*
b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.31, antiderivative size = 244, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4111, 4161, 4132, 3856, 2720, 4131, 2719}

$$\frac{2b(14a^2B+15aAb+3b^2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2(3a^3B+9a^2Ab+3ab^2B+Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(5a^3A-15a^2bB-15aAb^2-3b^3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2b^2(9aB+5Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{15d}+\frac{2bB\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^2}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*(5*a^3*A - 15*a*A*b^2 - 15*a^2*b*B - 3*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(9*a^2*A*b + A*b^3 + 3*a^
3*B + 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(3*d) + (2*b*(15*a*A*b + 14*a^2*B + 3*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(5*d) + (2*b^2*(5*A*b + 9*a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(
15*d) + (2*b*B*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(5*d
)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4161

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2
) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n
 + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &
&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{2bB\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2\sin(c+dx)}{5d} + \frac{2}{5}\int \frac{(\ldots}{\ldots}$$

$$= \frac{2b^2(5Ab+9aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2bB\sqrt{\sec(c+\ldots}}{\ldots}$$

$$= \frac{2b^2(5Ab+9aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{15d} + \frac{2bB\sqrt{\sec(c+\ldots}}{\ldots}$$

$$= \frac{2b(15aAb+14a^2B+3b^2B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5d} + \frac{2\ldots}{\ldots}$$

$$= \frac{2(9a^2Ab+Ab^3+3a^3B+3ab^2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+d\ldots\right.}{3d}$$

$$= \frac{2(5a^3A-15aAb^2-15a^2bB-3b^3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c\ldots\right.}{5d}$$

Mathematica [A]
time = 2.46, size = 190, normalized size = 0.78

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(12(5a^3A-15aAb^2-15a^2bB-3b^3B)\cos^{\frac{5}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(9a^2Ab+Ab^3+3a^3B+3ab^2B)\cos^{\frac{5}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+2b(15(3aAb+3a^2B+b^2B)+10b(Ab+3aB)\cos(c+dx)+9(5aAb+5a^2B+b^2B)\cos(2(c+dx)))\sin(c+dx)\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],
x]
```

```
[Out] (Sec[c + d*x]^(5/2)*(12*(5*a^3*A - 15*a*A*b^2 - 15*a^2*b*B - 3*b^3*B)*Cos[c
 + d*x]^(5/2)*EllipticE[(c + d*x)/2, 2] + 20*(9*a^2*A*b + A*b^3 + 3*a^3*B +
 3*a*b^2*B)*Cos[c + d*x]^(5/2)*EllipticF[(c + d*x)/2, 2] + 2*b*(15*(3*a*A*b
 + 3*a^2*B + b^2*B) + 10*b*(A*b + 3*a*B)*Cos[c + d*x] + 9*(5*a*A*b + 5*a^2*
B + b^2*B)*Cos[2*(c + d*x)])*Sin[c + d*x]))/(30*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 969 vs. $2(272) = 544$.
time = 5.30, size = 970, normalized size = 3.98

method	result	size
default	Expression too large to display	970

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*A*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+6*A*a^2*b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a^3*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+
2*b^2*(A*b+3*B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*B*b^3/(8*sin
(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2
*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+
12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)+6*b*a*(A*b+B*a)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1
/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*
c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.93, size = 326, normalized size = 1.34

$$-\frac{5\sqrt{2}\left(3i\,Ba^{3}+9i\,Aa^{2}b+3i\,Bab^{2}+i\,Ab^{3}\right)\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-3i\,Ba^{3}-9i\,Aa^{2}b-3i\,Bab^{2}-i\,Ab^{3}\right)\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+3\sqrt{2}\left(-5i\,Aa^{3}+15i\,Ba^{2}b+15i\,Aab^{2}+3i\,Bb^{3}\right)\cos\left(dx+c\right)^{2}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+3\sqrt{2}\left(5i\,Aa^{3}-15i\,Ba^{2}b-15i\,Aab^{2}-3i\,Bb^{3}\right)\cos\left(dx+c\right)^{2}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-\frac{2\left(3\,Bb^{3}+9\left(5\,Ba^{2}b+5\,Aab^{2}+Bb^{3}\right)\cos\left(dx+c\right)^{2}+5\left(3\,Bab^{2}+Ab^{3}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{15\,d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/15*(5*sqrt(2)*(3*I*B*a^3 + 9*I*A*a^2*b + 3*I*B*a*b^2 + I*A*b^3)*cos(d*x
+ c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2
)*(-3*I*B*a^3 - 9*I*A*a^2*b - 3*I*B*a*b^2 - I*A*b^3)*cos(d*x + c)^2*weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(-5*I*A*a^3
+ 15*I*B*a^2*b + 15*I*A*a*b^2 + 3*I*B*b^3)*cos(d*x + c)^2*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2
)*(5*I*A*a^3 - 15*I*B*a^2*b - 15*I*A*a*b^2 - 3*I*B*b^3)*cos(d*x + c)^2*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
)) - 2*(3*B*b^3 + 9*(5*B*a^2*b + 5*A*a*b^2 + B*b^3)*cos(d*x + c)^2 + 5*(3*B
*a*b^2 + A*b^3)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(d*cos(d*x +
 c)^2)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3061 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sqrt(sec(d*x + c)), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^3}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(1/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(1/2), x
)
```

3.410 $\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=239

$$\frac{2(3a^2Ab-Ab^3+a^3B-3ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{d}+\frac{2(a^3A+9aAb^2+9a^2bB+$$

```
[Out] -2/3*b^2*(A*a-B*b)*sec(d*x+c)^(3/2)*sin(d*x+c)/d+2/3*a*A*(a+b*sec(d*x+c))^2
*sin(d*x+c)/d/sec(d*x+c)^(1/2)-2/3*b*(2*A*a^2-3*A*b^2-9*B*a*b)*sin(d*x+c)*s
ec(d*x+c)^(1/2)/d+2*(3*A*a^2*b-A*b^3+B*a^3-3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(A*a^3+9*A*a*b^2+9*B*a^2*b+B*b^3)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.33, antiderivative size = 239, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4110, 4161, 4132, 3856, 2720, 4131, 2719}

$$-\frac{2b(2a^2A-9abB-3Ab^2)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d}+\frac{2(a^3A+9a^2bB+9aAb^2+b^3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(a^3B+3a^2Ab-3ab^2B-Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}-\frac{2b^2(aA-bB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3d}+\frac{2aA\sin(c+dx)(a+b\sec(c+dx))^2}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/d + (2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b^
3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*d)
 - (2*b*(2*a^2*A - 3*A*b^2 - 9*a*b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*d
) - (2*b^2*(a*A - b*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*d) + (2*a*A*(a +
 b*Sec[c + d*x])^2*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4161

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2
) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n
 + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &
&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA(a+b\sec(c+dx))^2\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{(a+b\sec(c+$$

$$= -\frac{2b^2(aA-bB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2aA(a+b\sec(}{3d\sqrt{}}$$

$$= -\frac{2b^2(aA-bB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d} + \frac{2aA(a+b\sec(}{3d\sqrt{}}$$

$$= -\frac{2b(2a^2A-3Ab^2-9abB)\sqrt{\sec(c+dx)}\sin(c+dx)}{3d} - \frac{2}{}$$

$$= \frac{2(a^3A+9aAb^2+9a^2bB+b^3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+d\right.}{3d}$$

$$= \frac{2(3a^2Ab-Ab^3+a^3B-3ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+d\right.}{d}$$

Mathematica [A]
time = 1.98, size = 166, normalized size = 0.69

$$\frac{\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\left(6(3a^2Ab-Ab^3+a^3B-3ab^2B)E\left(\frac{1}{2}(c+dx)\middle|2\right)+2(a^3A+9aAb^2+9a^2bB+b^3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{(a^3A+2b^3B+6b^2(Ab+3aB)\cos(c+dx)+a^3A\cos(2(c+dx)))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]]*(6*(3*a^2*A*b - A*b^3 + a^3*B - 3*a*
b^2*B)*EllipticE[(c + d*x)/2, 2] + 2*(a^3*A + 9*a*A*b^2 + 9*a^2*b*B + b^3*B
)*EllipticF[(c + d*x)/2, 2] + ((a^3*A + 2*b^3*B + 6*b^2*(A*b + 3*a*B)*Cos[c
 + d*x] + a^3*A*Cos[2*(c + d*x)])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1211 vs. 2(269) = 538.
time = 3.90, size = 1212, normalized size = 5.07

method	result	size
default	Expression too large to display	1212

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/3*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/(4*sin(1/2*d*
x+1/2*c)^4-4*sin(1/2*d*x+1/2*c)^2+1)/sin(1/2*d*x+1/2*c)^3*(8*A*cos(1/2*d*x+
1/2*c)*sin(1/2*d*x+1/2*c)^6*a^3-8*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4
*a^3-12*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*b^3+2*A*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*sin(1/2*d*x+1/2*c)^2*a^3+18*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+
1/2*c)^2*a*b^2-18*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*a^2*b+6*A
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*b^3-36*B*cos(1/2*d*x+1/2*c)*
sin(1/2*d*x+1/2*c)^4*a*b^2+18*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)
^2*a^2*b+2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*b^3-6*B*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2*a^3+18*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin
(1/2*d*x+1/2*c)^2*a*b^2+2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^3+6*A
*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3
-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b-3*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*b^3+18*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a
*b^2+2*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b^3-9*B*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*a^2*b-B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^3+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^
3-9*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.78, size = 298, normalized size = 1.25

$$\frac{\sqrt{2}\left(-i\,Aa^{3}-9i\,Ba^{2}b-9i\,Aab^{2}-i\,Bb^{3}\right)\cos\left(dx+c\right){\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\sqrt{2}\left(i\,Aa^{3}+9i\,Ba^{2}b+9i\,Aab^{2}+i\,Bb^{3}\right)\cos\left(dx+c\right){\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-3\sqrt{2}\left(-i\,Ba^{3}-3i\,Aa^{2}b+3i\,Bab^{2}+i\,Ab^{3}\right)\cos\left(dx+c\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-3\sqrt{2}\left(i\,Ba^{3}+3i\,Aa^{2}b-3i\,Bab^{2}-i\,Ab^{3}\right)\cos\left(dx+c\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+\frac{2\left(Aa^{3}\cos\left(dx+c\right)^{2}+Bb^{3}+3\left(3\,Bab^{2}+Ab^{3}\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{3\,d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A*a^3 - 9*I*B*a^2*b - 9*I*A*a*b^2 - I*B*b^3)*cos(d*x + c)*
weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A*a^
3 + 9*I*B*a^2*b + 9*I*A*a*b^2 + I*B*b^3)*cos(d*x + c)*weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(-I*B*a^3 - 3*I*A*a^2*b +
3*I*B*a*b^2 + I*A*b^3)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInve
rse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(I*B*a^3 + 3*I*A*a^2
*b - 3*I*B*a*b^2 - I*A*b^3)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(A*a^3*cos(d*x + c)^2 +
 B*b^3 + 3*(3*B*a*b^2 + A*b^3)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)
))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A+B\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)^{3}}{\sec^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3/sec(c + d*x)**(3/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sec(d*x + c)^(3/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(3/2),x)
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(3/2), x
)
```

3.411 $\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=236

$$\frac{2(3a^3A+15aAb^2+15a^2bB-5b^3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(3a^2Ab+3Ab^3+a^3$$

```
[Out] 2/5*a*A*(a+b*sec(d*x+c))^2*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/15*a^2*(9*A*b+5*
B*a)*sin(d*x+c)/d/sec(d*x+c)^(1/2)-2/5*b^2*(A*a-5*B*b)*sin(d*x+c)*sec(d*x+c
)^(1/2)/d+2/5*(3*A*a^3+15*A*a*b^2+15*B*a^2*b-5*B*b^3)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/d+2/3*(3*A*a^2*b+3*A*b^3+B*a^3+9*B*a*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.30, antiderivative size = 236, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4110, 4159, 4132, 3856, 2720, 4131, 2719}

$$\frac{2a^2(5aB+9Ab)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}}+\frac{2(a^3B+3a^2Ab+9ab^2B+3Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(3a^3A+15a^2bB+15aAb^2-5b^3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}-\frac{2b^2(aA-5bB)\sin(c+dx)\sqrt{\sec(c+dx)}}{5d}+\frac{2aA\sin(c+dx)(a+b\sec(c+dx))^2}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*(3*a^3*A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(3*a^2*A*b + 3*A*b^3 + a^
3*B + 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
d*x]])/(3*d) + (2*a^2*(9*A*b + 5*a*B)*Sin[c + d*x])/(15*d*Sqrt[Sec[c + d*x]
]) - (2*b^2*(a*A - 5*b*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(5*d) + (2*a*A*(
a + b*Sec[c + d*x])^2*Sin[c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
```

```
EqQ[n^2, 1/4]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA(a+b\sec(c+dx))^2\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{(a+b\sec(c+$$

$$= \frac{2a^2(9Ab+5aB)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2aA(a+b\sec(c+dx))^2\sin}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2(9Ab+5aB)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2aA(a+b\sec(c+dx))^2\sin}{5d\sec^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2a^2(9Ab+5aB)\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} - \frac{2b^2(aA-5bB)\sqrt{\sec(c+d}}{5d}$$

$$= \frac{2(3a^2Ab+3Ab^3+a^3B+9ab^2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+\right.}{3d}$$

$$= \frac{2(3a^3A+15aAb^2+15a^2bB-5b^3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(\right.}{5d}$$

Mathematica [A]
time = 1.54, size = 172, normalized size = 0.73

$$\frac{\sqrt{\sec(c+dx)}\left(12(3a^3A+15aAb^2+15a^2bB-5b^3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(3a^2Ab+3Ab^3+a^3B+9ab^2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+2(10a^2(3Ab+aB)\cos(c+dx)+3(a^3A+10b^3B+a^3A\cos(2(c+dx))))\sin(c+dx)\right)}{30d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(12*(3*a^3*A + 15*a*A*b^2 + 15*a^2*b*B - 5*b^3*B)*Sqrt[
Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 20*(3*a^2*A*b + 3*A*b^3 + a^3*B +
 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 2*(10*a^2*(3*A*b
 + a*B)*Cos[c + d*x] + 3*(a^3*A + 10*b^3*B + a^3*A*Cos[2*(c + d*x)]))*Sin[c
 + d*x]))/(30*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 640 vs. $2(264) = 528$.
time = 2.20, size = 641, normalized size = 2.72

method	result
default	$\frac{\frac{16A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}{5}-\frac{16A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3}{5}-8A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2b-\frac{8B\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}{3}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] 2/15*(24*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*a^3-24*A*cos(1/2*d*x+1/2
*c)*sin(1/2*d*x+1/2*c)^4*a^3-60*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a
^2*b-20*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^3+6*A*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2*a^3+30*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^2
*b-15*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-15*A*b^3*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(c
os(1/2*d*x+1/2*c),2^(1/2))*a^3+45*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2+10*B*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^3+30*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^2*b^3-5*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-45*B*a*b^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+45*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-15*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
b^3)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.77, size = 270, normalized size = 1.14

$$-\frac{5\sqrt{2}\left(i\,Ba^3+3i\,Aa^2b+9i\,Bab^2+3i\,Ab^3\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-i\,Ba^3-3i\,Aa^2b-9i\,Bab^2-3i\,Ab^3\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+3\sqrt{2}\left(-3i\,Aa^3-15i\,Ba^2b-15i\,Aab^2+5i\,Bb^3\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+3\sqrt{2}\left(3i\,Aa^3+15i\,Ba^2b+15i\,Aab^2-5i\,Bb^3\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-\frac{2\left(3\,Aa^3\cos\left(dx+c\right)^2+15\,Bb^3+5\,\left(Ba^3+3\,Aa^2b\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="fricas")
```

```
[Out] -1/15*(5*sqrt(2)*(I*B*a^3 + 3*I*A*a^2*b + 9*I*B*a*b^2 + 3*I*A*b^3)*weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*B*a^3 - 3
*I*A*a^2*b - 9*I*B*a*b^2 - 3*I*A*b^3)*weierstrassPInverse(-4, 0, cos(d*x +
```

```
c) - I*sin(d*x + c)) + 3*sqrt(2)*(-3*I*A*a^3 - 15*I*B*a^2*b - 15*I*A*a*b^2
+ 5*I*B*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c))) + 3*sqrt(2)*(3*I*A*a^3 + 15*I*B*a^2*b + 15*I*A*a*b^2 -
5*I*B*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) - 2*(3*A*a^3*cos(d*x + c)^2 + 15*B*b^3 + 5*(B*a^3 + 3*A*a
^2*b)*cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))(a + b\sec(c + dx))^3}{\sec^{\frac{5}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3/sec(c + d*x)**(5/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sec(d*x + c)^(5/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+d\,x)}\right)\,\left(a + \frac{b}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(5/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(5/2), x
)
```

3.412 $\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=245

$$\frac{2(9a^2Ab+5Ab^3+3a^3B+15ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5d}+\frac{2(5a^3A+21aAb^2+21a$$

```
[Out] 2/35*a^2*(11*A*b+7*B*a)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/7*a*A*(a+b*sec(d*x+
c))^2*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/21*a*(5*A*a^2+18*A*b^2+21*B*a*b)*sin(
d*x+c)/d/sec(d*x+c)^(1/2)+2/5*(9*A*a^2*b+5*A*b^3+3*B*a^3+15*B*a*b^2)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(5*A*a^3+21*A*a*b^2+21*B*a^2*
b+21*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.31, antiderivative size = 245, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4110, 4159, 4132, 3856, 2719, 4130, 2720}

$$\frac{2a(5a^2A+21abB+18Ab^2)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2a^2(7aB+11Ab)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(5a^3A+21a^2bB+21aAb^2+21b^3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(3a^3B+9a^2Ab+15ab^2B+5Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2aA\sin(c+dx)(a+b\sec(c+dx))^2}{7d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(9*a^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sqrt[Cos[c + d*x]]*Elliptic
E[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(5*d) + (2*(5*a^3*A + 21*a*A*b^2 + 21
*a^2*b*B + 21*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(21*d) + (2*a^2*(11*A*b + 7*a*B)*Sin[c + d*x])/(35*d*Sec[c + d*x
]^(3/2)) + (2*a*(5*a^2*A + 18*A*b^2 + 21*a*b*B)*Sin[c + d*x])/(21*d*Sqrt[Se
c[c + d*x]]) + (2*a*A*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(7*d*Sec[c + d*x
]^(5/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA(a+b\sec(c+dx))^2\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{(a+b\sec(c+d}{}$$

$$= \frac{2a^2(11Ab+7aB)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+b\sec(c+dx))^2\sin}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(11Ab+7aB)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2aA(a+b\sec(c+dx))^2\sin}{7d\sec^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2a^2(11Ab+7aB)\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2a(5a^2A+18Ab^2+21abB}{21d\sqrt{\sec(c+dx}}$$

$$= \frac{2(9a^2Ab+5Ab^3+3a^3B+15ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+\right.}{5d}$$

$$= \frac{2(9a^2Ab+5Ab^3+3a^3B+15ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+\right.}{5d}$$

Mathematica [A]
time = 1.39, size = 180, normalized size = 0.73

$$\frac{\sqrt{\sec(c+dx)}\left(84(9a^2Ab+5Ab^3+3a^3B+15ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+20(5a^3A+21aAb^2+21a^2bB+21b^3B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+a(42a(3Ab+aB)\cos(c+dx)+5(13a^2A+42Ab^2+42abB+3a^2A\cos(2(c+dx))))\sin(2(c+dx))\right)}{210d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(84*(9*a^2*A*b + 5*A*b^3 + 3*a^3*B + 15*a*b^2*B)*Sqrt[C
os[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 20*(5*a^3*A + 21*a*A*b^2 + 21*a^2*
b*B + 21*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + a*(42*a*(3*A
*b + a*B)*Cos[c + d*x] + 5*(13*a^2*A + 42*A*b^2 + 42*a*b*B + 3*a^2*A*Cos[2*
(c + d*x)]))*Sin[2*(c + d*x)]))/(210*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 663 vs. $2(273) = 546$.
time = 1.90, size = 664, normalized size = 2.71

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(240A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3+\left(-360A\,a^3-504A\,a^2b-168a^3B\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*A*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*a^3+(-360*A*a^3-504*A*a^2*b-168*B*a^3)*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a^3+504*A*a^2*b+420*A*a*b^2+1
68*B*a^3+420*B*a^2*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*A*a^3-12
6*A*a^2*b-210*A*a*b^2-42*B*a^3-210*B*a^2*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*
x+1/2*c)+25*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3+105*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a
*b^2-189*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b-105*A*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^
3+105*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b+105*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^3-6
3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3-315*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.67, size = 295, normalized size = 1.20

$$-\frac{5\sqrt{2}\left(5i\,Aa^3+21i\,Ba^2b+21i\,Aab^2+21i\,Bb^3\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+5\sqrt{2}\left(-5i\,Aa^3-21i\,Ba^2b-21i\,Aab^2-21i\,Bb^3\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+21\sqrt{2}\left(-3i\,Ba^3-9i\,Aa^2b-15i\,Bab^2-5i\,Ab^3\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+21\sqrt{2}\left(3i\,Ba^3+9i\,Aa^2b+15i\,Bab^2+5i\,Ab^3\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-\frac{2\left(15\,Aa^3\cos\left(dx+c\right)^3+21\left(Ba^3+3\,Aa^2b\right)\cos\left(dx+c\right)^2+5\left(5\,Aa^3+21\,Ba^2b+21\,Aab^2\right)\cos\left(dx+c\right)\right)\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(5*I*A*a^3 + 21*I*B*a^2*b + 21*I*A*a*b^2 + 21*I*B*b^3)*we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-5*I*A
```

```
*a^3 - 21*I*B*a^2*b - 21*I*A*a*b^2 - 21*I*B*b^3)*weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(-3*I*B*a^3 - 9*I*A*a^2*b - 15
*I*B*a*b^2 - 5*I*A*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(3*I*B*a^3 + 9*I*A*a^2*b + 15*I
*B*a*b^2 + 5*I*A*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c))) - 2*(15*A*a^3*cos(d*x + c)^3 + 21*(B*a^3 + 3*A
*a^2*b)*cos(d*x + c)^2 + 5*(5*A*a^3 + 21*B*a^2*b + 21*A*a*b^2)*cos(d*x + c)
)*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))(a+b\sec(c+dx))^3}{\sec^{\frac{7}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3/sec(c + d*x)**(7/2),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sec(d*x + c)^(7/2), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(7/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(7/2), x
)
```

3.413 $\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=295

$$\frac{2(7a^3A+27aAb^2+27a^2bB+15b^3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{2(15a^2Ab+7Ab^3+$$

```
[Out] 2/63*a^2*(13*A*b+9*B*a)*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/45*a*(7*A*a^2+22*A*
b^2+27*B*a*b)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/9*a*A*(a+b*sec(d*x+c))^2*sin(
d*x+c)/d/sec(d*x+c)^(7/2)+2/21*(15*A*a^2*b+7*A*b^3+5*B*a^3+21*B*a*b^2)*sin(
d*x+c)/d/sec(d*x+c)^(1/2)+2/15*(7*A*a^3+27*A*a*b^2+27*B*a^2*b+15*B*b^3)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/21*(15*A*a^2*b+7*A*b^3+5*B*a^
3+21*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.32, antiderivative size = 295, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4110, 4159, 4132, 3854, 3856, 2720, 4130, 2719}

$$\frac{2a(7a^2A+27abB+22Ab^2)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2a^2(9aB+13Ab)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{2(5a^3B+15a^2Ab+21ab^2B+7Ab^3)\sin(c+dx)}{21d\sqrt{\sec(c+dx)}}+\frac{2(5a^3B+15a^2Ab+21ab^2B+7Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(7a^3A+27a^2bB+27aAb^2+15b^3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2aA\sin(c+dx)(a+b\sec(c+dx))^2}{9d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*(7*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 15*b^3*B)*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(15*a^2*A*b + 7*A*b^3 +
 5*a^3*B + 21*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Se
c[c + d*x]])/(21*d) + (2*a^2*(13*A*b + 9*a*B)*Sin[c + d*x])/(63*d*Sec[c + d
*x]^(5/2)) + (2*a*(7*a^2*A + 22*A*b^2 + 27*a*b*B)*Sin[c + d*x])/(45*d*Sec[c
 + d*x]^(3/2)) + (2*(15*a^2*A*b + 7*A*b^3 + 5*a^3*B + 21*a*b^2*B)*Sin[c + d
*x])/(21*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + b*Sec[c + d*x])^2*Sin[c + d*x]
)/(9*d*Sec[c + d*x]^(7/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA(a+b\sec(c+dx))^2\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{2}{9}\int \frac{(a+b\sec(c+\cdots}{\cdots}$$

$$= \frac{2a^2(13Ab+9aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+b\sec(c+dx))^2\,\mathrm{s}\cdots}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^2(13Ab+9aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2aA(a+b\sec(c+dx))^2\,\mathrm{s}\cdots}{9d\sec^{\frac{7}{2}}(c+dx)}$$

$$= \frac{2a^2(13Ab+9aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(7a^2A+22Ab^2+27ab\cdots}{45d\sec^{\frac{3}{2}}(c+\cdots}$$

$$= \frac{2a^2(13Ab+9aB)\sin(c+dx)}{63d\sec^{\frac{5}{2}}(c+dx)} + \frac{2a(7a^2A+22Ab^2+27ab\cdots}{45d\sec^{\frac{3}{2}}(c+\cdots}$$

$$= \frac{2(7a^3A+27aAb^2+27a^2bB+15b^3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}\cdots\right.}{15d}$$

Mathematica [A]
time = 2.05, size = 219, normalized size = 0.74

$$\frac{\sqrt{\sec(c+dx)}\left(168(7a^3A+27aAb^2+27a^2bB+15b^3B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+120(15a^2Ab+7Ab^3+5a^3B+21ab^2B)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+(7a(43a^2A+108Ab^2+108abB)\cos(c+dx)+5(234a^2Ab+84Ab^3+78a^3B+252ab^2B+18a^2(3Ab+aB)\cos(2(c+dx))+7a^3A\cos(3(c+dx))))\sin(2(c+dx))\right)}{1260d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),
x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(168*(7*a^3*A + 27*a*A*b^2 + 27*a^2*b*B + 15*b^3*B)*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 120*(15*a^2*A*b + 7*A*b^3 + 5*a
^3*B + 21*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (7*a*(43*
a^2*A + 108*A*b^2 + 108*a*b*B)*Cos[c + d*x] + 5*(234*a^2*A*b + 84*A*b^3 + 7
8*a^3*B + 252*a*b^2*B + 18*a^2*(3*A*b + a*B)*Cos[2*(c + d*x)] + 7*a^3*A*Cos
[3*(c + d*x)]))*Sin[2*(c + d*x)]))/(1260*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 744 vs. $2(319) = 638$.
time = 1.94, size = 745, normalized size = 2.53

method	result

default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(-1120A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^3+\left(2240A\,a^3+2160A\,a^2b+72\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-1120*A*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10*a^3+(2240*A*a^3+2160*A*a^2*b+720*B*a^
3)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x+1/2*c)+(-2072*A*a^3-3240*A*a^2*b-1512*A
*a*b^2-1080*B*a^3-1512*B*a^2*b)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(95
2*A*a^3+2520*A*a^2*b+1512*A*a*b^2+420*A*b^3+840*B*a^3+1512*B*a^2*b+1260*B*a
*b^2)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-168*A*a^3-720*A*a^2*b-378*A
*a*b^2-210*A*b^3-240*B*a^3-378*B*a^2*b-630*B*a*b^2)*sin(1/2*d*x+1/2*c)^2*co
s(1/2*d*x+1/2*c)+225*A*a^2*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/
2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+105*A*b^3*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-147*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-567*A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*a*b^2+75*a^3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+315*B*a*b^2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-567*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-315*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
b^3)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c
)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.11, size = 332, normalized size = 1.13

$$-\frac{15\sqrt{2}\left(5i\,Ba^3+15i\,Aa^2b+21i\,Bab^2+7i\,Ab^3\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+15\sqrt{2}\left(-5i\,Ba^3-15i\,Aa^2b-21i\,Bab^2-7i\,Ab^3\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+21\sqrt{2}\left(-7i\,Aa^3-27i\,Ba^2b-27i\,Aab^2-15i\,Bb^3\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+21\sqrt{2}\left(7i\,Aa^3+27i\,Ba^2b+27i\,Aab^2+15i\,Bb^3\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-\frac{2\left(35\,Aa^3\cos(dx+c)^4+45\left(Ba^3+3\,Aa^2b\right)\cos(dx+c)^3+7\left(7\,Aa^3+27\,Ba^2b+27\,Aab^2\right)\cos(dx+c)^2+15\left(5\,Ba^3+15\,Aa^2b+21\,Bab^2+7\,Ab^3\right)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{315\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="fricas")
```

```
[Out] -1/315*(15*sqrt(2)*(5*I*B*a^3 + 15*I*A*a^2*b + 21*I*B*a*b^2 + 7*I*A*b^3)*we
ierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 15*sqrt(2)*(-5*I*
B*a^3 - 15*I*A*a^2*b - 21*I*B*a*b^2 - 7*I*A*b^3)*weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(-7*I*A*a^3 - 27*I*B*a^2*b - 2
7*I*A*a*b^2 - 15*I*B*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) + 21*sqrt(2)*(7*I*A*a^3 + 27*I*B*a^2*b + 2
7*I*A*a*b^2 + 15*I*B*b^3)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c))) - 2*(35*A*a^3*cos(d*x + c)^4 + 45*(B*a^3 +
 3*A*a^2*b)*cos(d*x + c)^3 + 7*(7*A*a^3 + 27*B*a^2*b + 27*A*a*b^2)*cos(d*x
+ c)^2 + 15*(5*B*a^3 + 15*A*a^2*b + 21*B*a*b^2 + 7*A*b^3)*cos(d*x + c))*sin
(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sec(d*x + c)^(9/2), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(9/2),x)
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(9/2), x
)
```

3.414 $\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=345

$$\frac{2(21a^2Ab+9Ab^3+7a^3B+27ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{15d}+\frac{2(45a^3A+165aAb^2+}{}$$

```
[Out] 2/99*a^2*(15*A*b+11*B*a)*sin(d*x+c)/d/sec(d*x+c)^(7/2)+2/77*a*(9*A*a^2+26*A
*b^2+33*B*a*b)*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/45*(21*A*a^2*b+9*A*b^3+7*B*a
^3+27*B*a*b^2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/11*a*A*(a+b*sec(d*x+c))^2*si
n(d*x+c)/d/sec(d*x+c)^(9/2)+2/231*(45*A*a^3+165*A*a*b^2+165*B*a^2*b+77*B*b^
3)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+2/15*(21*A*a^2*b+9*A*b^3+7*B*a^3+27*B*a*b^
2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+2/231*(45*A*a^3+165*A*a*b
^2+165*B*a^2*b+77*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.37, antiderivative size = 345, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4110, 4159, 4132, 3854, 3856, 2719, 4130, 2720}

$$\frac{2a(9a^2A+33abB+26Ab^2)\sin(c+dx)}{77d\sec^{\frac{5}{2}}(c+dx)}+\frac{2a^2(11aB+15Ab)\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)}+\frac{2(7a^3B+21a^2Ab+27ab^2B+9Ab^3)\sin(c+dx)}{45d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(45a^3A+165a^2bB+165aAb^2+77b^3B)\sin(c+dx)}{231d\sqrt{\sec(c+dx)}}+\frac{2(45a^3A+165a^2bB+165aAb^2+77b^3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{231d}+\frac{2(7a^3B+21a^2Ab+27ab^2B+9Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2aA\sin(c+dx)(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(11/2),x]
```

```
[Out] (2*(21*a^2*A*b + 9*A*b^3 + 7*a^3*B + 27*a*b^2*B)*Sqrt[Cos[c + d*x]]*Ellipti
cE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(15*d) + (2*(45*a^3*A + 165*a*A*b^2
+ 165*a^2*b*B + 77*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(231*d) + (2*a^2*(15*A*b + 11*a*B)*Sin[c + d*x])/(99*d*Sec[
c + d*x]^(7/2)) + (2*a*(9*a^2*A + 26*A*b^2 + 33*a*b*B)*Sin[c + d*x])/(77*d*
Sec[c + d*x]^(5/2)) + (2*(21*a^2*A*b + 9*A*b^3 + 7*a^3*B + 27*a*b^2*B)*Sin[
c + d*x])/(45*d*Sec[c + d*x]^(3/2)) + (2*(45*a^3*A + 165*a*A*b^2 + 165*a^2*
b*B + 77*b^3*B)*Sin[c + d*x])/(231*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + b*Se
c[c + d*x])^2*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 3854

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Csc[c + d*x])^(n + 1)/(b*d*n)), x] + Dist[(n + 1)/(b^2*n), Int[(b*Csc[c +
 d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n
]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4130

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[A*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*m)), x] +
Dist[(C*m + A*(m + 1))/(b^2*m), Int[(b*Csc[e + f*x])^(m + 2), x], x] /; Fre
eQ[{b, e, f, A, C}, x] && NeQ[C*m + A*(m + 1), 0] && LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4159

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Simp[A*a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n)), x] + Di
```

```
st[1/(d*n), Int[(d*Csc[e + f*x])^(n + 1)*Simp[n*(B*a + A*b) + (n*(a*C + B*b
) + A*a*(n + 1))*Csc[e + f*x] + b*C*n*Csc[e + f*x]^2, x], x], x] /; FreeQ[{
a, b, d, e, f, A, B, C}, x] && LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^3(A+B\sec(c+dx))}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA(a+b\sec(c+dx))^2\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} - \frac{2}{11}\int \frac{(a+b\sec(c+}{}$$

$$= \frac{2a^2(15Ab+11aB)\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2aA(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+d}$$

$$= \frac{2a^2(15Ab+11aB)\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2aA(a+b\sec(c+dx))^2}{11d\sec^{\frac{9}{2}}(c+d}$$

$$= \frac{2a^2(15Ab+11aB)\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a(9a^2A+26Ab^2+33a}{77d\sec^{\frac{5}{2}}(c+}$$

$$= \frac{2a^2(15Ab+11aB)\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a(9a^2A+26Ab^2+33a}{77d\sec^{\frac{5}{2}}(c+}$$

$$= \frac{2(21a^2Ab+9Ab^3+7a^3B+27ab^2B)\sqrt{\cos(c+dx)}\,E\left(\tfrac{1}{2}(c\right.}{15d}$$

$$= \frac{2(21a^2Ab+9Ab^3+7a^3B+27ab^2B)\sqrt{\cos(c+dx)}\,E\left(\tfrac{1}{2}(c\right.}{15d}$$

Mathematica [A]
time = 3.06, size = 256, normalized size = 0.74

$$\frac{\sqrt{\sec(c+dx)}\left(3696(21a^2Ab+9Ab^3+7a^3B+27ab^2B)\sqrt{\cos(c+dx)}\,E\left(\tfrac{1}{2}(c+dx)\middle|2\right)+240(45a^3A+165aAb^2+165a^2bB+77b^3B)\sqrt{\cos(c+dx)}\,F\left(\tfrac{1}{2}(c+dx)\middle|2\right)+(154(129a^2Ab+36Ab^3+43a^3B+108ab^2B)\cos(c+dx)+180a(16a^2A+33Ab^2+33abB)\cos(2(c+dx))+770a^2(3Ab+aB)\cos(3(c+dx))+15(531a^3A+1716aAb^2+1716a^2bB+616b^3B+21a^3A\cos(4(c+dx))))\sin(2(c+dx))\right)}{27720d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(11/2)
,x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(3696*(21*a^2*A*b + 9*A*b^3 + 7*a^3*B + 27*a*b^2*B)*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + 240*(45*a^3*A + 165*a*A*b^2 + 1
65*a^2*b*B + 77*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + (154*
(129*a^2*A*b + 36*A*b^3 + 43*a^3*B + 108*a*b^2*B)*Cos[c + d*x] + 180*a*(16*
a^2*A + 33*A*b^2 + 33*a*b*B)*Cos[2*(c + d*x)] + 770*a^2*(3*A*b + a*B)*Cos[3
*(c + d*x)] + 15*(531*a^3*A + 1716*a*A*b^2 + 1716*a^2*b*B + 616*b^3*B + 21*
a^3*A*Cos[4*(c + d*x)]))*Sin[2*(c + d*x)]))/(27720*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 824 vs. $2(365) = 730$.
time = 1.80, size = 825, normalized size = 2.39

method	result	size
default	Expression too large to display	825

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x,method=_RETURNV
ERBOSE)
```

```
[Out] -2/3465*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(20160*A*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^12*a^3+(-50400*A*a^3-36960*A*a^2*b-1232
0*B*a^3)*sin(1/2*d*x+1/2*c)^10*cos(1/2*d*x+1/2*c)+(56880*A*a^3+73920*A*a^2*
b+23760*A*a*b^2+24640*B*a^3+23760*B*a^2*b)*sin(1/2*d*x+1/2*c)^8*cos(1/2*d*x
+1/2*c)+(-34920*A*a^3-68376*A*a^2*b-35640*A*a*b^2-5544*A*b^3-22792*B*a^3-35
640*B*a^2*b-16632*B*a*b^2)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(13860*A
*a^3+31416*A*a^2*b+27720*A*a*b^2+5544*A*b^3+10472*B*a^3+27720*B*a^2*b+16632
*B*a*b^2+4620*B*b^3)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-2790*A*a^3-5
544*A*a^2*b-7920*A*a*b^2-1386*A*b^3-1848*B*a^3-7920*B*a^2*b-4158*B*a*b^2-23
10*B*b^3)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+675*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*a^3+2475*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2-4851*A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*a^2*b-2079*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^3+2475*B*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*a^2*b+1155*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*b^3-1617*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*a^3-6237*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)
/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorith
m="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.65, size = 369, normalized size = 1.07

$$-\frac{15\sqrt{2}\left(45i\,Aa^3+165i\,Ba^2b+165i\,Aab^2+77i\,Bb^3\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+15\sqrt{2}\left(-45i\,Aa^3-165i\,Ba^2b-165i\,Aab^2-77i\,Bb^3\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)+231\sqrt{2}\left(-7i\,Ba^3-21i\,Aa^2b-27i\,Bab^2-9i\,Ab^3\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)+231\sqrt{2}\left(7i\,Ba^3+21i\,Aa^2b+27i\,Bab^2+9i\,Ab^3\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)-\frac{2\left(315\,Aa^3\cos(dx+c)^5+385\left(Ba^3+3\,Aa^2b\right)\cos(dx+c)^4+135\left(3\,Aa^3+11\,Ba^2b+11\,Aab^2\right)\cos(dx+c)^3+77\left(7\,Ba^3+21\,Aa^2b+27\,Bab^2+9\,Ab^3\right)\cos(dx+c)^2+15\left(45\,Aa^3+165\,Ba^2b+165\,Aab^2+77\,Bb^3\right)\cos(dx+c)\right)\sin(dx+c)}{\sqrt{\cos\left(dx+c\right)}}}{3465\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorith
m="fricas")
```

```
[Out] -1/3465*(15*sqrt(2)*(45*I*A*a^3 + 165*I*B*a^2*b + 165*I*A*a*b^2 + 77*I*B*b^
3)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 15*sqrt(2)*(
-45*I*A*a^3 - 165*I*B*a^2*b - 165*I*A*a*b^2 - 77*I*B*b^3)*weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 231*sqrt(2)*(-7*I*B*a^3 - 21*I*A
*a^2*b - 27*I*B*a*b^2 - 9*I*A*b^3)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 231*sqrt(2)*(7*I*B*a^3 + 21*I*A
*a^2*b + 27*I*B*a*b^2 + 9*I*A*b^3)*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - 2*(315*A*a^3*cos(d*x + c)^5 + 3
85*(B*a^3 + 3*A*a^2*b)*cos(d*x + c)^4 + 135*(3*A*a^3 + 11*B*a^2*b + 11*A*a*
b^2)*cos(d*x + c)^3 + 77*(7*B*a^3 + 21*A*a^2*b + 27*B*a*b^2 + 9*A*b^3)*cos(
d*x + c)^2 + 15*(45*A*a^3 + 165*B*a^2*b + 165*A*a*b^2 + 77*B*b^3)*cos(d*x +
 c))*sin(d*x + c)/sqrt(cos(d*x + c)))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**3*(A+B*sec(d*x+c))/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^3*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algorith
m="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3/sec(d*x + c)^(11/2),
x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}{\left(\frac{1}{\cos(c+d\,x)}\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(11/2),x
)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3)/(1/cos(c + d*x))^(11/2),
x)
```

3.415 $\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=277

$$\frac{2(5aAb-5a^2B-3b^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5b^3d}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c\right.}{3b^2d}$$

```
[Out] 2/3*(A*b-B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/b^2/d+2/5*B*sec(d*x+c)^(5/2)*sin(
d*x+c)/b/d-2/5*(5*A*a*b-5*B*a^2-3*B*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^3/d+
2/5*(5*A*a*b-5*B*a^2-3*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
b^3/d+2/3*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/d+2*a^
2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(
1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^3/(a+
b)/d
```

Rubi [A]
time = 0.68, antiderivative size = 277, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4118, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{2a^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}-\frac{2(-5a^2B+5aAb-3b^2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{5b^3d}+\frac{2(-5a^2B+5aAb-3b^2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d}+\frac{2(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3b^2d}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d}+\frac{2B\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{5bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(5*a*A*b - 5*a^2*B - 3*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(5*b^3*d) + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*b^2*d) + (2*a^2*(A*b - a*B)*Sqrt
[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]]
)/(b^3*(a + b)*d) - (2*(5*a*A*b - 5*a^2*B - 3*b^2*B)*Sqrt[Sec[c + d*x]]*Sin
[c + d*x])/(5*b^3*d) + (2*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b
^2*d) + (2*B*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(5*b*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4118

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d^
2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 2)/(b*f*
(m + n))), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e +
 f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m +
n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n
, 0] &&  !IGtQ[m, 1]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
```

```
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \frac{2B\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5bd} + \frac{2\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3aB}{2}+\frac{3}{2}bB\sec(c+dx)+\frac{5}{2}(A}{a+b\sec(c+dx)}}{5b}$$

$$= \frac{2(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2d} + \frac{2B\sec^{\frac{5}{2}}(c+dx)\sin(c+}{5bd}$$

$$= -\frac{2(5aAb-5a^2B-3b^2B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5b^3d} + \frac{2(Ab-}{}$$

$$= -\frac{2(5aAb-5a^2B-3b^2B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5b^3d} + \frac{2(Ab-}{}$$

$$= -\frac{2(5aAb-5a^2B-3b^2B)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{5b^3d} + \frac{2(Ab-}{}$$

$$= \frac{2a^2(Ab-aB)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^3(a+b)d}$$

$$= \frac{2(5aAb-5a^2B-3b^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{5b^3d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 664 vs. 2(277) = 554.
time = 36.98, size = 664, normalized size = 2.40

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/30*((2*(-45*a^2*A*b - 10*A*b^3 + 45*a^3*B + 19*a*b^2*B)*Cos[c + d*x]^2*(
EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[
Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c +
d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-40*a*A*b^2 + 40*
a^2*b*B + 18*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d
*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(
b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-15*a^2*A*b + 15*a^3*B + 9*a*
b^2*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2
 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[
1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -
1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), A
rcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2]
- 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x
]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 -
 Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(b^3*d) + (Sqrt[
Sec[c + d*x]]*((2*(-5*a*A*b + 5*a^2*B + 3*b^2*B)*Sin[c + d*x])/(5*b^3) + (2
*Sec[c + d*x]*(A*b*Sin[c + d*x] - a*B*Sin[c + d*x]))/(3*b^2) + (2*B*Sec[c +
 d*x]*Tan[c + d*x])/(5*b)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 757 vs. $2(327) = 654$.
time = 6.20, size = 758, normalized size = 2.74

method	result	size
default	Expression too large to display	758

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b-B*a)/b^2
*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(A*b-B*a)*a^3/b^3/(a^2-a*b)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-
b),2^(1/2))+2/5*B/b/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1
/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*
x+1/2*c)-12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/
2*c)^4*cos(1/2*d*x+1/2*c)+12*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*sin(1/2*d*x+1/2*c)^2+
8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
```

```
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)-2*(A*b-B*a)/b^3*a/sin(1/2*d
*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + b/cos(c + d*x)),x)
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + b/cos(c + d*x)), x)
```

3.416 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=210

$$-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d}+\frac{2B\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{3bd}$$

```
[Out] 2/3*B*sec(d*x+c)^(3/2)*sin(d*x+c)/b/d+2*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/
2)/b^2/d-2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/d+2/3
*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/d-2*a*(A*b-B*a)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a
+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a+b)/d
```

Rubi [A]
time = 0.46, antiderivative size = 210, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4118, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{2(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2d}-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2B\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd}+\frac{2B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(b^2*d) + (2*B*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[
c + d*x]])/(3*b*d) - (2*a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(
a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^2*(a + b)*d) + (2*(A*b - a*B
)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*d) + (2*B*Sec[c + d*x]^(3/2)*Sin[c
+ d*x])/(3*b*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4118

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d^
2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 2)/(b*f*
(m + n))), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e +
 f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m +
n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n
, 0] &&  !IGtQ[m, 1]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3bd} + \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{aB}{2}+\frac{1}{2}bB\sec(c+dx\right.}{a+b\sec(c+dx)}}{3b}$$

$$= \frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2d} + \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+}{3bd}$$

$$= \frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2d} + \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+}{3bd}$$

$$= \frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2d} + \frac{2B\sec^{\frac{3}{2}}(c+dx)\sin(c+}{3bd}$$

$$= -\frac{2a(Ab-aB)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2(a+b)d}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2d} + \frac{2}{}$$

Mathematica [A]
time = 33.70, size = 225, normalized size = 1.07

$$-\frac{\cot(c+dx)\left(-b^2B\sec^{\frac{5}{2}}(c+dx)+b^2B\cos(2(c+dx))\sec^{\frac{5}{2}}(c+dx)-6b(Ab-aB)E\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}-2(3a^2B+b^2(-3A+B)+3ab(-A+B))F\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}-6aAb\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}+6a^2B\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}\right)}{3b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] -1/3*(Cot[c + d*x]*(-(b^2*B*Sec[c + d*x]^(5/2)) + b^2*B*Cos[2*(c + d*x)]*Se
c[c + d*x]^(5/2) - 6*b*(A*b - a*B)*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1
]*Sqrt[-Tan[c + d*x]^2] - 2*(3*a^2*B + b^2*(-3*A + B) + 3*a*b*(-A + B))*Ell
ipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 6*a*A*b*Elli
pticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 6*a^
2*B*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2
]))/(b^3*d)
```

Maple [A]
time = 3.74, size = 439, normalized size = 2.09

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2B\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\right.}}{6\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b-B*a)*a^2/b^2/(a^2-a*b)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)
)+2*(A*b-B*a)/b^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3063 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x)),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x)), x)
```

3.417 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=126

$$-\frac{2B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{bd}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c}}{b(a+b)d}$$

```
[Out] 2*B*sin(d*x+c)*sec(d*x+c)^(1/2)/b/d-2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/b/d+2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)
^(1/2)/b/(a+b)/d
```

Rubi [A]

time = 0.23, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4118, 4191, 3934, 2884, 12, 3856, 2719}

$$\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}+\frac{2B\sin(c+dx)\sqrt{\sec(c+dx)}}{bd}-\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*B*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*d
) + (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2
, 2]*Sqrt[Sec[c + d*x]])/(b*(a + b)*d) + (2*B*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(b*d)
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4118

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d^
2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 2)/(b*f*
(m + n))), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e +
 f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m +
n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n
, 0] &&  !IGtQ[m, 1]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \frac{2B\sqrt{\sec(c+dx)}\,\sin(c+dx)}{bd} + \frac{2\int \frac{-\frac{aB}{2}-\frac{1}{2}bB\sec(c+dx)+\frac{1}{2}(Ab-aB)\sec^2(}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx)}}{b}$$

$$= \frac{2B\sqrt{\sec(c+dx)}\,\sin(c+dx)}{bd} + \frac{2\int -\frac{a^2B}{2\sqrt{\sec(c+dx)}}\,dx}{a^2b} + \frac{(Ab-}{}$$

$$= \frac{2B\sqrt{\sec(c+dx)}\,\sin(c+dx)}{bd} - \frac{B\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{b} + \frac{\big((Ab-}{}$$

$$= \frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{b(a+b)d} + \frac{2}{}$$

$$= -\frac{2B\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{bd} + \frac{2(Ab-aB)}{}$$

Mathematica [A]
time = 31.40, size = 123, normalized size = 0.98

$$-\frac{2\cos(2(c+dx))\csc(c+dx)\left(bBE\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+(Ab-(a+b)B)F\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+(-Ab+aB)\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)\sec(c+dx)\sqrt{-\tan^2(c+dx)}}{b^2d\left(-2+\sec^2(c+dx)\right)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*Cos[2*(c + d*x)]*Csc[c + d*x]*(b*B*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]]
, -1] + (A*b - (a + b)*B)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + (-(A*
b) + a*B)*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sec[c + d*x]*
Sqrt[-Tan[c + d*x]^2])/(b^2*d*(-2 + Sec[c + d*x]^2))
```

Maple [A]
time = 2.25, size = 298, normalized size = 2.37

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2(Ab-Ba)a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}{b\left(a^2-ba\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(A*b-B*a)/b/
(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),2*a/(a-b),2^(1/2))+2/b*B/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)
^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\sec^{\frac{3}{2}}{(c + dx)}}{a + b\sec{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**(3/2)/(a + b*sec(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x)),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x)), x)
```

3.418 $\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=101

$$\frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a(a+b)d}$$

```
[Out] 2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d-2*(A*b-B*a)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+
b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a+b)/d
```

Rubi [A]
time = 0.13, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {4123, 3856, 2720, 3934, 2884}

$$\frac{2A\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*A*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
 - (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2]*Sqrt[Sec[c + d*x]])/(a*(a + b)*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4123

```
Int[((csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) +
(A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[A/a, Int[(
d*Csc[e + f*x])^n, x], x] - Dist[(A*b - a*B)/(a*d), Int[(d*Csc[e + f*x])^(n
 + 1)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] &&
 NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \frac{A\int\sqrt{\sec(c+dx)}\,dx}{a} - \frac{(Ab-aB)\int\frac{\sec^{\frac{3}{2}}(c+dx)}{a+b\sec(c+dx)}\,dx}{a}$$
$$= \frac{\left(A\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a} - \frac{\left((Ab\right.}{}$$
$$= \frac{2A\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{2(Ab-aB)}{}$$

Mathematica [A]
time = 20.64, size = 76, normalized size = 0.75

$$\frac{2\cot(c+dx)\left(aBF\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)+(Ab-aB)\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)\sqrt{-\tan^2(c+dx)}}{abd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*Cot[c + d*x]*(a*B*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] + (A*b - a*B
)*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*Sqrt[-Tan[c + d*x]^2]
)/(a*b*d)
```

Maple [A]
time = 1.46, size = 217, normalized size = 2.15

method	result

default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(A\,\mathrm{Ellipti}\right.}{a(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*a-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+A*EllipticPi(cos(1
/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b-B*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b
),2^(1/2))*a)/a/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int\frac{(A+B\sec(c+dx))\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(sec(c + d*x))/(a + b*sec(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{a+\frac{b}{\cos(c+d\,x)}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x)),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x)), x)
```

3.419 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=149

$$\frac{2A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad}-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}$$

[Out] 2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/d-2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/d+2*b*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a+b)/d

Rubi [A]

time = 0.18, antiderivative size = 149, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {4123, 3856, 2719, 3933, 2882, 2720, 2884}

$$-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*A*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*d)
 - (2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/(a^2*d) + (2*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a +
 b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a + b)*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2882

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
```

```
b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3933

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_)), x_Symbol] :> Dist[Sqrt[d*Sin[e + f*x]]*(Sqrt[d*Csc[e + f*x]]/d), Int[S
qrt[d*Sin[e + f*x]]/(b + a*Sin[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f},
x] && NeQ[a^2 - b^2, 0]
```

Rule 4123

```
Int[((csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) +
(A_)))/(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)), x_Symbol] :> Dist[A/a, Int[(
d*Csc[e + f*x])^n, x], x] - Dist[(A*b - a*B)/(a*d), Int[(d*Csc[e + f*x])^(n
 + 1)/(a + b*Csc[e + f*x]), x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] &&
 NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}\,dx = \frac{A\int \frac{1}{\sqrt{\sec(c+dx)}}\,dx}{a} - \frac{(Ab-aB)\int \frac{\sqrt{\sec(c+dx)}}{a+b\sec(c+dx)}\,dx}{a}$$

$$= \frac{\left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\cos(c+dx)}\,dx}{a} - \frac{\Big((Ab-$$

$$= \frac{2A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{\Big((Ab-aB)$$

$$= \frac{2A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{ad} - \frac{2(Ab-aB)$$

Mathematica [A]
time = 37.12, size = 220, normalized size = 1.48

$$\frac{\cot(c+dx)\left(-aA\sec^{\frac{3}{2}}(c+dx)-aA\cos(2(c+dx))\sec^{\frac{3}{2}}(c+dx)+aA\sec^{\frac{7}{2}}(c+dx)+aA\cos(2(c+dx))\sec^{\frac{7}{2}}(c+dx)-2aAE\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}+2aAF\left(\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}-2Ab\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}+2aB\Pi\left(-\frac{b}{a};\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{-\tan^2(c+dx)}\right)}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

```
[Out] (Cot[c + d*x]*(-(a*A*Sec[c + d*x]^(3/2)) - a*A*Cos[2*(c + d*x)]*Sec[c + d*x
]^(3/2) + a*A*Sec[c + d*x]^(7/2) + a*A*Cos[2*(c + d*x)]*Sec[c + d*x]^(7/2)
- 2*a*A*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] + 2
*a*A*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2] - 2*A*
b*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*x]^2]
+ 2*a*B*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[-Tan[c + d*
x]^2]))/(a^2*d)
```

Maple [A]
time = 1.52, size = 295, normalized size = 1.98

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(A\,\text{EllipticF}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a*b-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+A*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+A*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b^2-B*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*a^2+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-B*Ellipti
cPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*a*b)/a^2/(a-b)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c)^2 + a*sec(
d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec{\left(c + dx \right)}}{\left(a + b\sec{\left(c + dx \right)}\right)\sqrt{\sec{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/((a + b*sec(c + d*x))*sqrt(sec(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sqrt(sec(d*x + c))), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos\left(c+d\,x\right)}}{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2)),x)
[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(1/2)), x)
```

3.420 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=196

$$-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d}+\frac{2(a^2A+3Ab^2-3abB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c-\right.}{3a^3d}$$

```
[Out] 2/3*A*sin(d*x+c)/a/d/sec(d*x+c)^(1/2)-2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/a^2/d+2/3*(A*a^2+3*A*b^2-3*B*a*b)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/a^3/d-2*b^2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x
+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a+b)/d
```

Rubi [A]
time = 0.30, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4119, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$-\frac{2b^2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2(a^2A-3abB+3Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d}+\frac{2A\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (-2*(A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(a^2*d) + (2*(a^2*A + 3*A*b^2 - 3*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^3*d) - (2*b^2*(A*b - a*B)*Sqrt[Co
s[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a^3*(a + b)*d) + (2*A*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4119

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*n)), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \frac{2A\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{2\int \frac{\frac{3}{2}(Ab-aB)-\frac{1}{2}aA\sec(c+dx)-\frac{1}{2}Ab\sec^2(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}dx}{3a}$$

$$= \frac{2A\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{2\int \frac{\frac{3}{2}a(Ab-aB)-\left(\frac{a^2A}{2}+\frac{3}{2}b(Ab-aB)\right)\sec(c+dx)}{\sqrt{\sec(c+dx)}}dx}{3a^3} - \frac{(b}{}$$

$$= \frac{2A\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(Ab-aB)\int \frac{1}{\sqrt{\sec(c+dx)}}dx}{a^2} + \frac{(a^2A+3Ab}{}$$

$$= -\frac{2b^2(Ab-aB)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3(a+b)d}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2d} + \frac{2(a^2}{}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 540 vs. 2(196) = 392.

time = 36.85, size = 540, normalized size = 2.76

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] -1/6*((2*(A*b - 3*a*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]]
, -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c +
d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - C
os[c + d*x]^2)) - (4*A*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c
+ d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(
(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((3*A*b - 3*a*B)*Cos[2*(c + d*
x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[A
rcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2]
- 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]
]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d
*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-
(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c +
d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqr
t[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(a*d) + (A*Sqrt[Sec[c + d*x]]*Sin[2*
(c + d*x)])/(3*a*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 821 vs. $2(256) = 512$.
time = 1.96, size = 822, normalized size = 4.19

method	result	size
default	Expression too large to display	822

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^3-4*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
)^4*a^2*b-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^3+2*A*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^2*a^2*b+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^3-A*a^2*b*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a*b^2-3*A*b^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*a^2*b-3*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*a*b^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b^3-3*B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*a^2*b+3*B*a*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*a^3+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^
(1/2))*a*b^2)/a^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))\sec^{\frac{3}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/((a + b*sec(c + d*x))*sec(c + d*x)**(3/2)), x
)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(3/2)), x)
```

3.421 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=242

$$\frac{2(3a^2A+5Ab^2-5abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{5a^3d}-\frac{2(a^2+3b^2)(Ab-aB)\sqrt{\cos(c+a}}{3a}$$

```
[Out] 2/5*A*sin(d*x+c)/a/d/sec(d*x+c)^(3/2)-2/3*(A*b-B*a)*sin(d*x+c)/a^2/d/sec(d*
x+c)^(1/2)+2/5*(3*A*a^2+5*A*b^2-5*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/a^3/d-2/3*(a^2+3*b^2)*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/a^4/d+2*b^3*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x
+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*s
ec(d*x+c)^(1/2)/a^4/(a+b)/d
```

Rubi [A]

time = 0.50, antiderivative size = 242, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4119, 4189, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{2b^3(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^4d(a+b)}-\frac{2(Ab-aB)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}}-\frac{2(a^2+3b^2)(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^4d}+\frac{2(3a^2A-5abB+5Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d}+\frac{2A\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*(3*a^2*A + 5*A*b^2 - 5*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2,
2]*Sqrt[Sec[c + d*x]])/(5*a^3*d) - (2*(a^2 + 3*b^2)*(A*b - a*B)*Sqrt[Cos[c
+ d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^4*d) + (2*b^3*(A
*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt
[Sec[c + d*x]])/(a^4*(a + b)*d) + (2*A*Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3
/2)) - (2*(A*b - a*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Sec[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4119

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*n)), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \frac{2A\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2\int \frac{\frac{5}{2}(Ab-aB)-\frac{3}{2}aA\sec(c+dx)-\frac{3}{2}Ab\sec^2(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx}{5a}$$

$$= \frac{2A\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} + \frac{4\int \frac{3}{4}(3a^2A+5Ab^2-5abB \ldots}{\ldots}$$

$$= \frac{2A\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} + \frac{4\int \frac{3}{4}a(3a^2A+5Ab^2-5ab \ldots}{\ldots}$$

$$= \frac{2A\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(Ab-aB)\sin(c+dx)}{3a^2d\sqrt{\sec(c+dx)}} - \frac{((a^2+3b^2)(Ab-a \ldots}{\ldots}$$

$$= \frac{2b^3(Ab-aB)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^4(a+b)d} + \ldots$$

$$= \frac{2(3a^2A+5Ab^2-5abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+ \ldots}}{5a^3d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 612 vs. 2(242) = 484.
time = 36.97, size = 612, normalized size = 2.53

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] ((2*(9*a^2*A + 5*A*b^2 - 5*a*b*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec
[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a +
 b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d
*x])*(1 - Cos[c + d*x]^2)) + (2*(8*a*A*b + 10*a^2*B)*Cos[c + d*x]^2*Ellipti
```

```
cPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - S
ec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2))
+ ((9*a^2*A + 15*A*b^2 - 15*a*b*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-
4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]],
-1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[A
rcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2]
+ 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x
]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c +
d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2
*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c
+ d*x]^2)))/(30*a^2*d) + (Sqrt[Sec[c + d*x]]*((A*Sin[c + d*x])/(10*a) + ((-
(A*b) + a*B)*Sin[2*(c + d*x)])/(3*a^2) + (A*Sin[3*(c + d*x)])/(10*a)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1073 vs. $2(296) = 592$.

time = 1.95, size = 1074, normalized size = 4.44

method	result	size
default	Expression too large to display	1074

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-24*A*a^4+2
4*A*a^3*b)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*A*a^4-44*A*a^3*b+20*
A*a^2*b^2+20*B*a^4-20*B*a^3*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6*
A*a^4+16*A*a^3*b-10*A*a^2*b^2-10*B*a^4+10*B*a^3*b)*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)-5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+5*A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*a^2*b^2-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+15*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*b^4-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b
-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-
15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b^4+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a
^4-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
```

```
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2
-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-15
*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)
)*a*b^3)/a^4/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x
)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec{\left(c + dx \right)}}{\left(a + b\sec{\left(c + dx \right)}\right)\sec^{\frac{5}{2}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/((a + b*sec(c + d*x))*sec(c + d*x)**(5/2)), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))*(1/cos(c + d*x))^(5/2)), x)
```

3.422 $\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=406

$$-\frac{(3a^2Ab-2Ab^3-5a^3B+4ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^3(a^2-b^2)d}-\frac{(3aAb-5a^2B+2b^2B)}{}$$

```
[Out] -1/3*(3*A*a*b-5*B*a^2+2*B*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/b^2/(a^2-b^2)/d+
a*(A*b-B*a)*sec(d*x+c)^(5/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))+(3*A
*a^2*b-2*A*b^3-5*B*a^3+4*B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^3/(a^2-b^2)
/d-(3*A*a^2*b-2*A*b^3-5*B*a^3+4*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/b^3/(a^2-b^2)/d-1/3*(3*A*a*b-5*B*a^2+2*B*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x
+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d-a*(3*A*a^2*b-5*A*b^3-5*B*a^3+7*B
*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*
d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)/b^3/(
a+b)^2/d
```

Rubi [A]
time = 0.78, antiderivative size = 406, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4114, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{a(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{(-5a^2B+3aAb+2b^2B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3b^2d(a^2-b^2)}-\frac{(-5a^2B+3aAb+2b^2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d(a^2-b^2)}+\frac{(-5a^3B+3a^2Ab+4ab^2B-2Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{b^3d(a^2-b^2)}-\frac{(-5a^3B+3a^2Ab+4ab^2B-2Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a^2-b^2)}-\frac{a(-5a^3B+3a^2Ab+7ab^2B-5Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b^3*(a^2 - b^2)*d)) - ((3*a*A*b - 5*a
^2*B + 2*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d
*x]])/(3*b^2*(a^2 - b^2)*d) - (a*(3*a^2*A*b - 5*A*b^3 - 5*a^3*B + 7*a*b^2*B
)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c +
 d*x]])/((a - b)*b^3*(a + b)^2*d) + ((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b
^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^3*(a^2 - b^2)*d) - ((3*a*A*b - 5*
a^2*B + 2*b^2*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d) + (
a*(A*b - a*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[
c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = \frac{a(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(Ab-aB)-b(A\right.}{}}{}$$

$$= -\frac{\left(3aAb-5a^2B+2b^2B\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d} + \frac{a(Ab-aB)}{b\left(a^2-b\right.}$$

$$= \frac{\left(3a^2Ab-2Ab^3-5a^3B+4ab^2B\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^3\left(a^2-b^2\right)d} - \frac{(3}{}$$

$$= \frac{\left(3a^2Ab-2Ab^3-5a^3B+4ab^2B\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^3\left(a^2-b^2\right)d} - \frac{(3}{}$$

$$= \frac{\left(3a^2Ab-2Ab^3-5a^3B+4ab^2B\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^3\left(a^2-b^2\right)d} - \frac{(3}{}$$

$$= -\frac{a\left(3a^2Ab-5Ab^3-5a^3B+7ab^2B\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+\right.}{(a-b)b^3(a+b)^2d}$$

$$= -\frac{\left(3a^2Ab-2Ab^3-5a^3B+4ab^2B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\right|2}{b^3\left(a^2-b^2\right)d}$$

Mathematica [A]

time = 37.25, size = 733, normalized size = 1.81

$$\frac{\frac{2\left(-27a^3Ab+30aAb^3+45a^4B-44a^2b^2B-4b^4B\right)\cos^2(c+dx)\left(F\left(\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-\Pi\left(-\frac{b}{a};\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)(a+b\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)}{b(b+a\cos(c+dx))(1-\cos^2(c+dx))}+\frac{2\left(-24a^2Ab^2+12Ab^4+40a^3bB-28ab^3B\right)\cos^2(c+dx)\Pi\left(-\frac{b}{a};\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(a+b\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\sin(c+dx)}{a(b+a\cos(c+dx))(1-\cos^2(c+dx))}+\frac{\left(-9a^3Ab+6aAb^3+15a^4B-12a^2b^2B\right)\cos(2(c+dx))(a+b\sec(c+dx))\left(-4ab+4ab\sec^2(c+dx)-4abE\left(\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+dx)}-2a(a-2b)F\left(\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+dx)}+2a^2\Pi\left(-\frac{b}{a};\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+dx)}-4b^2\Pi\left(-\frac{b}{a};\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+dx)}\right)\sin(c+dx)}{a^2b(b+a\cos(c+dx))(1-\cos^2(c+dx))\sqrt{\sec(c+dx)}\left(2-\sec^2(c+dx)\right)}}{12(a-b)b^3(a+b)d}+\frac{\sqrt{\sec(c+dx)}\left(\frac{\left(-3a^2Ab+2Ab^3+5a^3B-4ab^2B\right)\sin(c+dx)}{b^3(-a^2+b^2)}+\frac{a^2Ab\sin(c+dx)-a^3B\sin(c+dx)}{b^2(-a^2+b^2)(b+a\cos(c+dx))}+\frac{2B\tan(c+dx)}{3b^2}\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,
x]
```

```
[Out] ((2*(-27*a^3*A*b + 30*a*A*b^3 + 45*a^4*B - 44*a^2*b^2*B - 4*b^4*B)*Cos[c +
d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcS
in[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*
Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-24*a^2*A
*b^2 + 12*A*b^4 + 40*a^3*b*B - 28*a*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(b/a)
, ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x
]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-9*a^3
*A*b + 6*a*A*b^3 + 15*a^4*B - 12*a^2*b^2*B)*Cos[2*(c + d*x)]*(a + b*Sec[c +
 d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*El
lipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c +
 d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Se
c[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt
[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d
*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2
 - Sec[c + d*x]^2)))/(12*(a - b)*b^3*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(((-3
*a^2*A*b + 2*A*b^3 + 5*a^3*B - 4*a*b^2*B)*Sin[c + d*x])/(b^3*(-a^2 + b^2))
+ (a^2*A*b*Sin[c + d*x] - a^3*B*Sin[c + d*x])/(b^2*(-a^2 + b^2)*(b + a*Cos[
c + d*x])) + (2*B*Tan[c + d*x])/(3*b^2)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 996 vs. 2(462) = 924.
time = 8.39, size = 997, normalized size = 2.46

method	result	size
default	Expression too large to display	997

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^2*(-1/6*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a^2*(A*b-2*B*a)/b^3/(a^2-a*b)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
```

```
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(
1/2))+2*(A*b-2*B*a)/b^3/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(A*b-B*a)*a/b^2*(1/b*a^2/(a^2
-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2
/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-
b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + b/cos(c + d*x))^2,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + b/cos(c + d*x))^2, x
)
```

3.423 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=315

$$\frac{(aAb-3a^2B+2b^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b^2(a^2-b^2)\,d}+\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+d\right.}{b(a^2-b^2)\,d}$$

```
[Out] a*(A*b-B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))-(A*a
*b-3*B*a^2+2*B*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d+(A*a*b-3*B*
a^2+2*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)/d+(
A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d+(A*a^2*
b-3*A*b^3-3*B*a^3+5*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/(a-b)/b^2/(a+b)^2/d
```

Rubi [A]
time = 0.55, antiderivative size = 315, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4114, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{a(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd(a^2-b^2)(a+b\sec(c+dx))}-\frac{(-3a^2B+aAb+2b^2B)\sin(c+dx)\sqrt{\sec(c+dx)}}{b^2d(a^2-b^2)}+\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd(a^2-b^2)}+\frac{(-3a^2B+aAb+2b^2B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a^2-b^2)}+\frac{(-3a^3B+a^2Ab+5ab^2B-3Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((a*A*b - 3*a^2*B + 2*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(b^2*(a^2 - b^2)*d) + ((A*b - a*B)*Sqrt[Cos[c + d*x]]*El
lipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b*(a^2 - b^2)*d) + ((a^2*A*b -
 3*A*b^3 - 3*a^3*B + 5*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b)
, (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/((a - b)*b^2*(a + b)^2*d) - ((a*A*b -
 3*a^2*B + 2*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d) +
(a*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec
[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
```

```
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = \frac{a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \sqrt{\sec(c+dx)}\,\left(\frac{1}{2}a(Ab-a\right.}{}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2\left(a^2-b^2\right)d} + \frac{a(Ab-aB}{b\left(a^2-\right.}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2\left(a^2-b^2\right)d} + \frac{a(Ab-aB}{b\left(a^2-\right.}$$

$$= -\frac{\left(aAb-3a^2B+2b^2B\right)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b^2\left(a^2-b^2\right)d} + \frac{a(Ab-aB}{b\left(a^2-\right.}$$

$$= \frac{\left(a^2Ab-3Ab^3-3a^3B+5ab^2B\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\right.}{(a-b)b^2(a+b)^2d}$$

$$= \frac{\left(aAb-3a^2B+2b^2B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+}}{b^2\left(a^2-b^2\right)d}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 680 vs. 2(315) = 630.
time = 37.06, size = 680, normalized size = 2.16

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,
x]
```

```
[Out] -1/4*((2*(-3*a^2*A*b + 4*A*b^3 + 9*a^3*B - 10*a*b^2*B)*Cos[c + d*x]^2*(Elli
pticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[
c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x]
)/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-4*a*A*b^2 + 8*a^2*b*
B - 4*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]],
-1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*C
os[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-(a^2*A*b) + 3*a^3*B - 2*a*b^2*B)*Co
s[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*
EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c
 + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[S
ec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqr
t[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*E
llipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1
 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c +
d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/((a - b)*b^2*(a + b)*d) +
 (Sqrt[Sec[c + d*x]]*(((a*A*b - 3*a^2*B + 2*b^2*B)*Sin[c + d*x])/(b^2*(-a^2
 + b^2)) + (-(a*A*b*Sin[c + d*x]) + a^2*B*Sin[c + d*x])/(b*(-a^2 + b^2)*(b
+ a*Cos[c + d*x]))))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 849 vs. $2(377) = 754$.
time = 4.35, size = 850, normalized size = 2.70

method	result	size
default	Expression too large to display	850

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*B/b^2/(a^
2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2*a/(a-b),2^(1/2))+2*B/b^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-
1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b-B*a)/b*(1/b*a^2/(a
^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1
```

```
/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^
2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3066 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x))^2,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x))^2, x
)
```

3.424 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=257

$$-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{b\left(a^2-b^2\right)d}-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{s}}{a\left(a^2-b^2\right)d}$$

```
[Out] a*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))-(A*b
-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x
+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d-(A*b-B*a)*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c
),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d+(A*a^2*b+A*b^3+B
*a^3-3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(
sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(
a-b)/b/(a+b)^2/d
```

Rubi [A]
time = 0.35, antiderivative size = 257, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4114, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)(a+b\sec(c+dx))}-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}+\frac{\left(a^3B+a^2Ab-3ab^2B+Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*
x]])/(b*(a^2 - b^2)*d)) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*(a^2 - b^2)*d) + ((a^2*A*b + A*b^3 + a^3*B
- 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(a*(a - b)*b*(a + b)^2*d) + (a*(A*b - a*B)*Sqrt[Sec[c +
d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{-\frac{1}{2}a(Ab-aB)-b(Ab-aB)\,\mathrm{se}}{\sqrt{\sec(c}}}{}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} + \frac{\int \frac{-\frac{1}{2}a^2(Ab-aB)-\frac{1}{2}ab(Ab-a}{\sqrt{\sec(c+dx}}}{a^2b\left(a^2-b^2\right.}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\left(a^2-b^2\right)d(a+b\sec(c+dx))} - \frac{(Ab-aB)\int \sqrt{\sec(c+}}{2a\left(a^2-b^2\right)}$$

$$= \frac{\left(a^2Ab+Ab^3+a^3B-3ab^2B\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|\,2\right.}{a(a-b)b(a+b)^2d}$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+dx)}}{b\left(a^2-b^2\right)d} - \frac{(A}{}$$

Mathematica [**B**] Leaf count is larger than twice the leaf count of optimal. 638 vs. 2(257) = 514.
time = 36.86, size = 638, normalized size = 2.48

$$\frac{\frac{2\left(-aAb-3a^2B+4b^2B\right)\cos^2(c+dx)\left(F\left(\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)-\Pi\left(-\frac{b}{a};\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\right)(a+b\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\,\sin(c+dx)}{b(b+a\cos(c+dx))\left(1-\cos^2(c+dx)\right)}+\frac{2\left(4Ab^2-4abB\right)\cos^2(c+dx)\,\Pi\left(-\frac{b}{a};\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)(a+b\sec(c+dx))\sqrt{1-\sec^2(c+dx)}\,\sin(c+dx)}{a(b+a\cos(c+dx))\left(1-\cos^2(c+dx)\right)}+\frac{\left(aAb-a^2B\right)\cos(2(c+dx))(a+b\sec(c+dx))\left(-4ab+4ab\sec^2(c+dx)-4abE\left(\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+dx)}-2a(a-2b)F\left(\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+dx)}+2a^2\Pi\left(-\frac{b}{a};\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+dx)}-4b^2\Pi\left(-\frac{b}{a};\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\middle|-1\right)\sqrt{\sec(c+dx)}\sqrt{1-\sec^2(c+dx)}\right)\sin(c+dx)}{a^2b(b+a\cos(c+dx))\left(1-\cos^2(c+dx)\right)\sqrt{\sec(c+dx)}\left(2-\sec^2(c+dx)\right)}}{4b(-a+b)(a+b)d}+\frac{\sqrt{\sec(c+dx)}\left(-\frac{(Ab-aB)\sin(c+dx)}{b\left(-a^2+b^2\right)}+\frac{Ab\sin(c+dx)-aB\sin(c+dx)}{\left(-a^2+b^2\right)(b+a\cos(c+dx))}\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,
x]
```

```
[Out] ((2*(-(a*A*b) - 3*a^2*B + 4*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Se
c[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a
+ b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c +
d*x])*(1 - Cos[c + d*x]^2)) + (2*(4*A*b^2 - 4*a*b*B)*Cos[c + d*x]^2*Ellipti
cPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - S
ec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2))
+ ((a*A*b - a^2*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Se
c[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c +
 d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c
 + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*Elliptic
Pi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[
c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt
[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c
+ d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(4*b
```

```
*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(-(((A*b - a*B)*Sin[c + d*x])/(b
*(-a^2 + b^2))) + (A*b*Sin[c + d*x] - a*B*Sin[c + d*x])/((-a^2 + b^2)*(b +
a*Cos[c + d*x]))))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 714 vs. $2(321) = 642$.

time = 3.53, size = 715, normalized size = 2.78

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{E}}{(a^2-ba)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*A/(a^2-a*b)*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/
(a-b),2^(1/2))+2*(-A*b+B*a)/a*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b
)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^
2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\sec^{\frac{3}{2}}(c+dx)}{(a+b\sec(c+dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sec(c + d*x)**(3/2)/(a + b*sec(c + d*x))**2,
x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x))^2,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x))^2, x
)
```

3.425 $\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=263

$$\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a\left(a^2-b^2\right)d}+\frac{\left(2a^2A-Ab^2-abB\right)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx\right.}{a^2\left(a^2-b^2\right)d}$$

```
[Out] -(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))+(A*b-B*
a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d+(2*A*a^2-A*b^
2-B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)/d-(3*A*
a^2*b-A*b^3-B*a^3-B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/a^2/(a-b)/(a+b)^2/d
```

Rubi [A]
time = 0.33, antiderivative size = 263, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4112, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)(a+b\sec(c+dx))}+\frac{\left(2a^2A-abB-Ab^2\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}+\frac{(Ab-aB)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{\left(a^3(-B)+3a^2Ab-ab^2B-Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((A*b - a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]
])/(a*(a^2 - b^2)*d) + ((2*a^2*A - A*b^2 - a*b*B)*Sqrt[Cos[c + d*x]]*Ellipt
icF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d) - ((3*a^2*A*b -
 A*b^3 - a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c +
 d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^2*(a - b)*(a + b)^2*d) - ((A*b - a*B)*Sq
rt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4112

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-d)*(A
*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*
Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) +
 d*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))} + \frac{\int \frac{\frac{1}{2}(-Ab+aB)-(aA-bB)\ldots}{\sqrt{\sec(c+\ldots}}\ldots}{\ldots}$$

$$= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{\int \frac{\frac{1}{2}a(-Ab+aB)-\left(\frac{1}{2}b(-Ab\ldots\right.}{\sqrt{\sec\ldots}}\ldots}{a^2\ldots}$$

$$= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d(a+b\sec(c+dx))} + \frac{(Ab-aB)\int \frac{\ldots}{\sqrt{\sec(c\ldots}}}{2a\,(a^2-b^2\ldots}$$

$$= -\frac{(3a^2Ab-Ab^3-a^3B-ab^2B)\,\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx\ldots\right.}{a^2(a-b)(a+b)^2d}$$

$$= \frac{(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a\,(a^2-b^2)\,d} + \frac{(2\ldots}{\ldots}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 722 vs. 2(263) = 526.
time = 36.96, size = 722, normalized size = 2.75

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,
x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*(-(A*b) + a*B
)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(
b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c
+ d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*
(4*a*A - 4*b*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]
], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b +
a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((A*b - a*B)*Cos[2*(c + d*x)]*(a +
b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqr
t[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a
- 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1
- Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1
]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), Ar
cSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*
Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c +
 d*x]]*(2 - Sec[c + d*x]^2))))/(4*(a - b)*(a + b)*d*(B + A*Cos[c + d*x])*(a
 + b*Sec[c + d*x])^2) + ((b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)*(A + B*S
```

```
ec[c + d*x])*(((-(A*b) + a*B)*Sin[c + d*x])/(a*(a^2 - b^2)) + (A*b^2*Sin[c
+ d*x] - a*b*B*Sin[c + d*x])/(a*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/(d*(B +
 A*Cos[c + d*x])*(a + b*Sec[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 801 vs. $2(327) = 654$.
time = 3.65, size = 802, normalized size = 3.05

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{Elli}}{a^2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*(
-2*A*b+B*a)/a/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi
(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*b*(A*b-B*a)/a^2*(1/b*a^2/(a^2-b^2)
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2
*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/
(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/
(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^2, x
)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec{(c+dx)})\sqrt{\sec{(c+dx)}}}{(a+b\sec{(c+dx)})^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**2, x
)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x))^2,x)
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x))^2, x
)
```

3.426 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=283

$$\frac{(2a^2A-3Ab^2+abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^2(a^2-b^2)d}-\frac{(4a^2Ab-3Ab^3-2a^3B+ab^2B)\sqrt{c}}{a^3(}$$

```
[Out] b*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))+(2*A
*a^2-3*A*b^2+B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipti
cE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b
^2)/d-(4*A*a^2*b-3*A*b^3-2*B*a^3+B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d
*x+c)^(1/2)/a^3/(a^2-b^2)/d+b*(5*A*a^2*b-3*A*b^3-3*B*a^3+B*a*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a
+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a-b)/(a+b)^2/d
```

Rubi [A]
time = 0.39, antiderivative size = 283, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {4115, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad(a^2-b^2)(a+b\sec(c+dx))}+\frac{(2a^2A+abB-3Ab^2)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a^2-b^2)}-\frac{(-2a^3B+4a^2Ab+ab^2B-3Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a^2-b^2)}+\frac{b(-3a^3B+5a^2Ab+ab^2B-3Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] ((2*a^2*A - 3*A*b^2 + a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*S
qrt[Sec[c + d*x]])/(a^2*(a^2 - b^2)*d) - ((4*a^2*A*b - 3*A*b^3 - 2*a^3*B +
a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
a^3*(a^2 - b^2)*d) + (b*(5*a^2*A*b - 3*A*b^3 - 3*a^3*B + a*b^2*B)*Sqrt[Cos[
c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^
3*(a - b)*(a + b)^2*d) + (b*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a
*(a^2 - b^2)*d*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2}\,dx = \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\,(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{\int \frac{\frac{1}{2}(-2a^2A+3Ab^2-abB)+a}{\sqrt{\sec(}}}{}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\,(a^2-b^2)\,d(a+b\sec(c+dx))} - \frac{\int \frac{1}{2}a(-2a^2A+3Ab^2-abB)-}{}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\,(a^2-b^2)\,d(a+b\sec(c+dx))} + \frac{(2a^2A-3Ab^2+abB)}{2a^2\,(a^2}$$

$$= \frac{b(5a^2Ab-3Ab^3-3a^3B+ab^2B)\,\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+d\right.}{a^3(a-b)(a+b)^2d}$$

$$= \frac{(2a^2A-3Ab^2+abB)\,\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{\sec(c+}}{a^2\,(a^2-b^2)\,d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 652 vs. $2(283) = 566$.

time = 36.99, size = 652, normalized size = 2.30

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^2),
x]
```

```
[Out] ((2*(-2*a^2*A + A*b^2 + a*b*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c
+ d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*
Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x]
)*(1 - Cos[c + d*x]^2)) + (2*(4*a*A*b - 4*a^2*B)*Cos[c + d*x]^2*EllipticPi[
-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c
 + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((
-2*a^2*A + 3*A*b^2 - a*b*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b +
 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqr
t[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[S
qrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2
*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt
[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]],
 -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b +
 a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^
```

```
2)))/(4*a*(-a + b)*(a + b)*d) + (Sqrt[Sec[c + d*x]]*(-((b*(A*b - a*B)*Sin[c
 + d*x])/(a^2*(-a^2 + b^2))) + (-(A*b^3*Sin[c + d*x]) + a*b^2*B*Sin[c + d*x
])/(a^2*(a^2 - b^2)*(b + a*Cos[c + d*x]))))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(347) = 694$.

time = 4.30, size = 843, normalized size = 2.98

method	result	size
default	Expression too large to display	843

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
b+A*a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*a*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))-2*b/a^2*(3*A*b-2*B*a)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*b^2*(A*b-B*a)/
a^3*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/
(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(
1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b^2*sec(d*x + c)^3 + 2*a*
b*sec(d*x + c)^2 + a^2*sec(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec(c + dx)}{(a + b\sec(c + dx))^2\sqrt{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))/((a + b*sec(c + d*x))**2*sqrt(sec(c + d*x))),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*sqrt(sec(d*x + c))),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+d\,x)}}{\left(a + \frac{b}{\cos(c+d\,x)}\right)^2\sqrt{\frac{1}{\cos(c + d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(1/2)), x
)
```

3.427 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=365

$$-\frac{(4a^2Ab-5Ab^3-2a^3B+3ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{a^3(a^2-b^2)d}+\frac{(2a^4A+16a^2Ab^2-15A$$

```
[Out] 1/3*(2*A*a^2-5*A*b^2+3*B*a*b)*sin(d*x+c)/a^2/(a^2-b^2)/d/sec(d*x+c)^(1/2)+b
*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2)-(4*A*
a^2*b-5*A*b^3-2*B*a^3+3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/
2)/a^3/(a^2-b^2)/d+1/3*(2*A*a^4+16*A*a^2*b^2-15*A*b^4-12*B*a^3*b+9*B*a*b^3)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*
c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^4/(a^2-b^2)/d-b^2*(7*A*a^2*
b-5*A*b^3-5*B*a^3+3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x
+c)^(1/2)/a^4/(a-b)/(a+b)^2/d
```

Rubi [A]
time = 0.57, antiderivative size = 365, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4115, 4189, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))}+\frac{(2a^2A+3abB-5Ab^2)\sin(c+dx)}{3a^2d(a^2-b^2)\sqrt{\sec(c+dx)}}-\frac{(-2a^3B+4a^2Ab+3ab^2B-5Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a^2-b^2)}-\frac{b^2(-5a^3B+7a^2Ab+3ab^2B-5Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^4d(a-b)(a+b)^2}+\frac{(2a^4A-12a^3bB+16a^2Ab^2+9ab^3B-15Ab^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^4d(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] -(((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^3*(a^2 - b^2)*d)) + ((2*a^4*A + 16*
a^2*A*b^2 - 15*A*b^4 - 12*a^3*b*B + 9*a*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF
[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(3*a^4*(a^2 - b^2)*d) - (b^2*(7*a^2*A*
b - 5*A*b^3 - 5*a^3*B + 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a +
 b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a^4*(a - b)*(a + b)^2*d) + ((2*a^
2*A - 5*A*b^2 + 3*a*b*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d*Sqrt[Sec[c + d*
x]]) + (b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a
+ b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))} - \frac{\int \frac{\frac{1}{2}\left(-2a^2A+5Ab^2-3a\right.}{}}{}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{b(Ab-aB)\sin(c}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{b(Ab-aB)\sin(c}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sin(c+dx)}{3a^2\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}} + \frac{b(Ab-aB)\sin(c}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a}$$

$$= -\frac{b^2\left(7a^2Ab-5Ab^3-5a^3B+3ab^2B\right)\sqrt{\cos(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+\right.}{a^4(a-b)(a+b)^2d}$$

$$= -\frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d}$$

Mathematica [A]
time = 37.18, size = 699, normalized size = 1.92

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),
x]
```

```
[Out] ((2*(-8*a^2*A*b + 5*A*b^3 + 6*a^3*B - 3*a*b^2*B)*Cos[c + d*x]^2*(EllipticF[
ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*
x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(
b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(4*a^3*A + 8*a*A*b^2 - 12*a^
2*b*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a
 + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c +
 d*x])*(1 - Cos[c + d*x]^2)) + ((-12*a^2*A*b + 15*A*b^3 + 6*a^3*B - 9*a*b^2
*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 -
4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 -
 Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*
Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcS
in[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4
*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*
Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Co
s[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(12*a^2*(a - b)*(a
+ b)*d) + (Sqrt[Sec[c + d*x]]*((b^2*(A*b - a*B)*Sin[c + d*x])/(a^3*(-a^2 +
b^2)) - (-(A*b^4*Sin[c + d*x]) + a*b^3*B*Sin[c + d*x])/(a^3*(a^2 - b^2)*(b
+ a*Cos[c + d*x])) + (A*Sin[2*(c + d*x)])/(3*a^2)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1058 vs. $2(423) = 846$.
time = 5.00, size = 1059, normalized size = 2.90

method	result	size
default	Expression too large to display	1059

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/a^4*(4*A*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^2-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^2*a^2+a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*b^2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+6*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-6*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*b^2/a^3*(4*A*b-3*B*a)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*b^3*(A*b-B*a)/a^4
```

```
*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^
2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2
))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)),
 x)
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b^2*sec(d*x + c)^4 + 2*a*
b*sec(d*x + c)^3 + a^2*sec(d*x + c)^2), x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*sec(d*x + c)^(3/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^2\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^2*(1/cos(c + d*x))^(3/2)), x
)
```

3.428 $\int \frac{\sec^{\frac{9}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=583

$$-\frac{(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^4(a^2-b^2)^2d}$$

```
[Out] -1/12*(15*A*a^3*b-33*A*a*b^3-35*B*a^4+61*B*a^2*b^2-8*B*b^4)*sec(d*x+c)^(3/2
)*sin(d*x+c)/b^3/(a^2-b^2)^2/d+1/2*a*(A*b-B*a)*sec(d*x+c)^(7/2)*sin(d*x+c)/
b/(a^2-b^2)/d/(a+b*sec(d*x+c))^2+1/4*a*(3*A*a^2*b-9*A*b^3-7*B*a^3+13*B*a*b^
2)*sec(d*x+c)^(5/2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))+1/4*(15*A
*a^4*b-29*A*a^2*b^3+8*A*b^5-35*B*a^5+65*B*a^3*b^2-24*B*a*b^4)*sin(d*x+c)*se
c(d*x+c)^(1/2)/b^4/(a^2-b^2)^2/d-1/4*(15*A*a^4*b-29*A*a^2*b^3+8*A*b^5-35*B*
a^5+65*B*a^3*b^2-24*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b
^4/(a^2-b^2)^2/d-1/12*(15*A*a^3*b-33*A*a*b^3-35*B*a^4+61*B*a^2*b^2-8*B*b^4)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*
c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^3/(a^2-b^2)^2/d-1/4*a*(15*A
*a^4*b-38*A*a^2*b^3+35*A*b^5-35*B*a^5+86*B*a^3*b^2-63*B*a*b^4)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b)
,2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)^2/b^4/(a+b)^3/d
```

Rubi [A]
time = 1.16, antiderivative size = 583, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {4114, 4183, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{a(Ab-aB)\sin(c+dx)\sec^{\frac{7}{2}}(c+dx)}{2bd(a^2-b^2)(a+b\sec(c+dx))^2}+\frac{a(-7a^3B+3a^2Ab+13ab^2B-9Ab^3)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{4b^2d(a^2-b^2)^2(a+b\sec(c+dx))}-\frac{(-35a^4B+15a^3Ab+61a^2b^2B-33aAb^3-8b^4B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{12b^3d(a^2-b^2)^2}-\frac{(-35a^4B+15a^3Ab+61a^2b^2B-33aAb^3-8b^4B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12b^3d(a^2-b^2)^2}+\frac{(-35a^5B+15a^4Ab+65a^3b^2B-29a^2Ab^3-24ab^4B+8Ab^5)\sin(c+dx)\sqrt{\sec(c+dx)}}{4b^4d(a^2-b^2)^2}-\frac{(-35a^5B+15a^4Ab+65a^3b^2B-29a^2Ab^3-24ab^4B+8Ab^5)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d(a^2-b^2)^2}-\frac{a(-35a^5B+15a^4Ab+86a^3b^2B-38a^2Ab^3-63ab^4B+35Ab^5)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(9/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/4*((15*a^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a^5*B + 65*a^3*b^2*B - 24*a
*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(b
^4*(a^2 - b^2)^2*d) - ((15*a^3*A*b - 33*a*A*b^3 - 35*a^4*B + 61*a^2*b^2*B -
 8*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/
(12*b^3*(a^2 - b^2)^2*d) - (a*(15*a^4*A*b - 38*a^2*A*b^3 + 35*A*b^5 - 35*a^
5*B + 86*a^3*b^2*B - 63*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b
), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a - b)^2*b^4*(a + b)^3*d) + ((15
*a^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a^5*B + 65*a^3*b^2*B - 24*a*b^4*B)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^4*(a^2 - b^2)^2*d) - ((15*a^3*A*b - 33
*a*A*b^3 - 35*a^4*B + 61*a^2*b^2*B - 8*b^4*B)*Sec[c + d*x]^(3/2)*Sin[c + d*
x])/(12*b^3*(a^2 - b^2)^2*d) + (a*(A*b - a*B)*Sec[c + d*x]^(7/2)*Sin[c + d*
x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2) + (a*(3*a^2*A*b - 9*A*b^3 -
```

```
7*a^3*B + 13*a*b^2*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2
*d*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
```

```
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4183

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
- b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n +
 b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}
, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{9}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \frac{a(Ab-aB)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{\int \sec^{\frac{5}{2}}(c+dx)\left(\frac{5}{2}a(Ab-aB)-2b(\ldots\right.}{\ldots}$$

$$= \frac{a(Ab-aB)\sec^{\frac{7}{2}}(c+dx)\sin(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{a(3a^2Ab-9Ab^3-7a^3B\ldots}{4b^2\,(a^2-b^2\ldots}$$

$$= -\frac{(15a^3Ab-33aAb^3-35a^4B+61a^2b^2B-8b^4B)\sec^{\frac{3}{2}}(c+dx)\,\mathrm{si}\ldots}{12b^3\,(a^2-b^2)^2\,d}$$

$$= \frac{(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B)\,\sqrt{\sec\ldots}}{4b^4\,(a^2-b^2)^2\,d}$$

$$= \frac{(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B)\,\sqrt{\sec\ldots}}{4b^4\,(a^2-b^2)^2\,d}$$

$$= \frac{(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B)\,\sqrt{\sec\ldots}}{4b^4\,(a^2-b^2)^2\,d}$$

$$= -\frac{a(15a^4Ab-38a^2Ab^3+35Ab^5-35a^5B+86a^3b^2B-63ab^4B)\ldots}{4(a-b)^2b^4(a+\ldots}$$

$$= -\frac{(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B)\,\sqrt{\ldots}}{4b^4\,(a^2-b^2)^2\,d}$$

Mathematica [A]
time = 37.65, size = 897, normalized size = 1.54

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(9/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,
x]

[Out] ((2*(-135*a^5*A*b + 285*a^3*A*b^3 - 168*a*A*b^5 + 315*a^6*B - 641*a^4*b^2*B
 + 328*a^2*b^4*B + 16*b^6*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c +
d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Se
c[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*
(1 - Cos[c + d*x]^2)) + (2*(-120*a^4*A*b^2 + 240*a^2*A*b^4 - 48*A*b^6 + 280
*a^5*b*B - 512*a^3*b^3*B + 160*a*b^5*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), A
rcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2
```

```
]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((-45*a^5*A
*b + 87*a^3*A*b^3 - 24*a*A*b^5 + 105*a^6*B - 195*a^4*b^2*B + 72*a^2*b^4*B)*
Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*
b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec
[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt
[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[S
qrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2
*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt
[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c
+ d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/(48*(a - b)^2*b^4*(a +
b)^2*d) + (Sqrt[Sec[c + d*x]]*(((15*a^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a
^5*B + 65*a^3*b^2*B - 24*a*b^4*B)*Sin[c + d*x])/(4*b^4*(-a^2 + b^2)^2) + (a
^2*A*b*Sin[c + d*x] - a^3*B*Sin[c + d*x])/(2*b^2*(-a^2 + b^2)*(b + a*Cos[c
+ d*x])^2) + (-5*a^4*A*b*Sin[c + d*x] + 11*a^2*A*b^3*Sin[c + d*x] + 9*a^5*B
*Sin[c + d*x] - 15*a^3*b^2*B*Sin[c + d*x])/(4*b^3*(-a^2 + b^2)^2*(b + a*Cos
[c + d*x])) + (2*B*Tan[c + d*x])/(3*b^3)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2150 vs. $2(627) = 1254$.
time = 18.16, size = 2151, normalized size = 3.69

method	result	size
default	Expression too large to display	2151

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^3*(-1/6*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a^2*(A*b-3*B*a)/b^4/(a^2-a*b)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(
1/2))+2*(A*b-3*B*a)/b^4/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(A*b-B*a)*a/b^2*(1/2/b*a^2/(a
^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*c
os(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b
```

```
^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-
b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b
)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2
-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2
^(1/2)))-2*a*(A*b-2*B*a)/b^3*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)
-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2
)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))
/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(9/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(9/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(9/2)/(b*sec(d*x + c) + a)^3, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{9/2}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(9/2))/(a + b/cos(c + d*x))^3,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(9/2))/(a + b/cos(c + d*x))^3, x
)
```

3.429 $\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=480

$$\frac{(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^3\left(a^2-b^2\right)^2d}+\frac{(a^2Ab-7Ab$$

```
[Out] 1/2*a*(A*b-B*a)*sec(d*x+c)^(5/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^
2+1/4*a*(A*a^2*b-7*A*b^3-5*B*a^3+11*B*a*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)/b^
2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))-1/4*(3*A*a^3*b-9*A*a*b^3-15*B*a^4+29*B*a^2
*b^2-8*B*b^4)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^3/(a^2-b^2)^2/d+1/4*(3*A*a^3*b-
9*A*a*b^3-15*B*a^4+29*B*a^2*b^2-8*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*
x+c)^(1/2)/b^3/(a^2-b^2)^2/d+1/4*(A*a^2*b-7*A*b^3-5*B*a^3+11*B*a*b^2)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(
1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d+1/4*(3*A*a^4*b-6*
A*a^2*b^3+15*A*b^5-15*B*a^5+38*B*a^3*b^2-35*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/(a-b)^2/b^3/(a+b)^3/d
```

Rubi [A]
time = 0.86, antiderivative size = 480, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {4114, 4183, 4187, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{a(Ab-aB)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{a(-5a^3B+a^2Ab+11ab^2B-7Ab^3)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{4b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{(-5a^3B+a^2Ab+11ab^2B-7Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{(-15a^4B+3a^3Ab+29a^2b^2B-9aAb^3-8b^4B)\sin(c+dx)\sqrt{\sec(c+dx)}}{4b^3d\left(a^2-b^2\right)^2}+\frac{(-15a^4B+3a^3Ab+29a^2b^2B-9aAb^3-8b^4B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\left(a^2-b^2\right)^2}+\frac{(-15a^5B+3a^4Ab+38a^3b^2B-6a^2Ab^3-35ab^4B+15Ab^5)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*Sqrt[Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^3*(a^2 - b^2)^2*d)
+ ((a^2*A*b - 7*A*b^3 - 5*a^3*B + 11*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^2*(a^2 - b^2)^2*d) + ((3*a^4*A*b -
 6*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 38*a^3*b^2*B - 35*a*b^4*B)*Sqrt[Cos[c
+ d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*(a
 - b)^2*b^3*(a + b)^3*d) - ((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*
B - 8*b^4*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*b^3*(a^2 - b^2)^2*d) + (a*
(A*b - a*B)*Sec[c + d*x]^(5/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a + b*Sec[
c + d*x])^2) + (a*(a^2*A*b - 7*A*b^3 - 5*a^3*B + 11*a*b^2*B)*Sec[c + d*x]^(
3/2)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4183

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
- b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n +
 b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}
, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \frac{a(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{\int \sec^{\frac{3}{2}}(c+dx)\left(\frac{3}{2}a(Ab-aB)-2b(Ab\right.}{}$$

$$= \frac{a(Ab-aB)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{a(a^2Ab-7Ab^3-5a^3B+}{4b^2\,(a^2-b^2)^2}$$

$$= -\frac{(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B)\sqrt{\sec(c+dx)}\,\sin}{4b^3\,(a^2-b^2)^2\,d}$$

$$= -\frac{(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B)\sqrt{\sec(c+dx)}\,\sin}{4b^3\,(a^2-b^2)^2\,d}$$

$$= -\frac{(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B)\sqrt{\sec(c+dx)}\,\sin}{4b^3\,(a^2-b^2)^2\,d}$$

$$= \frac{(3a^4Ab-6a^2Ab^3+15Ab^5-15a^5B+38a^3b^2B-35ab^4B)\sqrt{\cos(c}}{4(a-b)^2b^3(a+b)^3d}$$

$$= \frac{(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}\right.}{4b^3\,(a^2-b^2)^2\,d}$$

Mathematica [A]
time = 37.31, size = 842, normalized size = 1.75

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,
x]
```

```
[Out] -1/16*((2*(-9*a^4*A*b + 19*a^2*A*b^3 - 16*A*b^5 + 45*a^5*B - 95*a^3*b^2*B +
 56*a*b^4*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - El
lipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt
[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x
]^2)) + (2*(-8*a^3*A*b^2 + 32*a*A*b^4 + 40*a^4*b*B - 80*a^2*b^3*B + 16*b^5*
B)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b
*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x
])*(1 - Cos[c + d*x]^2)) + ((-3*a^4*A*b + 9*a^2*A*b^3 + 15*a^5*B - 29*a^3*b
^2*B + 8*a*b^4*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec
[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c +
```

```
d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c
+ d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a^2*EllipticP
i[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c
 + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[
Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c +
 d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x]^2)))/((a -
 b)^2*b^3*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(((-3*a^3*A*b + 9*a*A*b^3 + 15
*a^4*B - 29*a^2*b^2*B + 8*b^4*B)*Sin[c + d*x])/(4*b^3*(-a^2 + b^2)^2) + (-(
a*A*b*Sin[c + d*x]) + a^2*B*Sin[c + d*x])/(2*b*(-a^2 + b^2)*(b + a*Cos[c +
d*x])^2) + (a^3*A*b*Sin[c + d*x] - 7*a*A*b^3*Sin[c + d*x] - 5*a^4*B*Sin[c +
 d*x] + 11*a^2*b^2*B*Sin[c + d*x])/(4*b^2*(-a^2 + b^2)^2*(b + a*Cos[c + d*x
]))))/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1996 vs. $2(528) = 1056$.
time = 9.68, size = 1997, normalized size = 4.16

method	result	size
default	Expression too large to display	1997

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*B/b^3/(a^
2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2*a/(a-b),2^(1/2))+2*B/b^3/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-
1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b-B*a)/b*(1/2/b*a^2/
(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2
*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2
*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2
-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^
2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
```

```
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a
+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a
^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b)
,2^(1/2)))-2*a*B/b^2*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+
b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a
*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2
*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3879 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(7/2)/(b*sec(d*x + c) + a)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + b/cos(c + d*x))^3,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(7/2))/(a + b/cos(c + d*x))^3, x
)
```

3.430 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=402

$$\frac{\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4b^2\left(a^2-b^2\right)^2d}+\frac{\left(3a^2Ab+3Ab^3+a^3B-7a\right.}{}$$

```
[Out] 1/2*a*(A*b-B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^
2-1/4*a*(A*a^2*b+5*A*b^3+3*B*a^3-9*B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^2
/(a^2-b^2)^2/d/(a+b*sec(d*x+c))+1/4*(A*a^2*b+5*A*b^3+3*B*a^3-9*B*a*b^2)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d+1/4*(3*A*a^2*b+
3*A*b^3+B*a^3-7*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/b/(
a^2-b^2)^2/d+1/4*(A*a^4*b-10*A*a^2*b^3-3*A*b^5+3*B*a^5-6*B*a^3*b^2+15*B*a*b
^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+
1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/(a-b)^2/b^2/(
a+b)^3/d
```

Rubi [A]
time = 0.62, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4114, 4183, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{a(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}-\frac{a\left(3a^3B+a^2Ab-9ab^2B+5Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4b^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\left(a^2-b^2\right)^2}+\frac{\left(3a^3B+a^2Ab-9ab^2B+5Ab^3\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}+\frac{\left(3a^5B+a^4Ab-6a^3b^2B-10a^2Ab^3+15ab^4B-3Ab^5\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4ab^2d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*b^2*(a^2 - b^2)^2*d) + ((3*a^2*A*b + 3*
A*b^3 + a^3*B - 7*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqr
t[Sec[c + d*x]])/(4*a*b*(a^2 - b^2)^2*d) + ((a^4*A*b - 10*a^2*A*b^3 - 3*A*b
^5 + 3*a^5*B - 6*a^3*b^2*B + 15*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a
)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a*(a - b)^2*b^2*(a + b)^3
*d) + (a*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a
 + b*Sec[c + d*x])^2) - (a*(a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sqrt[S
ec[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4183

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
- b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n +
 b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}
, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \frac{a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} + \frac{\int \sqrt{\sec(c+dx)}\,\left(\frac{1}{2}a(Ab-aB)\right.}{}$$

$$= \frac{a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a\left(a^2Ab+5Ab^3+3a^3B-\right.}{4b^2\left(a^2-b^2\right)^2}$$

$$= \frac{a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a\left(a^2Ab+5Ab^3+3a^3B-\right.}{4b^2\left(a^2-b^2\right)^2}$$

$$= \frac{a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2b\left(a^2-b^2\right)d(a+b\sec(c+dx))^2} - \frac{a\left(a^2Ab+5Ab^3+3a^3B-\right.}{4b^2\left(a^2-b^2\right)^2}$$

$$= \frac{\left(a^4Ab-10a^2Ab^3-3Ab^5+3a^5B-6a^3b^2B+15ab^4B\right)\sqrt{\cos(c+}}{4a(a-b)^2b^2(a+b)^3d}$$

$$= \frac{\left(a^2Ab+5Ab^3+3a^3B-9ab^2B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{}}{4b^2\left(a^2-b^2\right)^2d}$$

Mathematica [A]
time = 37.15, size = 795, normalized size = 1.98

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,
x]
```

```
[Out] ((2*(3*a^3*A*b - 9*a*A*b^3 + 9*a^4*B - 19*a^2*b^2*B + 16*b^4*B)*Cos[c + d*x
]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[
Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin
[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(8*a^2*A*b^2
+ 16*A*b^4 + 8*a^3*b*B - 32*a*b^3*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcS
in[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*S
in[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((a^3*A*b + 5*
a*A*b^3 + 3*a^4*B - 9*a^2*b^2*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*
a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1
]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[Arc
Sin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] +
2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]
*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*
x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b
*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c +
d*x]^2)))/(16*(a - b)^2*b^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(-1/4*((a^2*
A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sin[c + d*x])/(b^2*(-a^2 + b^2)^2) + (
A*b*Sin[c + d*x] - a*B*Sin[c + d*x])/(2*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2
) + (3*a^2*A*b*Sin[c + d*x] + 3*A*b^3*Sin[c + d*x] + a^3*B*Sin[c + d*x] - 7
*a*b^2*B*Sin[c + d*x])/(4*b*(-a^2 + b^2)^2*(b + a*Cos[c + d*x]))))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1767 vs. $2(454) = 908$.

time = 7.40, size = 1768, normalized size = 4.40

method	result	size
default	Expression too large to display	1768

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-A*b+B*a)/a*
(1/2/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/
(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4
/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1
```

```
/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3
/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(
a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a
*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2*a/(a-b),2^(1/2)))+2*A/a*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-
1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)
/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*
d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^3, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x))^3,x)
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x))^3, x
)
```

3.431 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=402

$$-\frac{(5a^2Ab+Ab^3-a^3B-5ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4ab\left(a^2-b^2\right)^2d}-\frac{(7a^2Ab-Ab^3-3a^3B-3a$$

```
[Out] 1/2*a*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^
2+1/4*(3*A*a^2*b+3*A*b^3+B*a^3-7*B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^
2-b^2)^2/d/(a+b*sec(d*x+c))-1/4*(5*A*a^2*b+A*b^3-B*a^3-5*B*a*b^2)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)
)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a/b/(a^2-b^2)^2/d-1/4*(7*A*a^2*b-A*b^3-
3*B*a^3-3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
F(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^
2)^2/d+1/4*(3*A*a^4*b+10*A*a^2*b^3-A*b^5+B*a^5-10*B*a^3*b^2-3*B*a*b^4)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2
*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a-b)^2/b/(a+b)^3/d
```

Rubi [A]
time = 0.60, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4114, 4185, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2bd\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{(a^3B+3a^2Ab-7ab^2B+3Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{4bd\left(a^2-b^2\right)^2(a+b\sec(c+dx))}-\frac{(-3a^3B+7a^2Ab-3ab^2B-Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{(a^3(-B)+5a^2Ab-5ab^2B+Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\left(a^2-b^2\right)^2}+\frac{(a^5B+3a^4Ab-10a^3b^2B+10a^2Ab^3-3ab^4B-Ab^5)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^2bd(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/4*((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a*b*(a^2 - b^2)^2*d) - ((7*a^2*A*b - A
*b^3 - 3*a^3*B - 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sq
rt[Sec[c + d*x]])/(4*a^2*(a^2 - b^2)^2*d) + ((3*a^4*A*b + 10*a^2*A*b^3 - A*
b^5 + a^5*B - 10*a^3*b^2*B - 3*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)
/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a - b)^2*b*(a + b)^3*
d) + (a*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(a
+ b*Sec[c + d*x])^2) + ((3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B)*Sqrt[Sec[
c + d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{\int \frac{-\frac{1}{2}a(Ab-aB)-2b(Ab-aB)\sec}{\sqrt{\sec(c+}}}{2}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{(3a^2Ab+3Ab^3+a^3B-}{4b\,(a^2-b^2)}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{(3a^2Ab+3Ab^3+a^3B-}{4b\,(a^2-b^2)}$$

$$= \frac{a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2b\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{(3a^2Ab+3Ab^3+a^3B-}{4b\,(a^2-b^2)}$$

$$= \frac{(3a^4Ab+10a^2Ab^3-Ab^5+a^5B-10a^3b^2B-3ab^4B)\sqrt{\cos(c+dx}}{4a^2(a-b)^2b(a+b)^3d}$$

$$= -\frac{(5a^2Ab+Ab^3-a^3B-5ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)\sqrt{}}{4ab\,(a^2-b^2)^2\,d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 882 vs. 2(402) = 804.
time = 37.17, size = 882, normalized size = 2.19

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,
x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((2*(a^2*A*b +
5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c
+ d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*
Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x]
)*(1 - Cos[c + d*x]^2)) + (2*(-24*a*A*b^2 + 8*a^2*b*B + 16*b^3*B)*Cos[c + d
*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x
])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[
c + d*x]^2)) + ((-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)*Cos[2*(c + d*x)]*(
a + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin
[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a
*(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqr
t[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]]
, -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a)
, ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^
2])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec
[c + d*x]]*(2 - Sec[c + d*x]^2))))/(16*(a - b)^2*b*(a + b)^2*d*(B + A*Cos[c
 + d*x])*(a + b*Sec[c + d*x])^3) + ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/
2)*(A + B*Sec[c + d*x])*(((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*Sin[c + d
*x])/(4*a*b*(-a^2 + b^2)^2) - (-(A*b^2*Sin[c + d*x]) + a*b*B*Sin[c + d*x])/
(2*a*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (-7*a^2*A*b*Sin[c + d*x] + A*b^3
*Sin[c + d*x] + 3*a^3*B*Sin[c + d*x] + 3*a*b^2*B*Sin[c + d*x])/(4*a*(a^2 -
b^2)^2*(b + a*Cos[c + d*x]))))/(d*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])
^3)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1871 vs. $2(454) = 908$.
time = 7.73, size = 1872, normalized size = 4.66

method	result	size
default	Expression too large to display	1872

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*A/a/(a^2-a*b
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*
a/(a-b),2^(1/2))+2*b*(A*b-B*a)/a^2*(1/2/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^
2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1
```

```
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-
3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-
b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2
-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a
^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15
/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*(-2*A*b+B*a)/a^2
*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^
2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2
))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^3, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x))^3,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x))^3, x
)
```

3.432 $\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=402

$$\frac{(9a^2Ab-3Ab^3-5a^3B-ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^2(a^2-b^2)^2d}+\frac{(8a^4A-5a^2Ab^2+3Ab^4-7$$

```
[Out] -1/2*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^2-1
/4*(7*A*a^2*b-A*b^3-3*B*a^3-3*B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b
^2)^2/d/(a+b*sec(d*x+c))+1/4*(9*A*a^2*b-3*A*b^3-5*B*a^3-B*a*b^2)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2)^2/d+1/4*(8*A*a^4-5*A*a^2*b
^2+3*A*b^4-7*B*a^3*b+B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/
a^3/(a^2-b^2)^2/d-1/4*(15*A*a^4*b-6*A*a^2*b^3+3*A*b^5-3*B*a^5-10*B*a^3*b^2+
B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2
*d*x+1/2*c),2*a/(a+b),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a-b)^
2/(a+b)^3/d
```

Rubi [A]
time = 0.56, antiderivative size = 402, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4112, 4185, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$-\frac{(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2d(a^2-b^2)(a+b\sec(c+dx))^2}-\frac{(-3a^3B+7a^2Ab-3ab^2B-Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{4ad(a^2-b^2)^2(a+b\sec(c+dx))}+\frac{(-5a^3B+9a^2Ab-ab^2B-3Ab^3)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d(a^2-b^2)^2}+\frac{(8a^4A-7a^3bB-5a^2Ab^2+ab^3B+3Ab^4)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d(a^2-b^2)^2}-\frac{(-3a^5B+15a^4Ab-10a^3b^2B-6a^2Ab^3+ab^4B+3Ab^5)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^2*(a^2 - b^2)^2*d) + ((8*a^4*A - 5*a^
2*A*b^2 + 3*A*b^4 - 7*a^3*b*B + a*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c +
d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*d) - ((15*a^4*A*b - 6*a
^2*A*b^3 + 3*A*b^5 - 3*a^5*B - 10*a^3*b^2*B + a*b^4*B)*Sqrt[Cos[c + d*x]]*E
llipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a - b)
^2*(a + b)^3*d) - ((A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*(a^2 - b
^2)*d*(a + b*Sec[c + d*x])^2) - ((7*a^2*A*b - A*b^3 - 3*a^3*B - 3*a*b^2*B)*
Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(4*a*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4112

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-d)*(A
*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*
Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) +
 d*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} - \frac{\int \frac{\frac{1}{2}(-Ab+aB)-2(aA-bB)\,\mathrm{s}}{\sqrt{\sec(c+\mathrm{a}}}}{2\,(}$$

$$= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} - \frac{(7a^2Ab-Ab^3-3a^3B}{4a\,(a^2-}$$

$$= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} - \frac{(7a^2Ab-Ab^3-3a^3B}{4a\,(a^2-}$$

$$= -\frac{(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} - \frac{(7a^2Ab-Ab^3-3a^3B}{4a\,(a^2-}$$

$$= -\frac{(15a^4Ab-6a^2Ab^3+3Ab^5-3a^5B-10a^3b^2B+ab^4B)\sqrt{\cos(c}}{4a^3(a-b)^2(a+b)^3c}$$

$$= \frac{(9a^2Ab-3Ab^3-5a^3B-ab^2B)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\,(a^2-b^2)^2\,d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 885 vs. 2(402) = 804.

time = 37.19, size = 885, normalized size = 2.20

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,
x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((2*(-5*a^2*A*b
 - A*b^3 + a^3*B + 5*a*b^2*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c +
 d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*S
ec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])
*(1 - Cos[c + d*x]^2)) + (2*(16*a^3*A + 8*a*A*b^2 - 24*a^2*b*B)*Cos[c + d*x
]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])
*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c
+ d*x]^2)) + ((9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Cos[2*(c + d*x)]*(a
 + b*Sec[c + d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[
Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*
(a - 2*b)*EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt
[1 - Sec[c + d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]],
 -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a),
 ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2
])*Sin[c + d*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[
c + d*x]]*(2 - Sec[c + d*x]^2))))/(16*a*(a - b)^2*(a + b)^2*d*(B + A*Cos[c
+ d*x])*(a + b*Sec[c + d*x])^3) + ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2
)*(A + B*Sec[c + d*x])*(((-9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*Sin[c +
 d*x])/(4*a^2*(-a^2 + b^2)^2) - (A*b^3*Sin[c + d*x] - a*b^2*B*Sin[c + d*x])
/(2*a^2*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (11*a^2*A*b^2*Sin[c + d*x] -
5*A*b^4*Sin[c + d*x] - 7*a^3*b*B*Sin[c + d*x] + a*b^3*B*Sin[c + d*x])/(4*a^
2*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*(B + A*Cos[c + d*x])*(a + b*Sec[
c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1958 vs. $2(454) = 908$.
time = 8.07, size = 1959, normalized size = 4.87

method	result	size
default	Expression too large to display	1959

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-2*(
-3*A*b+B*a)/a^2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*b^2*(A*b-B*a)/a^3*(1/2/b*a^2/(a^
```

```
2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*co
s(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^
2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),
2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b
^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/
2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)
/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-
a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/
2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^
(1/2)))+2*b/a^3*(3*A*b-2*B*a)*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b
)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^
2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/
2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^3, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x))^3,x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x))^3, x
)
```

3.433 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=427

$$\frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^3\left(a^2-b^2\right)^2d}-\frac{(24a^4Ab-3$$

```
[Out] 1/2*b*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^
2+1/4*b*(11*A*a^2*b-5*A*b^3-7*B*a^3+B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a^
2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))+1/4*(8*A*a^4-29*A*a^2*b^2+15*A*b^4+9*B*a^3
*b-3*B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^3/(a^2-b^2)^2/
d-1/4*(24*A*a^4*b-33*A*a^2*b^3+15*A*b^5-8*B*a^5+5*B*a^3*b^2-3*B*a*b^4)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^
(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^4/(a^2-b^2)^2/d+1/4*b*(35*A*a^4*
b-38*A*a^2*b^3+15*A*b^5-15*B*a^5+6*B*a^3*b^2-3*B*a*b^4)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2
))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^4/(a-b)^2/(a+b)^3/d
```

Rubi [A]
time = 0.67, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {4115, 4185, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{2ad\left(a^2-b^2\right)(a+b\sec(c+dx))^2}+\frac{b\left(-7a^3B+11a^2Ab+ab^2B-5Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{4a^2d\left(a^2-b^2\right)^2(a+b\sec(c+dx))}+\frac{\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}-\frac{\left(-8a^5B+24a^4Ab+5a^3b^2B-33a^2Ab^3-3ab^4B+15Ab^5\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d\left(a^2-b^2\right)^2}+\frac{b\left(-15a^5B+35a^4Ab+6a^3b^2B-38a^2Ab^3-3ab^4B+15Ab^5\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*Sqrt[Cos[c + d
*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^3*(a^2 - b^2)^2*d)
- ((24*a^4*A*b - 33*a^2*A*b^3 + 15*A*b^5 - 8*a^5*B + 5*a^3*b^2*B - 3*a*b^4*
B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*
(a^2 - b^2)^2*d) + (b*(35*a^4*A*b - 38*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 6*
a^3*b^2*B - 3*a*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*
x)/2, 2]*Sqrt[Sec[c + d*x]])/(4*a^4*(a - b)^2*(a + b)^3*d) + (b*(A*b - a*B)
*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^2
) + (b*(11*a^2*A*b - 5*A*b^3 - 7*a^3*B + a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c
+ d*x])/(4*a^2*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^3}\,dx = \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} - \frac{\int \frac{\frac{1}{2}(-4a^2A+5Ab^2-abB)+2}{\sqrt{\sec(}}}{}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{b(11a^2Ab-5Ab^3-7a}{4a^2\,(a^2-}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{b(11a^2Ab-5Ab^3-7a}{4a^2\,(a^2-}$$

$$= \frac{b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{2a\,(a^2-b^2)\,d(a+b\sec(c+dx))^2} + \frac{b(11a^2Ab-5Ab^3-7a}{4a^2\,(a^2-}$$

$$= \frac{b(35a^4Ab-38a^2Ab^3+15Ab^5-15a^5B+6a^3b^2B-3ab^4B)\,\sqrt{c}}{4a^4(a-b)^2(a+b)}$$

$$= \frac{(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B)\,\sqrt{\cos(c+dx)}\,E(}{4a^3\,(a^2-b^2)^2\,d}$$

Mathematica [A]

time = 37.35, size = 818, normalized size = 1.92

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^3),
x]
```

```
[Out] ((2*(8*a^4*A - 7*a^2*A*b^2 + 5*A*b^4 - 5*a^3*b*B - a*b^3*B)*Cos[c + d*x]^2*
(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - EllipticPi[-(b/a), ArcSin[Sqrt
[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c +
 d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + (2*(-32*a^3*A*b + 8*
a*A*b^3 + 16*a^4*B + 8*a^2*b^2*B)*Cos[c + d*x]^2*EllipticPi[-(b/a), ArcSin[
Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[
c + d*x])/(a*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)) + ((8*a^4*A - 29*a^
2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*Cos[2*(c + d*x)]*(a + b*Sec[c +
 d*x])*(-4*a*b + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c +
 d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*El
lipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c +
 d*x]^2] + 2*a^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Se
c[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt
[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d
*x])/(a^2*b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2
 - Sec[c + d*x]^2)))/(16*a^2*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*(
-1/4*(b*(-13*a^2*A*b + 7*A*b^3 + 9*a^3*B - 3*a*b^2*B)*Sin[c + d*x])/(a^3*(-
a^2 + b^2)^2) - (-(A*b^4*Sin[c + d*x]) + a*b^3*B*Sin[c + d*x])/(2*a^3*(a^2
- b^2)*(b + a*Cos[c + d*x])^2) + (-15*a^2*A*b^3*Sin[c + d*x] + 9*A*b^5*Sin[
c + d*x] + 11*a^3*b^2*B*Sin[c + d*x] - 5*a*b^4*B*Sin[c + d*x])/(4*a^3*(a^2
- b^2)^2*(b + a*Cos[c + d*x]))))/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1999 vs. $2(479) = 958$.
time = 8.64, size = 2000, normalized size = 4.68

method	result	size
default	Expression too large to display	2000

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^4/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
b+A*a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*a*EllipticF(cos(1/2*d*x+1/2*c
```

),2^(1/2)))-6*b/a^3*(2*A*b-B*a)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*b^3*(A*b-B*a)/a^4*(1/2/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*b^2/a^4*(4*A*b-3*B*a)*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2

```
*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*sqrt(sec(d*x + c))),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)),x)
[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(1/2)), x
)
```

3.434 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=521

$$-\frac{\left(24a^4Ab-65a^2Ab^3+35Ab^5-8a^5B+29a^3b^2B-15ab^4B\right)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)\sqrt{\sec(c+dx)}}{4a^4\left(a^2-b^2\right)^2d}$$

```
[Out] 1/12*(8*A*a^4-61*A*a^2*b^2+35*A*b^4+33*B*a^3*b-15*B*a*b^3)*sin(d*x+c)/a^3/(
a^2-b^2)^2/d/sec(d*x+c)^(1/2)+1/2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b
*sec(d*x+c))^2/sec(d*x+c)^(1/2)+1/4*b*(13*A*a^2*b-7*A*b^3-9*B*a^3+3*B*a*b^2
)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))/sec(d*x+c)^(1/2)-1/4*(24*A*
a^4*b-65*A*a^2*b^3+35*A*b^5-8*B*a^5+29*B*a^3*b^2-15*B*a*b^4)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))*cos
(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^4/(a^2-b^2)^2/d+1/12*(8*A*a^6+128*A*a^4*b^
2-223*A*a^2*b^4+105*A*b^6-72*B*a^5*b+99*B*a^3*b^3-45*B*a*b^5)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^5/(a^2-b^2)^2/d-1/4*b^2*(63*A*a^4*b-86*A*
a^2*b^3+35*A*b^5-35*B*a^5+38*B*a^3*b^2-15*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^5/(a-b)^2/(a+b)^3/d
```

Rubi [A]
time = 0.95, antiderivative size = 521, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.303$, Rules used = {4115, 4185, 4189, 4191, 3934, 2884, 3872, 3856, 2719, 2720}

$$\frac{b(Ab-aB)\sin(c+dx)}{2ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2}+\frac{b\left(-9a^3B+13a^2Ab+3ab^2B-7Ab^3\right)\sin(c+dx)}{4a^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))}+\frac{\left(8a^4A+33a^3bB-61a^2Ab^2-15ab^3B+35Ab^4\right)\sin(c+dx)}{12a^3d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}}-\frac{\left(-8a^5B+24a^4Ab+29a^3b^2B-65a^2Ab^3-15ab^4B+35Ab^5\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d\left(a^2-b^2\right)^2}-\frac{b^2\left(-35a^5B+63a^4Ab+38a^3b^2B-86a^2Ab^3-15ab^4B+35Ab^5\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^5d(a-b)^2(a+b)^3}+\frac{\left(8a^6A-72a^5bB+128a^4Ab^2+99a^3b^3B-223a^2Ab^4-45ab^5B+105Ab^6\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12a^5d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] -1/4*((24*a^4*A*b - 65*a^2*A*b^3 + 35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a
*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(a
^4*(a^2 - b^2)^2*d) + ((8*a^6*A + 128*a^4*A*b^2 - 223*a^2*A*b^4 + 105*A*b^6
 - 72*a^5*b*B + 99*a^3*b^3*B - 45*a*b^5*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c
+ d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(12*a^5*(a^2 - b^2)^2*d) - (b^2*(63*a^4*A*
b - 86*a^2*A*b^3 + 35*A*b^5 - 35*a^5*B + 38*a^3*b^2*B - 15*a*b^4*B)*Sqrt[Co
s[c + d*x]]*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]*Sqrt[Sec[c + d*x]])/(
4*a^5*(a - b)^2*(a + b)^3*d) + ((8*a^4*A - 61*a^2*A*b^2 + 35*A*b^4 + 33*a^3
*b*B - 15*a*b^3*B)*Sin[c + d*x])/(12*a^3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]
) + (b*(A*b - a*B)*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a +
 b*Sec[c + d*x])^2) + (b*(13*a^2*A*b - 7*A*b^3 - 9*a^3*B + 3*a*b^2*B)*Sin[c
 + d*x])/(4*a^2*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3856

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Dist[(b*Csc[c + d*x]
)^n*Sin[c + d*x]^n, Int[1/Sin[c + d*x]^n, x], x] /; FreeQ[{b, c, d}, x] &&
EqQ[n^2, 1/4]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 3934

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[d*Sqrt[d*Sin[e + f*x]]*Sqrt[d*Csc[e + f*x]], Int[1
/(Sqrt[d*Sin[e + f*x]]*(b + a*Sin[e + f*x])), x], x] /; FreeQ[{a, b, d, e,
f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
```

```
Q[n, 0])
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4191

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))), x_Symbol] :> Dist[(A*b^2 - a*b*B + a^2*C)/(a^2*d^2), Int[(d*Csc[e + f
*x])^(3/2)/(a + b*Csc[e + f*x]), x], x] + Dist[1/a^2, Int[(a*A - (A*b - a*B
)*Csc[e + f*x])/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2} - \frac{\int \frac{1}{2}(-4a^2A+7Ab^2 \cdots}{\cdots}$$

$$= \frac{b(Ab-aB)\sin(c+dx)}{2a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^2} + \frac{b(13a^2Ab-7 \cdots}{4a^2\left(a^2-b^2\right)^2 \cdots}$$

$$= \frac{(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2d\sqrt{\sec(c+dx)}} + \frac{\cdots}{2a \cdots}$$

$$= \frac{(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2d\sqrt{\sec(c+dx)}} + \frac{\cdots}{2a \cdots}$$

$$= \frac{(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B)\sin(c+dx)}{12a^3\left(a^2-b^2\right)^2d\sqrt{\sec(c+dx)}} + \frac{\cdots}{2a \cdots}$$

$$= -\frac{b^2(63a^4Ab-86a^2Ab^3+35Ab^5-35a^5B+38a^3b^2B-15ab^4B) \cdots}{4a^5(a-b)^2(a+b \cdots}$$

$$= -\frac{(24a^4Ab-65a^2Ab^3+35Ab^5-8a^5B+29a^3b^2B-15ab^4B)\sqrt{c \cdots}}{4a^4\left(a^2-b^2\right)^2d \cdots}$$

Mathematica [**A**]
time = 37.56, size = 863, normalized size = 1.66

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),
x]
```

```
[Out] ((2*(-56*a^4*A*b + 73*a^2*A*b^3 - 35*A*b^5 + 24*a^5*B - 21*a^3*b^2*B + 15*a
*b^4*B)*Cos[c + d*x]^2*(EllipticF[ArcSin[Sqrt[Sec[c + d*x]]], -1] - Ellipti
cPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1])*(a + b*Sec[c + d*x])*Sqrt[1 -
Sec[c + d*x]^2]*Sin[c + d*x])/(b*(b + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2))
 + (2*(16*a^5*A + 112*a^3*A*b^2 - 56*a*A*b^4 - 96*a^4*b*B + 24*a^2*b^3*B)*C
os[c + d*x]^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*(a + b*Sec
[c + d*x])*Sqrt[1 - Sec[c + d*x]^2]*Sin[c + d*x])/(a*(b + a*Cos[c + d*x])*(
1 - Cos[c + d*x]^2)) + ((-72*a^4*A*b + 195*a^2*A*b^3 - 105*A*b^5 + 24*a^5*B
 - 87*a^3*b^2*B + 45*a*b^4*B)*Cos[2*(c + d*x)]*(a + b*Sec[c + d*x])*(-4*a*b
 + 4*a*b*Sec[c + d*x]^2 - 4*a*b*EllipticE[ArcSin[Sqrt[Sec[c + d*x]]], -1]*S
```

```
qrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] - 2*a*(a - 2*b)*EllipticF[ArcSin
[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2] + 2*a
^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]], -1]*Sqrt[Sec[c + d*x]]*Sq
rt[1 - Sec[c + d*x]^2] - 4*b^2*EllipticPi[-(b/a), ArcSin[Sqrt[Sec[c + d*x]]
], -1]*Sqrt[Sec[c + d*x]]*Sqrt[1 - Sec[c + d*x]^2])*Sin[c + d*x])/(a^2*b*(b
 + a*Cos[c + d*x])*(1 - Cos[c + d*x]^2)*Sqrt[Sec[c + d*x]]*(2 - Sec[c + d*x
]^2)))/(48*a^3*(a - b)^2*(a + b)^2*d) + (Sqrt[Sec[c + d*x]]*((b^2*(-17*a^2*
A*b + 11*A*b^3 + 13*a^3*B - 7*a*b^2*B)*Sin[c + d*x])/(4*a^4*(-a^2 + b^2)^2)
 - (A*b^5*Sin[c + d*x] - a*b^4*B*Sin[c + d*x])/(2*a^4*(a^2 - b^2)*(b + a*Co
s[c + d*x])^2) + (19*a^2*A*b^4*Sin[c + d*x] - 13*A*b^6*Sin[c + d*x] - 15*a^
3*b^3*B*Sin[c + d*x] + 9*a*b^5*B*Sin[c + d*x])/(4*a^4*(a^2 - b^2)^2*(b + a*
Cos[c + d*x])) + (A*Sin[2*(c + d*x)])/(3*a^3)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2215 vs. $2(569) = 1138$.
time = 9.87, size = 2216, normalized size = 4.25

method	result	size
default	Expression too large to display	2216

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/a^5*(4*A*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^2-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^2*a^2+a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+18*A*b^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-9*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)+4*b^2/a^4*(5*A*b-3*B*a)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*b^4*(A*b-B*a)/a^
5*(1/2/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^
2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1
/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
```

```
(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)
/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticP
i(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2
-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*
sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2
*c),2*a/(a-b),2^(1/2)))+2*b^3/a^5*(5*A*b-4*B*a)*(1/b*a^2/(a^2-b^2)*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*
d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2-b^2)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)
*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b^3*sec(d*x + c)^5 + 3*a*
b^2*sec(d*x + c)^4 + 3*a^2*b*sec(d*x + c)^3 + a^3*sec(d*x + c)^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*sec(d*x + c)^(3/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^3\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^3*(1/cos(c + d*x))^(3/2)), x
)
```

3.435 $\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=336

$$\frac{(4Ab+3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}}+\frac{(4aAb-a^2B+4b^2B)\sqrt{\frac{b+a\cos(c+\cdots}{a+b}}}{4bd\sqrt{a+\cdots}}$$

```
[Out] 1/4*(4*A*b+3*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)
*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+1/4*(4*A*a*b-B*a^2+4*B*b^2)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2
,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b
/d/(a+b*sec(d*x+c))^(1/2)+1/2*B*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c)
)^(1/2)/d-1/4*(4*A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)
/b/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+1/4*(4*A*b+B*a)*sin(d*
x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b/d
```

Rubi [A]
time = 0.73, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {4116, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(a^2(-B)+4aAb+4b^2B)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4bd\sqrt{a+b\sec(c+dx)}}+\frac{(aB+4Ab)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{4bd}+\frac{(3aB+4Ab)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}-\frac{(aB+4Ab)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4bd\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{B\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((4*A*b + 3*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) + ((4*a*A
*b - a^2*B + 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
 d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*b*d*Sqrt[a + b*Sec[c + d*x]]
) - ((4*A*b + a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c +
 d*x]])/(4*b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + ((4
*A*b + a*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*b*
d) + (B*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4116

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n
- 1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B
*m + A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}
, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
```

```
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \frac{B\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d}$$

$$= \frac{(4Ab+aB)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{4bd}$$

$$= \frac{(4Ab+aB)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{4bd}$$

$$= \frac{(4Ab+aB)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{4bd}$$

$$= \frac{(4Ab+aB)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{4bd}$$

$$= \frac{(4aAb-a^2B+4b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi(}{4bd\sqrt{a+b\sec(c+}}$$

$$= \frac{(4Ab+3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\right)}{4d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 15.49, size = 422, normalized size = 1.26

$$\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{8aBF\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{(a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}+\frac{2(4aAb-3a^2B+8b^2B)\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b(a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}-\frac{2i(4Ab+aB)\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\csc(c+dx)\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)\right)\right)}{a\sqrt{\frac{1}{a-b}}\,b^2\sqrt{b+a\cos(c+dx)}}+\frac{4(4Ab+aB)\tan(c+dx)}{b}+8B\sec(c+dx)\tan(c+dx)\right)}{16d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((8*a*B*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/((
a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(4*a*A*b - 3*a^2*B + 8*b^2*
B)*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b*(a + b)*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)]) - ((2*I)*(4*A*b + a*B)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a
+ b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*Elli
pticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a +
 b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x
]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]
*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*b^2*
Sqrt[b + a*Cos[c + d*x]]) + (4*(4*A*b + a*B)*Tan[c + d*x])/b + 8*B*Sec[c +
d*x]*Tan[c + d*x]))/(16*d*Sqrt[Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 12.97, size = 2521, normalized size = 7.50

method	result	size
default	Expression too large to display	2521

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/d*(-2*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+4*A*cos(d*x+c)^3*sin(d*x+c)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-8*A
*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b-B*cos(d*x+c)^3*sin(d*x+c)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-2*
B*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a*b+4*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-8*A*cos(d*x+c
)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/
(a-b),I/((a-b)/(a+b))^(1/2))*a*b-B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+2*B*((a-b)/(
a+b))^(1/2)*b^2-4*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b-2*B*cos(d*x+c)^3*(
(a-b)/(a+b))^(1/2)*a*b+4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-B*cos(d*x+c
)^2*((a-b)/(a+b))^(1/2)*a*b+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-2*B*cos(
```

```
d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2-B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2-4*A
*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2+B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^
2+4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-8*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))
^(1/2))*b^2-4*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+B*cos(d*x+c)^2*sin(d*x+c)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-2
*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^2+4*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+2*B*cos(d*x+
c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)
/(a-b),I/((a-b)/(a+b))^(1/2))*a^2-8*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))
*b^2-4*A*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+B*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-2*B*cos(
d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^2+4*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+2*B*cos(d*x+c)^3*si
n(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),
I/((a-b)/(a+b))^(1/2))*a^2)*(1/cos(d*x+c))^(3/2)*((b+a*cos(d*x+c))/cos(d*x+
c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2),
 x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2),
 x)
```

3.436 $\int \sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=253

$$\frac{(2aA+bB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}+\frac{(2Ab+aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c\right.}{d\sqrt{a+b\sec(c+}}$$

```
[Out] (2*A*a+B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d
*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+(2*A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/
2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2
)-B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(
a+b))^(1/2)/sec(d*x+c)^(1/2)+B*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))
^(1/2)/d
```

Rubi [A]
time = 0.52, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {4116, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(2aA+bB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{(aB+2Ab)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}-\frac{B\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((2*a*A + b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b + a
*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) - (B*EllipticE[(c +
d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]*Sqrt[Sec[c + d*x]]) + (B*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/d
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
```

```
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4116

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n
- 1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B
*m + A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}
, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \frac{B\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c}{d}$$

$$= \frac{B\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c}{d}$$

$$= \frac{B\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c}{d}$$

$$= \frac{B\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c}{d}$$

$$= \frac{(2Ab+aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\tfrac{1}{2}(c+\right.}{d\sqrt{a+b\sec(c+dx}}$$

$$= \frac{(2aA+bB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\tfrac{1}{2}(c+d\right.}{d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 15.01, size = 377, normalized size = 1.49

$$\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{8aAF\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{(a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}+\frac{2(4Ab+aB)\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{(a+b)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}-\frac{2iB\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\csc(c+dx)\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)\right)\right)}{a\sqrt{\frac{1}{a-b}}\,b\sqrt{b+a\cos(c+dx)}}+4B\tan(c+dx)\right)}{4d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((8*a*A*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/((
a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(4*A*b + a*B)*EllipticPi[2,
 (c + d*x)/2, (2*a)/(a + b)])/((a + b)*Sqrt[(b + a*Cos[c + d*x])/(a + b)])
- ((2*I)*B*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*
x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(
-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcS
inh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*Ell
```

```
ipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-
a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*b*Sqrt[b + a*Cos[c + d*x]]) + 4*B*
Tan[c + d*x]))/(4*d*Sqrt[Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 13.50, size = 1431, normalized size = 5.66

method	result	size
default	Expression too large to display	1431

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/d*(2*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-2*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+4*A*cos(d
*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a
+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b-B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+B*cos(d*x+c)
^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*b+2*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a+2*A*cos(d*x+c)*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a-2*A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+4*A*cos(d*x+c)*sin(d*x+c)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(
1/2))*b-B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a+B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+2*B*cos(d*x+c)*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b
),I/((a-b)/(a+b))^(1/2))*a+B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a-B*((a-b)/(a
+b))^(1/2)*cos(d*x+c)*a+B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b-B*((a-b)/(a+b))^
```

```
(1/2)*b)*(1/cos(d*x+c))^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/(b+a*cos(
d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)),
 x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec{(c + dx)})\,\sqrt{a + b\sec{(c + dx)}}\,\sqrt{\sec{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sqrt(sec(c + d*x)),
x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c)),
 x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2),
x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2),
 x)
```

3.437 $\int \frac{\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=208

$$\frac{2aB\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}+\frac{2bB\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{}}{d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*a*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^
(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*co
s(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2*A*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2
)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {4122, 3939, 3943, 2742, 2740, 3944, 2886, 2884, 3941, 2734, 2732}

$$\frac{2A\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2aB\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2bB\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] (2*a*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*b*B*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c
 + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*A*EllipticE[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sq
rt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3939

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[a, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f
*x]], x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4122

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B
_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] :> Dist[B/d, In
t[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]], x], x] + Dist[A, Int[Sqrt[
a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = A\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx + B\int \sqrt{\sec(c+dx)}\,\sqrt{a+b}$$

$$= (aB)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx + (bB)\int \frac{\sec^{\frac{3}{2}}(c+dx}{\sqrt{a+b\sec(c+}}$$

$$= \frac{\left(aB\sqrt{b+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+}}}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2AE\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c+dx)}} + \frac{\left(aB\sqrt{\frac{b+a\,c}{}}\right.}{}$$

$$= \frac{2aB\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A]
time = 2.55, size = 122, normalized size = 0.59

$$\frac{2\left(A(a+b)E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+B\left(aF\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+b\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\right)\right)\sqrt{a+b\sec(c+dx)}}{(a+b)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]
],x]
```

```
[Out] (2*(A*(a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + B*(a*EllipticF[(c + d
*x)/2, (2*a)/(a + b)] + b*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]))*Sqrt[
a + b*Sec[c + d*x]])/((a + b)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec
[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 13.15, size = 1549, normalized size = 7.45

method	result	size

default	Expression too large to display	1549

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] 2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a+A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b-B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a+B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b-2*B*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b+A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)-A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+c)-A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)+A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+c)-B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)+B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+c)-2*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b*sin(d*x+c)-A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a+A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a-A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b+A*((a-b)/(a+b))^(1/2)*b)/(1/cos(d*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sqrt{a + b\sec(c + dx)}}{\sqrt{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))/sqrt(sec(c + d*x)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(1/2
), x)
```

3.438 $\int \frac{\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=201

$$\frac{2A(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3ad\sqrt{a+b\sec(c+dx)}}+\frac{2(Ab+3aB)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

```
[Out] 2/3*A*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*s
ec(d*x+c)^(1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2/3*A*sin(d*x+c)*(a+b*sec(d*x+c)
)^(1/2)/d/sec(d*x+c)^(1/2)+2/3*(A*b+3*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*
sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {4117, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2A(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{a+b\sec(c+dx)}}+\frac{2(3aB+Ab)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2A\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*A*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(A
*b + 3*a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(3*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[
a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4117

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{2}{3}\int \frac{\frac{1}{2}(Ab+3aB)+}{\sqrt{\sec(c+dx}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{(A(a^2-b^2))\int \frac{\sqrt{}}{\sqrt{a+}}}{3a}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left(A(a^2-b^2)\sqrt{b+a}\right.}{}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} + \frac{\left(A(a^2-b^2)\sqrt{\frac{b+c}{}}\right.}{}$$

$$= \frac{2A(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c}}{3ad\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A]
time = 0.81, size = 165, normalized size = 0.82

$$\frac{2\sqrt{a+b\sec(c+dx)}\left((a+b)(Ab+3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+A(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+aA(b+a\cos(c+dx))\sin(c+dx)\right)}{3ad(b+a\cos(c+dx))\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2
),x]
```

```
[Out] (2*Sqrt[a + b*Sec[c + d*x]]*((a + b)*(A*b + 3*a*B)*Sqrt[(b + a*Cos[c + d*x]
)/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + A*(a^2 - b^2)*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*A*(b + a
*Cos[c + d*x])*Sin[c + d*x]))/(3*a*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]
])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1925 vs. $2(237) = 474$.
time = 13.13, size = 1926, normalized size = 9.58

method	result	size
default	Expression too large to display	1926

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-A*((a-b)/(a+b))^(1/2)*b^2-A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-3*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+3*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)-A*((a-b)/(a+b))^(1/2)*a*b-3*B*((a-b)/(a+b))^(1/2)*a*b+3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)+2*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+3*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-3*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)-A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)-3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)-3*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2+A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2+3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2+A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-A*((a-b)/(a+b))^(1/2)*cos(d*x+c

```
)*a*b+A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a^2*sin(d*x+c)+3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)-3*B*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c))*cos(d*x+c)^2*
(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(b+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.62, size = 452, normalized size = 2.25

$$\frac{6\,Aa^2\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\sqrt{2}\left(-3i\,Aa^2-3i\,Bab+2i\,Ab^2\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(3i\,Aa^2+3i\,Bab-2i\,Ab^2\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(-3i\,Ba^2-i\,Aab\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(3i\,Ba^2+i\,Aab\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)}{9\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/9*(6*A*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c) + sqrt(2)*(-3*I*A*a^2 - 3*I*B*a*b + 2*I*A*b^2)*sqrt(a)*weierstras
sPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*co
s(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*A*a^2 + 3*I*B*a*b
- 2*I*A*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*
a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) -
3*sqrt(2)*(-3*I*B*a^2 - I*A*a*b)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^
2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2
)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x +
c) + 2*b)/a)) - 3*sqrt(2)*(3*I*B*a^2 + I*A*a*b)*sqrt(a)*weierstrassZeta(-4/
3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3
*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3
*I*a*sin(d*x + c) + 2*b)/a)))/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\left(A + B\sec\left(c + dx\right)\right)\sqrt{a + b\sec\left(c + dx\right)}}{\sec^{\frac{3}{2}}\left(c + dx\right)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(3/2),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(3/2
), x)
```

3.439 $\int \frac{\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=267

$$-\frac{2(a^2-b^2)(2Ab-5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2A-2Ab^2+5abB)\,E}{15a^2d\sqrt{\frac{b+a\,\mathrm{co}}{a}}}$$

```
[Out] -2/15*(a^2-b^2)*(2*A*b-5*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/
(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2)+2/5*A*sin(d*x+c)
*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+2/15*(A*b+5*B*a)*sin(d*x+c)*(a+b
*sec(d*x+c))^(1/2)/a/d/sec(d*x+c)^(1/2)+2/15*(9*A*a^2-2*A*b^2+5*B*a*b)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b)
)^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.50, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4117, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{2(a^2-b^2)(2Ab-5aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2A+5abB-2Ab^2)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(5aB+Ab)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{15ad\sqrt{\sec(c+dx)}}+\frac{2A\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (-2*(a^2 - b^2)*(2*A*b - 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^2*d*Sqrt[a + b*Sec[
c + d*x]]) + (2*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*EllipticE[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a +
 b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d
*Sec[c + d*x]^(3/2)) + (2*(A*b + 5*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*
x])/(15*a*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4117

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{1}{2}(Ab+5aB)+}{\text{se}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+5aB)\sqrt{a+b}}{15ad\sqrt{\text{s}}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+5aB)\sqrt{a+b}}{15ad\sqrt{\text{s}}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+5aB)\sqrt{a+b}}{15ad\sqrt{\text{s}}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+5aB)\sqrt{a+b}}{15ad\sqrt{\text{s}}}$$

$$= -\frac{2(a^2-b^2)\,(2Ab-5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\;F\left(\frac{1}{2}(c+dx)\right.}{15a^2d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A]

time = 1.28, size = 200, normalized size = 0.75

$$\frac{2\sqrt{a+b\sec(c+dx)}\left((a+b)\left(9a^2A-2Ab^2+5abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+\left(a^2-b^2\right)(-2Ab+5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+a(b+a\cos(c+dx))(Ab+5aB+3aA\cos(c+dx))\sin(c+dx)\right)}{15a^2d(b+a\cos(c+dx))\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2
),x]
```

```
[Out] (2*Sqrt[a + b*Sec[c + d*x]]*((a + b)*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*Sqrt[(b
+ a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^2 - b
^2)*(-2*A*b + 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)
/2, (2*a)/(a + b)] + a*(b + a*Cos[c + d*x])*(A*b + 5*a*B + 3*a*A*Cos[c + d*
x])*Sin[c + d*x]))/(15*a^2*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2738 vs. $2(297) = 594$.
time = 13.33, size = 2739, normalized size = 10.26

method	result	size
default	Expression too large to display	2739

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(9*A*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)+2*A*((a-b)/
(a+b))^(1/2)*b^3-9*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+5*A*((a-b)/(a+b))^(1/2)*co
s(d*x+c)*a^2*b+2*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2-5*B*((a-b)/(a+b))^(
1/2)*cos(d*x+c)*a^2*b+5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2+4*A*((a-b)/(
a+b))^(1/2)*cos(d*x+c)^3*a^2*b+5*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-5*B*sin(d*x+
c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a*b^2-5*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-2*A*sin(d*x+c)*cos(d*x+
c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a*b^2+7*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
```

```
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+2*A*sin(d*x+c)*cos(d*x+c)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+9*
A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^3+2*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-9*A*sin(d*x+c)*co
s(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^3+5*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-9*A*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)-2*A*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*s
in(d*x+c)+7*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^2*b*sin(d*x+c)+2*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)+5*B*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)-5*
B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*b^2*sin(d*x+c)-5*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)+2*A*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)-9*A*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+
c)+5*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^3*sin(d*x+c)+5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3-5*B*((a-b)/(
a+b))^(1/2)*cos(d*x+c)*a^3+3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^3+6*A*((a
-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3-9*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3-2*
A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^3-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b
^2+10*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b-9*A*((a-b)/(a+b))^(1/2)*a^2*
b-A*((a-b)/(a+b))^(1/2)*a*b^2-5*B*((a-b)/(a+b))^(1/2)*a^2*b-5*B*((a-b)/(a+b
))^(1/2)*a*b^2)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*x+c
))/((a-b)/(a+b))^(1/2)/a^2
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.17, size = 519, normalized size = 1.94

$$\frac{\sqrt{2}\left(-15i\,Ba^3-3i\,Aa^2b+10i\,Bab^2-4i\,Ab^3\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(15i\,Ba^3+3i\,Aa^2b-10i\,Bab^2+4i\,Ab^3\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(-9i\,Aa^3-5i\,Ba^2b+2i\,Aab^2\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(9i\,Aa^3+5i\,Ba^2b-2i\,Aab^2\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)+\frac{6\left(3Aa^3\cos(dx+c)^2+(5Ba^3+Aa^2b)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{45\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/45*(sqrt(2)*(-15*I*B*a^3 - 3*I*A*a^2*b + 10*I*B*a*b^2 - 4*I*A*b^3)*sqrt(a
)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3,
 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(15*I*B*a^3
 + 3*I*A*a^2*b - 10*I*B*a*b^2 + 4*I*A*b^3)*sqrt(a)*weierstrassPInverse(-4/3
*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3
*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-9*I*A*a^3 - 5*I*B*a^2*b + 2*I*A*a
*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b
^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^
3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(
9*I*A*a^3 + 5*I*B*a^2*b - 2*I*A*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 -
 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(
d*x + c) + 2*b)/a)) + 6*(3*A*a^3*cos(d*x + c)^2 + (5*B*a^3 + A*a^2*b)*cos(d
*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
+ c)))/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\sqrt{a+b\sec(c+dx)}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))/sec(c + d*x)**(5/2),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(5/2
), x)
```

3.440 $\int \frac{\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=343

$$\frac{2(a^2-b^2)\left(25a^2A+8Ab^2-14abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{105a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2(19a^2Ab+8A$$

```
[Out] 2/105*(a^2-b^2)*(25*A*a^2+8*A*b^2-14*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+
a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^3/d/(a+b*sec(d*x+c))^(1/2)+2/
7*A*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+2/35*(A*b+7*B*a)*s
in(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d/sec(d*x+c)^(3/2)+2/105*(25*A*a^2-4*A*b
^2+7*B*a*b)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^2/d/sec(d*x+c)^(1/2)+2/105*
(19*A*a^2*b+8*A*b^3+63*B*a^3-14*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*se
c(d*x+c))^(1/2)/a^3/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.70, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4117, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2\left(25a^2A+7abB-4Ab^2\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{105a^2d\sqrt{\sec(c+dx)}}+\frac{2(a^2-b^2)\left(25a^2A-14abB+8Ab^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2\left(63a^3B+19a^2Ab-14ab^2B+8Ab^3\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^3d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(7aB+Ab)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{35ad\sec^{\frac{3}{2}}(c+dx)}+\frac{2A\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(a^2 - b^2)*(25*a^2*A + 8*A*b^2 - 14*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(105*a^3*d
*Sqrt[a + b*Sec[c + d*x]]) + (2*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2
*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*a^
3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a +
b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*(A*b + 7*a*B)*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*a*d*Sec[c + d*x]^(3/2)) + (2*(25*
a^2*A - 4*A*b^2 + 7*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(105*a^2*
d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4117

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{1}{2}(Ab+7aB)+}{\text{s}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a+b}}{35ad\,\text{se}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a+b}}{35ad\,\text{se}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a+b}}{35ad\,\text{se}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a+b}}{35ad\,\text{se}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+7aB)\sqrt{a+b}}{35ad\,\text{se}}$$

$$= \frac{2(a^2-b^2)\,(25a^2A+8Ab^2-14abB)\,\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F}{105a^3d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A]
time = 1.41, size = 208, normalized size = 0.61

$$\frac{\sqrt{a+b\sec(c+dx)}\left(\frac{4\left((19a^2Ab+8Ab^3+63a^3B-14ab^2B)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+(a-b)(25a^2A+8Ab^2-14abB)F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)}{\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}+a((115a^2A-16Ab^2+28abB)\sin(c+dx)+3a(2(Ab+7aB)\sin(2(c+dx))+5aA\sin(3(c+dx))))\right)}{210a^3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2
),x]
```

```
[Out] (Sqrt[a + b*Sec[c + d*x]]*((4*((19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2*
B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a - b)*(25*a^2*A + 8*A*b^2 - 14
*a*b*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))/Sqrt[(b + a*Cos[c + d*x])/(
a + b)] + a*((115*a^2*A - 16*A*b^2 + 28*a*b*B)*Sin[c + d*x] + 3*a*(2*(A*b +
```

```
7*a*B)*Sin[2*(c + d*x)] + 5*a*A*Sin[3*(c + d*x)]))))/(210*a^3*d*Sqrt[Sec[c
 + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3777 vs. $2(367) = 734$.
time = 14.10, size = 3778, normalized size = 11.01

method	result	size
default	Expression too large to display	3778

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/105/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-8*A*cos(d*x+c)*((a-b)/(a+b))
^(1/2)*a*b^3+35*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b+14*B*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*a^2*b^2-14*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+18*A*cos(
d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b-A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b
^2+28*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b-8*A*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*sin(d*x+c)-63*B*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*
sin(d*x+c)+63*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^4*sin(d*x+c)-19*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+25*A*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*si
n(d*x+c)-14*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+14*B*sin(d*x+c)*cos(d*x+c)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^
3+8*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4-63*B*cos(d*x+c)*((a-b)/(a+b))^(1/2
)*a^4+15*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4+10*A*cos(d*x+c)^3*((a-b)/(a
+b))^(1/2)*a^4-25*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4+21*B*cos(d*x+c)^4*((
a-b)/(a+b))^(1/2)*a^4+42*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4+2*A*sin(d*x
+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^2*b^2-8*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+19*A*sin(d*x+c)*cos(
d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
```

```
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^3*b-19*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+8*A*sin(d*x+c)*cos(d*x+c)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
*b^3+49*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+14*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-63
*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a^3*b-25*A*((a-b)/(a+b))^(1/2)*a^3*b-19*A*((a-b)/(
a+b))^(1/2)*a^2*b^2+4*A*((a-b)/(a+b))^(1/2)*a*b^3-63*B*((a-b)/(a+b))^(1/2)*
a^3*b-7*B*((a-b)/(a+b))^(1/2)*a^2*b^2+14*B*((a-b)/(a+b))^(1/2)*a*b^3+25*A*s
in(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a^4-8*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4-63*B*sin(d*x+c)*cos(
d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^4+63*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-19*A*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*sin(d*x+c)+2*A*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2
*sin(d*x+c)-8*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b^3*sin(d*x+c)+19*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*sin(d*x+c)-19*A*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.65, size = 588, normalized size = 1.71

$$\frac{\sqrt{2}\left(-75i\,Aa^4-21i\,Ba^3b+32i\,Aa^2b^2-28i\,Bab^3+16i\,Ab^4\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos\left(dx+c\right)+3i\,a\sin\left(dx+c\right)+2b}{3a}\right)+\sqrt{2}\left(75i\,Aa^4+21i\,Ba^3b-32i\,Aa^2b^2+28i\,Bab^3-16i\,Ab^4\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos\left(dx+c\right)-3i\,a\sin\left(dx+c\right)+2b}{3a}\right)-3\sqrt{2}\left(-63i\,Ba^4-19i\,Aa^3b+14i\,Ba^2b^2-8i\,Aab^3\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos\left(dx+c\right)+3i\,a\sin\left(dx+c\right)+2b}{3a}\right)\right)-3\sqrt{2}\left(63i\,Ba^4+19i\,Aa^3b-14i\,Ba^2b^2+8i\,Aab^3\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos\left(dx+c\right)-3i\,a\sin\left(dx+c\right)+2b}{3a}\right)\right)+\frac{6\left(15\,Aa^4\cos\left(dx+c\right)^3+3\left(7\,Ba^4+Aa^3b\right)\cos\left(dx+c\right)^2+\left(25\,Aa^4+7\,Ba^3b-4\,Aa^2b^2\right)\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\sin\left(dx+c\right)}{\sqrt{\cos\left(dx+c\right)}}}{315\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/315*(sqrt(2)*(-75*I*A*a^4 - 21*I*B*a^3*b + 32*I*A*a^2*b^2 - 28*I*B*a*b^3
+ 16*I*A*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) +
 sqrt(2)*(75*I*A*a^4 + 21*I*B*a^3*b - 32*I*A*a^2*b^2 + 28*I*B*a*b^3 - 16*I*
A*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b
- 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt
(2)*(-63*I*B*a^4 - 19*I*A*a^3*b + 14*I*B*a^2*b^2 - 8*I*A*a*b^3)*sqrt(a)*wei
erstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstr
assPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*
cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(63*I*B*a^4 + 19*I
*A*a^3*b - 14*I*B*a^2*b^2 + 8*I*A*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^
2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*si
n(d*x + c) + 2*b)/a)) + 6*(15*A*a^4*cos(d*x + c)^3 + 3*(7*B*a^4 + A*a^3*b)*
cos(d*x + c)^2 + (25*A*a^4 + 7*B*a^3*b - 4*A*a^2*b^2)*cos(d*x + c))*sqrt((a
*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2)/sec(d*x+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(7/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(7/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/(1/cos(c + d*x))^(7/2
), x)
```

3.441 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=421

$$\frac{(42aAb+17a^2B+16b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{24d\sqrt{a+b\sec(c+dx)}}+\frac{(6a^2Ab+8Ab^3-a^3B+$$

[Out] 1/24*(42*A*a*b+17*B*a^2+16*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+1/8*(6*A*a^2*b+8*A*b^3-B*a^3+12*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b/d/(a+b*sec(d*x+c))^(1/2)+1/12*(6*A*b+7*B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+1/3*b*B*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d-1/24*(30*A*a*b+3*B*a^2+16*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+1/24*(30*A*a*b+3*B*a^2+16*B*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b/d

Rubi [A]

time = 1.04, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {4111, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(3a^2B+30aAb+16b^2B)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{24bd}+\frac{(17a^2B+42aAb+16b^2B)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{a+b\sec(c+dx)}}-\frac{(3a^2B+30aAb+16b^2B)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24bd\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(a^3(-B)+6a^2Ab+12ab^2B+8Ab^3)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{8bd\sqrt{a+b\sec(c+dx)}}+\frac{(7aB+6Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{12d}+\frac{bB\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]

[Out] ((42*a*A*b + 17*a^2*B + 16*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(24*d*Sqrt[a + b*Sec[c + d*x]]) + ((6*a^2*A*b + 8*A*b^3 - a^3*B + 12*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(8*b*d*Sqrt[a + b*Sec[c + d*x]]) - ((30*a*A*b + 3*a^2*B + 16*b^2*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(24*b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + ((30*a*A*b + 3*a^2*B + 16*b^2*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*b*d) + ((6*A*b + 7*a*B)*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(12*d) + (b*B*Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
```

```
b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{bB\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+d}{3d}$$

$$= \frac{(6Ab+7aB)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{12d}$$

$$= \frac{(30aAb+3a^2B+16b^2B)\sqrt{\sec(c+dx)}\,\sqrt{a}}{24bd}$$

$$= \frac{(30aAb+3a^2B+16b^2B)\sqrt{\sec(c+dx)}\,\sqrt{a}}{24bd}$$

$$= \frac{(30aAb+3a^2B+16b^2B)\sqrt{\sec(c+dx)}\,\sqrt{a}}{24bd}$$

$$= \frac{(30aAb+3a^2B+16b^2B)\sqrt{\sec(c+dx)}\,\sqrt{a}}{24bd}$$

$$= \frac{(6a^2Ab+8Ab^3-a^3B+12ab^2B)\sqrt{\frac{b+a\cos(}{a+}}}{8bd\sqrt{a+b\,\mathrm{s}}}$$

$$= \frac{(42aAb+17a^2B+16b^2B)\sqrt{\frac{b+a\cos(c+dx}{a+b}}}{24d\sqrt{a+b\sec(c}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 16.82, size = 673, normalized size = 1.60

$$-\frac{(a+b\sec(c+dx))^{3/2}\left(\frac{2(-24aAb^2-28a^2bB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2(-6a^2Ab-48Ab^3+9a^3B-56ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2i(30a^2Ab+3a^3B+16ab^2B)\sqrt{\frac{a-a\cos(c+dx)}{a+b}}\sqrt{\frac{a+a\cos(c+dx)}{a-b}}\cos(2(c+dx))\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)\right)\right)\sin(c+dx)}{\sqrt{\frac{1}{a-b}}\,b\sqrt{1-\cos^2(c+dx)}\sqrt{\frac{a^2-a^2\cos^2(c+dx)}{a^2}}\left(-a^2+2b^2-4b(b+a\cos(c+dx))+2(b+a\cos(c+dx))^2\right)}\right)}{96bd(b+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}+\frac{(a+b\sec(c+dx))^{3/2}\left(\frac{1}{12}\sec^2(c+dx)(6Ab\sin(c+dx)+7aB\sin(c+dx))+\frac{\sec(c+dx)(30aAb\sin(c+dx)+3a^2B\sin(c+dx)+16b^2B\sin(c+dx))}{24b}+\frac{1}{3}bB\sec^2(c+dx)\tan(c+dx)\right)}{d(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] -1/96*((a + b*Sec[c + d*x])^(3/2)*((2*(-24*a*A*b^2 - 28*a^2*b*B)*Sqrt[(b +
a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*
Cos[c + d*x]] + (2*(-6*a^2*A*b - 48*A*b^3 + 9*a^3*B - 56*a*b^2*B)*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b
 + a*Cos[c + d*x]] + ((2*I)*(30*a^2*A*b + 3*a^3*B + 16*a*b^2*B)*Sqrt[(a - a
*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]
*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*
x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqr
t[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh
[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c +
d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c
+ d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c +
d*x])^2))))/(b*d*(b + a*Cos[c + d*x])^(3/2)*Sec[c + d*x]^(3/2)) + ((a + b*S
ec[c + d*x])^(3/2)*((Sec[c + d*x]^2*(6*A*b*Sin[c + d*x] + 7*a*B*Sin[c + d*x
]))/12 + (Sec[c + d*x]*(30*a*A*b*Sin[c + d*x] + 3*a^2*B*Sin[c + d*x] + 16*b
^2*B*Sin[c + d*x]))/(24*b) + (b*B*Sec[c + d*x]^2*Tan[c + d*x])/3))/(d*(b +
a*Cos[c + d*x])*Sec[c + d*x]^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 13.32, size = 4051, normalized size = 9.62

method	result	size
default	Expression too large to display	4051

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/24/d*(-12*A*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2+22*B*((a-b)/(a+b))^(1/2)*cos(d*
x+c)*a*b^2+30*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b-36*A*cos(d*x+c)^4*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a
+b))^(1/2))*sin(d*x+c)*a^2*b-3*B*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b+16*B*cos(d*x
```

```
+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*sin(d*x+c)*a*b^2-14*B*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b+20*B*cos(d*x+c)^4
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*sin(d*x+c)*a*b^2-72*B*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+30*A*cos
(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*sin(d*x+c)*a^2*b-30*A*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2-12*A*cos(d*x+
c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*sin(d*x+c)*a^2*b-12*A*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2-36*A*cos(d*x+c)^3*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elli
pticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/
(a+b))^(1/2))*sin(d*x+c)*a^2*b-3*B*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b+16*B*cos(d
*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*sin(d*x+c)*a*b^2-14*B*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b+20*B*cos(d*x+c)
^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*sin(d*x+c)*a*b^2-72*B*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2+30*A*c
os(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*sin(d*x+c)*a^2*b-30*A*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2-12*A*cos(d*
x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*sin(d*x+c)*a^2*b-30*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^2*b-12*A*co
s(d*x+c)^4*((a-b)/(a+b))^(1/2)*a*b^2-14*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*
a^2*b-16*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a*b^2-30*A*cos(d*x+c)^3*((a-b)/
```

```
(a+b))^(1/2)*a*b^2-3*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b-6*B*cos(d*x+c
)^3*((a-b)/(a+b))^(1/2)*a*b^2+24*A*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^3-48*A*cos(d*x
+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/(
(a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3+3*B*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^3-16*B*cos
(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*sin(d*x+c)*b^3-6*B*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2),
 x)
```

3.442 $\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=339

$$\frac{(8a^2A+4Ab^2+7abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}}+\frac{(12aAb+3a^2B+4b^2B)\sqrt{}}{}$$

```
[Out] 1/4*(8*A*a^2+4*A*b^2+7*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/
(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+1/4*(12*A*a*b+3*B*a^
2+4*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1
/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec
(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+1/2*b*B*sec(d*x+c)^(3/2)*sin(d*x+c)*
(a+b*sec(d*x+c))^(1/2)/d-1/4*(4*A*b+5*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*
sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+1/4*(4*
A*b+5*B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.80, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {4111, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(8a^2A+7abB+4Ab^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{(3a^2B+12aAb+4b^2B)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{(5aB+4Ab)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{4d}-\frac{(5aB+4Ab)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{bB\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((8*a^2*A + 4*A*b^2 + 7*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a + b*Sec[c + d*
x]]) + ((12*a*A*b + 3*a^2*B + 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[a +
b*Sec[c + d*x]]) - ((4*A*b + 5*a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*S
qrt[a + b*Sec[c + d*x]])/(4*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c
 + d*x]]) + ((4*A*b + 5*a*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/(4*d) + (b*B*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c
+ d*x])/(2*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
```

```
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \frac{bB\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+}{2d}$$

$$= \frac{(4Ab+5aB)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+}{4d}$$

$$= \frac{(4Ab+5aB)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+}{4d}$$

$$= \frac{(4Ab+5aB)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+}{4d}$$

$$= \frac{(4Ab+5aB)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+}{4d}$$

$$= \frac{(12aAb+3a^2B+4b^2B)\sqrt{\frac{b+a\cos(c+dx}{a+b}}}{4d\sqrt{a+b\sec(}}$$

$$= \frac{(8a^2A+4Ab^2+7abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}{4d\sqrt{a+b\sec(c}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.76, size = 595, normalized size = 1.76

$$\frac{(a+b\sec(c+dx))^{3/2}\left(\frac{2(16a^2A+4abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2(20aAb+a^2B+8b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2i(-4aAb-5a^2B)\sqrt{\frac{a-a\cos(c+dx)}{a+b}}\sqrt{\frac{a+a\cos(c+dx)}{a-b}}\cos(2(c+dx))\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)\right)\right)\sin(c+dx)}{\sqrt{\frac{1}{a-b}}\,b\sqrt{1-\cos^2(c+dx)}\sqrt{\frac{a^2-a^2\cos^2(c+dx)}{a^2}}\,(-a^2+2b^2-4b(b+a\cos(c+dx))+2(b+a\cos(c+dx))^2)}\right)}{16d(b+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}+\frac{(a+b\sec(c+dx))^{3/2}\left(\frac{1}{4}\sec(c+dx)(4Ab\sin(c+dx)+5aB\sin(c+dx))+\frac{1}{2}bB\sec(c+dx)\tan(c+dx)\right)}{d(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*((2*(16*a^2*A + 4*a*b*B)*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]
] + (2*(20*a*A*b + a^2*B + 8*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elli
pticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(-
4*a*A*b - 5*a^2*B)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d
*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a -
b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*
ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a
*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]]
, (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c +
d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos
[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(16*d*(b + a*Cos[c + d*x])^(3/2)*
Sec[c + d*x]^(3/2)) + ((a + b*Sec[c + d*x])^(3/2)*((Sec[c + d*x]*(4*A*b*Sin
[c + d*x] + 5*a*B*Sin[c + d*x]))/4 + (b*B*Sec[c + d*x]*Tan[c + d*x])/2))/(d
*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 12.97, size = 2947, normalized size = 8.69

method	result	size
default	Expression too large to display	2947

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/d*(2*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*A*cos(d*x+c)^3*sin(d*x+c)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+24*
A*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b+5*B*cos(d*x+c)^3*sin(d*x+c)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b
+2*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+24*A*cos(d
*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a
```

```
+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b+5*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-2*B*((
a-b)/(a+b))^(1/2)*b^2+4*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b+2*B*cos(d*x+
c)^3*((a-b)/(a+b))^(1/2)*a*b-4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+5*B*c
os(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-7*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-
8*A*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a*b-8*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+2*B*cos(d*x
+c)^2*((a-b)/(a+b))^(1/2)*b^2+5*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2+4*A*
cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2-5*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a
^2-4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2+8*B*cos(d*x+c)^3*sin(d*x+c)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticP
i((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b)
)^(1/2))*b^2+4*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-5*B*cos(d*x+c)^2*sin(d*x+c)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
2+2*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-4*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+6*B*cos(d
*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a
+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+8*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/
2))*b^2+4*A*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-5*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*B
*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a^2-4*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+6*B*cos(d*x+c)
^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(
a-b),I/((a-b)/(a+b))^(1/2))*a^2+8*A*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
```

```
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+8*A*cos(d*x
+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1/cos(d*x+c))^(1
/2)/(b+a*cos(d*x+c))/cos(d*x+c)/sin(d*x+c)/((a-...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)
), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4369 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c)
), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos \left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos \left(c+d\,x\right)}\right)}^{3/2}\,\sqrt{\frac{1}{\cos \left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2),
x)

[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2),
x)

3.443 $\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=272

$$\frac{(2aAb+2a^2B+b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}+\frac{b(2Ab+3aB)\sqrt{\frac{b+a\cos(c}{a+b}}}{d\sqrt{a}}$$

```
[Out] (2*A*a*b+2*B*a^2+B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))
^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+b*(2*A*b+3*B*a)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/
2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*
sec(d*x+c))^(1/2)+(2*A*a-B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(
1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+b*B*sin(d*x+c)*sec(d
*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.56, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {4111, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(2a^2B+2aAb+b^2B)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{(2aA-bB)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{b(3aB+2Ab)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] ((2*a*A*b + 2*a^2*B + b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
 + (b*(2*A*b + 3*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
 d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) +
((2*a*A - b*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (b*B*Sqrt[S
ec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/d
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{bB\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \int \frac{1}{2}c$$

$$= \frac{bB\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{1}{2}(b($$

$$= \frac{bB\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} + \frac{1}{2}(2a$$

$$= \frac{bB\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d} - \frac{((-2}{}$$

$$= \frac{b(2Ab+3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{}}{d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{(2aAb+2a^2B+b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\right.}{d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.78, size = 554, normalized size = 2.04

$$\frac{bB(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{d(b+a\cos(c+dx))\sqrt{\sec(c+dx)}} + \frac{(a+b\sec(c+dx))^{3/2}\left(\frac{2(8aAb+4a^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}} + \frac{2(2a^2A+4Ab^2+5abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}} + \frac{2i(2a^2A-abB)\sqrt{\frac{a-a\cos(c+dx)}{a+b}}\sqrt{\frac{a+a\cos(c+dx)}{a-b}}\cos(2(c+dx))\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)\right)\right)\sin(c+dx)}{\sqrt{\frac{1}{a-b}}\,b\sqrt{1-\cos^2(c+dx)}\sqrt{\frac{a^2-a^2\cos^2(c+dx)}{a^2}}\left(-a^2+2b^2-4b(b+a\cos(c+dx))+2(b+a\cos(c+dx))^2\right)}\right)}{4d(b+a\cos(c+dx))^{3/2}\sec^{\frac{3}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] (b*B*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(d*(b + a*Cos[c + d*x])*Sqrt[
Sec[c + d*x]]) + ((a + b*Sec[c + d*x])^(3/2)*((2*(8*a*A*b + 4*a^2*B)*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b
+ a*Cos[c + d*x]] + (2*(2*a^2*A + 4*A*b^2 + 5*a*b*B)*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c +
d*x]] + ((2*I)*(2*a^2*A - a*b*B)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a
 + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcS
inh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*
```

```
b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b
)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*
Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqr
t[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 -
4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(4*d*(b + a*Cos[c +
 d*x])^(3/2)*Sec[c + d*x]^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 13.55, size = 2595, normalized size = 9.54

method	result	size
default	Expression too large to display	2595

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(6*B*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(
d*x+c)^2*a*b+6*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b
)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b+4*A*cos(d*x+c)*sin
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a*b-B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-2*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-2*A*c
os(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a*b-2*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-2*A*cos(d*x+c)^2*s
in(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a*b-B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-B*((a-b)/(a+b))^(1/2)*b^2+2*A*c
os(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-
B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-2*A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*B*cos(
d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
```

```
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^2+2*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a^2-2*A*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*
x+c)^2*b^2+4*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-2*A*cos(d*x+c)^2*((a-b)/(a+b))^(
1/2)*a^2+B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2+2*A*((a-b)/(a+b))^(1/2)*cos(d
*x+c)^3*a^2+2*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-2*A*((a-b)/(a+b))^(1/2)*cos(d*x
+c)*a*b+4*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-
b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^2+B*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+
c)^2*b^2+2*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2-2*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2+4*A*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a
+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2+B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2-2*A*cos(d
*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^2)*(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))/((
a-b)/(a+b))^(1/2)
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)
), x)
```

Fricas [**F**]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sqrt(b*sec(d
*x + c) + a)/sqrt(sec(d*x + c)), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B \sec{(c + dx)}) (a + b \sec{(c + dx)})^{\frac{3}{2}}}{\sqrt{\sec{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(3/2)/sqrt(sec(c + d*x)
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sqrt(sec(d*x + c)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+d\,x)}\right)\,\left(a + \frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(1/2
), x)
```

3.444 $\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=276

$$\frac{2(a^2A-Ab^2+3abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3d\sqrt{a+b\sec(c+dx)}}+\frac{2b^2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi(2}{d\sqrt{a+b\mathrm{s}}}$$

```
[Out] 2/3*(A*a^2-A*b^2+3*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b
))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2*b^2*B*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/
(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*
x+c))^(1/2)+2/3*a*A*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+2/
3*(4*A*b+3*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*c
os(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.60, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {4110, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(a^2A+3abB-Ab^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{a+b\sec(c+dx)}}+\frac{2(3aB+4Ab)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2aA\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\sec(c+dx)}}+\frac{2b^2B\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] (2*(a^2*A - A*b^2 + 3*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[a + b*Sec[c + d*x]
]) + (2*b^2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*(4*A*
b + 3*a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/
(3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a
 + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{1}{2}a(4Ab+$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{-\frac{1}{2}a(4Ab+$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{1}{3}(-4Ab-3aB)$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{\Big((-a^2A+Ab^2-$$

$$= \frac{2b^2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+}{d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2(a^2A-Ab^2+3abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\right.}{3d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.45, size = 437, normalized size = 1.58

$$\frac{(a+b\sec(c+dx))^{3/2}\left(\frac{4(a^2A+3Ab^2+6abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{(b+a\cos(c+dx))^2}+\frac{2(4aAb+3a^2B+6b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{(b+a\cos(c+dx))^2}+\frac{2i(4Ab+3aB)\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\csc(c+dx)\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)\right)\right)}{a\sqrt{\frac{1}{a-b}}\,b(b+a\cos(c+dx))^{3/2}}+\frac{4aA\sin(c+dx)}{b+a\cos(c+dx)}\right)}{6d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*((4*(a^2*A + 3*A*b^2 + 6*a*b*B)*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d
*x])^2 + (2*(4*a*A*b + 3*a^2*B + 6*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b + a*Cos[c + d*x])^2 + ((2*I
)*(4*A*b + 3*a*B)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos
[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a
 - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF
[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]
```

```
+ a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x
]]], (-a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*b*(b + a*Cos[c + d*x])^(3/2
)) + (4*a*A*Sin[c + d*x])/(b + a*Cos[c + d*x])))/(6*d*Sec[c + d*x]^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 13.11, size = 2552, normalized size = 9.25

method	result	size
default	Expression too large to display	2552

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-4*A*((a-b)/(a+b))^(1/2)*b^2-4*
A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a*b-3*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+6*B*cos(d*x+c)*si
n(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a*b+4*A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*A*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)-A*((a-b)/(a+b
))^(1/2)*a*b-3*B*((a-b)/(a+b))^(1/2)*a*b+6*B*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)+5*A*cos(d*x+c)^
2*((a-b)/(a+b))^(1/2)*a*b+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-4*A*cos(d*
x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*b^2+A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+3*B*cos(d*x+c)*sin(d*x+c)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^2-3*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+4*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)-4*A*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)-3*B*
```

```
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a*b*sin(d*x+c)+6*B*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a
+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2-3*B*(1/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2*sin(d*x+
c)+6*B*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*b^2*sin(d*x+c)-3*B*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*sin(d*x+c)*cos(d*x+c)*b^2-3*B*((a-b
)/(a+b))^(1/2)*cos(d*x+c)*a^2-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2+A*((a-b)
/(a+b))^(1/2)*cos(d*x+c)^3*a^2+3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2+4*A
*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2+3*A*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2*sin(d*x+c)-4*A*((a-b)/(a+b))^(1
/2)*cos(d*x+c)*a*b+3*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2+A*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)+3*B*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*s
in(d*x+c)-3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^2*sin(d*x+c))*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/
(b+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2
), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))(a + b\sec(c + dx))^{\frac{3}{2}}}{\sec^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(3/
2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+d\,x)}\right)\,\left(a + \frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(3/2
), x)
```

3.445 $\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=266

$$\frac{2(a^2-b^2)\,(3Ab+5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\;F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15ad\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2A+3Ab^2+20abB)\,E}{15ad\sqrt{\frac{b+a\cos}{a}}}$$

```
[Out] 2/15*(a^2-b^2)*(3*A*b+5*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(
a+b))^(1/2)*sec(d*x+c)^(1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2/5*a*A*sin(d*x+c)*
(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+2/15*(6*A*b+5*B*a)*sin(d*x+c)*(a+
b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+2/15*(9*A*a^2+3*A*b^2+20*B*a*b)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^
(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.53, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4110, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(a^2-b^2)\,(5aB+3Ab)\sqrt{\sec(c+dx)}\,\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15ad\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2A+20abB+3Ab^2)\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15ad\sqrt{\sec(c+dx)}\,\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(5aB+6Ab)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{15d\sqrt{\sec(c+dx)}}+\frac{2aA\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5d\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*(a^2 - b^2)*(3*A*b + 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a*d*Sqrt[a + b*Sec[c +
 d*x]]) + (2*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*EllipticE[(c + d*x)/2, (2*a)/(a
 + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]
*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*d*S
ec[c + d*x]^(3/2)) + (2*(6*A*b + 5*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*
x])/(15*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{1}{2}a(6Ab+$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(6Ab+5aB)\sqrt{}}{15a}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(6Ab+5aB)\sqrt{}}{15a}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(6Ab+5aB)\sqrt{}}{15a}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(6Ab+5aB)\sqrt{}}{15a}$$

$$= \frac{2(a^2-b^2)(3Ab+5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx\right.}{15ad\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A]

time = 1.58, size = 201, normalized size = 0.76

$$\frac{2(a+b\sec(c+dx))^{3/2}\left((a+b)\left(9a^2A+3Ab^2+20abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+\left(a^2-b^2\right)(3Ab+5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+a(b+a\cos(c+dx))(6Ab+5aB+3aA\cos(c+dx))\sin(c+dx)\right)}{15ad(b+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5
/2),x]
```

```
[Out] (2*(a + b*Sec[c + d*x])^(3/2)*((a + b)*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*Sqrt[
(b + a*Cos[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^2
- b^2)*(3*A*b + 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*
x)/2, (2*a)/(a + b)] + a*(b + a*Cos[c + d*x])*(6*A*b + 5*a*B + 3*a*A*Cos[c
+ d*x])*Sin[c + d*x]))/(15*a*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2914 vs. 2(296) = 592.
time = 14.08, size = 2915, normalized size = 10.96

method	result	size
default	Expression too large to display	2915

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-3*A*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b
^2-9*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+3*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-20*B*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin
(d*x+c)*cos(d*x+c)*a^2*b+15*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2+20*B*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*c
os(d*x+c)*a^2*b-20*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-3*A*((a-b)/(a+b))^(1/2)*b^
3+12*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
```

2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-9*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)+20*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2+9*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b+3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^3+6*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3-9*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3+9*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3-3*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3+5*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3+12*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)-3*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)-9*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)+3*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)-20*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)+15*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)+20*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)-20*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)+5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3-5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3-9*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3+3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^3+9*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^2+25*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b-9*A*((a-b)/(a+b))^(1/2)*a^2*b-6*A*((a-b)/(a+b))^(1/2)*a*b^2-5*B*((a-b)/(a+b))^(1/2)*a^2*b-20*B*((a-b)/(a+b))^(1/2)*a*b^2+9*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)-3*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)+5*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),

```
(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)-3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b
^2-20*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b)*cos(d*x+c)^3*(1/cos(d*x+c))^(
5/2)/sin(d*x+c)/(b+a*cos(d*x+c))/((a-b)/(a+b))^...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.22, size = 520, normalized size = 1.95

$$\frac{\sqrt{2}\left(-15i\,Ba^3-18i\,Aa^2b-5i\,Bab^2+6i\,Ab^3\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(15i\,Ba^3+18i\,Aa^2b+5i\,Bab^2-6i\,Ab^3\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(-9i\,Aa^3-20i\,Ba^2b-3i\,Aab^2\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(9i\,Aa^3+20i\,Ba^2b+3i\,Aab^2\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)+\frac{6\left(3Aa^3\cos(dx+c)^2+(5Ba^3+6Aa^2b)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{45a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/45*(sqrt(2)*(-15*I*B*a^3 - 18*I*A*a^2*b - 5*I*B*a*b^2 + 6*I*A*b^3)*sqrt(a
)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3,
 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(15*I*B*a^3
 + 18*I*A*a^2*b + 5*I*B*a*b^2 - 6*I*A*b^3)*sqrt(a)*weierstrassPInverse(-4/3
*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3
*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-9*I*A*a^3 - 20*I*B*a^2*b - 3*I*A*
a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*
b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b
^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*
(9*I*A*a^3 + 20*I*B*a^2*b + 3*I*A*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^
2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*si
n(d*x + c) + 2*b)/a)) + 6*(3*A*a^3*cos(d*x + c)^2 + (5*B*a^3 + 6*A*a^2*b)*c
os(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(
d*x + c)))/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\,(a+b\sec(c+dx))^{\frac{3}{2}}}{\sec^{\frac{5}{2}}(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(3/2)/sec(c + d*x)**(5/
2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{3/2}}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(5/2
), x)
```

3.446 $\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=342

$$\frac{2(a^2-b^2)\left(25a^2A-6Ab^2+21abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{105a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2(82a^2Ab-6Ab}{}$$

```
[Out] 2/105*(a^2-b^2)*(25*A*a^2-6*A*b^2+21*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+
a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2)+2/
7*a*A*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+2/35*(8*A*b+7*B*
a)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+2/105*(25*A*a^2+3*A
*b^2+42*B*a*b)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d/sec(d*x+c)^(1/2)+2/105
*(82*A*a^2*b-6*A*b^3+63*B*a^3+21*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*s
ec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.74, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4110, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(25a^2A+42abB+3Ab^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{105ad\sqrt{\sec(c+dx)}}+\frac{2(a^2-b^2)\left(25a^2A+21abB-6Ab^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{105a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2(63a^3B+82a^2Ab+21ab^2B-6Ab^3)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{105a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(7aB+8Ab)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2aA\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(a^2 - b^2)*(25*a^2*A - 6*A*b^2 + 21*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(105*a^2*d
*Sqrt[a + b*Sec[c + d*x]]) + (2*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2
*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*a^
2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*A*Sqrt[a
+ b*Sec[c + d*x]]*Sin[c + d*x])/(7*d*Sec[c + d*x]^(5/2)) + (2*(8*A*b + 7*a*
B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) + (2*(2
5*a^2*A + 3*A*b^2 + 42*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(105*a
*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{1}{2}a(8Ab+$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(8Ab+7aB)\sqrt{}}{35}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(8Ab+7aB)\sqrt{}}{35}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(8Ab+7aB)\sqrt{}}{35}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(8Ab+7aB)\sqrt{}}{35}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(8Ab+7aB)\sqrt{}}{35}$$

$$= \frac{2(a^2-b^2)\,(25a^2A-6Ab^2+21abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}{105a^2d\sqrt{a+b\sec(c+d}}$$

Mathematica [A]
time = 1.96, size = 255, normalized size = 0.75

$$\frac{(a+b\sec(c+dx))^{3/2}\left(4\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\left(a^2(25a^2A+51Ab^2+84abB)\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+(82a^2Ab-6Ab^3+63a^3B+21ab^2B)\left((a+b)E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)-bF\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\right)\right)+a(b+a\cos(c+dx))\left((115a^2A+12Ab^2+168abB)\sin(c+dx)+3a(2(8Ab+7aB)\sin(2(c+dx))+5aA\sin(3(c+dx)))\right)\right)}{210a^2d(b+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7
/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*(4*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(a^2*(25*
a^2*A + 51*A*b^2 + 84*a*b*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + (82*a^
2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*((a + b)*EllipticE[(c + d*x)/2, (2
*a)/(a + b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)])) + a*(b + a*Cos[c +
 d*x])*((115*a^2*A + 12*A*b^2 + 168*a*b*B)*Sin[c + d*x] + 3*a*(2*(8*A*b + 7
```

```
*a*B)*Sin[2*(c + d*x)] + 5*a*A*Sin[3*(c + d*x)]))))/(210*a^2*d*(b + a*Cos[c
 + d*x])^2*Sec[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3751 vs. $2(366) = 732$.
time = 14.16, size = 3752, normalized size = 10.97

method	result	size
default	Expression too large to display	3752

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/105/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(6*A*cos(d*x+c)*((a-b)/(a+b))^
(1/2)*a*b^3-21*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+21*B*cos(d*x+c)*((a
-b)/(a+b))^(1/2)*a*b^3+39*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b+27*A*cos
(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+63*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)
*a^3*b+6*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*b^4*sin(d*x+c)-63*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*sin(d*x+c)+63*B*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*sin(d*x+c)-82*A*sin(
d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^3*b+25*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*sin(d*x+c)+21*B*sin(d*x+c)*cos(d*x+c)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*
b^2-21*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3-6*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4
-63*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4+15*A*cos(d*x+c)^5*((a-b)/(a+b))^(1
/2)*a^4+10*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4-25*A*cos(d*x+c)*((a-b)/(a
+b))^(1/2)*a^4+21*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4+42*B*cos(d*x+c)^2*
((a-b)/(a+b))^(1/2)*a^4+51*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+6*A*sin(d*x+c)*c
os(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a*b^3+82*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+
```

```
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-82*A*sin(d*x+c)*cos(d*x+c)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a^2*b^2-6*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+84*B*sin(d*x+c)*cos(d*x+c)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-2
1*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-63*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-25*A*((a
-b)/(a+b))^(1/2)*a^3*b-82*A*((a-b)/(a+b))^(1/2)*a^2*b^2-3*A*((a-b)/(a+b))^(
1/2)*a*b^3-63*B*((a-b)/(a+b))^(1/2)*a^3*b-42*B*((a-b)/(a+b))^(1/2)*a^2*b^2-
21*B*((a-b)/(a+b))^(1/2)*a*b^3+25*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+6*A*sin(d*x+c
)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*b^4-63*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+63*B*sin(d*x+c)*cos(d*x+c)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a^4-82*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^3*b*sin(d*x+c)+51*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*sin(d*x+c)+6*A*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*sin(d*x+c)+82*A
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a^3*b*sin(d*x+c)-82*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2
), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.02, size = 589, normalized size = 1.72

$$\frac{\sqrt{2}\left(-75i\,Aa^4-126i\,Ba^3b+11i\,Aa^2b^2+42i\,Bab^3-12i\,Ab^4\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(75i\,Aa^4+126i\,Ba^3b-11i\,Aa^2b^2-42i\,Bab^3+12i\,Ab^4\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(-63i\,Ba^4-82i\,Aa^3b-21i\,Ba^2b^2+6i\,Aab^3\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(63i\,Ba^4+82i\,Aa^3b+21i\,Ba^2b^2-6i\,Aab^3\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)+\frac{6\left(15\,Aa^4\cos(dx+c)^3+3\,(7\,Ba^4+8\,Aa^3b)\cos(dx+c)^2+(25\,Aa^4+42\,Ba^3b+3\,Aa^2b^2)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{315\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="fricas")

[Out] 1/315*(sqrt(2)*(-75*I*A*a^4 - 126*I*B*a^3*b + 11*I*A*a^2*b^2 + 42*I*B*a*b^3
 - 12*I*A*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)
+ sqrt(2)*(75*I*A*a^4 + 126*I*B*a^3*b - 11*I*A*a^2*b^2 - 42*I*B*a*b^3 + 12*
I*A*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*
b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sq
rt(2)*(-63*I*B*a^4 - 82*I*A*a^3*b - 21*I*B*a^2*b^2 + 6*I*A*a*b^3)*sqrt(a)*w
eierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weiers
trassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*
a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(63*I*B*a^4 + 82
*I*A*a^3*b + 21*I*B*a^2*b^2 - 6*I*A*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*
a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*
sin(d*x + c) + 2*b)/a)) + 6*(15*A*a^4*cos(d*x + c)^3 + 3*(7*B*a^4 + 8*A*a^3
*b)*cos(d*x + c)^2 + (25*A*a^4 + 42*B*a^3*b + 3*A*a^2*b^2)*cos(d*x + c))*sq
rt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3
*d)

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)

[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(7/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{3/2}}{{\left(\frac{1}{\cos(c+d\,x)}\right)}^{7/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(7/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(7/2
), x)
```

3.447 $\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=427

$$\frac{2(a^2-b^2)\left(39a^2Ab+8Ab^3+75a^3B-18ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{315a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2(14$$

```
[Out] 2/315*(a^2-b^2)*(39*A*a^2*b+8*A*b^3+75*B*a^3-18*B*a*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b)
)^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^3/d/(a+b*sec(d*x
+c))^(1/2)+2/9*a*A*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(7/2)+2/6
3*(10*A*b+9*B*a)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+2/315
*(49*A*a^2+3*A*b^2+72*B*a*b)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d/sec(d*x+
c)^(3/2)+2/315*(88*A*a^2*b-4*A*b^3+75*B*a^3+9*B*a*b^2)*sin(d*x+c)*(a+b*sec(
d*x+c))^(1/2)/a^2/d/sec(d*x+c)^(1/2)+2/315*(147*A*a^4+33*A*a^2*b^2+8*A*b^4+
246*B*a^3*b-18*B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a
^3/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 1.00, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4110, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(49a^2A+72abB+3Ab^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{315ad\sec^{\frac{3}{2}}(c+dx)}+\frac{2(75a^3B+88a^2Ab+9ab^2B-4Ab^3)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{315a^2d\sqrt{\sec(c+dx)}}+\frac{2(a^2-b^2)(75a^3B+39a^2Ab-18ab^2B+8Ab^3)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{315a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2(147a^4A+246a^3bB+33a^2Ab^2-18ab^3B+8Ab^4)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{315a^3d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(9aB+10Ab)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{63d\sec^{\frac{5}{2}}(c+dx)}+\frac{2aA\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*(a^2 - b^2)*(39*a^2*A*b + 8*A*b^3 + 75*a^3*B - 18*a*b^2*B)*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*
x]])/(315*a^3*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(147*a^4*A + 33*a^2*A*b^2 +
8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*S
qrt[a + b*Sec[c + d*x]])/(315*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt
[Sec[c + d*x]]) + (2*a*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(9*d*Sec[c
+ d*x]^(7/2)) + (2*(10*A*b + 9*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/
(63*d*Sec[c + d*x]^(5/2)) + (2*(49*a^2*A + 3*A*b^2 + 72*a*b*B)*Sqrt[a + b*S
ec[c + d*x]]*Sin[c + d*x])/(315*a*d*Sec[c + d*x]^(3/2)) + (2*(88*a^2*A*b -
4*A*b^3 + 75*a^3*B + 9*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315
*a^2*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
```

```
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{2}{9}\int \frac{-\frac{1}{2}a(10Ab \cdots}{\cdots}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(10Ab+9aB)\sqrt{\cdots}}{6\cdots}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(10Ab+9aB)\sqrt{\cdots}}{6\cdots}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(10Ab+9aB)\sqrt{\cdots}}{6\cdots}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(10Ab+9aB)\sqrt{\cdots}}{6\cdots}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(10Ab+9aB)\sqrt{\cdots}}{6\cdots}$$

$$= \frac{2aA\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(10Ab+9aB)\sqrt{\cdots}}{6\cdots}$$

$$= \frac{2(a^2-b^2)\left(39a^2Ab+8Ab^3+75a^3B-18ab^2B\right)\sqrt{\frac{b+a\cdots}{\cdots}}}{315a^3d\sqrt{a+b\sec(\cdots}}$$

Mathematica [A]

time = 2.11, size = 313, normalized size = 0.73

$$\frac{(a+b\sec(c+dx))^{3/2}\left(8\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\left(a^2(186a^2Ab+2Ab^3+75a^3B+153ab^2B)F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+(147a^4A+33a^2Ab^2+8Ab^4+246a^3bB-18ab^3B)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-bF\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)\right)+a(b+a\cos(c+dx))\left((804a^2Ab-32Ab^3+690a^3B+72ab^2B)\sin(c+dx)+a(2(133a^2A+6Ab^2+144abB)\sin(2(c+dx))+5a(2(10Ab+9aB)\sin(3(c+dx))+7aA\sin(4(c+dx))))\right)\right)}{1260a^3d(b+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9
/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(3/2)*(8*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(a^2*(186
*a^2*A*b + 2*A*b^3 + 75*a^3*B + 153*a*b^2*B)*EllipticF[(c + d*x)/2, (2*a)/(
```

```
a + b)] + (147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*(
(a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - b*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])) + a*(b + a*Cos[c + d*x])*((804*a^2*A*b - 32*A*b^3 + 690*a^3*
B + 72*a*b^2*B)*Sin[c + d*x] + a*(2*(133*a^2*A + 6*A*b^2 + 144*a*b*B)*Sin[2
*(c + d*x)] + 5*a*(2*(10*A*b + 9*a*B)*Sin[3*(c + d*x)] + 7*a*A*Sin[4*(c + d
*x)])))))/(1260*a^3*d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4845 vs. $2(445) = 890$.
time = 14.03, size = 4846, normalized size = 11.35

method	result	size
default	Expression too large to display	4846

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/315/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-33*A*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^
3*b^2+33*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^3-8*A*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^4+24
6*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*cos(d*x+c)*sin(d*x+c)*a^4*b-153*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b^2-18*B*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*co
s(d*x+c)*sin(d*x+c)*a^2*b^3-246*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4*b+246*B*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x
+c)*sin(d*x+c)*a^3*b^2+18*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^3-18*B*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*s
in(d*x+c)*a*b^4-186*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
```

```
(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4*b-30*B*cos(d*x+c)^3*((a-b)/
(a+b))^(1/2)*a^5+75*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5-35*A*cos(d*x+c)^6*
((a-b)/(a+b))^(1/2)*a^5-14*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^5-98*A*cos(
d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5+147*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5-8
*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^5-45*B*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)
*a^5+9*B*((a-b)/(a+b))^(1/2)*a^2*b^3-18*B*((a-b)/(a+b))^(1/2)*a*b^4+8*A*((a
-b)/(a+b))^(1/2)*b^5-52*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b+A*cos(d*x+
c)^3*((a-b)/(a+b))^(1/2)*a^2*b^3-81*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*
b^2-68*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b^2-4*A*cos(d*x+c)^2*((a-b)/(
a+b))^(1/2)*a*b^4-204*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*b+9*B*cos(d*x+
c)^2*((a-b)/(a+b))^(1/2)*a^2*b^3-10*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b+
33*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2-34*A*cos(d*x+c)*((a-b)/(a+b))^(
1/2)*a^2*b^3+8*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4+246*B*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*a^4*b-165*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2-18*B*cos
(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3+18*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b
^4-85*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4*b-53*A*cos(d*x+c)^4*((a-b)/(a+
b))^(1/2)*a^3*b^2-117*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4*b+147*A*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*si
n(d*x+c)+147*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^5-147*A*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^5+8*
A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*cos(d*x+c)*sin(d*x+c)*b^5-75*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^5-186*A*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b*sin(
d*x+c)+33*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a^3*b^2*sin(d*x+c)-2*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3*sin(d*x+c)+8*A*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4*sin(d*x+c)+
147*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(9/2
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.92, size = 666, normalized size = 1.56

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="fricas")
```

```
[Out] 1/945*(sqrt(2)*(-225*I*B*a^5 - 264*I*A*a^4*b + 33*I*B*a^3*b^2 + 60*I*A*a^2*
b^3 - 36*I*B*a*b^4 + 16*I*A*b^5)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 -
4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d
*x + c) + 2*b)/a) + sqrt(2)*(225*I*B*a^5 + 264*I*A*a^4*b - 33*I*B*a^3*b^2 -
 60*I*A*a^2*b^3 + 36*I*B*a*b^4 - 16*I*A*b^5)*sqrt(a)*weierstrassPInverse(-4
/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) -
 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-147*I*A*a^5 - 246*I*B*a^4*b - 3
3*I*A*a^3*b^2 + 18*I*B*a^2*b^3 - 8*I*A*a*b^4)*sqrt(a)*weierstrassZeta(-4/3*
(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(
3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I
*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(147*I*A*a^5 + 246*I*B*a^4*b + 33*I*
A*a^3*b^2 - 18*I*B*a^2*b^3 + 8*I*A*a*b^4)*sqrt(a)*weierstrassZeta(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^
2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*s
in(d*x + c) + 2*b)/a)) + 6*(35*A*a^5*cos(d*x + c)^4 + 5*(9*B*a^5 + 10*A*a^4
*b)*cos(d*x + c)^3 + (49*A*a^5 + 72*B*a^4*b + 3*A*a^3*b^2)*cos(d*x + c)^2 +
 (75*B*a^5 + 88*A*a^4*b + 9*B*a^3*b^2 - 4*A*a^2*b^3)*cos(d*x + c))*sqrt((a*
cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sec(d*x + c)^(9/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}}{{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{9/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(9/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/(1/cos(c + d*x))^(9/2
), x)
```

3.448 $\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=513

$$\frac{(472a^2Ab+128Ab^3+133a^3B+356ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{192d\sqrt{a+b\sec(c+dx)}}+\frac{(40a^3Ab\ \cdots}{}$$

```
[Out] 1/4*b*B*sec(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+1/192*(472*A*a
^2*b+128*A*b^3+133*B*a^3+356*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(
d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+1/64*(40*A*a
^3*b+160*A*a*b^3-5*B*a^4+120*B*a^2*b^2+48*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/
2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b/d/(a+b*sec(d*x+c))^(1
/2)+1/96*(104*A*a*b+59*B*a^2+36*B*b^2)*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec
(d*x+c))^(1/2)/d+1/24*b*(8*A*b+11*B*a)*sec(d*x+c)^(5/2)*sin(d*x+c)*(a+b*sec
(d*x+c))^(1/2)/d-1/192*(264*A*a^2*b+128*A*b^3+15*B*a^3+284*B*a*b^2)*(cos(1/
2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/
2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b/d/((b+a*cos(d*x+c))/(a+b))^(1/
2)/sec(d*x+c)^(1/2)+1/192*(264*A*a^2*b+128*A*b^3+15*B*a^3+284*B*a*b^2)*sin(
d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b/d
```

Rubi [A]

time = 1.33, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4111, 4181, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(59a^2B+104aAb+36b^2B)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{96d}+\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{192bd}+\frac{(133a^3B+472a^2Ab+356ab^2B+128Ab^3)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{192d\sqrt{a+b\sec(c+dx)}}-\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{192bd\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(-5a^4B+40a^3Ab+120a^2b^2B+160aAb^3+48b^4B)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{64bd\sqrt{a+b\sec(c+dx)}}+\frac{b(11aB+8Ab)\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{24d}+\frac{bB\sin(c+dx)\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}{4d}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((472*a^2*A*b + 128*A*b^3 + 133*a^3*B + 356*a*b^2*B)*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(192
*d*Sqrt[a + b*Sec[c + d*x]]) + ((40*a^3*A*b + 160*a*A*b^3 - 5*a^4*B + 120*a
^2*b^2*B + 48*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(64*b*d*Sqrt[a + b*Sec[c + d*x]]
) - ((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(192*b*d*Sqrt[(b + a*Cos[c + d
*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + ((264*a^2*A*b + 128*A*b^3 + 15*a^3*B +
```

```
284*a*b^2*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(192
*b*d) + ((104*a*A*b + 59*a^2*B + 36*b^2*B)*Sec[c + d*x]^(3/2)*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/(96*d) + (b*(8*A*b + 11*a*B)*Sec[c + d*x]^(5/2)*S
qrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*d) + (b*B*Sec[c + d*x]^(5/2)*(a +
 b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4181

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^n/(f*(m + n + 1))), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e +
```

```
 f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*
B)*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2
- b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{bB\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c+}{4d}$$

$$= \frac{b(8Ab+11aB)\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+d}}{24d}$$

$$= \frac{(104aAb+59a^2B+36b^2B)\sec^{\frac{3}{2}}(c+dx)\sqrt{a}}{96d}$$

$$= \frac{(264a^2Ab+128Ab^3+15a^3B+284ab^2B)\sqrt{s}}{192b}$$

$$= \frac{(264a^2Ab+128Ab^3+15a^3B+284ab^2B)\sqrt{s}}{192b}$$

$$= \frac{(264a^2Ab+128Ab^3+15a^3B+284ab^2B)\sqrt{s}}{192b}$$

$$= \frac{(264a^2Ab+128Ab^3+15a^3B+284ab^2B)\sqrt{s}}{192b}$$

$$= \frac{(40a^3Ab+160aAb^3-5a^4B+120a^2b^2B+48}{64bd}$$

$$= \frac{(472a^2Ab+128Ab^3+133a^3B+356ab^2B)\sqrt{}}{192d\sqrt{a}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.97, size = 768, normalized size = 1.50

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] -1/768*((a + b*Sec[c + d*x])^(5/2)*((2*(-416*a^2*A*b^2 - 236*a^3*b*B - 144*
a*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
 + b)])/Sqrt[b + a*Cos[c + d*x]] + (2*(24*a^3*A*b - 832*a*A*b^3 + 45*a^4*B
- 436*a^2*b^2*B - 288*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[
2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(264*a^3*
A*b + 128*a*A*b^3 + 15*a^4*B + 284*a^2*b^2*B)*Sqrt[(a - a*Cos[c + d*x])/(a
+ b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*Ell
ipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a
+ b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*
x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)
]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b
)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a
^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(b*d*(
b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + ((a + b*Sec[c + d*x])^(5/2)
*((Sec[c + d*x]^3*(8*A*b^2*Sin[c + d*x] + 17*a*b*B*Sin[c + d*x]))/24 + (Sec
[c + d*x]^2*(104*a*A*b*Sin[c + d*x] + 59*a^2*B*Sin[c + d*x] + 36*b^2*B*Sin[
c + d*x]))/96 + (Sec[c + d*x]*(264*a^2*A*b*Sin[c + d*x] + 128*A*b^3*Sin[c +
 d*x] + 15*a^3*B*Sin[c + d*x] + 284*a*b^2*B*Sin[c + d*x]))/(192*b) + (b^2*B
*Sec[c + d*x]^3*Tan[c + d*x])/4))/(d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5
/2))
```

Maple [C] Result contains complex when optimal does not.
time = 13.91, size = 5392, normalized size = 10.51

method	result	size
default	Expression too large to display	5392

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2),
x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2),
 x)
```

3.449 $\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=422

$$\frac{(48a^3A+66aAb^2+59a^2bB+16b^3B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{24d\sqrt{a+b\sec(c+dx)}}+\frac{(30a^2Ab+8.}{}$$

```
[Out] 1/3*b*B*sec(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+1/24*(48*A*a^3
+66*A*a*b^2+59*B*a^2*b+16*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c
))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+1/8*(30*A*a^2*b+8
*A*b^3+5*B*a^3+20*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/
(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+1/4*b*(2*A*b+3*B*a)*
sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d-1/24*(54*A*a*b+33*B*a^
2+16*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d
*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+1/24*(54*A*a*b+33*B*a^2+16*B*b^2)*sin(
d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 1.03, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {4111, 4181, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(33a^2B+54aAb+16b^2B)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{24d}-\frac{(33a^2B+54aAb+16b^2B)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(48a^3A+59a^2bB+66aAb^2+16b^3B)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{a+b\sec(c+dx)}}+\frac{(5a^3B+30a^2Ab+20ab^2B+8Ab^3)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{8d\sqrt{a+b\sec(c+dx)}}+\frac{b(3aB+2Ab)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{4d}+\frac{bB\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((48*a^3*A + 66*a*A*b^2 + 59*a^2*b*B + 16*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(24*d*Sq
rt[a + b*Sec[c + d*x]]) + ((30*a^2*A*b + 8*A*b^3 + 5*a^3*B + 20*a*b^2*B)*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*
Sqrt[Sec[c + d*x]])/(8*d*Sqrt[a + b*Sec[c + d*x]]) - ((54*a*A*b + 33*a^2*B
+ 16*b^2*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(24*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + ((54*a*A*b
+ 33*a^2*B + 16*b^2*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(24*d) + (b*(2*A*b + 3*a*B)*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x
```

```
]]*Sin[c + d*x])/(4*d) + (b*B*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)
*Sin[c + d*x])/(3*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4181

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^n/(f*(m + n + 1))), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e +
 f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*
B)*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[
```

```
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2
- b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \frac{bB\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\sin(}{3d}$$

$$= \frac{b(2Ab+3aB)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+}}{4d}$$

$$= \frac{(54aAb+33a^2B+16b^2B)\sqrt{\sec(c+dx)}}{24d}$$

$$= \frac{(54aAb+33a^2B+16b^2B)\sqrt{\sec(c+dx)}}{24d}$$

$$= \frac{(54aAb+33a^2B+16b^2B)\sqrt{\sec(c+dx)}}{24d}$$

$$= \frac{(54aAb+33a^2B+16b^2B)\sqrt{\sec(c+dx)}}{24d}$$

$$= \frac{(30a^2Ab+8Ab^3+5a^3B+20ab^2B)\sqrt{\frac{b+}{}}}{8d\sqrt{a-}}$$

$$= \frac{(48a^3A+66aAb^2+59a^2bB+16b^3B)\sqrt{\frac{b}{}}}{24d\sqrt{a}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.87, size = 678, normalized size = 1.61

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*((2*(96*a^3*A + 24*a*A*b^2 + 52*a^2*b*B)*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b
```

```
+ a*Cos[c + d*x]] + (2*(126*a^2*A*b + 48*A*b^3 - 3*a^3*B + 104*a*b^2*B)*Sqr
t[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/
Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(-54*a^2*A*b - 33*a^3*B - 16*a*b^2*B)*Sqr
t[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c
 + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Co
s[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(
-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I
*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*
Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^
2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*
Cos[c + d*x])^2))))/(96*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) +
((a + b*Sec[c + d*x])^(5/2)*((Sec[c + d*x]^2*(6*A*b^2*Sin[c + d*x] + 13*a*b
*B*Sin[c + d*x]))/12 + (Sec[c + d*x]*(54*a*A*b*Sin[c + d*x] + 33*a^2*B*Sin[
c + d*x] + 16*b^2*B*Sin[c + d*x]))/24 + (b^2*B*Sec[c + d*x]^2*Tan[c + d*x])
/3))/(d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 13.38, size = 4258, normalized size = 10.09

method	result	size
default	Expression too large to display	4258

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/24/d*(12*A*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2-34*B*((a-b)/(a+b))^(1/2)*cos(d*
x+c)*a*b^2-54*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b+180*A*cos(d*x+c)^4*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(
a+b))^(1/2))*sin(d*x+c)*a^2*b+33*B*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b-16*B*cos(d
*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*sin(d*x+c)*a*b^2+26*B*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b-44*B*cos(d*x+c)
^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*sin(d*x+c)*a*b^2+120*B*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-54*A*
```

cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*sin(d*x+c)*a^2*b+54*A*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2-36*A*cos(d
x+c)^3((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*sin(d*x+c)*a^2*b+12*A*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2+180*A*cos(d*x+c
)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a
-b)/(a+b))^(1/2))*sin(d*x+c)*a^2*b+33*B*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b-16*B*
cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*sin(d*x+c)*a*b^2+26*B*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b-44*B*cos(d
x+c)^3((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*sin(d*x+c)*a*b^2+120*B*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*a*b^2-
54*A*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b+54*A*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2-36*A*
cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*sin(d*x+c)*a^2*b+54*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^2*b+1
2*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a*b^2+26*B*cos(d*x+c)^4*((a-b)/(a+b))^
(1/2)*a^2*b+16*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a*b^2+54*A*cos(d*x+c)^3*(
(a-b)/(a+b))^(1/2)*a*b^2+33*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b+18*B*c
os(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2-24*A*cos(d*x+c)^4*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b^3+48*A
*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(
a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*b^3-33*B*cos(d*x+c)^4*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^3
+16*B*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(

```
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*sin(d*x+c)*b^3+18*B*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)...
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6190 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2),
x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2),
 x)
```

3.450 $\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx$

Optimal. Leaf size=359

$$\frac{(16a^2Ab+4Ab^3+8a^3B+11ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4d\sqrt{a+b\sec(c+dx)}}+\frac{b(20aAb+15a^2B}{}$$

```
[Out] 1/2*b*B*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)*sec(d*x+c)^(1/2)/d+1/4*(16*A*a^2*
b+4*A*b^3+8*B*a^3+11*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/
(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+1/4*b*(20*A*a*b+15*B
*a^2+4*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(si
n(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*
sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+1/4*(8*A*a^2-4*A*b^2-9*B*a*b)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(
1/2)/sec(d*x+c)^(1/2)+1/4*b*(4*A*b+7*B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*
sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.80, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {4111, 4181, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(8a^2A-9abB-4Ab^2)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{b(15a^2B+20aAb+4b^2B)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{(8a^3B+16a^2Ab+11ab^2B+4Ab^3)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{a+b\sec(c+dx)}}+\frac{b(7aB+4Ab)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{4d}+\frac{bB\sin(c+dx)\sqrt{\sec(c+dx)}(a+b\sec(c+dx))^{3/2}}{2d}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*x]],x]
```

```
[Out] ((16*a^2*A*b + 4*A*b^3 + 8*a^3*B + 11*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*d*Sqrt[
a + b*Sec[c + d*x]]) + (b*(20*a*A*b + 15*a^2*B + 4*b^2*B)*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*
x]])/(4*d*Sqrt[a + b*Sec[c + d*x]]) + ((8*a^2*A - 4*A*b^2 - 9*a*b*B)*Ellipt
icE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (b*(4*A*b + 7*a*B)*Sqrt[Sec[c
+ d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d) + (b*B*Sqrt[Sec[c + d*
x]]*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4181

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^n/(f*(m + n + 1))), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e +
 f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*
B)*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2
- b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
```

```
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sqrt{\sec(c+dx)}}\,dx = \frac{bB\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{2d} + \frac{1}{2}$$

$$= \frac{b(4Ab+7aB)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+}{4d}$$

$$= \frac{b(4Ab+7aB)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+}{4d}$$

$$= \frac{b(4Ab+7aB)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+}{4d}$$

$$= \frac{b(4Ab+7aB)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+}{4d}$$

$$= \frac{b(20aAb+15a^2B+4b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c\right.}{4d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{(16a^2Ab+4Ab^3+8a^3B+11ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}{4d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.87, size = 628, normalized size = 1.75

$$\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{2(48a^2Ab+16a^3B+4ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2(8a^3A+36aAb^2+21a^2bB+8b^3B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2i(8a^3A-4aAb^2-9a^2bB)\sqrt{\frac{a-a\cos(c+dx)}{a+b}}\sqrt{\frac{a+a\cos(c+dx)}{a-b}}\cos(2(c+dx))\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)\right)\right)\sin(c+dx)}{\sqrt{\frac{1}{a-b}}\,b\sqrt{1-\cos^2(c+dx)}\sqrt{\frac{a^2-a^2\cos^2(c+dx)}{a^2}}\left(-a^2+2b^2-4b(b+a\cos(c+dx))+2(b+a\cos(c+dx))^2\right)}\right)}{16d(b+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}+\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{1}{4}\sec(c+dx)\left(4Ab^2\sin(c+dx)+9abB\sin(c+dx)\right)+\frac{1}{2}b^2B\sec(c+dx)\tan(c+dx)\right)}{d(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Sec[c + d*
x]],x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*((2*(48*a^2*A*b + 16*a^3*B + 4*a*b^2*B)*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b +
 a*Cos[c + d*x]] + (2*(8*a^3*A + 36*a*A*b^2 + 21*a^2*b*B + 8*b^3*B)*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt
[b + a*Cos[c + d*x]] + ((2*I)*(8*a^3*A - 4*a*A*b^2 - 9*a^2*b*B)*Sqrt[(a - a
*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x)]
*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*
x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqr
t[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh
[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c +
d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c
+ d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c +
d*x])^2))))/(16*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + ((a + b*
Sec[c + d*x])^(5/2)*((Sec[c + d*x]*(4*A*b^2*Sin[c + d*x] + 9*a*b*B*Sin[c +
d*x]))/4 + (b^2*B*Sec[c + d*x]*Tan[c + d*x])/2))/(d*(b + a*Cos[c + d*x])^2*
Sec[c + d*x]^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 13.61, size = 3939, normalized size = 10.97

method	result	size
default	Expression too large to display	3939

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-8*A*cos(d*x+c)^3*((a-b)/(a+b))
^(1/2)*a^3+4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3-11*B*((a-b)/(a+b))^(1/2
)*cos(d*x+c)*a*b^2+8*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b+8*A*((a-b)/(a
+b))^(1/2)*cos(d*x+c)^4*a^3-8*A*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b-4*A*cos(d*x+c
)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*sin(d*x+c)*a*b^2+24*A*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b-16*A*cos(d*x+c)^3*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*s
in(d*x+c)*a*b^2-9*B*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b+9*B*cos(d*x+c)^3*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+
```

c)*a*b^2-6*B*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2*b+2*B*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b^2+4*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2+9*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b+2*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2+8*B*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^3-2*B*((a-b)/(a+b))^(1/2)*b^3+2*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3+40*A*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^2+30*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b+24*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-16*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-8*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-4*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+40*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^2-6*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+2*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-9*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+9*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+30*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b-4*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^3-4*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^2-9*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b-8*A*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-4*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x

```
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+8*B*cos(d
*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/sqrt(sec(d*x + c)
), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sqrt(sec(d*x + c)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/2}}{\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(1/2
), x)
```

3.451 $\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=349

$$\frac{(2a^3A+4aAb^2+12a^2bB+3b^3B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3d\sqrt{a+b\sec(c+dx)}}+\frac{b^2(2Ab+5aB)\sqrt{}}{}$$

```
[Out] 2/3*a*A*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(1/2)+1/3*(2*A*a^3+4
*A*a*b^2+12*B*a^2*b+3*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(
a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+b^2*(2*A*b+5*B*a)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),
2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/
d/(a+b*sec(d*x+c))^(1/2)+1/3*(14*A*a*b+6*B*a^2-3*B*b^2)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))
^(1/2))*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^
(1/2)-1/3*b*(2*A*a-3*B*b)*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2
)/d
```

Rubi [A]
time = 0.79, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {4110, 4181, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(6a^2B+14aAb-3b^2B)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(2a^3A+12a^2bB+4aAb^2+3b^3B)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{a+b\sec(c+dx)}}+\frac{b^2(5aB+2Ab)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{b(2aA-3bB)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{3d}+\frac{2aA\sin(c+dx)(a+b\sec(c+dx))^{3/2}}{3d\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3/2),x]
```

```
[Out] ((2*a^3*A + 4*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a
+ b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*d*Sqrt[a
 + b*Sec[c + d*x]]) + (b^2*(2*A*b + 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b
)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a
+ b*Sec[c + d*x]]) + ((14*a*A*b + 6*a^2*B - 3*b^2*B)*EllipticE[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*Sqrt[Sec[c + d*x]]) - (b*(2*a*A - 3*b*B)*Sqrt[Sec[c + d*x]]*Sqrt[a +
 b*Sec[c + d*x]]*Sin[c + d*x])/(3*d) + (2*a*A*(a + b*Sec[c + d*x])^(3/2)*Si
n[c + d*x])/(3*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4181

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^n/(f*(m + n + 1))), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e +
 f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*
B)*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2
- b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
```

```
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{3}{2}}(c+dx)}\,dx = \frac{2aA(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{3d\sqrt{\sec(c+dx)}} - \frac{2}{3}\int \frac{\sqrt{a+b\sec}}{}$$

$$= -\frac{b(2aA-3bB)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c}{3d}$$

$$= -\frac{b(2aA-3bB)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c}{3d}$$

$$= -\frac{b(2aA-3bB)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c}{3d}$$

$$= -\frac{b(2aA-3bB)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c}{3d}$$

$$= \frac{b^2(2Ab+5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{(2a^3A+4aAb^2+12a^2bB+3b^3B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}{3d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.92, size = 599, normalized size = 1.72

$$\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{2(4a^3A+36aAb^2+36a^2bB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2(14a^2Ab+12Ab^3+6a^3B+27ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2i(14a^2Ab+6a^3B-3ab^2B)\sqrt{\frac{a-a\cos(c+dx)}{a+b}}\sqrt{\frac{a+a\cos(c+dx)}{a-b}}\cos(2(c+dx))\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)\right)\right)\sin(c+dx)}{\sqrt{\frac{1}{a-b}}\,b\sqrt{1-\cos^2(c+dx)}\sqrt{\frac{a^2-a^2\cos^2(c+dx)}{a^2}}(-a^2+2b^2-4b(b+a\cos(c+dx))+2(b+a\cos(c+dx))^2)}\right)}{12d(b+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}+\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{2}{3}a^2A\sin(c+dx)+b^2B\tan(c+dx)\right)}{d(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(3
/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*((2*(4*a^3*A + 36*a*A*b^2 + 36*a^2*b*B)*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b +
 a*Cos[c + d*x]] + (2*(14*a^2*A*b + 12*A*b^3 + 6*a^3*B + 27*a*b^2*B)*Sqrt[(
b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqr
t[b + a*Cos[c + d*x]] + ((2*I)*(14*a^2*A*b + 6*a^3*B - 3*a*b^2*B)*Sqrt[(a -
 a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*Cos[2*(c + d*x
)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c +
d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*S
qrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSi
nh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)]))*Sin[c
+ d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[
c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*(b + a*Cos[c
+ d*x])^2))))/(12*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)) + ((a +
b*Sec[c + d*x])^(5/2)*((2*a^2*A*Sin[c + d*x])/3 + b^2*B*Tan[c + d*x]))/(d*(
b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 14.10, size = 3663, normalized size = 10.50

method	result	size
default	Expression too large to display	3663

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(18*A*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^
2+14*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-14*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2+18*B*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*si
n(d*x+c)*cos(d*x+c)*a^2*b-12*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-6*B*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*c
os(d*x+c)*a^2*b-3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-14*A*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*
```

```
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*
a^2*b-3*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2+16*A*((a-b)/(a+b))^(1/2)*cos
(d*x+c)^3*a^2*b+2*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^3-2*A*((a-b)/(a+b))^
(1/2)*cos(d*x+c)^2*a^3+2*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3-6*B*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x
+c)*a^3+30*B*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^2+30*B*sin(d*x+c)*co
s(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),
I/((a-b)/(a+b))^(1/2))*a*b^2-3*B*((a-b)/(a+b))^(1/2)*b^3-6*B*cos(d*x+c)^2*(
(a-b)/(a+b))^(1/2)*a^3+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3-14*A*cos(d*x+
c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^2*b+18*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+14*A*cos(d*x+c)^2*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^2*b-14*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+18*B*cos(d*x+c)^2*sin(d
*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^2*b-12*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-6*B*cos(d*x+c)^2*sin(d*x+c)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a^2*b-3*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+6*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^
3*a^3+14*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^2+6*B*((a-b)/(a+b))^(1/2)*c
os(d*x+c)^2*a^2*b-14*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2-6*B*((a-b)/(a+b
))^(1/2)*cos(d*x+c)*a^2*b+6*B*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+3*B*sin(d*x+c)*co
s(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*b^3-6*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+12*A*sin(d*x+c)*cos(d*x+c)*((b
```

```
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+
b))^(1/2))*b^3+6*B*sin(d*x+c)*cos(d*x+c)*((b+a*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(3/2
), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(3/2
), x)
```

3.452 $\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=342

$$\frac{2(8a^2Ab-8Ab^3+5a^3B+10ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15d\sqrt{a+b\sec(c+dx)}}+\frac{2b^3B\sqrt{\frac{b+a\cos}{a-}}}{}$$

```
[Out] 2/5*a*A*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(3/2)+2/15*(8*A*a^2*
b-8*A*b^3+5*B*a^3+10*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/
(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)+2*b^3*B*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)
*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*se
c(d*x+c))^(1/2)+2/15*a*(8*A*b+5*B*a)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/se
c(d*x+c)^(1/2)+2/15*(9*A*a^2+23*A*b^2+35*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*
(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.78, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {4110, 4179, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(9a^2A+35abB+23Ab^2)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(5a^3B+8a^2Ab+10ab^2B-8Ab^3)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{a+b\sec(c+dx)}}+\frac{2a(5aB+8Ab)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{15d\sqrt{\sec(c+dx)}}+\frac{2aA\sin(c+dx)(a+b\sec(c+dx))^{3/2}}{5d\sec^{\frac{3}{2}}(c+dx)}+\frac{2b^3B\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5/2),x]
```

```
[Out] (2*(8*a^2*A*b - 8*A*b^3 + 5*a^3*B + 10*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*d*Sqr
t[a + b*Sec[c + d*x]]) + (2*b^3*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c
 + d*x]]) + (2*(9*a^2*A + 23*A*b^2 + 35*a*b*B)*EllipticE[(c + d*x)/2, (2*a)
/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]*Sqrt[Sec[c + d*x]]) + (2*a*(8*A*b + 5*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c
 + d*x])/(15*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + b*Sec[c + d*x])^(3/2)*Sin[
c + d*x])/(5*d*Sec[c + d*x]^(3/2))
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4179

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
```

```
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{5}{2}}(c+dx)}\,dx = \frac{2aA(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{5d\sec^{\frac{3}{2}}(c+dx)} - \frac{2}{5}\int \sqrt{a+b\sec}$$

$$= \frac{2a(8Ab+5aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2aA(}{}$$

$$= \frac{2a(8Ab+5aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2aA(}{}$$

$$= \frac{2a(8Ab+5aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2aA(}{}$$

$$= \frac{2a(8Ab+5aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15d\sqrt{\sec(c+dx)}} + \frac{2aA(}{}$$

$$= \frac{2b^3B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+}}{d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2(8a^2Ab-8Ab^3+5a^3B+10ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}{15d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.88, size = 616, normalized size = 1.80

$$\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{2(34a^2Ab+30Ab^3+10a^3B+90ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2(9a^3A+23aAb^2+35a^2bB+30b^3B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2i(9a^3A+23aAb^2+35a^2bB)\sqrt{\frac{a-a\cos(c+dx)}{a+b}}\sqrt{\frac{a+a\cos(c+dx)}{a-b}}\cos(2(c+dx))\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)\middle|\frac{-a+b}{a+b}\right)\right)\right)\sin(c+dx)}{\sqrt{\frac{1}{a-b}}\,b\sqrt{1-\cos^2(c+dx)}\sqrt{\frac{a^2-a^2\cos^2(c+dx)}{a^2}}\left(-a^2+2b^2-4b(b+a\cos(c+dx))+2(b+a\cos(c+dx))^2\right)}\right)}{30d(b+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)}+\frac{(a+b\sec(c+dx))^{5/2}\left(\frac{2}{15}a(11Ab+5aB)\sin(c+dx)+\frac{1}{5}a^2A\sin(2(c+dx))\right)}{d(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(5
/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*((2*(34*a^2*A*b + 30*A*b^3 + 10*a^3*B + 90*a*b^
2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)])/Sqrt[b + a*Cos[c + d*x]] + (2*(9*a^3*A + 23*a*A*b^2 + 35*a^2*b*B + 30*b
^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(
a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(9*a^3*A + 23*a*A*b^2 + 35*a^2*b
*B)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[c + d*x])/(a - b)]*C
os[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b
 + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a
 - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 -
 a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a +
 b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos[c + d*x]^2]*Sqrt[(a
^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b + a*Cos[c + d*x]) + 2*
(b + a*Cos[c + d*x])^2))))/(30*d*(b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5
/2)) + ((a + b*Sec[c + d*x])^(5/2)*((2*a*(11*A*b + 5*a*B)*Sin[c + d*x])/15
+ (a^2*A*Sin[2*(c + d*x)])/5))/(d*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)
)
```

Maple [C] Result contains complex when optimal does not.
time = 13.69, size = 3564, normalized size = 10.42

method	result	size
default	Expression too large to display	3564

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-23*A*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*
b^2-9*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+23*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-35*B*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*s
in(d*x+c)*cos(d*x+c)*a^2*b+45*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2+35*B*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)
*cos(d*x+c)*a^2*b-35*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-23*A*((a-b)/(a+b))^(1/2)
```

```
*b^3+17*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-9*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)+35*B*((a-b)/(a+b))
^(1/2)*cos(d*x+c)*a*b^2+14*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b+3*A*((a
-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^3+6*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3-
9*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*sin(d*x+c)*cos(d*x+c)*a^3+9*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3-23*A*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)
*cos(d*x+c)*b^3+5*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3+17*A*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)-23*A*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*
b^2*sin(d*x+c)-9*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)+23*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)-35*B*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*
x+c)+45*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a*b^2*sin(d*x+c)+35*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)-35*B*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)+5*B*
((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3-5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3
-9*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3+23*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)
*b^3+34*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^2+40*B*((a-b)/(a+b))^(1/2)*c
os(d*x+c)^2*a^2*b-9*A*((a-b)/(a+b))^(1/2)*a^2*b-11*A*((a-b)/(a+b))^(1/2)*a*
b^2-5*B*((a-b)/(a+b))^(1/2)*a^2*b-35*B*((a-b)/(a+b))^(1/2)*a*b^2+9*A*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*si
n(d*x+c)-23*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*b^3*sin(d*x+c)+5*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
```

```
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)-23*A*((a-b)/(a+b))^(1/2)*co
s(d*x+c)*a*b^2-35*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b+15*A*sin(d*x+c)*co
s(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)/sec(d*x + c)^(5/2
), x)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(5/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(5/2
), x)
```

3.453 $\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx$

Optimal. Leaf size=340

$$\frac{2(a^2-b^2)\left(25a^2A+15Ab^2+56abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{105ad\sqrt{a+b\sec(c+dx)}}+\frac{2(145a^2Ab+1\ldots}{}$$

```
[Out] 2/7*a*A*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(5/2)+2/105*(a^2-b^2
)*(25*A*a^2+15*A*b^2+56*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))
/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2/35*a*(10*A*b+7*
B*a)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(3/2)+2/105*(25*A*a^2+4
5*A*b^2+77*B*a*b)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(1/2)+2/10
5*(145*A*a^2*b+15*A*b^3+63*B*a^3+161*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a
+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.74, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4110, 4179, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(25a^2A+77abB+45Ab^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{105d\sqrt{\sec(c+dx)}}+\frac{2(a^2-b^2)\left(25a^2A+56abB+15Ab^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105ad\sqrt{a+b\sec(c+dx)}}+\frac{2(63a^3B+145a^2Ab+161ab^2B+15Ab^3)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a(7aB+10Ab)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{35d\sec^{\frac{3}{2}}(c+dx)}+\frac{2aA\sin(c+dx)(a+b\sec(c+dx))^{3/2}}{7d\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7/2),x]
```

```
[Out] (2*(a^2 - b^2)*(25*a^2*A + 15*A*b^2 + 56*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(105*a*d*
Sqrt[a + b*Sec[c + d*x]]) + (2*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b
^2*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(105*
a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*(10*A*b +
 7*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(35*d*Sec[c + d*x]^(3/2)) +
(2*(25*a^2*A + 45*A*b^2 + 77*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/
(105*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x]
)/(7*d*Sec[c + d*x]^(5/2))
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4179

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{7}{2}}(c+dx)}\,dx = \frac{2aA(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{7d\sec^{\frac{5}{2}}(c+dx)} - \frac{2}{7}\int \frac{\sqrt{a+b\sec}}{}$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2aA}{}$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(25}{}$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(25}{}$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(25}{}$$

$$= \frac{2a(10Ab+7aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{35d\sec^{\frac{3}{2}}(c+dx)} + \frac{2(25}{}$$

$$= \frac{2(a^2-b^2)(25a^2A+15Ab^2+56abB)\sqrt{\frac{b+a\cos(c+dx}{a+b}}}{105ad\sqrt{a+b\sec(c+dx}}$$

<u>Mathematica</u> [A]

time = 1.89, size = 257, normalized size = 0.76

$$\frac{(a+b\sec(c+dx))^{5/2}\left(2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\left(a(25a^3A+135aAb^2+119a^2bB+105b^3B)F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+(145a^2Ab+15Ab^3+63a^3B+161ab^2B)\left((a+b)E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)-bF\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\right)\right)+a(b+a\cos(c+dx))(65a^2A+90Ab^2+154abB+6a(15Ab+7aB)\cos(c+dx)+15a^2A\cos(2(c+dx)))\sin(c+dx)\right)}{105ad(b+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(7
/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*(2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(a*(25*a^
3*A + 135*a*A*b^2 + 119*a^2*b*B + 105*b^3*B)*EllipticF[(c + d*x)/2, (2*a)/(
a + b)] + (145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*((a + b)*Ellipt
icE[(c + d*x)/2, (2*a)/(a + b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))
 + a*(b + a*Cos[c + d*x])*(65*a^2*A + 90*A*b^2 + 154*a*b*B + 6*a*(15*A*b +
7*a*B)*Cos[c + d*x] + 15*a^2*A*Cos[2*(c + d*x)])*Sin[c + d*x]))/(105*a*d*(b
 + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3979 vs. $2(364) = 728$.
time = 13.95, size = 3980, normalized size = 11.71

method	result	size
default	Expression too large to display	3980

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/105/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-15*A*cos(d*x+c)*((a-b)/(a+b)
)^(1/2)*a*b^3-35*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b-161*B*cos(d*x+c)*((
a-b)/(a+b))^(1/2)*a^2*b^2+161*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+60*A*c
os(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b+90*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)
*a^2*b^2+98*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b-15*A*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*sin(d*x+c)-63
*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^4*sin(d*x+c)+63*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a^4*sin(d*x+c)-145*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+25*
A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^4*sin(d*x+c)+161*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-161*B*sin(d*x+c)*cos(d
*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a*b^3+15*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4-63*B*cos(d*x+c)*((a-b)/
(a+b))^(1/2)*a^4+15*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4+10*A*cos(d*x+c)^
3*((a-b)/(a+b))^(1/2)*a^4-25*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4+21*B*cos(
d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4+42*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4+
135*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-15*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+145*A*
sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^3*b-145*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(
```

```
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+15*A*sin(d*
x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b^3+119*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-161*B*sin(d*x+c)*co
s(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^2*b^2-63*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-25*A*((a-b)/(a+b))^(1/2)*
a^3*b-145*A*((a-b)/(a+b))^(1/2)*a^2*b^2-45*A*((a-b)/(a+b))^(1/2)*a*b^3-63*B
*((a-b)/(a+b))^(1/2)*a^3*b-77*B*((a-b)/(a+b))^(1/2)*a^2*b^2-161*B*((a-b)/(a
+b))^(1/2)*a*b^3+25*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-15*A*sin(d*x+c)*cos(d*x+c)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
b^4-63*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+63*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-145*A*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3
*b*sin(d*x+c)+135*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^2*b^2*sin(d*x+c)-15*A*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*sin(d*x+c)+145*A*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*si
n(d*x+c)-145*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.56, size = 589, normalized size = 1.73

$$\frac{\sqrt{2}\left(-75i\,Aa^4-231i\,Ba^3b-115i\,Aa^2b^2+7i\,Bab^3+30i\,Ab^4\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(75i\,Aa^4+231i\,Ba^3b+115i\,Aa^2b^2-7i\,Bab^3-30i\,Ab^4\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(-63i\,Ba^4-145i\,Aa^3b-161i\,Ba^2b^2-15i\,Aab^3\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(63i\,Ba^4+145i\,Aa^3b+161i\,Ba^2b^2+15i\,Aab^3\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)+\frac{6\left(15\,Aa^4\cos(dx+c)^3+3\left(7\,Ba^4+15\,Aa^3b\right)\cos(dx+c)^2+\left(25\,Aa^4+77\,Ba^3b+45\,Aa^2b^2\right)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{315\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="fricas")
```

```
[Out] 1/315*(sqrt(2)*(-75*I*A*a^4 - 231*I*B*a^3*b - 115*I*A*a^2*b^2 + 7*I*B*a*b^3
 + 30*I*A*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)
+ sqrt(2)*(75*I*A*a^4 + 231*I*B*a^3*b + 115*I*A*a^2*b^2 - 7*I*B*a*b^3 - 30*
I*A*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*
b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sq
rt(2)*(-63*I*B*a^4 - 145*I*A*a^3*b - 161*I*B*a^2*b^2 - 15*I*A*a*b^3)*sqrt(a
)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, wei
erstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*
(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(63*I*B*a^4 +
 145*I*A*a^3*b + 161*I*B*a^2*b^2 + 15*I*A*a*b^3)*sqrt(a)*weierstrassZeta(-4
/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/
3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) -
3*I*a*sin(d*x + c) + 2*b)/a)) + 6*(15*A*a^4*cos(d*x + c)^3 + 3*(7*B*a^4 + 1
5*A*a^3*b)*cos(d*x + c)^2 + (25*A*a^4 + 77*B*a^3*b + 45*A*a^2*b^2)*cos(d*x
+ c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c
)))/(a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(7/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6191 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(7/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(7/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{7/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(7/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(7/2
), x)
```

3.454 $\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx$

Optimal. Leaf size=425

$$\frac{2(a^2-b^2)\left(114a^2Ab-10Ab^3+75a^3B+45ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{315a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2(}{}$$

```
[Out] 2/9*a*A*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(7/2)+2/315*(a^2-b^2
)*(114*A*a^2*b-10*A*b^3+75*B*a^3+45*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b
+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2)+2
/21*a*(4*A*b+3*B*a)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+2/
315*(49*A*a^2+75*A*b^2+135*B*a*b)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/sec(d
*x+c)^(3/2)+2/315*(163*A*a^2*b+5*A*b^3+75*B*a^3+135*B*a*b^2)*sin(d*x+c)*(a+
b*sec(d*x+c))^(1/2)/a/d/sec(d*x+c)^(1/2)+2/315*(147*A*a^4+279*A*a^2*b^2-10*
A*b^4+435*B*a^3*b+45*B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(
1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.99, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4110, 4179, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(49a^2A+135abB+75Ab^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{315d\sec^{\frac{3}{2}}(c+dx)}+\frac{2(75a^3B+163a^2Ab+135ab^2B+5Ab^3)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{315ad\sqrt{\sec(c+dx)}}+\frac{2(a^2-b^2)(75a^3B+114a^2Ab+45ab^2B-10Ab^3)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{315a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2(147a^4A+435a^3bB+279a^2Ab^2+45ab^3B-10Ab^4)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{315a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a(3aB+4Ab)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{21d\sec^{\frac{5}{2}}(c+dx)}+\frac{2aA\sin(c+dx)(a+b\sec(c+dx))^{3/2}}{9d\sec^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9/2),x]
```

```
[Out] (2*(a^2 - b^2)*(114*a^2*A*b - 10*A*b^3 + 75*a^3*B + 45*a*b^2*B)*Sqrt[(b + a
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c +
d*x]])/(315*a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(147*a^4*A + 279*a^2*A*b^2
 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b
)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*
Sqrt[Sec[c + d*x]]) + (2*a*(4*A*b + 3*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
 d*x])/(21*d*Sec[c + d*x]^(5/2)) + (2*(49*a^2*A + 75*A*b^2 + 135*a*b*B)*Sqr
t[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*d*Sec[c + d*x]^(3/2)) + (2*(163*a^
2*A*b + 5*A*b^3 + 75*a^3*B + 135*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
d*x])/(315*a*d*Sqrt[Sec[c + d*x]]) + (2*a*A*(a + b*Sec[c + d*x])^(3/2)*Sin[
c + d*x])/(9*d*Sec[c + d*x]^(7/2))
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
```

```
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4179

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{9}{2}}(c+dx)}\,dx = \frac{2aA(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{9d\sec^{\frac{7}{2}}(c+dx)} - \frac{2}{9}\int \frac{\sqrt{a+b\sec}}{}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2aA(}{}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(49a}{}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(49a}{}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(49a}{}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(49a}{}$$

$$= \frac{2a(4Ab+3aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{21d\sec^{\frac{5}{2}}(c+dx)} + \frac{2(49a}{}$$

$$= \frac{2(a^2-b^2)(114a^2Ab-10Ab^3+75a^3B+45ab^2B)\sqrt{\frac{b+}{}}}{315a^2d\sqrt{a+b\sec}}$$

Mathematica [A]
time = 2.57, size = 313, normalized size = 0.74

$$\frac{(a+b\sec(c+dx))^{5/2}\left(8\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\left(a^2(261a^2Ab+155Ab^3+75a^3B+405ab^2B)F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+(147a^4A+279a^2Ab^2-10Ab^4+435a^3bB+45ab^3B)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-bF\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)\right)+a(b+a\cos(c+dx))(2(747a^2Ab+20Ab^3+345a^3B+540ab^2B)\sin(c+dx)+a((266a^2A+300Ab^2+540abB)\sin(2(c+dx))+5a(2(19Ab+9aB)\sin(3(c+dx))+7aA\sin(4(c+dx)))))\right)}{1260a^2d(b+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(9
/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*(8*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(a^2*(261
*a^2*A*b + 155*A*b^3 + 75*a^3*B + 405*a*b^2*B)*EllipticF[(c + d*x)/2, (2*a)
/(a + b)] + (147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*
```

```
B)*((a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - b*EllipticF[(c + d*x)/2
, (2*a)/(a + b)])) + a*(b + a*Cos[c + d*x])*(2*(747*a^2*A*b + 20*A*b^3 + 34
5*a^3*B + 540*a*b^2*B)*Sin[c + d*x] + a*((266*a^2*A + 300*A*b^2 + 540*a*b*B
)*Sin[2*(c + d*x)] + 5*a*(2*(19*A*b + 9*a*B)*Sin[3*(c + d*x)] + 7*a*A*Sin[4
*(c + d*x)])))))/(1260*a^2*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4846 vs. $2(443) = 886$.
time = 15.12, size = 4847, normalized size = 11.40

method	result	size
default	Expression too large to display	4847

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/315/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(279*A*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a
^3*b^2-279*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^3-10*A*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^4
-435*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*cos(d*x+c)*sin(d*x+c)*a^4*b+405*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3*b^2-45*B
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*cos(d*x+c)*sin(d*x+c)*a^2*b^3+435*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4*b-435*B*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(
d*x+c)*sin(d*x+c)*a^3*b^2+45*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2*b^3-45*B*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c
)*sin(d*x+c)*a*b^4+261*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^4*b+30*B*cos(d*x+c)^3*((a-
```

```
b)/(a+b))^(1/2)*a^5-75*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5+35*A*cos(d*x+c)
^6*((a-b)/(a+b))^(1/2)*a^5+14*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^5+98*A*c
os(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5-147*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^
5-10*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^5+45*B*cos(d*x+c)^5*((a-b)/(a+b))^(
1/2)*a^5-135*B*((a-b)/(a+b))^(1/2)*a^2*b^3-45*B*((a-b)/(a+b))^(1/2)*a*b^4+1
0*A*((a-b)/(a+b))^(1/2)*b^5+82*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b+80*
A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^3+270*B*cos(d*x+c)^3*((a-b)/(a+b))
^(1/2)*a^3*b^2+272*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b^2-5*A*cos(d*x+c
)^2*((a-b)/(a+b))^(1/2)*a*b^4+330*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*b+
180*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^3-65*A*cos(d*x+c)*((a-b)/(a+b)
)^(1/2)*a^4*b-279*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2+199*A*cos(d*x+c)
*((a-b)/(a+b))^(1/2)*a^2*b^3+10*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4-435*
B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b+165*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)
*a^3*b^2-45*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3+45*B*cos(d*x+c)*((a-b)
/(a+b))^(1/2)*a*b^4+130*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4*b+170*A*cos(
d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b^2+180*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)
*a^4*b-147*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^5*sin(d*x+c)-147*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^5+147*A*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c
)*sin(d*x+c)*a^5+10*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^5+75*B*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*
a^5+261*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^4*b*sin(d*x+c)-279*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2*sin(d*x+c)+155*A*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3*sin(d*x+c
)+10*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a*b^4*sin(d*x+c)-147*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x...
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2), x)

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.90, size = 666, normalized size = 1.57

$$\frac{\sqrt{2}\left(-225i\,Ba^5-489i\,Aa^4b-345i\,Ba^3b^2+93i\,Aa^2b^3+90i\,Bab^4-20i\,Ab^5\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(225i\,Ba^5+489i\,Aa^4b+345i\,Ba^3b^2-93i\,Aa^2b^3-90i\,Bab^4+20i\,Ab^5\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(-147i\,Aa^5-435i\,Ba^4b-279i\,Aa^3b^2-45i\,Ba^2b^3+10i\,Aab^4\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(147i\,Aa^5+435i\,Ba^4b+279i\,Aa^3b^2+45i\,Ba^2b^3-10i\,Aab^4\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)\right)+\frac{6\left(35Aa^5\cos(dx+c)^4+5(9Ba^5+19Aa^4b)\cos(dx+c)^3+(49Aa^5+135Ba^4b+75Aa^3b^2)\cos(dx+c)^2+(75Ba^5+163Aa^4b+135Ba^3b^2+5Aa^2b^3)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{945a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algorithm="fricas")

[Out] 1/945*(sqrt(2)*(-225*I*B*a^5 - 489*I*A*a^4*b - 345*I*B*a^3*b^2 + 93*I*A*a^2*b^3 + 90*I*B*a*b^4 - 20*I*A*b^5)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(225*I*B*a^5 + 489*I*A*a^4*b + 345*I*B*a^3*b^2 - 93*I*A*a^2*b^3 - 90*I*B*a*b^4 + 20*I*A*b^5)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-147*I*A*a^5 - 435*I*B*a^4*b - 279*I*A*a^3*b^2 - 45*I*B*a^2*b^3 + 10*I*A*a*b^4)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(147*I*A*a^5 + 435*I*B*a^4*b + 279*I*A*a^3*b^2 + 45*I*B*a^2*b^3 - 10*I*A*a*b^4)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) + 6*(35*A*a^5*cos(d*x + c)^4 + 5*(9*B*a^5 + 19*A*a^4*b)*cos(d*x + c)^3 + (49*A*a^5 + 135*B*a^4*b + 75*A*a^3*b^2)*cos(d*x + c)^2 + (75*B*a^5 + 163*A*a^4*b + 135*B*a^3*b^2 + 5*A*a^2*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^3*d)

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(9/2),x)

[Out] Timed out

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(9/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(9/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{9/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(9/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(9/2
), x)
```

3.455 $\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{11}{2}}(c+dx)}\,dx$

Optimal. Leaf size=519

$$\frac{2(a^2-b^2)\left(675a^4A+285a^2Ab^2+40Ab^4+1254a^3bB-110ab^3B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\mathrm{s}}}{3465a^3d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2/11*a*A*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d/sec(d*x+c)^(9/2)+2/3465*(a^2-b
^2)*(675*A*a^4+285*A*a^2*b^2+40*A*b^4+1254*B*a^3*b-110*B*a*b^3)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(
a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^3/d/(a+b*
sec(d*x+c))^(1/2)+2/99*a*(14*A*b+11*B*a)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/
d/sec(d*x+c)^(7/2)+2/693*(81*A*a^2+113*A*b^2+209*B*a*b)*sin(d*x+c)*(a+b*sec
(d*x+c))^(1/2)/d/sec(d*x+c)^(5/2)+2/3465*(1145*A*a^2*b+15*A*b^3+539*B*a^3+8
25*B*a*b^2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d/sec(d*x+c)^(3/2)+2/3465*(
675*A*a^4+1025*A*a^2*b^2-20*A*b^4+1793*B*a^3*b+55*B*a*b^3)*sin(d*x+c)*(a+b*
sec(d*x+c))^(1/2)/a^2/d/sec(d*x+c)^(1/2)+2/3465*(3705*A*a^4*b+255*A*a^2*b^3
+40*A*b^5+1617*B*a^5+3069*B*a^3*b^2-110*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))
*(a+b*sec(d*x+c))^(1/2)/a^3/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/
2)
```

Rubi [A]
time = 1.27, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4110, 4179, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(81a^2A+209abB+113Ab^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{693d\sec^{\frac{5}{2}}(c+dx)}+\frac{2(539a^3B+1145a^2Ab+825ab^2B+15Ab^3)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3465ad\sec^{\frac{3}{2}}(c+dx)}+\frac{2(675a^4A+1793a^3bB+1025a^2Ab^2+55ab^3B-20Ab^4)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3465a^2d\sqrt{\sec(c+dx)}}+\frac{2(a^2-b^2)(675a^4A+1254a^3bB+285a^2Ab^2-110ab^3B+40Ab^4)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3465a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2(1617a^5B+3705a^4Ab+3069a^3b^2B+255a^2Ab^3-110ab^4B+40Ab^5)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3465a^3d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a(11aB+14Ab)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{99d\sec^{\frac{7}{2}}(c+dx)}+\frac{2aA\sin(c+dx)(a+b\sec(c+dx))^{3/2}}{11d\sec^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(11/2),x
]
```

```
[Out] (2*(a^2 - b^2)*(675*a^4*A + 285*a^2*A*b^2 + 40*A*b^4 + 1254*a^3*b*B - 110*a
*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(3465*a^3*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3705*
a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^
4*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3465*
a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*(14*A*b
 + 11*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(99*d*Sec[c + d*x]^(7/2))
 + (2*(81*a^2*A + 113*A*b^2 + 209*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/(693*d*Sec[c + d*x]^(5/2)) + (2*(1145*a^2*A*b + 15*A*b^3 + 539*a^3*B +
```

```
825*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3465*a*d*Sec[c + d*x]
^(3/2)) + (2*(675*a^4*A + 1025*a^2*A*b^2 - 20*A*b^4 + 1793*a^3*b*B + 55*a*b
^3*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3465*a^2*d*Sqrt[Sec[c + d*x]]
) + (2*a*A*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(11*d*Sec[c + d*x]^(9/2
))
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4179

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sec^{\frac{11}{2}}(c+dx)}\,dx = \frac{2aA(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{11d\sec^{\frac{9}{2}}(c+dx)} - \frac{2}{11}\int \frac{\sqrt{a+b\,\mathrm{se}}}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2a}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(8}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(8}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(8}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(8}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(8}{}$$

$$= \frac{2a(14Ab+11aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{99d\sec^{\frac{7}{2}}(c+dx)} + \frac{2(8}{}$$

$$= \frac{2(a^2-b^2)\left(675a^4A+285a^2Ab^2+40Ab^4+1254a^3bB-\right.}{3465a^3d\sqrt{}}$$

Mathematica [A]
time = 3.52, size = 380, normalized size = 0.73

$$\frac{(a+b\sec(c+dx))^{5/2}\left(16\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\left(a^2(675a^4A+3315a^2Ab^2+10Ab^4+2871a^3bB+1705ab^3B)F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+(3705a^4Ab+255a^2Ab^3+40Ab^5+1617a^5B+3069a^3b^2B-110ab^4B)\left((a+b)E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)-bF\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\right)\right)+a(b+a\cos(c+dx))\left(2(6525a^4A+9330a^2Ab^2-160Ab^4+16434a^3bB+440ab^3B)\sin(c+dx)+a(4(3095a^2Ab+30Ab^3+1463a^3B+1650ab^2B)\sin(2(c+dx))+5a((513a^2A+452Ab^2+836abB)\sin(3(c+dx))+7a((46Ab+22aB)\sin(4(c+dx))+9aA\sin(5(c+dx)))))\right)\right)}{27720a^3d(b+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sec[c + d*x]^(1
1/2),x]
```

```
[Out] ((a + b*Sec[c + d*x])^(5/2)*(16*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(a^2*(67
5*a^4*A + 3315*a^2*A*b^2 + 10*A*b^4 + 2871*a^3*b*B + 1705*a*b^3*B)*Elliptic
F[(c + d*x)/2, (2*a)/(a + b)] + (3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 +
1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*((a + b)*EllipticE[(c + d*x)/2,
(2*a)/(a + b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)])) + a*(b + a*Cos[c
 + d*x])*(2*(6525*a^4*A + 9330*a^2*A*b^2 - 160*A*b^4 + 16434*a^3*b*B + 440*
a*b^3*B)*Sin[c + d*x] + a*(4*(3095*a^2*A*b + 30*A*b^3 + 1463*a^3*B + 1650*a
*b^2*B)*Sin[2*(c + d*x)] + 5*a*((513*a^2*A + 452*A*b^2 + 836*a*b*B)*Sin[3*(
c + d*x)] + 7*a*((46*A*b + 22*a*B)*Sin[4*(c + d*x)] + 9*a*A*Sin[5*(c + d*x)
]))))))/(27720*a^3*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5945 vs. $2(531) = 1062$.
time = 14.38, size = 5946, normalized size = 11.46

method	result	size
default	Expression too large to display	5946

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x,method=_RET
URNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algo
rithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(11/
2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.50, size = 753, normalized size = 1.45

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algo
rithm="fricas")
```

```
[Out] 1/10395*(sqrt(2)*(-2025*I*A*a^6 - 5379*I*B*a^5*b - 2535*I*A*a^4*b^2 + 1023*
I*B*a^3*b^3 + 480*I*A*a^2*b^4 - 220*I*B*a*b^5 + 80*I*A*b^6)*sqrt(a)*weierst
rassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a
*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(2025*I*A*a^6 + 5379
*I*B*a^5*b + 2535*I*A*a^4*b^2 - 1023*I*B*a^3*b^3 - 480*I*A*a^2*b^4 + 220*I*
B*a*b^5 - 80*I*A*b^6)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2
*b)/a) - 3*sqrt(2)*(-1617*I*B*a^6 - 3705*I*A*a^5*b - 3069*I*B*a^4*b^2 - 255
*I*A*a^3*b^3 + 110*I*B*a^2*b^4 - 40*I*A*a*b^5)*sqrt(a)*weierstrassZeta(-4/3
*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*
(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*
I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(1617*I*B*a^6 + 3705*I*A*a^5*b + 30
69*I*B*a^4*b^2 + 255*I*A*a^3*b^3 - 110*I*B*a^2*b^4 + 40*I*A*a*b^5)*sqrt(a)*
weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weier
strassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3
*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) + 6*(315*A*a^6*cos(d*x + c)
^5 + 35*(11*B*a^6 + 23*A*a^5*b)*cos(d*x + c)^4 + 5*(81*A*a^6 + 209*B*a^5*b
+ 113*A*a^4*b^2)*cos(d*x + c)^3 + (539*B*a^6 + 1145*A*a^5*b + 825*B*a^4*b^2
 + 15*A*a^3*b^3)*cos(d*x + c)^2 + (675*A*a^6 + 1793*B*a^5*b + 1025*A*a^4*b^
2 + 55*B*a^3*b^3 - 20*A*a^2*b^4)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/(a^4*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)**(11/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/sec(d*x+c)^(11/2),x, algo
rithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sec(d*x + c)^(11/
2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}{\left(\frac{1}{\cos(c+d\,x)}\right)^{11/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(11/
2),x)
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/(1/cos(c + d*x))^(11/
2), x)
```

3.456 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=344

$$\frac{(4Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4bd\sqrt{a+b\sec(c+dx)}}-\frac{(4aAb-3a^2B-4b^2B)\sqrt{\frac{b+a\cos(c+\cdots}{a+b}}}{4b^2d\sqrt{a+b\cdots}}$$

```
[Out] 1/4*(4*A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*s
ec(d*x+c)^(1/2)/b/d/(a+b*sec(d*x+c))^(1/2)-1/4*(4*A*a*b-3*B*a^2-4*B*b^2)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c)
,2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)
/b^2/d/(a+b*sec(d*x+c))^(1/2)+1/2*B*sec(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*
x+c))^(1/2)/b/d-1/4*(4*A*b-3*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c
))^(1/2)/b^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)+1/4*(4*A*b-3
*B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b^2/d
```

Rubi [A]
time = 0.72, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {4118, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{(-3a^2B+4aAb-4b^2B)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{a+b\sec(c+dx)}}+\frac{(4Ab-3aB)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{4b^2d}-\frac{(4Ab-3aB)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(4Ab-aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4bd\sqrt{a+b\sec(c+dx)}}+\frac{B\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] ((4*A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*b*d*Sqrt[a + b*Sec[c + d*x]]) - ((4*a*A
*b - 3*a^2*B - 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(4*b^2*d*Sqrt[a + b*Sec[c + d
*x]]) - ((4*A*b - 3*a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*S
ec[c + d*x]])/(4*b^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]
]) + ((4*A*b - 3*a*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/(4*b^2*d) + (B*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x
])/(2*b*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4118

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d^
2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 2)/(b*f*
(m + n))), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e +
 f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m +
n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n
, 0] &&  !IGtQ[m, 1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
```

```
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{B\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2bd} + \frac{\int \frac{\sqrt{\sec(c+dx)}}{}}{}$$

$$= \frac{(4Ab-3aB)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4b^2d} + \frac{B\,\mathrm{s}}{}$$

$$= \frac{(4Ab-3aB)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4b^2d} + \frac{B\,\mathrm{s}}{}$$

$$= \frac{(4Ab-3aB)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4b^2d} + \frac{B\,\mathrm{s}}{}$$

$$= \frac{(4Ab-3aB)\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4b^2d} + \frac{B\,\mathrm{s}}{}$$

$$= -\frac{(4aAb-3a^2B-4b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{(4Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{4bd\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 13.91, size = 451, normalized size = 1.31

$$\frac{\sqrt{\sec(c+dx)}\left(\frac{8aB\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b}+\frac{2(-12aAb+9a^2B+8b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2}+\frac{2i(-4Ab+3aB)\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\sqrt{b+a\cos(c+dx)}\csc(c+dx)\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)\right)\right)}{a\sqrt{\frac{1}{a-b}}\,b^3}-\frac{4a(-4Ab+3aB)\sin(c+dx)}{b^2}+\frac{8aB\tan(c+dx)}{b}+\frac{4(4Ab-3aB)\tan(c+dx)}{b}+8B\sec(c+dx)\tan(c+dx)\right)}{16d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]
],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*((8*a*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c
 + d*x)/2, (2*a)/(a + b)])/b + (2*(-12*a*A*b + 9*a^2*B + 8*b^2*B)*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/b^2 +
((2*I)*(-4*A*b + 3*a*B)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1
 + Cos[c + d*x]))/(a - b)]*Sqrt[b + a*Cos[c + d*x]]*Csc[c + d*x]*(-2*b*(a +
 b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a +
 b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos
[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a -
b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-
1)]*b^3) - (4*a*(-4*A*b + 3*a*B)*Sin[c + d*x])/b^2 + (8*a*B*Tan[c + d*x])/b
 + (4*(4*A*b - 3*a*B)*Tan[c + d*x])/b + 8*B*Sec[c + d*x]*Tan[c + d*x]))/(16
*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 12.74, size = 2737, normalized size = 7.96

method	result	size
default	Expression too large to display	2737

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/d*(-4*A*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+2*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*
A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a*b-3*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-3*B*cos(d*x+c
)^3*((a-b)/(a+b))^(1/2)*a^2+4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2+2*B*co
s(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2-2*B*((a-b)/(a+b))^(1/2)*b^2-4*A*cos(d*x+
c)^2*((a-b)/(a+b))^(1/2)*a*b-3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b+B*cos
(d*x+c)*((a-b)/(a+b))^(1/2)*a*b+8*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+4*A*sin(d*x
+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*b^2-6*B*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
```

```
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-4*B*sin(d*x+c)*cos(d*x
+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*b^2+3*B*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+6*B*sin(d*x+c)*cos(d*x+c)^3*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticP
i((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b)
)^(1/2))*a^2+8*B*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2+4*A*sin(d*x+c)*c
os(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*b^2-4*B*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+3*B*sin(d*x+c)*cos(d*x+c)^2
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a^2+6*B*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+8*B*sin(d*x+c)*cos(d*x+c
)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a
-b)/(a+b))^(1/2))*b^2-8*A*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b+2*B*sin
(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a*b-3*B*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-8*A*sin(d*x+c)*cos
(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b)
,I/((a-b)/(a+b))^(1/2))*a*b+8*A*sin(d*x+c)*cos(d*x+c)^3*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+4*A*cos(d*x+c)^
3*((a-b)/(a+b))^(1/2)*a*b+2*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b+3*B*cos(
d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2-4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2-6*B
*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a^2)*cos(d*x+c)*(1/cos(d*x+c))^(5/2)*((b+a*cos(d*x
+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*cos(d*x+c))/b^2/((a-b)/(a+b))^(1/2)
```

Maxima [**F**]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a),
 x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x))^(1/2
),x)
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x))^(1/2
), x)
```

3.457 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=256

$$\frac{B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}+\frac{(2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\right.}{bd\sqrt{a+b\sec(c+dx)}}$$

```
[Out] B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2
)/d/(a+b*sec(d*x+c))^(1/2)+(2*A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*
cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b/d/(a+b*sec(d*x+c))^(1/2)-B*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b/d/((b+a*cos(d*x+c))/(a+b))^
(1/2)/sec(d*x+c)^(1/2)+B*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)
/b/d
```

Rubi [A]
time = 0.47, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {4118, 4194, 3944, 2886, 2884, 3947, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(2Ab-aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{bd\sqrt{a+b\sec(c+dx)}}+\frac{B\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{bd}+\frac{B\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}-\frac{B\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{bd\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b - a*B)*Sqrt[(b
+ a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[S
ec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) - (B*EllipticE[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*d*Sqrt[(b + a*Cos[c + d*x])/(a +
b)]*Sqrt[Sec[c + d*x]]) + (B*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/(b*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
```

```
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3947

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4118

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d^
2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 2)/(b*f*
(m + n))), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e +
 f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m +
n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n
, 0] &&  !IGtQ[m, 1]
```

Rule 4194

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[C/d^
2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[A, In
t[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b,
 d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = \frac{B\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{bd} + \frac{\int \frac{-\frac{aB}{2}+\frac{1}{2}}{\sqrt{\sec(c+dx}}}{}$$

$$= \frac{B\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{bd} - \frac{(aB)\int \frac{}{\sqrt{\sec(}}}{}$$

$$= \frac{B\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{bd} + \frac{1}{2}B\int \frac{\sqrt{\sec}}{\sqrt{a+b}}$$

$$= \frac{B\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{bd} + \frac{\left(B\sqrt{b+a\cos}\right.}{}$$

$$= \frac{(2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx}}{bd\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{(2Ab-}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.72, size = 339, normalized size = 1.32

$$\frac{\sqrt{\sec(c+dx)}\left(2(4Ab-3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-\frac{2iB\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\sqrt{b+a\cos(c+dx)}\csc(c+dx)\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)\right)\right)}{a\sqrt{\frac{1}{a-b}}\,b}+4B(b+a\cos(c+dx))\tan(c+dx)\right)}{4bd\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]
],x]
```

```
[Out] (Sqrt[Sec[c + d*x]]*(2*(4*A*b - 3*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticPi[2, (c + d*x)/2, (2*a)/(a + b)] - ((2*I)*B*Sqrt[-((a*(-1 + Cos[c +
 d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Sqrt[b + a*Cos[c + d
*x]]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt
[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[
(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1
```

```
 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a
 + b)])))/(a*Sqrt[(a - b)^(-1)]*b) + 4*B*(b + a*Cos[c + d*x])*Tan[c + d*x])
)/(4*b*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 13.73, size = 1439, normalized size = 5.62

method	result	size
default	Expression too large to display	1439

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*(2*A*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-4*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^
2*b-2*B*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a+2*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2
*a+B*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a-B*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+2*A*cos(d*x+c)*s
in(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*b-4*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b-2*B*sin(d*x+c)*cos(d*
x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*a+2*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a+B*sin(d*x+c)*cos(d*x+c)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-
B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*b-B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a+B*cos(d*x+c)
*((a-b)/(a+b))^(1/2)*a-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b+B*((a-b)/(a+b))^(
```

```
1/2)*b)*cos(d*x+c)*(1/cos(d*x+c))^(3/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)
/sin(d*x+c)/(b+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)/b
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a),
 x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x))^(1/2
), x)
```

3.458 $\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=138

$$\frac{2A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}+\frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1
/2)/d/(a+b*sec(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*
x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/d/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4121, 3943, 2742, 2740, 3944, 2886, 2884}

$$\frac{2A\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}+\frac{2B\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*A*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)]*Sqrt[Sec[c + d*x]])/(d*Sqrt[a + b*Sec[c + d*x]]) + (2*B*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d
*x]])/(d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4121

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (
A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A, Int[
Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B/d, Int[(d*Cs
c[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = A\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx + B\int \frac{\sec^{\frac{3}{2}}(c+dx)}{\sqrt{a+b\sec(c+dx)}}\,dx$$

$$= \frac{\left(A\sqrt{b+a\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,d}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{\left(A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c+dx)}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+}{a+b}}}}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}} + \frac{2B}{}$$

Mathematica [A]
time = 0.27, size = 91, normalized size = 0.66

$$\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\left(AF\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+B\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\right)\sqrt{\sec(c+dx)}}{d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]
],x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(A*EllipticF[(c + d*x)/2, (2*a)/(a +
b)] + B*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])*Sqrt[Sec[c + d*x]])/(d*S
qrt[a + b*Sec[c + d*x]])
```

Maple [C] Result contains complex when optimal does not.
time = 13.12, size = 265, normalized size = 1.92

method	result
default	$-\frac{2\left(A\,\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)-B\,\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)+2B\,\mathrm{EllipticPi}\left(\frac{(-1}{}\right.\right.}{d(b+a\cos(dx+c))\sqrt{\frac{1}{1+\cos(dx}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/d*(A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))-B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))+2*B*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)))*cos(d*x+c)*(1/cos(d*x+c))^(1/2)*((
b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b*sec(d*x + c) + a),
 x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sqrt{\sec(c + dx)}}{\sqrt{a + b\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(sec(c + d*x))/sqrt(a + b*sec(c + d*x)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/sqrt(b*sec(d*x + c) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x))^(1/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x))^(1/2
), x)
```

3.459 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=150

$$-\frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}}+\frac{2AE\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}}$$

```
[Out] -2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(
d*x+c)^(1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b
*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.20, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2A\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (-2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*A*Ell
ipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*d*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \frac{A\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}dx}{a} - \frac{(Ab-aB)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx}}}{a}$$

$$= -\frac{\left((Ab-aB)\sqrt{b+a\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{b+a\,\mathrm{c}}}}{a\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{\left((Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+dx)}\right)\int \frac{}{\sqrt{\frac{b}{a+b}}}}{a\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+}}{ad\sqrt{a+b\sec(c+dx)}}$$

Mathematica [**A**]
time = 3.78, size = 103, normalized size = 0.69

$$\frac{2\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\left(A(a+b)E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)+(-Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]
),x]
```

```
[Out] (2*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*(A*(a + b)*EllipticE[(c + d*x)/2, (2*
a)/(a + b)] + (-(A*b) + a*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)])*Sqrt[Se
c[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 939 vs. $2(196) = 392$.
time = 14.08, size = 940, normalized size = 6.27 Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-A*cos(d*x+c)*sin(d*x+c)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-
```

```
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+A*sin(
d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*a-A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b+B*sin(d*x+c)*cos(d*x+c)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a-A*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
*sin(d*x+c)+A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a*sin(d*x+c)-A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+c)+B*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)+A*cos(d*x+c)^2*((a
-b)/(a+b))^(1/2)*a-A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a+A*cos(d*x+c)*((a-b)/(
a+b))^(1/2)*b-A*((a-b)/(a+b))^(1/2)*b)/(1/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a
*cos(d*x+c))/((a-b)/(a+b))^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.86, size = 371, normalized size = 2.47

$$\frac{3i\sqrt{2}\,Aa^{\frac{3}{2}}\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right) - 3i\sqrt{2}\,Aa^{\frac{3}{2}}\mathrm{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right) + \sqrt{2}\,(-3i\,Ba+2i\,Ab)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right) + \sqrt{2}\,(3i\,Ba-2i\,Ab)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*I*sqrt(2)*A*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a))
- 3*I*sqrt(2)*A*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
```

```
^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^
2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) + s
qrt(2)*(-3*I*B*a + 2*I*A*b)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2
)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x +
c) + 2*b)/a) + sqrt(2)*(3*I*B*a - 2*I*A*b)*sqrt(a)*weierstrassPInverse(-4/3
*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3
*I*a*sin(d*x + c) + 2*b)/a))/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec(c + dx)}{\sqrt{a + b\sec(c + dx)}\,\sqrt{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(sqrt(a + b*sec(c + d*x))*sqrt(sec(c + d*x)))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sqrt(sec(d*x + c))
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+d\,x)}}{\sqrt{a + \frac{b}{\cos(c + d\,x)}}\,\sqrt{\frac{1}{\cos(c + d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)
),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(1/2)
), x)
```

3.460 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=212

$$\frac{2(a^2A+2Ab^2-3abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^2d\sqrt{a+b\sec(c+dx)}}-\frac{2(2Ab-3aB)E\left(\frac{1}{2}(c+dx)|\frac{2}{a}\right.}{3a^2d\sqrt{\frac{b+a\cos(c+d}{a+b}}}$$

```
[Out] 2/3*(A*a^2+2*A*b^2-3*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a
+b))^(1/2)*sec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2)+2/3*A*sin(d*x+c)*(
a+b*sec(d*x+c))^(1/2)/a/d/sec(d*x+c)^(1/2)-2/3*(2*A*b-3*B*a)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(
a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/se
c(d*x+c)^(1/2)
```

Rubi [A]

time = 0.32, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {4119, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(a^2A-3abB+2Ab^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{a+b\sec(c+dx)}}-\frac{2(2Ab-3aB)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2A\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3ad\sqrt{\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (2*(a^2*A + 2*A*b^2 - 3*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^2*d*Sqrt[a + b*Sec[c
+ d*x]]) - (2*(2*A*b - 3*a*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
+ b*Sec[c + d*x]])/(3*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c +
 d*x]]) + (2*A*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a*d*Sqrt[Sec[c + d
*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4119

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*n)), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(2Ab-3aB)-\frac{1}{2}aA\sec(}{\sqrt{\sec(c+dx)}\,\sqrt{a+b\,\mathrm{se}}}}{3a}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} - \frac{(2Ab-3aB)\int \frac{\sqrt{a+b\sec(}}{\sqrt{\sec(c+}}}{3a^2}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{\left(\left(A+\frac{b(2Ab-3aB)}{a^2}\right)\sqrt{b+a\,\mathrm{c}}\right.}{}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3ad\sqrt{\sec(c+dx)}} + \frac{\left(\left(A+\frac{b(2Ab-3aB)}{a^2}\right)\sqrt{\frac{b+a}{}}\right.}{}$$

$$= \frac{2\left(A+\frac{b(2Ab-3aB)}{a^2}\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c}}{3d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A]
time = 0.90, size = 161, normalized size = 0.76

$$\frac{2\sqrt{\sec(c+dx)}\left((a+b)(-2Ab+3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+(a^2A+2Ab^2-3abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+aA(b+a\cos(c+dx))\sin(c+dx)\right)}{3a^2d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]
),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*((a + b)*(-2*A*b + 3*a*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (a^2*A + 2*A*b^2 - 3*a*b*B)
*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] +
 a*A*(b + a*Cos[c + d*x])*Sin[c + d*x]))/(3*a^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1730 vs. $2(248) = 496$.
time = 14.43, size = 1731, normalized size = 8.17

method	result	size
default	Expression too large to display	1731

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(2*A*((a-b)/(a+b))^(1/2)*b^2+2*A
*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a*b-3*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-2*A*cos(d*x+c)*sin
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a*b-A*((a-b)/(a+b))^(1/2)*a*b-3*B*((a-b)/(a+b))^(1/2)*a*b-A*cos(d*x
+c)^2*((a-b)/(a+b))^(1/2)*a*b+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b+A*cos(
d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^2-3*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+A*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)-3*B*((
a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2+A*((a
-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2+3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2-
2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2+2*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a
*b+2*A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+3*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*A*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(
d*x+c)-2*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a*b*sin(d*x+c)-3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)+2*A*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2*sin(d*x+c)-3*B*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin
```

```
(d*x+c)+3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a^2*sin(d*x+c))*cos(d*x+c)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(b
+a*cos(d*x+c))/a^2/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.13, size = 452, normalized size = 2.13

$$\frac{6\,Aa^2\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\sqrt{2}\left(-3i\,Aa^2+6i\,Bab-4i\,Ab^2\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(3i\,Aa^2-6i\,Bab+4i\,Ab^2\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(-3i\,Ba^2+2i\,Aab\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(3i\,Ba^2-2i\,Aab\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)}{9\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/9*(6*A*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c) + sqrt(2)*(-3*I*A*a^2 + 6*I*B*a*b - 4*I*A*b^2)*sqrt(a)*weierstras
sPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*co
s(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*A*a^2 - 6*I*B*a*b
+ 4*I*A*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*
a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) -
3*sqrt(2)*(-3*I*B*a^2 + 2*I*A*a*b)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*
b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b
^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x
+ c) + 2*b)/a)) - 3*sqrt(2)*(3*I*B*a^2 - 2*I*A*a*b)*sqrt(a)*weierstrassZeta
(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(
-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
 - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec\left(c + dx\right)}{\sqrt{a + b\sec\left(c + dx\right)}\,\sec^{\frac{3}{2}}\left(c + dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(sqrt(a + b*sec(c + d*x))*sec(c + d*x)**(3/2)
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(3/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)
),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(3/2)
), x)
```

3.461 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=280

$$-\frac{2(7a^2Ab+8Ab^3-5a^3B-10ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2A+8Ab^2 \ldots}{15a \ldots}$$

```
[Out] -2/15*(7*A*a^2*b+8*A*b^3-5*B*a^3-10*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b
+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^3/d/(a+b*sec(d*x+c))^(1/2)+2
/5*A*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d/sec(d*x+c)^(3/2)-2/15*(4*A*b-5*B
*a)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^2/d/sec(d*x+c)^(1/2)+2/15*(9*A*a^2+
8*A*b^2-10*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^3/d/(
(b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.49, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4119, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{2(4Ab-5aB)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{15a^2d\sqrt{\sec(c+dx)}}+\frac{2(9a^2A-10abB+8Ab^2)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2(-5a^3B+7a^2Ab-10ab^2B+8Ab^3)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{a+b\sec(c+dx)}}+\frac{2A\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad\sec^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (-2*(7*a^2*A*b + 8*A*b^3 - 5*a^3*B - 10*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^3*
d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*EllipticE[(
c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^3*d*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*A*Sqrt[a + b*Sec[c + d*x]]*
Sin[c + d*x])/(5*a*d*Sec[c + d*x]^(3/2)) - (2*(4*A*b - 5*a*B)*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/(15*a^2*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4119

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*n)), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2\int \frac{\frac{1}{2}(4Ab-5aB)-\frac{3}{2}aA\sec(c+dx)-}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(}}}{5a}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(4Ab-5aB)\sqrt{a+b\sec(c}}{15a^2d\sqrt{\sec(c+}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(4Ab-5aB)\sqrt{a+b\sec(c}}{15a^2d\sqrt{\sec(c+}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(4Ab-5aB)\sqrt{a+b\sec(c}}{15a^2d\sqrt{\sec(c+}}$$

$$= \frac{2A\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5ad\sec^{\frac{3}{2}}(c+dx)} - \frac{2(4Ab-5aB)\sqrt{a+b\sec(c}}{15a^2d\sqrt{\sec(c+}}$$

$$= -\frac{2(7a^2Ab+8Ab^3-5a^3B-10ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c-\right.}{15a^3d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A]

time = 1.30, size = 198, normalized size = 0.71

$$\frac{2\sqrt{\sec(c+dx)}\left((a+b)\left(9a^2A+8Ab^2-10abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+\left(-7a^2Ab-8Ab^3+5a^3B+10ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+a(b+a\cos(c+dx))(-4Ab+5aB+3aA\cos(c+dx))\sin(c+dx)\right)}{15a^3d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]
),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*((a + b)*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)] + (-7*a^2*A*b -
 8*A*b^3 + 5*a^3*B + 10*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)] + a*(b + a*Cos[c + d*x])*(-4*A*b + 5*a*B + 3
*a*A*Cos[c + d*x])*Sin[c + d*x]))/(15*a^3*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2738 vs. $2(310) = 620$.
time = 13.66, size = 2739, normalized size = 9.78

method	result	size
default	Expression too large to display	2739

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-8*A*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b
^2-9*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+8*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2+10*B*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin
(d*x+c)*cos(d*x+c)*a^2*b-10*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+10*B*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*c
os(d*x+c)*a*b^2-8*A*((a-b)/(a+b))^(1/2)*b^3+2*A*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b
-9*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
```

```
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^3*sin(d*x+c)-10*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2-A*((a-b)/(a+b
))^(1/2)*cos(d*x+c)^3*a^2*b+3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^3+6*A*((
a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3-9*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3+9*A*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*
x+c)*cos(d*x+c)*a^3-8*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3+5*B*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)
*a^3+2*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^2*b*sin(d*x+c)-8*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)-9*A*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)+8*A*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b
^2*sin(d*x+c)+10*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(
d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)-10*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)+10*B*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*
x+c)+5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^3-5*B*((a-b)/(a+b))^(1/2)*cos(d
*x+c)*a^3-9*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3+8*A*((a-b)/(a+b))^(1/2)*co
s(d*x+c)*b^3+4*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^2-5*B*((a-b)/(a+b))^(
1/2)*cos(d*x+c)^2*a^2*b-9*A*((a-b)/(a+b))^(1/2)*a^2*b+4*A*((a-b)/(a+b))^(1/
2)*a*b^2-5*B*((a-b)/(a+b))^(1/2)*a^2*b+10*B*((a-b)/(a+b))^(1/2)*a*b^2+9*A*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^3*sin(d*x+c)-8*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*b^3*sin(d*x+c)+5*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)-8*A*((a-b)/(a+b))^(1/2)
*cos(d*x+c)*a*b^2+10*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b+10*A*cos(d*x+c)
*((a-b)/(a+b))^(1/2)*a^2*b)*cos(d*x+c)^3*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(b
+a*cos(d*x+c))/a^3/((a-b)/(a+b))^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.87, size = 520, normalized size = 1.86

$$\frac{\sqrt{2}\left(-15i\,Ba^3+12i\,Aa^2b-20i\,Bab^2+16i\,Ab^3\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(15i\,Ba^3-12i\,Aa^2b+20i\,Bab^2-16i\,Ab^3\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(-9i\,Aa^3+10i\,Ba^2b-8i\,Aab^2\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(9i\,Aa^3-10i\,Ba^2b+8i\,Aab^2\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)+\frac{6\left(3Aa^3\cos(dx+c)^2+\left(5Ba^3-4Aa^2b\right)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sin(dx+c)}{\sqrt{\cos(dx+c)}}}{45\,a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/45*(sqrt(2)*(-15*I*B*a^3 + 12*I*A*a^2*b - 20*I*B*a*b^2 + 16*I*A*b^3)*sqrt
(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^
3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(15*I*B*a
^3 - 12*I*A*a^2*b + 20*I*B*a*b^2 - 16*I*A*b^3)*sqrt(a)*weierstrassPInverse(
-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
 - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-9*I*A*a^3 + 10*I*B*a^2*b - 8*
I*A*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b
- 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt
(2)*(9*I*A*a^3 - 10*I*B*a^2*b + 8*I*A*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(
3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3
*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*
a*sin(d*x + c) + 2*b)/a)) + 6*(3*A*a^3*cos(d*x + c)^2 + (5*B*a^3 - 4*A*a^2*
b)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(
cos(d*x + c)))/(a^4*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sec(d*x + c)^(5/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)
),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(1/2)*(1/cos(c + d*x))^(5/2)
), x)
```

3.462 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=371

$$\frac{B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}}+\frac{(2Ab-3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\right.}{b^2d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*a*(A*b-B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1
/2)+B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^
(1/2)/b/d/(a+b*sec(d*x+c))^(1/2)+(2*A*b-3*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2)
)*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b^2/d/(a+b*sec(d*x+c))^(1
/2)+(2*A*a*b-3*B*a^2+B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/
2)/b^2/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)-(2*A*a*b
-3*B*a^2+B*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b^2/(a^2
-b^2)/d
```

Rubi [A]
time = 0.82, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {4114, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2a(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+dx)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}{b^2d\left(a^2-b^2\right)}+\frac{\left(-3a^2B+2aAb+b^2B\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(2Ab-3aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{a+b\sec(c+dx)}}+\frac{B\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[Sec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b - 3*a*B)*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)]*Sq
rt[Sec[c + d*x]])/(b^2*d*Sqrt[a + b*Sec[c + d*x]]) + ((2*a*A*b - 3*a^2*B +
b^2*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b^2
*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*
a*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*
Sec[c + d*x]]) - ((2*a*A*b - 3*a^2*B + b^2*B)*Sqrt[Sec[c + d*x]]*Sqrt[a + b
*Sec[c + d*x]]*Sin[c + d*x])/(b^2*(a^2 - b^2)*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
```

```
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{\sqrt{\sec(c+dx)}\left(\frac{1}{2}a(Ab-\right.}{}}{}$$

$$= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{(2aAb-3a^2B+b^2B)\,\sqrt{}}{}$$

$$= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{(2aAb-3a^2B+b^2B)\,\sqrt{}}{}$$

$$= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{(2aAb-3a^2B+b^2B)\,\sqrt{}}{}$$

$$= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{(2aAb-3a^2B+b^2B)\,\sqrt{}}{}$$

$$= \frac{(2Ab-3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+d}}{b^2d\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} + \frac{(2Ab-}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 15.68, size = 518, normalized size = 1.40

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(-\frac{(b+a\cos(c+dx))^{3/2}\left(\frac{8ab(-Ab+aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2\left(-6a^2Ab+4Ab^3+9a^3B-7ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2i\left(-2aAb+3a^2B-b^2B\right)\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\csc(c+dx)\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)\right)\right)}{a\sqrt{\frac{1}{a-b}}\,b}\right)}{(a-b)b^2(a+b)}+\frac{4(b+a\cos(c+dx))\left(b\left(-a^2+b^2\right)B+a\left(2aAb-3a^2B+b^2B\right)\cos(c+dx)\right)\tan(c+dx)}{-a^2b^2+b^4}\right)}{4d(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(-(((b + a*Cos[c + d*x])^(3/2)*((8*a*b*(-(A*b) + a*B)*S
qrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sq
rt[b + a*Cos[c + d*x]] + (2*(-6*a^2*A*b + 4*A*b^3 + 9*a^3*B - 7*a*b^2*B)*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])
/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(-2*a*A*b + 3*a^2*B - b^2*B)*Sqrt[-((a*(
-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*(1 + Cos[c + d*x]))/(a - b)]*Csc[c +
d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c
+ d*x]]], (-a + b)/(a + b)] + a*(2*b*EllipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]
*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*EllipticPi[1 - a/b, I*Arc
Sinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)])))/(a*
Sqrt[(a - b)^(-1)]*b)))/((a - b)*b^2*(a + b))) + (4*(b + a*Cos[c + d*x])*(b
*(-a^2 + b^2)*B + a*(2*a*A*b - 3*a^2*B + b^2*B)*Cos[c + d*x])*Tan[c + d*x])
/(-(a^2*b^2) + b^4)))/(4*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 20.76, size = 2656, normalized size = 7.16

method	result	size
default	Expression too large to display	2656

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*(6*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),
I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b+6*B*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*
x+c)*cos(d*x+c)*a*b+4*A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*B*cos(d*x+c)*sin(d*x+c)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a*b-2*A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+6*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*
a^2-4*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I
```

```
/((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b+B*((a-b)/(a+b))^(1/2)*a*b-4
*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a*b-2*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+B*((a-b)/(a+
b))^(1/2)*b^2+2*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b-B*cos(d*x+c)^2*((a-b
)/(a+b))^(1/2)*a*b-6*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*A*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^
2*b^2+4*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+3*B*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+6*B*s
in(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b
^2-4*A*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b+3*B*((a-b)/(a+b))^(1/2)*co
s(d*x+c)*a^2-3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2-6*B*cos(d*x+c)^2*sin(
d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^2-2*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b-4*A*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x
+c)*cos(d*x+c)^2*b^2-B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*b^2+2*A*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c
)*b^2-4*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b)
,I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^2-B*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b
^2+3*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d
*x+c)^2*(1/cos(d*x+c))^(5/2)/sin(d*x+c)/(b+a*cos(d*x+c))/((a-b)/(a+b))^(1/2
)/(a+b)/b^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(\frac{1}{\cos(c+d\,x)}\right)^{5/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x))^(3/2
), x)
```

3.463 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=220

$$\frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}}-\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+}}{b\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\sqrt{\sec(c+d}}$$

```
[Out] 2*a*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1
/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*
d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*
x+c)^(1/2)/b/d/(a+b*sec(d*x+c))^(1/2)-2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2)
)*(a+b*sec(d*x+c))^(1/2)/b/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d
*x+c)^(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4114, 4193, 3944, 2886, 2884, 21, 3941, 2734, 2732}

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2(Ab-aB)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2B\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
 + b)]*Sqrt[Sec[c + d*x]])/(b*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(A*b - a*B)*
EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*(a^2 - b
^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*a*(A*b -
a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d
*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
```

```
2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(Ab-aB)-\frac{1}{2}b(Ab-a}{\sqrt{\sec(c+d}}}{}$$

$$= \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2\int \frac{-\frac{1}{2}a(Ab-aB)-\frac{1}{2}b(}{\sqrt{\sec(c+dx)}\sqrt{a}}}{b\,(a^2-}$$

$$= \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{(Ab-aB)\int \frac{\sqrt{a+b\,\mathrm{s}}}{\sqrt{\mathrm{sec}}}}{b\,(a^2-b}$$

$$= \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{\left(B\sqrt{\frac{b+a\cos(c+d}{a+b}}\right.}{}$$

$$= \frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} + \frac{2a}{}$$

$$= \frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{bd\sqrt{a+b\sec(c+dx)}} - \frac{2(}{b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.44, size = 464, normalized size = 2.11

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(\frac{(b+a\cos(c+dx))^{3/2}\left(\frac{4b(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2\left(aAb-3a^2B+2b^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}-\frac{2i(-Ab+aB)\sqrt{-\frac{a(-1+\cos(c+dx))}{a+b}}\sqrt{\frac{a(1+\cos(c+dx))}{a-b}}\csc(c+dx)\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)\right)\right)}{a\sqrt{\frac{1}{a-b}}b}\right)}{(-a+b)(a+b)}+\frac{4a(Ab-aB)(b+a\cos(c+dx))\sin(c+dx)}{a^2-b^2}\right)}{2bd(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(((b + a*Cos[c + d*x])^(3/2)*((4*b*(A*b - a*B)*Sqrt[(b
+ a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b +
a*Cos[c + d*x]] + (2*(a*A*b - 3*a^2*B + 2*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]
] - ((2*I)*(-(A*b) + a*B)*Sqrt[-((a*(-1 + Cos[c + d*x]))/(a + b))]*Sqrt[(a*
(1 + Cos[c + d*x]))/(a - b)]*Csc[c + d*x]*(-2*b*(a + b)*EllipticE[I*ArcSinh
[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*E
llipticF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(
a + b)] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos
[c + d*x]]], (-a + b)/(a + b)])))/(a*Sqrt[(a - b)^(-1)]*b)))/((-a + b)*(a +
 b)) + (4*a*(A*b - a*B)*(b + a*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)))/(2
*b*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [C] Result contains complex when optimal does not.
time = 24.05, size = 1585, normalized size = 7.20

method	result	size
default	Expression too large to display	1585

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/d*(A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*b-A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b-2*B*sin(d*x+c)*co
s(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*a-B*sin(d*x+c)*cos(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b+B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
```

```
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+2*B*sin(d*x+
c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(
a-b),I/((a-b)/(a+b))^(1/2))*a+2*B*sin(d*x+c)*cos(d*x+c)*EllipticPi((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b+A*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b
*sin(d*x+c)-A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*b*sin(d*x+c)-2*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)-B*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+c)+B*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)+2
*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-
b)/(a+b))^(1/2))*a*sin(d*x+c)+2*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b*sin(d*x+c)+A*cos(d*x+c)*(
(a-b)/(a+b))^(1/2)*b-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a-A*((a-b)/(a+b))^(1/
2)*b+B*((a-b)/(a+b))^(1/2)*a)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c
)^2*(1/cos(d*x+c))^(3/2)/sin(d*x+c)/(b+a*cos(d*x+c))/((a-b)/(a+b))^(1/2)/(a
+b)/b
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(\frac{1}{\cos(c+d\,x)}\right)^{3/2}}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x))^(3/2
), x)
```

3.464 $\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=215

$$\frac{2A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}}+\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{a\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\sqrt{\sec(c+dx)}}$$

```
[Out] -2*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)
+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(
1/2)/a/d/(a+b*sec(d*x+c))^(1/2)+2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b
*sec(d*x+c))^(1/2)/a/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^
(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {4112, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2(Ab-aB)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2A\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{ad\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*A*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)]*Sqrt[Sec[c + d*x]])/(a*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(A*b - a*B)*Elli
pticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(a*(a^2 - b^2)*
d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*(A*b - a*B)*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4112

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-d)*(A
*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*
Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) +
 d*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
```

```
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(-Ab+aB)-}{\sqrt{\sec(c+dx)}}}{a^2}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{A\int \frac{\sqrt{\sec(c+d}}{\sqrt{a+b\sec(c+}}}{a}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{\left(A\sqrt{b+a\cos(c+\right.}{}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{\left(A\sqrt{\frac{b+a\cos(c+}{a+b}}\right.}{}$$

$$= \frac{2A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{ad\sqrt{a+b\sec(c+dx)}} + \frac{2(}{a}$$

Mathematica [A]
time = 0.74, size = 161, normalized size = 0.75

$$\frac{2\sqrt{\sec(c+dx)}\left(-\left((a+b)(-Ab+aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)+A(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+a(-Ab+aB)\sin(c+dx)\right)}{a(a-b)(a+b)d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (2*Sqrt[Sec[c + d*x]]*(-((a + b)*(-(A*b) + a*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]) + A*(a^2 - b^2)*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)] + a*(-(A*b) +
a*B)*Sin[c + d*x]))/(a*(a - b)*(a + b)*d*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 944 vs. $2(257) = 514$.
time = 20.41, size = 945, normalized size = 4.40

method	result
default	$\frac{2\left(-A\cos(dx+c)\sin(dx+c)\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\sqrt{\frac{1}{1+\cos(dx+c)}}\,\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)a-A\sin(\cdots}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/d*(-A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a-A*sin(d*x+c)*cos(d*x+c)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b-B*sin(d*x+c)*cos(
d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*a+B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a-A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*x+c)-A*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b*sin(d*x+c)-B*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*sin(d*
x+c)+B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a*sin(d*x+c)+A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b-B*cos(d*x+c)*((a-b)/
(a+b))^(1/2)*a-A*((a-b)/(a+b))^(1/2)*b+B*((a-b)/(a+b))^(1/2)*a)*cos(d*x+c)*
(1/cos(d*x+c))^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)/(b+a*co
s(d*x+c))/(a+b)/((a-b)/(a+b))^(1/2)/a
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(3/2
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.95, size = 606, normalized size = 2.82

$$\frac{6\,(Ba^3 - Aa^2b)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}\,\sin(dx+c) - \sqrt{2}\,(3i\,Aa^2b - i\,Bab^2 - 2i\,Ab^3 + (3i\,Aa^3 - i\,Ba^2b - 2i\,Aab^2)\cos(dx+c))\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2}, \frac{8\,(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right) - \sqrt{2}\,(-3i\,Aa^2b + i\,Bab^2 + 2i\,Ab^3 + (-3i\,Aa^3 + i\,Ba^2b + 2i\,Aab^2)\cos(dx+c))\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2}, \frac{8\,(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right) + 3\sqrt{2}\,(-i\,Ba^2b + i\,Aab^2 + (-i\,Ba^3 + i\,Aa^2b)\cos(dx+c))\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3a^2-4b^2)}{3a^2}, \frac{8\,(9a^2b-8b^3)}{27a^3}, \mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2}, \frac{8\,(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right) + 3\sqrt{2}\,(i\,Ba^2b - i\,Aab^2 + (i\,Ba^3 - i\,Aa^2b)\cos(dx+c))\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3a^2-4b^2)}{3a^2}, \frac{8\,(9a^2b-8b^3)}{27a^3}, \mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2}, \frac{8\,(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)}{3\,((a^5 - a^3b^2)d\cos(dx+c) + (a^4b - a^2b^3)d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(6*(B*a^3 - A*a^2*b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c) - sqrt(2)*(3*I*A*a^2*b - I*B*a*b^2 - 2*I*A*b^3 + (3*I
*A*a^3 - I*B*a^2*b - 2*I*A*a*b^2)*cos(d*x + c))*sqrt(a)*weierstrassPInverse
(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c
) + 3*I*a*sin(d*x + c) + 2*b)/a) - sqrt(2)*(-3*I*A*a^2*b + I*B*a*b^2 + 2*I*
A*b^3 + (-3*I*A*a^3 + I*B*a^2*b + 2*I*A*a*b^2)*cos(d*x + c))*sqrt(a)*weiers
trassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*
a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*sqrt(2)*(-I*B*a^2*b + I*A
*a*b^2 + (-I*B*a^3 + I*A*a^2*b)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*
(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(
3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I
*a*sin(d*x + c) + 2*b)/a)) + 3*sqrt(2)*(I*B*a^2*b - I*A*a*b^2 + (I*B*a^3 -
I*A*a^2*b)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b
)/a)))/((a^5 - a^3*b^2)*d*cos(d*x + c) + (a^4*b - a^2*b^3)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sqrt{\sec(c + dx)}}{(a + b\sec(c + dx))^{\frac{3}{2}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**(3/2
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x))^(3/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x))^(3/2
), x)
```

3.465 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=235

$$-\frac{2(2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{a^2d\sqrt{a+b\sec(c+dx)}}+\frac{2(a^2A-2Ab^2+abB)\,E\left(\frac{1}{2}(c+dx)|\right.}{a^2\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+}{a+b}}}$$

```
[Out] 2*b*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1
/2)-2*(2*A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)
*sec(d*x+c)^(1/2)/a^2/d/(a+b*sec(d*x+c))^(1/2)+2*(A*a^2-2*A*b^2+B*a*b)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/((b+a*cos(d*x
+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.36, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {4115, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2(a^2A+abB-2Ab^2)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2(2Ab-aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (-2*(2*A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(a^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(
a^2*A - 2*A*b^2 + a*b*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*S
ec[c + d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Se
c[c + d*x]]) + (2*b*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(a*(a^2 -
b^2)*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(-a^2A+2Ab^2-ab\ldots}{\sqrt{\sec(c+dx)}\ldots}}{a\,(\ldots}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{(2Ab-aB)\int \frac{\ldots}{\sqrt{a\ldots}}}{a\ldots}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left((2Ab-aB)\sqrt{b+\ldots}\right.}{\ldots}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left((2Ab-aB)\sqrt{\frac{b-\ldots}{\ldots}}\right.}{\ldots}$$

$$= -\frac{2(2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c\ldots}}{a^2d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A]
time = 1.07, size = 178, normalized size = 0.76

$$-\frac{2\sqrt{\sec(c+dx)}\left(-\left((a+b)\left(a^2A-2Ab^2+abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)-\left(a^2-b^2\right)(-2Ab+aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+ab(-Ab+aB)\sin(c+dx)\right)}{a^2(a-b)(a+b)d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] (-2*Sqrt[Sec[c + d*x]]*(-((a + b)*(a^2*A - 2*A*b^2 + a*b*B)*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]) - (a^2 - b^2)*(-
2*A*b + a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a
)/(a + b)] + a*b*(-(A*b) + a*B)*Sin[c + d*x]))/(a^2*(a - b)*(a + b)*d*Sqrt[
a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1447 vs. $2(277) = 554$.

time = 22.76, size = 1448, normalized size = 6.16

method	result	size
default	Expression too large to display	1448

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(A*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*a^2-2*A*c
os(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+co
s(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*b^2-A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-2*A*cos(d*x+c)*sin(d*x
+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a*b+B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+A*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d
*x+c)-2*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*b^2*sin(d*x+c)-A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^2*sin(d*x+c)-2*A*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+c)+B*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b*sin(d*x+
c)+B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^2*sin(d*x+c)+A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2+A*cos(d*x+c)^2*(
(a-b)/(a+b))^(1/2)*a*b-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2+2*A*cos(d*x+c)*
((a-b)/(a+b))^(1/2)*b^2-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b-A*((a-b)/(a+b)
)^(1/2)*a*b-2*A*((a-b)/(a+b))^(1/2)*b^2+B*((a-b)/(a+b))^(1/2)*a*b)/(1/cos(d
*x+c))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)/((a-b)/(a+b))^(1/2)/(a+b)/a^2
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.77, size = 679, normalized size = 2.89

$$-\frac{6\,(Ba^3b - Aa^2b^2)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c) + \sqrt{2}\,(3i\,Ba^3b - 5i\,Aa^2b^2 - 2i\,Bab^3 + 4i\,Ab^4 + (3i\,Ba^4 - 5i\,Aa^3b - 2i\,Ba^2b^2 + 4i\,Aab^3)\cos(dx+c))\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right) + \sqrt{2}\,(-3i\,Ba^3b + 5i\,Aa^2b^2 + 2i\,Bab^3 - 4i\,Ab^4 + (-3i\,Ba^4 + 5i\,Aa^3b + 2i\,Ba^2b^2 - 4i\,Aab^3)\cos(dx+c))\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right) - 3\sqrt{2}\,(i\,Aa^3b + i\,Ba^2b^2 - 2i\,Aab^3 + (i\,Aa^4 + i\,Ba^3b - 2i\,Aa^2b^2)\cos(dx+c))\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right) - 3\sqrt{2}\,(-i\,Aa^3b - i\,Ba^2b^2 + 2i\,Aab^3 + (-i\,Aa^4 - i\,Ba^3b + 2i\,Aa^2b^2)\cos(dx+c))\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2}, \frac{8(9a^2b-8b^3)}{27a^3}, \frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)}{3\left((a^6-a^4b^2)d\cos(dx+c)+(a^5b-a^3b^3)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] -1/3*(6*(B*a^3*b - A*a^2*b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) + sqrt(2)*(3*I*B*a^3*b - 5*I*A*a^2*b^2 - 2*I*B*a
*b^3 + 4*I*A*b^4 + (3*I*B*a^4 - 5*I*A*a^3*b - 2*I*B*a^2*b^2 + 4*I*A*a*b^3)*
cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) +
 sqrt(2)*(-3*I*B*a^3*b + 5*I*A*a^2*b^2 + 2*I*B*a*b^3 - 4*I*A*b^4 + (-3*I*B*
a^4 + 5*I*A*a^3*b + 2*I*B*a^2*b^2 - 4*I*A*a*b^3)*cos(d*x + c))*sqrt(a)*weie
rstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(
3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(I*A*a^3*b + I*
B*a^2*b^2 - 2*I*A*a*b^3 + (I*A*a^4 + I*B*a^3*b - 2*I*A*a^2*b^2)*cos(d*x + c
))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)
/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/
a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(-I*
A*a^3*b - I*B*a^2*b^2 + 2*I*A*a*b^3 + (-I*A*a^4 - I*B*a^3*b + 2*I*A*a^2*b^2
)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^
2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((
a^6 - a^4*b^2)*d*cos(d*x + c) + (a^5*b - a^3*b^3)*d)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec(c + dx)}{(a + b\sec(c + dx))^{\frac{3}{2}}\sqrt{\sec(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2)/sec(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/((a + b*sec(c + d*x))**(3/2)*sqrt(sec(c + d*x
))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*sqrt(sec(d*x + c
))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{3/2}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)
),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(1/2)
), x)
```

3.466 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=326

$$\frac{2(a^2A+8Ab^2-6abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^3d\sqrt{a+b\sec(c+dx)}}-\frac{2(5a^2Ab-8Ab^3-3a^3B+}{3a^3\left(a^2-b^2\right)d\sqrt{}}$$

```
[Out] 2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1
/2)+2/3*(A*a^2+8*A*b^2-6*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c)
)/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^3/d/(a+b*sec(d*x+c))^(1/2)+2/3*(A*a^2-4*A
*b^2+3*B*a*b)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/sec(d*x+c)^
(1/2)-2/3*(5*A*a^2*b-8*A*b^3-3*B*a^3+6*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*
(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d
*x+c)^(1/2)
```

Rubi [A]

time = 0.55, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4115, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(a^2A+3abB-4Ab^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}}+\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(a^2A-6abB+8Ab^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d\sqrt{a+b\sec(c+dx)}}-\frac{2(-3a^3B+5a^2Ab+6ab^2B-8Ab^3)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*(a^2*A + 8*A*b^2 - 6*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^3*d*Sqrt[a + b*Sec[c
+ d*x]]) - (2*(5*a^2*A*b - 8*A*b^3 - 3*a^3*B + 6*a*b^2*B)*EllipticE[(c + d*
x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)*d*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c +
d*x])/(a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(a
^2*A - 4*A*b^2 + 3*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^
2 - b^2)*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}\left(-a^2A+4A\right.}{}}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A-4Ab}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A-4Ab}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A-4Ab}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A-4Ab}{}$$

$$= \frac{2(a^2A+8Ab^2-6abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3a^3d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A]
time = 1.66, size = 252, normalized size = 0.77

$$\frac{2(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\left((a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\left(a^2(a^2A+2Ab^2-3abB)F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+(-5a^2Ab+8Ab^3+3a^3B-6ab^2B)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-bF\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)\right)+a(a-b)(a+b)\left(b(a^2A-4Ab^2+3abB)+aA(a^2-b^2)\cos(c+dx)\right)\sin(c+dx)\right)}{3a^3(a^2-b^2)^2d(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*((a^2 - b^2)*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]*(a^2*(a^2*A + 2*A*b^2 - 3*a*b*B)*EllipticF[(c + d*x)/2, (2*
a)/(a + b)] + (-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*((a + b)*Ellipti
cE[(c + d*x)/2, (2*a)/(a + b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))
+ a*(a - b)*(a + b)*(b*(a^2*A - 4*A*b^2 + 3*a*b*B) + a*A*(a^2 - b^2)*Cos[c
+ d*x])*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2286 vs. $2(358) = 716$.
time = 22.74, size = 2287, normalized size = 7.02

method	result	size
default	Expression too large to display	2287

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-8*A*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2
+5*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+6*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+6*B*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*
x+c)*cos(d*x+c)*a*b^2-8*A*((a-b)/(a+b))^(1/2)*b^3-6*A*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)
*a^2*b-3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^3-A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)-6*B*((a-b)/(a+b))^(1/
```

```
2)*cos(d*x+c)*a*b^2-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b-A*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*c
os(d*x+c)*a^3-8*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d
*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*b^3+3*B*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3-6
*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^2*b*sin(d*x+c)-8*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)+5*A*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)+6*B*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin
(d*x+c)+6*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a*b^2*sin(d*x+c)-3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+c)-A*cos(d*x+c)^3*((a-b)/(a+b)
)^(1/2)*a^3-3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3+3*B*((a-b)/(a+b))^(1/2
)*cos(d*x+c)*a^3+A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3+8*A*((a-b)/(a+b))^(1/
2)*cos(d*x+c)*b^3+4*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^2-3*B*((a-b)/(a+
b))^(1/2)*cos(d*x+c)^2*a^2*b+A*((a-b)/(a+b))^(1/2)*a^2*b-4*A*((a-b)/(a+b))^
(1/2)*a*b^2+3*B*((a-b)/(a+b))^(1/2)*a^2*b+6*B*((a-b)/(a+b))^(1/2)*a*b^2+4*A
*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b-8*A*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3*sin(d*x+c)+3*B*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*sin(d*x+
c)-4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b)*cos(d*x+c)^2*(1/cos(d*x+c))^(3
/2)/sin(d*x+c)/(b+a*cos(d*x+c))/a^3/(a+b)/((a-b)/(a+b))^(1/2)
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.05, size = 792, normalized size = 2.43

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/9*(sqrt(2)*(3*I*A*a^4*b - 15*I*B*a^3*b^2 + 16*I*A*a^2*b^3 + 12*I*B*a*b^4
 - 16*I*A*b^5 + (3*I*A*a^5 - 15*I*B*a^4*b + 16*I*A*a^3*b^2 + 12*I*B*a^2*b^3
 - 16*I*A*a*b^4)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*
b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x
 + c) + 2*b)/a) + sqrt(2)*(-3*I*A*a^4*b + 15*I*B*a^3*b^2 - 16*I*A*a^2*b^3 -
 12*I*B*a*b^4 + 16*I*A*b^5 + (-3*I*A*a^5 + 15*I*B*a^4*b - 16*I*A*a^3*b^2 -
12*I*B*a^2*b^3 + 16*I*A*a*b^4)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4
/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) -
 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(3*I*B*a^4*b - 5*I*A*a^3*b^2 - 6*
I*B*a^2*b^3 + 8*I*A*a*b^4 + (3*I*B*a^5 - 5*I*A*a^4*b - 6*I*B*a^3*b^2 + 8*I*
A*a^2*b^3)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b
)/a)) - 3*sqrt(2)*(-3*I*B*a^4*b + 5*I*A*a^3*b^2 + 6*I*B*a^2*b^3 - 8*I*A*a*b
^4 + (-3*I*B*a^5 + 5*I*A*a^4*b + 6*I*B*a^3*b^2 - 8*I*A*a^2*b^3)*cos(d*x + c
))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)
/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/
a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) - 6*((A*a^5 - A*
a^3*b^2)*cos(d*x + c)^2 + (A*a^4*b + 3*B*a^3*b^2 - 4*A*a^2*b^3)*cos(d*x + c
))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))
/((a^7 - a^5*b^2)*d*cos(d*x + c) + (a^6*b - a^4*b^3)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec{(c + dx)}}{(a + b\sec{(c + dx)})^{\frac{3}{2}}\sec^{\frac{3}{2}}{(c + dx)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/((a + b*sec(c + d*x))**(3/2)*sec(c + d*x)**(3
/2)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(3/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{3/2}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)
),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(3/2)
), x)
```

3.467 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=423

$$-\frac{2(12a^2Ab+48Ab^3-5a^3B-40ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{15a^4d\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^4A+24$$

[Out] 2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2)-2/15*(12*A*a^2*b+48*A*b^3-5*B*a^3-40*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^4/d/(a+b*sec(d*x+c))^(1/2)+2/5*(A*a^2-6*A*b^2+5*B*a*b)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)/d/sec(d*x+c)^(3/2)-2/15*(9*A*a^2*b-24*A*b^3-5*B*a^3+20*B*a*b^2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)/d/sec(d*x+c)^(1/2)+2/15*(9*A*a^4+24*A*a^2*b^2-48*A*b^4-25*B*a^3*b+40*B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^4/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)

Rubi [A]
time = 0.79, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.257, Rules used = {4115, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(a^2A+5abB-6Ab^2)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{5a^2d(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)}+\frac{2b(Ab-aB)\sin(c+dx)}{ad(a^2-b^2)\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}-\frac{2(-5a^3B+9a^2Ab+20ab^2B-24Ab^3)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{15a^3d(a^2-b^2)\sqrt{\sec(c+dx)}}-\frac{2(-5a^3B+12a^2Ab-40ab^2B+48Ab^3)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^4d\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^4A-25a^3bB+24a^2Ab^2+40ab^3B-48Ab^4)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^4d(a^2-b^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]

[Out] (-2*(12*a^2*A*b + 48*A*b^3 - 5*a^3*B - 40*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(15*a^4*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^4*A + 24*a^2*A*b^2 - 48*A*b^4 - 25*a^3*b*B + 40*a*b^3*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^4*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^2*A - 6*A*b^2 + 5*a*b*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d*Sec[c + d*x]^(3/2)) - (2*(9*a^2*A*b - 24*A*b^3 - 5*a^3*B + 20*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^3*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]])

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
```

```
Q[n, 0])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{a\,(a^2-b^2)\,d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{2\int \frac{\frac{1}{2}(-a^2A+6Ab^2}{}}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\,(a^2-b^2)\,d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A-6Ab^2}{5}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\,(a^2-b^2)\,d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A-6Ab^2}{5}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\,(a^2-b^2)\,d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A-6Ab^2}{5}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\,(a^2-b^2)\,d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A-6Ab^2}{5}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\,(a^2-b^2)\,d\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A-6Ab^2}{5}$$

$$= -\frac{2(12a^2Ab+48Ab^3-5a^3B-40ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F(}{15a^4d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A]
time = 2.31, size = 316, normalized size = 0.75

$$-\frac{(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\left(2(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\left(a^2(3a^2Ab+12Ab^3-5a^3B-10ab^2B)F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-(9a^4A+24a^2Ab^2-48Ab^4-25a^3bB+40ab^3B)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-bF\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)\right)+a(a-b)(a+b)\left(30b^3(-Ab+aB)\sin(c+dx)+2(a^2-b^2)(9Ab-5aB)(b+a\cos(c+dx))\sin(c+dx)-3aA(a^2-b^2)(b+a\cos(c+dx))\sin(2(c+dx))\right)\right)}{15a^4(a^2-b^2)^2d(a+b\sec(c+dx))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] -1/15*((b + a*Cos[c + d*x])*Sec[c + d*x]^(3/2)*(2*(a^2 - b^2)*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*(a^2*(3*a^2*A*b + 12*A*b^3 - 5*a^3*B - 10*a*b^2*B)*El
lipticF[(c + d*x)/2, (2*a)/(a + b)] - (9*a^4*A + 24*a^2*A*b^2 - 48*A*b^4 -
25*a^3*b*B + 40*a*b^3*B)*((a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - b
```

```
*EllipticF[(c + d*x)/2, (2*a)/(a + b)])) + a*(a - b)*(a + b)*(30*b^3*(-(A*b
) + a*B)*Sin[c + d*x] + 2*(a^2 - b^2)*(9*A*b - 5*a*B)*(b + a*Cos[c + d*x])*
Sin[c + d*x] - 3*a*A*(a^2 - b^2)*(b + a*Cos[c + d*x])*Sin[2*(c + d*x)])))/(
a^4*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3155 vs. $2(449) = 898$.
time = 22.51, size = 3156, normalized size = 7.46

method	result	size
default	Expression too large to display	3156

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(20*B*cos(d*x+c)*((a-b)/(a+b))^
(1/2)*a^3*b-40*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+3*A*cos(d*x+c)^4*((a-
b)/(a+b))^(1/2)*a^3*b-6*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2+5*B*cos(
d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b-48*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4*sin(d*x+c)+5*B*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*sin(d*x+c)-1
2*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^3*b-9*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*sin(d*x+c)+40*B*sin(d*x+c)*cos(d*x+
c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a*b^3+48*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4-5*B*cos(d*x+c)*((a-b)/(a+b
))^(1/2)*a^4-9*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4-36*A*sin(d*x+c)*cos(d*x
+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^2*b^2-48*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+24*A*sin(d*x+c)*cos(d*x+c)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2
*b^2+30*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+40*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-25
```

```
*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a^3*b-9*A*((a-b)/(a+b))^(1/2)*a^3*b-24*A*((a-b)/(a
+b))^(1/2)*a*b^3-5*B*((a-b)/(a+b))^(1/2)*a^3*b+20*B*((a-b)/(a+b))^(1/2)*a^2
*b^2+40*B*((a-b)/(a+b))^(1/2)*a*b^3+9*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+9*A*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*s
in(d*x+c)-20*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*b-9*A*sin(d*x+c)*cos(d*
x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^4-48*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+5*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-12*A*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a^3*b*sin(d*x+c)-36*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^2*b^2*sin(d*x+c)-48*A*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*sin(d*x+c)+24*A*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2
*sin(d*x+c)+30*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^3*b*sin(d*x+c)+40*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*sin(d*x+c)-25*B*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*sin(d*
x+c)+40*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a*b^3*sin(d*x+c)+5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^4+3*A*((a
-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^4+6*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^4-
48*A*((a-b)/(a+b))^(1/2)*b^4+6*A*cos(d*x+c)^2*(...
```

<u>**Maxima**</u> **[F(-1)]** Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.36, size = 916, normalized size = 2.17

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/45*(sqrt(2)*(15*I*B*a^5*b - 27*I*A*a^4*b^2 + 80*I*B*a^3*b^3 - 84*I*A*a^2
*b^4 - 80*I*B*a*b^5 + 96*I*A*b^6 + (15*I*B*a^6 - 27*I*A*a^5*b + 80*I*B*a^4*
b^2 - 84*I*A*a^3*b^3 - 80*I*B*a^2*b^4 + 96*I*A*a*b^5)*cos(d*x + c))*sqrt(a)
*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3,
1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(-15*I*B*a^5
*b + 27*I*A*a^4*b^2 - 80*I*B*a^3*b^3 + 84*I*A*a^2*b^4 + 80*I*B*a*b^5 - 96*I
*A*b^6 + (-15*I*B*a^6 + 27*I*A*a^5*b - 80*I*B*a^4*b^2 + 84*I*A*a^3*b^3 + 80
*I*B*a^2*b^4 - 96*I*A*a*b^5)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3
*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3
*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(9*I*A*a^5*b - 25*I*B*a^4*b^2 + 24*
I*A*a^3*b^3 + 40*I*B*a^2*b^4 - 48*I*A*a*b^5 + (9*I*A*a^6 - 25*I*B*a^5*b + 2
4*I*A*a^4*b^2 + 40*I*B*a^3*b^3 - 48*I*A*a^2*b^4)*cos(d*x + c))*sqrt(a)*weie
rstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstra
ssPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*c
os(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(-9*I*A*a^5*b + 25*
I*B*a^4*b^2 - 24*I*A*a^3*b^3 - 40*I*B*a^2*b^4 + 48*I*A*a*b^5 + (-9*I*A*a^6
+ 25*I*B*a^5*b - 24*I*A*a^4*b^2 - 40*I*B*a^3*b^3 + 48*I*A*a^2*b^4)*cos(d*x
+ c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b
^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^
3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) - 6*(3*(A*a^6
 - A*a^4*b^2)*cos(d*x + c)^3 + (5*B*a^6 - 6*A*a^5*b - 5*B*a^4*b^2 + 6*A*a^3
*b^3)*cos(d*x + c)^2 + (5*B*a^5*b - 9*A*a^4*b^2 - 20*B*a^3*b^3 + 24*A*a^2*b
^4)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt
(cos(d*x + c)))/((a^8 - a^6*b^2)*d*cos(d*x + c) + (a^7*b - a^5*b^3)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*sec(d*x + c)^(5/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{3/2}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)
),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(3/2)*(1/cos(c + d*x))^(5/2)
), x)
```

3.468 $\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=399

$$\frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}+\frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\ldots}{b^2d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2/3*a*(A*b-B*a)*sec(d*x+c)^(3/2)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^
(3/2)-2/3*a*(4*A*b^3+3*B*a^3-7*B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b^2/(a^
2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)+2/3*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*
((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+
c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(si
n(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*
sec(d*x+c)^(1/2)/b^2/d/(a+b*sec(d*x+c))^(1/2)+2/3*(4*A*b^3+3*B*a^3-7*B*a*b^
2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/b^2/(a^2-b^2)^2/d/((b+
a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.84, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {4114, 4183, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2a(Ab-aB)\sin(c+dx)\sec^{\frac{3}{2}}(c+dx)}{3bd\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3bd\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2a\left(3a^3B-7ab^2B+4Ab^3\right)\sin(c+dx)\sqrt{\sec(c+dx)}}{3b^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^3B-7ab^2B+4Ab^3\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2B\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*b*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]
]) + (2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*
a)/(a + b)]*Sqrt[Sec[c + d*x]])/(b^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(4*A*
b^3 + 3*a^3*B - 7*a*b^2*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b
*Sec[c + d*x]])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*S
qrt[Sec[c + d*x]]) + (2*a*(A*b - a*B)*Sec[c + d*x]^(3/2)*Sin[c + d*x])/(3*b
*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) - (2*a*(4*A*b^3 + 3*a^3*B - 7*a*
b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[a + b*S
ec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4183

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
- b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n +
 b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}
, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \sqrt{\sec(c+dx)}\,\left(\frac{1}{2}a(A\right.}{}$$

$$= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a(4Ab^3+3a^3B-7ab^2}{3b^2\left(a^2-b^2\right)^2 a}$$

$$= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a(4Ab^3+3a^3B-7ab^2}{3b^2\left(a^2-b^2\right)^2 a}$$

$$= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a(4Ab^3+3a^3B-7ab^2}{3b^2\left(a^2-b^2\right)^2 a}$$

$$= \frac{2a(Ab-aB)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3b\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{2a(4Ab^3+3a^3B-7ab^2}{3b^2\left(a^2-b^2\right)^2 a}$$

$$= \frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{b^2d\sqrt{a+b\sec(c+dx)}} + \frac{2a}{3}$$

$$= \frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3b\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.

time = 16.92, size = 726, normalized size = 1.82

$$\frac{(b+a\cos(c+dx))^{5/2}\sec^{\frac{5}{2}}(c+dx)\left(\frac{2(2a^2Ab^2+6Ab^4+4a^3bB-12ab^3B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2(4aAb^3+9a^4B-19a^2b^2B+6b^4B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{\sqrt{b+a\cos(c+dx)}}+\frac{2i(4aAb^3+3a^4B-7a^2b^2B)\sqrt{\frac{a-a\cos(c+dx)}{a+b}}\sqrt{\frac{a+a\cos(c+dx)}{a-b}}\cos(2(c+dx))\left(-2b(a+b)E\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\left(2bF\left(i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)+a\Pi\left(1-\frac{a}{b};i\sinh^{-1}\left(\sqrt{\frac{1}{a-b}}\sqrt{b+a\cos(c+dx)}\right)|\frac{-a+b}{a+b}\right)\right)\right)\sin(c+dx)}{\sqrt{\frac{1}{a-b}}\,b\sqrt{1-\cos^2(c+dx)}\sqrt{\frac{a^2-a^2\cos^2(c+dx)}{a^2}}(-a^2+2b^2-4b(b+a\cos(c+dx))+2(b+a\cos(c+dx))^2)}\right)}{6(a-b)^2b^2(a+b)^2d(a+b\sec(c+dx))^{5/2}}+\frac{(b+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)\left(-\frac{2(aAb\sin(c+dx)-a^2B\sin(c+dx))}{3b(-a^2+b^2)(b+a\cos(c+dx))^2}-\frac{2(4aAb^3\sin(c+dx)+3a^4B\sin(c+dx)-7a^2b^2B\sin(c+dx))}{3b^2(-a^2+b^2)^2(b+a\cos(c+dx))}\right)}{d(a+b\sec(c+dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sec[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5
/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^(5/2)*Sec[c + d*x]^(5/2)*((2*(2*a^2*A*b^2 + 6*A*b^4 +
 4*a^3*b*B - 12*a*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c +
d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + (2*(4*a*A*b^3 + 9*a^4*B
- 19*a^2*b^2*B + 6*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2,
(c + d*x)/2, (2*a)/(a + b)])/Sqrt[b + a*Cos[c + d*x]] + ((2*I)*(4*a*A*b^3 +
 3*a^4*B - 7*a^2*b^2*B)*Sqrt[(a - a*Cos[c + d*x])/(a + b)]*Sqrt[(a + a*Cos[
c + d*x])/(a - b)]*Cos[2*(c + d*x)]*(-2*b*(a + b)*EllipticE[I*ArcSinh[Sqrt[
(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)] + a*(2*b*Ellipti
cF[I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d*x]]], (-a + b)/(a + b)
] + a*EllipticPi[1 - a/b, I*ArcSinh[Sqrt[(a - b)^(-1)]*Sqrt[b + a*Cos[c + d
*x]]], (-a + b)/(a + b)]))*Sin[c + d*x])/(Sqrt[(a - b)^(-1)]*b*Sqrt[1 - Cos
[c + d*x]^2]*Sqrt[(a^2 - a^2*Cos[c + d*x]^2)/a^2]*(-a^2 + 2*b^2 - 4*b*(b +
a*Cos[c + d*x]) + 2*(b + a*Cos[c + d*x])^2))))/(6*(a - b)^2*b^2*(a + b)^2*d
*(a + b*Sec[c + d*x])^(5/2)) + ((b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2)*(
(-2*(a*A*b*Sin[c + d*x] - a^2*B*Sin[c + d*x]))/(3*b*(-a^2 + b^2)*(b + a*Cos
[c + d*x])^2) - (2*(4*a*A*b^3*Sin[c + d*x] + 3*a^4*B*Sin[c + d*x] - 7*a^2*b
^2*B*Sin[c + d*x]))/(3*b^2*(-a^2 + b^2)^2*(b + a*Cos[c + d*x]))))/(d*(a + b
*Sec[c + d*x])^(5/2))
```

Maple [C] Result contains complex when optimal does not.
time = 21.63, size = 5195, normalized size = 13.02

method	result	size
default	Expression too large to display	5195

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(5/2
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 4371 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(5/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(5/2))/(a + b/cos(c + d*x))^(5/2
), x)
```

3.469 $\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=329

$$-\frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}-\frac{2(3a^2A+Ab^2-4abB)\,E\left(\frac{1}{2}(c+dx)|\frac{2}{a+}\right.}{3a\,(a^2-b^2)^2\,d\sqrt{\frac{b+a\cos(c+dx}{a+b}}}$$

```
[Out] 2/3*a*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^
(3/2)+2/3*(2*A*a^2*b+2*A*b^3+B*a^3-5*B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/b
/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)-2/3*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/
2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(
d*x+c))^(1/2)-2/3*(3*A*a^2+A*b^2-4*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*s
ec(d*x+c))^(1/2)/a/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^
(1/2)
```

Rubi [A]
time = 0.55, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4114, 4185, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2a(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\,(a^2-b^2)\,(a+b\sec(c+dx))^{3/2}}-\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\,(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{2(3a^2A-4abB+Ab^2)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\,(a^2-b^2)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(a^3B+2a^2Ab-5ab^2B+2Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{3bd\,(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x
]]) - (2*(3*a^2*A + A*b^2 - 4*a*b*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*
Sqrt[a + b*Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*Sqrt[Sec[c + d*x]]) + (2*a*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*
x])/(3*b*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*(2*a^2*A*b + 2*A*b^
3 + a^3*B - 5*a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*b*(a^2 - b^2)^2*
d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rubi steps

$$\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2\int \frac{-\frac{1}{2}a(Ab-aB)-\frac{3}{2}b(Ab-a\cdots}{\sqrt{\sec\cdots}}}{\cdots}$$

$$= \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2(2a^2Ab+2Ab^3+a^3\cdots}{3b\,(a^2-\cdots}$$

$$= \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2(2a^2Ab+2Ab^3+a^3\cdots}{3b\,(a^2-\cdots}$$

$$= \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2(2a^2Ab+2Ab^3+a^3\cdots}{3b\,(a^2-\cdots}$$

$$= \frac{2a(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3b\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2(2a^2Ab+2Ab^3+a^3\cdots}{3b\,(a^2-\cdots}$$

$$= -\frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+d\cdots}}{3a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

<u>Mathematica</u> **[A]**
time = 1.87, size = 217, normalized size = 0.66

$$\frac{\sec^{\frac{5}{2}}(c+dx)\left(-\frac{2(a+b)\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{5/2}\left((3a^2A+Ab^2-4abB)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-(a-b)(-Ab+aB)F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)}{a(a-b)^2}+\frac{2(b+a\cos(c+dx))(2a^2Ab+2Ab^3+a^3B-5ab^2B+a(3a^2A+Ab^2-4abB)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2}\right)}{3d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sec[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5
/2),x]
```

```
[Out] (Sec[c + d*x]^(5/2)*((-2*(a + b)*((b + a*Cos[c + d*x])/(a + b))^(5/2)*((3*a
^2*A + A*b^2 - 4*a*b*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - (a - b)*(-(
A*b) + a*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))/(a*(a - b)^2) + (2*(b +
 a*Cos[c + d*x])*(2*a^2*A*b + 2*A*b^3 + a^3*B - 5*a*b^2*B + a*(3*a^2*A + A*
b^2 - 4*a*b*B)*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*d*(a + b*Sec[
c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3137 vs. $2(359) = 718$.
time = 21.48, size = 3138, normalized size = 9.54

method	result	size
default	Expression too large to display	3138

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3/d*(-B*((a-b)/(a+b))^(1/2)*a^3-A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-3*A*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*
x+c)*cos(d*x+c)*a^2*b-A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-2*B*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+
c)*a^2*b+4*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+4*B*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-A*
((a-b)/(a+b))^(1/2)*b^3+2*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(
1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-A*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(
d*x+c)*a*b^2-A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+
c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b+4*B*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^
2*b-3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1
/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^2*b-3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b^2-4*B*(
(a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2+3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^
3+3*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
```

```
1/2))*sin(d*x+c)*cos(d*x+c)*a^3-3*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3-A*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*
cos(d*x+c)*b^3+B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*
x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3+3*A*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)-A*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin
(d*x+c)-3*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c))
)^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*a^2*b*sin(d*x+c)+B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b*sin(d*x+c)+4*B*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2*sin(d*x+c)+B*cos(
d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3-3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3+A*(
(a-b)/(a+b))^(1/2)*cos(d*x+c)*b^3-3*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*
b-2*A*((a-b)/(a+b))^(1/2)*a^2*b+A*((a-b)/(a+b))^(1/2)*a*b^2-B*((a-b)/(a+b))
^(1/2)*a^2*b+4*B*((a-b)/(a+b))^(1/2)*a*b^2+B*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3-3*
B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*b^2*sin(d*x+c)-3*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x+c)*a^3+3*A*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)^2*sin(d*x
+c)*a^3-A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b-A*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.84, size = 951, normalized size = 2.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/9*(sqrt(2)*(-3*I*B*a^3*b^2 + 6*I*A*a^2*b^3 - I*B*a*b^4 - 2*I*A*b^5 + (-3*
I*B*a^5 + 6*I*A*a^4*b - I*B*a^3*b^2 - 2*I*A*a^2*b^3)*cos(d*x + c)^2 - 2*(3*
I*B*a^4*b - 6*I*A*a^3*b^2 + I*B*a^2*b^3 + 2*I*A*a*b^4)*cos(d*x + c))*sqrt(a
)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3,
 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*B*a^3*
b^2 - 6*I*A*a^2*b^3 + I*B*a*b^4 + 2*I*A*b^5 + (3*I*B*a^5 - 6*I*A*a^4*b + I*
B*a^3*b^2 + 2*I*A*a^2*b^3)*cos(d*x + c)^2 - 2*(-3*I*B*a^4*b + 6*I*A*a^3*b^2
 - I*B*a^2*b^3 - 2*I*A*a*b^4)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/
3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) -
3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(3*I*A*a^3*b^2 - 4*I*B*a^2*b^3 + I
*A*a*b^4 + (3*I*A*a^5 - 4*I*B*a^4*b + I*A*a^3*b^2)*cos(d*x + c)^2 + 2*(3*I*
A*a^4*b - 4*I*B*a^3*b^2 + I*A*a^2*b^3)*cos(d*x + c))*sqrt(a)*weierstrassZet
a(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse
(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c
) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(-3*I*A*a^3*b^2 + 4*I*B*a^2*b
^3 - I*A*a*b^4 + (-3*I*A*a^5 + 4*I*B*a^4*b - I*A*a^3*b^2)*cos(d*x + c)^2 +
2*(-3*I*A*a^4*b + 4*I*B*a^3*b^2 - I*A*a^2*b^3)*cos(d*x + c))*sqrt(a)*weiers
trassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrass
PInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos
(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) + 6*((3*A*a^5 - 4*B*a^4*b + A*a^3
*b^2)*cos(d*x + c)^2 + (B*a^5 + 2*A*a^4*b - 5*B*a^3*b^2 + 2*A*a^2*b^3)*cos(
d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x
 + c)))/((a^8 - 2*a^6*b^2 + a^4*b^4)*d*cos(d*x + c)^2 + 2*(a^7*b - 2*a^5*b^
3 + a^3*b^5)*d*cos(d*x + c) + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3010 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sec(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(3/2))/(a + b/cos(c + d*x))^(5/2
), x)
```

3.470 $\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=346

$$\frac{2(3a^2A-2Ab^2-abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^2\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}+\frac{2(6a^2Ab-2Ab^3-3a^3B-a}{3a^2\left(a^2-b^2\right)^2d\sqrt{}}$$

```
[Out] -2/3*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/
2)-2/3*(5*A*a^2*b-A*b^3-2*B*a^3-2*B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a
^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)+2/3*(3*A*a^2-2*A*b^2-B*a*b)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a
/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^2/(a^2-b^2
)/d/(a+b*sec(d*x+c))^(1/2)+2/3*(6*A*a^2*b-2*A*b^3-3*B*a^3-B*a*b^2)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)^2/d/((b+a*cos(d*x+c
))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 0.52, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4112, 4185, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{3d\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2(3a^2A-abB-2Ab^2)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2(-2a^3B+5a^2Ab-2ab^2B-Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2(-3a^3B+6a^2Ab-ab^2B-2Ab^3)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(3*a^2*A - 2*A*b^2 - a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^2*(a^2 - b^2)*d*Sqrt[
a + b*Sec[c + d*x]]) + (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*Ellipti
cE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^2*(a^2 - b^2)
^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) - (2*(A*b - a*B
)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3
/2)) - (2*(5*a^2*A*b - A*b^3 - 2*a^3*B - 2*a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[
c + d*x])/(3*a*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4112

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-d)*(A
*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*
Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) +
 d*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}(-Ab+aB)-\frac{3}{2}(aA-}{\sqrt{\sec(c}}}{}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2(5a^2Ab-Ab^3-2}{3a\,(a^2}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2(5a^2Ab-Ab^3-2}{3a\,(a^2}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2(5a^2Ab-Ab^3-2}{3a\,(a^2}$$

$$= -\frac{2(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2(5a^2Ab-Ab^3-2}{3a\,(a^2}$$

$$= \frac{2(3a^2A-2Ab^2-abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3a^2\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}}$$

Mathematica [A]
time = 2.19, size = 245, normalized size = 0.71

$$\frac{2(b+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\left(-\frac{\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}\left((-6a^2Ab+2Ab^3+3a^3B+ab^2B)E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)-(a-b)(3a^2A-2Ab^2-abB)F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\right)}{(a-b)^2}+\frac{a(b(-5a^2Ab+Ab^3+2a^3B+2ab^2B)+a(-6a^2Ab+2Ab^3+3a^3B+ab^2B)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2}\right)}{3a^2d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Sec[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5
/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*(-((((b + a*Cos[c + d*x])/(a + b
))^(3/2)*((-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*EllipticE[(c + d*x)/2,
 (2*a)/(a + b)] - (a - b)*(3*a^2*A - 2*A*b^2 - a*b*B)*EllipticF[(c + d*x)/2
, (2*a)/(a + b)]))/(a - b)^2) + (a*(b*(-5*a^2*A*b + A*b^3 + 2*a^3*B + 2*a*b
^2*B) + a*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Cos[c + d*x])*Sin[c +
d*x])/(a^2 - b^2)^2))/(3*a^2*d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3864 vs. $2(376) = 752$.
time = 22.30, size = 3865, normalized size = 11.17

method	result	size
default	Expression too large to display	3865

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3/d*(-2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3+3*B*cos(d*x+c)*((a-b)/(a+
b))^(1/2)*a^3*b-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2+B*cos(d*x+c)*((a-b
)/(a+b))^(1/2)*a*b^3-2*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*b^4*sin(d*x+c)-3*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+3*B
*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a^4-B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-B*sin(d*x+c)*c
os(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^
(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a*b^3+2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4-3*B*cos(d*x+c)*((a-b
)/(a+b))^(1/2)*a^4+3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4-5*A*sin(d*x+c)*
cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^2*b^2-2*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+6*A*sin(d*x+c)*cos(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^3*b+6*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-2*A*sin(d*x+c)*cos(d*x+c)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+2
*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(
1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a^3*b-B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-3*B*sin(d*x+
c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^3*b+5*A*((a-b)/(a+b))^(1/2)*a^2*b^2-A*((a-b)/(a+b))^(1/2)*a
```

```
*b^3-2*B*((a-b)/(a+b))^(1/2)*a^3*b+B*((a-b)/(a+b))^(1/2)*a^2*b^2-B*((a-b)/(
a+b))^(1/2)*a*b^3+3*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-B*((a-b)/(a+b))^(1/2)*cos
(d*x+c)^2*a^3*b+3*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-2*A*sin(d*x+c)*cos(d*x+c)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4
+3*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1
/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^4-3*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+3*A*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*sin(
d*x+c)-3*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^2*b^2*sin(d*x+c)-2*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3*sin(d*x+c)+6*A*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2*sin(d*x+c)+3
*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^3*b*sin(d*x+c)-B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^2*b^2*sin(d*x+c)-3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b*sin(d*x+c)-B*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*(1/(1+cos(d*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(5/2
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 1.50, size = 1071, normalized size = 3.10

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/9*(sqrt(2)*(-9*I*A*a^4*b^2 + 6*I*B*a^3*b^3 + 9*I*A*a^2*b^4 - 2*I*B*a*b^5
- 4*I*A*b^6 + (-9*I*A*a^6 + 6*I*B*a^5*b + 9*I*A*a^4*b^2 - 2*I*B*a^3*b^3 - 4
*I*A*a^2*b^4)*cos(d*x + c)^2 - 2*(9*I*A*a^5*b - 6*I*B*a^4*b^2 - 9*I*A*a^3*b
^3 + 2*I*B*a^2*b^4 + 4*I*A*a*b^5)*cos(d*x + c))*sqrt(a)*weierstrassPInverse
(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c
) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(9*I*A*a^4*b^2 - 6*I*B*a^3*b^3 -
 9*I*A*a^2*b^4 + 2*I*B*a*b^5 + 4*I*A*b^6 + (9*I*A*a^6 - 6*I*B*a^5*b - 9*I*A
*a^4*b^2 + 2*I*B*a^3*b^3 + 4*I*A*a^2*b^4)*cos(d*x + c)^2 - 2*(-9*I*A*a^5*b
+ 6*I*B*a^4*b^2 + 9*I*A*a^3*b^3 - 2*I*B*a^2*b^4 - 4*I*A*a*b^5)*cos(d*x + c)
)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b
^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(
3*I*B*a^4*b^2 - 6*I*A*a^3*b^3 + I*B*a^2*b^4 + 2*I*A*a*b^5 + (3*I*B*a^6 - 6*
I*A*a^5*b + I*B*a^4*b^2 + 2*I*A*a^3*b^3)*cos(d*x + c)^2 + 2*(3*I*B*a^5*b -
6*I*A*a^4*b^2 + I*B*a^3*b^3 + 2*I*A*a^2*b^4)*cos(d*x + c))*sqrt(a)*weierstr
assZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPI
nverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d
*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(-3*I*B*a^4*b^2 + 6*I*A
*a^3*b^3 - I*B*a^2*b^4 - 2*I*A*a*b^5 + (-3*I*B*a^6 + 6*I*A*a^5*b - I*B*a^4*
b^2 - 2*I*A*a^3*b^3)*cos(d*x + c)^2 + 2*(-3*I*B*a^5*b + 6*I*A*a^4*b^2 - I*B
*a^3*b^3 - 2*I*A*a^2*b^4)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2
 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin
(d*x + c) + 2*b)/a)) + 6*((3*B*a^6 - 6*A*a^5*b + B*a^4*b^2 + 2*A*a^3*b^3)*c
os(d*x + c)^2 + (2*B*a^5*b - 5*A*a^4*b^2 + 2*B*a^3*b^3 + A*a^2*b^4)*cos(d*x
 + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x +
c)))/((a^9 - 2*a^7*b^2 + a^5*b^4)*d*cos(d*x + c)^2 + 2*(a^8*b - 2*a^6*b^3 +
 a^4*b^5)*d*cos(d*x + c) + (a^7*b^2 - 2*a^5*b^4 + a^3*b^6)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sqrt{\sec(c + dx)}}{(a + b\sec(c + dx))^{\frac{5}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(sec(c + d*x))/(a + b*sec(c + d*x))**(5/2
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(sec(d*x + c))/(b*sec(d*x + c) + a)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x))^(5/2
),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(1/cos(c + d*x))^(1/2))/(a + b/cos(c + d*x))^(5/2
), x)
```

3.471 $\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=368

$$-\frac{2(9a^2Ab-8Ab^3-3a^3B+2ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^3\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^4A-15a^2A$$

```
[Out] 2/3*b*(A*b-B*a)*sin(d*x+c)*sec(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^
(3/2)+2/3*b*(8*A*a^2*b-4*A*b^3-5*B*a^3+B*a*b^2)*sin(d*x+c)*sec(d*x+c)^(1/2)
/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)-2/3*(9*A*a^2*b-8*A*b^3-3*B*a^3+2*
B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+
c)^(1/2)/a^3/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2/3*(3*A*a^4-15*A*a^2*b^2+8
*A*b^4+6*B*a^3*b-2*B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/
2)/a^3/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {4115, 4185, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\sec(c+dx)}}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2b(-5a^3B+8a^2Ab+ab^2B-4Ab^3)\sin(c+dx)\sqrt{\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}-\frac{2(-3a^3B+9a^2Ab+2ab^2B-8Ab^3)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^4A+6a^3bB-15a^2Ab^2-2ab^3B+8Ab^4)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (-2*(9*a^2*A*b - 8*A*b^3 - 3*a^3*B + 2*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x]])/(3*a^3*(a
^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^
4 + 6*a^3*b*B - 2*a*b^3*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b
*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*S
qrt[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a
*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*b*(8*a^2*A*b - 4*A*b^3 - 5*
a^3*B + a*b^2*B)*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sq
rt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{\frac{1}{2}(-3a^2A+4Ab^2-ab}{\sqrt{s}}}{}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2b(8a^2Ab-4Ab^3-}{3a^2\,(a}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2b(8a^2Ab-4Ab^3-}{3a^2\,(a}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2b(8a^2Ab-4Ab^3-}{3a^2\,(a}$$

$$= \frac{2b(Ab-aB)\sqrt{\sec(c+dx)}\,\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2b(8a^2Ab-4Ab^3-}{3a^2\,(a}$$

$$= -\frac{2(9a^2Ab-8Ab^3-3a^3B+2ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\big(\tfrac{1}{2}}{3a^3\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx}}$$

Mathematica [A]
time = 2.49, size = 297, normalized size = 0.81

$$\frac{2(b+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\left(-\frac{\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}\left(-a^2(-6a^2Ab+2Ab^3+3a^3B+ab^2B)F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-(3a^4A-15a^2Ab^2+8Ab^4+6a^3bB-2ab^3B)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-bF\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)\right)}{(a-b)^2(a+b)}-\frac{ab(b(-8a^2Ab+4Ab^3+5a^3B-ab^2B)+a(-9a^2Ab+5Ab^3+6a^3B-2ab^2B)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2}\right)}{3a^3d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*(-((((b + a*Cos[c + d*x])/(a + b
))^(3/2)*(-(a^2*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*EllipticF[(c + d
*x)/2, (2*a)/(a + b)]) - (3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*
a*b^3*B)*((a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - b*EllipticF[(c +
d*x)/2, (2*a)/(a + b)])))/((a - b)^2*(a + b))) - (a*b*(b*(-8*a^2*A*b + 4*A*
b^3 + 5*a^3*B - a*b^2*B) + a*(-9*a^2*A*b + 5*A*b^3 + 6*a^3*B - 2*a*b^2*B)*C
os[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2))/(3*a^3*d*(a + b*Sec[c + d*x])^(5
/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 5168 vs. $2(398) = 796$.
time = 24.29, size = 5169, normalized size = 14.05

method	result	size
default	Expression too large to display	5169

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c
))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.63, size = 1186, normalized size = 3.22

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/9*(sqrt(2)*(-9*I*B*a^5*b^2 + 24*I*A*a^4*b^3 + 9*I*B*a^3*b^4 - 36*I*A*a^2*
b^5 - 4*I*B*a*b^6 + 16*I*A*b^7 + (-9*I*B*a^7 + 24*I*A*a^6*b + 9*I*B*a^5*b^2
 - 36*I*A*a^4*b^3 - 4*I*B*a^3*b^4 + 16*I*A*a^2*b^5)*cos(d*x + c)^2 - 2*(9*I
*B*a^6*b - 24*I*A*a^5*b^2 - 9*I*B*a^4*b^3 + 36*I*A*a^3*b^4 + 4*I*B*a^2*b^5
- 16*I*A*a*b^6)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b
^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x
+ c) + 2*b)/a) + sqrt(2)*(9*I*B*a^5*b^2 - 24*I*A*a^4*b^3 - 9*I*B*a^3*b^4 +
36*I*A*a^2*b^5 + 4*I*B*a*b^6 - 16*I*A*b^7 + (9*I*B*a^7 - 24*I*A*a^6*b - 9*I
*B*a^5*b^2 + 36*I*A*a^4*b^3 + 4*I*B*a^3*b^4 - 16*I*A*a^2*b^5)*cos(d*x + c)^
2 - 2*(-9*I*B*a^6*b + 24*I*A*a^5*b^2 + 9*I*B*a^4*b^3 - 36*I*A*a^3*b^4 - 4*I
*B*a^2*b^5 + 16*I*A*a*b^6)*cos(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(
```

```
3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I
*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-3*I*A*a^5*b^2 - 6*I*B*a^4*b^3 + 15*
I*A*a^3*b^4 + 2*I*B*a^2*b^5 - 8*I*A*a*b^6 + (-3*I*A*a^7 - 6*I*B*a^6*b + 15*
I*A*a^5*b^2 + 2*I*B*a^4*b^3 - 8*I*A*a^3*b^4)*cos(d*x + c)^2 + 2*(-3*I*A*a^6
*b - 6*I*B*a^5*b^2 + 15*I*A*a^4*b^3 + 2*I*B*a^3*b^4 - 8*I*A*a^2*b^5)*cos(d*
x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8
*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*
b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)
*(3*I*A*a^5*b^2 + 6*I*B*a^4*b^3 - 15*I*A*a^3*b^4 - 2*I*B*a^2*b^5 + 8*I*A*a*
b^6 + (3*I*A*a^7 + 6*I*B*a^6*b - 15*I*A*a^5*b^2 - 2*I*B*a^4*b^3 + 8*I*A*a^3
*b^4)*cos(d*x + c)^2 + 2*(3*I*A*a^6*b + 6*I*B*a^5*b^2 - 15*I*A*a^4*b^3 - 2*
I*B*a^3*b^4 + 8*I*A*a^2*b^5)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*
a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*
sin(d*x + c) + 2*b)/a)) - 6*((6*B*a^6*b - 9*A*a^5*b^2 - 2*B*a^4*b^3 + 5*A*a
^3*b^4)*cos(d*x + c)^2 + (5*B*a^5*b^2 - 8*A*a^4*b^3 - B*a^3*b^4 + 4*A*a^2*b
^5)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt
(cos(d*x + c)))/((a^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*x + c)^2 + 2*(a^9*b -
 2*a^7*b^3 + a^5*b^5)*d*cos(d*x + c) + (a^8*b^2 - 2*a^6*b^4 + a^4*b^6)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2)/sec(d*x+c)**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/sec(d*x+c)^(1/2),x, algor
ithm="giac")
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*sqrt(sec(d*x + c
))), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/2}\,\sqrt{\frac{1}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)
),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(1/2)
), x)
```

3.472 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=472

$$\frac{2\left(a^4A+16a^2Ab^2-16Ab^4-9a^3bB+8ab^3B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\sec(c+dx)}}{3a^4\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}-\frac{2(8a^4\cdots}{}$$

```
[Out] 2/3*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)/sec(d*x+c)^
(1/2)+2/3*b*(10*A*a^2*b-6*A*b^3-7*B*a^3+3*B*a*b^2)*sin(d*x+c)/a^2/(a^2-b^2)
^2/d/sec(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*(A*a^4+16*A*a^2*b^2-16*A*b
^4-9*B*a^3*b+8*B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))
^(1/2)*sec(d*x+c)^(1/2)/a^4/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1/2)+2/3*(A*a^4-1
3*A*a^2*b^2+8*A*b^4+8*B*a^3*b-4*B*a*b^3)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/
a^3/(a^2-b^2)^2/d/sec(d*x+c)^(1/2)-2/3*(8*A*a^4*b-28*A*a^2*b^3+16*A*b^5-3*B
*a^5+15*B*a^3*b^2-8*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1
/2)/a^4/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]
time = 0.86, antiderivative size = 472, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4115, 4185, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}}+\frac{2b\left(-7a^3B+10a^2Ab+3ab^2B-6Ab^3\right)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^4A+8a^3bB-13a^2Ab^2-4ab^3B+8Ab^4\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3a^3d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}}+\frac{2\left(a^4A-9a^3bB+16a^2Ab^2+8ab^3B-16Ab^4\right)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^4d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}-\frac{2\left(-3a^5B+8a^4Ab+15a^3b^2B-28a^2Ab^3-8ab^4B+16Ab^5\right)\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^4d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*(a^4*A + 16*a^2*A*b^2 - 16*A*b^4 - 9*a^3*b*B + 8*a*b^3*B)*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[Sec[c + d*x
]])/(3*a^4*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(8*a^4*A*b - 28*a^2
*A*b^3 + 16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4*B)*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^4*(a^2 - b^2)^2*d*Sqrt[(b
 + a*Cos[c + d*x])/(a + b)]*Sqrt[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c +
d*x])/(3*a*(a^2 - b^2)*d*Sqrt[Sec[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (
2*b*(10*a^2*A*b - 6*A*b^3 - 7*a^3*B + 3*a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2
- b^2)^2*d*Sqrt[Sec[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^4*A - 13*a^
2*A*b^2 + 8*A*b^4 + 8*a^3*b*B - 4*a*b^3*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
 d*x])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
```

```
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{-\frac{3}{2}\left(a^2A-2\right.}{}}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{2b(10a^2Ab}{3a^2\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{2b(10a^2Ab}{3a^2\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{2b(10a^2Ab}{3a^2\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{2b(10a^2Ab}{3a^2\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sqrt{\sec(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{2b(10a^2Ab}{3a^2\left(a^2-b^2\right)}$$

$$= \frac{2(a^4A+16a^2Ab^2-16Ab^4-9a^3bB+8ab^3B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}{3a^4\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx}}$$

<u>Mathematica</u> [A]
time = 3.11, size = 353, normalized size = 0.75

$$\frac{2(b+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\left(\frac{\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}\left(a^2\left(a^4A+7a^2Ab^2-4Ab^4-6a^3bB+2ab^3B\right)F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)+\left(-8a^4Ab+28a^2Ab^3-16Ab^5+3a^5B-15a^3b^2B+8ab^4B\right)\left((a+b)E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)-bF\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\right)\right)}{(a-b)^2(a+b)}+\frac{a\left(a^6A-25a^2Ab^4+16Ab^6+16a^3b^3B-8ab^5B+2ab\left(2a^4A-16a^2Ab^2+10Ab^4+9a^3bB-5ab^3B\right)\cos(c+dx)+A\left(a^3-ab^2\right)^2\cos(2(c+dx))\right)\sin(c+dx)}{2\left(a^2-b^2\right)^2}\right)}{3a^4d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((((b + a*Cos[c + d*x])/(a + b))
^(3/2)*(a^2*(a^4*A + 7*a^2*A*b^2 - 4*A*b^4 - 6*a^3*b*B + 2*a*b^3*B)*Ellipti
cF[(c + d*x)/2, (2*a)/(a + b)] + (-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*
a^5*B - 15*a^3*b^2*B + 8*a*b^4*B)*((a + b)*EllipticE[(c + d*x)/2, (2*a)/(a
```

```
+ b)] - b*EllipticF[(c + d*x)/2, (2*a)/(a + b)]))/((a - b)^2*(a + b)) + (a
*(a^6*A - 25*a^2*A*b^4 + 16*A*b^6 + 16*a^3*b^3*B - 8*a*b^5*B + 2*a*b*(2*a^4
*A - 16*a^2*A*b^2 + 10*A*b^4 + 9*a^3*b*B - 5*a*b^3*B)*Cos[c + d*x] + A*(a^3
 - a*b^2)^2*Cos[2*(c + d*x)])*Sin[c + d*x])/(2*(a^2 - b^2)^2)))/(3*a^4*d*(a
 + b*Sec[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 6744 vs. $2(496) = 992$.
time = 22.66, size = 6745, normalized size = 14.29

method	result	size
default	Expression too large to display	6745

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/
2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.94, size = 1343, normalized size = 2.85

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/9*(sqrt(2)*(-3*I*A*a^6*b^2 + 24*I*B*a^5*b^3 - 37*I*A*a^4*b^4 - 36*I*B*a^3
*b^5 + 68*I*A*a^2*b^6 + 16*I*B*a*b^7 - 32*I*A*b^8 + (-3*I*A*a^8 + 24*I*B*a^
7*b - 37*I*A*a^6*b^2 - 36*I*B*a^5*b^3 + 68*I*A*a^4*b^4 + 16*I*B*a^3*b^5 - 3
2*I*A*a^2*b^6)*cos(d*x + c)^2 - 2*(3*I*A*a^7*b - 24*I*B*a^6*b^2 + 37*I*A*a^
5*b^3 + 36*I*B*a^4*b^4 - 68*I*A*a^3*b^5 - 16*I*B*a^2*b^6 + 32*I*A*a*b^7)*co
s(d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
```

```
^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + s
qrt(2)*(3*I*A*a^6*b^2 - 24*I*B*a^5*b^3 + 37*I*A*a^4*b^4 + 36*I*B*a^3*b^5 -
68*I*A*a^2*b^6 - 16*I*B*a*b^7 + 32*I*A*b^8 + (3*I*A*a^8 - 24*I*B*a^7*b + 37
*I*A*a^6*b^2 + 36*I*B*a^5*b^3 - 68*I*A*a^4*b^4 - 16*I*B*a^3*b^5 + 32*I*A*a^
2*b^6)*cos(d*x + c)^2 - 2*(-3*I*A*a^7*b + 24*I*B*a^6*b^2 - 37*I*A*a^5*b^3 -
 36*I*B*a^4*b^4 + 68*I*A*a^3*b^5 + 16*I*B*a^2*b^6 - 32*I*A*a*b^7)*cos(d*x +
 c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2
)*(-3*I*B*a^6*b^2 + 8*I*A*a^5*b^3 + 15*I*B*a^4*b^4 - 28*I*A*a^3*b^5 - 8*I*B
*a^2*b^6 + 16*I*A*a*b^7 + (-3*I*B*a^8 + 8*I*A*a^7*b + 15*I*B*a^6*b^2 - 28*I
*A*a^5*b^3 - 8*I*B*a^4*b^4 + 16*I*A*a^3*b^5)*cos(d*x + c)^2 + 2*(-3*I*B*a^7
*b + 8*I*A*a^6*b^2 + 15*I*B*a^5*b^3 - 28*I*A*a^4*b^4 - 8*I*B*a^3*b^5 + 16*I
*A*a^2*b^6)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*
b)/a)) - 3*sqrt(2)*(3*I*B*a^6*b^2 - 8*I*A*a^5*b^3 - 15*I*B*a^4*b^4 + 28*I*A
*a^3*b^5 + 8*I*B*a^2*b^6 - 16*I*A*a*b^7 + (3*I*B*a^8 - 8*I*A*a^7*b - 15*I*B
*a^6*b^2 + 28*I*A*a^5*b^3 + 8*I*B*a^4*b^4 - 16*I*A*a^3*b^5)*cos(d*x + c)^2
+ 2*(3*I*B*a^7*b - 8*I*A*a^6*b^2 - 15*I*B*a^5*b^3 + 28*I*A*a^4*b^4 + 8*I*B*
a^3*b^5 - 16*I*A*a^2*b^6)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2
 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin
(d*x + c) + 2*b)/a)) + 6*((A*a^8 - 2*A*a^6*b^2 + A*a^4*b^4)*cos(d*x + c)^3
+ (2*A*a^7*b + 9*B*a^6*b^2 - 16*A*a^5*b^3 - 5*B*a^4*b^4 + 10*A*a^3*b^5)*cos
(d*x + c)^2 + (A*a^6*b^2 + 8*B*a^5*b^3 - 13*A*a^4*b^4 - 4*B*a^3*b^5 + 8*A*a
^2*b^6)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sin(d*x + c)/
sqrt(cos(d*x + c)))/((a^11 - 2*a^9*b^2 + a^7*b^4)*d*cos(d*x + c)^2 + 2*(a^1
0*b - 2*a^8*b^3 + a^6*b^5)*d*cos(d*x + c) + (a^9*b^2 - 2*a^7*b^4 + a^5*b^6)
*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(3/
2)), x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/2}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)
),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(3/2)
), x)
```

3.473 $\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=588

$$-\frac{2(17a^4Ab+116a^2Ab^3-128Ab^5-5a^5B-80a^3b^2B+80ab^4B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{\mathrm{se}}}{15a^5\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2/3*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^
(3/2)+2/3*b*(12*A*a^2*b-8*A*b^3-9*B*a^3+5*B*a*b^2)*sin(d*x+c)/a^2/(a^2-b^2)
^2/d/sec(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2)-2/15*(17*A*a^4*b+116*A*a^2*b^3
-128*A*b^5-5*B*a^5-80*B*a^3*b^2+80*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+
a*cos(d*x+c))/(a+b))^(1/2)*sec(d*x+c)^(1/2)/a^5/(a^2-b^2)/d/(a+b*sec(d*x+c)
)^(1/2)+2/15*(3*A*a^4-71*A*a^2*b^2+48*A*b^4+50*B*a^3*b-30*B*a*b^3)*sin(d*x+
c)*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)^2/d/sec(d*x+c)^(3/2)-2/15*(14*A*a^4
*b-98*A*a^2*b^3+64*A*b^5-5*B*a^5+65*B*a^3*b^2-40*B*a*b^4)*sin(d*x+c)*(a+b*s
ec(d*x+c))^(1/2)/a^4/(a^2-b^2)^2/d/sec(d*x+c)^(1/2)+2/15*(9*A*a^6+55*A*a^4*
b^2-212*A*a^2*b^4+128*A*b^6-40*B*a^5*b+140*B*a^3*b^3-80*B*a*b^5)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*
(a/(a+b))^(1/2))*(a+b*sec(d*x+c))^(1/2)/a^5/(a^2-b^2)^2/d/((b+a*cos(d*x+c))
/(a+b))^(1/2)/sec(d*x+c)^(1/2)
```

Rubi [A]

time = 1.19, antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {4115, 4185, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad\left(a^2-b^2\right)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}+\frac{2b(-9a^3B+12a^2Ab+5ab^2B-8Ab^3)\sin(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^4A+50a^3bB-71a^2Ab^2-30ab^3B+48Ab^4)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{15a^3d\left(a^2-b^2\right)^2\sec^{\frac{3}{2}}(c+dx)}-\frac{2(-5a^5B+14a^4Ab+65a^3b^2B-98a^2Ab^3-40ab^4B+64Ab^5)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{15a^4d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}}-\frac{2(-5a^5B+17a^4Ab-80a^3b^2B+116a^2Ab^3+80ab^4B-128Ab^5)\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^5d\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^6A-40a^5bB+55a^4Ab^2+140a^3b^3B-212a^2Ab^4-80ab^5B+128Ab^6)\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^5d\left(a^2-b^2\right)^2\sqrt{\sec(c+dx)}\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (-2*(17*a^4*A*b + 116*a^2*A*b^3 - 128*A*b^5 - 5*a^5*B - 80*a^3*b^2*B + 80*a
*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[Sec[c + d*x]])/(15*a^5*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) +
 (2*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*
a^3*b^3*B - 80*a*b^5*B)*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Se
c[c + d*x]])/(15*a^5*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Sqr
t[Sec[c + d*x]]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sec[c
+ d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)) + (2*b*(12*a^2*A*b - 8*A*b^3 - 9*a
^3*B + 5*a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sec[c + d*x]^(3/2)*S
qrt[a + b*Sec[c + d*x]]) + (2*(3*a^4*A - 71*a^2*A*b^2 + 48*A*b^4 + 50*a^3*b
```

```
*B - 30*a*b^3*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^3*(a^2 - b^2)
^2*d*Sec[c + d*x]^(3/2)) - (2*(14*a^4*A*b - 98*a^2*A*b^3 + 64*A*b^5 - 5*a^5
*B + 65*a^3*b^2*B - 40*a*b^4*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*
a^4*(a^2 - b^2)^2*d*Sqrt[Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2\int \frac{1}{2}(-3a^2A+}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2b(12a^2Ab-}{3a^2\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2b(12a^2Ab-}{3a^2\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2b(12a^2Ab-}{3a^2\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2b(12a^2Ab-}{3a^2\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2b(12a^2Ab-}{3a^2\left(a^2-b^2\right)}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\left(a^2-b^2\right)d\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2b(12a^2Ab-}{3a^2\left(a^2-b^2\right)}$$

$$= -\frac{2(17a^4Ab+116a^2Ab^3-128Ab^5-5a^5B-80a^3b^2B+80ab^4B}{15a^5\left(a^2-b^2\right)d\sqrt{a+}}$$

Mathematica [A]
time = 3.97, size = 392, normalized size = 0.67

$$\frac{(b+a\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\left(-\frac{2\left(\frac{b+a\cos(c+dx)}{a+b}\right)^{3/2}\left(a^2(8a^4Ab+44a^2Ab^3-32Ab^5-5a^5B-35a^3b^2B+20ab^4B)F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)-(9a^6A+55a^4Ab^2-212a^2Ab^4+128Ab^6-40a^5bB+140a^3b^3B-80ab^5B)\left((a+b)E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)-bF\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\right)\right)}{(a-b)^2(a+b)}+a\left(\frac{10b^4(Ab-aB)\sin(c+dx)}{-a^2+b^2}-\frac{10b^3(-15a^2Ab+11Ab^3+12a^3B-8ab^2B)(b+a\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2}-2(14Ab-5aB)(b+a\cos(c+dx))^2\sin(c+dx)+3aA(b+a\cos(c+dx))^2\sin(2(c+dx))\right)\right)}{15a^5d(a+b\sec(c+dx))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sec[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])*Sec[c + d*x]^(5/2)*((-2*((b + a*Cos[c + d*x])/(a + b)
)^(3/2)*(a^2*(8*a^4*A*b + 44*a^2*A*b^3 - 32*A*b^5 - 5*a^5*B - 35*a^3*b^2*B
```

```
+ 20*a*b^4*B)*EllipticF[(c + d*x)/2, (2*a)/(a + b)] - (9*a^6*A + 55*a^4*A*b
^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*(
(a + b)*EllipticE[(c + d*x)/2, (2*a)/(a + b)] - b*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])))/((a - b)^2*(a + b)) + a*((10*b^4*(A*b - a*B)*Sin[c + d*x])/
(-a^2 + b^2) - (10*b^3*(-15*a^2*A*b + 11*A*b^3 + 12*a^3*B - 8*a*b^2*B)*(b +
 a*Cos[c + d*x])*Sin[c + d*x])/(a^2 - b^2)^2 - 2*(14*A*b - 5*a*B)*(b + a*Co
s[c + d*x])^2*Sin[c + d*x] + 3*a*A*(b + a*Cos[c + d*x])^2*Sin[2*(c + d*x)])
))/(15*a^5*d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 8250 vs. $2(606) = 1212$.
time = 16.87, size = 8251, normalized size = 14.03

method	result	size
default	Expression too large to display	8251

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/
2)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.47, size = 1531, normalized size = 2.60

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/45*(sqrt(2)*(-15*I*B*a^7*b^2 + 42*I*A*a^6*b^3 - 185*I*B*a^5*b^4 + 242*I*A
*a^4*b^5 + 340*I*B*a^3*b^6 - 520*I*A*a^2*b^7 - 160*I*B*a*b^8 + 256*I*A*b^9
+ (-15*I*B*a^9 + 42*I*A*a^8*b - 185*I*B*a^7*b^2 + 242*I*A*a^6*b^3 + 340*I*B
```

```
*a^5*b^4 - 520*I*A*a^4*b^5 - 160*I*B*a^3*b^6 + 256*I*A*a^2*b^7)*cos(d*x + c
)^2 - 2*(15*I*B*a^8*b - 42*I*A*a^7*b^2 + 185*I*B*a^6*b^3 - 242*I*A*a^5*b^4
- 340*I*B*a^4*b^5 + 520*I*A*a^3*b^6 + 160*I*B*a^2*b^7 - 256*I*A*a*b^8)*cos(
d*x + c))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2
*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqr
t(2)*(15*I*B*a^7*b^2 - 42*I*A*a^6*b^3 + 185*I*B*a^5*b^4 - 242*I*A*a^4*b^5 -
 340*I*B*a^3*b^6 + 520*I*A*a^2*b^7 + 160*I*B*a*b^8 - 256*I*A*b^9 + (15*I*B*
a^9 - 42*I*A*a^8*b + 185*I*B*a^7*b^2 - 242*I*A*a^6*b^3 - 340*I*B*a^5*b^4 +
520*I*A*a^4*b^5 + 160*I*B*a^3*b^6 - 256*I*A*a^2*b^7)*cos(d*x + c)^2 - 2*(-1
5*I*B*a^8*b + 42*I*A*a^7*b^2 - 185*I*B*a^6*b^3 + 242*I*A*a^5*b^4 + 340*I*B*
a^4*b^5 - 520*I*A*a^3*b^6 - 160*I*B*a^2*b^7 + 256*I*A*a*b^8)*cos(d*x + c))*
sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3
)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-9
*I*A*a^7*b^2 + 40*I*B*a^6*b^3 - 55*I*A*a^5*b^4 - 140*I*B*a^4*b^5 + 212*I*A*
a^3*b^6 + 80*I*B*a^2*b^7 - 128*I*A*a*b^8 + (-9*I*A*a^9 + 40*I*B*a^8*b - 55*
I*A*a^7*b^2 - 140*I*B*a^6*b^3 + 212*I*A*a^5*b^4 + 80*I*B*a^4*b^5 - 128*I*A*
a^3*b^6)*cos(d*x + c)^2 + 2*(-9*I*A*a^8*b + 40*I*B*a^7*b^2 - 55*I*A*a^6*b^3
 - 140*I*B*a^5*b^4 + 212*I*A*a^4*b^5 + 80*I*B*a^3*b^6 - 128*I*A*a^2*b^7)*co
s(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b
 - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b
- 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqr
t(2)*(9*I*A*a^7*b^2 - 40*I*B*a^6*b^3 + 55*I*A*a^5*b^4 + 140*I*B*a^4*b^5 - 2
12*I*A*a^3*b^6 - 80*I*B*a^2*b^7 + 128*I*A*a*b^8 + (9*I*A*a^9 - 40*I*B*a^8*b
 + 55*I*A*a^7*b^2 + 140*I*B*a^6*b^3 - 212*I*A*a^5*b^4 - 80*I*B*a^4*b^5 + 12
8*I*A*a^3*b^6)*cos(d*x + c)^2 + 2*(9*I*A*a^8*b - 40*I*B*a^7*b^2 + 55*I*A*a^
6*b^3 + 140*I*B*a^5*b^4 - 212*I*A*a^4*b^5 - 80*I*B*a^3*b^6 + 128*I*A*a^2*b^
7)*cos(d*x + c))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*
a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) +
6*(3*(A*a^9 - 2*A*a^7*b^2 + A*a^5*b^4)*cos(d*x + c)^4 + (5*B*a^9 - 8*A*a^8*
b - 10*B*a^7*b^2 + 16*A*a^6*b^3 + 5*B*a^5*b^4 - 8*A*a^4*b^5)*cos(d*x + c)^3
 + 5*(2*B*a^8*b - 5*A*a^7*b^2 - 16*B*a^6*b^3 + 25*A*a^5*b^4 + 10*B*a^4*b^5
- 16*A*a^3*b^6)*cos(d*x + c)^2 + (5*B*a^7*b^2 - 14*A*a^6*b^3 - 65*B*a^5*b^4
 + 98*A*a^4*b^5 + 40*B*a^3*b^6 - 64*A*a^2*b^7)*cos(d*x + c))*sqrt((a*cos(d*
x + c) + b)/cos(d*x + c))*sin(d*x + c)/sqrt(cos(d*x + c)))/((a^12 - 2*a^10*
b^2 + a^8*b^4)*d*cos(d*x + c)^2 + 2*(a^11*b - 2*a^9*b^3 + a^7*b^5)*d*cos(d*
x + c) + (a^10*b^2 - 2*a^8*b^4 + a^6*b^6)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6192 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/sec(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*sec(d*x + c)^(5/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/2}\,{\left(\frac{1}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2)
),x)
```

```
[Out] int((A + B/cos(c + d*x))/((a + b/cos(c + d*x))^(5/2)*(1/cos(c + d*x))^(5/2)
), x)
```

3.474 $\int (a+b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=126

$$\frac{\sqrt{2}\,BF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a+b\sec(c+dx))^{2/3}\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}}+A\mathrm{Int}\left((a+b\sec(\right.$$

```
[Out] B*AppellF1(1/2,-2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b
*sec(d*x+c))^(2/3)*2^(1/2)*tan(d*x+c)/d/((a+b*sec(d*x+c))/(a+b))^(2/3)/(1+s
ec(d*x+c))^(1/2)+A*Unintegrable((a+b*sec(d*x+c))^(2/3),x)
```

Rubi [A]
time = 0.11, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (a+b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, -2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[
c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(2/3)*Tan[c + d*x])/(d*Sqrt[1 + Se
c[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(2/3)) + A*Defer[Int][(a + b*Sec
[c + d*x])^(2/3), x]
```

Rubi steps

$$\int (a+b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx = A\int (a+b\sec(c+dx))^{2/3}\,dx + B\int \sec(c+dx)(a+b\sec$$

$$= A\int (a+b\sec(c+dx))^{2/3}\,dx - \frac{(B\tan(c+dx))\mathrm{Subst}\left(\int\right.}{d\sqrt{1-\sec(c}}$$

$$= A\int (a+b\sec(c+dx))^{2/3}\,dx - \frac{\left(B(a+b\sec(c+dx))^{2/3}\right.}{d\sqrt{1-\sec(}}$$

$$= \frac{\sqrt{2}\,BF_1\left(\frac{1}{2};\frac{1}{2},-\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)(a}{d\sqrt{1+\sec(c+dx)}\left(\frac{a+b\sec(c+d}{a+b}\right.}$$

Mathematica [A]
time = 41.89, size = 0, normalized size = 0.00

$$\int (a+b\sec(c+dx))^{2/3}(A+B\sec(c+dx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(2/3)*(A + B*Sec[c + d*x]), x]
```

Maple [A]
time = 0.30, size = 0, normalized size = 0.00

$$\int \left(a+b\sec\left(dx+c\right)\right)^{\frac{2}{3}}\left(A+B\sec\left(dx+c\right)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
[Out] int((a+b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(2/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A+B\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)^{\frac{2}{3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(2/3)*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(2/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(2/3)*(A+B*sec(d*x+c)),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(2/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{2/3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(2/3),x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(2/3), x)
```

3.475 $\int \sqrt[3]{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=126

$$\frac{\sqrt{2}\,BF_1\left(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{a+b\sec(c+dx)}\,\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\,\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}}+A\text{Int}\left(\sqrt[3]{a+b\sec(c\right.$$

```
[Out] B*AppellF1(1/2,-1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*(a+b
*sec(d*x+c))^(1/3)*2^(1/2)*tan(d*x+c)/d/((a+b*sec(d*x+c))/(a+b))^(1/3)/(1+s
ec(d*x+c))^(1/2)+A*Unintegrable((a+b*sec(d*x+c))^(1/3),x)
```

Rubi [A]
time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \sqrt[3]{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[(a + b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, -1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[
c + d*x]))/(a + b)]*(a + b*Sec[c + d*x])^(1/3)*Tan[c + d*x])/(d*Sqrt[1 + Se
c[c + d*x]]*((a + b*Sec[c + d*x])/(a + b))^(1/3)) + A*Defer[Int][(a + b*Sec
[c + d*x])^(1/3), x]
```

Rubi steps

$$\int \sqrt[3]{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = A\int \sqrt[3]{a+b\sec(c+dx)}\,dx + B\int \sec(c+dx)\sqrt[3]{a+b\sec}$$

$$= A\int \sqrt[3]{a+b\sec(c+dx)}\,dx - \frac{(B\tan(c+dx))\mathrm{Subst}\Big(\int -}{d\sqrt{1-\sec(c+}}$$

$$= A\int \sqrt[3]{a+b\sec(c+dx)}\,dx - \frac{\Big(B\sqrt[3]{a+b\sec(c+dx)}\,\tan}{d\sqrt{1-\sec(}}$$

$$= \frac{\sqrt{2}\,BF_1\Big(\frac{1}{2};\frac{1}{2},-\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\Big)\sqrt[3]{}}{d\sqrt{1+\sec(c+dx)}\,\sqrt[3]{\frac{a+b\sec(c}{a+b}}}$$

Mathematica [A]
time = 42.31, size = 0, normalized size = 0.00

$$\int \sqrt[3]{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(a + b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x]),x]
[Out] Integrate[(a + b*Sec[c + d*x])^(1/3)*(A + B*Sec[c + d*x]), x]
```

Maple [A]
time = 0.34, size = 0, normalized size = 0.00

$$\int (a+b\sec(dx+c))^{\frac{1}{3}}\,(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x)
[Out] int((a+b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="maxima")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="fricas")
[Out] Timed out
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))\sqrt[3]{a + b\sec(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(1/3)*(A+B*sec(d*x+c)),x)
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**(1/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(1/3)*(A+B*sec(d*x+c)),x, algorithm="giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(1/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A + \frac{B}{\cos(c + d\,x)}\right)\,\left(a + \frac{b}{\cos(c + d\,x)}\right)^{1/3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/3),x)
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/3), x)
```

3.476 $\int \frac{A+B\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=126

$$\frac{\sqrt{2}\,BF_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\,\sqrt[3]{a+b\sec(c+dx)}}+A\text{Int}\left(\frac{1}{\sqrt[3]{a+b\sec(}\right.$$

```
[Out] B*AppellF1(1/2,1/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b
*sec(d*x+c))/(a+b))^(1/3)*2^(1/2)*tan(d*x+c)/d/(a+b*sec(d*x+c))^(1/3)/(1+se
c(d*x+c))^(1/2)+A*Unintegrable(1/(a+b*sec(d*x+c))^(1/3),x)
```

Rubi [A]

time = 0.09, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{A+B\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(1/3),x]
```

```
[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, 1/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
 + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(1/3)*Tan[c + d*x])/(d*Sq
rt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(1/3)) + A*Defer[Int][(a + b*Sec[
c + d*x])^(-1/3), x]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx &= A\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx+B\int \frac{\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx\\
&= A\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx-\frac{(B\tan(c+dx))\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}\,\sqrt[3]{a}}\right.}{d\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec}}\\
&= A\int \frac{1}{\sqrt[3]{a+b\sec(c+dx)}}\,dx-\frac{\left(B\sqrt[3]{-\frac{a+b\sec(c+dx)}{-a-b}}\,\tan(c+dx)\right)\text{Subs}}{d\sqrt{1-\sec(c+dx)}}\\
&= \frac{\sqrt{2}\,BF_1\left(\frac{1}{2};\frac{1}{2},\frac{1}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\sqrt[3]{\frac{a+b\sec(c+dx)}{a+b}}\,\tan}{d\sqrt{1+\sec(c+dx)}\,\sqrt[3]{a+b\sec(c+dx)}}
\end{aligned}$$

Mathematica [A]
time = 12.53, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec(c+dx)}{\sqrt[3]{a+b\sec(c+dx)}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(1/3),x]
[Out] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(1/3), x]
```

Maple [A]
time = 0.29, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec\left(dx+c\right)}{\left(a+b\sec\left(dx+c\right)\right)^{\frac{1}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/3),x)
[Out] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/3),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/3),x, algorithm="maxima")
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(1/3), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/3),x, algorithm="fricas")
[Out] Timed out
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec\left(c+dx\right)}{\sqrt[3]{a+b\sec\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**(1/3), x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/3),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(1/3), x)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{1/3}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^(1/3),x)
```

```
[Out] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^(1/3), x)
```

3.477 $\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$

Optimal. Leaf size=126

$$\frac{\sqrt{2}\,BF_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\tan(c+dx)}{d\sqrt{1+\sec(c+dx)}\,(a+b\sec(c+dx))^{2/3}}+A\text{Int}\left(\frac{1}{(a+b\sec(c+dx)}\right.$$

```
[Out] B*AppellF1(1/2,2/3,1/2,3/2,b*(1-sec(d*x+c))/(a+b),1/2-1/2*sec(d*x+c))*((a+b
*sec(d*x+c))/(a+b))^(2/3)*2^(1/2)*tan(d*x+c)/d/(a+b*sec(d*x+c))^(2/3)/(1+se
c(d*x+c))^(1/2)+A*Unintegrable(1/(a+b*sec(d*x+c))^(2/3),x)
```

Rubi [A]

time = 0.10, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Int[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] (Sqrt[2]*B*AppellF1[1/2, 1/2, 2/3, 3/2, (1 - Sec[c + d*x])/2, (b*(1 - Sec[c
 + d*x]))/(a + b)]*((a + b*Sec[c + d*x])/(a + b))^(2/3)*Tan[c + d*x])/(d*Sq
rt[1 + Sec[c + d*x]]*(a + b*Sec[c + d*x])^(2/3)) + A*Defer[Int][(a + b*Sec[
c + d*x])^(-2/3), x]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx = A\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx + B\int \frac{\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$$

$$= A\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx - \frac{(B\tan(c+dx))\text{Subst}\left(\int \frac{1}{\sqrt{1-x}\,\sqrt{1+x}\,(a+\right.}{d\sqrt{1-\sec(c+dx)}\,\sqrt{1+\sec}}$$

$$= A\int \frac{1}{(a+b\sec(c+dx))^{2/3}}\,dx - \frac{\left(B\left(-\frac{a+b\sec(c+dx)}{-a-b}\right)^{2/3}\tan(c+dx)\right)\text{Subst}\left(\int\right.}{d\sqrt{1-\sec(c+dx)}\,\sqrt{1+}}$$

$$= \frac{\sqrt{2}\,BF_1\left(\frac{1}{2};\frac{1}{2},\frac{2}{3};\frac{3}{2};\frac{1}{2}(1-\sec(c+dx)),\frac{b(1-\sec(c+dx))}{a+b}\right)\left(\frac{a+b\sec(c+dx)}{a+b}\right)^{2/3}\tan(c+}{d\sqrt{1+\sec(c+dx)}\,(a+b\sec(c+dx))^{2/3}}$$

Mathematica [**A**]
time = 15.92, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^{2/3}}\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(2/3),x]
```

```
[Out] Integrate[(A + B*Sec[c + d*x])/(a + b*Sec[c + d*x])^(2/3), x]
```

Maple [**A**]
time = 0.30, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec\left(dx+c\right)}{\left(a+b\sec\left(dx+c\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(2/3),x)
```

```
[Out] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(2/3),x)
```

Maxima [**A**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(2/3),x, algorithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(2/3), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(2/3),x, algorithm="fricas")
```

```
[Out] Timed out
```

Sympy [**A**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec\left(c+dx\right)}{\left(a+b\sec\left(c+dx\right)\right)^{\frac{2}{3}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(2/3),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(a + b*sec(c + d*x))**(2/3), x)
```

Giac [**A**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(2/3),x, algorithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(b*sec(d*x + c) + a)^(2/3), x)
```

Mupad [**A**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{2/3}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^(2/3),x)
```

```
[Out] int((A + B/cos(c + d*x))/(a + b/cos(c + d*x))^(2/3), x)
```

3.478 $\int (c\sec(e+fx))^n(a+b\sec(e+fx))^m(A+B\sec(e+fx))\,dx$

Optimal. Leaf size=36

$$\text{Int}((c\sec(e+fx))^n(a+b\sec(e+fx))^m(A+B\sec(e+fx)),x)$$

```
[Out] Unintegrable((c*sec(f*x+e))^n*(a+b*sec(f*x+e))^m*(A+B*sec(f*x+e)),x)
```

Rubi [A]
time = 0.05, antiderivative size = 0, normalized size of antiderivative = 0.00, number of steps used = 0, number of rules used = 0, integrand size = 0, $\frac{\text{number of rules}}{\text{integrand size}} = 0.000$, Rules used = {}

$$\int (c\sec(e+fx))^n(a+b\sec(e+fx))^m(A+B\sec(e+fx))\,dx$$

Verification is not applicable to the result.

```
[In] Int[(c*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m*(A + B*Sec[e + f*x]),x]
```

```
[Out] Defer[Int][(c*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m*(A + B*Sec[e + f*x]),
x]
```

Rubi steps

$$\int (c\sec(e+fx))^n(a+b\sec(e+fx))^m(A+B\sec(e+fx))\,dx = \int (c\sec(e+fx))^n(a+b\sec(e+fx))^m(A$$

Mathematica [A]
time = 4.46, size = 0, normalized size = 0.00

$$\int (c\sec(e+fx))^n(a+b\sec(e+fx))^m(A+B\sec(e+fx))\,dx$$

Verification is not applicable to the result.

```
[In] Integrate[(c*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m*(A + B*Sec[e + f*x]),x]
```

```
[Out] Integrate[(c*Sec[e + f*x])^n*(a + b*Sec[e + f*x])^m*(A + B*Sec[e + f*x]), x
]
```

Maple [A]
time = 0.48, size = 0, normalized size = 0.00

$$\int (c\sec(fx+e))^n\,(a+b\sec(fx+e))^m\,(A+B\sec(fx+e))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((c*sec(f*x+e))^n*(a+b*sec(f*x+e))^m*(A+B*sec(f*x+e)),x)
```

```
[Out] int((c*sec(f*x+e))^n*(a+b*sec(f*x+e))^m*(A+B*sec(f*x+e)),x)
```

Maxima [A]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(f*x+e))^n*(a+b*sec(f*x+e))^m*(A+B*sec(f*x+e)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(f*x + e) + A)*(b*sec(f*x + e) + a)^m*(c*sec(f*x + e))^n, x
)
```

Fricas [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(f*x+e))^n*(a+b*sec(f*x+e))^m*(A+B*sec(f*x+e)),x, algorithm
="fricas")
```

```
[Out] integral((B*sec(f*x + e) + A)*(b*sec(f*x + e) + a)^m*(c*sec(f*x + e))^n, x)
```

Sympy [A]
time = 0.00, size = 0, normalized size = 0.00

$$\int (c\sec{(e+fx)})^n (A+B\sec{(e+fx)}) (a+b\sec{(e+fx)})^m dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(f*x+e))**n*(a+b*sec(f*x+e))**m*(A+B*sec(f*x+e)),x)
```

```
[Out] Integral((c*sec(e + f*x))**n*(A + B*sec(e + f*x))*(a + b*sec(e + f*x))**m,
x)
```

Giac [A]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((c*sec(f*x+e))^n*(a+b*sec(f*x+e))^m*(A+B*sec(f*x+e)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(f*x + e) + A)*(b*sec(f*x + e) + a)^m*(c*sec(f*x + e))^n, x
)
```

Mupad [A]
time = 0.00, size = -1, normalized size = -0.03

$$\int \left(A+\frac{B}{\cos\left(e+f\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(e+f\,x\right)}\right)}^m\,{\left(\frac{c}{\cos\left(e+f\,x\right)}\right)}^n \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(e + f*x))*(a + b/cos(e + f*x))^m*(c/cos(e + f*x))^n,x)
```

```
[Out] int((A + B/cos(e + f*x))*(a + b/cos(e + f*x))^m*(c/cos(e + f*x))^n, x)
```

3.479 $\int \sec^m(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=544

$$\frac{b(Ab^3(8+6m+m^2)+4ab^2B(8+6m+m^2)+2a^3B(19+8m+m^2)+a^2Ab(68+37m+5m^2))\sec^{1+m}(c+}{d(1+m)(3+m)(4+m)}$$

```
[Out] b*(A*b^3*(m^2+6*m+8)+4*a*b^2*B*(m^2+6*m+8)+2*a^3*B*(m^2+8*m+19)+a^2*A*b*(5*
m^2+37*m+68))*sec(d*x+c)^(1+m)*sin(d*x+c)/d/(4+m)/(m^2+4*m+3)+b^2*(b^2*B*(3
+m)^2+2*a*A*b*(4+m)^2+a^2*B*(m^2+9*m+26))*sec(d*x+c)^(2+m)*sin(d*x+c)/d/(4+
m)/(m^2+5*m+6)+b*(A*b*(4+m)+a*B*(7+m))*sec(d*x+c)^(1+m)*(a+b*sec(d*x+c))^2*
sin(d*x+c)/d/(3+m)/(4+m)+b*B*sec(d*x+c)^(1+m)*(a+b*sec(d*x+c))^3*sin(d*x+c)
/d/(4+m)-(A*b^4*m*(2+m)+4*a*b^3*B*m*(2+m)+6*a^2*A*b^2*m*(3+m)+4*a^3*b*B*m*(
3+m)+a^4*A*(m^2+4*m+3))*hypergeom([1/2, 1/2-1/2*m],[3/2-1/2*m],cos(d*x+c)^2
)*sec(d*x+c)^(-1+m)*sin(d*x+c)/d/(3+m)/(-m^2+1)/(sin(d*x+c)^2)^(1/2)+(b^4*B
*(m^2+4*m+3)+4*a*A*b^3*(m^2+5*m+4)+6*a^2*b^2*B*(m^2+5*m+4)+4*a^3*A*b*(m^2+6
*m+8)+a^4*B*(m^2+6*m+8))*hypergeom([1/2, -1/2*m],[1-1/2*m],cos(d*x+c)^2)*se
c(d*x+c)^m*sin(d*x+c)/d/m/(2+m)/(4+m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 1.07, antiderivative size = 544, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {4111, 4181, 4161, 4132, 3857, 2722, 4131}

$$\frac{b^2\sin(c+dx)\left(a^2B(m^2+9m+26)+2aAb(m+4)^2+b^2B(m+3)^2\right)\sec^{m+2}(c+dx)}{d(m+2)(m+3)(m+4)}+\frac{b\sin(c+dx)\left(2a^3B(m^2+8m+19)+a^2Ab(5m^2+37m+68)+4ab^2B(m^2+6m+8)+Ab^3(m^2+6m+8)\right)\sec^{m+1}(c+dx)}{d(m+1)(m+3)(m+4)}-\frac{\sin(c+dx)\left(a^4A(m^2+4m+3)+4a^3bBm(m+3)+6a^2Ab^2m(m+3)+4ab^3Bm(m+2)+Ab^4m(m+2)\right)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(c+dx)\right)}{d(1-m)(m+1)(m+3)\sqrt{\sin^2(c+dx)}}+\frac{\sin(c+dx)\left(a^4B(m^2+6m+8)+4a^3Ab(m^2+6m+8)+6a^2b^2B(m^2+5m+4)+4aAb^3(m^2+5m+4)+b^4B(m^2+4m+3)\right)\sec^m(c+dx)\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};\frac{2-m}{2};\cos^2(c+dx)\right)}{dm(m+2)(m+4)\sqrt{\sin^2(c+dx)}}+\frac{b\sin(c+dx)(aB(m+7)+Ab(m+4))\sec^{m+1}(c+dx)(a+b\sec(c+dx))^2}{d(m+3)(m+4)}+\frac{bB\sin(c+dx)\sec^{m+1}(c+dx)(a+b\sec(c+dx))^3}{d(m+4)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (b*(A*b^3*(8 + 6*m + m^2) + 4*a*b^2*B*(8 + 6*m + m^2) + 2*a^3*B*(19 + 8*m +
 m^2) + a^2*A*b*(68 + 37*m + 5*m^2))*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*
(1 + m)*(3 + m)*(4 + m)) + (b^2*(b^2*B*(3 + m)^2 + 2*a*A*b*(4 + m)^2 + a^2*
B*(26 + 9*m + m^2))*Sec[c + d*x]^(2 + m)*Sin[c + d*x])/(d*(2 + m)*(3 + m)*(
4 + m)) + (b*(A*b*(4 + m) + a*B*(7 + m))*Sec[c + d*x]^(1 + m)*(a + b*Sec[c
+ d*x])^2*Sin[c + d*x])/(d*(3 + m)*(4 + m)) + (b*B*Sec[c + d*x]^(1 + m)*(a
+ b*Sec[c + d*x])^3*Sin[c + d*x])/(d*(4 + m)) - ((A*b^4*m*(2 + m) + 4*a*b^3
*B*m*(2 + m) + 6*a^2*A*b^2*m*(3 + m) + 4*a^3*b*B*m*(3 + m) + a^4*A*(3 + 4*m
 + m^2))*Hypergeometric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[c + d*x]^2]*Sec[c
 + d*x]^(-1 + m)*Sin[c + d*x])/(d*(1 - m)*(1 + m)*(3 + m)*Sqrt[Sin[c + d*x]
^2]) + ((b^4*B*(3 + 4*m + m^2) + 4*a*A*b^3*(4 + 5*m + m^2) + 6*a^2*b^2*B*(4
 + 5*m + m^2) + 4*a^3*A*b*(8 + 6*m + m^2) + a^4*B*(8 + 6*m + m^2))*Hypergeo
metric2F1[1/2, -1/2*m, (2 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^m*Sin[c + d*
x])/(d*m*(2 + m)*(4 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4161

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2
) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n
 + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &
```

```
&  !LtQ[n, -1]
```

Rule 4181

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^n/(f*(m + n + 1))), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e +
 f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*
B)*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2
- b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rubi steps

$$\begin{aligned}\int \sec^m(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\,dx &= \frac{bB\sec^{1+m}(c+dx)(a+b\sec(c+dx))^3\sin(c+}{d(4+m)} \\ &= \frac{b(Ab(4+m)+aB(7+m))\sec^{1+m}(c+dx)(a+}{d(3+m)(4+m)} \\ &= \frac{b^2(b^2B(3+m)^2+2aAb(4+m)^2+a^2B(26+9}{d(2+m)\left(12+7m\right.} \\ &= \frac{b^2(b^2B(3+m)^2+2aAb(4+m)^2+a^2B(26+9}{d(2+m)\left(12+7m\right.} \\ &= \frac{b(Ab^3(8+6m+m^2)+4ab^2B(8+6m+m^2)+}{} \\ &= \frac{b(Ab^3(8+6m+m^2)+4ab^2B(8+6m+m^2)+}{} \\ &= \frac{b(Ab^3(8+6m+m^2)+4ab^2B(8+6m+m^2)+}{}\end{aligned}$$

Mathematica [A]
time = 4.83, size = 365, normalized size = 0.67

$$\frac{\csc(c+dx)\left(\frac{a^4A\cos^5(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};\sec^2(c+dx)\right)}{m}+\frac{a^3(4Ab+aB)\cos^4(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sec^2(c+dx)\right)}{1+m}+b\left(\frac{2a^2(3Ab+2aB)\cos^3(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\sec^2(c+dx)\right)}{2+m}+b\left(\frac{2a(2Ab+3aB)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3+m}{2};\frac{5+m}{2};\sec^2(c+dx)\right)}{3+m}+b\left(\frac{(Ab+4aB)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{4+m}{2};\frac{6+m}{2};\sec^2(c+dx)\right)}{4+m}+\frac{bB\,{}_2F_1\left(\frac{1}{2},\frac{5+m}{2};\frac{7+m}{2};\sec^2(c+dx)\right)}{5+m}\right)\right)\right)\right)\sec^{-1+m}(c+dx)(a+b\sec(c+dx))^4(A+B\sec(c+dx))\sqrt{-\tan^2(c+dx)}}{d(b+a\cos(c+dx))^4(B+A\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]*((a^4*A*Cos[c + d*x]^5*Hypergeometric2F1[1/2, m/2, (2 + m)/2,
 Sec[c + d*x]^2])/m + (a^3*(4*A*b + a*B)*Cos[c + d*x]^4*Hypergeometric2F1[1
```

```
/2, (1 + m)/2, (3 + m)/2, Sec[c + d*x]^2])/(1 + m) + b*((2*a^2*(3*A*b + 2*a
*B)*Cos[c + d*x]^3*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Sec[c + d*x
]^2])/(2 + m) + b*((2*a*(2*A*b + 3*a*B)*Cos[c + d*x]^2*Hypergeometric2F1[1/
2, (3 + m)/2, (5 + m)/2, Sec[c + d*x]^2])/(3 + m) + b*(((A*b + 4*a*B)*Cos[c
 + d*x]*Hypergeometric2F1[1/2, (4 + m)/2, (6 + m)/2, Sec[c + d*x]^2])/(4 +
m) + (b*B*Hypergeometric2F1[1/2, (5 + m)/2, (7 + m)/2, Sec[c + d*x]^2])/(5
+ m)))))*Sec[c + d*x]^(-1 + m)*(a + b*Sec[c + d*x])^4*(A + B*Sec[c + d*x])*
Sqrt[-Tan[c + d*x]^2])/(d*(b + a*Cos[c + d*x])^4*(B + A*Cos[c + d*x]))
```

Maple [F]
time = 0.42, size = 0, normalized size = 0.00

$$\int \left(\sec^{m}\left(dx+c\right)\right)\left(a+b\sec\left(dx+c\right)\right)^{4}\left(A+B\sec\left(dx+c\right)\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
[Out] int(sec(d*x+c)^m*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^4*sec(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
[Out] integral((B*b^4*sec(d*x + c)^5 + A*a^4 + (4*B*a*b^3 + A*b^4)*sec(d*x + c)^4
 + 2*(3*B*a^2*b^2 + 2*A*a*b^3)*sec(d*x + c)^3 + 2*(2*B*a^3*b + 3*A*a^2*b^2)
*sec(d*x + c)^2 + (B*a^4 + 4*A*a^3*b)*sec(d*x + c))*sec(d*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \left(A+B\sec\left(c+dx\right)\right)\left(a+b\sec\left(c+dx\right)\right)^{4}\sec^{m}\left(c+dx\right)dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(a+b*sec(d*x+c))**4*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**4*sec(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^4*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^4*sec(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^4\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^4*(1/cos(c + d*x))^m,x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^4*(1/cos(c + d*x))^m, x)
```

3.480 $\int \sec^m(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=366

$$\frac{b(b^2B(2+m)+3aAb(3+m)+2a^2B(4+m))\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+m)(3+m)}+\frac{b^2(Ab(3+m)+aB(5+m))\,\mathrm{se}}{d(2+m)(3}$$

```
[Out] b*(b^2*B*(2+m)+3*a*A*b*(3+m)+2*a^2*B*(4+m))*sec(d*x+c)^(1+m)*sin(d*x+c)/d/(
1+m)/(3+m)+b^2*(A*b*(3+m)+a*B*(5+m))*sec(d*x+c)^(2+m)*sin(d*x+c)/d/(2+m)/(3
+m)+b*B*sec(d*x+c)^(1+m)*(a+b*sec(d*x+c))^2*sin(d*x+c)/d/(3+m)-(b^3*B*m*(2+
m)+3*a*A*b^2*m*(3+m)+3*a^2*b*B*m*(3+m)+a^3*A*(m^2+4*m+3))*hypergeom([1/2, 1
/2-1/2*m],[3/2-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-1+m)*sin(d*x+c)/d/(3+m)/(-
m^2+1)/(sin(d*x+c)^2)^(1/2)+(A*b^3*(1+m)+3*a*b^2*B*(1+m)+3*a^2*A*b*(2+m)+a^
3*B*(2+m))*hypergeom([1/2, -1/2*m],[1-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^m*sin
(d*x+c)/d/m/(2+m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.52, antiderivative size = 366, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {4111, 4161, 4132, 3857, 2722, 4131}

$$\frac{b\sin(c+dx)\left(2a^2B(m+4)+3aAb(m+3)+b^2B(m+2)\right)\sec^{m+1}(c+dx)}{d(m+1)(m+3)}-\frac{\sin(c+dx)\left(a^3A(m^2+4m+3)+3a^2bBm(m+3)+3aAb^2m(m+3)+b^3Bm(m+2)\right)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(c+dx)\right)}{d(m+3)\left(1-m^2\right)\sqrt{\sin^2(c+dx)}}+\frac{\sin(c+dx)\left(a^3B(m+2)+3a^2Ab(m+2)+3ab^2B(m+1)+Ab^3(m+1)\right)\sec^m(c+dx)\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};\frac{2-m}{2};\cos^2(c+dx)\right)}{dm(m+2)\sqrt{\sin^2(c+dx)}}+\frac{b^2\sin(c+dx)(aB(m+5)+Ab(m+3))\sec^{m+2}(c+dx)}{d(m+2)(m+3)}+\frac{bB\sin(c+dx)\sec^{m+1}(c+dx)(a+b\sec(c+dx))^2}{d(m+3)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (b*(b^2*B*(2 + m) + 3*a*A*b*(3 + m) + 2*a^2*B*(4 + m))*Sec[c + d*x]^(1 + m)
*Sin[c + d*x])/(d*(1 + m)*(3 + m)) + (b^2*(A*b*(3 + m) + a*B*(5 + m))*Sec[c
 + d*x]^(2 + m)*Sin[c + d*x])/(d*(2 + m)*(3 + m)) + (b*B*Sec[c + d*x]^(1 +
m)*(a + b*Sec[c + d*x])^2*Sin[c + d*x])/(d*(3 + m)) - ((b^3*B*m*(2 + m) + 3
*a*A*b^2*m*(3 + m) + 3*a^2*b*B*m*(3 + m) + a^3*A*(3 + 4*m + m^2))*Hypergeom
etric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*S
in[c + d*x])/(d*(3 + m)*(1 - m^2)*Sqrt[Sin[c + d*x]^2]) + ((A*b^3*(1 + m) +
 3*a*b^2*B*(1 + m) + 3*a^2*A*b*(2 + m) + a^3*B*(2 + m))*Hypergeometric2F1[1
/2, -1/2*m, (2 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^m*Sin[c + d*x])/(d*m*(2
 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rule 4161

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_)), x_Symbol] :> Simp[(-b)*C*Csc[e + f*x]*Cot[e + f*x]*((d*Csc[e + f*x])^
n/(f*(n + 2))), x] + Dist[1/(n + 2), Int[(d*Csc[e + f*x])^n*Simp[A*a*(n + 2
) + (B*a*(n + 2) + b*(C*(n + 1) + A*(n + 2)))*Csc[e + f*x] + (a*C + B*b)*(n
 + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] &
&  !LtQ[n, -1]
```

Rubi steps

$$\int \sec^m(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\,dx = \frac{bB\sec^{1+m}(c+dx)(a+b\sec(c+dx))^2\sin(c+}{d(3+m)}$$

$$= \frac{b^2(Ab(3+m)+aB(5+m))\sec^{2+m}(c+dx)\,\mathrm{s}}{d(2+m)(3+m)}$$

$$= \frac{b^2(Ab(3+m)+aB(5+m))\sec^{2+m}(c+dx)\,\mathrm{s}}{d(2+m)(3+m)}$$

$$= \frac{b(b^2B(2+m)+3aAb(3+m)+2a^2B(4+m}{d(1+m)(3+m)}$$

$$= \frac{b(b^2B(2+m)+3aAb(3+m)+2a^2B(4+m}{d(1+m)(3+m)}$$

$$= \frac{b(b^2B(2+m)+3aAb(3+m)+2a^2B(4+m}{d(1+m)(3+m)}$$

Mathematica [A]
time = 2.36, size = 307, normalized size = 0.84

$$\frac{\csc(c+dx)\left(\frac{a^3A\cos^4(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};\sec^2(c+dx)\right)}{m}+\frac{a^2(3Ab+aB)\cos^3(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sec^2(c+dx)\right)}{1+m}+b\left(\frac{3a(Ab+aB)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\sec^2(c+dx)\right)}{2+m}+b\left(\frac{(Ab+3aB)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{3+m}{2};\frac{5+m}{2};\sec^2(c+dx)\right)}{3+m}+\frac{bB\,{}_2F_1\left(\frac{1}{2},\frac{4+m}{2};\frac{6+m}{2};\sec^2(c+dx)\right)}{4+m}\right)\right)\right)\sec^{-1+m}(c+dx)(a+b\sec(c+dx))^3(A+B\sec(c+dx))\sqrt{-\tan^2(c+dx)}}{d(b+a\cos(c+dx))^3(B+A\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(a + b*Sec[c + d*x])^3*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]*((a^3*A*Cos[c + d*x]^4*Hypergeometric2F1[1/2, m/2, (2 + m)/2,
 Sec[c + d*x]^2])/m + (a^2*(3*A*b + a*B)*Cos[c + d*x]^3*Hypergeometric2F1[1
/2, (1 + m)/2, (3 + m)/2, Sec[c + d*x]^2])/(1 + m) + b*((3*a*(A*b + a*B)*Co
s[c + d*x]^2*Hypergeometric2F1[1/2, (2 + m)/2, (4 + m)/2, Sec[c + d*x]^2])/
(2 + m) + b*(((A*b + 3*a*B)*Cos[c + d*x]*Hypergeometric2F1[1/2, (3 + m)/2,
(5 + m)/2, Sec[c + d*x]^2])/(3 + m) + (b*B*Hypergeometric2F1[1/2, (4 + m)/2
, (6 + m)/2, Sec[c + d*x]^2])/(4 + m))))*Sec[c + d*x]^(-1 + m)*(a + b*Sec[c
 + d*x])^3*(A + B*Sec[c + d*x])*Sqrt[-Tan[c + d*x]^2])/(d*(b + a*Cos[c + d*
x])^3*(B + A*Cos[c + d*x]))
```

Maple [F]
time = 0.33, size = 0, normalized size = 0.00

$$\int \left(\sec^m(dx+c)\right)(a+b\sec(dx+c))^3(A+B\sec(dx+c))\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^m*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3*sec(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
[Out] integral((B*b^3*sec(d*x + c)^4 + A*a^3 + (3*B*a*b^2 + A*b^3)*sec(d*x + c)^3
 + 3*(B*a^2*b + A*a*b^2)*sec(d*x + c)^2 + (B*a^3 + 3*A*a^2*b)*sec(d*x + c))
*sec(d*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec (c + dx)) (a + b\sec (c + dx))^3 \sec^m (c + dx) \, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(a+b*sec(d*x+c))**3*(A+B*sec(d*x+c)),x)
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**3*sec(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^3*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^3*sec(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^3\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3*(1/cos(c + d*x))^m,x)
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^3*(1/cos(c + d*x))^m, x)
```

3.481 $\int \sec^m(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=261

$$\frac{b(Ab(2+m)+aB(3+m))\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+m)(2+m)}+\frac{bB\sec^{1+m}(c+dx)(a+b\sec(c+dx))\sin(c+dx)}{d(2+m)}-$$

```
[Out] b*(A*b*(2+m)+a*B*(3+m))*sec(d*x+c)^(1+m)*sin(d*x+c)/d/(1+m)/(2+m)+b*B*sec(d
*x+c)^(1+m)*(a+b*sec(d*x+c))*sin(d*x+c)/d/(2+m)-(A*b^2*m+2*a*b*B*m+a^2*A*(1
+m))*hypergeom([1/2, 1/2-1/2*m],[3/2-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-1+m)
*sin(d*x+c)/d/(-m^2+1)/(sin(d*x+c)^2)^(1/2)+(b^2*B*(1+m)+a*(2*A*b+B*a)*(2+m
))*hypergeom([1/2, -1/2*m],[1-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^m*sin(d*x+c)/
d/m/(2+m)/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 261, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.161$, Rules used = {4111, 4132, 3857, 2722, 4131}

$$-\frac{\sin(c+dx)\left(a^2A(m+1)+2abBm+Ab^2m\right)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(c+dx)\right)}{d\left(1-m^2\right)\sqrt{\sin^2(c+dx)}}+\frac{\sin(c+dx)\left(a(m+2)(aB+2Ab)+b^2B(m+1)\right)\sec^m(c+dx)\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};\frac{2-m}{2};\cos^2(c+dx)\right)}{dm(m+2)\sqrt{\sin^2(c+dx)}}+\frac{b\sin(c+dx)(aB(m+3)+Ab(m+2))\sec^{m+1}(c+dx)}{d(m+1)(m+2)}+\frac{bB\sin(c+dx)\sec^{m+1}(c+dx)(a+b\sec(c+dx))}{d(m+2)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (b*(A*b*(2 + m) + a*B*(3 + m))*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(1 + m
)*(2 + m)) + (b*B*Sec[c + d*x]^(1 + m)*(a + b*Sec[c + d*x])*Sin[c + d*x])/(
d*(2 + m)) - ((A*b^2*m + 2*a*b*B*m + a^2*A*(1 + m))*Hypergeometric2F1[1/2,
(1 - m)/2, (3 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^(-1 + m)*Sin[c + d*x])/(
d*(1 - m^2)*Sqrt[Sin[c + d*x]^2]) + ((b^2*B*(1 + m) + a*(2*A*b + a*B)*(2 +
m))*Hypergeometric2F1[1/2, -1/2*m, (2 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^
m*Sin[c + d*x])/(d*m*(2 + m)*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
```

```
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4131

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]^2*(C_.)
+ (A_)), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*((b*Csc[e + f*x])^m/(f*(m + 1)
)), x] + Dist[(C*m + A*(m + 1))/(m + 1), Int[(b*Csc[e + f*x])^m, x], x] /;
FreeQ[{b, e, f, A, C, m}, x] && NeQ[C*m + A*(m + 1), 0] &&  !LeQ[m, -1]
```

Rule 4132

```
Int[(csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*((A_.) + csc[(e_.) + (f_.)*(x_)]*
(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.)), x_Symbol] :> Dist[B/b, Int[(b*Csc
[e + f*x])^(m + 1), x], x] + Int[(b*Csc[e + f*x])^m*(A + C*Csc[e + f*x]^2),
 x] /; FreeQ[{b, e, f, A, B, C, m}, x]
```

Rubi steps

$$\int \sec^m(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \frac{bB\sec^{1+m}(c+dx)(a+b\sec(c+dx))\sin(c+}{d(2+m)}$$

$$= \frac{bB\sec^{1+m}(c+dx)(a+b\sec(c+dx))\sin(c+}{d(2+m)}$$

$$= \frac{b(Ab(2+m)+aB(3+m))\sec^{1+m}(c+dx)\,\mathrm{si}}{d(1+m)(2+m)}$$

$$= \frac{b(Ab(2+m)+aB(3+m))\sec^{1+m}(c+dx)\,\mathrm{si}}{d(1+m)(2+m)}$$

$$= \frac{b(Ab(2+m)+aB(3+m))\sec^{1+m}(c+dx)\,\mathrm{si}}{d(1+m)(2+m)}$$

Mathematica [A]
time = 1.04, size = 239, normalized size = 0.92

$$\frac{\csc(c+dx)\left(a^2A(6+11m+6m^2+m^3)\cos^3(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};\sec^2(c+dx)\right)+a(2Ab+aB)m(6+5m+m^2)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sec^2(c+dx)\right)+bm(1+m)\left((Ab+2aB)(3+m)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\sec^2(c+dx)\right)+bB(2+m)\,{}_2F_1\left(\frac{1}{2},\frac{3+m}{2};\frac{5+m}{2};\sec^2(c+dx)\right)\right)\right)\sec^{2+m}(c+dx)\sqrt{-\tan^2(c+dx)}}{dm(1+m)(2+m)(3+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]*(a^2*A*(6 + 11*m + 6*m^2 + m^3)*Cos[c + d*x]^3*Hypergeometric
2F1[1/2, m/2, (2 + m)/2, Sec[c + d*x]^2] + a*(2*A*b + a*B)*m*(6 + 5*m + m^2
)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, (1 + m)/2, (3 + m)/2, Sec[c + d*x]^
2] + b*m*(1 + m)*((A*b + 2*a*B)*(3 + m)*Cos[c + d*x]*Hypergeometric2F1[1/2,
 (2 + m)/2, (4 + m)/2, Sec[c + d*x]^2] + b*B*(2 + m)*Hypergeometric2F1[1/2,
 (3 + m)/2, (5 + m)/2, Sec[c + d*x]^2]))*Sec[c + d*x]^(2 + m)*Sqrt[-Tan[c +
 d*x]^2])/(d*m*(1 + m)*(2 + m)*(3 + m))
```

Maple [F]
time = 0.32, size = 0, normalized size = 0.00

$$\int (\sec^m(dx+c))\,(a+b\sec(dx+c))^2\,(A+B\sec(dx+c))\;dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^m*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="ma
xima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*sec(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="fr
icas")
```

```
[Out] integral((B*b^2*sec(d*x + c)^3 + A*a^2 + (2*B*a*b + A*b^2)*sec(d*x + c)^2 +
 (B*a^2 + 2*A*a*b)*sec(d*x + c))*sec(d*x + c)^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\sec{(c+dx)})\,(a+b\sec{(c+dx)})^2\sec^m{(c+dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*sec(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm="gi
ac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*sec(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \left(A+\frac{B}{\cos{(c+d\,x)}}\right)\,\left(a+\frac{b}{\cos{(c+d\,x)}}\right)^2\,\left(\frac{1}{\cos{(c+d\,x)}}\right)^m\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2*(1/cos(c + d*x))^m,x)
```

```
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2*(1/cos(c + d*x))^m, x)
```

3.482 $\int \sec^m(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=177

$$\frac{bB\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+m)}-\frac{(bBm+aA(1+m))\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(c+dx)\right)\sec^{-1+m}(c+dx)\sin(c+dx)}{d(1-m^2)\sqrt{\sin^2(c+dx)}}$$

```
[Out] b*B*sec(d*x+c)^(1+m)*sin(d*x+c)/d/(1+m)-(b*B*m+a*A*(1+m))*hypergeom([1/2, 1
/2-1/2*m],[3/2-1/2*m],cos(d*x+c)^2)*sec(d*x+c)^(-1+m)*sin(d*x+c)/d/(-m^2+1)
/(sin(d*x+c)^2)^(1/2)+(A*b+B*a)*hypergeom([1/2, -1/2*m],[1-1/2*m],cos(d*x+c
)^2)*sec(d*x+c)^m*sin(d*x+c)/d/m/(sin(d*x+c)^2)^(1/2)
```

Rubi [A]
time = 0.14, antiderivative size = 177, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 29, $\frac{\text{number of rules}}{\text{integrand size}} = 0.138$, Rules used = {4082, 3872, 3857, 2722}

$$-\frac{\sin(c+dx)(aA(m+1)+bBm)\sec^{m-1}(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1-m}{2};\frac{3-m}{2};\cos^2(c+dx)\right)}{d(1-m^2)\sqrt{\sin^2(c+dx)}}+\frac{(aB+Ab)\sin(c+dx)\sec^m(c+dx)\,{}_2F_1\left(\frac{1}{2},-\frac{m}{2};\frac{2-m}{2};\cos^2(c+dx)\right)}{dm\sqrt{\sin^2(c+dx)}}+\frac{bB\sin(c+dx)\sec^{m+1}(c+dx)}{d(m+1)}$$

Antiderivative was successfully verified.

```
[In] Int[Sec[c + d*x]^m*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (b*B*Sec[c + d*x]^(1 + m)*Sin[c + d*x])/(d*(1 + m)) - ((b*B*m + a*A*(1 + m)
)*Hypergeometric2F1[1/2, (1 - m)/2, (3 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]
^(-1 + m)*Sin[c + d*x])/(d*(1 - m^2)*Sqrt[Sin[c + d*x]^2]) + ((A*b + a*B)*H
ypergeometric2F1[1/2, -1/2*m, (2 - m)/2, Cos[c + d*x]^2]*Sec[c + d*x]^m*Sin
[c + d*x])/(d*m*Sqrt[Sin[c + d*x]^2])
```

Rule 2722

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1)*Sqrt[Cos[c + d*x]^2]))*Hypergeometric2
F1[1/2, (n + 1)/2, (n + 3)/2, Sin[c + d*x]^2], x] /; FreeQ[{b, c, d, n}, x]
 &&  !IntegerQ[2*n]
```

Rule 3857

```
Int[(csc[(c_.) + (d_.)*(x_)]*(b_.))^(n_), x_Symbol] :> Simp[(b*Csc[c + d*x]
)^(n - 1)*((Sin[c + d*x]/b)^(n - 1)*Int[1/(Sin[c + d*x]/b)^n, x]), x] /; Fr
eeQ[{b, c, d, n}, x] &&  !IntegerQ[n]
```

Rule 3872

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_)), x_Symbol] :> Dist[a, Int[(d*Csc[e + f*x])^n, x], x] + Dist[b/d, Int[
```

```
(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, n}, x]
```

Rule 4082

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_.)*(csc[(e_.) + (f_.)*(x_)]*(b_.) +
(a_))*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*Cot[
e + f*x]*((d*Csc[e + f*x])^n/(f*(n + 1))), x] + Dist[1/(n + 1), Int[(d*Csc[
e + f*x])^n*Simp[A*a*(n + 1) + B*b*n + (A*b + B*a)*(n + 1)*Csc[e + f*x], x]
, x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&  !LeQ[n
, -1]
```

Rubi steps

$$\int \sec^m(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx = \frac{bB\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+m)} + \frac{\int \sec^m(c+d}{}$$
$$= \frac{bB\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+m)} + (Ab+aB)\int$$
$$= \frac{bB\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+m)} + ((Ab+aB)\,c$$
$$= \frac{bB\sec^{1+m}(c+dx)\sin(c+dx)}{d(1+m)} - \frac{\left(aA+\frac{bBm}{1+m}\right)}{}$$

Mathematica [A]
time = 0.43, size = 168, normalized size = 0.95

$$\frac{\csc(c+dx)\left(aA(2+3m+m^2)\cos^2(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{m}{2};\frac{2+m}{2};\sec^2(c+dx)\right)+(Ab+aB)m(2+m)\cos(c+dx)\,{}_2F_1\left(\frac{1}{2},\frac{1+m}{2};\frac{3+m}{2};\sec^2(c+dx)\right)+bBm(1+m)\,{}_2F_1\left(\frac{1}{2},\frac{2+m}{2};\frac{4+m}{2};\sec^2(c+dx)\right)\right)\sec^{1+m}(c+dx)\sqrt{-\tan^2(c+dx)}}{dm(1+m)(2+m)}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sec[c + d*x]^m*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Csc[c + d*x]*(a*A*(2 + 3*m + m^2)*Cos[c + d*x]^2*Hypergeometric2F1[1/2, m/
2, (2 + m)/2, Sec[c + d*x]^2] + (A*b + a*B)*m*(2 + m)*Cos[c + d*x]*Hypergeo
metric2F1[1/2, (1 + m)/2, (3 + m)/2, Sec[c + d*x]^2] + b*B*m*(1 + m)*Hyperg
eometric2F1[1/2, (2 + m)/2, (4 + m)/2, Sec[c + d*x]^2])*Sec[c + d*x]^(1 + m
)*Sqrt[-Tan[c + d*x]^2])/(d*m*(1 + m)*(2 + m))
```

Maple [F]
time = 0.23, size = 0, normalized size = 0.00

$$\int \left(\sec^m(dx+c)\right)\left(a+b\sec(dx+c)\right)\left(A+B\sec(dx+c)\right)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(sec(d*x+c)^m*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] int(sec(d*x+c)^m*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxi
ma")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sec(d*x + c)^m, x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="fric
as")
```

```
[Out] integral((B*b*sec(d*x + c)^2 + A*a + (B*a + A*b)*sec(d*x + c))*sec(d*x + c)
^m, x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\sec(c+dx))\,(a+b\sec(c+dx))\sec^m(c+dx)\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)**m*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*sec(c + d*x)**m, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(sec(d*x+c)^m*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="giac
")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sec(d*x + c)^m, x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)\,{\left(\frac{1}{\cos\left(c+d\,x\right)}\right)}^m \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))*(1/cos(c + d*x))^m,x)
[Out] int((A + B/cos(c + d*x))*(a + b/cos(c + d*x))*(1/cos(c + d*x))^m, x)
```

3.483 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=132

$$\frac{6a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5A+7B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a(5A+7B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2a(A$$

```
[Out] 6/5*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))/d+2/21*a*(5*A+7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*(A+B)*cos(d*x+
c)^(3/2)*sin(d*x+c)/d+2/7*a*A*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/21*a*(5*A+7*B
)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3033, 3047, 3102, 2827, 2715, 2720, 2719}

$$\frac{2a(5A+7B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2a(5A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2aA\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (6*a*(A + B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(5*A + 7*B)*EllipticF[
(c + d*x)/2, 2])/(21*d) + (2*a*(5*A + 7*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(21*d) + (2*a*(A + B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*a*A*Cos[
c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))(B+A\cos(c$$

$$= \int \cos^{\frac{3}{2}}(c+dx)\left(aB+(aA+aB)\cos(c+dx)+\right.$$

$$= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2}{7}\int \cos^{\frac{3}{2}}(c+d$$

$$= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + (a(A+B))\int c$$

$$= \frac{2a(5A+7B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2a(A}{}$$

$$= \frac{6a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(5A+7B)F\left(\frac{1}{2}\right.}{21d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.31, size = 872, normalized size = 6.61

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*((-3*(A + B)*
Cot[c])/(5*d) + ((23*A + 28*B)*Cos[d*x]*Sin[c])/(84*d) + ((A + B)*Cos[2*d*x
]*Sin[2*c])/(10*d) + (A*Cos[3*d*x]*Sin[3*c])/(28*d) + ((23*A + 28*B)*Cos[c]
*Sin[d*x])/(84*d) + ((A + B)*Cos[2*c]*Sin[2*d*x])/(10*d) + (A*Cos[3*c]*Sin[
3*d*x])/(28*d)) - (5*A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/
2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcT
an[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*S
in[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*
d*Sqrt[1 + Cot[c]^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4
, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x -
ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^
2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/
(3*d*Sqrt[1 + Cot[c]^2]) - (3*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2
]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[
1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqr
t[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])
```

```
/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[
c]^2]]))/(10*d) - (3*B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hyp
ergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x +
ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d
*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]
^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan
[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^
2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/
(10*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 382 vs. $2(168) = 336$.
time = 1.99, size = 383, normalized size = 2.90

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(240A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-528A-168B)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(240*A*cos
(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-528*A-168*B)*sin(1/2*d*x+1/2*c)^6*co
s(1/2*d*x+1/2*c)+(448*A+308*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-12
2*A-112*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+35*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 179, normalized size = 1.36

$$\frac{-5i\sqrt{2}\,(5A+7B)a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}\,(5A+7B)a\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+63i\sqrt{2}\,(A+B)a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-63i\sqrt{2}\,(A+B)a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\left(15Aa\cos(dx+c)^2+21(A+B)a\cos(dx+c)+5(5A+7B)a\right)\sqrt{\cos(dx+c)}\sin(dx+c)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/105*(-5*I*sqrt(2)*(5*A + 7*B)*a*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) + 5*I*sqrt(2)*(5*A + 7*B)*a*weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c)) + 63*I*sqrt(2)*(A + B)*a*weierstrassZeta(-4, 0,
 weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 63*I*sqrt(2)*
(A + B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) + 2*(15*A*a*cos(d*x + c)^2 + 21*(A + B)*a*cos(d*x + c) + 5
*(5*A + 7*B)*a)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Mupad [B]
time = 0.89, size = 166, normalized size = 1.26

$$\frac{2Ba\left(\sqrt{\cos(c+dx)}\,\sin(c+dx)+\mathrm{F}\left(\frac{c}{2}+\frac{dx}{2}\middle|2\right)\right)}{3d}-\frac{2Aa\cos(c+dx)^{7/2}\sin(c+dx)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+dx)^2\right)}{7d\sqrt{\sin(c+dx)^2}}-\frac{2Aa\cos(c+dx)^{9/2}\sin(c+dx)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos(c+dx)^2\right)}{9d\sqrt{\sin(c+dx)^2}}-\frac{2Ba\cos(c+dx)^{7/2}\sin(c+dx)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos(c+dx)^2\right)}{7d\sqrt{\sin(c+dx)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x)),x)
```

```
[Out] (2*B*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*
d) - (2*A*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos
(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*a*cos(c + d*x)^(9/2)*sin(
c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)
^(1/2)) - (2*B*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4
, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.484 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=101

$$\frac{2a(3A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A+B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2aA\cos$$

```
[Out] 2/5*a*(3*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*A*cos(d*x+c)^(3
/2)*sin(d*x+c)/d+2/3*a*(A+B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.14, antiderivative size = 101, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3033, 3047, 3102, 2827, 2719, 2715, 2720}

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(3A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2aA\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*(3*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + B)*EllipticF[
(c + d*x)/2, 2])/(3*d) + (2*a*(A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d
) + (2*a*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))(B+A\cos$$

$$= \int \sqrt{\cos(c+dx)}\,(aB+(aA+aB)\cos(c+dx$$

$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + \frac{2}{5}\int \sqrt{\cos(c+$$

$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d} + (a(A+B))\int$$

$$= \frac{2a(3A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(A+B)\sqrt{}}{}$$

$$= \frac{2a(3A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(A+B)F(}{3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.23, size = 830, normalized size = 8.22

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(-1/5*((3*A +
 5*B)*Cot[c])/d + ((A + B)*Cos[d*x]*Sin[c])/(3*d) + (A*Cos[2*d*x]*Sin[2*c])
/(10*d) + ((A + B)*Cos[c]*Sin[d*x])/(3*d) + (A*Cos[2*c]*Sin[2*d*x])/(10*d))
 - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*
x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[
1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - A
rcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^
2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[
c]^2]) - (3*A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeometr
icPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Ta
n[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcT
an[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt
[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] +
(2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c
]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d) -
(B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2
, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[
c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]
]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]
^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2
*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[
Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 354 vs. $2(141) = 282$.
time = 1.85, size = 355, normalized size = 3.51

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a\left(-24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(44A+20B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(-24*A*cos(
1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(44*A+20*B)*sin(1/2*d*x+1/2*c)^4*cos(1/
2*d*x+1/2*c)+(-16*A-10*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+5*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.60, size = 161, normalized size = 1.59

$$\frac{-5i\sqrt{2}(A+B)a\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5i\sqrt{2}(A+B)a\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}(3A+5B)a\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}(3A+5B)a\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2(3Aa\cos(dx+c)+5(A+B)a)\sqrt{\cos(dx+c)}\sin(dx+c)}{15d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c)) + 5*I*sqrt(2)*(A + B)*a*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) + 3*I*sqrt(2)*(3*A + 5*B)*a*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(3*A +
5*B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))) + 2*(3*A*a*cos(d*x + c) + 5*(A + B)*a)*sqrt(cos(d*x + c))*sin
(d*x + c))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Mupad [**B**]
time = 0.36, size = 128, normalized size = 1.27

$$\frac{2\,A\,a\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,B\,a\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,B\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}-\frac{2\,A\,a\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x)),x)
```

```
[Out] (2*A*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*
d) + (2*B*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2))
)/(3*d) + (2*B*a*ellipticE(c/2 + (d*x)/2, 2))/d - (2*A*a*cos(c + d*x)^(7/2)
*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*
x)^2)^(1/2))
```

3.485 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=70

$$\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aA\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 2*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+2/3*a*(A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*A*sin(d*x+c)*cos(d*
x+c)^(1/2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 70, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3033, 3047, 3102, 2827, 2720, 2719}

$$\frac{2a(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*(A + B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + 3*B)*EllipticF[(c + d
*x)/2, 2])/(3*d) + (2*a*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx = \int \frac{(a+a\cos(c+dx))(B+A\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \int \frac{aB+(aA+aB)\cos(c+dx)+aA\cos^2(c+a}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2aA\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{\frac{1}{2}a(A+3}{}$$

$$= \frac{2aA\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+(a(A+B))\int$$

$$= \frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+3B)F\left(\frac{1}{2}(\right.}{3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.97, size = 309, normalized size = 4.41

$$\frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(-6(A+B)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\text{ArcTan}(\tan(c)))\right)\sec(c)\sin(dx+\text{ArcTan}(\tan(c)))+\left(9(A+B)\cos(c-dx-\text{ArcTan}(\tan(c)))\csc(c)\sec(c)+3A\cos(c+dx+\text{ArcTan}(\tan(c)))\csc(c)\sec(c)+3B\cos(c+dx+\text{ArcTan}(\tan(c)))\csc(c)\sec(c)-12A\cos(c+dx)\cot(c)\sqrt{\sec^2(c)}-12B\cos(c+dx)\cot(c)\sqrt{\sec^2(c)}-4(A+3B)\cos(c+dx)\sqrt{\cos^2(dx-\text{ArcTan}(\cot(c)))}\sqrt{\csc^2(c)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\text{ArcTan}(\cot(c)))\right)\sqrt{\sec^2(c)}\sec(dx-\text{ArcTan}(\cot(c)))\sin(c)+4A\cos(c+dx)\sqrt{\sec^2(c)}\sin(c+dx)\right)\sqrt{\sin^2(dx+\text{ArcTan}(\tan(c)))}\right)}{12d\sqrt{\cos(c+dx)}\sqrt{\sec^2(c)}\sqrt{\sin^2(dx+\text{ArcTan}(\tan(c)))}}$$

Warning: Unable to verify antiderivative.

[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]

[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*(-6*(A + B)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sec[c]*Sin[d*x + ArcTan[Tan[c]]] + (9*(A + B)*Cos[c - d*x - ArcTan[Tan[c]]]*Csc[c]*Sec[c] + 3*A*Cos[c + d*x + ArcTan[Tan[c]]]*Csc[c]*Sec[c] + 3*B*Cos[c + d*x + ArcTan[Tan[c]]]*Csc[c]*Sec[c] - 12*A*Cos[c + d*x]*Cot[c]*Sqrt[Sec[c]^2] - 12*B*Cos[c + d*x]*Cot[c]*Sqrt[Sec[c]^2] - 4*(A + 3*B)*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Sec[c]^2]*Sec[d*x - ArcTan[Cot[c]]]*Sin[c] + 4*A*Cos[c + d*x]*Sqrt[Sec[c]^2]*Sin[c + d*x])*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2]))/(12*d*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c]^2]*Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 320 vs. $2(116) = 232$.
time = 1.77, size = 321, normalized size = 4.59

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a \left(4A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(4*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="maxima")

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.91, size = 142, normalized size = 2.03

$$\frac{2\,Aa\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-i\,\sqrt{2}\,(A+3\,B)a\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+i\,\sqrt{2}\,(A+3\,B)a\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))+3i\,\sqrt{2}\,(A+B)a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-3i\,\sqrt{2}\,(A+B)a\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/3*(2*A*a*sqrt(cos(d*x + c))*sin(d*x + c) - I*sqrt(2)*(A + 3*B)*a*weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(A + 3*B)*a*w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*(A +
 B)*a*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*si
n(d*x + c))) - 3*I*sqrt(2)*(A + B)*a*weierstrassZeta(-4, 0, weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Mupad [B]
time = 2.66, size = 85, normalized size = 1.21

$$\frac{2\,A\,a\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,A\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x)),x)
```

```
[Out] (2*A*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*
d) + (2*A*a*ellipticE(c/2 + (d*x)/2, 2))/d + (2*B*a*ellipticE(c/2 + (d*x)/2
, 2))/d + (2*B*a*ellipticF(c/2 + (d*x)/2, 2))/d
```

3.486 $\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=66

$$\frac{2a(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 2*a*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/d+2*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*a*B*sin(d*x+c)/d/cos(d*x+c)
^(1/2)
```

Rubi [A]
time = 0.13, antiderivative size = 66, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.194, Rules used = {3033, 3047, 3100, 2827, 2720, 2719}

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*(A - B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(A + B)*EllipticF[(c + d*x
)/2, 2])/d + (2*a*B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\begin{aligned}\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))(A+B\sec(c+dx))\,dx &= \int \frac{(a+a\cos(c+dx))(B+A\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= \int \frac{aB+(aA+aB)\cos(c+dx)+aA\cos^2(c+}{\cos^{\frac{3}{2}}(c+dx)}\\ &= \frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+2\int \frac{\frac{1}{2}a(A+B)+\frac{1}{2}a(A-}{\sqrt{\cos(c+}}\\ &= \frac{2aB\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+(a(A-B))\int \sqrt{\cos(c+d}\\ &= \frac{2a(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+B)F\left(\frac{1}{2}\right.}{d}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 5.94, size = 252, normalized size = 3.82

$$\frac{a(1+\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)\left(\frac{\csc(c)\left(3(A-B)\cos(c-dx-\text{ArcTan}(\tan(c)))\sec(c)+(A-B)\cos(c+dx+\text{ArcTan}(\tan(c)))\sec(c)-2((A-2B)\cos(dx)+A\cos(2c+dx))\sqrt{\sec^2(c)}\right)}{\sqrt{\sec^2(c)}}-4(A+B)\cos(c+dx)\sqrt{\cos^2(dx-\text{ArcTan}(\cot(c)))}\sqrt{\csc^2(c)}\,{}_2F_1\left(\frac{1}{4},\frac{1}{2};\frac{5}{4};\sin^2(dx-\text{ArcTan}(\cot(c)))\right)\sec(dx-\text{ArcTan}(\cot(c)))\sin(c)-\frac{2(A-B)\,{}_2F_1\left(-\frac{1}{2},-\frac{1}{4};\frac{3}{4};\cos^2(dx+\text{ArcTan}(\tan(c)))\right)\sec(c)\sin(dx+\text{ArcTan}(\tan(c)))}{\sqrt{\sec^2(c)}\sqrt{\sin^2(dx+\text{ArcTan}(\tan(c)))}}\right)}{4d\sqrt{\cos(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a*(1 + Cos[c + d*x])*Sec[(c + d*x)/2]^2*((Csc[c]*(3*(A - B)*Cos[c - d*x -
ArcTan[Tan[c]]]*Sec[c] + (A - B)*Cos[c + d*x + ArcTan[Tan[c]]]*Sec[c] - 2*(
(A - 2*B)*Cos[d*x] + A*Cos[2*c + d*x])*Sqrt[Sec[c]^2]))/Sqrt[Sec[c]^2] - 4*
(A + B)*Cos[c + d*x]*Sqrt[Cos[d*x - ArcTan[Cot[c]]]^2]*Sqrt[Csc[c]^2]*Hyper
geometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[d*x - ArcT
an[Cot[c]]]*Sin[c] - (2*(A - B)*HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[
d*x + ArcTan[Tan[c]]]^2]*Sec[c]*Sin[d*x + ArcTan[Tan[c]]])/(Sqrt[Sec[c]^2]*
Sqrt[Sin[d*x + ArcTan[Tan[c]]]^2])))/(4*d*Sqrt[Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 239 vs. $2(116) = 232$.
time = 1.92, size = 240, normalized size = 3.64

method	result
default	$-\frac{2a\left(A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\text{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -2*a*(A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-2*B*cos(1/2*d*x
+1/2*c)*sin(1/2*d*x+1/2*c)^2+B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2
^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.68, size = 173, normalized size = 2.62

$$\frac{-i\sqrt{2}\,(A+B)a\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,(A+B)a\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+i\sqrt{2}\,(A-B)a\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-i\sqrt{2}\,(A-B)a\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\,Ba\sqrt{\cos(dx+c)}\,\sin(dx+c)}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] (-I*sqrt(2)*(A + B)*a*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + I*sqrt(2)*(A + B)*a*cos(d*x + c)*weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c)) + I*sqrt(2)*(A - B)*a*cos(d*x + c)*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
)) - I*sqrt(2)*(A - B)*a*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPIn
verse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*B*a*sqrt(cos(d*x + c))*sin
(d*x + c))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int A\sqrt{\cos(c+dx)}\,dx+\int A\sqrt{\cos(c+dx)}\,\sec(c+dx)\,dx+\int B\sqrt{\cos(c+dx)}\,\sec(c+dx)\,dx+\int B\sqrt{\cos(c+dx)}\,\sec^2(c+dx)\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(A*sqrt(cos(c + d*x)), x) + Integral(A*sqrt(cos(c + d*x))*sec(c
+ d*x), x) + Integral(B*sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(B*sq
rt(cos(c + d*x))*sec(c + d*x)**2, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 3.06, size = 96, normalized size = 1.45

$$\frac{2\,A\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a\,\sin(c+d\,x)\;{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\;\frac{3}{4};\;\cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x)),x)
```

```
[Out] (2*A*a*ellipticE(c/2 + (d*x)/2, 2))/d + (2*A*a*ellipticF(c/2 + (d*x)/2, 2))
/d + (2*B*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*a*sin(c + d*x)*hypergeom(
[-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(
1/2))
```

3.487 $\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=95

$$-\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(A+B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/d+2/3*a*(3*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*B*sin(d*x+c)/d/cos
(d*x+c)^(3/2)+2*a*(A+B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 95, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3033, 3047, 3100, 2827, 2716, 2719, 2720}

$$\frac{2a(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a(A+B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2aB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*a*(A + B)*EllipticE[(c + d*x)/2, 2])/d + (2*a*(3*A + B)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a*B*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(A
 + B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(a+a\cos(c+dx))(B+A\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \int \frac{aB+(aA+aB)\cos(c+dx)+aA\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2aB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{3}{2}a(A+B)+\frac{1}{2}a(3A+B)\cos(c+d}{\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2aB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + (a(A+B))\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{3}(a(3A$$

$$= \frac{2a(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2aB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(A+}{d\sqrt{co}}$$

$$= -\frac{2a(A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2a(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right.}{3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.37, size = 813, normalized size = 8.56

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((A + B)*Csc
[c]*Sec[c])/d + (B*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (Sec[c]*Sec[c +
d*x]*(B*Sin[c] + 3*A*Sin[d*x] + 3*B*Sin[d*x]))/(3*d)) - (A*(1 + Cos[c + d*x
])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]
*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[C
ot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[
1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*Sqrt[1 + Cot[c]^2]) - (B*(1 + Cos[c + d*
x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2
]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[
Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt
[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot[c]^2]) + (A*(1 + Cos[c +
 d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4},
 Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcT
an[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x
```

```
+ ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) + (B*(1 + Cos[c + d*x])*Csc[c
]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + A
rcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Arc
Tan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[T
an[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*
Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 398 vs. $2(139) = 278$.

time = 3.52, size = 399, normalized size = 4.20

method	result
default	$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a\left(\frac{A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}{2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/2*A*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1
/2*B*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+(1/2*A+1/2*B)/sin(1/2*d*x+1/2
*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.52, size = 196, normalized size = 2.06

$$\frac{-i\sqrt{2}\,(3A+B)a\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+i\sqrt{2}\,(3A+B)a\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}\,(A+B)a\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}\,(A+B)a\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))+2\,(3\,(A+B)a\cos(dx+c)+Ba)\sqrt{\cos(dx+c)}\,\sin(dx+c)}{3\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/3*(-I*sqrt(2)*(3*A + B)*a*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c)) + I*sqrt(2)*(3*A + B)*a*cos(d*x + c)^2*weierstras
sPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(A + B)*a*cos
(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c))) + 3*I*sqrt(2)*(A + B)*a*cos(d*x + c)^2*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(3*(A
+ B)*a*cos(d*x + c) + B*a)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)
^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{A}{\sqrt{\cos(c+dx)}}\,dx+\int \frac{A\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx+\int \frac{B\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx+\int \frac{B\sec^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] a*(Integral(A/sqrt(cos(c + d*x)), x) + Integral(A*sec(c + d*x)/sqrt(cos(c +
 d*x)), x) + Integral(B*sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(B*se
c(c + d*x)**2/sqrt(cos(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 3.29, size = 150, normalized size = 1.58

$$\frac{2\,A\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,B\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,B\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*A*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a*sin(c + d*x)*hypergeom([-1/4
, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
 + (2*B*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(
c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a*sin(c + d*x)*hypergeom([-3/
4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/
2))
```

3.488 $\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=132

$$-\frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aB\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2a(A+B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a}{}$$

```
[Out] -2/5*a*(5*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*B*sin(d*x+c)/d
/cos(d*x+c)^(5/2)+2/3*a*(A+B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*a*(5*A+3*B)
*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 132, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3033, 3047, 3100, 2827, 2716, 2720, 2719}

$$\frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(A+B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(5A+3B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2aB\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*a*(5*A + 3*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*a*(A + B)*EllipticF
[(c + d*x)/2, 2])/(3*d) + (2*a*B*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (
2*a*(A + B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*a*(5*A + 3*B)*Sin[c
 + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(a+a\cos(c+dx))(B+A\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \int \frac{aB+(aA+aB)\cos(c+dx)+aA\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}\,dx$$

$$= \frac{2aB\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{5}{2}a(A+B)+\frac{1}{2}a(5A+3B)\cos(c+}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2aB\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + (a(A+B))\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx + \frac{1}{5}(a(5A$$

$$= \frac{2aB\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2a(A+B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2a(5A+3B)\mathrm{si}}{5d\sqrt{\cos(c-}}$$

$$= -\frac{2a(5A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2a(A+B)F\left(\frac{1}{2}(c+dx)\middle|\right.}{3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.43, size = 865, normalized size = 6.55

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] a*(Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sec[c/2 + (d*x)/2]^2*(((5*A + 3*B)
*Csc[c]*Sec[c])/(5*d) + (B*Sec[c]*Sec[c + d*x]^3*Sin[d*x])/(5*d) + (Sec[c]*
Sec[c + d*x]^2*(3*B*Sin[c] + 5*A*Sin[d*x] + 5*B*Sin[d*x]))/(15*d) + (Sec[c]
*Sec[c + d*x]*(5*A*Sin[c] + 5*B*Sin[c] + 15*A*Sin[d*x] + 9*B*Sin[d*x]))/(15
*d)) - (A*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Si
n[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 + Cot
[c]^2]) - (B*(1 + Cos[c + d*x])*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4},
 Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^2*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*Sqrt[1 +
Cot[c]^2]) + (A*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((Hypergeome
tricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[
Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + Ar
cTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sq
rt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2]
```

+ (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d) + (3*B*(1 + Cos[c + d*x])*Csc[c]*Sec[c/2 + (d*x)/2]^2*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d))

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 633 vs. $2(168) = 336$.
time = 5.14, size = 634, normalized size = 4.80

method	result
default	$-\frac{4\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a \left(\frac{B\left(24\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)-12\,\mathrm{EllipticE}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right.\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVERBOSE)

[Out] -4*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a*(1/10*B/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(1/2*A+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+1/2*A/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.58, size = 219, normalized size = 1.66

$$\frac{-5i\sqrt{2}(A+B)a\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5i\sqrt{2}(A+B)a\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-3i\sqrt{2}(5A+3B)a\cos(dx+c)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+3i\sqrt{2}(5A+3B)a\cos(dx+c)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))+2\left(3(5A+3B)a\cos(dx+c)^2+5(A+B)a\cos(dx+c)+3Ba\right)\sqrt{\cos(dx+c)}\,\sin(dx+c)}{15d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] 1/15*(-5*I*sqrt(2)*(A + B)*a*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c)) + 5*I*sqrt(2)*(A + B)*a*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(5*A + 3*B)*
a*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c))) + 3*I*sqrt(2)*(5*A + 3*B)*a*cos(d*x + c)^3*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) +
 2*(3*(5*A + 3*B)*a*cos(d*x + c)^2 + 5*(A + B)*a*cos(d*x + c) + 3*B*a)*sqrt
(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a\left(\int \frac{A}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{A\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{B\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] a*(Integral(A/cos(c + d*x)**(3/2), x) + Integral(A*sec(c + d*x)/cos(c + d*x
)**(3/2), x) + Integral(B*sec(c + d*x)/cos(c + d*x)**(3/2), x) + Integral(B
*sec(c + d*x)**2/cos(c + d*x)**(3/2), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Mupad [B]
time = 3.55, size = 177, normalized size = 1.34

$$\frac{2\,A\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,A\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,B\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,B\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{5\,d\,\cos\left(c+d\,x\right)^{5/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x)))/cos(c + d*x)^(3/2),x)
```

```
[Out] (2*A*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c +
 d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*A*a*sin(c + d*x)*hypergeom([-3/4,
1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
 + (2*B*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*co
s(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a*sin(c + d*x)*hypergeom([-
5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^
(1/2))
```

3.489 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=194

$$\frac{4a^2(8A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{4a^2(5A+6B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(5A+6B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d}+$$

```
[Out] 4/15*a^2*(8*A+9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^2*(5*A+6*B)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/45*a^2*(8
*A+9*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/63*a^2*(11*A+9*B)*cos(d*x+c)^(5/2)*
sin(d*x+c)/d+2/9*A*cos(d*x+c)^(5/2)*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d+4/21*
a^2*(5*A+6*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.26, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3033, 3055, 3047, 3102, 2827, 2715, 2720, 2719}

$$\frac{4a^2(5A+6B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(8A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{15d}+\frac{2a^2(11A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d}+\frac{4a^2(8A+9B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{45d}+\frac{4a^2(5A+6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2A\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (4*a^2*(8*A + 9*B)*EllipticE[(c + d*x)/2, 2])/(15*d) + (4*a^2*(5*A + 6*B)*E
llipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(5*A + 6*B)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(21*d) + (4*a^2*(8*A + 9*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4
5*d) + (2*a^2*(11*A + 9*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(63*d) + (2*A*C
os[c + d*x]^(5/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(9*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \int \cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2(B+A\cos$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)\,(a^2+a^2\cos(c+dx))\sin(c+d}{9d}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)\,(a^2+a^2\cos(c+dx))\sin(c+d}{9d}$$

$$= \frac{2a^2(11A+9B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2A}{}$$

$$= \frac{2a^2(11A+9B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d} + \frac{2A}{}$$

$$= \frac{4a^2(5A+6B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{4a^2}{}$$

$$= \frac{4a^2(8A+9B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{15d} + \frac{4a^2(5A+6B}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.32, size = 1086, normalized size = 5.60

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[
c + d*x])*(-1/15*((8*A + 9*B)*Cot[c])/d + ((46*A + 51*B)*Cos[d*x]*Sin[c])/(
168*d) + ((37*A + 36*B)*Cos[2*d*x]*Sin[2*c])/(360*d) + ((2*A + B)*Cos[3*d*x
]*Sin[3*c])/(56*d) + (A*Cos[4*d*x]*Sin[4*c])/(144*d) + ((46*A + 51*B)*Cos[c
]*Sin[d*x])/(168*d) + ((37*A + 36*B)*Cos[2*c]*Sin[2*d*x])/(360*d) + ((2*A +
 B)*Cos[3*c]*Sin[3*d*x])/(56*d) + (A*Cos[4*c]*Sin[4*d*x])/(144*d)))/(B + A*
Cos[c + d*x]) - (5*A*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5
/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])
^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan
[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqr
t[1 + Sin[d*x - ArcTan[Cot[c]]]])/(21*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c
]^2]) - (2*B*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin
[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A +
B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]
]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Si
```

```
n[d*x - ArcTan[Cot[c]]]])/(7*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (
4*A*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A +
B*Sec[c + d*x])*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[T
an[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan
[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]
*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1
+ Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*S
qrt[1 + Tan[c]^2]]))/(15*d*(B + A*Cos[c + d*x])) - (3*B*Cos[c + d*x]^3*Csc[
c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((Hyper
geometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c
]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
+ Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(1
0*d*(B + A*Cos[c + d*x]))
```

Maple [A]
time = 1.77, size = 413, normalized size = 2.13

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^{2}\left(-560 A \cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(1840 A+360 B)\left(\sin^{8}\left(\frac{dx}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/315*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-560*A*
cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^10+(1840*A+360*B)*sin(1/2*d*x+1/2*c)^
8*cos(1/2*d*x+1/2*c)+(-2368*A-1044*B)*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*
c)+(1568*A+1134*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-387*A-351*B)*s
in(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+75*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-168*A*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))+90*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-189*B*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.80, size = 223, normalized size = 1.15

$$-\frac{2\left(15i\sqrt{2}(5A+6B)a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)) - 15i\sqrt{2}(5A+6B)a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)) - 21i\sqrt{2}(8A+9B)a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))) + 21i\sqrt{2}(8A+9B)a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))) - \left(35Aa^2\cos(dx+c)^3 + 45(2A+B)a^2\cos(dx+c)^2 + 14(8A+9B)a^2\cos(dx+c) + 30(5A+6B)a^2\right)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{315d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] -2/315*(15*I*sqrt(2)*(5*A + 6*B)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) - 15*I*sqrt(2)*(5*A + 6*B)*a^2*weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(2)*(8*A + 9*B)*a^2*weierstras
sZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 2
1*I*sqrt(2)*(8*A + 9*B)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4,
0, cos(d*x + c) - I*sin(d*x + c))) - (35*A*a^2*cos(d*x + c)^3 + 45*(2*A + B
)*a^2*cos(d*x + c)^2 + 14*(8*A + 9*B)*a^2*cos(d*x + c) + 30*(5*A + 6*B)*a^2
)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(9/2), x
)
```

Mupad [B]
time = 3.19, size = 266, normalized size = 1.37

$$\frac{2\,B\,a^2\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}-\frac{2\,A\,a^2\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{4\,A\,a^2\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,A\,a^2\,{\cos\left(c+d\,x\right)}^{11/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{11}{4};\ \frac{15}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{11\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{4\,B\,a^2\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,B\,a^2\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*B*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(
3*d) - (2*A*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4,
 cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (4*A*a^2*cos(c + d*x)^(9/2
)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d
*x)^2)^(1/2)) - (2*A*a^2*cos(c + d*x)^(11/2)*sin(c + d*x)*hypergeom([1/2, 1
1/4], 15/4, cos(c + d*x)^2))/(11*d*(sin(c + d*x)^2)^(1/2)) - (4*B*a^2*cos(c
 + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*
d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hyperg
eom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2))
```

3.490 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=161

$$\frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2(6A+7B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(6A+7B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d}+$$

```
[Out] 4/5*a^2*(3*A+4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^2*(6*A+7*B)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/35*a^2*(9*
A+7*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*A*cos(d*x+c)^(3/2)*(a^2+a^2*cos(d*
x+c))*sin(d*x+c)/d+4/21*a^2*(6*A+7*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.24, antiderivative size = 161, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3033, 3055, 3047, 3102, 2827, 2719, 2715, 2720}

$$\frac{4a^2(6A+7B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(9A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d}+\frac{4a^2(6A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (4*a^2*(3*A + 4*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(6*A + 7*B)*El
lipticF[(c + d*x)/2, 2])/(21*d) + (4*a^2*(6*A + 7*B)*Sqrt[Cos[c + d*x]]*Sin
[c + d*x])/(21*d) + (2*a^2*(9*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35
*d) + (2*A*Cos[c + d*x]^(3/2)*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(7*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \int \sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2(B+A\,\mathrm{c}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\,(a^2+a^2\cos(c+dx))\sin(c+d}{7d}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\,(a^2+a^2\cos(c+dx))\sin(c+d}{7d}$$

$$= \frac{2a^2(9A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2A\,\mathrm{c}}{}$$

$$= \frac{2a^2(9A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2A\,\mathrm{c}}{}$$

$$= \frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{4a^2(6A+7B}{}$$

$$= \frac{4a^2(3A+4B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{4a^2(6A+7B}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.28, size = 1040, normalized size = 6.46

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[
c + d*x])*(-1/5*((3*A + 4*B)*Cot[c])/d + ((51*A + 56*B)*Cos[d*x]*Sin[c])/(1
68*d) + ((2*A + B)*Cos[2*d*x]*Sin[2*c])/(20*d) + (A*Cos[3*d*x]*Sin[3*c])/(5
6*d) + ((51*A + 56*B)*Cos[c]*Sin[d*x])/(168*d) + ((2*A + B)*Cos[2*c]*Sin[2*
d*x])/(20*d) + (A*Cos[3*c]*Sin[3*d*x])/(56*d)))/(B + A*Cos[c + d*x]) - (2*A
*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTa
n[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*
x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sq
rt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcT
an[Cot[c]]]])/(7*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (B*Cos[c + d*
x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^
2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x
 - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[
c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]
])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (3*A*Cos[c + d*x]^3*Csc[
```

```
c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((Hyper
geometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + Ar
cTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x
 + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2
]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c
]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2
+ Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(1
0*d*(B + A*Cos[c + d*x])) - (2*B*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^4
*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[{-1/2, -1/
4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(
Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqr
t[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2])
- ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[
d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c
]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(B + A*Cos[c + d*x])
)
```

Maple [A]
time = 1.74, size = 385, normalized size = 2.39

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,a^2\left(120A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(-348A-84B)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(120*A*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8+(-348*A-84*B)*sin(1/2*d*x+1/2*c)^6*c
os(1/2*d*x+1/2*c)+(378*A+224*B)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-1
17*A-91*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+30*A*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-63*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))+35*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-84*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.80, size = 203, normalized size = 1.26

$$-\frac{2\left(5i\sqrt{2}\,(6A+7B)a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}\,(6A+7B)a^2\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-21i\sqrt{2}\,(3A+4B)a^2\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+21i\sqrt{2}\,(3A+4B)a^2\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\left(15Aa^2\cos(dx+c)^2+21(2A+B)a^2\cos(dx+c)+10(6A+7B)a^2\right)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{105d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] -2/105*(5*I*sqrt(2)*(6*A + 7*B)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c)
 + I*sin(d*x + c)) - 5*I*sqrt(2)*(6*A + 7*B)*a^2*weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c)) - 21*I*sqrt(2)*(3*A + 4*B)*a^2*weierstrassZ
eta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 21*
I*sqrt(2)*(3*A + 4*B)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) - I*sin(d*x + c))) - (15*A*a^2*cos(d*x + c)^2 + 21*(2*A + B)*
a^2*cos(d*x + c) + 10*(6*A + 7*B)*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x
)
```

Mupad [B]

time = 3.01, size = 231, normalized size = 1.43

$$\frac{2\,B\,a^2\,\left(\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{2\,A\,a^2\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,B\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}-\frac{4\,A\,a^2\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,A\,a^2\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,B\,a^2\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*B*a^2*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/
2, 2))/3))/d + (2*A*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 +
(d*x)/2, 2)))/(3*d) + (2*B*a^2*ellipticE(c/2 + (d*x)/2, 2))/d - (4*A*a^2*co
s(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/
(7*d*(sin(c + d*x)^2)^(1/2)) - (2*A*a^2*cos(c + d*x)^(9/2)*sin(c + d*x)*hyp
ergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (2
*B*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c +
d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.491 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=126

$$\frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(7A+5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d}+\frac{2}{}$$

```
[Out] 4/5*a^2*(4*A+5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a^2*(A+2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/15*a^2*(7*A+5
*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d+2/5*A*(a^2+a^2*cos(d*x+c))*sin(d*x+c)*cos
(d*x+c)^(1/2)/d
```

Rubi [A]

time = 0.23, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3033, 3055, 3047, 3102, 2827, 2720, 2719}

$$\frac{4a^2(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(7A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}\left(a^2\cos(c+dx)+a^2\right)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (4*a^2*(4*A + 5*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(A + 2*B)*Elli
pticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(7*A + 5*B)*Sqrt[Cos[c + d*x]]*Sin[c
+ d*x])/(15*d) + (2*A*Sqrt[Cos[c + d*x]]*(a^2 + a^2*Cos[c + d*x])*Sin[c + d
*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3055

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(n
 + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e + f
*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d*(m + n + 1) + B*(a*c*(m - 1)
 + b*d*(n + 1)) + (A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[e + f
*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c - a*d,
0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
 && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \int \frac{(a+a\cos(c+dx))^2(B+A\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{2A\sqrt{\cos(c+dx)}\,(a^2+a^2\cos(c+dx))\sin(c+}{5d}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\,(a^2+a^2\cos(c+dx))\sin(c+}{5d}$$

$$= \frac{2a^2(7A+5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2A}{}$$

$$= \frac{2a^2(7A+5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d} + \frac{2A}{}$$

$$= \frac{4a^2(4A+5B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{4a^2(A+2B)}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 6.32, size = 994, normalized size = 7.89

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[
c + d*x])*(-1/5*((4*A + 5*B)*Cot[c])/d + ((2*A + B)*Cos[d*x]*Sin[c])/(6*d)
+ (A*Cos[2*d*x]*Sin[2*c])/(20*d) + ((2*A + B)*Cos[c]*Sin[d*x])/(6*d) + (A*C
os[2*c]*Sin[2*d*x])/(20*d)))/(B + A*Cos[c + d*x]) - (A*Cos[c + d*x]^3*Csc[c
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[
Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[
c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B
 + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[c + d*x]^3*Csc[c]*Hyperge
ometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/
2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[
c + d*x])*Sqrt[1 + Cot[c]^2]) - (2*A*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/
2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[{-1/2,
 -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c
])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]
```

```
*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^
2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*
Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[C
os[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(B + A*Cos[c + d
*x])) - (B*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^
2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + A
rcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + Arc
Tan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + Arc
Tan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[T
an[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*
Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan
[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(B + A*Cos[c + d*x]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 356 vs. $2(166) = 332$.
time = 1.86, size = 357, normalized size = 2.83

method	result
default	$-\frac{4\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^2\left(-12A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+(32A+10B)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(-12*A*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(32*A+10*B)*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+(-13*A-5*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+5*A*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-12*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+10*B*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.60, size = 179, normalized size = 1.42

$$-\frac{2\left(5i\sqrt{2}\,(A+2\,B)a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))-5i\sqrt{2}\,(A+2\,B)a^2\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-3i\sqrt{2}\,(4\,A+5\,B)a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+3i\sqrt{2}\,(4\,A+5\,B)a^2\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-(3\,Aa^2\cos(dx+c)+5\,(2\,A+B)a^2)\sqrt{\cos(dx+c)}\,\sin(dx+c)\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] -2/15*(5*I*sqrt(2)*(A + 2*B)*a^2*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) - 5*I*sqrt(2)*(A + 2*B)*a^2*weierstrassPInverse(-4, 0, cos(
d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*(4*A + 5*B)*a^2*weierstrassZeta(-4
, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*I*sqrt(
2)*(4*A + 5*B)*a^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))) - (3*A*a^2*cos(d*x + c) + 5*(2*A + B)*a^2)*sqrt(c
os(d*x + c))*sin(d*x + c))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x
)
```

Mupad [B]
time = 2.91, size = 153, normalized size = 1.21

$$\frac{2\,A\,a^2\left(\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{2\,B\,a^2\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+6\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,A\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}-\frac{2\,A\,a^2\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{{}}_2{\mathrm{F}}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*A*a^2*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/
2, 2))/3))/d + (2*B*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + 6*ellipticE(c/2
+ (d*x)/2, 2) + 4*ellipticF(c/2 + (d*x)/2, 2)))/(3*d) + (2*A*a^2*ellipticE(
c/2 + (d*x)/2, 2))/d - (2*A*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([
1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.492 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=116

$$\frac{4a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4a^2(2A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(A-3B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2B(a^2+a\ldots}{\ldots}$$

```
[Out] 4*a^2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/d+4/3*a^2*(2*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*B*(a^2+a^2*cos(d*x+c)
)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+2/3*a^2*(A-3*B)*sin(d*x+c)*cos(d*x+c)^(1/2)
/d
```

Rubi [A]
time = 0.22, antiderivative size = 116, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3033, 3054, 3047, 3102, 2827, 2720, 2719}

$$\frac{4a^2(2A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{4a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (4*a^2*A*EllipticE[(c + d*x)/2, 2])/d + (4*a^2*(2*A + 3*B)*EllipticF[(c + d
*x)/2, 2])/(3*d) + (2*a^2*(A - 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
+ (2*B*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \int \frac{(a+a\cos(c+dx))^2(B+A\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$

$$= \frac{2B(a^2+a^2\cos(c+dx))\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{(a -}{}$$

$$= \frac{2B(a^2+a^2\cos(c+dx))\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{1}{2}a^2}{}$$

$$= \frac{2a^2(A-3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2B(}{}$$

$$= \frac{2a^2(A-3B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3d} + \frac{2B(}{}$$

$$= \frac{4a^2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{4a^2(2A+3B)F\left(\frac{1}{2}(c -\right.}{3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.41, size = 735, normalized size = 6.34

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[
c + d*x])*(-1/4*((2*A - B + 2*A*Cos[2*c] + B*Cos[2*c])*Csc[c]*Sec[c])/d + (
A*Cos[d*x]*Sin[c])/(6*d) + (A*Cos[c]*Sin[d*x])/(6*d) + (B*Sec[c]*Sec[c + d*
x]*Sin[d*x])/(2*d)))/(B + A*Cos[c + d*x]) - (2*A*Cos[c + d*x]^3*Csc[c]*Hype
rgeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*
x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]
]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin
[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*C
os[c + d*x])*Sqrt[1 + Cot[c]^2]) - (B*Cos[c + d*x]^3*Csc[c]*HypergeometricP
FQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a
+ a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*
Sqrt[1 + Cot[c]^2]) - (A*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*
Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[{-1/2, -1/4}, {3/4
}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 -
 Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]
```

```
*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[
d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + Ar
cTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*
x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(B + A*Cos[c + d*x]))
```

Maple [A]
time = 1.86, size = 245, normalized size = 2.11

method	result
default	$-\frac{4a^2\left(2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/3*a^2*(2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-sin(1/2*d*x+1/2*c)^2*
cos(1/2*d*x+1/2*c)*A+2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))-3*B*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2+3*B*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.69, size = 198, normalized size = 1.71

$$-\frac{2\left(i\sqrt{2}\,(2A+3B)a^2\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-i\sqrt{2}\,(2A+3B)a^2\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3i\sqrt{2}\,Aa^2\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3i\sqrt{2}\,Aa^2\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-(Aa^2\cos(dx+c)+3Ba^2)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{3d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] -2/3*(I*sqrt(2)*(2*A + 3*B)*a^2*cos(d*x + c)*weierstrassPInverse(-4, 0, cos
(d*x + c) + I*sin(d*x + c)) - I*sqrt(2)*(2*A + 3*B)*a^2*cos(d*x + c)*weiers
trassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*I*sqrt(2)*A*a^2*cos
(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c))) + 3*I*sqrt(2)*A*a^2*cos(d*x + c)*weierstrassZeta(-4, 0, we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (A*a^2*cos(d*x +
 c) + 3*B*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x
)
```

Mupad [B]
time = 3.16, size = 134, normalized size = 1.16

$$\frac{2\,A\,a^2\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+6\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)+4\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,B\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,B\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*A*a^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + 6*ellipticE(c/2 + (d*x)/2, 2) +
 4*ellipticF(c/2 + (d*x)/2, 2)))/(3*d) + (2*B*a^2*ellipticE(c/2 + (d*x)/2,
2))/d + (4*B*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*a^2*sin(c + d*x)*hyp
ergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*
x)^2)^(1/2))
```

3.493 $\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=120

$$-\frac{4a^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{4a^2(3A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(3A+5B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}+\frac{2B(a^2+a^2\cos(c+\cdots}{3d\cos^{\frac{3}{2}}(c\cdots}$$

```
[Out] -4*a^2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/d+4/3*a^2*(3*A+2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*B*(a^2+a^2*cos(d*x
+c))*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/3*a^2*(3*A+5*B)*sin(d*x+c)/d/cos(d*x+c
)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 120, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3033, 3054, 3047, 3100, 2827, 2720, 2719}

$$\frac{4a^2(3A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(3A+5B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}}-\frac{4a^2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2B\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-4*a^2*B*EllipticE[(c + d*x)/2, 2])/d + (4*a^2*(3*A + 2*B)*EllipticF[(c +
d*x)/2, 2])/(3*d) + (2*a^2*(3*A + 5*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]
]) + (2*B*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \int \frac{(a+a\cos(c+dx))^2(B+A\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{2B(a^2+a^2\cos(c+dx))\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int$$

$$= \frac{2B(a^2+a^2\cos(c+dx))\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int$$

$$= \frac{2a^2(3A+5B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2B(a^2+a^2\text{co}}{3d\,\text{c}}$$

$$= \frac{2a^2(3A+5B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}} + \frac{2B(a^2+a^2\text{co}}{3d\,\text{c}}$$

$$= -\frac{4a^2BE\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{d} + \frac{4a^2(3A+2B)F}{3d}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.47, size = 736, normalized size = 6.13

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[
c + d*x])*(-1/4*((-A - 4*B + A*Cos[2*c])*Csc[c]*Sec[c])/d + (B*Sec[c]*Sec[c
 + d*x]^2*Sin[d*x])/(6*d) + (Sec[c]*Sec[c + d*x]*(B*Sin[c] + 3*A*Sin[d*x] +
 6*B*Sin[d*x]))/(6*d)))/(B + A*Cos[c + d*x]) - (A*Cos[c + d*x]^3*Csc[c]*Hyp
ergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d
*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Co
s[c + d*x])*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[c + d*x]^3*Csc[c]*Hypergeometric
PFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a
 + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
- Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Arc
Tan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x
])*Sqrt[1 + Cot[c]^2]) + (B*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a +
 a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[{-1/2, -1/4}, {
3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[
1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos
```

```
[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((S
in[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos
[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(B + A*Cos[c + d*x]))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 512 vs. $2(162) = 324$.
time = 2.24, size = 513, normalized size = 4.28

method	result
default	$-\frac{4\left(6\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,(A+2B)\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\mathrm{si}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -4/3*(6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A+2*B)*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)*(3*A+7*B)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(
1/2)))*sin(1/2*d*x+1/2*c)^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+3*B*(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*a^2/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2-1)^(3/2)/sin(1/2*d*
x+1/2*c)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.80, size = 210, normalized size = 1.75

$$-\frac{2\left(i\sqrt{2}\,(3A+2B)a^{2}\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)-i\sqrt{2}\,(3A+2B)a^{2}\cos\left(dx+c\right)^{2}\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+3i\sqrt{2}\,Ba^{2}\cos\left(dx+c\right)^{2}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-3i\sqrt{2}\,Ba^{2}\cos\left(dx+c\right)^{2}\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)-\left(3(A+2B)a^{2}\cos\left(dx+c\right)+Ba^{2}\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)\right)}{3d\cos\left(dx+c\right)^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -2/3*(I*sqrt(2)*(3*A + 2*B)*a^2*cos(d*x + c)^2*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) - I*sqrt(2)*(3*A + 2*B)*a^2*cos(d*x + c)^2*we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*B*a^2
*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
 c) + I*sin(d*x + c))) - 3*I*sqrt(2)*B*a^2*cos(d*x + c)^2*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) - (3*(A +
2*B)*a^2*cos(d*x + c) + B*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x
+ c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^{2}\left(\int A\sqrt{\cos\left(c+dx\right)}\,dx+\int 2A\sqrt{\cos\left(c+dx\right)}\sec\left(c+dx\right)dx+\int A\sqrt{\cos\left(c+dx\right)}\sec^{2}\left(c+dx\right)dx+\int B\sqrt{\cos\left(c+dx\right)}\sec\left(c+dx\right)dx+\int 2B\sqrt{\cos\left(c+dx\right)}\sec^{2}\left(c+dx\right)dx+\int B\sqrt{\cos\left(c+dx\right)}\sec^{3}\left(c+dx\right)dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] a**2*(Integral(A*sqrt(cos(c + d*x)), x) + Integral(2*A*sqrt(cos(c + d*x))*s
ec(c + d*x), x) + Integral(A*sqrt(cos(c + d*x))*sec(c + d*x)**2, x) + Integ
ral(B*sqrt(cos(c + d*x))*sec(c + d*x), x) + Integral(2*B*sqrt(cos(c + d*x))
*sec(c + d*x)**2, x) + Integral(B*sqrt(cos(c + d*x))*sec(c + d*x)**3, x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x
)
```

Mupad [B]
time = 3.94, size = 196, normalized size = 1.63

$$\frac{2\,A\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,A\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{4\,B\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,B\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2,x)
```

```
[Out] (2*A*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (4*A*a^2*ellipticF(c/2 + (d*x)/2,
 2))/d + (2*B*a^2*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*a^2*sin(c + d*x)*hy
pergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d
*x)^2)^(1/2)) + (4*B*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d
*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*a^2*sin(c + d*
x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(si
n(c + d*x)^2)^(1/2))
```

3.494 $\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=159

$$-\frac{4a^2(5A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a^2(5A+7B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2(5A+4B)}{5d\sqrt{\cos(}}$$

```
[Out] -4/5*a^2*(5*A+4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/3*a^2*(2*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/15*a^2*(5*A+
7*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*B*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d/
cos(d*x+c)^(5/2)+4/5*a^2*(5*A+4*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 159, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3033, 3054, 3047, 3100, 2827, 2716, 2719, 2720}

$$\frac{4a^2(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{4a^2(5A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a^2(5A+7B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}+\frac{4a^2(5A+4B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-4*a^2*(5*A + 4*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(2*A + B)*Ell
ipticF[(c + d*x)/2, 2])/(3*d) + (2*a^2*(5*A + 7*B)*Sin[c + d*x])/(15*d*Cos[
c + d*x]^(3/2)) + (4*a^2*(5*A + 4*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
 + (2*B*(a^2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx &= \int \frac{(a+a\cos(c+dx))^2(B+A\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx \\
&= \frac{2B(a^2+a^2\cos(c+dx))\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{(a+a\cos(c+ \\
&= \frac{2B(a^2+a^2\cos(c+dx))\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{1}{2}a^2(5A+7B)}{} \\
&= \frac{2a^2(5A+7B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2B(a^2+a^2\cos(c+dx))\sin(}{5d\cos^{\frac{5}{2}}(c+dx)} \\
&= \frac{2a^2(5A+7B)\sin(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)} + \frac{2B(a^2+a^2\cos(c+dx))\sin(}{5d\cos^{\frac{5}{2}}(c+dx)} \\
&= \frac{4a^2(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2a^2(5A+7B)\sin(c+dx}{15d\cos^{\frac{3}{2}}(c+dx)} \\
&= -\frac{4a^2(5A+4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(2A+B)F\left(\frac{1}{2}(c+\right.}{3d}
\end{aligned}
$$

<u>Mathematica</u> [**C**] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.55, size = 1025, normalized size = 6.45

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],
x]
```

```
[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[
c + d*x])*(((5*A + 4*B)*Csc[c]*Sec[c])/(5*d) + (B*Sec[c]*Sec[c + d*x]^3*Sin
[d*x])/(10*d) + (Sec[c]*Sec[c + d*x]^2*(3*B*Sin[c] + 5*A*Sin[d*x] + 10*B*Si
n[d*x]))/(30*d) + (Sec[c]*Sec[c + d*x]*(5*A*Sin[c] + 10*B*Sin[c] + 30*A*Sin
[d*x] + 24*B*Sin[d*x]))/(30*d)))/(B + A*Cos[c + d*x]) - (2*A*Cos[c + d*x]^3
*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*S
ec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - A
rcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2
]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(
3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (B*Cos[c + d*x]^3*Csc[c]*Hyp
ergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2 + (d
*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c
]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Si
n[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*
```

```
Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) + (A*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x
)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[{-1/
2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan
[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]
]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c
]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^
2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt
[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(2*d*(B + A*Cos[c +
 d*x])) + (2*B*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*
x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x
 + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x +
 ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x +
 ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcT
an[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c
]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan
[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(5*d*(B + A*Cos[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 713 vs. $2(195) = 390$.
time = 4.90, size = 714, normalized size = 4.49

method	result
default	$-\frac{8\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\, a^2 \left(\frac{A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\cdots}{4\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\cdots\right.}}\right.}{\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/4*A*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
+1/20*B/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c
)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1
/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1
/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*
x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(1/4*A+1/2*B)*(-1/6*cos(1/2*d*x+1/2*c)*
```

```
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-
1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2)))+(1/2*A+1/4*B)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-
1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.72, size = 239, normalized size = 1.50

$$-\frac{2\left(5i\sqrt{2}(2A+B)a^2\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(2A+B)a^2\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3i\sqrt{2}(5A+4B)a^2\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3i\sqrt{2}(5A+4B)a^2\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\left(6(5A+4B)a^2\cos(dx+c)^2+5(A+2B)a^2\cos(dx+c)+3Ba^2\right)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{15d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -2/15*(5*I*sqrt(2)*(2*A + B)*a^2*cos(d*x + c)^3*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(2*A + B)*a^2*cos(d*x + c)^3*w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*I*sqrt(2)*(5*A
 + 4*B)*a^2*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) + I*sin(d*x + c))) - 3*I*sqrt(2)*(5*A + 4*B)*a^2*cos(d*x + c
)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(
d*x + c))) - (6*(5*A + 4*B)*a^2*cos(d*x + c)^2 + 5*(A + 2*B)*a^2*cos(d*x +
c) + 3*B*a^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int \frac{A}{\sqrt{\cos(c+dx)}}\,dx+\int \frac{2A\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx+\int \frac{A\sec^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx+\int \frac{B\sec(c+dx)}{\sqrt{\cos(c+dx)}}\,dx+\int \frac{2B\sec^2(c+dx)}{\sqrt{\cos(c+dx)}}\,dx+\int \frac{B\sec^3(c+dx)}{\sqrt{\cos(c+dx)}}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] a**2*(Integral(A/sqrt(cos(c + d*x)), x) + Integral(2*A*sec(c + d*x)/sqrt(co
s(c + d*x)), x) + Integral(A*sec(c + d*x)**2/sqrt(cos(c + d*x)), x) + Integ
ral(B*sec(c + d*x)/sqrt(cos(c + d*x)), x) + Integral(2*B*sec(c + d*x)**2/sq
rt(cos(c + d*x)), x) + Integral(B*sec(c + d*x)**3/sqrt(cos(c + d*x)), x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x
)
```

Mupad [B]
time = 4.15, size = 229, normalized size = 1.44

$$\frac{6\,B\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)+20\,B\,a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)+30\,B\,a^2\,\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{15\,d\,\cos\left(c+d\,x\right)^{5/2}\,\sqrt{1-\cos\left(c+d\,x\right)^2}}+\frac{2\,A\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,A\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,A\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/cos(c + d*x)^(1/2),x)
```

```
[Out] (6*B*a^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 20*B*a
^2*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) +
30*B*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*
x)^2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2)) + (2*A*a^2*elli
pticF(c/2 + (d*x)/2, 2))/d + (4*A*a^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3
/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*A*a
^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d
*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
```

3.495 $\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=194

$$-\frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2(7A+6B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2a^2(7A+9B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{4a^2(7A+6\ldots}{21d\cos\ldots}$$

```
[Out] -4/5*a^2*(4*A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+4/21*a^2*(7*A+6*B)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/35*a^2*(7
*A+9*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+4/21*a^2*(7*A+6*B)*sin(d*x+c)/d/cos(d
*x+c)^(3/2)+2/7*B*(a^2+a^2*cos(d*x+c))*sin(d*x+c)/d/cos(d*x+c)^(7/2)+4/5*a^
2*(4*A+3*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 194, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3033, 3054, 3047, 3100, 2827, 2716, 2720, 2719}

$$\frac{4a^2(7A+6B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{4a^2(7A+6B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2a^2(7A+9B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)}+\frac{4a^2(4A+3B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2B\sin(c+dx)\left(a^2\cos(c+dx)+a^2\right)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-4*a^2*(4*A + 3*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (4*a^2*(7*A + 6*B)*E
llipticF[(c + d*x)/2, 2])/(21*d) + (2*a^2*(7*A + 9*B)*Sin[c + d*x])/(35*d*C
os[c + d*x]^(5/2)) + (4*a^2*(7*A + 6*B)*Sin[c + d*x])/(21*d*Cos[c + d*x]^(3
/2)) + (4*a^2*(4*A + 3*B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]) + (2*B*(a^
2 + a^2*Cos[c + d*x])*Sin[c + d*x])/(7*d*Cos[c + d*x]^(7/2))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3054

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(-b^2)*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[
e + f*x])^(n + 1)/(d*f*(n + 1)*(b*c + a*d))), x] - Dist[b/(d*(n + 1)*(b*c +
 a*d)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[
a*A*d*(m - n - 2) - B*(a*c*(m - 1) + b*d*(n + 1)) - (A*b*d*(m + n + 1) - B*
(b*c*m - a*d*(n + 1)))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f,
 A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &
& GtQ[m, 1/2] && LtQ[n, -1] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0]
)
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
```

```
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+a\sec(c+dx))^2(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx &= \int \frac{(a+a\cos(c+dx))^2(B+A\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx \\
&= \frac{2B(a^2+a^2\cos(c+dx))\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{(a+a\cos(c+
\\
&= \frac{2B(a^2+a^2\cos(c+dx))\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2}{7}\int \frac{\frac{1}{2}a^2(7A+9B
\\
&= \frac{2a^2(7A+9B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2B(a^2+a^2\cos(c+dx))\sin(}{7d\cos^{\frac{7}{2}}(c+dx)} \\
&= \frac{2a^2(7A+9B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{2B(a^2+a^2\cos(c+dx))\sin(}{7d\cos^{\frac{7}{2}}(c+dx)} \\
&= \frac{2a^2(7A+9B)\sin(c+dx)}{35d\cos^{\frac{5}{2}}(c+dx)} + \frac{4a^2(7A+6B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)} + \\
&= -\frac{4a^2(4A+3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{4a^2(7A+6B)F\left(\frac{1}{2}(c\right.}{21d}
\end{aligned}
$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.64, size = 1067, normalized size = 5.50

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] (Cos[c + d*x]^(7/2)*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[
c + d*x])*(((4*A + 3*B)*Csc[c]*Sec[c])/(5*d) + (B*Sec[c]*Sec[c + d*x]^4*Sin
[d*x])/(14*d) + (Sec[c]*Sec[c + d*x]^3*(5*B*Sin[c] + 7*A*Sin[d*x] + 14*B*Si
n[d*x]))/(70*d) + (Sec[c]*Sec[c + d*x]^2*(21*A*Sin[c] + 42*B*Sin[c] + 70*A*
Sin[d*x] + 60*B*Sin[d*x]))/(210*d) + (Sec[c]*Sec[c + d*x]*(35*A*Sin[c] + 30
*B*Sin[c] + 84*A*Sin[d*x] + 63*B*Sin[d*x]))/(105*d)))/(B + A*Cos[c + d*x])
- (A*Cos[c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - A
rcTan[Cot[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c
+ d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[
-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x -
```

```
ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) - (2*B*Cos[
c + d*x]^3*Csc[c]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot
[c]]]^2]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*S
ec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1
+ Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Co
t[c]]]])/(7*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]) + (2*A*Cos[c + d*x]^
3*Csc[c]*Sec[c/2 + (d*x)/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*(
(HypergeometricPFQ[{-1/2, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*
x + ArcTan[Tan[c]]]*Tan[c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + C
os[d*x + ArcTan[Tan[c]]]]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Ta
n[c]^2]]*Sqrt[1 + Tan[c]^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 +
 Tan[c]^2] + (2*Cos[c]^2*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos
[c]^2 + Sin[c]^2))/Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]
]))/(5*d*(B + A*Cos[c + d*x])) + (3*B*Cos[c + d*x]^3*Csc[c]*Sec[c/2 + (d*x)
/2]^4*(a + a*Sec[c + d*x])^2*(A + B*Sec[c + d*x])*((HypergeometricPFQ[{-1/2
, -1/4}, {3/4}, Cos[d*x + ArcTan[Tan[c]]]^2]*Sin[d*x + ArcTan[Tan[c]]]*Tan[
c])/(Sqrt[1 - Cos[d*x + ArcTan[Tan[c]]]]*Sqrt[1 + Cos[d*x + ArcTan[Tan[c]]]
]*Sqrt[Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]*Sqrt[1 + Tan[c]
^2]) - ((Sin[d*x + ArcTan[Tan[c]]]*Tan[c])/Sqrt[1 + Tan[c]^2] + (2*Cos[c]^2
*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2])/(Cos[c]^2 + Sin[c]^2))/Sqrt[
Cos[c]*Cos[d*x + ArcTan[Tan[c]]]*Sqrt[1 + Tan[c]^2]]))/(10*d*(B + A*Cos[c +
 d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 823 vs. $2(226) = 452$.

time = 6.45, size = 824, normalized size = 4.25

method	result	size
default	Expression too large to display	824

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -8*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*a^2*(1/5*(1/4*
A+1/2*B)/sin(1/2*d*x+1/2*c)^2/(8*sin(1/2*d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)
^4+6*sin(1/2*d*x+1/2*c)^2-1)*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(
1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(
1/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)
^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d
*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+(1/2*A+1/4*B)*(-1/6*cos(1/2*d*x+1/2*c)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2
```

```
-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2)))+1/4*B*(-1/56*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^4-5/42*cos(1/2*d*x+1/
2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*
c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2)))+1/4*A/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)
*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)
^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.94, size = 263, normalized size = 1.36

$$-\frac{2\left(5i\sqrt{2}(7A+6B)a^2\cos(dx+c)^4\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5i\sqrt{2}(7A+6B)a^2\cos(dx+c)^4\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+21i\sqrt{2}(4A+3B)a^2\cos(dx+c)^4\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-21i\sqrt{2}(4A+3B)a^2\cos(dx+c)^4\operatorname{weierstrassZeta}(-4,0,\operatorname{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-\left(42(4A+3B)a^2\cos(dx+c)^3+10(7A+6B)a^2\cos(dx+c)^2+21(A+2B)a^2\cos(dx+c)+15Ba^2\right)\sqrt{\cos(dx+c)}\sin(dx+c)\right)}{105d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] -2/105*(5*I*sqrt(2)*(7*A + 6*B)*a^2*cos(d*x + c)^4*weierstrassPInverse(-4,
0, cos(d*x + c) + I*sin(d*x + c)) - 5*I*sqrt(2)*(7*A + 6*B)*a^2*cos(d*x + c
)^4*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 21*I*sqrt(2
)*(4*A + 3*B)*a^2*cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 21*I*sqrt(2)*(4*A + 3*B)*a^2*cos(
d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
 I*sin(d*x + c))) - (42*(4*A + 3*B)*a^2*cos(d*x + c)^3 + 10*(7*A + 6*B)*a^2
*cos(d*x + c)^2 + 21*(A + 2*B)*a^2*cos(d*x + c) + 15*B*a^2)*sqrt(cos(d*x +
c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$a^2\left(\int\frac{A}{\cos^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{2A\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{A\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{2B\sec^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx+\int\frac{B\sec^3(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\,dx\right)$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**2*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] a**2*(Integral(A/cos(c + d*x)**(3/2), x) + Integral(2*A*sec(c + d*x)/cos(c
+ d*x)**(3/2), x) + Integral(A*sec(c + d*x)**2/cos(c + d*x)**(3/2), x) + In
tegral(B*sec(c + d*x)/cos(c + d*x)**(3/2), x) + Integral(2*B*sec(c + d*x)**
2/cos(c + d*x)**(3/2), x) + Integral(B*sec(c + d*x)**3/cos(c + d*x)**(3/2),
 x))
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x
)
```

Mupad [B]
time = 4.69, size = 235, normalized size = 1.21

$$\frac{6\,A\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)+20\,A\,a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)+30\,A\,a^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{15\,d\cos\left(c+d\,x\right)^{5/2}\,\sqrt{1-\cos\left(c+d\,x\right)^2}}+\frac{30\,B\,a^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)+84\,B\,a^2\,\cos\left(c+d\,x\right)\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)+70\,B\,a^2\cos\left(c+d\,x\right)^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{105\,d\cos\left(c+d\,x\right)^{7/2}\,\sqrt{1-\cos\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^2)/cos(c + d*x)^(3/2),x)
```

```
[Out] (6*A*a^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 20*A*a
^2*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) +
30*A*a^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*
x)^2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2)) + (30*B*a^2*sin
(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2) + 84*B*a^2*cos(c + d
*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 70*B*a^2*co
s(c + d*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(105
*d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))
```

3.496 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=157

$$\frac{3(7A-5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{5(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{5(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad}+\frac{(7A-5B}{}$$

```
[Out] 3/5*(7*A-5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))/a/d-5/3*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+1/5*(7*A-5*B)*cos(d*x
+c)^(3/2)*sin(d*x+c)/a/d-(A-B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c
))-5/3*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d
```

Rubi [A]

time = 0.18, antiderivative size = 157, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3033, 3056, 2827, 2715, 2720, 2719}

$$-\frac{5(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{3(7A-5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad}-\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(7A-5B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5ad}-\frac{5(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (3*(7*A - 5*B)*EllipticE[(c + d*x)/2, 2])/(5*a*d) - (5*(A - B)*EllipticF[(c
 + d*x)/2, 2])/(3*a*d) - (5*(A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d
) + ((7*A - 5*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*a*d) - ((A - B)*Cos[c
+ d*x]^(5/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+A\cos(c+dx))}{a+a\cos(c+dx)}\,dx$$
$$= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \cos^{\frac{3}{2}}(c+dx)\left(-\frac{5}{2}a(A-B\right.}{}$$
$$= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{(7A-5B)\int \cos^{\frac{5}{2}}(c+dx)\,d}{2a}$$
$$= -\frac{5(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} + \frac{(7A-5B)\cos^{\frac{3}{2}}(c+dx)\,s}{5ad}$$
$$= \frac{3(7A-5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5ad} - \frac{5(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} - \frac{5(A-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 6.69, size = 1292, normalized size = 8.23

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (((21*I)/20)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*
((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c]
+ I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d
*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*S
in[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Si
n[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Si
n[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*S
in[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c
]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/(
(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) - (((3*I)/4)*B*Cos[c/2 + (d*x)/2
]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric
2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^(
(2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
 E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)
*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/
4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*
d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2
*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*
Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[
c + d*x])) + (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x])*
((2*(-5*A + 5*B - 16*A*Cos[c] + 10*B*Cos[c])*Csc[c])/(5*d) + (4*(-A + B)*Co
s[d*x]*Sin[c])/(3*d) + (2*A*Cos[2*d*x]*Sin[2*c])/(5*d) + (2*Sec[c/2]*Sec[c/
2 + (d*x)/2]*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/d + (4*(-A + B)*Cos[c]*S
in[d*x])/(3*d) + (2*A*Cos[2*c]*Sin[2*d*x])/(5*d)))/((B + A*Cos[c + d*x])*(a
 + a*Sec[c + d*x])) + (5*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[
{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x
])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqr
t[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTa
n[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*
x])) - (5*B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/
4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - Ar
cTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]
*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3
*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x]))
```

Maple [A]
time = 1.72, size = 282, normalized size = 1.80

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-cos(1/2*d*x
+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(25*A
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+63*A*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))-25*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-45*B*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2)))+48*A*sin(1/2*d*x+1/2*c)^8+(-56*A-40*B)*sin(1/2*d*x+1/2*c
)^6+(-30*A+90*B)*sin(1/2*d*x+1/2*c)^4+(23*A-35*B)*sin(1/2*d*x+1/2*c)^2)/a/c
os(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(
1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.79, size = 269, normalized size = 1.71

$$\frac{2\left(6A\cos(dx+c)^2-2(2A-5B)\cos(dx+c)-25A+25B\right)\sqrt{\cos(dx+c)}\sin(dx+c)-25\left(\sqrt{2}(-iA+iB)\cos(dx+c)+\sqrt{2}(-iA+iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-25\left(\sqrt{2}(iA-iB)\cos(dx+c)+\sqrt{2}(iA-iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-9\left(\sqrt{2}(-7iA+5iB)\cos(dx+c)+\sqrt{2}(-7iA+5iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-9\left(\sqrt{2}(7iA-5iB)\cos(dx+c)+\sqrt{2}(7iA-5iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{30\left(ad\cos(dx+c)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/30*(2*(6*A*cos(d*x + c)^2 - 2*(2*A - 5*B)*cos(d*x + c) - 25*A + 25*B)*sqr
t(cos(d*x + c))*sin(d*x + c) - 25*(sqrt(2)*(-I*A + I*B)*cos(d*x + c) + sqrt
(2)*(-I*A + I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
 - 25*(sqrt(2)*(I*A - I*B)*cos(d*x + c) + sqrt(2)*(I*A - I*B))*weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 9*(sqrt(2)*(-7*I*A + 5*I*B)
```

```
*cos(d*x + c) + sqrt(2)*(-7*I*A + 5*I*B))*weierstrassZeta(-4, 0, weierstras
sPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 9*(sqrt(2)*(7*I*A - 5*I*
B)*cos(d*x + c) + sqrt(2)*(7*I*A - 5*I*B))*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c) + a*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{a+\frac{a}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x)), x)
```

3.497 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=124

$$-\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(5A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}+\frac{(5A-3B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad}-\frac{(A-B)\,c}{d(c}$$

```
[Out] -3*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/a/d+1/3*(5*A-3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-(A-B)*cos(d*x+c)^(3/2)
*sin(d*x+c)/d/(a+a*cos(d*x+c))+1/3*(5*A-3*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/
d
```

Rubi [A]
time = 0.16, antiderivative size = 124, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3033, 3056, 2827, 2719, 2715, 2720}

$$\frac{(5A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{d(a\cos(c+dx)+a)}+\frac{(5A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (-3*(A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((5*A - 3*B)*EllipticF[(c +
d*x)/2, 2])/(3*a*d) + ((5*A - 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a*d)
 - ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(d*(a + a*Cos[c + d*x]))
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)(B+A\cos(c+dx))}{a+a\cos(c+dx)}\,dx$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \sqrt{\cos(c+dx)}\,\left(-\frac{3}{2}a(A \right.}{}$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{(5A-3B)\int\cos^{\frac{3}{2}}(c+dx)}{2a}$$

$$= -\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(5A-3B)\sqrt{\cos(c+dx)}\,\sin(c+}{3ad}$$

$$= -\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(5A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} + \frac{(5A}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.60, size = 1239, normalized size = 9.99

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (((-3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*(
(2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] +
 I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*
x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Si
n[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin
[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin
[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Si
n[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]
])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((
B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) + (((3*I)/4)*B*Cos[c/2 + (d*x)/2]
^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2
F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*
d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4
, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d
*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*
I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*C
os[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c
 + d*x])) + (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x])*(
(-2*(-A + B)*(1 + 2*Cos[c])*Csc[c])/d + (4*A*Cos[d*x]*Sin[c])/(3*d) - (2*Se
c[c/2]*Sec[c/2 + (d*x)/2]*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/d + (4*A*Co
s[c]*Sin[d*x])/(3*d)))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) - (5*A*C
os[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x -
 ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]
*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d
*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos
[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) + (B*Cos[c/2 + (d*x)/2]
^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^
2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
 - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c
]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*Sqrt[1 +
 Cot[c]^2]*(a + a*Sec[c + d*x]))
```

Maple [A]
time = 1.94, size = 262, normalized size = 2.11

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\left(5\right.\right.}{3a\cos\left(\frac{dx}{2}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -1/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1
/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(5*A*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))-3*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*B*EllipticE(cos(1/2*d*x+1/2*
c),2^(1/2)))-8*A*sin(1/2*d*x+1/2*c)^6+(18*A-6*B)*sin(1/2*d*x+1/2*c)^4+(-7*A
+3*B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1
/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.66, size = 250, normalized size = 2.02

$$\frac{2 \left(2 A \cos \left(dx+c\right)+5 A-3 B\right) \sqrt{\cos \left(dx+c\right)} \sin \left(dx+c\right)+\left(\sqrt{2} \left(-5 i A+3 i B\right) \cos \left(dx+c\right)+\sqrt{2} \left(-5 i A+3 i B\right)\right) \mathrm{weierstrassPInverse}\left(-4,0,\cos \left(dx+c\right)+i \sin \left(dx+c\right)\right)+\left(\sqrt{2} \left(5 i A-3 i B\right) \cos \left(dx+c\right)+\sqrt{2} \left(5 i A-3 i B\right)\right) \mathrm{weierstrassPInverse}\left(-4,0,\cos \left(dx+c\right)-i \sin \left(dx+c\right)\right)-9 \left(\sqrt{2} \left(i A-i B\right) \cos \left(dx+c\right)+\sqrt{2} \left(i A-i B\right)\right) \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos \left(dx+c\right)+i \sin \left(dx+c\right)\right)\right)-9 \left(\sqrt{2} \left(-i A+i B\right) \cos \left(dx+c\right)+\sqrt{2} \left(-i A+i B\right)\right) \mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos \left(dx+c\right)-i \sin \left(dx+c\right)\right)\right)}{6 \left(ad \cos \left(dx+c\right)+ad\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/6*(2*(2*A*cos(d*x + c) + 5*A - 3*B)*sqrt(cos(d*x + c))*sin(d*x + c) + (sq
rt(2)*(-5*I*A + 3*I*B)*cos(d*x + c) + sqrt(2)*(-5*I*A + 3*I*B))*weierstrass
PInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + (sqrt(2)*(5*I*A - 3*I*B)*c
os(d*x + c) + sqrt(2)*(5*I*A - 3*I*B))*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) - 9*(sqrt(2)*(I*A - I*B)*cos(d*x + c) + sqrt(2)*(I*A
- I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) - 9*(sqrt(2)*(-I*A + I*B)*cos(d*x + c) + sqrt(2)*(-I*A + I*B
))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c))))/(a*d*cos(d*x + c) + a*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{a+\frac{a}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x)), x)
```

3.498 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx$

Optimal. Leaf size=88

$$\frac{(3A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))}$$

```
[Out] (3*A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/a/d-(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-(A-B)*sin(d*x+c)*cos(d*x+c)^(1/
2)/d/(a+a*cos(d*x+c))
```

Rubi [A]

time = 0.14, antiderivative size = 88, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3033, 3056, 2827, 2720, 2719}

$$-\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(3A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] ((3*A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) - ((A - B)*EllipticF[(c + d*x)/
2, 2])/(a*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c +
d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{a+a\sec(c+dx)}\,dx &= \int \frac{\sqrt{\cos(c+dx)}\,(B+A\cos(c+dx))}{a+a\cos(c+dx)}\,dx\\ &= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))}+\frac{\int \frac{-\frac{1}{2}a(A-B)+\frac{1}{2}a(3A-B)\cos(}{\sqrt{\cos(c+dx)}}}{a^2}\\ &= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))}-\frac{(A-B)\int \frac{1}{\sqrt{\cos(c+dx}}}{2a}\\ &= \frac{(3A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-}{}\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.49, size = 1208, normalized size = 13.73

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x]),x]
```

```
[Out] (((3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((
2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] +
```

```
I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x
))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin
[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[
c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[
c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin
[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]]
)/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B
 + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) - ((I/4)*B*Cos[c/2 + (d*x)/2]^2*Cs
c[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/
2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*
d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2
*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))
*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2
, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*
Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*
x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c]
 + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*
x])) + (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x])*((-2*(
A - B + 2*A*Cos[c])*Csc[c])/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-(A*Sin[(d*
x)/2]) + B*Sin[(d*x)/2]))/d))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) +
 (A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[
d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot
[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*
Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*
Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) - (B*Cos[c/2 + (d*x)
/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]
]]^2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*Sqrt[
1 + Cot[c]^2]*(a + a*Sec[c + d*x]))
```

Maple [A]
time = 1.60, size = 244, normalized size = 2.77

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\left(A\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(s\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] ((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c)
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF
```

```
(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
)+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x
+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/
2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.85, size = 241, normalized size = 2.74

$$-\frac{2\,(A-B)\sqrt{\cos(dx+c)}\,\sin(dx+c)-\left(\sqrt{2}\,(iA-iB)\cos(dx+c)+\sqrt{2}\,(iA-iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))-\left(\sqrt{2}\,(-iA+iB)\cos(dx+c)+\sqrt{2}\,(-iA+iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))-\left(\sqrt{2}\,(3iA-iB)\cos(dx+c)+\sqrt{2}\,(3iA-iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))-\left(\sqrt{2}\,(-3iA+iB)\cos(dx+c)+\sqrt{2}\,(-3iA+iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))}{2\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] -1/2*(2*(A - B)*sqrt(cos(d*x + c))*sin(d*x + c) - (sqrt(2)*(I*A - I*B)*cos(
d*x + c) + sqrt(2)*(I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I
*sin(d*x + c)) - (sqrt(2)*(-I*A + I*B)*cos(d*x + c) + sqrt(2)*(-I*A + I*B))
*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - (sqrt(2)*(3*I*
A - I*B)*cos(d*x + c) + sqrt(2)*(3*I*A - I*B))*weierstrassZeta(-4, 0, weier
strassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - (sqrt(2)*(-3*I*A +
I*B)*cos(d*x + c) + sqrt(2)*(-3*I*A + I*B))*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c) + a*d
)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sqrt{\cos{(c+dx)}}}{\sec{(c+dx)}+1}\,dx+\int \frac{B\sqrt{\cos{(c+dx)}}\sec{(c+dx)}}{\sec{(c+dx)}+1}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A*sqrt(cos(c + d*x))/(sec(c + d*x) + 1), x) + Integral(B*sqrt(cos
(c + d*x))*sec(c + d*x)/(sec(c + d*x) + 1), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x)), x)
```

3.499 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=83

$$-\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))}$$

```
[Out] -(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*
x+1/2*c),2^(1/2))/a/d+(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d+(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2
)/d/(a+a*cos(d*x+c))
```

Rubi [A]
time = 0.14, antiderivative size = 83, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3033, 3057, 2827, 2720, 2719}

$$\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{d(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] -(((A - B)*EllipticE[(c + d*x)/2, 2])/(a*d)) + ((A + B)*EllipticF[(c + d*x)
/2, 2])/(a*d) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(d*(a + a*Cos[c +
 d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))}\,dx = \int \frac{B+A\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}\,dx$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}a(A+B)-\frac{1}{2}a(A-B)\cos(c+dx}{\sqrt{\cos(c+dx)}}}{a^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{d(a+a\cos(c+dx))} - \frac{(A-B)\int\sqrt{\cos(c+dx)}}{2a}$$

$$= -\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} + \frac{(A-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.52, size = 1204, normalized size = 14.51

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])),x]
```

```
[Out] ((-1/4*I)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2
*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I
```

```
*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x)
)*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[
2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c
]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c
])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[
c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])
/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B
+ A*Cos[c + d*x])*(a + a*Sec[c + d*x])) + ((I/4)*B*Cos[c/2 + (d*x)/2]^2*Csc
[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2
, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d
*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*
I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*
Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2,
 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*C
os[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x
)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c]
+ d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x
])) + (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x])*((-2*(-
A + B)*Csc[c])/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-(A*Sin[(d*x)/2]) + B*Si
n[(d*x)/2]))/d))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) - (A*Cos[c/2 +
 (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[
Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1
- Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - Arc
Tan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])
*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) - (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[
Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt
[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]
*(a + a*Sec[c + d*x]))
```

Maple [A]
time = 1.60, size = 243, normalized size = 2.93

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(\right.\right.}{a\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\mathrm{s}\right.}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(cos(1/2*d*x+1/2*c
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*Elliptic
```

```
F(cos(1/2*d*x+1/2*c),2^(1/2))+A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+B*Ell
ipticF(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))
+(2*A-2*B)*sin(1/2*d*x+1/2*c)^4+(-A+B)*sin(1/2*d*x+1/2*c)^2)/a/cos(1/2*d*x+
1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2
*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.75, size = 237, normalized size = 2.86

$$\frac{2\,(A-B)\sqrt{\cos(dx+c)}\,\sin(dx+c)+\left(\sqrt{2}\,(-iA-iB)\cos(dx+c)+\sqrt{2}\,(-iA-iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\left(\sqrt{2}\,(iA+iB)\cos(dx+c)+\sqrt{2}\,(iA+iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+\left(\sqrt{2}\,(-iA+iB)\cos(dx+c)+\sqrt{2}\,(-iA+iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+\left(\sqrt{2}\,(iA-iB)\cos(dx+c)+\sqrt{2}\,(iA-iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))}{2\,(ad\cos(dx+c)+ad)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/2*(2*(A - B)*sqrt(cos(d*x + c))*sin(d*x + c) + (sqrt(2)*(-I*A - I*B)*cos(
d*x + c) + sqrt(2)*(-I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) +
I*sin(d*x + c)) + (sqrt(2)*(I*A + I*B)*cos(d*x + c) + sqrt(2)*(I*A + I*B))*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + (sqrt(2)*(-I*A
+ I*B)*cos(d*x + c) + sqrt(2)*(-I*A + I*B))*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + (sqrt(2)*(I*A - I*B)*c
os(d*x + c) + sqrt(2)*(I*A - I*B))*weierstrassZeta(-4, 0, weierstrassPInver
se(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c) + a*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sqrt{\cos(c+dx)}\sec(c+dx)+\sqrt{\cos(c+dx)}}\,dx+\int \frac{B\sec(c+dx)}{\sqrt{\cos(c+dx)}\sec(c+dx)+\sqrt{\cos(c+dx)}}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] (Integral(A/(sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))), x) + In
tegral(B*sec(c + d*x)/(sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))
), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\sqrt{\cos\left(c+d\,x\right)}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))), x)
```

3.500 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=113

$$\frac{(A-3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-3B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}+\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\cos(c$$

```
[Out] (A-3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/a/d+(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-(A-3*B)*sin(d*x+c)/a/d/cos(d*x+
c)^(1/2)+(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.16, antiderivative size = 113, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3033, 3057, 2827, 2716, 2719, 2720}

$$\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-3B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}+\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+a)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] ((A - 3*B)*EllipticE[(c + d*x)/2, 2])/(a*d) + ((A - B)*EllipticF[(c + d*x)/
2, 2])/(a*d) - ((A - 3*B)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) + ((A - B)
*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x]))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))}\,dx=\int \frac{B+A\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$$

$$=\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}+\frac{\int \frac{-\frac{1}{2}a(A-3B)+\frac{1}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}$$

$$=\frac{(A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))}-\frac{(A-3B)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx}{2a}+$$

$$=\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-3B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}+\frac{(A-B}{d\sqrt{\cos(c+dx}}$$

$$=\frac{(A-3B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(A-3B)}{ad\sqrt{\cos}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 6.73, size = 1240, normalized size = 10.97

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] ((I/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^
((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Si
n[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*S
in[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c
]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c])
- (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^
2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])
/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((
-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A
*Cos[c + d*x])*(a + a*Sec[c + d*x])) - (((3*I)/4)*B*Cos[c/2 + (d*x)/2]^2*Cs
c[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/
2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*
d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2
*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))
*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2
, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*
Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*
x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c]
 + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*
x])) + (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x])*(((2*B
 - A*Cos[c] + B*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/d + (2*Sec[c/2]*Sec[c/2 +
 (d*x)/2]*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/d + (4*B*Sec[c]*Sec[c + d*x
]*Sin[d*x])/d))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) - (A*Cos[c/2 +
(d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[C
ot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*
Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) + (B*Cos[c/2 + (d*x)/2]^2*Csc[c/2]
*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]
*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[C
ot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[
1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*
(a + a*Sec[c + d*x]))
```

Maple [A]
time = 2.56, size = 318, normalized size = 2.81

method	result

default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*(-cos(1/2*d*x+
1/2*c)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+3*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A-3*B)*sin(1/2*d*x+1/2*c)^4-(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(A-5*B)*sin(1/2*d*x+1/2*c)^
2)/cos(1/2*d*x+1/2*c)/sin(1/2*d*x+1/2*c)^3/(2*sin(1/2*d*x+1/2*c)^2-1)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.09, size = 292, normalized size = 2.58

$$-\frac{2\left((A-3B)\cos(dx+c)-2B\right)\sqrt{\cos(dx+c)}\sin(dx+c)-\left(\sqrt{2}(-iA+iB)\cos(dx+c)^2+\sqrt{2}(-iA+iB)\cos(dx+c)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-\left(\sqrt{2}(iA-iB)\cos(dx+c)^2+\sqrt{2}(iA-iB)\cos(dx+c)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-\left(\sqrt{2}(iA-3iB)\cos(dx+c)^2+\sqrt{2}(iA-3iB)\cos(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-\left(\sqrt{2}(-iA+3iB)\cos(dx+c)^2+\sqrt{2}(-iA+3iB)\cos(dx+c)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{2\left(ad\cos(dx+c)^2+ad\cos(dx+c)\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] -1/2*(2*((A - 3*B)*cos(d*x + c) - 2*B)*sqrt(cos(d*x + c))*sin(d*x + c) - (s
qrt(2)*(-I*A + I*B)*cos(d*x + c)^2 + sqrt(2)*(-I*A + I*B)*cos(d*x + c))*wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - (sqrt(2)*(I*A - I*
B)*cos(d*x + c)^2 + sqrt(2)*(I*A - I*B)*cos(d*x + c))*weierstrassPInverse(-
```

```
4, 0, cos(d*x + c) - I*sin(d*x + c)) - (sqrt(2)*(I*A - 3*I*B)*cos(d*x + c)^
2 + sqrt(2)*(I*A - 3*I*B)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - (sqrt(2)*(-I*A + 3*I*B)*co
s(d*x + c)^2 + sqrt(2)*(-I*A + 3*I*B)*cos(d*x + c))*weierstrassZeta(-4, 0,
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x +
c)^2 + a*d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\cos^{\frac{3}{2}}(c+dx)\sec(c+dx)+\cos^{\frac{3}{2}}(c+dx)}\,dx + \int \frac{B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sec(c+dx)+\cos^{\frac{3}{2}}(c+dx)}\,dx}{a}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] (Integral(A/(cos(c + d*x)**(3/2)*sec(c + d*x) + cos(c + d*x)**(3/2)), x) +
Integral(B*sec(c + d*x)/(cos(c + d*x)**(3/2)*sec(c + d*x) + cos(c + d*x)**(
3/2)), x))/a
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\cos\left(c+d\,x\right)}^{3/2}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))), x)
```

3.501 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx$

Optimal. Leaf size=152

$$-\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{(3A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{(3A-5B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}+\frac{3(A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}+$$

```
[Out] -3*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2))/a/d-1/3*(3*A-5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/d-1/3*(3*A-5*B)*sin(d*x+
c)/a/d/cos(d*x+c)^(3/2)+(A-B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c)
)+3*(A-B)*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 152, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3033, 3057, 2827, 2716, 2720, 2719}

$$-\frac{(3A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad}-\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}+\frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)}-\frac{(3A-5B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)}+\frac{3(A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (-3*(A - B)*EllipticE[(c + d*x)/2, 2])/(a*d) - ((3*A - 5*B)*EllipticF[(c +
d*x)/2, 2])/(3*a*d) - ((3*A - 5*B)*Sin[c + d*x])/(3*a*d*Cos[c + d*x]^(3/2))
 + (3*(A - B)*Sin[c + d*x])/(a*d*Sqrt[Cos[c + d*x]]) + ((A - B)*Sin[c + d*x
])/(d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x]))
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))}\,dx = \int \frac{B+A\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} + \frac{\int \frac{-\frac{1}{2}a(3A-5B)+\frac{3}{2}a(A-B)\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}}{a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))} - \frac{(3A-5B)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}\,dx}{2a} + $$

$$= -\frac{(3A-5B)\sin(c+dx)}{3ad\cos^{\frac{3}{2}}(c+dx)} + \frac{3(A-B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}} + \frac{(A-B}{d\cos^{\frac{3}{2}}(c+dx}$$

$$= -\frac{3(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(3A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3ad} - \frac{(3A}{3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 7.16, size = 1277, normalized size = 8.40

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])),x]
```

```
[Out] (((-3*I)/4)*A*Cos[c/2 + (d*x)/2]^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*(
(2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] +
 I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*
x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Si
n[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin
[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin
[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Si
n[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]
])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((
B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) + (((3*I)/4)*B*Cos[c/2 + (d*x)/2]
^2*Csc[c/2]*Sec[c/2]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2
F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((
2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 +
E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*
d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4
, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d
*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*
I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*C
os[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c
 + d*x])) + (Cos[c/2 + (d*x)/2]^2*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x])*(
-(((-A + B)*(2 + Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/d) - (2*Sec[c/2]*Sec[c/2
 + (d*x)/2]*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/d + (4*B*Sec[c]*Sec[c + d
*x]^2*Sin[d*x])/(3*d) + (4*Sec[c]*Sec[c + d*x]*(B*Sin[c] + 3*A*Sin[d*x] - 3
*B*Sin[d*x]))/(3*d)))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])) + (A*Cos[
c/2 + (d*x)/2]^2*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - Ar
cTan[Cot[c]]]^2]*Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c +
d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])) - (5*B*Cos[c/2 + (d*x)/2]^2*
Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
Sec[c/2]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x -
ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]]
)]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 +
Cot[c]^2]*(a + a*Sec[c + d*x]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 465 vs. $2(194) = 388$.
time = 4.28, size = 466, normalized size = 3.07

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{(-A+B)\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\sqrt{\frac{1}{2}-}\right.}{\cos\left(\frac{dx}{2}+\frac{c}{2}\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a*((-A+B)*(cos(1
/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
)))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2*B*(-1/6*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-
1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2)))+(2*A-2*B)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.80, size = 318, normalized size = 2.09

$$\frac{2\left(9\,(A-B)\cos\left(dx+c\right)^2+2\,(3\,A-2\,B)\cos\left(dx+c\right)+2\,B\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(\sqrt{2}\,(3i\,A-5i\,B)\cos\left(dx+c\right)^3+\sqrt{2}\,(3i\,A-5i\,B)\cos\left(dx+c\right)^2\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\left(\sqrt{2}\,(-3i\,A+5i\,B)\cos\left(dx+c\right)^3+\sqrt{2}\,(-3i\,A+5i\,B)\cos\left(dx+c\right)^2\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-9\left(\sqrt{2}\,(i\,A-i\,B)\cos\left(dx+c\right)^3+\sqrt{2}\,(i\,A-i\,B)\cos\left(dx+c\right)^2\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-9\left(\sqrt{2}\,(-i\,A+i\,B)\cos\left(dx+c\right)^3+\sqrt{2}\,(-i\,A+i\,B)\cos\left(dx+c\right)^2\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)}{6\left(ad\cos\left(dx+c\right)^3+ad\cos\left(dx+c\right)^2\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/6*(2*(9*(A - B)*cos(d*x + c)^2 + 2*(3*A - 2*B)*cos(d*x + c) + 2*B)*sqrt(c
os(d*x + c))*sin(d*x + c) + (sqrt(2)*(3*I*A - 5*I*B)*cos(d*x + c)^3 + sqrt(
2)*(3*I*A - 5*I*B)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + (sqrt(2)*(-3*I*A + 5*I*B)*cos(d*x + c)^3 + sqrt(2)*(-3*
I*A + 5*I*B)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c)) - 9*(sqrt(2)*(I*A - I*B)*cos(d*x + c)^3 + sqrt(2)*(I*A - I*B)*c
os(d*x + c)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) - 9*(sqrt(2)*(-I*A + I*B)*cos(d*x + c)^3 + sqrt(2)*(-
I*A + I*B)*cos(d*x + c)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0
, cos(d*x + c) - I*sin(d*x + c))))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2
)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\cos\left(c+d\,x\right)}^{5/2}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))), x)
```

3.502 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=204

$$\frac{7(8A-5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{5(3A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{5(3A-2B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d}+\frac{7(8A$$

```
[Out] 7/5*(8*A-5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))/a^2/d-5/3*(3*A-2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+7/15*(8*A-5*B
)*cos(d*x+c)^(3/2)*sin(d*x+c)/a^2/d-(3*A-2*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/a
^2/d/(1+cos(d*x+c))-1/3*(A-B)*cos(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c)
)^2-5/3*(3*A-2*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.27, antiderivative size = 204, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3033, 3056, 2827, 2715, 2720, 2719}

$$-\frac{5(3A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{7(8A-5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d}-\frac{(3A-2B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{a^2d(\cos(c+dx)+1)}+\frac{7(8A-5B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{15a^2d}-\frac{5(3A-2B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d}-\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] (7*(8*A - 5*B)*EllipticE[(c + d*x)/2, 2])/(5*a^2*d) - (5*(3*A - 2*B)*Ellipt
icF[(c + d*x)/2, 2])/(3*a^2*d) - (5*(3*A - 2*B)*Sqrt[Cos[c + d*x]]*Sin[c +
d*x])/(3*a^2*d) + (7*(8*A - 5*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(15*a^2*d
) - ((3*A - 2*B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x])
) - ((A - B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^{\frac{7}{2}}(c+dx)(B+A\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)\left(-\frac{7}{2}a(A-B)+\frac{1}{2}a(11A}{a+a\cos(c+dx)}}{3a^2}$$

$$= -\frac{(3A-2B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin}{3d(a+a\cos(c+dx}$$

$$= -\frac{(3A-2B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin}{3d(a+a\cos(c+dx}$$

$$= -\frac{5(3A-2B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d} + \frac{7(8A-5B)\cos^{\frac{3}{2}}(c+}{15a^2d}$$

$$= \frac{7(8A-5B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^2d} - \frac{5(3A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{5(3}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.93, size = 1396, normalized size = 6.84

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,
x]
```

```
[Out] (((28*I)/5)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Se
c[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d
*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1
+ E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^(
(2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2
*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(C
os[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((
2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*
d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))
*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - (((7*I)/2)*B*Cos
[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E
^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*S
in[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*
Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*
c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c])
 - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])
^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c]
)/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/(
(-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B +
A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + (10*A*Cos[c/2 + (d*x)/2]^4*Csc[c/
2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/
2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin
[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[C
ot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*Sqrt
[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) - (20*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[
d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Co
t[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqr
t[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*(A + B*Sec[
c + d*x])*((4*(-20*A + 15*B - 36*A*Cos[c] + 20*B*Cos[c])*Csc[c])/(5*d) + (8
*(-2*A + B)*Cos[d*x]*Sin[c])/(3*d) + (4*A*Cos[2*d*x]*Sin[2*c])/(5*d) - (2*S
ec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(3*d) +
(4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-4*A*Sin[(d*x)/2] + 3*B*Sin[(d*x)/2]))/d +
(8*(-2*A + B)*Cos[c]*Sin[d*x])/(3*d) + (4*A*Cos[2*c]*Sin[2*d*x])/(5*d) - (2
```

```
*(-A + B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(Sqrt[Cos[c + d*x]]*(B + A
*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [A]
time = 2.02, size = 465, normalized size = 2.28

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(96A\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-352A\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+80B\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/30*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(96*A*cos(1/2
*d*x+1/2*c)^10-352*A*cos(1/2*d*x+1/2*c)^8+80*B*cos(1/2*d*x+1/2*c)^8+120*A*c
os(1/2*d*x+1/2*c)^6-150*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-33
6*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+60*B*cos(1/2*d*x+1/2*c)^
6+100*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Elli
pticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+210*B*cos(1/2*d*x+1/
2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))+266*A*cos(1/2*d*x+1/2*c)^4-240*B*cos(1/2*d
*x+1/2*c)^4-135*A*cos(1/2*d*x+1/2*c)^2+105*B*cos(1/2*d*x+1/2*c)^2+5*A-5*B)/
a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.55, size = 384, normalized size = 1.88

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] 1/30*(2*(6*A*cos(d*x + c)^3 - 2*(4*A - 5*B)*cos(d*x + c)^2 - (94*A - 65*B)*
cos(d*x + c) - 75*A + 50*B)*sqrt(cos(d*x + c))*sin(d*x + c) - 25*(sqrt(2)*(
-3*I*A + 2*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(-3*I*A + 2*I*B)*cos(d*x + c) +
sqrt(2)*(-3*I*A + 2*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c)) - 25*(sqrt(2)*(3*I*A - 2*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(3*I*A -
2*I*B)*cos(d*x + c) + sqrt(2)*(3*I*A - 2*I*B))*weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c)) - 21*(sqrt(2)*(-8*I*A + 5*I*B)*cos(d*x + c)^2
 + 2*sqrt(2)*(-8*I*A + 5*I*B)*cos(d*x + c) + sqrt(2)*(-8*I*A + 5*I*B))*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
)) - 21*(sqrt(2)*(8*I*A - 5*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(8*I*A - 5*I*B)
*cos(d*x + c) + sqrt(2)*(8*I*A - 5*I*B))*weierstrassZeta(-4, 0, weierstrass
PInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*
a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^2, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^2, x)
```

3.503 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=171

$$-\frac{(7A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{5(2A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{5(2A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d}-\frac{(7A-}{}$$

```
[Out] -(7*A-4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/a^2/d+5/3*(2*A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-1/3*(7*A-4*B)*cos(
d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(1+cos(d*x+c))-1/3*(A-B)*cos(d*x+c)^(5/2)*sin
(d*x+c)/d/(a+a*cos(d*x+c))^2+5/3*(2*A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.26, antiderivative size = 171, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3033, 3056, 2827, 2719, 2715, 2720}

$$\frac{5(2A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(7A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(7A-4B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d(\cos(c+dx)+1)}+\frac{5(2A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d}-\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] -(((7*A - 4*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + (5*(2*A - B)*EllipticF
[(c + d*x)/2, 2])/(3*a^2*d) + (5*(2*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])
/(3*a^2*d) - ((7*A - 4*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*d*(1 + Co
s[c + d*x])) - ((A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*d*(a + a*Cos[c
+ d*x])^2)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+A\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(-\frac{5}{2}a(A-B)+\frac{3}{2}a(3A-\right.}{a+a\cos(c+dx)}}{3a^2}$$

$$= -\frac{(7A-4B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin}{3d(a+a\cos(c+dx}$$

$$= -\frac{(7A-4B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin}{3d(a+a\cos(c+dx}$$

$$= -\frac{(7A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{5(2A-B)\sqrt{\cos(c+dx)}\,\sin(c+}{3a^2d}$$

$$= -\frac{(7A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{5(2A-B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{5(2}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.84, size = 1352, normalized size = 7.91

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,
x]
```

```
[Out] (((-7*I)/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Se
c[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d
*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1
+ E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^(
(2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2
*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(C
os[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((
2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*
d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))
*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + ((2*I)*B*Cos[c/2
 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E^((2
*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c
])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[
c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])
/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (
2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]
*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^
(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)
*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Co
s[c + d*x])*(a + a*Sec[c + d*x])^2) - (20*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*H
ypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*S
ec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
 - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c
]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1
 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (10*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*
Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[
1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*(A + B*Sec[c
+ d*x])*((-4*(-3*A + 2*B - 4*A*Cos[c] + 2*B*Cos[c])*Csc[c])/d + (8*A*Cos[d*
x]*Sin[c])/(3*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*
Sin[(d*x)/2]))/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-3*A*Sin[(d*x)/2] +
2*B*Sin[(d*x)/2]))/d + (8*A*Cos[c]*Sin[d*x])/(3*d) + (2*(-A + B)*Sec[c/2 +
```

```
(d*x)/2]^2*Tan[c/2])/(3*d)))/(Sqrt[Cos[c + d*x]]*(B + A*Cos[c + d*x])*(a +
a*Sec[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 434 vs. $2(209) = 418$.
time = 2.30, size = 435, normalized size = 2.54

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(16A\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+12A\left(\cos^{6}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+20A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(16*A*cos(1/2*
d*x+1/2*c)^8+12*A*cos(1/2*d*x+1/2*c)^6+20*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1
/2*d*x+1/2*c)^3+42*A*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-24*B*co
s(1/2*d*x+1/2*c)^6-10*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-24*B
*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-48*A*cos(1/2*d*x+1/2*c)^4+3
8*B*cos(1/2*d*x+1/2*c)^4+21*A*cos(1/2*d*x+1/2*c)^2-15*B*cos(1/2*d*x+1/2*c)^
2-A+B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.87, size = 366, normalized size = 2.14

$$\frac{2\left(2A\cos\left(dx+c\right)^{2}+\left(13A-6B\right)\cos\left(dx+c\right)+10A-5B\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\left(\sqrt{2}\left(2iA-iB\right)\cos\left(dx+c\right)^{2}+2\sqrt{2}\left(2iA-iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(2iA-iB\right)\right){\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)-5\left(\sqrt{2}\left(-2iA+iB\right)\cos\left(dx+c\right)^{2}+2\sqrt{2}\left(-2iA+iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-2iA+iB\right)\right){\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-3\left(\sqrt{2}\left(7iA-4iB\right)\cos\left(dx+c\right)^{2}+2\sqrt{2}\left(7iA-4iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(7iA-4iB\right)\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-3\left(\sqrt{2}\left(-7iA+4iB\right)\cos\left(dx+c\right)^{2}+2\sqrt{2}\left(-7iA+4iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-7iA+4iB\right)\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{6\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] 1/6*(2*(2*A*cos(d*x + c)^2 + (13*A - 6*B)*cos(d*x + c) + 10*A - 5*B)*sqrt(c
os(d*x + c))*sin(d*x + c) - 5*(sqrt(2)*(2*I*A - I*B)*cos(d*x + c)^2 + 2*sqr
t(2)*(2*I*A - I*B)*cos(d*x + c) + sqrt(2)*(2*I*A - I*B))*weierstrassPInvers
e(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*(sqrt(2)*(-2*I*A + I*B)*cos(d*x
 + c)^2 + 2*sqrt(2)*(-2*I*A + I*B)*cos(d*x + c) + sqrt(2)*(-2*I*A + I*B))*w
eierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(sqrt(2)*(7*I*
A - 4*I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(7*I*A - 4*I*B)*cos(d*x + c) + sqrt(2
)*(7*I*A - 4*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) + I*sin(d*x + c))) - 3*(sqrt(2)*(-7*I*A + 4*I*B)*cos(d*x + c)^2 + 2*
sqrt(2)*(-7*I*A + 4*I*B)*cos(d*x + c) + sqrt(2)*(-7*I*A + 4*I*B))*weierstra
ssZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(
a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^2, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^2,x)
[Out] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^2, x)
```

3.504 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=137

$$\frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(5A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(5A-2B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d(1+\cos(c+dx))}-\frac{(A-B)\,c}{3d(a}$$

```
[Out] (4*A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/a^2/d-1/3*(5*A-2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-1/3*(A-B)*cos(d*x+c
)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2-1/3*(5*A-2*B)*sin(d*x+c)*cos(d*x+c)
^(1/2)/a^2/d/(1+cos(d*x+c))
```

Rubi [A]
time = 0.24, antiderivative size = 137, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3033, 3056, 2827, 2720, 2719}

$$-\frac{(5A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(5A-2B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(\cos(c+dx)+1)}-\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,x]
```

```
[Out] ((4*A - B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) - ((5*A - 2*B)*EllipticF[(c +
 d*x)/2, 2])/(3*a^2*d) - ((5*A - 2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a
^2*d*(1 + Cos[c + d*x])) - ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*d*(
a + a*Cos[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\begin{aligned}
\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{(a+a\sec(c+dx))^2}\,dx &= \int \frac{\cos^{\frac{3}{2}}(c+dx)(B+A\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx \\
&= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\sqrt{\cos(c+dx)}\,\left(-\frac{3}{2}a(A-B\right.}{a+a\cos(c}}{3a^2} \\
&= -\frac{(5A-2B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)}{3d(a+a\cos(c+} \\
&= -\frac{(5A-2B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d(1+\cos(c+dx))} - \frac{(A-B)\cos^{\frac{3}{2}}(c+dx)}{3d(a+a\cos(c+} \\
&= \frac{(4A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{(5A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - (5A
\end{aligned}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.71, size = 1318, normalized size = 9.62

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^2,
x]
```

```
[Out] ((2*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c +
 d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(
Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^(
(2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)
*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d
*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c]
 + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*
d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*
Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[
c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - ((I/2)*B*Cos[c/2 + (d
*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d
*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)
]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E
^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*
I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyp
ergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1
 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Cos[c +
 d*x])*(a + a*Sec[c + d*x])^2) + (10*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c
+ d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Ar
cTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]
*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Co
t[c]^2]*(a + a*Sec[c + d*x])^2) - (4*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c
+ d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Ar
cTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]
*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Co
t[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*(A + B*Sec[c + d*x]
)*((-4*(2*A - B + 2*A*Cos[c])*Csc[c])/d + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-
2*A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-
(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(3*d) - (2*(-A + B)*Sec[c/2 + (d*x)/2]^
2*Tan[c/2])/(3*d)))/(Sqrt[Cos[c + d*x]]*(B + A*Cos[c + d*x])*(a + a*Sec[c +
 d*x])^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 420 vs. $2(179) = 358$.
time = 2.05, size = 421, normalized size = 3.07

method	result

default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(24A\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(24*A*cos(1/2*d
*x+1/2*c)^6+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+24*A*cos(1/
2*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^6-4*B*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/
2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-6*B*cos(1/2*d*x+1/2*c)^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))-38*A*cos(1/2*d*x+1/2*c)^4+20*B*cos(1/2*d*x+1/2*c)^4+15*A
*cos(1/2*d*x+1/2*c)^2-9*B*cos(1/2*d*x+1/2*c)^2-A+B)/a^2/cos(1/2*d*x+1/2*c)^
3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(
2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^2, x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.82, size = 356, normalized size = 2.60

$$-\frac{2\left(3\left(2A-B\right)\cos\left(dx+c\right)+5A-2B\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\left(\sqrt{2}\left(5iA-2iB\right)\cos\left(dx+c\right)^2-2\sqrt{2}\left(-5iA+2iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(5iA-2iB\right)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)-\left(\sqrt{2}\left(-5iA+2iB\right)\cos\left(dx+c\right)^2-2\sqrt{2}\left(5iA-2iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-5iA+2iB\right)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+3\left(\sqrt{2}\left(-4iA+iB\right)\cos\left(dx+c\right)^2+2\sqrt{2}\left(-4iA+iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-4iA+iB\right)\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+3\left(\sqrt{2}\left(4iA-iB\right)\cos\left(dx+c\right)^2+2\sqrt{2}\left(4iA-iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(4iA-iB\right)\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{6\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] -1/6*(2*(3*(2*A - B)*cos(d*x + c) + 5*A - 2*B)*sqrt(cos(d*x + c))*sin(d*x +
 c) - (sqrt(2)*(5*I*A - 2*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(-5*I*A + 2*I*B)*
```

```
cos(d*x + c) + sqrt(2)*(5*I*A - 2*I*B))*weierstrassPInverse(-4, 0, cos(d*x
+ c) + I*sin(d*x + c)) - (sqrt(2)*(-5*I*A + 2*I*B)*cos(d*x + c)^2 - 2*sqrt(
2)*(5*I*A - 2*I*B)*cos(d*x + c) + sqrt(2)*(-5*I*A + 2*I*B))*weierstrassPInv
erse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*(sqrt(2)*(-4*I*A + I*B)*cos(
d*x + c)^2 + 2*sqrt(2)*(-4*I*A + I*B)*cos(d*x + c) + sqrt(2)*(-4*I*A + I*B)
)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c))) + 3*(sqrt(2)*(4*I*A - I*B)*cos(d*x + c)^2 + 2*sqrt(2)*(4*I*A - I*B
)*cos(d*x + c) + sqrt(2)*(4*I*A - I*B))*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a
^2*d*cos(d*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sqrt{\cos\left(c+dx\right)}}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx+\int \frac{B\sqrt{\cos\left(c+dx\right)}\sec\left(c+dx\right)}{\sec^2\left(c+dx\right)+2\sec\left(c+dx\right)+1}\,dx}{a^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] (Integral(A*sqrt(cos(c + d*x))/(sec(c + d*x)**2 + 2*sec(c + d*x) + 1), x) +
 Integral(B*sqrt(cos(c + d*x))*sec(c + d*x)/(sec(c + d*x)**2 + 2*sec(c + d*
x) + 1), x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^2, x)
```

3.505 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=121

$$-\frac{AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{A\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))}-\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(}{3d(a+a\cos(c+dx))}$$

```
[Out] -A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2))/a^2/d+1/3*(2*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+A*sin(d*x+c)*cos(d*x+c)^(1
/2)/a^2/d/(1+cos(d*x+c))-1/3*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d
*x+c))^2
```

Rubi [A]
time = 0.23, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3033, 3056, 3057, 2827, 2720, 2719}

$$\frac{(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{A\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] -((A*EllipticE[(c + d*x)/2, 2])/(a^2*d)) + ((2*A + B)*EllipticF[(c + d*x)/2
, 2])/(3*a^2*d) + (A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d
*x])) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])
^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos(c+dx)}\,(B+A\cos(c+dx))}{(a+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{-\frac{1}{2}a(A-B)+\frac{1}{2}a(5A+B)\cos(}{\sqrt{\cos(c+dx)}\,(a+a\cos(}}{3a^2}$$

$$= \frac{A\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+d}{3d(a+a\cos(c+dx))^2}$$

$$= \frac{A\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))} - \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+d}{3d(a+a\cos(c+dx))^2}$$

$$= -\frac{AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(2A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} + \frac{A\sqrt{\cos(c+}}{a^2d(1+}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.57, size = 921, normalized size = 7.61

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^2),
x]
```

```
[Out] ((-1/2*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[
c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x
)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2
*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I
)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos
[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*
I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*
x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*S
in[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - (4*A*Cos[c/2 + (d*
x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[
c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Co
s[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) - (2*B*Cos[c/2 + (d*
x)/2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[
c]]]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]
```

```
]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[
d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Co
s[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2
]^4*(A + B*Sec[c + d*x])*((4*A*Csc[c])/d + (4*A*Sec[c/2]*Sec[c/2 + (d*x)/2]
*Sin[(d*x)/2])/d + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*
Sin[(d*x)/2]))/(3*d) + (2*(-A + B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(
Sqrt[Cos[c + d*x]]*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(165) = 330$.
time = 1.86, size = 350, normalized size = 2.89

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(12A\left(\cos^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+4A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^6+4*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3+6*A*cos(1/2
*d*x+1/2*c)^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1
/2*d*x+1/2*c)^3-20*A*cos(1/2*d*x+1/2*c)^4+2*B*cos(1/2*d*x+1/2*c)^4+9*A*cos(
1/2*d*x+1/2*c)^2-3*B*cos(1/2*d*x+1/2*c)^2-A+B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.01, size = 314, normalized size = 2.60

$$\frac{2\left(3A\cos\left(dx+c\right)+2A+B\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(\sqrt{2}\left(-2iA-iB\right)\cos\left(dx+c\right)^{2}-2\sqrt{2}\left(2iA+iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-2iA-iB\right)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+\left(\sqrt{2}\left(2iA+iB\right)\cos\left(dx+c\right)^{2}-2\sqrt{2}\left(-2iA-iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(2iA+iB\right)\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-3\left(i\sqrt{2}A\cos\left(dx+c\right)^{2}+2i\sqrt{2}A\cos\left(dx+c\right)+i\sqrt{2}A\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-3\left(-i\sqrt{2}A\cos\left(dx+c\right)^{2}-2i\sqrt{2}A\cos\left(dx+c\right)-i\sqrt{2}A\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{6\left(a^{2}d\cos\left(dx+c\right)^{2}+2a^{2}d\cos\left(dx+c\right)+a^{2}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/6*(2*(3*A*cos(d*x + c) + 2*A + B)*sqrt(cos(d*x + c))*sin(d*x + c) + (sqrt
(2)*(-2*I*A - I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(2*I*A + I*B)*cos(d*x + c) +
sqrt(2)*(-2*I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x
 + c)) + (sqrt(2)*(2*I*A + I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(-2*I*A - I*B)*c
os(d*x + c) + sqrt(2)*(2*I*A + I*B))*weierstrassPInverse(-4, 0, cos(d*x + c
) - I*sin(d*x + c)) - 3*(I*sqrt(2)*A*cos(d*x + c)^2 + 2*I*sqrt(2)*A*cos(d*x
 + c) + I*sqrt(2)*A)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(
d*x + c) + I*sin(d*x + c))) - 3*(-I*sqrt(2)*A*cos(d*x + c)^2 - 2*I*sqrt(2)*
A*cos(d*x + c) - I*sqrt(2)*A)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d
*x + c) + a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sqrt{\cos\left(c+dx\right)}\sec^{2}\left(c+dx\right)+2\sqrt{\cos\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{\cos\left(c+dx\right)}}\,dx+\int \frac{B\sec\left(c+dx\right)}{\sqrt{\cos\left(c+dx\right)}\sec^{2}\left(c+dx\right)+2\sqrt{\cos\left(c+dx\right)}\sec\left(c+dx\right)+\sqrt{\cos\left(c+dx\right)}}\,dx}{a^{2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)
```

```
[Out] (Integral(A/(sqrt(cos(c + d*x))*sec(c + d*x)**2 + 2*sqrt(cos(c + d*x))*sec(
c + d*x) + sqrt(cos(c + d*x))), x) + Integral(B*sec(c + d*x)/(sqrt(cos(c +
d*x))*sec(c + d*x)**2 + 2*sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*
x))), x))/a**2
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))),
 x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\sqrt{\cos\left(c+d\,x\right)}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^2),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^2), x)
```

3.506 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=121

$$\frac{BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))}+\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2}$$

```
[Out] B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2
*c),2^(1/2))/a^2/d+1/3*(A+2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-B*sin(d*x+c)*cos(d*x+c)^(1/
2)/a^2/d/(1+cos(d*x+c))+1/3*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*
x+c))^2
```

Rubi [A]

time = 0.23, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3033, 3057, 2827, 2720, 2719}

$$\frac{(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{B\sin(c+dx)\sqrt{\cos(c+dx)}}{a^2d(\cos(c+dx)+1)}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] (B*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((A + 2*B)*EllipticF[(c + d*x)/2, 2
])/(3*a^2*d) - (B*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a^2*d*(1 + Cos[c + d*x]
)) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*(a + a*Cos[c + d*x])^2)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{B+A\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2}\,dx$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d(a+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}a(A+5B)+\frac{1}{2}a(A-B)\cos(c+d}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+}}{3a^2}$$

$$= -\frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))} + \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+}{3d(a+a\cos(c+dx))^2}$$

$$= -\frac{B\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a^2d(1+\cos(c+dx))} + \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+}{3d(a+a\cos(c+dx))^2}$$

$$= \frac{BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(A+2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{B\sqrt{\cos(c+d}}{a^2d(1+\mathrm{co}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.59, size = 921, normalized size = 7.61

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^2),
x]
```

```
[Out] ((I/2)*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c +
 d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(
Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^(
(2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)
*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d
*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c]
 + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*
d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*
Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[
c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - (2*A*Cos[c/2 + (d*x)/
2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]
]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c
 + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) - (4*B*Cos[c/2 + (d*x)/
2]^4*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]
]^2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c
 + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4
*(A + B*Sec[c + d*x])*((-4*B*Csc[c])/d - (4*B*Sec[c/2]*Sec[c/2 + (d*x)/2]*S
in[(d*x)/2])/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*Si
n[(d*x)/2]))/(3*d) - (2*(-A + B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(3*d)))/(Sq
rt[Cos[c + d*x]]*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 349 vs. $2(165) = 330$.
time = 2.19, size = 350, normalized size = 2.89

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \text{EllipticF}}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/6*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-12*B*cos(1/2*d*x+1/2*c)^6+4*B*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d
```

```
*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^3-6*B*cos(1/2*d*x+1/2*c)^3*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+
1/2*c),2^(1/2))+2*A*cos(1/2*d*x+1/2*c)^4+16*B*cos(1/2*d*x+1/2*c)^4-3*A*cos(
1/2*d*x+1/2*c)^2-3*B*cos(1/2*d*x+1/2*c)^2+A-B)/a^2/cos(1/2*d*x+1/2*c)^3/(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos
(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.88, size = 318, normalized size = 2.63

$$-\frac{2\,(3\,B\cos(dx+c)-A+4\,B)\sqrt{\cos(dx+c)}\,\sin(dx+c)-\left(\sqrt{2}\,(-i\,A-2i\,B)\cos(dx+c)^2-2\,\sqrt{2}\,(i\,A+2i\,B)\cos(dx+c)+\sqrt{2}\,(-i\,A-2i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))-\left(\sqrt{2}\,(i\,A+2i\,B)\cos(dx+c)^2-2\,\sqrt{2}\,(-i\,A-2i\,B)\cos(dx+c)+\sqrt{2}\,(i\,A+2i\,B)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+3\left(-i\,\sqrt{2}\,B\cos(dx+c)^2-2i\,\sqrt{2}\,B\cos(dx+c)-i\,\sqrt{2}\,B\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+3\left(i\,\sqrt{2}\,B\cos(dx+c)^2+2i\,\sqrt{2}\,B\cos(dx+c)+i\,\sqrt{2}\,B\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))}{6\,(a^2d\cos(dx+c)^2+2\,a^2d\cos(dx+c)+a^2d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] -1/6*(2*(3*B*cos(d*x + c) - A + 4*B)*sqrt(cos(d*x + c))*sin(d*x + c) - (sqr
t(2)*(-I*A - 2*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(I*A + 2*I*B)*cos(d*x + c) +
 sqrt(2)*(-I*A - 2*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*
x + c)) - (sqrt(2)*(I*A + 2*I*B)*cos(d*x + c)^2 - 2*sqrt(2)*(-I*A - 2*I*B)*
cos(d*x + c) + sqrt(2)*(I*A + 2*I*B))*weierstrassPInverse(-4, 0, cos(d*x +
c) - I*sin(d*x + c)) + 3*(-I*sqrt(2)*B*cos(d*x + c)^2 - 2*I*sqrt(2)*B*cos(d
*x + c) - I*sqrt(2)*B)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, co
s(d*x + c) + I*sin(d*x + c))) + 3*(I*sqrt(2)*B*cos(d*x + c)^2 + 2*I*sqrt(2)
*B*cos(d*x + c) + I*sqrt(2)*B)*weierstrassZeta(-4, 0, weierstrassPInverse(-
4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(
d*x + c) + a^2*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^{3/2}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^2), x)
```

3.507 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=164

$$\frac{(A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(2A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(A-4B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{(2A-5B)\sin(c+}{3a^2d\sqrt{\cos(c+dx)}\,(1+c}$$

```
[Out] (A-4*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d
*x+1/2*c),2^(1/2))/a^2/d+1/3*(2*A-5*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-(A-4*B)*sin(d*x+c)/
a^2/d/cos(d*x+c)^(1/2)+1/3*(2*A-5*B)*sin(d*x+c)/a^2/d/(1+cos(d*x+c))/cos(d*
x+c)^(1/2)+1/3*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 164, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3033, 3057, 2827, 2716, 2719, 2720}

$$\frac{(2A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}+\frac{(A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{(A-4B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{(2A-5B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(\cos(c+dx)+1)}+\frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] ((A - 4*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + ((2*A - 5*B)*EllipticF[(c +
 d*x)/2, 2])/(3*a^2*d) - ((A - 4*B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]
) + ((2*A - 5*B)*Sin[c + d*x])/(3*a^2*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x
])) + ((A - B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2
)
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{B+A\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^2} + \frac{\int \frac{-\frac{1}{2}a(A-7B)+\frac{3}{2}a(A-B)\cos(c+}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))}}{3a^2}$$

$$= \frac{(2A-5B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(1+\cos(c+dx))} + \frac{(A-B)\sin(c+d}{3d\sqrt{\cos(c+dx)}\,(a+a\,co}$$

$$= \frac{(2A-5B)\sin(c+dx)}{3a^2d\sqrt{\cos(c+dx)}\,(1+\cos(c+dx))} + \frac{(A-B)\sin(c+d}{3d\sqrt{\cos(c+dx)}\,(a+a\,co}$$

$$= \frac{(2A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{(A-4B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{(2A}{3a^2d\sqrt{\cos(}}$$

$$= \frac{(A-4B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{(2A-5B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{(A-4}{a^2d\sqrt{}}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.85, size = 1351, normalized size = 8.24

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^2),
x]
```

```
[Out] ((I/2)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c +
 d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(
Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^(
(2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)
*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d
*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c]
 + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*
d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*
Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[
c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) - ((2*I)*B*Cos[c/2 + (d
*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E^((2*I)*d
*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)
]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E
^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*
I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hyp
ergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt
[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*
x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1
 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Cos[c +
 d*x])*(a + a*Sec[c + d*x])^2) - (4*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hyperge
ometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c +
 d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Arc
Tan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*
Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot
[c]^2]*(a + a*Sec[c + d*x])^2) + (10*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Hyperg
eometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c
+ d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - Ar
cTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]
*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Co
t[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*(A + B*Sec[c + d*x]
)*((2*(2*B - A*Cos[c] + 2*B*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/d + (2*Sec[c/
2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(3*d) + (4*Se
c[c/2]*Sec[c/2 + (d*x)/2]*(-(A*Sin[(d*x)/2]) + 2*B*Sin[(d*x)/2]))/d + (8*B*
Sec[c]*Sec[c + d*x]*Sin[d*x])/d + (2*(-A + B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2]
)/(3*d)))/(Sqrt[Cos[c + d*x]]*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 491 vs. $2(204) = 408$.
time = 2.63, size = 492, normalized size = 3.00

method	result
default	$\frac{2\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\left(2A\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/6*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1
/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-5*B*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+12*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2
*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2-2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*
(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))-5*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+1/2*c)-12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*(A-4*B)*sin(1/2*d*x+1/2*c)^6+2*(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(10*A-43*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(7*A-37*B)*sin(1/2*d*x+1/2*c)^2)/a
^2/cos(1/2*d*x+1/2*c)^3/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.61, size = 407, normalized size = 2.48

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] -1/6*(2*(3*(A - 4*B)*cos(d*x + c)^2 + (4*A - 19*B)*cos(d*x + c) - 6*B)*sqrt
(cos(d*x + c))*sin(d*x + c) - (sqrt(2)*(-2*I*A + 5*I*B)*cos(d*x + c)^3 - 2*
sqrt(2)*(2*I*A - 5*I*B)*cos(d*x + c)^2 + sqrt(2)*(-2*I*A + 5*I*B)*cos(d*x +
 c))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - (sqrt(2)*(
2*I*A - 5*I*B)*cos(d*x + c)^3 - 2*sqrt(2)*(-2*I*A + 5*I*B)*cos(d*x + c)^2 +
 sqrt(2)*(2*I*A - 5*I*B)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x +
 c) - I*sin(d*x + c)) + 3*(sqrt(2)*(-I*A + 4*I*B)*cos(d*x + c)^3 + 2*sqrt(2
)*(-I*A + 4*I*B)*cos(d*x + c)^2 + sqrt(2)*(-I*A + 4*I*B)*cos(d*x + c))*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)
)) + 3*(sqrt(2)*(I*A - 4*I*B)*cos(d*x + c)^3 + 2*sqrt(2)*(I*A - 4*I*B)*cos(
d*x + c)^2 + sqrt(2)*(I*A - 4*I*B)*cos(d*x + c))*weierstrassZeta(-4, 0, wei
erstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c
)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5009 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^2), x)
```

3.508 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx$

Optimal. Leaf size=197

$$-\frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}-\frac{5(A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{5(A-2B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{(4A-7B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}$$

```
[Out] -(4*A-7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/a^2/d-5/3*(A-2*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d-5/3*(A-2*B)*sin(d*
x+c)/a^2/d/cos(d*x+c)^(3/2)+1/3*(4*A-7*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(3/2)
/(1+cos(d*x+c))+1/3*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*cos(d*x+c))^2+
(4*A-7*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.27, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3033, 3057, 2827, 2716, 2720, 2719}

$$-\frac{5(A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d}-\frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{(4A-7B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(\cos(c+dx)+1)}-\frac{5(A-2B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{(4A-7B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}}+\frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2),x]
```

```
[Out] -(((4*A - 7*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d)) - (5*(A - 2*B)*EllipticF
[(c + d*x)/2, 2])/(3*a^2*d) - (5*(A - 2*B)*Sin[c + d*x])/(3*a^2*d*Cos[c + d
*x]^(3/2)) + ((4*A - 7*B)*Sin[c + d*x])/(a^2*d*Sqrt[Cos[c + d*x]]) + ((4*A
- 7*B)*Sin[c + d*x])/(3*a^2*d*Cos[c + d*x]^(3/2)*(1 + Cos[c + d*x])) + ((A
- B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)*(a + a*Cos[c + d*x])^2)
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^2}\,dx = \int \frac{B+A\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))^2}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^2} + \frac{\int \frac{-\frac{3}{2}a(A-3B)+\frac{5}{2}a(A-B)\cos(c+dx}{\cos^{\frac{5}{2}}(c+dx)(a+a\cos(c+dx))}}{3a^2}$$

$$= \frac{(4A-7B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))} + \frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c}$$

$$= \frac{(4A-7B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)(1+\cos(c+dx))} + \frac{(A-B)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c}$$

$$= -\frac{5(A-2B)\sin(c+dx)}{3a^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4A-7B)\sin(c+dx)}{a^2d\sqrt{\cos(c+dx)}} + \frac{(4A-}{3a^2d\cos^{\frac{3}{2}}(c+}$$

$$= -\frac{(4A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} - \frac{5(A-2B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^2d} - \frac{5(A}{3}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.49, size = 1392, normalized size = 7.07

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^2),
x]
```

```
[Out] ((-2*I)*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c
+ d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*
(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^
((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I
)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*
d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c
] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)
*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)
*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin
[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^2) + (((7*I)/2)*B*Cos[c/2
 + (d*x)/2]^4*Csc[c/2]*Sec[c/2]*Sec[c + d*x]*(A + B*Sec[c + d*x])*((2*E^((2
*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c
])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[
c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])
```

```
/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (
2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]
*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^
(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)
*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Co
s[c + d*x])*(a + a*Sec[c + d*x])^2) + (10*A*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*H
ypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*S
ec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x
 - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c
]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1
 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) - (20*B*Cos[c/2 + (d*x)/2]^4*Csc[c/2]*
HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*
Sec[c + d*x]*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*
x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[
c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[
1 + Cot[c]^2]*(a + a*Sec[c + d*x])^2) + (Cos[c/2 + (d*x)/2]^4*(A + B*Sec[c
+ d*x])*((-2*(-2*A + 4*B - 2*A*Cos[c] + 3*B*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c
])/d - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]
))/(3*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-2*A*Sin[(d*x)/2] + 3*B*Sin[(d*x
)/2]))/d + (8*B*Sec[c]*Sec[c + d*x]^2*Sin[d*x])/(3*d) + (8*Sec[c]*Sec[c + d
*x]*(B*Sin[c] + 3*A*Sin[d*x] - 6*B*Sin[d*x]))/(3*d) - (2*(-A + B)*Sec[c/2 +
 (d*x)/2]^2*Tan[c/2])/(3*d)))/(Sqrt[Cos[c + d*x]]*(B + A*Cos[c + d*x])*(a +
 a*Sec[c + d*x])^2)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(233) = 466$.
time = 5.57, size = 723, normalized size = 3.67

method	result	size
default	Expression too large to display	723

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/2*(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)/a^2*((-2*A+4
*B)*(cos(1/2*d*x+1/2*c)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-EllipticE(cos(1/2*d*x+1/2
*c),2^(1/2)))-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)/cos(1/2*d*x+1/2*
c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+4*B*(-1/6*cos(1/2*d
*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x
+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2)))+1/3*(-A+B)*(2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*Ellip
```

```
ticE(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-3*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1
/2*d*x+1/2*c)-12*sin(1/2*d*x+1/2*c)^6+20*sin(1/2*d*x+1/2*c)^4-7*sin(1/2*d*x
+1/2*c)^2)/cos(1/2*d*x+1/2*c)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)/(sin(1/2*d*x+1/2*c)^2-1)+(4*A-8*B)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*
d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin
(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x
+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.26, size = 436, normalized size = 2.21

$$\frac{2\left(3\left(4A-7B\right)\cos\left(dx+c\right)^{3}+\left(19A-32B\right)\cos\left(dx+c\right)^{2}+2\left(3A-4B\right)\cos\left(dx+c\right)+2B\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-5\left(\sqrt{2}\left(-iA+2iB\right)\cos\left(dx+c\right)^{4}+2\sqrt{2}\left(-iA+2iB\right)\cos\left(dx+c\right)^{3}+\sqrt{2}\left(-iA+2iB\right)\cos\left(dx+c\right)^{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)-5\left(\sqrt{2}\left(iA-2iB\right)\cos\left(dx+c\right)^{4}+2\sqrt{2}\left(iA-2iB\right)\cos\left(dx+c\right)^{3}+\sqrt{2}\left(iA-2iB\right)\cos\left(dx+c\right)^{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-3\left(\sqrt{2}\left(4iA-7iB\right)\cos\left(dx+c\right)^{4}+2\sqrt{2}\left(4iA-7iB\right)\cos\left(dx+c\right)^{3}+\sqrt{2}\left(4iA-7iB\right)\cos\left(dx+c\right)^{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-3\left(\sqrt{2}\left(-4iA+7iB\right)\cos\left(dx+c\right)^{4}+2\sqrt{2}\left(-4iA+7iB\right)\cos\left(dx+c\right)^{3}+\sqrt{2}\left(-4iA+7iB\right)\cos\left(dx+c\right)^{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{6\left(a^{2}d\cos\left(dx+c\right)^{4}+2a^{2}d\cos\left(dx+c\right)^{3}+a^{2}d\cos\left(dx+c\right)^{2}\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] 1/6*(2*(3*(4*A - 7*B)*cos(d*x + c)^3 + (19*A - 32*B)*cos(d*x + c)^2 + 2*(3*
A - 4*B)*cos(d*x + c) + 2*B)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(sqrt(2)*(
-I*A + 2*I*B)*cos(d*x + c)^4 + 2*sqrt(2)*(-I*A + 2*I*B)*cos(d*x + c)^3 + sq
rt(2)*(-I*A + 2*I*B)*cos(d*x + c)^2)*weierstrassPInverse(-4, 0, cos(d*x + c
) + I*sin(d*x + c)) - 5*(sqrt(2)*(I*A - 2*I*B)*cos(d*x + c)^4 + 2*sqrt(2)*(
I*A - 2*I*B)*cos(d*x + c)^3 + sqrt(2)*(I*A - 2*I*B)*cos(d*x + c)^2)*weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(sqrt(2)*(4*I*A - 7*
I*B)*cos(d*x + c)^4 + 2*sqrt(2)*(4*I*A - 7*I*B)*cos(d*x + c)^3 + sqrt(2)*(4
*I*A - 7*I*B)*cos(d*x + c)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4
, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(sqrt(2)*(-4*I*A + 7*I*B)*cos(d*x
+ c)^4 + 2*sqrt(2)*(-4*I*A + 7*I*B)*cos(d*x + c)^3 + sqrt(2)*(-4*I*A + 7*I*
B)*cos(d*x + c)^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))))/(a^2*d*cos(d*x + c)^4 + 2*a^2*d*cos(d*x + c)^3 +
 a^2*d*cos(d*x + c)^2)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2)),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^{7/2}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^2),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^2), x)
```

3.509 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$-\frac{7(17A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(33A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(33A-13B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{6a^3d}-($$

```
[Out] -7/10*(17*A-7*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(
sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(33*A-13*B)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*(A-B
)*cos(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-1/3*(2*A-B)*cos(d*x+c)^(
5/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2-7/30*(17*A-7*B)*cos(d*x+c)^(3/2)*sin
(d*x+c)/d/(a^3+a^3*cos(d*x+c))+1/6*(33*A-13*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/
a^3/d
```

Rubi [A]
time = 0.37, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3033, 3056, 2827, 2719, 2715, 2720}

$$\frac{(33A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{7(17A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{7(17A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{30d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(33A-13B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6a^3d}-\frac{(A-B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{(2A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{3ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] (-7*(17*A - 7*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((33*A - 13*B)*Ell
ipticF[(c + d*x)/2, 2])/(6*a^3*d) + ((33*A - 13*B)*Sqrt[Cos[c + d*x]]*Sin[c
 + d*x])/(6*a^3*d) - ((A - B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(5*d*(a + a*
Cos[c + d*x])^3) - ((2*A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(3*a*d*(a +
a*Cos[c + d*x])^2) - (7*(17*A - 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(30*d
*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos^{\frac{7}{2}}(c+dx)(B+A\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{5}{2}}(c+dx)(-\frac{7}{2}a(A-B)+\frac{1}{2}a(13A}{(a+a\cos(c+dx))^2}}{5a^2}$$

$$= -\frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c}{3ad(a+a\cos(c+dx))}$$

$$= -\frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c}{3ad(a+a\cos(c+dx))}$$

$$= -\frac{(A-B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(2A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c}{3ad(a+a\cos(c+dx))}$$

$$= -\frac{7(17A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(33A-13B)\sqrt{\cos(c+dx)}\ \sin}{6a^3d}$$

$$= -\frac{7(17A-7B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(33A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} +$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.07, size = 1448, normalized size = 6.55

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,
x]
```

```
[Out] (((-119*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A +
 B*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2
*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)
*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] +
I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 +
E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*
x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
 E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((
2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*
d*x))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (((49*I)/10
)*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*
x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos
```

```
[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*
I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*
x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x)
)*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] +
I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x
))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin
[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])
))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (22*A*Cos[c/2 + (d*x)/2]
^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^
2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*S
qrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
 - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c +
 d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (26*B*Cos[c/2 + (d*x)/2
]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]
^2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*
Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*
x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[
c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^
6*(A + B*Sec[c + d*x])*((-4*(-59*A + 29*B - 60*A*Cos[c] + 20*B*Cos[c])*Csc[
c])/(5*d) + (16*A*Cos[d*x]*Sin[c])/(3*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5
*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/
2]^3*(-19*A*Sin[(d*x)/2] + 14*B*Sin[(d*x)/2]))/(15*d) - (4*Sec[c/2]*Sec[c/2
 + (d*x)/2]*(-59*A*Sin[(d*x)/2] + 29*B*Sin[(d*x)/2]))/(5*d) + (16*A*Cos[c]*
Sin[d*x])/(3*d) + (4*(-19*A + 14*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) -
 (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Cos[c + d*x]^(3/2)*(B
+ A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [A]
time = 2.40, size = 465, normalized size = 2.10

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(160A\left(\cos^{10}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+468A\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+330A\sqrt{\frac{1}{2}-\frac{\cos(d}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(160*A*cos(1/
2*d*x+1/2*c)^10+468*A*cos(1/2*d*x+1/2*c)^8+330*A*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
cos(1/2*d*x+1/2*c)^5+714*A*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3
```

```
48*B*cos(1/2*d*x+1/2*c)^8-130*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c
)^5-294*B*cos(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1058*A*cos(1/2*d*x
+1/2*c)^6+578*B*cos(1/2*d*x+1/2*c)^6+474*A*cos(1/2*d*x+1/2*c)^4-264*B*cos(1
/2*d*x+1/2*c)^4-47*A*cos(1/2*d*x+1/2*c)^2+37*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B
)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.30, size = 478, normalized size = 2.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] 1/60*(2*(20*A*cos(d*x + c)^3 + 3*(79*A - 29*B)*cos(d*x + c)^2 + 2*(188*A -
73*B)*cos(d*x + c) + 165*A - 65*B)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(sqr
t(2)*(33*I*A - 13*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(33*I*A - 13*I*B)*cos(d*x
 + c)^2 + 3*sqrt(2)*(33*I*A - 13*I*B)*cos(d*x + c) + sqrt(2)*(33*I*A - 13*I
*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*(sqrt(2)
*(-33*I*A + 13*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-33*I*A + 13*I*B)*cos(d*x +
 c)^2 + 3*sqrt(2)*(-33*I*A + 13*I*B)*cos(d*x + c) + sqrt(2)*(-33*I*A + 13*I
*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*(sqrt(2
)*(17*I*A - 7*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(17*I*A - 7*I*B)*cos(d*x + c)
^2 + 3*sqrt(2)*(17*I*A - 7*I*B)*cos(d*x + c) + sqrt(2)*(17*I*A - 7*I*B))*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 21*(sqrt(2)*(-17*I*A + 7*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-17*I*A +
7*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-17*I*A + 7*I*B)*cos(d*x + c) + sqrt(2)*
(-17*I*A + 7*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 +
 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^3, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^3, x)
```

3.510 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=188

$$\frac{(49A-9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(13A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(8A-3B)\,c}{15ad(c}$$

```
[Out] 1/10*(49*A-9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/6*(13*A-3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*(A-B)*
cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-1/15*(8*A-3*B)*cos(d*x+c)^
(3/2)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2-1/6*(13*A-3*B)*sin(d*x+c)*cos(d*x+c
)^(1/2)/d/(a^3+a^3*cos(d*x+c))
```

Rubi [A]
time = 0.35, antiderivative size = 188, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.152, Rules used = {3033, 3056, 2827, 2720, 2719}

$$-\frac{(13A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(49A-9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(13A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6d\,(a^3\cos(c+dx)+a^3)}-\frac{(A-B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{(8A-3B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{15ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,x]
```

```
[Out] ((49*A - 9*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) - ((13*A - 3*B)*Ellipti
cF[(c + d*x)/2, 2])/(6*a^3*d) - ((A - B)*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(
5*d*(a + a*Cos[c + d*x])^3) - ((8*A - 3*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])
/(15*a*d*(a + a*Cos[c + d*x])^2) - ((13*A - 3*B)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(6*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+A\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\cos^{\frac{3}{2}}(c+dx)\left(-\frac{5}{2}a(A-B)+\frac{1}{2}a\right.}{(a+a\cos(c+d}}{5a^2}$$

$$= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(8A-3B)\cos^{\frac{3}{2}}(c+dx)}{15ad(a+a\cos(c+}$$

$$= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(8A-3B)\cos^{\frac{3}{2}}(c+dx)}{15ad(a+a\cos(c+}$$

$$= -\frac{(A-B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(8A-3B)\cos^{\frac{3}{2}}(c+dx)}{15ad(a+a\cos(c+}$$

$$= \frac{(49A-9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} - \frac{(13A-3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} -$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.95, size = 1415, normalized size = 7.53

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^3,
x]
```

```
[Out] (((49*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B
*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I
)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(
-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*
E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^
((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)
*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E
^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*
I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*
x))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (((9*I)/10)*B
*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])
*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c]
 + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*
d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*
Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*S
in[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*S
in[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*
Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*
c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/
((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (26*A*Cos[c/2 + (d*x)/2]^6*
Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*
Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt
[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x -
ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c +
d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) - (2*B*Cos[c/2 + (d*x)/2]^
6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2
]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sq
rt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x
- ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c +
d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A
 + B*Sec[c + d*x])*((-4*(29*A - 9*B + 20*A*Cos[c])*Csc[c])/(5*d) + (2*Sec[c
/2]*Sec[c/2 + (d*x)/2]^5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) + (4*S
ec[c/2]*Sec[c/2 + (d*x)/2]*(-29*A*Sin[(d*x)/2] + 9*B*Sin[(d*x)/2]))/(5*d) -
 (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-14*A*Sin[(d*x)/2] + 9*B*Sin[(d*x)/2]))/
(15*d) - (4*(-14*A + 9*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(-A +
B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Cos[c + d*x]^(3/2)*(B + A*Cos[c
+ d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(224) = 448$.
time = 2.35, size = 451, normalized size = 2.40

method	result
default	$\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(348A\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+130A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{dx}{2}\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(348*A*cos(1/2
*d*x+1/2*c)^8+130*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+294*A*co
s(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-108*B*cos(1/2*d*x+1/2*c)^8-30*
B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-54*B*cos(1/2*d*x+1/2*c)^5*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(co
s(1/2*d*x+1/2*c),2^(1/2))-578*A*cos(1/2*d*x+1/2*c)^6+198*B*cos(1/2*d*x+1/2*
c)^6+264*A*cos(1/2*d*x+1/2*c)^4-114*B*cos(1/2*d*x+1/2*c)^4-37*A*cos(1/2*d*x
+1/2*c)^2+27*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

<u>Maxima</u> **[F]**
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^3, x
)
```

<u>Fricas</u> **[C]** Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.72, size = 467, normalized size = 2.48

$$-\frac{2\left(3\left(29A-9B\right)\cos\left(dx+c\right)^{2}+2\left(73A-18B\right)\cos\left(dx+c\right)+65A-15B\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+5\left(\sqrt{2}\left(-13iA+3iB\right)\cos\left(dx+c\right)^{3}+3\sqrt{2}\left(-13iA+3iB\right)\cos\left(dx+c\right)^{2}+3\sqrt{2}\left(-13iA+3iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-13iA+3iB\right)\right){\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+5\left(\sqrt{2}\left(13iA-3iB\right)\cos\left(dx+c\right)^{3}+3\sqrt{2}\left(13iA-3iB\right)\cos\left(dx+c\right)^{2}+3\sqrt{2}\left(13iA-3iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(13iA-3iB\right)\right){\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+3\left(\sqrt{2}\left(-49iA+9iB\right)\cos\left(dx+c\right)^{3}+3\sqrt{2}\left(-49iA+9iB\right)\cos\left(dx+c\right)^{2}+3\sqrt{2}\left(-49iA+9iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-49iA+9iB\right)\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+3\left(\sqrt{2}\left(49iA-9iB\right)\cos\left(dx+c\right)^{3}+3\sqrt{2}\left(49iA-9iB\right)\cos\left(dx+c\right)^{2}+3\sqrt{2}\left(49iA-9iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(49iA-9iB\right)\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{60\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] -1/60*(2*(3*(29*A - 9*B)*cos(d*x + c)^2 + 2*(73*A - 18*B)*cos(d*x + c) + 65
*A - 15*B)*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(sqrt(2)*(-13*I*A + 3*I*B)*c
os(d*x + c)^3 + 3*sqrt(2)*(-13*I*A + 3*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-13
*I*A + 3*I*B)*cos(d*x + c) + sqrt(2)*(-13*I*A + 3*I*B))*weierstrassPInverse
(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*(sqrt(2)*(13*I*A - 3*I*B)*cos(d*
x + c)^3 + 3*sqrt(2)*(13*I*A - 3*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(13*I*A -
3*I*B)*cos(d*x + c) + sqrt(2)*(13*I*A - 3*I*B))*weierstrassPInverse(-4, 0,
cos(d*x + c) - I*sin(d*x + c)) + 3*(sqrt(2)*(-49*I*A + 9*I*B)*cos(d*x + c)^
3 + 3*sqrt(2)*(-49*I*A + 9*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-49*I*A + 9*I*B
)*cos(d*x + c) + sqrt(2)*(-49*I*A + 9*I*B))*weierstrassZeta(-4, 0, weierstr
assPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(sqrt(2)*(49*I*A - 9
*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(49*I*A - 9*I*B)*cos(d*x + c)^2 + 3*sqrt(2
)*(49*I*A - 9*I*B)*cos(d*x + c) + sqrt(2)*(49*I*A - 9*I*B))*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*d*
cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A\sqrt{\cos{(c+dx)}}}{\sec^3{(c+dx)}+3\sec^2{(c+dx)}+3\sec{(c+dx)}+1}\,dx+\int \frac{B\sqrt{\cos{(c+dx)}}\sec{(c+dx)}}{\sec^3{(c+dx)}+3\sec^2{(c+dx)}+3\sec{(c+dx)}+1}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] (Integral(A*sqrt(cos(c + d*x))/(sec(c + d*x)**3 + 3*sec(c + d*x)**2 + 3*sec
(c + d*x) + 1), x) + Integral(B*sqrt(cos(c + d*x))*sec(c + d*x)/(sec(c + d*
x)**3 + 3*sec(c + d*x)**2 + 3*sec(c + d*x) + 1), x))/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos{(c+d\,x)}}\,\left(A+\frac{B}{\cos(c+d\,x)}\right)}{{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^3,x)
[Out] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^3, x)
```

3.511 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=182

$$-\frac{(9A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3}-\frac{(6A-B)\sqrt{\mathrm{co}}}{15ad(a+}$$

```
[Out] -1/10*(9*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(3*A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*(A-B)*cos(d
*x+c)^(3/2)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3-1/15*(6*A-B)*sin(d*x+c)*cos(d*x
+c)^(1/2)/a/d/(a+a*cos(d*x+c))^2+1/10*(9*A+B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
/(a^3+a^3*cos(d*x+c))
```

Rubi [A]
time = 0.36, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3033, 3056, 3057, 2827, 2720, 2719}

$$\frac{(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(9A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}-\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d(a\cos(c+dx)+a)^3}-\frac{(6A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a\cos(c+dx)+a)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] -1/10*((9*A + B)*EllipticE[(c + d*x)/2, 2])/(a^3*d) + ((3*A + B)*EllipticF[
(c + d*x)/2, 2])/(6*a^3*d) - ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d
*(a + a*Cos[c + d*x])^3) - ((6*A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*
a*d*(a + a*Cos[c + d*x])^2) + ((9*A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
10*d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^3}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)(B+A\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\sqrt{\cos(c+dx)}\,\left(-\frac{3}{2}a(A-B\right.}{(a+a\cos(c+}}{5a^2}$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(6A-B)\sqrt{\cos(c+dx)}\,\mathrm{s}}{15ad(a+a\cos(c+}$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(6A-B)\sqrt{\cos(c+dx)}\,\mathrm{s}}{15ad(a+a\cos(c+}$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d(a+a\cos(c+dx))^3} - \frac{(6A-B)\sqrt{\cos(c+dx)}\,\mathrm{s}}{15ad(a+a\cos(c+}$$

$$= -\frac{(9A+B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(3A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(A}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.90, size = 1407, normalized size = 7.73

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^3),
x]
```

```
[Out] (((-9*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B
*Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I
)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(
-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*
E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^
((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)
*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E
^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*
I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*
x))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - ((I/10)*B*Cos
[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((2
*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I
*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x)
)*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[
```

```
2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c
]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c
])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[
c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])
/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B
+ A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (2*A*Cos[c/2 + (d*x)/2]^6*Csc[c
/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c
/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*S
qrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c
/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c
/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])
*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + B*
Sec[c + d*x])*((4*(9*A + B)*Csc[c])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^
5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)
/2]*(9*A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x
)/2]^3*(-9*A*Sin[(d*x)/2] + 4*B*Sin[(d*x)/2]))/(15*d) + (4*(-9*A + 4*B)*Sec
[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c
/2])/(5*d)))/(Cos[c + d*x]^(3/2)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^
3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(218) = 436$.
time = 2.10, size = 451, normalized size = 2.48

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(108A\left(\cos^{8}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+30A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^{2}\left(\frac{d}{2}\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(108*A*cos(1/
2*d*x+1/2*c)^8+30*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*A*cos
(1/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+12*B*cos(1/2*d*x+1/2*c)^8+10*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*B*cos(1/2*d*x+1/2*c)^5*(si
```

```
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-198*A*cos(1/2*d*x+1/2*c)^6-2*B*cos(1/2*d*x+1/2*c)^6+
114*A*cos(1/2*d*x+1/2*c)^4-24*B*cos(1/2*d*x+1/2*c)^4-27*A*cos(1/2*d*x+1/2*c
)^2+17*B*cos(1/2*d*x+1/2*c)^2+3*A-3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x
+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.80, size = 465, normalized size = 2.55

$$\frac{2\left(3(9A+B)\cos(dx+c)^2+2(18A+7B)\cos(dx+c)+15A+5B\right)\sqrt{\cos(dx+c)}\,\sin(dx+c)-5\left(\sqrt{2}(3iA+iB)\cos(dx+c)^3+3\sqrt{2}(3iA+iB)\cos(dx+c)^2+3\sqrt{2}(3iA+iB)\cos(dx+c)+\sqrt{2}(3iA+iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5\left(\sqrt{2}(-3iA-iB)\cos(dx+c)^3+3\sqrt{2}(-3iA-iB)\cos(dx+c)^2+3\sqrt{2}(-3iA-iB)\cos(dx+c)+\sqrt{2}(-3iA-iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3\left(\sqrt{2}(9iA+iB)\cos(dx+c)^3+3\sqrt{2}(9iA+iB)\cos(dx+c)^2+3\sqrt{2}(9iA+iB)\cos(dx+c)+\sqrt{2}(9iA+iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3\left(\sqrt{2}(-9iA-iB)\cos(dx+c)^3+3\sqrt{2}(-9iA-iB)\cos(dx+c)^2+3\sqrt{2}(-9iA-iB)\cos(dx+c)+\sqrt{2}(-9iA-iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{60\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] 1/60*(2*(3*(9*A + B)*cos(d*x + c)^2 + 2*(18*A + 7*B)*cos(d*x + c) + 15*A +
5*B)*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(sqrt(2)*(3*I*A + I*B)*cos(d*x + c
)^3 + 3*sqrt(2)*(3*I*A + I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(3*I*A + I*B)*cos(
d*x + c) + sqrt(2)*(3*I*A + I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) +
 I*sin(d*x + c)) - 5*(sqrt(2)*(-3*I*A - I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-3
*I*A - I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-3*I*A - I*B)*cos(d*x + c) + sqrt(2
)*(-3*I*A - I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))
 - 3*(sqrt(2)*(9*I*A + I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(9*I*A + I*B)*cos(d*
x + c)^2 + 3*sqrt(2)*(9*I*A + I*B)*cos(d*x + c) + sqrt(2)*(9*I*A + I*B))*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 3*(sqrt(2)*(-9*I*A - I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-9*I*A - I*B)*
cos(d*x + c)^2 + 3*sqrt(2)*(-9*I*A - I*B)*cos(d*x + c) + sqrt(2)*(-9*I*A -
I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
n(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(
d*x + c) + a^3*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\frac{\int \frac{A}{\sqrt{\cos(c+dx)}\sec^3(c+dx)+3\sqrt{\cos(c+dx)}\sec^2(c+dx)+3\sqrt{\cos(c+dx)}\sec(c+dx)+\sqrt{\cos(c+dx)}}\,dx+\int \frac{B\sec(c+dx)}{\sqrt{\cos(c+dx)}\sec^3(c+dx)+3\sqrt{\cos(c+dx)}\sec^2(c+dx)+3\sqrt{\cos(c+dx)}\sec(c+dx)+\sqrt{\cos(c+dx)}}\,dx}{a^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] (Integral(A/(sqrt(cos(c + d*x))*sec(c + d*x)**3 + 3*sqrt(cos(c + d*x))*sec(
c + d*x)**2 + 3*sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c + d*x))), x) +
 Integral(B*sec(c + d*x)/(sqrt(cos(c + d*x))*sec(c + d*x)**3 + 3*sqrt(cos(c
 + d*x))*sec(c + d*x)**2 + 3*sqrt(cos(c + d*x))*sec(c + d*x) + sqrt(cos(c +
 d*x))), x))/a**3
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^3),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^3), x)
```

3.512 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=178

$$-\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(4A+B)\sqrt{\cos}}{15ad(a+}$$

```
[Out] -1/10*(A-B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(A+B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/5*(A-B)*sin(d*x+c
)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^3+1/15*(4*A+B)*sin(d*x+c)*cos(d*x+c)^
(1/2)/a/d/(a+a*cos(d*x+c))^2+1/10*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a^3+
a^3*cos(d*x+c))
```

Rubi [A]

time = 0.34, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3033, 3056, 3057, 2827, 2720, 2719}

$$\frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\left(a^3\cos(c+dx)+a^3\right)}+\frac{(4A+B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a\cos(c+dx)+a)^2}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] -1/10*((A - B)*EllipticE[(c + d*x)/2, 2])/(a^3*d) + ((A + B)*EllipticF[(c +
 d*x)/2, 2])/(6*a^3*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a
+ a*Cos[c + d*x])^3) + ((4*A + B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*d*
(a + a*Cos[c + d*x])^2) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*d*(
a^3 + a^3*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3056

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^n/(
a*f*(2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Sin[e + f*x])^(m +
 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[A*(a*d*n - b*c*(m + 1)) - B*(a*c*m +
b*d*n) - d*(a*B*(m - n) + A*b*(m + n + 1))*Sin[e + f*x], x], x], x] /; Free
Q[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && EqQ[a^2 - b^2, 0] &&
 NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)] && GtQ[n, 0] && IntegerQ[2*m] && (Int
egerQ[2*n] || EqQ[c, 0])
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\cos(c+dx)}\,(B+A\cos(c+dx))}{(a+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{-\frac{1}{2}a(A-B)+\frac{1}{2}a(7A+3B)\cos(c\cdots}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+\cdots}}{5a^2}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4A+B)\sqrt{\cos(c+dx)}\,\mathrm{si}\cdots}{15ad(a+a\cos(c+d\cdots}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4A+B)\sqrt{\cos(c+dx)}\,\mathrm{si}\cdots}{15ad(a+a\cos(c+d\cdots}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(4A+B)\sqrt{\cos(c+dx)}\,\mathrm{si}\cdots}{15ad(a+a\cos(c+d\cdots}$$

$$= -\frac{(A-B)E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{10a^3d} + \frac{(A+B)F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{6a^3d} - \frac{(A-B)\cdots}{5a\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 6.81, size = 1406, normalized size = 7.90

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^3),
x]
```

```
[Out] ((-1/10*I)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*S
ec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*
d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1
 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^
((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((
2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(
Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^(
(2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)
*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x)
)*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + ((I/10)*B*Cos[c
/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((2*E
^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*S
in[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*
```

Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*
c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c])
 - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])
^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c]
)/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/(
(-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B +
A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2
]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2
]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Si
n[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[
Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*S
qrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[c
/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c
/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTa
n[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])
Sqrt[1 + Cot[c]^2](a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + B*
Sec[c + d*x])*((-4*(-A + B)*Csc[c])/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*
(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2
]^5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*
x)/2]^3*(4*A*Sin[(d*x)/2] + B*Sin[(d*x)/2]))/(15*d) + (4*(4*A + B)*Sec[c/2
+ (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/
(5*d)))/(Cos[c + d*x]^(3/2)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(214) = 428$.
time = 2.06, size = 451, normalized size = 2.53

method	result
default	$-\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12A\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}\right.\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)

[Out] -1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2
*d*x+1/2*c)^8+10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1
/2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-12*B*cos(1/2*d*x+1/2*c)^8+10*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5-6*B*cos(1/2*d*x+1/2*c)^5*(sin(

```
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))-2*A*cos(1/2*d*x+1/2*c)^6+22*B*cos(1/2*d*x+1/2*c)^6-24*
A*cos(1/2*d*x+1/2*c)^4-6*B*cos(1/2*d*x+1/2*c)^4+17*A*cos(1/2*d*x+1/2*c)^2-7
*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.82, size = 465, normalized size = 2.61

$$\frac{2\left(3(A-B)\cos(dx+c)^2+2(7A-2B)\cos(dx+c)+5A+5B\right)\sqrt{\cos(dx+c)}\sin(dx+c)-5\left(\sqrt{2}(iA+iB)\cos(dx+c)^3+3\sqrt{2}(iA+iB)\cos(dx+c)^2+3\sqrt{2}(iA+iB)\cos(dx+c)+\sqrt{2}(iA+iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))-5\left(\sqrt{2}(-iA-iB)\cos(dx+c)^3+3\sqrt{2}(-iA-iB)\cos(dx+c)^2+3\sqrt{2}(-iA-iB)\cos(dx+c)+\sqrt{2}(-iA-iB)\right)\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))-3\left(\sqrt{2}(iA-iB)\cos(dx+c)^3+3\sqrt{2}(iA-iB)\cos(dx+c)^2+3\sqrt{2}(iA-iB)\cos(dx+c)+\sqrt{2}(iA-iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))-3\left(\sqrt{2}(-iA+iB)\cos(dx+c)^3+3\sqrt{2}(-iA+iB)\cos(dx+c)^2+3\sqrt{2}(-iA+iB)\cos(dx+c)+\sqrt{2}(-iA+iB)\right)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))}{60\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] 1/60*(2*(3*(A - B)*cos(d*x + c)^2 + 2*(7*A - 2*B)*cos(d*x + c) + 5*A + 5*B)
*sqrt(cos(d*x + c))*sin(d*x + c) - 5*(sqrt(2)*(I*A + I*B)*cos(d*x + c)^3 +
3*sqrt(2)*(I*A + I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(I*A + I*B)*cos(d*x + c) +
 sqrt(2)*(I*A + I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
 c)) - 5*(sqrt(2)*(-I*A - I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-I*A - I*B)*cos(
d*x + c)^2 + 3*sqrt(2)*(-I*A - I*B)*cos(d*x + c) + sqrt(2)*(-I*A - I*B))*we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*(sqrt(2)*(I*A -
 I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(I*A - I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(I*
A - I*B)*cos(d*x + c) + sqrt(2)*(I*A - I*B))*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*(sqrt(2)*(-I*A + I*
B)*cos(d*x + c)^3 + 3*sqrt(2)*(-I*A + I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-I*A
 + I*B)*cos(d*x + c) + sqrt(2)*(-I*A + I*B))*weierstrassZeta(-4, 0, weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/(a^3*d*cos(d*x + c)^3
+ 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^3),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^3), x)
```

3.513 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=180

$$\frac{(A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3}+\frac{(A-6B)\sqrt{\cos}}{15ad(a+}$$

```
[Out] 1/10*(A+9*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(A+3*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/5*(A-B)*sin(d*
x+c)*cos(d*x+c)^(1/2)/d/(a+a*cos(d*x+c))^3+1/15*(A-6*B)*sin(d*x+c)*cos(d*x+
c)^(1/2)/a/d/(a+a*cos(d*x+c))^2-1/10*(A+9*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/
(a^3+a^3*cos(d*x+c))
```

Rubi [A]

time = 0.35, antiderivative size = 180, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.152$, Rules used = {3033, 3057, 2827, 2720, 2719}

$$\frac{(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(A+9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{10d\,(a^3\cos(c+dx)+a^3)}+\frac{(A-6B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15ad(a\cos(c+dx)+a)^2}+\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{5d(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),x]
```

```
[Out] ((A + 9*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((A + 3*B)*EllipticF[(c
+ d*x)/2, 2])/(6*a^3*d) + ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(5*d*(a
 + a*Cos[c + d*x])^3) + ((A - 6*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*a*d
*(a + a*Cos[c + d*x])^2) - ((A + 9*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(10*
d*(a^3 + a^3*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{B+A\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3}\,dx$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{\int \frac{\frac{1}{2}a(A+9B)+\frac{3}{2}a(A-B)\cos(c+}{\sqrt{\cos(c+dx)}\,(a+a\cos(c+}}{5a^2}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-6B)\sqrt{\cos(c+dx)}\,\mathrm{si}}{15ad(a+a\cos(c+d}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-6B)\sqrt{\cos(c+dx)}\,\mathrm{si}}{15ad(a+a\cos(c+d}$$

$$= \frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{5d(a+a\cos(c+dx))^3} + \frac{(A-6B)\sqrt{\cos(c+dx)}\,\mathrm{si}}{15ad(a+a\cos(c+d}$$

$$= \frac{(A+9B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(A+3B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} + \frac{(A-}{}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.

time = 6.85, size = 1407, normalized size = 7.82

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^3),
x]
```

```
[Out] ((I/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[
c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x
)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 +
E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2
*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I
)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos
[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*
I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*
x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*S
in[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) + (((9*I)/10)*B*Cos[
c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*((2*
E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*
Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))
*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2
*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]
) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c]
)^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c
])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/
((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B +
 A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/
2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/
2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - S
in[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan
[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*
Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) - (2*B*Cos[c/2 + (d*x)/2]^6*Csc[
c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[
c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 -
 Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcT
an[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*
Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + B*S
ec[c + d*x])*((-4*(A + 9*B)*Csc[c])/(5*d) - (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^
5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) - (4*Sec[c/2]*Sec[c/2 + (d*x)
/2]^3*(-(A*Sin[(d*x)/2]) + 6*B*Sin[(d*x)/2]))/(15*d) - (4*Sec[c/2]*Sec[c/2
+ (d*x)/2]*(A*Sin[(d*x)/2] + 9*B*Sin[(d*x)/2]))/(5*d) - (4*(-A + 6*B)*Sec[c
/2 + (d*x)/2]^2*Tan[c/2])/(15*d) - (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2
])/(5*d)))/(Cos[c + d*x]^(3/2)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 450 vs. $2(216) = 432$.
time = 2.24, size = 451, normalized size = 2.51

method	result
default	$\frac{\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(12A\left(\cos^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-10A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] 1/60*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(12*A*cos(1/2*
d*x+1/2*c)^8-10*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+6*A*cos(1/
2*d*x+1/2*c)^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/
2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+108*B*cos(1/2*d*x+1/2*c)^8-30*B*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))*cos(1/2*d*x+1/2*c)^5+54*B*cos(1/2*d*x+1/2*c)^5*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))-22*A*cos(1/2*d*x+1/2*c)^6-138*B*cos(1/2*d*x+1/2*c)^6+
6*A*cos(1/2*d*x+1/2*c)^4+24*B*cos(1/2*d*x+1/2*c)^4+7*A*cos(1/2*d*x+1/2*c)^2
+3*B*cos(1/2*d*x+1/2*c)^2-3*A+3*B)/a^3/cos(1/2*d*x+1/2*c)^5/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2
*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.84, size = 465, normalized size = 2.58

$$-\frac{2\left(3\left(A+9B\right)\cos\left(dx+c\right)^{2}+2\left(2A+33B\right)\cos\left(dx+c\right)-5A+45B\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+5\left(\sqrt{2}\left(iA+3iB\right)\cos\left(dx+c\right)^{3}+3\sqrt{2}\left(iA+3iB\right)\cos\left(dx+c\right)^{2}+3\sqrt{2}\left(iA+3iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(iA+3iB\right)\right){\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+5\left(\sqrt{2}\left(-iA-3iB\right)\cos\left(dx+c\right)^{3}+3\sqrt{2}\left(-iA-3iB\right)\cos\left(dx+c\right)^{2}+3\sqrt{2}\left(-iA-3iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-iA-3iB\right)\right){\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)+3\left(\sqrt{2}\left(-iA-9iB\right)\cos\left(dx+c\right)^{3}+3\sqrt{2}\left(-iA-9iB\right)\cos\left(dx+c\right)^{2}+3\sqrt{2}\left(-iA-9iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(-iA-9iB\right)\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)+3\left(\sqrt{2}\left(iA+9iB\right)\cos\left(dx+c\right)^{3}+3\sqrt{2}\left(iA+9iB\right)\cos\left(dx+c\right)^{2}+3\sqrt{2}\left(iA+9iB\right)\cos\left(dx+c\right)+\sqrt{2}\left(iA+9iB\right)\right){\rm weierstrassZeta}\left(-4,0,{\rm weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)}{60\left(a^{3}d\cos\left(dx+c\right)^{3}+3a^{3}d\cos\left(dx+c\right)^{2}+3a^{3}d\cos\left(dx+c\right)+a^{3}d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] -1/60*(2*(3*(A + 9*B)*cos(d*x + c)^2 + 2*(2*A + 33*B)*cos(d*x + c) - 5*A +
45*B)*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(sqrt(2)*(I*A + 3*I*B)*cos(d*x +
c)^3 + 3*sqrt(2)*(I*A + 3*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(I*A + 3*I*B)*cos
(d*x + c) + sqrt(2)*(I*A + 3*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) + 5*(sqrt(2)*(-I*A - 3*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-
I*A - 3*I*B)*cos(d*x + c)^2 + 3*sqrt(2)*(-I*A - 3*I*B)*cos(d*x + c) + sqrt(
2)*(-I*A - 3*I*B))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
) + 3*(sqrt(2)*(-I*A - 9*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-I*A - 9*I*B)*cos
(d*x + c)^2 + 3*sqrt(2)*(-I*A - 9*I*B)*cos(d*x + c) + sqrt(2)*(-I*A - 9*I*B
))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d
*x + c))) + 3*(sqrt(2)*(I*A + 9*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(I*A + 9*I*
B)*cos(d*x + c)^2 + 3*sqrt(2)*(I*A + 9*I*B)*cos(d*x + c) + sqrt(2)*(I*A + 9
*I*B))*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos
(d*x + c) + a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2)),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^3),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^3), x)
```

3.514 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx$

Optimal. Leaf size=221

$$\frac{(9A-49B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}+\frac{(3A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}-\frac{(9A-49B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}+\frac{(A-B)\sin(}{5d\sqrt{\cos(c+dx)}\,(a+}$$

[Out] 1/10*(9*A-49*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d+1/6*(3*A-13*B)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-1/10*(9*A-49*B)*sin(d*x+c)/a^3/d/cos(d*x+c)^(1/2)+1/5*(A-B)*sin(d*x+c)/d/(a+a*cos(d*x+c))^3/cos(d*x+c)^(1/2)+1/15*(3*A-8*B)*sin(d*x+c)/a/d/(a+a*cos(d*x+c))^2/cos(d*x+c)^(1/2)+1/6*(3*A-13*B)*sin(d*x+c)/d/(a^3+a^3*cos(d*x+c))/cos(d*x+c)^(1/2)

Rubi [A]
time = 0.39, antiderivative size = 221, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3033, 3057, 2827, 2716, 2719, 2720}

$$\frac{(3A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d}+\frac{(9A-49B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d}-\frac{(9A-49B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}}+\frac{(3A-13B)\sin(c+dx)}{6d\sqrt{\cos(c+dx)}\,(a^3\cos(c+dx)+a^3)}+\frac{(3A-8B)\sin(c+dx)}{15ad\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+a)^2}+\frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+a)^3}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3),x]

[Out] ((9*A - 49*B)*EllipticE[(c + d*x)/2, 2])/(10*a^3*d) + ((3*A - 13*B)*EllipticF[(c + d*x)/2, 2])/(6*a^3*d) - ((9*A - 49*B)*Sin[c + d*x])/(10*a^3*d*Sqrt[Cos[c + d*x]]) + ((A - B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^3) + ((3*A - 8*B)*Sin[c + d*x])/(15*a*d*Sqrt[Cos[c + d*x]]*(a + a*Cos[c + d*x])^2) + ((3*A - 13*B)*Sin[c + d*x])/(6*d*Sqrt[Cos[c + d*x]]*(a^3 + a^3*Cos[c + d*x]))

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), Int[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] && IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3057

```
Int[((a_) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
(f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Sim
p[b*(A*b - a*B)*Cos[e + f*x]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(
n + 1)/(a*f*(2*m + 1)*(b*c - a*d))), x] + Dist[1/(a*(2*m + 1)*(b*c - a*d)),
 Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[B*(a*c*m + b*
d*(n + 1)) + A*(b*c*(m + 1) - a*d*(2*m + n + 2)) + d*(A*b - a*B)*(m + n + 2
)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[
b*c - a*d, 0] && EqQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[m, -2^(-1)]
&&  !GtQ[n, 0] && IntegerQ[2*m] && (IntegerQ[2*n] || EqQ[c, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^3}\,dx = \int \frac{B+A\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))^3}\,dx$$

$$= \frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3} + \frac{\int \frac{-\frac{1}{2}a(A-11B)+\frac{5}{2}a(A-B)\cos(c\cdots}{\cos^{\frac{3}{2}}(c+dx)(a+a\cos(c+dx))\cdots}}{5a^2}$$

$$= \frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3} + \frac{(3A-8B)\sin(c\cdots}{15ad\sqrt{\cos(c+dx)}\,(a+a\cdots}$$

$$= \frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3} + \frac{(3A-8B)\sin(c\cdots}{15ad\sqrt{\cos(c+dx)}\,(a+a\cdots}$$

$$= \frac{(A-B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}\,(a+a\cos(c+dx))^3} + \frac{(3A-8B)\sin(c\cdots}{15ad\sqrt{\cos(c+dx)}\,(a+a\cdots}$$

$$= \frac{(3A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(9A-49B)\sin(c+dx)}{10a^3d\sqrt{\cos(c+dx)}} + \frac{\cdots}{5d\sqrt{\cos\cdots}}$$

$$= \frac{(9A-49B)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{10a^3d} + \frac{(3A-13B)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{6a^3d} - \frac{(9\cdots}{1\cdots}$$

Mathematica [C] Result contains higher order function than in optimal. Order 5 vs. order 4 in optimal.
time = 7.17, size = 1447, normalized size = 6.55

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^3),
x]
```

```
[Out] (((9*I)/10)*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*
Sec[c + d*x])*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)
*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-
1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E
^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^(
(2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*
(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^
((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I
)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x
))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (((49*I)/10)*B
*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])
```

*((2*E^((2*I)*d*x)*Hypergeometric2F1[1/2, 3/4, 7/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((3*I)*d*(1 + E^((2*I)*d*x))*Cos[c] - 3*d*(-1 + E^((2*I)*d*x))*Sin[c]) - (2*Hypergeometric2F1[-1/4, 1/2, 3/4, -(E^((2*I)*d*x)*(Cos[c] + I*Sin[c])^2)]*Sqrt[(2*(1 + E^((2*I)*d*x))*Cos[c] + (2*I)*(-1 + E^((2*I)*d*x))*Sin[c])/E^(I*d*x)]*Sqrt[1 + E^((2*I)*d*x)*Cos[2*c] + I*E^((2*I)*d*x)*Sin[2*c]])/((-I)*d*(1 + E^((2*I)*d*x))*Cos[c] + d*(-1 + E^((2*I)*d*x))*Sin[c])))/((B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3) - (2*A*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (26*B*Cos[c/2 + (d*x)/2]^6*Csc[c/2]*HypergeometricPFQ[{1/4, 1/2}, {5/4}, Sin[d*x - ArcTan[Cot[c]]]^2]*Sec[c/2]*Sec[c + d*x]^2*(A + B*Sec[c + d*x])*Sec[d*x - ArcTan[Cot[c]]]*Sqrt[1 - Sin[d*x - ArcTan[Cot[c]]]]*Sqrt[-(Sqrt[1 + Cot[c]^2]*Sin[c]*Sin[d*x - ArcTan[Cot[c]]])]*Sqrt[1 + Sin[d*x - ArcTan[Cot[c]]]])/(3*d*(B + A*Cos[c + d*x])*Sqrt[1 + Cot[c]^2]*(a + a*Sec[c + d*x])^3) + (Cos[c/2 + (d*x)/2]^6*(A + B*Sec[c + d*x])*((2*(20*B - 9*A*Cos[c] + 29*B*Cos[c])*Csc[c/2]*Sec[c/2]*Sec[c])/(5*d) + (2*Sec[c/2]*Sec[c/2 + (d*x)/2]^5*(-(A*Sin[(d*x)/2]) + B*Sin[(d*x)/2]))/(5*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]^3*(-6*A*Sin[(d*x)/2] + 11*B*Sin[(d*x)/2]))/(15*d) + (4*Sec[c/2]*Sec[c/2 + (d*x)/2]*(-9*A*Sin[(d*x)/2] + 29*B*Sin[(d*x)/2]))/(5*d) + (16*B*Sec[c]*Sec[c + d*x]*Sin[d*x])/d + (4*(-6*A + 11*B)*Sec[c/2 + (d*x)/2]^2*Tan[c/2])/(15*d) + (2*(-A + B)*Sec[c/2 + (d*x)/2]^4*Tan[c/2])/(5*d)))/(Cos[c + d*x]^(3/2)*(B + A*Cos[c + d*x])*(a + a*Sec[c + d*x])^3)

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 684 vs. $2(253) = 506$.
time = 3.14, size = 685, normalized size = 3.10

method	result
default	$\frac{-2\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\left(15A\,\mathrm{EllipticF}\left(\cos\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x,method=_RETURNVERBOSE)

[Out] 1/60*(-2*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*EllipticF

```
(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*c
os(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4+4*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(15*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-27*A*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))-65*B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+147*B*Elliptic
E(cos(1/2*d*x+1/2*c),2^(1/2)))*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-2*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(15*A*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))-27*A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-65*B*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))+147*B*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*cos(1/2*d*x+
1/2*c)+12*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(9*A-49*B)*s
in(1/2*d*x+1/2*c)^8-2*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
(147*A-817*B)*sin(1/2*d*x+1/2*c)^6+6*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*(43*A-248*B)*sin(1/2*d*x+1/2*c)^4-(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*(69*A-439*B)*sin(1/2*d*x+1/2*c)^2)/a^3/cos(1/2*d
*x+1/2*c)^5/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*
x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.56, size = 521, normalized size = 2.36

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] -1/60*(2*(3*(9*A - 49*B)*cos(d*x + c)^3 + 2*(33*A - 188*B)*cos(d*x + c)^2 +
 5*(9*A - 59*B)*cos(d*x + c) - 60*B)*sqrt(cos(d*x + c))*sin(d*x + c) + 5*(s
qrt(2)*(3*I*A - 13*I*B)*cos(d*x + c)^4 + 3*sqrt(2)*(3*I*A - 13*I*B)*cos(d*x
 + c)^3 + 3*sqrt(2)*(3*I*A - 13*I*B)*cos(d*x + c)^2 + sqrt(2)*(3*I*A - 13*I
*B)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))
 + 5*(sqrt(2)*(-3*I*A + 13*I*B)*cos(d*x + c)^4 + 3*sqrt(2)*(-3*I*A + 13*I*B
)*cos(d*x + c)^3 + 3*sqrt(2)*(-3*I*A + 13*I*B)*cos(d*x + c)^2 + sqrt(2)*(-3
*I*A + 13*I*B)*cos(d*x + c))*weierstrassPInverse(-4, 0, cos(d*x + c) - I*si
```

```
n(d*x + c)) + 3*(sqrt(2)*(-9*I*A + 49*I*B)*cos(d*x + c)^4 + 3*sqrt(2)*(-9*I
*A + 49*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(-9*I*A + 49*I*B)*cos(d*x + c)^2 +
sqrt(2)*(-9*I*A + 49*I*B)*cos(d*x + c))*weierstrassZeta(-4, 0, weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*(sqrt(2)*(9*I*A - 49*I*B
)*cos(d*x + c)^4 + 3*sqrt(2)*(9*I*A - 49*I*B)*cos(d*x + c)^3 + 3*sqrt(2)*(9
*I*A - 49*I*B)*cos(d*x + c)^2 + sqrt(2)*(9*I*A - 49*I*B)*cos(d*x + c))*weie
rstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)
)))/(a^3*d*cos(d*x + c)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2
 + a^3*d*cos(d*x + c))
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2)),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\cos\left(c+d\,x\right)}^{7/2}\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^3),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^3), x)
```

3.515 $\int \cos^{\frac{9}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=220

$$\frac{32a(8A+9B)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}}+\frac{16a(8A+9B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}+\frac{4a(8A+9B)\cos^{\frac{3}{2}}(c+}{105d\sqrt{a+a\sec(}$$

```
[Out] 4/105*a*(8*A+9*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/63
*a*(8*A+9*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/9*a*A*c
os(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+32/315*a*(8*A+9*B)*sin(
d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16/315*a*(8*A+9*B)*sin(d*x
+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.31, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.114, Rules used = {3034, 4100, 3890, 3889}

$$\frac{2a(8A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{4a(8A+9B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{16a(8A+9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{32a(8A+9B)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}+\frac{2aA\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{9d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (32*a*(8*A + 9*B)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c
+ d*x]]) + (16*a*(8*A + 9*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(315*d*Sqrt[a
 + a*Sec[c + d*x]]) + (4*a*(8*A + 9*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(10
5*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*(8*A + 9*B)*Cos[c + d*x]^(5/2)*Sin[c +
 d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^(7/2)*Sin[c +
d*x])/(9*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\,\mathrm{s}\cdots}}{\cdots}$$

$$= \frac{2aA\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{9d\sqrt{a+a\sec(c+dx)}} + \frac{1}{9}\Big((8A+9B\cdots$$

$$= \frac{2a(8A+9B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\sec(c+dx)}} + \frac{2aA\cdots}{9\cdots}$$

$$= \frac{4a(8A+9B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} + \frac{2a(\cdots}{\cdots}$$

$$= \frac{16a(8A+9B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}} + \frac{4\cdots}{\cdots}$$

$$= \frac{32a(8A+9B)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{1\cdots}{\cdots}$$

Mathematica [A]
time = 0.50, size = 119, normalized size = 0.54

$$\frac{\sqrt{\cos(c+dx)}\,(1321A+1368B+94(8A+9B)\cos(c+dx)+4(83A+54B)\cos(2(c+dx))+80A\cos(3(c+dx))+90B\cos(3(c+dx))+35A\cos(4(c+dx)))\sqrt{a(1+\sec(c+dx))}\,\sin(c+dx)}{1260d(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(1321*A + 1368*B + 94*(8*A + 9*B)*Cos[c + d*x] + 4*(83*
A + 54*B)*Cos[2*(c + d*x)] + 80*A*Cos[3*(c + d*x)] + 90*B*Cos[3*(c + d*x)]
+ 35*A*Cos[4*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(1260*d*(
1 + Cos[c + d*x]))
```

Maple [A]
time = 12.10, size = 130, normalized size = 0.59

method	result
default	$-\frac{2(-1+\cos(dx+c))(35A(\cos^4(dx+c))+40A(\cos^3(dx+c))+45B(\cos^3(dx+c))+48A(\cos^2(dx+c))+54B(\cos^2(dx+c))+64A\cos(dx+c}{315d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/315/d*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+40*A*cos(d*x+c)^3+45*B*cos(d*x+
c)^3+48*A*cos(d*x+c)^2+54*B*cos(d*x+c)^2+64*A*cos(d*x+c)+72*B*cos(d*x+c)+12
8*A+144*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(190) = 380$.
time = 0.85, size = 547, normalized size = 2.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(1890*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9
/2*c)))*sin(9/2*d*x + 9/2*c) + 420*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c), co
s(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 252*cos(4/9*arctan2(sin(9/2*d*x
 + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 45*cos(2/9*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) - 1890*c
os(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2
*c))) - 420*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x + 9/2*c), cos(
9/2*d*x + 9/2*c))) - 252*cos(9/2*d*x + 9/2*c)*sin(4/9*arctan2(sin(9/2*d*x +
 9/2*c), cos(9/2*d*x + 9/2*c))) - 45*cos(9/2*d*x + 9/2*c)*sin(2/9*arctan2(s
in(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*sin(9/2*d*x + 9/2*c) + 45*
sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 252*sin(5/9*
arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 420*sin(1/3*arctan2(
sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 1890*sin(1/9*arctan2(sin(9/2
*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*A*sqrt(a) - 18*sqrt(2)*(7*(15*sin(3*
d*x + 3*c) + 5*sin(2*d*x + 2*c) + sin(d*x + c))*cos(7/2*arctan2(sin(d*x + c
```

```
), cos(d*x + c))) - (105*cos(3*d*x + 3*c) + 35*cos(2*d*x + 2*c) + 7*cos(d*x
 + c) + 10)*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 7*sin(5/2*arctan
2(sin(d*x + c), cos(d*x + c))) - 35*sin(3/2*arctan2(sin(d*x + c), cos(d*x +
 c))) - 105*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*B*sqrt(a))/d
```

Fricas [A]
time = 2.22, size = 116, normalized size = 0.53

$$\frac{2\,(35\,A\cos{(dx+c)}^4+5\,(8\,A+9\,B)\cos{(dx+c)}^3+6\,(8\,A+9\,B)\cos{(dx+c)}^2+8\,(8\,A+9\,B)\cos{(dx+c)}+128\,A+144\,B)\sqrt{\frac{a\cos{(dx+c)}+a}{\cos{(dx+c)}}}\,\sqrt{\cos{(dx+c)}}\,\sin{(dx+c)}}{315\,(d\cos{(dx+c)}+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 2/315*(35*A*cos(d*x + c)^4 + 5*(8*A + 9*B)*cos(d*x + c)^3 + 6*(8*A + 9*B)*c
os(d*x + c)^2 + 8*(8*A + 9*B)*cos(d*x + c) + 128*A + 144*B)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) +
d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(9/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\cos\left(c+d\,x\right)}^{9/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)
```

3.516 $\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=175

$$\frac{16a(6A+7B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}}+\frac{8a(6A+7B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}+\frac{2a(6A+7B)\cos^{\frac{3}{2}}(c+d}{35d\sqrt{a+a\sec(c}$$

```
[Out] 2/35*a*(6*A+7*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/7*a
*A*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+16/105*a*(6*A+7*B)*
sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+8/105*a*(6*A+7*B)*sin(
d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.27, antiderivative size = 175, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3034, 4100, 3890, 3889}

$$\frac{2a(6A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}+\frac{8a(6A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{16a(6A+7B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}+\frac{2aA\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (16*a*(6*A + 7*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c
+ d*x]]) + (8*a*(6*A + 7*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d*Sqrt[a
+ a*Sec[c + d*x]]) + (2*a*(6*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*
d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*
Sqrt[a + a*Sec[c + d*x]])
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\,\mathrm{s}}}{} \\ &= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}}+\frac{1}{7}\Big((6A+7B \\ &= \frac{2a(6A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}}+\frac{2aA}{} \\ &= \frac{8a(6A+7B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}+\frac{2a}{} \\ &= \frac{16a(6A+7B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}}+\frac{8}{}\end{aligned}$$

Mathematica [A]
time = 0.35, size = 96, normalized size = 0.55

$$\frac{\sqrt{\cos(c+dx)}\,(228A+266B+(141A+112B)\cos(c+dx)+6(6A+7B)\cos(2(c+dx))+15A\cos(3(c+dx)))\sqrt{a(1+\sec(c+dx))}\,\sin(c+dx)}{210d(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(228*A + 266*B + (141*A + 112*B)*Cos[c + d*x] + 6*(6*A
+ 7*B)*Cos[2*(c + d*x)] + 15*A*Cos[3*(c + d*x)])*Sqrt[a*(1 + Sec[c + d*x])]
*Sin[c + d*x])/(210*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 11.26, size = 108, normalized size = 0.62

method	result
default	$-\frac{2(-1+\cos(dx+c))(15A(\cos^3(dx+c))+18A(\cos^2(dx+c))+21B(\cos^2(dx+c))+24A\cos(dx+c)+28B\cos(dx+c)+48A+56B)\sqrt{\frac{a(1+}{\mathrm{c}}}}{105d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/105/d*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+18*A*cos(d*x+c)^2+21*B*cos(d*x+
c)^2+24*A*cos(d*x+c)+28*B*cos(d*x+c)+48*A+56*B)*(a*(1+cos(d*x+c))/cos(d*x+c
))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 418 vs. 2(151) = 302.
time = 0.79, size = 418, normalized size = 2.39

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/840*(3*sqrt(2)*(105*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7
/2*c)))*sin(7/2*d*x + 7/2*c) + 35*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 7*cos(2/7*arctan2(sin(7/2*d*x +
7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 105*cos(7/2*d*x + 7/2
*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 35*cos(7
/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))
) - 7*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*
x + 7/2*c))) + 10*sin(7/2*d*x + 7/2*c) + 7*sin(5/7*arctan2(sin(7/2*d*x + 7/
2*c), cos(7/2*d*x + 7/2*c))) + 35*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos
(7/2*d*x + 7/2*c))) + 105*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c))))*A*sqrt(a) - 14*sqrt(2)*(5*(6*sin(2*d*x + 2*c) + sin(d*x + c))*
cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - (30*cos(2*d*x + 2*c) + 5*cos
(d*x + c) + 6)*sin(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 5*sin(3/2*arc
tan2(sin(d*x + c), cos(d*x + c))) - 30*sin(1/2*arctan2(sin(d*x + c), cos(d*
x + c))))*B*sqrt(a))/d
```

Fricas [A]
time = 1.80, size = 99, normalized size = 0.57

$$\frac{2\left(15\,A\cos\left(dx+c\right)^{3}+3\left(6\,A+7\,B\right)\cos\left(dx+c\right)^{2}+4\left(6\,A+7\,B\right)\cos\left(dx+c\right)+48\,A+56\,B\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 2/105*(15*A*cos(d*x + c)^3 + 3*(6*A + 7*B)*cos(d*x + c)^2 + 4*(6*A + 7*B)*c
os(d*x + c) + 48*A + 56*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos
(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(7/2),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{7/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)
```

3.517 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=130

$$\frac{4a(4A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{2a(4A+5B)\sqrt{\cos(c+dx)}\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}}+\frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2/5*a*A*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+4/15*a*(4*A+5*
B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/15*a*(4*A+5*B)*si
n(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.21, antiderivative size = 130, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3034, 4100, 3890, 3889}

$$\frac{2a(4A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{4a(4A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2aA\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (4*a*(4*A + 5*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c +
d*x]]) + (2*a*(4*A + 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a + a
*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*Sqrt[a + a*S
ec[c + d*x]])
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\,\mathrm{s}}}{}$$

$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{1}{5}\Big((4A+5B$$

$$= \frac{2a(4A+5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2a}{}$$

$$= \frac{4a(4A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{2a}{}$$

Mathematica [A]
time = 0.17, size = 79, normalized size = 0.61

$$\frac{2\sqrt{\cos(c+dx)}\,(8A+10B+(4A+5B)\cos(c+dx)+3A\cos^2(c+dx))\,\sqrt{a(1+\sec(c+dx))}\,\sin(c+dx)}{15d(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(8*A + 10*B + (4*A + 5*B)*Cos[c + d*x] + 3*A*Cos[c +
d*x]^2)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(15*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 12.56, size = 86, normalized size = 0.66

method	result	size
default	$-\frac{2(-1+\cos(dx+c))\left(3A\left(\cos^2(dx+c)\right)+4A\cos(dx+c)+5B\cos(dx+c)+8A+10B\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,\left(\sqrt{\cos}\,(dx+c)\right)}{15d\sin(dx+c)}$	86

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*(-1+cos(d*x+c))*(3*A*cos(d*x+c)^2+4*A*cos(d*x+c)+5*B*cos(d*x+c)+8*A
+10*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 296 vs. $2(112) = 224$.
time = 0.80, size = 296, normalized size = 2.28

$$\frac{\sqrt{2}\left(30\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)-30\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\sin\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)-5\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\sin\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+6\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5\sin\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+30\sin\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\right)A\sqrt{a}-10\left(3\sqrt{2}\cos\left(\frac{3}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)\sin\left(dx+c\right)-\left(3\sqrt{2}\cos\left(dx+c\right)+2\sqrt{2}\right)\sin\left(\frac{3}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)-3\sqrt{2}\sin\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)\right)B\sqrt{a}}{60\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/60*(sqrt(2)*(30*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) + 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*
d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 30*cos(5/2*d*x + 5/2*c)*sin(4/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*cos(5/2*d*x + 5/2*c)*si
n(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 6*sin(5/2*d*x
+ 5/2*c) + 5*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) +
 30*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*A*sqrt(a)
 - 10*(3*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(d*x + c)
- (3*sqrt(2)*cos(d*x + c) + 2*sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*
x + c))) - 3*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*B*sqrt(a
))/d
```

Fricas [A]
time = 2.07, size = 81, normalized size = 0.62

$$\frac{2\left(3A\cos\left(dx+c\right)^{2}+\left(4A+5B\right)\cos\left(dx+c\right)+8A+10B\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 2/15*(3*A*cos(d*x + c)^2 + (4*A + 5*B)*cos(d*x + c) + 8*A + 10*B)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x +
 c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{5/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)
```

3.518 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=82

$$\frac{2a(A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 2/3*a*(A+3*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/3*A*si
n(d*x+c)*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.17, antiderivative size = 82, normalized size of antiderivative = 1.00, number of steps used = 3, number of rules used = 3, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.086$, Rules used = {3034, 4098, 3889}

$$\frac{2a(A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*(A + 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x
]]) + (2*A*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
```

```
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+a\,\mathrm{s}}}{}$$
$$= \frac{2A\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,\sin(c}{3d}$$
$$= \frac{2a(A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{2A}{}$$

Mathematica [A]
time = 0.20, size = 56, normalized size = 0.68

$$\frac{2\sqrt{\cos(c+dx)}\,(2A+3B+A\cos(c+dx))\sqrt{a(1+\sec(c+dx))}\,\tan\left(\frac{1}{2}(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(2*A + 3*B + A*Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x]
)]*Tan[(c + d*x)/2])/(3*d)
```

Maple [A]
time = 11.90, size = 65, normalized size = 0.79

method	result	size
default	$-\frac{2(-1+\cos(dx+c))(A\cos(dx+c)+2A+3B)\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}}{3d\sin(dx+c)}$	65

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(A*cos(d*x+c)+2*A+3*B)*cos(d*x+c)^(1/2)*(a*(1+cos(d*
x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 141 vs. $2(70) = 140$.
time = 0.85, size = 141, normalized size = 1.72

$$\frac{\sqrt{2}\left(3\cos\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)-3\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\sin\left(\frac{2}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)+2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+3\sin\left(\frac{1}{3}\arctan\left(\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right),\cos\left(\frac{3}{2}dx+\frac{3}{2}c\right)\right)\right)\right)A\sqrt{a}+12\sqrt{2}B\sqrt{a}\sin\left(\frac{1}{2}\arctan\left(\sin\left(dx+c\right),\cos\left(dx+c\right)\right)\right)}{6d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/6*(sqrt(2)*(3*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)*sin(3/2*d*x + 3/2*c) - 3*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(3/2*d*x + 3/2*c) + 3*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A*sqrt(a) + 12*sqrt(2)*B*
sqrt(a)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))/d
```

Fricas [A]
time = 2.18, size = 64, normalized size = 0.78

$$\frac{2\left(A\cos\left(dx+c\right)+2\,A+3\,B\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 2/3*(A*cos(d*x + c) + 2*A + 3*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{3/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)
```

3.519 $\int \sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=96

$$\frac{2\sqrt{a}\,B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}+\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*B*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d+2*a*A*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)
)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 96, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3034, 4100, 3886, 221}

$$\frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2\sqrt{a}\,B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*Sqrt[a]*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a*A*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+}}{}$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \Big(B$$

$$\Big(2$$

$$= \frac{2aA\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} - $$

$$= \frac{2\sqrt{a}\,B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{c}}{d}$$

Mathematica [A]
time = 0.33, size = 94, normalized size = 0.98

$$\frac{2\sqrt{\cos(c+dx)}\left(A\sqrt{1-\sec(c+dx)}-B\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\sqrt{\sec(c+dx)}\right)\sqrt{a(1+\sec(c+dx))}\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*(A*Sqrt[1 - Sec[c + d*x]] - B*ArcSin[Sqrt[Sec[c + d*x
]]]*Sqrt[Sec[c + d*x]])*Sqrt[a*(1 + Sec[c + d*x])]*Tan[(c + d*x)/2])/(d*Sqr
t[1 - Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 168 vs. $2(82) = 164$.
time = 11.96, size = 169, normalized size = 1.76

method	result
default	$-\frac{(-1+\cos(dx+c))\left(2A\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}+B\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-B\sqrt{\ldots}\right.}{d\sin(dx+c)^2\sqrt{-\frac{2}{1+\cos(d\ldots}}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/d*(-1+cos(d*x+c))*(2*A*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)+B*2^(1/2)*ar
ctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))-B*2^(
1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2)
))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^2/(-2/(1
+cos(d*x+c)))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 262 vs. $2(82) = 164$.
time = 0.77, size = 262, normalized size = 2.73

$$\frac{4\sqrt{2}A\sqrt{a}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+B\sqrt{a}\left(\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)-\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)+\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)-\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)\right)}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*(4*sqrt(2)*A*sqrt(a)*sin(1/2*d*x + 1/2*c) + B*sqrt(a)*(log(2*cos(1/2*d*
x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) +
2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
 + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log(
2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x
 + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)))/d
```

Fricas [A]
time = 2.41, size = 298, normalized size = 3.10

$$\left[\frac{4A\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+(B\cos(dx+c)+B)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7a\cos(dx+c)^2+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{2(d\cos(dx+c)+d)}, \frac{2A\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+(B\cos(dx+c)+B)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)}{d\cos(dx+c)+d}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(4*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + (B*cos(d*x + c) + B)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqr
t((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*
sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2))
)/(d*cos(d*x + c) + d), (2*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c) + (B*cos(d*x + c) + B)*sqrt(-a)*arctan(2*sqrt(-a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a
*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \sqrt{a\left(\sec\left(c+dx\right)+1\right)}\left(A+B\sec\left(c+dx\right)\right)\sqrt{\cos\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)*(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))*sqrt(cos(c + d*x))
, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)*sqrt(cos(d*x + c)),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos\left(c+d\,x\right)}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}} \,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2), x)
```

3.520 $\int \frac{\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=98

$$\frac{\sqrt{a}\,(2A+B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{d}+\frac{aB\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}}$$

```
[Out] (2*A+B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)*cos(d*x+
c)^(1/2)*sec(d*x+c)^(1/2)/d+a*B*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+
c))^(1/2)
```

Rubi [A]

time = 0.17, antiderivative size = 98, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3034, 4101, 3886, 221}

$$\frac{\sqrt{a}\,(2A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{aB\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (Sqrt[a]*(2*A + B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]
*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a*B*Sin[c + d*x])/(d*Cos[c + d
*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,\sqrt{a+$$

$$= \frac{aB\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{2}\Big((2A+B)\sqrt{\cos($$

$$\Big((2A+B)\sqrt{\cos(c$$

$$= \frac{aB\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - $$

$$= \frac{\sqrt{a}\,(2A+B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+d}{d}$$

Mathematica [A]
time = 0.41, size = 89, normalized size = 0.91

$$\frac{\sqrt{\cos(c+dx)}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{2}\,(2A+B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2B\sec(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)\right)}{2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]
],x]
```

[Out] (Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(2
*A + B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*B*Sec[c + d*x]*Sin[(c + d*x)/
2]))/(2*d)

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 273 vs. $2(84) = 168$.
time = 12.61, size = 274, normalized size = 2.80

method	result
default	$-\frac{(-1+\cos(dx+c))\left(2A\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(-1-\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\cos(dx+c)\sqrt{2}+2A\arctan\left(\sqrt{-\frac{2}{1+\cos(d}}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)

[Out] -1/2/d*(-1+cos(d*x+c))*(2*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*
x+c)+sin(d*x+c))*2^(1/2))*cos(d*x+c)*2^(1/2)+2*A*arctan(1/4*(-2/(1+cos(d*x+
c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*x+c)*2^(1/2)+B*arctan(1
/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*x+c)
*2^(1/2)+B*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2
^(1/2))*cos(d*x+c)*2^(1/2)+2*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c))*(a*(1+
cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^2/(-2/(1+cos(d*x+c)))^(1/2)/cos(d*
x+c)^(1/2)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 905 vs. $2(84) = 168$.
time = 0.91, size = 905, normalized size = 9.23

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")

[Out] 1/4*(2*A*sqrt(a)*(log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - log
(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*
x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c

```
) + 2)) - (4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
 + 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
+ 2*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)
*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos
(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
 (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*c
os(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
 c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*
d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*a
rctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos
(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*
sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(s
in(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*
sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x +
2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*B*
sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1))
/d
```

Fricas [A]
time = 2.76, size = 351, normalized size = 3.58

$$\left[\frac{4 B \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right) + \left(\left(2A+B\right)\cos\left(dx+c\right)^2 + \left(2A+B\right)\cos\left(dx+c\right)\right)\sqrt{a} \log\left(\frac{a\cos\left(dx+c\right)^3 - 4\sqrt{a}\sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\left(\cos\left(dx+c\right)-2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right) - 7a\cos\left(dx+c\right)^2 + 8a}{\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2}\right)}{4\left(d\cos\left(dx+c\right)^2 + d\cos\left(dx+c\right)\right)}, \frac{2 B \sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}} \sqrt{\cos\left(dx+c\right)} \sin\left(dx+c\right) + \left(\left(2A+B\right)\cos\left(dx+c\right)^2 + \left(2A+B\right)\cos\left(dx+c\right)\right)\sqrt{-a} \arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a \cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{a\cos\left(dx+c\right)^2 - a\cos\left(dx+c\right) - 2a}\right)}{2\left(d\cos\left(dx+c\right)^2 + d\cos\left(dx+c\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(4*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) + ((2*A + B)*cos(d*x + c)^2 + (2*A + B)*cos(d*x + c))*sqrt(a)*log((a
*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*
x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(co
s(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)), 1/2*(
2*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)
 + ((2*A + B)*cos(d*x + c)^2 + (2*A + B)*cos(d*x + c))*sqrt(-a)*arctan(2*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
 c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^2 + d*cos(d
*x + c))]
```

Sympy [F]

time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\left(A+B\sec\left(c+dx\right)\right)}{\sqrt{\cos\left(c+dx\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))/sqrt(cos(c + d*x))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/sqrt(cos(d*x + c)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2), x
)
```

3.521 $\int \frac{\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=151

$$\frac{\sqrt{a}\,(4A+3B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{4d}+\frac{aB\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(}}$$

[Out] 1/4*(4*A+3*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+1/2*a*B*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+1/4*a*(4*A+3*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)

Rubi [A]
time = 0.21, antiderivative size = 151, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.143, Rules used = {3034, 4101, 3888, 3886, 221}

$$\frac{a(4A+3B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}\,(4A+3B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{aB\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]

[Out] (Sqrt[a]*(4*A + 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a*B*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(4*A + 3*B)*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{se}$$

$$= \frac{aB\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{4}\Big((4A+3B)\sqrt{\cos}$$

$$= \frac{aB\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(4A+3B)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= \frac{aB\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(4A+3B)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a}}$$

$$= \frac{\sqrt{a}\,(4A+3B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+}}{4d}$$

Mathematica [**A**]

time = 0.65, size = 106, normalized size = 0.70

$$\frac{\sqrt{\cos(c+dx)}\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{2}(4A+3B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sec(c+dx)(4A+3B+2B\sec(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2
),x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*(4
*A + 3*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(4*A + 3*B + 2
*B*Sec[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 341 vs. 2(127) = 254.
time = 12.50, size = 342, normalized size = 2.26

method	result
default	$-\frac{(-1+\cos(dx+c))\left(4A\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\left(\cos^2(dx+c)\right)\sqrt{2}-4A\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/8/d*(-1+cos(d*x+c))*(4*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x
+c)+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-4*A*arctan(1/4*(-2/(1+cos(d*x
+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)+3*B*arc
tan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*
x+c)^2*2^(1/2)-3*B*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d
*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)+8*A*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+
c)*cos(d*x+c)+6*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+4*B*(-2/(
1+cos(d*x+c)))^(1/2)*sin(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/(-2/(1
+cos(d*x+c)))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1927 vs. 2(127) = 254.
time = 1.01, size = 1927, normalized size = 12.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] -1/16*(4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
+ 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x +
 2*c) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) +
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*co
s(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(
d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*s
qrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 4*(sqrt(2)*cos(2*d*x + 2
*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*co
s(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*A*s
qrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1) +
 (12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/2*arctan
2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*si
n(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2)*s
in(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d*x + c),
 cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c)
)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*x + 2*c) +
 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*
x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4
*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arc
tan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos
(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x +
 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log
(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d
*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x
 + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 3*(
2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*
d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4
*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x
 + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2
```

```
- 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)
*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x +
 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*co
s(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 +
2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan
2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), co
s(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*
c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqrt(2)*cos
(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arctan2(sin(d
*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x
 + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 12*(sqrt(
2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))))*B*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*
d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2
 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x
 + 2*c) + 1))/d
```

Fricas [A]
time = 2.84, size = 401, normalized size = 2.66

$$\left[\frac{4\,\left(\left(4\,A+3\,B\right)\cos\left(dx+c\right)+2\,B\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+\left(\left(4\,A+3\,B\right)\cos\left(dx+c\right)^{3}+\left(4\,A+3\,B\right)\cos\left(dx+c\right)^{2}\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-4\,\sqrt{a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,(\cos(dx+c)-2)\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-7\,a\cos(dx+c)^{2}+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)}{16\,\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)},\ \frac{2\,\left(\left(4\,A+3\,B\right)\cos\left(dx+c\right)+2\,B\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+\left(\left(4\,A+3\,B\right)\cos\left(dx+c\right)^{3}+\left(4\,A+3\,B\right)\cos\left(dx+c\right)^{2}\right)\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)}{a\cos(dx+c)^{2}-a\cos(dx+c)-2\,a}\right)}{8\,\left(d\cos\left(dx+c\right)^{3}+d\cos\left(dx+c\right)^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(4*((4*A + 3*B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x
 + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((4*A + 3*B)*cos(d*x + c)^3 + (4*A
 + 3*B)*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*
x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*c
os(d*x + c)^3 + d*cos(d*x + c)^2), 1/8*(2*((4*A + 3*B)*cos(d*x + c) + 2*B)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (
(4*A + 3*B)*cos(d*x + c)^3 + (4*A + 3*B)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*
sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^3 + d*cos
(d*x + c)^2)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{\sqrt{a\left(\sec\left(c+dx\right)+1\right)}\,\left(A+B\sec\left(c+dx\right)\right)}{\cos^{\frac{3}{2}}\left(c+dx\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral(sqrt(a*(sec(c + d*x) + 1))*(A + B*sec(c + d*x))/cos(c + d*x)**(3/2
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/cos(d*x + c)^(3/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}{{\cos\left(c+d\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2), x
)
```

3.522 $\int \frac{\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=196

$$\frac{\sqrt{a}\,(6A+5B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{8d}+\frac{aB\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(}}$$

```
[Out] 1/8*(6*A+5*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*a^(1/2)*co
s(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+1/3*a*B*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+
a*sec(d*x+c))^(1/2)+1/12*a*(6*A+5*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec
(d*x+c))^(1/2)+1/8*a*(6*A+5*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c
))^(1/2)
```

Rubi [A]

time = 0.26, antiderivative size = 196, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3034, 4101, 3888, 3886, 221}

$$\frac{a(6A+5B)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a(6A+5B)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{a}\,(6A+5B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{aB\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (Sqrt[a]*(6*A + 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a*B*Sin[c + d*x])/(3*d*C
os[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(6*A + 5*B)*Sin[c + d*x])/
(12*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*(6*A + 5*B)*Sin[c +
 d*x])/(8*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+a\sec(c+dx)}\,(A+B\sec(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\,\mathrm{s}$$

$$= \frac{aB\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{1}{6}\Big((6A+5B)\sqrt{\mathrm{c}}$$

$$= \frac{aB\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(6A+5B}{12d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{aB\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(6A+5B}{12d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{aB\sin(c+dx)}{3d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a(6A+5B}{12d\cos^{\frac{5}{2}}(c+dx)}$$

$$= \frac{\sqrt{a}\,(6A+5B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+}}{8d}$$

Mathematica [A]
time = 1.05, size = 131, normalized size = 0.67

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(3\sqrt{2}\,(6A+5B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3(c+dx)+(18A+31B+4(6A+5B)\cos(c+dx)+3(6A+5B)\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{48d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[a + a*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(5/2
),x]
```

```
[Out] (Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(6*A + 5*B)*ArcTanh
[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (18*A + 31*B + 4*(6*A + 5*B)*Co
s[c + d*x] + 3*(6*A + 5*B)*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*d*Cos[c
 + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(166) = 332.
time = 12.10, size = 404, normalized size = 2.06

method	result

default	$-\frac{(-1+\cos(dx+c))\left(18A\sqrt{2}\,(\cos^3(dx+c))\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(-1-\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)+18A\sqrt{2}\,(\cos^3(dx+c)}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/48/d*(-1+cos(d*x+c))*(18*A*2^(1/2)*cos(d*x+c)^3*arctan(1/4*(-2/(1+cos(d*
x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))+18*A*2^(1/2)*cos(d*x+c)^3*
arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))+15*
B*2^(1/2)*cos(d*x+c)^3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+
sin(d*x+c))*2^(1/2))+15*B*2^(1/2)*cos(d*x+c)^3*arctan(1/4*(-2/(1+cos(d*x+c)
))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))+36*A*sin(d*x+c)*cos(d*x+c)^2*(-
2/(1+cos(d*x+c)))^(1/2)+30*B*sin(d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1
/2)+24*A*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)+20*B*(-2/(1+cos(d*
x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+16*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c
))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^2/(-2/(1+cos(d*x+c)))^(1/
2)/cos(d*x+c)^(5/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3342 vs. $2(166) = 332$.
time = 1.05, size = 3342, normalized size = 17.05

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] -1/96*(6*(12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/
2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sq
rt(2)*sin(2*d*x + 2*c))*cos(5/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(s
qrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(d
*x + c), cos(d*x + c))) - 12*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*
x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 3*(2*(2*cos(2*d*x
+ 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 +
sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*
c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x
+ c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos
(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d
*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*
```

c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin
(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c)
+ 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arcta
n2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c),
cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
2) - 3*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 +
4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2
(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x +
c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2
)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + 3*(2*(2*cos(2*d*x + 2
*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin
(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^
2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(2)*cos(1/
2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2) - 12*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*
d*x + 2*c) + sqrt(2))*sin(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 4*(sqr
t(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/2*arcta
n2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*c
os(2*d*x + 2*c) + sqrt(2))*sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 1
2*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(1/2
*arctan2(sin(d*x + c), cos(d*x + c))))*A*sqrt(a)/(2*(2*cos(2*d*x + 2*c) + 1
)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x
+ 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*c
os(2*d*x + 2*c) + 1) + (60*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x
+ 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(11/2*arctan2(sin(d*x + c), cos(d*x
+ c))) + 20*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqr
t(2)*sin(2*d*x + 2*c))*cos(9/2*arctan2(sin(d*x + c), cos(d*x + c))) + 168*(
sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x
+ 2*c))*cos(7/2*arctan2(sin(d*x + c), cos(d*x + c))) - 168*(sqrt(2)*sin(6*
d*x + 6*c) + 3*sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(5
/2*arctan2(sin(d*x + c), cos(d*x + c))) - 20*(sqrt(2)*sin(6*d*x + 6*c) + 3*
sqrt(2)*sin(4*d*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(3/2*arctan2(sin(
d*x + c), cos(d*x + c))) - 60*(sqrt(2)*sin(6*d*x + 6*c) + 3*sqrt(2)*sin(4*d
*x + 4*c) + 3*sqrt(2)*sin(2*d*x + 2*c))*cos(1/2*arctan2(sin(d*x + c), cos(d
x + c))) - 15(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x +
6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) +
9*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d
*x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 1
8*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x +
2*c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*
arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x
+ c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) + 2) + 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6

```
*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*
cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*x
 + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18*
sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2*
c) + 1)*log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*ar
ctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x +
c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))
 + 2) - 15*(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x + 6*c
) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9*co
s(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(si...
```

Fricas [A]
time = 3.09, size = 439, normalized size = 2.24

$$\left[\frac{4\,\left(3\,\left(6\,A+5\,B\right)\cos\left(dx+c\right)^{2}+2\,\left(6\,A+5\,B\right)\cos\left(dx+c\right)+8\,B\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+3\,\left(\left(6\,A+5\,B\right)\cos\left(dx+c\right)^{4}+\left(6\,A+5\,B\right)\cos\left(dx+c\right)^{3}\right)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^{3}-4\,\sqrt{a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,(\cos(dx+c)-2)\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)-7\,a\cos(dx+c)^{2}+8\,a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)}{96\,\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}, \frac{2\,\left(3\,\left(6\,A+5\,B\right)\cos\left(dx+c\right)^{2}+2\,\left(6\,A+5\,B\right)\cos\left(dx+c\right)+8\,B\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+3\,\left(\left(6\,A+5\,B\right)\cos\left(dx+c\right)^{4}+\left(6\,A+5\,B\right)\cos\left(dx+c\right)^{3}\right)\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{-a}\,\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin(dx+c)}{a\cos(dx+c)^{2}-a\cos(dx+c)-2\,a}\right)}{48\,\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/96*(4*(3*(6*A + 5*B)*cos(d*x + c)^2 + 2*(6*A + 5*B)*cos(d*x + c) + 8*B)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3
*((6*A + 5*B)*cos(d*x + c)^4 + (6*A + 5*B)*cos(d*x + c)^3)*sqrt(a)*log((a*c
os(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x
+ c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(
d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3), 1/48*
(2*(3*(6*A + 5*B)*cos(d*x + c)^2 + 2*(6*A + 5*B)*cos(d*x + c) + 8*B)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((6*A
 + 5*B)*cos(d*x + c)^4 + (6*A + 5*B)*cos(d*x + c)^3)*sqrt(-a)*arctan(2*sqrt
(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^4 + d*cos(d*x
 + c)^3)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))**(1/2)/cos(d*x+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+a*sec(d*x+c))^(1/2)/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(a*sec(d*x + c) + a)/cos(d*x + c)^(5/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}{{\cos\left(c+d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/cos(c + d*x)^(5/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(1/2))/cos(c + d*x)^(5/2), x
)
```

3.523 $\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=275

$$\frac{32a^2(168A+187B)\sin(c+dx)}{3465d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{16a^2(168A+187B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3465d\sqrt{a+a\sec(c+dx)}}+\frac{4a^2(168A+187}{1155d\sqrt{}}$$

```
[Out] 4/1155*a^2*(168*A+187*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/
2)+2/693*a^2*(168*A+187*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(
1/2)+2/99*a^2*(12*A+11*B)*cos(d*x+c)^(7/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1
/2)+32/3465*a^2*(168*A+187*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c)
)^(1/2)+16/3465*a^2*(168*A+187*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*
x+c))^(1/2)+2/11*a*A*cos(d*x+c)^(9/2)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.47, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3034, 4102, 4100, 3890, 3889}

$$\frac{2a^2(12A+11B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)}{99d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(168A+187B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{4a^2(168A+187B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{1155d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(168A+187B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3465d\sqrt{a\sec(c+dx)+a}}+\frac{32a^2(168A+187B)\sin(c+dx)}{3465d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2aA\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (32*a^2*(168*A + 187*B)*Sin[c + d*x])/(3465*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a
*Sec[c + d*x]]) + (16*a^2*(168*A + 187*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/
(3465*d*Sqrt[a + a*Sec[c + d*x]]) + (4*a^2*(168*A + 187*B)*Cos[c + d*x]^(3/
2)*Sin[c + d*x])/(1155*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(168*A + 187*B)
*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(693*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2
*(12*A + 11*B)*Cos[c + d*x]^(7/2)*Sin[c + d*x])/(99*d*Sqrt[a + a*Sec[c + d*
x]]) + (2*a*A*Cos[c + d*x]^(9/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(11
*d)
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\mathrm{se}}{}$$

$$= \frac{2aA\cos^{\frac{9}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c}{11d}$$

$$= \frac{2a^2(12A+11B)\cos^{\frac{7}{2}}(c+dx)\sin(c+dx)}{99d\sqrt{a+a\sec(c+dx)}} +$$

$$= \frac{2a^2(168A+187B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{4a^2(168A+187B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{1155d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{16a^2(168A+187B)\sqrt{\cos(c+dx)}\,\sin(c+d}{3465d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{32a^2(168A+187B)\sin(c+dx)}{3465d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} +$$

Mathematica [A]
time = 0.54, size = 131, normalized size = 0.48

$$\frac{2a\sqrt{\cos(c+dx)}\,(2688A+2992B+8(168A+187B)\cos(c+dx)+6(168A+187B)\cos^2(c+dx)+(840A+935B)\cos^3(c+dx)+35(21A+11B)\cos^4(c+dx)+315A\cos^5(c+dx))\sqrt{a(1+\sec(c+dx))}\,\sin(c+dx)}{3465d(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x
]),x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*(2688*A + 2992*B + 8*(168*A + 187*B)*Cos[c + d*x] +
 6*(168*A + 187*B)*Cos[c + d*x]^2 + (840*A + 935*B)*Cos[c + d*x]^3 + 35*(21
*A + 11*B)*Cos[c + d*x]^4 + 315*A*Cos[c + d*x]^5)*Sqrt[a*(1 + Sec[c + d*x])
]*Sin[c + d*x])/(3465*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 13.45, size = 153, normalized size = 0.56

method	result
default	$-\frac{2a(-1+\cos(dx+c))\left(315A\left(\cos^5(dx+c)\right)+735A\left(\cos^4(dx+c)\right)+385B\left(\cos^4(dx+c)\right)+840A\left(\cos^3(dx+c)\right)+935B\left(\cos^3(dx+c)\right)+1008A}{3465d\,\mathrm{si}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RET
URNVERBOSE)
```

```
[Out] -2/3465/d*a*(-1+cos(d*x+c))*(315*A*cos(d*x+c)^5+735*A*cos(d*x+c)^4+385*B*co
s(d*x+c)^4+840*A*cos(d*x+c)^3+935*B*cos(d*x+c)^3+1008*A*cos(d*x+c)^2+1122*B
*cos(d*x+c)^2+1344*A*cos(d*x+c)+1496*B*cos(d*x+c)+2688*A+2992*B)*cos(d*x+c)
^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 703 vs. $2(239) = 478$.
time = 0.97, size = 703, normalized size = 2.56

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algo
rithm="maxima")
```

```
[Out] 1/110880*(21*sqrt(2)*(3630*a*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(
11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 990*a*cos(8/11*arctan2(sin(11
/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 429*a*c
os(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d
*x + 11/2*c) + 165*a*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x
+ 11/2*c)))*sin(11/2*d*x + 11/2*c) + 55*a*cos(2/11*arctan2(sin(11/2*d*x + 1
1/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) - 3630*a*cos(11/2*d
*x + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*
c))) - 990*a*cos(11/2*d*x + 11/2*c)*sin(8/11*arctan2(sin(11/2*d*x + 11/2*c)
, cos(11/2*d*x + 11/2*c))) - 429*a*cos(11/2*d*x + 11/2*c)*sin(6/11*arctan2(
sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 165*a*cos(11/2*d*x + 11/
2*c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 55
*a*cos(11/2*d*x + 11/2*c)*sin(2/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2
*d*x + 11/2*c))) + 30*a*sin(11/2*d*x + 11/2*c) + 55*a*sin(9/11*arctan2(sin(
11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 165*a*sin(7/11*arctan2(sin(1
1/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 429*a*sin(5/11*arctan2(sin(11
/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 990*a*sin(3/11*arctan2(sin(11/
2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 3630*a*sin(1/11*arctan2(sin(11/
2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))*A*sqrt(a) - 44*sqrt(2)*(189*(10*
a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) - 7*(270*a*cos(4*d*x + 4*c) + 27*a*cos(2*d*x + 2*c) + 5*
a)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 135*a*sin(7/4*arc
tan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 189*a*sin(5/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - 1050*a*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(
```

```
2*d*x + 2*c))) - 1890*a*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
))*B*sqrt(a))/d
```

Fricas [A]
time = 1.96, size = 142, normalized size = 0.52

$$\frac{2\,\left(315\,Aa\cos\left(dx+c\right)^{5}+35\,\left(21\,A+11\,B\right)a\cos\left(dx+c\right)^{4}+5\,\left(168\,A+187\,B\right)a\cos\left(dx+c\right)^{3}+6\,\left(168\,A+187\,B\right)a\cos\left(dx+c\right)^{2}+8\,\left(168\,A+187\,B\right)a\cos\left(dx+c\right)+16\,\left(168\,A+187\,B\right)a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3465\,\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algo
rithm="fricas")
```

```
[Out] 2/3465*(315*A*a*cos(d*x + c)^5 + 35*(21*A + 11*B)*a*cos(d*x + c)^4 + 5*(168
*A + 187*B)*a*cos(d*x + c)^3 + 6*(168*A + 187*B)*a*cos(d*x + c)^2 + 8*(168*
A + 187*B)*a*cos(d*x + c) + 16*(168*A + 187*B)*a)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algo
rithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(11/
2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\cos\left(c+d\,x\right)}^{11/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(11/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(11/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)
```

3.524 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=228

$$\frac{16a^2(34A+39B)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{8a^2(34A+39B)\sqrt{\cos(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(34A+39B)\cos}{105d\sqrt{a+}}$$

```
[Out] 2/105*a^2*(34*A+39*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+
2/63*a^2*(10*A+9*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+16
/315*a^2*(34*A+39*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+8
/315*a^2*(34*A+39*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2
/9*a*A*cos(d*x+c)^(7/2)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.41, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3034, 4102, 4100, 3890, 3889}

$$\frac{2a^2(10A+9B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{63d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(34A+39B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{105d\sqrt{a\sec(c+dx)+a}}+\frac{8a^2(34A+39B)\sin(c+dx)\sqrt{\cos(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(34A+39B)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2aA\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (16*a^2*(34*A + 39*B)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Se
c[c + d*x]]) + (8*a^2*(34*A + 39*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(315*d
*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(34*A + 39*B)*Cos[c + d*x]^(3/2)*Sin[c
+ d*x])/(105*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(10*A + 9*B)*Cos[c + d*x]
^(5/2)*Sin[c + d*x])/(63*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^
(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(9*d)
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec}{}$$

$$= \frac{2aA\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\sin(c+}{9d}$$

$$= \frac{2a^2(10A+9B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{63d\sqrt{a+a\sec(c+dx)}}+\frac{2a}{}$$

$$= \frac{2a^2(34A+39B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}+\frac{2}{}$$

$$= \frac{8a^2(34A+39B)\sqrt{\cos(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}+$$

$$= \frac{16a^2(34A+39B)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{8a}{}$$

Mathematica [A]
time = 0.43, size = 118, normalized size = 0.52

$$\frac{2a\sqrt{\cos(c+dx)}\,(8(34A+39B)+4(34A+39B)\cos(c+dx)+3(34A+39B)\cos^2(c+dx)+5(17A+9B)\cos^3(c+dx)+35A\cos^4(c+dx))\,\sqrt{a(1+\sec(c+dx))}\,\sin(c+dx)}{315d(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*(8*(34*A + 39*B) + 4*(34*A + 39*B)*Cos[c + d*x] + 3
*(34*A + 39*B)*Cos[c + d*x]^2 + 5*(17*A + 9*B)*Cos[c + d*x]^3 + 35*A*Cos[c
+ d*x]^4)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(315*d*(1 + Cos[c + d*x]
))
```

Maple [A]
time = 12.28, size = 131, normalized size = 0.57

method	result
default	$-\frac{2a(-1+\cos(dx+c))\left(35A\left(\cos^4(dx+c)\right)+85A\left(\cos^3(dx+c)\right)+45B\left(\cos^3(dx+c)\right)+102A\left(\cos^2(dx+c)\right)+117B\left(\cos^2(dx+c)\right)+136A\,\mathrm{co}\right.}{315d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/315/d*a*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+85*A*cos(d*x+c)^3+45*B*cos(d*
x+c)^3+102*A*cos(d*x+c)^2+117*B*cos(d*x+c)^2+136*A*cos(d*x+c)+156*B*cos(d*x
+c)+272*A+312*B)*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d
*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 558 vs. 2(198) = 396.
time = 0.91, size = 558, normalized size = 2.45

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(3780*a*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x +
 9/2*c)))*sin(9/2*d*x + 9/2*c) + 1050*a*cos(2/3*arctan2(sin(9/2*d*x + 9/2*c
), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 378*a*cos(4/9*arctan2(sin(
9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 135*a*cos(2
```

```
/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c
) - 3780*a*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9
/2*d*x + 9/2*c))) - 1050*a*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2(sin(9/2*d*x
 + 9/2*c), cos(9/2*d*x + 9/2*c))) - 378*a*cos(9/2*d*x + 9/2*c)*sin(4/9*arct
an2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 135*a*cos(9/2*d*x + 9/2*
c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) + 70*a*sin(
9/2*d*x + 9/2*c) + 135*a*sin(7/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
+ 9/2*c))) + 378*a*sin(5/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*
c))) + 1050*a*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))
+ 3780*a*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))))*A*sq
rt(a) - 6*sqrt(2)*(175*a*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))*sin(2*d*x + 2*c) - 5*(35*a*cos(2*d*x + 2*c) + 6*a)*sin(7/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) - 126*a*sin(5/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))) - 175*a*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 1470*a*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(a
))/d
```

Fricas [A]
time = 2.33, size = 124, normalized size = 0.54

$$\frac{2\,{\left(35\,A a \cos\left(d x + c\right)^{4} + 5\,{\left(17\,A + 9\,B\right)} a \cos\left(d x + c\right)^{3} + 3\,{\left(34\,A + 39\,B\right)} a \cos\left(d x + c\right)^{2} + 4\,{\left(34\,A + 39\,B\right)} a \cos\left(d x + c\right) + 8\,{\left(34\,A + 39\,B\right)} a\right)} \sqrt{\frac{a \cos\left(d x + c\right) + a}{\cos\left(d x + c\right)}} \sqrt{\cos\left(d x + c\right)} \sin\left(d x + c\right)}{315\,{\left(d \cos\left(d x + c\right) + d\right)}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 2/315*(35*A*a*cos(d*x + c)^4 + 5*(17*A + 9*B)*a*cos(d*x + c)^3 + 3*(34*A +
39*B)*a*cos(d*x + c)^2 + 4*(34*A + 39*B)*a*cos(d*x + c) + 8*(34*A + 39*B)*a
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(
d*cos(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(9/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{9/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)
```

3.525 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=181

$$\frac{4a^2(52A+63B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(52A+63B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}+\frac{2a^2(8A+7B)\cos^{\frac{3}{2}}(c}{35d\sqrt{a+a\,\mathrm{se}}}$$

```
[Out] 2/35*a^2*(8*A+7*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+4/1
05*a^2*(52*A+63*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/1
05*a^2*(52*A+63*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/7
*a*A*cos(d*x+c)^(5/2)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.36, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3034, 4102, 4100, 3890, 3889}

$$\frac{2a^2(8A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(52A+63B)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}+\frac{4a^2(52A+63B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}+\frac{2aA\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (4*a^2*(52*A + 63*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec
[c + d*x]]) + (2*a^2*(52*A + 63*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d*
Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(8*A + 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d
*x])/(35*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Cos[c + d*x]^(5/2)*Sqrt[a + a
*Sec[c + d*x]]*Sin[c + d*x])/(7*d)
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\mathrm{s}}{}$$

$$= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}\,\sin(c}{7d}$$

$$= \frac{2a^2(8A+7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2}{}$$

$$= \frac{2a^2(52A+63B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{4a^2(52A+63B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} +$$

Mathematica [A]

time = 0.32, size = 100, normalized size = 0.55

$$\frac{2a\sqrt{\cos(c+dx)}\,(2(52A+63B)+(52A+63B)\cos(c+dx)+3(13A+7B)\cos^2(c+dx)+15A\cos^3(c+dx))\,\sqrt{a(1+\sec(c+dx))}\,\sin(c+dx)}{105d(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*(2*(52*A + 63*B) + (52*A + 63*B)*Cos[c + d*x] + 3*(
13*A + 7*B)*Cos[c + d*x]^2 + 15*A*Cos[c + d*x]^3)*Sqrt[a*(1 + Sec[c + d*x])
]*Sin[c + d*x])/(105*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 11.85, size = 109, normalized size = 0.60

method	result
default	$-\frac{2a(-1+\cos(dx+c))(15A(\cos^3(dx+c))+39A(\cos^2(dx+c))+21B(\cos^2(dx+c))+52A\cos(dx+c)+63B\cos(dx+c)+104A+126B)\left(\sqrt{\right.}{105d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/105/d*a*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+39*A*cos(d*x+c)^2+21*B*cos(d*
x+c)^2+52*A*cos(d*x+c)+63*B*cos(d*x+c)+104*A+126*B)*cos(d*x+c)^(1/2)*(a*(1+
cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 451 vs. 2(157) = 314.
time = 0.90, size = 451, normalized size = 2.49

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/840*(sqrt(2)*(735*a*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7
/2*c)))*sin(7/2*d*x + 7/2*c) + 175*a*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c),
cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 63*a*cos(2/7*arctan2(sin(7/2*
d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 735*a*cos(7/2*d
*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) -
175*a*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*
x + 7/2*c))) - 63*a*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*
```

```
c), cos(7/2*d*x + 7/2*c))) + 30*a*sin(7/2*d*x + 7/2*c) + 63*a*sin(5/7*arcta
n2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*a*sin(3/7*arctan2(sin
(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 735*a*sin(1/7*arctan2(sin(7/2*d
*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A*sqrt(a) - 84*(10*sqrt(2)*a*cos(5/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - 5*sqrt(2)*a*
sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 10*sqrt(2)*a*sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - (10*sqrt(2)*a*cos(2*d*x + 2
*c) + sqrt(2)*a)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sq
rt(a))/d
```

Fricas [A]
time = 2.53, size = 105, normalized size = 0.58

$$\frac{2\,(15\,Aa\cos{(dx+c)}^3+3\,(13\,A+7\,B)a\cos{(dx+c)}^2+(52\,A+63\,B)a\cos{(dx+c)}+2\,(52\,A+63\,B)a)\sqrt{\frac{a\cos{(dx+c)}+a}{\cos{(dx+c)}}}\,\sqrt{\cos{(dx+c)}}\,\sin{(dx+c)}}{105\,(d\cos{(dx+c)}+d)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 2/105*(15*A*a*cos(d*x + c)^3 + 3*(13*A + 7*B)*a*cos(d*x + c)^2 + (52*A + 63
*B)*a*cos(d*x + c) + 2*(52*A + 63*B)*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{7/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)
```

3.526 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=131

$$\frac{8a^2(3A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{2a(3A+5B)\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c+dx)}{15d}+\frac{2A$$

```
[Out] 2/5*A*cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+8/15*a^2*(3*A+5*
B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/15*a*(3*A+5*B)*si
n(d*x+c)*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.23, antiderivative size = 131, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3034, 4098, 3894, 3889}

$$\frac{8a^2(3A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a(3A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (8*a^2*(3*A + 5*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c
+ d*x]]) + (2*a*(3*A + 5*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin
[c + d*x])/(15*d) + (2*A*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[
c + d*x])/(5*d)
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
```

```
*((d*Csc[e + f*x])^n/(f*m)), x] + Dist[b*((2*m - 1)/(d*m)), Int[(a + b*Csc[
e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ
[2*m]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec}{}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c+}{5d}$$

$$= \frac{2a(3A+5B)\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+}}{15d}$$

$$= \frac{8a^2(3A+5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{2a}{}$$

Mathematica [A]
time = 0.27, size = 80, normalized size = 0.61

$$\frac{2a\sqrt{\cos(c+dx)}\,(18A+25B+(9A+5B)\cos(c+dx)+3A\cos^2(c+dx))\sqrt{a(1+\sec(c+dx))}\,\sin(c+dx)}{15d(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (2*a*Sqrt[Cos[c + d*x]]*(18*A + 25*B + (9*A + 5*B)*Cos[c + d*x] + 3*A*Cos[c
 + d*x]^2)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(15*d*(1 + Cos[c + d*x]
))
```

Maple [A]
time = 11.78, size = 87, normalized size = 0.66

method	result	size
default	$-\frac{2a(-1+\cos(dx+c))\left(3A\left(\cos^{2}(dx+c)\right)+9A\cos(dx+c)+5B\cos(dx+c)+18A+25B\right)\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}}{15d\sin(dx+c)}$	87

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*a*(-1+cos(d*x+c))*(3*A*cos(d*x+c)^2+9*A*cos(d*x+c)+5*B*cos(d*x+c)+1
8*A+25*B)*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 276 vs. $2(113) = 226$.
time = 0.89, size = 276, normalized size = 2.11

$$\frac{3\sqrt{2}\left(20a\cos\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5a\cos\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)-20a\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\sin\left(\frac{4}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)-5a\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\sin\left(\frac{2}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+2a\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+5a\sin\left(\frac{3}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)+20a\sin\left(\frac{1}{5}\arctan\left(\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right),\cos\left(\frac{5}{2}dx+\frac{5}{2}c\right)\right)\right)\right)A\sqrt{a}+20\left(\sqrt{2}a\sin\left(\frac{3}{4}\arctan\left(\sin\left(2dx+2c\right),\cos\left(2dx+2c\right)\right)\right)+9\sqrt{2}a\sin\left(\frac{1}{4}\arctan\left(\sin\left(2dx+2c\right),\cos\left(2dx+2c\right)\right)\right)\right)B\sqrt{a}}{60d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/60*(3*sqrt(2)*(20*a*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5
/2*c)))*sin(5/2*d*x + 5/2*c) + 5*a*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), co
s(5/2*d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 20*a*cos(5/2*d*x + 5/2*c)*sin(4
/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 5*a*cos(5/2*d*x +
 5/2*c)*sin(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 2*a*
sin(5/2*d*x + 5/2*c) + 5*a*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*
x + 5/2*c))) + 20*a*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2
*c))))*A*sqrt(a) + 20*(sqrt(2)*a*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 9*sqrt(2)*a*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*B*sqrt(a))/d
```

Fricas [A]
time = 2.87, size = 86, normalized size = 0.66

$$\frac{2\left(3\,Aa\cos\left(dx+c\right)^{2}+\left(9\,A+5\,B\right)a\cos\left(dx+c\right)+\left(18\,A+25\,B\right)a\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 2/15*(3*A*a*cos(d*x + c)^2 + (9*A + 5*B)*a*cos(d*x + c) + (18*A + 25*B)*a)*
sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*
cos(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{5/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)
```

3.527 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=145

$$\frac{2a^{3/2}B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}+\frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2*a^(3/2)*B*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(
1/2)*sec(d*x+c)^(1/2)/d+2/3*a^2*(4*A+3*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+
a*sec(d*x+c))^(1/2)+2/3*a*A*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1
/2)/d
```

Rubi [A]
time = 0.28, antiderivative size = 145, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3034, 4102, 4100, 3886, 221}

$$\frac{2a^{3/2}B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2aA\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^(3/2)*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^2*(4*A + 3*B)*Sin[c + d*x])/(3*d
*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a*A*Sqrt[Cos[c + d*x]]*S
qrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec}{}$$

$$= \frac{2aA\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}\sin(c}{3d}$$

$$= \frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2aA}{}$$

$$= \frac{2a^2(4A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2aA}{}$$

$$= \frac{2a^{3/2}B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos}}{d}$$

Mathematica [A]
time = 0.45, size = 101, normalized size = 0.70

$$\frac{2a^2\left((5A+3B+A\cos(c+dx))\sqrt{1-\sec(c+dx)}+3B\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)\sqrt{\sec(c+dx)}\right)\sin(c+dx)}{3d\sqrt{-1+\cos(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (2*a^2*((5*A + 3*B + A*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] + 3*B*ArcSin[Sq
rt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*Sin[c + d*x])/(3*d*Sqrt[-1 + Cos[
c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 12.48, size = 201, normalized size = 1.39

method	result
default	$-\frac{a\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3B\sqrt{2}\,\sin(dx+c)\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/6/d*a*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*B*2^(1/2)*
sin(d*x+c)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2
^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)-3*B*2^(1/2)*sin(d*x+c)*(-2/(1+cos(d*x+c))
)^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1
/2))+4*A*cos(d*x+c)^2+16*A*cos(d*x+c)+12*B*cos(d*x+c)-20*A-12*B)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 583 vs. 2(123) = 246.
time = 0.92, size = 583, normalized size = 4.02

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/30*(10*(sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 9*sqrt(2)*a*sin(1/2*d*x + 1/2*c)
)*A*sqrt(a) + 3*(2*sqrt(2)*a*sin(5/2*d*x + 5/2*c) + 40*sqrt(2)*a*sin(1/2*d*
```

```
x + 1/2*c) - 2*sqrt(2)*a*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) - 20*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 5*a*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ 2) - 5*a*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2
*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*s
qrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) +
5*a*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2
)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*
sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 5*a*log
(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2))*B*sqrt(a))/d
```

Fricas [A]
time = 2.82, size = 343, normalized size = 2.37

$$\left[\frac{4\,(Aa\cos(dx+c)+(5\,A+3\,B)a)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)+3\,(Ba\cos(dx+c)+Ba)\sqrt{a}\,\log\left(\frac{a\cos(dx+c)^3-4\,\sqrt{a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\,\sin(dx+c)-7\,a\cos(dx+c)^2+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{6\,(d\cos(dx+c)+d)},\ \frac{2\,(Aa\cos(dx+c)+(5\,A+3\,B)a)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)+3\,(Ba\cos(dx+c)+Ba)\sqrt{-a}\,\arctan\left(\frac{2\,\sqrt{-a}\,\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\,\sqrt{\cos(dx+c)}\,\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2\,a}\right)}{3\,(d\cos(dx+c)+d)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] [1/6*(4*(A*a*cos(d*x + c) + (5*A + 3*B)*a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*(B*a*cos(d*x + c) + B*a)*sqrt(a
)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 +
8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c) + d), 1/3*(2*(A*a*
cos(d*x + c) + (5*A + 3*B)*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) + 3*(B*a*cos(d*x + c) + B*a)*sqrt(-a)*arctan(2*s
qrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
+ c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c) + d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{3/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)
```

3.528 $\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=144

$$\frac{a^{3/2}(2A+3B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{d}+\frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}}$$

```
[Out] a^(3/2)*(2*A+3*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*
x+c)^(1/2)*sec(d*x+c)^(1/2)/d+a^2*(2*A-B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+
a*sec(d*x+c))^(1/2)+a*B*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2
)
```

Rubi [A]
time = 0.27, antiderivative size = 144, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3034, 4103, 4100, 3886, 221}

$$\frac{a^{3/2}(2A+3B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}+\frac{aB\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(3/2)*(2*A + 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^2*(2*A - B)*Sin[c + d*x])/
(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (a*B*Sqrt[a + a*Sec[c + d
*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\ldots}{\ldots}$$

$$= \frac{aB\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \Big(\ldots$$

$$= \frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{a\ldots}{\ldots}$$

$$= \frac{a^2(2A-B)\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{a\ldots}{\ldots}$$

$$= \frac{a^{3/2}(2A+3B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+d\ldots}}\right.}{d}$$

Mathematica [A]
time = 0.82, size = 133, normalized size = 0.92

$$\frac{a\sqrt{\cos(c+dx)}\,\sqrt{a(1+\sec(c+dx))}\left(2A\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)\sqrt{\sec(c+dx)}-3B\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\sqrt{\sec(c+dx)}+\sqrt{1-\sec(c+dx)}\,(2A+B\sec(c+dx))\right)\tan\left(\frac{1}{2}(c+dx)\right)}{d\sqrt{1-\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])]*(2*A*ArcSin[Sqrt[1 - Sec[c
 + d*x]]]*Sqrt[Sec[c + d*x]] - 3*B*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[Sec[c +
d*x]] + Sqrt[1 - Sec[c + d*x]]*(2*A + B*Sec[c + d*x]))*Tan[(c + d*x)/2])/(d
*Sqrt[1 - Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 305 vs. 2(126) = 252.
time = 12.97, size = 306, normalized size = 2.12

method	result
default	$-\frac{a(-1+\cos(dx+c))\left(2A\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(-1-\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\cos(dx+c)\sqrt{2}+2A\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(}}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2/d*a*(-1+cos(d*x+c))*(2*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(
d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*x+c)*2^(1/2)+2*A*arctan(1/4*(-2/(1+cos(d*
x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*x+c)*2^(1/2)+4*A*(-2/
(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)+3*B*arctan(1/4*(-2/(1+cos(d*x+c
)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*x+c)*2^(1/2)+3*B*arctan
(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*x+c
)*2^(1/2)+2*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c))*(a*(1+cos(d*x+c))/cos(d
*x+c))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)^2/(-2/(1+cos(d*x+c)))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1417 vs. 2(126) = 252.
time = 0.86, size = 1417, normalized size = 9.84

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="maxima")

[Out] 1/4*(sqrt(2)*(sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - sqrt(2)*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sq
rt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*
a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1
/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8*a*sin(1/2*d*x + 1/2*c))*A*sq
rt(a) + (3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*d*
x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2*d*x
+ 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*a*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - 4*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))*sin(
2*d*x + 2*c))*B*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1))/d
```

Fricas [A]
time = 3.15, size = 389, normalized size = 2.70

$$\left[\frac{4\left(2Aa\cos\left(dx+c\right)+Ba\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(\left(2A+3B\right)a\cos\left(dx+c\right)^{2}+\left(2A+3B\right)a\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-4\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}(\cos(dx+c)-2)\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-7a\cos(dx+c)^{2}+8a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)}{4\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}, \frac{2\left(2Aa\cos\left(dx+c\right)+Ba\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(\left(2A+3B\right)a\cos\left(dx+c\right)^{2}+\left(2A+3B\right)a\cos\left(dx+c\right)\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{a\cos(dx+c)^{2}-a\cos(dx+c)-2a}\right)}{2\left(d\cos\left(dx+c\right)^{2}+d\cos\left(dx+c\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(4*(2*A*a*cos(d*x + c) + B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c) + ((2*A + 3*B)*a*cos(d*x + c)^2 + (2*A + 3*
B)*a*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c
) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*
x + c)^2 + d*cos(d*x + c)), 1/2*(2*(2*A*a*cos(d*x + c) + B*a)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((2*A + 3*B)*a
*cos(d*x + c)^2 + (2*A + 3*B)*a*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*co
s(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^2 + d*cos(d*x + c))]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \sqrt{\cos\left(c+d\,x\right)}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2), x)
```

3.529 $\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=153

$$\frac{a^{3/2}(12A+7B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4d}+\frac{a^2(4A+5B)\sin(c+dx}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c}$$

```
[Out] 1/4*a^(3/2)*(12*A+7*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*c
os(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+1/4*a^2*(4*A+5*B)*sin(d*x+c)/d/cos(d*x+c
)^(3/2)/(a+a*sec(d*x+c))^(1/2)+1/2*a*B*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/
cos(d*x+c)^(3/2)
```

Rubi [A]
time = 0.29, antiderivative size = 153, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3034, 4103, 4101, 3886, 221}

$$\frac{a^{3/2}(12A+7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a^2(4A+5B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{aB\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{2d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a^(3/2)*(12*A + 7*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x
]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^2*(4*A + 5*B)*Sin[c +
 d*x])/(4*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*B*Sqrt[a + a*
Sec[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,(a+a$$

$$= \frac{aB\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{2}\Big(\sqrt{\cos(c+dx)}$$

$$= \frac{a^2(4A+5B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{aB\sqrt{a+a\sec(c}{2d\cos}$$

$$= \frac{a^2(4A+5B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{aB\sqrt{a+a\sec(c}{2d\cos}$$

$$= \frac{a^{3/2}(12A+7B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c}{4d}$$

Mathematica [A]
time = 0.83, size = 107, normalized size = 0.70

$$\frac{a\sqrt{\cos(c+dx)}\,\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(\sqrt{2}\,(12A+7B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)+2\sec(c+dx)(4A+7B+2B\sec(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{8d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] (a*Sqrt[Cos[c + d*x]]*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(Sqrt[2]*
(12*A + 7*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]] + 2*Sec[c + d*x]*(4*A + 7*B
+ 2*B*Sec[c + d*x])*Sin[(c + d*x)/2]))/(8*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(129) = 258.
time = 12.65, size = 343, normalized size = 2.24

method	result
default	$-\frac{a\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(12A\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)(\cos^2(dx+c))\sqrt{2}-1\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/8/d*a*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(12*A*arctan(1
/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*x+c)^
2*2^(1/2)-12*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c
))*2^(1/2))*cos(d*x+c)^2*2^(1/2)+7*B*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(
1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-7*B*arctan(1/4*(-2/(
1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2
)+8*A*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)+14*B*(-2/(1+cos(d*x+c
)))^(1/2)*cos(d*x+c)*sin(d*x+c)+4*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c))/s
in(d*x+c)^2/(-2/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 3389 vs. 2(129) = 258.
time = 1.09, size = 3389, normalized size = 22.15

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")

[Out] 1/16*(4*(3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*s
qrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2))*cos(2*d*x + 2*c)^2 + 3*(a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a*lo
g(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d
*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(3/2*d*
x + 3/2*c) - 4*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 2*(2*sqrt(2)*a*sin(3/2*d*x
+ 3/2*c) - 2*sqrt(2)*a*sin(1/2*d*x + 1/2*c) + 3*a*log(2*cos(1/2*d*x + 1/2*c
)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)
*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/
2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2
*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*l
og(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*
d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) + 3*a*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - 3*a*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2)
 - 4*(sqrt(2)*a*cos(3/2*d*x + 3/2*c) - sqrt(2)*a*cos(1/2*d*x + 1/2*c))*sin(
2*d*x + 2*c))*A*sqrt(a)/(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*
d*x + 2*c) + 1) - (56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) - 24*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt
(2)*a*sin(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(
2)*a*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(
2)*a*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(
2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*
```

```
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3
/2*d*x + 3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) - 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(
8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/
2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
 + 7*(a*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*
a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),...
```

Fricas [A]
time = 2.48, size = 409, normalized size = 2.67

$$\left[\frac{4\left((4A+7B)a\cos\left(dx+c\right)+2Ba\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left((12A+7B)a\cos\left(dx+c\right)^3+(12A+7B)a\cos\left(dx+c\right)^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}(\cos(dx+c)-2)\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-7a\cos(dx+c)^2+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{16\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)}, \frac{2\left((4A+7B)a\cos\left(dx+c\right)+2Ba\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left((12A+7B)a\cos\left(dx+c\right)^3+(12A+7B)a\cos\left(dx+c\right)^2\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)}{8\left(d\cos\left(dx+c\right)^3+d\cos\left(dx+c\right)^2\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(4*((4*A + 7*B)*a*cos(d*x + c) + 2*B*a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((12*A + 7*B)*a*cos(d*x + c)^3
 + (12*A + 7*B)*a*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x +
c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)
^2)))/(d*cos(d*x + c)^3 + d*cos(d*x + c)^2), 1/8*(2*((4*A + 7*B)*a*cos(d*x
+ c) + 2*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c) + ((12*A + 7*B)*a*cos(d*x + c)^3 + (12*A + 7*B)*a*cos(d*x + c)^2
)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
```

```
os(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*co
s(d*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{3/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2), x
)
```

3.530 $\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=200

$$\frac{a^{3/2}(14A+11B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{8d}+\frac{a^2(6A+7B)\sin(c+}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec}}$$

```
[Out] 1/8*a^(3/2)*(14*A+11*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*
cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+1/12*a^2*(6*A+7*B)*sin(d*x+c)/d/cos(d*x
+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+1/8*a^2*(14*A+11*B)*sin(d*x+c)/d/cos(d*x+c
)^(3/2)/(a+a*sec(d*x+c))^(1/2)+1/3*a*B*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/
cos(d*x+c)^(5/2)
```

Rubi [A]

time = 0.35, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3034, 4103, 4101, 3888, 3886, 221}

$$\frac{a^{3/2}(14A+11B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^2(14A+11B)\sin(c+dx)}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(6A+7B)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{aB\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a^(3/2)*(14*A + 11*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^2*(6*A + 7*B)*Sin[c
+ d*x])/(12*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(14*A + 1
1*B)*Sin[c + d*x])/(8*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a*B
*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec$$

$$= \frac{aB\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\right.$$

$$= \frac{a^2(6A+7B)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{aB\sqrt{a+a\sec(c}{3d\cos^{\frac{5}{2}}}$$

$$= \frac{a^2(6A+7B)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(14A+11}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{}}$$

$$= \frac{a^2(6A+7B)\sin(c+dx)}{12d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(14A+11}{8d\cos^{\frac{3}{2}}(c+dx)\sqrt{}}$$

$$= \frac{a^{3/2}(14A+11B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c}}{8d}$$

Mathematica [A]
time = 1.26, size = 134, normalized size = 0.67

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(3\sqrt{2}(14A+11B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3(c+dx)+(7(6A+7B)+4(6A+11B)\cos(c+dx)+(42A+33B)\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{48d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(14*A + 11*B)*Arc
Tanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (7*(6*A + 7*B) + 4*(6*A + 1
1*B)*Cos[c + d*x] + (42*A + 33*B)*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*
d*Cos[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 404 vs. $2(170) = 340$.
time = 13.18, size = 405, normalized size = 2.02

method	result

default	$-\frac{a\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(42A\sqrt{2}\,(\cos^3(dx+c))\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/48/d*a*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(42*A*2^(1/2)
*cos(d*x+c)^3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c)
)*2^(1/2))-42*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+
c))*2^(1/2))*cos(d*x+c)^3*2^(1/2)+33*B*2^(1/2)*cos(d*x+c)^3*arctan(1/4*(-2/
(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))-33*B*arctan(1/4*(-
2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*cos(d*x+c)^3*2^(
1/2)+84*A*sin(d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+66*B*sin(d*x+c)
*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+24*A*(-2/(1+cos(d*x+c)))^(1/2)*sin(
d*x+c)*cos(d*x+c)+44*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+16*B
*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c))/(-2/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)
^(5/2)/sin(d*x+c)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 4606 vs. $2(170) = 340$.
time = 1.23, size = 4606, normalized size = 23.03

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] -1/96*(6*(56*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 24*
sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(
4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*sqrt(2)*a*sin
(3/2*d*x + 3/2*c) + 28*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) - 4*(3*sqrt(2)*a*sin(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*sin(
7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*sin(
5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 8*(3*sqrt(2)*a*sin(3/2*d*x +
3/2*c) - 7*sqrt(2)*a*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 7*(a
```

```
*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 + a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(
2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*c
os(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
a)*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 7*(a*cos
(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)
*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*c
os(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 7*(a*cos(8
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*cos(4/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a*sin(8/3*arctan2(sin
(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*a*sin(8/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 4*a*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c)))^2 + 2*(2*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) + a)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
) + 4*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a)*l
og(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin
```

```
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*(3*sqrt(2)
*a*cos(3/2*d*x + 3/2*c) + 7*sqrt(2)*a*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*a*cos(5/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) - 7*sqrt(2)*a*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
+ 3/2*c))) - 28*(2*sqrt(2)*a*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))) + sqrt(2)*a)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))) + 12*(2*sqrt(2)*a*cos(4/3*arctan2...
```

Fricas [A]

time = 2.67, size = 449, normalized size = 2.24

$$\left[\frac{4\left(3\left(14A+11B\right)a\cos\left(dx+c\right)^2+2\left(6A+11B\right)a\cos\left(dx+c\right)+8Ba\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\left(\left(14A+11B\right)a\cos\left(dx+c\right)^4+\left(14A+11B\right)a\cos\left(dx+c\right)^3\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}(\cos(dx+c)-2)\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-7a\cos(dx+c)^2+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{96\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}, \frac{2\left(3\left(14A+11B\right)a\cos\left(dx+c\right)^2+2\left(6A+11B\right)a\cos\left(dx+c\right)+8Ba\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\left(\left(14A+11B\right)a\cos\left(dx+c\right)^4+\left(14A+11B\right)a\cos\left(dx+c\right)^3\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)}{48\left(d\cos\left(dx+c\right)^4+d\cos\left(dx+c\right)^3\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/96*(4*(3*(14*A + 11*B)*a*cos(d*x + c)^2 + 2*(6*A + 11*B)*a*cos(d*x + c)
+ 8*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x
 + c) + 3*((14*A + 11*B)*a*cos(d*x + c)^4 + (14*A + 11*B)*a*cos(d*x + c)^3)
*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x +
c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^4 + d*cos(d
*x + c)^3), 1/48*(2*(3*(14*A + 11*B)*a*cos(d*x + c)^2 + 2*(6*A + 11*B)*a*co
s(d*x + c) + 8*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
c))*sin(d*x + c) + 3*((14*A + 11*B)*a*cos(d*x + c)^4 + (14*A + 11*B)*a*cos(
d*x + c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*
a)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]
```

Sympy [F(-2)]

time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F]

time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2), x
)
```

3.531 $\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=247

$$\frac{a^{3/2}(88A+75B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{64d}+\frac{a^2(8A+9B)\sin(c-}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\,\mathrm{s}}}$$

```
[Out] 1/64*a^(3/2)*(88*A+75*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))
*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+1/24*a^2*(8*A+9*B)*sin(d*x+c)/d/cos(d*
x+c)^(7/2)/(a+a*sec(d*x+c))^(1/2)+1/96*a^2*(88*A+75*B)*sin(d*x+c)/d/cos(d*x
+c)^(5/2)/(a+a*sec(d*x+c))^(1/2)+1/64*a^2*(88*A+75*B)*sin(d*x+c)/d/cos(d*x+
c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+1/4*a*B*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
/cos(d*x+c)^(7/2)
```

Rubi [A]
time = 0.40, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3034, 4103, 4101, 3888, 3886, 221}

$$\frac{a^{3/2}(88A+75B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^2(88A+75B)\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(88A+75B)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(8A+9B)\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{aB\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{4d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (a^(3/2)*(88*A + 75*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^2*(8*A + 9*B)*Sin[c
 + d*x])/(24*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(88*A +
75*B)*Sin[c + d*x])/(96*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a
^2*(88*A + 75*B)*Sin[c + d*x])/(64*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c +
d*x]]) + (a*B*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2
))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
```

```
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)(a+a\,\mathrm{se}$$

$$= \frac{aB\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)} + \frac{1}{4}\Big(\sqrt{\cos(c+dx)}$$

$$= \frac{a^2(8A+9B)\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{aB\sqrt{a+a\sec(}}{4d\cos}$$

$$= \frac{a^2(8A+9B)\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(88A+}{96d\cos^{\frac{5}{2}}(c+dx}$$

$$= \frac{a^2(8A+9B)\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(88A+}{96d\cos^{\frac{5}{2}}(c+dx}$$

$$= \frac{a^2(8A+9B)\sin(c+dx)}{24d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(88A+}{96d\cos^{\frac{5}{2}}(c+dx}$$

$$= \frac{a^{3/2}(88A+75B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(}}{64d}$$

Mathematica [A]
time = 1.83, size = 153, normalized size = 0.62

$$\frac{a\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(6\sqrt{2}\,(88A+75B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^4(c+dx)+(352A+492B+(1048A+1155B)\cos(c+dx)+4(88A+75B)\cos(2(c+dx))+264A\cos(3(c+dx))+225B\cos(3(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{768d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(5
/2),x]
```

```
[Out] (a*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(88*A + 75*B)*Arc
Tanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (352*A + 492*B + (1048*A +
1155*B)*Cos[c + d*x] + 4*(88*A + 75*B)*Cos[2*(c + d*x)] + 264*A*Cos[3*(c +
d*x)] + 225*B*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(768*d*Cos[c + d*x]^(7/2
))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 466 vs. $2(211) = 422$.
time = 12.55, size = 467, normalized size = 1.89

method	result
default	$-\frac{a\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(264A\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(-1-\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)(\cos^4(dx+c))\sqrt{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/384/d*a*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(264*A*arcta
n(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*x
+c)^4*2^(1/2)+264*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(
d*x+c))*2^(1/2))*cos(d*x+c)^4*2^(1/2)+225*B*arctan(1/4*(-2/(1+cos(d*x+c)))^
(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*x+c)^4*2^(1/2)+225*B*arctan
(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*x+c
)^4*2^(1/2)+528*A*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^3+450*B*(
-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^3+352*A*sin(d*x+c)*cos(d*x+c
)^2*(-2/(1+cos(d*x+c)))^(1/2)+300*B*sin(d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x+
c)))^(1/2)+128*A*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)+240*B*(-2/
(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+96*B*(-2/(1+cos(d*x+c)))^(1/2)*
sin(d*x+c))/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^2/cos(d*x+c)^(7/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 5879 vs. $2(211) = 422$.
time = 1.47, size = 5879, normalized size = 23.80

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] -1/768*(8*(132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) + 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 216*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
 3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c))) - 44*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
```

```
 2*c))) - 132*(sqrt(2)*a*sin(6*d*x + 6*c) + 3*sqrt(2)*a*sin(4*d*x + 4*c) +
3*sqrt(2)*a*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x
 + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x
+ 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c)
+ 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)
*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*
d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sq
rt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 33*(a*cos
(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6
*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2
*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) +
 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d
*x + 6*c) + a)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 33*(a*cos(6*d*x + 6*c)^2 + 9
*a*cos(4*d*x + 4*c)^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a
*sin(4*d*x + 4*c)^2 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*
x + 2*c)^2 + 2*(3*a*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x
+ 6*c) + 6*(3*a*cos(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*
c) + 6*(a*sin(4*d*x + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(
2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 2) + 33*(a*cos(6*d*x + 6*c)^2 + 9*a*cos(4*d*x + 4*c)
^2 + 9*a*cos(2*d*x + 2*c)^2 + a*sin(6*d*x + 6*c)^2 + 9*a*sin(4*d*x + 4*c)^2
 + 18*a*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*a*sin(2*d*x + 2*c)^2 + 2*(3*a
*cos(4*d*x + 4*c) + 3*a*cos(2*d*x + 2*c) + a)*cos(6*d*x + 6*c) + 6*(3*a*cos
(2*d*x + 2*c) + a)*cos(4*d*x + 4*c) + 6*a*cos(2*d*x + 2*c) + 6*(a*sin(4*d*x
 + 4*c) + a*sin(2*d*x + 2*c))*sin(6*d*x + 6*c) + a)*log(2*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 2) - 132*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3
*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(11/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 44*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*
x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(9/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 216*(sqrt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2
)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*sin(7/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 216*(sqrt(2)*a*cos(6*d*x + 6*c
) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a
```

```
)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 44*(sqrt(2)*a*cos(
6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2*d*x + 2*c)
+ sqrt(2)*a)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 132*(sq
rt(2)*a*cos(6*d*x + 6*c) + 3*sqrt(2)*a*cos(4*d*x + 4*c) + 3*sqrt(2)*a*cos(2
*d*x + 2*c) + sqrt(2)*a)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*A*sqrt(a)/(2*(3*cos(4*d*x + 4*c) + 3*cos(2*d*x + 2*c) + 1)*cos(6*d*x +
6*c) + cos(6*d*x + 6*c)^2 + 6*(3*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + 9
*cos(4*d*x + 4*c)^2 + 9*cos(2*d*x + 2*c)^2 + 6*(sin(4*d*x + 4*c) + sin(2*d*
x + 2*c))*sin(6*d*x + 6*c) + sin(6*d*x + 6*c)^2 + 9*sin(4*d*x + 4*c)^2 + 18
*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 9*sin(2*d*x + 2*c)^2 + 6*cos(2*d*x + 2
*c) + 1) + 3*(300*(sqrt(2)*a*sin(8*d*x + 8*c) + 4*sqrt(2)*a*sin(6*d*x + 6*c
) + 6*sqrt(2)*a*sin(4*d*x + 4*c) + 4*sqrt(2)*a*...
```

Fricas [A]
time = 2.84, size = 485, normalized size = 1.96

$$\left[\frac{4\,{\left(3\,{\left(88\,A + 75\,B\right)}a\cos\left(dx + c\right)^{3} + 2\,{\left(88\,A + 75\,B\right)}a\cos\left(dx + c\right)^{2} + 8\,{\left(8\,A + 15\,B\right)}a\cos\left(dx + c\right) + 48\,Ba\right)}\sqrt{\frac{a\cos\left(dx + c\right) + a}{\cos\left(dx + c\right)}}\sqrt{\cos\left(dx + c\right)}\sin\left(dx + c\right) + 3\,{\left({\left(88\,A + 75\,B\right)}a\cos\left(dx + c\right)^{5} + {\left(88\,A + 75\,B\right)}a\cos\left(dx + c\right)^{4}\right)}\sqrt{a}\log\left(\frac{a\cos\left(dx+c\right)^{3} - 4\,\sqrt{a}\sqrt{\frac{a\cos\left(dx + c\right) + a}{\cos\left(dx + c\right)}}{\left(\cos\left(dx+c\right) - 2\right)}\sqrt{\cos\left(dx + c\right)}\sin\left(dx+c\right) - 7\,a\cos\left(dx+c\right)^{2} + 8\,a}{\cos\left(dx+c\right)^{3} + \cos\left(dx+c\right)^{2}}\right)}{768\,{\left(d\cos\left(dx + c\right)^{5} + d\cos\left(dx + c\right)^{4}\right)}},\ \frac{2\,{\left(3\,{\left(88\,A + 75\,B\right)}a\cos\left(dx + c\right)^{3} + 2\,{\left(88\,A + 75\,B\right)}a\cos\left(dx + c\right)^{2} + 8\,{\left(8\,A + 15\,B\right)}a\cos\left(dx + c\right) + 48\,Ba\right)}\sqrt{\frac{a\cos\left(dx + c\right) + a}{\cos\left(dx + c\right)}}\sqrt{\cos\left(dx + c\right)}\sin\left(dx + c\right) + 3\,{\left({\left(88\,A + 75\,B\right)}a\cos\left(dx + c\right)^{5} + {\left(88\,A + 75\,B\right)}a\cos\left(dx + c\right)^{4}\right)}\sqrt{-a}\arctan\left(\frac{2\,\sqrt{-a}\sqrt{\frac{a\cos\left(dx + c\right) + a}{\cos\left(dx + c\right)}}\sqrt{\cos\left(dx + c\right)}\sin\left(dx+c\right)}{a\cos\left(dx+c\right)^{2} - a\cos\left(dx+c\right) - 2\,a}\right)}{384\,{\left(d\cos\left(dx + c\right)^{5} + d\cos\left(dx + c\right)^{4}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/768*(4*(3*(88*A + 75*B)*a*cos(d*x + c)^3 + 2*(88*A + 75*B)*a*cos(d*x + c
)^2 + 8*(8*A + 15*B)*a*cos(d*x + c) + 48*B*a)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((88*A + 75*B)*a*cos(d*x + c
)^5 + (88*A + 75*B)*a*cos(d*x + c)^4)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqr
t(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*
x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x
+ c)^2)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4), 1/384*(2*(3*(88*A + 75*B)*
a*cos(d*x + c)^3 + 2*(88*A + 75*B)*a*cos(d*x + c)^2 + 8*(8*A + 15*B)*a*cos(
d*x + c) + 48*B*a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c) + 3*((88*A + 75*B)*a*cos(d*x + c)^5 + (88*A + 75*B)*a*cos(d
*x + c)^4)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a
)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(3/2))/cos(c + d*x)^(5/2), x
)
```

3.532 $\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=275

$$\frac{16a^3(710A+803B)\sin(c+dx)}{3465d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{8a^3(710A+803B)\sqrt{\cos(c+dx)}\sin(c+dx)}{3465d\sqrt{a+a\sec(c+dx)}}+\frac{2a^3(710A+803B}{1155d\sqrt{}}$$

```
[Out] 2/11*a*A*cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/1155*a^3*(7
10*A+803*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/693*a^3*
(194*A+209*B)*cos(d*x+c)^(5/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+16/3465*
a^3*(710*A+803*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+8/34
65*a^3*(710*A+803*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2
/99*a^2*(14*A+11*B)*cos(d*x+c)^(7/2)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]
time = 0.55, antiderivative size = 275, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3034, 4102, 4100, 3890, 3889}

$$\frac{2a^3(194A+209B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{693d\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(710A+803B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{1155d\sqrt{a\sec(c+dx)+a}}+\frac{8a^3(710A+803B)\sin(c+dx)\sqrt{\cos(c+dx)}}{3465d\sqrt{a\sec(c+dx)+a}}+\frac{16a^3(710A+803B)\sin(c+dx)}{3465d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(14A+11B)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{99d}+\frac{2aA\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (16*a^3*(710*A + 803*B)*Sin[c + d*x])/(3465*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a
*Sec[c + d*x]]) + (8*a^3*(710*A + 803*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(
3465*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(710*A + 803*B)*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(1155*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(194*A + 209*B)*
Cos[c + d*x]^(5/2)*Sin[c + d*x])/(693*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*
(14*A + 11*B)*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(99
*d) + (2*a*A*Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(1
1*d)
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{11}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\mathrm{se}}{}$$

$$= \frac{2aA\cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c}{11d}$$

$$= \frac{2a^2(14A+11B)\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c-}}{99d}$$

$$= \frac{2a^3(194A+209B)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{693d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^3(710A+803B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{1155d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{8a^3(710A+803B)\sqrt{\cos(c+dx)}\,\sin(c+dx}{3465d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{16a^3(710A+803B)\sin(c+dx)}{3465d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} +$$

Mathematica [A]

time = 0.58, size = 137, normalized size = 0.50

$$\frac{2a^2\sqrt{\cos(c+dx)}\,(8(710A+803B)+4(710A+803B)\cos(c+dx)+3(710A+803B)\cos^2(c+dx)+5(355A+286B)\cos^3(c+dx)+35(32A+11B)\cos^4(c+dx)+315A\cos^5(c+dx))\sqrt{a(1+\sec(c+dx))}\,\sin(c+dx)}{3465d(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(11/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x
]),x]
```

```
[Out] (2*a^2*Sqrt[Cos[c + d*x]]*(8*(710*A + 803*B) + 4*(710*A + 803*B)*Cos[c + d*
x] + 3*(710*A + 803*B)*Cos[c + d*x]^2 + 5*(355*A + 286*B)*Cos[c + d*x]^3 +
35*(32*A + 11*B)*Cos[c + d*x]^4 + 315*A*Cos[c + d*x]^5)*Sqrt[a*(1 + Sec[c +
 d*x])]*Sin[c + d*x])/(3465*d*(1 + Cos[c + d*x]))
```

Maple [A]

time = 12.64, size = 155, normalized size = 0.56

method	result
default	$-\frac{2a^2(-1+\cos(dx+c))(315A(\cos^5(dx+c))+1120A(\cos^4(dx+c))+385B(\cos^4(dx+c))+1775A(\cos^3(dx+c))+1430B(\cos^3(dx+c))+21}{3465a}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] -2/3465/d*a^2*(-1+cos(d*x+c))*(315*A*cos(d*x+c)^5+1120*A*cos(d*x+c)^4+385*B*cos(d*x+c)^4+1775*A*cos(d*x+c)^3+1430*B*cos(d*x+c)^3+2130*A*cos(d*x+c)^2+2409*B*cos(d*x+c)^2+2840*A*cos(d*x+c)+3212*B*cos(d*x+c)+5680*A+6424*B)*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 754 vs. $2(239) = 478$.
time = 1.08, size = 754, normalized size = 2.74

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algorithm="maxima")

[Out] 1/110880*(5*sqrt(2)*(31878*a^2*cos(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 8778*a^2*cos(8/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 3465*a^2*cos(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 1287*a^2*cos(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) + 385*a^2*cos(2/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c)))*sin(11/2*d*x + 11/2*c) - 31878*a^2*cos(11/2*d*x + 11/2*c)*sin(10/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 8778*a^2*cos(11/2*d*x + 11/2*c)*sin(8/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 3465*a^2*cos(11/2*d*x + 11/2*c)*sin(6/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 1287*a^2*cos(11/2*d*x + 11/2*c)*sin(4/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) - 385*a^2*cos(11/2*d*x + 11/2*c)*sin(2/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 126*a^2*sin(11/2*d*x + 11/2*c) + 385*a^2*sin(9/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 1287*a^2*sin(7/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 3465*a^2*sin(5/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 8778*a^2*sin(3/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))) + 31878*a^2*sin(1/11*arctan2(sin(11/2*d*x + 11/2*c), cos(11/2*d*x + 11/2*c))))*A*sqrt(a) + 44*sqrt(2)*(225*a^2*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 378*a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2100*a^2*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4095*a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 63*(65*a^2*sin(4*d*x + 4*c) + 6*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x +

```
2*c), cos(2*d*x + 2*c))) + 7*(585*a^2*cos(4*d*x + 4*c) + 54*a^2*cos(2*d*x +
 2*c) + 5*a^2)*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt
(a))/d
```

Fricas [A]
time = 2.38, size = 154, normalized size = 0.56

$$\frac{2\,\left(315\,Aa^{2}\cos\left(dx+c\right)^{5}+35\,\left(32\,A+11\,B\right)a^{2}\cos\left(dx+c\right)^{4}+5\,\left(355\,A+286\,B\right)a^{2}\cos\left(dx+c\right)^{3}+3\,\left(710\,A+803\,B\right)a^{2}\cos\left(dx+c\right)^{2}+4\,\left(710\,A+803\,B\right)a^{2}\cos\left(dx+c\right)+8\,\left(710\,A+803\,B\right)a^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{3465\,\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algo
rithm="fricas")
```

```
[Out] 2/3465*(315*A*a^2*cos(d*x + c)^5 + 35*(32*A + 11*B)*a^2*cos(d*x + c)^4 + 5*
(355*A + 286*B)*a^2*cos(d*x + c)^3 + 3*(710*A + 803*B)*a^2*cos(d*x + c)^2 +
 4*(710*A + 803*B)*a^2*cos(d*x + c) + 8*(710*A + 803*B)*a^2)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) +
 d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algo
rithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(11/
2), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\cos\left(c+d\,x\right)}^{11/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(11/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(11/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)
```

3.533 $\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=228

$$\frac{4a^3(292A+345B)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}+\frac{2a^3(292A+345B)\sqrt{\cos(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}+\frac{2a^3(124A+135B}{315d\sqrt{c}}$$

```
[Out] 2/9*a*A*cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/315*a^3*(124
*A+135*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+4/315*a^3*(2
92*A+345*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/315*a^3*
(292*A+345*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)+2/21*a^2
*(4*A+3*B)*cos(d*x+c)^(5/2)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.48, antiderivative size = 228, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3034, 4102, 4100, 3890, 3889}

$$\frac{2a^3(124A+135B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{315d\sqrt{a\sec(c+dx)+a}}+\frac{2a^3(292A+345B)\sin(c+dx)\sqrt{\cos(c+dx)}}{315d\sqrt{a\sec(c+dx)+a}}+\frac{4a^3(292A+345B)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(4A+3B)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}{21d}+\frac{2aA\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (4*a^3*(292*A + 345*B)*Sin[c + d*x])/(315*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*S
ec[c + d*x]]) + (2*a^3*(292*A + 345*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(31
5*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^3*(124*A + 135*B)*Cos[c + d*x]^(3/2)*S
in[c + d*x])/(315*d*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(4*A + 3*B)*Cos[c +
d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*A*Cos[c + d
*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(9*d)
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3890

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[a*Cot[e + f*x]*((d*Csc[e + f*x])^n/(f*n*Sqrt[a
+ b*Csc[e + f*x]])), x] + Dist[a*((2*n + 1)/(2*b*d*n)), Int[Sqrt[a + b*Csc[
e + f*x]]*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f}, x] &&
EqQ[a^2 - b^2, 0] && LtQ[n, -2^(-1)] && IntegerQ[2*n]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec}{}$$

$$= \frac{2aA\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c}{9d}$$

$$= \frac{2a^2(4A+3B)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+d}}{21d}$$

$$= \frac{2a^3(124A+135B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2a^3(292A+345B)\sqrt{\cos(c+dx)}\sin(c+dx)}{315d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{4a^3(292A+345B)\sin(c+dx)}{315d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a}{}$$

Mathematica [A]
time = 0.46, size = 116, normalized size = 0.51

$$\frac{2a^2\sqrt{\cos(c+dx)}\,(584A+690B+(292A+345B)\cos(c+dx)+3(73A+60B)\cos^2(c+dx)+5(26A+9B)\cos^3(c+dx)+35A\cos^4(c+dx))\,\sqrt{a(1+\sec(c+dx))}\,\sin(c+dx)}{315d(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (2*a^2*Sqrt[Cos[c + d*x]]*(584*A + 690*B + (292*A + 345*B)*Cos[c + d*x] + 3
*(73*A + 60*B)*Cos[c + d*x]^2 + 5*(26*A + 9*B)*Cos[c + d*x]^3 + 35*A*Cos[c
+ d*x]^4)*Sqrt[a*(1 + Sec[c + d*x])]*Sin[c + d*x])/(315*d*(1 + Cos[c + d*x]
))
```

Maple [A]
time = 12.63, size = 133, normalized size = 0.58

method	result
default	$-\frac{2a^2(-1+\cos(dx+c))(35A(\cos^4(dx+c))+130A(\cos^3(dx+c))+45B(\cos^3(dx+c))+219A(\cos^2(dx+c))+180B(\cos^2(dx+c))+292A}{315d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/315/d*a^2*(-1+cos(d*x+c))*(35*A*cos(d*x+c)^4+130*A*cos(d*x+c)^3+45*B*cos
(d*x+c)^3+219*A*cos(d*x+c)^2+180*B*cos(d*x+c)^2+292*A*cos(d*x+c)+345*B*cos(
d*x+c)+584*A+690*B)*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/si
n(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 596 vs. $2(198) = 396$.
time = 0.91, size = 596, normalized size = 2.61

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/5040*(sqrt(2)*(8190*a^2*cos(8/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x
 + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 2100*a^2*cos(2/3*arctan2(sin(9/2*d*x + 9
/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 756*a^2*cos(4/9*arctan
2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x + 9/2*c) + 225*a
```

```
^2*cos(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c)))*sin(9/2*d*x
 + 9/2*c) - 8190*a^2*cos(9/2*d*x + 9/2*c)*sin(8/9*arctan2(sin(9/2*d*x + 9/2
*c), cos(9/2*d*x + 9/2*c))) - 2100*a^2*cos(9/2*d*x + 9/2*c)*sin(2/3*arctan2
(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 756*a^2*cos(9/2*d*x + 9/2*c
)*sin(4/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*c))) - 225*a^2*co
s(9/2*d*x + 9/2*c)*sin(2/9*arctan2(sin(9/2*d*x + 9/2*c), cos(9/2*d*x + 9/2*
c))) + 70*a^2*sin(9/2*d*x + 9/2*c) + 225*a^2*sin(7/9*arctan2(sin(9/2*d*x +
9/2*c), cos(9/2*d*x + 9/2*c))) + 756*a^2*sin(5/9*arctan2(sin(9/2*d*x + 9/2*
c), cos(9/2*d*x + 9/2*c))) + 2100*a^2*sin(1/3*arctan2(sin(9/2*d*x + 9/2*c),
 cos(9/2*d*x + 9/2*c))) + 8190*a^2*sin(1/9*arctan2(sin(9/2*d*x + 9/2*c), co
s(9/2*d*x + 9/2*c))))*A*sqrt(a) - 30*sqrt(2)*(77*a^2*cos(7/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - 42*a^2*sin(5/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - 77*a^2*sin(3/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) - 630*a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) - (77*a^2*cos(2*d*x + 2*c) + 6*a^2)*sin(7/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*B*sqrt(a))/d
```

Fricas [A]
time = 2.60, size = 133, normalized size = 0.58

$$\frac{2\,\left(35\,Aa^{2}\cos\left(dx+c\right)^{4}+5\,(26\,A+9\,B)a^{2}\cos\left(dx+c\right)^{3}+3\,(73\,A+60\,B)a^{2}\cos\left(dx+c\right)^{2}+(292\,A+345\,B)a^{2}\cos\left(dx+c\right)+2\,(292\,A+345\,B)a^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\,\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)}{315\,\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 2/315*(35*A*a^2*cos(d*x + c)^4 + 5*(26*A + 9*B)*a^2*cos(d*x + c)^3 + 3*(73*
A + 60*B)*a^2*cos(d*x + c)^2 + (292*A + 345*B)*a^2*cos(d*x + c) + 2*(292*A
+ 345*B)*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*si
n(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{9/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)
```

3.534 $\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=178

$$\frac{64a^3(5A+7B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}}+\frac{16a^2(5A+7B)\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{105d}+\frac{2a}{}$$

```
[Out] 2/35*a*(5*A+7*B)*cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/7*A
*cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*sin(d*x+c)/d+64/105*a^3*(5*A+7*B)*
sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+16/105*a^2*(5*A+7*B)*s
in(d*x+c)*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1/2)/d
```

Rubi [A]

time = 0.29, antiderivative size = 178, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3034, 4098, 3894, 3889}

$$\frac{64a^3(5A+7B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}+\frac{16a^2(5A+7B)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}{105d}+\frac{2a(5A+7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}{35d}+\frac{2A\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (64*a^3*(5*A + 7*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[
c + d*x]]) + (16*a^2*(5*A + 7*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]
]*Sin[c + d*x])/(105*d) + (2*a*(5*A + 7*B)*Cos[c + d*x]^(3/2)*(a + a*Sec[c
+ d*x])^(3/2)*Sin[c + d*x])/(35*d) + (2*A*Cos[c + d*x]^(5/2)*(a + a*Sec[c +
 d*x])^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3889

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Simp[-2*a*(Cot[e + f*x]/(f*Sqrt[a + b*Csc[e + f*x]]*S
qrt[d*Csc[e + f*x]])), x] /; FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3894

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[(-a)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)
*((d*Csc[e + f*x])^n/(f*m)), x] + Dist[b*((2*m - 1)/(d*m)), Int[(a + b*Csc[
e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f,
 m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && GtQ[m, 1/2] && IntegerQ
[2*m]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\mathrm{se}}{}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}\sin(c}{7d}$$

$$= \frac{2a(5A+7B)\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx}{35d}$$

$$= \frac{16a^2(5A+7B)\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c}}{105d}$$

$$= \frac{64a^3(5A+7B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} +$$

Mathematica [A]
time = 0.36, size = 99, normalized size = 0.56

$$\frac{2a^2\sqrt{\cos(c+dx)}\,(230A+301B+(115A+98B)\cos(c+dx)+3(20A+7B)\cos^2(c+dx)+15A\cos^3(c+dx))\,\sqrt{a(1+\sec(c+dx))}\,\sin(c+dx)}{105d(1+\cos(c+dx))}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (2*a^2*Sqrt[Cos[c + d*x]]*(230*A + 301*B + (115*A + 98*B)*Cos[c + d*x] + 3*
(20*A + 7*B)*Cos[c + d*x]^2 + 15*A*Cos[c + d*x]^3)*Sqrt[a*(1 + Sec[c + d*x]
)]*Sin[c + d*x])/(105*d*(1 + Cos[c + d*x]))
```

Maple [A]
time = 12.73, size = 111, normalized size = 0.62

method	result
default	$-\frac{2a^{2}(-1+\cos(dx+c))(15A(\cos^{3}(dx+c))+60A(\cos^{2}(dx+c))+21B(\cos^{2}(dx+c))+115A\cos(dx+c)+98B\cos(dx+c)+230A+301B)\left(\right.}{105d\sin(dx+c)}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/105/d*a^2*(-1+cos(d*x+c))*(15*A*cos(d*x+c)^3+60*A*cos(d*x+c)^2+21*B*cos(
d*x+c)^2+115*A*cos(d*x+c)+98*B*cos(d*x+c)+230*A+301*B)*cos(d*x+c)^(1/2)*(a*
(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 482 vs. 2(154) = 308.
time = 0.89, size = 482, normalized size = 2.71

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/840*(5*sqrt(2)*(315*a^2*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x
 + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 77*a^2*cos(4/7*arctan2(sin(7/2*d*x + 7/2
*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*a^2*cos(2/7*arctan2(s
in(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 315*a^2*
cos(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/
2*c))) - 77*a^2*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c),
cos(7/2*d*x + 7/2*c))) - 21*a^2*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/
2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 6*a^2*sin(7/2*d*x + 7/2*c) + 21*a^
2*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 77*a^2*sin
(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 315*a^2*sin(1/7
*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A*sqrt(a) - 28*(75*s
qrt(2)*a^2*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +
 2*c) - 25*sqrt(2)*a^2*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 - 75*sqrt(2)*a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*
(25*sqrt(2)*a^2*cos(2*d*x + 2*c) + sqrt(2)*a^2)*sin(5/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))))*B*sqrt(a))/d
```

Fricas [A]
time = 2.14, size = 112, normalized size = 0.63

$$\frac{2\left(15\,Aa^{2}\cos\left(dx+c\right)^{3}+3\left(20\,A+7\,B\right)a^{2}\cos\left(dx+c\right)^{2}+\left(115\,A+98\,B\right)a^{2}\cos\left(dx+c\right)+\left(230\,A+301\,B\right)a^{2}\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{105\left(d\cos\left(dx+c\right)+d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 2/105*(15*A*a^2*cos(d*x + c)^3 + 3*(20*A + 7*B)*a^2*cos(d*x + c)^2 + (115*A
 + 98*B)*a^2*cos(d*x + c) + (230*A + 301*B)*a^2)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(d*cos(d*x + c) + d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\text{could not integrate}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{7/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)
```

3.535 $\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=192

$$\frac{2a^{5/2}B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{d}+\frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}$$

```
[Out] 2/5*a*A*cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2*a^(5/2)*B*ar
csinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c
)^(1/2)/d+2/15*a^3*(32*A+35*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c
))^(1/2)+2/15*a^2*(8*A+5*B)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+a*sec(d*x+c))^(1
/2)/d
```

Rubi [A]
time = 0.38, antiderivative size = 192, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3034, 4102, 4100, 3886, 221}

$$\frac{2a^{5/2}B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}+\frac{2a^2(8A+5B)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}{15d}+\frac{2aA\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a^(5/2)*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (2*a^3*(32*A + 35*B)*Sin[c + d*x])/(1
5*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (2*a^2*(8*A + 5*B)*Sqrt[
Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*A*Cos[c
+ d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\sec}{}$$

$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}\sin(c}{5d}$$

$$= \frac{2a^2(8A+5B)\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+}}{15d}$$

$$= \frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a}{}$$

$$= \frac{2a^3(32A+35B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2a}{}$$

$$= \frac{2a^{5/2}B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos}}{d}$$

Mathematica [A]

time = 0.66, size = 118, normalized size = 0.61

$$\frac{2a^3\left((43A+40B+(14A+5B)\cos(c+dx)+3A\cos^2(c+dx))\sqrt{1-\sec(c+dx)}+15B\,\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)\sqrt{\sec(c+dx)}\right)\sin(c+dx)}{15d\sqrt{-1+\cos(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (2*a^3*((43*A + 40*B + (14*A + 5*B)*Cos[c + d*x] + 3*A*Cos[c + d*x]^2)*Sqrt
[1 - Sec[c + d*x]] + 15*B*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]]
)*Sin[c + d*x])/(15*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]

time = 13.63, size = 225, normalized size = 1.17

method	result
default	$-\frac{a^2\left(\sqrt{\cos}(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(-15B\sqrt{2}\sin(dx+c)\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}(1+\cos(dx+c)-\sin(dx+c))\sqrt{2}}{4}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/30/d*a^2*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-15*B*2^(
1/2)*sin(d*x+c)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+
c))*2^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)+15*B*2^(1/2)*sin(d*x+c)*(-2/(1+cos(d
*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c)
)*2^(1/2))+12*A*cos(d*x+c)^3+44*A*cos(d*x+c)^2+20*B*cos(d*x+c)^2+116*A*cos(
d*x+c)+140*B*cos(d*x+c)-172*A-160*B)/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 352 vs. $2(164) = 328$.
time = 0.82, size = 352, normalized size = 1.83

$$\frac{\left(3\sqrt{2}a^2\sin\left(\frac{5}{2}dx+\frac{5}{2}c\right)+25\sqrt{2}a^2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+150\sqrt{2}a^2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A\sqrt{a}+5\left(2\sqrt{2}a^2\sin\left(\frac{3}{2}dx+\frac{3}{2}c\right)+30\sqrt{2}a^2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+3a^2\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)-3a^2\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)+3a^2\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)-3a^2\log\left(2\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sqrt{2}\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)-2\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+2\right)\right)B\sqrt{a}}{30\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/30*((3*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 25*sqrt(2)*a^2*sin(3/2*d*x + 3/
2*c) + 150*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c))*A*sqrt(a) + 5*(2*sqrt(2)*a^2*s
in(3/2*d*x + 3/2*c) + 30*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 3*a^2*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 3*a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) + 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/
2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
 1/2*c) + 2) - 3*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^
2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*B
*sqrt(a))/d
```

Fricas [A]
time = 2.07, size = 399, normalized size = 2.08

$$\left[\frac{4\left(3Aa^2\cos\left(dx+c\right)^2+(14A+5B)a^2\cos\left(dx+c\right)+(43A+40B)a^2\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+15\left(Ba^2\cos\left(dx+c\right)+Ba^2\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}(\cos(dx+c)-2)\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-7a\cos(dx+c)^2+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{30\left(d\cos\left(dx+c\right)+d\right)}, \frac{2\left(3Aa^2\cos\left(dx+c\right)^2+(14A+5B)a^2\cos\left(dx+c\right)+(43A+40B)a^2\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+15\left(Ba^2\cos\left(dx+c\right)+Ba^2\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)}{15\left(d\cos\left(dx+c\right)+d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] [1/30*(4*(3*A*a^2*cos(d*x + c)^2 + (14*A + 5*B)*a^2*cos(d*x + c) + (43*A +
40*B)*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c) + 15*(B*a^2*cos(d*x + c) + B*a^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4
*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(co
s(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(
d*x + c)^2)))/(d*cos(d*x + c) + d), 1/15*(2*(3*A*a^2*cos(d*x + c)^2 + (14*A
 + 5*B)*a^2*cos(d*x + c) + (43*A + 40*B)*a^2)*sqrt((a*cos(d*x + c) + a)/cos
(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*(B*a^2*cos(d*x + c) + B*a^2
)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*co
s(d*x + c) + d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{5/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)
```

3.536 $\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=197

$$\frac{a^{5/2}(2A+5B)\sinh^{-1}\left(\frac{\sqrt{a}\ \tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\ \sqrt{\sec(c+dx)}}{d}+\frac{a^3(14A+3B)\sin(c+}{3d\sqrt{\cos(c+dx)}\ \sqrt{a+a\sec}}$$

```
[Out] 2/3*a*A*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d+a^(5/2)*(2*A+5
*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d+1/3*a^3*(14*A+3*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d
*x+c))^(1/2)-1/3*a^2*(2*A-3*B)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/cos(d*x+
c)^(1/2)
```

Rubi [A]

time = 0.40, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3034, 4102, 4103, 4100, 3886, 221}

$$\frac{a^{5/2}(2A+5B)\sqrt{\cos(c+dx)}\ \sqrt{\sec(c+dx)}\ \sinh^{-1}\left(\frac{\sqrt{a}\ \tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{d}+\frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\ \sqrt{a\sec(c+dx)+a}}-\frac{a^2(2A-3B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{3d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)\sqrt{\cos(c+dx)}\ (a\sec(c+dx)+a)^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(5/2)*(2*A + 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]
]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/d + (a^3*(14*A + 3*B)*Sin[c + d*x
])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - (a^2*(2*A - 3*B)*Sqr
t[a + a*Sec[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*a*A*Sqrt[
Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4102

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] - Dis
t[b/(a*d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp
[a*A*(m - n - 1) - b*B*n - (a*B*n + A*b*(m + n))*Csc[e + f*x], x], x], x] /
; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
 && GtQ[m, 1/2] && LtQ[n, -1]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\mathrm{se}}{}$$

$$= \frac{2aA\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}\sin}{3d}$$

$$= -\frac{a^2(2A-3B)\sqrt{a+a\sec(c+dx)}\,\sin(c+}{3d\sqrt{\cos(c+dx)}}$$

$$= \frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} - \frac{a^2}{}$$

$$= \frac{a^3(14A+3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} - \frac{a^2}{}$$

$$= \frac{a^{5/2}(2A+5B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right.}{d}$$

Mathematica [A]
time = 0.68, size = 117, normalized size = 0.59

$$\frac{a^3\left(3(2A+5B)\mathrm{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)\sqrt{\sec(c+dx)}+\sqrt{1-\sec(c+dx)}\,(16A+6B+2A\cos(c+dx)+3B\sec(c+dx))\right)\sin(c+dx)}{3d\sqrt{-1+\cos(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (a^3*(3*(2*A + 5*B)*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]] + Sqr
t[1 - Sec[c + d*x]]*(16*A + 6*B + 2*A*Cos[c + d*x] + 3*B*Sec[c + d*x]))*Sin
[c + d*x])/(3*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 367 vs. 2(169) = 338.
time = 12.91, size = 368, normalized size = 1.87

method	result

default	$-\frac{a^2\left(6A\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}\ \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\ (1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\cos(dx+c)\sqrt{2}-6A\sin(dx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/12/d*a^2*(6*A*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos
(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*x+c)*2^(1/2)-6*A*s
in(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1
+cos(d*x+c)-sin(d*x+c))*2^(1/2))*cos(d*x+c)*2^(1/2)+15*B*(-2/(1+cos(d*x+c))
)^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1
/2))*cos(d*x+c)*sin(d*x+c)*2^(1/2)-15*B*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/
4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*cos(d*x+c)*s
in(d*x+c)*2^(1/2)+8*A*cos(d*x+c)^3+56*A*cos(d*x+c)^2+24*B*cos(d*x+c)^2-64*A
*cos(d*x+c)-12*B*cos(d*x+c)-12*B)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d
*x+c)/cos(d*x+c)^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2589 vs. $2(169) = 338$.
time = 0.98, size = 2589, normalized size = 13.14

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] 1/12*(sqrt(2)*(30*a^2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))*sin(3/2*d*x + 3/2*c) - 30*a^2*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 3*sqrt(2)*a^2*log(2*cos(1/3*
arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2*cos(1
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 3*sqrt(2)*a^2*log(2*co
s(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*ar
```

ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*sin(1/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 3*sqrt(2)*a^2*log(2
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + 4*a^2*sin(3/2*d*
x + 3/2*c) + 30*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c))))*A*sqrt(a) + 3*(4*sqrt(2)*a^2*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sq
rt(2)*a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*a^2*sin(1/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^3 - 4*sqrt(2)*a^2*cos(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c))) + 4*sqrt(2)*a^2*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(2*sqrt(2)*a^
2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - sqrt(2)*a^2*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 5*(a^2*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*a^2
*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c))) + a^2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
)^2 + 2*a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + a^2*sin(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2
)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 5*(a^2*cos(5/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*a^2*cos(5/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + a^2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*a^2*sin(5/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 5*(a^2*cos(5/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2 + 2*a^2*cos(5/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + a^2*
cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + a^2*sin(5/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*a^2*sin(5/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2)*log(2*co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(si

```
n(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 2) - 5*(a^2*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + 2*a^2*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*cos
(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + a^2*cos(1/4*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + a^2*sin(5/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + 2*a^2*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + a^2*sin(1/4*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2)...
```

Fricas [A]
time = 2.15, size = 449, normalized size = 2.28

$$\left[\frac{4\,{\left(2\,A a^{2} \cos\left(d x + c\right)^{2} + 2\,{\left(8\,A + 3\,B\right)} a^{2} \cos\left(d x + c\right) + 3\,B a^{2}\right)} \sqrt{\frac{a \cos\left(d x + c\right) + a}{\cos\left(d x + c\right)}} \sqrt{\cos\left(d x + c\right)} \sin\left(d x + c\right) + 3\,{\left({\left(2\,A + 5\,B\right)} a^{2} \cos\left(d x + c\right)^{2} + {\left(2\,A + 5\,B\right)} a^{2} \cos\left(d x + c\right)\right)} \sqrt{a} \log\left(\frac{a \cos\left(d x + c\right)^{3} - 4\,\sqrt{a} \sqrt{\frac{a \cos\left(d x + c\right) + a}{\cos\left(d x + c\right)}} {\left(\cos\left(d x + c\right) - 2\right)} \sqrt{\cos\left(d x + c\right)} \sin\left(d x + c\right) - 7\,a \cos\left(d x + c\right)^{2} + 8\,a}{\cos\left(d x + c\right)^{3} + \cos\left(d x + c\right)^{2}}\right)}{12\,{\left(d \cos\left(d x + c\right)^{2} + d \cos\left(d x + c\right)\right)}}, \frac{2\,{\left(2\,A a^{2} \cos\left(d x + c\right)^{2} + 2\,{\left(8\,A + 3\,B\right)} a^{2} \cos\left(d x + c\right) + 3\,B a^{2}\right)} \sqrt{\frac{a \cos\left(d x + c\right) + a}{\cos\left(d x + c\right)}} \sqrt{\cos\left(d x + c\right)} \sin\left(d x + c\right) + 3\,{\left({\left(2\,A + 5\,B\right)} a^{2} \cos\left(d x + c\right)^{2} + {\left(2\,A + 5\,B\right)} a^{2} \cos\left(d x + c\right)\right)} \sqrt{-a} \arctan\left(\frac{2\,\sqrt{-a} \sqrt{\frac{a \cos\left(d x + c\right) + a}{\cos\left(d x + c\right)}} \sqrt{\cos\left(d x + c\right)} \sin\left(d x + c\right)}{a \cos\left(d x + c\right)^{2} - a \cos\left(d x + c\right) - 2\,a}\right)}{6\,{\left(d \cos\left(d x + c\right)^{2} + d \cos\left(d x + c\right)\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] [1/12*(4*(2*A*a^2*cos(d*x + c)^2 + 2*(8*A + 3*B)*a^2*cos(d*x + c) + 3*B*a^2
)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) +
 3*((2*A + 5*B)*a^2*cos(d*x + c)^2 + (2*A + 5*B)*a^2*cos(d*x + c))*sqrt(a)*
log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(
cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*
a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^2 + d*cos(d*x + c)),
 1/6*(2*(2*A*a^2*cos(d*x + c)^2 + 2*(8*A + 3*B)*a^2*cos(d*x + c) + 3*B*a^2)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) +
3*((2*A + 5*B)*a^2*cos(d*x + c)^2 + (2*A + 5*B)*a^2*cos(d*x + c))*sqrt(-a)*
arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^
2 + d*cos(d*x + c))]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \cos\left(c+d\,x\right)^{3/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)
```

3.537 $\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=200

$$\frac{a^{5/2}(20A+19B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{4d}+\frac{a^3(4A-9B)\sin(c+}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+a\,\mathrm{se}}}$$

```
[Out] 1/2*a*B*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+1/4*a^(5/2)*(2
0*A+19*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/
2)*sec(d*x+c)^(1/2)/d+1/4*a^3*(4*A-9*B)*sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*
sec(d*x+c))^(1/2)+1/4*a^2*(4*A+7*B)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/cos
(d*x+c)^(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3034, 4103, 4100, 3886, 221}

$$\frac{a^{5/2}(20A+19B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4d}+\frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}+\frac{a^2(4A+7B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{4d\sqrt{\cos(c+dx)}}+\frac{aB\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (a^(5/2)*(20*A + 19*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*
x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(4*d) + (a^3*(4*A - 9*B)*Sin[c
+ d*x])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(4*A + 7*B
)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + (a*B*(a
 + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 4100

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*b^2*Co
t[e + f*x]*((d*Csc[e + f*x])^n/(a*f*n*Sqrt[a + b*Csc[e + f*x]])), x] + Dist
[(A*b*(2*n + 1) + 2*a*B*n)/(2*a*d*n), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e
 + f*x])^(n + 1), x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*
B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] && LtQ[n, 0]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+a\,\mathrm{s}}{}$$

$$= \frac{aB(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{2d\sqrt{\cos(c+dx)}} + \frac{1}{2}\Big($$

$$= \frac{a^2(4A+7B)\sqrt{a+a\sec(c+dx)}\,\sin(c+d}{4d\sqrt{\cos(c+dx)}}$$

$$= \frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{a^2}{}$$

$$= \frac{a^3(4A-9B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{a^2}{}$$

$$= \frac{a^{5/2}(20A+19B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+}}\right.}{4d}$$

Mathematica [A]
time = 0.99, size = 173, normalized size = 0.86

$$\frac{a^3\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))\left(20A\text{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)\sqrt{\sec(c+dx)}-19B\text{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\sqrt{\sec(c+dx)}+\sqrt{1-\sec(c+dx)}\,(8A+(4A+11B)\sec(c+dx)+2B\sec^2(c+dx))\right)\sin(c+dx)}{4d(B+A\cos(c+dx))\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (a^3*Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x])*(20*A*ArcSin[Sqrt[1 - Sec[c +
d*x]]]*Sqrt[Sec[c + d*x]] - 19*B*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[Sec[c + d*
x]] + Sqrt[1 - Sec[c + d*x]]*(8*A + (4*A + 11*B)*Sec[c + d*x] + 2*B*Sec[c +
 d*x]^2))*Sin[c + d*x])/(4*d*(B + A*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]]*Sq
rt[a*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 375 vs. 2(170) = 340.
time = 12.98, size = 376, normalized size = 1.88

method	result

default	$-\frac{a^{2}(-1+\cos(dx+c))\left(20A\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)(\cos^{2}(dx+c))\sqrt{2}+20A\sqrt{2}\,(\cos^{2}(dx+$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/8/d*a^2*(-1+cos(d*x+c))*(20*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+co
s(d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)+20*A*2^(1/2)*cos(d*x+c)^
2*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))+
16*A*sin(d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+19*B*arctan(1/4*(-2/
(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/
2)+19*B*2^(1/2)*cos(d*x+c)^2*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d
*x+c)+sin(d*x+c))*2^(1/2))+8*A*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x
+c)+22*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+4*B*(-2/(1+cos(d*x
+c)))^(1/2)*sin(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^(3/2
)/sin(d*x+c)^2/(-2/(1+cos(d*x+c)))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 14322 vs. $2(170) = 340$.
time = 4.69, size = 14322, normalized size = 71.61

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/16*(4*(8*a^2*cos(1/2*d*x + 1/2*c)^4*sin(1/2*d*x + 1/2*c) + 16*a^2*cos(1/2
*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^3 + 8*a^2*sin(1/2*d*x + 1/2*c)^5 + 5*(
sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqr
t(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a
^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*
cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x +
 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d
*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) -
 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^4 + 10*(sqrt(2)*
a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2
*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x
```

```
+ 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*cos(1/2*
d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c)
+ 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2
*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(
2)*sin(1/2*d*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2
 + 5*(sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 +
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqr
t(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(1/2*d*x + 1/2*c)^4 + (8*a^2
*sin(1/2*d*x + 1/2*c)^3 + (5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*s
in(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*
d*x + 1/2*c) + 2) - 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1
/2*c) + 2) + 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1
/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) +
 2) - 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 8
*a^2*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c)^2 + 5*(sqrt(2)*a^2*log(2*co
s(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1
/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*
sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d
*x + 1/2*c) + 2))*cos(1/2*d*x + 1/2*c)^2 + (5*sqrt(2)*a^2*log(2*cos(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2
*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2
)*sin(1/2*d*x + 1/2*c) + 2) + 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2) - 5*sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) + 8*a^2*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 5*(sqr
t(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2
)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*
log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2
*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*cos
(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/
2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*
```

```
sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(1/2*d*x + 1/2*c)^2 + 2*(8*a^2*cos(1/
2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c) + 5*(sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt
(2)*sin(1/2*d*x + 1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1
/2*d*x + 1/2*c) + 2) + sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x +
1/2*c) + 2) - sqrt(2)*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqr...
```

Fricas [A]
time = 2.37, size = 453, normalized size = 2.26

$$\left[\frac{4\,{\left(8\,A a^{2} \cos\left(d x + c\right)^{2} + {\left(4\,A + 11\,B\right)} a^{2} \cos\left(d x + c\right) + 2\,B a^{2}\right)} \sqrt{\frac{a \cos\left(d x + c\right) + a}{\cos\left(d x + c\right)}} \sqrt{\cos\left(d x + c\right)} \sin\left(d x + c\right) + {\left({\left(20\,A + 19\,B\right)} a^{2} \cos\left(d x + c\right)^{3} + {\left(20\,A + 19\,B\right)} a^{2} \cos\left(d x + c\right)^{2}\right)} \sqrt{a} \log\left(\frac{a \cos\left(d x + c\right)^{3} - 4\,\sqrt{a} \sqrt{\frac{a \cos\left(d x + c\right) + a}{\cos\left(d x + c\right)}} {\left(\cos\left(d x + c\right) - 2\right)} \sqrt{\cos\left(d x + c\right)} \sin\left(d x + c\right) - 7\,a \cos\left(d x + c\right)^{2} + 8\,a}{\cos\left(d x + c\right)^{3} + \cos\left(d x + c\right)^{2}}\right)}{16\,{\left(d \cos\left(d x + c\right)^{3} + d \cos\left(d x + c\right)^{2}\right)}}, \frac{2\,{\left(8\,A a^{2} \cos\left(d x + c\right)^{2} + {\left(4\,A + 11\,B\right)} a^{2} \cos\left(d x + c\right) + 2\,B a^{2}\right)} \sqrt{\frac{a \cos\left(d x + c\right) + a}{\cos\left(d x + c\right)}} \sqrt{\cos\left(d x + c\right)} \sin\left(d x + c\right) + {\left({\left(20\,A + 19\,B\right)} a^{2} \cos\left(d x + c\right)^{3} + {\left(20\,A + 19\,B\right)} a^{2} \cos\left(d x + c\right)^{2}\right)} \sqrt{-a} \arctan\left(\frac{2\,\sqrt{-a} \sqrt{\frac{a \cos\left(d x + c\right) + a}{\cos\left(d x + c\right)}} \sqrt{\cos\left(d x + c\right)} \sin\left(d x + c\right)}{a \cos\left(d x + c\right)^{2} - a \cos\left(d x + c\right) - 2\,a}\right)}{8\,{\left(d \cos\left(d x + c\right)^{3} + d \cos\left(d x + c\right)^{2}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(4*(8*A*a^2*cos(d*x + c)^2 + (4*A + 11*B)*a^2*cos(d*x + c) + 2*B*a^2)
*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) +
((20*A + 19*B)*a^2*cos(d*x + c)^3 + (20*A + 19*B)*a^2*cos(d*x + c)^2)*sqrt(
a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c)
)*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 +
 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^3 + d*cos(d*x + c
)^2), 1/8*(2*(8*A*a^2*cos(d*x + c)^2 + (4*A + 11*B)*a^2*cos(d*x + c) + 2*B*
a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
) + ((20*A + 19*B)*a^2*cos(d*x + c)^3 + (20*A + 19*B)*a^2*cos(d*x + c)^2)*s
qrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d
*x + c)^3 + d*cos(d*x + c)^2)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos(c+d\,x)}\,\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2), x)
```

3.538 $\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=200

$$\frac{a^{5/2}(38A+25B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{8d}+\frac{a^3(54A+49B)\sin(c}{24d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{s}}}$$

[Out] 1/3*a*B*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+1/8*a^(5/2)*(38*A+25*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d+1/24*a^3*(54*A+49*B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+1/4*a^2*(2*A+3*B)*sin(d*x+c)*(a+a*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)

Rubi [A]
time = 0.41, antiderivative size = 200, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3034, 4103, 4101, 3886, 221}

$$\frac{a^{5/2}(38A+25B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{8d}+\frac{a^3(54A+49B)\sin(c+dx)}{24d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(2A+3B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{4d\cos^{\frac{3}{2}}(c+dx)}+\frac{aB\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (a^(5/2)*(38*A + 25*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(8*d) + (a^3*(54*A + 49*B)*Sin[c + d*x])/(24*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^2*(2*A + 3*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/2)) + (a*B*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dist[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g, m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && IntegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,(a+a$$

$$= \frac{aB(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\Big(\sqrt{\cos(c+dx}$$

$$= \frac{a^2(2A+3B)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{aB(a+}{}$$

$$= \frac{a^3(54A+49B)\sin(c+dx)}{24d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(2A+3B)\sqrt{}}{4}$$

$$= \frac{a^3(54A+49B)\sin(c+dx)}{24d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(2A+3B)\sqrt{}}{4}$$

$$= \frac{a^{5/2}(38A+25B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(}}{8d}$$

Mathematica [A]
time = 1.32, size = 133, normalized size = 0.66

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(3\sqrt{2}\,(38A+25B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3(c+dx)+(66A+91B+4(6A+17B)\cos(c+dx)+(66A+75B)\cos(2(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{48d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(3*Sqrt[2]*(38*A + 25*B)*A
rcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^3 + (66*A + 91*B + 4*(6*A + 1
7*B)*Cos[c + d*x] + (66*A + 75*B)*Cos[2*(c + d*x)])*Sin[(c + d*x)/2]))/(48*
d*Cos[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 406 vs. 2(170) = 340.
time = 13.45, size = 407, normalized size = 2.04

method	result

default	$-\frac{a^{2}\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(114A\sqrt{2}\,(\cos^{3}(dx+c))\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(-1-\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/48/d*a^2*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(114*A*2^(1
/2)*cos(d*x+c)^3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*
x+c))*2^(1/2))+114*A*2^(1/2)*cos(d*x+c)^3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1
/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))+75*B*2^(1/2)*cos(d*x+c)^3*arctan(1/4
*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))+75*B*2^(1/2)
*cos(d*x+c)^3*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c)
)*2^(1/2))+132*A*sin(d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+150*B*si
n(d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)+24*A*(-2/(1+cos(d*x+c)))^(1
/2)*sin(d*x+c)*cos(d*x+c)+68*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x
+c)+16*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c))/sin(d*x+c)^2/cos(d*x+c)^(5/2
)/(-2/(1+cos(d*x+c)))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 6297 vs. $2(170) = 340$.
time = 4.78, size = 6297, normalized size = 31.48

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/96*(6*(88*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c)*sin(2*d*x + 2*c) - 56*sqrt(2)
*a^2*cos(5/2*d*x + 5/2*c)*sin(2*d*x + 2*c) - 28*sqrt(2)*a^2*sin(3/2*d*x + 3
/2*c) + 44*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) - 19*(a^2*log(2*cos(1/2*d*x + 1
/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqr
t(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x
+ 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2
 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a
^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(
1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(4*d*x + 4*c)^2
- 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(
2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*c
```

os(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c)^2 - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(4*d*x + 4*c)^2 - 76*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*sin(2*d*x + 2*c)^2 - 2*(22*sqrt(2)*a^2*sin(7/2*d*x + 7/2*c) - 14*sqrt(2)*a^2*sin(5/2*d*x + 5/2*c) + 14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) - 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + 38*(a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) + a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c))*cos(4*d*x + 4*c) - 4*(14*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) -

```
 22*sqrt(2)*a^2*sin(1/2*d*x + 1/2*c) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2
+ 2*sin(1/2*d*x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin
(1/2*d*x + 1/2*c) + 2) - 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*
x + 1/2*c)^2 + 2*sqrt(2)*cos(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2
*c) + 2) + 19*a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 -
 2*sqrt(2)*cos(1/2*d*x + 1/2*c) + 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2) - 19*
a^2*log(2*cos(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c)^2 - 2*sqrt(2)*cos
(1/2*d*x + 1/2*c) - 2*sqrt(2)*sin(1/2*d*x + 1/2*c) + 2))*cos(2*d*x + 2*c) +
 4*(11*sqrt(2)*a^2*cos(7/2*d*x + 7/2*c) - 7*sqrt(2)*a^2*cos(5/2*d*x + 5/2*c
) + 7*sqrt(2)*a^2*cos(3/2*d*x + 3/2*c) - 11*sqr...
```

Fricas [**A**]
time = 2.09, size = 469, normalized size = 2.34

$$\left[\frac{4\,{\left(3\,{\left(22\,A+25\,B\right)}a^{2}\cos\left(dx+c\right)^{2}+2\,{\left(6\,A+17\,B\right)}a^{2}\cos\left(dx+c\right)+8\,Ba^{2}\right)}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\,{\left({\left(38\,A+25\,B\right)}a^{2}\cos\left(dx+c\right)^{4}+{\left(38\,A+25\,B\right)}a^{2}\cos\left(dx+c\right)^{3}\right)}\sqrt{a}\log\left(\frac{a\cos\left(dx+c\right)^{3}-4\,\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}{\left(\cos\left(dx+c\right)-2\right)}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-7\,a\cos\left(dx+c\right)^{2}+8\,a}{\cos\left(dx+c\right)^{3}+\cos\left(dx+c\right)^{2}}\right)}{96\,{\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}},\ \frac{2\,{\left(3\,{\left(22\,A+25\,B\right)}a^{2}\cos\left(dx+c\right)^{2}+2\,{\left(6\,A+17\,B\right)}a^{2}\cos\left(dx+c\right)+8\,Ba^{2}\right)}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\,{\left({\left(38\,A+25\,B\right)}a^{2}\cos\left(dx+c\right)^{4}+{\left(38\,A+25\,B\right)}a^{2}\cos\left(dx+c\right)^{3}\right)}\sqrt{-a}\arctan\left(\frac{2\,\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{a\cos\left(dx+c\right)^{2}-a\cos\left(dx+c\right)-2\,a}\right)}{48\,{\left(d\cos\left(dx+c\right)^{4}+d\cos\left(dx+c\right)^{3}\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/96*(4*(3*(22*A + 25*B)*a^2*cos(d*x + c)^2 + 2*(6*A + 17*B)*a^2*cos(d*x +
 c) + 8*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c) + 3*((38*A + 25*B)*a^2*cos(d*x + c)^4 + (38*A + 25*B)*a^2*cos(d
*x + c)^3)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*
cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^4
 + d*cos(d*x + c)^3), 1/48*(2*(3*(22*A + 25*B)*a^2*cos(d*x + c)^2 + 2*(6*A
+ 17*B)*a^2*cos(d*x + c) + 8*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((38*A + 25*B)*a^2*cos(d*x + c)^4 + (3
8*A + 25*B)*a^2*cos(d*x + c)^3)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 -
 a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^4 + d*cos(d*x + c)^3)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2), x
)
```

3.539 $\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=247

$$\frac{a^{5/2}(200A+163B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{64d}+\frac{a^3(104A+95B)\sin(}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a}}$$

```
[Out] 1/4*a*B*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+1/64*a^(5/2)*(
200*A+163*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^
(1/2)*sec(d*x+c)^(1/2)/d+1/96*a^3*(104*A+95*B)*sin(d*x+c)/d/cos(d*x+c)^(5/2
)/(a+a*sec(d*x+c))^(1/2)+1/64*a^3*(200*A+163*B)*sin(d*x+c)/d/cos(d*x+c)^(3/
2)/(a+a*sec(d*x+c))^(1/2)+1/24*a^2*(8*A+11*B)*sin(d*x+c)*(a+a*sec(d*x+c))^(
1/2)/d/cos(d*x+c)^(5/2)
```

Rubi [A]

time = 0.48, antiderivative size = 247, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3034, 4103, 4101, 3888, 3886, 221}

$$\frac{a^{5/2}(200A+163B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{64d}+\frac{a^3(200A+163B)\sin(c+dx)}{64d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(104A+95B)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(8A+11B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{24d\cos^{\frac{5}{2}}(c+dx)}+\frac{aB\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{4d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (a^(5/2)*(200*A + 163*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(64*d) + (a^3*(104*A + 95*B)*
Sin[c + d*x])/(96*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^3*(20
0*A + 163*B)*Sin[c + d*x])/(64*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]
]) + (a^2*(8*A + 11*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c + d*x])/(24*d*Cos[c +
 d*x]^(5/2)) + (a*B*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(4*d*Cos[c + d
*x]^(5/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
```

```
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+a\sec$$

$$= \frac{aB(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{4}\left(\sqrt{\cos(c+dx)}\right.$$

$$= \frac{a^2(8A+11B)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{aB(a+}{}$$

$$= \frac{a^3(104A+95B)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(8A+11B)\sqrt{}}{24}$$

$$= \frac{a^3(104A+95B)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^3(200A+1}{64d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^3(104A+95B)\sin(c+dx)}{96d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^3(200A+1}{64d\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{a^{5/2}(200A+163B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos}}{64d}$$

Mathematica [A]
time = 1.90, size = 154, normalized size = 0.62

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(6\sqrt{2}\,(200A+163B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^4(c+dx)+(544A+844B+(2056A+2203B)\cos(c+dx)+(544A+652B)\cos(2(c+dx))+600A\cos(3(c+dx))+489B\cos(3(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{768d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(6*Sqrt[2]*(200*A + 163*B)
*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^4 + (544*A + 844*B + (2056*
A + 2203*B)*Cos[c + d*x] + (544*A + 652*B)*Cos[2*(c + d*x)] + 600*A*Cos[3*(
c + d*x)] + 489*B*Cos[3*(c + d*x)])*Sin[(c + d*x)/2]))/(768*d*Cos[c + d*x]^
(7/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 468 vs. $2(211) = 422$.
time = 12.58, size = 469, normalized size = 1.90

method	result
default	$-\frac{a^{2} \sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c)) \left(600 A \arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)(\cos^{4}(dx+c))\sqrt{2}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/384/d*a^2*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(600*A*arc
tan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*
x+c)^4*2^(1/2)-600*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin
(d*x+c))*2^(1/2))*cos(d*x+c)^4*2^(1/2)+489*B*arctan(1/4*(-2/(1+cos(d*x+c)))
^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*x+c)^4*2^(1/2)-489*B*arctan
(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*cos(d*x+c
)^4*2^(1/2)+1200*A*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^3+978*B*
(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^3+544*A*sin(d*x+c)*cos(d*x+
c)^2*(-2/(1+cos(d*x+c)))^(1/2)+652*B*sin(d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x
+c)))^(1/2)+128*A*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)+368*B*(-2
/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*sin(d*x+c)+96*B*(-2/(1+cos(d*x+c)))^(1/2)
*sin(d*x+c))/(-2/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(7/2)/sin(d*x+c)^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 7331 vs. $2(211) = 422$.
time = 1.65, size = 7331, normalized size = 29.68

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/768*(8*(300*sqrt(2)*a^2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c)))*sin(6*d*x + 6*c) - 28*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) + 28*sqr
t(2)*a^2*sin(3/2*d*x + 3/2*c) - 28*(sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - sqrt
(2)*a^2*sin(3/2*d*x + 3/2*c))*cos(6*d*x + 6*c) - 300*(sqrt(2)*a^2*sin(6*d*x
 + 6*c) + 3*sqrt(2)*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))))*cos(11/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
- 12*(7*sqrt(2)*a^2*sin(9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*
c) - 114*sqrt(2)*a^2*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c))) + 114*sqrt(2)*a^2*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
```

+ 3/2*c))) + 75*sqrt(2)*a^2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
*x + 3/2*c))))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
 - 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a^2*sin(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(7/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + 456*(sqrt(2)*a^2*sin(6*d*x + 6*c) + 3*sqrt(2)*a
^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 12*(7*sqrt(2)*a^2*sin(
9/2*d*x + 9/2*c) - 7*sqrt(2)*a^2*sin(3/2*d*x + 3/2*c) + 75*sqrt(2)*a^2*sin(
1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 75*(a^2*cos(6*d*x + 6*c)^2 +
9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^
2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(
6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c)
+ 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)
*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(
6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
c))) + 6(a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2)
 - 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c)^2 + 9*a^2*sin(8/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 6*a^2*sin(6*d*x + 6*c)*sin(4/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 9*a^2*sin(4/3*arcta
n2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*a^2*cos(6*d*x + 6*c)
+ a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c) + a^2)*cos(4/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*sin(6*d*x + 6*c) + 3*a^2*sin
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*log(2*cos(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3
/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 2) + 75*(a^2*cos(6*d*x + 6*c)^2 + 9*a^2*cos(8
/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 9*a^2*cos(4/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + a^2*sin(6*d*x + 6*c
)^2 + 9*a^2*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
+ 6*a^2*sin(6*d*x + 6*c)*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x

```
+ 3/2*c))) + 9*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c)))^2 + 2*a^2*cos(6*d*x + 6*c) + a^2 + 6*(a^2*cos(6*d*x + 6*c) + 3*a^2*cos
(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + a^2)*cos(8/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(a^2*cos(6*d*x + 6*c
) + a^2)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 6*(
a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
))*log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2
*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)
*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*s
in(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) - 75*(a^2*
cos(6*d*x + 6*c)^2 + 9*a^2*cos(8/3*arctan2(sin(...
```

Fricas [A]
time = 2.86, size = 509, normalized size = 2.06

$$\left[\frac{4\left(3\left(200A+163B\right)a^2\cos\left(dx+c\right)^3+2\left(136A+163B\right)a^2\cos\left(dx+c\right)^2+8\left(8A+23B\right)a^2\cos\left(dx+c\right)+48Ba^2\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\left(\left(200A+163B\right)a^2\cos\left(dx+c\right)^5+\left(200A+163B\right)a^2\cos\left(dx+c\right)^4\right)\sqrt{a}\log\left(\frac{a\cos\left(dx+c\right)^3-4\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\left(\cos\left(dx+c\right)-2\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-7a\cos\left(dx+c\right)^2+8a}{\cos\left(dx+c\right)^3+\cos\left(dx+c\right)^2}\right)}{768\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)}, \frac{2\left(3\left(200A+163B\right)a^2\cos\left(dx+c\right)^3+2\left(136A+163B\right)a^2\cos\left(dx+c\right)^2+8\left(8A+23B\right)a^2\cos\left(dx+c\right)+48Ba^2\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+3\left(\left(200A+163B\right)a^2\cos\left(dx+c\right)^5+\left(200A+163B\right)a^2\cos\left(dx+c\right)^4\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{a\cos\left(dx+c\right)^2-a\cos\left(dx+c\right)-2a}\right)}{384\left(d\cos\left(dx+c\right)^5+d\cos\left(dx+c\right)^4\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] [1/768*(4*(3*(200*A + 163*B)*a^2*cos(d*x + c)^3 + 2*(136*A + 163*B)*a^2*cos
(d*x + c)^2 + 8*(8*A + 23*B)*a^2*cos(d*x + c) + 48*B*a^2)*sqrt((a*cos(d*x +
 c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((200*A + 163*B)
*a^2*cos(d*x + c)^5 + (200*A + 163*B)*a^2*cos(d*x + c)^4)*sqrt(a)*log((a*co
s(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x +
 c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d
*x + c)^3 + cos(d*x + c)^2)))/(d*cos(d*x + c)^5 + d*cos(d*x + c)^4), 1/384*
(2*(3*(200*A + 163*B)*a^2*cos(d*x + c)^3 + 2*(136*A + 163*B)*a^2*cos(d*x +
c)^2 + 8*(8*A + 23*B)*a^2*cos(d*x + c) + 48*B*a^2)*sqrt((a*cos(d*x + c) + a
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 3*((200*A + 163*B)*a^2*co
s(d*x + c)^5 + (200*A + 163*B)*a^2*cos(d*x + c)^4)*sqrt(-a)*arctan(2*sqrt(-
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/
(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c)^5 + d*cos(d*x +
 c)^4)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

[Out] Timed out

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{5/2}}{\cos\left(c+d\,x\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2), x
)
```

3.540 $\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$

Optimal. Leaf size=294

$$\frac{a^{5/2}(326A+283B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{128d}+\frac{a^3(170A+157B)\,\mathrm{s}}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+}}$$

```
[Out] 1/5*a*B*(a+a*sec(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(7/2)+1/128*a^(5/2)*
(326*A+283*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)
^(1/2)*sec(d*x+c)^(1/2)/d+1/240*a^3*(170*A+157*B)*sin(d*x+c)/d/cos(d*x+c)^(
7/2)/(a+a*sec(d*x+c))^(1/2)+1/192*a^3*(326*A+283*B)*sin(d*x+c)/d/cos(d*x+c)
^(5/2)/(a+a*sec(d*x+c))^(1/2)+1/128*a^3*(326*A+283*B)*sin(d*x+c)/d/cos(d*x+
c)^(3/2)/(a+a*sec(d*x+c))^(1/2)+1/40*a^2*(10*A+13*B)*sin(d*x+c)*(a+a*sec(d*
x+c))^(1/2)/d/cos(d*x+c)^(7/2)
```

Rubi [A]

time = 0.54, antiderivative size = 294, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3034, 4103, 4101, 3888, 3886, 221}

$$\frac{a^{5/2}(326A+283B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{128d}+\frac{a^3(326A+283B)\sin(c+dx)}{128d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(326A+283B)\sin(c+dx)}{192d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^3(170A+157B)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{a^2(10A+13B)\sin(c+dx)\sqrt{a\sec(c+dx)+a}}{40d\cos^{\frac{7}{2}}(c+dx)}+\frac{aB\sin(c+dx)(a\sec(c+dx)+a)^{3/2}}{5d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(5/2),x]
```

```
[Out] (a^(5/2)*(326*A + 283*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c +
d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(128*d) + (a^3*(170*A + 157*B
)*Sin[c + d*x])/(240*d*Cos[c + d*x]^(7/2)*Sqrt[a + a*Sec[c + d*x]]) + (a^3*
(326*A + 283*B)*Sin[c + d*x])/(192*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c +
d*x]]) + (a^3*(326*A + 283*B)*Sin[c + d*x])/(128*d*Cos[c + d*x]^(3/2)*Sqrt[
a + a*Sec[c + d*x]]) + (a^2*(10*A + 13*B)*Sqrt[a + a*Sec[c + d*x]]*Sin[c +
d*x])/(40*d*Cos[c + d*x]^(7/2)) + (a*B*(a + a*Sec[c + d*x])^(3/2)*Sin[c + d
*x])/(5*d*Cos[c + d*x]^(7/2))
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
```

```
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3888

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)], x_Symbol] :> Simp[-2*b*d*Cot[e + f*x]*((d*Csc[e + f*x])^(n - 1)/(
f*(2*n - 1)*Sqrt[a + b*Csc[e + f*x]])), x] + Dist[2*a*d*((n - 1)/(b*(2*n -
1))), Int[Sqrt[a + b*Csc[e + f*x]]*(d*Csc[e + f*x])^(n - 1), x], x] /; Free
Q[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[n, 1] && IntegerQ[2*n]
```

Rule 4101

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
 + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[-2*b*B*C
ot[e + f*x]*((d*Csc[e + f*x])^n/(f*(2*n + 1)*Sqrt[a + b*Csc[e + f*x]])), x]
 + Dist[(A*b*(2*n + 1) + 2*a*B*n)/(b*(2*n + 1)), Int[Sqrt[a + b*Csc[e + f*x
]]*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[
A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && NeQ[A*b*(2*n + 1) + 2*a*B*n, 0] &&  !
LtQ[n, 0]
```

Rule 4103

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(d*(m + n)), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])
^n*Simp[a*A*d*(m + n) + B*(b*d*n) + (A*b*d*(m + n) + a*B*d*(2*m + n - 1))*C
sc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b -
a*B, 0] && EqQ[a^2 - b^2, 0] && GtQ[m, 1/2] &&  !LtQ[n, -1]
```

Rubi steps

$$\int \frac{(a+a\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{5}{2}}(c+dx)(a+a\,\mathrm{se}$$

$$= \frac{aB(a+a\sec(c+dx))^{3/2}\sin(c+dx)}{5d\cos^{\frac{7}{2}}(c+dx)} + \frac{1}{5}\Big(\sqrt{\cos(c+dx}$$

$$= \frac{a^2(10A+13B)\sqrt{a+a\sec(c+dx)}\,\sin(c+dx)}{40d\cos^{\frac{7}{2}}(c+dx)} + \frac{aB(a}{}$$

$$= \frac{a^3(170A+157B)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^2(10A+13B}{}$$

$$= \frac{a^3(170A+157B)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^3(326A}{192d\cos^{\frac{5}{2}}(c+}$$

$$= \frac{a^3(170A+157B)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^3(326A}{192d\cos^{\frac{5}{2}}(c+}$$

$$= \frac{a^3(170A+157B)\sin(c+dx)}{240d\cos^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{a^3(326A}{192d\cos^{\frac{5}{2}}(c+}$$

$$= \frac{a^{5/2}(326A+283B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\mathrm{co}}}{128d}$$

Mathematica [A]
time = 2.75, size = 178, normalized size = 0.61

$$\frac{a^2\sec\left(\frac{1}{2}(c+dx)\right)\sqrt{a(1+\sec(c+dx))}\left(60\sqrt{2}(326A+283B)\tanh^{-1}\left(\sqrt{2}\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^5(c+dx)+(22030A+24863B+36(650A+781B)\cos(c+dx)+4(6730A+6509B)\cos(2(c+dx))+6520A\cos(3(c+dx))+5660B\cos(3(c+dx))+4890A\cos(4(c+dx))+4245B\cos(4(c+dx)))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{15360d\cos^{\frac{9}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + a*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(5
/2),x]
```

```
[Out] (a^2*Sec[(c + d*x)/2]*Sqrt[a*(1 + Sec[c + d*x])]*(60*Sqrt[2]*(326*A + 283*B
)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^5 + (22030*A + 24863*B + 3
6*(650*A + 781*B)*Cos[c + d*x] + 4*(6730*A + 6509*B)*Cos[2*(c + d*x)] + 652
0*A*Cos[3*(c + d*x)] + 5660*B*Cos[3*(c + d*x)] + 4890*A*Cos[4*(c + d*x)] +
4245*B*Cos[4*(c + d*x)])*Sin[(c + d*x)/2]))/(15360*d*Cos[c + d*x]^(9/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(252) = 504$.
time = 12.72, size = 531, normalized size = 1.81

method	result
default	$\frac{a^{2}\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(4890A\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)-\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}\,(\cos^{5}(dx+c))-\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/3840/d*a^2*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(4890*A*ar
ctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*2^(1/
2)*cos(d*x+c)^5-4890*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+s
in(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)^5+4245*B*arctan(1/4*(-2/(1+cos(d*x+c
)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)^5-4245*B*ar
ctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/
2)*cos(d*x+c)^5-9780*A*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^4-84
90*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^4*sin(d*x+c)-6520*A*(-2/(1+cos(d*
x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^3-5660*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d
*x+c)*cos(d*x+c)^3-3680*A*sin(d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)
-4528*B*sin(d*x+c)*cos(d*x+c)^2*(-2/(1+cos(d*x+c)))^(1/2)-960*A*(-2/(1+cos(
d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)-2784*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d
*x+c)*sin(d*x+c)-768*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c))/cos(d*x+c)^(9/
2)/sin(d*x+c)^2/(-2/(1+cos(d*x+c)))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 9242 vs. $2(252) = 504$.
time = 2.48, size = 9242, normalized size = 31.44

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="maxima")
```

```
[Out] -1/7680*(10*(1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x +
 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*co
s(15/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 652*(sqrt(2)*a^2*sin(
8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4
*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(13/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6
*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*
c))*cos(11/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2060*(sqrt(2)*a
```

^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2060*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6204*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 652*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 1956*(sqrt(2)*a^2*sin(8*d*x + 8*c) + 4*sqrt(2)*a^2*sin(6*d*x + 6*c) + 6*sqrt(2)*a^2*sin(4*d*x + 4*c) + 4*sqrt(2)*a^2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 489*(a^2*cos(8*d*x + 8*c)^2 + 16*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x + 2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a

```
^2*sin(6*d*x + 6*c) + 3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(
8*d*x + 8*c) + 16*(3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c))*sin(6*d
*x + 6*c))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 489*(a^2*cos(8*d*x + 8*c)^2 + 16
*a^2*cos(6*d*x + 6*c)^2 + 36*a^2*cos(4*d*x + 4*c)^2 + 16*a^2*cos(2*d*x + 2*
c)^2 + a^2*sin(8*d*x + 8*c)^2 + 16*a^2*sin(6*d*x + 6*c)^2 + 36*a^2*sin(4*d*
x + 4*c)^2 + 48*a^2*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 16*a^2*sin(2*d*x +
2*c)^2 + 8*a^2*cos(2*d*x + 2*c) + a^2 + 2*(4*a^2*cos(6*d*x + 6*c) + 6*a^2*c
os(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(8*d*x + 8*c) + 8*(6*a^2
*cos(4*d*x + 4*c) + 4*a^2*cos(2*d*x + 2*c) + a^2)*cos(6*d*x + 6*c) + 12*(4*
a^2*cos(2*d*x + 2*c) + a^2)*cos(4*d*x + 4*c) + 4*(2*a^2*sin(6*d*x + 6*c) +
3*a^2*sin(4*d*x + 4*c) + 2*a^2*sin(2*d*x + 2*c)...
```

Fricas [A]
time = 3.36, size = 549, normalized size = 1.87

$$\left[\frac{4\left(15\,(326\,A+283\,B)a^2\cos(dx+c)^4+10\,(326\,A+283\,B)a^2\cos(dx+c)^3+8\,(230\,A+283\,B)a^2\cos(dx+c)^2+48\,(10\,A+29\,B)a^2\cos(dx+c)+384\,Ba^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+15\left((326\,A+283\,B)a^2\cos(dx+c)^6+(326\,A+283\,B)a^2\cos(dx+c)^5\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7\,a\cos(dx+c)^2+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{7680\left(d\cos(dx+c)^6+d\cos(dx+c)^5\right)},\ \frac{2\left(15\,(326\,A+283\,B)a^2\cos(dx+c)^4+10\,(326\,A+283\,B)a^2\cos(dx+c)^3+8\,(230\,A+283\,B)a^2\cos(dx+c)^2+48\,(10\,A+29\,B)a^2\cos(dx+c)+384\,Ba^2\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+15\left((326\,A+283\,B)a^2\cos(dx+c)^6+(326\,A+283\,B)a^2\cos(dx+c)^5\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2\,a}\right)}{3840\left(d\cos(dx+c)^6+d\cos(dx+c)^5\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/7680*(4*(15*(326*A + 283*B)*a^2*cos(d*x + c)^4 + 10*(326*A + 283*B)*a^2*
cos(d*x + c)^3 + 8*(230*A + 283*B)*a^2*cos(d*x + c)^2 + 48*(10*A + 29*B)*a^
2*cos(d*x + c) + 384*B*a^2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c) + 15*((326*A + 283*B)*a^2*cos(d*x + c)^6 + (326*A
+ 283*B)*a^2*cos(d*x + c)^5)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*s
in(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))
/(d*cos(d*x + c)^6 + d*cos(d*x + c)^5), 1/3840*(2*(15*(326*A + 283*B)*a^2*c
os(d*x + c)^4 + 10*(326*A + 283*B)*a^2*cos(d*x + c)^3 + 8*(230*A + 283*B)*a
^2*cos(d*x + c)^2 + 48*(10*A + 29*B)*a^2*cos(d*x + c) + 384*B*a^2)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + 15*((326*
A + 283*B)*a^2*cos(d*x + c)^6 + (326*A + 283*B)*a^2*cos(d*x + c)^5)*sqrt(-a
)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(d*cos(d*x + c
)^6 + d*cos(d*x + c)^5)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+a*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(a*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/cos(c + d*x)^(5/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + a/cos(c + d*x))^(5/2))/cos(c + d*x)^(5/2), x
)
```

3.541 $\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=250

$$\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}-\frac{2(43A-91B)\sin(}{105d\sqrt{\cos(c+dx)}\,\sqrt{a+}}$$

```
[Out] (A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+
c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)-2/35*(A-7*B)
*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/7*A*cos(d*x+c)^(5/2
)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)-2/105*(43*A-91*B)*sin(d*x+c)/d/cos(d*
x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/105*(31*A-7*B)*sin(d*x+c)*cos(d*x+c)^(1
/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.54, antiderivative size = 250, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3034, 4107, 4098, 3893, 212}

$$-\frac{2(A-7B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d\sqrt{a\sec(c+dx)+a}}+\frac{2(31A-7B)\sin(c+dx)\sqrt{\cos(c+dx)}}{105d\sqrt{a\sec(c+dx)+a}}-\frac{2(43A-91B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2A\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]
*d) - (2*(43*A - 91*B)*Sin[c + d*x])/(105*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*S
ec[c + d*x]]) + (2*(31*A - 7*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(105*d*Sqr
t[a + a*Sec[c + d*x]]) - (2*(A - 7*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*
d*Sqrt[a + a*Sec[c + d*x]]) + (2*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d*Sq
rt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
```

```
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{7}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{7}{2}}(c+dx)\sqrt{a+a\sec(c+d}}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d\sqrt{a+a\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int}{7c}$$

$$= -\frac{2(A-7B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d\sqrt{a+a\sec(c+dx)}} + \frac{2A\cos^{\frac{5}{2}}(c+dx)\sin(c+d}{7d\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{2(31A-7B)\sqrt{\cos(c+dx)}\sin(c+dx)}{105d\sqrt{a+a\sec(c+dx)}} - \frac{2(A-7B)\cos^{\frac{3}{2}}(c+dx}{35d\sqrt{a+a\sec(c}}$$

$$= -\frac{2(43A-91B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2(31A-7B)\sqrt{\cos(c}}{105d\sqrt{a+a\,\mathrm{s}}}$$

$$= -\frac{2(43A-91B)\sin(c+dx)}{105d\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}} + \frac{2(31A-7B)\sqrt{\cos(c}}{105d\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{\sqrt{2}(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}}{\sqrt{a}\,d}$$

Mathematica [A]
time = 1.16, size = 170, normalized size = 0.68

$$\frac{\cos^{\frac{5}{2}}(c+dx)\left(-105\sqrt{2}(A-B)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec^{\frac{7}{2}}(c+dx)+2\sqrt{1-\sec(c+dx)}\left(15A-3(A-7B)\sec(c+dx)+(31A-7B)\sec^2(c+dx)+(-43A+91B)\sec^3(c+dx)\right)\right)\sin(c+dx)}{105d\sqrt{1-\sec(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(7/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]
],x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(-105*Sqrt[2]*(A - B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]
])/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^(7/2) + 2*Sqrt[1 - Sec[c + d*x]]*(1
5*A - 3*(A - 7*B)*Sec[c + d*x] + (31*A - 7*B)*Sec[c + d*x]^2 + (-43*A + 91*
B)*Sec[c + d*x]^3))*Sin[c + d*x])/(105*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 +
 Sec[c + d*x])])
```

Maple [A]
time = 13.34, size = 217, normalized size = 0.87

method	result
default	$-\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(30A(\cos^4(dx+c))+105\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/105/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(30*A*cos(d*x+c)^4+105*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*A*sin(d*x+c)-36*A*cos(d*x+c)^3-105*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c)+42*B*cos(d*x+c)^3+68*A*cos(d*x+c)^2-56*B*cos(d*x+c)^2-148*A*cos(d*x+c)+196*B*cos(d*x+c)+86*A-182*B)/a/sin(d*x+c)

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 749 vs. $2(211) = 422$.
time = 0.92, size = 749, normalized size = 3.00

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algorithm="maxima")

[Out] -1/840*(sqrt(2)*(525*cos(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 175*cos(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) + 21*cos(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))*sin(7/2*d*x + 7/2*c) - 525*cos(7/2*d*x + 7/2*c)*sin(6/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 175*cos(7/2*d*x + 7/2*c)*sin(4/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 21*cos(7/2*d*x + 7/2*c)*sin(2/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 420*log(cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))^2 + sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))^2 + 2*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 1) + 420*log(cos(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))^2 + sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c)))^2 - 2*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 1) - 30*sin(7/2*d*x + 7/2*c) + 21*sin(5/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) - 175*sin(3/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))) + 525*sin(1/7*arctan2(sin(7/2*d*x + 7/2*c), cos(7/2*d*x + 7/2*c))))*A/sqrt(a) + 28*(30*sqrt(2)*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x +

```
2*c) - 3*(10*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*sin(5/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 15*sqrt(2)*log(cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 15*s
qrt(2)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 1) + 5*sqrt(2)*sin(3/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 30*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*B/sqrt(a))/d
```

Fricas [A]
time = 2.79, size = 400, normalized size = 1.60

$$\left[\frac{4\,(15\,A\cos(dx+c)^3-3\,(A-7\,B)\cos(dx+c)^2+(31\,A-7\,B)\cos(dx+c)-43\,A+91\,B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\frac{105\sqrt{2}\,((A-B)a\cos(dx+c)+(A-B)a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}}{210\,(ad\cos(dx+c)+ad)},\ -\frac{105\sqrt{2}\,((A-B)a\cos(dx+c)+(A-B)a)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-2\,(15\,A\cos(dx+c)^3-3\,(A-7\,B)\cos(dx+c)^2+(31\,A-7\,B)\cos(dx+c)-43\,A+91\,B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{105\,(ad\cos(dx+c)+ad)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/210*(4*(15*A*cos(d*x + c)^3 - 3*(A - 7*B)*cos(d*x + c)^2 + (31*A - 7*B)*
cos(d*x + c) - 43*A + 91*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c) - 105*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)
*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^
2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), -1/105*(105*sqr
t(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)
) - 2*(15*A*cos(d*x + c)^3 - 3*(A - 7*B)*cos(d*x + c)^2 + (31*A - 7*B)*cos(
d*x + c) - 43*A + 91*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(7/2)/sqrt(a*sec(d*x + c) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{7/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(7/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^(7/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2), x
)
```

3.542 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=207

$$-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}+\frac{2(13A-5B)\sin}{15d\sqrt{\cos(c+dx)}\,\sqrt{a-}}$$

```
[Out] -(A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x
+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)+2/5*A*cos(d
*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(1/2)+2/15*(13*A-5*B)*sin(d*x+c)/
d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)-2/15*(A-5*B)*sin(d*x+c)*cos(d*x+c
)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.41, antiderivative size = 207, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3034, 4107, 4098, 3893, 212}

$$-\frac{2(A-5B)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d\sqrt{a\sec(c+dx)+a}}+\frac{2(13A-5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[
a]*d)) + (2*(13*A - 5*B)*Sin[c + d*x])/(15*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*
Sec[c + d*x]]) - (2*(A - 5*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d*Sqrt[a
 + a*Sec[c + d*x]]) + (2*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d*Sqrt[a + a
*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
```

```
tegerQ[n])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+d}}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d\sqrt{a+a\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int}{5d}$$

$$= -\frac{2(A-5B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d\sqrt{a+a\sec(c+dx)}} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+}{5d\sqrt{a+a\sec(c+dx}}$$

$$= \frac{2(13A-5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} - \frac{2(A-5B)\sqrt{\cos(c+dx}}{15d\sqrt{a+a\sec(c}}$$

$$= \frac{2(13A-5B)\sin(c+dx)}{15d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} - \frac{2(A-5B)\sqrt{\cos(c+dx}}{15d\sqrt{a+a\sec(c}}$$

$$= -\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx}}{\sqrt{a}\,d}$$

Mathematica [A]

time = 0.77, size = 154, normalized size = 0.74

$$\frac{\cos^{\frac{3}{2}}(c+dx)\left(15\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sec^{\frac{5}{2}}(c+dx)+2\sqrt{1-\sec(c+dx)}\,(3A-(A-5B)\sec(c+dx)+(13A-5B)\sec^2(c+dx))\right)\sin(c+dx)}{15d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]
],x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(15*Sqrt[2]*(A - B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])
/Sqrt[1 - Sec[c + d*x]]]*Sec[c + d*x]^(5/2) + 2*Sqrt[1 - Sec[c + d*x]]*(3*A
 - (A - 5*B)*Sec[c + d*x] + (13*A - 5*B)*Sec[c + d*x]^2))*Sin[c + d*x])/(15
*d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]

time = 13.45, size = 195, normalized size = 0.94

method	result

default	$-\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(6A(\cos^3(dx+c))-15\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,A\,s\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/15/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(6*A*cos(d*x+c
)^3-15*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))
^(1/2)*A*sin(d*x+c)+15*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2
/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c)-8*A*cos(d*x+c)^2+10*B*cos(d*x+c)^2+28*A
*cos(d*x+c)-20*B*cos(d*x+c)-26*A+10*B)/a/sin(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 581 vs. $2(174) = 348$.
time = 0.92, size = 581, normalized size = 2.81

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/60*(sqrt(2)*(60*cos(4/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))*sin(5/2*d*x + 5/2*c) - 5*cos(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*
d*x + 5/2*c)))*sin(5/2*d*x + 5/2*c) - 60*cos(5/2*d*x + 5/2*c)*sin(4/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 5*cos(5/2*d*x + 5/2*c)*si
n(2/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 30*log(cos(1/5
*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(s
in(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + 2*sin(1/5*arctan2(sin(5/2*d
*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 1) + 30*log(cos(1/5*arctan2(sin(5/2*d
*x + 5/2*c), cos(5/2*d*x + 5/2*c)))^2 + sin(1/5*arctan2(sin(5/2*d*x + 5/2*c
), cos(5/2*d*x + 5/2*c)))^2 - 2*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c))) + 1) + 6*sin(5/2*d*x + 5/2*c) - 5*sin(3/5*arctan2(sin(5/2
*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 60*sin(1/5*arctan2(sin(5/2*d*x + 5/
2*c), cos(5/2*d*x + 5/2*c))))*A/sqrt(a) + 10*(3*sqrt(2)*log(cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 1) - 3*sqrt(2)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*sqrt(2)*sin(3/4*arctan
```

```
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 6*sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*B/sqrt(a))/d
```

Fricas [A]
time = 2.57, size = 368, normalized size = 1.78

$$\left[\frac{4\,{\left(3\,A\cos\left(dx+c\right)^{2}-{\left(A-5\,B\right)}\cos\left(dx+c\right)+13\,A-5\,B\right)}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\frac{15\,\sqrt{2}\,{\left({\left(A-B\right)}a\cos\left(dx+c\right)+{\left(A-B\right)}a\right)}\log\left(-\frac{\cos\left(dx+c\right)^{2}-\frac{2\,\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{\sqrt{a}}-2\,\cos\left(dx+c\right)-3}{\cos\left(dx+c\right)^{2}+2\,\cos\left(dx+c\right)+1}\right)}{\sqrt{a}}}{30\,{\left(ad\cos\left(dx+c\right)+ad\right)}},\ \frac{15\,\sqrt{2}\,{\left({\left(A-B\right)}a\cos\left(dx+c\right)+{\left(A-B\right)}a\right)}\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{-\frac{1}{a}}\sqrt{\cos\left(dx+c\right)}}{\sin\left(dx+c\right)}\right)+2\,{\left(3\,A\cos\left(dx+c\right)^{2}-{\left(A-5\,B\right)}\cos\left(dx+c\right)+13\,A-5\,B\right)}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{15\,{\left(ad\cos\left(dx+c\right)+ad\right)}}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/30*(4*(3*A*cos(d*x + c)^2 - (A - 5*B)*cos(d*x + c) + 13*A - 5*B)*sqrt((a
*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 15*sqrt(
2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sq
rt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(
a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a
*d*cos(d*x + c) + a*d), 1/15*(15*sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*
a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-
1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*(3*A*cos(d*x + c)^2 - (A - 5*B)*c
os(d*x + c) + 13*A - 5*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/sqrt(a*sec(d*x + c) + a),
 x)
```

Mupad [F]

time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2),x)
[Out] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2), x
)
```

3.543 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=162

$$\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}-\frac{2(A-3B)\sin(c+}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a}}$$

```
[Out] (A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+
c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)-2/3*(A-3*B)*
sin(d*x+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+2/3*A*sin(d*x+c)*cos(d
*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]

time = 0.29, antiderivative size = 162, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3034, 4107, 4098, 3893, 212}

$$-\frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}}{3d\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]
*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]
*d) - (2*(A - 3*B)*Sin[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c +
 d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d*Sqrt[a + a*Sec[c + d*x
]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d\sqrt{a+a\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right.}{}$$

$$= -\frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\,\sin(}{3d\sqrt{a+a\sec(c+}}$$

$$= -\frac{2(A-3B)\sin(c+dx)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \frac{2A\sqrt{\cos(c+dx)}\,\sin(}{3d\sqrt{a+a\sec(c+}}$$

$$= \frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx}}{\sqrt{a}\,d}$$

Mathematica [A]
time = 0.33, size = 124, normalized size = 0.77

$$\frac{\left(2(-A+3B+A\cos(c+dx))\sqrt{1-\sec(c+dx)}-3\sqrt{2}(A-B)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sqrt{\sec(c+dx)}\right)\sin(c+dx)}{3d\sqrt{-1+\cos(c+dx)}\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]
],x]
```

```
[Out] ((2*(-A + 3*B + A*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x]] - 3*Sqrt[2]*(A - B)*
ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*
x]])*Sin[c + d*x])/(3*d*Sqrt[-1 + Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 13.04, size = 173, normalized size = 1.07

method	result
default	$-\frac{\left(\sqrt{\cos}(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(3\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\,A\sin(dx+c)-3\arctan\left(\right.\right.}{3da\sin(}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/3/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(3*arctan(1/2*s
in(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*A*sin(d*x+c)
-3*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/
2)*B*sin(d*x+c)+2*A*cos(d*x+c)^2-4*A*cos(d*x+c)+6*B*cos(d*x+c)+2*A-6*B)/a/s
in(d*x+c)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 478 vs. $2(135) = 270$.
time = 0.86, size = 478, normalized size = 2.95

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/6*((3*sqrt(2)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))*sin(3/2*d*x + 3/2*c) - 3*sqrt(2)*cos(3/2*d*x + 3/2*c)*sin(2/3*arctan2(si
```

```
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*sqrt(2)*log(cos(1/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*
x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 1) + 3*sqrt(2)*log(cos(1/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))) + 1) - 2*sqrt(2)*sin(3/2*d*x + 3/2*c) + 3*sqrt(2)*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*A/sqrt(a) + 3*(sqrt(2)
*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + 1) - sqrt(2)*log(cos(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*sqrt
(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B/sqrt(a))/d
```

Fricas [**A**]
time = 2.69, size = 336, normalized size = 2.07

$$\left[\frac{4\,(A\cos(dx+c)-A+3\,B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\frac{3\sqrt{2}\,((A-B)a\cos(dx+c)+(A-B)a)\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}}{6\,(ad\cos(dx+c)+ad)},\ -\frac{3\sqrt{2}\,((A-B)a\cos(dx+c)+(A-B)a)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-2\,(A\cos(dx+c)-A+3\,B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{3\,(ad\cos(dx+c)+ad)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/6*(4*(A*cos(d*x + c) - A + 3*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(cos(d*x + c))*sin(d*x + c) - 3*sqrt(2)*((A - B)*a*cos(d*x + c) + (A -
B)*a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x
+ c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c) + a*d), -1/3*(3*sq
rt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c
)) - 2*(A*cos(d*x + c) - A + 3*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(a*sec(d*x + c) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2), x
)
```

3.544 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=119

$$-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}+\frac{2A\sin(c+}{d\sqrt{\cos(c+dx)}\,\sqrt{a+}}$$

```
[Out] -(A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x
+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)+2*A*sin(d*x
+c)/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.19, antiderivative size = 119, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3034, 4098, 3893, 212}

$$\frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}-\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]],x]
```

```
[Out] -((Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[
2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[
a]*d)) + (2*A*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,\sqrt{a+a\sec(c}$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} + \left((-A+B)\sqrt{\cos(c+dx}\right.$$

$$= \frac{2A\sin(c+dx)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+a\sec(c+dx)}} - \frac{\left(2(-A+B)\sqrt{\cos(c+}\right.}{}$$

$$= -\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+}}{\sqrt{a}\,d}$$

Mathematica [A]
time = 0.31, size = 140, normalized size = 1.18

$$\frac{\sqrt{\cos(c+dx)}\left(2A\sqrt{1-\sec(c+dx)}+\sqrt{2}\,(A-B)\text{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\sqrt{\sec(c+dx)}\right)(A+B\sec(c+dx))\sin(c+dx)}{d(B+A\cos(c+dx))\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + a*Sec[c + d*x]
],x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*(2*A*Sqrt[1 - Sec[c + d*x]] + Sqrt[2]*(A - B)*ArcTan[(S
qrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Sec[c + d*x]])*(A +
```

```
B*Sec[c + d*x])*Sin[c + d*x])/(d*(B + A*Cos[c + d*x])*Sqrt[1 - Sec[c + d*x
]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 13.44, size = 142, normalized size = 1.19

method	result
default	$\frac{\left(A\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}-A\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)+B\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\right)(\sqrt{}}{da\sin(dx+c)^2}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*(A*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)-A*arctan(1/2*sin(d*x+c)*(-2/(1+
cos(d*x+c)))^(1/2))+B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)))*cos
(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-2/(1+cos(d*x+c)))^(1/2)
/a/sin(d*x+c)^2*(cos(d*x+c)^2-1)
```

Maxima [A]
time = 0.86, size = 195, normalized size = 1.64

$$-\frac{\frac{\left(\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-4\sqrt{2}\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)\right)A}{\sqrt{a}}-\frac{\left(\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)-\sqrt{2}\log\left(\cos\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2+\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)^2-2\sin\left(\frac{1}{2}dx+\frac{1}{2}c\right)+1\right)\right)B}{\sqrt{a}}}{2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/2*((sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(
1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 4*sqrt(2)*sin(1/2*d*x + 1/2*c))*A/
sqrt(a) - (sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*
sin(1/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*
x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*B/sqrt(a))/d
```

Fricas [A]
time = 1.88, size = 306, normalized size = 2.57

$$\left[\frac{4A\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\frac{\sqrt{2}\left((A-B)a\cos(dx+c)+(A-B)a\right)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}}{2\left(ad\cos(dx+c)+ad\right)},\ \frac{\sqrt{2}\left((A-B)a\cos(dx+c)+(A-B)a\right)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)+2A\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{ad\cos(dx+c)+ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/2*(4*A*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) - sqrt(2)*((A - B)*a*cos(d*x + c) + (A - B)*a)*log(-(cos(d*x + c)^2
- 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(
d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1
))/sqrt(a))/(a*d*cos(d*x + c) + a*d), (sqrt(2)*((A - B)*a*cos(d*x + c) + (A
 - B)*a)*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*
sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*A*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a*d*cos(d*x + c) + a*d)]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sqrt{\cos(c + dx)}}{\sqrt{a(\sec(c + dx) + 1)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(cos(c + d*x))/sqrt(a*(sec(c + d*x) + 1))
, x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(a*sec(d*x + c) + a),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c + d\,x)}\,\left(A + \frac{B}{\cos(c+d\,x)}\right)}{\sqrt{a + \frac{a}{\cos(c + d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(1/2), x
)
```

3.545 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=140

$$\frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}+\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\ldots}{\sqrt{2}\sqrt{a+a\sec(\ldots}}\right)\ldots}{\sqrt{\ldots}}$$

```
[Out] 2*B*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/d/a^(1/2)+(A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/
2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2
)/d/a^(1/2)
```

Rubi [A]

time = 0.22, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3034, 4108, 3893, 212, 3886, 221}

$$\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTanh[(Sqrt[a]*S
qrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d)
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
```

```
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c}{\sqrt{a+a\sec(c+dx)}}$$

$$= \left((A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+a\sec(c+dx)}}$$

$$= -\frac{\left(2(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\text{Subst}\left(\int \frac{1}{2a-x^2}\,dx,\right.}{d}$$

$$= \frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}$$

Mathematica [A]
time = 0.20, size = 115, normalized size = 0.82

$$\frac{\left(-2B\,\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)+\sqrt{2}\,(-A+B)\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\right)\sqrt{\cos(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)}{d\sqrt{1-\sec(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]
),x]
```

```
[Out] ((-2*B*ArcSin[Sqrt[Sec[c + d*x]]] + Sqrt[2]*(-A + B)*ArcTan[(Sqrt[2]*Sqrt[S
ec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]])*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2
)*Sin[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 12.64, size = 201, normalized size = 1.44

method	result
default	$-\frac{(-1+\cos(dx+c))\left(B\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)-B\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\cdots}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/d*(-1+cos(d*x+c))*(B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos
(d*x+c)+sin(d*x+c))*2^(1/2))-B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)
*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))+2*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*
x+c)))^(1/2))-2*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)))*cos(d*x
+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^2/a/(-2/(1+cos(d*x
+c)))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 699 vs. $2(115) = 230$.
time = 0.89, size = 699, normalized size = 4.99

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/2*((sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1
/2*d*x + 1/2*c) + 1) - sqrt(2)*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*A/sqrt(a) - (sqrt(2)*log(cos(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - sqrt(2)*log(cos(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 1) - log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c)))^2 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
 3/2*c))) + 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 2) + log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/
2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 2*sqrt(2)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) -
 2*sqrt(2)*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2
) - log(2*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2
)*cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sqrt(2)*
sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2) + log(2*c
os(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 - 2*sqrt(2)*cos(1/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 2*sqrt(2)*sin(1/3*arc
tan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2))*B/sqrt(a))/d
```

Fricas [A]
time = 2.02, size = 357, normalized size = 2.55

$$\left[-\frac{\sqrt{2}(A-B)\sqrt{a}\log\left(-\frac{\cos(dx+c)^2+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-B\sqrt{a}\log\left(\frac{a\cos(dx+c)^3-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7a\cos(dx+c)^2+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{2ad},\ -\frac{\sqrt{2}(A-B)a\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-B\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)}{ad}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [-1/2*(sqrt(2)*(A - B)*sqrt(a)*log(-(cos(d*x + c)^2 + 2*sqrt(2)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*co
s(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - B*sqrt(a)*log((a*c
os(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x
+ c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(
d*x + c)^3 + cos(d*x + c)^2)))/(a*d), -(sqrt(2)*(A - B)*a*sqrt(-1/a)*arctan
(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x +
c))/sin(d*x + c)) - B*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/
cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x
 + c) - 2*a)))/(a*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec(c + dx)}{\sqrt{a(\sec(c + dx) + 1)}\,\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(1/2),x)
[Out] Integral((A + B*sec(c + d*x))/(sqrt(a*(sec(c + d*x) + 1))*sqrt(cos(c + d*x)
)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*sqrt(cos(d*x + c))
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+d\,x)}}{\sqrt{\cos(c + d\,x)}\,\sqrt{a + \frac{a}{\cos(c + d\,x)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(1/2)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(1/2)), x
)
```

3.546 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=181

$$\frac{(2A-B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{\sqrt{a}\,d}-\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c\ \cdots}}{\sqrt{2}\sqrt{a+\cdots}}\right.\cdots}{\cdots}$$

```
[Out] (2*A-B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)
*sec(d*x+c)^(1/2)/d/a^(1/2)-(A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)
^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(d*x+c)^
(1/2)/d/a^(1/2)+B*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.33, antiderivative size = 181, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3034, 4106, 4108, 3893, 212, 3886, 221}

$$-\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{(2A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}+\frac{B\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] ((2*A - B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Co
s[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) - (Sqrt[2]*(A - B)*ArcTanh[(Sqr
t[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*S
qrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (B*Sin[c + d*x])/(d*Cos
[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
```

```
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4106

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fre
eQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& GtQ[n, 1]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx)}{\sqrt{a+a\sec(c+dx)}}$$

$$= \frac{B\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right.}{}$$

$$= \frac{B\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} - \left((A-B)\sqrt{\cos(c+dx)}\,\sqrt{s\right.$$

$$= \frac{B\sin(c+dx)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{\left(2(A-B)\sqrt{\cos(c+dx)}\,\sqrt{\right.}{}$$

$$= \frac{(2A-B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+}}{\sqrt{a}\,d}$$

Mathematica [A]
time = 0.52, size = 114, normalized size = 0.63

$$-\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(2(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos(c+dx)-\sqrt{2}\,(2A-B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos(c+dx)-2B\sin\left(\frac{1}{2}(c+dx)\right)\right)}{d\cos^{\frac{3}{2}}(c+dx)\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]]
),x]
```

```
[Out] -((Cos[(c + d*x)/2]*(2*(A - B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[c + d*x] - Sqr
t[2]*(2*A - B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x] - 2*B*Sin[(c
+ d*x)/2]))/(d*Cos[c + d*x]^(3/2)*Sqrt[a*(1 + Sec[c + d*x])]))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 342 vs. 2(152) = 304.
time = 12.68, size = 343, normalized size = 1.90

method	result

default	$-\frac{(-1+\cos(dx+c))\left(2A\cos(dx+c)\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)+2A\cos(dx+c)\sqrt{2}\,\arctan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2/d*(-1+cos(d*x+c))*(2*A*cos(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)
))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))+2*A*cos(d*x+c)*2^(1/2)*arctan(1
/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))-B*cos(d*x+
c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2
^(1/2))-B*cos(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d
*x+c)+sin(d*x+c))*2^(1/2))-4*A*cos(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(
d*x+c)))^(1/2))+2*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)+4*B*cos(d*x+c)*arc
tan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)))*(a*(1+cos(d*x+c))/cos(d*x+c)
)^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)^2/(-2/(1+cos(d*x+c)))^(1/2)/a
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 1509 vs. $2(152) = 304$.
time = 0.98, size = 1509, normalized size = 8.34

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/4*(2*(sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1
/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c),
 cos(d*x + c))) + 1) - sqrt(2)*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x +
c)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 1) - log(2*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(
2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sqrt(
2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 2) - log(2*cos(1/2*arctan2(sin(d*x + c), co
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(
2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 2) + log(2*cos(1/2*arctan2(sin(d*x + c), co
```

```
s(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sqrt(
2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*sin(1/2*arctan2
(sin(d*x + c), cos(d*x + c))) + 2))*A/sqrt(a) + (4*sqrt(2)*cos(3/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) - 4*sqrt(2)*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) + (cos(2*d*x +
 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 2) - (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) +
 1)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 2) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^
2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 -
2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*
sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - (cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqr
t(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 +
 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 2*(sqrt(2)*c
os(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c)
 + sqrt(2))*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + si
n(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqr
t(2))*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos
(2*d*x + 2*c) + sqrt(2))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)))*B/((cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*s
qrt(a)))/d
```

<u>Fricas</u> [A]
time = 3.18, size = 575, normalized size = 3.18

$$\left[\frac{4\,B\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)-\left(\left(2\,A-B\right)\cos\left(dx+c\right)^2+\left(2\,A-B\right)\cos\left(dx+c\right)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3+4\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}(\cos(dx+c)-2)\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-7\,a\cos(dx+c)^2+8\,a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)-\frac{2\sqrt{2}\left((A-B)a\cos(dx+c)^2+(A-B)a\cos(dx+c)\right)\log\left(-\frac{\cos(dx+c)^2-\frac{2\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)}{\sqrt{a}}}{4\left(ad\cos\left(dx+c\right)^2+ad\cos\left(dx+c\right)\right)}, \frac{2\sqrt{2}\left((A-B)a\cos\left(dx+c\right)^2+(A-B)a\cos\left(dx+c\right)\right)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{-\frac{1}{a}}\sqrt{\cos\left(dx+c\right)}}{\sin(dx+c)}\right)+2\,B\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\left(\left(2\,A-B\right)\cos\left(dx+c\right)^2+\left(2\,A-B\right)\cos\left(dx+c\right)\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2\,a}\right)}{2\left(ad\cos\left(dx+c\right)^2+ad\cos\left(dx+c\right)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/4*(4*B*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c) - ((2*A - B)*cos(d*x + c)^2 + (2*A - B)*cos(d*x + c))*sqrt(a)*log((a
*cos(d*x + c)^3 + 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*
x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(co
s(d*x + c)^3 + cos(d*x + c)^2)) - 2*sqrt(2)*((A - B)*a*cos(d*x + c)^2 + (A
- B)*a*cos(d*x + c))*log(-(cos(d*x + c)^2 - 2*sqrt(2)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c)
 - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1))/sqrt(a))/(a*d*cos(d*x + c)^2 +
 a*d*cos(d*x + c)), 1/2*(2*sqrt(2)*((A - B)*a*cos(d*x + c)^2 + (A - B)*a*co
s(d*x + c))*sqrt(-1/a)*arctan(sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(-1/a)*sqrt(cos(d*x + c))/sin(d*x + c)) + 2*B*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((2*A - B)*cos(d*x + c)
^2 + (2*A - B)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*
cos(d*x + c) - 2*a)))/(a*d*cos(d*x + c)^2 + a*d*cos(d*x + c))]
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec{\left(c + dx \right)}}{\sqrt{a \left(\sec{\left(c + dx \right)} + 1\right)} \cos^{\frac{3}{2}}{\left(c + dx \right)}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(sqrt(a*(sec(c + d*x) + 1))*cos(c + d*x)**(3/
2)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(3/2)
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos\left(c+d\,x\right)}}{{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(1/2)), x
)
```

3.547 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx$

Optimal. Leaf size=230

$$-\frac{(4A-7B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4\sqrt{a}\,d}+\frac{\sqrt{2}\,(A-B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\text{se}}}{\sqrt{2}\sqrt{\ldots}}\right.}{}$$

```
[Out] -1/4*(4*A-7*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c
)^(1/2)*sec(d*x+c)^(1/2)/d/a^(1/2)+(A-B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec
(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*2^(1/2)*cos(d*x+c)^(1/2)*sec(
d*x+c)^(1/2)/d/a^(1/2)+1/2*B*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))
^(1/2)+1/4*(4*A-B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 230, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3034, 4106, 4108, 3893, 212, 3886, 221}

$$\frac{(4A-B)\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{\sqrt{2}\,(A-B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{\sqrt{a}\,d}-\frac{(4A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4\sqrt{a}\,d}+\frac{B\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]),x]
```

```
[Out] -1/4*((4*A - 7*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (Sqrt[2]*(A - B)*ArcTa
nh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*
x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[a]*d) + (B*Sin[c + d*x])
/(2*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + ((4*A - B)*Sin[c + d*x
])/(4*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4106

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fre
eQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& GtQ[n, 1]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+$$

$$= \frac{B\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c-\right.}{}$$

$$= \frac{B\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{(4A-B)\sin(c+}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{B\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{(4A-B)\sin(c+}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{s}}}$$

$$= \frac{B\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+a\sec(c+dx)}} + \frac{(4A-B)\sin(c+}{4d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a\,\mathrm{s}}}$$

$$= -\frac{(4A-7B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{se}}}{4\sqrt{a}\,d}$$

Mathematica [A]
time = 0.99, size = 137, normalized size = 0.60

$$\frac{\cos\left(\frac{1}{2}(c+dx)\right)\left(8(A-B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^2(c+dx)-\sqrt{2}\,(4A-7B)\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^2(c+dx)+2(2B+(4A-B)\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{4d\cos^{\frac{5}{2}}(c+dx)\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]
),x]
```

```
[Out] (Cos[(c + d*x)/2]*(8*(A - B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[c + d*x]^2 - Sqr
t[2]*(4*A - 7*B)*ArcTanh[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[c + d*x]^2 + 2*(2*B
+ (4*A - B)*Cos[c + d*x])*Sin[(c + d*x)/2]))/(4*d*Cos[c + d*x]^(5/2)*Sqrt[a
*(1 + Sec[c + d*x])])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(191) = 382$.
time = 12.52, size = 413, normalized size = 1.80

method	result
default	$\frac{(-1+\cos(dx+c))\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(4A\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)(\cos^2(dx+c))\sqrt{2}-4A\,\mathrm{arc}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/8/d*(-1+cos(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(4*A*arctan(1/4*(
-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^
(1/2)-4*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^
(1/2))*cos(d*x+c)^2*2^(1/2)-7*B*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos
(d*x+c)+sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)+7*B*arctan(1/4*(-2/(1+cos
(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c))*2^(1/2))*cos(d*x+c)^2*2^(1/2)-16*
A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^2-8*A*(-2/(1+
cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)+16*B*arctan(1/2*sin(d*x+c)*(-2/(1+
cos(d*x+c)))^(1/2))*cos(d*x+c)^2+2*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)*s
in(d*x+c)-4*B*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c))/a/(-2/(1+cos(d*x+c)))^(
1/2)/sin(d*x+c)^2/cos(d*x+c)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2704 vs. $2(191) = 382$.
time = 0.99, size = 2704, normalized size = 11.76

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] -1/16*(4*(4*sqrt(2)*cos(3/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x
+ 2*c) - 4*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))*sin(2*d*x +
 2*c) + (cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*
log(2*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(
d*x + c))) + 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) -
(cos(2*d*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*co
s(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x +
c), cos(d*x + c)))^2 + 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)
)) - 2*sqrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) + (cos(2*d
*x + 2*c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*ar
```

ctan2(sin(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(
d*x + c)))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2*s
qrt(2)*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - (cos(2*d*x + 2*c
)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*log(2*cos(1/2*arctan2(si
n(d*x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)
))^2 - 2*sqrt(2)*cos(1/2*arctan2(sin(d*x + c), cos(d*x + c))) - 2*sqrt(2)*s
in(1/2*arctan2(sin(d*x + c), cos(d*x + c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2
*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 + sin(1/2*arctan2(sin(d*
x + c), cos(d*x + c)))^2 + 2*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))) +
 1) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 2*sqrt(2
)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/2*arctan2(sin(d*x + c), cos(d*x + c
)))^2 + sin(1/2*arctan2(sin(d*x + c), cos(d*x + c)))^2 - 2*sin(1/2*arctan2(
sin(d*x + c), cos(d*x + c))) + 1) - 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*
sin(3/2*arctan2(sin(d*x + c), cos(d*x + c))) + 4*(sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*sin(1/2*arctan2(sin(d*x + c), cos(d*x + c))))*A/((cos(2*d*x + 2*
c)^2 + sin(2*d*x + 2*c)^2 + 2*cos(2*d*x + 2*c) + 1)*sqrt(a)) - (4*(sqrt(2)*
sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(7/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) - 20*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d
*x + 2*c))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 20*(sqrt(
2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*cos(3/4*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) - 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2
*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 7*(2*(2
*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(2*d*x
+ 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sin
(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 7
(2(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 + 4*cos(
2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) +
 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
2) + 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c)^2 +
4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*d*x +
2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2
*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 2) - 7*(2*(2*cos(2*d*x + 2*c) + 1)*cos(4*d*x + 4*c) + cos(4*d*x + 4*c
)^2 + 4*cos(2*d*x + 2*c)^2 + sin(4*d*x + 4*c)^2 + 4*sin(4*d*x + 4*c)*sin(2*
d*x + 2*c) + 4*sin(2*d*x + 2*c)^2 + 4*cos(2*d*x + 2*c) + 1)*log(2*cos(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x

```
 + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 2) - 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)
^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*
c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2)
)*cos(4*d*x + 4*c) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 1) + 8*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2
+ sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c)
+ 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 2*(2*sqrt(2)*c...
```

Fricas [A]
time = 3.46, size = 621, normalized size = 2.70

$$\left[\frac{4\left((4A-B)\cos(dx+c)+2B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\left((4A-7B)\cos(dx+c)^{3}+(4A-7B)\cos(dx+c)^{2}\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^{3}-4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7a\cos(dx+c)^{2}+8a}{\cos(dx+c)^{3}+\cos(dx+c)^{2}}\right)-\frac{8\sqrt{2}\left((A-B)a\cos(dx+c)^{3}+(A-B)a\cos(dx+c)^{2}\right)\log\left(-\frac{\cos(dx+c)^{2}+\frac{2\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{\sqrt{a}}-2\cos(dx+c)-3}{\cos(dx+c)^{2}+2\cos(dx+c)+1}\right)}{\sqrt{a}}}{16\left(ad\cos(dx+c)^{3}+ad\cos(dx+c)^{2}\right)},-\frac{8\sqrt{2}\left((A-B)a\cos(dx+c)^{3}+(A-B)a\cos(dx+c)^{2}\right)\sqrt{-\frac{1}{a}}\arctan\left(\frac{\sqrt{2}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{-\frac{1}{a}}\sqrt{\cos(dx+c)}}{\sin(dx+c)}\right)-2\left((4A-B)\cos(dx+c)+2B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\left((4A-7B)\cos(dx+c)^{3}+(4A-7B)\cos(dx+c)^{2}\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^{2}-a\cos(dx+c)-2a}\right)}{8\left(ad\cos(dx+c)^{3}+ad\cos(dx+c)^{2}\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/16*(4*((4*A - B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))*sin(d*x + c) - ((4*A - 7*B)*cos(d*x + c)^3 + (4*A -
 7*B)*cos(d*x + c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x
+ c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)) - 8*sqr
t(2)*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*x + c)^2)*log(-(cos(d*x +
c)^2 + 2*sqrt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c)/sqrt(a) - 2*cos(d*x + c) - 3)/(cos(d*x + c)^2 + 2*cos(d*x + c
) + 1))/sqrt(a))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2), -1/8*(8*sqrt(2)
*((A - B)*a*cos(d*x + c)^3 + (A - B)*a*cos(d*x + c)^2)*sqrt(-1/a)*arctan(sq
rt(2)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(-1/a)*sqrt(cos(d*x + c))
/sin(d*x + c)) - 2*((4*A - B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((4*A - 7*B)*cos(d*x + c)^
3 + (4*A - 7*B)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*cos(d*x
+ c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x + c)^2 -
 a*cos(d*x + c) - 2*a)))/(a*d*cos(d*x + c)^3 + a*d*cos(d*x + c)^2)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6439 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(a*sec(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^{5/2}\,\sqrt{a+\frac{a}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(1/2)), x
)
```

3.548 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=270

$$-\frac{(15A-11B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{2d(a+a\sec(c+d}$$

```
[Out] -1/2*(A-B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)-1/4*(15*A-1
1*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c
))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)+1/10*(9*A-5*B
)*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(1/2)+1/30*(147*A-95*B)*
sin(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)-1/30*(39*A-35*B)*sin
(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3034, 4105, 4107, 4098, 3893, 212}

$$-\frac{(15A-11B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(9A-5B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{10ad\sqrt{a\sec(c+dx)+a}}-\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2d(a\sec(c+dx)+a)^{3/2}}-\frac{(39A-35B)\sin(c+dx)\sqrt{\cos(c+dx)}}{30ad\sqrt{a\sec(c+dx)+a}}+\frac{(147A-95B)\sin(c+dx)}{30ad\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] -1/2*((15*A - 11*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt
[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt
[2]*a^(3/2)*d) - ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*d*(a + a*Sec[
c + d*x])^(3/2)) + ((147*A - 95*B)*Sin[c + d*x])/(30*a*d*Sqrt[Cos[c + d*x]]
*Sqrt[a + a*Sec[c + d*x]]) - ((39*A - 35*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x]
)/(30*a*d*Sqrt[a + a*Sec[c + d*x]]) + ((9*A - 5*B)*Cos[c + d*x]^(3/2)*Sin[c
 + d*x])/(10*a*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx)}$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+d\right.}{}$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(9A-5B)\cos^{\frac{3}{2}}(c+dx)\sin}{10ad\sqrt{a+a\sec(c+d}}$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(39A-35B)\sqrt{\cos(c+dx)}}{30ad\sqrt{a+a\sec(c+}}$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(147A-95B)\sin(c}{30ad\sqrt{\cos(c+dx)}\,\sqrt{a+a}}$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(147A-95B)\sin(c}{30ad\sqrt{\cos(c+dx)}\,\sqrt{a+a}}$$

$$= -\frac{(15A-11B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx}}{2\sqrt{2}\,a^{3/2}d}$$

Mathematica [A]
time = 1.26, size = 178, normalized size = 0.66

$$\frac{30\sqrt{2}(15A-11B)\text{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)+2(141A-85B+3(39A-20B)\cos(c+dx)+(-6A+10B)\cos(2(c+dx))+3A\cos(3(c+dx)))\sqrt{1-\sec(c+dx)}\tan(c+dx)}{60d\sqrt{-1+\cos(c+dx)}(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (30*Sqrt[2]*(15*A - 11*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[
c + d*x]]]*Cos[(c + d*x)/2]^2*Sec[c + d*x]^(3/2)*Sin[c + d*x] + 2*(141*A -
85*B + 3*(39*A - 20*B)*Cos[c + d*x] + (-6*A + 10*B)*Cos[2*(c + d*x)] + 3*A*
Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Tan[c + d*x])/(60*d*Sqrt[-1 + Cos[
c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A]
time = 13.67, size = 329, normalized size = 1.22

method	result
default	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))\left(24A\left(\cos^4(dx+c)\right)-225A\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c}}}{2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/60/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
*(24*A*cos(d*x+c)^4-225*A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c
)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+165*B*cos(d*x+c)*sin(d*x+c)
*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))
-48*A*cos(d*x+c)^3-225*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2
/(1+cos(d*x+c)))^(1/2)*A*sin(d*x+c)+40*B*cos(d*x+c)^3+165*arctan(1/2*sin(d*
x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c)+240*
A*cos(d*x+c)^2-160*B*cos(d*x+c)^2+78*A*cos(d*x+c)-70*B*cos(d*x+c)-294*A+190
*B)/sin(d*x+c)^3/a^2
```

Maxima [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: RuntimeError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Exception raised: RuntimeError >> ECL says: THROW: The catch RAT-ERR is und
efined.
```

Fricas [A]
time = 1.74, size = 480, normalized size = 1.78

$$\left[-\frac{15\sqrt{2}\left((15A-11B)\cos(dx+c)^2+2(15A-11B)\cos(dx+c)+15A-11B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\left(12A\cos(dx+c)^3-4(3A-5B)\cos(dx+c)^2+12(9A-5B)\cos(dx+c)+147A-95B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{120\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}, \frac{15\sqrt{2}\left((15A-11B)\cos(dx+c)^2+2(15A-11B)\cos(dx+c)+15A-11B\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)+2\left(12A\cos(dx+c)^3-4(3A-5B)\cos(dx+c)^2+12(9A-5B)\cos(dx+c)+147A-95B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{60\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/120*(15*sqrt(2)*((15*A - 11*B)*cos(d*x + c)^2 + 2*(15*A - 11*B)*cos(d*x
 + c) + 15*A - 11*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqr
```

```
t((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*
cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(12*A*cos(d*
x + c)^3 - 4*(3*A - 5*B)*cos(d*x + c)^2 + 12*(9*A - 5*B)*cos(d*x + c) + 147
*A - 95*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/60*(15*sq
rt(2)*((15*A - 11*B)*cos(d*x + c)^2 + 2*(15*A - 11*B)*cos(d*x + c) + 15*A -
 11*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x +
 c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(12*A*cos(d*x + c)^3 - 4*(3*A
 - 5*B)*cos(d*x + c)^2 + 12*(9*A - 5*B)*cos(d*x + c) + 147*A - 95*B)*sqrt((
a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*c
os(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^{5/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(3/2), x
)
```

3.549 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=223

$$\frac{(11A-7B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{2\sqrt{2}a^{3/2}d}-\frac{(A-B)\sqrt{\cos(c+dx)}\,\mathrm{s}}{2d(a+a\sec(c+dx}$$

```
[Out] -1/2*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(3/2)+1/4*(11*A-7
*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c)
)^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)-1/6*(19*A-15*B
)*sin(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+1/6*(7*A-3*B)*sin(
d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3034, 4105, 4107, 4098, 3893, 212}

$$\frac{(11A-7B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}+\frac{(7A-3B)\sin(c+dx)\sqrt{\cos(c+dx)}}{6ad\sqrt{a\sec(c+dx)+a}}-\frac{(19A-15B)\sin(c+dx)}{6ad\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] ((11*A - 7*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*
a^(3/2)*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(2*d*(a + a*Sec[c +
d*x])^(3/2)) - ((19*A - 15*B)*Sin[c + d*x])/(6*a*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + a*Sec[c + d*x]]) + ((7*A - 3*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(6*a*d
*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
```

```
tegerQ[n])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx &= \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+d
\\
&= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c
\\
&= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} + \frac{(7A-3B)\sqrt{\cos(c+dx)}}{6ad\sqrt{a+a\sec(c}}
\\
&= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(19A-15B)\sin(}{6ad\sqrt{\cos(c+dx)}\,\sqrt{a+}}
\\
&= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2d(a+a\sec(c+dx))^{3/2}} - \frac{(19A-15B)\sin(}{6ad\sqrt{\cos(c+dx)}\,\sqrt{a+}}
\\
&= \frac{(11A-7B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}}{2\sqrt{2}\,a^{3/2}d}
\end{aligned}
$$

Mathematica [A]
time = 1.27, size = 155, normalized size = 0.70

$$
\frac{\left(-3\sqrt{2}\,(11A-7B)\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{3}{2}}(c+dx)+\sqrt{1-\sec(c+dx)}\,(12(-A+B)+(-17A+15B+2A\cos(2(c+dx)))\sec(c+dx))\right)\sin(c+dx)}{6d\sqrt{-1+\cos(c+dx)}\,(a(1+\sec(c+dx)))^{3/2}}
$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3
/2),x]
```

```
[Out] ((-3*Sqrt[2]*(11*A - 7*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[
c + d*x]]]*Cos[(c + d*x)/2]^2*Sec[c + d*x]^(3/2) + Sqrt[1 - Sec[c + d*x]]*(
12*(-A + B) + (-17*A + 15*B + 2*A*Cos[2*(c + d*x)])*Sec[c + d*x]))*Sin[c +
d*x])/(6*d*Sqrt[-1 + Cos[c + d*x]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A]
time = 12.92, size = 307, normalized size = 1.38

method	result

default	$\frac{(-1+\cos(dx+c))\left(33A\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(dx+c)}}\cos(dx+c)-21B\cos(dx+c)\sin(dx+c\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/12/d*(-1+cos(d*x+c))*(33*A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*
x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-21*B*cos(d*x+c)*sin(d*x+
c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2
))+8*A*cos(d*x+c)^3+33*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2
/(1+cos(d*x+c)))^(1/2)*A*sin(d*x+c)-21*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x
+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c)-32*A*cos(d*x+c)^2+24*B*
cos(d*x+c)^2-14*A*cos(d*x+c)+6*B*cos(d*x+c)+38*A-30*B)*(a*(1+cos(d*x+c))/co
s(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/a^2/sin(d*x+c)^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 36231 vs. $2(188) = 376$.
time = 1.81, size = 36231, normalized size = 162.47

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/12*(3*(8*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 32*cos(3/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 8*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 32*sin(3/4*arctan2(sin(2*d*x
+ 2*c), cos(2*d*x + 2*c)))^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 8*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^3 + 4*(8*co
s(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*
x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - sin(2*d*x
 + 2*c) - 5*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(4*cos(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) - sin(2*d*x + 2*c) - 5*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 7*(2*(2*cos
(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/4*arctan2(sin(2*d
```

*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(2*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) + 7*(2*(2*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(2*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2)*log(cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1) - 4*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(2*d*x + 2*c) + 4*(8*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + cos(2*d*x + 2*c) + 5*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2)*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(4*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + cos(2*d*x + 2*c) + 5*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 20*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + cos(2*d*x + 2*c) + 2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 20*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*B*sqrt(a)/(sqrt(2)*a^2*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*a^2*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*a^2*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a^2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a^2

```
*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*a^2*sin
(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*a^2*sin(3/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))) + sqrt(2)*a^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*a^2*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + sqrt(2)*a^2*cos(1/4*arctan2(si...
```

Fricas [A]
time = 1.72, size = 442, normalized size = 1.98

$$\left[-\frac{3\sqrt{2}\left((11A-7B)\cos(dx+c)^2+2(11A-7B)\cos(dx+c)+11A-7B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\left(4A\cos(dx+c)^2-12(A-B)\cos(dx+c)-19A+15B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{24\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}, -\frac{3\sqrt{2}\left((11A-7B)\cos(dx+c)^2+2(11A-7B)\cos(dx+c)+11A-7B\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-2\left(4A\cos(dx+c)^2-12(A-B)\cos(dx+c)-19A+15B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{12\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/24*(3*sqrt(2)*((11*A - 7*B)*cos(d*x + c)^2 + 2*(11*A - 7*B)*cos(d*x + c
) + 11*A - 7*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d
*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(4*A*cos(d*x + c)
^2 - 12*(A - B)*cos(d*x + c) - 19*A + 15*B)*sqrt((a*cos(d*x + c) + a)/cos(d
*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*c
os(d*x + c) + a^2*d), -1/12*(3*sqrt(2)*((11*A - 7*B)*cos(d*x + c)^2 + 2*(11
*A - 7*B)*cos(d*x + c) + 11*A - 7*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt(
(a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2
*(4*A*cos(d*x + c)^2 - 12*(A - B)*cos(d*x + c) - 19*A + 15*B)*sqrt((a*cos(d
*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x
+ c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(3/2), x
)
```

3.550 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=176

$$-\frac{(7A-3B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\sin(c+}{2d\sqrt{\cos(c+dx)}\,(a+a\,\mathrm{se}}$$

```
[Out] -1/2*(A-B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)-1/4*(7*A-3*
B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)+1/2*(5*A-B)*sin
(d*x+c)/a/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.32, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3034, 4105, 4098, 3893, 212}

$$-\frac{(7A-3B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(5A-B)\sin(c+dx)}{2ad\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}-\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3/2),x]
```

```
[Out] -1/2*((7*A - 3*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2
]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqrt[2
]*a^(3/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c
+ d*x])^(3/2)) + ((5*A - B)*Sin[c + d*x])/(2*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a
+ a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+\cdots}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{s}\cdots}\right.}{\cdots}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}} + \frac{(5A-B)\,\mathrm{s}\cdots}{2ad\sqrt{\cos(c+dx)}\,\cdots}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}} + \frac{(5A-B)\,\mathrm{s}\cdots}{2ad\sqrt{\cos(c+dx)}\,\cdots}$$

$$= -\frac{(7A-3B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+d\cdots}}{2\sqrt{2}\,a^{3/2}d}$$

Mathematica [A]
time = 1.93, size = 198, normalized size = 1.12

$$\frac{4\sqrt{2}\,(7A-3B)\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^3\left(\tfrac{1}{2}(c+dx)\right)(B+A\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\sin\left(\tfrac{1}{2}(c+dx)\right)+2(2A^2+5AB-B^2+A(5A+3B)\cos(c+dx)+2A^2\cos(2(c+dx)))\sqrt{1-\sec(c+dx)}\,\tan(c+dx)}{4d\sqrt{-1+\cos(c+dx)}\,(B+A\cos(c+dx))(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (4*Sqrt[2]*(7*A - 3*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c +
 d*x]]]*Cos[(c + d*x)/2]^3*(B + A*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sin[(c +
 d*x)/2] + 2*(2*A^2 + 5*A*B - B^2 + A*(5*A + 3*B)*Cos[c + d*x] + 2*A^2*Cos[
2*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Tan[c + d*x])/(4*d*Sqrt[-1 + Cos[c + d
*x]]*(B + A*Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A]
time = 12.32, size = 235, normalized size = 1.34

method	result

default	$\frac{(-1+\cos(dx+c))\left(4A\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos^2(dx+c))+7A\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)+A\sqrt{-\frac{2}{1+\cos(\ldots}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/d*(-1+cos(d*x+c))*(4*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+7*A*sin(d
*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))+A*(-2/(1+cos(d*x+c))
)^(1/2)*cos(d*x+c)-3*B*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))
^(1/2))-B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-5*A*(-2/(1+cos(d*x+c)))^(1/2
)+B*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c
))^(1/2)/a^2/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 8208 vs. $2(147) = 294$.
time = 1.01, size = 8208, normalized size = 46.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] -1/4*((4*(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c
)^4 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4 + 4*(7*log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)
- 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^4 + 70*(log(cos(1/2*d*x + 1/
2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2
*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos
(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^2 + 7*(log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*
x + 1/2*c)^4 - 8*sin(1/2*d*x + 1/2*c)^5 + 28*(7*(log(cos(1/2*d*x + 1/2*c)^2
 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x +
 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d
```

*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/
2*c)^3 + 4*(21*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^
2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 24*sin(1/2*d*x + 1/
2*c)^2 - 20)*sin(3/2*d*x + 3/2*c)^3 - 8*(10*cos(1/2*d*x + 1/2*c)^2 + 3)*sin
(1/2*d*x + 1/2*c)^3 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*cos(3
/2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (7*log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
 - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c
) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(log(cos(1
/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) -
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c
) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))
*cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 8*sin
(1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x + 3/2*c) - 8*(9*cos(1/2*d*x + 1/2*c)^2
 + 2)*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c)^2 + (427*(log(cos(1/2*d*x
+ 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))
*cos(1/2*d*x + 1/2*c)^2 + 35*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/
2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 40*
sin(1/2*d*x + 1/2*c)^3 - 8*(61*cos(1/2*d*x + 1/2*c)^2 + 9)*sin(1/2*d*x + 1/
2*c))*cos(3/2*d*x + 3/2*c)^2 + ((7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 +
sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*
c))*cos(3/2*d*x + 3/2*c)^2 + 63*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 +
(7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c) + 1) - 7*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1
/2*d*x + 1/2*c) + 1) - 8*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 7*(
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*
x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 8*sin(1/2*d*x + 1/2*c)^3 + 6*(7*(
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*

```
x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x
+ 1/2*c))*cos(3/2*d*x + 3/2*c) + 2*(7*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2
*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2
+ sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c
) - 8*sin(1/2*d*x + 1/2*c)^2 - 8)*sin(3/2*d*x +...
```

Fricas [A]
time = 1.61, size = 410, normalized size = 2.33

$$\left[-\frac{\sqrt{2}\left((7A-3B)\cos\left(dx+c\right)^2+2\,(7A-3B)\cos\left(dx+c\right)+7A-3B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\left(4A\cos\left(dx+c\right)+5A-B\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{8\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}, \frac{\sqrt{2}\left((7A-3B)\cos\left(dx+c\right)^2+2\,(7A-3B)\cos\left(dx+c\right)+7A-3B\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}}{a\sin(dx+c)}\right)+2\left(4A\cos\left(dx+c\right)+5A-B\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{4\left(a^2d\cos\left(dx+c\right)^2+2a^2d\cos\left(dx+c\right)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((7*A - 3*B)*cos(d*x + c)^2 + 2*(7*A - 3*B)*cos(d*x + c) + 7
*A - 3*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*
x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c
) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(4*A*cos(d*x + c) + 5*A
 - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), 1/4*(sqrt(2)*((7
*A - 3*B)*cos(d*x + c)^2 + 2*(7*A - 3*B)*cos(d*x + c) + 7*A - 3*B)*sqrt(-a)
*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))/(a*sin(d*x + c))) + 2*(4*A*cos(d*x + c) + 5*A - B)*sqrt((a*cos(d*x
 + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x +
c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\sqrt{\cos{(c + dx)}}}{(a(\sec{(c + dx)} + 1))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(cos(c + d*x))/(a*(sec(c + d*x) + 1))**(3
/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(3/2), x
)
```

3.551 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=127

$$\frac{(3A+B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}$$

```
[Out] -1/2*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+1/4*(3*A+B)
*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(
1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)
```

Rubi [A]

time = 0.20, antiderivative size = 127, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.114$, Rules used = {3034, 4097, 3893, 212}

$$\frac{(3A+B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}-\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] ((3*A + B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[
a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(
3/2)*d) - ((A - B)*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x
])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4097

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n},
 x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m
, -1]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(\cdots}{(a+a\sec(c+dx))^{3/\cdots}}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left((3A+B)\sqrt{\cos(c+\cdots}\right.}{\cdots}$$

$$= -\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left((3A+B)\sqrt{\cos(c+\cdots}\right.}{\cdots}$$

$$= \frac{(3A+B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\cdots}{2\sqrt{2}\,a^{3/2}d}$$

Mathematica [**A**]
time = 0.55, size = 86, normalized size = 0.68

$$\frac{(3A+B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+\frac{1}{2}(-A+B)\sin(c+dx)}{ad\sqrt{\cos(c+dx)}\,(1+\cos(c+dx))\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(3/
2)),x]
```

[Out] ((3*A + B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]^3 + ((-A + B)*Sin[c + d*x])/2)/(a*d*Sqrt[Cos[c + d*x]]*(1 + Cos[c + d*x])*Sqrt[a*(1 + Sec[c + d*x])])

Maple [A]
time = 13.14, size = 209, normalized size = 1.65

method	result
default	$-\frac{(-1+\cos(dx+c))\left(A\sqrt{-\frac{2}{1+\cos(dx+c)}}\,\cos(dx+c)+3A\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)-B\sqrt{-\frac{2}{1+\cos(dx+c)}}\right.}{2\cdots}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x,method=_RETURNVERBOSE)

[Out] -1/2/d*(-1+cos(d*x+c))*(A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+3*A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+B*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-A*(-2/(1+cos(d*x+c)))^(1/2)+B*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/a^2/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 2166 vs. $2(104) = 208$.
time = 0.92, size = 2166, normalized size = 17.06

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algorithm="maxima")

[Out] 1/4*((3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c)^2 + 3*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 12*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin

(d*x + c)^2 + 2*(6*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 3*log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 2
*sin(3/2*d*x + 3/2*c) + 2*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 4*(3*log
(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
 1) - 3*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
 + 1/2*c) + 1) + 2*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(3*(log(cos(1/2*d
*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) +
1))*sin(d*x + c) + cos(3/2*d*x + 3/2*c) - cos(1/2*d*x + 1/2*c))*sin(2*d*x +
 2*c) - 4*(2*cos(d*x + c) + 1)*sin(3/2*d*x + 3/2*c) + 8*cos(3/2*d*x + 3/2*c
)*sin(d*x + c) - 8*cos(1/2*d*x + 1/2*c)*sin(d*x + c) + 3*log(cos(1/2*d*x +
1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 3*log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1)
+ 4*sin(1/2*d*x + 1/2*c))*A/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*co
s(d*x + c)^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*a*sin(2*d*x + 2*c)*
sin(d*x + c) + 4*sqrt(2)*a*sin(d*x + c)^2 + 4*sqrt(2)*a*cos(d*x + c) + 2*(2
*sqrt(2)*a*cos(d*x + c) + sqrt(2)*a)*cos(2*d*x + 2*c) + sqrt(2)*a)*sqrt(a))
 + (4*(sin(3/2*d*x + 3/2*c) - sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c))))*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)) + 8*(sin(3/2*d*x + 3/2*c) - sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))))*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + (2*(2*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) +
1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(4/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(2/3*arctan2(s
in(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(4/3*arctan2(sin(3/2*d*x
 + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 4*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))
)^2 + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*l
og(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3
*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 2*sin(1/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1) - (2*(2*cos(2/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c)))^2 + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2
*d*x + 3/2*c)))^2 + sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2
*c)))^2 + 4*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*si
n(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sin(2/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*cos(2/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*log(cos(1/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c
), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3

```
/2*d*x + 3/2*c))) + 1) - 4*(cos(3/2*d*x + 3/2*c) - cos(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) - 8*(cos(3/2*d*x + 3/2*c) - cos(1/3*arctan2(sin(3/2
*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c)
, cos(3/2*d*x + 3/2*c))) + 4*sin(3/2*d*x + 3/2*c) - 4*sin(1/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*B/((sqrt(2)*a*cos(4/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*sqrt(2)*a*cos(2/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sqrt(2)*a*sin(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*sqrt(2)*a*sin(4/3*arctan2(
sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 4*sqrt(2)*a*sin(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 4*sqrt(2)*a*...
```

Fricas [A]
time = 1.78, size = 376, normalized size = 2.96

$$\left[\frac{\sqrt{2}\left((3A+B)\cos(dx+c)^2+2(3A+B)\cos(dx+c)+3A+B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4(A-B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{8\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}, -\frac{\sqrt{2}\left((3A+B)\cos(dx+c)^2+2(3A+B)\cos(dx+c)+3A+B\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)+2(A-B)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{4\left(a^2d\cos(dx+c)^2+2a^2d\cos(dx+c)+a^2d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/8*(sqrt(2)*((3*A + B)*cos(d*x + c)^2 + 2*(3*A + B)*cos(d*x + c) + 3*A +
B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a
)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(A - B)*sqrt((a*cos(d*x + c) +
 a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^2*d*cos(d*x + c)^2 +
2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)*((3*A + B)*cos(d*x + c)^2 + 2*
(3*A + B)*cos(d*x + c) + 3*A + B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*
cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(A
 - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(a^2*d*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec(c+dx)}{(a(\sec(c+dx)+1))^{\frac{3}{2}}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/((a*(sec(c + d*x) + 1))**(3/2)*sqrt(cos(c + d
*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c
))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\sqrt{\cos\left(c+d\,x\right)}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(3/2)), x
)
```

3.552 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=185

$$\frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{3/2}d}+\frac{(A-5B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin}{\sqrt{2}\sqrt{a+a\sec(c+}}\right.}{2\sqrt{2}\,a}$$

[Out] 1/2*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+2*B*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d+1/4*(A-5*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)

Rubi [A]

time = 0.34, antiderivative size = 185, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.200$, Rules used = {3034, 4104, 4108, 3893, 212, 3886, 221}

$$\frac{(A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}+\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2)),x]

[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((A - 5*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
```

```
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+}{(a+a\sec(c+dx))^{3/2}}$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(\right.}{}$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left((A-5B)\sqrt{\cos(c+d\right.}{}$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{\left((A-5B)\sqrt{\cos(c+d\right.}{}$$

$$= \frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx}}{a^{3/2}d}$$

Mathematica [A]
time = 0.96, size = 113, normalized size = 0.61

$$\frac{(A-5B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)+4\sqrt{2}\,B\tanh^{-1}\left(\sqrt{2}\,\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos\left(\frac{1}{2}(c+dx)\right)+(A-B)\tan\left(\frac{1}{2}(c+dx)\right)}{2ad\sqrt{\cos(c+dx)}\,\sqrt{a(1+\sec(c+dx))}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] ((A - 5*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + 4*Sqrt[2]*B*ArcTanh
[Sqrt[2]*Sin[(c + d*x)/2]]*Cos[(c + d*x)/2] + (A - B)*Tan[(c + d*x)/2])/(2*
a*d*Sqrt[Cos[c + d*x]]*Sqrt[a*(1 + Sec[c + d*x])])
```

Maple [A]
time = 13.27, size = 303, normalized size = 1.64

method	result

default	$-\frac{\left(\sqrt{\cos}\,(dx+c)\right)(-1+\cos(dx+c))\left(2B\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}\,\sin(dx+c)+2B\arctan\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/2/d*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))*(2*B*arctan(1/4*(-2/(1+cos(d*x+c)))
^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/2)*sin(d*x+c)+2*B*arctan(1/4
*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/2)*sin(
d*x+c)+A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-A*(-2/
(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-5*B*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(
1+cos(d*x+c)))^(1/2))+B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+A*(-2/(1+cos(d
*x+c)))^(1/2)-B*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1
/2)/sin(d*x+c)^3/(-2/(1+cos(d*x+c)))^(1/2)/a^2
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 2.32, size = 592, normalized size = 3.20

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((A - 5*B)*cos(d*x + c)^2 + 2*(A - 5*B)*cos(d*x + c) + A - 5
*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c)
 + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*
a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) - 4*(A - B)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 4*(B*cos(d*x + c)^2 +
```

```
2*B*cos(d*x + c) + B)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x
+ c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a^2*d
*cos(d*x + c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d), -1/4*(sqrt(2)*((A - 5*B)*c
os(d*x + c)^2 + 2*(A - 5*B)*cos(d*x + c) + A - 5*B)*sqrt(-a)*arctan(sqrt(2)
*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin
(d*x + c))) - 2*(A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))*sin(d*x + c) - 4*(B*cos(d*x + c)^2 + 2*B*cos(d*x + c) + B)*sqrt(-a)
*arctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^2*d*cos(d*x
+ c)^2 + 2*a^2*d*cos(d*x + c) + a^2*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3436 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(3/2)), x
)
```

3.553 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=237

$$\frac{(2A-3B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{a^{3/2}d}-\frac{(5A-9B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c-}}{\sqrt{2}\,\sqrt{a+a}}\right.}{}$$

[Out] 1/2*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2)+(2*A-3*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d-1/4*(5*A-9*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(3/2)/d*2^(1/2)-1/2*(A-3*B)*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(1/2)

Rubi [A]
time = 0.47, antiderivative size = 237, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3034, 4104, 4106, 4108, 3893, 212, 3886, 221}

$$-\frac{(5A-9B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}\,a^{3/2}d}+\frac{(2A-3B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{3/2}d}-\frac{(A-3B)\sin(c+dx)}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)),x]

[Out] ((2*A - 3*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) - ((5*A - 9*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B)*Sin[c + d*x])/(2*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)) - ((A - 3*B)*Sin[c + d*x])/(2*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]])

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4106

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fre
eQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& GtQ[n, 1]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
```

```
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.}{}$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{(A-3B)\sin(c-}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a}}$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{(A-3B)\sin(c-}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a}}$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{(A-3B)\sin(c-}{2ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a+a}}$$

$$= \frac{(2A-3B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c}{a^{3/2}d}$$

Mathematica [A]
time = 2.31, size = 288, normalized size = 1.22

$$-\frac{\sqrt{\cos(c+dx)}\sec^{\frac{5}{2}}(c+dx)\left(4(A-3B)\mathrm{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)+20A\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)-36B\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)-2\sqrt{2}(5A-9B)\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)-4B\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}+2A\cos(c+dx)\sqrt{(-1+\cos(c+dx))\sec^2(c+dx)}-6B\cos(c+dx)\sqrt{(-1+\cos(c+dx))\sec^2(c+dx)}\right)\sin(c+dx)}{4d\sqrt{1-\sec(c+dx)}\,(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] -1/4*(Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(5/2)*(4*(A - 3*B)*ArcSin[Sqrt[1 - Se
c[c + d*x]]]*Cos[(c + d*x)/2]^2 + 20*A*ArcSin[Sqrt[Sec[c + d*x]]]*Cos[(c +
d*x)/2]^2 - 36*B*ArcSin[Sqrt[Sec[c + d*x]]]*Cos[(c + d*x)/2]^2 - 2*Sqrt[2]*
```

```
(5*A - 9*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos
[(c + d*x)/2]^2 - 4*B*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])] + 2*A*Cos[c
 + d*x]*Sqrt[(-1 + Cos[c + d*x])*Sec[c + d*x]^2] - 6*B*Cos[c + d*x]*Sqrt[(-
1 + Cos[c + d*x])*Sec[c + d*x]^2])*Sin[c + d*x])/(d*Sqrt[1 - Sec[c + d*x]]*
(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 467 vs. $2(198) = 396$.

time = 13.02, size = 468, normalized size = 1.97

method	result
default	$\frac{(-1+\cos(dx+c))\left(2A\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)-\sin(dx+c))\sqrt{2}}{4}\right)\sin(dx+c)\sqrt{2}\,\cos(dx+c)-2A\arctan\left(\frac{\sqrt{-}}{}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/2/d*(-1+cos(d*x+c))*(2*A*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+
c)-sin(d*x+c))*2^(1/2))*sin(d*x+c)*2^(1/2)*cos(d*x+c)-2*A*arctan(1/4*(-2/(1
+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*sin(d*x+c)*2^(1/2)*c
os(d*x+c)-3*B*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c)
)*2^(1/2))*2^(1/2)*cos(d*x+c)*sin(d*x+c)+3*B*arctan(1/4*(-2/(1+cos(d*x+c)))
^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)*sin(d*x+c)+5*A
*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+c)-A*(
-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-9*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(
d*x+c)))^(1/2))*cos(d*x+c)*sin(d*x+c)+3*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x
+c)^2+A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-B*(-2/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)-2*B*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)
/cos(d*x+c)^(1/2)/sin(d*x+c)^3/(-2/(1+cos(d*x+c)))^(1/2)/a^2
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 7057 vs. $2(198) = 396$.

time = 1.05, size = 7057, normalized size = 29.78

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] 1/4*((4*(sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*(sqrt(2)*
```

```
cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
+ 2*c)))^2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt
(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2
*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqr
t(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x
 + 2*c)^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(
1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*
c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2
+ 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 2*(sqrt(2)*cos(2*d*x + 2*c)^2
 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(
2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt
(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arcta
n2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 2) - 2*(sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt
(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(2*
d*x + 2*c)^2 + 4*sqrt(2)*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + 4*(sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(
2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 2) - 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1
)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x +
 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(co
s(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + 1) + 5*(cos(2*d*x + 2*c)^2 + 4*(cos(2*d*x + 2*c) + 1)*co
s(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(1/2*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(2*d*x + 2*c)^2 + 4*sin(2*d*x + 2*c
```

```
)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*cos(2*d*x + 2*c) + 1)*log(cos(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sin(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))) + 1) - 4*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*
sin(2*d*x + 2*c) - 4*(cos(2*d*x + 2*c) + 2*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 1)*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 8*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(cos(2*d*x + 2*c) + 1)*sin(1/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*cos(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))))*A/((sqrt(2)*a*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*a*cos(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*a*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*
a*sin(2*d*x + 2*c)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4
*sqrt(2)*a*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(
2)*a*cos(2*d*x + 2*c) + 4*(sqrt(2)*a*cos(2*d*x + 2*c) + sqrt(2)*a)*cos(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2)*a)*sqrt(a)) - (12*(s
in(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(3/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
)*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +...
```

<u>Fricas</u> [A]
time = 2.55, size = 716, normalized size = 3.02

$$\left[-\frac{\sqrt{2}\left((5A-9B)\cos(dx+c)^3+2(5A-9B)\cos(dx+c)^2+(5A-9B)\cos(dx+c)\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\left((A-3B)\cos(dx+c)-2B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+2\left((2A-3B)\cos(dx+c)^3+2(2A-3B)\cos(dx+c)^2+(2A-3B)\cos(dx+c)\right)\sqrt{a}\log\left(\frac{a\cos(dx+c)^3+4\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}(\cos(dx+c)-2)\sqrt{\cos(dx+c)}\sin(dx+c)-7a\cos(dx+c)^2+8a}{\cos(dx+c)^3+\cos(dx+c)^2}\right)}{8\left(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2+a^2d\cos(dx+c)\right)}, \frac{\sqrt{2}\left((5A-9B)\cos(dx+c)^3+2(5A-9B)\cos(dx+c)^2+(5A-9B)\cos(dx+c)\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-2\left((A-3B)\cos(dx+c)-2B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+2\left((2A-3B)\cos(dx+c)^3+2(2A-3B)\cos(dx+c)^2+(2A-3B)\cos(dx+c)\right)\sqrt{-a}\arctan\left(\frac{2\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{a\cos(dx+c)^2-a\cos(dx+c)-2a}\right)}{4\left(a^2d\cos(dx+c)^3+2a^2d\cos(dx+c)^2+a^2d\cos(dx+c)\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/8*(sqrt(2)*((5*A - 9*B)*cos(d*x + c)^3 + 2*(5*A - 9*B)*cos(d*x + c)^2 +
 (5*A - 9*B)*cos(d*x + c))*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(
a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)
- 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((A -
3*B)*cos(d*x + c) - 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c) + 2*((2*A - 3*B)*cos(d*x + c)^3 + 2*(2*A - 3*B)*cos(d
*x + c)^2 + (2*A - 3*B)*cos(d*x + c))*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqr
t(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*
x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x
+ c)^2)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)^2 + a^2*d*cos(d*x +
c)), 1/4*(sqrt(2)*((5*A - 9*B)*cos(d*x + c)^3 + 2*(5*A - 9*B)*cos(d*x + c)^
2 + (5*A - 9*B)*cos(d*x + c))*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(
d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*((A -
3*B)*cos(d*x + c) - 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c) + 2*((2*A - 3*B)*cos(d*x + c)^3 + 2*(2*A - 3*B)*cos(d
```

```
*x + c)^2 + (2*A - 3*B)*cos(d*x + c))*sqrt(-a)*arctan(2*sqrt(-a)*sqrt((a*co
s(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c)/(a*cos(d*x +
c)^2 - a*cos(d*x + c) - 2*a)))/(a^2*d*cos(d*x + c)^3 + 2*a^2*d*cos(d*x + c)
^2 + a^2*d*cos(d*x + c))]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/
2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^{5/2}\,\left(a+\frac{a}{\cos(c+d\,x)}\right)^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(3/2)), x
)
```

3.554 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=287

$$-\frac{(12A-19B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{4a^{3/2}d}+\frac{(9A-13B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{}}{\sqrt{2}\sqrt{}}\right.}{}$$

```
[Out] 1/2*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2)-1/4*(12*A-19
*B)*arcsinh(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec
(d*x+c)^(1/2)/a^(3/2)/d+1/4*(9*A-13*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d
*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/
2)/a^(3/2)/d*2^(1/2)-1/2*(A-2*B)*sin(d*x+c)/a/d/cos(d*x+c)^(5/2)/(a+a*sec(d
*x+c))^(1/2)+1/4*(6*A-7*B)*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))
^(1/2)
```

Rubi [**A**]
time = 0.59, antiderivative size = 287, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3034, 4104, 4106, 4108, 3893, 212, 3886, 221}

$$\frac{(9A-13B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{2\sqrt{2}a^{3/2}d}-\frac{(12A-19B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{4a^{3/2}d}+\frac{(6A-7B)\sin(c+dx)}{4ad\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}-\frac{(A-2B)\sin(c+dx)}{2ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)),x]
```

```
[Out] -1/4*((12*A - 19*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]
]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(3/2)*d) + ((9*A - 13*B)*ArcTan
h[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x
]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(2*Sqrt[2]*a^(3/2)*d) + ((A - B
)*Sin[c + d*x])/(2*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/2)) - ((A -
 2*B)*Sin[c + d*x])/(2*a*d*Cos[c + d*x]^(5/2)*Sqrt[a + a*Sec[c + d*x]]) + (
(6*A - 7*B)*Sin[c + d*x])/(4*a*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c + d*x]
])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4106

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fre
eQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& GtQ[n, 1]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+ \ldots}{(a+a\sec(c+dx))^{3/2}}$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(\ldots}{\ldots}$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{(A-2B)\sin(c \ldots}{2ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+ \ldots}}$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{(A-2B)\sin(c \ldots}{2ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+ \ldots}}$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{(A-2B)\sin(c \ldots}{2ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+ \ldots}}$$

$$= \frac{(A-B)\sin(c+dx)}{2d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{3/2}} - \frac{(A-2B)\sin(c \ldots}{2ad\cos^{\frac{5}{2}}(c+dx)\sqrt{a+ \ldots}}$$

$$= -\frac{(12A-19B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\ldots}}{4a^{3/2}d}$$

Mathematica [A]
time = 3.36, size = 328, normalized size = 1.14

$$\frac{\sec^{\frac{3}{2}}(c+dx)\left(2(6A-7B)\mathrm{ArcSin}\left(\sqrt{1-\sec(c+dx)}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)+36A\,\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)-52B\,\mathrm{ArcSin}\left(\sqrt{\sec(c+dx)}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)-2\sqrt{2}(9A-13B)\mathrm{ArcTan}\left(\frac{\sqrt{2}\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^2\left(\frac{1}{2}(c+dx)\right)+2B\sqrt{1-\sec(c+dx)}\,\sec^{\frac{3}{2}}(c+dx)+4A\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}-3B\sqrt{-((-1+\sec(c+dx))\sec(c+dx))}+6A\cos(c+dx)\sqrt{(-1+\cos(c+dx))\sec^2(c+dx)}-7B\cos(c+dx)\sqrt{(-1+\cos(c+dx))\sec^2(c+dx)}\right)\sin(c+dx)}{4d\sqrt{-1+\cos(c+dx)}\,(a(1+\sec(c+dx)))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] (Sec[c + d*x]^(3/2)*(2*(6*A - 7*B)*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Cos[(c +
d*x)/2]^2 + 36*A*ArcSin[Sqrt[Sec[c + d*x]]]*Cos[(c + d*x)/2]^2 - 52*B*ArcSi
n[Sqrt[Sec[c + d*x]]]*Cos[(c + d*x)/2]^2 - 2*Sqrt[2]*(9*A - 13*B)*ArcTan[(S
qrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Cos[(c + d*x)/2]^2 + 2*B
*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]^(3/2) + 4*A*Sqrt[-((-1 + Sec[c + d*x])
*Sec[c + d*x])] - 3*B*Sqrt[-((-1 + Sec[c + d*x])*Sec[c + d*x])] + 6*A*Cos[c
 + d*x]*Sqrt[(-1 + Cos[c + d*x])*Sec[c + d*x]^2] - 7*B*Cos[c + d*x]*Sqrt[(-
1 + Cos[c + d*x])*Sec[c + d*x]^2])*Sin[c + d*x])/(4*d*Sqrt[-1 + Cos[c + d*x
]]*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 530 vs. $2(240) = 480$.
time = 13.46, size = 531, normalized size = 1.85

method	result
default	$\frac{(-1+\cos(dx+c))\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\left(-12A\left(\cos^2(dx+c)\right)\sin(dx+c)\sqrt{2}\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}(1+\cos(dx+c)-\sin(dx+c))\sqrt{}}{4}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/8/d*(-1+cos(d*x+c))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-12*A*cos(d*x+c)
^2*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-si
n(d*x+c))*2^(1/2))+12*A*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+c
os(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))+19*B*cos(d*x+c)^2*sin(
d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)-sin(d*x+c
))*2^(1/2))-19*B*cos(d*x+c)^2*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+
c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))+12*A*cos(d*x+c)^3*(-2/(1+cos(
d*x+c)))^(1/2)-36*A*cos(d*x+c)^2*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+co
s(d*x+c)))^(1/2))-14*B*cos(d*x+c)^3*(-2/(1+cos(d*x+c)))^(1/2)+52*B*cos(d*x+
c)^2*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-4*A*(-2/(1
+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+8*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2
-8*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+10*B*(-2/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)-4*B*(-2/(1+cos(d*x+c)))^(1/2))/a^2/(-2/(1+cos(d*x+c)))^(1/2)/sin(d
*x+c)^3/cos(d*x+c)^(3/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 13364 vs. $2(240) = 480$.
time = 2.12, size = 13364, normalized size = 46.56

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] -1/16*(4*(12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(3/2*arctan2(sin
(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 8*(
sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - sin(3/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 3*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(s
in(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) -
4*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c) + 2*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) - 12*(sin(4*d*x + 4*c) + 2*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*
d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
))^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + s
qrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4
*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4
*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2
)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt
(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(
2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 3*(s
qrt(2)*cos(4*d*x + 4*c)^2 + 4*sqrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/
2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan
2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*s
qrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4
*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)
*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(
2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*
sqrt(2)*cos(2*d*x + 2*c) + 2*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(
2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*c
```

```
os(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x
+ 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c) + 2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x +
 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*
c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sqrt(2)*cos(2*d*x + 2*c) + s
qrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*si
n(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*ar
ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 3*(sqrt(2)*cos(4*d*x + 4*c)^2 + 4*s
qrt(2)*cos(2*d*x + 2*c)^2 + 4*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))^2 + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 4*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d
*x + 2*c) + 4*sqrt(2)*sin(2*d*x + 2*c)^2 + 4*sqrt(2)*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c)))^2 + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))^2 + 2*(2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d
*x + 4*c) + 4*(sqrt(2)*cos(4*d*x + 4*c) + 2*sqrt(2)*cos(2*d*x + 2*c) + 2*sq
rt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3
/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*cos(4*d*x + 4*
c) + 2*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c)
, cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4*c) + 2*sqrt(2)*sin(2*d*x +
2*c) + 2*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(
3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*(sqrt(2)*sin(4*d*x + 4
*c) + 2*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 4*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d...
```

Fricas [A]
time = 1.63, size = 764, normalized size = 2.66

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] [-1/16*(2*sqrt(2)*((9*A - 13*B)*cos(d*x + c)^4 + 2*(9*A - 13*B)*cos(d*x + c
)^3 + (9*A - 13*B)*cos(d*x + c)^2)*sqrt(a)*log(-(a*cos(d*x + c)^2 + 2*sqrt(
2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d
*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) + 1)) -
4*((6*A - 7*B)*cos(d*x + c)^2 + (4*A - 3*B)*cos(d*x + c) + 2*B)*sqrt((a*cos
(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + ((12*A - 19*
B)*cos(d*x + c)^4 + 2*(12*A - 19*B)*cos(d*x + c)^3 + (12*A - 19*B)*cos(d*x
+ c)^2)*sqrt(a)*log((a*cos(d*x + c)^3 - 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)
```

```
/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos
(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a^2*d*cos(d*x + c)^
4 + 2*a^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2), -1/8*(2*sqrt(2)*((9*A -
 13*B)*cos(d*x + c)^4 + 2*(9*A - 13*B)*cos(d*x + c)^3 + (9*A - 13*B)*cos(d*
x + c)^2)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*((6*A - 7*B)*cos(d*x + c)^
2 + (4*A - 3*B)*cos(d*x + c) + 2*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sqrt(cos(d*x + c))*sin(d*x + c) + ((12*A - 19*B)*cos(d*x + c)^4 + 2*(12*A
- 19*B)*cos(d*x + c)^3 + (12*A - 19*B)*cos(d*x + c)^2)*sqrt(-a)*arctan(2*sq
rt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
 c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^2*d*cos(d*x + c)^4 + 2*a
^2*d*cos(d*x + c)^3 + a^2*d*cos(d*x + c)^2)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/
2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\cos\left(c+d\,x\right)}^{7/2}\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(3/2)), x
)
```

3.555 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=317

$$-\frac{(283A-163B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{16\sqrt{2}\,a^{5/2}d}-\frac{(A-B)\cos^{\frac{3}{2}}(c+dx}{4d(a+a\sec(c+}$$

```
[Out] -1/4*(A-B)*cos(d*x+c)^(3/2)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)-1/16*(21*A-
13*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)-1/32*(283*A-16
3*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c
))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)+1/80*(157*A-8
5*B)*cos(d*x+c)^(3/2)*sin(d*x+c)/a^2/d/(a+a*sec(d*x+c))^(1/2)+1/240*(2671*A
-1495*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)-1/240*(78
7*A-475*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.71, antiderivative size = 317, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3034, 4105, 4107, 4098, 3893, 212}

$$-\frac{(283A-163B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(157A-85B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{80a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(787A-475B)\sin(c+dx)\sqrt{\cos(c+dx)}}{240a^2d\sqrt{a\sec(c+dx)+a}}+\frac{(2671A-1495B)\sin(c+dx)}{240a^2d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}-\frac{(21A-13B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -1/16*((283*A - 163*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(S
qrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(S
qrt[2]*a^(5/2)*d) - ((A - B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4*d*(a + a*S
ec[c + d*x])^(5/2)) - ((21*A - 13*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(16*a
*d*(a + a*Sec[c + d*x])^(3/2)) + ((2671*A - 1495*B)*Sin[c + d*x])/(240*a^2*
d*Sqrt[Cos[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) - ((787*A - 475*B)*Sqrt[Cos[
c + d*x]]*Sin[c + d*x])/(240*a^2*d*Sqrt[a + a*Sec[c + d*x]]) + ((157*A - 85
*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(80*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
```

```
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] && !LeQ[m, -1]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx)$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+d}\right.}{}$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B)\cos^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{16ad(a+a\sec(c+dx}$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B)\cos^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{16ad(a+a\sec(c+dx}$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B)\cos^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{16ad(a+a\sec(c+dx}$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B)\cos^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{16ad(a+a\sec(c+dx}$$

$$= -\frac{(A-B)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(21A-13B)\cos^{\frac{3}{2}}(c+dx)\,\mathrm{s}}{16ad(a+a\sec(c+dx}$$

$$= -\frac{(283A-163B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+}}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [A]
time = 2.12, size = 207, normalized size = 0.65

$$\frac{60\sqrt{2}\,(283A-163B)\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^4\left(\tfrac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)+2(3491A-1895B+5(887A-479B)\cos(c+dx)+16(52A-25B)\cos(2(c+dx))-40A\cos(3(c+dx))+40B\cos(3(c+dx))+12A\cos(4(c+dx)))\sqrt{1-\sec(c+dx)}\,\sec(c+dx)\tan(c+dx)}{480d\sqrt{-1+\cos(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5
/2),x]
```

```
[Out] (60*Sqrt[2]*(283*A - 163*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Se
c[c + d*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 2*(3491*A
 - 1895*B + 5*(887*A - 479*B)*Cos[c + d*x] + 16*(52*A - 25*B)*Cos[2*(c + d*
x)] - 40*A*Cos[3*(c + d*x)] + 40*B*Cos[3*(c + d*x)] + 12*A*Cos[4*(c + d*x)]
)*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(480*d*Sqrt[-1 + Cos[c
+ d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [A]
time = 13.86, size = 461, normalized size = 1.45

method	result
default	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2\left(4245A\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{}{1+\cos}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/480/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)^2*(4245*A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2
/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-192*A*cos(d*x+c)^5-2445*B*arctan(1/2*si
n(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*co
s(d*x+c)^2+8490*A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2
))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+512*A*cos(d*x+c)^4-4890*B*cos(d*x+c
)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x
+c)))^(1/2))-320*B*cos(d*x+c)^4+4245*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c
)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*A*sin(d*x+c)-3456*A*cos(d*x+c)^3-2445*
arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*
B*sin(d*x+c)+1920*B*cos(d*x+c)^3-5974*A*cos(d*x+c)^2+3430*B*cos(d*x+c)^2+37
68*A*cos(d*x+c)-2040*B*cos(d*x+c)+5342*A-2990*B)/sin(d*x+c)^5/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 244922 vs. $2(270) = 540$.
time = 8.04, size = 244922, normalized size = 772.62

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/480*((16*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) + 3*sin(5*d*x + 5*c)^
2*sin(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(3
/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 300*(cos(5*d*x +
5*c)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*
d*x + 5/2*c))))*cos(13/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
))^4 + 4096*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) + 3*sin(5*d*x + 5*c)
^2*sin(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(
3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 300*(cos(5*d*x +
 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
```

```
*d*x + 5/2*c))))*cos(11/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c
)))^4 + 20736*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) + 3*sin(5*d*x + 5*
c)^2*sin(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*si
n(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 300*(cos(5*d*x
 + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c))))*cos(9/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*
c)))^4 + 4096*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) + 3*sin(5*d*x + 5*
c)^2*sin(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*si
n(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 300*(cos(5*d*x
 + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5
/2*d*x + 5/2*c))))*cos(7/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*
c)))^4 + 48*sin(5/2*d*x + 5/2*c)^5 + 16*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*x +
 5/2*c) + 3*sin(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*c)^
2 + sin(5*d*x + 5*c)^2)*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x +
 5/2*c))) + 300*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan2(s
in(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sin(13/5*arctan2(sin(5/2*d*x +
 5/2*c), cos(5/2*d*x + 5/2*c)))^4 + 4096*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*x
+ 5/2*c) + 3*sin(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*c)
^2 + sin(5*d*x + 5*c)^2)*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x
+ 5/2*c))) + 300*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan2(
sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sin(11/5*arctan2(sin(5/2*d*x
+ 5/2*c), cos(5/2*d*x + 5/2*c)))^4 + 20736*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*
x + 5/2*c) + 3*sin(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*
c)^2 + sin(5*d*x + 5*c)^2)*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*
x + 5/2*c))) + 300*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sin(9/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c)))^4 + 4096*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*
x + 5/2*c) + 3*sin(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*
c)^2 + sin(5*d*x + 5*c)^2)*sin(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*
x + 5/2*c))) + 300*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan
2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))))*sin(7/5*arctan2(sin(5/2*d*x
 + 5/2*c), cos(5/2*d*x + 5/2*c)))^4 + 4*(24*cos(5*d*x + 5*c)^2*cos(5/2*d*x
+ 5/2*c)*sin(5/2*d*x + 5/2*c) + 24*cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c)^2*
sin(5/2*d*x + 5/2*c) + 24*cos(5*d*x + 5*c)*cos(5/2*d*x + 5/2*c)*sin(5/2*d*x
 + 5/2*c) + 64*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) + 3*sin(5*d*x + 5
*c)^2*sin(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*s
in(3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 300*(cos(5*d*
x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(
5/2*d*x + 5/2*c))))*cos(11/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/
2*c))) + 96*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) + 3*sin(5*d*x + 5*c)
^2*sin(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(
3/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 300*(cos(5*d*x +
 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2
*d*x + 5/2*c))))*cos(9/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c)
)) + 64*(3*cos(5*d*x + 5*c)^2*sin(5/2*d*x + 5/2*c) + 3*sin(5*d*x + 5*c)^2*s
```

```
in(5/2*d*x + 5/2*c) - 25*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(3/5*
arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) + 300*(cos(5*d*x + 5*c
)^2 + sin(5*d*x + 5*c)^2)*sin(1/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x
 + 5/2*c))))*cos(7/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) +
 (24*sin(5/2*d*x + 5/2*c)^2 - 4265)*sin(5*d*x + 5*c) - 645*(cos(5*d*x + 5*c
)^2 + sin(5*d*x + 5*c)^2)*sin(12/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*
x + 5/2*c))) - 7247*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(8/5*arcta
n2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 4803*(cos(5*d*x + 5*c)^2
+ sin(5*d*x + 5*c)^2)*sin(6/5*arctan2(sin(5/2*d*x + 5/2*c), cos(5/2*d*x + 5
/2*c))) - 872*(cos(5*d*x + 5*c)^2 + sin(5*d*x + 5*c)^2)*sin(4/5*arctan2(sin
(5/2*d*x + 5/2*c), cos(5/2*d*x + 5/2*c))) - 200*(cos(5*d*x + 5*c)^2*cos(5/2
*d*x + 5/2*c) + cos(5/2*d*x + 5/2*c)*sin(5*d*x + 5*c)^2 + cos(5*d*x + 5*c)*
cos(5/2*d*x + 5/2*c) + sin(5*d*x + 5*c)*sin(5/2...
```

Fricas [**A**]
time = 2.29, size = 572, normalized size = 1.80

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/960*(15*sqrt(2)*((283*A - 163*B)*cos(d*x + c)^3 + 3*(283*A - 163*B)*cos
(d*x + c)^2 + 3*(283*A - 163*B)*cos(d*x + c) + 283*A - 163*B)*sqrt(a)*log(-
(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c
))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^
2 + 2*cos(d*x + c) + 1)) - 4*(96*A*cos(d*x + c)^4 - 160*(A - B)*cos(d*x + c
)^3 + 32*(49*A - 25*B)*cos(d*x + c)^2 + 5*(911*A - 503*B)*cos(d*x + c) + 26
71*A - 1495*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d), 1/480*(15*sqrt(2)*((283*A - 163*B)*cos(d*x + c)^3 + 3*(28
3*A - 163*B)*cos(d*x + c)^2 + 3*(283*A - 163*B)*cos(d*x + c) + 283*A - 163*
B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))
*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*(96*A*cos(d*x + c)^4 - 160*(A - B
)*cos(d*x + c)^3 + 32*(49*A - 25*B)*cos(d*x + c)^2 + 5*(911*A - 503*B)*cos(
d*x + c) + 2671*A - 1495*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(co
s(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 +
3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(a*sec(d*x + c) + a)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^{5/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(5/2), x
)
```

3.556 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=270

$$\frac{(163A-75B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}\,a^{5/2}d}-\frac{(A-B)\sqrt{\cos(c+dx)}}{4d(a+a\sec(c+}$$

```
[Out] -1/4*(A-B)*sin(d*x+c)*cos(d*x+c)^(1/2)/d/(a+a*sec(d*x+c))^(5/2)-1/16*(17*A-
9*B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/d/(a+a*sec(d*x+c))^(3/2)+1/32*(163*A-75*
B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))
^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)-1/48*(299*A-147
*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)+1/48*(95*A-39*
B)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 270, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3034, 4105, 4107, 4098, 3893, 212}

$$\frac{(163A-75B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(95A-39B)\sin(c+dx)\sqrt{\cos(c+dx)}}{48a^2d\sqrt{a\sec(c+dx)+a}}-\frac{(299A-147B)\sin(c+dx)}{48a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{(17A-9B)\sin(c+dx)\sqrt{\cos(c+dx)}}{16ad(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)\sqrt{\cos(c+dx)}}{4d(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] ((163*A - 75*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*
Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[
2]*a^(5/2)*d) - ((A - B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*d*(a + a*Sec[c
 + d*x])^(5/2)) - ((17*A - 9*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(16*a*d*(a
 + a*Sec[c + d*x])^(3/2)) - ((299*A - 147*B)*Sin[c + d*x])/(48*a^2*d*Sqrt[C
os[c + d*x]]*Sqrt[a + a*Sec[c + d*x]]) + ((95*A - 39*B)*Sqrt[Cos[c + d*x]]*
Sin[c + d*x])/(48*a^2*d*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rule 4107

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(b*d
*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*A*m - b*B*n
 - A*b*(m + n + 1)*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[n, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.}{}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B)\sqrt{\cos(c+dx}}{16ad(a+a\sec(c+}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B)\sqrt{\cos(c+dx}}{16ad(a+a\sec(c+}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B)\sqrt{\cos(c+dx}}{16ad(a+a\sec(c+}$$

$$= -\frac{(A-B)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{4d(a+a\sec(c+dx))^{5/2}} - \frac{(17A-9B)\sqrt{\cos(c+dx}}{16ad(a+a\sec(c+}$$

$$= \frac{(163A-75B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+d}}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [A]
time = 1.73, size = 183, normalized size = 0.68

$$\frac{-12\sqrt{2}\,(163A-75B)\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^4\left(\frac{1}{2}(c+dx)\right)\sec^{\frac{5}{2}}(c+dx)\sin(c+dx)+2(-379A+195B+(-479A+255B)\cos(c+dx)+(-80A+48B)\cos(2(c+dx))+8A\cos(3(c+dx)))\sqrt{1-\sec(c+dx)}\,\sec(c+dx)\tan(c+dx)}{96d\sqrt{-1+\cos(c+dx)}\,(a(1+\sec(c+dx)))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5
/2),x]
```

```
[Out] (-12*Sqrt[2]*(163*A - 75*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Se
c[c + d*x]]]*Cos[(c + d*x)/2]^4*Sec[c + d*x]^(5/2)*Sin[c + d*x] + 2*(-379*A
 + 195*B + (-479*A + 255*B)*Cos[c + d*x] + (-80*A + 48*B)*Cos[2*(c + d*x)]
+ 8*A*Cos[3*(c + d*x)])*Sqrt[1 - Sec[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(
96*d*Sqrt[-1 + Cos[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [A]
time = 13.26, size = 439, normalized size = 1.63

method	result
default	$-\frac{\left(\sqrt{\cos}\left(dx+c\right)\right)\sqrt{\frac{a(1+\cos(dx+c))}{\cos(dx+c)}}\,(-1+\cos(dx+c))^2\left(489A\sin(dx+c)\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sqrt{-\frac{2}{1+\cos(}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x,method=_RETURNVERBOSE)

[Out] -1/96/d*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))^2*(489*A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-225*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*sin(d*x+c)*cos(d*x+c)^2+978*A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+64*A*cos(d*x+c)^4-450*B*cos(d*x+c)*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))+489*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*A*sin(d*x+c)-384*A*cos(d*x+c)^3-225*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*(-2/(1+cos(d*x+c)))^(1/2)*B*sin(d*x+c)+192*B*cos(d*x+c)^3-686*A*cos(d*x+c)^2+318*B*cos(d*x+c)^2+408*A*cos(d*x+c)-216*B*cos(d*x+c)+598*A-294*B)/a^3/sin(d*x+c)^5

<u>Maxima</u> [B] Leaf count of result is larger than twice the leaf count of optimal. 154245 vs. $2(229) = 458$.
time = 5.27, size = 154245, normalized size = 571.28

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algorithm="maxima")

[Out] 1/96*(3*(64*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1024*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2304*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1024*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 64*sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1024*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2304*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1024*sin(3/4*ar

ctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2*sin(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c))) + 64*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
c)))^3 + 4(128*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1
/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 192*cos(5/4*arctan2(sin(2
*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
 + 2*c))) + 128*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/
4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 32*cos(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 5*sin(4*d*x + 4*c) - 109*sin(2*d*x + 2*c) - 51*sin(3/2*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c))) - 85*sin(1/2*arctan2(sin(2*d*x + 2*c), c
os(2*d*x + 2*c))))*cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 1
6*(192*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2
(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 128*cos(3/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 32*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 5*sin(4*d*x + 4*c) - 109*sin(2*d*x +
2*c) - 51*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 85*sin(1/2
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))) + 204*(6*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d
*x + 2*c))) + 4*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(
1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))) + 24*(128*cos(3/4*arctan2(sin(2*d*x + 2*c), cos
(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 32*c
os(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))) + 5*sin(4*d*x + 4*c) - 109*sin(2*d*x + 2*c) -
 85*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(32*cos(1/4*arctan2(sin(2*d*x + 2*
c), cos(2*d*x + 2*c)))*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))
 + 5*sin(4*d*x + 4*c) - 109*sin(2*d*x + 2*c) - 85*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 4*(5*sin(4*d*x + 4*c) - 109*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c))) - 75*(2*(4*cos(7/4*arctan2(sin(2*d*x + 2*c
), cos(2*d*x + 2*c))) + 6*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) + 4*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/4*arct
an2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*cos(9/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + cos(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^
2 + 8*(6*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*cos(3/4*a
rctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/4*arctan2(sin(2*d*x + 2
*c), cos(2*d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)
)) + 16*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 12*(4*cos(
3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + cos(1/4*arctan2(sin(2*d*
x + 2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
 2*c))) + 36*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*co
s(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*cos(3/4*arctan2(si
n(2*d*x + 2*c), cos(2*d*x + 2*c)))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*

```
d*x + 2*c))) + cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(
4*sin(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 6*sin(5/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(3/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c))) + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*
sin(9/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sin(9/4*arctan2(sin(
2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 8*(6*sin(5/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))) + 4*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))
) + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(7/4*arctan2(s
in(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*sin(7/4*arctan2(sin(2*d*x + 2*c),
cos(2*d*x + 2*c)))^2 + 12*(4*sin(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + sin(1/4*arctan2(sin(2*d*x + 2*c), cos(...
```

Fricas [A]
time = 1.85, size = 542, normalized size = 2.01

$$\left[-\frac{3\sqrt{2}\left((163A-75B)\cos(dx+c)^3+3(163A-75B)\cos(dx+c)^2+3(163A-75B)\cos(dx+c)+163A-75B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2+2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\left(32A\cos(dx+c)^3-32(5A-3B)\cos(dx+c)^2-(503A-255B)\cos(dx+c)-299A+147B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{192\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}, -\frac{3\sqrt{2}\left((163A-75B)\cos(dx+c)^3+3(163A-75B)\cos(dx+c)^2+3(163A-75B)\cos(dx+c)+163A-75B\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-2\left(32A\cos(dx+c)^3-32(5A-3B)\cos(dx+c)^2-(503A-255B)\cos(dx+c)-299A+147B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{96\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/192*(3*sqrt(2)*((163*A - 75*B)*cos(d*x + c)^3 + 3*(163*A - 75*B)*cos(d*
x + c)^2 + 3*(163*A - 75*B)*cos(d*x + c) + 163*A - 75*B)*sqrt(a)*log(-(a*co
s(d*x + c)^2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sq
rt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2
*cos(d*x + c) + 1)) - 4*(32*A*cos(d*x + c)^3 - 32*(5*A - 3*B)*cos(d*x + c)^
2 - (503*A - 255*B)*cos(d*x + c) - 299*A + 147*B)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a
^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/96*(3*sqrt(2)*((163
*A - 75*B)*cos(d*x + c)^3 + 3*(163*A - 75*B)*cos(d*x + c)^2 + 3*(163*A - 75
*B)*cos(d*x + c) + 163*A - 75*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*c
os(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*(32
*A*cos(d*x + c)^3 - 32*(5*A - 3*B)*cos(d*x + c)^2 - (503*A - 255*B)*cos(d*x
 + c) - 299*A + 147*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
 + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3
*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(a*sec(d*x + c) + a)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(5/2), x
)
```

3.557 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=223

$$-\frac{(75A-19B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}\,a^{5/2}d}-\frac{(A-B)\sin(c}{4d\sqrt{\cos(c+dx)}\,(a+c}$$

```
[Out] -1/4*(A-B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2)-1/16*(13*A-
5*B)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)-1/32*(75*A-19*B
)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^
(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)+1/16*(49*A-9*B)*
sin(d*x+c)/a^2/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3034, 4105, 4098, 3893, 212}

$$-\frac{(75A-19B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{(49A-9B)\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\sqrt{a\sec(c+dx)+a}}-\frac{(13A-5B)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5/2),x]
```

```
[Out] -1/16*((75*A - 19*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqr
t[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(Sqr
t[2]*a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]*(a + a*Sec
[c + d*x])^(5/2)) - ((13*A - 5*B)*Sin[c + d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*
(a + a*Sec[c + d*x])^(3/2)) + ((49*A - 9*B)*Sin[c + d*x])/(16*a^2*d*Sqrt[Co
s[c + d*x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
```

```
tegerQ[n])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+a\sec(c+$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{s}\right.}{}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B}{16ad\sqrt{\cos(c+dx)}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B}{16ad\sqrt{\cos(c+dx)}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}} - \frac{(13A-5B}{16ad\sqrt{\cos(c+dx)}}$$

$$= -\frac{(75A-19B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [A]
time = 2.61, size = 228, normalized size = 1.02

$$\frac{8\sqrt{2}\,(75A-19B)\mathrm{ArcTan}\left(\frac{\sqrt{2}\,\sqrt{\sec(c+dx)}}{\sqrt{1-\sec(c+dx)}}\right)\cos^5\left(\frac{1}{2}(c+dx)\right)(B+A\cos(c+dx))\sec^{\frac{5}{2}}(c+dx)\sin\left(\frac{1}{2}(c+dx)\right)+(85A^2+117AB-18B^2+2(73A^2+76AB-13B^2)\cos(c+dx)+A(85A+19B)\cos(2(c+dx))+16A^2\cos(3(c+dx)))\sqrt{1-\sec(c+dx)}\,\sec(c+dx)\tan(c+dx)}{32d\sqrt{-1+\cos(c+dx)}\,(B+A\cos(c+dx))(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + a*Sec[c + d*x])^(5
/2),x]
```

```
[Out] (8*Sqrt[2]*(75*A - 19*B)*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c
 + d*x]]]*Cos[(c + d*x)/2]^5*(B + A*Cos[c + d*x])*Sec[c + d*x]^(5/2)*Sin[(c
 + d*x)/2] + (85*A^2 + 117*A*B - 18*B^2 + 2*(73*A^2 + 76*A*B - 13*B^2)*Cos[
c + d*x] + A*(85*A + 19*B)*Cos[2*(c + d*x)] + 16*A^2*Cos[3*(c + d*x)])*Sqrt
[1 - Sec[c + d*x]]*Sec[c + d*x]*Tan[c + d*x])/(32*d*Sqrt[-1 + Cos[c + d*x]]
*(B + A*Cos[c + d*x])*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [A]
time = 13.00, size = 365, normalized size = 1.64

method	result

default	$-\frac{(-1+\cos(dx+c))^2\left(32A(\cos^3(dx+c))\sqrt{-\frac{2}{1+\cos(dx+c)}}+75A\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sin(dx+c)\cos(dx+c)+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/16/d*(-1+cos(d*x+c))^2*(32*A*cos(d*x+c)^3*(-2/(1+cos(d*x+c)))^(1/2)+75*A
*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+c)+53*
A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-19*B*arctan(1/2*sin(d*x+c)*(-2/(1+
cos(d*x+c)))^(1/2))*cos(d*x+c)*sin(d*x+c)-13*B*(-2/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)^2+75*A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))
-36*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-19*B*sin(d*x+c)*arctan(1/2*sin(d
*x+c)*(-2/(1+cos(d*x+c)))^(1/2))+4*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-4
9*A*(-2/(1+cos(d*x+c)))^(1/2)+9*B*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^(1/
2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/a^3/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*
x+c)^5
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 261506 vs. $2(188) = 376$.
time = 3.74, size = 261506, normalized size = 1172.67

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] -1/32*((576*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin
(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x +
 5/2*c)^6 + 14400*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(3/2
*d*x + 3/2*c)^6 + 187500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)
^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
 + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^6 + 576*(75
*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*
c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/
2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)^6 + 518
4*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x +
```

1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*s
in(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^6
+ 262500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d
*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*
sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^4*sin(1/2*d*x + 1/2*c)^2 +
77700*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x
+ 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin
(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2*sin(1/2*d*x + 1/2*c)^4 + 270
0*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/
2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2
*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^6 - 2304*sin(1/2*d*x + 1/2*c)^7 +
96*(86*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2
- 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*
c) + 10275*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2
*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 -
2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c) - 8768*cos(1/2*d*x + 1/2*
c)*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x + 5/2*c)^5 + 88800*(75*(log(cos(1/2*d*
x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1
))*cos(1/2*d*x + 1/2*c) - 64*cos(1/2*d*x + 1/2*c)*sin(1/2*d*x + 1/2*c))*cos
(3/2*d*x + 3/2*c)^5 + 96*(62*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + s
in(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*
c))*sin(3/2*d*x + 3/2*c) + 2625*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(
1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 22
40*sin(1/2*d*x + 1/2*c)^2 - 1996)*sin(5/2*d*x + 5/2*c)^5 + 864*(1275*(log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1
) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1
/2*c) + 1))*sin(1/2*d*x + 1/2*c) - 1088*sin(1/2*d*x + 1/2*c)^2 - 920)*sin(3
/2*d*x + 3/2*c)^5 - 16*(4144*cos(1/2*d*x + 1/2*c)^2 + 675)*sin(1/2*d*x + 1/
2*c)^5 + 16*((75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*si
n(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2
*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(5/2*d*x
+ 5/2*c)^2 + 25*(75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2
*sin(1/2*d*x + 1/2*c) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x +
1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(3/2*d
*x + 3/2*c)^2 + 7500*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(1/2*d*x + 1/2*c)^2 + (75*log(cos
(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
- 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(5/2*d*x + 5/2*c)^2 + 9*(75*log(c
os(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1

```
) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x
+ 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*sin(3/2*d*x + 3/2*c)^2 + 300*(log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x +
1/2*c) + 1))*sin(1/2*d*x + 1/2*c)^2 - 256*sin(1/2*d*x + 1/2*c)^3 + 10*((75*
log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c
) + 1) - 75*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2
*d*x + 1/2*c) + 1) - 64*sin(1/2*d*x + 1/2*c))*cos(3/2*d*x + 3/2*c) + 150*(l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c...
```

Fricas [A]
time = 1.66, size = 504, normalized size = 2.26

$$\left[-\frac{\sqrt{2}\left((75A-19B)\cos(dx+c)^3+3(75A-19B)\cos(dx+c)^2+3(75A-19B)\cos(dx+c)+75A-19B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\left(32A\cos(dx+c)^2+(85A-13B)\cos(dx+c)+49A-9B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}, \frac{\sqrt{2}\left((75A-19B)\cos(dx+c)^3+3(75A-19B)\cos(dx+c)^2+3(75A-19B)\cos(dx+c)+75A-19B\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)+2\left(32A\cos(dx+c)^2+(85A-13B)\cos(dx+c)+49A-9B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((75*A - 19*B)*cos(d*x + c)^3 + 3*(75*A - 19*B)*cos(d*x + c
)^2 + 3*(75*A - 19*B)*cos(d*x + c) + 75*A - 19*B)*sqrt(a)*log(-(a*cos(d*x +
 c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(
d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*
x + c) + 1)) - 4*(32*A*cos(d*x + c)^2 + (85*A - 13*B)*cos(d*x + c) + 49*A -
 9*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) +
 a^3*d), 1/32*(sqrt(2)*((75*A - 19*B)*cos(d*x + c)^3 + 3*(75*A - 19*B)*cos(
d*x + c)^2 + 3*(75*A - 19*B)*cos(d*x + c) + 75*A - 19*B)*sqrt(-a)*arctan(sq
rt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(
a*sin(d*x + c))) + 2*(32*A*cos(d*x + c)^2 + (85*A - 13*B)*cos(d*x + c) + 49
*A - 9*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*
x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x +
c) + a^3*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(a*sec(d*x + c) + a)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{a}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + a/cos(c + d*x))^(5/2), x
)
```

3.558 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=223

$$\frac{(19A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{16\sqrt{2}\,a^{5/2}d}+\frac{(A-B)\sin(c+}{4d\sqrt{\cos(c+dx)}\,(a+a\,\mathrm{s}}$$

```
[Out] 1/4*(A-B)*sin(d*x+c)/d/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2)+1/16*(5*A+3*
B)*sin(d*x+c)/a/d/(a+a*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2)+1/32*(19*A+5*B)*a
rctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/
2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)-1/16*(9*A-B)*sin(d*
x+c)/a^2/d/cos(d*x+c)^(1/2)/(a+a*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.171$, Rules used = {3034, 4104, 4105, 4098, 3893, 212}

$$\frac{(19A+5B)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\,\tanh^{-1}\left(\frac{\sqrt{a}\,\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\,\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}-\frac{(9A-B)\sin(c+dx)}{16a^2d\sqrt{\cos(c+dx)}\,\sqrt{a\sec(c+dx)+a}}+\frac{(5A+3B)\sin(c+dx)}{16ad\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] ((19*A + 5*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sq
rt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]
*a^(5/2)*d) + ((A - B)*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]*(a + a*Sec[c +
 d*x])^(5/2)) + ((5*A + 3*B)*Sin[c + d*x])/(16*a*d*Sqrt[Cos[c + d*x]]*(a +
a*Sec[c + d*x])^(3/2)) - ((9*A - B)*Sin[c + d*x])/(16*a^2*d*Sqrt[Cos[c + d*
x]]*Sqrt[a + a*Sec[c + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
```

```
tegerQ[n])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4098

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[(a*A*m
 - b*B*n)/(b*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1), x],
 x] /; FreeQ[{a, b, d, e, f, A, B, m, n}, x] && NeQ[A*b - a*B, 0] && EqQ[a^
2 - b^2, 0] && EqQ[m + n + 1, 0] &&  !LeQ[m, -1]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4105

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] - Dist[1/(a^2*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc
[e + f*x])^n*Simp[b*B*n - a*A*(2*m + n + 1) + (A*b - a*B)*(m + n + 1)*Csc[e
 + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B,
 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] &&  !GtQ[n, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec}{(a+a\sec(c+dx))}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\right.}{}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B}{16ad\sqrt{\cos(c+dx)}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B}{16ad\sqrt{\cos(c+dx)}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\sqrt{\cos(c+dx)}\,(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B}{16ad\sqrt{\cos(c+dx)}}$$

$$= \frac{(19A+5B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+}}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [A]
time = 1.04, size = 108, normalized size = 0.48

$$\frac{\sec\left(\frac{1}{2}(c+dx)\right)\left(8(19A+5B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^4\left(\frac{1}{2}(c+dx)\right)+4(-9A+B+(-13A+5B)\cos(c+dx))\sin\left(\frac{1}{2}(c+dx)\right)\right)}{64ad\cos^{\frac{3}{2}}(c+dx)(a(1+\sec(c+dx)))^{3/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + a*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] (Sec[(c + d*x)/2]*(8*(19*A + 5*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2
]^4 + 4*(-9*A + B + (-13*A + 5*B)*Cos[c + d*x])*Sin[(c + d*x)/2]))/(64*a*d*
Cos[c + d*x]^(3/2)*(a*(1 + Sec[c + d*x]))^(3/2))
```

Maple [A]
time = 13.57, size = 339, normalized size = 1.52

method	result

default	$\frac{(-1+\cos(dx+c))^2\left(13A\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\cos^2(dx+c))+19A\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sin(dx+c)\cos(dx+c)-5B\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/16/d*(-1+cos(d*x+c))^2*(13*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+19*A*
arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+c)-5*B*
(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+5*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos
(d*x+c)))^(1/2))*cos(d*x+c)*sin(d*x+c)-4*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*
x+c)+19*A*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))+4*B*(
-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+5*B*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-
2/(1+cos(d*x+c)))^(1/2))-9*A*(-2/(1+cos(d*x+c)))^(1/2)+B*(-2/(1+cos(d*x+c))
)^(1/2))*cos(d*x+c)^(1/2)*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/a^3/sin(d*x+c
)^5/(-2/(1+cos(d*x+c)))^(1/2)
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 5924 vs. $2(188) = 376$.
time = 1.51, size = 5924, normalized size = 26.57

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*((19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*
d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2
*sin(1/2*d*x + 1/2*c) + 1))*cos(4*d*x + 4*c)^2 + 304*(log(cos(1/2*d*x + 1/2
*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*
d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(
3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 +
 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1
/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c)^2 + 304*(log(cos(1/
2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - l
og(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c)
 + 1))*cos(d*x + c)^2 + 19*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*
c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d
*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(4*d*x + 4*c)^2 + 304*(log(
cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) +
```

1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c)^2 + 684*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c)^2 + 304*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c)^2 + 2*(76*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(3*d*x + 3*c) + 114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 26*sin(7/2*d*x + 7/2*c) - 10*sin(5/2*d*x + 5/2*c) + 10*sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(4*d*x + 4*c) + 104*(2*sin(3*d*x + 3*c) + 3*sin(2*d*x + 2*c) + 2*sin(d*x + c))*cos(7/2*d*x + 7/2*c) + 8*(114*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(2*d*x + 2*c) + 76*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) - 10*sin(5/2*d*x + 5/2*c) + 10*sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(3*d*x + 3*c) + 40*(3*sin(2*d*x + 2*c) + 2*sin(d*x + c))*cos(5/2*d*x + 5/2*c) + 12*(76*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*cos(d*x + c) + 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 10*sin(3/2*d*x + 3/2*c) + 26*sin(1/2*d*x + 1/2*c))*cos(2*d*x + 2*c) + 8*(19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - 19*log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1) + 26*sin(1/2*d*x + 1/2*c))*cos(d*x + c) + 4*(38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(3*d*x + 3*c) + 57*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + 1))*sin(d*x + c)

```
+ 13*cos(7/2*d*x + 7/2*c) + 5*cos(5/2*d*x + 5/2*c) - 5*cos(3/2*d*x + 3/2*c)
 - 13*cos(1/2*d*x + 1/2*c))*sin(4*d*x + 4*c) - 52*(4*cos(3*d*x + 3*c) + 6*c
os(2*d*x + 2*c) + 4*cos(d*x + c) + 1)*sin(7/2*d*x + 7/2*c) + 16*(57*(log(co
s(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1)
 - log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/
2*c) + 1))*sin(2*d*x + 2*c) + 38*(log(cos(1/2*d*x + 1/2*c)^2 + sin(1/2*d*x
+ 1/2*c)^2 + 2*sin(1/2*d*x + 1/2*c) + 1) - log(cos(1/2*d*x + 1/2*c)^2 + sin
(1/2*d*x + 1/2*c)^2 - 2*sin(1/2*d*x + 1/2*c) + ...
```

Fricas [A]
time = 2.25, size = 482, normalized size = 2.16

$$\left[\frac{\sqrt{2}\left((19\,A+5\,B)\cos\left(dx+c\right)^3+3\,(19\,A+5\,B)\cos\left(dx+c\right)^2+3\,(19\,A+5\,B)\cos\left(dx+c\right)+19\,A+5\,B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin(dx+c)-2\,a\cos(dx+c)-3\,a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)-4\left((13\,A-5\,B)\cos\left(dx+c\right)+9\,A-B\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{64\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)},\ -\frac{\sqrt{2}\left((19\,A+5\,B)\cos\left(dx+c\right)^3+3\,(19\,A+5\,B)\cos\left(dx+c\right)^2+3\,(19\,A+5\,B)\cos\left(dx+c\right)+19\,A+5\,B\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}}{a\sin(dx+c)}\right)+2\left((13\,A-5\,B)\cos\left(dx+c\right)+9\,A-B\right)\sqrt{\frac{a\cos\left(dx+c\right)+a}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{32\left(a^3d\cos\left(dx+c\right)^3+3\,a^3d\cos\left(dx+c\right)^2+3\,a^3d\cos\left(dx+c\right)+a^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((19*A + 5*B)*cos(d*x + c)^3 + 3*(19*A + 5*B)*cos(d*x + c)^2
 + 3*(19*A + 5*B)*cos(d*x + c) + 19*A + 5*B)*sqrt(a)*log(-(a*cos(d*x + c)^2
 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
 c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c
) + 1)) - 4*((13*A - 5*B)*cos(d*x + c) + 9*A - B)*sqrt((a*cos(d*x + c) + a)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a
^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((19*A
+ 5*B)*cos(d*x + c)^3 + 3*(19*A + 5*B)*cos(d*x + c)^2 + 3*(19*A + 5*B)*cos(
d*x + c) + 19*A + 5*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c
) + a)/cos(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) + 2*((13*A - 5*B)
*cos(d*x + c) + 9*A - B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a
^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6439 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+a*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c
))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + a/cos(c + d*x))^(5/2)), x
)
```

3.559 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=176

$$\frac{(5A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\sin(c+dx)}{\sqrt{2}\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{16\sqrt{2}a^{5/2}d}-\frac{(A-B)\sin(c+dx}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c}$$

```
[Out] -1/4*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2)+1/16*(5*A+3
*B)*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+1/32*(5*A+3*B)*a
rctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/
2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 176, normalized size of antiderivative = 1.00, number of steps used = 5, number of rules used = 5, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.143$, Rules used = {3034, 4097, 3895, 3893, 212}

$$\frac{(5A+3B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(5A+3B)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}-\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] ((5*A + 3*B)*ArcTanh[(Sqrt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqr
t[a + a*Sec[c + d*x]])]*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*
a^(5/2)*d) - ((A - B)*Sin[c + d*x])/(4*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c +
d*x])^(5/2)) + ((5*A + 3*B)*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a
*Sec[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 3895

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_), x_Symbol] :> Simp[b*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^(n - 1)/(a*f*(2*m + 1))), x] + Dist[d*((m + 1)/(b*(2*m + 1))),
Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1), x], x] /; FreeQ[
{a, b, d, e, f, m, n}, x] && EqQ[a^2 - b^2, 0] && EqQ[m + n, 0] && LtQ[m, -
2^(-1)] && IntegerQ[2*m]
```

Rule 4097

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-(A*b
- a*B))*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(b*f*(2*m +
 1))), x] + Dist[(a*A*m + b*B*(m + 1))/(a^2*(2*m + 1)), Int[(a + b*Csc[e +
f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b, d, e, f, A, B, n},
 x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && EqQ[m + n + 1, 0] && LeQ[m
, -1]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+}{(a+a\sec(c+dx))^{5/2}}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{\left((5A+3B)\sqrt{\cos(c}\right.}{}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\,\mathrm{s}}{16ad\cos^{\frac{3}{2}}(c+dx)(a}$$

$$= -\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(5A+3B)\,\mathrm{s}}{16ad\cos^{\frac{3}{2}}(c+dx)(a}$$

$$= \frac{(5A+3B)\tanh^{-1}\left(\frac{\sqrt{a}\,\sqrt{\sec(c+dx)}\,\sin(c+dx)}{\sqrt{2}\,\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}}{16\sqrt{2}\,a^{5/2}d}$$

Mathematica [A]
time = 0.89, size = 108, normalized size = 0.61

$$\frac{\cos^2\left(\frac{1}{2}(c+dx)\right)\left((5A+3B)\tanh^{-1}\left(\sin\left(\frac{1}{2}(c+dx)\right)\right)\cos^3\left(\frac{1}{2}(c+dx)\right)+\frac{1}{2}(A+7B+(5A+3B)\cos(c+dx))\tan\left(\frac{1}{2}(c+dx)\right)\right)}{4d\cos^{\frac{5}{2}}(c+dx)(a(1+\sec(c+dx)))^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] (Cos[(c + d*x)/2]^2*((5*A + 3*B)*ArcTanh[Sin[(c + d*x)/2]]*Cos[(c + d*x)/2]
^3 + ((A + 7*B + (5*A + 3*B)*Cos[c + d*x])*Tan[(c + d*x)/2])/2))/(4*d*Cos[c
 + d*x]^(5/2)*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 338 vs. 2(147) = 294.
time = 13.45, size = 339, normalized size = 1.93

method	result
default	$\frac{\left(\sqrt{\cos}\,(dx+c)\right)(-1+\cos(dx+c))^2\left(5A\arctan\left(\frac{\sin(dx+c)\sqrt{-\frac{2}{1+\cos(dx+c)}}}{2}\right)\sin(dx+c)\cos(dx+c)-5A\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(\mathrm{co}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/16/d*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))^2*(5*A*arctan(1/2*sin(d*x+c)*(-2/(1
+cos(d*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+c)-5*A*(-2/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)^2+3*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)*
sin(d*x+c)-3*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+5*A*sin(d*x+c)*arctan
(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))+4*A*(-2/(1+cos(d*x+c)))^(1/2)*co
s(d*x+c)+3*B*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-4*
B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+A*(-2/(1+cos(d*x+c)))^(1/2)+7*B*(-2/
(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d*x+c))/cos(d*x+c))^(1/2)/sin(d*x+c)^5/(-2
/(1+cos(d*x+c)))^(1/2)/a^3
```

Maxima [B] Leaf count of result is larger than twice the leaf count of optimal. 5356 vs. 2(147) = 294.
time = 1.29, size = 5356, normalized size = 30.43

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*((4*(3*sin(3/2*d*x + 3/2*c) + 5*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) - 3*sin(5/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
)))*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 40*(2*si
n(3*d*x + 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*
c))) + 2*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(
7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(2*sin(3*d*x
+ 3*c) + 3*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2
*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(5/3*arct
an2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 24*(3*sin(3/2*d*x + 3/2*
c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*cos(4/
3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 16*(3*sin(3/2*d*x
+ 3/2*c) - 5*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*
cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 5*(16*cos(3*
d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/
2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 +
12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*
x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c
))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + 8
*(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x
 + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)
))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3*sin(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos
(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), co
s(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2
 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x +
3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^
2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3
/2*c)))^2 + 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))
+ 1) - 5*(16*cos(3*d*x + 3*c)^2 + 2*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2
(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d
*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(8/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + cos(8/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*
d*x + 3/2*c)))^2 + 12*(4*cos(3*d*x + 3*c) + 4*cos(2/3*arctan2(sin(3/2*d*x +
 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c),
cos(3/2*d*x + 3/2*c))) + 36*cos(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d
```

```
*x + 3/2*c)))^2 + 8*(4*cos(3*d*x + 3*c) + 1)*cos(2/3*arctan2(sin(3/2*d*x +
3/2*c), cos(3/2*d*x + 3/2*c))) + 16*cos(2/3*arctan2(sin(3/2*d*x + 3/2*c), c
os(3/2*d*x + 3/2*c)))^2 + 16*sin(3*d*x + 3*c)^2 + 4*(2*sin(3*d*x + 3*c) + 3
*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 2*sin(2/3*a
rctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + sin(8/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^2 + 48*(sin(3*d*x + 3*c) + sin(2/3*arctan2(sin(3
/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(4/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c))) + 36*sin(4/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3
/2*d*x + 3/2*c)))^2 + 32*sin(3*d*x + 3*c)*sin(2/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 16*sin(2/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c)))^2 + 8*cos(3*d*x + 3*c) + 1)*log(cos(1/3*arctan2(sin(3/2*
d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))^2 + sin(1/3*arctan2(sin(3/2*d*x + 3/2*
c), cos(3/2*d*x + 3/2*c)))^2 - 2*sin(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(
3/2*d*x + 3/2*c))) + 1) - 48*cos(3/2*d*x + 3/2*c)*sin(3*d*x + 3*c) + 80*cos
(1/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c)))*sin(3*d*x + 3*c)
+ 48*cos(3*d*x + 3*c)*sin(3/2*d*x + 3/2*c) - 4*(3*cos(3/2*d*x + 3/2*c) + 5*
cos(7/3*arctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 3*cos(5/3*ar
ctan2(sin(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) - 5*cos(1/3*arctan2(sin(
3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))))*sin(8/3*arctan2(sin(3/2*d*x + 3/2
*c), cos(3/2*d*x + 3/2*c))) + 20*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(si
n(3/2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(2/3*arctan2(sin(3/2*d*x
+ 3/2*c), cos(3/2*d*x + 3/2*c))) + 1)*sin(7/3*arctan2(sin(3/2*d*x + 3/2*c),
 cos(3/2*d*x + 3/2*c))) - 12*(4*cos(3*d*x + 3*c) + 6*cos(4/3*arctan2(sin(3/
2*d*x + 3/2*c), cos(3/2*d*x + 3/2*c))) + 4*cos(...
```

Fricas [A]
time = 2.35, size = 478, normalized size = 2.72

$$\left[\frac{\sqrt{2}\left((5A+3B)\cos(dx+c)^3+3(5A+3B)\cos(dx+c)^2+3(5A+3B)\cos(dx+c)+5A+3B\right)\sqrt{a}\log\left(-\frac{a\cos(dx+c)^2-2\sqrt{2}\sqrt{a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-2a\cos(dx+c)-3a}{\cos(dx+c)^2+2\cos(dx+c)+1}\right)+4\left((5A+3B)\cos(dx+c)+A+7B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{64\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}, -\frac{\sqrt{2}\left((5A+3B)\cos(dx+c)^3+3(5A+3B)\cos(dx+c)^2+3(5A+3B)\cos(dx+c)+5A+3B\right)\sqrt{-a}\arctan\left(\frac{\sqrt{2}\sqrt{-a}\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}}{a\sin(dx+c)}\right)-2\left((5A+3B)\cos(dx+c)+A+7B\right)\sqrt{\frac{a\cos(dx+c)+a}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)}{32\left(a^3d\cos(dx+c)^3+3a^3d\cos(dx+c)^2+3a^3d\cos(dx+c)+a^3d\right)}\right]$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [1/64*(sqrt(2)*((5*A + 3*B)*cos(d*x + c)^3 + 3*(5*A + 3*B)*cos(d*x + c)^2 +
 3*(5*A + 3*B)*cos(d*x + c) + 5*A + 3*B)*sqrt(a)*log(-(a*cos(d*x + c)^2 - 2
*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))
*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x + c) +
1)) + 4*((5*A + 3*B)*cos(d*x + c) + A + 7*B)*sqrt((a*cos(d*x + c) + a)/cos(
d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*
cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d), -1/32*(sqrt(2)*((5*A + 3*B)
*cos(d*x + c)^3 + 3*(5*A + 3*B)*cos(d*x + c)^2 + 3*(5*A + 3*B)*cos(d*x + c)
 + 5*A + 3*B)*sqrt(-a)*arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))/(a*sin(d*x + c))) - 2*((5*A + 3*B)*cos(d*x +
```

```
 c) + A + 7*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*s
in(d*x + c))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d
*x + c) + a^3*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/
2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^{3/2}\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + a/cos(c + d*x))^(5/2)), x
)
```

3.560 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=234

$$\frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d}+\frac{(3A-43B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec(c+dx)}\,\mathrm{s}}{\sqrt{2}\sqrt{a+a\sec(c-}}\right.}{16\sqrt{2}\,a}$$

```
[Out] 1/4*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2)+1/16*(3*A-11
*B)*sin(d*x+c)/a/d/cos(d*x+c)^(3/2)/(a+a*sec(d*x+c))^(3/2)+2*B*arcsinh(a^(1
/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^
(5/2)/d+1/32*(3*A-43*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(1/2)*2^(
1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5/2)/d*2^
(1/2)
```

Rubi [A]
time = 0.47, antiderivative size = 234, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.200, Rules used = {3034, 4104, 4108, 3893, 212, 3886, 221}

$$\frac{(3A-43B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}\,a^{5/2}d}+\frac{2B\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}+\frac{(3A-11B)\sin(c+dx)}{16ad\cos^{\frac{3}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*B*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[Cos[c +
d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) + ((3*A - 43*B)*ArcTanh[(Sqrt[a]*Sqrt
[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*Sqrt[Cos[c
 + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Sin[c + d*x]
)/(4*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/2)) + ((3*A - 11*B)*Sin[c
 + d*x])/(16*a*d*Cos[c + d*x]^(3/2)*(a + a*Sec[c + d*x])^(3/2))
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+}{(a+a\sec(c+dx))^{5/2}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c}{}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(3A-11B)\sin(}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(3A-11B)\sin(}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(3A-11B)\sin(}{16ad\cos^{\frac{3}{2}}(c+dx)(a+a}$$

$$= \frac{2B\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}}{a^{5/2}d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 965 vs. 2(234) = 468.

time = 6.17, size = 965, normalized size = 4.12

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] -1/4*(B*Sin[c + d*x])/(d*Cos[c + d*x]^(9/2)*(a*(1 + Sec[c + d*x]))^(5/2)) -
 (A*Sin[c + d*x])/(4*d*Cos[c + d*x]^(7/2)*(a*(1 + Sec[c + d*x]))^(5/2)) + (
3*B*(1 + Sec[c + d*x])*Sin[c + d*x])/(16*d*Cos[c + d*x]^(9/2)*(a*(1 + Sec[c
 + d*x]))^(5/2)) - (A*(1 + Sec[c + d*x])*Sin[c + d*x])/(16*d*Cos[c + d*x]^(
7/2)*(a*(1 + Sec[c + d*x]))^(5/2)) - (3*B*(1 + Sec[c + d*x])^2*Sin[c + d*x]
)/(16*d*Cos[c + d*x]^(7/2)*(a*(1 + Sec[c + d*x]))^(5/2)) + (A*(1 + Sec[c +
d*x])^2*Sin[c + d*x])/(16*d*Cos[c + d*x]^(5/2)*(a*(1 + Sec[c + d*x]))^(5/2)
) + (7*B*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Cos[c + d*x]^(5/2)*(a*(1
+ Sec[c + d*x]))^(5/2)) + (3*A*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Cos
```

```
[c + d*x]^(3/2)*(a*(1 + Sec[c + d*x]))^(5/2)) - (11*B*(1 + Sec[c + d*x])^2*
Sin[c + d*x])/(16*d*Cos[c + d*x]^(3/2)*(a*(1 + Sec[c + d*x]))^(5/2)) + (3*A
*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(1 +
Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c +
d*x]))^(5/2)) - (11*B*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sec
[c + d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Sqrt[1 - Sec[c + d
*x]]*(a*(1 + Sec[c + d*x]))^(5/2)) + (3*A*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[C
os[c + d*x]]*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Sq
rt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)) - (43*B*ArcSin[Sqrt[Sec[
c + d*x]]]*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c
 + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)) - (3*A*
ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Cos[c + d*
x]]*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*Sqrt[2]*d*Sqr
t[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)) + (43*B*ArcTan[(Sqrt[2]*S
qrt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^
(3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*Sqrt[2]*d*Sqrt[1 - Sec[c + d*x
]]*(a*(1 + Sec[c + d*x]))^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 539 vs. $2(195) = 390$.

time = 14.40, size = 540, normalized size = 2.31

method	result
default	$-\frac{\left(\sqrt{\cos}\,(dx+c)\right)(-1+\cos(dx+c))^2\left(-16B\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(1+\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sqrt{2}\,\cos(dx+c)\sin(d\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/16/d*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))^2*(-16*B*arctan(1/4*(-2/(1+cos(d*x
+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/2)*cos(d*x+c)*sin(d*x+c
)-16*B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+
c))*2^(1/2))*sin(d*x+c)*cos(d*x+c)+3*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)
^2-3*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x+c)*cos(d*x+
c)-11*B*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2-16*B*arctan(1/4*(-2/(1+cos(d
*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1/2)*sin(d*x+c)-16*B*ar
ctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))*2^(1
/2)*sin(d*x+c)+43*B*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*
x+c)*sin(d*x+c)+4*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)-3*A*sin(d*x+c)*arc
tan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-4*B*(-2/(1+cos(d*x+c)))^(1/2)
*cos(d*x+c)+43*B*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2)
```

```
)-7*A*(-2/(1+cos(d*x+c)))^(1/2)+15*B*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1+cos(d
*x+c))/cos(d*x+c))^(1/2)/(-2/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^5/a^3
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [**A**]
time = 2.40, size = 720, normalized size = 3.08

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((3*A - 43*B)*cos(d*x + c)^3 + 3*(3*A - 43*B)*cos(d*x + c)^
2 + 3*(3*A - 43*B)*cos(d*x + c) + 3*A - 43*B)*sqrt(a)*log(-(a*cos(d*x + c)^
2 + 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x
+ c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(d*x + c)^2 + 2*cos(d*x +
c) + 1)) - 4*((3*A - 11*B)*cos(d*x + c) + 7*A - 15*B)*sqrt((a*cos(d*x + c)
+ a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 32*(B*cos(d*x + c)^3 +
 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c) + B)*sqrt(a)*log((a*cos(d*x + c)^3 -
 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*(cos(d*x + c) - 2)*sqrt(
cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x + c)^2 + 8*a)/(cos(d*x + c)^3 + co
s(d*x + c)^2)))/(a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*co
s(d*x + c) + a^3*d), -1/32*(sqrt(2)*((3*A - 43*B)*cos(d*x + c)^3 + 3*(3*A -
 43*B)*cos(d*x + c)^2 + 3*(3*A - 43*B)*cos(d*x + c) + 3*A - 43*B)*sqrt(-a)*
arctan(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*
x + c))/(a*sin(d*x + c))) - 2*((3*A - 11*B)*cos(d*x + c) + 7*A - 15*B)*sqrt
((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 32*(B
*cos(d*x + c)^3 + 3*B*cos(d*x + c)^2 + 3*B*cos(d*x + c) + B)*sqrt(-a)*arcta
n(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^3*d*cos(d*x + c)^3
 + 3*a^3*d*cos(d*x + c)^2 + 3*a^3*d*cos(d*x + c) + a^3*d)]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+a*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\cos\left(c+d\,x\right)}^{5/2}\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + a/cos(c + d*x))^(5/2)), x
)
```

3.561 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=286

$$\frac{(2A-5B)\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}}{a^{5/2}d}-\frac{(43A-115B)\tanh^{-1}\left(\frac{\sqrt{a}\sqrt{\sec}}{\sqrt{2}\sqrt{a}}\right.}{}$$

```
[Out] 1/4*(A-B)*sin(d*x+c)/d/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2)+1/16*(7*A-15
*B)*sin(d*x+c)/a/d/cos(d*x+c)^(5/2)/(a+a*sec(d*x+c))^(3/2)+(2*A-5*B)*arcsin
h(a^(1/2)*tan(d*x+c)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1
/2)/a^(5/2)/d-1/32*(43*A-115*B)*arctanh(1/2*sin(d*x+c)*a^(1/2)*sec(d*x+c)^(
1/2)*2^(1/2)/(a+a*sec(d*x+c))^(1/2))*cos(d*x+c)^(1/2)*sec(d*x+c)^(1/2)/a^(5
/2)/d*2^(1/2)-1/16*(11*A-35*B)*sin(d*x+c)/a^2/d/cos(d*x+c)^(3/2)/(a+a*sec(d
*x+c))^(1/2)
```

Rubi [A]

time = 0.61, antiderivative size = 286, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3034, 4104, 4106, 4108, 3893, 212, 3886, 221}

$$-\frac{(43A-115B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\tanh^{-1}\left(\frac{\sqrt{a}\sin(c+dx)\sqrt{\sec(c+dx)}}{\sqrt{2}\sqrt{a\sec(c+dx)+a}}\right)}{16\sqrt{2}a^{5/2}d}+\frac{(2A-5B)\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\sinh^{-1}\left(\frac{\sqrt{a}\tan(c+dx)}{\sqrt{a\sec(c+dx)+a}}\right)}{a^{5/2}d}-\frac{(11A-35B)\sin(c+dx)}{16a^2d\cos^{\frac{3}{2}}(c+dx)\sqrt{a\sec(c+dx)+a}}+\frac{(7A-15B)\sin(c+dx)}{16ad\cos^{\frac{5}{2}}(c+dx)(a\sec(c+dx)+a)^{3/2}}+\frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a\sec(c+dx)+a)^{5/2}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)),x]
```

```
[Out] ((2*A - 5*B)*ArcSinh[(Sqrt[a]*Tan[c + d*x])/Sqrt[a + a*Sec[c + d*x]]]*Sqrt[
Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(a^(5/2)*d) - ((43*A - 115*B)*ArcTanh[(Sq
rt[a]*Sqrt[Sec[c + d*x]]*Sin[c + d*x])/(Sqrt[2]*Sqrt[a + a*Sec[c + d*x]])]*
Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(16*Sqrt[2]*a^(5/2)*d) + ((A - B)*Si
n[c + d*x])/(4*d*Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/2)) + ((7*A - 1
5*B)*Sin[c + d*x])/(16*a*d*Cos[c + d*x]^(5/2)*(a + a*Sec[c + d*x])^(3/2)) -
 ((11*A - 35*B)*Sin[c + d*x])/(16*a^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + a*Sec[c
 + d*x]])
```

Rule 212

```
Int[((a_) + (b_.)*(x_)^2)^(-1), x_Symbol] :> Simp[(1/(Rt[a, 2]*Rt[-b, 2]))*
ArcTanh[Rt[-b, 2]*(x/Rt[a, 2])], x] /; FreeQ[{a, b}, x] && NegQ[a/b] && (Gt
Q[a, 0] || LtQ[b, 0])
```

Rule 221

```
Int[1/Sqrt[(a_) + (b_.)*(x_)^2], x_Symbol] :> Simp[ArcSinh[Rt[b, 2]*(x/Sqrt
[a])]/Rt[b, 2], x] /; FreeQ[{a, b}, x] && GtQ[a, 0] && PosQ[b]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3886

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*(a/(b*f))*Sqrt[a*(d/b)], Subst[Int[1/Sqrt[1 +
 x^2/a], x], x, b*(Cot[e + f*x]/Sqrt[a + b*Csc[e + f*x]])], x] /; FreeQ[{a,
 b, d, e, f}, x] && EqQ[a^2 - b^2, 0] && GtQ[a*(d/b), 0]
```

Rule 3893

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[-2*b*(d/(a*f)), Subst[Int[1/(2*b - d*x^2), x], x
, b*(Cot[e + f*x]/(Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]]))], x] /;
FreeQ[{a, b, d, e, f}, x] && EqQ[a^2 - b^2, 0]
```

Rule 4104

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[d*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(a*f*(
2*m + 1))), x] - Dist[1/(a*b*(2*m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(
d*Csc[e + f*x])^(n - 1)*Simp[A*(a*d*(n - 1)) - B*(b*d*(n - 1)) - d*(a*B*(m
- n + 1) + A*b*(m + n))*Csc[e + f*x], x], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] && LtQ[m, -2^(-1)] && Gt
Q[n, 0]
```

Rule 4106

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n -
1)*Simp[b*B*(n - 1) + (A*b*(m + n) + a*B*m)*Csc[e + f*x], x], x], x] /; Fre
eQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0] &
& GtQ[n, 1]
```

Rule 4108

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Dist[(A*b -
a*B)/b, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^n, x], x] + Dist[B/b, I
nt[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^n, x], x] /; FreeQ[{a, b,
d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0] && EqQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+\ldots}{(a+a\sec(c+dx))^{5/2}}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\ldots}{\ldots}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(7A-15B)\sin(\ldots}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\ldots}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(7A-15B)\sin(\ldots}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\ldots}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(7A-15B)\sin(\ldots}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\ldots}$$

$$= \frac{(A-B)\sin(c+dx)}{4d\cos^{\frac{7}{2}}(c+dx)(a+a\sec(c+dx))^{5/2}} + \frac{(7A-15B)\sin(\ldots}{16ad\cos^{\frac{5}{2}}(c+dx)(a+a\ldots}$$

$$= \frac{(2A-5B)\sinh^{-1}\left(\frac{\sqrt{a}\,\tan(c+dx)}{\sqrt{a+a\sec(c+dx)}}\right)\sqrt{\cos(c+dx)}\,\sqrt{\sec(\ldots}{a^{5/2}d}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 1061 vs. $2(286) = 572$.
time = 6.18, size = 1061, normalized size = 3.71

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + a*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] -1/4*(B*Sin[c + d*x])/(d*Cos[c + d*x]^(11/2)*(a*(1 + Sec[c + d*x]))^(5/2))
- (A*Sin[c + d*x])/(4*d*Cos[c + d*x]^(9/2)*(a*(1 + Sec[c + d*x]))^(5/2)) +
(7*B*(1 + Sec[c + d*x])*Sin[c + d*x])/(16*d*Cos[c + d*x]^(11/2)*(a*(1 + Sec
[c + d*x]))^(5/2)) + (3*A*(1 + Sec[c + d*x])*Sin[c + d*x])/(16*d*Cos[c + d*
x]^(9/2)*(a*(1 + Sec[c + d*x]))^(5/2)) - (7*B*(1 + Sec[c + d*x])^2*Sin[c +
d*x])/(16*d*Cos[c + d*x]^(9/2)*(a*(1 + Sec[c + d*x]))^(5/2)) - (3*A*(1 + Se
c[c + d*x])^2*Sin[c + d*x])/(16*d*Cos[c + d*x]^(7/2)*(a*(1 + Sec[c + d*x]))
^(5/2)) + (11*B*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Cos[c + d*x]^(7/2)
*(a*(1 + Sec[c + d*x]))^(5/2)) + (7*A*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(1
6*d*Cos[c + d*x]^(5/2)*(a*(1 + Sec[c + d*x]))^(5/2)) - (15*B*(1 + Sec[c + d
*x])^2*Sin[c + d*x])/(16*d*Cos[c + d*x]^(5/2)*(a*(1 + Sec[c + d*x]))^(5/2))
 - (11*A*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Cos[c + d*x]^(3/2)*(a*(1
+ Sec[c + d*x]))^(5/2)) + (35*B*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Co
s[c + d*x]^(3/2)*(a*(1 + Sec[c + d*x]))^(5/2)) - (11*A*ArcSin[Sqrt[1 - Sec[
c + d*x]]]*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c
 + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)) + (35*B
*ArcSin[Sqrt[1 - Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(1 +
Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c +
d*x]))^(5/2)) - (43*A*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sec[c +
 d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Sqrt[1 - Sec[c + d*x]]
*(a*(1 + Sec[c + d*x]))^(5/2)) + (115*B*ArcSin[Sqrt[Sec[c + d*x]]]*Sqrt[Cos
[c + d*x]]*Sec[c + d*x]^(3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*d*Sqrt
[1 - Sec[c + d*x]]*(a*(1 + Sec[c + d*x]))^(5/2)) + (43*A*ArcTan[(Sqrt[2]*Sq
rt[Sec[c + d*x]])/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(
3/2)*(1 + Sec[c + d*x])^2*Sin[c + d*x])/(16*Sqrt[2]*d*Sqrt[1 - Sec[c + d*x]
]*(a*(1 + Sec[c + d*x]))^(5/2)) - (115*B*ArcTan[(Sqrt[2]*Sqrt[Sec[c + d*x]]
)/Sqrt[1 - Sec[c + d*x]]]*Sqrt[Cos[c + d*x]]*Sec[c + d*x]^(3/2)*(1 + Sec[c
+ d*x])^2*Sin[c + d*x])/(16*Sqrt[2]*d*Sqrt[1 - Sec[c + d*x]]*(a*(1 + Sec[c
+ d*x]))^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 820 vs. $2(241) = 482$.

time = 14.12, size = 821, normalized size = 2.87

method	result
default	$\frac{(-1+\cos(dx+c))^2\left(16A\sqrt{2}\,\arctan\left(\frac{\sqrt{-\frac{2}{1+\cos(dx+c)}}\,(-1-\cos(dx+c)+\sin(dx+c))\sqrt{2}}{4}\right)\sin(dx+c)\left(\cos^2(dx+c)\right)+16A\left(\cos^2(d\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/16/d*(-1+cos(d*x+c))^2*(16*A*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)
*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))*sin(d*x+c)*cos(d*x+c)^2+16*A*cos(d*x+c
)^2*sin(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+s
in(d*x+c))*2^(1/2))-40*B*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-c
os(d*x+c)+sin(d*x+c))*2^(1/2))*sin(d*x+c)*cos(d*x+c)^2-40*B*cos(d*x+c)^2*si
n(d*x+c)*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x
+c))*2^(1/2))+16*A*2^(1/2)*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x
+c)+sin(d*x+c))*2^(1/2))*sin(d*x+c)*cos(d*x+c)+16*A*2^(1/2)*arctan(1/4*(-2/
(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c))*2^(1/2))*sin(d*x+c)*cos(d*x
+c)-43*A*cos(d*x+c)^2*sin(d*x+c)*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^
(1/2))+11*A*cos(d*x+c)^3*(-2/(1+cos(d*x+c)))^(1/2)-40*B*2^(1/2)*arctan(1/4*
(-2/(1+cos(d*x+c)))^(1/2)*(-1-cos(d*x+c)+sin(d*x+c))*2^(1/2))*sin(d*x+c)*co
s(d*x+c)-40*B*arctan(1/4*(-2/(1+cos(d*x+c)))^(1/2)*(1+cos(d*x+c)+sin(d*x+c)
)*2^(1/2))*2^(1/2)*cos(d*x+c)*sin(d*x+c)+115*B*cos(d*x+c)^2*sin(d*x+c)*arct
an(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))-35*B*cos(d*x+c)^3*(-2/(1+cos(d
*x+c)))^(1/2)-43*A*arctan(1/2*sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*sin(d*x
+c)*cos(d*x+c)+4*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)^2+115*B*arctan(1/2*
sin(d*x+c)*(-2/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)*sin(d*x+c)-20*B*(-2/(1+cos
(d*x+c)))^(1/2)*cos(d*x+c)^2-15*A*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+39*B
*(-2/(1+cos(d*x+c)))^(1/2)*cos(d*x+c)+16*B*(-2/(1+cos(d*x+c)))^(1/2))*(a*(1
+cos(d*x+c))/cos(d*x+c))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)^5/(-2/(1+cos(d*x
+c)))^(1/2)/a^3
```

Maxima [**B**] Leaf count of result is larger than twice the leaf count of optimal. 14037 vs. $2(241) = 482$.
time = 4.95, size = 14037, normalized size = 49.08

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] 1/32*((44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(3/2*arctan2(sin(2*
d*x + 2*c), cos(2*d*x + 2*c))) + 4*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*
d*x + 2*c))))*cos(7/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 16*(19
*sin(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 19*sin(3/4*arctan2(
sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 11*sin(1/4*arctan2(sin(2*d*x + 2*c),
 cos(2*d*x + 2*c))))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +
 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d*x +
2*c), cos(2*d*x + 2*c))))*cos(5/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c
))) - 76*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c) + 4*sin(1/2*arctan2(sin(2*d
*x + 2*c), cos(2*d*x + 2*c))))*cos(3/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x
```

+ 2*c))) - 44*(sin(4*d*x + 4*c) + 6*sin(2*d*x + 2*c))*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) - 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*sqrt(2)*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) - 2*sqrt(2)*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 2) + 16*(sqrt(2)*cos(4*d*x + 4*c)^2 + 36*sqrt(2)*cos(2*d*x + 2*c)^2 + 16*sqrt(2)*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + sqrt(2)*sin(4*d*x + 4*c)^2 + 12*sqrt(2)*sin(4*d*x + 4*c)*sin(2*d*x + 2*c) + 36*sqrt(2)*sin(2*d*x + 2*c)^2 + 16*sqrt(2)*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 16*sqrt(2)*sin(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2*(6*sqrt(2)*co

```
s(2*d*x + 2*c) + sqrt(2))*cos(4*d*x + 4*c) + 8*(sqrt(2)*cos(4*d*x + 4*c) +
6*sqrt(2)*cos(2*d*x + 2*c) + 4*sqrt(2)*cos(1/2*arctan2(sin(2*d*x + 2*c), co
s(2*d*x + 2*c))) + sqrt(2))*cos(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2
*c))) + 8*(sqrt(2)*cos(4*d*x + 4*c) + 6*sqrt(2)*cos(2*d*x + 2*c) + sqrt(2))
*cos(1/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 8*(sqrt(2)*sin(4*d*
x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c) + 4*sqrt(2)*sin(1/2*arctan2(sin(2*d*x
 + 2*c), cos(2*d*x + 2*c))))*sin(3/2*arctan2(sin(2*d*x + 2*c), cos(2*d*x +
2*c))) + 8*(sqrt(2)*sin(4*d*x + 4*c) + 6*sqrt(2)*sin(2*d*x + 2*c))*sin(1/2*
arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) + 12*sqrt(2)*cos(2*d*x + 2*c)
+ sqrt(2))*log(2*cos(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 + 2
*sin(1/4*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c)))^2 - 2*sqrt(2)*cos(1/4
*arctan2(sin(2*d*x + 2*c), cos(2*d*x + 2*c))) +...
```

Fricas [A]
time = 2.43, size = 850, normalized size = 2.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] [-1/64*(sqrt(2)*((43*A - 115*B)*cos(d*x + c)^4 + 3*(43*A - 115*B)*cos(d*x +
 c)^3 + 3*(43*A - 115*B)*cos(d*x + c)^2 + (43*A - 115*B)*cos(d*x + c))*sqrt
(a)*log(-(a*cos(d*x + c)^2 - 2*sqrt(2)*sqrt(a)*sqrt((a*cos(d*x + c) + a)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 2*a*cos(d*x + c) - 3*a)/(cos(
d*x + c)^2 + 2*cos(d*x + c) + 1)) + 4*((11*A - 35*B)*cos(d*x + c)^2 + 5*(3*
A - 11*B)*cos(d*x + c) - 16*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt
(cos(d*x + c))*sin(d*x + c) + 16*((2*A - 5*B)*cos(d*x + c)^4 + 3*(2*A - 5*B
)*cos(d*x + c)^3 + 3*(2*A - 5*B)*cos(d*x + c)^2 + (2*A - 5*B)*cos(d*x + c))
*sqrt(a)*log((a*cos(d*x + c)^3 + 4*sqrt(a)*sqrt((a*cos(d*x + c) + a)/cos(d*
x + c))*(cos(d*x + c) - 2)*sqrt(cos(d*x + c))*sin(d*x + c) - 7*a*cos(d*x +
c)^2 + 8*a)/(cos(d*x + c)^3 + cos(d*x + c)^2)))/(a^3*d*cos(d*x + c)^4 + 3*a
^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c)), 1/32*(s
qrt(2)*((43*A - 115*B)*cos(d*x + c)^4 + 3*(43*A - 115*B)*cos(d*x + c)^3 + 3
*(43*A - 115*B)*cos(d*x + c)^2 + (43*A - 115*B)*cos(d*x + c))*sqrt(-a)*arct
an(sqrt(2)*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x +
c))/(a*sin(d*x + c))) - 2*((11*A - 35*B)*cos(d*x + c)^2 + 5*(3*A - 11*B)*co
s(d*x + c) - 16*B)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c
))*sin(d*x + c) + 16*((2*A - 5*B)*cos(d*x + c)^4 + 3*(2*A - 5*B)*cos(d*x +
c)^3 + 3*(2*A - 5*B)*cos(d*x + c)^2 + (2*A - 5*B)*cos(d*x + c))*sqrt(-a)*ar
ctan(2*sqrt(-a)*sqrt((a*cos(d*x + c) + a)/cos(d*x + c))*sqrt(cos(d*x + c))*
sin(d*x + c)/(a*cos(d*x + c)^2 - a*cos(d*x + c) - 2*a)))/(a^3*d*cos(d*x + c
)^4 + 3*a^3*d*cos(d*x + c)^3 + 3*a^3*d*cos(d*x + c)^2 + a^3*d*cos(d*x + c))
]
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+a*sec(d*x+c))**(5/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+a*sec(d*x+c))^(5/2),x, algor
ithm="giac")
[Out] integrate((B*sec(d*x + c) + A)/((a*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/
2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\cos\left(c+d\,x\right)}^{7/2}\,{\left(a+\frac{a}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(5/2)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + a/cos(c + d*x))^(5/2)), x
)
```

3.562 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=140

$$\frac{6(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(5aA+7bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2(5aA+7bB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d}+\frac{2(\cdots}{}$$

```
[Out] 6/5*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin
(1/2*d*x+1/2*c),2^(1/2))/d+2/21*(5*A*a+7*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*(A*b+B*a)*co
s(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*a*A*cos(d*x+c)^(5/2)*sin(d*x+c)/d+2/21*(5*A
*a+7*B*b)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.16, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3033, 3047, 3102, 2827, 2715, 2720, 2719}

$$\frac{2(5aA+7bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{6(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}+\frac{2(5aA+7bB)\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2aA\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (6*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*a*A + 7*b*B)*Ellipt
icF[(c + d*x)/2, 2])/(21*d) + (2*(5*a*A + 7*b*B)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(21*d) + (2*(A*b + a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d) + (2*
a*A*Cos[c + d*x]^(5/2)*Sin[c + d*x])/(7*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx = \int \cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))(B+A\cos(c$$

$$= \int \cos^{\frac{3}{2}}(c+dx)\left(bB+(Ab+aB)\cos(c+dx)+\right.$$

$$= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + \frac{2}{7}\int \cos^{\frac{3}{2}}(c+d$$

$$= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{7d} + (Ab+aB)\int \mathrm{co}$$

$$= \frac{2(5aA+7bB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{21d} + \frac{2(A}{}$$

$$= \frac{6(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(5aA+7bB)F}{21d}$$

Mathematica [A]
time = 0.95, size = 103, normalized size = 0.74

$$\frac{126(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(5aA+7bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(65aA+70bB+42(Ab+aB)\cos(c+dx)+15aA\cos(2(c+dx)))\sin(c+dx)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (126*(A*b + a*B)*EllipticE[(c + d*x)/2, 2] + 10*(5*a*A + 7*b*B)*EllipticF[(
c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(65*a*A + 70*b*B + 42*(A*b + a*B)*Cos[c
 + d*x] + 15*a*A*Cos[2*(c + d*x)])*Sin[c + d*x])/(105*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 412 vs. $2(176) = 352$.
time = 1.83, size = 413, normalized size = 2.95

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+(-360Aa-168Ab-168Ba)\left(\sin^6\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*A*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*a+(-360*A*a-168*A*b-168*B*a)*sin(1/2*d*x
```

```
+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a+168*A*b+168*B*a+140*B*b)*sin(1/2*d*x+
1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*A*a-42*A*b-42*B*a-70*B*b)*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)+25*A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c
),2^(1/2))*b+35*B*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*
a)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/
(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.72, size = 192, normalized size = 1.37

$$\frac{2\left(15\,Aa\cos\left(dx+c\right)^{2}+25\,Aa+35\,Bb+21\,\left(Ba+Ab\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-5\,\sqrt{2}\,\left(5i\,Aa+7i\,Bb\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)-5\,\sqrt{2}\,\left(-5i\,Aa-7i\,Bb\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-63\,\sqrt{2}\,\left(-i\,Ba-i\,Ab\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-63\,\sqrt{2}\,\left(i\,Ba+i\,Ab\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/105*(2*(15*A*a*cos(d*x + c)^2 + 25*A*a + 35*B*b + 21*(B*a + A*b)*cos(d*x
+ c))*sqrt(cos(d*x + c))*sin(d*x + c) - 5*sqrt(2)*(5*I*A*a + 7*I*B*b)*weier
strassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-5*I*A*a
- 7*I*B*b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 63*s
qrt(2)*(-I*B*a - I*A*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c))) - 63*sqrt(2)*(I*B*a + I*A*b)*weierstrassZeta
(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(d*x + c)^(7/2), x)
```

Mupad [**B**]
time = 0.77, size = 166, normalized size = 1.19

$$\frac{2\,B\,b\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}-\frac{2\,A\,a\cos\left(c+d\,x\right)^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ \cos\left(c+d\,x\right)^2\right)}{9\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{2\,A\,b\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}-\frac{2\,B\,a\cos\left(c+d\,x\right)^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ \cos\left(c+d\,x\right)^2\right)}{7\,d\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*B*b*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*
d) - (2*A*a*cos(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos
(c + d*x)^2))/(9*d*(sin(c + d*x)^2)^(1/2)) - (2*A*b*cos(c + d*x)^(7/2)*sin(
c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)
^(1/2)) - (2*B*a*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4
, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.563 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=108

$$\frac{2(3aA+5bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(Ab+aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2aA}{}$$

```
[Out] 2/5*(3*A*a+5*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*a*A*cos(d*x+c
)^(3/2)*sin(d*x+c)/d+2/3*(A*b+B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.15, antiderivative size = 108, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3033, 3047, 3102, 2827, 2719, 2715, 2720}

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(3aA+5bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2aA\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(3*a*A + 5*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(A*b + a*B)*Ellipt
icF[(c + d*x)/2, 2])/(3*d) + (2*(A*b + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x]
)/(3*d) + (2*a*A*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx = \int \sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))(B+A\cos($$

$$= \int \sqrt{\cos(c+dx)}\,(bB+(Ab+aB)\cos(c+dx)$$

$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+\frac{2}{5}\int \sqrt{\cos(c+}$$

$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5d}+(Ab+aB)\int \text{co}$$

$$= \frac{2(3aA+5bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(Ab+aB)\sqrt{}}{}$$

$$= \frac{2(3aA+5bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(Ab+aB)F}{3}$$

Mathematica [A]
time = 0.46, size = 86, normalized size = 0.80

$$\frac{2\left(3(3aA+5bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\sqrt{\cos(c+dx)}\,(5Ab+5aB+3aA\cos(c+dx))\sin(c+dx)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(3*(3*a*A + 5*b*B)*EllipticE[(c + d*x)/2, 2] + 5*(A*b + a*B)*EllipticF[(
c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(5*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Si
n[c + d*x]))/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 370 vs. $2(148) = 296$.
time = 1.80, size = 371, normalized size = 3.44

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+(24Aa+20Ab+20Ba)\left(\sin^4\left(\right.\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*a+(24*A*a+20*A*b+20*B*a)*sin(1/2*d*x+1/2*
c)^4*cos(1/2*d*x+1/2*c)+(-6*A*a-10*A*b-10*B*a)*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)+5*A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a+5
*B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
F(cos(1/2*d*x+1/2*c),2^(1/2))-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+
1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.62, size = 175, normalized size = 1.62

$$\frac{2\left(3\,Aa\cos\left(dx+c\right)+5\,Ba+5\,Ab\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-5\,\sqrt{2}\left(i\,Ba+i\,Ab\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)-5\,\sqrt{2}\left(-i\,Ba-i\,Ab\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-3\,\sqrt{2}\left(-3i\,Aa-5i\,Bb\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-3\,\sqrt{2}\left(3i\,Aa+5i\,Bb\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/15*(2*(3*A*a*cos(d*x + c) + 5*B*a + 5*A*b)*sqrt(cos(d*x + c))*sin(d*x + c
) - 5*sqrt(2)*(I*B*a + I*A*b)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*s
in(d*x + c)) - 5*sqrt(2)*(-I*B*a - I*A*b)*weierstrassPInverse(-4, 0, cos(d*
x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(-3*I*A*a - 5*I*B*b)*weierstrassZeta(-
4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2
)*(3*I*A*a + 5*I*B*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos
(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2), x)
```

Mupad [**B**]
time = 0.62, size = 128, normalized size = 1.19

$$\frac{2\,A\,b\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,B\,a\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,B\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}-\frac{2\,A\,a\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*A*b*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*
d) + (2*B*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2))
)/(3*d) + (2*B*b*ellipticE(c/2 + (d*x)/2, 2))/d - (2*A*a*cos(c + d*x)^(7/2)
*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*
x)^2)^(1/2))
```

3.564 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=75

$$\frac{2(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(aA+3bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2aA\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}$$

```
[Out] 2*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))/d+2/3*(A*a+3*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*a*A*sin(d*x+c)*co
s(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.13, antiderivative size = 75, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3033, 3047, 3102, 2827, 2720, 2719}

$$\frac{2(aA+3bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2aA\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(a*A + 3*b*B)*EllipticF[(c
 + d*x)/2, 2])/(3*d) + (2*a*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
```

```
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx = \int \frac{(b+a\cos(c+dx))(B+A\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \int \frac{bB+(Ab+aB)\cos(c+dx)+aA\cos^2(c+}{\sqrt{\cos(c+dx)}}$$

$$= \frac{2aA\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+\frac{2}{3}\int \frac{1}{2}(aA+$$

$$= \frac{2aA\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3d}+(Ab+aB)\int$$

$$= \frac{2(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(aA+3bB)F}{3}$$

Mathematica [A]
time = 0.25, size = 67, normalized size = 0.89

$$\frac{2\left(3(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+(aA+3bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)+aA\sqrt{\cos(c+dx)}\,\sin(c+dx)\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(3*(A*b + a*B)*EllipticE[(c + d*x)/2, 2] + (a*A + 3*b*B)*EllipticF[(c +
d*x)/2, 2] + a*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x]))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 325 vs. $2(121) = 242$.
time = 1.74, size = 326, normalized size = 4.35

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(4A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^
2*a+A*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b+3*B*b*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^
2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)
^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.62, size = 156, normalized size = 2.08

$$\frac{2\,Aa\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)+\sqrt{2}\,(-i\,Aa-3i\,Bb)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right))+\sqrt{2}\,(i\,Aa+3i\,Bb)\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right))-3\,\sqrt{2}\,(-i\,Ba-i\,Ab)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)))-3\,\sqrt{2}\,(i\,Ba+i\,Ab)\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)))}{3\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] 1/3*(2*A*a*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-I*A*a - 3*I*B*b)*wei
erstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A*a + 3
*I*B*b)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(
2)*(-I*B*a - I*A*b)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(I*B*a + I*A*b)*weierstrassZeta(-4,
0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3005 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2), x)
```

Mupad [**B**]
time = 0.62, size = 85, normalized size = 1.13

$$\frac{2\,A\,a\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,A\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*A*a*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(3*
d) + (2*A*b*ellipticE(c/2 + (d*x)/2, 2))/d + (2*B*a*ellipticE(c/2 + (d*x)/2
, 2))/d + (2*B*b*ellipticF(c/2 + (d*x)/2, 2))/d
```

3.565 $\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=71

$$\frac{2(aA-bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bB\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] 2*(A*a-B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2))/d+2*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2*b*B*sin(d*x+c)/d/cos(d*
x+c)^(1/2)
```

Rubi [A]

time = 0.13, antiderivative size = 71, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.194$, Rules used = {3033, 3047, 3100, 2827, 2720, 2719}

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(aA-bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2bB\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(a*A - b*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(A*b + a*B)*EllipticF[(c +
 d*x)/2, 2])/d + (2*b*B*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))(A+B\sec(c+dx))\,dx = \int \frac{(b+a\cos(c+dx))(B+A\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$$
$$= \int \frac{bB+(Ab+aB)\cos(c+dx)+aA\cos^2(c+$$
$$= \frac{2bB\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + 2\int \frac{\frac{1}{2}(Ab+aB)+\frac{1}{2}(aA}{\sqrt{\cos(c+}}$$
$$= \frac{2bB\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + (Ab+aB)\int \frac{1}{\sqrt{\cos(c+d}}$$
$$= \frac{2(aA-bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(Ab+aB)F}{}$$

Mathematica [A]
time = 0.38, size = 64, normalized size = 0.90

$$\frac{2\left((aA-bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)+(Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{bB\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*((a*A - b*B)*EllipticE[(c + d*x)/2, 2] + (A*b + a*B)*EllipticF[(c + d*x)
/2, 2] + (b*B*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 243 vs. $2(121) = 242$.
time = 2.31, size = 244, normalized size = 3.44

method	result
default	$-\frac{2\left(Ab\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1}\,\mathrm{EllipticF}\left(\cos\left(\frac{dx}{2}+\frac{c}{2}\right),\sqrt{2}\right)-A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\right.}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2*(A*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2))-A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a-2*B*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*b+B*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/
2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+B*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*b)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.75, size = 185, normalized size = 2.61

$$\frac{2\,Bb\sqrt{\cos(dx+c)}\,\sin(dx+c)+\sqrt{2}\,(-i\,Ba-i\,Ab)\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+\sqrt{2}\,(i\,Ba+i\,Ab)\cos(dx+c)\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+\sqrt{2}\,(i\,Aa-i\,Bb)\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+\sqrt{2}\,(-i\,Aa+i\,Bb)\cos(dx+c)\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))}{d\cos(dx+c)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] (2*B*b*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-I*B*a - I*A*b)*cos(d*x +
 c)*weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*
B*a + I*A*b)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c)) + sqrt(2)*(I*A*a - I*B*b)*cos(d*x + c)*weierstrassZeta(-4, 0, weie
rstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) + sqrt(2)*(-I*A*a +
I*B*b)*cos(d*x + c)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c))))/(d*cos(d*x + c))
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A+B\sec (c+dx)) (a+b\sec (c+dx)) \sqrt{\cos (c+dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))*sqrt(cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))*(A+B*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 3.34, size = 96, normalized size = 1.35

$$\frac{2\,A\,a\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x)),x)
```

```
[Out] (2*A*a*ellipticE(c/2 + (d*x)/2, 2))/d + (2*A*b*ellipticF(c/2 + (d*x)/2, 2))
/d + (2*B*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*B*b*sin(c + d*x)*hypergeom(
[-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(
1/2))
```

3.566 $\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=103

$$-\frac{2(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(3aA+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2bB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(Ab+aB)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

```
[Out] -2*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))/d+2/3*(3*A*a+B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*b*B*sin(d*x+c)/d
/cos(d*x+c)^(3/2)+2*(A*b+B*a)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.15, antiderivative size = 103, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3033, 3047, 3100, 2827, 2716, 2719, 2720}

$$\frac{2(3aA+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(aB+Ab)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(aB+Ab)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2bB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*(A*b + a*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(3*a*A + b*B)*EllipticF[(
c + d*x)/2, 2])/(3*d) + (2*b*B*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*
(A*b + a*B)*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \int \frac{(b+a\cos(c+dx))(B+A\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \int \frac{bB+(Ab+aB)\cos(c+dx)+aA\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}\,dx$$

$$= \frac{2bB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2}{3}\int \frac{\frac{3}{2}(Ab+aB)+\frac{1}{2}(3aA+bB)\cos(c+}{\cos^{\frac{3}{2}}(c+dx)}$$

$$= \frac{2bB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + (Ab+aB)\int \frac{1}{\cos^{\frac{3}{2}}(c+dx)}\,dx + \frac{1}{3}(3aA+$$

$$= \frac{2(3aA+bB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2bB\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(Ab+a}{d\sqrt{c}}$$

$$= -\frac{2(Ab+aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2(3aA+bB)F\left(\frac{1}{2}(c+dx)\right|}{3d}$$

Mathematica [A]
time = 0.54, size = 107, normalized size = 1.04

$$\frac{2\left(-3(Ab+aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+(3aA+bB)\sqrt{\cos(c+dx)}\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+3Ab\sin(c+dx)+3aB\sin(c+dx)+bB\tan(c+dx)\right)}{3d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (2*(-3*(A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, 2] + (3*a*A +
b*B)*Sqrt[Cos[c + d*x]]*EllipticF[(c + d*x)/2, 2] + 3*A*b*Sin[c + d*x] + 3*
a*B*Sin[c + d*x] + b*B*Tan[c + d*x]))/(3*d*Sqrt[Cos[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 400 vs. $2(147) = 294$.
time = 4.25, size = 401, normalized size = 3.89

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2Aa\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \mathrm{El}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A*a*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*B*b
*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b+B*a)/sin(1/2*d*x+1/2*c)^2/
(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.60, size = 213, normalized size = 2.07

$$\frac{\sqrt{2}\,(-3i\,Aa - i\,Bb)\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\,\sin(dx+c)) + \sqrt{2}\,(3i\,Aa + i\,Bb)\cos(dx+c)^2\,\mathrm{weierstrassPInverse}(-4, 0, \cos(dx+c) - i\,\sin(dx+c)) - 3\sqrt{2}\,(i\,Ba + i\,Ab)\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4, 0, \mathrm{weierstrassPInverse}(-4, 0, \cos(dx+c) + i\,\sin(dx+c))) - 3\sqrt{2}\,(-i\,Ba - i\,Ab)\cos(dx+c)^2\,\mathrm{weierstrassZeta}(-4, 0, \mathrm{weierstrassPInverse}(-4, 0, \cos(dx+c) - i\,\sin(dx+c))) + 2\,(Bb + 3\,(Ba + Ab)\cos(dx+c))\sqrt{\cos(dx+c)}\,\sin(dx+c)}{3\,d\cos(dx+c)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*A*a - I*B*b)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*A*a + I*B*b)*cos(d*x + c)^2*we
ierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqrt(2)*(I*B*a
+ I*A*b)*cos(d*x + c)^2*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(-I*B*a - I*A*b)*cos(d*x + c)^2*
weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x
+ c))) + 2*(B*b + 3*(B*a + A*b)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x +
c))/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\,(a + b\sec(c + dx))}{\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))/sqrt(cos(c + d*x)), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)), x)
```

Mupad [B]
time = 3.86, size = 150, normalized size = 1.46

$$\frac{2\,A\,a\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,B\,a\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}+\frac{2\,B\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos\left(c+d\,x\right)^2\right)}{3\,d\,\cos\left(c+d\,x\right)^{3/2}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/cos(c + d*x)^(1/2),x)
```

```
[Out] (2*A*a*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*b*sin(c + d*x)*hypergeom([-1/4
, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
 + (2*B*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(
c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*b*sin(c + d*x)*hypergeom([-3/
4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/
2))
```

3.567 $\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=140

$$-\frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(Ab+aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2bB\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2(Ab+aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] -2/5*(5*A*a+3*B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
E(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/5*b*B*sin(d*x+
c)/d/cos(d*x+c)^(5/2)+2/3*(A*b+B*a)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2/5*(5*A*
a+3*B*b)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.16, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 31, $\frac{\text{number of rules}}{\text{integrand size}} = 0.226$, Rules used = {3033, 3047, 3100, 2827, 2716, 2720, 2719}

$$\frac{2(aB+Ab)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}-\frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(aB+Ab)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(5aA+3bB)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2bB\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(5*a*A + 3*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(A*b + a*B)*Ellip
ticF[(c + d*x)/2, 2])/(3*d) + (2*b*B*Sin[c + d*x])/(5*d*Cos[c + d*x]^(5/2))
 + (2*(A*b + a*B)*Sin[c + d*x])/(3*d*Cos[c + d*x]^(3/2)) + (2*(5*a*A + 3*b*
B)*Sin[c + d*x])/(5*d*Sqrt[Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3047

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Int[(a
 + b*Sin[e + f*x])^m*(A*c + (B*c + A*d)*Sin[e + f*x] + B*d*Sin[e + f*x]^2),
 x] /; FreeQ[{a, b, c, d, e, f, A, B, m}, x] && NeQ[b*c - a*d, 0]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}dx = \int \frac{(b+a\cos(c+dx))(B+A\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}dx$$
$$= \int \frac{bB+(Ab+aB)\cos(c+dx)+aA\cos^2(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}dx$$
$$= \frac{2bB\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2}{5}\int \frac{\frac{5}{2}(Ab+aB)+\frac{1}{2}(5aA+3bB)\cos(c}{\cos^{\frac{5}{2}}(c+dx)}$$
$$= \frac{2bB\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + (Ab+aB)\int \frac{1}{\cos^{\frac{5}{2}}(c+dx)}dx + \frac{1}{5}(5aA$$
$$= \frac{2bB\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(Ab+aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2(5aA+3bB}{5d\sqrt{\cos(}}$$
$$= -\frac{2(5aA+3bB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(Ab+aB)F\left(\frac{1}{2}(c+d\right.}{3d}$$

Mathematica [A]
time = 0.87, size = 134, normalized size = 0.96

$$\frac{-6(5aA+3bB)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(Ab+aB)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+10Ab\sin(c+dx)+10aB\sin(c+dx)+15aA\sin(2(c+dx))+9bB\sin(2(c+dx))+6bB\tan(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
[Out] (-6*(5*a*A + 3*b*B)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2, 2] + 10*(A*b
+ a*B)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x)/2, 2] + 10*A*b*Sin[c + d*x] +
 10*a*B*Sin[c + d*x] + 15*a*A*Sin[2*(c + d*x)] + 9*b*B*Sin[2*(c + d*x)] + 6
*b*B*Tan[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 635 vs. 2(176) = 352.
time = 5.90, size = 636, normalized size = 4.54

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(2(Ab+Ba)\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin}}{6\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^2}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b+B*a)*(-1
/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*B*b/(8*sin(1/2*d*x+1/2*c)^6-12*si
n(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*sin(1
/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+
1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*EllipticE(cos(1/2*d*
x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*
d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)+2
*A*a/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/
2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.04, size = 235, normalized size = 1.68

$$-\frac{5\sqrt{2}(iBa+iAb)\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c))+5\sqrt{2}(-iBa-iAb)\cos(dx+c)^3\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c))+3\sqrt{2}(5iAa+3iBb)\cos(dx+c)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\sin(dx+c)))+3\sqrt{2}(-5iAa-3iBb)\cos(dx+c)^3\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\sin(dx+c)))-2\left(3(5Aa+3Bb)\cos(dx+c)^2+3Bb+5(Ba+Ab)\cos(dx+c)\right)\sqrt{\cos(dx+c)}\sin(dx+c)}{15d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
fricas")
```

```
[Out] -1/15*(5*sqrt(2)*(I*B*a + I*A*b)*cos(d*x + c)^3*weierstrassPInverse(-4, 0,
cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-I*B*a - I*A*b)*cos(d*x + c)^3*
weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) + 3*sqrt(2)*(5*I*
A*a + 3*I*B*b)*cos(d*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4
```

```
, 0, cos(d*x + c) + I*sin(d*x + c))) + 3*sqrt(2)*(-5*I*A*a - 3*I*B*b)*cos(d
*x + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) -
I*sin(d*x + c))) - 2*(3*(5*A*a + 3*B*b)*cos(d*x + c)^2 + 3*B*b + 5*(B*a + A
*b)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\,(a + b\sec{(c + dx)})}{\cos^{\frac{3}{2}}{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))/cos(c + d*x)**(3/2), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2), x)
```

Mupad [B]
time = 4.30, size = 177, normalized size = 1.26

$$\frac{2\,A\,a\,\sin{(c+d\,x)}\;{}_2\mathrm{F}_1\!\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos{(c+d\,x)}^2\right)}{d\,\sqrt{\cos{(c+d\,x)}}\,\sqrt{\sin{(c+d\,x)}^2}}+\frac{2\,A\,b\,\sin{(c+d\,x)}\;{}_2\mathrm{F}_1\!\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos{(c+d\,x)}^2\right)}{3\,d\cos{(c+d\,x)}^{3/2}\,\sqrt{\sin{(c+d\,x)}^2}}+\frac{2\,B\,a\,\sin{(c+d\,x)}\;{}_2\mathrm{F}_1\!\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos{(c+d\,x)}^2\right)}{3\,d\cos{(c+d\,x)}^{3/2}\,\sqrt{\sin{(c+d\,x)}^2}}+\frac{2\,B\,b\,\sin{(c+d\,x)}\;{}_2\mathrm{F}_1\!\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos{(c+d\,x)}^2\right)}{5\,d\cos{(c+d\,x)}^{5/2}\,\sqrt{\sin{(c+d\,x)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x)))/cos(c + d*x)^(3/2),x)
```

```
[Out] (2*A*a*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c +
 d*x)^(1/2)*(sin(c + d*x)^2)^(1/2)) + (2*A*b*sin(c + d*x)*hypergeom([-3/4,
1/2], 1/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2))
 + (2*B*a*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2))/(3*d*co
s(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (2*B*b*sin(c + d*x)*hypergeom([-
5/4, 1/2], -1/4, cos(c + d*x)^2))/(5*d*cos(c + d*x)^(5/2)*(sin(c + d*x)^2)^
(1/2))
```

3.568 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=182

$$\frac{2(6aAb+3a^2B+5b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}+\frac{2(5a^2A+7b(Ab+2aB))\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21d}+\frac{2(5a^2A+7b(Ab+2a$$

```
[Out] 2/5*(6*A*a*b+3*B*a^2+5*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*(5*a^2*A+7*b*(A*b+2*B*a))*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2))/d+2/35*a*(9*A*b+7*B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)/d+2/7*a*A*cos(
d*x+c)^(3/2)*(b+a*cos(d*x+c))*sin(d*x+c)/d+2/21*(5*a^2*A+7*b*(A*b+2*B*a))*s
in(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.24, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3033, 3069, 3102, 2827, 2719, 2715, 2720}

$$\frac{2(3a^2B+6aAb+5b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}+\frac{2(5a^2A+7b(2aB+Ab))\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{21d}+\frac{2(5a^2A+7b(2aB+Ab))\sin(c+dx)\sqrt{\cos(c+dx)}}{21d}+\frac{2a(7aB+9Ab)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{35d}+\frac{2aA\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(6*a*A*b + 3*a^2*B + 5*b^2*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(5*a
^2*A + 7*b*(A*b + 2*a*B))*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*(5*a^2*A +
 7*b*(A*b + 2*a*B))*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*(9*A*b +
 7*a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(35*d) + (2*a*A*Cos[c + d*x]^(3/2)
*(b + a*Cos[c + d*x])*Sin[c + d*x])/(7*d)
```

Rule 2715

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[(-b)*Cos[c + d*
x]*((b*Sin[c + d*x])^(n - 1)/(d*n)), x] + Dist[b^2*((n - 1)/n), Int[(b*Sin[
c + d*x])^(n - 2), x], x] /; FreeQ[{b, c, d}, x] && GtQ[n, 1] && IntegerQ[2
*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \int \sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))^2(B+A\,\mathrm{co}$$

$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))\sin(c+dx}{7d}$$

$$= \frac{2a(9Ab+7aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2aA}{}$$

$$= \frac{2a(9Ab+7aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{35d} + \frac{2aA}{}$$

$$= \frac{2(6aAb+3a^2B+5b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{2(5}{}$$

$$= \frac{2(6aAb+3a^2B+5b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d} + \frac{2(5}{}$$

Mathematica [A]
time = 1.26, size = 139, normalized size = 0.76

$$\frac{42(6aAb+3a^2B+5b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+10(5a^2A+7Ab^2+14abB)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+\sqrt{\cos(c+dx)}\,(42a(2Ab+aB)\cos(c+dx)+5(13a^2A+14Ab^2+28abB+3a^2A\cos(2(c+dx))))\sin(c+dx)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (42*(6*a*A*b + 3*a^2*B + 5*b^2*B)*EllipticE[(c + d*x)/2, 2] + 10*(5*a^2*A +
 7*A*b^2 + 14*a*b*B)*EllipticF[(c + d*x)/2, 2] + Sqrt[Cos[c + d*x]]*(42*a*(
2*A*b + a*B)*Cos[c + d*x] + 5*(13*a^2*A + 14*A*b^2 + 28*a*b*B + 3*a^2*A*Cos
[2*(c + d*x)]))*Sin[c + d*x])/(105*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 547 vs. $2(218) = 436$.
time = 2.21, size = 548, normalized size = 3.01

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\left(240A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^8\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2+\left(-360a^2A-336Aba-168a^2B\right)\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/105*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(240*A*cos(1
/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^8*a^2+(-360*A*a^2-336*A*a*b-168*B*a^2)*sin
(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)+(280*A*a^2+336*A*a*b+140*A*b^2+168*B*a
^2+280*B*a*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-80*A*a^2-84*A*a*b-7
0*A*b^2-42*B*a^2-140*B*a*b)*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)+25*a^2*
A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(c
os(1/2*d*x+1/2*c),2^(1/2))+35*A*b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-126*A*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x
+1/2*c),2^(1/2))*a*b+70*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-63*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))*a^2-105*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(
1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.74, size = 243, normalized size = 1.34

$$\frac{2\left(15\,Aa^{2}\cos\left(dx+c\right)^{2}+25\,Aa^{2}+70\,Bab+35\,Ab^{2}+21\left(Ba^{2}+2\,Aab\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-5\sqrt{2}\left(5i\,Aa^{2}+14i\,Bab+7i\,Ab^{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)-5\sqrt{2}\left(-5i\,Aa^{2}-14i\,Bab-7i\,Ab^{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-21\sqrt{2}\left(-3i\,Ba^{2}-6i\,Aab-5i\,Bb^{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-21\sqrt{2}\left(3i\,Ba^{2}+6i\,Aab+5i\,Bb^{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)}{105\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/105*(2*(15*A*a^2*cos(d*x + c)^2 + 25*A*a^2 + 70*B*a*b + 35*A*b^2 + 21*(B*
a^2 + 2*A*a*b)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - 5*sqrt(2)*(5
*I*A*a^2 + 14*I*B*a*b + 7*I*A*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c)
+ I*sin(d*x + c)) - 5*sqrt(2)*(-5*I*A*a^2 - 14*I*B*a*b - 7*I*A*b^2)*weierst
rassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 21*sqrt(2)*(-3*I*B*a^2
 - 6*I*A*a*b - 5*I*B*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0,
 cos(d*x + c) + I*sin(d*x + c))) - 21*sqrt(2)*(3*I*B*a^2 + 6*I*A*a*b + 5*I*
```

```
B*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2), x
)
```

Mupad [B]
time = 3.17, size = 229, normalized size = 1.26

$$\frac{2\,A\,b^2\,\left(\sqrt{\cos\left(c+d\,x\right)}\,\sin\left(c+d\,x\right)+\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)\right)}{3\,d}+\frac{2\,B\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a\,b\,\left(\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}-\frac{2\,A\,a^2\,{\cos\left(c+d\,x\right)}^{9/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{9}{4};\ \frac{13}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{9\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{2\,B\,a^2\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}-\frac{4\,A\,a\,b\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*A*b^2*(cos(c + d*x)^(1/2)*sin(c + d*x) + ellipticF(c/2 + (d*x)/2, 2)))/(
3*d) + (2*B*b^2*ellipticE(c/2 + (d*x)/2, 2))/d + (2*B*a*b*((2*cos(c + d*x)^
(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2, 2))/3))/d - (2*A*a^2*co
s(c + d*x)^(9/2)*sin(c + d*x)*hypergeom([1/2, 9/4], 13/4, cos(c + d*x)^2))/
(9*d*(sin(c + d*x)^2)^(1/2)) - (2*B*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hyp
ergeom([1/2, 7/4], 11/4, cos(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2)) - (4
*A*a*b*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, cos(c +
d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.569 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=140

$$\frac{2(3a^2A+5b(Ab+2aB))\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(2aAb+a^2B+3b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2a(7Ab+5aB)\sqrt{\cos}}{15}$$

```
[Out] 2/5*(3*a^2*A+5*b*(A*b+2*B*a))*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(2*A*a*b+B*a^2+3*B*b^2)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^
(1/2))/d+2/15*a*(7*A*b+5*B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/d+2/5*a*A*(b+a*co
s(d*x+c))*sin(d*x+c)*cos(d*x+c)^(1/2)/d
```

Rubi [A]
time = 0.23, antiderivative size = 140, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3033, 3069, 3102, 2827, 2720, 2719}

$$\frac{2(a^2B+2aAb+3b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(3a^2A+5b(2aB+Ab))\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2a(5aB+7Ab)\sin(c+dx)\sqrt{\cos(c+dx)}}{15d}+\frac{2aA\sin(c+dx)\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+b)}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(3*a^2*A + 5*b*(A*b + 2*a*B))*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(2*a
*A*b + a^2*B + 3*b^2*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*a*(7*A*b + 5*
a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*A*Sqrt[Cos[c + d*x]]*(b
 + a*Cos[c + d*x])*Sin[c + d*x])/(5*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \int \frac{(b+a\cos(c+dx))^2(B+A\cos(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$$

$$= \frac{2aA\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))\sin(c+d}{5d}$$

$$= \frac{2a(7Ab+5aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d}+\frac{2a}{}$$

$$= \frac{2a(7Ab+5aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{15d}+\frac{2a}{}$$

$$= \frac{2(3a^2A+5b(Ab+2aB))\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}+\frac{2(}{}$$

Mathematica [A]
time = 0.68, size = 106, normalized size = 0.76

$$\frac{2\left(3(3a^2A+5Ab^2+10abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)+5(2aAb+a^2B+3b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)+a\sqrt{\cos(c+dx)}\,(10Ab+5aB+3aA\cos(c+dx))\sin(c+dx)\right)}{15d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(3*(3*a^2*A + 5*A*b^2 + 10*a*b*B)*EllipticE[(c + d*x)/2, 2] + 5*(2*a*A*b
 + a^2*B + 3*b^2*B)*EllipticF[(c + d*x)/2, 2] + a*Sqrt[Cos[c + d*x]]*(10*A*
b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[c + d*x]))/(15*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 486 vs. $2(180) = 360$.
time = 2.49, size = 487, normalized size = 3.48

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-24A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^6\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2+\left(24a^2A+40Aba+20a^2B\right)\left(\mathrm{si}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-24*A*cos(1/
2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6*a^2+(24*A*a^2+40*A*a*b+20*B*a^2)*sin(1/2*
d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6*A*a^2-20*A*a*b-10*B*a^2)*sin(1/2*d*x+1/
2*c)^2*cos(1/2*d*x+1/2*c)+10*A*b*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9*A*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/
2*c),2^(1/2))*a^2-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2
-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+5*a^2*B*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))+15*b^2*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-30*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a
*b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)
/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.46, size = 216, normalized size = 1.54

$$\frac{2\left(3\,Aa^{2}\cos\left(dx+c\right)+5\,Ba^{2}+10\,Aab\right)\sqrt{\cos\left(dx+c\right)}\,\sin\left(dx+c\right)-5\,\sqrt{2}\left(i\,Ba^{2}+2i\,Aab+3i\,Bb^{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)-5\,\sqrt{2}\left(-i\,Ba^{2}-2i\,Aab-3i\,Bb^{2}\right)\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-3\,\sqrt{2}\left(-3i\,Aa^{2}-10i\,Bab-5i\,Ab^{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-3\,\sqrt{2}\left(3i\,Aa^{2}+10i\,Bab+5i\,Ab^{2}\right)\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)}{15\,d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/15*(2*(3*A*a^2*cos(d*x + c) + 5*B*a^2 + 10*A*a*b)*sqrt(cos(d*x + c))*sin(
d*x + c) - 5*sqrt(2)*(I*B*a^2 + 2*I*A*a*b + 3*I*B*b^2)*weierstrassPInverse(
-4, 0, cos(d*x + c) + I*sin(d*x + c)) - 5*sqrt(2)*(-I*B*a^2 - 2*I*A*a*b - 3
*I*B*b^2)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c)) - 3*sqr
t(2)*(-3*I*A*a^2 - 10*I*B*a*b - 5*I*A*b^2)*weierstrassZeta(-4, 0, weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) - 3*sqrt(2)*(3*I*A*a^2 +
10*I*B*a*b + 5*I*A*b^2)*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, c
os(d*x + c) - I*sin(d*x + c))))/d
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2), x
)
```

Mupad [B]
time = 3.04, size = 177, normalized size = 1.26

$$\frac{B\,a^2\,\left(\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{2\,A\,b^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,a\,b\,\left(\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{4\,B\,a\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}-\frac{2\,A\,a^2\,{\cos\left(c+d\,x\right)}^{7/2}\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(\frac{1}{2},\frac{7}{4};\ \frac{11}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{7\,d\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (B*a^2*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2,
 2))/3))/d + (2*A*b^2*ellipticE(c/2 + (d*x)/2, 2))/d + (2*B*b^2*ellipticF(c
/2 + (d*x)/2, 2))/d + (2*A*a*b*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*
ellipticF(c/2 + (d*x)/2, 2))/3))/d + (4*B*a*b*ellipticE(c/2 + (d*x)/2, 2))/
d - (2*A*a^2*cos(c + d*x)^(7/2)*sin(c + d*x)*hypergeom([1/2, 7/4], 11/4, co
s(c + d*x)^2))/(7*d*(sin(c + d*x)^2)^(1/2))
```

3.570 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=121

$$\frac{2(2aAb+a^2B-b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(a^2A+3Ab^2+6abB)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b^2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2a^2A\sqrt{}}{}$$

```
[Out] 2*(2*A*a*b+B*a^2-B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(A*a^2+3*A*b^2+6*B*a*b)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
/d+2*b^2*B*sin(d*x+c)/d/cos(d*x+c)^(1/2)+2/3*a^2*A*sin(d*x+c)*cos(d*x+c)^(1
/2)/d
```

Rubi [A]
time = 0.22, antiderivative size = 121, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3033, 3067, 3102, 2827, 2720, 2719}

$$\frac{2(a^2A+6abB+3Ab^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(a^2B+2aAb-b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2a^2A\sin(c+dx)\sqrt{\cos(c+dx)}}{3d}+\frac{2b^2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(2*a*A*b + a^2*B - b^2*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(a^2*A + 3*A
*b^2 + 6*a*b*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b^2*B*Sin[c + d*x])/(
d*Sqrt[Cos[c + d*x]]) + (2*a^2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3102

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Co
s[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 2))), x] + Dist[1/(b*(m
+ 2)), Int[(a + b*Sin[e + f*x])^m*Simp[A*b*(m + 2) + b*C*(m + 1) + (b*B*(m
+ 2) - a*C)*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B, C, m}, x]
&&  !LtQ[m, -1]
```

Rubi steps

$$\begin{aligned}\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx &= \int \frac{(b+a\cos(c+dx))^2(B+A\cos(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx\\ &= \frac{2b^2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}-2\int \frac{-\frac{1}{2}b(Ab+2aB)-\frac{1}{2}(\\ &= \frac{2b^2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2a^2A\sqrt{\cos(c+dx)}\sin(c}{3d}\\ &= \frac{2b^2B\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2a^2A\sqrt{\cos(c+dx)}\sin(c}{3d}\\ &= \frac{2(2aAb+a^2B-b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(a}{}\end{aligned}$$

Mathematica [A]
time = 0.70, size = 102, normalized size = 0.84

$$\frac{2\left(3(2aAb+a^2B-b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+(a^2A+3Ab^2+6abB)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{(3b^2B+a^2A\cos(c+dx))\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(3*(2*a*A*b + a^2*B - b^2*B)*EllipticE[(c + d*x)/2, 2] + (a^2*A + 3*A*b^
2 + 6*a*b*B)*EllipticF[(c + d*x)/2, 2] + ((3*b^2*B + a^2*A*Cos[c + d*x])*Si
n[c + d*x])/Sqrt[Cos[c + d*x]]))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 403 vs. 2(165) = 330.
time = 2.73, size = 404, normalized size = 3.34

method	result
default	$-\frac{2\left(4A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2-2A\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a^2+a^2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{2\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -2/3*(4*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^2-2*A*cos(1/2*d*x+1/2*c
)*sin(1/2*d*x+1/2*c)^2*a^2+a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*
x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3*A*b^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))-6*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-6*B*cos(1/2*d*x+1/2*c)*sin
(1/2*d*x+1/2*c)^2*b^2+6*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))*a^2+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1
/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*b^2)/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.81, size = 240, normalized size = 1.98

$$\frac{\sqrt{2}\left(-iAa^{2}-6iBab-3iAb^{2}\right)\cos\left(dx+c\right)\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)+\sqrt{2}\left(iAa^{2}+6iBab+3iAb^{2}\right)\cos\left(dx+c\right)\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)-3\sqrt{2}\left(-iBa^{2}-2iAab+iBb^{2}\right)\cos\left(dx+c\right)\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\sin\left(dx+c\right)\right)\right)-3\sqrt{2}\left(iBa^{2}+2iAab-iBb^{2}\right)\cos\left(dx+c\right)\text{weierstrassZeta}\left(-4,0,\text{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\sin\left(dx+c\right)\right)\right)+2\left(Aa^{2}\cos\left(dx+c\right)+3Bb^{2}\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3d\cos\left(dx+c\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-I*A*a^2 - 6*I*B*a*b - 3*I*A*b^2)*cos(d*x + c)*weierstrassPIn
verse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(I*A*a^2 + 6*I*B*a*b
+ 3*I*A*b^2)*cos(d*x + c)*weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d
*x + c)) - 3*sqrt(2)*(-I*B*a^2 - 2*I*A*a*b + I*B*b^2)*cos(d*x + c)*weierstr
assZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c))) -
 3*sqrt(2)*(I*B*a^2 + 2*I*A*a*b - I*B*b^2)*cos(d*x + c)*weierstrassZeta(-4,
 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*(A*a^2*c
os(d*x + c) + 3*B*b^2)*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c))
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5007 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2), x
)
```

Mupad [B]
time = 3.31, size = 158, normalized size = 1.31

$$\frac{A\,a^2\,\left(\frac{2\,\sqrt{\cos\left(c+d\,x\right)}\,\sin(c+d\,x)}{3}+\frac{2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{3}\right)}{d}+\frac{2\,B\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,b^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,A\,a\,b\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,B\,a\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,b^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos\left(c+d\,x\right)^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{\sin\left(c+d\,x\right)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (A*a^2*((2*cos(c + d*x)^(1/2)*sin(c + d*x))/3 + (2*ellipticF(c/2 + (d*x)/2,
 2))/3))/d + (2*B*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (2*A*b^2*ellipticF(c
/2 + (d*x)/2, 2))/d + (4*A*a*b*ellipticE(c/2 + (d*x)/2, 2))/d + (4*B*a*b*el
lipticF(c/2 + (d*x)/2, 2))/d + (2*B*b^2*sin(c + d*x)*hypergeom([-1/4, 1/2],
 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.571 $\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=126

$$\frac{2(a^2A-Ab^2-2abB)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2(6aAb+3a^2B+b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2b^2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2b(A$$

```
[Out] 2*(A*a^2-A*b^2-2*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(6*A*a*b+3*B*a^2+B*b^2)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))
/d+2/3*b^2*B*sin(d*x+c)/d/cos(d*x+c)^(3/2)+2*b*(A*b+2*B*a)*sin(d*x+c)/d/cos
(d*x+c)^(1/2)
```

Rubi [A]
time = 0.23, antiderivative size = 126, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3033, 3067, 3100, 2827, 2720, 2719}

$$\frac{2(3a^2B+6aAb+b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d}+\frac{2(a^2A-2abB-Ab^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d}+\frac{2b(2aB+Ab)\sin(c+dx)}{d\sqrt{\cos(c+dx)}}+\frac{2b^2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2*A - A*b^2 - 2*a*b*B)*EllipticE[(c + d*x)/2, 2])/d + (2*(6*a*A*b + 3
*a^2*B + b^2*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b^2*B*Sin[c + d*x])/(
3*d*Cos[c + d*x]^(3/2)) + (2*b*(A*b + 2*a*B)*Sin[c + d*x])/(d*Sqrt[Cos[c +
d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2(A+B\sec(c+dx))\,dx = \int \frac{(b+a\cos(c+dx))^2(B+A\cos(c+dx))}{\cos^{\frac{5}{2}}(c+dx)}$$
$$= \frac{2b^2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{2}{3}\int \frac{-\frac{3}{2}b(Ab+2aB) -}{}$$
$$= \frac{2b^2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+d}{d\sqrt{\cos(c+dx)}}$$
$$= \frac{2b^2B\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+d}{d\sqrt{\cos(c+dx)}}$$
$$= \frac{2(a^2A-Ab^2-2abB)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{d} + \frac{2}{}$$

Mathematica [A]
time = 1.27, size = 105, normalized size = 0.83

$$\frac{2\left(3(a^2A-Ab^2-2abB)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+(6aAb+3a^2B+b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{b(bB+3(Ab+2aB)\cos(c+dx))\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(3*(a^2*A - A*b^2 - 2*a*b*B)*EllipticE[(c + d*x)/2, 2] + (6*a*A*b + 3*a^
2*B + b^2*B)*EllipticF[(c + d*x)/2, 2] + (b*(b*B + 3*(A*b + 2*a*B)*Cos[c +
d*x])*Sin[c + d*x])/Cos[c + d*x]^(3/2)))/(3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 649 vs. $2(170) = 340$.
time = 4.48, size = 650, normalized size = 5.16

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2a^2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right.\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+4*A*b*a*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*a^2*B*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*
b^2*B*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b*(A*b+2*B*a)/sin(1/2*d*x+
1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2
*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^
(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.85, size = 255, normalized size = 2.02

$$\frac{\sqrt{2}\left(-3i\,Ba^2-6i\,Aab-i\,Bb^2\right)\cos\left(dx+c\right)^2\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)+\sqrt{2}\left(3i\,Ba^2+6i\,Aab+i\,Bb^2\right)\cos\left(dx+c\right)^2\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)-3\sqrt{2}\left(-i\,Aa^2+2i\,Bab+i\,Ab^2\right)\cos\left(dx+c\right)^2\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)+i\,\sin\left(dx+c\right)\right)\right)-3\sqrt{2}\left(i\,Aa^2-2i\,Bab-i\,Ab^2\right)\cos\left(dx+c\right)^2\mathrm{weierstrassZeta}\left(-4,0,\mathrm{weierstrassPInverse}\left(-4,0,\cos\left(dx+c\right)-i\,\sin\left(dx+c\right)\right)\right)+2\left(Bb^2+3\left(2\,Bab+Ab^2\right)\cos\left(dx+c\right)\right)\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)}{3\,d\cos\left(dx+c\right)^2}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="fricas")
```

```
[Out] 1/3*(sqrt(2)*(-3*I*B*a^2 - 6*I*A*a*b - I*B*b^2)*cos(d*x + c)^2*weierstrassP
Inverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + sqrt(2)*(3*I*B*a^2 + 6*I*A*
a*b + I*B*b^2)*cos(d*x + c)^2*weierstrassPInverse(-4, 0, cos(d*x + c) - I*s
in(d*x + c)) - 3*sqrt(2)*(-I*A*a^2 + 2*I*B*a*b + I*A*b^2)*cos(d*x + c)^2*we
ierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x +
c))) - 3*sqrt(2)*(I*A*a^2 - 2*I*B*a*b - I*A*b^2)*cos(d*x + c)^2*weierstrass
```

```
Zeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin(d*x + c))) + 2*
(B*b^2 + 3*(2*B*a*b + A*b^2)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))
/(d*cos(d*x + c)^2)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))(a + b\sec(c + dx))^2 \sqrt{\cos(c + dx)}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**2*(A+B*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2*sqrt(cos(c + d*x)), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^2*(A+B*sec(d*x+c)),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c)), x
)
```

Mupad [B]
time = 4.27, size = 194, normalized size = 1.54

$$\frac{2\,A\,a^2\,\mathrm{E}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,B\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{4\,A\,a\,b\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,b^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{2\,B\,b^2\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{3\,d\,{\cos\left(c+d\,x\right)}^{3/2}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}+\frac{4\,B\,a\,b\,\sin\left(c+d\,x\right)\,{}_2\mathrm{F}_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ {\cos\left(c+d\,x\right)}^2\right)}{d\,\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{{\sin\left(c+d\,x\right)}^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2,x)
```

```
[Out] (2*A*a^2*ellipticE(c/2 + (d*x)/2, 2))/d + (2*B*a^2*ellipticF(c/2 + (d*x)/2,
 2))/d + (4*A*a*b*ellipticF(c/2 + (d*x)/2, 2))/d + (2*A*b^2*sin(c + d*x)*hy
pergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(sin(c + d
*x)^2)^(1/2)) + (2*B*b^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d
*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (4*B*a*b*sin(c +
d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d*x)^(1/2)*(si
n(c + d*x)^2)^(1/2))
```

3.572 $\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=172

$$-\frac{2(10aAb+5a^2B+3b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}+\frac{2(3a^2A+Ab^2+2abB)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d}+\frac{2b^2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2}{}$$

```
[Out] -2/5*(10*A*a*b+5*B*a^2+3*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/3*(3*A*a^2+A*b^2+2*B*a*b)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),
2^(1/2))/d+2/5*b^2*B*sin(d*x+c)/d/cos(d*x+c)^(5/2)+2/3*b*(A*b+2*B*a)*sin(d*
x+c)/d/cos(d*x+c)^(3/2)+2/5*(10*A*a*b+5*B*a^2+3*B*b^2)*sin(d*x+c)/d/cos(d*x
+c)^(1/2)
```

Rubi [A]
time = 0.25, antiderivative size = 172, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.212, Rules used = {3033, 3067, 3100, 2827, 2716, 2719, 2720}

$$\frac{2(3a^2A+2abB+Ab^2)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{3d}-\frac{2(5a^2B+10aAb+3b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5d}+\frac{2(5a^2B+10aAb+3b^2B)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2b(2aB+Ab)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)}+\frac{2b^2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] (-2*(10*a*A*b + 5*a^2*B + 3*b^2*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(3
*a^2*A + A*b^2 + 2*a*b*B)*EllipticF[(c + d*x)/2, 2])/(3*d) + (2*b^2*B*Sin[c
 + d*x])/(5*d*Cos[c + d*x]^(5/2)) + (2*b*(A*b + 2*a*B)*Sin[c + d*x])/(3*d*C
os[c + d*x]^(3/2)) + (2*(10*a*A*b + 5*a^2*B + 3*b^2*B)*Sin[c + d*x])/(5*d*S
qrt[Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$
\begin{aligned}
\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx &= \int \frac{(b+a\cos(c+dx))^2(B+A\cos(c+dx))}{\cos^{\frac{7}{2}}(c+dx)}\,dx \\
&= \frac{2b^2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{2}{5}\int \frac{-\frac{5}{2}b(Ab+2aB) - \frac{1}{2}(10aAb+5a^2\ldots}{\text{co}\ldots} \\
&= \frac{2b^2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{4}{15}\int \frac{-\frac{3}{4}(\ldots}{\ldots} \\
&= \frac{2b^2B\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} - \frac{1}{3}(-3a^2A \ldots \\
&= \frac{2(3a^2A+Ab^2+2abB)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3d} + \frac{2b^2B\sin(c+dx\ldots}{5d\cos^{\frac{5}{2}}(c+dx)\ldots} \\
&= -\frac{2(10aAb+5a^2B+3b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(3a^2A+A\ldots}{\ldots}
\end{aligned}
$$

Mathematica [A]
time = 1.21, size = 175, normalized size = 1.02

$$
\frac{-6(10aAb+5a^2B+3b^2B)\cos^{\frac{3}{2}}(c+dx)E\left(\frac{1}{2}(c+dx)\middle|2\right)+10(3a^2A+Ab^2+2abB)\cos^{\frac{3}{2}}(c+dx)F\left(\frac{1}{2}(c+dx)\middle|2\right)+10Ab^2\sin(c+dx)+20abB\sin(c+dx)+30aAb\sin(2(c+dx))+15a^2B\sin(2(c+dx))+9b^2B\sin(2(c+dx))+6b^2B\tan(c+dx)}{15d\cos^{\frac{3}{2}}(c+dx)}
$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],
x]

[Out] (-6*(10*a*A*b + 5*a^2*B + 3*b^2*B)*Cos[c + d*x]^(3/2)*EllipticE[(c + d*x)/2
, 2] + 10*(3*a^2*A + A*b^2 + 2*a*b*B)*Cos[c + d*x]^(3/2)*EllipticF[(c + d*x
)/2, 2] + 10*A*b^2*Sin[c + d*x] + 20*a*b*B*Sin[c + d*x] + 30*a*A*b*Sin[2*(c
 + d*x)] + 15*a^2*B*Sin[2*(c + d*x)] + 9*b^2*B*Sin[2*(c + d*x)] + 6*b^2*B*T
an[c + d*x])/(15*d*Cos[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 722 vs. $2(208) = 416$.
time = 6.58, size = 723, normalized size = 4.20

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2a^2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\ \text{E}\ldots}{\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\ldots\right.}}\right.\ldots}{\ldots}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*A*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*b
*(A*b+2*B*a)*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*b^2*B/(8*sin(1/2*
d*x+1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+
1/2*c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d
*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2
*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*
d*x+1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)+2*a*(2*A*b+B*a)/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)
^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x
)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.63, size = 286, normalized size = 1.66

$$-\frac{5\sqrt{2}\,(3i\,Aa^2+2i\,Bab+i\,Ab^2)\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}\,(-3i\,Aa^2-2i\,Bab-i\,Ab^2)\cos(dx+c)^3\,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+3\sqrt{2}\,(5i\,Ba^2+10i\,Aab+3i\,Bb^2)\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+3\sqrt{2}\,(-5i\,Ba^2-10i\,Aab-3i\,Bb^2)\cos(dx+c)^3\,\mathrm{weierstrassZeta}(-4,0,\mathrm{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-2\,(3\,Bb^2+3\,(5\,Ba^2+10\,Aab+3\,Bb^2)\cos(dx+c)^2+5\,(2\,Bab+Ab^2)\cos(dx+c))\sqrt{\cos(dx+c)}\,\sin(dx+c)}{15\,d\cos(dx+c)^3}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] -1/15*(5*sqrt(2)*(3*I*A*a^2 + 2*I*B*a*b + I*A*b^2)*cos(d*x + c)^3*weierstra
ssPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-3*I*A*a^2 -
2*I*B*a*b - I*A*b^2)*cos(d*x + c)^3*weierstrassPInverse(-4, 0, cos(d*x + c)
 - I*sin(d*x + c)) + 3*sqrt(2)*(5*I*B*a^2 + 10*I*A*a*b + 3*I*B*b^2)*cos(d*x
 + c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) + I*
sin(d*x + c))) + 3*sqrt(2)*(-5*I*B*a^2 - 10*I*A*a*b - 3*I*B*b^2)*cos(d*x +
c)^3*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c) - I*sin
(d*x + c))) - 2*(3*B*b^2 + 3*(5*B*a^2 + 10*A*a*b + 3*B*b^2)*cos(d*x + c)^2
+ 5*(2*B*a*b + A*b^2)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos
(d*x + c)^3)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))(a + b\sec(c + dx))^2}{\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2/sqrt(cos(c + d*x)), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/sqrt(cos(d*x + c)), x
)
```

Mupad [B]
time = 4.64, size = 227, normalized size = 1.32

$$\frac{6\,B\,b^2\,\sin(c+d\,x)\,{}_2F_1\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos(c+d\,x)^2\right)+30\,B\,a^2\cos(c+d\,x)^2\,\sin(c+d\,x)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)+20\,B\,a\,b\,\cos(c+d\,x)\,\sin(c+d\,x)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)}{15\,d\cos(c+d\,x)^{5/2}\,\sqrt{1-\cos(c+d\,x)^2}}+\frac{2\,A\,a^2\,\mathrm{F}\left(\frac{c}{2}+\frac{d\,x}{2}\middle|2\right)}{d}+\frac{2\,A\,b^2\,\sin(c+d\,x)\,{}_2F_1\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)}{3\,d\cos(c+d\,x)^{3/2}\,\sqrt{\sin(c+d\,x)^2}}+\frac{4\,A\,a\,b\,\sin(c+d\,x)\,{}_2F_1\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)}{d\,\sqrt{\cos(c+d\,x)}\,\sqrt{\sin(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/cos(c + d*x)^(1/2),x)
```

```
[Out] (6*B*b^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 30*B*a
^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2)
+ 20*B*a*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*
x)^2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2)) + (2*A*a^2*elli
pticF(c/2 + (d*x)/2, 2))/d + (2*A*b^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1
/4, cos(c + d*x)^2))/(3*d*cos(c + d*x)^(3/2)*(sin(c + d*x)^2)^(1/2)) + (4*A
*a*b*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2))/(d*cos(c + d
*x)^(1/2)*(sin(c + d*x)^2)^(1/2))
```

3.573 $\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=214

$$-\frac{2(5a^2A+3Ab^2+6abB)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(14aAb+7a^2B+5b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}+\frac{2b^2B\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}+$$

```
[Out] -2/5*(5*A*a^2+3*A*b^2+6*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/d+2/21*(14*A*a*b+7*B*a^2+5*B*b^2)
*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*
c),2^(1/2))/d+2/7*b^2*B*sin(d*x+c)/d/cos(d*x+c)^(7/2)+2/5*b*(A*b+2*B*a)*sin
(d*x+c)/d/cos(d*x+c)^(5/2)+2/21*(14*A*a*b+7*B*a^2+5*B*b^2)*sin(d*x+c)/d/cos
(d*x+c)^(3/2)+2/5*(5*A*a^2+3*A*b^2+6*B*a*b)*sin(d*x+c)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 214, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3033, 3067, 3100, 2827, 2716, 2720, 2719}

$$\frac{2(7a^2B+14aAb+5b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{21d}-\frac{2(5a^2A+6abB+3Ab^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d}+\frac{2(7a^2B+14aAb+5b^2B)\sin(c+dx)}{21d\cos^{\frac{3}{2}}(c+dx)}+\frac{2(5a^2A+6abB+3Ab^2)\sin(c+dx)}{5d\sqrt{\cos(c+dx)}}+\frac{2b(2aB+Ab)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)}+\frac{2b^2B\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] (-2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticE[(c + d*x)/2, 2])/(5*d) + (2*(14
*a*A*b + 7*a^2*B + 5*b^2*B)*EllipticF[(c + d*x)/2, 2])/(21*d) + (2*b^2*B*Si
n[c + d*x])/(7*d*Cos[c + d*x]^(7/2)) + (2*b*(A*b + 2*a*B)*Sin[c + d*x])/(5*
d*Cos[c + d*x]^(5/2)) + (2*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Sin[c + d*x])/(21
*d*Cos[c + d*x]^(3/2)) + (2*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*Sin[c + d*x])/(5*
d*Sqrt[Cos[c + d*x]])
```

Rule 2716

```
Int[((b_.)*sin[(c_.) + (d_.)*(x_)])^(n_), x_Symbol] :> Simp[Cos[c + d*x]*((
b*Sin[c + d*x])^(n + 1)/(b*d*(n + 1))), x] + Dist[(n + 2)/(b^2*(n + 1)), In
t[(b*Sin[c + d*x])^(n + 2), x], x] /; FreeQ[{b, c, d}, x] && LtQ[n, -1] &&
IntegerQ[2*n]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2827

```
Int[((b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_) + (d_.)*sin[(e_.) + (f_.)*(x
_)]), x_Symbol] :> Dist[c, Int[(b*Sin[e + f*x])^m, x], x] + Dist[d/b, Int[(
b*Sin[e + f*x])^(m + 1), x], x] /; FreeQ[{b, c, d, e, f, m}, x]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3067

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^2*((A_.) + (B_.)*sin[(e_.) + (f
_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Simp[
(B*c - A*d)*(b*c - a*d)^2*Cos[e + f*x]*((c + d*Sin[e + f*x])^(n + 1)/(f*d^2
*(n + 1)*(c^2 - d^2))), x] - Dist[1/(d^2*(n + 1)*(c^2 - d^2)), Int[(c + d*S
in[e + f*x])^(n + 1)*Simp[d*(n + 1)*(B*(b*c - a*d)^2 - A*d*(a^2*c + b^2*c -
 2*a*b*d)) - ((B*c - A*d)*(a^2*d^2*(n + 2) + b^2*(c^2 + d^2*(n + 1))) + 2*a
*b*d*(A*c*d*(n + 2) - B*(c^2 + d^2*(n + 1))))*Sin[e + f*x] - b^2*B*d*(n + 1
)*(c^2 - d^2)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && LtQ[n
, -1]
```

Rule 3100

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2
 - a*b*B + a^2*C))*Cos[e + f*x]*((a + b*Sin[e + f*x])^(m + 1)/(b*f*(m + 1)*
(a^2 - b^2))), x] + Dist[1/(b*(m + 1)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x]
)^(m + 1)*Simp[b*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C + b*(A*
b - a*B + b*C)*(m + 1))*Sin[e + f*x], x], x], x] /; FreeQ[{a, b, e, f, A, B
, C}, x] && LtQ[m, -1] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^2(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \int \frac{(b+a\cos(c+dx))^2(B+A\cos(c+dx))}{\cos^{\frac{9}{2}}(c+dx)}\,dx$$

$$= \frac{2b^2B\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} - \frac{2}{7}\int \frac{-\frac{7}{2}b(Ab+2aB) - \frac{1}{2}(14aAb+7a^2 \cdots}{\text{co}\cdots}$$

$$= \frac{2b^2B\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{4}{35}\int \frac{-\frac{5}{4}(\cdots}{\cdots}$$

$$= \frac{2b^2B\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} - \frac{1}{5}(-5a^2A \cdots$$

$$= \frac{2b^2B\sin(c+dx)}{7d\cos^{\frac{7}{2}}(c+dx)} + \frac{2b(Ab+2aB)\sin(c+dx)}{5d\cos^{\frac{5}{2}}(c+dx)} + \frac{2(14aAb+\cdots}{\cdots}$$

$$= -\frac{2(5a^2A+3Ab^2+6abB)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5d} + \frac{2(14aAb+7a\cdots}{\cdots}$$

Mathematica [A]
time = 4.80, size = 191, normalized size = 0.89

$$\frac{2\left(-21(5a^2A+3Ab^2+6abB)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)+5(14aAb+7a^2B+5b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)+\frac{15b^2B\sin(c+dx)}{\cos^{\frac{7}{2}}(c+dx)}+\frac{21b(Ab+2aB)\sin(c+dx)}{\cos^{\frac{5}{2}}(c+dx)}+\frac{5(14aAb+7a^2B+5b^2B)\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)}+\frac{21(5a^2A+3Ab^2+6abB)\sin(c+dx)}{\sqrt{\cos(c+dx)}}\right)}{105d}$$

Antiderivative was successfully verified.

```
[In] Integrate[((a + b*Sec[c + d*x])^2*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),
x]
```

```
[Out] (2*(-21*(5*a^2*A + 3*A*b^2 + 6*a*b*B)*EllipticE[(c + d*x)/2, 2] + 5*(14*a*A
*b + 7*a^2*B + 5*b^2*B)*EllipticF[(c + d*x)/2, 2] + (15*b^2*B*Sin[c + d*x])
/Cos[c + d*x]^(7/2) + (21*b*(A*b + 2*a*B)*Sin[c + d*x])/Cos[c + d*x]^(5/2)
+ (5*(14*a*A*b + 7*a^2*B + 5*b^2*B)*Sin[c + d*x])/Cos[c + d*x]^(3/2) + (21*
(5*a^2*A + 3*A*b^2 + 6*a*b*B)*Sin[c + d*x])/Sqrt[Cos[c + d*x]]))/(105*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 831 vs. 2(246) = 492.
time = 8.92, size = 832, normalized size = 3.89

method	result	size
default	Expression too large to display	832

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a*(2*A*b+B*a)
*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*b*(A*b+2*B*a)/(8*sin(1/2*d*x+
1/2*c)^6-12*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*
c)^2*(24*sin(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1
/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2
)*sin(1/2*d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x
+1/2*c)^2-1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)-3*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)+2*a^2*A/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(
1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*b^2*B*(-1/56*cos(1/2*d*x+1/2*c)*
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-
1/2)^4-5/42*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+5/21*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(
1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.59, size = 314, normalized size = 1.47

$$-\frac{5\sqrt{2}\,(7i\,Ba^2+14i\,Aab+5i\,Bb^2)\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c))+5\sqrt{2}\,(-7i\,Ba^2-14i\,Aab-5i\,Bb^2)\cos(dx+c)^4\,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c))+21\sqrt{2}\,(5i\,Aa^2+6i\,Bab+3i\,Ab^2)\cos(dx+c)^4\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)+i\,\sin(dx+c)))+21\sqrt{2}\,(-5i\,Aa^2-6i\,Bab-3i\,Ab^2)\cos(dx+c)^4\,\text{weierstrassZeta}(-4,0,\text{weierstrassPInverse}(-4,0,\cos(dx+c)-i\,\sin(dx+c)))-2\,(21\,(5\,Aa^2+6\,Bab+3\,Ab^2)\cos(dx+c)^3+15\,Bb^2+5\,(7\,Ba^2+14\,Aab+5\,Bb^2)\cos(dx+c)^2+21\,(2\,Bab+Ab^2)\cos(dx+c))\sqrt{\cos(dx+c)}\,\sin(dx+c)}{105\,d\cos(dx+c)^4}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="fricas")
```

```
[Out] -1/105*(5*sqrt(2)*(7*I*B*a^2 + 14*I*A*a*b + 5*I*B*b^2)*cos(d*x + c)^4*weier
strassPInverse(-4, 0, cos(d*x + c) + I*sin(d*x + c)) + 5*sqrt(2)*(-7*I*B*a^
```

```
2 - 14*I*A*a*b - 5*I*B*b^2)*cos(d*x + c)^4*weierstrassPInverse(-4, 0, cos(d
*x + c) - I*sin(d*x + c)) + 21*sqrt(2)*(5*I*A*a^2 + 6*I*B*a*b + 3*I*A*b^2)*
cos(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x +
c) + I*sin(d*x + c))) + 21*sqrt(2)*(-5*I*A*a^2 - 6*I*B*a*b - 3*I*A*b^2)*cos
(d*x + c)^4*weierstrassZeta(-4, 0, weierstrassPInverse(-4, 0, cos(d*x + c)
- I*sin(d*x + c))) - 2*(21*(5*A*a^2 + 6*B*a*b + 3*A*b^2)*cos(d*x + c)^3 + 1
5*B*b^2 + 5*(7*B*a^2 + 14*A*a*b + 5*B*b^2)*cos(d*x + c)^2 + 21*(2*B*a*b + A
*b^2)*cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c))/(d*cos(d*x + c)^4)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\,(a + b\sec(c + dx))^2}{\cos^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**2*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*(a + b*sec(c + d*x))**2/cos(c + d*x)**(3/2),
x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^2*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^2/cos(d*x + c)^(3/2), x
)
```

Mupad [**B**]
time = 4.98, size = 233, normalized size = 1.09

$$\frac{6\,A\,b^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\!\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos(c+d\,x)^2\right)+30\,A\,a^2\cos(c+d\,x)^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\!\left(-\frac{1}{4},\frac{1}{2};\ \frac{3}{4};\ \cos(c+d\,x)^2\right)+20\,A\,a\,b\,\cos(c+d\,x)\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\!\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)}{15\,d\cos(c+d\,x)^{5/2}\,\sqrt{1-\cos(c+d\,x)^2}}+\frac{30\,B\,b^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\!\left(-\frac{7}{4},\frac{1}{2};\ -\frac{3}{4};\ \cos(c+d\,x)^2\right)+70\,B\,a^2\cos(c+d\,x)^2\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\!\left(-\frac{3}{4},\frac{1}{2};\ \frac{1}{4};\ \cos(c+d\,x)^2\right)+84\,B\,a\,b\,\cos(c+d\,x)\,\sin(c+d\,x)\,{}_2\mathrm{F}_1\!\left(-\frac{5}{4},\frac{1}{2};\ -\frac{1}{4};\ \cos(c+d\,x)^2\right)}{105\,d\cos(c+d\,x)^{7/2}\,\sqrt{1-\cos(c+d\,x)^2}}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^2)/cos(c + d*x)^(3/2),x)
```

```
[Out] (6*A*b^2*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2) + 30*A*a
^2*cos(c + d*x)^2*sin(c + d*x)*hypergeom([-1/4, 1/2], 3/4, cos(c + d*x)^2)
+ 20*A*a*b*cos(c + d*x)*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*
x)^2))/(15*d*cos(c + d*x)^(5/2)*(1 - cos(c + d*x)^2)^(1/2)) + (30*B*b^2*sin
(c + d*x)*hypergeom([-7/4, 1/2], -3/4, cos(c + d*x)^2) + 70*B*a^2*cos(c + d
*x)^2*sin(c + d*x)*hypergeom([-3/4, 1/2], 1/4, cos(c + d*x)^2) + 84*B*a*b*c
os(c + d*x)*sin(c + d*x)*hypergeom([-5/4, 1/2], -1/4, cos(c + d*x)^2))/(105
*d*cos(c + d*x)^(7/2)*(1 - cos(c + d*x)^2)^(1/2))
```

3.574 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=182

$$\frac{2(3a^2A+5Ab^2-5abB)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d}-\frac{2(a^2+3b^2)\,(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^4d}+\frac{2b^3(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}\right.}{a^4(a+b)d}$$

```
[Out] 2/5*(3*A*a^2+5*A*b^2-5*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-2/3*(a^2+3*b^2)*(A*b-B*a)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),
2^(1/2))/a^4/d+2*b^3*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^4/(a+b)/d+2/5*A*cos(
d*x+c)^(3/2)*sin(d*x+c)/a/d-2/3*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/d
```

Rubi [A]
time = 0.55, antiderivative size = 182, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3033, 3069, 3128, 3138, 2719, 3081, 2720, 2884}

$$\frac{2b^3(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^4d(a+b)}-\frac{2(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d}-\frac{2(a^2+3b^2)\,(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^4d}+\frac{2(3a^2A-5abB+5Ab^2)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5a^3d}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*(3*a^2*A + 5*A*b^2 - 5*a*b*B)*EllipticE[(c + d*x)/2, 2])/(5*a^3*d) - (2*
(a^2 + 3*b^2)*(A*b - a*B)*EllipticF[(c + d*x)/2, 2])/(3*a^4*d) + (2*b^3*(A*
b - a*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^4*(a + b)*d) - (2*(A
*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(3*a^2*d) + (2*A*Cos[c + d*x]^(3
/2)*Sin[c + d*x])/(5*a*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+A\cos(c+dx))}{b+a\cos(c+dx)}\,dx$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{5ad} + \frac{2\int \frac{\sqrt{\cos(c+dx)}\left(\frac{3Ab}{2}+\frac{3}{2}aA\cos(c+d\right.}{b+a\cos(c+dx}}{5a}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c}{5ad}$$

$$= -\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a^2d} + \frac{2A\cos^{\frac{3}{2}}(c+dx)\sin(c}{5ad}$$

$$= \frac{2(3a^2A+5Ab^2-5abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5a^3d} - \frac{2(Ab-aB)\sqrt{\cos(c}}{3a^2}$$

$$= \frac{2(3a^2A+5Ab^2-5abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5a^3d} - \frac{2(a^2+3b^2)\,(Ab-aI}{3a^4}$$

Mathematica [A]
time = 12.58, size = 260, normalized size = 1.43

$$\frac{\frac{2a^2(9a^2A+5Ab^2-5abB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + 2a^2(4Ab+5aB)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right) - \frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right) + 4a^2\sqrt{\cos(c+dx)}\,(-5Ab+5aB+3aA\cos(c+dx))\sin(c+dx) + \frac{6(3a^2A+5Ab^2-5abB)\left(-2abE\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{b\sqrt{\sin^2(c+dx)}}}{30a^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] ((2*a^2*(9*a^2*A + 5*A*b^2 - 5*a*b*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2
, 2])/(a + b) + 2*a^2*(4*A*b + 5*a*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*E
llipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)) + 4*a^2*Sqrt[Cos[c + d*x
]]*(-5*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[c + d*x] + (6*(3*a^2*A + 5*A*b
^2 - 5*a*b*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a +
b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a
```

```
/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(b*Sqrt[Sin[c + d*x]^2]
))/(30*a^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1073 vs. $2(248) = 496$.
time = 3.04, size = 1074, normalized size = 5.90

method	result	size
default	Expression too large to display	1074

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/15*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*((-24*A*a^4+2
4*A*a^3*b)*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^6+(24*A*a^4-44*A*a^3*b+20*
A*a^2*b^2+20*B*a^4-20*B*a^3*b)*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+(-6*
A*a^4+16*A*a^3*b-10*A*a^2*b^2-10*B*a^4+10*B*a^3*b)*sin(1/2*d*x+1/2*c)^2*cos
(1/2*d*x+1/2*c)-5*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+5*A*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))*a^2*b^2-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+15*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))*b^4-9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^4+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b
-15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2
*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3-
15*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b^4+5*B*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a
^4-5*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipt
icF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*
sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2
-15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^3+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*s
in(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^3*b-15
*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b^2+15*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*si
n(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)
)*a*b^3)/a^4/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin
(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+\frac{B}{\cos(c+d\,x)}\right)}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x)),x)
[Out] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x)), x)
```

3.575 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=136

$$-\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2(a^2A+3Ab^2-3abB)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d}-\frac{2b^2(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\right|}{a^3(a+b)d}$$

```
[Out] -2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))/a^2/d+2/3*(A*a^2+3*A*b^2-3*B*a*b)*(cos(1/2*d*x+1/2*
c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/d-
2*b^2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(
sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^3/(a+b)/d+2/3*A*sin(d*x+c)*cos(d*x+
c)^(1/2)/a/d
```

Rubi [A]
time = 0.37, antiderivative size = 136, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3033, 3069, 3138, 2719, 3081, 2720, 2884}

$$-\frac{2b^2(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^3d(a+b)}-\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2(a^2A-3abB+3Ab^2)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^3d}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (-2*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(a^2*d) + (2*(a^2*A + 3*A*b^2 -
3*a*b*B)*EllipticF[(c + d*x)/2, 2])/(3*a^3*d) - (2*b^2*(A*b - a*B)*Elliptic
Pi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^3*(a + b)*d) + (2*A*Sqrt[Cos[c + d*x]
]*Sin[c + d*x])/(3*a*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3069

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-b)*B*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c + d*Sin[e + f*x])^(
n + 1)/(d*f*(m + n + 1))), x] + Dist[1/(d*(m + n + 1)), Int[(a + b*Sin[e +
f*x])^(m - 2)*(c + d*Sin[e + f*x])^n*Simp[a^2*A*d*(m + n + 1) + b*B*(b*c*(m
 - 1) + a*d*(n + 1)) + (a*d*(2*A*b + a*B)*(m + n + 1) - b*B*(a*c - b*d*(m +
 n)))*Sin[e + f*x] + b*(A*b*d*(m + n + 1) - B*(b*c*m - a*d*(2*m + n)))*Sin[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}, x] && NeQ[b*c
- a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] &&  !(IGtQ
[n, 1] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)(B+A\cos(c+dx))}{b+a\cos(c+dx)}\,dx$$

$$= \frac{2A\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} + \frac{2\int \frac{\frac{Ab}{2}+\frac{1}{2}aA\cos(c+dx)-\frac{3}{2}(Ab-aB)\cos^2(}{\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx}}{3a}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} - \frac{2\int \frac{-\frac{1}{2}aAb-\frac{1}{2}(a^2A+3Ab^2-3abB)\cos(c+}{\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx)}}{3a^2}$$

$$= -\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2A\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3ad} -$$

$$= -\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2(a^2A+3Ab^2-3abB)\,F\left(\frac{1}{2}(c-\right.}{3a^3d}$$

Mathematica [A]
time = 11.54, size = 207, normalized size = 1.52

$$\frac{\frac{(-Ab+3aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)|2\right)}{a+b}+A\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)|2\right)}{a+b}\right)+2A\sqrt{\cos(c+dx)}\,\sin(c+dx)+\frac{3(-Ab+aB)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a^2b\sqrt{\sin^2(c+dx)}}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (((-(A*b) + 3*a*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + A*(
2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2
])/(a + b)) + 2*A*Sqrt[Cos[c + d*x]]*Sin[c + d*x] + (3*(-(A*b) + a*B)*(-2*a
*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin
[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Co
s[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/(3*a*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 821 vs. $2(208) = 416$.
time = 2.70, size = 822, normalized size = 6.04

method	result	size
default	Expression too large to display	822

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2/3*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(4*A*cos(1/2*d
*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^3-4*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c
```

```
)^4*a^2*b-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^2*a^3+2*A*cos(1/2*d*x+1
/2*c)*sin(1/2*d*x+1/2*c)^2*a^2*b+A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*
d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^3-A*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+
1/2*c),2^(1/2))*a^2*b+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c
)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-3*A*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))*b^3+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^
(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*A*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/
2))*a*b^2+3*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b^3-3*B*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2
^(1/2))*a^2*b+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(
1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b^2-3*B*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2
))*a^3+3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a^2*b-3*B*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^
(1/2))*a*b^2)/a^3/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
maxima")
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
fricas")
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+\frac{B}{\cos(c+d\,x)}\right)}{a+\frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x)), x)
```

3.576 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx$

Optimal. Leaf size=89

$$\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2b(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2(a+b)d}$$

```
[Out] 2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1
/2*c),2^(1/2))/a/d-2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/d+2*b*(A*b-B*a)*(cos(1/2*d*x+
1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),
2^(1/2))/a^2/(a+b)/d
```

Rubi [A]
time = 0.18, antiderivative size = 89, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3033, 3081, 2719, 2882, 2720, 2884}

$$-\frac{2(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d}+\frac{2b(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a+b)}+\frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (2*A*EllipticE[(c + d*x)/2, 2])/(a*d) - (2*(A*b - a*B)*EllipticF[(c + d*x)/
2, 2])/(a^2*d) + (2*b*(A*b - a*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
)/(a^2*(a + b)*d)
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2882

```
Int[Sqrt[(c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]]/((a_.) + (b_.)*sin[(e_.) +
(f_.)*(x_)]), x_Symbol] :> Dist[d/b, Int[1/Sqrt[c + d*Sin[e + f*x]], x], x]
 + Dist[(b*c - a*d)/b, Int[1/((a + b*Sin[e + f*x])*Sqrt[c + d*Sin[e + f*x]]
), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 -
 b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{a+b\sec(c+dx)}\,dx = \int \frac{\sqrt{\cos(c+dx)}\,(B+A\cos(c+dx))}{b+a\cos(c+dx)}\,dx$$

$$= \frac{A\int\sqrt{\cos(c+dx)}\,dx}{a} - \frac{(Ab-aB)\int\frac{\sqrt{\cos(c+dx)}}{b+a\cos(c+dx)}\,dx}{a}$$

$$= \frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{(Ab-aB)\int\frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a^2} + \frac{(b(Ab$$

$$= \frac{2AE\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d} + \frac{2b(Ab-$$

Mathematica [A]
time = 10.99, size = 128, normalized size = 1.44

$$\frac{aB\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)-\frac{2A\left(aE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)-(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+b\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{\sqrt{\sin^2(c+dx)}}}{a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x]),x]
```

```
[Out] (a*B*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x
)/2, 2])/(a + b)) - (2*A*(a*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] - (a
+ b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + b*EllipticPi[-(a/b), ArcSi
n[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/Sqrt[Sin[c + d*x]^2])/(a^2*d)
```

Maple [A]
time = 2.20, size = 295, normalized size = 3.31

method	result
default	$\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(A\,\mathrm{EllipticF}\left(\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] 2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2*
c),2^(1/2))*a*b-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b^2+A*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2-A*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b+A*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b^2-B*EllipticF(cos(1/2*d*
x+1/2*c),2^(1/2))*a^2+B*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-B*Ellipti
cPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*a*b)/a^2/(a-b)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)
^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sqrt{\cos(c + dx)}}{a + b\sec(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(cos(c + d*x))/(a + b*sec(c + d*x)), x)
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(1/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos(c + d\,x)}\,\left(A + \frac{B}{\cos(c+d\,x)}\right)}{a + \frac{b}{\cos(c+d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x)),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x)), x)
```

3.577 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=61

$$\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a+b)d}$$

```
[Out] 2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))/a/d-2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a/(a+b)/d
```

Rubi [A]
time = 0.14, antiderivative size = 61, normalized size of antiderivative = 1.00, number of steps used = 4, number of rules used = 4, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}}$ = 0.121, Rules used = {3033, 3081, 2720, 2884}

$$\frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad}-\frac{2(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{ad(a+b)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*A*EllipticF[(c + d*x)/2, 2])/(a*d) - (2*(A*b - a*B)*EllipticPi[(2*a)/(a
+ b), (c + d*x)/2, 2])/(a*(a + b)*d)
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))}\,dx = \int \frac{B+A\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))}\,dx$$
$$= \frac{A\int \frac{1}{\sqrt{\cos(c+dx)}}\,dx}{a} - \frac{(Ab-aB)\int \frac{1}{\sqrt{\cos(c+dx)}\,(b+a\cos(c+d}$$
$$= \frac{2AF\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad} - \frac{2(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a(a+b)d}$$

Mathematica [A]
time = 0.23, size = 58, normalized size = 0.95

$$\frac{2\left(A(a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)+(-Ab+aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a(a+b)d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*(A*(a + b)*EllipticF[(c + d*x)/2, 2] + (-(A*b) + a*B)*EllipticPi[(2*a)/(
a + b), (c + d*x)/2, 2]))/(a*(a + b)*d)
```

Maple [A]
time = 1.82, size = 217, normalized size = 3.56

method	result
default	$-\frac{2\sqrt{\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)-1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\,\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\,\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\left(A\,\mathrm{Elliptic}\right.}{a(a-b)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin}}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -2*((2*cos(1/2*d*x+1/2*c)^2-1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)*(A*EllipticF(cos(1/2*d*x+1/2
*c),2^(1/2))*a-A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*b+A*EllipticPi(cos(1
/2*d*x+1/2*c),2*a/(a-b),2^(1/2))*b-B*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b
),2^(1/2))*a)/a/(a-b)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x
)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec{\left(c + dx \right)}}{\left(a + b\sec{\left(c + dx \right)}\right)\sqrt{\cos{\left(c + dx \right)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/((a + b*sec(c + d*x))*sqrt(cos(c + d*x))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*sqrt(cos(d*x + c))), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.02

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\sqrt{\cos\left(c+d\,x\right)}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))), x)
```

3.578 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=86

$$-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{2(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d}+\frac{2B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}$$

```
[Out] -2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+
1/2*c),2^(1/2))/b/d+2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/b/(a+b)/d+2*B*sin(d*x
+c)/b/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.26, antiderivative size = 86, normalized size of antiderivative = 1.00, number of steps used = 6, number of rules used = 6, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.182$, Rules used = {3033, 3079, 3138, 2719, 12, 2884}

$$\frac{2(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{bd(a+b)}-\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd}+\frac{2B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (-2*B*EllipticE[(c + d*x)/2, 2])/(b*d) + (2*(A*b - a*B)*EllipticPi[(2*a)/(a
 + b), (c + d*x)/2, 2])/(b*(a + b)*d) + (2*B*Sin[c + d*x])/(b*d*Sqrt[Cos[c
+ d*x]])
```

Rule 12

```
Int[(a_)*(u_), x_Symbol] :> Dist[a, Int[u, x], x] /; FreeQ[a, x] &&  !Match
Q[u, (b_)*(v_) /; FreeQ[b, x]]
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{B+A\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))}\,dx$$

$$= \frac{2B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{2\int \frac{\frac{1}{2}(Ab-aB)-\frac{1}{2}bB\cos(c+dx)-\frac{1}{2}aB\cos^2(c+dx)}{\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))}dx}{b}$$

$$= \frac{2B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} - \frac{2\int -\frac{a(Ab-aB)}{2\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))}dx}{ab} - \frac{B\int \sqrt{\cdots}}{\cdots}$$

$$= -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} + \frac{2B\sin(c+dx)}{bd\sqrt{\cos(c+dx)}} + \frac{(Ab-aB)\int \frac{\cdots}{\sqrt{\cos(c+\cdots}}}{b\cdots}$$

$$= -\frac{2BE\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd} + \frac{2(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b(a+b)d} + \frac{2B\sin\cdots}{bd\sqrt{\cos\cdots}}$$

Mathematica [B] Leaf count is larger than twice the leaf count of optimal. 206 vs. 2(86) = 172.
time = 12.56, size = 206, normalized size = 2.40

$$\frac{\frac{2(2Ab-3aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)|2\right)}{a+b} - \frac{2bB\left(2F\left(\frac{1}{2}(c+dx)|2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)|2\right)}{a+b}\right)}{a} + \frac{4B\sin(c+dx)}{\sqrt{\cos(c+dx)}} - \frac{2B\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)|-1\right)\right)\sin(c+dx)}{ab\sqrt{\sin^2(c+dx)}}}{2bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] ((2*(2*A*b - 3*a*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) - (2
*b*B*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x
)/2, 2])/(a + b)))/a + (4*B*Sin[c + d*x])/Sqrt[Cos[c + d*x]] - (2*B*(-2*a*b
*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[S
qrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[
c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(2*b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 297 vs. 2(136) = 272.
time = 2.79, size = 298, normalized size = 3.47

method	result

default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(-\frac{2(Ab-Ba)a\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right.}}{b\left(a^2-ba\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*(A*b-B*a)/b/
(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/
(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x
+1/2*c),2*a/(a-b),2^(1/2))+2*B/b/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)
^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1
/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2
*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x
)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec(c + dx)}{(a + b\sec(c + dx))\cos^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/((a + b*sec(c + d*x))*cos(c + d*x)**(3/2)), x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A + \frac{B}{\cos(c+d\,x)}}{\cos(c + d\,x)^{3/2}\,\left(a + \frac{b}{\cos(c+d\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))), x)
```

3.579 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=150

$$-\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd}-\frac{2a(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2(a+b)d}+\frac{2B\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)}$$

```
[Out] -2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(
1/2*d*x+1/2*c),2^(1/2))/b^2/d+2/3*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*
x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b/d-2*a*(A*b-B*a)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+
b),2^(1/2))/b^2/(a+b)/d+2/3*B*sin(d*x+c)/b/d/cos(d*x+c)^(3/2)+2*(A*b-B*a)*s
in(d*x+c)/b^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.54, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3033, 3079, 3134, 3138, 2719, 3081, 2720, 2884}

$$-\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d}-\frac{2a(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a+b)}+\frac{2(Ab-aB)\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}}+\frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd}+\frac{2B\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (-2*(A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(b^2*d) + (2*B*EllipticF[(c + d*
x)/2, 2])/(3*b*d) - (2*a*(A*b - a*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2])/(b^2*(a + b)*d) + (2*B*Sin[c + d*x])/(3*b*d*Cos[c + d*x]^(3/2)) + (2*(
A*b - a*B)*Sin[c + d*x])/(b^2*d*Sqrt[Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
```

```
0] && GtQ[c + d, 0]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
```

```
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\begin{aligned}
\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))}\,dx &= \int \frac{B+A\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b+a\cos(c+dx))}\,dx \\
&= \frac{2B\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} + \frac{2\int \frac{\frac{3}{2}(Ab-aB)+\frac{1}{2}bB\cos(c+dx)+\frac{1}{2}aB\cos^2(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))}\,dx}{3b} \\
&= \frac{2B\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} + \frac{2(Ab-aB)\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}} + \frac{4\int \frac{\frac{1}{4}(-3aAb+3a^2B+b^2\cdots}{\cdots}}{\cdots} \\
&= \frac{2B\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} + \frac{2(Ab-aB)\sin(c+dx)}{b^2d\sqrt{\cos(c+dx)}} - \frac{4\int \frac{\frac{1}{4}a(3aAb-3a^2B-b^2\cdots}{\sqrt{\cos(c+dx}\cdots}}{3a\cdots} \\
&= -\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2B\sin(c+dx)}{3bd\cos^{\frac{3}{2}}(c+dx)} + \frac{2(Ab-aB)\cdots}{b^2d\sqrt{\cos(\cdots}} \\
&= -\frac{2(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d} + \frac{2BF\left(\frac{1}{2}(c+dx)\middle|2\right)}{3bd} - \frac{2a(Ab-\cdots}{\cdots}
\end{aligned}$$

Mathematica [A]

time = 12.26, size = 260, normalized size = 1.73

$$\frac{\frac{2b(-9aAb+9a^2B+2b^2B)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{b(-6Ab^2+8abB)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{a} + \frac{4b^2B\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} + \frac{12b(Ab-aB)\sin(c+dx)}{\sqrt{\cos(c+dx)}} + \frac{6(-Ab+aB)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a\sqrt{\sin^2(c+dx)}}}{6b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] ((2*b*(-9*a*A*b + 9*a^2*B + 2*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2,
 2])/(a + b) + (b*(-6*A*b^2 + 8*a*b*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*
EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a + (4*b^2*B*Sin[c + d
*x])/Cos[c + d*x]^(3/2) + (12*b*(A*b - a*B)*Sin[c + d*x])/Sqrt[Cos[c + d*x]
] + (6*(-(A*b) + a*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2
*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*Ellipt
icPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c +
 d*x]^2]))/(6*b^3*d)
```

Maple [A]
time = 5.36, size = 439, normalized size = 2.93

method	result
default	$-\frac{\sqrt{-\left(-2\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2B\left(-\frac{\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^{4}\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^{2}\left(\frac{dx}{2}+\right.}}{6\left(\cos^{2}\left(\frac{dx}{2}+\frac{c}{2}\right)-\frac{1}{2}\right)^{2}}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b*(-1/6*cos
(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos(1
/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b-B*a)*a^2/b^2/(a^2-a*b)*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)
)+2*(A*b-B*a)/b^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2
*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/
2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x
)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3007 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)), x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\cos\left(c+d\,x\right)}^{5/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))), x)
```

3.580 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx$

Optimal. Leaf size=217

$$\frac{2(5aAb-5a^2B-3b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d}+\frac{2(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d}+\frac{2a^2(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3(a+b)d}$$

```
[Out] 2/5*(5*A*a*b-5*B*a^2-3*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/d+2/3*(A*b-B*a)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^2
/d+2*a^2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Elliptic
Pi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/b^3/(a+b)/d+2/5*B*sin(d*x+c)/b/d/c
os(d*x+c)^(5/2)+2/3*(A*b-B*a)*sin(d*x+c)/b^2/d/cos(d*x+c)^(3/2)-2/5*(5*A*a*
b-5*B*a^2-3*B*b^2)*sin(d*x+c)/b^3/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.76, antiderivative size = 217, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3033, 3079, 3134, 3138, 2719, 3081, 2720, 2884}

$$\frac{2a^2(Ab-aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a+b)}+\frac{2(-5a^2B+5aAb-3b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{5b^3d}-\frac{2(-5a^2B+5aAb-3b^2B)\sin(c+dx)}{5b^3d\sqrt{\cos(c+dx)}}+\frac{2(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d}+\frac{2(Ab-aB)\sin(c+dx)}{3b^2d\cos^{\frac{3}{2}}(c+dx)}+\frac{2B\sin(c+dx)}{5bd\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] (2*(5*a*A*b - 5*a^2*B - 3*b^2*B)*EllipticE[(c + d*x)/2, 2])/(5*b^3*d) + (2*
(A*b - a*B)*EllipticF[(c + d*x)/2, 2])/(3*b^2*d) + (2*a^2*(A*b - a*B)*Ellip
ticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(b^3*(a + b)*d) + (2*B*Sin[c + d*x])/
(5*b*d*Cos[c + d*x]^(5/2)) + (2*(A*b - a*B)*Sin[c + d*x])/(3*b^2*d*Cos[c +
d*x]^(3/2)) - (2*(5*a*A*b - 5*a^2*B - 3*b^2*B)*Sin[c + d*x])/(5*b^3*d*Sqrt[
Cos[c + d*x]])
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
```

```
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))}\,dx = \int \frac{B+A\cos(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(b+a\cos(c+dx))}\,dx$$

$$= \frac{2B\sin(c+dx)}{5bd\cos^{\frac{5}{2}}(c+dx)} + \frac{2\int \frac{\frac{5}{2}(Ab-aB)+\frac{3}{2}bB\cos(c+dx)+\frac{3}{2}aB\cos^2(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b+a\cos(c+dx))}\,dx}{5b}$$

$$= \frac{2B\sin(c+dx)}{5bd\cos^{\frac{5}{2}}(c+dx)} + \frac{2(Ab-aB)\sin(c+dx)}{3b^2d\cos^{\frac{3}{2}}(c+dx)} + \frac{4\int \frac{-\frac{3}{4}(5aAb-5a^2B-3b^2B}{}}{}$$

$$= \frac{2B\sin(c+dx)}{5bd\cos^{\frac{5}{2}}(c+dx)} + \frac{2(Ab-aB)\sin(c+dx)}{3b^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{2(5aAb-5a^2B-3b}{5b^3d\sqrt{\cos(c}}$$

$$= \frac{2B\sin(c+dx)}{5bd\cos^{\frac{5}{2}}(c+dx)} + \frac{2(Ab-aB)\sin(c+dx)}{3b^2d\cos^{\frac{3}{2}}(c+dx)} - \frac{2(5aAb-5a^2B-3b}{5b^3d\sqrt{\cos(c}}$$

$$= \frac{2(5aAb-5a^2B-3b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5b^3d} + \frac{2B\sin(c+dx)}{5bd\cos^{\frac{5}{2}}(c+dx)} + \frac{2(A}{}$$

$$= \frac{2(5aAb-5a^2B-3b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{5b^3d} + \frac{2(Ab-aB)F\left(\frac{1}{2}(c+d\right.}{3b^2d}$$

Mathematica [A]
time = 14.60, size = 326, normalized size = 1.50

$$\frac{\frac{b(45a^2Ab+10Ab^3-45a^3B-19ab^2B)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} - \frac{b^2(-20aAb+20a^2B+9b^2B)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{a} + \frac{6b^3B\sin(c+dx)}{\cos^{\frac{5}{2}}(c+dx)} + \frac{10b^2(Ab-aB)\sin(c+dx)}{\cos^{\frac{3}{2}}(c+dx)} + \frac{6b(-5aAb+5a^2B+3b^2B)\sin(c+dx)}{\sqrt{\cos(c+dx)}} - \frac{3(-5aAb+5a^2B+3b^2B)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a\sqrt{\sin^2(c+dx)}}}{15b^4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])),x]
```

```
[Out] ((b*(45*a^2*A*b + 10*A*b^3 - 45*a^3*B - 19*a*b^2*B)*EllipticPi[(2*a)/(a + b
), (c + d*x)/2, 2])/(a + b) - (b^2*(-20*a*A*b + 20*a^2*B + 9*b^2*B)*(2*Elli
pticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a
+ b)))/a + (6*b^3*B*Sin[c + d*x])/Cos[c + d*x]^(5/2) + (10*b^2*(A*b - a*B)*
Sin[c + d*x])/Cos[c + d*x]^(3/2) + (6*b*(-5*a*A*b + 5*a^2*B + 3*b^2*B)*Sin[
c + d*x])/Sqrt[Cos[c + d*x]] - (3*(-5*a*A*b + 5*a^2*B + 3*b^2*B)*(-2*a*b*El
lipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt
[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c +
 d*x]]], -1])*Sin[c + d*x])/(a*Sqrt[Sin[c + d*x]^2]))/(15*b^4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 757 vs. $2(279) = 558$.
time = 8.67, size = 758, normalized size = 3.49

method	result	size
default	Expression too large to display	758

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x,method=_RETURNVERB
OSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(A*b-B*a)/b^2
*(-1/6*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)/(cos(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2/5*B/b/(8*sin(1/2*d*x+1/2*c)^6-1
2*sin(1/2*d*x+1/2*c)^4+6*sin(1/2*d*x+1/2*c)^2-1)/sin(1/2*d*x+1/2*c)^2*(24*s
in(1/2*d*x+1/2*c)^6*cos(1/2*d*x+1/2*c)-12*EllipticE(cos(1/2*d*x+1/2*c),2^(1
/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*sin(1/2*
d*x+1/2*c)^4-24*sin(1/2*d*x+1/2*c)^4*cos(1/2*d*x+1/2*c)+12*EllipticE(cos(1/
2*d*x+1/2*c),2^(1/2))*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-
1)^(1/2)*sin(1/2*d*x+1/2*c)^2+8*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-3*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2)))*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)-2*(A*b-B*a)*a^3/b^3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d
*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*E
llipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*(A*b-B*a)/b^3*a/sin(1/2*d
*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+
1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c)
,2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="
maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x
)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="
fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c)),x, algorithm="
giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)*cos(d*x + c)^(7/2)), x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\cos\left(c+d\,x\right)}^{7/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))), x)
```

3.581 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=305

$$-\frac{(4a^2Ab-5Ab^3-2a^3B+3ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\,(a^2-b^2)\,d}+\frac{(2a^4A+16a^2Ab^2-15Ab^4-12a^3bB+9ab^3B)\,F\left(\frac{1}{2}(c\right.}{3a^4\,(a^2-b^2)\,d}$$

```
[Out] -(4*A*a^2*b-5*A*b^3-2*B*a^3+3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)/d+1/3*(2*A*
a^4+16*A*a^2*b^2-15*A*b^4-12*B*a^3*b+9*B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^4/(a^2-b^2)/d-
b^2*(7*A*a^2*b-5*A*b^3-5*B*a^3+3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^4/(a-b)/(
a+b)^2/d+b*(A*b-B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*cos(d*x
+c))+1/3*(2*A*a^2-5*A*b^2+3*B*a*b)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2/(a^2-b^2
)/d
```

Rubi [A]
time = 0.66, antiderivative size = 305, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3033, 3068, 3128, 3138, 2719, 3081, 2720, 2884}

$$\frac{b(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{ad\,(a^2-b^2)\,(a\cos(c+dx)+b)}+\frac{(2a^2A+3abB-5Ab^2)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d\,(a^2-b^2)}-\frac{(-2a^3B+4a^2Ab+3ab^2B-5Ab^3)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3d\,(a^2-b^2)}-\frac{b^2(-5a^3B+7a^2Ab+3ab^2B-5Ab^3)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^4d(a-b)(a+b)^2}+\frac{(2a^4A-12a^3bB+16a^2Ab^2+9ab^3B-15Ab^4)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3a^4d\,(a^2-b^2)}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] -(((4*a^2*A*b - 5*A*b^3 - 2*a^3*B + 3*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(
a^3*(a^2 - b^2)*d)) + ((2*a^4*A + 16*a^2*A*b^2 - 15*A*b^4 - 12*a^3*b*B + 9*
a*b^3*B)*EllipticF[(c + d*x)/2, 2])/(3*a^4*(a^2 - b^2)*d) - (b^2*(7*a^2*A*b
 - 5*A*b^3 - 5*a^3*B + 3*a*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]
)/(a^4*(a - b)*(a + b)^2*d) + ((2*a^2*A - 5*A*b^2 + 3*a*b*B)*Sqrt[Cos[c + d
*x]]*Sin[c + d*x])/(3*a^2*(a^2 - b^2)*d) + (b*(A*b - a*B)*Cos[c + d*x]^(3/2
)*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
```

```
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+A\cos(c+dx))}{(b+a\cos(c+dx))^2}\,dx$$

$$= \frac{b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d(b+a\cos(c+dx))} + \frac{\int \sqrt{\cos(c+dx)}\ \left(\tfrac{3}{2}b(Ab-aB\right.}{}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sqrt{\cos(c+dx)}\ \sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\,c}{a\left(a^2-b^2\right.}$$

$$= \frac{\left(2a^2A-5Ab^2+3abB\right)\sqrt{\cos(c+dx)}\ \sin(c+dx)}{3a^2\left(a^2-b^2\right)d} + \frac{b(Ab-aB)\,c}{a\left(a^2-b^2\right.}$$

$$= -\frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^2A-5A\right.}{}$$

$$= -\frac{\left(4a^2Ab-5Ab^3-2a^3B+3ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^3\left(a^2-b^2\right)d} + \frac{\left(2a^4A+16\right.}{}$$

Mathematica [A]
time = 13.38, size = 318, normalized size = 1.04

$$\frac{4\sqrt{\cos(c+dx)}\ \left(2A+\frac{3b^2(Ab-aB)}{(-a^2+b^2)(b+a\cos(c+dx))}\right)\sin(c+dx)-\frac{\frac{2\left(-8a^2Ab+5Ab^3+6a^3B-3ab^2B\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8\left(a^2A+2Ab^2-3abB\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{6\left(-4a^2Ab+5Ab^3+2a^3B-3ab^2B\right)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a^2b\sqrt{\sin^2(c+dx)}}}{(-a+b)(a+b)}}{12a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,
x]
```

```
[Out] (4*Sqrt[Cos[c + d*x]]*(2*A + (3*b^2*(A*b - a*B))/((-a^2 + b^2)*(b + a*Cos[c
 + d*x])))*Sin[c + d*x] - ((2*(-8*a^2*A*b + 5*A*b^3 + 6*a^3*B - 3*a*b^2*B)*
EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*(a^2*A + 2*A*b^2 -
3*a*b*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (
c + d*x)/2, 2]))/(a + b) + (6*(-4*a^2*A*b + 5*A*b^3 + 2*a^3*B - 3*a*b^2*B)*
(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[A
rcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sq
rt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((-a +
b)*(a + b)))/(12*a^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1058 vs. $2(375) = 750$.
time = 6.83, size = 1059, normalized size = 3.47

method	result	size
default	Expression too large to display	1059

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/a^4*(4*A*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^2-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^2*a^2+a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+9*A*b^2*(sin(1/2*d*x+1/2*c)^2
)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/
2))+6*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-6*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos(
1/2*d*x+1/2*c),2^(1/2))*a^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)+2*b^2/a^3*(4*A*b-3*B*a)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*b^3*(A*b-B*a)/a^4
*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^
2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x
```

```
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2
))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x
+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ell
ipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1
/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(cos(d*x + c))/
(b^2*sec(d*x + c)^2 + 2*a*b*sec(d*x + c) + a^2), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^2, x)
```

3.582 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=223

$$\frac{(2a^2A-3Ab^2+abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^2\,(a^2-b^2)\,d}-\frac{(4a^2Ab-3Ab^3-2a^3B+ab^2B)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^3\,(a^2-b^2)\,d}+\frac{b(5a^2Ab-3Ab^3}{}$$

```
[Out] (2*A*a^2-3*A*b^2+B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)/d-(4*A*a^2*b-3*A*b^3-2*B*a
^3+B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)/d+b*(5*A*a^2*b-3*A*b^3-3*B*a^3+B*a*b^2
)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/
2*c),2*a/(a+b),2^(1/2))/a^3/(a-b)/(a+b)^2/d+b*(A*b-B*a)*sin(d*x+c)*cos(d*x+
c)^(1/2)/a/(a^2-b^2)/d/(b+a*cos(d*x+c))
```

Rubi [A]

time = 0.45, antiderivative size = 223, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3033, 3068, 3138, 2719, 3081, 2720, 2884}

$$\frac{(2a^2A+abB-3Ab^2)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^2d\,(a^2-b^2)}+\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\,(a^2-b^2)\,(a\cos(c+dx)+b)}-\frac{(-2a^3B+4a^2Ab+ab^2B-3Ab^3)\,F\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^3d\,(a^2-b^2)}+\frac{b(-3a^3B+5a^2Ab+ab^2B-3Ab^3)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|\,2\right)}{a^3d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,x]
```

```
[Out] ((2*a^2*A - 3*A*b^2 + a*b*B)*EllipticE[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d)
 - ((4*a^2*A*b - 3*A*b^3 - 2*a^3*B + a*b^2*B)*EllipticF[(c + d*x)/2, 2])/(a
^3*(a^2 - b^2)*d) + (b*(5*a^2*A*b - 3*A*b^3 - 3*a^3*B + a*b^2*B)*EllipticPi
[(2*a)/(a + b), (c + d*x)/2, 2])/(a^3*(a - b)*(a + b)^2*d) + (b*(A*b - a*B)
*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*(b + a*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
```

```
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{(a+b\sec(c+dx))^2}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)(B+A\cos(c+dx))}{(b+a\cos(c+dx))^2}\,dx$$

$$= \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\,(a^2-b^2)\,d(b+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}b(Ab-aB)-a(Ab-aB)\cos}{\sqrt{\cos(c}}}{}$$

$$= \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\,(a^2-b^2)\,d(b+a\cos(c+dx))} - \frac{\int \frac{-\frac{1}{2}ab(Ab-aB)+\frac{1}{2}(4a^2Ab-}{\sqrt{\cos(c+d}}}{a^2\,(}$$

$$= \frac{(2a^2A-3Ab^2+abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^2\,(a^2-b^2)\,d} + \frac{b(Ab-aB)\sqrt{\cos(c+}}{a\,(a^2-b^2)\,d(b+a}$$

$$= \frac{(2a^2A-3Ab^2+abB)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{a^2\,(a^2-b^2)\,d} - \frac{(4a^2Ab-3Ab^3-2a^3}{a^3\,(}$$

Mathematica [A]
time = 12.89, size = 281, normalized size = 1.26

$$\frac{\frac{4b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{(a^2-b^2)(b+a\cos(c+dx))}+\frac{\frac{2(2a^2A-Ab^2-abB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8(-Ab+aB)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{2(2a^2A-3Ab^2+abB)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a^2b\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}}{4ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^2,
x]
```

```
[Out] ((4*b*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[
c + d*x])) + ((2*(2*a^2*A - A*b^2 - a*b*B)*EllipticPi[(2*a)/(a + b), (c + d
*x)/2, 2])/(a + b) + (8*(-(A*b) + a*B)*((a + b)*EllipticF[(c + d*x)/2, 2] -
 b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a + b) + (2*(2*a^2*A - 3*A*
b^2 + a*b*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b
)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/
b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]
^2]))/((a - b)*(a + b)))/(4*a*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 842 vs. $2(299) = 598$.
time = 5.76, size = 843, normalized size = 3.78

method	result	size
default	Expression too large to display	843

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^3/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(2*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
b+A*a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*a*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))-2*b^2*(A*b-B*a)/a^3*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a
+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-
b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)
))-2*b/a^2*(3*A*b-2*B*a)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*co
s(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^2, x
)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b^2*sec(d*x + c)^2 + 2*a*
b*sec(d*x + c) + a^2), x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sqrt{\cos(c + dx)}}{(a + b\sec(c + dx))^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**2, x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^2, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos(c + d\,x)}\,\left(A + \frac{B}{\cos(c+d\,x)}\right)}{\left(a + \frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^2,x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^2, x)
```

3.583 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=203

$$\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d}+\frac{\left(2a^2A-Ab^2-abB\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2\left(a^2-b^2\right)d}-\frac{\left(3a^2Ab-Ab^3-a^3B-ab^2B\right)\Pi\left(\frac{2a}{a+b};\right.}{a^2(a-b)(a+b)^2d}$$

```
[Out] (A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2
*d*x+1/2*c),2^(1/2))/a/(a^2-b^2)/d+(2*A*a^2-A*b^2-B*a*b)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^
2-b^2)/d-(3*A*a^2*b-A*b^3-B*a^3-B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^2/(a-b)/(a
+b)^2/d-(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d/(b+a*cos(d*x+c))
```

Rubi [A]

time = 0.39, antiderivative size = 203, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3033, 3078, 3138, 2719, 3081, 2720, 2884}

$$\frac{\left(2a^2A-abB-Ab^2\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a^2d\left(a^2-b^2\right)}+\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{d\left(a^2-b^2\right)(a\cos(c+dx)+b)}-\frac{\left(a^3(-B)+3a^2Ab-ab^2B-Ab^3\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] ((A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) + ((2*a^2*A - A*b
^2 - a*b*B)*EllipticF[(c + d*x)/2, 2])/(a^2*(a^2 - b^2)*d) - ((3*a^2*A*b -
A*b^3 - a^3*B - a*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a^2*(a
 - b)*(a + b)^2*d) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 -
b^2)*d*(b + a*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3078

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^2}\,dx = \int \frac{\sqrt{\cos(c+dx)}\,(B+A\cos(c+dx))}{(b+a\cos(c+dx))^2}\,dx$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} + \frac{\int \frac{\frac{1}{2}(-Ab+aB)+(aA-bB)\,c}{\sqrt{\cos(c+}}}{}$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{(a^2-b^2)\,d(b+a\cos(c+dx))} - \frac{\int \frac{\frac{1}{2}a(Ab-aB)-\frac{1}{2}(2a^2A-}{\sqrt{\cos(c+dx)}\,(}}{a\,(a^2-}$$

$$= \frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\,(a^2-b^2)\,d} - \frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+}{(a^2-b^2)\,d(b+a\cos(c+dx)}$$

$$= \frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\,(a^2-b^2)\,d} + \frac{(2a^2A-Ab^2-abB)\,F\left(\frac{1}{2}(c+dx\right.}{a^2\,(a^2-b^2)\,d}$$

Mathematica [A]
time = 12.47, size = 260, normalized size = 1.28

$$\frac{\frac{4(-Ab+aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{(a^2-b^2)(b+a\cos(c+dx))} - \frac{\frac{2(-Ab+aB)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b} + \frac{(4aA-4bB)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{a} + \frac{2(Ab-aB)\left(-2abE\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a^2b\sqrt{\sin^2(c+dx)}}}{(-a+b)(a+b)}}{4d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^2),
x]
```

```
[Out] ((4*(-(A*b) + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos
[c + d*x])) - ((2*(-(A*b) + a*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])
/(a + b) + ((4*a*A - 4*b*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[
(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a + (2*(A*b - a*B)*(-2*a*b*Ellipt
icE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos
[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x
]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((-a + b)*(a + b)))/(
4*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 801 vs. $2(279) = 558$.
time = 5.37, size = 802, normalized size = 3.95

method	result

default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2A\sqrt{\frac{1}{2}-\frac{\cos(dx+c)}{2}}\sqrt{-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1}\,\mathrm{Elli}}{a^2\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin^2\left(\frac{dx}{2}\right.}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^2*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*b
*(A*b-B*a)/a^2*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos
(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*
a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*(-2*A*b+B*a)
/a/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*
x+1/2*c),2*a/(a-b),2^(1/2)))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^
(1/2)/d
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm ="fricas")

[Out] Timed out

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec{(c + dx)}}{(a + b\sec{(c + dx)})^2\sqrt{\cos{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**2/cos(d*x+c)**(1/2),x)

[Out] Integral((A + B*sec(c + d*x))/((a + b*sec(c + d*x))**2*sqrt(cos(c + d*x))), x)

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^2/cos(d*x+c)^(1/2),x, algorithm ="giac")

[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*sqrt(cos(d*x + c))), x)

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+d\,x)}}{\sqrt{\cos(c + d\,x)}\,\left(a + \frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^2),x)

[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^2), x)

3.584 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=197

$$-\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\left(a^2-b^2\right)d}-\frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\left(a^2-b^2\right)d}+\frac{(a^2Ab+Ab^3+a^3B-3ab^2B)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\right|}{a(a-b)b(a+b)^2d}$$

```
[Out] -(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/
2*d*x+1/2*c),2^(1/2))/b/(a^2-b^2)/d-(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/(a^2-b^2)/d+(A*a
^2*b+A*b^3+B*a^3-3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a/(a-b)/b/(a+b)^2/d+a*(A*
b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2-b^2)/d/(b+a*cos(d*x+c))
```

Rubi [A]
time = 0.43, antiderivative size = 197, normalized size of antiderivative = 1.00, number of steps used = 7, number of rules used = 7, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.212$, Rules used = {3033, 3079, 3138, 2719, 3081, 2720, 2884}

$$-\frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{ad\left(a^2-b^2\right)}-\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\left(a^2-b^2\right)}+\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{bd\left(a^2-b^2\right)(a\cos(c+dx)+b)}+\frac{(a^3B+a^2Ab-3ab^2B+Ab^3)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{abd(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] -(((A*b - a*B)*EllipticE[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d)) - ((A*b - a*B)
*EllipticF[(c + d*x)/2, 2])/(a*(a^2 - b^2)*d) + ((a^2*A*b + A*b^3 + a^3*B -
 3*a*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a*(a - b)*b*(a + b)
^2*d) + (a*(A*b - a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(b*(a^2 - b^2)*d*(b
 + a*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
```

```
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx=\int \frac{B+A\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))^2}\,dx$$

$$=\frac{a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b\,(a^2-b^2)\,d(b+a\cos(c+dx))}-\frac{\int \frac{\frac{1}{2}(-aAb-a^2B+2b^2B)+b(Ab-}{\sqrt{\cos(c+}}}{b}$$

$$=\frac{a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{b\,(a^2-b^2)\,d(b+a\cos(c+dx))}+\frac{\int \frac{\frac{1}{2}a(aAb+a^2B-2b^2B)-\frac{1}{2}ab(Ab}{\sqrt{\cos(c+dx)}\,(b+a}}{ab\,(a^2-b^2}$$

$$=-\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\,(a^2-b^2)\,d}+\frac{a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+}{b\,(a^2-b^2)\,d(b+a\cos(c+dx))}$$

$$=-\frac{(Ab-aB)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\,(a^2-b^2)\,d}-\frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{a\,(a^2-b^2)\,d}+\frac{(a^2A}{}$$

Mathematica [A]
time = 12.67, size = 273, normalized size = 1.39

$$\frac{-\frac{4a(-Ab+aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{(a^2-b^2)(b+a\cos(c+dx))}+\frac{\frac{2\left(aAb+3a^2B-4b^2B\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{4b(-Ab+aB)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{a}+\frac{2(-Ab+aB)\left(-2abE\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}}{4bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^2),
x]
```

```
[Out] ((-4*a*(-(A*b) + a*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/((a^2 - b^2)*(b + a*
Cos[c + d*x])) + ((2*(a*A*b + 3*a^2*B - 4*b^2*B)*EllipticPi[(2*a)/(a + b),
(c + d*x)/2, 2])/(a + b) + (4*b*(-(A*b) + a*B)*(2*EllipticF[(c + d*x)/2, 2]
 - (2*b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)))/a + (2*(-(A*b)
 + a*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*Ell
ipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), A
rcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/(
(a - b)*(a + b)))/(4*b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 714 vs. $2(273) = 546$.
time = 4.70, size = 715, normalized size = 3.63

method	result

default	$-\frac{\sqrt{-\left(-2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+1\right)\left(\sin^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)}\left(\frac{2(-Ab+Ba)\left(\frac{a^2\cos\left(\frac{dx}{2}+\frac{c}{2}\right)\sqrt{-2\left(\sin^4\left(\frac{dx}{2}+\frac{c}{2}\right)\right)+\sin}}{b\left(a^2-b^2\right)\left(2\left(\cos^2\left(\frac{dx}{2}+\frac{c}{2}\right)\right)a-a+b\right)}\right.}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-A*b+B*a)/a*
(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2
-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/
2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)
)+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*A/(a^2-a*b)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))
/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(3/2)),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^{3/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^2),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^2), x)
```

3.585 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=255

$$\frac{(aAb-3a^2B+2b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\,(a^2-b^2)\,d}+\frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{b\,(a^2-b^2)\,d}+\frac{(a^2Ab-3Ab^3-3a^3B+5ab^2B)\,\Pi\left(\frac{2}{a\cdots}\right.}{(a-b)b^2(a+b)^2d}$$

```
[Out] (A*a*b-3*B*a^2+2*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)/d+(A*b-B*a)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b/(
a^2-b^2)/d+(A*a^2*b-3*A*b^3-3*B*a^3+5*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/(a-b)/
b^2/(a+b)^2/d-(A*a*b-3*B*a^2+2*B*b^2)*sin(d*x+c)/b^2/(a^2-b^2)/d/cos(d*x+c)
^(1/2)+a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(b+a*cos(d*x+c))/cos(d*x+c)^(1/
2)
```

Rubi [A]

time = 0.62, antiderivative size = 255, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3033, 3079, 3134, 3138, 2719, 3081, 2720, 2884}

$$\frac{(Ab-aB)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{bd\,(a^2-b^2)}+\frac{(-3a^2B+aAb+2b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2d\,(a^2-b^2)}-\frac{(-3a^2B+aAb+2b^2B)\sin(c+dx)}{b^2d\,(a^2-b^2)\sqrt{\cos(c+dx)}}+\frac{a(Ab-aB)\sin(c+dx)}{bd\,(a^2-b^2)\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+b)}+\frac{(-3a^3B+a^2Ab+5ab^2B-3Ab^3)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^2d(a-b)(a+b)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] ((a*A*b - 3*a^2*B + 2*b^2*B)*EllipticE[(c + d*x)/2, 2])/(b^2*(a^2 - b^2)*d)
 + ((A*b - a*B)*EllipticF[(c + d*x)/2, 2])/(b*(a^2 - b^2)*d) + ((a^2*A*b -
3*A*b^3 - 3*a^3*B + 5*a*b^2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(
(a - b)*b^2*(a + b)^2*d) - ((a*A*b - 3*a^2*B + 2*b^2*B)*Sin[c + d*x])/(b^2*
(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + (a*(A*b - a*B)*Sin[c + d*x])/(b*(a^2 -
b^2)*d*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
```

```
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{B+A\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))^2}\,dx$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))} - \frac{\int \frac{1}{2}(aAb-3a^2B+2b^2$$

$$= -\frac{(aAb-3a^2B+2b^2B)\sin(c+dx)}{b^2\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}} + \frac{a(Ab-aB)\sin}{b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}}$$

$$= -\frac{(aAb-3a^2B+2b^2B)\sin(c+dx)}{b^2\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}} + \frac{a(Ab-aB)\sin}{b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}}$$

$$= \frac{(aAb-3a^2B+2b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\,(a^2-b^2)\,d} - \frac{(aAb-3a^2B+2b^2B)\,\mathrm{s}}{b^2\,(a^2-b^2)\,d\sqrt{\cos(c}}$$

$$= \frac{(aAb-3a^2B+2b^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^2\,(a^2-b^2)\,d} + \frac{(Ab-aB)F\left(\frac{1}{2}(c+dx\right.}{b\,(a^2-b^2)\,d}$$

Mathematica [A]
time = 14.43, size = 317, normalized size = 1.24

$$\frac{-\frac{\frac{2\left(-3a^2Ab+4Ab^3+9a^3B-10ab^2B\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}-\frac{8b\left(aAb-2a^2B+b^2B\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a(a+b)}+\frac{2\left(-aAb+3a^2B-2b^2B\right)\left(-2abE\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab\sqrt{\sin^2(c+dx)}}}{(a-b)(a+b)}+4\sqrt{\cos(c+dx)}\left(\frac{a^2(-Ab+aB)\sin(c+dx)}{(a^2-b^2)(b+a\cos(c+dx))}+2B\tan(c+dx)\right)}{4b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^2),
x]
```

```
[Out] (-(((2*(-3*a^2*A*b + 4*A*b^3 + 9*a^3*B - 10*a*b^2*B)*EllipticPi[(2*a)/(a +
b), (c + d*x)/2, 2])/(a + b) - (8*b*(a*A*b - 2*a^2*B + b^2*B)*((a + b)*Elli
pticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a*(a
 + b)) + (2*(-(a*A*b) + 3*a^2*B - 2*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Co
s[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] +
 (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c +
d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)*(a + b))) + 4*Sqrt[Cos[c + d*x]]
*((a^2*(-(A*b) + a*B)*Sin[c + d*x])/((a^2 - b^2)*(b + a*Cos[c + d*x])) + 2*
B*Tan[c + d*x]))/(4*b^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 849 vs. $2(329) = 658$.
time = 6.09, size = 850, normalized size = 3.33

method	result	size
default	Expression too large to display	850

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*a^2*B/b^2/(a^
2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2*a/(a-b),2^(1/2))+2*B/b^2/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-
1)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*
c)^2*cos(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b-B*a)/b*(1/b*a^2/(a
^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2
)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/
2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1
/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2
*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^
2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
```

```
1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c
)^2-1)^(1/2)/d
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 5009 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(5/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^{5/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^2),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^2), x)
```

3.586 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx$

Optimal. Leaf size=346

$$-\frac{(3a^2Ab-2Ab^3-5a^3B+4ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3(a^2-b^2)\,d}-\frac{(3aAb-5a^2B+2b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2(a^2-b^2)\,d}-\frac{a(3a^2Ab-$$

```
[Out] -(3*A*a^2*b-2*A*b^3-5*B*a^3+4*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/(a^2-b^2)/d-1/3*(3*A*
a*b-5*B*a^2+2*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellipt
icF(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)/d-a*(3*A*a^2*b-5*A*b^3-5*B*a^
3+7*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin
(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/(a-b)/b^3/(a+b)^2/d-1/3*(3*A*a*b-5*B*a^2
+2*B*b^2)*sin(d*x+c)/b^2/(a^2-b^2)/d/cos(d*x+c)^(3/2)+a*(A*b-B*a)*sin(d*x+c
)/b/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(b+a*cos(d*x+c))+(3*A*a^2*b-2*A*b^3-5*B*a^
3+4*B*a*b^2)*sin(d*x+c)/b^3/(a^2-b^2)/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.85, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3033, 3079, 3134, 3138, 2719, 3081, 2720, 2884}

$$-\frac{(-5a^2B+3aAb+2b^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{3b^2d(a^2-b^2)}+\frac{a(Ab-aB)\sin(c+dx)}{bd(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)}-\frac{(-5a^2B+3aAb+2b^2B)\sin(c+dx)}{3b^2d(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)}-\frac{(-5a^3B+3a^2Ab+4ab^2B-2Ab^3)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a^2-b^2)}-\frac{a(-5a^3B+3a^2Ab+7ab^2B-5Ab^3)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{b^3d(a-b)(a+b)^2}+\frac{(-5a^3B+3a^2Ab+4ab^2B-2Ab^3)\sin(c+dx)}{b^3d(a^2-b^2)\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2),x]
```

```
[Out] -(((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(
b^3*(a^2 - b^2)*d)) - ((3*a*A*b - 5*a^2*B + 2*b^2*B)*EllipticF[(c + d*x)/2,
 2])/(3*b^2*(a^2 - b^2)*d) - (a*(3*a^2*A*b - 5*A*b^3 - 5*a^3*B + 7*a*b^2*B)
*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/((a - b)*b^3*(a + b)^2*d) - ((3
*a*A*b - 5*a^2*B + 2*b^2*B)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)*d*Cos[c + d*x]
^(3/2)) + ((3*a^2*A*b - 2*A*b^3 - 5*a^3*B + 4*a*b^2*B)*Sin[c + d*x])/(b^3*(
a^2 - b^2)*d*Sqrt[Cos[c + d*x]]) + (a*(A*b - a*B)*Sin[c + d*x])/(b*(a^2 - b
^2)*d*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
```

```
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^2}\,dx = \int \frac{B+A\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b+a\cos(c+dx))^2}\,dx$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))} - \frac{\int \frac{1}{2}(3aAb-5a^2B+2b^2B$$

$$= -\frac{(3aAb-5a^2B+2b^2B)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} + \frac{a(Ab-aB)\sin}{b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)}$$

$$= -\frac{(3aAb-5a^2B+2b^2B)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} + \frac{(3a^2Ab-2Ab^3-5a^3B+}{b^3\left(a^2-b^2\right)d\sqrt{c}}$$

$$= -\frac{(3aAb-5a^2B+2b^2B)\sin(c+dx)}{3b^2\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)} + \frac{(3a^2Ab-2Ab^3-5a^3B+}{b^3\left(a^2-b^2\right)d\sqrt{c}}$$

$$= -\frac{(3a^2Ab-2Ab^3-5a^3B+4ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} - \frac{(3aAb-5}{3b^2\left(a^2\right.}$$

$$= -\frac{(3a^2Ab-2Ab^3-5a^3B+4ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{b^3\left(a^2-b^2\right)d} - \frac{(3aAb-5}{}$$

Mathematica [**A**]
time = 16.96, size = 427, normalized size = 1.23

$$\frac{\frac{2(-27a^3Ab+30aAb^3+45a^4B-44a^2b^2B-4b^4B)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{(-24a^2Ab^2+12Ab^4+40a^3bB-28ab^3B)\left(2F\left(\frac{1}{2}(c+dx)\middle|2\right)-\frac{2b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}\right)}{a}+\frac{2(-9a^3Ab+6aAb^3+15a^4B-12a^2b^2B)\cos(2(c+dx))\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a^2b\sqrt{1-\cos^2(c+dx)}\,(-1+2\cos^2(c+dx))}}{12(a-b)b^3(a+b)d}+\frac{\sqrt{\cos(c+dx)}\left(\frac{2\sec(c+dx)(Ab\sin(c+dx)-2aB\sin(c+dx))}{b^3}+\frac{-a^3Ab\sin(c+dx)+a^4B\sin(c+dx)}{b^3(-a^2+b^2)(b+a\cos(c+dx))}+\frac{2B\sec(c+dx)\tan(c+dx)}{3b^2}\right)}{d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^2),
x]
```

```
[Out] ((2*(-27*a^3*A*b + 30*a*A*b^3 + 45*a^4*B - 44*a^2*b^2*B - 4*b^4*B)*Elliptic
Pi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + ((-24*a^2*A*b^2 + 12*A*b^4 + 4
0*a^3*b*B - 28*a*b^3*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a
)/(a + b), (c + d*x)/2, 2])/(a + b)))/a + (2*(-9*a^3*A*b + 6*a*A*b^3 + 15*a
^4*B - 12*a^2*b^2*B)*Cos[2*(c + d*x)]*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c +
 d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2
 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])
/(a^2*b*Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos[c + d*x]^2)))/(12*(a - b)*b^3*
(a + b)*d) + (Sqrt[Cos[c + d*x]]*((2*Sec[c + d*x]*(A*b*Sin[c + d*x] - 2*a*B
*Sin[c + d*x]))/b^3 + (-(a^3*A*b*Sin[c + d*x]) + a^4*B*Sin[c + d*x])/(b^3*(
-a^2 + b^2)*(b + a*Cos[c + d*x])) + (2*B*Sec[c + d*x]*Tan[c + d*x])/(3*b^2)
))/d
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 996 vs. $2(414) = 828$.
time = 12.05, size = 997, normalized size = 2.88

method	result	size
default	Expression too large to display	997

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^2*(-1/6*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(A*b-B*a)*a/b^2*(1/b*a^2/(a^2-b^2)*cos(
1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(
1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/
2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2
)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2-
b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin
```

```
(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),
2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos
(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-
a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*
c),2*a/(a-b),2^(1/2)))+2*a^2*(A*b-2*B*a)/b^3/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*(A*
b-2*B*a)/b^3/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+
1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipt
icE(cos(1/2*d*x+1/2*c),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^
2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

$$\text{Timed out}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**2,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^2,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^2*cos(d*x + c)^(7/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\cos\left(c+d\,x\right)}^{7/2}\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^2),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^2), x)
```

3.587 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=461

$$-\frac{(24a^4Ab-65a^2Ab^3+35Ab^5-8a^5B+29a^3b^2B-15ab^4B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4\left(a^2-b^2\right)^2d}+\frac{(8a^6A+128a^4Ab^2-223a^2$$

```
[Out] -1/4*(24*A*a^4*b-65*A*a^2*b^3+35*A*b^5-8*B*a^5+29*B*a^3*b^2-15*B*a*b^4)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))/a^4/(a^2-b^2)^2/d+1/12*(8*A*a^6+128*A*a^4*b^2-223*A*a^2*b^4+105*A*b
^6-72*B*a^5*b+99*B*a^3*b^3-45*B*a*b^5)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^5/(a^2-b^2)^2/d-1/4*b^2
*(63*A*a^4*b-86*A*a^2*b^3+35*A*b^5-35*B*a^5+38*B*a^3*b^2-15*B*a*b^4)*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a
/(a+b),2^(1/2))/a^5/(a-b)^2/(a+b)^3/d+1/2*b*(A*b-B*a)*cos(d*x+c)^(5/2)*sin(
d*x+c)/a/(a^2-b^2)/d/(b+a*cos(d*x+c))^2+1/4*b*(13*A*a^2*b-7*A*b^3-9*B*a^3+3
*B*a*b^2)*cos(d*x+c)^(3/2)*sin(d*x+c)/a^2/(a^2-b^2)^2/d/(b+a*cos(d*x+c))+1/
12*(8*A*a^4-61*A*a^2*b^2+35*A*b^4+33*B*a^3*b-15*B*a*b^3)*sin(d*x+c)*cos(d*x
+c)^(1/2)/a^3/(a^2-b^2)^2/d
```

Rubi [A]
time = 1.04, antiderivative size = 461, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.273$, Rules used = {3033, 3068, 3126, 3128, 3138, 2719, 3081, 2720, 2884}

$$\frac{b(Ab-aB)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}+\frac{b\left(-9a^3B+13a^2Ab+3ab^2B-7Ab^3\right)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{4a^2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{\left(8a^4A+33a^3bB-61a^2Ab^2-15ab^3B+35Ab^4\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{12a^3d\left(a^2-b^2\right)^2}-\frac{\left(-8a^5B+24a^4Ab+29a^3b^2B-65a^2Ab^3-15ab^4B+35Ab^5\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d\left(a^2-b^2\right)^2}-\frac{b^2\left(-35a^5B+63a^4Ab+38a^3b^2B-86a^2Ab^3-15ab^4B+35Ab^5\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^5d(a-b)^2(a+b)^3}+\frac{\left(8a^6A-72a^5bB+128a^4Ab^2+99a^3b^3B-223a^2Ab^4-45ab^5B+105Ab^6\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12a^5d\left(a^2-b^2\right)^2}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] -1/4*((24*a^4*A*b - 65*a^2*A*b^3 + 35*A*b^5 - 8*a^5*B + 29*a^3*b^2*B - 15*a
*b^4*B)*EllipticE[(c + d*x)/2, 2])/(a^4*(a^2 - b^2)^2*d) + ((8*a^6*A + 128*
a^4*A*b^2 - 223*a^2*A*b^4 + 105*A*b^6 - 72*a^5*b*B + 99*a^3*b^3*B - 45*a*b^
5*B)*EllipticF[(c + d*x)/2, 2])/(12*a^5*(a^2 - b^2)^2*d) - (b^2*(63*a^4*A*b
 - 86*a^2*A*b^3 + 35*A*b^5 - 35*a^5*B + 38*a^3*b^2*B - 15*a*b^4*B)*Elliptic
Pi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a^5*(a - b)^2*(a + b)^3*d) + ((8*a^4*
A - 61*a^2*A*b^2 + 35*A*b^4 + 33*a^3*b*B - 15*a*b^3*B)*Sqrt[Cos[c + d*x]]*S
in[c + d*x])/(12*a^3*(a^2 - b^2)^2*d) + (b*(A*b - a*B)*Cos[c + d*x]^(5/2)*S
in[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + (b*(13*a^2*A*b -
7*A*b^3 - 9*a^3*B + 3*a*b^2*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(4*a^2*(a^2
 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3128

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_.)*((c_.) + (d_.)*sin[(e_.)
+ (f_.)*(x_)])^(n_.)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_
.) + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-C)*Cos[e + f*x]*(a + b*Sin[e + f*x
])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(m + n + 2))), x] + Dist[1/(d*(m +
n + 2)), Int[(a + b*Sin[e + f*x])^(m - 1)*(c + d*Sin[e + f*x])^n*Simp[a*A*d
*(m + n + 2) + C*(b*c*m + a*d*(n + 1)) + (d*(A*b + a*B)*(m + n + 2) - C*(a*
c - b*d*(m + n + 1)))*Sin[e + f*x] + (C*(a*d*m - b*c*(m + 1)) + b*B*d*(m +
n + 2))*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C, n},
 x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m
, 0] &&  !(IGtQ[n, 0] && ( !IntegerQ[m] || (EqQ[a, 0] && NeQ[c, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos^{\frac{7}{2}}(c+dx)(B+A\cos(c+dx))}{(b+a\cos(c+dx))^3}\,dx$$

$$= \frac{b(Ab-aB)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\,(a^2-b^2)\,d(b+a\cos(c+dx))^2} + \frac{\int \cos^{\frac{3}{2}}(c+dx)\left(\frac{5}{2}b(Ab-aB)-2a(Ab\right.}{}$$

$$= \frac{b(Ab-aB)\cos^{\frac{5}{2}}(c+dx)\sin(c+dx)}{2a\,(a^2-b^2)\,d(b+a\cos(c+dx))^2} + \frac{b(13a^2Ab-7Ab^3-9a^3B}{4a^2\,(a^2-b^2)^2}$$

$$= \frac{\left(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B\right)\sqrt{\cos(c+dx)}\,\sin}{12a^3\,(a^2-b^2)^2\,d}$$

$$= \frac{\left(8a^4A-61a^2Ab^2+35Ab^4+33a^3bB-15ab^3B\right)\sqrt{\cos(c+dx)}\,\sin}{12a^3\,(a^2-b^2)^2\,d}$$

$$= -\frac{\left(24a^4Ab-65a^2Ab^3+35Ab^5-8a^5B+29a^3b^2B-15ab^4B\right)E\left(\frac{1}{2}\right.}{4a^4\,(a^2-b^2)^2\,d}$$

$$= -\frac{\left(24a^4Ab-65a^2Ab^3+35Ab^5-8a^5B+29a^3b^2B-15ab^4B\right)E\left(\frac{1}{2}\right.}{4a^4\,(a^2-b^2)^2\,d}$$

Mathematica [A]
time = 15.84, size = 461, normalized size = 1.00

$$\frac{\frac{4\sqrt{\cos(c+dx)}\left(4a^6A-57a^2Ab^4+35Ab^6+33a^3b^3B-15ab^5B+ab(16a^4A-83a^2Ab^2+49Ab^4+39a^3bB-21ab^3B)\cos(c+dx)+4A(a^3-ab^2)^2\cos(2(c+dx))\right)\sin(c+dx)}{(a^2-b^2)^2(b+a\cos(c+dx))^2}+\frac{\frac{2(-56a^4Ab+73a^2Ab^3-35Ab^5+24a^5B-21a^3b^2B+15ab^4B)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{16(2a^4A+14a^2Ab^2-7Ab^4-12a^3bB+3ab^3B)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{6(-24a^4Ab+65a^2Ab^3-35Ab^5+8a^5B-29a^3b^2B+15ab^4B)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a^2b\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{48a^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,
x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(4*a^6*A - 57*a^2*A*b^4 + 35*A*b^6 + 33*a^3*b^3*B -
15*a*b^5*B + a*b*(16*a^4*A - 83*a^2*A*b^2 + 49*A*b^4 + 39*a^3*b*B - 21*a*b^
3*B)*Cos[c + d*x] + 4*A*(a^3 - a*b^2)^2*Cos[2*(c + d*x)])*Sin[c + d*x])/((a
^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + ((2*(-56*a^4*A*b + 73*a^2*A*b^3 - 35*
A*b^5 + 24*a^5*B - 21*a^3*b^2*B + 15*a*b^4*B)*EllipticPi[(2*a)/(a + b), (c
+ d*x)/2, 2])/(a + b) + (16*(2*a^4*A + 14*a^2*A*b^2 - 7*A*b^4 - 12*a^3*b*B
+ 3*a*b^3*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b
), (c + d*x)/2, 2]))/(a + b) + (6*(-24*a^4*A*b + 65*a^2*A*b^3 - 35*A*b^5 +
8*a^5*B - 29*a^3*b^2*B + 15*a*b^4*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c +
d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2
- 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/
(a^2*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(48*a^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2215 vs. $2(521) = 1042$.

time = 14.20, size = 2216, normalized size = 4.81

method	result	size
default	Expression too large to display	2216

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2/3/a^5*(4*A*co
s(1/2*d*x+1/2*c)*sin(1/2*d*x+1/2*c)^4*a^2-2*A*cos(1/2*d*x+1/2*c)*sin(1/2*d*
x+1/2*c)^2*a^2+a^2*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+18*A*b^2*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1
/2))+9*A*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*Elli
pticE(cos(1/2*d*x+1/2*c),2^(1/2))*a*b-9*B*a*b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*
(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3*B*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*EllipticE(cos
(1/2*d*x+1/2*c),2^(1/2))*a^2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)+2*b^3/a^5*(5*A*b-4*B*a)*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a
+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(
1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1
/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-
b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+
1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+
sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)
))+4*b^2/a^4*(5*A*b-3*B*a)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*b^4*(A*b-B*a)/a^5*(1/
2/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^
2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a
```

```
+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9
/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/
(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2
-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos
(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)
*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] integral((B*cos(d*x + c)*sec(d*x + c) + A*cos(d*x + c))*sqrt(cos(d*x + c))/
(b^3*sec(d*x + c)^3 + 3*a*b^2*sec(d*x + c)^2 + 3*a^2*b*sec(d*x + c) + a^3),
 x)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^3, x
)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^3, x)
```

3.588 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=367

$$\frac{\left(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2d}-\frac{\left(24a^4Ab-33a^2Ab^3+15Ab^5-8a^5B+5a^3b\right.}{4a^4\left(a^2-b^2\right)^2d}$$

```
[Out] 1/4*(8*A*a^4-29*A*a^2*b^2+15*A*b^4+9*B*a^3*b-3*B*a*b^3)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2
-b^2)^2/d-1/4*(24*A*a^4*b-33*A*a^2*b^3+15*A*b^5-8*B*a^5+5*B*a^3*b^2-3*B*a*b
^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2))/a^4/(a^2-b^2)^2/d+1/4*b*(35*A*a^4*b-38*A*a^2*b^3+15*A*b^5-15
*B*a^5+6*B*a^3*b^2-3*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^4/(a-b)^2/(a+b)^3/d+1
/2*b*(A*b-B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(b+a*cos(d*x+c))^2
+1/4*b*(11*A*a^2*b-5*A*b^3-7*B*a^3+B*a*b^2)*sin(d*x+c)*cos(d*x+c)^(1/2)/a^2
/(a^2-b^2)^2/d/(b+a*cos(d*x+c))
```

Rubi [A]
time = 0.71, antiderivative size = 367, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3033, 3068, 3126, 3138, 2719, 3081, 2720, 2884}

$$\frac{b(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{2ad\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}+\frac{b\left(-7a^3B+11a^2Ab+ab^2B-5Ab^3\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{4a^2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{\left(8a^4A+9a^3bB-29a^2Ab^2-3ab^3B+15Ab^4\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}-\frac{\left(-8a^5B+24a^4Ab+5a^3b^2B-33a^2Ab^3-3ab^4B+15Ab^5\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d\left(a^2-b^2\right)^2}+\frac{b\left(-15a^5B+35a^4Ab+6a^3b^2B-38a^2Ab^3-3ab^4B+15Ab^5\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^4d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,x]
```

```
[Out] ((8*a^4*A - 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*EllipticE[(c +
 d*x)/2, 2])/(4*a^3*(a^2 - b^2)^2*d) - ((24*a^4*A*b - 33*a^2*A*b^3 + 15*A*b
^5 - 8*a^5*B + 5*a^3*b^2*B - 3*a*b^4*B)*EllipticF[(c + d*x)/2, 2])/(4*a^4*(
a^2 - b^2)^2*d) + (b*(35*a^4*A*b - 38*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 6*a
^3*b^2*B - 3*a*b^4*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(4*a^4*(a
- b)^2*(a + b)^3*d) + (b*(A*b - a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(2*a*
(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + (b*(11*a^2*A*b - 5*A*b^3 - 7*a^3*B
+ a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*a^2*(a^2 - b^2)^2*d*(b + a*C
os[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3126

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(c^2*C - B*c*d + A*d^2))*Cos[e + f*x
```

```
]*(a + b*Sin[e + f*x])^m*((c + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 -
d^2))), x] + Dist[1/(d*(n + 1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m -
1)*(c + d*Sin[e + f*x])^(n + 1)*Simp[A*d*(b*d*m + a*c*(n + 1)) + (c*C - B*d
)*(b*c*m + a*d*(n + 1)) - (d*(A*(a*d*(n + 2) - b*c*(n + 1)) + B*(b*d*(n + 1
) - a*c*(n + 2))) - C*(b*c*d*(n + 1) - a*(c^2 + d^2*(n + 1))))*Sin[e + f*x]
 + b*(d*(B*c - A*d)*(m + n + 2) - C*(c^2*(m + 1) + d^2*(n + 1)))*Sin[e + f*
x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d,
 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 0] && LtQ[n, -1]
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos^{\frac{5}{2}}(c+dx)(B+A\cos(c+dx))}{(b+a\cos(c+dx))^3}\,dx$$

$$= \frac{b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{\int \sqrt{\cos(c+dx)}\,\left(\tfrac{3}{2}b(Ab-\ldots\right.}{\ldots}$$

$$= \frac{b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{b(11a^2Ab-5Ab^3-7a^3\ldots}{4a^2\left(a^2-b\ldots\right.}$$

$$= \frac{b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{2a\left(a^2-b^2\right)d(b+a\cos(c+dx))^2} + \frac{b(11a^2Ab-5Ab^3-7a^3\ldots}{4a^2\left(a^2-b\ldots\right.}$$

$$= \frac{(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2d} + \ldots$$

$$= \frac{(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2d} - \ldots$$

Mathematica [A]
time = 15.07, size = 390, normalized size = 1.06

$$\frac{-\frac{2b\sqrt{\cos(c+dx)}\,(b(-11a^2Ab+5Ab^3+7a^3B-ab^2B)+a(-13a^2Ab+7Ab^3+9a^3B-3ab^2B)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2(b+a\cos(c+dx))^2}+\frac{\frac{(8a^4A-7a^2Ab^2+5Ab^4-5a^3bB-ab^3B)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8(-4a^2Ab+Ab^3+2a^3B+ab^2B)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}+\frac{(8a^4A-29a^2Ab^2+15Ab^4+9a^3bB-3ab^3B)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a^2b\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{8a^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^3,
x]
```

```
[Out] ((-2*b*Sqrt[Cos[c + d*x]]*(b*(-11*a^2*A*b + 5*A*b^3 + 7*a^3*B - a*b^2*B) +
a*(-13*a^2*A*b + 7*A*b^3 + 9*a^3*B - 3*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])
/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + (((8*a^4*A - 7*a^2*A*b^2 + 5*A*b^
4 - 5*a^3*b*B - a*b^3*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b)
 + (8*(-4*a^2*A*b + A*b^3 + 2*a^3*B + a*b^2*B)*((a + b)*EllipticF[(c + d*x)
/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a + b) + ((8*a^4*A
- 29*a^2*A*b^2 + 15*A*b^4 + 9*a^3*b*B - 3*a*b^3*B)*(-2*a*b*EllipticE[ArcSin
[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]
], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*
Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(8*a^2*d
)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1999 vs. $2(431) = 862$.
time = 12.32, size = 2000, normalized size = 5.45

method	result	size
default	Expression too large to display	2000

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2/a^4/(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2-1)^(1/
2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(3*A*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
b+A*a*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-B*a*EllipticF(cos(1/2*d*x+1/2*c
),2^(1/2)))-2*b^2/a^4*(4*A*b-3*B*a)*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(
-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2
*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(
1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(
a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^
2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(
cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(
```

```
1/2)))-6*b/a^3*(2*A*b-B*a)/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1
/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/
2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+2*b^3*(A*b-B*a)/a^4*(1/
2/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^
2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1
/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+s
in(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a
+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*
cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2
)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(
1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1
/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9
/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(
1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2
*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(
-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c
)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/
(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*co
s(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^
(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2
-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos
(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)
*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2
*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\sqrt{\cos{(c + dx)}}}{(a + b\sec{(c + dx)})^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**3, x
)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^3, x
)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^3,x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^3, x)
```

3.589 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=346

$$\frac{\left(9a^2Ab-3Ab^3-5a^3B-ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2d}+\frac{\left(8a^4A-5a^2Ab^2+3Ab^4-7a^3bB+ab^3B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3\left(a^2-b^2\right)^2d}$$

```
[Out] 1/4*(9*A*a^2*b-3*A*b^3-5*B*a^3-B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)^2/d+1/4*(8
*A*a^4-5*A*a^2*b^2+3*A*b^4-7*B*a^3*b+B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^3/(a^2-b^2)^2/d-
1/4*(15*A*a^4*b-6*A*a^2*b^3+3*A*b^5-3*B*a^5-10*B*a^3*b^2+B*a*b^4)*(cos(1/2*
d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a
+b),2^(1/2))/a^3/(a-b)^2/(a+b)^3/d+1/2*b*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1
/2)/a/(a^2-b^2)/d/(b+a*cos(d*x+c))^2-1/4*(9*A*a^2*b-3*A*b^3-5*B*a^3-B*a*b^2
)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)^2/d/(b+a*cos(d*x+c))
```

Rubi [A]

time = 0.73, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3033, 3068, 3134, 3138, 2719, 3081, 2720, 2884}

$$\frac{b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2ad\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}+\frac{\left(-5a^3B+9a^2Ab-ab^2B-3Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{\left(-5a^3B+9a^2Ab-ab^2B-3Ab^3\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{4ad\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{\left(8a^4A-7a^3bB-5a^2Ab^2+ab^3B+3Ab^4\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d\left(a^2-b^2\right)^2}-\frac{\left(-3a^5B+15a^4Ab-10a^3b^2B-6a^2Ab^3+ab^4B+3Ab^5\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^3d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((9*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(4*a^
2*(a^2 - b^2)^2*d) + ((8*a^4*A - 5*a^2*A*b^2 + 3*A*b^4 - 7*a^3*b*B + a*b^3*
B)*EllipticF[(c + d*x)/2, 2])/(4*a^3*(a^2 - b^2)^2*d) - ((15*a^4*A*b - 6*a^
2*A*b^3 + 3*A*b^5 - 3*a^5*B - 10*a^3*b^2*B + a*b^4*B)*EllipticPi[(2*a)/(a +
 b), (c + d*x)/2, 2])/(4*a^3*(a - b)^2*(a + b)^3*d) + (b*(A*b - a*B)*Sqrt[C
os[c + d*x]]*Sin[c + d*x])/(2*a*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) - ((9
*a^2*A*b - 3*A*b^3 - 5*a^3*B - a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4
*a*(a^2 - b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3068

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(b*c - a*d))*(B*c - A*d)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m - 1)*((c
 + d*Sin[e + f*x])^(n + 1)/(d*f*(n + 1)*(c^2 - d^2))), x] + Dist[1/(d*(n +
1)*(c^2 - d^2)), Int[(a + b*Sin[e + f*x])^(m - 2)*(c + d*Sin[e + f*x])^(n +
 1)*Simp[b*(b*c - a*d)*(B*c - A*d)*(m - 1) + a*d*(a*A*c + b*B*c - (A*b + a*
B)*d)*(n + 1) + (b*(b*d*(B*c - A*d) + a*(A*c*d + B*(c^2 - 2*d^2)))*(n + 1)
- a*(b*c - a*d)*(B*c - A*d)*(n + 2))*Sin[e + f*x] + b*(d*(A*b*c + a*B*c - a
*A*d)*(m + n + 1) - b*B*(c^2*m + d^2*(n + 1)))*Sin[e + f*x]^2, x], x], x] /
; FreeQ[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2,
 0] && NeQ[c^2 - d^2, 0] && GtQ[m, 1] && LtQ[n, -1]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
```

```
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^3}\,dx = \int \frac{\cos^{\frac{3}{2}}(c+dx)(B+A\cos(c+dx))}{(b+a\cos(c+dx))^3}\,dx$$

$$= \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a\,(a^2-b^2)\,d(b+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}b(Ab-aB)-2a(Ab-aB)\,\mathrm{co}}{\sqrt{\cos(c}}}{}$$

$$= \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a\,(a^2-b^2)\,d(b+a\cos(c+dx))^2} - \frac{(9a^2Ab-3Ab^3-5a^3B}{4a\,(a^2-b}$$

$$= \frac{b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2a\,(a^2-b^2)\,d(b+a\cos(c+dx))^2} - \frac{(9a^2Ab-3Ab^3-5a^3B}{4a\,(a^2-b}$$

$$= \frac{(9a^2Ab-3Ab^3-5a^3B-ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\,(a^2-b^2)^2\,d} + \frac{b(Ab-aB)}{2a\,(a^2-b}$$

$$= \frac{(9a^2Ab-3Ab^3-5a^3B-ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\,(a^2-b^2)^2\,d} + \frac{(8a^4A-5a^2}{}$$

Mathematica [A]
time = 14.73, size = 361, normalized size = 1.04

$$\frac{\frac{4\sqrt{\cos(c+dx)}\,(b(-7a^2Ab+Ab^3+3a^3B+3ab^2B)+a(-9a^2Ab+3Ab^3+5a^3B+ab^2B)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2(b+a\cos(c+dx))^2}+\frac{\frac{2(-5a^2Ab-Ab^3+a^3B+5ab^2B)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{16(2a^2A+Ab^2-3abB)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a+b}-\frac{2(-9a^2Ab+3Ab^3+5a^3B+ab^2B)\left(-2abE\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\mathrm{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a^2b\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{16ad}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^3),
x]
```

```
[Out] ((4*Sqrt[Cos[c + d*x]]*(b*(-7*a^2*A*b + A*b^3 + 3*a^3*B + 3*a*b^2*B) + a*(-
9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/((a^2
- b^2)^2*(b + a*Cos[c + d*x])^2) + ((2*(-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^
2*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (16*(2*a^2*A + A*
b^2 - 3*a*b*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a +
 b), (c + d*x)/2, 2]))/(a + b) - (2*(-9*a^2*A*b + 3*A*b^3 + 5*a^3*B + a*b^2
*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*Ellipti
cF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSi
n[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((a
 - b)^2*(a + b)^2))/(16*a*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1958 vs. $2(410) = 820$.
time = 10.93, size = 1959, normalized size = 5.66

method	result	size
default	Expression too large to display	1959

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a^3*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+2*b
/a^3*(3*A*b-2*B*a)*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)
/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1
/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^
(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF
(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b
)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c
),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*(-3*A*b+
B*a)/a^2/(a^2-a*b)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(
```

```
1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-2*b^2*(A*b-B*a)/a^3*(1/2/b*a^2/(a^2-b^2)*
cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*
cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d
*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d
*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(s
in(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*
x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2)
)*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(
cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(
sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d
*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2
))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2
*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/
2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1
/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b
^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(c
os(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^
3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*
a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))
)/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec{(c+dx)}}{(a+b\sec{(c+dx)})^3\sqrt{\cos{(c+dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**3/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/((a + b*sec(c + d*x))**3*sqrt(cos(c + d*x))),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^3/cos(d*x+c)^(1/2),x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*sqrt(cos(d*x + c))),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\sqrt{\cos\left(c+d\,x\right)}\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^3),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^3), x)
```

3.590 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=338

$$-\frac{\left(5a^2Ab+Ab^3-a^3B-5ab^2B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab\left(a^2-b^2\right)^2d}-\frac{\left(7a^2Ab-Ab^3-3a^3B-3ab^2B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2\left(a^2-b^2\right)^2d}+\frac{(3a^4A$$

```
[Out] -1/4*(5*A*a^2*b+A*b^3-B*a^3-5*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/a/b/(a^2-b^2)^2/d-1/4*(7*
A*a^2*b-A*b^3-3*B*a^3-3*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a^2/(a^2-b^2)^2/d+1/4*(3*A*a^4*
b+10*A*a^2*b^3-A*b^5+B*a^5-10*B*a^3*b^2-3*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/a^
2/(a-b)^2/b/(a+b)^3/d-1/2*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/(a^2-b^2)/d
/(b+a*cos(d*x+c))^2+1/4*(5*A*a^2*b+A*b^3-B*a^3-5*B*a*b^2)*sin(d*x+c)*cos(d*
x+c)^(1/2)/b/(a^2-b^2)^2/d/(b+a*cos(d*x+c))
```

Rubi [A]

time = 0.66, antiderivative size = 338, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3033, 3078, 3134, 3138, 2719, 3081, 2720, 2884}

$$-\frac{(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2d\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}-\frac{\left(-3a^3B+7a^2Ab-3ab^2B-Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4a^2d\left(a^2-b^2\right)^2}-\frac{\left(a^3(-B)+5a^2Ab-5ab^2B+Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\left(a^2-b^2\right)^2}+\frac{\left(a^3(-B)+5a^2Ab-5ab^2B+Ab^3\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{4bd\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{\left(a^5B+3a^4Ab-10a^3b^2B+10a^2Ab^3-3ab^4B-Ab^5\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4a^2bd(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] -1/4*((5*a^2*A*b + A*b^3 - a^3*B - 5*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(a
*b*(a^2 - b^2)^2*d) - ((7*a^2*A*b - A*b^3 - 3*a^3*B - 3*a*b^2*B)*EllipticF[
(c + d*x)/2, 2])/(4*a^2*(a^2 - b^2)^2*d) + ((3*a^4*A*b + 10*a^2*A*b^3 - A*b
^5 + a^5*B - 10*a^3*b^2*B - 3*a*b^4*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/
2, 2])/(4*a^2*(a - b)^2*b*(a + b)^3*d) - ((A*b - a*B)*Sqrt[Cos[c + d*x]]*Si
n[c + d*x])/(2*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) + ((5*a^2*A*b + A*b^3
- a^3*B - 5*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*b*(a^2 - b^2)^2*d*
(b + a*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3078

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(B*a - A*b)*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*
x])^n/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a +
 b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^(n - 1)*Simp[c*(a*A - b*B)*(m
 + 1) + d*n*(A*b - a*B) + (d*(a*A - b*B)*(m + 1) - c*(A*b - a*B)*(m + 2))*S
in[e + f*x] - d*(A*b - a*B)*(m + n + 2)*Sin[e + f*x]^2, x], x], x] /; FreeQ
[{a, b, c, d, e, f, A, B}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] &&
NeQ[c^2 - d^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
```

```
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{\sqrt{\cos(c+dx)}\,(B+A\cos(c+dx))}{(b+a\cos(c+dx))^3}\,dx$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2\,(a^2-b^2)\,d(b+a\cos(c+dx))^2} + \frac{\int \frac{\frac{1}{2}(-Ab+aB)+2(aA-bB)\cos(\cdots}{\sqrt{\cos(c+dx}\cdots}}{2\,(a^2\cdots}$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2\,(a^2-b^2)\,d(b+a\cos(c+dx))^2} + \frac{(5a^2Ab+Ab^3-a^3B-\cdots}{4b\,(a^2-b^2)\cdots}$$

$$= -\frac{(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2\,(a^2-b^2)\,d(b+a\cos(c+dx))^2} + \frac{(5a^2Ab+Ab^3-a^3B-\cdots}{4b\,(a^2-b^2)\cdots}$$

$$= -\frac{(5a^2Ab+Ab^3-a^3B-5ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4ab\,(a^2-b^2)^2\,d} - \frac{(Ab-aB)\sqrt{\cdots}}{2\,(a^2-b^2)\cdots}$$

$$= -\frac{(5a^2Ab+Ab^3-a^3B-5ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4ab\,(a^2-b^2)^2\,d} - \frac{(7a^2Ab-Ab^3\cdots}{\cdots}$$

Mathematica [A]
time = 14.86, size = 364, normalized size = 1.08

$$\frac{\frac{2\sqrt{\cos(c+dx)}\,(b(3a^2Ab+3Ab^3+a^3B-7ab^2B)-a(-5a^2Ab-Ab^3+a^3B+5ab^2B)\cos(c+dx))\sin(c+dx)}{(a^2-b^2)^2(b+a\cos(c+dx))^2}+\frac{\frac{(a^2Ab+5Ab^3+3a^3B-9ab^2B)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8b\left(-3aAb+a^2B+2b^2B\right)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a(a+b)}+\frac{\left(-5a^2Ab-Ab^3+a^3B+5ab^2B\right)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+\left(a^2-2b^2\right)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{a^2b\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{8bd}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^3),
x]
```

```
[Out] ((2*Sqrt[Cos[c + d*x]]*(b*(3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B) - a*(-5
*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/((a^2 - b
^2)^2*(b + a*Cos[c + d*x])^2) + (((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)
*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*b*(-3*a*A*b + a^2*
B + 2*b^2*B)*((a + b)*EllipticF[(c + d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b
), (c + d*x)/2, 2]))/(a*(a + b)) + ((-5*a^2*A*b - A*b^3 + a^3*B + 5*a*b^2*B
)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF
[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[
Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[Sin[c + d*x]^2]))/((a -
 b)^2*(a + b)^2))/(8*b*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1871 vs. $2(402) = 804$.
time = 10.99, size = 1872, normalized size = 5.54

method	result	size
default	Expression too large to display	1872

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*(-2*A*b+B*a)/
a^2*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*
d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x
+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4
+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/
(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/
2*c),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(
1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*
d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*
EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))-2*A/a/(a^2-a*b)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(
1/2))+2*b*(A*b-B*a)/a^2*(1/2/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2
*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+
3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(
```

```
a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2
*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^
2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/
8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))
+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2
*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*
c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^
(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a
+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/2*d*x+1/2*c)/(2*cos(1/
2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(3/2)),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^{3/2}\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^3),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^3), x)
```

3.591 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=342

$$\frac{(a^2Ab+5Ab^3+3a^3B-9ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2\left(a^2-b^2\right)^2d}+\frac{(3a^2Ab+3Ab^3+a^3B-7ab^2B)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4ab\left(a^2-b^2\right)^2d}+\frac{(a^4Ab$$

```
[Out] 1/4*(A*a^2*b+5*A*b^3+3*B*a^3-9*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/
2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)^2/d+1/4*(3
*A*a^2*b+3*A*b^3+B*a^3-7*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/a/b/(a^2-b^2)^2/d+1/4*(A*a^4*b
-10*A*a^2*b^3-3*A*b^5+3*B*a^5-6*B*a^3*b^2+15*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))
/a/(a-b)^2/b^2/(a+b)^3/d+1/2*a*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/b/(a^2
-b^2)/d/(b+a*cos(d*x+c))^2-1/4*a*(A*a^2*b+5*A*b^3+3*B*a^3-9*B*a*b^2)*sin(d*
x+c)*cos(d*x+c)^(1/2)/b^2/(a^2-b^2)^2/d/(b+a*cos(d*x+c))
```

Rubi [A]
time = 0.71, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3033, 3079, 3134, 3138, 2719, 3081, 2720, 2884}

$$\frac{a(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{2bd\left(a^2-b^2\right)(a\cos(c+dx)+b)^2}+\frac{\left(a^3B+3a^2Ab-7ab^2B+3Ab^3\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4abd\left(a^2-b^2\right)^2}+\frac{\left(3a^3B+a^2Ab-9ab^2B+5Ab^3\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}-\frac{a\left(3a^3B+a^2Ab-9ab^2B+5Ab^3\right)\sin(c+dx)\sqrt{\cos(c+dx)}}{4b^2d\left(a^2-b^2\right)^2(a\cos(c+dx)+b)}+\frac{\left(3a^5B+a^4Ab-6a^3b^2B-10a^2Ab^3+15ab^4B-3Ab^5\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4ab^2d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*EllipticE[(c + d*x)/2, 2])/(4*b^
2*(a^2 - b^2)^2*d) + ((3*a^2*A*b + 3*A*b^3 + a^3*B - 7*a*b^2*B)*EllipticF[(
c + d*x)/2, 2])/(4*a*b*(a^2 - b^2)^2*d) + ((a^4*A*b - 10*a^2*A*b^3 - 3*A*b^
5 + 3*a^5*B - 6*a^3*b^2*B + 15*a*b^4*B)*EllipticPi[(2*a)/(a + b), (c + d*x)
/2, 2])/(4*a*(a - b)^2*b^2*(a + b)^3*d) + (a*(A*b - a*B)*Sqrt[Cos[c + d*x]]
*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*(b + a*Cos[c + d*x])^2) - (a*(a^2*A*b + 5
*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c + d*x])/(4*b^2*(a^2
- b^2)^2*d*(b + a*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
```

```
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{B+A\cos(c+dx)}{\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))^3}\,dx$$

$$= \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2b\,(a^2-b^2)\,d(b+a\cos(c+dx))^2} - \frac{\int \frac{\frac{1}{2}(-aAb-3a^2B+4b^2B)+2b(A\cdots}{\sqrt{\cos(c+\cdots}}}{2\cdots}$$

$$= \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2b\,(a^2-b^2)\,d(b+a\cos(c+dx))^2} - \frac{a(a^2Ab+5Ab^3+3a^3B-\cdots}{4b^2\,(a^2-b^2\cdots}$$

$$= \frac{a(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{2b\,(a^2-b^2)\,d(b+a\cos(c+dx))^2} - \frac{a(a^2Ab+5Ab^3+3a^3B-\cdots}{4b^2\,(a^2-b^2\cdots}$$

$$= \frac{(a^2Ab+5Ab^3+3a^3B-9ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4b^2\,(a^2-b^2)^2\,d} + \frac{a(Ab-aB)\sqrt{\cdots}}{2b\,(a^2-b^2)\cdots}$$

$$= \frac{(a^2Ab+5Ab^3+3a^3B-9ab^2B)\,E\left(\frac{1}{2}(c+dx)\middle|\,2\right)}{4b^2\,(a^2-b^2)^2\,d} + \frac{(3a^2Ab+3Ab^3\cdots}{}$$

Mathematica [A]

time = 14.90, size = 383, normalized size = 1.12

$$\frac{-\frac{2a\sqrt{\cos(c+dx)}\,\left(b(-a^2Ab+7Ab^3+5a^3B-11ab^2B)+a(a^2Ab+5Ab^3+3a^3B-9ab^2B)\cos(c+dx)\right)\sin(c+dx)}{(a^2-b^2)^2(b+a\cos(c+dx))^2}+\frac{\frac{(3a^3Ab-9aAb^3+9a^4B-19a^2b^2B+16b^4B)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8b(a^2Ab+2Ab^3+a^3B-4ab^2B)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a(a+b)}+\frac{(a^2Ab+5Ab^3+3a^3B-9ab^2B)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}}{8b^2d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^3),
x]
```

```
[Out] ((-2*a*Sqrt[Cos[c + d*x]]*(b*(-(a^2*A*b) + 7*A*b^3 + 5*a^3*B - 11*a*b^2*B)
+ a*(a^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/(
(a^2 - b^2)^2*(b + a*Cos[c + d*x])^2) + (((3*a^3*A*b - 9*a*A*b^3 + 9*a^4*B
- 19*a^2*b^2*B + 16*b^4*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a +
b) + (8*b*(a^2*A*b + 2*A*b^3 + a^3*B - 4*a*b^2*B)*((a + b)*EllipticF[(c + d
*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a*(a + b)) + ((a
^2*A*b + 5*A*b^3 + 3*a^3*B - 9*a*b^2*B)*(-2*a*b*EllipticE[ArcSin[Sqrt[Cos[c
 + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[c + d*x]]], -1] + (a
^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]]], -1])*Sin[c + d*x
])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2))/(8*b^2*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1767 vs. $2(406) = 812$.
time = 11.06, size = 1768, normalized size = 5.17

method	result	size
default	Expression too large to display	1768

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*A/a*(1/b*a^2/
(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)
^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d
*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(
a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+
1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(co
s(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*(-A*b+B*a)/a*(1/2/b*a^2/(a^2-b^2)*co
s(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*co
s(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x
+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x
+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
```

```
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin
(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+
1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*
a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2
+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(co
s(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1
/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+
1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(si
n(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))
-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c
)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE
(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)
*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2
*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2
)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1
)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos
(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*
(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*
d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/
(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/
sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(5/2)),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^{5/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^3}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^3),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^3), x)
```

3.592 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=420

$$\frac{(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2d}+\frac{(a^2Ab-7Ab^3-5a^3B+11ab^2B)\,F\left(\frac{1}{2}(c+$$

```
[Out] 1/4*(3*A*a^3*b-9*A*a*b^3-15*B*a^4+29*B*a^2*b^2-8*B*b^4)*(cos(1/2*d*x+1/2*c)
^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2))/b^3/(a^2
-b^2)^2/d+1/4*(A*a^2*b-7*A*b^3-5*B*a^3+11*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(
1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2))/b^2/(a^2-b^2)
^2/d+1/4*(3*A*a^4*b-6*A*a^2*b^3+15*A*b^5-15*B*a^5+38*B*a^3*b^2-35*B*a*b^4)*
(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*
c),2*a/(a+b),2^(1/2))/(a-b)^2/b^3/(a+b)^3/d-1/4*(3*A*a^3*b-9*A*a*b^3-15*B*a
^4+29*B*a^2*b^2-8*B*b^4)*sin(d*x+c)/b^3/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)+1/2*
a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(b+a*cos(d*x+c))^2/cos(d*x+c)^(1/2)+1/
4*a*(A*a^2*b-7*A*b^3-5*B*a^3+11*B*a*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/(b+a*
cos(d*x+c))/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 0.97, antiderivative size = 420, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3033, 3079, 3134, 3138, 2719, 3081, 2720, 2884}

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+b)^2}+\frac{(-5a^3B+a^2Ab+11ab^2B-7Ab^3)\,F\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^2d\left(a^2-b^2\right)^2}+\frac{a(-5a^3B+a^2Ab+11ab^2B-7Ab^3)\sin(c+dx)}{4b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,(a\cos(c+dx)+b)}+\frac{(-15a^4B+3a^3Ab+29a^2b^2B-9aAb^3-8b^4B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d\left(a^2-b^2\right)^2}-\frac{(-15a^4B+3a^3Ab+29a^2b^2B-9aAb^3-8b^4B)\sin(c+dx)}{4b^3d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}}+\frac{(-15a^5B+3a^4Ab+38a^3b^2B-6a^2Ab^3-35ab^4B+15Ab^5)\,\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4b^3d(a-b)^2(a+b)^3}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] ((3*a^3*A*b - 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*EllipticE[(c +
 d*x)/2, 2])/(4*b^3*(a^2 - b^2)^2*d) + ((a^2*A*b - 7*A*b^3 - 5*a^3*B + 11*a
*b^2*B)*EllipticF[(c + d*x)/2, 2])/(4*b^2*(a^2 - b^2)^2*d) + ((3*a^4*A*b -
6*a^2*A*b^3 + 15*A*b^5 - 15*a^5*B + 38*a^3*b^2*B - 35*a*b^4*B)*EllipticPi[(
2*a)/(a + b), (c + d*x)/2, 2])/(4*(a - b)^2*b^3*(a + b)^3*d) - ((3*a^3*A*b
- 9*a*A*b^3 - 15*a^4*B + 29*a^2*b^2*B - 8*b^4*B)*Sin[c + d*x])/(4*b^3*(a^2
- b^2)^2*d*Sqrt[Cos[c + d*x]]) + (a*(A*b - a*B)*Sin[c + d*x])/(2*b*(a^2 - b
^2)*d*Sqrt[Cos[c + d*x]]*(b + a*Cos[c + d*x])^2) + (a*(a^2*A*b - 7*A*b^3 -
5*a^3*B + 11*a*b^2*B)*Sin[c + d*x])/(4*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x
]]*(b + a*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{B+A\cos(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))^3}\,dx$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))^2} - \frac{\int \frac{\frac{1}{2}(aAb-5a^2B+\cdots}{\cdots}}{\cdots}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(b+a\cos(c+dx))^2} + \frac{a(a^2Ab-7A\cdots}{4b^2\left(a^2-b^2\right)^2\cdots}$$

$$= -\frac{(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B)\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}} + \frac{\cdots}{2b\cdots}$$

$$= -\frac{(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B)\sin(c+dx)}{4b^3\left(a^2-b^2\right)^2d\sqrt{\cos(c+dx)}} + \frac{\cdots}{2b\cdots}$$

$$= \frac{(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2d} - \cdots$$

$$= \frac{(3a^3Ab-9aAb^3-15a^4B+29a^2b^2B-8b^4B)\,E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^3\left(a^2-b^2\right)^2d} + \cdots$$

Mathematica [A]
time = 15.85, size = 458, normalized size = 1.09

$$-\frac{\frac{\frac{(-9a^4Ab+19a^2Ab^3-16Ab^5+45a^5B-95a^3b^2B+56ab^4B)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{a+b}+\frac{8b(-a^3Ab+4aAb^3+5a^4B-10a^2b^2B+2b^4B)\left((a+b)F\left(\frac{1}{2}(c+dx)\middle|2\right)-b\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)\right)}{a(a+b)}+\frac{(-3a^3Ab+9aAb^3+15a^4B-29a^2b^2B+8b^4B)\left(-2abE\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+2b(a+b)F\left(\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)+(a^2-2b^2)\Pi\left(-\frac{a}{b};\text{ArcSin}\left(\sqrt{\cos(c+dx)}\right)\middle|-1\right)\right)\sin(c+dx)}{ab\sqrt{\sin^2(c+dx)}}}{(a-b)^2(a+b)^2}+\frac{\sqrt{\cos(c+dx)}\left(2ab(-5a^3Ab+11aAb^3+25a^4B-47a^2b^2B+16b^4B)\sin(c+dx)+a^2(-3a^3Ab+9aAb^3+15a^4B-29a^2b^2B+8b^4B)\sin(2(c+dx))+16(-a^2b+b^3)^2B\tan(c+dx)\right)}{(a^2-b^2)^2(b+a\cos(c+dx))^2}}{8b^3d}$$

Antiderivative was successfully verified.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^3),
x]
```

```
[Out] (-((((-9*a^4*A*b + 19*a^2*A*b^3 - 16*A*b^5 + 45*a^5*B - 95*a^3*b^2*B + 56*a
*b^4*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a + b) + (8*b*(-(a^3*A*
b) + 4*a*A*b^3 + 5*a^4*B - 10*a^2*b^2*B + 2*b^4*B)*((a + b)*EllipticF[(c +
d*x)/2, 2] - b*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2]))/(a*(a + b)) + ((
-3*a^3*A*b + 9*a*A*b^3 + 15*a^4*B - 29*a^2*b^2*B + 8*b^4*B)*(-2*a*b*Ellipti
cE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[Sqrt[Cos[
c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos[c + d*x]
]], -1])*Sin[c + d*x])/(a*b*Sqrt[Sin[c + d*x]^2]))/((a - b)^2*(a + b)^2)) +
 (Sqrt[Cos[c + d*x]]*(2*a*b*(-5*a^3*A*b + 11*a*A*b^3 + 25*a^4*B - 47*a^2*b^
2*B + 16*b^4*B)*Sin[c + d*x] + a^2*(-3*a^3*A*b + 9*a*A*b^3 + 15*a^4*B - 29*
```

```
a^2*b^2*B + 8*b^4*B)*Sin[2*(c + d*x)] + 16*(-(a^2*b) + b^3)^2*B*Tan[c + d*x
]))/((a^2 - b^2)^2*(b + a*Cos[c + d*x])^2))/(8*b^3*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1996 vs. $2(480) = 960$.
time = 13.97, size = 1997, normalized size = 4.75

method	result	size
default	Expression too large to display	1997

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*a*B/b^2*(1/b
*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*
c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^
2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2
*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2
)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/
2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(
1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)
^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(
cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*
c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(
1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/
2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*
c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elliptic
Pi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))+2*a^2*B/b^3/(a^2-a*b)*(sin(1/2*d*
x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^
4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/
2))+2*B/b^3/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2*sin(1/2*d*x
+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*cos(1/2*d*x+1
/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellipti
cE(cos(1/2*d*x+1/2*c),2^(1/2)))+2*(A*b-B*a)/b*(1/2/b*a^2/(a^2-b^2)*cos(1/2*
d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*
d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*cos(1/2*d*x+1/2*c
)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c
)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b^2)/b*(sin(1/2*d
*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)
^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a+7/8/
(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*
```

```
d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-
2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)
^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-b^2)^2*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-3/8*a
^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)
^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1
/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*c
os(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)
^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b)/(a^2-b^2)/b^2/
(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2
)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d
*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2-a*b)*a^3*(sin(1
/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/
2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),
2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/2*d*x+1/2*c)^2)^
(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*
x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))))/sin(1/
2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(7/2)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^{7/2}\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^3),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^3), x)
```

3.593 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx$

Optimal. Leaf size=523

$$-\frac{\left(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4\left(a^2-b^2\right)^2d}-\frac{(15a^3Ab-33aAb^3-35a^4}{12}$$

```
[Out] -1/4*(15*A*a^4*b-29*A*a^2*b^3+8*A*b^5-35*B*a^5+65*B*a^3*b^2-24*B*a*b^4)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2
^(1/2))/b^4/(a^2-b^2)^2/d-1/12*(15*A*a^3*b-33*A*a*b^3-35*B*a^4+61*B*a^2*b^2
-8*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2
*d*x+1/2*c),2^(1/2))/b^3/(a^2-b^2)^2/d-1/4*a*(15*A*a^4*b-38*A*a^2*b^3+35*A*
b^5-35*B*a^5+86*B*a^3*b^2-63*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2*a/(a+b),2^(1/2))/(a-b)^2/b^4/(a+
b)^3/d-1/12*(15*A*a^3*b-33*A*a*b^3-35*B*a^4+61*B*a^2*b^2-8*B*b^4)*sin(d*x+c
)/b^3/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)+1/2*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)
/d/cos(d*x+c)^(3/2)/(b+a*cos(d*x+c))^2+1/4*a*(3*A*a^2*b-9*A*b^3-7*B*a^3+13*
B*a*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/cos(d*x+c)^(3/2)/(b+a*cos(d*x+c))+1/4
*(15*A*a^4*b-29*A*a^2*b^3+8*A*b^5-35*B*a^5+65*B*a^3*b^2-24*B*a*b^4)*sin(d*x
+c)/b^4/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)
```

Rubi [A]
time = 1.29, antiderivative size = 523, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 8, integrand size = 33, $\frac{\text{number of rules}}{\text{integrand size}} = 0.242$, Rules used = {3033, 3079, 3134, 3138, 2719, 3081, 2720, 2884}

$$\frac{a(Ab-aB)\sin(c+dx)}{2bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)^2}+\frac{a\left(-7a^3B+3a^2Ab+13ab^2B-9Ab^3\right)\sin(c+dx)}{4b^2d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)(a\cos(c+dx)+b)}-\frac{\left(-35a^4B+15a^3Ab+61a^2b^2B-33aAb^3-8b^4B\right)F\left(\frac{1}{2}(c+dx)\middle|2\right)}{12b^3d\left(a^2-b^2\right)^2}-\frac{\left(-35a^4B+15a^3Ab+61a^2b^2B-33aAb^3-8b^4B\right)\sin(c+dx)}{12b^3d\left(a^2-b^2\right)^2\cos^{\frac{3}{2}}(c+dx)}-\frac{\left(-35a^5B+15a^4Ab+65a^3b^2B-29a^2Ab^3-24ab^4B+8Ab^5\right)E\left(\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d\left(a^2-b^2\right)^2}-\frac{a\left(-35a^5B+15a^4Ab+86a^3b^2B-38a^2Ab^3-63ab^4B+35Ab^5\right)\Pi\left(\frac{2a}{a+b};\frac{1}{2}(c+dx)\middle|2\right)}{4b^4d(a-b)^2(a+b)^3}+\frac{\left(-35a^5B+15a^4Ab+65a^3b^2B-29a^2Ab^3-24ab^4B+8Ab^5\right)\sin(c+dx)}{4b^4d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3),x]
```

```
[Out] -1/4*((15*a^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a^5*B + 65*a^3*b^2*B - 24*a
*b^4*B)*EllipticE[(c + d*x)/2, 2])/(b^4*(a^2 - b^2)^2*d) - ((15*a^3*A*b - 3
3*a*A*b^3 - 35*a^4*B + 61*a^2*b^2*B - 8*b^4*B)*EllipticF[(c + d*x)/2, 2])/(
12*b^3*(a^2 - b^2)^2*d) - (a*(15*a^4*A*b - 38*a^2*A*b^3 + 35*A*b^5 - 35*a^5
*B + 86*a^3*b^2*B - 63*a*b^4*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/
(4*(a - b)^2*b^4*(a + b)^3*d) - ((15*a^3*A*b - 33*a*A*b^3 - 35*a^4*B + 61*a
^2*b^2*B - 8*b^4*B)*Sin[c + d*x])/(12*b^3*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2
)) + ((15*a^4*A*b - 29*a^2*A*b^3 + 8*A*b^5 - 35*a^5*B + 65*a^3*b^2*B - 24*a
*b^4*B)*Sin[c + d*x])/(4*b^4*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]) + (a*(A*b
- a*B)*Sin[c + d*x])/(2*b*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d
*x])^2) + (a*(3*a^2*A*b - 9*A*b^3 - 7*a^3*B + 13*a*b^2*B)*Sin[c + d*x])/(4*
b^2*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x]))
```

Rule 2719

```
Int[Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticE[(1/2)*
(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2720

```
Int[1/Sqrt[sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/d)*EllipticF[(1/2
)*(c - Pi/2 + d*x), 2], x] /; FreeQ[{c, d}, x]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 3033

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[g^(m + n), Int[(g*Sin[e + f*x])^(p - m - n)*(b + a*Sin[e + f*x])^m*(d + c
*Sin[e + f*x])^n, x], x] /; FreeQ[{a, b, c, d, e, f, g, p}, x] && NeQ[b*c -
 a*d, 0] &&  !IntegerQ[p] && IntegerQ[m] && IntegerQ[n]
```

Rule 3079

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.) +
 (f_.)*(x_)])*((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)])^(n_), x_Symbol] :> Si
mp[(-(A*b^2 - a*b*B))*Cos[e + f*x]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin
[e + f*x])^(1 + n)/(f*(m + 1)*(b*c - a*d)*(a^2 - b^2))), x] + Dist[1/((m +
1)*(b*c - a*d)*(a^2 - b^2)), Int[(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e
+ f*x])^n*Simp[(a*A - b*B)*(b*c - a*d)*(m + 1) + b*d*(A*b - a*B)*(m + n + 2
) + (A*b - a*B)*(a*d*(m + 1) - b*c*(m + 2))*Sin[e + f*x] - b*d*(A*b - a*B)*
(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b, c, d, e, f, A, B, n}
, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] && Rati
onalQ[m] && m < -1 && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[n]) ||  !(In
tegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) || EqQ[a, 0]))
)
```

Rule 3081

```
Int[(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((A_.) + (B_.)*sin[(e_.)
+ (f_.)*(x_)]))/((c_.) + (d_.)*sin[(e_.) + (f_.)*(x_)]), x_Symbol] :> Dist[
B/d, Int[(a + b*Sin[e + f*x])^m, x], x] - Dist[(B*c - A*d)/d, Int[(a + b*Si
n[e + f*x])^m/(c + d*Sin[e + f*x]), x], x] /; FreeQ[{a, b, c, d, e, f, A, B
```

```
, m}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0]
```

Rule 3134

```
Int[((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])^(m_)*((c_.) + (d_.)*sin[(e_.) +
 (f_.)*(x_)])^(n_)*((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.)
 + (f_.)*(x_)]^2), x_Symbol] :> Simp[(-(A*b^2 - a*b*B + a^2*C))*Cos[e + f*x
]*(a + b*Sin[e + f*x])^(m + 1)*((c + d*Sin[e + f*x])^(n + 1)/(f*(m + 1)*(b*
c - a*d)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(b*c - a*d)*(a^2 - b^2)), Int[
(a + b*Sin[e + f*x])^(m + 1)*(c + d*Sin[e + f*x])^n*Simp[(m + 1)*(b*c - a*d
)*(a*A - b*B + a*C) + d*(A*b^2 - a*b*B + a^2*C)*(m + n + 2) - (c*(A*b^2 - a
*b*B + a^2*C) + (m + 1)*(b*c - a*d)*(A*b - a*B + b*C))*Sin[e + f*x] - d*(A*
b^2 - a*b*B + a^2*C)*(m + n + 3)*Sin[e + f*x]^2, x], x], x] /; FreeQ[{a, b,
 c, d, e, f, A, B, C, n}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && N
eQ[c^2 - d^2, 0] && LtQ[m, -1] && ((EqQ[a, 0] && IntegerQ[m] &&  !IntegerQ[
n]) ||  !(IntegerQ[2*n] && LtQ[n, -1] && ((IntegerQ[n] &&  !IntegerQ[m]) ||
 EqQ[a, 0])))
```

Rule 3138

```
Int[((A_.) + (B_.)*sin[(e_.) + (f_.)*(x_)] + (C_.)*sin[(e_.) + (f_.)*(x_)]^
2)/(Sqrt[(a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)]]*((c_.) + (d_.)*sin[(e_.) +
(f_.)*(x_)])), x_Symbol] :> Dist[C/(b*d), Int[Sqrt[a + b*Sin[e + f*x]], x],
 x] - Dist[1/(b*d), Int[Simp[a*c*C - A*b*d + (b*c*C - b*B*d + a*C*d)*Sin[e
+ f*x], x]/(Sqrt[a + b*Sin[e + f*x]]*(c + d*Sin[e + f*x])), x], x] /; FreeQ
[{a, b, c, d, e, f, A, B, C}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0]
&& NeQ[c^2 - d^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^3}\,dx = \int \frac{B+A\cos(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(b+a\cos(c+dx))^3}\,dx$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))^2} - \frac{\int \frac{1}{2}(3aAb-7a^2B+4b^2}{}$$

$$= \frac{a(Ab-aB)\sin(c+dx)}{2b\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))^2} + \frac{a(3a^2Ab-9Ab^3}{4b^2\left(a^2-b^2\right)^2 d\,\mathrm{co}}$$

$$= -\frac{\left(15a^3Ab-33aAb^3-35a^4B+61a^2b^2B-8b^4B\right)\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)} + \frac{}{2b}$$

$$= -\frac{\left(15a^3Ab-33aAb^3-35a^4B+61a^2b^2B-8b^4B\right)\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)} + \frac{(1}{}$$

$$= -\frac{\left(15a^3Ab-33aAb^3-35a^4B+61a^2b^2B-8b^4B\right)\sin(c+dx)}{12b^3\left(a^2-b^2\right)^2 d\cos^{\frac{3}{2}}(c+dx)} + \frac{(1}{}$$

$$= -\frac{\left(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B\right)E\left(\frac{1}{2}\right.}{4b^4\left(a^2-b^2\right)^2 d}$$

$$= -\frac{\left(15a^4Ab-29a^2Ab^3+8Ab^5-35a^5B+65a^3b^2B-24ab^4B\right)E\left(\frac{1}{2}\right.}{4b^4\left(a^2-b^2\right)^2 d}$$

Mathematica [A]
time = 17.35, size = 570, normalized size = 1.09

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^3),
x]
```

```
[Out] ((2*(-135*a^5*A*b + 285*a^3*A*b^3 - 168*a*A*b^5 + 315*a^6*B - 641*a^4*b^2*B
 + 328*a^2*b^4*B + 16*b^6*B)*EllipticPi[(2*a)/(a + b), (c + d*x)/2, 2])/(a
+ b) + ((-120*a^4*A*b^2 + 240*a^2*A*b^4 - 48*A*b^6 + 280*a^5*b*B - 512*a^3*
b^3*B + 160*a*b^5*B)*(2*EllipticF[(c + d*x)/2, 2] - (2*b*EllipticPi[(2*a)/(
a + b), (c + d*x)/2, 2])/(a + b)))/a + (2*(-45*a^5*A*b + 87*a^3*A*b^3 - 24*
a*A*b^5 + 105*a^6*B - 195*a^4*b^2*B + 72*a^2*b^4*B)*Cos[2*(c + d*x)]*(-2*a*
b*EllipticE[ArcSin[Sqrt[Cos[c + d*x]]], -1] + 2*b*(a + b)*EllipticF[ArcSin[
Sqrt[Cos[c + d*x]]], -1] + (a^2 - 2*b^2)*EllipticPi[-(a/b), ArcSin[Sqrt[Cos
```

```
[c + d*x]]], -1])*Sin[c + d*x])/(a^2*b*Sqrt[1 - Cos[c + d*x]^2]*(-1 + 2*Cos
[c + d*x]^2)))/(48*(a - b)^2*b^4*(a + b)^2*d) + (Sqrt[Cos[c + d*x]]*((2*Sec
[c + d*x]*(A*b*Sin[c + d*x] - 3*a*B*Sin[c + d*x]))/b^4 + (-(a^3*A*b*Sin[c +
 d*x]) + a^4*B*Sin[c + d*x])/(2*b^3*(-a^2 + b^2)*(b + a*Cos[c + d*x])^2) +
(7*a^5*A*b*Sin[c + d*x] - 13*a^3*A*b^3*Sin[c + d*x] - 11*a^6*B*Sin[c + d*x]
 + 17*a^4*b^2*B*Sin[c + d*x])/(4*b^4*(-a^2 + b^2)^2*(b + a*Cos[c + d*x])) +
 (2*B*Sec[c + d*x]*Tan[c + d*x])/(3*b^3)))/d
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2150 vs. $2(579) = 1158$.
time = 25.92, size = 2151, normalized size = 4.11

method	result	size
default	Expression too large to display	2151

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x,method=_RETURNVE
RBOSE)
```

```
[Out] -(-(-2*cos(1/2*d*x+1/2*c)^2+1)*sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*B/b^3*(-1/6*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(cos
(1/2*d*x+1/2*c)^2-1/2)^2+1/3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1
/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Ellip
ticF(cos(1/2*d*x+1/2*c),2^(1/2)))+2*a^2*(A*b-3*B*a)/b^4/(a^2-a*b)*(sin(1/2*
d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c
)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(
1/2))+2*(A*b-3*B*a)/b^4/sin(1/2*d*x+1/2*c)^2/(2*sin(1/2*d*x+1/2*c)^2-1)*(-2
*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*(2*sin(1/2*d*x+1/2*c)^2*c
os(1/2*d*x+1/2*c)-(2*sin(1/2*d*x+1/2*c)^2-1)^(1/2)*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2)))-2*(A*b-B*a)*a/b^2*(1/2/b*a^2/(a
^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(
1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)^2+3/4*a^2*(a^2-3*b^2)/b^2/(a^2-b^2)^2*c
os(1/2*d*x+1/2*c)*(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*c
os(1/2*d*x+1/2*c)^2*a-a+b)-3/8/(a+b)/(a^2-b^2)/b^2*(sin(1/2*d*x+1/2*c)^2)^(
1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x
+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))*a^2-1/4/(a+b)/(a^2-b
^2)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*si
n(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c)
,2^(1/2))*a+7/8/(a+b)/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*
x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*El
lipticF(cos(1/2*d*x+1/2*c),2^(1/2))+3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/
2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+si
n(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-9/8*a/(a^2-
b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*s
in(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x+1/2*c
```

```
),2^(1/2))-3/8*a^3/b^2/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))+9/8*a/(a^2-b^2)^2*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-3/8/(a-b)/(a+b
)/(a^2-b^2)/b^2/(a^2-a*b)*a^5*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+
1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*Elli
pticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/4/(a-b)/(a+b)/(a^2-b^2)/(a^2
-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-
2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1
/2*c),2*a/(a-b),2^(1/2))-15/8/(a-b)/(a+b)/(a^2-b^2)*b^2/(a^2-a*b)*a*(sin(1/
2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2
*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2
^(1/2)))-2*a*(A*b-2*B*a)/b^3*(1/b*a^2/(a^2-b^2)*cos(1/2*d*x+1/2*c)*(-2*sin(
1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)/(2*cos(1/2*d*x+1/2*c)^2*a-a+b)
-1/2/(a+b)/b*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)
/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticF(cos(1/2*d*x
+1/2*c),2^(1/2))+1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2
*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)
*EllipticF(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b*a/(a^2-b^2)*(sin(1/2*d*x+1/2*c
)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1
/2*d*x+1/2*c)^2)^(1/2)*EllipticE(cos(1/2*d*x+1/2*c),2^(1/2))-1/2/b/(a^2-b^2
)/(a^2-a*b)*a^3*(sin(1/2*d*x+1/2*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1
/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2
*d*x+1/2*c),2*a/(a-b),2^(1/2))+3/2*b/(a^2-b^2)/(a^2-a*b)*a*(sin(1/2*d*x+1/2
*c)^2)^(1/2)*(-2*cos(1/2*d*x+1/2*c)^2+1)^(1/2)/(-2*sin(1/2*d*x+1/2*c)^4+sin
(1/2*d*x+1/2*c)^2)^(1/2)*EllipticPi(cos(1/2*d*x+1/2*c),2*a/(a-b),2^(1/2)))
/sin(1/2*d*x+1/2*c)/(2*cos(1/2*d*x+1/2*c)^2-1)^(1/2)/d
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm
="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm
="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(9/2)/(a+b*sec(d*x+c))**3,x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(9/2)/(a+b*sec(d*x+c))^3,x, algorithm
="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^3*cos(d*x + c)^(9/2)),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\cos\left(c+d\,x\right)}^{9/2}\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^3} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^3),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(9/2)*(a + b/cos(c + d*x))^3), x)
```

3.594 $\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=343

$$\frac{2(a^2-b^2)\,(25a^2A+8Ab^2-14abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(19a^2Ab+8Ab^3+63a^3B-14}{}$$

```
[Out] 2/105*(a^2-b^2)*(25*A*a^2+8*A*b^2-14*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+
a*cos(d*x+c))/(a+b))^(1/2)/a^3/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/
35*(A*b+7*B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d+2/7*A
*cos(d*x+c)^(5/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/105*(25*A*a^2-4*A*b
^2+7*B*a*b)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/d+2/105*
(19*A*a^2*b+8*A*b^3+63*B*a^3-14*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1
/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x
+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^3/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.78, antiderivative size = 343, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3034, 4117, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(25a^2A+7abB-4Ab^2)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{105a^2d}+\frac{2(a^2-b^2)\,(25a^2A-14abB+8Ab^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(63a^3B+19a^2Ab-14ab^2B+8Ab^3)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^3d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(7aB+Ab)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{35ad}+\frac{2A\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2 - b^2)*(25*a^2*A + 8*A*b^2 - 14*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(105*a^3*d*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Sec[c + d*x]]) + (2*(19*a^2*A*b + 8*A*b^3 + 63*a^3*B - 14*a*b^2
*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec
[c + d*x]])/(105*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(25*a^2*A -
 4*A*b^2 + 7*a*b*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x
])/(105*a^2*d) + (2*(A*b + 7*a*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/(35*a*d) + (2*A*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]
]*Sin[c + d*x])/(7*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4117

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\,\mathrm{s}}}{}$$

$$= \frac{2A\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+}{7d}$$

$$= \frac{2(Ab+7aB)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx}}{35ad}$$

$$= \frac{2(25a^2A-4Ab^2+7abB)\,\sqrt{\cos(c+dx)}\,\sqrt{a}}{105a^2d}$$

$$= \frac{2(25a^2A-4Ab^2+7abB)\,\sqrt{\cos(c+dx)}\,\sqrt{a}}{105a^2d}$$

$$= \frac{2(25a^2A-4Ab^2+7abB)\,\sqrt{\cos(c+dx)}\,\sqrt{a}}{105a^2d}$$

$$= \frac{2(25a^2A-4Ab^2+7abB)\,\sqrt{\cos(c+dx)}\,\sqrt{a}}{105a^2d}$$

$$= \frac{2(a^2-b^2)\,(25a^2A+8Ab^2-14abB)\sqrt{\frac{b+a}{}}}{105a^3d\sqrt{\cos(c+dx)}\,\sqrt{a+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.68, size = 455, normalized size = 1.33

$$\frac{\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\left(\frac{(115a^2A-16Ab^2+28abB)\sin(c+dx)}{210a^2}+\frac{(Ab+7aB)\sin(2(c+dx))}{35a}+\frac{1}{14}A\sin(3(c+dx))\right)}{d}-\frac{2\cos^{\frac{3}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\sqrt{a+b\sec(c+dx)}\left(-i(a+b)\left(19a^2Ab+8Ab^3+63a^3B-14ab^2B\right)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+ia(a+b)\left(8Ab^2-2ab(3A+7B)+a^2(25A+63B)\right)F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-\left(19a^2Ab+8Ab^3+63a^3B-14ab^2B\right)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{105a^3d(b+a\cos(c+dx))\sqrt{\sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(((115*a^2*A - 16*A*b^2 + 28*a
*b*B)*Sin[c + d*x])/(210*a^2) + ((A*b + 7*a*B)*Sin[2*(c + d*x)])/(35*a) + (
A*Sin[3*(c + d*x)])/14))/d - (2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[
c + d*x])^(3/2)*Sqrt[a + b*Sec[c + d*x]]*((-I)*(a + b)*(19*a^2*A*b + 8*A*b^
```

```
3 + 63*a^3*B - 14*a*b^2*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/
(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/
(a + b)] + I*a*(a + b)*(8*A*b^2 - 2*a*b*(3*A + 7*B) + a^2*(25*A + 63*B))*El
lipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*S
qrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (19*a^2*A*b + 8*A*
b^3 + 63*a^3*B - 14*a*b^2*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2
)*Tan[(c + d*x)/2]))/(105*a^3*d*(b + a*Cos[c + d*x])*Sqrt[Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2363 vs. $2(367) = 734$.
time = 19.07, size = 2364, normalized size = 6.89

method	result	size
default	Expression too large to display	2364

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/105/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c)
)*(1+cos(d*x+c))*(-14*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+
c)))^(1/2)-25*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*(1/(1+cos(d*x+c)))^(1/2)
+25*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-8*A*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+21*B*cos(d
*x+c)^4*((a-b)/(a+b))^(1/2)*a^4*(1/(1+cos(d*x+c)))^(1/2)+42*B*cos(d*x+c)^2*
((a-b)/(a+b))^(1/2)*a^4*(1/(1+cos(d*x+c)))^(1/2)+8*A*cos(d*x+c)*((a-b)/(a+b
))^(1/2)*b^4*(1/(1+cos(d*x+c)))^(1/2)-63*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a
^4*(1/(1+cos(d*x+c)))^(1/2)-25*A*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c)
))^(1/2)-19*A*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)+4*A*((a-
b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)-63*B*((a-b)/(a+b))^(1/2)*a^3
*b*(1/(1+cos(d*x+c)))^(1/2)-7*B*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c
)))^(1/2)+14*B*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)-8*A*((a-b
)/(a+b))^(1/2)*b^4*(1/(1+cos(d*x+c)))^(1/2)+63*B*sin(d*x+c)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-63*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+15*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4
*(1/(1+cos(d*x+c)))^(1/2)+10*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*(1/(1+c
os(d*x+c)))^(1/2)+18*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x
+c)))^(1/2)-A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))^(
1/2)+28*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c)))^(1/2)+2
6*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c)))^(1/2)+4*A*cos
(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)-7*B*cos(d*x+c)
```

```
^2*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)-19*A*cos(d*x+c)*((a
-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c)))^(1/2)+20*A*cos(d*x+c)*((a-b)/(a+b
))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)-8*A*cos(d*x+c)*((a-b)/(a+b))^(1/2
)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)+35*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b*
(1/(1+cos(d*x+c)))^(1/2)+14*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+
cos(d*x+c)))^(1/2)+8*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a*b^3-19*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^3*b+2*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^2*b^2-8*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a*b^3+19*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3
*b-19*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^
2-63*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-1
4*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+14
*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+49*B*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+14*B*sin
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2)/a^3/((a-
b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(7/2),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.88, size = 578, normalized size = 1.69

$$\frac{6\left(15\,Aa^{4}\cos\left(dx+c\right)^{2}+25\,Aa^{4}+7\,Ba^{3}b-4\,Aa^{2}b^{2}+3\left(7\,Ba^{4}+Aa^{3}b\right)\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\sqrt{2}\left(-75i\,Aa^{4}-21i\,Ba^{3}b+32i\,Aa^{2}b^{2}-28i\,Bab^{3}+16i\,Ab^{4}\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos\left(dx+c\right)+3ia\sin\left(dx+c\right)+2b}{3a}\right)+\sqrt{2}\left(75i\,Aa^{4}+21i\,Ba^{3}b-32i\,Aa^{2}b^{2}+28i\,Bab^{3}-16i\,Ab^{4}\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos\left(dx+c\right)-3ia\sin\left(dx+c\right)+2b}{3a}\right)-3\sqrt{2}\left(-63i\,Ba^{4}-19i\,Aa^{3}b+14i\,Ba^{2}b^{2}-8i\,Aab^{3}\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos\left(dx+c\right)+3ia\sin\left(dx+c\right)+2b}{3a}\right)\right)-3\sqrt{2}\left(63i\,Ba^{4}+19i\,Aa^{3}b-14i\,Ba^{2}b^{2}+8i\,Aab^{3}\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos\left(dx+c\right)-3ia\sin\left(dx+c\right)+2b}{3a}\right)\right)}{315\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/315*(6*(15*A*a^4*cos(d*x + c)^2 + 25*A*a^4 + 7*B*a^3*b - 4*A*a^2*b^2 + 3*
(7*B*a^4 + A*a^3*b)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*s
qrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-75*I*A*a^4 - 21*I*B*a^3*b + 32*I
*A*a^2*b^2 - 28*I*B*a*b^3 + 16*I*A*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3
*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*
a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(75*I*A*a^4 + 21*I*B*a^3*b - 32*I*A*a^2*
b^2 + 28*I*B*a*b^3 - 16*I*A*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 -
4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d
*x + c) + 2*b)/a) - 3*sqrt(2)*(-63*I*B*a^4 - 19*I*A*a^3*b + 14*I*B*a^2*b^2
- 8*I*A*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^
2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2
*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*
sqrt(2)*(63*I*B*a^4 + 19*I*A*a^3*b - 14*I*B*a^2*b^2 + 8*I*A*a*b^3)*sqrt(a)*
weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weier
strassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3
*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^4*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(7/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int {\cos\left(c+d\,x\right)}^{7/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2),x)
[Out] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2), x)
```

3.595 $\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=267

$$-\frac{2(a^2-b^2)(2Ab-5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2A-2Ab^2+5abB)\sqrt{\cos(c+dx)}\,E(}{15a^2d\sqrt{\frac{b+a\cos}{a}}}$$

```
[Out] -2/15*(a^2-b^2)*(2*A*b-5*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/
(a+b))^(1/2)/a^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/5*A*cos(d*x+c)
^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/15*(A*b+5*B*a)*sin(d*x+c)*cos(
d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/d+2/15*(9*A*a^2-2*A*b^2+5*B*a*b)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*
cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 267, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3034, 4117, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{2(a^2-b^2)(2Ab-5aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2A+5abB-2Ab^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{15a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(5aB+Ab)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{15ad}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (-2*(a^2 - b^2)*(2*A*b - 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elliptic
F[(c + d*x)/2, (2*a)/(a + b)])/(15*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[
c + d*x]]) + (2*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^2*d*Sqrt[(b + a
*Cos[c + d*x])/(a + b)]) + (2*(A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*S
ec[c + d*x]]*Sin[c + d*x])/(15*a*d) + (2*A*Cos[c + d*x]^(3/2)*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/(5*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4117

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n
```

```
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\,\mathrm{se}}}{}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+}{5d}$$

$$= \frac{2(Ab+5aB)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+d}}{15ad}$$

$$= \frac{2(Ab+5aB)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+d}}{15ad}$$

$$= \frac{2(Ab+5aB)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+d}}{15ad}$$

$$= \frac{2(Ab+5aB)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+d}}{15ad}$$

$$= -\frac{2(a^2-b^2)(2Ab-5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}{15a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.40, size = 353, normalized size = 1.32

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\left(a(Ab+5aB+3aA\cos(c+dx))\sin(c+dx)-\frac{(\cos^2(\frac{1}{2}(c+dx))\sec(c+dx))^{3/2}\left(-i(a+b)(9a^2A-2Ab^2+5abB)E\left(i\sinh^{-1}(\tan(\frac{1}{2}(c+dx)))|\frac{-a+b}{a+b}\right)\sec^2(\frac{1}{2}(c+dx))\sqrt{\frac{(b+a\cos(c+dx))\sec^2(\frac{1}{2}(c+dx))}{a+b}}+ia(a+b)(9aA-2Ab+5aB)F\left(i\sinh^{-1}(\tan(\frac{1}{2}(c+dx)))|\frac{-a+b}{a+b}\right)\sec^2(\frac{1}{2}(c+dx))\sqrt{\frac{(b+a\cos(c+dx))\sec^2(\frac{1}{2}(c+dx))}{a+b}}-(9a^2A-2Ab^2+5abB)(b+a\cos(c+dx))\sec^2(\frac{1}{2}(c+dx))^{3/2}\tan(\frac{1}{2}(c+dx))\right)}{(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\right)}{15a^2d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(a*(A*b + 5*a*B + 3*a*A*Cos[
c + d*x])*Sin[c + d*x] - ((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(a
+ b)*(9*a^2*A - 2*A*b^2 + 5*a*b*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (
-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x
)/2]^2)/(a + b)] + I*a*(a + b)*(9*a*A - 2*A*b + 5*a*B)*EllipticF[I*ArcSinh[
Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
```

```
+ d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (9*a^2*A - 2*A*b^2 + 5*a*b*B)*(b + a
*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/((b + a*Cos[c
+ d*x])*Sec[c + d*x]^(3/2))))/(15*a^2*d)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1700 vs. $2(297) = 594$.
time = 19.05, size = 1701, normalized size = 6.37

method	result	size
default	Expression too large to display	1701

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/15/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*cos(d*x+c)^(1/2)*(-1+cos(d*x+c))
*(1+cos(d*x+c))*(2*A*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)-9*A*s
in(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+9*A*sin(d*x
+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+2*A*sin(d*x+c)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+4*A*((a-b)/(a+b))^(1/2)
*cos(d*x+c)^3*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+3*A*((a-b)/(a+b))^(1/2)*cos(d*
x+c)^4*a^3*(1/(1+cos(d*x+c)))^(1/2)+5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*(1
/(1+cos(d*x+c)))^(1/2)*a^3-5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(1+cos
(d*x+c)))^(1/2)+5*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^3+6*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*(1/(1+cos(d*x+c)))^(1/2)
-9*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(1+cos(d*x+c)))^(1/2)-2*A*((a-b)
/(a+b))^(1/2)*cos(d*x+c)*b^3*(1/(1+cos(d*x+c)))^(1/2)-9*A*((a-b)/(a+b))^(1/
2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-A*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x
+c)))^(1/2)-5*B*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-5*B*((a-
b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-A*((a-b)/(a+b))^(1/2)*cos(d*
x+c)^2*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+10*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2
*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+5*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1
/(1+cos(d*x+c)))^(1/2)+2*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+cos(d
*x+c)))^(1/2)-5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1/(1+cos(d*x+c)))^(
1/2)+5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+7*A*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+2*A*sin(
d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-9*A*sin(d*x+
c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-2*A*sin(d*x+c)*(
```

```
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+5*B*sin(d*x+c)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-5*B*sin(d*x+c)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-5*B*sin(d*x+c)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b)/a^2/((a-b)/(a+b))^(1/2)/(b+a*co
s(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.31, size = 509, normalized size = 1.91

$$\frac{6\left(3\,Aa^3\cos\left(dx+c\right)+5\,Ba^3+Aa^2b\right)\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\sqrt{2}\left(-15i\,Ba^3-3i\,Aa^2b+10i\,Bab^2-4i\,Ab^3\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(15i\,Ba^3+3i\,Aa^2b-10i\,Bab^2+4i\,Ab^3\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(-9i\,Aa^3-5i\,Ba^2b+2i\,Aab^2\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(9i\,Aa^3+5i\,Ba^2b-2i\,Aab^2\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)\right)}{45\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/45*(6*(3*A*a^3*cos(d*x + c) + 5*B*a^3 + A*a^2*b)*sqrt((a*cos(d*x + c) + b
)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-15*I*B*a^3 - 3*
I*A*a^2*b + 10*I*B*a*b^2 - 4*I*A*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*
sin(d*x + c) + 2*b)/a) + sqrt(2)*(15*I*B*a^3 + 3*I*A*a^2*b - 10*I*B*a*b^2 +
 4*I*A*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3
*sqrt(2)*(-9*I*A*a^3 - 5*I*B*a^2*b + 2*I*A*a*b^2)*sqrt(a)*weierstrassZeta(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4
/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) +
 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(9*I*A*a^3 + 5*I*B*a^2*b - 2*I*A
*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8
*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*
b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{5/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2), x)
```

3.596 $\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=201

$$\frac{2A(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(Ab+3aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b}}{3ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

```
[Out] 2/3*A*(a^2-b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a
/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*A*sin(d*x+c)*cos(d*x+c)^(1/2
)*(a+b*sec(d*x+c))^(1/2)/d+2/3*(A*b+3*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos
(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d
*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.39, antiderivative size = 201, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3034, 4117, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2A(a^2-b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(3aB+Ab)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*A*(a^2 - b^2)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(3*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(A
*b + 3*a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
 + b*Sec[c + d*x]])/(3*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*A*Sqrt[
Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4117

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^n/(f*n)), x] - Dist[1/(d*n
), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n + 1)*Simp[A*b*m - a
*B*n - (b*B*n + a*A*(n + 1))*Csc[e + f*x] - A*b*(m + n + 1)*Csc[e + f*x]^2,
 x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[
```

```
a^2 - b^2, 0] && LtQ[0, m, 1] && LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\,\mathrm{se}}}{}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+}{3d}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+}{3d}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+}{3d}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+}{3d}$$

$$= \frac{2A(a^2-b^2)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx\right.}{3ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 8.49, size = 305, normalized size = 1.52

$$\frac{2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\left(A\sin(c+dx)+\frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(i(a+b)(Ab+3aB)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-ia(a+b)(A+3B)F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+(Ab+3aB)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a(b+a\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)}\right)}{3d}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A*Sin[c + d*x] + ((Cos[(c +
 d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*(a + b)*(A*b + 3*a*B)*EllipticE[I*ArcSinh
[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c
 + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a + b)*(A + 3*B)*EllipticF[I*A
rcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a
*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (A*b + 3*a*B)*(b + a*Cos[c +
d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(a*(b + a*Cos[c + d*x])
*Sec[c + d*x]^(3/2))))/(3*d)
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 1161 vs. $2(237) = 474$.
time = 19.82, size = 1162, normalized size = 5.78

method	result	size
default	Expression too large to display	1162

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/3/d*(-1+cos(d*x+c))*(1+cos(d*x+c))*(A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^
2*(1/(1+cos(d*x+c)))^(1/2)+2*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(1+c
os(d*x+c)))^(1/2)+3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)
))^(1/2)-A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)-A*co
s(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)+A*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)+A*sin(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a*b+A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a*b-A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*b^2-3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2
)+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)-3*B*sin(d
*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2+3*B*sin(d*x+c)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+3*B*sin(d*x+c)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-3*B*sin(d*x+c)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-A*((a-b)/(a+b))^(1/2)*a*b*(1/(1+c
```

```
os(d*x+c)))^(1/2)-A*((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)-3*B*((
a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^(1/2)*((b+a*cos(
d*x+c))/cos(d*x+c))^(1/2)/a/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(
d*x+c)))^(1/2)/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.69, size = 452, normalized size = 2.25

$$\frac{6\,Aa^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\sqrt{2}\,(-3i\,Aa^2-3i\,Bab+2i\,Ab^2)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\,(3i\,Aa^2+3i\,Bab-2i\,Ab^2)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\,(-3i\,Ba^2-i\,Aab)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\,(3i\,Ba^2+i\,Aab)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)}{9\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/9*(6*A*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c) + sqrt(2)*(-3*I*A*a^2 - 3*I*B*a*b + 2*I*A*b^2)*sqrt(a)*weierstras
sPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*co
s(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*A*a^2 + 3*I*B*a*b
- 2*I*A*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*
a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) -
3*sqrt(2)*(-3*I*B*a^2 - I*A*a*b)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^
2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2
)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x +
c) + 2*b)/a)) - 3*sqrt(2)*(3*I*B*a^2 + I*A*a*b)*sqrt(a)*weierstrassZeta(-4/
3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3
*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3
*I*a*sin(d*x + c) + 2*b)/a)))/(a^2*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{3/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2), x)
```

3.597 $\int \sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=208

$$\frac{2aB\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2bB\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2A\sqrt{\cos(c+d}}{}$$

```
[Out] 2*a*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c
)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*b*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d
*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*co
s(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*A*(cos(1
/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1
/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*
x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.45, antiderivative size = 208, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 12, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.343$, Rules used = {3034, 4122, 3939, 3943, 2742, 2740, 3944, 2886, 2884, 3941, 2734, 2732}

$$\frac{2A\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2aB\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2bB\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*a*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a +
 b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*b*B*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*A*Sqrt[Cos[c + d*x]]*EllipticE[
(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c
+ d*x])/(a + b)])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3939

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[a, Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f
```

```
*x]], x], x] + Dist[b/d, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]
], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4122

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]*(csc[(e_.) + (f_.)*(x_)]*(B
_.) + (A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)], x_Symbol] :> Dist[B/d, In
t[Sqrt[a + b*Csc[e + f*x]]*Sqrt[d*Csc[e + f*x]], x], x] + Dist[A, Int[Sqrt[
a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f, A
, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\,\mathrm{s}}}{}$$

$$= \left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+}}{\sqrt{\mathrm{se}}}$$

$$= \left(aB\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{}}{\sqrt{a+}}$$

$$= \frac{\left(aB\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+}}}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)\sqrt{a+b}}{d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

$$= \frac{2aB\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 29.90, size = 25347, normalized size = 121.86

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]),
x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 19.52, size = 1563, normalized size = 7.51

method	result	size
default	Expression too large to display	1563

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/d*(-1+cos(d*x+c))*(1+cos(d*x+c))*(-A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(1+cos(
d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a+A*cos(d*x+c)
*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*b+A*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(1+cos(d*x+c)))^(1/2)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a-A*cos(d*x+c)*sin(d*x+c)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(1+c
os(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b+2*B*cos(d
*x+c)*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*b*(1/(1+cos(d*x+c)))^(1/2)+B*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*(1/(1+cos(d*x+c)))^(1/2)-B*cos(d*x+c)*sin
(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b*(1/(1+cos(d*x+c
)))^(1/2)-A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*
(1/(1+cos(d*x+c)))^(1/2)+A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*b*(1/(1+cos(d*x+c)))^(1/2)+A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a*(1/(1+cos(d*x+c)))^(1/2)-A*sin(d*x+c)*Ellipti
cE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/
(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b+2*B*s
in(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(
a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b
*(1/(1+cos(d*x+c)))^(1/2)+B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*a*(1/(1+cos(d*x+c)))^(1/2)-B*sin(d*x+c)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*b*(1/(1+cos(d*x+c)))^(1/2)+A*cos(d*x+c)^2*((a-
b)/(a+b))^(1/2)*a-A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a+A*cos(d*x+c)*((a-b)/(a
+b))^(1/2)*b-A*((a-b)/(a+b))^(1/2)*b)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/co
s(d*x+c))^(1/2)/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^3
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)),
 x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] integral((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)),
x)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int (A + B\sec(c + dx))\sqrt{a + b\sec(c + dx)}\sqrt{\cos(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)*(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))*sqrt(cos(c + d*x)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos(c + d\,x)}\left(A + \frac{B}{\cos(c + d\,x)}\right)\sqrt{a + \frac{b}{\cos(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2), x)
```

3.598 $\int \frac{\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=253

$$\frac{(2aA+bB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{(2Ab+aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-$$

```
[Out] (2*A*a+B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1
/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos
(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+(2*A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/
2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2
)+B*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-B*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+
b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b
))^(1/2)
```

Rubi [A]
time = 0.60, antiderivative size = 253, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3034, 4116, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(2aA+bB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{(aB+2Ab)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{B\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\cos(c+dx)}}-\frac{B\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((2*a*A + b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b + a
*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
+ b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (B*Sqrt[Cos[c + d*
x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(d*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]) + (B*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])
/(d*Sqrt[Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4116

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n
- 1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B
*m + A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}
, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,\sqrt{a+b\,\mathrm{s}$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{sec}\right.$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\cos(c+dx)}\,\sqrt{\mathrm{sec}\right.$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{1}{2}\left(B\sqrt{\cos(c+dx)}\,\sqrt{\right.$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{\left((2aA+bB)\sqrt{b+a\right.}{2\sqrt{\cos(c}$$

$$= \frac{(2Ab+aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{B}{}$$

$$= \frac{(2aA+bB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{(2Ab}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 32.47, size = 52603, normalized size = 207.92

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]
],x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 19.06, size = 774, normalized size = 3.06

method	result
default	$-\frac{\left(2A\sin(dx+c)\cos(dx+c)\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\ \text{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)a-2A\sin(dx+c)\cos(dx+\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/d*(2*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a-2*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*b+4*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a
-b),I/((a-b)/(a+b))^(1/2))*b+2*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a-B*sin(d*x+c)*cos(d*x+c)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a+B*sin(d*x+c)*cos(d*x+c)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b+B*cos(d*x+c)^2*((a-b)/(a+b)
)^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*(1/(1+c
os(d*x+c)))^(1/2)*a+B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/
2)*b-B*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b)*((b+a*cos(d*x+c))/co
s(d*x+c))^(1/2)/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/
2)/cos(d*x+c)^(1/2)/sin(d*x+c)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)),
 x)
```

Fricas [F(-1)] Timed out

time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sqrt{a + b\sec(c + dx)}}{\sqrt{\cos(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))/sqrt(cos(c + d*x)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/sqrt(cos(d*x + c)),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A + \frac{B}{\cos(c+d\,x)}\right)\,\sqrt{a + \frac{b}{\cos(c + d\,x)}}}{\sqrt{\cos(c + d\,x)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x)^(1/2), x
)
```

3.599 $\int \frac{\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=336

$$\frac{(4Ab+3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{(4aAb-a^2B+4b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c-\right.}{4bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 1/4*(4*A*b+3*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)
/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/4*(4*A*a*b-B*a^2+4*B*b^2)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2
,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b/d/cos(d*x+c)^(1/
2)/(a+b*sec(d*x+c))^(1/2)+1/2*B*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x
+c)^(3/2)+1/4*(4*A*b+B*a)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/b/d/cos(d*x+c)^
(1/2)-1/4*(4*A*b+B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(
d*x+c))^(1/2)/b/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]

time = 0.80, antiderivative size = 336, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3034, 4116, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(a^2(-B)+4aAb+4b^2B)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{(aB+4Ab)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4bd\sqrt{\cos(c+dx)}}+\frac{(3aB+4Ab)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{(aB+4Ab)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{4bd\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{B\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{2d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] ((4*A*b + 3*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((4*a*A
*b - a^2*B + 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
 d*x)/2, (2*a)/(a + b)])/(4*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]
) - ((4*A*b + a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]
*Sqrt[a + b*Sec[c + d*x]])/(4*b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (B*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(3/2)) + ((4*A*b +
 a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*b*d*Sqrt[Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4116

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d*
Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e + f*x])^(n - 1)/(f*(m + n))),
 x] + Dist[d/(m + n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*Csc[e + f*x])^(n
- 1)*Simp[a*B*(n - 1) + (b*B*(m + n - 1) + a*A*(m + n))*Csc[e + f*x] + (a*B
*m + A*b*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}
, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[0, m, 1] && GtQ[n, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
```

```
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{a+b\sec(c+dx)}\,(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\,\mathrm{s}\cdots$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\,\sqrt{\cdots}\right.$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab+aB)\sqrt{a+b\cdots}}{4bd\sqrt{\mathrm{c}\cdots}}$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab+aB)\sqrt{a+b\cdots}}{4bd\sqrt{\mathrm{c}\cdots}}$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab+aB)\sqrt{a+b\cdots}}{4bd\sqrt{\mathrm{c}\cdots}}$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab+aB)\sqrt{a+b\cdots}}{4bd\sqrt{\mathrm{c}\cdots}}$$

$$= \frac{(4aAb-a^2B+4b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\tfrac{1}{2}(c+dx\cdots\right.}{4bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{(4Ab+3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{(4\cdots}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 33.19, size = 77879, normalized size = 231.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[a + b*Sec[c + d*x]]*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2
),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 19.67, size = 1474, normalized size = 4.39

method	result	size
default	Expression too large to display	1474

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/d*(-1+cos(d*x+c))*(1+cos(d*x+c))*(4*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*
a*b*(1/(1+cos(d*x+c)))^(1/2)-4*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+4*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+8*A*cos(d*x+c)^2*sin(d*x+c)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b+B*cos(d*x+c
)^3*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)+2*B*cos(d*x+c)^3*((a-b
)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)-B*cos(d*x+c)^2*sin(d*x+c)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+B*cos(d*x+c)^2*sin(d*x+c)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+2*B*cos(d*x+c)^2*sin(
d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2+2*B*cos(d*x+c)
^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-4*B*cos
(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2-
2*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-
b)/(a+b))^(1/2))*a^2+8*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2-4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1
/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)+4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2*(
1/(1+cos(d*x+c)))^(1/2)-B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*
x+c)))^(1/2)+B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2
)+2*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)-4*A*cos
(d*x+c)*((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)-3*B*cos(d*x+c)*((a
-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)-2*B*((a-b)/(a+b))^(1/2)*(1/(1
+cos(d*x+c)))^(1/2)*b^2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/b/((a-b)/(a+b)
)^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3/cos(d*x+c)^(
3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2),
 x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec(c + dx))\sqrt{a + b\sec(c + dx)}}{\cos^{\frac{3}{2}}(c + dx)}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))**(1/2)/cos(d*x+c)**(3/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(a + b*sec(c + d*x))/cos(c + d*x)**(3/2),
 x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*(a+b*sec(d*x+c))^(1/2)/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(b*sec(d*x + c) + a)/cos(d*x + c)^(3/2),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}{{\cos\left(c+d\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2),x)
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(1/2))/cos(c + d*x)^(3/2), x
)
```

3.600 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=427

$$\frac{2(a^2-b^2)\left(39a^2Ab+8Ab^3+75a^3B-18ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{315a^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(147a^4A+33a^2A$$

```
[Out] 2/315*(a^2-b^2)*(39*A*a^2*b+8*A*b^3+75*B*a^3-18*B*a*b^2)*(cos(1/2*d*x+1/2*c
)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b)
)^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a^3/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x
+c))^(1/2)+2/315*(49*A*a^2+3*A*b^2+72*B*a*b)*cos(d*x+c)^(3/2)*sin(d*x+c)*(a
+b*sec(d*x+c))^(1/2)/a/d+2/63*(10*A*b+9*B*a)*cos(d*x+c)^(5/2)*sin(d*x+c)*(a
+b*sec(d*x+c))^(1/2)/d+2/9*a*A*cos(d*x+c)^(7/2)*sin(d*x+c)*(a+b*sec(d*x+c))
^(1/2)/d+2/315*(88*A*a^2*b-4*A*b^3+75*B*a^3+9*B*a*b^2)*sin(d*x+c)*cos(d*x+c
)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/d+2/315*(147*A*a^4+33*A*a^2*b^2+8*A*b^4+
246*B*a^3*b-18*B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*se
c(d*x+c))^(1/2)/a^3/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 1.09, antiderivative size = 427, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3034, 4110, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(49a^2A+72abB+3Ab^2)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{315ad}+\frac{2(75a^3B+88a^2Ab+9ab^2B-4Ab^3)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{315a^2d}+\frac{2(a^2-b^2)\left(75a^3B+39a^2Ab-18ab^2B+8Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{315a^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(147a^4A+246a^3bB+33a^2Ab^2-18ab^3B+8Ab^4)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{315a^3d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(9aB+10Ab)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{63d}+\frac{2aA\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2 - b^2)*(39*a^2*A*b + 8*A*b^3 + 75*a^3*B - 18*a*b^2*B)*Sqrt[(b + a*C
os[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(315*a^3*d*Sqr
t[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(147*a^4*A + 33*a^2*A*b^2 +
8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/
2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^3*d*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]) + (2*(88*a^2*A*b - 4*A*b^3 + 75*a^3*B + 9*a*b^2*B)*Sqrt[Cos
[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a^2*d) + (2*(49*a^2*
A + 3*A*b^2 + 72*a*b*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c +
 d*x])/(315*a*d) + (2*(10*A*b + 9*a*B)*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c
+ d*x]]*Sin[c + d*x])/(63*d) + (2*a*A*Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c +
 d*x]]*Sin[c + d*x])/(9*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec}{}$$

$$= \frac{2aA\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+}{9d}$$

$$= \frac{2(10Ab+9aB)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+}{63d}$$

$$= \frac{2(49a^2A+3Ab^2+72abB)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+}{315ad}$$

$$= \frac{2(88a^2Ab-4Ab^3+75a^3B+9ab^2B)\sqrt{\cos(c}}{315a^2d}$$

$$= \frac{2(88a^2Ab-4Ab^3+75a^3B+9ab^2B)\sqrt{\cos(c}}{315a^2d}$$

$$= \frac{2(88a^2Ab-4Ab^3+75a^3B+9ab^2B)\sqrt{\cos(c}}{315a^2d}$$

$$= \frac{2(88a^2Ab-4Ab^3+75a^3B+9ab^2B)\sqrt{\cos(c}}{315a^2d}$$

$$= \frac{2(a^2-b^2)(39a^2Ab+8Ab^3+75a^3B-18ab^2}{315a^3d\sqrt{\cos(c+dx)}\,\sqrt{}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 18.72, size = 540, normalized size = 1.26

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(((402*a^2*A*b - 16*A*b^3 +
345*a^3*B + 36*a*b^2*B)*Sin[c + d*x])/(630*a^2) + ((133*a^2*A + 6*A*b^2 + 1
```

```
44*a*b*B)*Sin[2*(c + d*x)])/(630*a) + ((10*A*b + 9*a*B)*Sin[3*(c + d*x)])/1
26 + (a*A*Sin[4*(c + d*x)])/36))/(d*(b + a*Cos[c + d*x])) - (2*Cos[c + d*x]
^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(
(-I)*(a + b)*(147*a^4*A + 33*a^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B
)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]
^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(8
*A*b^3 - 6*a*b^2*(A + 3*B) + 3*a^3*(49*A + 25*B) + 3*a^2*b*(13*A + 57*B))*E
llipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*
Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (147*a^4*A + 33*a
^2*A*b^2 + 8*A*b^4 + 246*a^3*b*B - 18*a*b^3*B)*(b + a*Cos[c + d*x])*(Sec[(c
 + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(315*a^3*d*(b + a*Cos[c + d*x])^2*Se
c[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3068 vs. $2(445) = 890$.
time = 20.94, size = 3069, normalized size = 7.19

method	result	size
default	Expression too large to display	3069

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/315/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)*(1+cos(d*x+c))*(85*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4*b*(1/(1+cos(d*x
+c)))^(1/2)-8*A*((a-b)/(a+b))^(1/2)*b^5*(1/(1+cos(d*x+c)))^(1/2)+35*A*cos(d
*x+c)^6*((a-b)/(a+b))^(1/2)*a^5*(1/(1+cos(d*x+c)))^(1/2)+45*B*cos(d*x+c)^5*
((a-b)/(a+b))^(1/2)*a^5*(1/(1+cos(d*x+c)))^(1/2)+30*B*cos(d*x+c)^3*((a-b)/(
a+b))^(1/2)*a^5*(1/(1+cos(d*x+c)))^(1/2)-75*B*cos(d*x+c)*((a-b)/(a+b))^(1/2
)*a^5*(1/(1+cos(d*x+c)))^(1/2)+75*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^5+14*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^5*(1/(1+c
os(d*x+c)))^(1/2)+98*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5*(1/(1+cos(d*x+c
)))^(1/2)-147*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5*(1/(1+cos(d*x+c)))^(1/2)
+8*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^5*(1/(1+cos(d*x+c)))^(1/2)-147*A*((a-
b)/(a+b))^(1/2)*a^4*b*(1/(1+cos(d*x+c)))^(1/2)-88*A*((a-b)/(a+b))^(1/2)*a^3
*b^2*(1/(1+cos(d*x+c)))^(1/2)-33*A*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(1+cos(d*
x+c)))^(1/2)+4*A*((a-b)/(a+b))^(1/2)*a*b^4*(1/(1+cos(d*x+c)))^(1/2)-75*B*((
a-b)/(a+b))^(1/2)*a^4*b*(1/(1+cos(d*x+c)))^(1/2)-246*B*((a-b)/(a+b))^(1/2)*
a^3*b^2*(1/(1+cos(d*x+c)))^(1/2)-9*B*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(1+cos(
d*x+c)))^(1/2)+18*B*((a-b)/(a+b))^(1/2)*a*b^4*(1/(1+cos(d*x+c)))^(1/2)-147*
A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5+147*A*si
n(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
```

x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5-8*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5+53*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(1+cos(d*x+c)))^(1/2)+117*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4*b*(1/(1+cos(d*x+c)))^(1/2)+52*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b*(1/(1+cos(d*x+c)))^(1/2)-A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(1+cos(d*x+c)))^(1/2)-246*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b+153*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+18*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3+246*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b-246*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2-18*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3+18*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+81*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(1+cos(d*x+c)))^(1/2)+68*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(1+cos(d*x+c)))^(1/2)+204*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*b*(1/(1+cos(d*x+c)))^(1/2)+10*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b*(1/(1+cos(d*x+c)))^(1/2)-33*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(1+cos(d*x+c)))^(1/2)+34*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(1+cos(d*x+c)))^(1/2)-8*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4*(1/(1+cos(d*x+c)))^(1/2)-246*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b*(1/(1+cos(d*x+c)))^(1/2)+165*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(1+cos(d*x+c)))^(1/2)+18*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(1+cos(d*x+c)))^(1/2)-18*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4*(1/(1+cos(d*x+c)))^(1/2)+4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^4-9*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^3+186*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b-33*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+2*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3-8*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4-147*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b+33*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2-33*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d...

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(9/2
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.81, size = 656, normalized size = 1.54

$$\frac{6\left(35Aa^5\cos(dx+c)^3+75Ba^5+88Aa^4b+9Ba^3b^2-4Aa^2b^3+5\left(9Ba^5+10Aa^4b\right)\cos(dx+c)^2+\left(49Aa^5+72Ba^4b+3Aa^3b^2\right)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\sqrt{2}\left(-225iBa^5-264iAa^4b+33iBa^3b^2+60iAa^2b^3-36iBab^4+16iAb^5\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(225iBa^5+264iAa^4b-33iBa^3b^2-60iAa^2b^3+36iBab^4-16iAb^5\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(-147iAa^5-246iBa^4b-33iAa^3b^2+18iBa^2b^3-8iAab^4\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(147iAa^5+246iBa^4b+33iAa^3b^2-18iBa^2b^3+8iAab^4\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)\right)}{945a^4d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/945*(6*(35*A*a^5*cos(d*x + c)^3 + 75*B*a^5 + 88*A*a^4*b + 9*B*a^3*b^2 - 4
*A*a^2*b^3 + 5*(9*B*a^5 + 10*A*a^4*b)*cos(d*x + c)^2 + (49*A*a^5 + 72*B*a^4
*b + 3*A*a^3*b^2)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqr
t(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-225*I*B*a^5 - 264*I*A*a^4*b + 33*I
*B*a^3*b^2 + 60*I*A*a^2*b^3 - 36*I*B*a*b^4 + 16*I*A*b^5)*sqrt(a)*weierstras
sPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*co
s(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(225*I*B*a^5 + 264*I*A*
a^4*b - 33*I*B*a^3*b^2 - 60*I*A*a^2*b^3 + 36*I*B*a*b^4 - 16*I*A*b^5)*sqrt(a
)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3,
 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-147*I*A
*a^5 - 246*I*B*a^4*b - 33*I*A*a^3*b^2 + 18*I*B*a^2*b^3 - 8*I*A*a*b^4)*sqrt(
a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, we
ierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3
*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(147*I*A*a^5
 + 246*I*B*a^4*b + 33*I*A*a^3*b^2 - 18*I*B*a^2*b^3 + 8*I*A*a*b^4)*sqrt(a)*w
eierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weiers
trassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*
a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^4*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(9/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{9/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2), x)
```

3.601 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=342

$$\frac{2(a^2-b^2)\left(25a^2A-6Ab^2+21abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(82a^2Ab-6Ab^3+63a^3B+}{}$$

```
[Out] 2/105*(a^2-b^2)*(25*A*a^2-6*A*b^2+21*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+
a*cos(d*x+c))/(a+b))^(1/2)/a^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/
35*(8*A*b+7*B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/7*a
*A*cos(d*x+c)^(5/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/105*(25*A*a^2+3*A
*b^2+42*B*a*b)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/d+2/105
*(82*A*a^2*b-6*A*b^3+63*B*a^3+21*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*
x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.84, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3034, 4110, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(25a^2A+42abB+3Ab^2)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{105ad}+\frac{2(a^2-b^2)\left(25a^2A+21abB-6Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(63a^3B+82a^2Ab+21ab^2B-6Ab^3)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(7aB+8Ab)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{35d}+\frac{2aA\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2 - b^2)*(25*a^2*A - 6*A*b^2 + 21*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(105*a^2*d*Sqrt[Cos[c + d*x]]
*Sqrt[a + b*Sec[c + d*x]]) + (2*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2
*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec
[c + d*x]])/(105*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(25*a^2*A +
 3*A*b^2 + 42*a*b*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*
x])/(105*a*d) + (2*(8*A*b + 7*a*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(35*d) + (2*a*A*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x
]]*Sin[c + d*x])/(7*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec}{}$$

$$= \frac{2aA\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+}{7d}$$

$$= \frac{2(8Ab+7aB)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx}}{35d}$$

$$= \frac{2(25a^2A+3Ab^2+42abB)\sqrt{\cos(c+dx)}\,\sqrt{a}}{105ad}$$

$$= \frac{2(25a^2A+3Ab^2+42abB)\sqrt{\cos(c+dx)}\,\sqrt{a}}{105ad}$$

$$= \frac{2(25a^2A+3Ab^2+42abB)\sqrt{\cos(c+dx)}\,\sqrt{a}}{105ad}$$

$$= \frac{2(25a^2A+3Ab^2+42abB)\sqrt{\cos(c+dx)}\,\sqrt{a}}{105ad}$$

$$= \frac{2(a^2-b^2)(25a^2A-6Ab^2+21abB)\sqrt{\frac{b+a\,c}{}}}{105a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.39, size = 466, normalized size = 1.36

$$\frac{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(\frac{(115a^2A+12Ab^2+168abB)\sin(c+dx)}{210a}+\frac{1}{35}(8Ab+7aB)\sin(2(c+dx))+\frac{1}{14}aA\sin(3(c+dx))\right)}{d(b+a\cos(c+dx))}-\frac{2\cos^{\frac{3}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}(a+b\sec(c+dx))^{3/2}\left(-i(a+b)\left(82a^2Ab-6Ab^3+63a^3B+21ab^2B\right)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+ia(a+b)\left(-6Ab^2+3ab(19A+7B)+a^2(25A+63B)\right)F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-\left(82a^2Ab-6Ab^3+63a^3B+21ab^2B\right)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{105a^2d(b+a\cos(c+dx))^2\sec^{\frac{3}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(((115*a^2*A + 12*A*b^2 + 16
8*a*b*B)*Sin[c + d*x])/(210*a) + ((8*A*b + 7*a*B)*Sin[2*(c + d*x)])/35 + (a
*A*Sin[3*(c + d*x)])/14))/(d*(b + a*Cos[c + d*x])) - (2*Cos[c + d*x]^(3/2)*
(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b*Sec[c + d*x])^(3/2)*((-I)*(a
```

```
 + b)*(82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*EllipticE[I*ArcSinh[Ta
n[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c +
d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-6*A*b^2 + 3*a*b*(19*A +
7*B) + a^2*(25*A + 63*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(
a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(
a + b)] - (82*a^2*A*b - 6*A*b^3 + 63*a^3*B + 21*a*b^2*B)*(b + a*Cos[c + d*x
])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(105*a^2*d*(b + a*Cos[c +
d*x])^2*Sec[c + d*x]^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2325 vs. $2(366) = 732$.
time = 19.04, size = 2326, normalized size = 6.80

method	result	size
default	Expression too large to display	2326

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/105/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)*(1+cos(d*x+c))*(21*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c
)))^(1/2)-25*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*(1/(1+cos(d*x+c)))^(1/2)+
25*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+6*A*s
in(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+21*B*cos(d*
x+c)^4*((a-b)/(a+b))^(1/2)*a^4*(1/(1+cos(d*x+c)))^(1/2)+42*B*cos(d*x+c)^2*(
(a-b)/(a+b))^(1/2)*a^4*(1/(1+cos(d*x+c)))^(1/2)-6*A*cos(d*x+c)*((a-b)/(a+b)
)^(1/2)*b^4*(1/(1+cos(d*x+c)))^(1/2)-63*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^
4*(1/(1+cos(d*x+c)))^(1/2)-25*A*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c))
)^(1/2)-82*A*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)-3*A*((a-b
)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)-63*B*((a-b)/(a+b))^(1/2)*a^3*
b*(1/(1+cos(d*x+c)))^(1/2)-42*B*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c
)))^(1/2)-21*B*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)+6*A*((a-b
)/(a+b))^(1/2)*b^4*(1/(1+cos(d*x+c)))^(1/2)+63*B*sin(d*x+c)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-63*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+15*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4
*(1/(1+cos(d*x+c)))^(1/2)+10*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*(1/(1+c
os(d*x+c)))^(1/2)+39*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x
+c)))^(1/2)+27*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c))
)^(1/2)+63*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c)))^(1/2
)+68*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c)))^(1/2)-3*A*
```

```
cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)+63*B*cos(d*
x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)-82*A*cos(d*x+c)
*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c)))^(1/2)+55*A*cos(d*x+c)*((a-b)/
(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)+6*A*cos(d*x+c)*((a-b)/(a+b))^
(1/2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)-21*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^
2*b^2*(1/(1+cos(d*x+c)))^(1/2)-6*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a*b^3-82*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^3*b+51*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^2*b^2+6*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a*b^3+82*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^3*b-82*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^2*b^2-63*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^3*b+21*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^2*b^2-21*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a*b^3+84*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^3*b-21*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*
b^2)/a^2/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/sin(
d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.58, size = 579, normalized size = 1.69

$$\frac{6\left(15\,Aa^4\cos\left(dx+c\right)^2+25\,Aa^4+42\,Ba^3b+3\,Aa^2b^2+3\left(7\,Ba^4+8\,Aa^3b\right)\cos\left(dx+c\right)\right)\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\sqrt{2}\left(-75i\,Aa^4-126i\,Ba^3b+11i\,Aa^2b^2+42i\,Bab^3-12i\,Ab^4\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos\left(dx+c\right)+3i\,a\sin\left(dx+c\right)+2b}{3a}\right)+\sqrt{2}\left(75i\,Aa^4+126i\,Ba^3b-11i\,Aa^2b^2-42i\,Bab^3+12i\,Ab^4\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos\left(dx+c\right)-3i\,a\sin\left(dx+c\right)+2b}{3a}\right)-3\sqrt{2}\left(-63i\,Ba^4-82i\,Aa^3b-21i\,Ba^2b^2+6i\,Aab^3\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos\left(dx+c\right)+3i\,a\sin\left(dx+c\right)+2b}{3a}\right)\right)-3\sqrt{2}\left(63i\,Ba^4+82i\,Aa^3b+21i\,Ba^2b^2-6i\,Aab^3\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos\left(dx+c\right)-3i\,a\sin\left(dx+c\right)+2b}{3a}\right)\right)}{315\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/315*(6*(15*A*a^4*cos(d*x + c)^2 + 25*A*a^4 + 42*B*a^3*b + 3*A*a^2*b^2 + 3
*(7*B*a^4 + 8*A*a^3*b)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-75*I*A*a^4 - 126*I*B*a^3*b +
11*I*A*a^2*b^2 + 42*I*B*a*b^3 - 12*I*A*b^4)*sqrt(a)*weierstrassPInverse(-4/
3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) +
3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(75*I*A*a^4 + 126*I*B*a^3*b - 11*I*A
*a^2*b^2 - 42*I*B*a*b^3 + 12*I*A*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3*a
^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*
sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-63*I*B*a^4 - 82*I*A*a^3*b - 21*I*B*a^2
*b^2 + 6*I*A*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*
(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a))
 - 3*sqrt(2)*(63*I*B*a^4 + 82*I*A*a^3*b + 21*I*B*a^2*b^2 - 6*I*A*a*b^3)*sqr
t(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3,
weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1
/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^3*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{7/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2), x)
```

3.602 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=266

$$\frac{2(a^2-b^2)\,(3Ab+5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\;F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15ad\sqrt{\cos(c+dx)}\;\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2A+3Ab^2+20abB)\;\sqrt{\cos(c+dx)}\;E}{15ad\sqrt{\frac{b+a\cos}{a}}}$$

```
[Out] 2/15*(a^2-b^2)*(3*A*b+5*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(
a+b))^(1/2)/a/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/5*a*A*cos(d*x+c)^
(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/15*(6*A*b+5*B*a)*sin(d*x+c)*cos
(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d+2/15*(9*A*a^2+3*A*b^2+20*B*a*b)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/d/((b+a*co
s(d*x+c))/(a+b))^(1/2)
```

Rubi [A]

time = 0.62, antiderivative size = 266, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3034, 4110, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(a^2-b^2)\,(5aB+3Ab)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\;F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15ad\sqrt{\cos(c+dx)}\;\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2A+20abB+3Ab^2)\;\sqrt{\cos(c+dx)}\;\sqrt{a+b\sec(c+dx)}\;E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(5aB+6Ab)\sin(c+dx)\sqrt{\cos(c+dx)}\;\sqrt{a+b\sec(c+dx)}}{15d}+\frac{2aA\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{5d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2 - b^2)*(3*A*b + 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(15*a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c +
 d*x]]) + (2*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c
 + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a*d*Sqrt[(b + a*Cos
[c + d*x])/(a + b)]) + (2*(6*A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec
[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*A*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[
c + d*x]]*Sin[c + d*x])/(5*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
```

```
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec}{}$$

$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+}{5d}$$

$$= \frac{2(6Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}{15d}$$

$$= \frac{2(6Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}{15d}$$

$$= \frac{2(6Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}{15d}$$

$$= \frac{2(6Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}{15d}$$

$$= \frac{2(a^2-b^2)(3Ab+5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,E}{15ad\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c-}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 14.62, size = 369, normalized size = 1.39

$$\frac{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\left(2(b+a\cos(c+dx))(6Ab+5aB+3aA\cos(c+dx))\sin(c+dx)-\frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-i(a+b)\left(9a^2A+3Ab^2+20abB\right)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+ia(a+b)(3b(A+5B)+a(9A+5B))F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-\left(9a^2A+3Ab^2+20abB\right)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{a\sec^{\frac{3}{2}}(c+dx)}\right)}{15d(b+a\cos(c+dx))^2}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(2*(b + a*Cos[c + d*x])*(6*A
*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[c + d*x] - (2*(Cos[(c + d*x)/2]^2*Sec[
c + d*x])^(3/2)*((-I)*(a + b)*(9*a^2*A + 3*A*b^2 + 20*a*b*B)*EllipticE[I*Ar
cSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*
Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(3*b*(A + 5*B) + a
*(9*A + 5*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[
```

```
(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (9
*a^2*A + 3*A*b^2 + 20*a*b*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2
)*Tan[(c + d*x)/2]))/(a*Sec[c + d*x]^(3/2))))/(15*d*(b + a*Cos[c + d*x])^2)
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1748 vs. $2(296) = 592$.

time = 20.33, size = 1749, normalized size = 6.58

method	result	size
default	Expression too large to display	1749

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/15/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
*(1+cos(d*x+c))*(-3*A*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)-9*A*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+9*A*sin(d*
x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-3*A*sin(d*x+c)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+9*A*((a-b)/(a+b))^(1/2
)*cos(d*x+c)^3*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+3*A*((a-b)/(a+b))^(1/2)*cos(d
*x+c)^4*a^3*(1/(1+cos(d*x+c)))^(1/2)+5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*(
1/(1+cos(d*x+c)))^(1/2)*a^3-5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(1+co
s(d*x+c)))^(1/2)+5*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^3+6*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*(1/(1+cos(d*x+c)))^(1/2
)-9*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(1+cos(d*x+c)))^(1/2)+3*A*((a-b
)/(a+b))^(1/2)*cos(d*x+c)*b^3*(1/(1+cos(d*x+c)))^(1/2)-9*A*((a-b)/(a+b))^(1
/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-6*A*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(
d*x+c)))^(1/2)-5*B*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-20*B*
((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+9*A*((a-b)/(a+b))^(1/2)*
cos(d*x+c)^2*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+25*B*((a-b)/(a+b))^(1/2)*cos(d*
x+c)^2*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*
b^2*(1/(1+cos(d*x+c)))^(1/2)-20*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1/(
1+cos(d*x+c)))^(1/2)+20*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+cos(d*
x+c)))^(1/2)+12*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^2*b-3*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a*b^2-9*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*
b+3*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
```

```
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+20
*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-20*B*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-20*B*sin
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+15*B*sin(d*
x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2)/a/((a-b)/(a+b
))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.34, size = 510, normalized size = 1.92

$$\frac{6\left(3\,Aa^3\cos\left(dx+c\right)+5\,Ba^3+6\,Aa^2b\right)\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\sqrt{2}\left(-15i\,Ba^3-18i\,Aa^2b-5i\,Bab^2+6i\,Ab^3\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(15i\,Ba^3+18i\,Aa^2b+5i\,Bab^2-6i\,Ab^3\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(-9i\,Aa^3-20i\,Ba^2b-3i\,Aab^2\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)+3ia\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(9i\,Aa^3+20i\,Ba^2b+3i\,Aab^2\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^2-4b^2\right)}{3a^2},\frac{8\left(9a^2b-8b^3\right)}{27a^3},\frac{3a\cos(dx+c)-3ia\sin(dx+c)+2b}{3a}\right)\right)}{45\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/45*(6*(3*A*a^3*cos(d*x + c) + 5*B*a^3 + 6*A*a^2*b)*sqrt((a*cos(d*x + c) +
 b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-15*I*B*a^3 -
18*I*A*a^2*b - 5*I*B*a*b^2 + 6*I*A*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3
*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*
a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(15*I*B*a^3 + 18*I*A*a^2*b + 5*I*B*a*b^2
 - 6*I*A*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) -
 3*sqrt(2)*(-9*I*A*a^3 - 20*I*B*a^2*b - 3*I*A*a*b^2)*sqrt(a)*weierstrassZet
a(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse
(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c
) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(9*I*A*a^3 + 20*I*B*a^2*b + 3
*I*A*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b
 - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b
```

```
- 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^2*d
)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{5/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2), x)
```

3.603 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=276

$$\frac{2(a^2A-Ab^2+3abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2b^2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2/3*(A*a^2-A*b^2+3*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*E
llipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b
))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*b^2*B*(cos(1/2*d*x+1/2
*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/
(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*
x+c))^(1/2)+2/3*a*A*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d+2/
3*(4*A*b+3*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(s
in(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c)
)^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]

time = 0.72, antiderivative size = 276, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3034, 4110, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(a^2A+3abB-Ab^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(3aB+4Ab)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2aA\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{3d}+\frac{2b^2B\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2*A - A*b^2 + 3*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]
]) + (2*b^2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2,
 (2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(4*A*
b + 3*a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
+ b*Sec[c + d*x]])/(3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a*A*Sqrt[C
os[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
```

```
b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{se}}{}$$

$$= \frac{2aA\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(}{3d}$$

$$= \frac{2aA\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(}{3d}$$

$$= \frac{2aA\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(}{3d}$$

$$= \frac{2aA\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(}{3d}$$

$$= \frac{2b^2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+}\right.}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2(a^2A-Ab^2+3abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\ldots}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 34.43, size = 45958, normalized size = 166.51

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 20.47, size = 1429, normalized size = 5.18

method	result	size
default	Expression too large to display	1429

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/3/d*(-1+cos(d*x+c))*(1+cos(d*x+c))*(A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^
2*(1/(1+cos(d*x+c)))^(1/2)+5*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(1+c
os(d*x+c)))^(1/2)+3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)
))^(1/2)-A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)-4*A*
cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)+4*A*cos(d*x+c)*
((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)+A*sin(d*x+c)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-4*A*sin(d*x+c)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+3*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*b^2+4*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a*b-4*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*b^2-3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^
(1/2)+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)-3*B*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2+6*B*sin(d*x
+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-3*B*sin(d*x+c)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+3*B*sin(d*x+c)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-3*B*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b+6*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2-A*((a-b)/(a+b))^(1/2)*a*b*(
1/(1+cos(d*x+c)))^(1/2)-4*A*((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2
)-3*B*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2))*((b+a*cos(d*x+c))/c
os(d*x+c))^(1/2)*cos(d*x+c)^(1/2)/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/sin(
d*x+c)^3/(1/(1+cos(d*x+c)))^(1/2)
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{3/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2), x)
```

3.604 $\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=272

$$\frac{(2aAb+2a^2B+b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{b(2Ab+3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c\right.}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx}}$$

```
[Out] (2*A*a*b+2*B*a^2+B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))
^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+b*(2*A*b+3*B*a)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/
2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*
sec(d*x+c))^(1/2)+b*B*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+
(2*A*a-B*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1
/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1
/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.65, antiderivative size = 272, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3034, 4111, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(2a^2B+2aAb+b^2B)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{(2aA-bB)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{b(3aB+2Ab)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{bB\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((2*a*A*b + 2*a^2*B + b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(
c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
 + (b*(2*A*b + 3*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
 d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) +
((2*a*A - b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqr
t[a + b*Sec[c + d*x]])/(d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b*B*Sqrt[a
 + b*Sec[c + d*x]]*Sin[c + d*x])/(d*Sqrt[Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
```

b^2, 0]

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\mathrm{s}}{}$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\mathrm{c}}\right.$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \left(\sqrt{\mathrm{c}}\right.$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} + \frac{1}{2}\Big((2$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{d\sqrt{\cos(c+dx)}} - \frac{\Big((-2}{}$$

$$= \frac{b(2Ab+3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c\right.}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx}}$$

$$= \frac{(2aAb+2a^2B+b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 33.23, size = 66581, normalized size = 244.78

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 19.38, size = 1409, normalized size = 5.18

method	result	size
default	Expression too large to display	1409

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1+cos(d*x+c))*(2*A
*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)-2*A*cos(d*x+
c)^2*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)+2*A*cos(d*x+c)^2*((a-
b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)+2*A*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a^2-2*A*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b+4*A*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*cos(d*x+c)*sin(d*x+c)*b^2-2
*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*
a^2+4*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*
x+c)*a*b-2*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*s
in(d*x+c)*b^2+B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/
2)-B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*x+c
)*a*b+B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*cos(d*x+c)*sin(d*
x+c)*b^2+6*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)
)*cos(d*x+c)*sin(d*x+c)*a*b+2*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*cos(d*x+c)*sin(d*x+c)*a^2-2*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b)
)^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a
-b))^(1/2))*cos(d*x+c)*sin(d*x+c)*a*b-2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*
b*(1/(1+cos(d*x+c)))^(1/2)-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d
*x+c)))^(1/2)+B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)
-B*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2)/((a-b)/(a+b))^(1/2)/(b
+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(1/2)/sin(d*x+c)^3
```

Maxima [F]

time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6437 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c)
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos\left(c+d\,x\right)}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2),x)
[Out] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2), x)
```

3.605 $\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=339

$$\frac{(8a^2A+4Ab^2+7abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{(12aAb+3a^2B+4b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c}$$

```
[Out] 1/4*(8*A*a^2+4*A*b^2+7*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/
(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/4*(12*A*a*b+3*B*a^
2+4*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1
/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/c
os(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/2*b*B*sin(d*x+c)*(a+b*sec(d*x+c))^
(1/2)/d/cos(d*x+c)^(3/2)+1/4*(4*A*b+5*B*a)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2
)/d/cos(d*x+c)^(1/2)-1/4*(4*A*b+5*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c
)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.89, antiderivative size = 339, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3034, 4111, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(8a^2A+7abB+4Ab^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{(3a^2B+12aAb+4b^2B)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{(5aB+4Ab)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4d\sqrt{\cos(c+dx)}}-\frac{(5aB+4Ab)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{bB\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{2d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((8*a^2*A + 4*A*b^2 + 7*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*
x]]) + ((12*a*A*b + 3*a^2*B + 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*E
llipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[a +
b*Sec[c + d*x]]) - ((4*A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/
2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*d*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]) + (b*B*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*d*Cos[c + d*x]^(
3/2)) + ((4*A*b + 5*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[C
os[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
```

```
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,(a+b\,\mathrm{s}$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{2}\left(\sqrt{\cos(c+dx)}\,\sqrt{\right.$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab+5aB)\sqrt{a+}}{4d\sqrt{c}}$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab+5aB)\sqrt{a+}}{4d\sqrt{c}}$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab+5aB)\sqrt{a+}}{4d\sqrt{c}}$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2d\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab+5aB)\sqrt{a+}}{4d\sqrt{c}}$$

$$= \frac{(12aAb+3a^2B+4b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+d\right.}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{(8a^2A+4Ab^2+7abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\right|}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 33.27, size = 79375, normalized size = 234.14

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*
x]],x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 20.91, size = 1659, normalized size = 4.89

method	result	size
default	Expression too large to display	1659

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1+cos(d*x+c))*(
4*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a*b-4*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*b^2-8*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x
+c)*cos(d*x+c)^2*a^2+8*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
sin(d*x+c)*cos(d*x+c)^2*a*b-24*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b-4*A*cos(d*x+c)^3*((a-b)/(
a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)+5*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-5*B*cos(d*x+c)^2*sin(d*x+c)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-2*B*cos(d*x+c)^2*sin
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-2*B*cos(d*x+c
)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+4*B*co
s(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2
-6*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a
-b)/(a+b))^(1/2))*a^2-8*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^2-5*B*cos(d*x+c)^3*((a-b)/(a+b))^(
1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)-2*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b*
(1/(1+cos(d*x+c)))^(1/2)+4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos
(d*x+c)))^(1/2)-4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))
^(1/2)+5*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)-5*
B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)-2*B*cos(d*x
+c)^2*((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)+4*A*cos(d*x+c)*((a-b
```

```
)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)+7*B*cos(d*x+c)*((a-b)/(a+b))^(1
/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)+2*B*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))
^(1/2)*b^2)/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/s
in(d*x+c)^3/cos(d*x+c)^(3/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/sqrt(cos(d*x + c)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/cos(c + d*x)^(1/2), x
)
```

3.606 $\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=421

$$\frac{(42aAb+17a^2B+16b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(6a^2Ab+8Ab^3-a^3B+12ab^2B)\sqrt{b+}}{8bd\sqrt{\cos(c+dx)}\sqrt{a}}$$

```
[Out] 1/24*(42*A*a*b+17*B*a^2+16*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+
c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/8*(6*A*a^2*b+8
*A*b^3-B*a^3+12*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*El
lipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a
+b))^(1/2)/b/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/3*b*B*sin(d*x+c)*(
a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(5/2)+1/12*(6*A*b+7*B*a)*sin(d*x+c)*(a+b
*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/24*(30*A*a*b+3*B*a^2+16*B*b^2)*sin(
d*x+c)*(a+b*sec(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)-1/24*(30*A*a*b+3*B*a^2+1
6*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*
d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)
/b/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 1.16, antiderivative size = 421, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3034, 4111, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(3a^2B+30aAb+16b^2B)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24bd\sqrt{\cos(c+dx)}}+\frac{(17a^2B+42aAb+16b^2B)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{(3a^2B+30aAb+16b^2B)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24bd\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(a^3(-B)+6a^2Ab+12ab^2B+8Ab^3)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{8bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(7aB+6Ab)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{12d\cos^{\frac{3}{2}}(c+dx)}+\frac{bB\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] ((42*a*A*b + 17*a^2*B + 16*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icF[(c + d*x)/2, (2*a)/(a + b)])/(24*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]) + ((6*a^2*A*b + 8*A*b^3 - a^3*B + 12*a*b^2*B)*Sqrt[(b + a*Cos[c +
d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(8*b*d*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((30*a*A*b + 3*a^2*B + 16*b^2*B)*Sqrt[
Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]
])/(24*b*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b*B*Sqrt[a + b*Sec[c + d*
x]]*Sin[c + d*x])/(3*d*Cos[c + d*x]^(5/2)) + ((6*A*b + 7*a*B)*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/(12*d*Cos[c + d*x]^(3/2)) + ((30*a*A*b + 3*a^2*B
+ 16*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24*b*d*Sqrt[Cos[c + d*x
]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
```

```
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{3/2}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\,\sqrt{\right.$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{(6Ab+7aB)\sqrt{a+}}{12d\,\mathrm{c}}$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{(6Ab+7aB)\sqrt{a+}}{12d\,\mathrm{c}}$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{(6Ab+7aB)\sqrt{a+}}{12d\,\mathrm{c}}$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{(6Ab+7aB)\sqrt{a+}}{12d\,\mathrm{c}}$$

$$= \frac{bB\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{3d\cos^{\frac{5}{2}}(c+dx)} + \frac{(6Ab+7aB)\sqrt{a+}}{12d\,\mathrm{c}}$$

$$= \frac{(6a^2Ab+8Ab^3-a^3B+12ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi(}{8bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{(42aAb+17a^2B+16b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+\right.}{24d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 33.91, size = 104716, normalized size = 248.73

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(3/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 19.64, size = 2351, normalized size = 5.58

method	result	size
default	Expression too large to display	2351

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/24/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1+cos(d*x+c))*(
30*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-30*A*(
(a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-3*B*((a-b)/(
a+b))^(1/2)*cos(d*x+c)^3*(1/(1+cos(d*x+c)))^(1/2)*a^3-12*A*((a-b)/(a+b))^(1
/2)*cos(d*x+c)*b^3*(1/(1+cos(d*x+c)))^(1/2)-8*B*((a-b)/(a+b))^(1/2)*b^3*(1/
(1+cos(d*x+c)))^(1/2)-42*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^2*(1/(1+cos
(d*x+c)))^(1/2)-17*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b*(1/(1+cos(d*x+c
)))^(1/2)-22*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+cos(d*x+c)))^(1/2
)+3*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^2*b-16*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*b^2+72*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^2+14*B*cos(d*x+c)^3*sin(d*x+c)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+12*A*cos(d*x+c)^4*((
a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+14*B*cos(d*x+c)^4*((a-b)/(
a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+16*B*cos(d*x+c)^4*((a-b)/(a+b))^
(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-20*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+12*A*cos(d*x+c)^3*sin(d*x+c)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+12*A*cos(d*x+c)^3*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-30*A*cos
(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*
b+30*A*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a*b^2+36*A*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
```

```
a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)
/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b+12*A*cos(d*x+c)^3*(1/(1+cos(d*x+c)))^(1
/2)*((a-b)/(a+b))^(1/2)*b^3+3*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*(1/(1+
cos(d*x+c)))^(1/2)+16*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+
c)))^(1/2)-8*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2
)+30*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+3*B*
cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+6*B*cos(d*x
+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-24*A*cos(d*x+c)^3*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+48*A*cos(d
*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))
^(1/2))*b^3-3*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^3+16*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*b^3-6*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^3+6*B*cos(d*x+c)^3*sin(d*x+c)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3)/b/((a-b)/(a+b))^(1/
2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(5/2)/sin(d*x+c)^3
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(3/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(3/2)/cos(d*x + c)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{3/2}}{{\cos\left(c+d\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2),x)
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(3/2))/cos(c + d*x)^(3/2), x
)
```

3.607 $\int \cos^{\frac{11}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=519

$$\frac{2(a^2-b^2)\left(675a^4A+285a^2Ab^2+40Ab^4+1254a^3bB-110ab^3B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3465a^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+2$$

```
[Out] 2/11*a*A*cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/3465*(a^2-b
^2)*(675*A*a^4+285*A*a^2*b^2+40*A*b^4+1254*B*a^3*b-110*B*a*b^3)*(cos(1/2*d*
x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(
a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a^3/d/cos(d*x+c)^(1/2)/(a+b*
sec(d*x+c))^(1/2)+2/3465*(1145*A*a^2*b+15*A*b^3+539*B*a^3+825*B*a*b^2)*cos(
d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d+2/693*(81*A*a^2+113*A*b^
2+209*B*a*b)*cos(d*x+c)^(5/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/99*a*(1
4*A*b+11*B*a)*cos(d*x+c)^(7/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/3465*(
675*A*a^4+1025*A*a^2*b^2-20*A*b^4+1793*B*a^3*b+55*B*a*b^3)*sin(d*x+c)*cos(d
*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/d+2/3465*(3705*A*a^4*b+255*A*a^2*b^3
+40*A*b^5+1617*B*a^5+3069*B*a^3*b^2-110*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))
*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^3/d/((b+a*cos(d*x+c))/(a+b))^(1/
2)
```

Rubi [A]
time = 1.42, antiderivative size = 519, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3034, 4110, 4179, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(81a^2A+209abB+113Ab^2)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{693d}+\frac{2(539a^3B+1145a^2Ab+825ab^2B+15Ab^3)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{3465ad}+\frac{2(675a^4A+1793a^3bB+1025a^2Ab^2+55ab^3B-20Ab^4)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{3465a^2d}+\frac{2(a^2-b^2)(675a^4A+1254a^3bB+285a^2Ab^2-110ab^3B+40Ab^4)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3465a^3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(1617a^5B+3705a^4Ab+3069a^3b^2B+255a^2Ab^3-110ab^4B+40Ab^5)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3465a^3d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a(11aB+14Ab)\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{99d}+\frac{2aA\sin(c+dx)\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}{11d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(11/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2 - b^2)*(675*a^4*A + 285*a^2*A*b^2 + 40*A*b^4 + 1254*a^3*b*B - 110*a
*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
+ b)])/(3465*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(3705*
a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^
4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Se
c[c + d*x]])/(3465*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(675*a^4*
A + 1025*a^2*A*b^2 - 20*A*b^4 + 1793*a^3*b*B + 55*a*b^3*B)*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3465*a^2*d) + (2*(1145*a^2*A*b +
 15*A*b^3 + 539*a^3*B + 825*a*b^2*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c +
```

```
d*x]]*Sin[c + d*x])/(3465*a*d) + (2*(81*a^2*A + 113*A*b^2 + 209*a*b*B)*Cos[
c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(693*d) + (2*a*(14*A*
b + 11*a*B)*Cos[c + d*x]^(7/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(99*d
) + (2*a*A*Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(11*
d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4179

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{11}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\mathrm{s}}{}$$

$$= \frac{2aA\cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\sin(}{11d}$$

$$= \frac{2a(14Ab+11aB)\cos^{\frac{7}{2}}(c+dx)\sqrt{a+b\sec(}}{99d}$$

$$= \frac{2(81a^2A+113Ab^2+209abB)\cos^{\frac{5}{2}}(c+dx)}{693d}$$

$$= \frac{2(1145a^2Ab+15Ab^3+539a^3B+825ab^2B}{34}$$

$$= \frac{2(675a^4A+1025a^2Ab^2-20Ab^4+1793a^3b}{}$$

$$= \frac{2(675a^4A+1025a^2Ab^2-20Ab^4+1793a^3b}{}$$

$$= \frac{2(675a^4A+1025a^2Ab^2-20Ab^4+1793a^3b}{}$$

$$= \frac{2(675a^4A+1025a^2Ab^2-20Ab^4+1793a^3b}{}$$

$$= \frac{2(a^2-b^2)\left(675a^4A+285a^2Ab^2+40Ab^4+\right.}{3465a^3d\sqrt{\mathrm{co}}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 20.53, size = 626, normalized size = 1.21

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(11/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x
]),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*(((6525*a^4*A + 9330*a^2*A*b
^2 - 160*A*b^4 + 16434*a^3*b*B + 440*a*b^3*B)*Sin[c + d*x])/(13860*a^2) + (
(3095*a^2*A*b + 30*A*b^3 + 1463*a^3*B + 1650*a*b^2*B)*Sin[2*(c + d*x)])/(69
30*a) + ((513*a^2*A + 452*A*b^2 + 836*a*b*B)*Sin[3*(c + d*x)])/5544 + (a*(2
3*A*b + 11*a*B)*Sin[4*(c + d*x)])/396 + (a^2*A*Sin[5*(c + d*x)])/88))/(d*(b
 + a*Cos[c + d*x])^2) - (2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d
*x])^(3/2)*(a + b*Sec[c + d*x])^(5/2)*((-I)*(a + b)*(3705*a^4*A*b + 255*a^2
*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*B - 110*a*b^4*B)*EllipticE[I*
ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b +
a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(40*A*b^4 - 10*a
*b^3*(3*A + 11*B) + 15*a^2*b^2*(19*A + 121*B) + 6*a^3*b*(505*A + 209*B) + 3
*a^4*(225*A + 539*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a +
b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a +
b)] - (3705*a^4*A*b + 255*a^2*A*b^3 + 40*A*b^5 + 1617*a^5*B + 3069*a^3*b^2*
B - 110*a*b^4*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d
*x)/2]))/(3465*a^3*d*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3815 vs. $2(531) = 1062$.
time = 21.13, size = 3816, normalized size = 7.35

method	result	size
default	Expression too large to display	3816

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RET
URNVERBOSE)
```

```
[Out] 2/3465/d*(-1+cos(d*x+c))*(1+cos(d*x+c))*(430*A*cos(d*x+c)^4*((a-b)/(a+b))^(
1/2)*a^5*b*(1/(1+cos(d*x+c)))^(1/2)-675*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^
6*(1/(1+cos(d*x+c)))^(1/2)+20*A*((a-b)/(a+b))^(1/2)*a*b^5*(1/(1+cos(d*x+c))
)^(1/2)-1617*B*((a-b)/(a+b))^(1/2)*a^5*b*(1/(1+cos(d*x+c)))^(1/2)-1793*B*((
a-b)/(a+b))^(1/2)*a^4*b^2*(1/(1+cos(d*x+c)))^(1/2)-3069*B*((a-b)/(a+b))^(1/
2)*a^3*b^3*(1/(1+cos(d*x+c)))^(1/2)-55*B*((a-b)/(a+b))^(1/2)*a^2*b^4*(1/(1+
cos(d*x+c)))^(1/2)+110*B*((a-b)/(a+b))^(1/2)*a*b^5*(1/(1+cos(d*x+c)))^(1/2)
+385*B*cos(d*x+c)^6*((a-b)/(a+b))^(1/2)*a^6*(1/(1+cos(d*x+c)))^(1/2)+154*B*
cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^6*(1/(1+cos(d*x+c)))^(1/2)+1078*B*cos(d*
x+c)^2*((a-b)/(a+b))^(1/2)*a^6*(1/(1+cos(d*x+c)))^(1/2)+40*A*cos(d*x+c)*((a
-b)/(a+b))^(1/2)*b^6*(1/(1+cos(d*x+c)))^(1/2)-1617*B*cos(d*x+c)*((a-b)/(a+b
))^(1/2)*a^6*(1/(1+cos(d*x+c)))^(1/2)-675*A*((a-b)/(a+b))^(1/2)*a^5*b*(1/(1
+cos(d*x+c)))^(1/2)-3705*A*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/(1+cos(d*x+c)))^(
1/2)-1025*A*((a-b)/(a+b))^(1/2)*a^3*b^3*(1/(1+cos(d*x+c)))^(1/2)-255*A*((a-
b)/(a+b))^(1/2)*a^2*b^4*(1/(1+cos(d*x+c)))^(1/2)+2992*B*cos(d*x+c)^2*((a-b)
```

/(a+b))^(1/2)*a^4*b^2*(1/(1+cos(d*x+c)))^(1/2)-55*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^4*(1/(1+cos(d*x+c)))^(1/2)-5*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^4*(1/(1+cos(d*x+c)))^(1/2)+902*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^5*b*(1/(1+cos(d*x+c)))^(1/2)+880*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b^3*(1/(1+cos(d*x+c)))^(1/2)-3705*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5*b*(1/(1+cos(d*x+c)))^(1/2)+1535*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/(1+cos(d*x+c)))^(1/2)+255*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3-255*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4+40*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^5-3705*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b+3315*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2-255*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3+10*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4-40*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^5+3705*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b-3705*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2+1120*A*cos(d*x+c)^6*((a-b)/(a+b))^(1/2)*a^5*b*(1/(1+cos(d*x+c)))^(1/2)+800*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/(1+cos(d*x+c)))^(1/2)-255*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^3*(1/(1+cos(d*x+c)))^(1/2)+260*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^4*(1/(1+cos(d*x+c)))^(1/2)-40*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^5*(1/(1+cos(d*x+c)))^(1/2)-715*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5*b*(1/(1+cos(d*x+c)))^(1/2)-3069*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/(1+cos(d*x+c)))^(1/2)+2189*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^3*(1/(1+cos(d*x+c)))^(1/2)+110*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^4*(1/(1+cos(d*x+c)))^(1/2)-110*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^5*(1/(1+cos(d*x+c)))^(1/2)-1617*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b+3069*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2-3069*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3-110*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4+110*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(

```
d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^5+2871*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b-3069*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2+1705*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3+110*B*sin(d...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algo
rithm="maxima")

[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(11/
2), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.17, size = 743, normalized size = 1.43

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algo
rithm="fricas")

[Out] 1/10395*(6*(315*A*a^6*cos(d*x + c)^4 + 675*A*a^6 + 1793*B*a^5*b + 1025*A*a^
4*b^2 + 55*B*a^3*b^3 - 20*A*a^2*b^4 + 35*(11*B*a^6 + 23*A*a^5*b)*cos(d*x +
c)^3 + 5*(81*A*a^6 + 209*B*a^5*b + 113*A*a^4*b^2)*cos(d*x + c)^2 + (539*B*a
^6 + 1145*A*a^5*b + 825*B*a^4*b^2 + 15*A*a^3*b^3)*cos(d*x + c))*sqrt((a*cos
(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-20
25*I*A*a^6 - 5379*I*B*a^5*b - 2535*I*A*a^4*b^2 + 1023*I*B*a^3*b^3 + 480*I*A
*a^2*b^4 - 220*I*B*a*b^5 + 80*I*A*b^6)*sqrt(a)*weierstrassPInverse(-4/3*(3*
a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a
*sin(d*x + c) + 2*b)/a) + sqrt(2)*(2025*I*A*a^6 + 5379*I*B*a^5*b + 2535*I*A
*a^4*b^2 - 1023*I*B*a^3*b^3 - 480*I*A*a^2*b^4 + 220*I*B*a*b^5 - 80*I*A*b^6)
*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^
3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-
1617*I*B*a^6 - 3705*I*A*a^5*b - 3069*I*B*a^4*b^2 - 255*I*A*a^3*b^3 + 110*I*
B*a^2*b^4 - 40*I*A*a*b^5)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*
b)/a)) - 3*sqrt(2)*(1617*I*B*a^6 + 3705*I*A*a^5*b + 3069*I*B*a^4*b^2 + 255*
```

```
I*A*a^3*b^3 - 110*I*B*a^2*b^4 + 40*I*A*a*b^5)*sqrt(a)*weierstrassZeta(-4/3*
(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(
3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I
*a*sin(d*x + c) + 2*b)/a)))/(a^4*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(11/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(11/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algo
rithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(11/
2), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^{11/2}\,\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(11/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(11/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)
```

3.608 $\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=425

$$\frac{2(a^2-b^2)\left(114a^2Ab-10Ab^3+75a^3B+45ab^2B\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{315a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(147a^4A+279a$$

```
[Out] 2/9*a*A*cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/315*(a^2-b^2
)*(114*A*a^2*b-10*A*b^3+75*B*a^3+45*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b
+a*cos(d*x+c))/(a+b))^(1/2)/a^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2
/315*(49*A*a^2+75*A*b^2+135*B*a*b)*cos(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x
+c))^(1/2)/d+2/21*a*(4*A*b+3*B*a)*cos(d*x+c)^(5/2)*sin(d*x+c)*(a+b*sec(d*x+
c))^(1/2)/d+2/315*(163*A*a^2*b+5*A*b^3+75*B*a^3+135*B*a*b^2)*sin(d*x+c)*cos
(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/d+2/315*(147*A*a^4+279*A*a^2*b^2-10*
A*b^4+435*B*a^3*b+45*B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(
a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 1.11, antiderivative size = 425, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3034, 4110, 4179, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(49a^2A+135abB+75Ab^2)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{315d}+\frac{2(75a^3B+163a^2Ab+135ab^2B+5Ab^3)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{315ad}+\frac{2(a^2-b^2)\left(75a^3B+114a^2Ab+45ab^2B-10Ab^3\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{315a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(147a^4A+435a^3bB+279a^2Ab^2+45ab^3B-10Ab^4)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{315a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a(3aB+4Ab)\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{21d}+\frac{2aA\sin(c+dx)\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}{9d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2 - b^2)*(114*a^2*A*b - 10*A*b^3 + 75*a^3*B + 45*a*b^2*B)*Sqrt[(b + a
*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(315*a^2*d*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(147*a^4*A + 279*a^2*A*b^2
 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d
*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(315*a^2*d*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]) + (2*(163*a^2*A*b + 5*A*b^3 + 75*a^3*B + 135*a*b^2*B)*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(315*a*d) + (2*(49
*a^2*A + 75*A*b^2 + 135*a*b*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*
Sin[c + d*x])/(315*d) + (2*a*(4*A*b + 3*a*B)*Cos[c + d*x]^(5/2)*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(21*d) + (2*a*A*Cos[c + d*x]^(7/2)*(a + b*Sec[c
 + d*x])^(3/2)*Sin[c + d*x])/(9*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4179

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{9}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{se}\cdots}{\cdots}$$

$$= \frac{2aA\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c\cdots}{9d}$$

$$= \frac{2a(4Ab+3aB)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+\cdots}}{21d}$$

$$= \frac{2(49a^2A+75Ab^2+135abB)\cos^{\frac{3}{2}}(c+dx)\sqrt{\cdots}}{315d}$$

$$= \frac{2(163a^2Ab+5Ab^3+75a^3B+135ab^2B)\sqrt{\cdots}}{315\cdots}$$

$$= \frac{2(163a^2Ab+5Ab^3+75a^3B+135ab^2B)\sqrt{\cdots}}{315\cdots}$$

$$= \frac{2(163a^2Ab+5Ab^3+75a^3B+135ab^2B)\sqrt{\cdots}}{315\cdots}$$

$$= \frac{2(163a^2Ab+5Ab^3+75a^3B+135ab^2B)\sqrt{\cdots}}{315\cdots}$$

$$= \frac{2(a^2-b^2)\left(114a^2Ab-10Ab^3+75a^3B+45\cdots\right)}{315a^2d\sqrt{\cos(c+dx)\cdots}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 19.00, size = 542, normalized size = 1.28

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(9/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*(((747*a^2*A*b + 20*A*b^3 +
345*a^3*B + 540*a*b^2*B)*Sin[c + d*x])/(630*a) + ((133*a^2*A + 150*A*b^2 +
```

```
270*a*b*B)*Sin[2*(c + d*x)])/630 + (a*(19*A*b + 9*a*B)*Sin[3*(c + d*x)])/12
6 + (a^2*A*Sin[4*(c + d*x)])/36))/(d*(b + a*Cos[c + d*x])^2) - (2*Cos[c + d
*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b*Sec[c + d*x])^(5/2
)*((-I)*(a + b)*(147*a^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*
b^3*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*
x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a +
b)*(-10*A*b^3 + 15*a*b^2*(11*A + 3*B) + 3*a^3*(49*A + 25*B) + 6*a^2*b*(19*A
 + 60*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c +
 d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (147*a
^4*A + 279*a^2*A*b^2 - 10*A*b^4 + 435*a^3*b*B + 45*a*b^3*B)*(b + a*Cos[c +
d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(315*a^2*d*(b + a*Cos[c
 + d*x])^3*Sec[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 3068 vs. $2(443) = 886$.
time = 21.03, size = 3069, normalized size = 7.22

method	result	size
default	Expression too large to display	3069

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/315/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)*(1+cos(d*x+c))*(130*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4*b*(1/(1+cos(d*
x+c)))^(1/2)+10*A*((a-b)/(a+b))^(1/2)*b^5*(1/(1+cos(d*x+c)))^(1/2)+35*A*cos
(d*x+c)^6*((a-b)/(a+b))^(1/2)*a^5*(1/(1+cos(d*x+c)))^(1/2)+45*B*cos(d*x+c)^
5*((a-b)/(a+b))^(1/2)*a^5*(1/(1+cos(d*x+c)))^(1/2)+30*B*cos(d*x+c)^3*((a-b)
/(a+b))^(1/2)*a^5*(1/(1+cos(d*x+c)))^(1/2)-75*B*cos(d*x+c)*((a-b)/(a+b))^(1
/2)*a^5*(1/(1+cos(d*x+c)))^(1/2)+75*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a^5+14*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^5*(1/(1
+cos(d*x+c)))^(1/2)+98*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5*(1/(1+cos(d*x
+c)))^(1/2)-147*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5*(1/(1+cos(d*x+c)))^(1/
2)-10*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^5*(1/(1+cos(d*x+c)))^(1/2)-147*A*(
(a-b)/(a+b))^(1/2)*a^4*b*(1/(1+cos(d*x+c)))^(1/2)-163*A*((a-b)/(a+b))^(1/2)
*a^3*b^2*(1/(1+cos(d*x+c)))^(1/2)-279*A*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(1+c
os(d*x+c)))^(1/2)-5*A*((a-b)/(a+b))^(1/2)*a*b^4*(1/(1+cos(d*x+c)))^(1/2)-75
*B*((a-b)/(a+b))^(1/2)*a^4*b*(1/(1+cos(d*x+c)))^(1/2)-435*B*((a-b)/(a+b))^(
1/2)*a^3*b^2*(1/(1+cos(d*x+c)))^(1/2)-135*B*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/
(1+cos(d*x+c)))^(1/2)-45*B*((a-b)/(a+b))^(1/2)*a*b^4*(1/(1+cos(d*x+c)))^(1/
2)-147*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5+1
47*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
```

+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5+10*A*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5+170*A*cos(
d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(1+cos(d*x+c)))^(1/2)+180*B*cos(d*x
+c)^4*((a-b)/(a+b))^(1/2)*a^4*b*(1/(1+cos(d*x+c)))^(1/2)+82*A*cos(d*x+c)^3*
((a-b)/(a+b))^(1/2)*a^4*b*(1/(1+cos(d*x+c)))^(1/2)+80*A*cos(d*x+c)^3*((a-b)
/(a+b))^(1/2)*a^2*b^3*(1/(1+cos(d*x+c)))^(1/2)-435*B*sin(d*x+c)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b+405*B*sin(d*x+c)*((b+a*cos(d*x
+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1
/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2-45*B*sin(d*x+c)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3+435*B*sin(d*x+c)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b-435*B*sin(d*x+c)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+45*B*sin(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3-45*B*sin(d*x+c)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4+270*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2
)*a^3*b^2*(1/(1+cos(d*x+c)))^(1/2)+272*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a
^3*b^2*(1/(1+cos(d*x+c)))^(1/2)+330*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*
b*(1/(1+cos(d*x+c)))^(1/2)-65*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b*(1/(1+
cos(d*x+c)))^(1/2)-279*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(1+cos(d
*x+c)))^(1/2)+199*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(1+cos(d*x+c)
))^(1/2)+10*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4*(1/(1+cos(d*x+c)))^(1/2)
-435*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b*(1/(1+cos(d*x+c)))^(1/2)+165*B*
cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(1+cos(d*x+c)))^(1/2)-45*B*cos(d*
x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(1+cos(d*x+c)))^(1/2)+45*B*cos(d*x+c)*(
(a-b)/(a+b))^(1/2)*a*b^4*(1/(1+cos(d*x+c)))^(1/2)-5*A*cos(d*x+c)^2*((a-b)/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a*b^4+180*B*cos(d*x+c)^2*((a-b)/(a+b))
^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b^3+261*A*sin(d*x+c)*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/s
in(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b-279*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2+155*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^3+10*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4-147*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^4*b+279*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a^3*b^2-279*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(

```
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.88, size = 656, normalized size = 1.54

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/945*(6*(35*A*a^5*cos(d*x + c)^3 + 75*B*a^5 + 163*A*a^4*b + 135*B*a^3*b^2
+ 5*A*a^2*b^3 + 5*(9*B*a^5 + 19*A*a^4*b)*cos(d*x + c)^2 + (49*A*a^5 + 135*B
*a^4*b + 75*A*a^3*b^2)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c)
)*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-225*I*B*a^5 - 489*I*A*a^4*b -
 345*I*B*a^3*b^2 + 93*I*A*a^2*b^3 + 90*I*B*a*b^4 - 20*I*A*b^5)*sqrt(a)*weie
rstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(
3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(225*I*B*a^5 + 48
9*I*A*a^4*b + 345*I*B*a^3*b^2 - 93*I*A*a^2*b^3 - 90*I*B*a*b^4 + 20*I*A*b^5)
*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^
3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-
147*I*A*a^5 - 435*I*B*a^4*b - 279*I*A*a^3*b^2 - 45*I*B*a^2*b^3 + 10*I*A*a*b
^4)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3
)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)
/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(14
7*I*A*a^5 + 435*I*B*a^4*b + 279*I*A*a^3*b^2 + 45*I*B*a^2*b^3 - 10*I*A*a*b^4
)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/
a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a
^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^3*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(9/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(9/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(9/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{9/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(9/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)
```

3.609 $\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=340

$$\frac{2(a^2-b^2)\left(25a^2A+15Ab^2+56abB\right)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(145a^2Ab+15Ab^3+63a^3B\cdots}{}$$

```
[Out] 2/7*a*A*cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/105*(a^2-b^2
)*(25*A*a^2+15*A*b^2+56*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2
*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))
/(a+b))^(1/2)/a/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/35*a*(10*A*b+7*
B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d+2/105*(25*A*a^2+4
5*A*b^2+77*B*a*b)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d+2/10
5*(145*A*a^2*b+15*A*b^3+63*B*a^3+161*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/
cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*co
s(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.85, antiderivative size = 340, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3034, 4110, 4179, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(25a^2A+77abB+45Ab^2)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{105d}+\frac{2(a^2-b^2)\left(25a^2A+56abB+15Ab^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(63a^3B+145a^2Ab+161ab^2B+15Ab^3)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{105ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2a(7aB+10Ab)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{35d}+\frac{2aA\sin(c+dx)\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}{7d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(a^2 - b^2)*(25*a^2*A + 15*A*b^2 + 56*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(105*a*d*Sqrt[Cos[c + d*x]]*
Sqrt[a + b*Sec[c + d*x]]) + (2*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b
^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*S
ec[c + d*x]])/(105*a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*(25*a^2*A +
 45*A*b^2 + 77*a*b*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d
*x])/(105*d) + (2*a*(10*A*b + 7*a*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(35*d) + (2*a*A*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])
^(3/2)*Sin[c + d*x])/(7*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4179

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\,\mathrm{se}}{}$$

$$= \frac{2aA\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c}{7d}$$

$$= \frac{2a(10Ab+7aB)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c}}{35d}$$

$$= \frac{2(25a^2A+45Ab^2+77abB)\sqrt{\cos(c+dx)}}{105d}$$

$$= \frac{2(25a^2A+45Ab^2+77abB)\sqrt{\cos(c+dx)}}{105d}$$

$$= \frac{2(25a^2A+45Ab^2+77abB)\sqrt{\cos(c+dx)}}{105d}$$

$$= \frac{2(25a^2A+45Ab^2+77abB)\sqrt{\cos(c+dx)}}{105d}$$

$$= \frac{2(a^2-b^2)(25a^2A+15Ab^2+56abB)\sqrt{\frac{b+}{}}}{105ad\sqrt{\cos(c+dx)}\,\sqrt{a+}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 17.82, size = 470, normalized size = 1.38

$$\frac{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}\left(\frac{1}{210}(115a^2A+180Ab^2+308abB)\sin(c+dx)+\frac{1}{35}a(15Ab+7aB)\sin(2(c+dx))+\frac{1}{14}a^2A\sin(3(c+dx))\right)}{d(b+a\cos(c+dx))^2} - \frac{2\cos^{\frac{3}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}(a+b\sec(c+dx))^{5/2}\left(-i(a+b)(145a^2Ab+15Ab^3+63a^3B+161ab^2B)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+ia(a+b)(15b^2(A+7B)+8ab(15A+7B)+a^2(25A+63B))F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-(145a^2Ab+15Ab^3+63a^3B+161ab^2B)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{105ad(b+a\cos(c+dx))^3\sec^{\frac{5}{2}}(c+dx)}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] (Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*(((115*a^2*A + 180*A*b^2 + 3
08*a*b*B)*Sin[c + d*x])/210 + (a*(15*A*b + 7*a*B)*Sin[2*(c + d*x)])/35 + (a
^2*A*Sin[3*(c + d*x)])/14))/(d*(b + a*Cos[c + d*x])^2) - (2*Cos[c + d*x]^(3
/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(a + b*Sec[c + d*x])^(5/2)*((-I
```

```
)*(a + b)*(145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*EllipticE[I*Arc
Sinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(15*b^2*(A + 7*B) +
 8*a*b*(15*A + 7*B) + a^2*(25*A + 63*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2)/(a + b)] - (145*a^2*A*b + 15*A*b^3 + 63*a^3*B + 161*a*b^2*B)*
(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(105*a*d
*(b + a*Cos[c + d*x])^3*Sec[c + d*x]^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2449 vs. $2(364) = 728$.
time = 20.95, size = 2450, normalized size = 7.21

method	result	size
default	Expression too large to display	2450

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/105/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)*(1+cos(d*x+c))*(161*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+
c)))^(1/2)+105*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a*b^3-25*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*(1/(1+cos(d*x+c)))^(1/2)+2
5*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-15*A*s
in(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+21*B*cos(d*
x+c)^4*((a-b)/(a+b))^(1/2)*a^4*(1/(1+cos(d*x+c)))^(1/2)+42*B*cos(d*x+c)^2*(
(a-b)/(a+b))^(1/2)*a^4*(1/(1+cos(d*x+c)))^(1/2)+15*A*cos(d*x+c)*((a-b)/(a+b
))^(1/2)*b^4*(1/(1+cos(d*x+c)))^(1/2)-63*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a
^4*(1/(1+cos(d*x+c)))^(1/2)-25*A*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c)
))^(1/2)-145*A*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)-45*A*((
a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)-63*B*((a-b)/(a+b))^(1/2)*a
^3*b*(1/(1+cos(d*x+c)))^(1/2)-77*B*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*
x+c)))^(1/2)-161*B*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)-15*A*
((a-b)/(a+b))^(1/2)*b^4*(1/(1+cos(d*x+c)))^(1/2)+63*B*sin(d*x+c)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-63*B*sin(d*x+c)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+15*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2
)*a^4*(1/(1+cos(d*x+c)))^(1/2)+10*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*(1
/(1+cos(d*x+c)))^(1/2)+60*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+co
s(d*x+c)))^(1/2)+90*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*
```

```
x+c)))^(1/2)+98*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c)))
^(1/2)+110*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c)))^(1/2
)+60*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)+238*
B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)-145*A*c
os(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c)))^(1/2)+55*A*cos(d*x+c
)*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)-15*A*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)-35*B*cos(d*x+c)*((a-b)/(a+b)
)^(1/2)*a^3*b*(1/(1+cos(d*x+c)))^(1/2)-161*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)
*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)+15*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a*b^3-145*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a^3*b+135*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a^2*b^2-15*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a*b^3+145*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^3*b-145*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^2*b^2-63*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^3*b+161*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^2*b^2-161*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a*b^3+119*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^3*b-161*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^2*b^2)/a/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))
^(1/2)/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.

time = 0.61, size = 579, normalized size = 1.70

$$\frac{6\left(15Aa^4\cos(dx+c)^2+25Aa^4+77Ba^3b+45Aa^2b^2+3\left(7Ba^4+15Aa^3b\right)\cos(dx+c)\right)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\sqrt{2}\left(-75i\,Aa^4-231i\,Ba^3b-115i\,Aa^2b^2+7i\,Bab^3+30i\,Ab^4\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(75i\,Aa^4+231i\,Ba^3b+115i\,Aa^2b^2-7i\,Bab^3-30i\,Ab^4\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(-63i\,Ba^4-145i\,Aa^3b-161i\,Ba^2b^2-15i\,Aab^3\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(63i\,Ba^4+145i\,Aa^3b+161i\,Ba^2b^2+15i\,Aab^3\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4(3a^2-4b^2)}{3a^2},\frac{8(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)}{315\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] 1/315*(6*(15*A*a^4*cos(d*x + c)^2 + 25*A*a^4 + 77*B*a^3*b + 45*A*a^2*b^2 +
3*(7*B*a^4 + 15*A*a^3*b)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x +
c))*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-75*I*A*a^4 - 231*I*B*a^3*b
- 115*I*A*a^2*b^2 + 7*I*B*a*b^3 + 30*I*A*b^4)*sqrt(a)*weierstrassPInverse(-
4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
+ 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(75*I*A*a^4 + 231*I*B*a^3*b + 115*
I*A*a^2*b^2 - 7*I*B*a*b^3 - 30*I*A*b^4)*sqrt(a)*weierstrassPInverse(-4/3*(3
*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*
a*sin(d*x + c) + 2*b)/a) - 3*sqrt(2)*(-63*I*B*a^4 - 145*I*A*a^3*b - 161*I*B
*a^2*b^2 - 15*I*A*a*b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2,
8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8
/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b
)/a)) - 3*sqrt(2)*(63*I*B*a^4 + 145*I*A*a^3*b + 161*I*B*a^2*b^2 + 15*I*A*a*
b^3)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^
3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3
)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^2*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(7/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(7/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^{7/2}\left(A+\frac{B}{\cos(c+d\,x)}\right)\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^(7/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)
```

3.610 $\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=342

$$\frac{2(8a^2Ab-8Ab^3+5a^3B+10ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2b^3B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b})}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2/5*a*A*cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d+2/15*(8*A*a^2*
b-8*A*b^3+5*B*a^3+10*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/
(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*b^3*B*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)
*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*se
c(d*x+c))^(1/2)+2/15*a*(8*A*b+5*B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d
*x+c))^(1/2)/d+2/15*(9*A*a^2+23*A*b^2+35*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*
cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.91, antiderivative size = 342, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3034, 4110, 4179, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(9a^2A+35abB+23Ab^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(5a^3B+8a^2Ab+10ab^2B-8Ab^3)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2a(5aB+8Ab)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{15d}+\frac{2aA\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}{5d}+\frac{2b^3B\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] (2*(8*a^2*A*b - 8*A*b^3 + 5*a^3*B + 10*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(15*d*Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Sec[c + d*x]]) + (2*b^3*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Ellipt
icPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
 + d*x]]) + (2*(9*a^2*A + 23*A*b^2 + 35*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE
[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15*d*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]) + (2*a*(8*A*b + 5*a*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(15*d) + (2*a*A*Cos[c + d*x]^(3/2)*(a + b*Sec[c
 + d*x])^(3/2)*Sin[c + d*x])/(5*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4179

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Csc[e
 + f*x])^n/(f*n)), x] - Dist[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 1)*(d*C
```

```
sc[e + f*x])^(n + 1)*Simp[A*b*m - a*B*n - (b*B*n + a*(C*n + A*(n + 1)))*Csc
[e + f*x] - b*(C*n + A*(m + n + 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a,
 b, d, e, f, A, B, C}, x] && NeQ[a^2 - b^2, 0] && GtQ[m, 0] && LeQ[n, -1]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec}{}$$

$$= \frac{2aA\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}\sin(c-}{5d}$$

$$= \frac{2a(8Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}{15d}$$

$$= \frac{2a(8Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}{15d}$$

$$= \frac{2a(8Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}{15d}$$

$$= \frac{2a(8Ab+5aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+}}{15d}$$

$$= \frac{2b^3B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{2(8a^2Ab-8Ab^3+5a^3B+10ab^2B)\sqrt{\frac{b+a\,c}{a}}}{15d\sqrt{\cos(c+dx)}\sqrt{a+b\,s}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 35.14, size = 49609, normalized size = 145.06

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 19.46, size = 2052, normalized size = 6.00

method	result	size
default	Expression too large to display	2052

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/15/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))
*(1+cos(d*x+c))*(-23*A*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)-9*A
*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+9*A*sin(d
*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-23*A*sin(d*x+c)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+14*A*((a-b)/(a+b))^(
1/2)*cos(d*x+c)^3*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+3*A*((a-b)/(a+b))^(1/2)*co
s(d*x+c)^4*a^3*(1/(1+cos(d*x+c)))^(1/2)+5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^
3*(1/(1+cos(d*x+c)))^(1/2)*a^3-5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(1
+cos(d*x+c)))^(1/2)+5*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^3+6*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*(1/(1+cos(d*x+c)))^(
1/2)-9*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(1+cos(d*x+c)))^(1/2)+23*A*(
(a-b)/(a+b))^(1/2)*cos(d*x+c)*b^3*(1/(1+cos(d*x+c)))^(1/2)-9*A*((a-b)/(a+b)
)^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-11*A*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1
+cos(d*x+c)))^(1/2)-5*B*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-
35*B*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+34*A*((a-b)/(a+b))^
(1/2)*cos(d*x+c)^2*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+40*B*((a-b)/(a+b))^(1/2)*
cos(d*x+c)^2*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-5*A*((a-b)/(a+b))^(1/2)*cos(d*x
+c)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-23*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^
2*(1/(1+cos(d*x+c)))^(1/2)-35*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1/(1+
cos(d*x+c)))^(1/2)+35*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+cos(d*x+
c)))^(1/2)+17*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2)
)*a^2*b-23*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
*b^2-9*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b
+23*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+35
*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-35*B*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-35*B*sin
```

```
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-15*B*sin(d*
x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+30*B*sin(d*x+c)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^3+15*A*si
n(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+45*B*sin(d*x
+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2)/((a-b)/(a+b))^
(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2
), x)
```

Fricas [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] integral((B*b^2*cos(d*x + c)^2*sec(d*x + c)^3 + A*a^2*cos(d*x + c)^2 + (2*B
*a*b + A*b^2)*cos(d*x + c)^2*sec(d*x + c)^2 + (B*a^2 + 2*A*a*b)*cos(d*x + c
)^2*sec(d*x + c))*sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c)), x)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos\left(c+d\,x\right)^{5/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2),x)
[Out] int(cos(c + d*x)^(5/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)
```

3.611 $\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=349

$$\frac{(2a^3A+4aAb^2+12a^2bB+3b^3B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{b^2(2Ab+5aB)\sqrt{\frac{b+a\cos(c+d}{a+b}}}{d\sqrt{\cos(c+dx)}\sqrt{a+}}$$

```
[Out] 2/3*a*A*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)*cos(d*x+c)^(1/2)/d+1/3*(2*A*a^3+4
*A*a*b^2+12*B*a^2*b+3*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(
a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+b^2*(2*A*b+5*B*a)*(co
s(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),
2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2
)/(a+b*sec(d*x+c))^(1/2)-1/3*b*(2*A*a-3*B*b)*sin(d*x+c)*(a+b*sec(d*x+c))^(1
/2)/d/cos(d*x+c)^(1/2)+1/3*(14*A*a*b+6*B*a^2-3*B*b^2)*(cos(1/2*d*x+1/2*c)^2
)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(
1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1
/2)
```

Rubi [A]

time = 0.90, antiderivative size = 349, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3034, 4110, 4181, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(6a^2B+14aAb-3b^2B)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(2a^3A+12a^2bB+4aAb^2+3b^3B)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{b^2(5aB+2Ab)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{b(2aA-3bB)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3d\sqrt{\cos(c+dx)}}+\frac{2aA\sin(c+dx)\sqrt{\cos(c+dx)}(a+b\sec(c+dx))^{3/2}}{3d}$$

Antiderivative was successfully verified.

```
[In] Int[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((2*a^3*A + 4*a*A*b^2 + 12*a^2*b*B + 3*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/(a
+ b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*d*Sqrt[Cos[c + d*x]]*Sqrt[a
 + b*Sec[c + d*x]]) + (b^2*(2*A*b + 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b
)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a
+ b*Sec[c + d*x]]) + ((14*a*A*b + 6*a^2*B - 3*b^2*B)*Sqrt[Cos[c + d*x]]*Ell
ipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*d*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]) - (b*(2*a*A - 3*b*B)*Sqrt[a + b*Sec[c + d*x]]*Si
n[c + d*x])/(3*d*Sqrt[Cos[c + d*x]]) + (2*a*A*Sqrt[Cos[c + d*x]]*(a + b*Sec
[c + d*x])^(3/2)*Sin[c + d*x])/(3*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4110

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*A*Cot
[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*n)), x] + Dis
t[1/(d*n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^(n + 1)*Simp[a
*(a*B*n - A*b*(m - n - 1)) + (2*a*b*B*n + A*(b^2*n + a^2*(1 + n)))*Csc[e +
f*x] + b*(b*B*n + a*A*(m + n))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d
, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[m, 1] &&
 LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4181

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^n/(f*(m + n + 1))), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e +
 f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*
B)*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2
- b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\sec}{}$$

$$= \frac{2aA\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}\sin(c}{3d}$$

$$= -\frac{b(2aA-3bB)\sqrt{a+b\sec(c+dx)}\,\sin(c+d}{3d\sqrt{\cos(c+dx)}}$$

$$= -\frac{b(2aA-3bB)\sqrt{a+b\sec(c+dx)}\,\sin(c+d}{3d\sqrt{\cos(c+dx)}}$$

$$= -\frac{b(2aA-3bB)\sqrt{a+b\sec(c+dx)}\,\sin(c+d}{3d\sqrt{\cos(c+dx)}}$$

$$= -\frac{b(2aA-3bB)\sqrt{a+b\sec(c+dx)}\,\sin(c+d}{3d\sqrt{\cos(c+dx)}}$$

$$= \frac{b^2(2Ab+5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi(2;\tfrac{1}{2}(c}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{(2a^3A+4aAb^2+12a^2bB+3b^3B)\sqrt{\frac{b+a\cos}{a}}}{3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 33.96, size = 73332, normalized size = 210.12

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 19.81, size = 2073, normalized size = 5.94

method	result	size
default	Expression too large to display	2073

Verification of antiderivative is not currently implemented for this CAS.

[In] int(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x,method=_RETURNVERBOSE)

[Out] 1/3/d*(-1+cos(d*x+c))*(1+cos(d*x+c))*(18*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-12*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-6*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2+30*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2-14*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b+18*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2+14*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^2*b-14*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a*b^2+16*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+2*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^4*a^3*(1/(1+cos(d*x+c)))^(1/2)+6*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*(1/(1+cos(d*x+c)))^(1/2)*a^3-2*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*(1/(1+cos(d*x+c)))^(1/2)-6*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*(1/(1+cos(d*x+c)))^(1/2)+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)-3*B*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)-14*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+14*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+6*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-2*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-14*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-6*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-3*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-6*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3+6*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(

```
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3+2*A*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)*a^3
-6*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c
)*b^3+12*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*
sin(d*x+c)*cos(d*x+c)*b^3+3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*sin(d*x+c)*cos(d*x+c)*b^3)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)/((a-b)/
(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3/cos(d*x
+c)^(1/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2
), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c)),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c)),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \cos(c+d\,x)^{3/2}\,\left(A+\frac{B}{\cos(c+d\,x)}\right)\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(3/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)
```

3.612 $\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx$

Optimal. Leaf size=359

$$\frac{(16a^2Ab+4Ab^3+8a^3B+11ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{b(20aAb+15a^2B+4b^2B)\sqrt{\frac{b\cdots}{\cdots}}}{4d\sqrt{\cos(c+dx)}\cdots}$$

```
[Out] 1/2*b*B*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(1/2)+1/4*(16*A*a^2*
b+4*A*b^3+8*B*a^3+11*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*
c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/
(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/4*b*(20*A*a*b+15*B
*a^2+4*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(si
n(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/
d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/4*b*(4*A*b+7*B*a)*sin(d*x+c)*(a
+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)+1/4*(8*A*a^2-4*A*b^2-9*B*a*b)*(cos(
1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(
1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/d/((b+a*cos(d
*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.92, antiderivative size = 359, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3034, 4111, 4181, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(8a^2A-9abB-4Ab^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{b(15a^2B+20aAb+4b^2B)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{(8a^3B+16a^2Ab+11ab^2B+4Ab^3)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{b(7aB+4Ab)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4d\sqrt{\cos(c+dx)}}+\frac{bB\sin(c+dx)(a+b\sec(c+dx))^{3/2}}{2d\sqrt{\cos(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]),x]
```

```
[Out] ((16*a^2*A*b + 4*A*b^3 + 8*a^3*B + 11*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]) + (b*(20*a*A*b + 15*a^2*B + 4*b^2*B)*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(4*d*Sqrt[Cos[
c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((8*a^2*A - 4*A*b^2 - 9*a*b*B)*Sqrt[C
os[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]]
)/(4*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b*(4*A*b + 7*a*B)*Sqrt[a + b*
Sec[c + d*x]]*Sin[c + d*x])/(4*d*Sqrt[Cos[c + d*x]]) + (b*B*(a + b*Sec[c +
d*x])^(3/2)*Sin[c + d*x])/(2*d*Sqrt[Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4181

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
```

```
_))^(m_), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^n/(f*(m + n + 1))), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e +
 f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*
B)*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2
- b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{(a+b\text{s}}{}$$

$$= \frac{bB(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{2d\sqrt{\cos(c+dx)}} + \frac{1}{2}\Big(\surd$$

$$= \frac{b(4Ab+7aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+d}{4d\sqrt{\cos(c+dx)}}$$

$$= \frac{b(4Ab+7aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+d}{4d\sqrt{\cos(c+dx)}}$$

$$= \frac{b(4Ab+7aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+d}{4d\sqrt{\cos(c+dx)}}$$

$$= \frac{b(4Ab+7aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+d}{4d\sqrt{\cos(c+dx)}}$$

$$= \frac{b(20aAb+15a^2B+4b^2B)\sqrt{\frac{b+a\cos(c+}{a+b}}}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec}}$$

$$= \frac{(16a^2Ab+4Ab^3+8a^3B+11ab^2B)\sqrt{\frac{b+a}{}}}{4d\sqrt{\cos(c+dx)}\,\sqrt{a+b}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 32.82, size = 97208, normalized size = 270.77

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]
),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 19.69, size = 2216, normalized size = 6.17

method	result	size
default	Expression too large to display	2216

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1+cos(d*x+c))*(-
8*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*(1/(1+cos(d*x+c)))^(1/2)+4*A*cos(d
*x+c)^2*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)+8*A*((a-b)/(a+b))^
(1/2)*cos(d*x+c)^3*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+8*A*((a-b)/(a+b))^(1/2)*c
os(d*x+c)^4*a^3*(1/(1+cos(d*x+c)))^(1/2)-4*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)
*b^3*(1/(1+cos(d*x+c)))^(1/2)+24*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-16*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-8*A*cos(d*x+c)^2*sin(d*x+
c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-4*A*cos(d*x+c)^2
*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+40*A*co
s(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+
b))^(1/2))*a*b^2-6*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^2*b+2*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+30*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b-9*B*cos(d*x+c)^2
*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+9*B*cos
(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^
2-2*B*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)-8*A*cos(d*x+c)^2*((a
-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+9*B*cos(d*x+c)^2*((a-b)/(a+
b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-4*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)
^2*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-9*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*
b*(1/(1+cos(d*x+c)))^(1/2)-11*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+
cos(d*x+c)))^(1/2)+2*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c
)))^(1/2)-8*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^3+8*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c
```

```
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^3+4*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*b^3+8*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)
/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-4*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+8*B*cos(d*x+c)^2*sin(d*x+c)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^3+4*A*cos(d
*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+9*B*cos(d*x+c)^3
*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+2*B*cos(d*x+c)^3*((a-b)
/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2))/((a-b)/(a+b))^(1/2)/(b+a*cos(
d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(3/2)/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)
), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)**(1/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))*cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \sqrt{\cos\left(c+d\,x\right)}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)\,{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int(cos(c + d*x)^(1/2)*(A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2), x)
```

3.613 $\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx$

Optimal. Leaf size=422

$$\frac{(48a^3A+66aAb^2+59a^2bB+16b^3B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(30a^2Ab+8Ab^3+5a^3B+20}{8d\sqrt{\cos(c}$$

```
[Out] 1/3*b*B*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(3/2)+1/24*(48*A*a^3
+66*A*a*b^2+59*B*a^2*b+16*B*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c
))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/8*(30*A*a^2*b+8
*A*b^3+5*B*a^3+20*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*
EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/
(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/4*b*(2*A*b+3*B*a)*
sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(3/2)+1/24*(54*A*a*b+33*B*a^
2+16*B*b^2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(1/2)-1/24*(54*A
*a*b+33*B*a^2+16*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*se
c(d*x+c))^(1/2)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]

time = 1.16, antiderivative size = 422, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 15, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3034, 4111, 4181, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(33a^2B+54aAb+16b^2B)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24d\sqrt{\cos(c+dx)}}-\frac{(33a^2B+54aAb+16b^2B)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(48a^3A+59a^2bB+66aAb^2+16b^3B)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{24d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(5a^3B+30a^2Ab+20ab^2B+8Ab^3)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{8d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{b(3aB+2Ab)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4d\cos^{\frac{3}{2}}(c+dx)}+\frac{bB\sin(c+dx)(a+b\sec(c+dx))^{3/2}}{3d\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*x]],x]
```

```
[Out] ((48*a^3*A + 66*a*A*b^2 + 59*a^2*b*B + 16*b^3*B)*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(24*d*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Sec[c + d*x]]) + ((30*a^2*A*b + 8*A*b^3 + 5*a^3*B + 20*a*b^2*B)*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])
/(8*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((54*a*A*b + 33*a^2*B
+ 16*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
 + b*Sec[c + d*x]])/(24*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (b*(2*A*b +
 3*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*d*Cos[c + d*x]^(3/2)) + (
(54*a*A*b + 33*a^2*B + 16*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(24
*d*Sqrt[Cos[c + d*x]]) + (b*B*(a + b*Sec[c + d*x])^(3/2)*Sin[c + d*x])/(3*d
*Cos[c + d*x]^(3/2))
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
```

```
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] &&  !(IGtQ[n, 1] &&  !IntegerQ[m])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4181

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^n/(f*(m + n + 1))), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e +
 f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*
B)*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2
- b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\sqrt{\cos(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \sqrt{\sec(c+dx)}\,(a+b\,\mathrm{s}$$

$$= \frac{bB(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{3d\cos^{\frac{3}{2}}(c+dx)} + \frac{1}{3}\left(\sqrt{\cos(c+dx)}\right.$$

$$= \frac{b(2Ab+3aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{bB(a+}{}$$

$$= \frac{b(2Ab+3aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{(54aAb}{}$$

$$= \frac{b(2Ab+3aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{(54aAb}{}$$

$$= \frac{b(2Ab+3aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{(54aAb}{}$$

$$= \frac{b(2Ab+3aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{4d\cos^{\frac{3}{2}}(c+dx)} + \frac{(54aAb}{}$$

$$= \frac{(30a^2Ab+8Ab^3+5a^3B+20ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\mathrm{I}}{8d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{(48a^3A+66aAb^2+59a^2bB+16b^3B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}{24d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 32.67, size = 106199, normalized size = 251.66

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[Cos[c + d*
x]],x]
```

[Out] Result too large to show

Maple [C] Result contains complex when optimal does not.
time = 20.02, size = 2441, normalized size = 5.78

method	result	size
default	Expression too large to display	2441

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/24/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1+cos(d*x+c))*(
54*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-54*A*(
(a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-33*B*((a-b)/
(a+b))^(1/2)*cos(d*x+c)^3*(1/(1+cos(d*x+c)))^(1/2)*a^3-12*A*((a-b)/(a+b))^(
1/2)*cos(d*x+c)*b^3*(1/(1+cos(d*x+c)))^(1/2)-8*B*((a-b)/(a+b))^(1/2)*b^3*(1
/(1+cos(d*x+c)))^(1/2)-66*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a*b^2*(1/(1+co
s(d*x+c)))^(1/2)-59*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^2*b*(1/(1+cos(d*x+
c)))^(1/2)-34*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+cos(d*x+c)))^(1/
2)+33*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^2*b-16*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a*b^2+120*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^2+26*B*cos(d*x+c)^3*sin(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+12*A*cos(d*x+c)^4
*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+26*B*cos(d*x+c)^4*((a-b
)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+16*B*cos(d*x+c)^4*((a-b)/(a+b
))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-44*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-36*A*cos(d*x+c)^3*sin(d*x
+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+12*A*cos(d*x+c)
^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-54*A*
cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^2*b+54*A*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a*b^2+180*A*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b+12*A*cos(d*x+c)^3*(1/(1+cos(d*x+c))
```

```
)^(1/2)*((a-b)/(a+b))^(1/2)*b^3+33*B*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*(
1/(1+cos(d*x+c)))^(1/2)+16*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos
(d*x+c)))^(1/2)-8*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))
^(1/2)+54*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)
+33*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+18*B*
cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-24*A*cos(d*
x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+48*
A*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)
/(a+b))^(1/2))*b^3-33*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a^3+16*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+30*B*cos(d*x+c)^3*sin(d*x+c)*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^3+18*B*cos(d*x+c)^3*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+48*A*cos(d
*x+c)^3*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3)/(
(a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/cos(d*x+c)^(5/
2)/sin(d*x+c)^3
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(1/2),x)
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(1/2),x, algor
ithm="giac")
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/sqrt(cos(d*x + c)
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/2}}{\sqrt{\cos\left(c+d\,x\right)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2),x)
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/cos(c + d*x)^(1/2), x
)
```

3.614 $\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx$

Optimal. Leaf size=513

$$\frac{(472a^2Ab+128Ab^3+133a^3B+356ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{192d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(40a^3Ab+160aAb^3-5a$$

```
[Out] 1/4*b*B*(a+b*sec(d*x+c))^(3/2)*sin(d*x+c)/d/cos(d*x+c)^(5/2)+1/192*(472*A*a
^2*b+128*A*b^3+133*B*a^3+356*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*
d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(
d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/64*(40*A*a
^3*b+160*A*a*b^3-5*B*a^4+120*B*a^2*b^2+48*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/
2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/
2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1
/2)+1/24*b*(8*A*b+11*B*a)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/cos(d*x+c)^(5
/2)+1/96*(104*A*a*b+59*B*a^2+36*B*b^2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/d/
cos(d*x+c)^(3/2)+1/192*(264*A*a^2*b+128*A*b^3+15*B*a^3+284*B*a*b^2)*sin(d*x
+c)*(a+b*sec(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)-1/192*(264*A*a^2*b+128*A*b^
3+15*B*a^3+284*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*se
c(d*x+c))^(1/2)/b/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 1.42, antiderivative size = 513, normalized size of antiderivative = 1.00, number of steps used = 16, number of rules used = 15, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3034, 4111, 4181, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(59a^2B+104aAb+36b^2B)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{96d\cos^{\frac{3}{2}}(c+dx)}+\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{192bd\sqrt{\cos(c+dx)}}+\frac{(133a^3B+472a^2Ab+356ab^2B+128Ab^3)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{192d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{(15a^3B+264a^2Ab+284ab^2B+128Ab^3)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{192bd\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(-5a^4B+40a^3Ab+120a^2b^2B+160aAb^3+48b^4B)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{64bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{b(11aB+8Ab)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{24d\cos^{\frac{5}{2}}(c+dx)}+\frac{bB\sin(c+dx)(a+b\sec(c+dx))^{3/2}}{4d\cos^{\frac{5}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3/2),x]
```

```
[Out] ((472*a^2*A*b + 128*A*b^3 + 133*a^3*B + 356*a*b^2*B)*Sqrt[(b + a*Cos[c + d*
x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(192*d*Sqrt[Cos[c + d*x
]]*Sqrt[a + b*Sec[c + d*x]]) + ((40*a^3*A*b + 160*a*A*b^3 - 5*a^4*B + 120*a
^2*b^2*B + 48*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c +
d*x)/2, (2*a)/(a + b)])/(64*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]
) - ((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(192*b*d*Sq
rt[(b + a*Cos[c + d*x])/(a + b)]) + (b*(8*A*b + 11*a*B)*Sqrt[a + b*Sec[c +
d*x]]*Sin[c + d*x])/(24*d*Cos[c + d*x]^(5/2)) + ((104*a*A*b + 59*a^2*B + 36
```

```
*b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(96*d*Cos[c + d*x]^(3/2)) +
((264*a^2*A*b + 128*A*b^3 + 15*a^3*B + 284*a*b^2*B)*Sqrt[a + b*Sec[c + d*x]
]*Sin[c + d*x])/(192*b*d*Sqrt[Cos[c + d*x]]) + (b*B*(a + b*Sec[c + d*x])^(3
/2)*Sin[c + d*x])/(4*d*Cos[c + d*x]^(5/2))
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4111

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-b)*B*
Cot[e + f*x]*(a + b*Csc[e + f*x])^(m - 1)*((d*Csc[e + f*x])^n/(f*(m + n))),
 x] + Dist[1/(m + n), Int[(a + b*Csc[e + f*x])^(m - 2)*(d*Csc[e + f*x])^n*S
imp[a^2*A*(m + n) + a*b*B*n + (a*(2*A*b + a*B)*(m + n) + b^2*B*(m + n - 1))
*Csc[e + f*x] + b*(A*b*(m + n) + a*B*(2*m + n - 1))*Csc[e + f*x]^2, x], x],
 x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 -
 b^2, 0] && GtQ[m, 1] && !(IGtQ[n, 1] && !IntegerQ[m])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
```

```
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4181

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*Cot[e + f*x]*(a + b*Csc[e + f*x])^m*((d*Cs
c[e + f*x])^n/(f*(m + n + 1))), x] + Dist[1/(m + n + 1), Int[(a + b*Csc[e +
 f*x])^(m - 1)*(d*Csc[e + f*x])^n*Simp[a*A*(m + n + 1) + a*C*n + ((A*b + a*
B)*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) + a*C*m)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x] && NeQ[a^2
- b^2, 0] && GtQ[m, 0] &&  !LeQ[n, -1]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{(a+b\sec(c+dx))^{5/2}(A+B\sec(c+dx))}{\cos^{\frac{3}{2}}(c+dx)}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \sec^{\frac{3}{2}}(c+dx)(a+b\sec$$

$$= \frac{bB(a+b\sec(c+dx))^{3/2}\sin(c+dx)}{4d\cos^{\frac{5}{2}}(c+dx)} + \frac{1}{4}\left(\sqrt{\cos(c+dx)}\right.$$

$$= \frac{b(8Ab+11aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{bB(a+$$

$$= \frac{b(8Ab+11aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{(104aA$$

$$= \frac{b(8Ab+11aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{(104aA$$

$$= \frac{b(8Ab+11aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{(104aA$$

$$= \frac{b(8Ab+11aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{(104aA$$

$$= \frac{b(8Ab+11aB)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{24d\cos^{\frac{5}{2}}(c+dx)} + \frac{(104aA$$

$$= \frac{(40a^3Ab+160aAb^3-5a^4B+120a^2b^2B+48b^4B)\sqrt{\frac{b+}{}}}{64bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(}}$$

$$= \frac{(472a^2Ab+128Ab^3+133a^3B+356ab^2B)\sqrt{\frac{b+a\cos(c-}{a+b}}}{192d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+d}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 33.10, size = 131553, normalized size = 256.44

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[((a + b*Sec[c + d*x])^(5/2)*(A + B*Sec[c + d*x]))/Cos[c + d*x]^(3
/2),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 20.07, size = 3175, normalized size = 6.19

method	result	size
default	Expression too large to display	3175

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/192/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1+cos(d*x+c))*
(264*A*sin(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^2*b^2-128*A*sin(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a*b^3+240*A*sin(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^3*b+960*A*sin(d*x+c)*cos(d*x+
c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^3-
184*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)+128*A*s
in(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^
4-48*B*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^4-15*B*cos(d*x+c)^4*(
(a-b)/(a+b))^(1/2)*a^4*(1/(1+cos(d*x+c)))^(1/2)-64*A*cos(d*x+c)*((a-b)/(a+b
))^(1/2)*b^4*(1/(1+cos(d*x+c)))^(1/2)+118*B*sin(d*x+c)*cos(d*x+c)^4*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-76*B*sin(d*x+c)*cos(d*x+c)
^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+72*B*sin(d*x+c
)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+15*B
*sin(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a^3*b-284*B*sin(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a^2*b^2+284*B*sin(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a*b^3+720*B*sin(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c
```

```
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b^2+144*A*sin(d*x+c)*c
os(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+208*A*s
in(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
2*b^2-352*A*sin(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-
b))^(1/2))*a*b^3-264*A*sin(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^3*b-264*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b*(1
/(1+cos(d*x+c)))^(1/2)-472*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1
+cos(d*x+c)))^(1/2)-133*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(
d*x+c)))^(1/2)-272*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c
)))^(1/2)-254*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))
^(1/2)+15*B*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^4*(1/(1+cos(d*x+c)))^(1/2)+1
28*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*b^4*(1/(1+cos(d*x+c)))^(1/2)-64*A*cos
(d*x+c)^3*((a-b)/(a+b))^(1/2)*b^4*(1/(1+cos(d*x+c)))^(1/2)+72*B*cos(d*x+c)^
4*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^4-24*B*cos(d*x+c)^2*((a-b)
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^4+30*B*sin(d*x+c)*cos(d*x+c)^4*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-144*B*sin(d*x+c)*cos(d*x
+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c
))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4-15*B*sin(d*x+c)
*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-30*B*si
n(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipti
cPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+
b))^(1/2))*a^4+288*B*sin(d*x+c)*cos(d*x+c)^4*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^4+264*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2
)*a^3*b*(1/(1+cos(d*x+c)))^(1/2)+208*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^2
*b^2*(1/(1+cos(d*x+c)))^(1/2)+128*A*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a*b^3*
(1/(1+cos(d*x+c)))^(1/2)+118*B*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1
+cos(d*x+c)))^(1/2)+284*B*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+co
s(d*x+c)))^(1/2)+72*B*cos(d*x+c)^5*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+
c)))^(1/2)+264*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c))
)^(1/2)+144*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))**(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)**(3/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((a+b*sec(d*x+c))^(5/2)*(A+B*sec(d*x+c))/cos(d*x+c)^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*(b*sec(d*x + c) + a)^(5/2)/cos(d*x + c)^(3/2
), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\left(A+\frac{B}{\cos(c+d\,x)}\right)\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/2}}{{\cos\left(c+d\,x\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2),x)
```

```
[Out] int(((A + B/cos(c + d*x))*(a + b/cos(c + d*x))^(5/2))/cos(c + d*x)^(3/2), x
)
```

3.615 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=280

$$-\frac{2(7a^2Ab+8Ab^3-5a^3B-10ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^2A+8Ab^2-10abB)\sqrt{\mathrm{c}}}{15a}$$

```
[Out] -2/15*(7*A*a^2*b+8*A*b^3-5*B*a^3-10*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b
+a*cos(d*x+c))/(a+b))^(1/2)/a^3/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2
/5*A*cos(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a/d-2/15*(4*A*b-5*B
*a)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/d+2/15*(9*A*a^2+
8*A*b^2-10*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE
(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+
c))^(1/2)/a^3/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.59, antiderivative size = 280, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3034, 4119, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{2(4Ab-5aB)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{15a^2d}+\frac{2(9a^2A-10abB+8Ab^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2(-5a^3B+7a^2Ab-10ab^2B+8Ab^3)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2A\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{5ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*(7*a^2*A*b + 8*A*b^3 - 5*a^3*B - 10*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/
(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(15*a^3*d*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^2*A + 8*A*b^2 - 10*a*b*B)*Sqrt[Cos[c
+ d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(15
*a^3*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*(4*A*b - 5*a*B)*Sqrt[Cos[c
+ d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^2*d) + (2*A*Cos[c + d*
x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(5*a*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4119

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*n)), x] + Dis
```

```
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+ \ldots}}$$

$$= \frac{2A\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{5ad} - \frac{\left(2\sqrt{\cos(c+ \ldots}}{}$$

$$= -\frac{2(4Ab-5aB)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15a^2d} + \ldots$$

$$= -\frac{2(4Ab-5aB)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15a^2d} + \ldots$$

$$= -\frac{2(4Ab-5aB)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15a^2d} + \ldots$$

$$= -\frac{2(4Ab-5aB)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{15a^2d} + \ldots$$

$$= -\frac{2(7a^2Ab+8Ab^3-5a^3B-10ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c \ldots\right.}{15a^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.55, size = 363, normalized size = 1.30

$$\frac{2a(b+a\cos(c+dx))(-4Ab+5aB+3aA\cos(c+dx))\sin(c+dx)+\frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(i(a+b)(9a^2A+8Ab^2-10abB)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-ia(8Ab^2+2ab(A-5B)+a^2(9A+5B))F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+(9a^2A+8Ab^2-10abB)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sec^{\frac{3}{2}}(c+dx)}}{15a^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]
],x]
```

```
[Out] (2*a*(b + a*Cos[c + d*x])*(-4*A*b + 5*a*B + 3*a*A*Cos[c + d*x])*Sin[c + d*x
] + (2*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*(a + b)*(9*a^2*A + 8*A*b^
2 - 10*a*b*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[
(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*
a*(8*A*b^2 + 2*a*b*(A - 5*B) + a^2*(9*A + 5*B))*EllipticF[I*ArcSinh[Tan[(c
+ d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])
```

```
*Sec[(c + d*x)/2]^2)/(a + b)] + (9*a^2*A + 8*A*b^2 - 10*a*b*B)*(b + a*Cos[c
 + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2))/
(15*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1699 vs. $2(310) = 620$.
time = 20.53, size = 1700, normalized size = 6.07

method	result	size
default	Expression too large to display	1700

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/15/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c)
)*(1+cos(d*x+c))*(8*A*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)+9*A*
sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(
d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-9*A*sin(d*
x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+8*A*sin(d*x+c)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+A*((a-b)/(a+b))^(1/2)*
cos(d*x+c)^3*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-3*A*((a-b)/(a+b))^(1/2)*cos(d*x
+c)^4*a^3*(1/(1+cos(d*x+c)))^(1/2)-5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*(1/
(1+cos(d*x+c)))^(1/2)*a^3+5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(1+cos(
d*x+c)))^(1/2)-5*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^3-6*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*(1/(1+cos(d*x+c)))^(1/2)+
9*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(1+cos(d*x+c)))^(1/2)-8*A*((a-b)/
(a+b))^(1/2)*cos(d*x+c)*b^3*(1/(1+cos(d*x+c)))^(1/2)+9*A*((a-b)/(a+b))^(1/2
)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-4*A*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*
x+c)))^(1/2)+5*B*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-10*B*((
a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-4*A*((a-b)/(a+b))^(1/2)*co
s(d*x+c)^2*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+5*B*((a-b)/(a+b))^(1/2)*cos(d*x+c
)^2*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-10*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*
b*(1/(1+cos(d*x+c)))^(1/2)+8*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+c
os(d*x+c)))^(1/2)-10*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1/(1+cos(d*x+c
)))^(1/2)+10*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+cos(d*x+c)))^(1/2
)-2*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+8*
A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+9*A*si
n(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-8*A*sin(d*
```

```
x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+10*B*sin(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-10*B*sin(d*x+c)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-10*B*sin(d*x+c)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b)/a^3/((a-b)/(a+b))^(1/2
)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.90, size = 510, normalized size = 1.82

$$\frac{6\left(3\,Aa^{3}\cos\left(dx+c\right)+5\,Ba^{3}-4\,Aa^{2}b\right)\sqrt{\frac{a\cos\left(dx+c\right)+b}{\cos\left(dx+c\right)}}\sqrt{\cos\left(dx+c\right)}\sin\left(dx+c\right)+\sqrt{2}\left(-15i\,Ba^{3}+12i\,Aa^{2}b-20i\,Bab^{2}+16i\,Ab^{3}\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\left(15i\,Ba^{3}-12i\,Aa^{2}b+20i\,Bab^{2}-16i\,Ab^{3}\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\left(-9i\,Aa^{3}+10i\,Ba^{2}b-8i\,Aab^{2}\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\left(9i\,Aa^{3}-10i\,Ba^{2}b+8i\,Aab^{2}\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\mathrm{weierstrassPInverse}\left(-\frac{4\left(3a^{2}-4b^{2}\right)}{3a^{2}},\frac{8\left(9a^{2}b-8b^{3}\right)}{27a^{3}},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)}{45\,a^{4}d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/45*(6*(3*A*a^3*cos(d*x + c) + 5*B*a^3 - 4*A*a^2*b)*sqrt((a*cos(d*x + c) +
 b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + sqrt(2)*(-15*I*B*a^3 +
12*I*A*a^2*b - 20*I*B*a*b^2 + 16*I*A*b^3)*sqrt(a)*weierstrassPInverse(-4/3*
(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*
I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(15*I*B*a^3 - 12*I*A*a^2*b + 20*I*B*a*
b^2 - 16*I*A*b^3)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/2
7*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/
a) - 3*sqrt(2)*(-9*I*A*a^3 + 10*I*B*a^2*b - 8*I*A*a*b^2)*sqrt(a)*weierstras
sZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInv
erse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
 + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*sqrt(2)*(9*I*A*a^3 - 10*I*B*a^2*b
 + 8*I*A*a*b^2)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^
2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a
^4*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/sqrt(b*sec(d*x + c) + a),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(1/2), x
)
```

3.616 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=212

$$\frac{2(a^2A+2Ab^2-3abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{2(2Ab-3aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\right.}{3a^2d\sqrt{\frac{b+a\cos(c+}{a+b}}}$$

```
[Out] 2/3*(A*a^2+2*A*b^2-3*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a
+b))^(1/2)/a^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*A*sin(d*x+c)*c
os(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/d-2/3*(2*A*b-3*B*a)*(cos(1/2*d*x+1
/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(
a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/d/((b+a*cos(d*x+c)
)/(a+b))^(1/2)
```

Rubi [A]

time = 0.41, antiderivative size = 212, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3034, 4119, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(a^2A-3abB+2Ab^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{2(2Ab-3aB)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2A\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{3ad}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (2*(a^2*A + 2*A*b^2 - 3*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(3*a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]) - (2*(2*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*
a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^2*d*Sqrt[(b + a*Cos[c + d*x])/(a
 + b)]) + (2*A*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3
*a*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4119

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[A*Cot[e
 + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*n)), x] + Dis
t[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n
 - A*b*(m + n + 1) + A*a*(n + 1)*Csc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x
```

```
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, m}, x] && NeQ[A*b - a*B, 0]
&& NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+ \cdots}}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} - \frac{\left(2\sqrt{\cos(c \cdots\right.}{\cdots}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} - \frac{\left((2Ab-3a \cdots\right.}{\cdots}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} + \frac{\left(2\left(\frac{a^2A}{2}+\frac{1}{2}\cdots\right.\right.}{\cdots}$$

$$= \frac{2A\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\sin(c+dx)}{3ad} + \frac{\left(2\left(\frac{a^2A}{2}+\frac{1}{2}\cdots\right.\right.}{\cdots}$$

$$= \frac{2(a^2A+2Ab^2-3abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} - \cdots$$

Mathematica [C] Result contains complex when optimal does not.
time = 5.20, size = 311, normalized size = 1.47

$$\frac{2\left(aA(b+a\cos(c+dx))\sin(c+dx)-\frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-i(a+b)(-2Ab+3aB)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+ia(-2Ab+a(A+3B))F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+(2Ab-3aB)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sec^{\frac{3}{2}}(c+dx)}\right)}{3a^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]
],x]
```

```
[Out] (2*(a*A*(b + a*Cos[c + d*x])*Sin[c + d*x] - ((Cos[(c + d*x)/2]^2*Sec[c + d*
x])^(3/2)*((-I)*(a + b)*(-2*A*b + 3*a*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2)/(a + b)] + I*a*(-2*A*b + a*(A + 3*B))*EllipticF[I*ArcSinh[Tan
[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (2*A*b - 3*a*B)*(b + a*Cos[c + d*x])*(S
ec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2)))/(3*a^2*d*S
qrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1079 vs. $2(248) = 496$.
time = 20.46, size = 1080, normalized size = 5.09

method	result	size
default	Expression too large to display	1080

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/3/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*
(1+cos(d*x+c))*(A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(
1/2)-A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)+3*B*co
s(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)-A*cos(d*x+c)*((
a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)+2*A*cos(d*x+c)*((a-b)/(a+b))
^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)-2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^2*
(1/(1+cos(d*x+c)))^(1/2)-2*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a*b+2*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*b^2+A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a^2+2*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-3
*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)+3*B*cos(d*x+
c)*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)+3*B*sin(d*x+c)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-3*B*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b-3*B*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
```

```
a+b)/(a-b))^(1/2))*sin(d*x+c)*a^2-A*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c
)))^(1/2)+2*A*((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)-3*B*((a-b)/(
a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2))/a^2/((a-b)/(a+b))^(1/2)/(b+a*cos(
d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.78, size = 452, normalized size = 2.13

$$\frac{6\,Aa^2\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)+\sqrt{2}\,(-3i\,Aa^2+6i\,Bab-4i\,Ab^2)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)+\sqrt{2}\,(3i\,Aa^2-6i\,Bab+4i\,Ab^2)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)-3\sqrt{2}\,(-3i\,Ba^2+2i\,Aab)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)-3\sqrt{2}\,(3i\,Ba^2-2i\,Aab)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)}{9\,a^3d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/9*(6*A*a^2*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin
(d*x + c) + sqrt(2)*(-3*I*A*a^2 + 6*I*B*a*b - 4*I*A*b^2)*sqrt(a)*weierstras
sPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*co
s(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + sqrt(2)*(3*I*A*a^2 - 6*I*B*a*b
+ 4*I*A*b^2)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*
a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) -
3*sqrt(2)*(-3*I*B*a^2 + 2*I*A*a*b)*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*
b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b
^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x
+ c) + 2*b)/a)) - 3*sqrt(2)*(3*I*B*a^2 - 2*I*A*a*b)*sqrt(a)*weierstrassZeta
(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(
-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c)
 - 3*I*a*sin(d*x + c) + 2*b)/a)))/(a^3*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/sqrt(b*sec(d*x + c) + a),
 x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^{3/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(1/2), x
)
```

3.617 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=150

$$-\frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2A\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{ad\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

```
[Out] -2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(
1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a/d/
cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(
d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.28, antiderivative size = 150, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3034, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2A\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2(Ab-aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]],x]
```

```
[Out] (-2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*a)/(a + b)])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*A*Sqr
t[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*
x]])/(a*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(}}$$

$$= \frac{\left(A\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{a+b\sec(c+dx)}}{\sqrt{\sec(c+dx)}}\,dx}{a} -$$

$$= -\frac{\left((Ab-aB)\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{a\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{\left((Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+d}{a+b}}}}{a\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

$$= -\frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2A\sqrt{c}}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 4.20, size = 260, normalized size = 1.73

$$\frac{2\sqrt{\cos(c+dx)}\sqrt{\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)}\,(A+B\sec(c+dx))\left(iA(a+b)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-ia(A+B)F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+A(b+a\cos(c+dx))\sqrt{\sec^2\left(\frac{1}{2}(c+dx)\right)}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{ad(B+A\cos(c+dx))\sqrt{\sec(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/Sqrt[a + b*Sec[c + d*x]
],x]
```

```
[Out] (2*Sqrt[Cos[c + d*x]]*Sqrt[Cos[(c + d*x)/2]^2*Sec[c + d*x]]*(A + B*Sec[c +
d*x])*(I*A*(a + b)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(A + B)*Elli
pticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sqrt[((b + a*Cos[c + d
*x])*Sec[(c + d*x)/2]^2)/(a + b)] + A*(b + a*Cos[c + d*x])*Sqrt[Sec[(c + d*
x)/2]^2]*Tan[(c + d*x)/2]))/(a*d*(B + A*Cos[c + d*x])*Sqrt[Sec[c + d*x]]*Sq
rt[a + b*Sec[c + d*x]])
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 953 vs. $2(196) = 392$.
time = 21.04, size = 954, normalized size = 6.36

method	result
default	Expression too large to display
risch	$-\frac{iA\left(a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a\right)\sqrt{2}\,\sqrt{\left(e^{2i(dx+c)}+1\right)e^{-i(dx+c)}}}{ad\sqrt{\frac{a\,e^{2i(dx+c)}+2b\,e^{i(dx+c)}+a}{e^{2i(dx+c)}+1}}\,\left(e^{2i(dx+c)}+1\right)}-\frac{i\left(\frac{2B\left(b+\sqrt{-a^2+b^2}\right)\sqrt{\frac{\left(e^{i(dx+c)}+\frac{b+\sqrt{-a}}{a}\right.}{b+\sqrt{-a^2+}}}}{}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETU RNVERBOSE)

[Out] 2/d*(-1+cos(d*x+c))*(1+cos(d*x+c))*(-A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a+A*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a-A*cos(d*x+c)*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b+B*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*(1/(1+cos(d*x+c)))^(1/2)-A*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*(1/(1+cos(d*x+c)))^(1/2)+A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*(1/(1+cos(d*x+c)))^(1/2)-A*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*(1/(1+cos(d*x+c)))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b+B*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+c

```
os(d*x+c))/(a+b))^(1/2)*a*(1/(1+cos(d*x+c)))^(1/2)+A*cos(d*x+c)^2*((a-b)/(a
+b))^(1/2)*a-A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a+A*cos(d*x+c)*((a-b)/(a+b))^
(1/2)*b-A*((a-b)/(a+b))^(1/2)*b)*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x
+c))^(1/2)/a/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a),
 x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.66, size = 371, normalized size = 2.47

$$\frac{3i\sqrt{2}\,Aa^{\frac{3}{2}}\operatorname{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \operatorname{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right) - 3i\sqrt{2}\,Aa^{\frac{3}{2}}\operatorname{weierstrassZeta}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \operatorname{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)\right) + \sqrt{2}\,(-3i\,Ba+2i\,Ab)\sqrt{a}\operatorname{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \frac{3\,a\cos(dx+c)+3i\,a\sin(dx+c)+2\,b}{3\,a}\right) + \sqrt{2}\,(3i\,Ba-2i\,Ab)\sqrt{a}\operatorname{weierstrassPInverse}\left(-\frac{4\,(3\,a^2-4\,b^2)}{3\,a^2}, \frac{8\,(9\,a^2b-8\,b^3)}{27\,a^3}, \frac{3\,a\cos(dx+c)-3i\,a\sin(dx+c)+2\,b}{3\,a}\right)}{3\,a^2d}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(3*I*sqrt(2)*A*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(
9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a))
- 3*I*sqrt(2)*A*a^(3/2)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a
^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^
2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)) + s
qrt(2)*(-3*I*B*a + 2*I*A*b)*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2
)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x +
c) + 2*b)/a) + sqrt(2)*(3*I*B*a - 2*I*A*b)*sqrt(a)*weierstrassPInverse(-4/3
*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3
*I*a*sin(d*x + c) + 2*b)/a))/(a^2*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A + B\sec{(c + dx)})\sqrt{\cos{(c + dx)}}}{\sqrt{a + b\sec{(c + dx)}}}\, dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))*sqrt(cos(c + d*x))/sqrt(a + b*sec(c + d*x)),
x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/sqrt(b*sec(d*x + c) + a),
 x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(1/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(1/2), x
)
```

3.618 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=138

$$\frac{2A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1
/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^
(1/2)/(a+b*sec(d*x+c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1
/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*
x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
```

Rubi [A]
time = 0.34, antiderivative size = 138, normalized size of antiderivative = 1.00, number of steps used = 8, number of rules used = 8, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.229$, Rules used = {3034, 4121, 3943, 2742, 2740, 3944, 2886, 2884}

$$\frac{2A\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2B\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (2*A*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)])/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*B*Sqrt[(b + a*Cos[
c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(d*Sqrt[Cos[c
 + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4121

```
Int[(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (
A_)))/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)], x_Symbol] :> Dist[A, Int[
Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[B/d, Int[(d*Cs
c[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(}{\sqrt{a+b\sec(c+dx)}}$$

$$= \left(A\sqrt{\cos(c+dx)}\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}}{\sqrt{a+b\sec(c+dx)}}\,dx +$$

$$= \frac{\left(A\sqrt{b+a\cos(c+dx)}\right)\int \frac{1}{\sqrt{b+a\cos(c+dx)}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{\left(B\sqrt{b\right.}}{}$$

$$= \frac{\left(A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\right)\int \frac{1}{\sqrt{\frac{b}{a+b}+\frac{a\cos(c+dx)}{a+b}}}\,dx}{\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + -$$

$$= \frac{2A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}} + \frac{2B\sqrt{\frac{b+a\cos(c}{a+b}}}{d\sqrt{\cos(c+dx}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 26.50, size = 9363, normalized size = 67.85

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]
),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 20.15, size = 273, normalized size = 1.98

method	result
default	$\frac{2(-1+\cos(dx+c))\left(A\,\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)-B\,\mathrm{EllipticF}\left(\frac{(-1+\cos(dx+c))\sqrt{\frac{a-b}{a+b}}}{\sin(dx+c)},\sqrt{-\frac{a+b}{a-b}}\right)+2B\right.}{d\sqrt{\frac{a-b}{a+b}}\,(b+a\cos(dx+c))\left(\frac{}{1+c}\right.}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/d*(-1+cos(d*x+c))*(A*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))-B*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))+2*B*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2)))*cos(d*x+c)^(1/2)*((
b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(3/2)/sin(d*x+
c)^2
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F**]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec{(c + dx)}}{\sqrt{a + b\sec{(c + dx)}}\,\sqrt{\cos{(c + dx)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(sqrt(a + b*sec(c + d*x))*sqrt(cos(c + d*x)))
, x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*sqrt(cos(d*x + c))
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.01

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\sqrt{\cos\left(c+d\,x\right)}\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(1/2)), x
)
```

3.619 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=256

$$\frac{B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{B\sqrt{\cos(c\cdots}}{}$$

```
[Out] B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2
*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/d/cos(d*x+c)^(1
/2)/(a+b*sec(d*x+c))^(1/2)+(2*A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2
*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*
cos(d*x+c))/(a+b))^(1/2)/b/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+B*sin(
d*x+c)*(a+b*sec(d*x+c))^(1/2)/b/d/cos(d*x+c)^(1/2)-B*(cos(1/2*d*x+1/2*c)^2)
^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1
/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b/d/((b+a*cos(d*x+c))/(a+b))^(
1/2)
```

Rubi [A]
time = 0.57, antiderivative size = 256, normalized size of antiderivative = 1.00, number of steps used = 13, number of rules used = 13, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.371$, Rules used = {3034, 4118, 4194, 3944, 2886, 2884, 3947, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{(2Ab-aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{B\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{bd\sqrt{\cos(c+dx)}}+\frac{B\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{B\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{bd\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] (B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b - a*B)*Sqrt[(b
+ a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b*d*
Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (B*Sqrt[Cos[c + d*x]]*Ellipt
icE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(b*d*Sqrt[(b + a*
Cos[c + d*x])/(a + b)]) + (B*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(b*d*Sq
rt[Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
```

```
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3947

```
Int[1/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_
.) + (a_)]), x_Symbol] :> Dist[1/a, Int[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc
[e + f*x]], x], x] - Dist[b/(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[
e + f*x]], x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4118

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d^
2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 2)/(b*f*
(m + n))), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e +
 f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m +
n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n
, 0] &&  !IGtQ[m, 1]
```

Rule 4194

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.))/(Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[C/d^
2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Csc[e + f*x]], x], x] + Dist[A, In
t[1/(Sqrt[d*Csc[e + f*x]]*Sqrt[a + b*Csc[e + f*x]]), x], x] /; FreeQ[{a, b,
 d, e, f, A, C}, x] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx=\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int\frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+d}{\sqrt{a+b\sec(c+dx)}}$$

$$=\frac{B\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}+\frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+d\right.}{}$$

$$=\frac{B\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}-\frac{\left(aB\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.}{}$$

$$=\frac{B\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}+\frac{1}{2}\left(B\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.$$

$$=\frac{B\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{bd\sqrt{\cos(c+dx)}}+\frac{\left(B\sqrt{b+a\cos(c+dx)}\right)\int}{2\sqrt{\cos(c+dx)}\,\sqrt{a}$$

$$=\frac{(2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{B\sqrt{a+}{}$$

$$=\frac{B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{(2Ab-aB)\sqrt{\frac{b+a}{}}}{bd\sqrt{\cos(c+}$$

Mathematica [C] Result contains complex when optimal does not.
time = 30.18, size = 37262, normalized size = 145.55

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]
),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 21.50, size = 689, normalized size = 2.69

method	result
default	$-\frac{(-1+\cos(dx+c))\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\left(B\cos(dx+c)\sin(dx+c)\sqrt{\frac{a-b}{a+b}}\left(\frac{1}{1+\cos(dx+c)}\right)^{\frac{3}{2}}a+B\sin(dx+c)\sqrt{\frac{a-b}{a+b}}\left(\frac{1}{1+\cos(dx+c)}\right)^{\frac{3}{2}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/d*(-1+cos(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(B*cos(d*x+c)*sin(
d*x+c)*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(3/2)*a+B*sin(d*x+c)*((a-b)/(
a+b))^(1/2)*(1/(1+cos(d*x+c)))^(3/2)*b+2*A*cos(d*x+c)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*b-4*A*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+
c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b-2*B*cos(d*x+c)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a+2*B*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a
+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a+B*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a-B*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*b)/b/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(
3/2)/sin(d*x+c)^2/cos(d*x+c)^(1/2)
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)
), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A + B\sec{(c + dx)}}{\sqrt{a + b\sec{(c + dx)}}\cos^{\frac{3}{2}}{(c + dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/(sqrt(a + b*sec(c + d*x))*cos(c + d*x)**(3/2)
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(3/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A + \frac{B}{\cos(c+d\,x)}}{{\cos\left(c + d\,x\right)}^{3/2}\,\sqrt{a + \frac{b}{\cos\left(c + d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(1/2)), x
)
```

3.620 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx$

Optimal. Leaf size=344

$$\frac{(4Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{(4aAb-3a^2B-4b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+\right.}{4b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 1/4*(4*A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(s
in(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b
/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-1/4*(4*A*a*b-3*B*a^2-4*B*b^2)*(c
os(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c)
,2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b^2/d/cos(d*x+c)
^(1/2)/(a+b*sec(d*x+c))^(1/2)+1/2*B*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/b/d/c
os(d*x+c)^(3/2)+1/4*(4*A*b-3*B*a)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/b^2/d/c
os(d*x+c)^(1/2)-1/4*(4*A*b-3*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+
1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/
2)*(a+b*sec(d*x+c))^(1/2)/b^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.81, antiderivative size = 344, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3034, 4118, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{(-3a^2B+4aAb-4b^2B)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{(4Ab-3aB)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4b^2d\sqrt{\cos(c+dx)}}-\frac{(4Ab-3aB)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(4Ab-aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{B\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{2bd\cos^{\frac{3}{2}}(c+dx)}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]),x]
```

```
[Out] ((4*A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(4*b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((4*a*A
*b - 3*a^2*B - 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c
 + d*x)/2, (2*a)/(a + b)])/(4*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d
*x]]) - ((4*A*b - 3*a*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a
 + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*b^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)
]) + (B*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*b*d*Cos[c + d*x]^(3/2)) +
 ((4*A*b - 3*a*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*b^2*d*Sqrt[Cos[
c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4118

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-B)*d^
2*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 2)/(b*f*
(m + n))), x] + Dist[d^2/(b*(m + n)), Int[(a + b*Csc[e + f*x])^m*(d*Csc[e +
 f*x])^(n - 2)*Simp[a*B*(n - 2) + B*b*(m + n - 1)*Csc[e + f*x] + (A*b*(m +
n) - a*B*(n - 1))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B,
 m}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && GtQ[n, 1] && NeQ[m + n
, 0] &&  !IGtQ[m, 1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
```

```
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx}{\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} + \frac{\left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx}\right.}{}$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab-3aB)\sqrt{a+b\sec(c+}}{4b^2d\sqrt{\cos(c+d}}$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab-3aB)\sqrt{a+b\sec(c+}}{4b^2d\sqrt{\cos(c+d}}$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab-3aB)\sqrt{a+b\sec(c+}}{4b^2d\sqrt{\cos(c+d}}$$

$$= \frac{B\sqrt{a+b\sec(c+dx)}\,\sin(c+dx)}{2bd\cos^{\frac{3}{2}}(c+dx)} + \frac{(4Ab-3aB)\sqrt{a+b\sec(c+}}{4b^2d\sqrt{\cos(c+d}}$$

$$= -\frac{(4aAb-3a^2B-4b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{(4Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{(4aAb-3a^2}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 31.89, size = 77909, normalized size = 226.48

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]
),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 20.50, size = 1568, normalized size = 4.56

method	result	size
default	Expression too large to display	1568

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 1/4/d*(-1+cos(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))*(4
*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)+8*A*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*cos(d*x+c)^2*a*b-4*A
*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a*b+4*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*b^2-8*A*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(
a-b),I/((a-b)/(a+b))^(1/2))*a*b-3*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*(1
/(1+cos(d*x+c)))^(1/2)+2*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d
*x+c)))^(1/2)-6*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^2+2*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d
*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c
),(-(a+b)/(a-b))^(1/2))*a*b-4*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(
1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/si
n(d*x+c),(-(a+b)/(a-b))^(1/2))*b^2+3*B*cos(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(
1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-3*B*cos(d*x+c)^2*sin(d*x+c)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b+6*B*cos(d*x+c)^2*sin(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+8*B*c
os(d*x+c)^2*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a
+b))^(1/2))*b^2-4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))
^(1/2)+4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)+3*
B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)-3*B*cos(d*x
+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)+2*B*cos(d*x+c)^2*((a
-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)-4*A*cos(d*x+c)*((a-b)/(a+b))^
(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)+B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/
```

```
(1+cos(d*x+c)))^(1/2)-2*B*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2)
/b^2/((a-b)/(a+b))^(1/2)/(b+a*cos(d*x+c))/(1/(1+cos(d*x+c)))^(1/2)/sin(d*x+
c)^3/cos(d*x+c)^(3/2)
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6439 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/(sqrt(b*sec(d*x + c) + a)*cos(d*x + c)^(5/2)
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^{5/2}\,\sqrt{a+\frac{b}{\cos\left(c+d\,x\right)}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(1/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(1/2)), x
)
```

3.621 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=423

$$-\frac{2(12a^2Ab+48Ab^3-5a^3B-40ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^4A+24a^2Ab^2-48Ab^4$$

```
[Out] 2*b*(A*b-B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1
/2)-2/15*(12*A*a^2*b+48*A*b^3-5*B*a^3-40*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2)
)*((b+a*cos(d*x+c))/(a+b))^(1/2)/a^4/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1
/2)+2/5*(A*a^2-6*A*b^2+5*B*a*b)*cos(d*x+c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c)
)^(1/2)/a^2/(a^2-b^2)/d-2/15*(9*A*a^2*b-24*A*b^3-5*B*a^3+20*B*a*b^2)*sin(d*
x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)/d+2/15*(9*A*a^4+
24*A*a^2*b^2-48*A*b^4-25*B*a^3*b+40*B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/c
os(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos
(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^4/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b
))^(1/2)
```

Rubi [A]
time = 0.91, antiderivative size = 423, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3034, 4115, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(a^2A+5abB-6Ab^2)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{5a^2d(a^2-b^2)}+\frac{2b(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{ad(a^2-b^2)\sqrt{a+b\sec(c+dx)}}-\frac{2(-5a^3B+9a^2Ab+20ab^2B-24Ab^3)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{15a^3d(a^2-b^2)}-\frac{2(-5a^3B+12a^2Ab-40ab^2B+48Ab^3)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^4d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^4A-25a^3bB+24a^2Ab^2+40ab^3B-48Ab^4)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^4d(a^2-b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(12*a^2*A*b + 48*A*b^3 - 5*a^3*B - 40*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x]
)/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(15*a^4*d*Sqrt[Cos[c + d*
x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(9*a^4*A + 24*a^2*A*b^2 - 48*A*b^4 - 25*
a^3*b*B + 40*a*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a +
b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^4*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]) + (2*b*(A*b - a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(a*(a^2 -
b^2)*d*Sqrt[a + b*Sec[c + d*x]]) - (2*(9*a^2*A*b - 24*A*b^3 - 5*a^3*B + 20*
a*b^2*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(15*a^3*
(a^2 - b^2)*d) + (2*(a^2*A - 6*A*b^2 + 5*a*b*B)*Cos[c + d*x]^(3/2)*Sqrt[a +
 b*Sec[c + d*x]]*Sin[c + d*x])/(5*a^2*(a^2 - b^2)*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(\right.}{}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A-6Ab^2+5abB)}{}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2(9a^2Ab-24Ab^3-5a^3}{}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2(9a^2Ab-24Ab^3-5a^3}{}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2(9a^2Ab-24Ab^3-5a^3}{}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{a+b\sec(c+dx)}} - \frac{2(9a^2Ab-24Ab^3-5a^3}{}$$

$$= -\frac{2(12a^2Ab+48Ab^3-5a^3B-40ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\right.}{15a^4d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 15.23, size = 533, normalized size = 1.26

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3
/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*((2*(-9*A*b + 5*a*B)*Sin[c + d*x])/(15*a^3) + (2*(A
*b^4*Sin[c + d*x] - a*b^3*B*Sin[c + d*x]))/(a^3*(a^2 - b^2)*(b + a*Cos[c +
```

```
d*x])) + (A*Sin[2*(c + d*x)])/(5*a^2)))/(d*Cos[c + d*x]^(3/2)*(a + b*Sec[c
+ d*x])^(3/2)) - (2*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])*Sec[c + d*x]^(3
/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(a + b)*(9*a^4*A + 24*a^2
*A*b^2 - 48*A*b^4 - 25*a^3*b*B + 40*a*b^3*B)*EllipticE[I*ArcSinh[Tan[(c + d
*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Se
c[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(-48*A*b^3 - 6*a^2*b*(2*A + 5*B) +
 a^3*(9*A + 5*B) + 4*a*b^2*(9*A + 10*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/
2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c
 + d*x)/2]^2)/(a + b)] - (9*a^4*A + 24*a^2*A*b^2 - 48*A*b^4 - 25*a^3*b*B +
40*a*b^3*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2
]))/(15*a^4*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2083 vs. $2(449) = 898$.
time = 37.36, size = 2084, normalized size = 4.93

method	result	size
default	Expression too large to display	2084

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/15/d*(-1+cos(d*x+c))*(1+cos(d*x+c))^2*(-40*B*cos(d*x+c)*((a-b)/(a+b))^(1/
2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)+20*B*(1/(1+cos(d*x+c)))^(1/2)*((a-b)/(a+b
))^(1/2)*a^2*b^2+3*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^4*(1/(1+cos(d*x+c))
)^(1/2)+6*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*(1/(1+cos(d*x+c)))^(1/2)-9
*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4+9*A*sin
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4-48*A*sin(d*x+
c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4+5*B*(1/(1+cos(d*x+
c)))^(1/2)*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4-9*A*cos(d*x+c)*((a-b)/(a+b)
)^(1/2)*a^4*(1/(1+cos(d*x+c)))^(1/2)+48*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^
4*(1/(1+cos(d*x+c)))^(1/2)-5*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*(1/(1+cos
(d*x+c)))^(1/2)-5*B*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c)))^(1/2)+40*B
*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)-48*A*((a-b)/(a+b))^(1/2
)*b^4*(1/(1+cos(d*x+c)))^(1/2)+5*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
-(a+b)/(a-b))^(1/2))*a^4-9*A*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c)))^(
1/2)-24*A*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)-12*A*sin(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-36*A*sin(d*x+c)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
```

```
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-48*A*sin(d*x+c)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b
)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+24*A*sin(d*x+c)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-6*A*cos(d*x+c)^3*((a-b
)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c)))^(1/2)+6*A*(1/(1+cos(d*x+c)))^(1/2)*
cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b+30*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1
+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+40*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-25*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^3*b+40*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a*b^3+24*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2*
(1/(1+cos(d*x+c)))^(1/2)-20*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+
cos(d*x+c)))^(1/2)+3*A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x
+c)))^(1/2)-6*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))
^(1/2)+5*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c)))^(1/2)+
24*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)-20*B*c
os(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)+6*A*cos(d*
x+c)*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c)))^(1/2)-18*A*cos(d*x+c)*((a
-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)+20*B*cos(d*x+c)*((a-b)/(a
+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c
))/cos(d*x+c))^(1/2)*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)/a^4/(b+a*
cos(d*x+c))/(a-b)/sin(d*x+c)^3
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 2.01, size = 928, normalized size = 2.19

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/45*(6*(5*B*a^5*b - 9*A*a^4*b^2 - 20*B*a^3*b^3 + 24*A*a^2*b^4 + 3*(A*a^6 -
 A*a^4*b^2)*cos(d*x + c)^2 + (5*B*a^6 - 6*A*a^5*b - 5*B*a^4*b^2 + 6*A*a^3*b
^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c)
)*sin(d*x + c) - (sqrt(2)*(15*I*B*a^6 - 27*I*A*a^5*b + 80*I*B*a^4*b^2 - 84*
I*A*a^3*b^3 - 80*I*B*a^2*b^4 + 96*I*A*a*b^5)*cos(d*x + c) + sqrt(2)*(15*I*B
*a^5*b - 27*I*A*a^4*b^2 + 80*I*B*a^3*b^3 - 84*I*A*a^2*b^4 - 80*I*B*a*b^5 +
96*I*A*b^6))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*
a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) -
(sqrt(2)*(-15*I*B*a^6 + 27*I*A*a^5*b - 80*I*B*a^4*b^2 + 84*I*A*a^3*b^3 + 80
*I*B*a^2*b^4 - 96*I*A*a*b^5)*cos(d*x + c) + sqrt(2)*(-15*I*B*a^5*b + 27*I*A
*a^4*b^2 - 80*I*B*a^3*b^3 + 84*I*A*a^2*b^4 + 80*I*B*a*b^5 - 96*I*A*b^6))*sq
rt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/
a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*(sqrt(2)*(9*I
*A*a^6 - 25*I*B*a^5*b + 24*I*A*a^4*b^2 + 40*I*B*a^3*b^3 - 48*I*A*a^2*b^4)*c
os(d*x + c) + sqrt(2)*(9*I*A*a^5*b - 25*I*B*a^4*b^2 + 24*I*A*a^3*b^3 + 40*I
*B*a^2*b^4 - 48*I*A*a*b^5))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
2*b)/a)) + 3*(sqrt(2)*(-9*I*A*a^6 + 25*I*B*a^5*b - 24*I*A*a^4*b^2 - 40*I*B*
a^3*b^3 + 48*I*A*a^2*b^4)*cos(d*x + c) + sqrt(2)*(-9*I*A*a^5*b + 25*I*B*a^4
*b^2 - 24*I*A*a^3*b^3 - 40*I*B*a^2*b^4 + 48*I*A*a*b^5))*sqrt(a)*weierstrass
Zeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInve
rse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
+ c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^8 - a^6*b^2)*d*cos(d*x + c) + (a^
7*b - a^5*b^3)*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{5/2}\,\left(A+\frac{B}{\cos(c+d\,x)}\right)}{\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2), x
)
```

3.622 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=326

$$\frac{2(a^2A+8Ab^2-6abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{2(5a^2Ab-8Ab^3-3a^3B+6ab^2B)\,\sqrt{\cos(c+}}{3a^3\left(a^2-b^2\right)d\sqrt{}}$$

```
[Out] 2*b*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(1
/2)+2/3*(A*a^2+8*A*b^2-6*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/
2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c)
)/(a+b))^(1/2)/a^3/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*(A*a^2-4*A
*b^2+3*B*a*b)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b
^2)/d-2/3*(5*A*a^2*b-8*A*b^3-3*B*a^3+6*B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*
cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)/d/((b+a*cos(d*x+c))/(
a+b))^(1/2)
```

Rubi [A]

time = 0.65, antiderivative size = 326, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3034, 4115, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2(a^2A+3abB-4Ab^2)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{3a^2d\left(a^2-b^2\right)}+\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{ad\left(a^2-b^2\right)\sqrt{a+b\sec(c+dx)}}+\frac{2(a^2A-6abB+8Ab^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{2(-3a^3B+5a^2Ab+6ab^2B-8Ab^3)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (2*(a^2*A + 8*A*b^2 - 6*a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(3*a^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c
+ d*x]]) - (2*(5*a^2*A*b - 8*A*b^3 - 3*a^3*B + 6*a*b^2*B)*Sqrt[Cos[c + d*x]
]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a
^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*b*(A*b - a*B)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(a
^2*A - 4*A*b^2 + 3*a*b*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]*Sin[c
 + d*x])/(3*a^2*(a^2 - b^2)*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
```

```
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\text{se}}\right.}{}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A-4Ab^2+3abB}{}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A-4Ab^2+3abB}{}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A-4Ab^2+3abB}{}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{a\,(a^2-b^2)\,d\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A-4Ab^2+3abB}{}$$

$$= \frac{2(a^2A+8Ab^2-6abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3a^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} -$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.41, size = 417, normalized size = 1.28

$$\frac{2(b+a\cos(c+dx))\left(a(b(a^2A-4Ab^2+3abB)+aA(a^2-b^2)\cos(c+dx))\sec^{\frac{3}{2}}(c+dx)\sin(c+dx)+\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(i(a+b)\left(-5a^2Ab+8Ab^3+3a^3B-6ab^2B\right)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-ia(a^2-ab-2b^2)(-4Ab+a(A+3B))F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+\left(-5a^2Ab+8Ab^3+3a^3B-6ab^2B\right)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)\right)}{3a^3\left(a^2-b^2\right)d\cos^{\frac{3}{2}}(c+dx)\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*(a*(b*(a^2*A - 4*A*b^2 + 3*a*b*B) + a*A*(a^2 - b^2)
*Cos[c + d*x])*Sec[c + d*x]^(3/2)*Sin[c + d*x] + (Cos[(c + d*x)/2]^2*Sec[c
+ d*x])^(3/2)*(I*(a + b)*(-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*Ellip
ticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt
[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a^2 - a*b - 2*b^
```

```
2)*(-4*A*b + a*(A + 3*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(
a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(
a + b)] + (-5*a^2*A*b + 8*A*b^3 + 3*a^3*B - 6*a*b^2*B)*(b + a*Cos[c + d*x])
*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2])))/(3*a^3*(a^2 - b^2)*d*Cos[c
+ d*x]^(3/2)*Sec[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1459 vs. $2(358) = 716$.
time = 35.34, size = 1460, normalized size = 4.48

method	result	size
default	Expression too large to display	1460

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/3/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*
(1+cos(d*x+c))^2*(8*A*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)+A*si
n(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+8*A*sin(d*x+
c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3+A*((a-b)/(a+b))^(1
/2)*cos(d*x+c)^3*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-3*B*((a-b)/(a+b))^(1/2)*cos
(d*x+c)*a^3*(1/(1+cos(d*x+c)))^(1/2)-3*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+co
s(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*
x+c),(-(a+b)/(a-b))^(1/2))*a^3-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(1+c
os(d*x+c)))^(1/2)-8*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^3*(1/(1+cos(d*x+c)))
^(1/2)-A*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+4*A*((a-b)/(a+b
))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-3*B*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1
+cos(d*x+c)))^(1/2)-6*B*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+
3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*(1/(1+cos(d*x+c)))^(1/2)-4*A*cos(d
*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-4*A*((a-b)/(a+b)
)^(1/2)*cos(d*x+c)^2*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+3*B*((a-b)/(a+b))^(1/2)
*cos(d*x+c)^2*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+4*A*((a-b)/(a+b))^(1/2)*cos(d*
x+c)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+6*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^
2*(1/(1+cos(d*x+c)))^(1/2)+6*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/
(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*a^2*b+8*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a*b^2-5*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^2*b-6*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a*b^2-6*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
```

```
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a^2*b+A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*(1/(1+cos(d*x+c)))^(1/2)+3*B*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a^3)*((a-b)/(a
+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)/a^3/(b+a*cos(d*x+c))/(a-b)/sin(d*x+c)^3
```

Maxima [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.80, size = 804, normalized size = 2.47

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] 1/9*(6*(A*a^4*b + 3*B*a^3*b^2 - 4*A*a^2*b^3 + (A*a^5 - A*a^3*b^2)*cos(d*x +
 c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c
) - (sqrt(2)*(3*I*A*a^5 - 15*I*B*a^4*b + 16*I*A*a^3*b^2 + 12*I*B*a^2*b^3 -
16*I*A*a*b^4)*cos(d*x + c) + sqrt(2)*(3*I*A*a^4*b - 15*I*B*a^3*b^2 + 16*I*A
*a^2*b^3 + 12*I*B*a*b^4 - 16*I*A*b^5))*sqrt(a)*weierstrassPInverse(-4/3*(3*
a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a
*sin(d*x + c) + 2*b)/a) - (sqrt(2)*(-3*I*A*a^5 + 15*I*B*a^4*b - 16*I*A*a^3*
b^2 - 12*I*B*a^2*b^3 + 16*I*A*a*b^4)*cos(d*x + c) + sqrt(2)*(-3*I*A*a^4*b +
 15*I*B*a^3*b^2 - 16*I*A*a^2*b^3 - 12*I*B*a*b^4 + 16*I*A*b^5))*sqrt(a)*weie
rstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(
3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*(sqrt(2)*(3*I*B*a^5 - 5
*I*A*a^4*b - 6*I*B*a^3*b^2 + 8*I*A*a^2*b^3)*cos(d*x + c) + sqrt(2)*(3*I*B*a
^4*b - 5*I*A*a^3*b^2 - 6*I*B*a^2*b^3 + 8*I*A*a*b^4))*sqrt(a)*weierstrassZet
a(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse
(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c
) + 3*I*a*sin(d*x + c) + 2*b)/a)) + 3*(sqrt(2)*(-3*I*B*a^5 + 5*I*A*a^4*b +
6*I*B*a^3*b^2 - 8*I*A*a^2*b^3)*cos(d*x + c) + sqrt(2)*(-3*I*B*a^4*b + 5*I*A
*a^3*b^2 + 6*I*B*a^2*b^3 - 8*I*A*a*b^4))*sqrt(a)*weierstrassZeta(-4/3*(3*a^
2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2
 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*si
```

```
n(d*x + c) + 2*b)/a)))/((a^7 - a^5*b^2)*d*cos(d*x + c) + (a^6*b - a^4*b^3)*
d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\cos\left(c+d\,x\right)^{3/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2),x)
```

```
[Out] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2), x
)
```

3.623 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=235

$$-\frac{2(2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(a^2A-2Ab^2+abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\right.}{a^2(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+}{a+b}}}$$

```
[Out] 2*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1
/2)-2*(2*A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF
(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)
/a^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*(A*a^2-2*A*b^2+B*a*b)*(cos
(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^
(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2
)/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.46, antiderivative size = 235, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3034, 4115, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2b(Ab-aB)\sin(c+dx)}{ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2\left(a^2A+abB-2Ab^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}-\frac{2(2Ab-aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3/2),x]
```

```
[Out] (-2*(2*A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
 (2*a)/(a + b)])/(a^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(
a^2*A - 2*A*b^2 + a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a
 + b)]*Sqrt[a + b*Sec[c + d*x]])/(a^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x
])/(a + b)]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(a*(a^2 - b^2)*d*Sqrt[Cos[c +
 d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
```

```
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
```

```
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c+}\right.}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{\left(\left((2Ab-aB\right.\right.}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{\left(\left((2Ab-aB\right.\right.}{}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{\left(\left((2Ab-aB\right.\right.}{}$$

$$= -\frac{2(2Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{a^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2(a^2A}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 9.45, size = 365, normalized size = 1.55

$$\frac{2(b+a\cos(c+dx))(A+B\sec(c+dx))\left(-ab(-Ab+aB)\sin(c+dx)+\frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(i(a+b)\left(a^2A-2Ab^2+abB\right)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-ia(a+b)(-2Ab+a(A+B))F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+\left(a^2A-2Ab^2+abB\right)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{\sec^{\frac{3}{2}}(c+dx)}\right)}{a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(B+A\cos(c+dx))(a+b\sec(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(3
/2),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*(A + B*Sec[c + d*x])*(-(a*b*(-(A*b) + a*B)*Sin[c +
d*x]) + ((Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*(I*(a + b)*(a^2*A - 2*A*b^
2 + a*b*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c
+ d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(
a + b)*(-2*A*b + a*(A + B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)
/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)
/(a + b)] + (a^2*A - 2*A*b^2 + a*b*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2
]^2)^(3/2)*Tan[(c + d*x)/2]))/Sec[c + d*x]^(3/2)))/(a^2*(a^2 - b^2)*d*Sqrt[
Cos[c + d*x]]*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 888 vs. $2(277) = 554$.
time = 33.05, size = 889, normalized size = 3.78

method	result
default	$\frac{2\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\,(-1+\cos(dx+c))(1+\cos(dx+c))^2\left(A\left(\cos^2(dx+c)\right)\sqrt{\frac{a-b}{a+b}}\,a^2\sqrt{\frac{1}{1+\cos(dx+c)}}+A\left(\cos^2(dx+c)\right)\sqrt{\frac{a-b}{a+b}}\,ab\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1+cos(d*x+c))^2*(A
*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)+A*cos(d*x+c)
^2*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)-A*cos(d*x+c)*((a-b)/(a+
b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)+2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b
^2*(1/(1+cos(d*x+c)))^(1/2)+A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*sin(d*x+c)*a^2-2*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*s
in(d*x+c)*b^2-A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c
)*a^2-2*A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a*b-
B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)+B*sin(d*x+c)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+B*sin(d*x+c)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2-A*((a-b)/(a+b))^(1/2)*a*b*(1/
```

```
(1+cos(d*x+c)))^(1/2)-2*A*((a-b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)+
B*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^(1/2)*((a-b)
/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)/a^2/(b+a*cos(d*x+c))/(a-b)/sin(d*x+c
)^3
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.64, size = 699, normalized size = 2.97

[illegible]

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] -1/3*(6*(B*a^3*b - A*a^2*b^2)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(
cos(d*x + c))*sin(d*x + c) + (sqrt(2)*(3*I*B*a^4 - 5*I*A*a^3*b - 2*I*B*a^2*
b^2 + 4*I*A*a*b^3)*cos(d*x + c) + sqrt(2)*(3*I*B*a^3*b - 5*I*A*a^2*b^2 - 2*
I*B*a*b^3 + 4*I*A*b^4))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^
2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
 2*b)/a) + (sqrt(2)*(-3*I*B*a^4 + 5*I*A*a^3*b + 2*I*B*a^2*b^2 - 4*I*A*a*b^3
)*cos(d*x + c) + sqrt(2)*(-3*I*B*a^3*b + 5*I*A*a^2*b^2 + 2*I*B*a*b^3 - 4*I*
A*b^4))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b
 - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*(sq
rt(2)*(I*A*a^4 + I*B*a^3*b - 2*I*A*a^2*b^2)*cos(d*x + c) + sqrt(2)*(I*A*a^3
*b + I*B*a^2*b^2 - 2*I*A*a*b^3))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^
2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2
)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x +
c) + 2*b)/a)) - 3*(sqrt(2)*(-I*A*a^4 - I*B*a^3*b + 2*I*A*a^2*b^2)*cos(d*x +
 c) + sqrt(2)*(-I*A*a^3*b - I*B*a^2*b^2 + 2*I*A*a*b^3))*sqrt(a)*weierstrass
Zeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInve
rse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
+ c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^6 - a^4*b^2)*d*cos(d*x + c) + (a^
5*b - a^3*b^3)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{(A+B\sec(c+dx))\sqrt{\cos(c+dx)}}{(a+b\sec(c+dx))^{\frac{3}{2}}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(3/2),x)
[Out] Integral((A + B*sec(c + d*x))*sqrt(cos(c + d*x))/(a + b*sec(c + d*x))**(3/2
), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(3/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos(c+d\,x)}\,\left(A+\frac{B}{\cos(c+d\,x)}\right)}{{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2),x)
[Out] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(3/2), x
)
```

3.624 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=215

$$\frac{2A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+dx)}}{a\left(a^2-b^2\right)d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

```
[Out] -2*(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)
+2*A*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+
1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a/d/cos(d*x+
c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(
d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b))^
(1/2)
```

Rubi [A]
time = 0.42, antiderivative size = 215, normalized size of antiderivative = 1.00, number of steps used = 9, number of rules used = 9, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.257$, Rules used = {3034, 4112, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{2(Ab-aB)\sin(c+dx)}{d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2A\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*A*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b
)])/(a*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(A*b - a*B)*Sqrt
[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x
]])/(a*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*(A*b - a*B)*S
in[c + d*x])/((a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
```

```
0] && !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] && !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4112

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-d)*(A
*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*
Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) +
 d*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x
```

```
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec \cdots}{(a+b\sec(c+dx))^{\cdots}}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{\left(2\sqrt{\cos(c \cdots}{\cdots}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\left(A\sqrt{\cos(c \cdots}{\cdots}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\left(A\sqrt{b+a \cdots}{a\sqrt{\cdots}}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\left(A\sqrt{\frac{b+\cdots}{\cdots}}\right.\cdots}{\cdots}$$

$$= \frac{2A\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{ad\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2(Ab-aB)\sqrt{\mathrm{co}\cdots}}{\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 6.56, size = 328, normalized size = 1.53

$$\frac{2(b+a\cos(c+dx))\left(\frac{(-Ab+aB)\sin(c+dx)}{a^2-b^2}+\frac{\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-i(a+b)(-Ab+aB)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-ia(a+b)(A-B)F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+(Ab-aB)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^3-ab^2)\sec^{\frac{3}{2}}(c+dx)}\right)}{d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] (2*(b + a*Cos[c + d*x])*(((-(A*b) + a*B)*Sin[c + d*x])/(a^2 - b^2) + ((Cos[
(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*((-I)*(a + b)*(-(A*b) + a*B)*EllipticE[I
*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b +
 a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a + b)*(A - B)*Ellipti
cF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[(
(b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + (A*b - a*B)*(b + a*Cos[
c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/((a^3 - a*b^2)*Sec[
c + d*x]^(3/2))))/(d*Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 565 vs. $2(257) = 514$.
time = 33.62, size = 566, normalized size = 2.63

method	result
default	$-\frac{2(-1+\cos(dx+c))(1+\cos(dx+c))^2\left(A\cos(dx+c)\sqrt{\frac{a-b}{a+b}}\,b\sqrt{\frac{1}{1+\cos(dx+c)}}-A\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))(a+b)}}\,\text{EllipticF}\left(\frac{(-1+\cos(dx}{\sin(}\right.\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/d*(-1+cos(d*x+c))*(1+cos(d*x+c))^2*(A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b*(
1/(1+cos(d*x+c)))^(1/2)-A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*sin(d*x+c)*a-A*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c
)*b-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a-B*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*a+B*sin(d*x+c)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a-A*((a-b)/(a+b))^(1/2)*b*(1/(1+c
os(d*x+c)))^(1/2)+B*((a-b)/(a+b))^(1/2)*a*(1/(1+cos(d*x+c)))^(1/2))*cos(d*x
+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*((a-b)/(a+b))^(1/2)*(1/(1+cos
(d*x+c)))^(1/2)/a/(b+a*cos(d*x+c))/(a-b)/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c
))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.13, size = 626, normalized size = 2.91

$$\frac{6\,(Ba^3-Aa^2b)\sqrt{\frac{a\cos(dx+c)+b}{\cos(dx+c)}}\sqrt{\cos(dx+c)}\sin(dx+c)-\left(\sqrt{2}\,(3i\,Aa^3-i\,Ba^2b-2i\,Aab^2)\cos(dx+c)+\sqrt{2}\,(3i\,Aa^2b-i\,Bab^2-2i\,Ab^3)\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)-\left(\sqrt{2}\,(-3i\,Aa^3+i\,Ba^2b+2i\,Aab^2)\cos(dx+c)+\sqrt{2}\,(-3i\,Aa^2b+i\,Bab^2+2i\,Ab^3)\right)\sqrt{a}\,\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)+3\left(\sqrt{2}\,(-i\,Ba^3+i\,Aa^2b)\cos(dx+c)+\sqrt{2}\,(-i\,Ba^2b+i\,Aab^2)\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)+3i\,a\sin(dx+c)+2b}{3a}\right)\right)+3\left(\sqrt{2}\,(i\,Ba^3-i\,Aa^2b)\cos(dx+c)+\sqrt{2}\,(i\,Ba^2b-i\,Aab^2)\right)\sqrt{a}\,\mathrm{weierstrassZeta}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\mathrm{weierstrassPInverse}\left(-\frac{4\,(3a^2-4b^2)}{3a^2},\frac{8\,(9a^2b-8b^3)}{27a^3},\frac{3a\cos(dx+c)-3i\,a\sin(dx+c)+2b}{3a}\right)\right)}{3\left((a^5-a^3b^2)d\cos(dx+c)+(a^4b-a^2b^3)d\right)}$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/3*(6*(B*a^3 - A*a^2*b)*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(cos(d
*x + c))*sin(d*x + c) - (sqrt(2)*(3*I*A*a^3 - I*B*a^2*b - 2*I*A*a*b^2)*cos(
d*x + c) + sqrt(2)*(3*I*A*a^2*b - I*B*a*b^2 - 2*I*A*b^3))*sqrt(a)*weierstra
ssPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*c
os(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) - (sqrt(2)*(-3*I*A*a^3 + I*B*a^2
*b + 2*I*A*a*b^2)*cos(d*x + c) + sqrt(2)*(-3*I*A*a^2*b + I*B*a*b^2 + 2*I*A*
b^3))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b -
 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*(sqrt
(2)*(-I*B*a^3 + I*A*a^2*b)*cos(d*x + c) + sqrt(2)*(-I*B*a^2*b + I*A*a*b^2))
*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a
^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^
3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) + 3*(sqrt(2)*(I*B*
a^3 - I*A*a^2*b)*cos(d*x + c) + sqrt(2)*(I*B*a^2*b - I*A*a*b^2))*sqrt(a)*we
ierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierst
rassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a
*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^5 - a^3*b^2)*d*cos(d*x +
 c) + (a^4*b - a^2*b^3)*d)
```

Sympy [F]
time = 0.00, size = 0, normalized size = 0.00

$$\int \frac{A+B\sec(c+dx)}{(a+b\sec(c+dx))^{\frac{3}{2}}\sqrt{\cos(c+dx)}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(3/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Integral((A + B*sec(c + d*x))/((a + b*sec(c + d*x))**(3/2)*sqrt(cos(c + d*x
))), x)
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*sqrt(cos(d*x + c
))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\sqrt{\cos\left(c+d\,x\right)}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(3/2)), x
)
```

3.625 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=220

$$\frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)\sqrt{a+b\sec(c+}}{b\,(a^2-b^2)\,d\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}$$

```
[Out] 2*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1
/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*
d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b/d/co
s(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-2*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1
/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2)
)*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b/(a^2-b^2)/d/((b+a*cos(d*x+c))/(
a+b))^(1/2)
```

Rubi [A]

time = 0.50, antiderivative size = 220, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3034, 4114, 4193, 3944, 2886, 2884, 21, 3941, 2734, 2732}

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\,(a^2-b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{2(Ab-aB)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\,(a^2-b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2B\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a
 + b)])/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(A*b - a*B)*
Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c +
 d*x]])/(b*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a*(A*b -
a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d
*x]])
```

Rule 21

```
Int[(u_.)*((a_) + (b_.)*(v_))^(m_.)*((c_) + (d_.)*(v_))^(n_.), x_Symbol] :>
 Dist[(b/d)^m, Int[u*(c + d*v)^(m + n), x], x] /; FreeQ[{a, b, c, d, n}, x]
 && EqQ[b*c - a*d, 0] && IntegerQ[m] && ( !IntegerQ[n] || SimplerQ[c + d*x,
 a + b*x])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
```

```
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+d}{(a+b\sec(c+dx))^{3/2}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+d}\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+d}\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\left((-Ab+aB)\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{\left(B\sqrt{\frac{b+a\cos}{a}}\right.}{b\sqrt{}}$$

$$= \frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2a(Ab}{b\,(a^2-b^2)\,d\sqrt{\cos}}$$

$$= \frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{2(Ab-aB)\sqrt{\cos}}{b}$$

Mathematica [C] Result contains complex when optimal does not.
time = 30.53, size = 37303, normalized size = 169.56

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 21.44, size = 840, normalized size = 3.82

method	result
default	$\frac{2(-1+\cos(dx+c))\sqrt{\frac{b+a\cos(dx+c)}{\cos(dx+c)}}\,(1+\cos(dx+c))^2\left(A\cos(dx+c)\sqrt{\frac{a-b}{a+b}}\,b\sqrt{\frac{1}{1+\cos(dx+c)}}+A\sin(dx+c)\sqrt{\frac{b+a\cos(dx+c)}{(1+\cos(dx+c))}}\right.}{}$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/d*(-1+cos(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(A
*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b*(1/(1+cos(d*x+c)))^(1/2)+A*sin(d*x+c)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b-A*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*sin(d*x+c)*b-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*(1/
(1+cos(d*x+c)))^(1/2)*a+2*B*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a
+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*x+c))/
(1+cos(d*x+c))/(a+b))^(1/2)*a+2*B*sin(d*x+c)*EllipticPi((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b-2*B*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*sin(d*x+c)*a-B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*b+B*sin(d*x+c)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*a-A*((a-b)/(a+b))^(1/2)*b*(1/(1+cos(d*x+c)))^(1/2)+B*((a-b)/(a+b))^(1
/2)*a*(1/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^(1/2)*((a-b)/(a+b))^(1/2)*(1/(1+
cos(d*x+c)))^(1/2)/b/(b+a*cos(d*x+c))/(a-b)/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/
2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3436 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(3/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^{3/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(3/2)), x
)
```

3.626 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=371

$$\frac{B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{(2Ab-3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{(2aAb-$$

```
[Out] 2*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1
/2)+B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x
+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b/d/cos(d*x
+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+(2*A*b-3*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)
/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2)
)*((b+a*cos(d*x+c))/(a+b))^(1/2)/b^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1
/2)-(2*A*a*b-3*B*a^2+B*b^2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/b^2/(a^2-b^2)
/d/cos(d*x+c)^(1/2)+(2*A*a*b-3*B*a^2+B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(
d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/((b+a*cos(d*x+c))/(a+b)
)^(1/2)
```

Rubi [A]
time = 0.91, antiderivative size = 371, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3034, 4114, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2a(Ab-aB)\sin(c+dx)}{bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}-\frac{\left(-3a^2B+2aAb+b^2B\right)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{b^2d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}}+\frac{\left(-3a^2B+2aAb+b^2B\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\left(a^2-b^2\right)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(2Ab-3aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{B\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] (B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)]
)/(b*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*A*b - 3*a*B)*Sqrt
[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(
b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + ((2*a*A*b - 3*a^2*B +
b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*
Sec[c + d*x]])/(b^2*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*
a*(A*b - a*B)*Sin[c + d*x])/(b*(a^2 - b^2)*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*
Sec[c + d*x]]) - ((2*a*A*b - 3*a^2*B + b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[
c + d*x])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
```

```
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
```

```
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+}{(a+b\sec(c+dx))^{3/2}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+d}\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{(2aAb-3a^2B}{b^2}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{(2aAb-3a^2B}{b^2}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{(2aAb-3a^2B}{b^2}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{(2aAb-3a^2B}{b^2}$$

$$= \frac{(2Ab-3aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{}{b\,(a^2-}$$

$$= \frac{B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{bd\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{(2Ab-3aB)\sqrt{\frac{b+}{}}}{b^2d\sqrt{\cos(c}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 32.48, size = 95694, normalized size = 257.94

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 21.50, size = 1441, normalized size = 3.88

method	result	size
default	Expression too large to display	1441

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/d*(-1+cos(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*(
2*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)-4*A*cos(d
*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(
1/2))*a*b-4*A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))
^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b
),I/((a-b)/(a+b))^(1/2))*b^2+4*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c)
)/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+2*A*cos(d*x+c)*sin(d*x+c)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*((b+a*cos(d
*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*b^2-2*A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b-3*B*cos(d*x+c)^2*((a-b)/(a+b))^
(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)-B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b*(
1/(1+cos(d*x+c)))^(1/2)+6*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2+6*B*cos(d*x+c)*sin(d*x+c)*((b+a*
cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b-6*B*cos(d*x+c
)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+
b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a^2-4*B*cos(
d*x+c)*sin(d*x+c)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*a*b+3*B
*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
2-B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*b^2-2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b*(1/(1+cos(d*x+c)))^(1/2)+3*B*co
s(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*(1/(1+cos(d*x+c)))^(1/2)-B*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*b^2*(1/(1+cos(d*x+c)))^(1/2)+B*((a-b)/(a+b))^(1/2)*a*b*(1/(
1+cos(d*x+c)))^(1/2)+B*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*b^2)*((
a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)/b^2/(b+a*cos(d*x+c))/cos(d*x+c)^
(1/2)/(a-b)/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/
2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(5/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^{5/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{3/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(3/2)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(3/2)), x
)
```

3.627 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx$

Optimal. Leaf size=487

$$\frac{(4Ab-5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{(12aAb-15a^2B-4b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\right.}{4b^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

```
[Out] 2*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(1
/2)+1/4*(4*A*b-5*B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ellip
ticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(
1/2)/b^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-1/4*(12*A*a*b-15*B*a^2-4
*B*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*
d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b^3/d/
cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-1/2*(4*A*a*b-5*B*a^2+B*b^2)*sin(d*x
+c)*(a+b*sec(d*x+c))^(1/2)/b^2/(a^2-b^2)/d/cos(d*x+c)^(3/2)+1/4*(12*A*a^2*b
-4*A*b^3-15*B*a^3+7*B*a*b^2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/b^3/(a^2-b^2
)/d/cos(d*x+c)^(1/2)-1/4*(12*A*a^2*b-4*A*b^3-15*B*a^3+7*B*a*b^2)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*
(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b^3/(a^2-b^2)/d/((
b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]

time = 1.20, antiderivative size = 487, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3034, 4114, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{(-5a^2B+4aAb+b^2B)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{2b^2d(a^2-b^2)\cos^{\frac{3}{2}}(c+dx)}+\frac{2a(Ab-aB)\sin(c+dx)}{bd(a^2-b^2)\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}-\frac{(-15a^2B+12aAb-4b^2B)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{4b^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{(-15a^3B+12a^2Ab+7ab^2B-4Ab^3)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{4b^3d(a^2-b^2)\sqrt{\cos(c+dx)}}-\frac{(-15a^3B+12a^2Ab+7ab^2B-4Ab^3)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{4b^3d(a^2-b^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(4Ab-5aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/2)),x]
```

```
[Out] ((4*A*b - 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2,
(2*a)/(a + b)])/(4*b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - ((1
2*a*A*b - 15*a^2*B - 4*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi
[2, (c + d*x)/2, (2*a)/(a + b)])/(4*b^3*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec
[c + d*x]]) - ((12*a^2*A*b - 4*A*b^3 - 15*a^3*B + 7*a*b^2*B)*Sqrt[Cos[c + d
*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(4*b^3
*(a^2 - b^2)*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*a*(A*b - a*B)*Sin[c
 + d*x])/(b*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)*Sqrt[a + b*Sec[c + d*x]]) - ((
4*a*A*b - 5*a^2*B + b^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(2*b^2*(a
^2 - b^2)*d*Cos[c + d*x]^(3/2)) + ((12*a^2*A*b - 4*A*b^3 - 15*a^3*B + 7*a*b
```

```
^2*B)*Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(4*b^3*(a^2 - b^2)*d*Sqrt[Cos[
c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
```

```
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+}{(a+b\sec(c+dx))^{3/2}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} + \frac{\left(2\sqrt{\cos(c+d}\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{(4aAb-5a^2B}{2}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{(4aAb-5a^2B}{2}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{(4aAb-5a^2B}{2}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{(4aAb-5a^2B}{2}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{b\left(a^2-b^2\right)d\cos^{\frac{5}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}} - \frac{(4aAb-5a^2B}{2}$$

$$= -\frac{(12aAb-15a^2B-4b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)\right.}{4b^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

$$= \frac{(4Ab-5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{4b^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{(12aAb}{}$$

Mathematica [C] Result contains complex when optimal does not.
time = 33.42, size = 140027, normalized size = 287.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(3/
2)),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 20.72, size = 2295, normalized size = 4.71

method	result	size
default	Expression too large to display	2295

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -1/4/d*(-1+cos(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2
*(-12*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+15*
B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^3*(1/(1+cos(d*x+c)))^(1/2)*a^3+4*A*((a-b)/
(a+b))^(1/2)*cos(d*x+c)*b^3*(1/(1+cos(d*x+c)))^(1/2)+2*B*((a-b)/(a+b))^(1/2
)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-15*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*
(1/(1+cos(d*x+c)))^(1/2)+2*B*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/
2)+12*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+5*B
*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+4*A*((a-b)
/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-5*B*((a-b)/(a+b))^(
1/2)*cos(d*x+c)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-5*B*((a-b)/(a+b))^(1/2)*cos(
d*x+c)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)+24*A*sin(d*x+c)*cos(d*x+c)^2*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b+24*A*sin(d*x+
c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1
/2))*a*b^2-30*B*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/
(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b-8*B*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*
x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^
(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a*b^2+20*B*sin(d*x+c)*c
os(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos
(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+2*B*sin
(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^
2+7*B*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a*b^2-24*A*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
/(a-b))^(1/2))*a^2*b-16*A*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(
d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+
c),(-(a+b)/(a-b))^(1/2))*a*b^2+12*A*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c
```

```
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-4*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/
2)*b^3*(1/(1+cos(d*x+c)))^(1/2)-2*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*b^3*(1
/(1+cos(d*x+c)))^(1/2)-2*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos
(d*x+c)))^(1/2)-4*A*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*b^3-4*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c
)))^(1/2)*a*b^2-30*B*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(
a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^3-8*B*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*b^3+30*B*sin(d*x+c)*
cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3+4*B*sin(
d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF
((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^3-1
5*B*sin(d*x+c)*cos(d*x+c)^2*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^3+5*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^2*b)
*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)/b^3/(b+a*cos(d*x+c))/cos(d*x+
c)^(3/2)/(a-b)/sin(d*x+c)^3
```

Maxima [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="maxima")
```

```
[Out] Timed out
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(3/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(3/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(3/2)*cos(d*x + c)^(7/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\cos\left(c+d\,x\right)}^{7/2}\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{3/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(3/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(3/2)), x
)
```

3.628 $\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=588

$$-\frac{2(17a^4Ab+116a^2Ab^3-128Ab^5-5a^5B-80a^3b^2B+80ab^4B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^5\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+$$

```
[Out] 2/3*b*(A*b-B*a)*cos(d*x+c)^(3/2)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^
(3/2)+2/3*b*(12*A*a^2*b-8*A*b^3-9*B*a^3+5*B*a*b^2)*cos(d*x+c)^(3/2)*sin(d*x
+c)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)-2/15*(17*A*a^4*b+116*A*a^2*b^3
-128*A*b^5-5*B*a^5-80*B*a^3*b^2+80*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/co
s(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+
a*cos(d*x+c))/(a+b))^(1/2)/a^5/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c)
)^(1/2)+2/15*(3*A*a^4-71*A*a^2*b^2+48*A*b^4+50*B*a^3*b-30*B*a*b^3)*cos(d*x+
c)^(3/2)*sin(d*x+c)*(a+b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)^2/d-2/15*(14*A*a^4
*b-98*A*a^2*b^3+64*A*b^5-5*B*a^5+65*B*a^3*b^2-40*B*a*b^4)*sin(d*x+c)*cos(d*
x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^4/(a^2-b^2)^2/d+2/15*(9*A*a^6+55*A*a^4*
b^2-212*A*a^2*b^4+128*A*b^6-40*B*a^5*b+140*B*a^3*b^3-80*B*a*b^5)*(cos(1/2*d
*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*
(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^5/(a^2-b^2)^2/d/
((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 1.33, antiderivative size = 588, normalized size of antiderivative = 1.00, number of steps used = 12, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3034, 4115, 4185, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2b(Ab-aB)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3ad\left(a^2-b^2\right)(a+b\sec(c+dx))^{3/2}}+\frac{2b(-9a^3B+12a^2Ab+5ab^2B-8Ab^3)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)}{3a^2d\left(a^2-b^2\right)^2\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^4A+50a^3bB-71a^2Ab^2-30ab^3B+48Ab^4)\sin(c+dx)\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}{15a^3d\left(a^2-b^2\right)^2}-\frac{2(-5a^5B+14a^4Ab+65a^3b^2B-98a^2Ab^3-40ab^4B+64Ab^5)\sin(c+dx)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}{15a^4d\left(a^2-b^2\right)^2}-\frac{2(-5a^5B+17a^4Ab-80a^3b^2B+116a^2Ab^3+80ab^4B-128Ab^5)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^5d\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(9a^6A-40a^5bB+55a^4Ab^2+140a^3b^3B-212a^2Ab^4-80ab^5B+128Ab^6)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{15a^5d\left(a^2-b^2\right)^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*(17*a^4*A*b + 116*a^2*A*b^3 - 128*A*b^5 - 5*a^5*B - 80*a^3*b^2*B + 80*a
*b^4*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a
+ b)])/(15*a^5*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) +
 (2*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*
a^3*b^3*B - 80*a*b^5*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a
+ b)]*Sqrt[a + b*Sec[c + d*x]])/(15*a^5*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c +
 d*x])/(a + b)]) + (2*b*(A*b - a*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a*(
a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (2*b*(12*a^2*A*b - 8*A*b^3 - 9*a
^3*B + 5*a*b^2*B)*Cos[c + d*x]^(3/2)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*S
qrt[a + b*Sec[c + d*x]]) - (2*(14*a^4*A*b - 98*a^2*A*b^3 + 64*A*b^5 - 5*a^5
```

```
*B + 65*a^3*b^2*B - 40*a*b^4*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(15*a^4*(a^2 - b^2)^2*d) + (2*(3*a^4*A - 71*a^2*A*b^2 + 48*A
*b^4 + 50*a^3*b*B - 30*a*b^3*B)*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[c + d*x]]
*Sin[c + d*x])/(15*a^3*(a^2 - b^2)^2*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
```

```
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^{\frac{5}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx)}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec(c\right.}{}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b(12a^2Ab-8Ab^3-9a^3}{3a^2\left(a^2-b^2\right.}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b(12a^2Ab-8Ab^3-9a^3}{3a^2\left(a^2-b^2\right.}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b(12a^2Ab-8Ab^3-9a^3}{3a^2\left(a^2-b^2\right.}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b(12a^2Ab-8Ab^3-9a^3}{3a^2\left(a^2-b^2\right.}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b(12a^2Ab-8Ab^3-9a^3}{3a^2\left(a^2-b^2\right.}$$

$$= \frac{2b(Ab-aB)\cos^{\frac{3}{2}}(c+dx)\sin(c+dx)}{3a\left(a^2-b^2\right)d(a+b\sec(c+dx))^{3/2}} + \frac{2b(12a^2Ab-8Ab^3-9a^3}{3a^2\left(a^2-b^2\right.}$$

$$= -\frac{2(17a^4Ab+116a^2Ab^3-128Ab^5-5a^5B-80a^3b^2B+80ab^4B)\,\sqrt{}}{15a^5\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 22.04, size = 4179, normalized size = 7.11

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(5/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5
/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*((2*(-14*A*b + 5*a*B)*Sin[c + d*x])/(15*a^4) - (2*(
A*b^5*Sin[c + d*x] - a*b^4*B*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)*(b + a*Cos[c
 + d*x])^2) - (2*(-15*a^2*A*b^4*Sin[c + d*x] + 11*A*b^6*Sin[c + d*x] + 12*a
^3*b^3*B*Sin[c + d*x] - 8*a*b^5*B*Sin[c + d*x]))/(3*a^4*(a^2 - b^2)^2*(b +
a*Cos[c + d*x])) + (A*Sin[2*(c + d*x)])/(5*a^3)))/(d*Cos[c + d*x]^(5/2)*(a
+ b*Sec[c + d*x])^(5/2)) - (2*Cos[c + d*x]^(3/2)*(b + a*Cos[c + d*x])^2*((3
*a^2*A*Sqrt[Cos[c + d*x]])/(5*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[S
ec[c + d*x]]) + (11*A*b^2*Sqrt[Cos[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[b + a*C
os[c + d*x]]*Sqrt[Sec[c + d*x]]) - (212*A*b^4*Sqrt[Cos[c + d*x]])/(15*a^2*(
a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) + (128*A*b^6*Sqrt
[Cos[c + d*x]])/(15*a^4*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c +
 d*x]]) - (8*a*b*B*Sqrt[Cos[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c +
d*x]]*Sqrt[Sec[c + d*x]]) + (28*b^3*B*Sqrt[Cos[c + d*x]])/(3*a*(a^2 - b^2)^
2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (16*b^5*B*Sqrt[Cos[c + d*x
]])/(3*a^3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]*Sqrt[Sec[c + d*x]]) - (8*
a*A*b*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(15*(a^2 - b^2)^2*Sqrt[b + a*C
os[c + d*x]]) - (44*A*b^3*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(15*a*(a^2
 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (32*A*b^5*Sqrt[Cos[c + d*x]]*Sqrt[Sec
[c + d*x]])/(15*a^3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]) + (a^2*B*Sqrt[C
os[c + d*x]]*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]])
 + (7*b^2*B*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(3*(a^2 - b^2)^2*Sqrt[b
+ a*Cos[c + d*x]]) - (4*b^4*B*Sqrt[Cos[c + d*x]]*Sqrt[Sec[c + d*x]])/(3*a^2
*(a^2 - b^2)^2*Sqrt[b + a*Cos[c + d*x]]))*Sec[c + d*x]^(5/2)*(Cos[(c + d*x)
/2]^2*Sec[c + d*x])^(3/2)*((-I)*(a + b)*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A
*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*EllipticE[I*Arc
Sinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*C
os[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(128*A*b^5 - 16*a*b
^4*(6*A + 5*B) + a^5*(9*A + 5*B) + 8*a^3*b^2*(9*A + 10*B) + 4*a^2*b^3*(-29*
A + 15*B) - a^4*b*(17*A + 45*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a
 + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/
2]^2)/(a + b)] - (9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a
^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]
^2)^(3/2)*Tan[(c + d*x)/2]))/(15*a^5*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(
5/2)*(-1/15*(Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sin
[c + d*x]*((-I)*(a + b)*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6
 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*EllipticE[I*ArcSinh[Tan[(c + d*
x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec
[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(128*A*b^5 - 16*a*b^4*(6*A + 5*B) +
 a^5*(9*A + 5*B) + 8*a^3*b^2*(9*A + 10*B) + 4*a^2*b^3*(-29*A + 15*B) - a^4*
b*(17*A + 45*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*S
```

```
ec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] -
 (9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3
*b^3*B - 80*a*b^5*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c
 + d*x)/2]))/(a^4*(a^2 - b^2)^2*(b + a*Cos[c + d*x])^(3/2)) + (Sqrt[Cos[c +
 d*x]]*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/2)*Sin[c + d*x]*((-I)*(a + b)*(
9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b
^3*B - 80*a*b^5*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]
*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]
 + I*a*(a + b)*(128*A*b^5 - 16*a*b^4*(6*A + 5*B) + a^5*(9*A + 5*B) + 8*a^3*
b^2*(9*A + 10*B) + 4*a^2*b^3*(-29*A + 15*B) - a^4*b*(17*A + 45*B))*Elliptic
F[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((
b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (9*a^6*A + 55*a^4*A*b^2
- 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*(b +
 a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(5*a^5*(a^2
- b^2)^2*Sqrt[b + a*Cos[c + d*x]]) - (2*Cos[c + d*x]^(3/2)*(Cos[(c + d*x)/2
]^2*Sec[c + d*x])^(3/2)*(-1/2*((9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^4 + 12
8*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*(b + a*Cos[c + d*x])*(Se
c[(c + d*x)/2]^2)^(5/2)) - I*(a + b)*(9*a^6*A + 55*a^4*A*b^2 - 212*a^2*A*b^
4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*EllipticE[I*ArcSin
h[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[
c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] + I*a*(a + b)*(128*
A*b^5 - 16*a*b^4*(6*A + 5*B) + a^5*(9*A + 5*B) + 8*a^3*b^2*(9*A + 10*B) + 4
*a^2*b^3*(-29*A + 15*B) - a^4*b*(17*A + 45*B))*EllipticF[I*ArcSinh[Tan[(c +
 d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*
Sec[(c + d*x)/2]^2)/(a + b)]*Tan[(c + d*x)/2] + a*(9*a^6*A + 55*a^4*A*b^2 -
 212*a^2*A*b^4 + 128*A*b^6 - 40*a^5*b*B + 140*a^3*b^3*B - 80*a*b^5*B)*(Sec[
(c + d*x)/2]^2)^(3/2)*Sin[c + d*x]*Tan[(c + d*x...
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 5674 vs. $2(606) = 1212$.
time = 22.74, size = 5675, normalized size = 9.65

method	result	size
default	Expression too large to display	5675

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(5/2
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.52, size = 1553, normalized size = 2.64

Too large to display

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/45*(6*(5*B*a^7*b^2 - 14*A*a^6*b^3 - 65*B*a^5*b^4 + 98*A*a^4*b^5 + 40*B*a^
3*b^6 - 64*A*a^2*b^7 + 3*(A*a^9 - 2*A*a^7*b^2 + A*a^5*b^4)*cos(d*x + c)^3 +
 (5*B*a^9 - 8*A*a^8*b - 10*B*a^7*b^2 + 16*A*a^6*b^3 + 5*B*a^5*b^4 - 8*A*a^4
*b^5)*cos(d*x + c)^2 + 5*(2*B*a^8*b - 5*A*a^7*b^2 - 16*B*a^6*b^3 + 25*A*a^5
*b^4 + 10*B*a^4*b^5 - 16*A*a^3*b^6)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)
/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (sqrt(2)*(-15*I*B*a^9 + 42
*I*A*a^8*b - 185*I*B*a^7*b^2 + 242*I*A*a^6*b^3 + 340*I*B*a^5*b^4 - 520*I*A*
a^4*b^5 - 160*I*B*a^3*b^6 + 256*I*A*a^2*b^7)*cos(d*x + c)^2 - 2*sqrt(2)*(15
*I*B*a^8*b - 42*I*A*a^7*b^2 + 185*I*B*a^6*b^3 - 242*I*A*a^5*b^4 - 340*I*B*a
^4*b^5 + 520*I*A*a^3*b^6 + 160*I*B*a^2*b^7 - 256*I*A*a*b^8)*cos(d*x + c) +
sqrt(2)*(-15*I*B*a^7*b^2 + 42*I*A*a^6*b^3 - 185*I*B*a^5*b^4 + 242*I*A*a^4*b
^5 + 340*I*B*a^3*b^6 - 520*I*A*a^2*b^7 - 160*I*B*a*b^8 + 256*I*A*b^9))*sqrt
(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^
3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + (sqrt(2)*(15*I*B*
a^9 - 42*I*A*a^8*b + 185*I*B*a^7*b^2 - 242*I*A*a^6*b^3 - 340*I*B*a^5*b^4 +
520*I*A*a^4*b^5 + 160*I*B*a^3*b^6 - 256*I*A*a^2*b^7)*cos(d*x + c)^2 - 2*sqr
t(2)*(-15*I*B*a^8*b + 42*I*A*a^7*b^2 - 185*I*B*a^6*b^3 + 242*I*A*a^5*b^4 +
340*I*B*a^4*b^5 - 520*I*A*a^3*b^6 - 160*I*B*a^2*b^7 + 256*I*A*a*b^8)*cos(d*
x + c) + sqrt(2)*(15*I*B*a^7*b^2 - 42*I*A*a^6*b^3 + 185*I*B*a^5*b^4 - 242*I
*A*a^4*b^5 - 340*I*B*a^3*b^6 + 520*I*A*a^2*b^7 + 160*I*B*a*b^8 - 256*I*A*b^
9))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8
*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*(sqrt(2
)*(-9*I*A*a^9 + 40*I*B*a^8*b - 55*I*A*a^7*b^2 - 140*I*B*a^6*b^3 + 212*I*A*a
^5*b^4 + 80*I*B*a^4*b^5 - 128*I*A*a^3*b^6)*cos(d*x + c)^2 + 2*sqrt(2)*(-9*I
*A*a^8*b + 40*I*B*a^7*b^2 - 55*I*A*a^6*b^3 - 140*I*B*a^5*b^4 + 212*I*A*a^4*
b^5 + 80*I*B*a^3*b^6 - 128*I*A*a^2*b^7)*cos(d*x + c) + sqrt(2)*(-9*I*A*a^7*
b^2 + 40*I*B*a^6*b^3 - 55*I*A*a^5*b^4 - 140*I*B*a^4*b^5 + 212*I*A*a^3*b^6 +
 80*I*B*a^2*b^7 - 128*I*A*a*b^8))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b
^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^
```

```
2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x +
 c) + 2*b)/a)) - 3*(sqrt(2)*(9*I*A*a^9 - 40*I*B*a^8*b + 55*I*A*a^7*b^2 + 14
0*I*B*a^6*b^3 - 212*I*A*a^5*b^4 - 80*I*B*a^4*b^5 + 128*I*A*a^3*b^6)*cos(d*x
 + c)^2 + 2*sqrt(2)*(9*I*A*a^8*b - 40*I*B*a^7*b^2 + 55*I*A*a^6*b^3 + 140*I*
B*a^5*b^4 - 212*I*A*a^4*b^5 - 80*I*B*a^3*b^6 + 128*I*A*a^2*b^7)*cos(d*x + c
) + sqrt(2)*(9*I*A*a^7*b^2 - 40*I*B*a^6*b^3 + 55*I*A*a^5*b^4 + 140*I*B*a^4*
b^5 - 212*I*A*a^3*b^6 - 80*I*B*a^2*b^7 + 128*I*A*a*b^8))*sqrt(a)*weierstras
sZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInv
erse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
 + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^12 - 2*a^10*b^2 + a^8*b^4)*d*cos
(d*x + c)^2 + 2*(a^11*b - 2*a^9*b^3 + a^7*b^5)*d*cos(d*x + c) + (a^10*b^2 -
 2*a^8*b^4 + a^6*b^6)*d)
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(5/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(5/2)/(b*sec(d*x + c) + a)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^{5/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(5/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(5/2), x
)
```

3.629 $\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=472

$$\frac{2(a^4A+16a^2Ab^2-16Ab^4-9a^3bB+8ab^3B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^4(a^2-b^2)\,d\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2(8a^4Ab-28a^2Ab^3}{}$$

```
[Out] 2/3*b*(A*b-B*a)*sin(d*x+c)*cos(d*x+c)^(1/2)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^
(3/2)+2/3*b*(10*A*a^2*b-6*A*b^3-7*B*a^3+3*B*a*b^2)*sin(d*x+c)*cos(d*x+c)^(1
/2)/a^2/(a^2-b^2)^2/d/(a+b*sec(d*x+c))^(1/2)+2/3*(A*a^4+16*A*a^2*b^2-16*A*b
^4-9*B*a^3*b+8*B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*Ell
ipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))
^(1/2)/a^4/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*(A*a^4-1
3*A*a^2*b^2+8*A*b^4+8*B*a^3*b-4*B*a*b^3)*sin(d*x+c)*cos(d*x+c)^(1/2)*(a+b*s
ec(d*x+c))^(1/2)/a^3/(a^2-b^2)^2/d-2/3*(8*A*a^4*b-28*A*a^2*b^3+16*A*b^5-3*B
*a^5+15*B*a^3*b^2-8*B*a*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c
)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a
+b*sec(d*x+c))^(1/2)/a^4/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.98, antiderivative size = 472, normalized size of antiderivative = 1.00, number of steps used = 11, number of rules used = 11, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.314$, Rules used = {3034, 4115, 4185, 4189, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2b(Ab-aB)\sin(c+dx)\sqrt{\cos(c+dx)}}{3ad(a^2-b^2)(a+b\sec(c+dx))^{3/2}}+\frac{2b(-7a^3B+10a^2Ab+3ab^2B-6Ab^3)\sin(c+dx)\sqrt{\cos(c+dx)}}{3a^2d(a^2-b^2)^2\sqrt{a+b\sec(c+dx)}}+\frac{2(a^4A+8a^3bB-13a^2Ab^2-4ab^3B+8Ab^4)\sin(c+dx)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}{3a^3d(a^2-b^2)^2}+\frac{2(a^4A-9a^3bB+16a^2Ab^2+8ab^3B-16Ab^4)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^4d(a^2-b^2)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}}-\frac{2(-3a^5B+8a^4Ab+15a^3b^2B-28a^2Ab^3-8ab^4B+16Ab^5)\sqrt{\cos(c+dx)}\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^4d(a^2-b^2)^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (2*(a^4*A + 16*a^2*A*b^2 - 16*A*b^4 - 9*a^3*b*B + 8*a*b^3*B)*Sqrt[(b + a*Co
s[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*a^4*(a^2 - b
^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) - (2*(8*a^4*A*b - 28*a^2
*A*b^3 + 16*A*b^5 - 3*a^5*B + 15*a^3*b^2*B - 8*a*b^4*B)*Sqrt[Cos[c + d*x]]*
EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^4*(a^2
 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) + (2*b*(A*b - a*B)*Sqrt[Cos
[c + d*x]]*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*(a + b*Sec[c + d*x])^(3/2)) + (
2*b*(10*a^2*A*b - 6*A*b^3 - 7*a^3*B + 3*a*b^2*B)*Sqrt[Cos[c + d*x]]*Sin[c +
 d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[a + b*Sec[c + d*x]]) + (2*(a^4*A - 13*a^
2*A*b^2 + 8*A*b^4 + 8*a^3*b*B - 4*a*b^3*B)*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Se
c[c + d*x]]*Sin[c + d*x])/(3*a^3*(a^2 - b^2)^2*d)
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rule 4189

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[A*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d
*Csc[e + f*x])^n/(a*f*n)), x] + Dist[1/(a*d*n), Int[(a + b*Csc[e + f*x])^m*
(d*Csc[e + f*x])^(n + 1)*Simp[a*B*n - A*b*(m + n + 1) + a*(A + A*n + C*n)*C
sc[e + f*x] + A*b*(m + n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d,
 e, f, A, B, C, m}, x] && NeQ[a^2 - b^2, 0] && LeQ[n, -1]
```

Rubi steps

$$\int \frac{\cos^{\frac{3}{2}}(c+dx)(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sec^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx)}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c+dx)}\,\sqrt{\sec\right.}{}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2b(10a^2Ab-6Ab^3-7a}{3a^2\,(a^2-}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2b(10a^2Ab-6Ab^3-7a}{3a^2\,(a^2-}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2b(10a^2Ab-6Ab^3-7a}{3a^2\,(a^2-}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2b(10a^2Ab-6Ab^3-7a}{3a^2\,(a^2-}$$

$$= \frac{2b(Ab-aB)\sqrt{\cos(c+dx)}\,\sin(c+dx)}{3a\,(a^2-b^2)\,d(a+b\sec(c+dx))^{3/2}} + \frac{2b(10a^2Ab-6Ab^3-7a}{3a^2\,(a^2-}$$

$$= \frac{2(a^4A+16a^2Ab^2-16Ab^4-9a^3bB+8ab^3B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}}{3a^4\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 16.95, size = 626, normalized size = 1.33

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Cos[c + d*x]^(3/2)*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5
/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*((2*A*Sin[c + d*x])/(3*a^3) + (2*(A*b^4*Sin[c + d*x
] - a*b^3*B*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) + (2*
```

```
(-12*a^2*A*b^3*Sin[c + d*x] + 8*A*b^5*Sin[c + d*x] + 9*a^3*b^2*B*Sin[c + d*
x] - 5*a*b^4*B*Sin[c + d*x]))/(3*a^3*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/
(d*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)) - (2*Cos[c + d*x]^(3/2)*(
b + a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^
(3/2)*((-I)*(a + b)*(-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^
3*b^2*B + 8*a*b^4*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b
)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b
)] + I*a*(a + b)*(-16*A*b^4 + 2*a^2*b^2*(8*A - 3*B) - 9*a^3*b*(A + B) + 4*a
*b^3*(3*A + 2*B) + a^4*(A + 3*B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-
a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)
/2]^2)/(a + b)] - (-8*a^4*A*b + 28*a^2*A*b^3 - 16*A*b^5 + 3*a^5*B - 15*a^3*
b^2*B + 8*a*b^4*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c +
 d*x)/2]))/(3*a^4*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 4479 vs. $2(496) = 992$.
time = 23.26, size = 4480, normalized size = 9.49

method	result	size
default	Expression too large to display	4480

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/3/d*cos(d*x+c)^(1/2)*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*
(1+cos(d*x+c))^2*(A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^5*b*(1/(1+cos(d*x+c)
))^(1/2)-A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^6+A*
cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)*a^6+3*B*cos(d*x+c
)^3*((a-b)/(a+b))^(1/2)*a^6*(1/(1+cos(d*x+c)))^(1/2)-8*A*((a-b)/(a+b))^(1/2
)*a*b^5*(1/(1+cos(d*x+c)))^(1/2)-3*B*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/(1+cos(
d*x+c)))^(1/2)-11*B*((a-b)/(a+b))^(1/2)*a^3*b^3*(1/(1+cos(d*x+c)))^(1/2)+4*
B*((a-b)/(a+b))^(1/2)*a^2*b^4*(1/(1+cos(d*x+c)))^(1/2)+8*B*((a-b)/(a+b))^(1
/2)*a*b^5*(1/(1+cos(d*x+c)))^(1/2)-3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^6
*(1/(1+cos(d*x+c)))^(1/2)+16*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^6*(1/(1+cos
(d*x+c)))^(1/2)-A*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/(1+cos(d*x+c)))^(1/2)+7*A*
((a-b)/(a+b))^(1/2)*a^3*b^3*(1/(1+cos(d*x+c)))^(1/2)+20*A*((a-b)/(a+b))^(1/
2)*a^2*b^4*(1/(1+cos(d*x+c)))^(1/2)+18*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a
^4*b^2*(1/(1+cos(d*x+c)))^(1/2)-12*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b
^4*(1/(1+cos(d*x+c)))^(1/2)+6*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^2*b^4*(1
/(1+cos(d*x+c)))^(1/2)+3*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^5*b*(1/(1+cos
(d*x+c)))^(1/2)-3*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b^3*(1/(1+cos(d*x+
c)))^(1/2)-2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5*b*(1/(1+cos(d*x+c)))^(1/2
)+14*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/(1+cos(d*x+c)))^(1/2)+28*A
*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos
```

(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4+A*sin
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b+9*A*sin(d*x
+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2+16*A*sin(d*x+
c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3-12*A*sin(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4-16*A*sin(d*x+c)
*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((
a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^5-8*A*sin(d*x+c)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2-6*A*cos(d*x+c)^3*((a
-b)/(a+b))^(1/2)*a^4*b^2*(1/(1+cos(d*x+c)))^(1/2)+22*A*cos(d*x+c)*((a-b)/(a
+b))^(1/2)*a^3*b^3*(1/(1+cos(d*x+c)))^(1/2)-34*A*cos(d*x+c)*((a-b)/(a+b))^(
1/2)*a^2*b^4*(1/(1+cos(d*x+c)))^(1/2)-16*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a
b^5(1/(1+cos(d*x+c)))^(1/2)-6*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^5*b*(1/(
1+cos(d*x+c)))^(1/2)-12*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/(1+cos(
d*x+c)))^(1/2)+18*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^3*(1/(1+cos(d*x+c)
))^(1/2)+8*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^4*(1/(1+cos(d*x+c)))^(1/2
)-8*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^5*(1/(1+cos(d*x+c)))^(1/2)+3*B*sin
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b-15*B*sin(d*
x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c)
)*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3+8*B*sin(d*x+
c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^5-3*B*sin(d*x+c)*(
(b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5*b-9*B*sin(d*x+c)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b^2+6*B*sin(d*x+c)*((b+a*c
os(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+
b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^3+8*B*sin(d*x+c)*((b+a*cos
(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b)
)^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^4-16*A*sin(d*x+c)*((b+a*cos(
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^6-16*A*((a-b)/(a+b))^(1/2)*b^6*(1
/(1+cos(d*x+c)))^(1/2)-A*cos(d*x+c)^4*((a-b)/(a+b))^(1/2)*a^3*b^3*(1/(1+cos
(d*x+c)))^(1/2)+7*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^5*b*(1/(1+cos(d*x+c)
))^(1/2)-34*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b^3*(1/(1+cos(d*x+c)))^(
1/2)+24*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^5*(1/(1+cos(d*x+c)))^(1/2)-3
*B*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/(1+cos(d*x+c)))^(1/2)-6*A*co
s(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*b^2*(1/(1+cos(d*x+c)))^(1/2)+8*A*cos(d*x
+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^4*(1/(1+cos(d*x+c)))^(1/2)+3*B*cos(d*x+c)^2
*((a-b)/(a+b))^(1/2)*a^5*b*(1/(1+cos(d*x+c)))^(1/2)-4*B*cos(d*x+c)^2*((a-b)

```
/(a+b))^(1/2)*a^3*b^3*(1/(1+cos(d*x+c)))^(1/2)-3*B*cos(d*x+c)*sin(d*x+c)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.54, size = 1365, normalized size = 2.89

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/9*(6*(A*a^6*b^2 + 8*B*a^5*b^3 - 13*A*a^4*b^4 - 4*B*a^3*b^5 + 8*A*a^2*b^6
+ (A*a^8 - 2*A*a^6*b^2 + A*a^4*b^4)*cos(d*x + c)^2 + (2*A*a^7*b + 9*B*a^6*b
^2 - 16*A*a^5*b^3 - 5*B*a^4*b^4 + 10*A*a^3*b^5)*cos(d*x + c))*sqrt((a*cos(d
*x + c) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (sqrt(2)*(-3*I
*A*a^8 + 24*I*B*a^7*b - 37*I*A*a^6*b^2 - 36*I*B*a^5*b^3 + 68*I*A*a^4*b^4 +
16*I*B*a^3*b^5 - 32*I*A*a^2*b^6)*cos(d*x + c)^2 - 2*sqrt(2)*(3*I*A*a^7*b -
24*I*B*a^6*b^2 + 37*I*A*a^5*b^3 + 36*I*B*a^4*b^4 - 68*I*A*a^3*b^5 - 16*I*B*
a^2*b^6 + 32*I*A*a*b^7)*cos(d*x + c) + sqrt(2)*(-3*I*A*a^6*b^2 + 24*I*B*a^5
*b^3 - 37*I*A*a^4*b^4 - 36*I*B*a^3*b^5 + 68*I*A*a^2*b^6 + 16*I*B*a*b^7 - 32
*I*A*b^8))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^
2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a) + (s
qrt(2)*(3*I*A*a^8 - 24*I*B*a^7*b + 37*I*A*a^6*b^2 + 36*I*B*a^5*b^3 - 68*I*A
*a^4*b^4 - 16*I*B*a^3*b^5 + 32*I*A*a^2*b^6)*cos(d*x + c)^2 - 2*sqrt(2)*(-3*
I*A*a^7*b + 24*I*B*a^6*b^2 - 37*I*A*a^5*b^3 - 36*I*B*a^4*b^4 + 68*I*A*a^3*b
^5 + 16*I*B*a^2*b^6 - 32*I*A*a*b^7)*cos(d*x + c) + sqrt(2)*(3*I*A*a^6*b^2 -
 24*I*B*a^5*b^3 + 37*I*A*a^4*b^4 + 36*I*B*a^3*b^5 - 68*I*A*a^2*b^6 - 16*I*B
*a*b^7 + 32*I*A*b^8))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2,
 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2
*b)/a) - 3*(sqrt(2)*(-3*I*B*a^8 + 8*I*A*a^7*b + 15*I*B*a^6*b^2 - 28*I*A*a^5
*b^3 - 8*I*B*a^4*b^4 + 16*I*A*a^3*b^5)*cos(d*x + c)^2 + 2*sqrt(2)*(-3*I*B*a
^7*b + 8*I*A*a^6*b^2 + 15*I*B*a^5*b^3 - 28*I*A*a^4*b^4 - 8*I*B*a^3*b^5 + 16
*I*A*a^2*b^6)*cos(d*x + c) + sqrt(2)*(-3*I*B*a^6*b^2 + 8*I*A*a^5*b^3 + 15*I
```

```
*B*a^4*b^4 - 28*I*A*a^3*b^5 - 8*I*B*a^2*b^6 + 16*I*A*a*b^7))*sqrt(a)*weiers
trassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrass
PInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos
(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*(sqrt(2)*(3*I*B*a^8 - 8*I*A*a
^7*b - 15*I*B*a^6*b^2 + 28*I*A*a^5*b^3 + 8*I*B*a^4*b^4 - 16*I*A*a^3*b^5)*co
s(d*x + c)^2 + 2*sqrt(2)*(3*I*B*a^7*b - 8*I*A*a^6*b^2 - 15*I*B*a^5*b^3 + 28
*I*A*a^4*b^4 + 8*I*B*a^3*b^5 - 16*I*A*a^2*b^6)*cos(d*x + c) + sqrt(2)*(3*I*
B*a^6*b^2 - 8*I*A*a^5*b^3 - 15*I*B*a^4*b^4 + 28*I*A*a^3*b^5 + 8*I*B*a^2*b^6
 - 16*I*A*a*b^7))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9
*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*
a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/
((a^11 - 2*a^9*b^2 + a^7*b^4)*d*cos(d*x + c)^2 + 2*(a^10*b - 2*a^8*b^3 + a^
6*b^5)*d*cos(d*x + c) + (a^9*b^2 - 2*a^7*b^4 + a^5*b^6)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)**(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2),x)
[Out] Exception raised: SystemError >> excessive stack use: stack is 6438 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate(cos(d*x+c)^(3/2)*(A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
[Out] integrate((B*sec(d*x + c) + A)*cos(d*x + c)^(3/2)/(b*sec(d*x + c) + a)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{{\cos\left(c+d\,x\right)}^{3/2}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(5/2),x)
[Out] int((cos(c + d*x)^(3/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(5/2), x
)
```

3.630 $\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=368

$$-\frac{2(9a^2Ab-8Ab^3-3a^3B+2ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^4A-15a^2Ab^2+8Ab^4+\cdots}{}$$

```
[Out] 2/3*b*(A*b-B*a)*sin(d*x+c)/a/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^
(1/2)+2/3*b*(8*A*a^2*b-4*A*b^3-5*B*a^3+B*a*b^2)*sin(d*x+c)/a^2/(a^2-b^2)^2/
d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-2/3*(9*A*a^2*b-8*A*b^3-3*B*a^3+2*
B*a*b^2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*
d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a^3/(a^2
-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*(3*A*a^4-15*A*a^2*b^2+8
*A*b^4+6*B*a^3*b-2*B*a*b^3)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)
*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+
b*sec(d*x+c))^(1/2)/a^3/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.70, antiderivative size = 368, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3034, 4115, 4185, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2b(Ab-aB)\sin(c+dx)}{3ad(a^2-b^2)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}}+\frac{2b(-5a^3B+8a^2Ab+ab^2B-4Ab^3)\sin(c+dx)}{3a^2d(a^2-b^2)^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{2(-3a^3B+9a^2Ab+2ab^2B-8Ab^3)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d(a^2-b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(3a^4A+6a^3bB-15a^2Ab^2-2ab^3B+8Ab^4)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^3d(a^2-b^2)^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5/2),x]
```

```
[Out] (-2*(9*a^2*A*b - 8*A*b^3 - 3*a^3*B + 2*a*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(
a + b)]*EllipticF[(c + d*x)/2, (2*a)/(a + b)])/(3*a^3*(a^2 - b^2)*d*Sqrt[Co
s[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(3*a^4*A - 15*a^2*A*b^2 + 8*A*b^
4 + 6*a^3*b*B - 2*a*b^3*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]) + (2*b*(A*b - a*B)*Sin[c + d*x])/(3*a*(a^2 - b^2)*d*Sqrt
[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (2*b*(8*a^2*A*b - 4*A*b^3 - 5*
a^3*B + a*b^2*B)*Sin[c + d*x])/(3*a^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4115

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[b*(A*b
```

```
- a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(
m + 1)*(a^2 - b^2))), x] + Dist[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e
 + f*x])^(m + 1)*(d*Csc[e + f*x])^n*Simp[A*(a^2*(m + 1) - b^2*(m + n + 1))
+ a*b*B*n - a*(A*b - a*B)*(m + 1)*Csc[e + f*x] + b*(A*b - a*B)*(m + n + 2)*
Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, n}, x] && NeQ[A*b
 - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILt
Q[n, 0])
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rubi steps

$$\int \frac{\sqrt{\cos(c+dx)}\,(A+B\sec(c+dx))}{(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{A+B\sec(c+dx)}{\sqrt{\sec(c+dx)}\,(a+b\sec(c+\cdots}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(c\cdots\right.}{\cdots}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{2b(8a^2Ab\cdots}{3a^2\,(a^2-b\cdots}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{2b(8a^2Ab\cdots}{3a^2\,(a^2-b\cdots}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{2b(8a^2Ab\cdots}{3a^2\,(a^2-b\cdots}$$

$$= \frac{2b(Ab-aB)\sin(c+dx)}{3a\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{2b(8a^2Ab\cdots}{3a^2\,(a^2-b\cdots}$$

$$= -\frac{2(9a^2Ab-8Ab^3-3a^3B+2ab^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c\cdots\right.}{3a^3\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\cdots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 15.62, size = 621, normalized size = 1.69

[illegible]

Warning: Unable to verify antiderivative.

```
[In] Integrate[(Sqrt[Cos[c + d*x]]*(A + B*Sec[c + d*x]))/(a + b*Sec[c + d*x])^(5
/2),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*(A + B*Sec[c + d*x])*((-2*(A*b^3*Sin[c + d*x] - a*b
^2*B*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)*(b + a*Cos[c + d*x])^2) - (2*(-9*a^2
*A*b^2*Sin[c + d*x] + 5*A*b^4*Sin[c + d*x] + 6*a^3*b*B*Sin[c + d*x] - 2*a*b
^3*B*Sin[c + d*x]))/(3*a^2*(a^2 - b^2)^2*(b + a*Cos[c + d*x]))))/(d*Cos[c +
 d*x]^(3/2)*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^(5/2)) - (2*Cos[c + d
```

```
*x]^(3/2)*(b + a*Cos[c + d*x])^2*Sec[c + d*x]^(3/2)*(Cos[(c + d*x)/2]^2*Sec
[c + d*x])^(3/2)*(A + B*Sec[c + d*x])*((-I)*(a + b)*(3*a^4*A - 15*a^2*A*b^2
 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*EllipticE[I*ArcSinh[Tan[(c + d*x)/2]],
(-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*
x)/2]^2)/(a + b)] + I*a*(a + b)*(8*A*b^3 + 3*a^2*b*(-3*A + B) + 3*a^3*(A +
B) - 2*a*b^2*(3*A + B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a
+ b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a
+ b)] - (3*a^4*A - 15*a^2*A*b^2 + 8*A*b^4 + 6*a^3*b*B - 2*a*b^3*B)*(b + a*C
os[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3*a*(a^3 - a*b^
2)^2*d*(B + A*Cos[c + d*x])*(a + b*Sec[c + d*x])^(5/2))
```

Maple [**B**] Leaf count of result is larger than twice the leaf count of optimal. 3336 vs. $2(398) = 796$.
time = 22.66, size = 3337, normalized size = 9.07

method	result	size
default	Expression too large to display	3337

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3/d*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(-1+cos(d*x+c))*(1+cos(d*x+c))^2
*(-3*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^5*(1/(1+cos(d*x+c)))^(1/2)+3*A*co
s(d*x+c)^3*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(1+cos(d*x+c)))^(1/2)-3*A*cos(d*x
+c)^2*((a-b)/(a+b))^(1/2)*a^4*b*(1/(1+cos(d*x+c)))^(1/2)-3*B*sin(d*x+c)*cos
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x
+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5+3*A*sin(d*x+c
)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5-3*A*sin(
d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^5+4*A
*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(1+cos(d*x+c)))^(1/2)-B*cos(d*
x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(1+cos(d*x+c)))^(1/2)-8*A*((a-b)/(a+b
))^(1/2)*b^5*(1/(1+cos(d*x+c)))^(1/2)+3*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*
a^5*(1/(1+cos(d*x+c)))^(1/2)+8*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^5*(1/(1+c
os(d*x+c)))^(1/2)+3*A*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(1+cos(d*x+c)))^(1/2)+
11*A*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(1+cos(d*x+c)))^(1/2)-4*A*((a-b)/(a+b))
^(1/2)*a*b^4*(1/(1+cos(d*x+c)))^(1/2)-5*B*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(1
+cos(d*x+c)))^(1/2)+B*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(1+cos(d*x+c)))^(1/2)+
2*B*((a-b)/(a+b))^(1/2)*a*b^4*(1/(1+cos(d*x+c)))^(1/2)-8*A*sin(d*x+c)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^5+3*B*sin(d*x+c)*cos(d*x+c)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b+2*B*sin(d*x+c)*cos(
```

```
d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2-6*B*sin(d*
x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1
+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^4*b+2*B
*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellipt
icE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^
2*b^3+9*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^4*b-6*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^3*b^2-8*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*a^2*b^3+15*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))
/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/
sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b^2-8*A*sin(d*x+c)*cos(d*x+c)*((b+a*co
s(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b
))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^4-3*A*cos(d*x+c)^3*((a-b)/(a+
b))^(1/2)*a^4*b*(1/(1+cos(d*x+c)))^(1/2)+3*A*cos(d*x+c)^3*((a-b)/(a+b))^(1/
2)*a^2*b^3*(1/(1+cos(d*x+c)))^(1/2)-3*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos
(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x
+c),(-(a+b)/(a-b))^(1/2))*a^4*b+3*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x
+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),
(-(a+b)/(a-b))^(1/2))*a^3*b^2+2*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^2*b^3-6*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))
/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a
+b)/(a-b))^(1/2))*a^3*b^2+2*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a*b^4-18*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(1+c
os(d*x+c)))^(1/2)+6*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*b*(1/(1+cos(d*x+
c)))^(1/2)+6*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*b*(1/(1+cos(d*x+c)))^(1/2
)+12*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b^2*(1/(1+cos(d*x+c)))^(1/2)-18*A
*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^2*b^3*(1/(1+cos(d*x+c)))^(1/2)-8*A*cos(d*
x+c)*((a-b)/(a+b))^(1/2)*a*b^4*(1/(1+cos(d*x+c)))^(1/2)-6*B*cos(d*x+c)*((a-
b)/(a+b))^(1/2)*a^4*b*(1/(1+cos(d*x+c)))^(1/2)+6*B*cos(d*x+c)*((a-b)/(a+b))
^(1/2)*a^3*b^2*(1/(1+cos(d*x+c)))^(1/2)+2*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*
a^2*b^3*(1/(1+cos(d*x+c)))^(1/2)-2*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^4*(
1/(1+cos(d*x+c)))^(1/2)+12*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x
+c)))^(1/2)*a*b^4-3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(
1/2)*a^2*b^3+3*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^4*b+9*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^3*b^2-6*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(1+cos(...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(5/2
), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.93, size = 1210, normalized size = 3.29

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] -1/9*(6*(5*B*a^5*b^2 - 8*A*a^4*b^3 - B*a^3*b^4 + 4*A*a^2*b^5 + (6*B*a^6*b -
 9*A*a^5*b^2 - 2*B*a^4*b^3 + 5*A*a^3*b^4)*cos(d*x + c))*sqrt((a*cos(d*x + c
) + b)/cos(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) - (sqrt(2)*(-9*I*B*a^7
 + 24*I*A*a^6*b + 9*I*B*a^5*b^2 - 36*I*A*a^4*b^3 - 4*I*B*a^3*b^4 + 16*I*A*a
^2*b^5)*cos(d*x + c)^2 - 2*sqrt(2)*(9*I*B*a^6*b - 24*I*A*a^5*b^2 - 9*I*B*a^
4*b^3 + 36*I*A*a^3*b^4 + 4*I*B*a^2*b^5 - 16*I*A*a*b^6)*cos(d*x + c) + sqrt(
2)*(-9*I*B*a^5*b^2 + 24*I*A*a^4*b^3 + 9*I*B*a^3*b^4 - 36*I*A*a^2*b^5 - 4*I*
B*a*b^6 + 16*I*A*b^7))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2
, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) +
2*b)/a) - (sqrt(2)*(9*I*B*a^7 - 24*I*A*a^6*b - 9*I*B*a^5*b^2 + 36*I*A*a^4*b
^3 + 4*I*B*a^3*b^4 - 16*I*A*a^2*b^5)*cos(d*x + c)^2 - 2*sqrt(2)*(-9*I*B*a^6
*b + 24*I*A*a^5*b^2 + 9*I*B*a^4*b^3 - 36*I*A*a^3*b^4 - 4*I*B*a^2*b^5 + 16*I
*A*a*b^6)*cos(d*x + c) + sqrt(2)*(9*I*B*a^5*b^2 - 24*I*A*a^4*b^3 - 9*I*B*a^
3*b^4 + 36*I*A*a^2*b^5 + 4*I*B*a*b^6 - 16*I*A*b^7))*sqrt(a)*weierstrassPInv
erse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x
 + c) - 3*I*a*sin(d*x + c) + 2*b)/a) + 3*(sqrt(2)*(-3*I*A*a^7 - 6*I*B*a^6*b
 + 15*I*A*a^5*b^2 + 2*I*B*a^4*b^3 - 8*I*A*a^3*b^4)*cos(d*x + c)^2 + 2*sqrt(
2)*(-3*I*A*a^6*b - 6*I*B*a^5*b^2 + 15*I*A*a^4*b^3 + 2*I*B*a^3*b^4 - 8*I*A*a
^2*b^5)*cos(d*x + c) + sqrt(2)*(-3*I*A*a^5*b^2 - 6*I*B*a^4*b^3 + 15*I*A*a^3
*b^4 + 2*I*B*a^2*b^5 - 8*I*A*a*b^6))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 -
4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4
*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*
x + c) + 2*b)/a)) + 3*(sqrt(2)*(3*I*A*a^7 + 6*I*B*a^6*b - 15*I*A*a^5*b^2 -
2*I*B*a^4*b^3 + 8*I*A*a^3*b^4)*cos(d*x + c)^2 + 2*sqrt(2)*(3*I*A*a^6*b + 6*
```

```
I*B*a^5*b^2 - 15*I*A*a^4*b^3 - 2*I*B*a^3*b^4 + 8*I*A*a^2*b^5)*cos(d*x + c)
+ sqrt(2)*(3*I*A*a^5*b^2 + 6*I*B*a^4*b^3 - 15*I*A*a^3*b^4 - 2*I*B*a^2*b^5 +
 8*I*A*a*b^6))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^
2*b - 8*b^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2
*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a
^10 - 2*a^8*b^2 + a^6*b^4)*d*cos(d*x + c)^2 + 2*(a^9*b - 2*a^7*b^3 + a^5*b^
5)*d*cos(d*x + c) + (a^8*b^2 - 2*a^6*b^4 + a^4*b^6)*d)
```

Sympy [F(-2)]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)**(1/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 3435 deep
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))*cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)*sqrt(cos(d*x + c))/(b*sec(d*x + c) + a)^(5/2
), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{\sqrt{\cos\left(c+d\,x\right)}\,\left(A+\frac{B}{\cos\left(c+d\,x\right)}\right)}{{\left(a+\frac{b}{\cos\left(c+d\,x\right)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(5/2),x)
```

```
[Out] int((cos(c + d*x)^(1/2)*(A + B/cos(c + d*x)))/(a + b/cos(c + d*x))^(5/2), x
)
```

3.631 $\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=346

$$\frac{2(3a^2A-2Ab^2-abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(6a^2Ab-2Ab^3-3a^3B-ab^2B)\sqrt{\cos(c-}{3a^2(a^2-b^2)^2\,d\sqrt{}}$$

```
[Out] -2/3*(A*b-B*a)*sin(d*x+c)/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^(1/
2)-2/3*(5*A*a^2*b-A*b^3-2*B*a^3-2*B*a*b^2)*sin(d*x+c)/a/(a^2-b^2)^2/d/cos(d
*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*(3*A*a^2-2*A*b^2-B*a*b)*(cos(1/2*d*x
+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a
/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a^2/(a^2-b^2)/d/cos(d*x+c)^(1
/2)/(a+b*sec(d*x+c))^(1/2)+2/3*(6*A*a^2*b-2*A*b^3-3*B*a^3-B*a*b^2)*(cos(1/2
*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2
)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a^2/(a^2-b^2)^2/
d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]
time = 0.64, antiderivative size = 346, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3034, 4112, 4185, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$-\frac{2(Ab-aB)\sin(c+dx)}{3d(a^2-b^2)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}}+\frac{2(3a^2A-abB-2Ab^2)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d(a^2-b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{2(-2a^3B+5a^2Ab-2ab^2B-Ab^3)\sin(c+dx)}{3ad(a^2-b^2)^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(-3a^3B+6a^2Ab-ab^2B-2Ab^3)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2d(a^2-b^2)^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*(3*a^2*A - 2*A*b^2 - a*b*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF
[(c + d*x)/2, (2*a)/(a + b)])/(3*a^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[
a + b*Sec[c + d*x]]) + (2*(6*a^2*A*b - 2*A*b^3 - 3*a^3*B - a*b^2*B)*Sqrt[Co
s[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])
/(3*a^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b)]) - (2*(A*b - a*B
)*Sin[c + d*x])/(3*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3
/2)) - (2*(5*a^2*A*b - A*b^3 - 2*a^3*B - 2*a*b^2*B)*Sin[c + d*x])/(3*a*(a^2
 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4112

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[(-d)*(A
```

```
*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
1)/(f*(m + 1)*(a^2 - b^2))), x] + Dist[1/((m + 1)*(a^2 - b^2)), Int[(a + b*
Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 1)*Simp[d*(n - 1)*(A*b - a*B) +
 d*(a*A - b*B)*(m + 1)*Csc[e + f*x] - d*(A*b - a*B)*(m + n + 1)*Csc[e + f*x
]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] &&
NeQ[a^2 - b^2, 0] && LtQ[m, -1] && LtQ[0, n, 1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sqrt{\sec(c+dx)}\,(A+B\sec(c}{(a+b\sec(c+dx))^{5/}}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{\left(2\sqrt{\cos(}\right.}{}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{2(5a^2A}{3a\left(a^2-\right.}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{2(5a^2A}{3a\left(a^2-\right.}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{2(5a^2A}{3a\left(a^2-\right.}$$

$$= -\frac{2(Ab-aB)\sin(c+dx)}{3\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} - \frac{2(5a^2A}{3a\left(a^2-\right.}$$

$$= \frac{2(3a^2A-2Ab^2-abB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a^2\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 11.02, size = 463, normalized size = 1.34

$$\frac{(b+a\cos(c+dx))^2\left(\frac{2(b(-5a^2Ab+Ab^3+2a^3B+2ab^2B)+a(-6a^2Ab+2Ab^3+3a^3B+ab^2B)\cos(c+dx))\sin(c+dx)}{a(a^2-b^2)^2(b+a\cos(c+dx))}+\frac{2\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-i(a+b)(-6a^2Ab+2Ab^3+3a^3B+ab^2B)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-ia(a+b)(-2Ab^2+3a^2(A-B)+ab(3A-B))F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-(-6a^2Ab+2Ab^3+3a^3B+ab^2B)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{(a^3-ab^2)^2\sec^{\frac{3}{2}}(c+dx)}\right)}{3d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])^2*((2*(b*(-5*a^2*A*b + A*b^3 + 2*a^3*B + 2*a*b^2*B) +
 a*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*Cos[c + d*x])*Sin[c + d*x])/(
a*(a^2 - b^2)^2*(b + a*Cos[c + d*x])) + (2*(Cos[(c + d*x)/2]^2*Sec[c + d*x]
)^(3/2)*((-I)*(a + b)*(-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*EllipticE[
```

```
I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b
+ a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - I*a*(a + b)*(-2*A*b^2 + 3*
a^2*(A - B) + a*b*(3*A - B))*EllipticF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b
)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2
)/(a + b)] - (-6*a^2*A*b + 2*A*b^3 + 3*a^3*B + a*b^2*B)*(b + a*Cos[c + d*x]
)*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/((a^3 - a*b^2)^2*Sec[c + d*
x]^(3/2))))/(3*d*Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 2415 vs. $2(376) = 752$.
time = 22.77, size = 2416, normalized size = 6.98

method	result	size
default	Expression too large to display	2416

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*(1+cos(d*x+c))^2*(2*A*((a-b)/(a+b))^(1/2)*b^4*(1/(1+
cos(d*x+c)))^(1/2)-B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c))
)^(1/2)-2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*b^4*(1/(1+cos(d*x+c)))^(1/2)+3*B
*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*(1/(1+cos(d*x+c)))^(1/2)-5*A*((a-b)/(a+
b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)+B*((a-b)/(a+b))^(1/2)*a*b^3*(1/(
1+cos(d*x+c)))^(1/2)+B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c
))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-
(a+b)/(a-b))^(1/2))*a^2*b^2+3*A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(
d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+2*A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+
c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/
2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2-6*A*cos(d*x+c)*sin(d*x+c)*((b+a
*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(
a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+2*A*cos(d*x+c)*sin(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+B*cos(d*x+c)*sin(
d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b+B*cos(d*x+c)
*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)+6*A*cos(d*x+c)^2*((a-
b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c)))^(1/2)-A*cos(d*x+c)^2*((a-b)/(a+b))
^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)+B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*
a^3*b*(1/(1+cos(d*x+c)))^(1/2)-6*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b*(1/
(1+cos(d*x+c)))^(1/2)+2*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*
x+c)))^(1/2)-3*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*a^4+3*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
```

```
)/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^4-3*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b
))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(
a-b))^(1/2))*a^3*b+B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/
2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^
(1/2))*a^2*b^2+3*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^3*b+B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
*b^3-3*A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2
)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(
1/2))*a^4-3*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*
a^3*b+3*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
F((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*
b^2+2*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF(
(-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3-
6*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+
cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+2*
A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+co
s(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*b^4-3*B*cos(
d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*(1/(1+cos(d*x+c)))^(1/2)+A*((a-b)/(a+b))^(
1/2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)+2*B*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos
(d*x+c)))^(1/2)-B*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)-3*A*
cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)+6*A*cos(d*x
+c)*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)-3*B*cos(d*x+c)*((a
-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c)))^(1/2))*cos(d*x+c)^(1/2)*((b+a*cos
(d*x+c))/cos(d*x+c))^(1/2)*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)/a^2
/(a+b)/(a-b)^2/(b+a*cos(d*x+c))^2/sin(d*x+c)^3
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c
))), x)
```

Fricas [C] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 0.83, size = 1093, normalized size = 3.16

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algor
ithm="fricas")
```

```
[Out] 1/9*(6*(2*B*a^5*b - 5*A*a^4*b^2 + 2*B*a^3*b^3 + A*a^2*b^4 + (3*B*a^6 - 6*A*
a^5*b + B*a^4*b^2 + 2*A*a^3*b^3)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/co
s(d*x + c))*sqrt(cos(d*x + c))*sin(d*x + c) + (sqrt(2)*(-9*I*A*a^6 + 6*I*B*
a^5*b + 9*I*A*a^4*b^2 - 2*I*B*a^3*b^3 - 4*I*A*a^2*b^4)*cos(d*x + c)^2 - 2*s
qrt(2)*(9*I*A*a^5*b - 6*I*B*a^4*b^2 - 9*I*A*a^3*b^3 + 2*I*B*a^2*b^4 + 4*I*A
*a*b^5)*cos(d*x + c) + sqrt(2)*(-9*I*A*a^4*b^2 + 6*I*B*a^3*b^3 + 9*I*A*a^2*
b^4 - 2*I*B*a*b^5 - 4*I*A*b^6))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4
*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*
x + c) + 2*b)/a) + (sqrt(2)*(9*I*A*a^6 - 6*I*B*a^5*b - 9*I*A*a^4*b^2 + 2*I*
B*a^3*b^3 + 4*I*A*a^2*b^4)*cos(d*x + c)^2 - 2*sqrt(2)*(-9*I*A*a^5*b + 6*I*B
*a^4*b^2 + 9*I*A*a^3*b^3 - 2*I*B*a^2*b^4 - 4*I*A*a*b^5)*cos(d*x + c) + sqrt
(2)*(9*I*A*a^4*b^2 - 6*I*B*a^3*b^3 - 9*I*A*a^2*b^4 + 2*I*B*a*b^5 + 4*I*A*b^
6))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8
*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a) - 3*(sqrt(2
)*(3*I*B*a^6 - 6*I*A*a^5*b + I*B*a^4*b^2 + 2*I*A*a^3*b^3)*cos(d*x + c)^2 +
2*sqrt(2)*(3*I*B*a^5*b - 6*I*A*a^4*b^2 + I*B*a^3*b^3 + 2*I*A*a^2*b^4)*cos(d
*x + c) + sqrt(2)*(3*I*B*a^4*b^2 - 6*I*A*a^3*b^3 + I*B*a^2*b^4 + 2*I*A*a*b^
5))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3
)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)
/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*(sqrt(2)*(-
3*I*B*a^6 + 6*I*A*a^5*b - I*B*a^4*b^2 - 2*I*A*a^3*b^3)*cos(d*x + c)^2 + 2*s
qrt(2)*(-3*I*B*a^5*b + 6*I*A*a^4*b^2 - I*B*a^3*b^3 - 2*I*A*a^2*b^4)*cos(d*x
 + c) + sqrt(2)*(-3*I*B*a^4*b^2 + 6*I*A*a^3*b^3 - I*B*a^2*b^4 - 2*I*A*a*b^5
))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)
/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/
a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^9 - 2*a^7*b
^2 + a^5*b^4)*d*cos(d*x + c)^2 + 2*(a^8*b - 2*a^6*b^3 + a^4*b^5)*d*cos(d*x
+ c) + (a^7*b^2 - 2*a^5*b^4 + a^3*b^6)*d)
```

Sympy [**F(-2)**]
time = 0.00, size = 0, normalized size = 0.00

Exception raised: SystemError

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))**(5/2)/cos(d*x+c)**(1/2),x)
```

```
[Out] Exception raised: SystemError >> excessive stack use: stack is 6439 deep
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/(a+b*sec(d*x+c))^(5/2)/cos(d*x+c)^(1/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*sqrt(cos(d*x + c
))), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\sqrt{\cos\left(c+d\,x\right)}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(1/2)*(a + b/cos(c + d*x))^(5/2)), x
)
```

3.632 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=329

$$-\frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3a\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{2(3a^2A+Ab^2-4abB)\sqrt{\cos(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\right.}{3a\left(a^2-b^2\right)^2d\sqrt{\frac{b+a\cos(c+}{a+b}}}$$

```
[Out] 2/3*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/(a+b*sec(d*x+c))^(3/2)/cos(d*x+c)^
(1/2)+2/3*(2*A*a^2*b+2*A*b^3+B*a^3-5*B*a*b^2)*sin(d*x+c)/b/(a^2-b^2)^2/d/co
s(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)-2/3*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/
2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/a/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(
d*x+c))^(1/2)-2/3*(3*A*a^2+A*b^2-4*B*a*b)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*
x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/a/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^
(1/2)
```

Rubi [A]
time = 0.67, antiderivative size = 329, normalized size of antiderivative = 1.00, number of steps used = 10, number of rules used = 10, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.286$, Rules used = {3034, 4114, 4185, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}}-\frac{2(Ab-aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{2(3a^2A-4abB+Ab^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3ad\left(a^2-b^2\right)^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2(a^3B+2a^2Ab-5ab^2B+2Ab^3)\sin(c+dx)}{3bd\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (-2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (
2*a)/(a + b)])/(3*a*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x
]]) - (2*(3*a^2*A + A*b^2 - 4*a*b*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)
/2, (2*a)/(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*a*(a^2 - b^2)^2*d*Sqrt[(b +
 a*Cos[c + d*x])/(a + b)]) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*b*(a^2 - b^2
)*d*Sqrt[Cos[c + d*x]]*(a + b*Sec[c + d*x])^(3/2)) + (2*(2*a^2*A*b + 2*A*b^
3 + a^3*B - 5*a*b^2*B)*Sin[c + d*x])/(3*b*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]
]*Sqrt[a + b*Sec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
```

```
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4185

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a + b*Csc
[e + f*x])^(m + 1)*((d*Csc[e + f*x])^n/(a*f*(m + 1)*(a^2 - b^2))), x] + Dis
t[1/(a*(m + 1)*(a^2 - b^2)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*
x])^n*Simp[a*(a*A - b*B + a*C)*(m + 1) - (A*b^2 - a*b*B + a^2*C)*(m + n + 1
) - a*(A*b - a*B + b*C)*(m + 1)*Csc[e + f*x] + (A*b^2 - a*b*B + a^2*C)*(m +
 n + 2)*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, n}, x]
 && NeQ[a^2 - b^2, 0] && LtQ[m, -1] &&  !(ILtQ[m + 1/2, 0] && ILtQ[n, 0])
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{3}{2}}(c+dx)(A+B\sec(c+\ldots}{(a+b\sec(c+dx))^{5/2}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+\ldots\right.}{\ldots}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{2(2a^2Ab+\ldots}{3b\,(a^2-b^2)^2\ldots}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{2(2a^2Ab+\ldots}{3b\,(a^2-b^2)^2\ldots}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{2(2a^2Ab+\ldots}{3b\,(a^2-b^2)^2\ldots}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,(a+b\sec(c+dx))^{3/2}} + \frac{2(2a^2Ab+\ldots}{3b\,(a^2-b^2)^2\ldots}$$

$$= -\frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)\middle|\frac{2a}{a+b}\right)}{3a\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} - \frac{2(3a^2A+\ldots}{\ldots}$$

Mathematica [C] Result contains complex when optimal does not.
time = 12.37, size = 487, normalized size = 1.48

$$\frac{(b+a\cos(c+dx))^3\left(\frac{2(Ab\sin(c+dx)-aB\sin(c+dx))}{3(-a^2+b^2)(b+a\cos(c+dx))^2}+\frac{2(3a^2A\sin(c+dx)+Ab^2\sin(c+dx)-4abB\sin(c+dx))}{3(-a^2+b^2)^2(b+a\cos(c+dx))}\right)}{d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}} + \frac{2\cos^{\frac{3}{2}}(c+dx)(b+a\cos(c+dx))^2\sec^{\frac{5}{2}}(c+dx)\left(\cos^2\left(\frac{1}{2}(c+dx)\right)\sec(c+dx)\right)^{3/2}\left(-i(a+b)\left(3a^2A+Ab^2-4abB\right)E\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}+ia(a+b)(3aA+Ab-aB-3bB)F\left(i\sinh^{-1}\left(\tan\left(\frac{1}{2}(c+dx)\right)\right)\middle|\frac{-a+b}{a+b}\right)\sec^2\left(\frac{1}{2}(c+dx)\right)\sqrt{\frac{(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)}{a+b}}-\left(3a^2A+Ab^2-4abB\right)(b+a\cos(c+dx))\sec^2\left(\frac{1}{2}(c+dx)\right)^{3/2}\tan\left(\frac{1}{2}(c+dx)\right)\right)}{3a\left(a^2-b^2\right)^2d(a+b\sec(c+dx))^{5/2}}$$

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] ((b + a*Cos[c + d*x])^3*((2*(A*b*Sin[c + d*x] - a*B*Sin[c + d*x]))/(3*(-a^2
 + b^2)*(b + a*Cos[c + d*x])^2) + (2*(3*a^2*A*Sin[c + d*x] + A*b^2*Sin[c +
d*x] - 4*a*b*B*Sin[c + d*x]))/(3*(-a^2 + b^2)^2*(b + a*Cos[c + d*x]))))/(d*
Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)) + (2*Cos[c + d*x]^(3/2)*(b +
 a*Cos[c + d*x])^2*Sec[c + d*x]^(5/2)*(Cos[(c + d*x)/2]^2*Sec[c + d*x])^(3/
```

```
2)*((-I)*(a + b)*(3*a^2*A + A*b^2 - 4*a*b*B)*EllipticE[I*ArcSinh[Tan[(c + d
*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[((b + a*Cos[c + d*x])*Se
c[(c + d*x)/2]^2)/(a + b)] + I*a*(a + b)*(3*a*A + A*b - a*B - 3*b*B)*Ellipt
icF[I*ArcSinh[Tan[(c + d*x)/2]], (-a + b)/(a + b)]*Sec[(c + d*x)/2]^2*Sqrt[
((b + a*Cos[c + d*x])*Sec[(c + d*x)/2]^2)/(a + b)] - (3*a^2*A + A*b^2 - 4*a
*b*B)*(b + a*Cos[c + d*x])*(Sec[(c + d*x)/2]^2)^(3/2)*Tan[(c + d*x)/2]))/(3
*a*(a^2 - b^2)^2*d*(a + b*Sec[c + d*x])^(5/2))
```

Maple [B] Leaf count of result is larger than twice the leaf count of optimal. 1920 vs. $2(359) = 718$.
time = 21.08, size = 1921, normalized size = 5.84

method	result	size
default	Expression too large to display	1921

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] -2/3/d*(-1+cos(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2
*(-3*A*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*a^3+A*((a-b)/(a+b))^(1/2)*b^3*(1/(1+cos(d*x+c)))^(1/2)+B*((a-b)/(a+b))^
(1/2)*a^3*(1/(1+cos(d*x+c)))^(1/2)+A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*
x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c)
,(-(a+b)/(a-b))^(1/2))*b^3-3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)^2*a^3*(1/(1+c
os(d*x+c)))^(1/2)+3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^3*(1/(1+cos(d*x+c)))
^(1/2)-A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*b^3*(1/(1+cos(d*x+c)))^(1/2)+2*A*((
a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-A*((a-b)/(a+b))^(1/2)*a*b^
2*(1/(1+cos(d*x+c)))^(1/2)+B*((a-b)/(a+b))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(
1/2)-4*B*((a-b)/(a+b))^(1/2)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-B*cos(d*x+c)^2*
((a-b)/(a+b))^(1/2)*a^3*(1/(1+cos(d*x+c)))^(1/2)+A*cos(d*x+c)^2*((a-b)/(a+b
))^(1/2)*a^2*b*(1/(1+cos(d*x+c)))^(1/2)+3*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)^
2*a^2*b*(1/(1+cos(d*x+c)))^(1/2)-3*A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(
1/(1+cos(d*x+c)))^(1/2)+A*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+cos(d*
x+c)))^(1/2)-4*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a^2*b*(1/(1+cos(d*x+c)))^(1
/2)+4*B*((a-b)/(a+b))^(1/2)*cos(d*x+c)*a*b^2*(1/(1+cos(d*x+c)))^(1/2)-3*A*s
in(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d
*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+A*sin(d*x
+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))
*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+3*A*sin(d*x+c)*
((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a
-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-4*B*sin(d*x+c)*((b+
a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/
(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2-B*sin(d*x+c)*((b+a*cos(
```

```
d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))
^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b-4*B*sin(d*x+c)*cos(d*x+c)*((b
+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)
/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+A*sin(d*x+c)*cos(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*(
(a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+A*sin(d*x+c)*cos(
d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+
c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2+3*B*sin(d*x+
c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+c
os(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b+3*A*s
in(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elliptic
E((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3-
B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
^3+3*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^2)*
cos(d*x+c)^(1/2)*((a-b)/(a+b))^(1/2)*(1/(1+cos(d*x+c)))^(1/2)/(a+b)/(a-b)^2
/a/(b+a*cos(d*x+c))^2/sin(d*x+c)^3
```

Maxima [**F**]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/
2)), x)
```

Fricas [**C**] Result contains higher order function than in optimal. Order 9 vs. order 4.
time = 1.48, size = 973, normalized size = 2.96

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] 1/9*(6*(B*a^5 + 2*A*a^4*b - 5*B*a^3*b^2 + 2*A*a^2*b^3 + (3*A*a^5 - 4*B*a^4*
b + A*a^3*b^2)*cos(d*x + c))*sqrt((a*cos(d*x + c) + b)/cos(d*x + c))*sqrt(c
os(d*x + c))*sin(d*x + c) + (sqrt(2)*(-3*I*B*a^5 + 6*I*A*a^4*b - I*B*a^3*b^
2 - 2*I*A*a^2*b^3)*cos(d*x + c)^2 - 2*sqrt(2)*(3*I*B*a^4*b - 6*I*A*a^3*b^2
+ I*B*a^2*b^3 + 2*I*A*a*b^4)*cos(d*x + c) + sqrt(2)*(-3*I*B*a^3*b^2 + 6*I*A
*a^2*b^3 - I*B*a*b^4 - 2*I*A*b^5))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2
```

```
- 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) + 3*I*a*sin
(d*x + c) + 2*b)/a) + (sqrt(2)*(3*I*B*a^5 - 6*I*A*a^4*b + I*B*a^3*b^2 + 2*I
*A*a^2*b^3)*cos(d*x + c)^2 - 2*sqrt(2)*(-3*I*B*a^4*b + 6*I*A*a^3*b^2 - I*B*
a^2*b^3 - 2*I*A*a*b^4)*cos(d*x + c) + sqrt(2)*(3*I*B*a^3*b^2 - 6*I*A*a^2*b^
3 + I*B*a*b^4 + 2*I*A*b^5))*sqrt(a)*weierstrassPInverse(-4/3*(3*a^2 - 4*b^2
)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x +
c) + 2*b)/a) - 3*(sqrt(2)*(3*I*A*a^5 - 4*I*B*a^4*b + I*A*a^3*b^2)*cos(d*x +
 c)^2 + 2*sqrt(2)*(3*I*A*a^4*b - 4*I*B*a^3*b^2 + I*A*a^2*b^3)*cos(d*x + c)
+ sqrt(2)*(3*I*A*a^3*b^2 - 4*I*B*a^2*b^3 + I*A*a*b^4))*sqrt(a)*weierstrassZ
eta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, weierstrassPInver
se(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^3)/a^3, 1/3*(3*a*cos(d*x +
 c) + 3*I*a*sin(d*x + c) + 2*b)/a)) - 3*(sqrt(2)*(-3*I*A*a^5 + 4*I*B*a^4*b
- I*A*a^3*b^2)*cos(d*x + c)^2 + 2*sqrt(2)*(-3*I*A*a^4*b + 4*I*B*a^3*b^2 - I
*A*a^2*b^3)*cos(d*x + c) + sqrt(2)*(-3*I*A*a^3*b^2 + 4*I*B*a^2*b^3 - I*A*a*
b^4))*sqrt(a)*weierstrassZeta(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b
^3)/a^3, weierstrassPInverse(-4/3*(3*a^2 - 4*b^2)/a^2, 8/27*(9*a^2*b - 8*b^
3)/a^3, 1/3*(3*a*cos(d*x + c) - 3*I*a*sin(d*x + c) + 2*b)/a)))/((a^8 - 2*a^
6*b^2 + a^4*b^4)*d*cos(d*x + c)^2 + 2*(a^7*b - 2*a^5*b^3 + a^3*b^5)*d*cos(d
*x + c) + (a^6*b^2 - 2*a^4*b^4 + a^2*b^6)*d)
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(3/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(3/
2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{{\cos\left(c+d\,x\right)}^{3/2}\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(5/2)),x)
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(3/2)*(a + b/cos(c + d*x))^(5/2)), x
)
```

3.633 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=399

$$\frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3b\left(a^2-b^2\right)d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2(4Ab^3}{}$$

```
[Out] 2/3*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^
(3/2)-2/3*a*(4*A*b^3+3*B*a^3-7*B*a*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^2/d/cos(d*
x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*(A*b-B*a)*(cos(1/2*d*x+1/2*c)^2)^(1/2
)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*
((b+a*cos(d*x+c))/(a+b))^(1/2)/b/(a^2-b^2)/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+
c))^(1/2)+2*B*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(si
n(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/
b^2/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+2/3*(4*A*b^3+3*B*a^3-7*B*a*b^
2)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/
2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b^2/(
a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b))^(1/2)
```

Rubi [A]

time = 0.97, antiderivative size = 399, normalized size of antiderivative = 1.00, number of steps used = 14, number of rules used = 14, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.400$, Rules used = {3034, 4114, 4183, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd\left(a^2-b^2\right)\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}+\frac{2(Ab-aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3bd\left(a^2-b^2\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}-\frac{2a\left(3a^3B-7ab^2B+4Ab^3\right)\sin(c+dx)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2\left(3a^3B-7ab^2B+4Ab^3\right)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3b^2d\left(a^2-b^2\right)^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{2B\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] (2*(A*b - a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticF[(c + d*x)/2, (2
*a)/(a + b)])/(3*b*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]
]) + (2*B*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*EllipticPi[2, (c + d*x)/2, (2*
a)/(a + b)])/(b^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*Sec[c + d*x]]) + (2*(4*A*
b^3 + 3*a^3*B - 7*a*b^2*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/
(a + b)]*Sqrt[a + b*Sec[c + d*x]])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c
 + d*x])/(a + b)]) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*Cos[
c + d*x]^(3/2)*(a + b*Sec[c + d*x])^(3/2)) - (2*a*(4*A*b^3 + 3*a^3*B - 7*a*
b^2*B)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Sqrt[Cos[c + d*x]]*Sqrt[a + b*S
ec[c + d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
```

```
m, n, p}, x] && NeQ[b*c - a*d, 0] && !IntegerQ[p] && !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4183

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
- b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n +
 b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}
, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{5}{2}}(c+dx)(A+B\sec(c+\cdots}{(a+b\sec(c+dx))^{5/2}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+\cdots}{\cdots}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2a(4A\cdots}{3b^2\,(a^2-b^2)\cdots}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2a(4A\cdots}{3b^2\,(a^2-b^2)\cdots}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2a(4A\cdots}{3b^2\,(a^2-b^2)\cdots}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\,(a^2-b^2)\,d\cos^{\frac{3}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} - \frac{2a(4A\cdots}{3b^2\,(a^2-b^2)\cdots}$$

$$= \frac{2B\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^2d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2a(A\cdots}{3b\,(a^2-b^2)\,d\cos\cdots}$$

$$= \frac{2(Ab-aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3b\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{2B\sqrt{\frac{b+\cdots}{\cdots}}}{b^2d\sqrt{c\cdots}}$$

<u>**Mathematica**</u> [C] Result contains complex when optimal does not.
time = 32.58, size = 97528, normalized size = 244.43

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(5/2)*(a + b*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 20.50, size = 3159, normalized size = 7.92

method	result	size
default	Expression too large to display	3159

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] 2/3/d*(-1+cos(d*x+c))*((b+a*cos(d*x+c))/cos(d*x+c))^(1/2)*(1+cos(d*x+c))^2*
(4*A*((a-b)/(a+b))^(1/2)*b^4*(1/(1+cos(d*x+c)))^(1/2)+7*B*cos(d*x+c)*((a-b)
/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)-4*A*cos(d*x+c)*((a-b)/(a+b))^(
1/2)*b^4*(1/(1+cos(d*x+c)))^(1/2)+3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^4*(1
/(1+cos(d*x+c)))^(1/2)-A*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))^(1/
2)-7*B*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)+6*B*sin(d*x+c)*((
b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-
b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^3*b+6*B*sin
(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*
x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2
*b^2-6*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticP
i((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b)
)^(1/2))*a*b^3-7*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(
a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b
)/(a-b))^(1/2))*a^2*b^2-3*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+
b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/
(a-b))^(1/2))*b^4+3*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*b^4-6*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*El
lipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b
)/(a+b))^(1/2))*b^4+A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c)
)/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(
a+b)/(a-b))^(1/2))*a^2*b^2+4*A*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+c
os(d*x+c))/(a+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d
*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3-4*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c
))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2
)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^3*b-7*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)
*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)+A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^
2*(1/(1+cos(d*x+c)))^(1/2)-B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+c
os(d*x+c)))^(1/2)+4*A*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c)
))^(1/2)-6*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^
(1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b
))^(1/2))*a^4+3*B*cos(d*x+c)*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a
+b))^(1/2)*EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)
```

```
/(a-b))^(1/2))*a^4-6*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(
1/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b)
)^(1/2))*a^3*b-4*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)
*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1
/2))*a^2*b^2+3*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*E
llipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2
))*a^3*b-7*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ellip
ticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a
*b^3+A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((
-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a*b^3+6
*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Elli
pticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((a-b)/
(a+b))^(1/2))*a^4+9*B*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))
^(1/2))*a*b^3+4*A*sin(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*
EllipticE((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/
2))*b^4-3*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^4*(1/(1+cos(d*x+c)))^(1/2)-A
*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c)))^(1/2)+4*B*((a-b)/(a+b))^(1/2)
*a^3*b*(1/(1+cos(d*x+c)))^(1/2)+B*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x
+c)))^(1/2)-3*A*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a*b^3*(1/(1+cos(d*x+c)))^(
1/2)+6*B*cos(d*x+c)^2*((a-b)/(a+b))^(1/2)*a^2*b^2*(1/(1+cos(d*x+c)))^(1/2)-
3*B*cos(d*x+c)*((a-b)/(a+b))^(1/2)*a^3*b*(1/(1+cos(d*x+c)))^(1/2)+9*B*sin(d
*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticF((-
1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))*a^2*b^2+
3*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*Ell
ipticF((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(-(a+b)/(a-b))^(1/2))
*a*b^3+6*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1
/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I
/((a-b)/(a+b))^(1/2))*a^3*b-6*B*sin(d*x+c)*cos(d*x+c)*((b+a*cos(d*x+c))/(1+
cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*((a-b)/(a+b))^(1/2)/sin
(d*x+c),(a+b)/(a-b),I/((a-b)/(a+b))^(1/2))*a^2*b^2-6*B*sin(d*x+c)*cos(d*x+c
)*((b+a*cos(d*x+c))/(1+cos(d*x+c))/(a+b))^(1/2)*EllipticPi((-1+cos(d*x+c))*
((a-b)/(a+b))^(1/2)/sin(d*x+c),(a+b)/(a-b),I/((...
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/
2)), x)
```

Fricas [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [**F(-1)**] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(5/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [**F**]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(5/
2)), x)
```

Mupad [**F**]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^{5/2}\,\left(a+\frac{b}{\cos(c+d\,x)}\right)^{5/2}}\,dx$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(5/2)*(a + b/cos(c + d*x))^(5/2)), x
)
```

3.634 $\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx$

Optimal. Leaf size=526

$$-\frac{(2aAb-5a^2B+3b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3b^2(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{(2Ab-5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\,\Pi\left(2;\frac{1}{2}\right.}{b^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+}}$$

```
[Out] 2/3*a*(A*b-B*a)*sin(d*x+c)/b/(a^2-b^2)/d/cos(d*x+c)^(5/2)/(a+b*sec(d*x+c))^
(3/2)+2/3*a*(2*A*a^2*b-6*A*b^3-5*B*a^3+9*B*a*b^2)*sin(d*x+c)/b^2/(a^2-b^2)^
2/d/cos(d*x+c)^(3/2)/(a+b*sec(d*x+c))^(1/2)-1/3*(2*A*a*b-5*B*a^2+3*B*b^2)*(
cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(1/2*d*x+1/2*c)*EllipticF(sin(1/2*d*x+1/2*c)
,2^(1/2)*(a/(a+b))^(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b^2/(a^2-b^2)/d/co
s(d*x+c)^(1/2)/(a+b*sec(d*x+c))^(1/2)+(2*A*b-5*B*a)*(cos(1/2*d*x+1/2*c)^2)^
(1/2)/cos(1/2*d*x+1/2*c)*EllipticPi(sin(1/2*d*x+1/2*c),2,2^(1/2)*(a/(a+b))^
(1/2))*((b+a*cos(d*x+c))/(a+b))^(1/2)/b^3/d/cos(d*x+c)^(1/2)/(a+b*sec(d*x+c
))^(1/2)-1/3*(6*A*a^3*b-14*A*a*b^3-15*B*a^4+26*B*a^2*b^2-3*B*b^4)*sin(d*x+c
)*(a+b*sec(d*x+c))^(1/2)/b^3/(a^2-b^2)^2/d/cos(d*x+c)^(1/2)+1/3*(6*A*a^3*b-
14*A*a*b^3-15*B*a^4+26*B*a^2*b^2-3*B*b^4)*(cos(1/2*d*x+1/2*c)^2)^(1/2)/cos(
1/2*d*x+1/2*c)*EllipticE(sin(1/2*d*x+1/2*c),2^(1/2)*(a/(a+b))^(1/2))*cos(d*
x+c)^(1/2)*(a+b*sec(d*x+c))^(1/2)/b^3/(a^2-b^2)^2/d/((b+a*cos(d*x+c))/(a+b)
)^(1/2)
```

Rubi [A]
time = 1.26, antiderivative size = 526, normalized size of antiderivative = 1.00, number of steps used = 15, number of rules used = 15, integrand size = 35, $\frac{\text{number of rules}}{\text{integrand size}} = 0.429$, Rules used = {3034, 4114, 4183, 4187, 4193, 3944, 2886, 2884, 4120, 3941, 2734, 2732, 3943, 2742, 2740}

$$\frac{2a(Ab-aB)\sin(c+dx)}{3bd(a^2-b^2)\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}}-\frac{(-5a^2B+2aAb+3b^2B)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3b^2d(a^2-b^2)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}+\frac{2a(-5a^3B+2a^2Ab+9ab^2B-6Ab^3)\sin(c+dx)}{3b^2d(a^2-b^2)^2\cos^{\frac{3}{2}}(c+dx)\sqrt{a+b\sec(c+dx)}}-\frac{(-15a^4B+6a^3Ab+26a^2b^2B-14aAb^3-3b^4B)\sin(c+dx)\sqrt{a+b\sec(c+dx)}}{3b^3d(a^2-b^2)^2\sqrt{\cos(c+dx)}}+\frac{(-15a^4B+6a^3Ab+26a^2b^2B-14aAb^3-3b^4B)\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}\,E\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{3b^3d(a^2-b^2)^2\sqrt{\frac{a\cos(c+dx)+b}{a+b}}}+\frac{(2Ab-5aB)\sqrt{\frac{a\cos(c+dx)+b}{a+b}}\,\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Antiderivative was successfully verified.

```
[In] Int[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/2)),x]
```

```
[Out] -1/3*((2*a*A*b - 5*a^2*B + 3*b^2*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b)]*Elli
pticF[(c + d*x)/2, (2*a)/(a + b)])/(b^2*(a^2 - b^2)*d*Sqrt[Cos[c + d*x]]*Sq
rt[a + b*Sec[c + d*x]]) + ((2*A*b - 5*a*B)*Sqrt[(b + a*Cos[c + d*x])/(a + b
)]*EllipticPi[2, (c + d*x)/2, (2*a)/(a + b)])/(b^3*d*Sqrt[Cos[c + d*x]]*Sqr
t[a + b*Sec[c + d*x]]) + ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B
 - 3*b^4*B)*Sqrt[Cos[c + d*x]]*EllipticE[(c + d*x)/2, (2*a)/(a + b)]*Sqrt[a
 + b*Sec[c + d*x]])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[(b + a*Cos[c + d*x])/(a + b
)]) + (2*a*(A*b - a*B)*Sin[c + d*x])/(3*b*(a^2 - b^2)*d*Cos[c + d*x]^(5/2)*
(a + b*Sec[c + d*x])^(3/2)) + (2*a*(2*a^2*A*b - 6*A*b^3 - 5*a^3*B + 9*a*b^2
```

```
*B)*Sin[c + d*x])/(3*b^2*(a^2 - b^2)^2*d*Cos[c + d*x]^(3/2)*Sqrt[a + b*Sec[
c + d*x]]) - ((6*a^3*A*b - 14*a*A*b^3 - 15*a^4*B + 26*a^2*b^2*B - 3*b^4*B)*
Sqrt[a + b*Sec[c + d*x]]*Sin[c + d*x])/(3*b^3*(a^2 - b^2)^2*d*Sqrt[Cos[c +
d*x]])
```

Rule 2732

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[2*(Sqrt[a
 + b]/d)*EllipticE[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[{a,
b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2734

```
Int[Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[a +
b*Sin[c + d*x]]/Sqrt[(a + b*Sin[c + d*x])/(a + b)], Int[Sqrt[a/(a + b) + (b
/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 - b^2,
 0] &&  !GtQ[a + b, 0]
```

Rule 2740

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Simp[(2/(d*S
qrt[a + b]))*EllipticF[(1/2)*(c - Pi/2 + d*x), 2*(b/(a + b))], x] /; FreeQ[
{a, b, c, d}, x] && NeQ[a^2 - b^2, 0] && GtQ[a + b, 0]
```

Rule 2742

```
Int[1/Sqrt[(a_) + (b_.)*sin[(c_.) + (d_.)*(x_)]], x_Symbol] :> Dist[Sqrt[(a
 + b*Sin[c + d*x])/(a + b)]/Sqrt[a + b*Sin[c + d*x]], Int[1/Sqrt[a/(a + b)
+ (b/(a + b))*Sin[c + d*x]], x], x] /; FreeQ[{a, b, c, d}, x] && NeQ[a^2 -
b^2, 0] &&  !GtQ[a + b, 0]
```

Rule 2884

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Simp[(2/(f*(a + b)*Sqrt[c + d]))*EllipticPi[
2*(b/(a + b)), (1/2)*(e - Pi/2 + f*x), 2*(d/(c + d))], x] /; FreeQ[{a, b, c
, d, e, f}, x] && NeQ[b*c - a*d, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2,
0] && GtQ[c + d, 0]
```

Rule 2886

```
Int[1/(((a_.) + (b_.)*sin[(e_.) + (f_.)*(x_)])*Sqrt[(c_.) + (d_.)*sin[(e_.)
 + (f_.)*(x_)]]), x_Symbol] :> Dist[Sqrt[(c + d*Sin[e + f*x])/(c + d)]/Sqrt
[c + d*Sin[e + f*x]], Int[1/((a + b*Sin[e + f*x])*Sqrt[c/(c + d) + (d/(c +
d))*Sin[e + f*x]]), x], x] /; FreeQ[{a, b, c, d, e, f}, x] && NeQ[b*c - a*d
, 0] && NeQ[a^2 - b^2, 0] && NeQ[c^2 - d^2, 0] &&  !GtQ[c + d, 0]
```

Rule 3034

```
Int[((a_.) + csc[(e_.) + (f_.)*(x_)]*(b_.))^(m_.)*(csc[(e_.) + (f_.)*(x_)]*
(d_.) + (c_))^(n_.)*((g_.)*sin[(e_.) + (f_.)*(x_)])^(p_.), x_Symbol] :> Dis
t[(g*Csc[e + f*x])^p*(g*Sin[e + f*x])^p, Int[(a + b*Csc[e + f*x])^m*((c + d
*Csc[e + f*x])^n/(g*Csc[e + f*x])^p), x], x] /; FreeQ[{a, b, c, d, e, f, g,
 m, n, p}, x] && NeQ[b*c - a*d, 0] &&  !IntegerQ[p] &&  !(IntegerQ[m] && In
tegerQ[n])
```

Rule 3941

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]/Sqrt[csc[(e_.) + (f_.)*(x_)]
*(d_.)], x_Symbol] :> Dist[Sqrt[a + b*Csc[e + f*x]]/(Sqrt[d*Csc[e + f*x]]*S
qrt[b + a*Sin[e + f*x]]), Int[Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{a,
 b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3943

```
Int[Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)], x_Symbol] :> Dist[Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x]]/S
qrt[a + b*Csc[e + f*x]]), Int[1/Sqrt[b + a*Sin[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 3944

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(3/2)/Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.
) + (a_)], x_Symbol] :> Dist[d*Sqrt[d*Csc[e + f*x]]*(Sqrt[b + a*Sin[e + f*x
]]/Sqrt[a + b*Csc[e + f*x]]), Int[1/(Sin[e + f*x]*Sqrt[b + a*Sin[e + f*x]])
, x], x] /; FreeQ[{a, b, d, e, f}, x] && NeQ[a^2 - b^2, 0]
```

Rule 4114

```
Int[(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (
a_))^(m_)*(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_)), x_Symbol] :> Simp[a*d^2*(
A*b - a*B)*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n -
 2)/(b*f*(m + 1)*(a^2 - b^2))), x] - Dist[d/(b*(m + 1)*(a^2 - b^2)), Int[(a
 + b*Csc[e + f*x])^(m + 1)*(d*Csc[e + f*x])^(n - 2)*Simp[a*d*(A*b - a*B)*(n
 - 2) + b*d*(A*b - a*B)*(m + 1)*Csc[e + f*x] - (a*A*b*d*(m + n) - d*B*(a^2*
(n - 1) + b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f,
 A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n
, 1]
```

Rule 4120

```
Int[(csc[(e_.) + (f_.)*(x_)]*(B_.) + (A_))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d
_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.) + (a_)]), x_Symbol] :> Dist[A/a, In
```

```
t[Sqrt[a + b*Csc[e + f*x]]/Sqrt[d*Csc[e + f*x]], x], x] - Dist[(A*b - a*B)/
(a*d), Int[Sqrt[d*Csc[e + f*x]]/Sqrt[a + b*Csc[e + f*x]], x], x] /; FreeQ[{
a, b, d, e, f, A, B}, x] && NeQ[A*b - a*B, 0] && NeQ[a^2 - b^2, 0]
```

Rule 4183

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-d)*(A*b^2 - a*b*B + a^2*C)*Cot[e + f*x]*(a +
b*Csc[e + f*x])^(m + 1)*((d*Csc[e + f*x])^(n - 1)/(b*f*(a^2 - b^2)*(m + 1))
), x] + Dist[d/(b*(a^2 - b^2)*(m + 1)), Int[(a + b*Csc[e + f*x])^(m + 1)*(d
*Csc[e + f*x])^(n - 1)*Simp[A*b^2*(n - 1) - a*(b*B - a*C)*(n - 1) + b*(a*A
- b*B + a*C)*(m + 1)*Csc[e + f*x] - (b*(A*b - a*B)*(m + n + 1) + C*(a^2*n +
 b^2*(m + 1)))*Csc[e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C}
, x] && NeQ[a^2 - b^2, 0] && LtQ[m, -1] && GtQ[n, 0]
```

Rule 4187

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))*(csc[(e_.) + (f_.)*(x_)]*(d_.))^(n_)*(csc[(e_.) + (f_.)*(x_)]*(b_.) + (a
_))^(m_), x_Symbol] :> Simp[(-C)*d*Cot[e + f*x]*(a + b*Csc[e + f*x])^(m + 1
)*((d*Csc[e + f*x])^(n - 1)/(b*f*(m + n + 1))), x] + Dist[d/(b*(m + n + 1))
, Int[(a + b*Csc[e + f*x])^m*(d*Csc[e + f*x])^(n - 1)*Simp[a*C*(n - 1) + (A
*b*(m + n + 1) + b*C*(m + n))*Csc[e + f*x] + (b*B*(m + n + 1) - a*C*n)*Csc[
e + f*x]^2, x], x], x] /; FreeQ[{a, b, d, e, f, A, B, C, m}, x] && NeQ[a^2
- b^2, 0] && GtQ[n, 0]
```

Rule 4193

```
Int[((A_.) + csc[(e_.) + (f_.)*(x_)]*(B_.) + csc[(e_.) + (f_.)*(x_)]^2*(C_.
))/(Sqrt[csc[(e_.) + (f_.)*(x_)]*(d_.)]*Sqrt[csc[(e_.) + (f_.)*(x_)]*(b_.)
+ (a_)]), x_Symbol] :> Dist[C/d^2, Int[(d*Csc[e + f*x])^(3/2)/Sqrt[a + b*Cs
c[e + f*x]], x], x] + Int[(A + B*Csc[e + f*x])/(Sqrt[d*Csc[e + f*x]]*Sqrt[a
 + b*Csc[e + f*x]]), x] /; FreeQ[{a, b, d, e, f, A, B, C}, x] && NeQ[a^2 -
b^2, 0]
```

Rubi steps

$$\int \frac{A+B\sec(c+dx)}{\cos^{\frac{7}{2}}(c+dx)(a+b\sec(c+dx))^{5/2}}\,dx = \left(\sqrt{\cos(c+dx)}\,\sqrt{\sec(c+dx)}\right)\int \frac{\sec^{\frac{7}{2}}(c+dx)(A+B\sec(c+}{(a+b\sec(c+dx))^{5/2}}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\,(a^2-b^2)\,d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{\left(2\sqrt{\cos(c+\right.}{}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\,(a^2-b^2)\,d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2a(2a^2Ab-}{3b^2\,(a^2-b^2)}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\,(a^2-b^2)\,d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2a(2a^2Ab-}{3b^2\,(a^2-b^2)}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\,(a^2-b^2)\,d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2a(2a^2Ab-}{3b^2\,(a^2-b^2)}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\,(a^2-b^2)\,d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2a(2a^2Ab-}{3b^2\,(a^2-b^2)}$$

$$= \frac{2a(Ab-aB)\sin(c+dx)}{3b\,(a^2-b^2)\,d\cos^{\frac{5}{2}}(c+dx)(a+b\sec(c+dx))^{3/2}} + \frac{2a(2a^2Ab-}{3b^2\,(a^2-b^2)}$$

$$= \frac{(2Ab-5aB)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\;\Pi\left(2;\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right)}{b^3d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}} + \frac{}{3b\,(a^2}$$

$$= -\frac{(2aAb-5a^2B+3b^2B)\sqrt{\frac{b+a\cos(c+dx)}{a+b}}\;F\left(\frac{1}{2}(c+dx)|\frac{2a}{a+b}\right.}{3b^2\,(a^2-b^2)\,d\sqrt{\cos(c+dx)}\,\sqrt{a+b\sec(c+dx)}}$$

Mathematica [C] Result contains complex when optimal does not.
time = 34.32, size = 184379, normalized size = 350.53

Result too large to show

Warning: Unable to verify antiderivative.

```
[In] Integrate[(A + B*Sec[c + d*x])/(Cos[c + d*x]^(7/2)*(a + b*Sec[c + d*x])^(5/
2)),x]
```

```
[Out] Result too large to show
```

Maple [C] Result contains complex when optimal does not.
time = 22.55, size = 5358, normalized size = 10.19

method	result	size
default	Expression too large to display	5358

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x,method=_RETU
RNVERBOSE)
```

```
[Out] result too large to display
```

Maxima [F]
time = 0.00, size = 0, normalized size = 0.00

Failed to integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="maxima")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/
2)), x)
```

Fricas [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="fricas")
```

```
[Out] Timed out
```

Sympy [F(-1)] Timed out
time = 0.00, size = 0, normalized size = 0.00

Timed out

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)**(7/2)/(a+b*sec(d*x+c))**(5/2),x)
```

```
[Out] Timed out
```

Giac [F]
time = 0.00, size = 0, normalized size = 0.00

could not integrate

Verification of antiderivative is not currently implemented for this CAS.

```
[In] integrate((A+B*sec(d*x+c))/cos(d*x+c)^(7/2)/(a+b*sec(d*x+c))^(5/2),x, algor
ithm="giac")
```

```
[Out] integrate((B*sec(d*x + c) + A)/((b*sec(d*x + c) + a)^(5/2)*cos(d*x + c)^(7/
2)), x)
```

Mupad [F]
time = 0.00, size = -1, normalized size = -0.00

$$\int \frac{A+\frac{B}{\cos(c+d\,x)}}{\cos\left(c+d\,x\right)^{7/2}\,{\left(a+\frac{b}{\cos(c+d\,x)}\right)}^{5/2}} \,d x$$

Verification of antiderivative is not currently implemented for this CAS.

```
[In] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(5/2)),x)
```

```
[Out] int((A + B/cos(c + d*x))/(cos(c + d*x)^(7/2)*(a + b/cos(c + d*x))^(5/2)), x
)
```

Chapter 4

Appendix

Local contents

4.1 Download section

The following zip files contain the raw integrals used in this test.

Mathematica format Mathematica_syntax.zip

Maple and Mupad format Maple_syntax.zip

Sympy format SYMPY_syntax.zip

Sage math format SAGE_syntax.zip

4.2 Listing of Grading functions

The following are the current version of the grading functions used for grading the quality of the antiderivative with reference to the optimal antiderivative included in the test suite.

There is a version for Maple and for Mathematica/Rubi. There is a version for grading Sympy and version for use with Sagemath.

The following are links to the current source code.

The following are the listings of source code of the grading functions.

4.2.1 Mathematica and Rubi grading function

```
(* Original version thanks to Albert Rich emailed on 03/21/2017 *)
(* ::Package:: *)

(* Nasser: April 7,2022. add second output which gives reason for the grade *)
(*                       Small rewrite of logic in main function to make it*)
(*                       match Maple's logic. No change in functionality otherwise*)

(* ::Subsection:: *)
(*GradeAntiderivative[result,optimal]*)


(* ::Text:: *)
(*If result and optimal are mathematical expressions, *)
(*         GradeAntiderivative[result,optimal] returns*)
(*  "F" if the result fails to integrate an expression that*)
(*      is integrable*)
(*  "C" if result involves higher level functions than necessary*)
(*  "B" if result is more than twice the size of the optimal*)
(*      antiderivative*)
(*  "A" if result can be considered optimal*)
```

```
GradeAntiderivative[result_,optimal_] := Module[{expnResult,expnOptimal,leafCountResult,leafC
    expnResult  = ExpnType[result];
    expnOptimal = ExpnType[optimal];
    leafCountResult = LeafCount[result];
    leafCountOptimal = LeafCount[optimal];

    (*Print["expnResult=",expnResult," expnOptimal=",expnOptimal];*)
    If[expnResult<=expnOptimal,
        If[Not[FreeQ[result,Complex]], (*result contains complex*)
            If[Not[FreeQ[optimal,Complex]], (*optimal contains complex*)
                If[leafCountResult<=2*leafCountOptimal,
                    finalresult={"A","none"}
                    ,(*ELSE*)
                    finalresult={"B","Both result and optimal contain complex but leaf count
                  ]
            ,(*ELSE*)
                finalresult={"C","Result contains complex when optimal does not."}
            ]
        ,(*ELSE*)(*result does not contains complex*)
            If[leafCountResult<=2*leafCountOptimal,
               finalresult={"A","none"}
            ,(*ELSE*)
               finalresult={"B","Leaf count is larger than twice the leaf count of optimal. $
              ]
        ]
    ,(*ELSE*) (*expnResult>expnOptimal*)
        If[FreeQ[result,Integrate] && FreeQ[result,Int],
            finalresult={"C","Result contains higher order function than in optimal. Order "<
            ,
            finalresult={"F","Contains unresolved integral."}
        ]
    ];

    finalresult
]

(* ::Text:: *)
(*The following summarizes the type number assigned an *)
(*expression based on the functions it involves*)
(*1 = rational function*)
(*2 = algebraic function*)
(*3 = elementary function*)
(*4 = special function*)
(*5 = hyperpergeometric function*)
(*6 = appell function*)
(*7 = rootsum function*)
(*8 = integrate function*)
```

```
(*9 = unknown function*)


ExpnType[expn_] :=
  If[AtomQ[expn],
    1,
  If[ListQ[expn],
    Max[Map[ExpnType,expn]],
  If[Head[expn]===Power,
    If[IntegerQ[expn[[2]]],
      ExpnType[expn[[1]]],
    If[Head[expn[[2]]]===Rational,
      If[IntegerQ[expn[[1]]] || Head[expn[[1]]]===Rational,
        1,
      Max[ExpnType[expn[[1]]],2]],
    Max[ExpnType[expn[[1]]],ExpnType[expn[[2]]],3]]],
  If[Head[expn]===Plus || Head[expn]===Times,
    Max[ExpnType[First[expn]],ExpnType[Rest[expn]]],
  If[ElementaryFunctionQ[Head[expn]],
    Max[3,ExpnType[expn[[1]]]],
  If[SpecialFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],4]],
  If[HypergeometricFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],5]],
  If[AppellFunctionQ[Head[expn]],
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],6]],
  If[Head[expn]===RootSum,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
  If[Head[expn]===Integrate || Head[expn]===Int,
    Apply[Max,Append[Map[ExpnType,Apply[List,expn]],8]],
  9]]]]]]]]]]


ElementaryFunctionQ[func_] :=
  MemberQ[{
  Exp,Log,
  Sin,Cos,Tan,Cot,Sec,Csc,
  ArcSin,ArcCos,ArcTan,ArcCot,ArcSec,ArcCsc,
  Sinh,Cosh,Tanh,Coth,Sech,Csch,
  ArcSinh,ArcCosh,ArcTanh,ArcCoth,ArcSech,ArcCsch
},func]


SpecialFunctionQ[func_] :=
  MemberQ[{
  Erf, Erfc, Erfi,
  FresnelS, FresnelC,
```

```
  ExpIntegralE, ExpIntegralEi, LogIntegral,
  SinIntegral, CosIntegral, SinhIntegral, CoshIntegral,
  Gamma, LogGamma, PolyGamma,
  Zeta, PolyLog, ProductLog,
  EllipticF, EllipticE, EllipticPi
},func]


HypergeometricFunctionQ[func_] :=
  MemberQ[{Hypergeometric1F1,Hypergeometric2F1,HypergeometricPFQ},func]


AppellFunctionQ[func_] :=
  MemberQ[{AppellF1},func]
```

4.2.2 Maple grading function

```
# File: GradeAntiderivative.mpl
# Original version thanks to Albert Rich emailed on 03/21/2017

#Nasser 03/22/2017  Use Maple leaf count instead since buildin
#Nasser 03/23/2017  missing 'ln' for ElementaryFunctionQ added
#Nasser 03/24/2017  corrected the check for complex result
#Nasser 10/27/2017  check for leafsize and do not call ExpnType()
#                   if leaf size is "too large". Set at 500,000
#Nasser 12/22/2019  Added debug flag, added 'dilog' to special functions
#                   see problem 156, file Apostol_Problems
#Nasser 4/07/2022   add second output which gives reason for the grade

GradeAntiderivative := proc(result,optimal)
local leaf_count_result,
        leaf_count_optimal,
        ExpnType_result,
        ExpnType_optimal,
        debug:=false;

        leaf_count_result:=leafcount(result);
        #do NOT call ExpnType() if leaf size is too large. Recursion problem
        if leaf_count_result > 500000 then
            return "B","result has leaf size over 500,000. Avoiding possible recursion issues
        fi;

        leaf_count_optimal := leafcount(optimal);
        ExpnType_result  := ExpnType(result);
        ExpnType_optimal := ExpnType(optimal);
```

```
        if debug then
            print("ExpnType_result",ExpnType_result," ExpnType_optimal=",ExpnType_optimal);
        fi;

# If result and optimal are mathematical expressions,
#  GradeAntiderivative[result,optimal] returns
#   "F" if the result fails to integrate an expression that
#       is integrable
#   "C" if result involves higher level functions than necessary
#   "B" if result is more than twice the size of the optimal
#       antiderivative
#   "A" if result can be considered optimal

    #This check below actually is not needed, since I only
    #call this grading only for passed integrals. i.e. I check
    #for "F" before calling this. But no harm of keeping it here.
    #just in case.


    if not type(result,freeof('int')) then
        return "F","Result contains unresolved integral";
    fi;


    if ExpnType_result<=ExpnType_optimal then
        if debug then
             print("ExpnType_result<=ExpnType_optimal");
        fi;
        if is_contains_complex(result) then
            if is_contains_complex(optimal) then
                if debug then
                        print("both result and optimal complex");
                fi;
                if leaf_count_result<=2*leaf_count_optimal then
                    return "A","";
                else
                   return "B",cat("Both result and optimal contain complex but leaf count of re
                                convert(leaf_count_result,string)," vs. $2 (",
                                convert(leaf_count_optimal,string)," ) = ",convert(2*leaf_co
                end if
            else #result contains complex but optimal is not
                if debug then
                        print("result contains complex but optimal is not");
                fi;
                return "C","Result contains complex when optimal does not.";
            fi;
        else # result do not contain complex
```

```
             # this assumes optimal do not as well. No check is needed here.
            if debug then
                   print("result do not contain complex, this assumes optimal do not as well"
            fi;
            if leaf_count_result<=2*leaf_count_optimal then
                if debug then
                    print("leaf_count_result<=2*leaf_count_optimal");
                fi;
                return "A","";
            else
                if debug then
                    print("leaf_count_result>2*leaf_count_optimal");
                fi;
                return "B",cat("Leaf count of result is larger than twice the leaf count of o
                               convert(leaf_count_result,string),"$ vs. $2(",
                               convert(leaf_count_optimal,string),")=",convert(2*leaf_cou
            fi;
        fi;
    else #ExpnType(result) > ExpnType(optimal)
        if debug then
            print("ExpnType(result) > ExpnType(optimal)");
        fi;
        return "C",cat("Result contains higher order function than in optimal. Order ",
                       convert(ExpnType_result,string)," vs. order ",
                       convert(ExpnType_optimal,string),".");
    fi;

end proc:

#
# is_contains_complex(result)
# takes expressions and returns true if it contains "I" else false
#
#Nasser 032417
is_contains_complex:= proc(expression)
  return (has(expression,I));
end proc:

# The following summarizes the type number assigned an expression
# based on the functions it involves
# 1 = rational function
# 2 = algebraic function
# 3 = elementary function
# 4 = special function
# 5 = hyperpergeometric function
# 6 = appell function
# 7 = rootsum function
```

```
# 8 = integrate function
# 9 = unknown function

ExpnType := proc(expn)
  if type(expn,'atomic') then
    1
  elif type(expn,'list') then
    apply(max,map(ExpnType,expn))
  elif type(expn,'sqrt') then
    if type(op(1,expn),'rational') then
       1
    else
       max(2,ExpnType(op(1,expn)))
    end if
  elif type(expn,'`^`') then
    if type(op(2,expn),'integer') then
      ExpnType(op(1,expn))
    elif type(op(2,expn),'rational') then
      if type(op(1,expn),'rational') then
         1
      else
         max(2,ExpnType(op(1,expn)))
      end if
    else
         max(3,ExpnType(op(1,expn)),ExpnType(op(2,expn)))
    end if
  elif type(expn,'`+`') or type(expn,'`*`') then
    max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
  elif ElementaryFunctionQ(op(0,expn)) then
    max(3,ExpnType(op(1,expn)))
  elif SpecialFunctionQ(op(0,expn)) then
    max(4,apply(max,map(ExpnType,[op(expn)])))
  elif HypergeometricFunctionQ(op(0,expn)) then
    max(5,apply(max,map(ExpnType,[op(expn)])))
  elif AppellFunctionQ(op(0,expn)) then
    max(6,apply(max,map(ExpnType,[op(expn)])))
  elif op(0,expn)='int' then
    max(8,apply(max,map(ExpnType,[op(expn)]))) else
  9
  end if
end proc:


ElementaryFunctionQ := proc(func)
  member(func,[
        exp,log,ln,
        sin,cos,tan,cot,sec,csc,
```

```
        arcsin,arccos,arctan,arccot,arcsec,arccsc,
        sinh,cosh,tanh,coth,sech,csch,
        arcsinh,arccosh,arctanh,arccoth,arcsech,arccsch])
end proc:

SpecialFunctionQ := proc(func)
  member(func,[
        erf,erfc,erfi,
        FresnelS,FresnelC,
        Ei,Ei,Li,Si,Ci,Shi,Chi,
        GAMMA,lnGAMMA,Psi,Zeta,polylog,dilog,LambertW,
        EllipticF,EllipticE,EllipticPi])
end proc:

HypergeometricFunctionQ := proc(func)
  member(func,[Hypergeometric1F1,hypergeom,HypergeometricPFQ])
end proc:

AppellFunctionQ := proc(func)
  member(func,[AppellF1])
end proc:

# u is a sum or product.  rest(u) returns all but the
# first term or factor of u.
rest := proc(u) local v;
  if nops(u)=2 then
     op(2,u)
  else
     apply(op(0,u),op(2..nops(u),u))
  end if
end proc:

#leafcount(u) returns the number of nodes in u.
#Nasser 3/23/17  Replaced by build-in leafCount from package in Maple
leafcount := proc(u)
   MmaTranslator[Mma][LeafCount](u);
end proc:
```

4.2.3 Sympy grading function

```
#Dec 24, 2019. Nasser M. Abbasi:
#             Port of original Maple grading function by
#             Albert Rich to use with Sympy/Python
#Dec 27, 2019  Nasser. Added `RootSum`. See problem 177, Timofeev file
#             added 'exp_polar'
from sympy import *

def leaf_count(expr):
   #sympy do not have leaf count function. This is approximation
   return round(1.7*count_ops(expr))

def is_sqrt(expr):
   if isinstance(expr,Pow):
      if expr.args[1] == Rational(1,2):
         return True
      else:
         return False
   else:
      return False

def is_elementary_function(func):
   return func in [exp,log,ln,sin,cos,tan,cot,sec,csc,
         asin,acos,atan,acot,asec,acsc,sinh,cosh,tanh,coth,sech,csch,
         asinh,acosh,atanh,acoth,asech,acsch
      ]

def is_special_function(func):
   return func in [ erf,erfc,erfi,
          fresnels,fresnelc,Ei,Ei,Li,Si,Ci,Shi,Chi,
          gamma,loggamma,digamma,zeta,polylog,LambertW,
          elliptic_f,elliptic_e,elliptic_pi,exp_polar
       ]

def is_hypergeometric_function(func):
   return func in [hyper]

def is_appell_function(func):
   return func in [appellf1]

def is_atom(expn):
   try:
      if expn.isAtom or isinstance(expn,int) or isinstance(expn,float):
        return True
      else:
        return False
```

```
    except AttributeError as error:
        return False

def expnType(expn):
    debug=False
    if debug:
        print("expn=",expn,"type(expn)=",type(expn))

    if is_atom(expn):
       return 1
    elif isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
            return 1
        else:
            return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
    elif isinstance(expn,Pow):    #type(expn,'`^`')
        if isinstance(expn.args[1],Integer):   #type(op(2,expn),'integer')
            return expnType(expn.args[0])    #ExpnType(op(1,expn))
        elif isinstance(expn.args[1],Rational):   #type(op(2,expn),'rational')
            if isinstance(expn.args[0],Rational): #type(op(1,expn),'rational')
                return 1
            else:
                return max(2,expnType(expn.args[0]))   #max(2,ExpnType(op(1,expn)))
        else:
            return max(3,expnType(expn.args[0]),expnType(expn.args[1])) #max(3,ExpnType(op(1,expn)),ExpnT
    elif isinstance(expn,Add) or isinstance(expn,Mul): #type(expn,'`+`') or type(expn,'`*`')
        m1 = expnType(expn.args[0])
        m2 = expnType(list(expn.args[1:]))
        return max(m1,m2)   #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
    elif is_elementary_function(expn.func):   #ElementaryFunctionQ(op(0,expn))
        return max(3,expnType(expn.args[0]))   #max(3,ExpnType(op(1,expn)))
    elif is_special_function(expn.func): #SpecialFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(4,m1)    #max(4,apply(max,map(ExpnType,[op(expn)])))
    elif is_hypergeometric_function(expn.func): #HypergeometricFunctionQ(op(0,expn))
        m1 = max(map(expnType, list(expn.args)))
        return max(5,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif is_appell_function(expn.func):
        m1 = max(map(expnType, list(expn.args)))
        return max(6,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    elif isinstance(expn,RootSum):
        m1 = max(map(expnType, list(expn.args))) #Apply[Max,Append[Map[ExpnType,Apply[List,expn]],7]],
        return max(7,m1)
    elif str(expn).find("Integral") != -1:
```

```
        m1 = max(map(expnType, list(expn.args)))
        return max(8,m1)    #max(5,apply(max,map(ExpnType,[op(expn)])))
    else:
        return 9

#main function
def grade_antiderivative(result,optimal):

    #print ("Enter grade_antiderivative for sagemath")
    #print("Enter grade_antiderivative, result=",result," optimal=",optimal)

    leaf_count_result  = leaf_count(result)
    leaf_count_optimal = leaf_count(optimal)

    #print("leaf_count_result=",leaf_count_result)
    #print("leaf_count_optimal=",leaf_count_optimal)

    expnType_result  = expnType(result)
    expnType_optimal = expnType(optimal)

    if str(result).find("Integral") != -1:
        grade = "F"
        grade_annotation =""
    else:
        if expnType_result <= expnType_optimal:
            if result.has(I):
                if optimal.has(I): #both result and optimal complex
                    if leaf_count_result <= 2*leaf_count_optimal:
                        grade = "A"
                        grade_annotation =""
                    else:
                        grade = "B"
                        grade_annotation ="Both result and optimal contain complex but leaf count of result is larger
                else: #result contains complex but optimal is not
                    grade = "C"
                    grade_annotation ="Result contains complex when optimal does not."
            else: # result do not contain complex, this assumes optimal do not as well
                if leaf_count_result <= 2*leaf_count_optimal:
                    grade = "A"
                    grade_annotation =""
                else:
                    grade = "B"
                    grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(lea
        else:
            grade = "C"
            grade_annotation ="Result contains higher order function than in optimal. Order "+str(ExpnType_re
```

```
    #print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

    return grade, grade_annotation
```

4.2.4 SageMath grading function

```
#Dec 24, 2019. Nasser: Ported original Maple grading function by
#              Albert Rich to use with Sagemath. This is used to
#              grade Fricas, Giac and Maxima results.
#Dec 24, 2019. Nasser: Added 'exp_integral_e' and 'sng', 'sin_integral'
#              'arctan2','floor','abs','log_integral'
#June 4, 2022  Made default grade_annotation "none" instead of "" due
#              issue later when reading the file.
#July 14, 2022. Added ellipticF. This is until they fix sagemath, then remove it.

from sage.all import *
from sage.symbolic.operators import add_vararg, mul_vararg

debug=False;

def tree_size(expr):
    r"""
    Return the tree size of this expression.
    """
    #print("Enter tree_size, expr is ",expr)

    if expr not in SR:
        # deal with lists, tuples, vectors
        return 1 + sum(tree_size(a) for a in expr)
    expr = SR(expr)
    x, aa = expr.operator(), expr.operands()
    if x is None:
        return 1
    else:
        return 1 + sum(tree_size(a) for a in aa)

def is_sqrt(expr):
    if expr.operator() == operator.pow:   #isinstance(expr,Pow):
        if expr.operands()[1]==1/2: #expr.args[1] == Rational(1,2):
            if debug: print ("expr is sqrt")
            return True
        else:
            return False
    else:
        return False
```

```
def is_elementary_function(func):
    #debug=False
    m = func.name() in ['exp','log','ln',
            'sin','cos','tan','cot','sec','csc',
            'arcsin','arccos','arctan','arccot','arcsec','arccsc',
            'sinh','cosh','tanh','coth','sech','csch',
            'arcsinh','arccosh','arctanh','arccoth','arcsech','arccsch','sgn',
        'arctan2','floor','abs'
        ]
    if debug:
        if m:
            print ("func ", func , " is elementary_function")
        else:
            print ("func ", func , " is NOT elementary_function")


    return m

def is_special_function(func):
    #debug=False
    if debug:
        print ("type(func)=", type(func))

    m= func.name() in ['erf','erfc','erfi','fresnel_sin','fresnel_cos','Ei',
          'Ei','Li','Si','sin_integral','Ci','cos_integral','Shi','sinh_integral'
          'Chi','cosh_integral','gamma','log_gamma','psi,zeta',
          'polylog','lambert_w','elliptic_f','elliptic_e','ellipticF',
          'elliptic_pi','exp_integral_e','log_integral']

    if debug:
        print ("m=",m)
        if m:
            print ("func ", func ," is special_function")
        else:
            print ("func ", func ," is NOT special_function")


    return m


def is_hypergeometric_function(func):
    return func.name() in ['hypergeometric','hypergeometric_M','hypergeometric_U']

def is_appell_function(func):
    return func.name() in ['hypergeometric']    #[appellf1] can't find this in sagemath
```

```
def is_atom(expn):

    #debug=False
    if debug:
         print ("Enter is_atom, expn=",expn)

    if not hasattr(expn, 'parent'):
        return False


    #thanks to answer at https://ask.sagemath.org/question/49179/what-is-sagemath-equivalent-to-atomic-
    try:
        if expn.parent() is SR:
            return expn.operator() is None
        if expn.parent() in (ZZ, QQ, AA, QQbar):
            return expn in expn.parent() # Should always return True
        if hasattr(expn.parent(),"base_ring") and hasattr(expn.parent(),"gens"):
            return expn in expn.parent().base_ring() or expn in expn.parent().gens()

        return False

    except AttributeError as error:
        print("Exception,AttributeError in  is_atom")
        print ("cought exception" , type(error).__name__ )
        return False


def expnType(expn):

    if debug:
        print (">>>>>Enter expnType, expn=", expn)
        print (">>>>>is_atom(expn)=", is_atom(expn))

    if is_atom(expn):
        return 1
    elif type(expn)==list:   #isinstance(expn,list):
        return max(map(expnType, expn))    #apply(max,map(ExpnType,expn))
    elif  is_sqrt(expn):
        if  type(expn.operands()[0])==Rational:  #type(isinstance(expn.args[0],Rational):
            return 1
        else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
    elif expn.operator() == operator.pow:   #isinstance(expn,Pow)
        if type(expn.operands()[1])==Integer:  #isinstance(expn.args[1],Integer)
            return expnType(expn.operands()[0])   #expnType(expn.args[0])
        elif type(expn.operands()[1])==Rational:  #isinstance(expn.args[1],Rational)
            if type(expn.operands()[0])==Rational:  #isinstance(expn.args[0],Rational)
```

```
            return 1
         else:
            return max(2,expnType(expn.operands()[0]))  #max(2,expnType(expn.args[0]))
      else:
         return max(3,expnType(expn.operands()[0]),expnType(expn.operands()[1])) #max(3,expnType(expn.op
   elif expn.operator() == add_vararg or expn.operator() == mul_vararg: #isinstance(expn,Add) or isinstan
      m1 = expnType(expn.operands()[0]) #expnType(expn.args[0])
      m2 = expnType(expn.operands()[1:]) #expnType(list(expn.args[1:]))
      return max(m1,m2)  #max(ExpnType(op(1,expn)),max(ExpnType(rest(expn))))
   elif is_elementary_function(expn.operator()):  #is_elementary_function(expn.func)
      return max(3,expnType(expn.operands()[0]))
   elif is_special_function(expn.operator()): #is_special_function(expn.func)
      m1 = max(map(expnType, expn.operands()))      #max(map(expnType, list(expn.args)))
      return max(4,m1)   #max(4,m1)
   elif is_hypergeometric_function(expn.operator()): #is_hypergeometric_function(expn.func)
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(5,m1)   #max(5,m1)
   elif is_appell_function(expn.operator()):
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(6,m1)   #max(6,m1)
   elif str(expn).find("Integral") != -1: #this will never happen, since it
            #is checked before calling the grading function that is passed.
            #but kept it here.
      m1 = max(map(expnType, expn.operands()))       #max(map(expnType, list(expn.args)))
      return max(8,m1)   #max(5,apply(max,map(ExpnType,[op(expn)])))
   else:
      return 9

#main function
def grade_antiderivative(result,optimal):


   if debug:
      print ("Enter grade_antiderivative for sagemath")
      print("Enter grade_antiderivative, result=",result)
      print("Enter grade_antiderivative, optimal=",optimal)
      print("type(anti)=",type(result))
      print("type(optimal)=",type(optimal))

   leaf_count_result  = tree_size(result) #leaf_count(result)
   leaf_count_optimal = tree_size(optimal) #leaf_count(optimal)

   #if debug: print ("leaf_count_result=", leaf_count_result, "leaf_count_optimal=",leaf_count_optimal)


   expnType_result  = expnType(result)
   expnType_optimal = expnType(optimal)
```

```
if debug: print ("expnType_result=", expnType_result, "expnType_optimal=",expnType_optimal)

if expnType_result <= expnType_optimal:
    if result.has(I):
        if optimal.has(I): #both result and optimal complex
            if leaf_count_result <= 2*leaf_count_optimal:
                grade = "A"
                grade_annotation ="none"
            else:
                grade = "B"
                grade_annotation ="Both result and optimal contain complex but leaf count of result is larger t
        else: #result contains complex but optimal is not
            grade = "C"
            grade_annotation ="Result contains complex when optimal does not."
    else: # result do not contain complex, this assumes optimal do not as well
        if leaf_count_result <= 2*leaf_count_optimal:
            grade = "A"
            grade_annotation ="none"
        else:
            grade = "B"
            grade_annotation ="Leaf count of result is larger than twice the leaf count of optimal. "+str(leaf_
else:
    grade = "C"
    grade_annotation ="Result contains higher order function than in optimal. Order "+str(expnType_resu


print("Before returning. grade=",grade, " grade_annotation=",grade_annotation)

return grade, grade_annotation
```